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## **Defined benefit pension plans and regulation**

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# Defined benefit pension plans and regulation \*

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## Abstract

In this paper, it is investigated to what extent optimal investment policy by defined benefit pension funds is affected by an increased emphasis on market valuation in regulation. A policy based solely on market interest rates turns out to lead to higher costs than one based on a fixed actuarial rate, as high pension premiums are to be payed exactly when expected future returns are low. In practice, this timing disadvantage hardly affects Dutch pension funds however, as Dutch solvency requirements are only applied to guaranteed pension rights, whereas a major part of pension benefits (indexation) is conditional. Moreover, a fixed discount rate may still be used to calculate pension premiums. Regarding the asset mix, the optimal duration of bonds in portfolio seems to be much higher than currently observed, both under market valuation and under a fixed discount rate method. The new regulatory rules only slightly reduce the attractiveness of equity investment.

*Keywords:* pension valuation, equity investment, optimal duration

*J.E.L. Code:* C15, G11, G23, G28

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# 1 Introduction

The Dutch pension system is experiencing hectic times at the moment. The bad stock market returns over the period 2000 – 2002 have severely affected pension fund assets, whereas at the same time interest rates have dropped. These lower interest rates imply lower expected future returns, making the discounted costs of future pension benefits higher. Starting in 2007, these higher costs will have to be taken explicitly into account by the pension funds as under the new regulatory regime (*Financieel Toetsings Kader, FTK*), guaranteed pension liabilities are to be calculated using the actual market interest rate instead of a fixed actuarial interest rate. If the coverage ratio (ratio of assets to liabilities) of a pension fund drops below 105%, they have only one year to recover. In equilibrium, the funds should hold asset buffers high enough to keep the probability of underfunding below 2.5%. If buffers drop below this target, funds have 15 years to recover.

The new regulatory regime is much more in line with modern finance viewpoints in the sense that market valuation is not only applied to assets but also to liabilities, and that more risk taking results in higher required buffers. Notwithstanding these improved risk management tools, the new method is said to have a major impact on optimal pension fund policy, as there would be less room to exploit the long run perspective of pension funds. Especially the one year recovery period in case of underfunding is thought to be too strict. It would force pension funds to reduce the mismatch between assets and liabilities by increasing the duration of bonds in their portfolio and decreasing risky investments in stocks. Whether this is indeed optimal policy remains to be seen however. Less equity investment also reduces the average return which might hamper the indexation ambition of the fund.

In this paper, we try to shed some more light on the optimal policy for Dutch pension funds and the impact of regulation. Policy decisions have to be taken in at least three dimensions: Investment policy, contribution policy and indexation policy. Moreover, the number of conditions affecting optimal policy are also numerous, for instance the initial funding ratio, the current term structure of interest rates and the actual inflation rate. The optimization problem is further hampered by the heterogeneity of the stakeholders of the fund, which makes the definition of the objective of the fund a challenging task. Consequently, *the* optimal policy for pension funds can hardly be derived. Instead, we will use the pension asset and liability model PALMNET (Van Rooij, Siegmann, and Vlaar 2004), for some partial analysis. In particular, we will concentrate on the impact of changes in the asset mix, first with respect to the percentage equity in portfolio and second regarding the duration of

bonds. The indexation and contribution rules are kept constant in the analysis. Another important dimension of our analysis will be the valuation method used to calculate total pension liabilities (including conditional indexation) and pension premiums. We will compare a fixed actuarial rate method with a market valuation approach using either the current actual term structure of interest rates (plus a risk supplement in case of conditional rights) or a ten year moving average rate. All three methods are allowed for to calculate pension premiums and conditional pension liabilities under the new regulatory regime. Unconditional liabilities on the other hand should always be based on the actual term structure of interest rates.

The rest of this paper is organized as follows: Section 2 describes the Dutch pension system and the regulatory regime, Section 3 discusses ways to evaluate policy of a pension fund, Section 4 describes some features of PALMNET, Section 5 shows the results of the policy simulations, Section 6 concludes, and the appendix provides some more details on the modeled inflation and interest rate block.

## **2 The Dutch pension system**

The Dutch system of old age provision is based on three pillars. The first is the basic pension for everyone aged 65 and older, under the Old Age Pensions Act [Algemene Ouderdomswet] (AOW). This benefit is funded by the government via the pay-as-you-go method. The second pillar concerns employees' compulsory membership of their employer's group pension scheme, whereby employees save for a pension in addition to the AOW benefit according to a capital funding system. These savings are managed by pension funds which collect the premiums and pay out the pensions, deal with the administration of pension rights and invest the resources, principally in equities, bonds and property. The third pillar of the old age provision comprises schemes which people arrange individually in addition to the first and second pillar schemes. Regarding the importance of the three pillars, AOW and employee pensions are currently about equally important, whereas the third pillar is still relatively unimportant. There is a clear trend though of growing importance of the employee pension pillar.

Under Dutch law, membership of the employer's group pension scheme is compulsory. Over the years an increasing number of professions have created pension arrangements. The pension funds are separate legal entities (we have more than 700 of them in the Netherlands) that are either linked to an individual company or a whole industry. At the moment more than 90% of the workforce participates in a pension system. Generally, both the employer and the employee pay pension contributions, the employer bearing the greater part of the

cost. Pension premiums are charged on gross wages of employees after deduction of a specific amount, known as the statutory offset ('franchise'). That offset is generally linked to the level of the AOW benefit. The amount on which pension contributions are paid is called pensionable salary or premium base. Almost all (97% in 2003) pension schemes in the Netherlands are 'defined benefit' schemes, which means that a particular amount of pension is promised for the contribution paid. This contrasts with 'defined contribution' schemes in which only the financial contribution is fixed and the eventual benefit depends on the return on the funds invested. As regards the level of benefit promised, most pension contracts only guarantee a nominal pension. However, they also aim to link pensions to the wage or price index, though this indexation may be cut if the pension fund managers consider that desirable (in practice, if the pension capital is insufficient). The Dutch situation can therefore best be described as a defined benefit nominal pension combined with a 'target benefit' index-linked pension.

Up till recently, the majority of defined benefit schemes in The Netherlands were of the final salary type. In such a system the pension benefit promised is a certain percentage of the salary in the final working year, where this percentage is related to the number of years pension premiums are paid. After the three bad investment years 2000 – 2002, many pension funds shifted towards an average earnings system. The proportion of final salary schemes dropped from 67% in 1998 to 54% in 2003 and 14% in 2004. In the average earnings system, the pension promise is not related to the final wage, but to the wage at the time the pension premium is paid. As these promises are usually only guaranteed in nominal terms, the indexation cutting instrument is much more effective in an average earnings system since it can also be applied to the rights of active workers<sup>1</sup>. This increased flexibility becomes more and more important as the ratio of pension liabilities to the premium base is increasing due to the aging population. Consequently, the premium instrument becomes less effective.

## 2.1 Regulation

Pension funds in the Netherlands are obliged to keep their pension promises fully funded. Up till now, the liabilities related to these expected future pension benefits (depending on rights built up and the life expectancy of members) are to be calculated on the basis of a fixed actuarial interest rate of at most 4%. The 4% can be thought of as a conservative estimate of the expected long run return on the asset portfolio. This actuarial accounting method has got a very bad name in recent years for at least three reasons. The first reason is related

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<sup>1</sup>Indexation already given in previous years can not be reversed.

to the link of the discount rate with expected returns. In the US, pension funds that were underfunded could reduce their liabilities by taking more risk in the asset portfolio, thereby increasing the discount rate. For the Netherlands, this argument is hardly valid as there was always a cap of 4% on the discount rate. A second argument against the method is that it is impossible to determine an objective measure for expected long run returns. Consequently, there is a tendency to be too optimistic about future returns as this seems to lower the pension costs. Indeed, the actuarial rate of 4% for pension liabilities has not been changed since 1969, even though interest rates have dropped considerably. We will investigate the consequences of systematically overestimating future returns later on. A third disadvantage of the method is that the long run emphasis of the method supposes continuity. As the recent past has shown, this assumption is not always appropriate, especially for company pension funds (Enron). For industry wide pension funds, this presumption does not seem to be problematic.

Within the new regulatory framework, to be effective as from January 2007, the fixed actuarial interest rate to calculate liabilities will be replaced by the market rate. That is to say, expected future benefit payments are to be discounted by the zero coupon term structure of interest rates. For guaranteed pension rights, this discounting method has the clear advantage that pension promises can indeed be met with certainty (apart from unavoidable mortality risk), provided a market interest rate is available. Pension funds should always maintain a coverage ratio for vested pension rights of at least 105%. If assets drop below 105% of guaranteed liabilities, the fund has only one year to recover<sup>2</sup>. In exceptional cases, the supervisor can prolong this recovery period. In order to limit the risk of underfunding, pension funds should always aim at a buffer high enough to keep the probability of underfunding one year ahead lower than 2.5%. For a standard pension fund this means a coverage ratio of about 130%. If the buffer is too low, pension funds should present a recovery plan to the supervisor aiming at an appropriate buffer to be established in at most fifteen years. Both solvency tests only apply to guaranteed (in practice nominal) pension rights.

For conditional rights (indexation), funds are not obliged to reserve extra capital, provided the conditional nature of the indexation is made clear to members so that they cannot gain

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<sup>2</sup>Although this requirement is often blamed for being too tough, it is actually a relaxation of current rules. Under the old regulatory regime pension funds have only thirteen weeks to regain solvency if they are undercovered. In international perspective, the one year recovery period seems short though. In the UK for instance, the one-year recovery prescriptions in case of underfunding were relaxed in March 2002 to a three-year term. The definition of underfunding is different though. The critical *real* funding ratio (with a maximum inflation compensation of 5%) should be at least 90%. In the US, it is even allowed to amortize a shortfall over 30 years, although a shortening to 18 years has been proposed.

the false impression that they are entitled to it. Moreover, in order to maintain consistency between the fund's ambition, its communication and its policy, pension premiums should rise with the indexation ambition.

Regarding premiums, under normal conditions, pension funds should levy a so-called cost effective premium. This premium comprises of four elements: (1) normal costs of guaranteed rights, (2) administrative costs, (3) normal costs of conditional rights, and (4) costs to build up the solvency buffer. The actual premium will be higher if the solvency buffer is too small, and it is allowed to be lower if the coverage ratio is high enough to guarantee both conditional and unconditional pension rights without endangering the solvency of the fund. In practice, this means 100% of future benefits discounted by the real term structure of interest rates, plus about 30% (the buffer) discounted by the nominal term structure. Pension funds have some freedom in the way they calculate the normal costs of pension rights. They can either use the actual market rate, a moving average of past market rates *or returns* with a maximum smoothing period of ten years, or a fixed rate. Smoothing or fixing is allowed for (though not for solvency requirements) to mitigate the volatility of premiums. Although no strict rules are prescribed with respect to the calculation of the fixed rate or average returns, we will assume that the average discount rate can not be higher than the average expected long run return.

### **3 Pension fund utility**

In order to evaluate pension fund policy, one needs to define a utility measure for the pension fund. This is a far from trivial exercise as the different members of a pension fund might very well have different preferences. Pensioners do not pay premiums any more and are hardly able to offset benefit cuts, so they will prefer a relatively high premium and save investments. Plan sponsors or young workers on the other hand might prefer more risk taking resulting in on average lower premiums.

In the literature, several loss functions have been used to analyze pension fund policy. Boulier, Trussant, and Florens (1995) optimize the contribution rate and investment policy to minimize discounted quadratic contributions. They conclude that both contributions and risk taking decrease with the funding level. Siegmann and Lucas (1999) generalize this result for general loss functions with constant relative risk aversion or constant absolute risk aversion. A problem with these loss functions is that in equilibrium funding ratios converge to such high levels that premiums and stock market investments can be reduced to zero as the further

discounted costs will be zero in that case.

Instead of total quadratic premium payments, quadratic supplementary costs (in excess to normal cost) are also used, often combined with quadratic unfunded liability (Josa-Fombellida and Rincón-Zapatero (2001, 2004), Haberman and Sung (2002, 2005), Owadally and Haberman (2004)). In these specifications, the contribution rate converges to the normal cost and the coverage ratio to 100%, which seems more in accordance with reality. However, this result is driven by the assumption that having too much money is just as negative as having not enough. Adding linear terms in the utility function (Chang, Tzeng, and Miao 2003) introduces asymmetries, but the perverse incentives in case of high coverage ratios remain. Haberman, Khorasane, Ngwira, and Wright (2003) only consider positive supplementary costs thereby eliminating the perverse incentives, though the benefits of overfunding are still not accounted for.

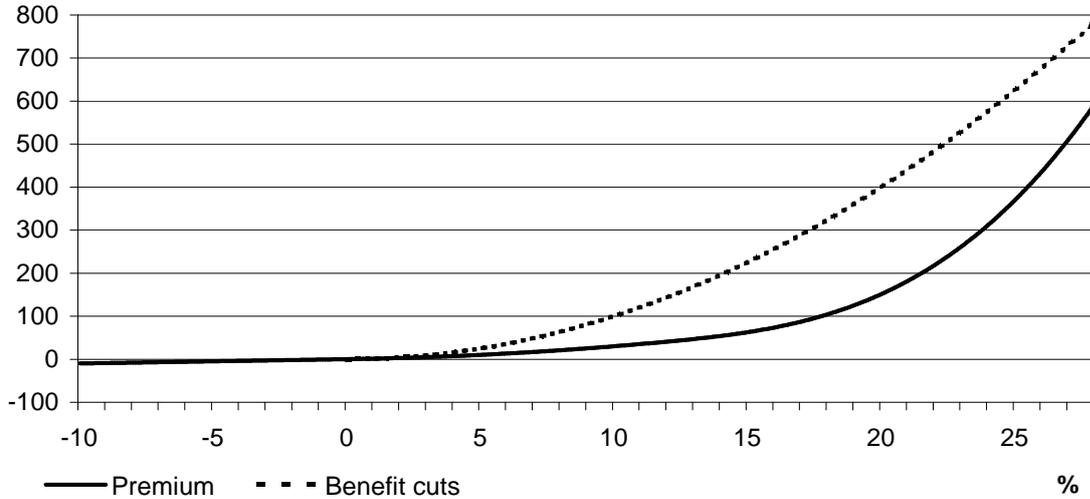
Berkelaar, Kouwenberg, and Post (2004) use prospect theory (Kahneman and Tversky 1979) to analyze optimal investment decisions. They show that loss aversion has a considerable impact on the optimal investment strategy. With respect to premium payments, we will also use a utility concept that depends on the level of contributions. The premium loss function consists of three terms:

$$L_{P,t} = P_t + \alpha_2(P_t > 0) P_t^2 + \alpha_3(P_t > NC_t)(P_t - NC_t)^3 \quad (1)$$

where  $L_{P,t}$ ,  $P_t$  and  $NC_t$  are the loss due to premium payments, the premium, and the normal costs of pension benefits (about 12% of gross wages) at time  $t$  respectively. The linear term establishes the positive utility of premium restitutions and negative of premium payments. The quadratic term establishes the increasing costs of high pension contributions. Implicitly, this quadratic term also penalizes for volatility in the pension premium. The cubic term emphasizes the danger of underfunding. If pension funds are highly underfunded such that huge shortfall premiums become necessary, young workers might try to avoid becoming a member as the costs they have to pay are much higher than the benefits they can expect later on. By including a cubic term, the precautionary savings motive is also incorporated (Sargent 1979). Figure 1 shows the loss for the parameter values  $\alpha_2 = 0.2$  and  $\alpha_3 = 0.1$ , which will be used later on.

Apart from the contributions, benefit cuts also result in a loss. We will include these costs quadratically. As the appropriate weights for contributions and benefit cuts can hardly be established, we will show results separately as well as together. As Figure 1 shows, a benefit

Figure 1: Loss due to premium payments or benefit cuts



cut of  $x\%$  is assumed to result in a bigger loss than a contribution rate of  $x\%$ . We think this is reasonable as young people are better able to correct unfavorable income shocks. Moreover, high benefit cuts may result in a loss of confidence in the pension system, which is bad for young members as well.

Solvency risk as such is not given a weight in the loss function as our model implicitly assumes the pension fund will always survive. Consequently, the costs of underfunding are already taken into account via higher (future) premiums and benefit cuts. Although discontinuity risk might be of interest as well, this risk category can better be measured separately. In this paper, we focus primarily on the total discounted loss ( $TDL_t$ ) assuming continuity:

$$TDL_t = \sum_{i=0}^{\infty} (1 + \beta)^{-i} (L_{P,t+i} + Cuts_{t+i}^2) \quad (2)$$

where  $Cuts_t$  represent the time  $t$  cumulative benefit cuts as a percentage of the fully indexed pensions, and  $\beta$  is the time preference parameter. It measures the extent to which a higher premium now is worse than a higher premium next year. As this parameter is rather subjective, we will present results both for a relatively high value of 3.1% (expected real return on representative portfolio) and a very low value of 0.75% (mean real short term interest rate in a low interest rate environment).

## 4 PALMNET

In order to simulate the consequences of pension fund policy, we will use the pension asset and liability model PALMNET (Van Rooij, Siegmann, and Vlaar 2004). The view that stochastic simulation models can provide a very useful tool for pension funds has long been recognized. Early examples are Wilkie (1986, 1995) for the UK, and Boenders (1997) for the Netherlands. PALMNET represents the average Dutch pension fund, assuming an average earnings defined benefit system, where only the nominal benefits are guaranteed, but the ambition is to index benefits to wage inflation. As the ambition is to index benefits to wage inflation, the real funding ratio (assets divided by total discounted liabilities) is given a central role in pension fund policy. This real funding ratio (as well as the premium payments) can either be based on a fixed actuarial rate or on the (moving average of) actual real term structure of interest rates, possibly supplemented by a risk factor. Usually the discount rate is chosen equal to the expected real return on the portfolio, but other choices are possible. Contribution policy is geared towards establishing a target real funding ratio high enough to keep the probability of underfunding in one year time lower than 2.5%. If the buffers are too low, shortfall premiums are levied such that recovery is expected to take place in fifteen years. If the real funding ratio drops below 105%, benefits are cut as well. Below 85% indexation is stopped completely. If the real funding ratio recovers to at least 125%, previous indexation cuts are made good.

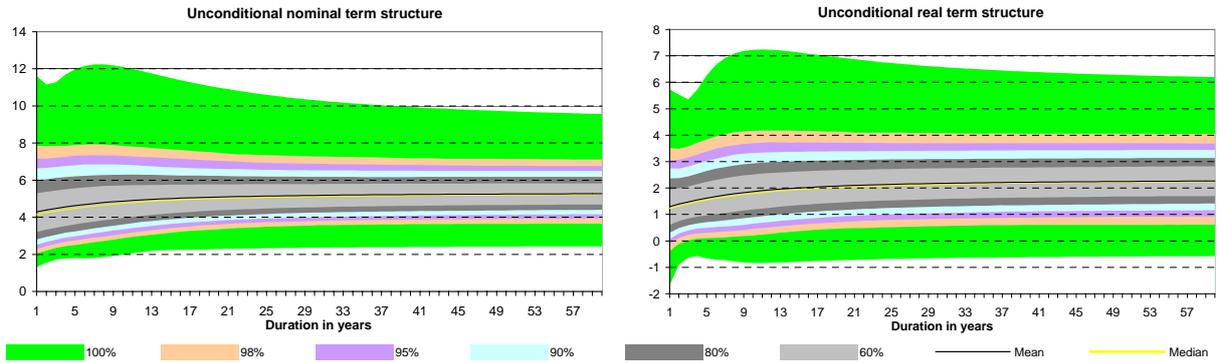
The nominal funding ratio is always computed using the actual nominal term structure of interest rates as discount factor. In the standard version of the model, this ratio does not affect pension fund policy. In the FTK-version of the model, which is fully compatible with the new regulatory rules, both the nominal and the real funding ratio determine pension fund indexation and premium policy. If the nominal funding ratio drops below 105%, premiums have to rise to regain solvency in one year. Fifteen year recovery plans become active if the probability of either real or nominal underfunding in one year time rises above 2.5%. Indexation cuts start if the nominal funding ratio is below target or the real funding ratio is too low<sup>3</sup>, and are complete for a nominal funding ratio below 105% or a real funding ratio below 85%. To add realism, we assume that premiums can never be higher than 50% of the premium base (about 28% of gross wages), and that premium changes from one year to another can not be bigger than three percentage points of the premium base. This latter restriction is not applied if the nominal funding ratio drops below 105%.

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<sup>3</sup>This parameter is used to make the cuts similar for different discounting methods for a fund with a standard portfolio.

Interest rates in the model are based on a two factor term structure model, where the factors are the short-term nominal interest rate and expected inflation (see the appendix). Figure 2 shows some percentiles of the unconditional probability distribution of the nominal and real term structure. The real interest rates are hereby constructed as the nominal ones minus expected long run price inflation (also derived from the term structure model) minus expected real wage increases (fixed at 1.1% per year).

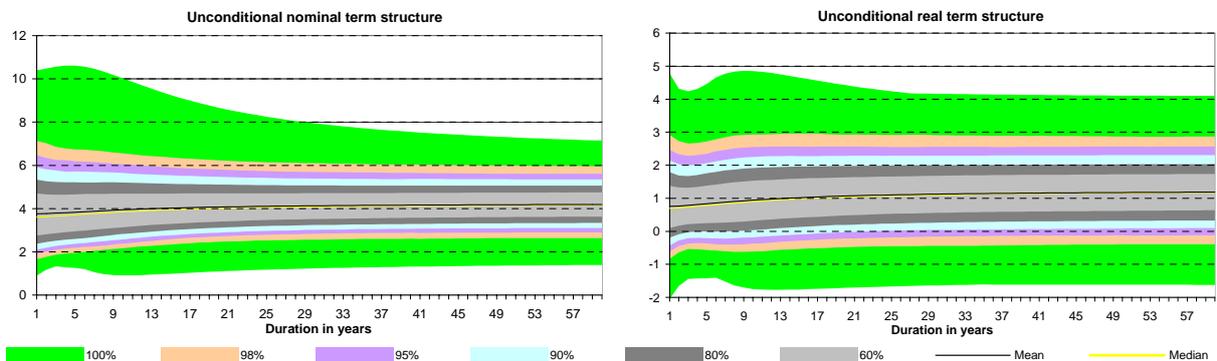
Figure 2: Probability distribution standard nominal and real term structure



The distributions are asymmetric due to a level effect in volatility. Volatility is higher during high inflation periods. This effect is important as neglecting it would lead to unrealistically low interest rates every now and then. Under market valuation these low rates have a huge effect on the pension liabilities.

In order to analyze the sensitivity of our results to the assumed term structure, we also simulated some scenario's with a permanently lower and flatter yield curve. The unconditional distributions of these term structures are shown in figure 3.

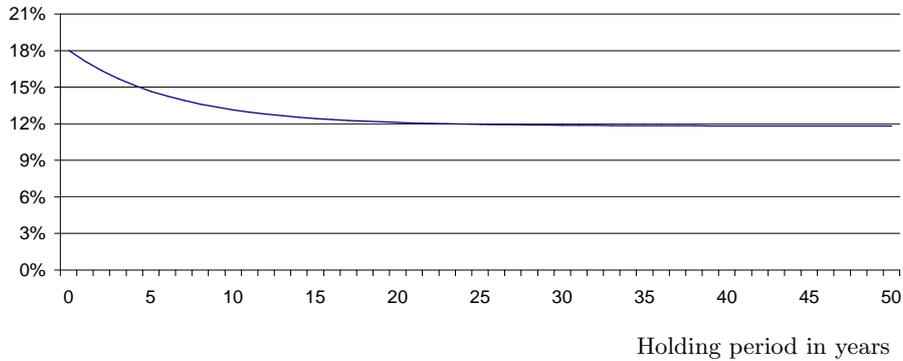
Figure 3: Probability distribution nominal and real term structure in low interest environment



Pension funds can invest in nominal bonds, the duration of which can be freely chosen, and stocks. The portfolio is rebalanced annually. Regarding the stock market, it is assumed

that the expected annual return is three percentage points higher than the five year interest rate. The volatility is 18% per year (in log terms). Shocks to stock market returns are assumed not to be correlated with interest rate or inflation shocks. In the standard version of the model, we do not assume mean reversion in the stock market. In order to investigate its impact, mean reversion can be included though. Figure 4 shows the average annual volatility in case of 5% mean reversion per year, where the overvaluation is measured as 90% of last years value times the current unexpected return. This mean reversion pattern is still modest. Campbell and Viceira (2005), for instance, find the long run real stock market volatility to be less than half of short run volatility.

Figure 4: Annualized stock market volatility in case of mean reversion



## 5 Simulation results

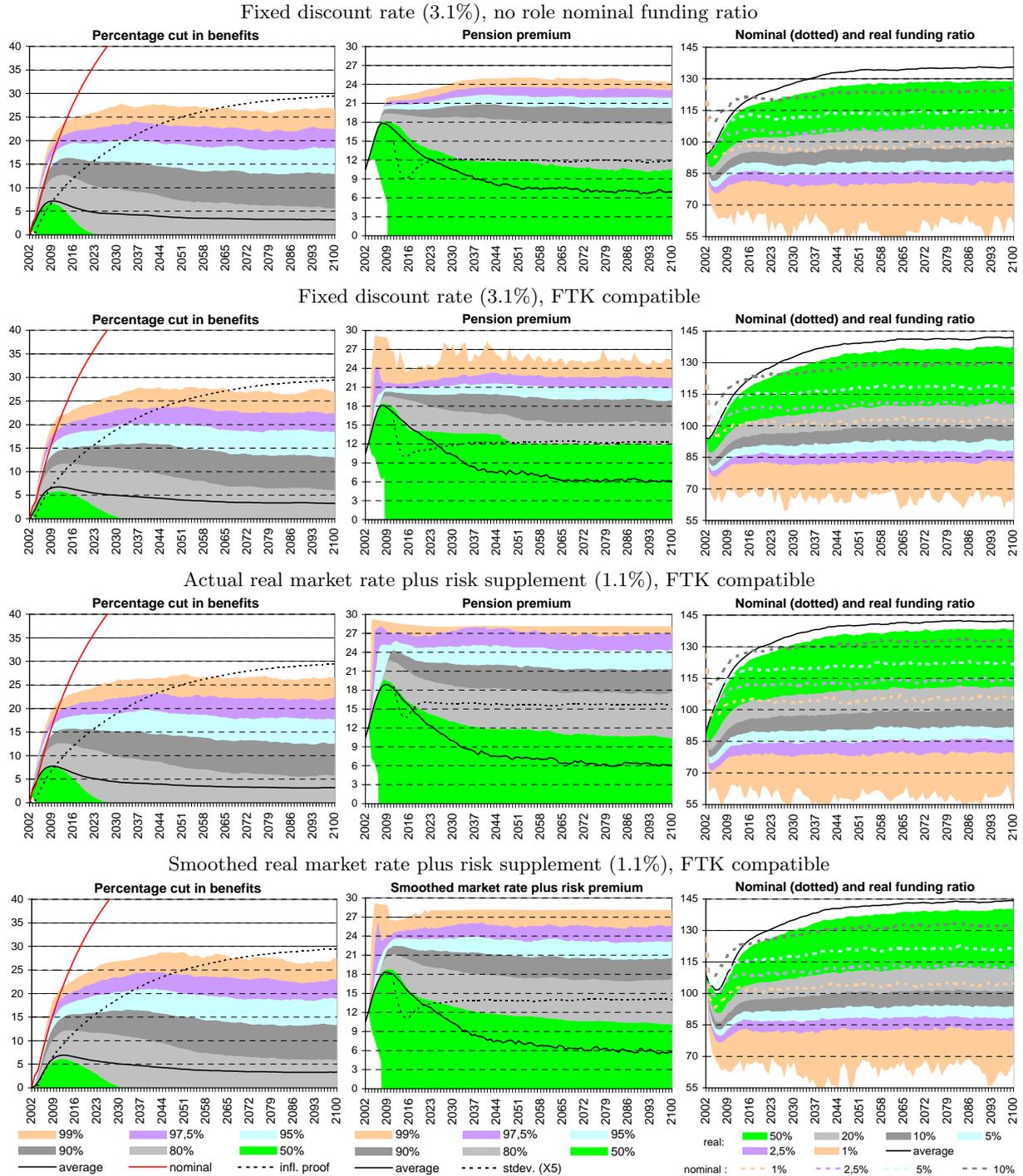
The simulations are performed by means of 10000 stochastic Monte Carlo simulations over the period 2003 – 2100, taking end 2002 as a starting point. In order to calculate the infinite sum in the discounted losses, it is assumed that the situation in 2100 is representative for the indefinite future<sup>4</sup>.

### 5.1 Accounting method

Before discussing the optimal asset mix of pension funds, we will first analyze the influence of the new regulatory regime (FTK) on the premium and indexation of pension funds. Figure 5 shows the time path for several percentiles of the distribution of the benefit cuts relative to a prosperity-linked pension, the contributions as a percentage of gross wages and the nominal and real funding ratio. In these simulations, the real target funding ratio is chosen such

<sup>4</sup>The contribution of costs after 2100 to total discounted costs is less than 3.5% if time preference is high, but about 58% if it is low.

Figure 5: Impact FTK for standard pension fund (50% stocks, duration bonds 5 years)



that the probability of real underfunding is 2.5%. The average real discount rate is in all simulations equal to the average expected real return<sup>5</sup>. This rate is used to calculate the real

<sup>5</sup>In case of market valuation a fixed mark-up is used equal to expected equilibrium real return minus the equilibrium real 16 year (average duration of liabilities) interest rate.

funding ratio, the cost-effective pension premium, as well as the shortfall premium in case of insufficient buffers. The nominal funding ratio (representing guaranteed pension rights) is always computed using the actual nominal 16 year interest rate.

In order to be better able to compare different discounting techniques, the real funding ratio below which benefits are cut is adjusted somewhat, to give similar indexation results. In the pre-FTK situation it is 105%, under the new regulatory regime it is 98.5% using a fixed rate, 107% using the actual market rate and 105% under a 40 quarter smoothed market rate. On average, the cuts are very minor: the average pension benefit around 2010 is only about 8% less than it would have been if there were no cuts at all. Thereafter, this percentage gradually declines to less than 3.5%. However, in bad circumstances, things look much worse. The 97.5 percentile for instance indicates cuts between 20% and 25%, which means that during the first 45 years pension benefits can not keep up with average price increases<sup>6</sup>.

Regarding the premiums, substantial differences are apparent between accounting methods. The introduction of the new regulatory regime has only minor effects if the fixed actuarial rate is maintained for premium policy and the calculation of the real funding ratio. There is a small probability that the nominal funding ratio drops below 105%, which results in substantial premium jumps. Consequently, the 99 percentile is higher than before<sup>7</sup>. However, already for the 97.5 percentile, premiums are marginally lower under the FTK regime. Also, the 95, 90, and 80 percentiles are lower, whereas the median premium is somewhat higher. This is due to the fact that the FTK forces pension funds to levy the cost-effective premium up to a relatively high funding ratio.

An important advantage of the FTK is that discontinuity risk is diminished. As nominal pension rights are unconditional, these rights should preferably always be transferrable if necessary (in case of bankruptcy of the sponsor). The emphasis of the FTK on the market value of nominal pension rights reduces the probability of nominal underfunding.

The premium pattern looks much worse if the actual real market rate is used as a discount rate. Due to the volatility of the liabilities, the cost-effective premium and recovery costs are much more volatile and much higher in the extremes<sup>8</sup>. In about 2% of all cases, the maximum premium (50% of the premium base) is charged. Regarding the average premium,

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<sup>6</sup>Both the figures for the nominal pension and the inflation-proof pension are calculated for the average price inflation rate of 1.9% per year. During high inflation regimes, the cuts can be larger.

<sup>7</sup>Also under the old regulatory rules underfunding, using the 4% discount rate, was not allowed and had to be remedied immediately. This requirement is not modeled in the pre-FTK regime however.

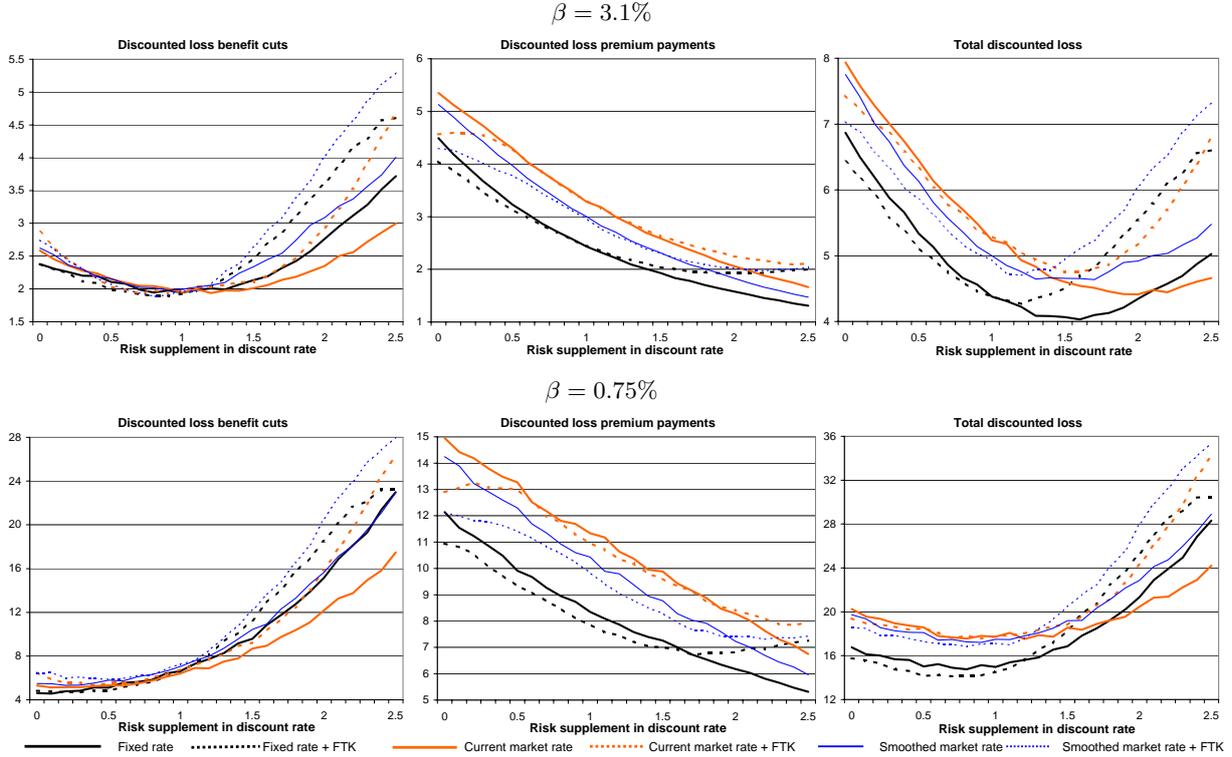
<sup>8</sup>The dotted line in the premium graphs shows the average scenario-specific ten year standard deviation of positive premium payments (in case of premium restitutions, the premium is set to zero).

the initial rise has to be slightly higher than under a fixed rate, whereas in the long run the two methods give almost identical results. The main reason for the even on average higher cost of the market valuation method is timing. Under market valuation decreasing interest rates negatively affect the funding ratio as the duration of the liabilities (about 16 years) is much higher than the one of the assets. Consequently, low funding ratios are more likely during low interest rates. The recovery costs on the other hand are the highest with low interest rates as expected future returns are lower in that case. Even if funding ratios are still sufficient, higher premiums will be charged during periods of low interest rates as the cost-effective premium also rises if interest rates decline. If a fixed actuarial rate is used instead, the charged cost-effective premium and recovery costs do not depend on the actual market interest rate and the liabilities are given a duration of zero. Consequently, low funding ratios are more likely after a rise in interest rates as the bonds in portfolio will lose value. This means high premiums are more likely if interest rates — and thereby expected future returns — are high. Ignoring the (hopefully temporary) higher costs in case of low interest rates comes not completely without costs though. If discontinuation of the pension fund is a serious threat, the hope for better times later on might be idle as the fund might not survive. The probability of nominal underfunding is slightly higher under a fixed actuarial rate than under market valuation.

The smoothed real market rate method seems to inherit primarily the negative aspects of both other methods. Despite the smoothing of real interest rates over forty quarters (which lowers the annual volatility of the 16 year rate from 61 to 11 base points), the premium volatility is only slightly lower than under the current rate. Moreover, according to the smoothing method, pension funds only had a small problem in 2002 as the high interest rates of the early nineties still kept the discount rate relatively high. As with a fixed rate method, a long smoothing period carries the danger of postponing necessary recovery measures for too long, because of too optimistic return expectations.

So far, we have calibrated the real discount rate in such a way that on average it is the same as the expected real return. Although this is common for the actuarial fixed rate method, it is not for the market valuation method. According to market valuation, a higher probability of indexation would have to lead to a lower discount rate and thereby a higher liability. This however, is exactly opposite to the desired situation from a pension security point of view. It would mean that a solid sponsor, who could easily donate a substantial shortfall premium if necessary, has a higher liability and therefore should have more reserves than a weak sponsor. In other words, the capital buffer today should be the highest for a

Figure 6: Optimal risk supplement in discount rate



sponsor who, in the absence of a buffer, is already most likely to fulfill its indexation ambition tomorrow.

The impact of the size of the risk supplement in the discount rate, above the (average) real 16 year market rate, is shown in Figure 6. The higher three graphs show the discounted utility loss of the benefit cuts, the premium payments and the two added together under a time preference parameter equal to the expected real return (3.1%), whereas the lower three graphs give a much higher weight to future cuts and contributions with a time preference parameter of only 0.75%. The loss for the three discounting methods are shown both before (straight lines) and after (dotted lines) the new regulatory rules become effective.

As to the contributions, a higher risk supplement substantially reduces the discounted costs, even if future costs are given a high weight. Under the FTK, a very high risk supplement would again increase losses due to contributions, as nominal underfunding becomes more likely. This only happens however, at very high risk supplement levels that would imply a discount rate higher than expected real returns. In all cases, using a fixed actuarial rate leads to substantially lower discounted premium costs, both for high and low time preference. This result is robust to changes in the utility function as market valuation increases

both the average contribution rate (due to the worse timing) and its volatility<sup>9</sup>. The more aggressive adjustments under the FTK lead to somewhat lower discounted premium costs if time preference is low, but about equal costs if it is high.

With respect to indexation, a low risk supplement is to be preferred, although a very low value leads again to higher losses if time preference is high. This is due to the large initial cuts resulting from the lower starting funding ratio. High risk supplements lead to substantial cuts, especially under FTK as nominal buffer deficiencies become increasingly likely. Taken the results for benefits and contributions together, a high time preference parameter would lead to an optimal discount factor that is even higher than expected returns. For low time preference, a risk supplement of about 1.1% (leading to an average real discount rate equal to expected real returns) seems only slightly too high for the actuarial rate FTK combination, and about optimal for the other cases. Therefore, we will proceed with discount rates that are on average equal to expected real returns.

## 5.2 Optimal stock bond allocation

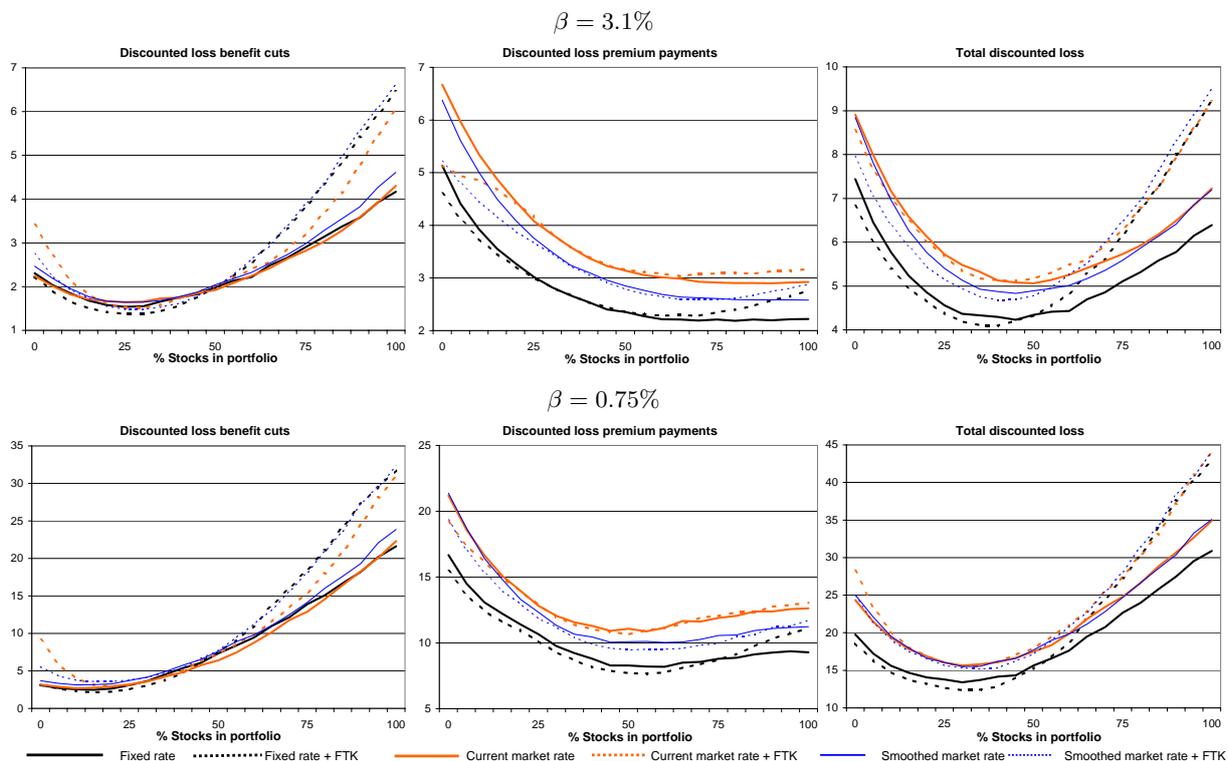
The focus of the new regulatory regime on short term solvency is often blamed to discourage risk taking. In order to investigate whether the optimal asset mix is indeed seriously affected by regulation, Figure 7 shows the optimal percentage of stocks in portfolio for all possible discounting methods. In this subsection, it is assumed that the duration of bonds in portfolio is always 5 years.

As before, the difference in premium costs between the fixed rate method and the market valuation methods is outspoken. With respect to premiums, total discounted utility loss decreases with the stock market allocation up to about 60% after which the costs stabilize in the pre-FTK area and increase somewhat after the new regime is effective. Regarding benefit cuts, about 25% investment in stocks seems optimal, after which the loss increases substantially, especially under the new regulatory regime. Taken together, the optimal percentage of stocks assuming high time preference is about 45 without taking the FTK into account and about 40 including it. For low time preference, this percentage is somewhat lower, about 30, both before and after the introduction of the FTK. Consequently, the new regulatory regime indeed encourages pension funds to take slightly less risk if time preference is relatively high,

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<sup>9</sup>Van Binsbergen and Brandt (2005) on the other hand find that using the actual market rate leads to higher utility than using a smoothed or fixed discount rate. The main reason for this result is the inclusion of the funding ratio based on the same actual interest rate in their utility function. Moreover, only linear contribution costs are included.

Figure 7: Optimal % stocks in portfolio

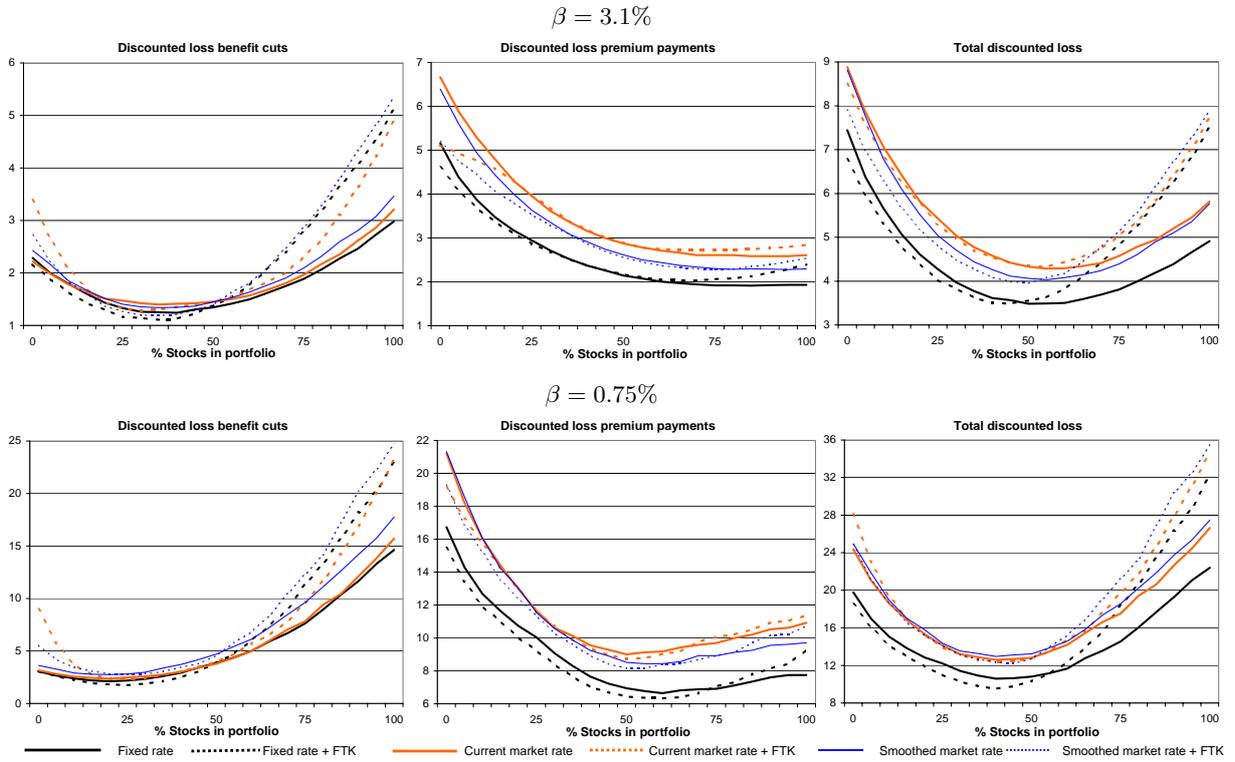


but this only amounts to about 5 percentage points less equity. Moreover, total costs do not rise under the FTK, as a matter of fact they even decline somewhat<sup>10</sup>!

An optimal asset allocation with only 30% to 45% stocks seems somewhat low given the observed allocation nowadays (about 50%). There might be several reasons why pension funds take more risk than seems optimal from these calculations. First of all, they might of course have a different utility function than the one assumed here. Less weight to either benefit cuts or extreme premium payments would increase the optimal stock allocation. If cuts and contributions are both measured quadratically and with equal weights, the optimal fraction of stocks would increase to 40% to 60%, for instance. Second, the assumed risk premium on the stock market (3%) is rather conservative. Historically, higher risk premiums are observed in most countries (Dimson, Marsh, and Staunton 2002). Third, these results are obtained assuming no mean reversion in the stock market. Although mean reversion in the stock market is certainly not a well established statistical phenomenon (Jorion (2003)),

<sup>10</sup>In principle, supervision can hardly lead to lower losses as pension funds can voluntarily follow the rules enforced by regulation. In case of pension funds, external pressure might be necessary however, as optimal policy is not the same for all participants. Without regulation, it would for instance be very hard to levy a cost-effective premium if funding ratios are already high above 100%.

Figure 8: Optimal % stocks in portfolio if the stock market is mean reverting



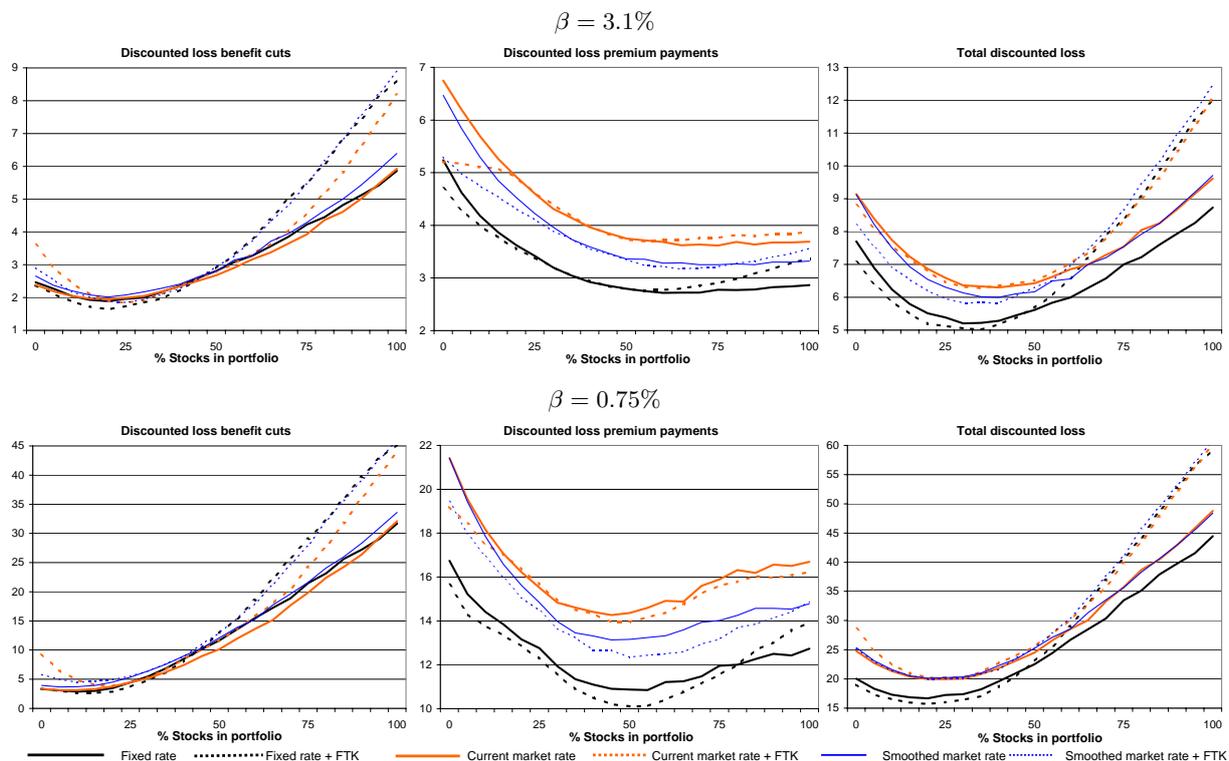
Malkiel (2004)), some indication for its existence is certainly documented (Campbell and Shiller (1988), Fama and French (1988), Poterba and Summers (1988), Campbell and Viceira (2002, 2005)). Mean reversion strongly increases the attractiveness of the stock market for pension funds as low returns are followed by higher expected future returns.

Figure 8 shows the impact of moderate mean reversion (see Figure 4). Although long run stock market volatility is still assumed to be two third of short run volatility (Campbell and Viceira (2005) assume it to be less than half), optimal stock market investment already increases to 40 to 50%.

Figure 9 shows the impact of a lower than expected equity risk premium. Although policy is based on the assumption that the risk premium is 3%, in reality it turns out to be only 2%. The lower equity returns naturally decreases the attractiveness of stocks. From a premium perspective though, the optimal equity allocation is still 45% or more. However, the loss due to benefit cuts increases more rapidly with stock market investment. Consequently, the optimal stock market allocation will be about 10 percentage points lower. The costs of overinvestment in stocks are modest however.

Next, we are going to investigate to what extent the new regulatory rules force pension

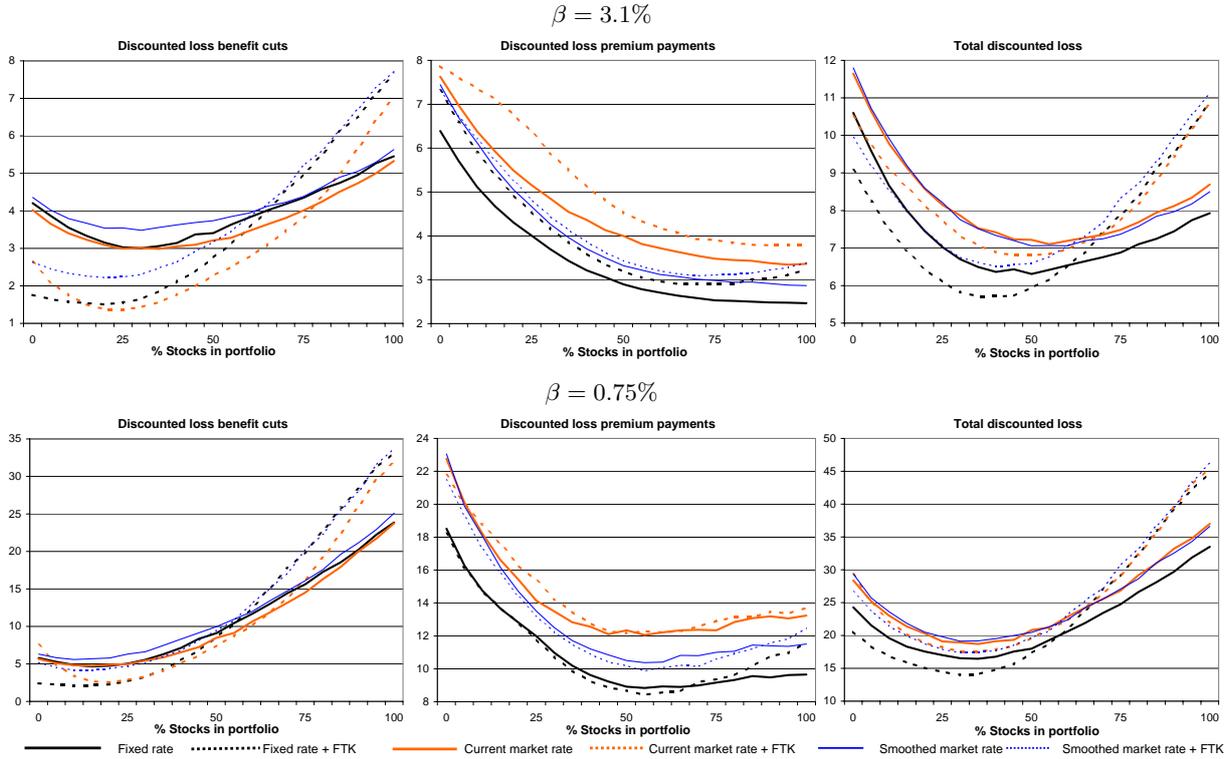
Figure 9: Optimal % stocks in portfolio if equity risk premium is unexpectedly lower



funds out of risky investments if the funding ratio becomes too low. The high penalty for nominal underfunding can be better avoided by taking less risk. On the other hand, less risk taking also reduces expected returns and thereby the expected speed of recovery. From the literature, the picture emerges that in the absence of a large penalty, lower funding ratios increase optimal risk taking, see for instance Boulier, Trussant, and Florens (1995) and Siegmann and Lucas (1999). Boulier, Michel, and Wisnia (1996) add a maximum contribution rate, using a linear quadratic loss function, and find optimal risk taking to decrease for low funding ratios. Cairns (1995) finds that constant proportion portfolio insurance (Black and Perold 1992), in which risk taking declines with the probability of underfunding, is optimal if there is a hard lower limit on the funding ratio. Dybvig (1999) adds a spending rule to the portfolio insurance asset allocation rule, and finds that portfolio insurance outperforms traditional strategies if returns are persistently high or low, but underperforms if high and low returns alternate. Siegmann (2006) uses a mean-shortfall model to show that the optimal investment strategy is V-shaped. Both large shortfalls and large buffers should lead to more risk taking. Empirically, he finds no clear evidence of this behavior for Dutch pension funds however.

Figure 10 shows the discounted losses if the initial asset position of pension funds in

Figure 10: Optimal % stocks in portfolio if starting funding ratio is lower



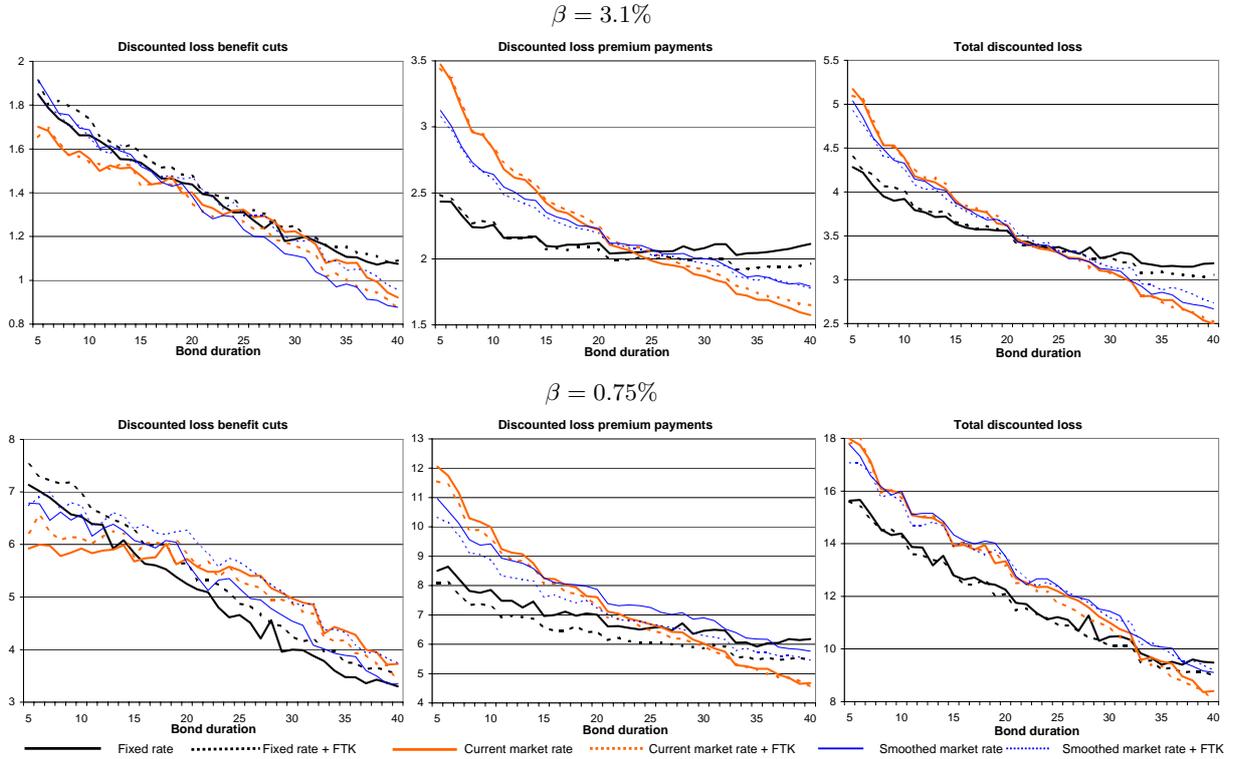
2002 was 20% lower. The optimal asset mix seems not very much affected by the initial funding ratio<sup>11</sup>. The new regulatory regime does not give more incentives to avoid the stock market in case of low funding ratios. On the one hand, the costs of benefit cuts increase more if the percentage of stocks rises above 25%, but on the other hand the discounted loss of contributions drops quicker with risk taking. Surprisingly, the more aggressive recovery policy under the new regulatory regime seems to lead to lower total discounted loss, even if time preference is relatively high. This result is due to the lower benefit cuts resulting from the quicker recovery.

### 5.3 Optimal duration

Apart from the optimal percentage stocks, another important aspect of the asset mix is the duration of bonds in portfolio. With a fixed actuarial rate, the assumed duration of the liabilities is zero, whereas under a market valuation approach it is about 16 years for the

<sup>11</sup>For very high starting funding ratios on the other hand, our model shows a clear preference towards less risk taking, primarily to reduce the probability of indexation cuts. Cairns (2000) and Owadally and Haberman (2004) also find optimal risk taking to decrease with the funding ratio. Sundaresan and Zapatero (1997) on the other hand find the opposite. They assume one time funding in a complete market setting. As liabilities can be fully hedged, any funding surplus can be used to take more risk.

Figure 11: Optimal duration if starting position is equilibrium standard term structure



average pension fund. Consequently, extending the duration of bonds in portfolio increases the duration gap in a fixed rate environment and decreases it under market valuation. Indeed, the target funding ratio rises with the bond duration under actuarial accounting, whereas it decreases under market valuation. In this subsection, we will investigate the impact of bond duration, thereby assuming the asset mix contains 50% equity.

Figure 11 shows the impact of extending bond duration, assuming the initial term structure and inflation rate are identical to their equilibrium values (see Figure 2). Surprisingly, extending the duration not only improves results under market valuation, but also if a fixed actuarial rate is used for conditional rights. The main reason for this result probably is that interest rates are mean reverting. Rising interest rates deteriorate the funding ratio more under actuarial accounting if duration is high. At the same time however, future return expectations improve. Consequently, a higher duration forces to high contributions exactly when return prospects are good. Therefore, deteriorations of the funding ratio are less likely to be long lasting, leading to an especially improved benefit performance. Another reason for the lower costs is the on average somewhat higher returns on long term bonds due to the term premium. The impact of the new regulatory regime is only very minor. What matters

is the way pension premiums and the real funding ratio are calculated, not so much whether the nominal funding ratio is given a prominent role.

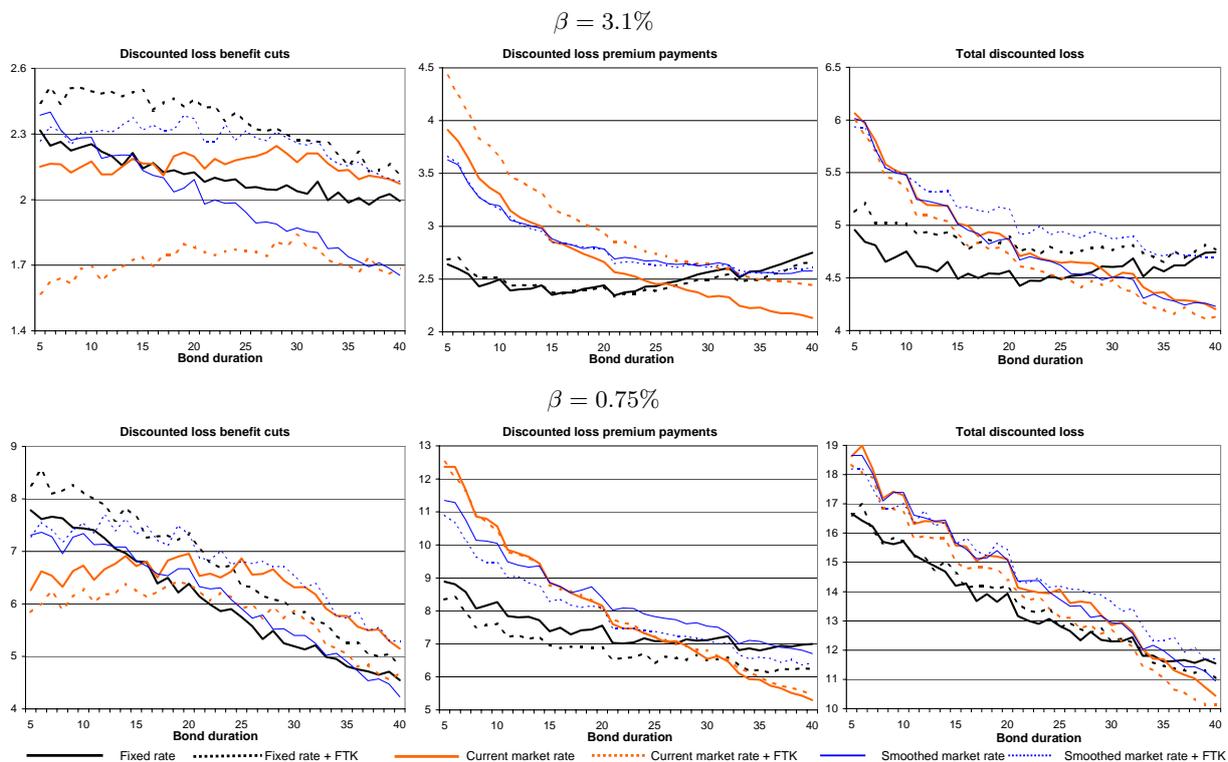
The very positive impact of duration extension contrasts with De Jong (2003), who finds long term nominal bonds to be a very poor hedge due to inflation risk. There are at least three reasons for the difference in results. First, De Jong optimizes utility of terminal wealth given initial wealth, without taking additional contributions into account. Consequently, the timing issue does not arise in his study. Second, De Jong uses the parameter values of Brennan and Xia (2002). Their AR(1) parameter for inflation is as high as 0.973, whereas our estimate is only 0.953. Apart from the different area (US versus Germany), the higher persistence is probably due to the use of a one-step estimation procedure. Third, we allow for interaction of interest rates and inflation, whereas this is not modeled in De Jong (2003).

If market valuation is also applied to the real discount rate, the gains of duration extension are even larger. The main improvement now takes place in the contribution costs. A duration mismatch of zero is certainly not optimal: even if the bond duration is already 32 years, extending the duration further still significantly improves the funding costs. The most important reason for this result is probably again timing. Due to the mean reversion in interest rates, it is optimal to pay higher premiums if interest rates are high. If the duration mismatch is zero, this is still not the case as lower interest rates increase the cost-effective premium. Although a high duration improves the market valuation results more than the fixed rate results, for durations up to 22 or even 32 years (depending on time preference) the fixed rate method still leads to lower costs. As market valuation increases both the mean contribution rate and its volatility, this result is quite robust with respect to the specification of the utility function. Given the limited supply of long term bonds, these results suggest a clear preference for fixed rate accounting for conditional rights for every feasible duration.

The next issue investigated is whether it is still beneficial to extend the duration if initially interest rates are very low. In July 2005, the 16 year zero coupon rate had dropped to only 3.79%, whereas the equilibrium value according to our term structure model is 5.02%. This lower interest rate is taken as new starting value (for 2002), to examine whether the long run better performance compensates enough for the short term foreseeable decline in bond value due to an interest rate rise. This is analyzed in three different settings. First, it is assumed that our standard term structure model (Figure 2) is still appropriate. Second, the impact of a permanently lower and flatter yield curve (see Figure 3) is investigated, both under the assumption that this lower equilibrium curve comes as a surprise, and if it is anticipated.

Figure 12 shows the impact of only temporarily lower interest rates. Compared to the

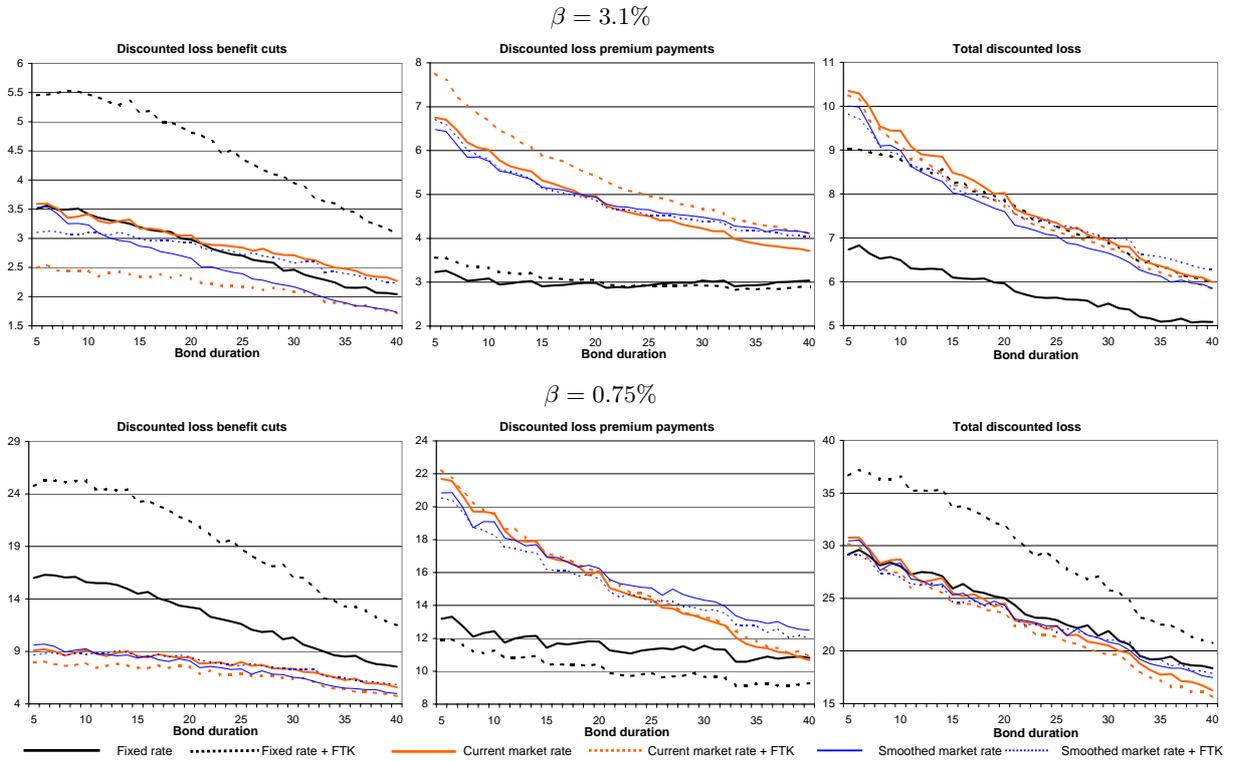
Figure 12: Optimal duration if interest rates are temporarily low



equilibrium starting position both indexation results and premium costs improve less for higher durations. Nevertheless, overall discounted costs are still lower for high durations than for low ones. This does of course not mean that performance can not be improved further if the duration extension is postponed somewhat. Regarding the influence of the new regulations, there is a clear distinction between the fixed rate and the market rate method. The FTK leads to a much more aggressive premium policy, especially under market valuation, as the maximum premium increase of three percentage points is not applied in case of nominal underfunding. Under market valuation premiums remain high, as the real funding ratio is very low as well, whereas the real funding ratio under actuarial accounting is not affected by the low initial interest rates. The more aggressive premium policy under market valuation leads to higher contribution losses but lower benefit cuts. Overall, the fixed actuarial rate method still seems to be preferred for every feasible bond duration.

If interest rates remain permanently low, the fixed actuarial rate method does not correctly signal real funding problems (Figure 13). As the low interest rate environment is not anticipated, the fixed actuarial rate is not affected by this change. Consequently, under actuarial accounting, the implicit target buffers are lower resulting in a higher probability of

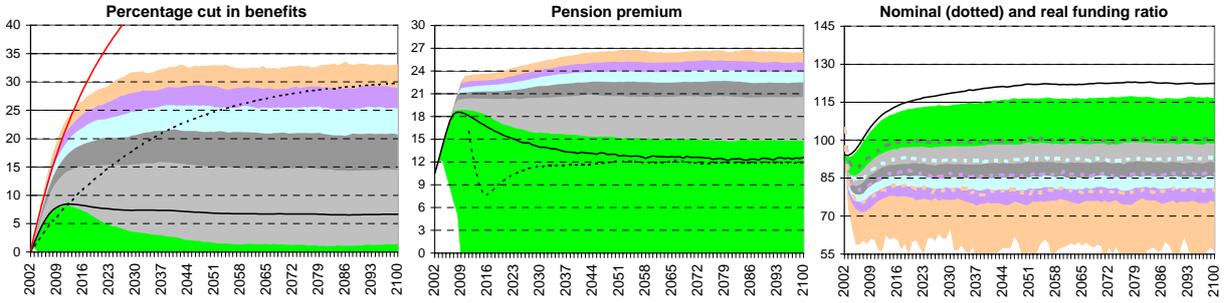
Figure 13: Optimal duration if interest rates are unexpectedly permanently low



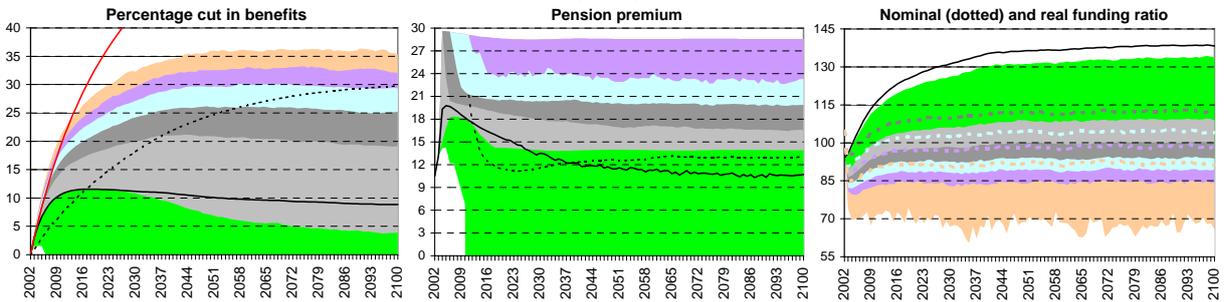
indexation cuts. In terms of contributions on the other hand, the (too) high discount rate leads to less aggressive premium setting and therefore lower losses. For the market valuation method, the opposite holds as the risk supplement in the discount rate is slightly too low (about 20 base points if the bond duration is 5 years) due to the unexpectedly flatter yield curve. Despite the flattening of the yield curve, a higher duration is still optimal, both under market valuation and actuarial accounting. If time preference is high, actuarial accounting still seems optimal, although the lower costs vanish under the FTK. The higher discounted loss under the FTK can be justified by the higher probability that guaranteed pension rights can indeed be fulfilled.

Figure 14 gives some more details for a standard pension fund with a duration of only 5 years, and 50% equity in the asset mix. As returns are on average 76 base points lower than expected, the probability of nominal underfunding remains higher than 10% under actuarial accounting. Under the FTK, this leads to substantial premium jumps and also higher indexation cuts. Although the indexation and premium pattern may look more attractive in the pre-FTK era, one has to bear in mind that these results do rely on the continuity assumption. If a plan sponsor fails, nominal underfunding will be a serious problem.

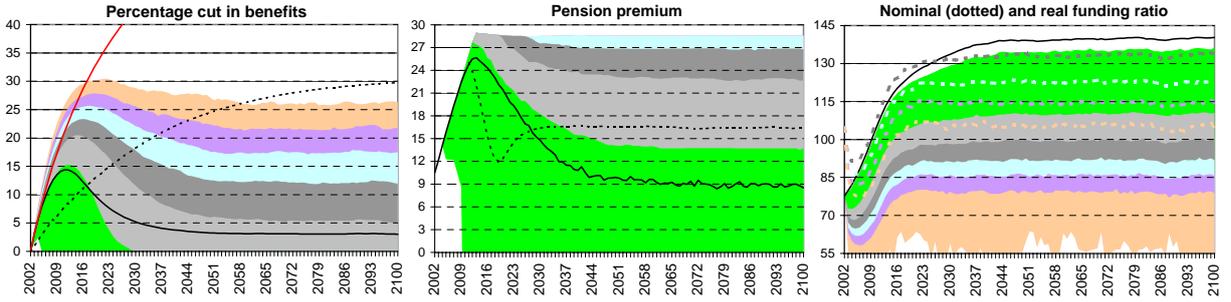
Figure 14: Impact unexpected permanently lower interest rates for a standard pension fund  
 Fixed discount rate (3.1%), no role nominal funding ratio



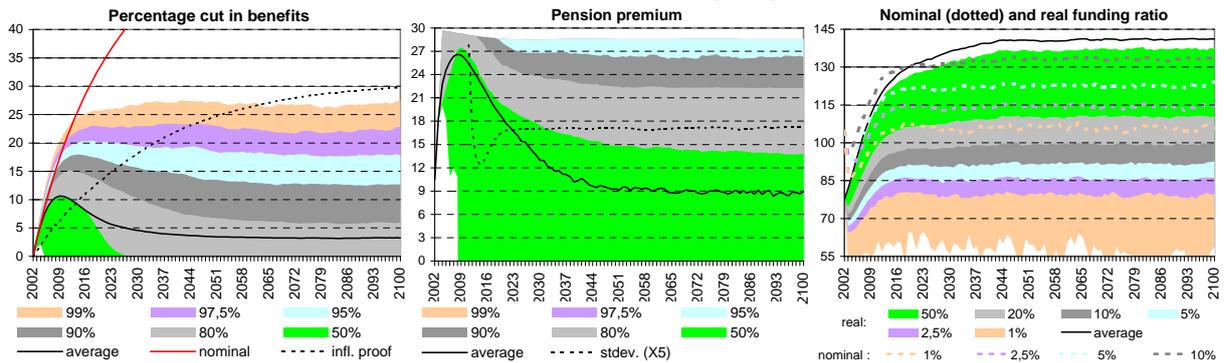
Fixed discount rate (3.1%), FTK compatible



Actual real market rate plus risk supplement (1.1%), no role nominal funding ratio

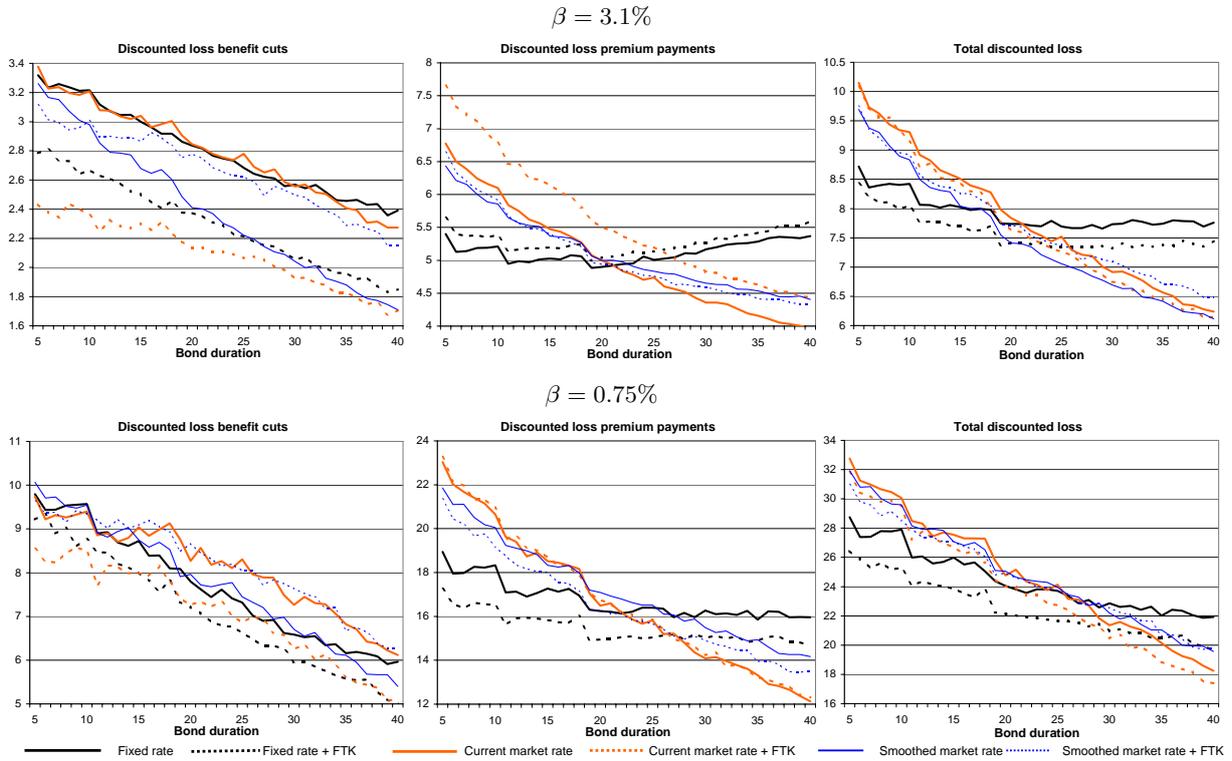


Actual real market rate plus risk supplement (1.1%), FTK compatible



Under market valuation, the premium setting is much more aggressive. Even though nominal underfunding is much less likely in this setting, there is a high probability that the maximum premium is charged. In the long run this probability is about 6.75% pre-FTK,

Figure 15: Optimal duration if interest rates are anticipated to remain low



6.25% under FTK, whereas it is about 3.25% under FTK using a fixed rate. The advantages of the more aggressive premium policy are lower indexation cuts and a much lower probability of nominal underfunding.

Of course, the assumption that a permanent lowering and flattening of the yield curve is not observed for 100 years is somewhat unrealistic. Therefore, Figure 15 shows the impact of the lower yield curve if it is anticipated. For the market valuation methods nothing much changes. Due to the flattening of the yield curve, the risk supplement in the discount rate (equal to the average expected real return minus the average expected 16 year real rate) is increased somewhat. The discounted costs hardly change however. For the fixed rate method the consequences are more material. The lower actuarial rate diminishes the probability of extended periods with low buffers, thereby decreasing the indexation cuts. On the other hand, the lower discount rate requires more aggressive premium increases, which are not costless either. Also in this scenario, using a fixed rate method seems optimal for all feasible durations.

## 6 Conclusions

In this paper, it is investigated to what extent the new regulatory rules regarding pension supervision in the Netherlands (FTK) influence optimal investment policy. The main conclusions are:

- The focus of the FTK on market valuation and risk related capital buffers substantially reduces discontinuity risk.
- The costs of a defined benefit pension plan increase if policy is *fully* based on market interest rates instead of a fixed actuarial rate. The only exceptions to this rule are funds with an extreme duration of bonds in portfolio (more than 22 years) or funds which systematically overestimate future returns and at the same time have low time preference. The higher costs are due to the mean reversion in interest rates. Under market valuation premiums are to be the highest exactly when expected returns are the lowest.
- As the new Dutch regulatory rules do allow for the use of a fixed rate for premium policy and as no solvency requirements are prescribed for conditional pension rights (indexation), this disadvantage of market valuation does not seriously affect Dutch pension funds.
- If time preference is relatively high, the optimal percentage of stocks in portfolio is about five percentage points lower under the FTK, than under a regime without a severe penalty for underfunding. For low time preference, the new regulatory regime does not affect optimal risk taking.
- Optimal risk taking of pension funds under the FTK is not lower in case of worse funding ratios.
- The optimal duration of bonds in portfolio of pension funds seems to be much higher than currently observed. Although the gain of extending the duration is the highest under market valuation, costs are also lower for high durations under actuarial accounting.

## Appendix: The inflation and interest rate block

Long term interest rates in PALMNET are based on an affine two-factor term structure model (Duffie and Kan (1996), Duffee (2002)). The determining variables are the short term interest rate and expected (short term) inflation. The model we use assumes a first order vector autoregressive structure for expected inflation and 3-month interest rates. The model is heteroscedastic in the sense that volatility is rising with the level of inflation or interest rates. This way, it is guaranteed that nominal interest rates can not become negative. The same heteroscedasticity factor is used in all model equations in order to preserve the affine (that is linear) term structure relationship without having to rely on independence between shocks to inflation and interest rates. Longer term interest rates are a linear function of future short term interest rates and the price of risk. The former follow directly from the current short term interest rate and expected inflation, whereas the latter is determined by the covariance of the pricing kernel with expected inflation and short term interest rates respectively. The model is calibrated on quarterly data for Germany. We use German data as their monetary policy is most representative for the euro system monetary policy, which is relevant for the interaction between interest rates and inflation. The quarterly frequency (as opposed the annual frequency of the rest of the model) is used first to increase the number of observations, second to be able to perform simulations with smoothed interest rates, and third to reduce the probability that short term interest rates become negative.

In principle, the model for inflation, short and long term interest rates can be estimated simultaneously. However, it turns out that the expected time series behavior of short term interest rates and expected inflation according to the term structure of interest rates is not the same as the one actually observed in the past. As we simulate over a forecast horizon of 100 years, including proper time series parameters is most important. Therefore, a four-step procedure was used instead. In the first step the dynamics of short term interest rates ( $i_t^s$ ) and (short-term) expected inflation ( $\pi_t^e$ ) is estimated, using a Kalman filter approach (Harvey 1989) to decompose actual inflation into expected inflation, surprise inflation and seasonal pattern, on quarterly data for 1960-I until 2004-II:

$$\begin{bmatrix} i_t^s \\ \pi_t^e \end{bmatrix} = \begin{bmatrix} \bar{i}^s \\ \bar{\pi}^e \end{bmatrix} + \begin{bmatrix} 0.90 & 0.11 \\ -0.00 & 0.95 \end{bmatrix} \begin{bmatrix} i_{t-1}^s - \bar{i}^s \\ \pi_{t-1}^e - \bar{\pi}^e \end{bmatrix} + \sqrt{0.03i_{t-1}^s + 0.15\pi_{t-1}^e} \begin{bmatrix} \varepsilon_t^i \\ \varepsilon_t^\pi \end{bmatrix} \quad (3)$$

where  $\bar{i}^s$  and  $\bar{\pi}^e$  represent sample averages, and the disturbance terms  $\varepsilon_t^i$  and  $\varepsilon_t^\pi$  follow an

i.i.d normal distribution:

$$\begin{bmatrix} \varepsilon_t^i \\ \varepsilon_t^\pi \end{bmatrix} \sim \text{i.i.d. } N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.27 \\ 0.27 & 0.59 \end{bmatrix} \right) \quad (4)$$

In the second step the covariances of the pricing kernel with shocks to the short term interest rate and expected inflation are estimated, given the estimated parameters and the optimal prediction of expected inflation from the first step. Hereby, the measurement errors of the bonds of different maturities are allowed to be correlated. The measurement errors represent factors in longer term yields that are independent from inflation and short term rates. For instance, long term interest rates in Europe are, at least in the short run, influenced by bond returns in the US.

In the third step, these measurement errors are examined. It turns out that they are indeed highly correlated, both cross-sectionally and over time. The measurement errors are very similar for all maturities, though slightly bigger for longer ones. Therefore, we decided to model longer term yields as the yield following from the term structure model plus a measurement error that is identical for all maturities apart from a scaling factor. This measurement error follows a first order autoregressive process with an AR(1)-parameter of 0.9.

Finally, the problem arises that the time series pattern of the past need not be representative for the future. For instance, the average inflation rate over the sample was 3%, whereas for the future 1.9% is assumed. For the short rate, an equilibrium value of 4.2% is assumed. Moreover the volatility of interest rates and inflation in the seventies was much higher than in recent times. Therefore, we decided to rescale volatilities of inflation and short rates to 55% of historical values. The volatility of innovations to the long-term measurement errors was calibrated at 85% of the short rate volatility, in accordance with results over the last 20 years. In order to remain a reasonable fit for long term rates the covariances with the kernel are adjusted somewhat as well, such that the fit for 2003 was reasonably good.

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