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Risk management of pensions from the perspective of loss aversion ☆

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Abstract

This paper studies two interrelated questions. First, is a pay-as-you-go (PAYG) pension component beneficial from a risk management point of view? Second, does optimal risk management of old-age consumption differ between different income groups? The analysis is based on so-called lexicographic loss aversion preferences. Interest in these preferences stems from the fact that they explain the cross-section of individuals' savings and asset allocation choices better than alternative models. I find that all income groups benefit from the presence of a substantial PAYG component. Only for the two highest income quintiles old-age provision should heavily rely on equity investments.

JEL classification: D91; G11; H55
Keywords: Pay-as-you-go pension system; Old-age provision; Risk management; Income heterogeneity

1. Introduction

Decreasing birth rates as well as increasing longevity have shifted the implicit returns of pay-as-you-go (PAYG) pension systems downwards. It has become very popular to take this as an argument for more radical pension reforms, aiming to substantially reduce (or even eliminate) the

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reliance on a PAYG component. It has often been claimed that a privatized system would lead to a higher "money's worth" due to higher returns. President George W. Bush's current reform plans for the U.S. are heavily influenced by this idea.[1]

The focus on rate-of-return comparisons overlooks, however, two crucial issues. First, the seemingly advantageous returns of a privatized system disappear more or less when Social Security's unfunded liability is taken into account (Geanakoplos et al., 1998, 1999; Shiller, 2005). Second, different pension systems imply very different risk exposures of old-age income. A fully mature PAYG system can be understood as an asset whose returns are linked to the (stochastic) growth rate of aggregate wage income. In the absence of complete financial markets, the provision of such an asset through the government might be valuable. In particular, as will be shown, the returns of such a "PAYG asset" have a very limited downside risk in comparison to bonds and stocks.

This paper focuses on the second issue. Its main aim is to address the question whether, indeed, having a PAYG pension component is beneficial from a risk management point of view. Given the incompleteness of real-world financial markets, it would not be surprising to find that the introduction of a PAYG asset has positive value, however. The relevant question to ask is, therefore, whether the gains from having such an asset are quantitatively important. A particular issue on which I focus in this paper is whether the quantitative benefits from a PAYG system differ substantially between different income groups. Possibly, only low-income individuals, who may be much more averse to down-side risks of old-age consumption than high-income earners, will benefit from a PAYG component. In order to address this issue in a most transparent way, the analysis in this paper will abstract from any form of intragenerational distribution.

Empirically, individuals' saving rates as well as the share of savings invested in stocks depend strongly on income.[2] In particular, members of the two bottom quintiles of the income distribution do typically not invest in stocks, neither directly nor indirectly. In contrast, members of the highest quintile invest about 55% of their portfolio in stocks. This suggests that optimal risk management of old-age provision varies, indeed, substantially with income. With respect to this it is essential to note that in the literature risk management of old-age income has predominantly been analyzed from the perspective of constant relative risk aversion (CRRA) preferences. This model is severely at odds with observed savings and investment choices, as it predicts that saving rates and equity shares are identical across all (permanent) income levels.

In order to avoid undue paternalism, this paper aims to study risk management of old-age provision in such a way that the evaluation of risk-return trade-offs is consistent with the observed cross-section of individual savings and portfolio choices. A priori, one might expect that standard hyperbolic risk aversion (HARA) preferences, habit formation or standard loss aversion preferences deliver a good quantitative explanation of the empirical income expansion paths for saving rates and equity shares. However, it is shown in previous work (Binswanger, 2005) that this is, in fact, not true. As demonstrated there, the lexicographic loss aversion (LLA) model is the

[1] One out of many examples documenting such reasoning is a quote from a speech held on June 8, 2005 in Washington, D.C.: "Right now, in the Social Security system, we get about 1.8% on your money for you, which is really low. A conservative mix of bonds and stocks is expected to pay about 4.6% annually over time. (...) Heck, you can put your money in T-bills alone and do better than the 1.8% we get you. And over time, that money grows. The difference between what we can get on your money and what you can get in your own personal savings account, if you decide to set one up, is pretty darn significant." See Geanakoplos et al. (1998, 1999) for a thorough discussion of whether privatizing Social Security allows to increase its "money's worth."

[2] See Section 4.

only one so far that is consistent with the cross-section of observed savings and asset allocation choices. This preference model will underlie the analysis of this paper.

At first sight, it might seem odd to use a "behavioral" model, exhibiting loss aversion, for a normative evaluation of old-age provision and pension design. However, this would only be inconsistent if "behavioral" is understood as "irrational." Are loss aversion preferences irrational? Tversky and Kahneman (1991, p. 1057) conclude that this is not the case whenever actual experiences indeed correspond to the feeling of loss aversion. I adopt this position, accepting from a normative point of view that people legitimately are particularly averse to outcomes that undercut some specific consumption threshold levels.

Apart from the above mentioned preference models, there exists by now a sufficiently large amount of other nonstandard models aiming to describe individuals' decision making in an intertemporal environment and in the presence of uncertainty. These range from the hyperbolic discounting model over Kreps–Porteus/Epstein–Zin preferences, (rank-dependent) probability weighting models to models with disappointment and ambiguity aversion.[3] None of these preference models, however, is a natural candidate to explain how saving and risk taking vary with income levels. In particular, hyperbolic discounting preferences have been introduced to examine how a taste for immediate gratification affects individuals' total savings. Kreps–Porteus and Epstein–Zin preferences allow for a preference for earlier or later resolution of uncertainty in a decision tree. Furthermore, risk trade-offs may be evaluated differently from intertemporal trade-offs. Preferences with probability weighting allow for deviations of subjective from objective probabilities and thus for optimism or pessimism. Models with disappointment aversion allow for a feeling of regret if outcomes from a decision are worse than expected. Finally, models with ambiguity aversion have been introduced to study how individuals make decisions in situations where they do not know the true probability distribution of events.

The analysis of this paper takes place within a simple partial equilibrium setting where individuals live for two periods. This allows a focus on the question of how old-age provision differs between different income groups. I assume that each individual faces the choice of how much to invest in bonds, stocks as well as in a PAYG asset whose returns are given by the growth rate of aggregate wage income. While it is natural to think of this asset as a PAYG system, it should be kept in mind that such an asset may also be created in an alternative way, e.g. through "macro markets" (Shiller, 1998). In the case of a PAYG system, a social planner would implement an optimal old-age provision plan by setting contribution rates equal to the individual expenditure shares for PAYG investments. In a stationary state, these contribution rates would be constant over time/generations, but they may differ between income groups.

The main findings are as follows. A PAYG system offers a particularly favorable down-side risk protection of old-age income. PAYG investment rates decline with income, but are substantial for all income groups. Optimal equity investment rates are zero or very low for the three bottom quintiles of the income distribution, while they are substantial for the two top quintiles. Bond investments are only valued at very low incomes as a contribution to portfolio diversification. Overall, the analysis makes a case against a privatization of Social Security. Rather, it suggests a system where pension income is financed purely out of a PAYG system for lower-income earners. In contrast, old-age provision for higher-income earners should also substantially rely on equity investments. To put these results into perspective, I also examine

[3] See Rabin (1998), Starmer (2000) and Backus et al. (2004) for overviews. See Laibson et al. (1998), Haliassos and Hassapis (2001), Ang et al. (2005), Gomes and Michaelides (2005) and Epstein and Schneider (2006) for applications of nonstandard preferences to savings and portfolio choice.

optimal pension plans under standard HARA preferences, which are the natural "competing" candidates to LLA preferences when analyzing risk management of pensions. It turns out that these plans are problematic.

Other studies focusing on the risk management of old-age provision are Baxter and King (2001), Campbell et al. (2001), Poterba (2004), Poterba et al. (2005) and Gollier (2005). My analysis departs from this literature in two ways. First, I explicitly consider the contribution of a PAYG system to risk management of old-age income. Second, I do not assume CRRA preferences, which imply constant saving rates and constant equity shares over (permanent) income. Rather, I assume that preferences are of the LLA type. This preference model has been developed and tested in Binswanger (2005).

The rest of the paper is organized as follows. Section 2 discusses the empirical distribution of the growth rate of aggregate wage income and of bond and stock returns. Section 3 explores theoretically the conditions under which a PAYG system is beneficial from a risk-management point of view. Section 4 provides calibrations for optimal old-age provision plans. Section 5 presents calibrations for an extended version of LLA preferences, taking into account a more general pattern of risk aversion. For the sake of comparison, Section 6 considers optimal pension plans under HARA preferences. Section 7 concludes.

2. Return distributions for alternative pension components

This section discusses the return distributions for a PAYG system, stocks and bonds, as they can be taken as representative for future investments, at least for the U.S. These return distributions will then underly the simulation analysis in Sections 4, 5 and 6. As discussed in the Introduction, I focus on a stylized PAYG system where the implicit returns are simply given by the growth rates of aggregate wage income (see below). I will henceforth refer to the growth rates of aggregate wage income simply as PAYG returns. A PAYG system with such returns may potentially be replicated by the use of "macro markets" (Shiller, 1998). The discussion in this section has a particular focus on the downside risk exposure of the potential pension components, apart from their expected returns. The reason is that downside risk, rather than the variance, is a first-order measure of riskiness under LLA preferences.

The derivation of the return distributions is based on historical data of annual stock and bond returns as well as historical growth rates of aggregate wage income. Data source for the growth rates of aggregate wage income are the National Income and Product Accounts (NIPA Table 1.10). Aggregate wage income is measured by compensation of employees and deflated by the GDP deflator. Data source for bond and stock returns is Shiller (2000).[4] The sample period ranges from 1929, the first year with entries in the NIPA Tables, to 2003. For bond returns I use historical real returns of non-inflation-protected bonds rather than returns of Treasury inflation protected securities (TIPS). The reason is that there is very little historical knowledge regarding the range of returns of the latter. Their current (04/10/06) long-term returns are about 2.4% p.a. (source: U.S. Department of the Treasury). The geometric mean of annual real returns on long-term bonds has been 1.8% from 1929 to 2003 and only 1.2% from 1938 to 2003. For an average of long-term bonds and money market rates the corresponding figures are 1.6% and 1.1%, respectively. Hence, current returns on TIPS exceed by far the average of historical bond returns. It is important to note that TIPS are less risky than ordinary bonds, as they are not exposed to inflation risk. Thus, if anything, this suggests that, on average, TIPS returns should be lower than

[4] Updates are found at http://www.irrationalexuberance.com/ShillerSocSec.xls.

the returns on ordinary bonds. This suggests that the current level of return on TIPS should be taken as exceptionally high and therefore less appropriate to gage the return distribution for future bond investments. After potential adjustments of the level of the historical data (see below) I estimate autoregression equations for the logs of each return series.[5] I make the usual assumption that the disturbance terms in the autoregressions are normally distributed. This implies that log-returns are normally distributed and level returns are hence lognormal. Drawing randomly a series of disturbance terms for each of T years, the autoregression equations can be used to calculate a series of annual returns and, finally, the return of an investment over T years. Repeating this procedure many times, a simulated return distribution for investments over a horizon of T years is obtained. Realizations of the annual disturbance terms are drawn in a way that allows for contemporaneous correlation between the different returns. The underlying covariance matrix has been estimated from the data. The details of the derivation of the return distributions are described in Appendix A.

It is the simulated distribution of investment returns over T years that underlies the discussion here and the simulations presented in Sections 4, 5 and 6. For all baseline simulations of optimal old-age provision plans, the length of the time horizon T is set to 32 years. This number is obtained in the following way. I assume that working age lasts from age 21 to 65. Furthermore, death takes place with certainty at the age of 87.[6] This corresponds to the projected average life-expectancy of men and women at age 65 for the year 2080 (Board of Trustees, 2005, Table V. A3). The average investment horizon for old-age provision can then be roughly approximated by the distance between the middle of 65 and 87 and the middle of 21 and 65, minus one. The subtraction of one year refers to the assumption that savings are invested at the end of a particular year during working age, but are withdrawn at the beginning of a particular year during retirement. Under these assumptions, the typical old-age provision dollar is invested for 32 years.

When the return simulations are based on unadjusted historical return data, the annualized expected return obtained is 4.1% annually for the PAYG system. For both, long-term bonds as well as an average of long-term bonds and money market rates, expected returns amount to 1.2%. (I will henceforth refer to the latter as average bonds, or simply bonds when there is no danger of confusion). For stocks the corresponding number is 7.2%. The downside risk is measured by the bottom percentiles of the return distributions.[7] On the basis of unadjusted historical return data one obtains first percentiles of 2.3% annually for PAYG returns, −2.2 and −2.8% for average and long-term bonds, respectively, and −.8% for stock returns. From these figures one would conclude that the difference in downside risks between a PAYG system and the other assets is overwhelming.

For two reasons, however, these return distributions cannot be accepted as representative for future investments. First, the returns from a PAYG system are likely to be grossly overestimated, since the growth rate of the labor force is expected to be much lower in the future. Second, it is

[5] Alternatively, one might estimate VARs. There are two reasons why I do not pursue this approach. First, model selection procedures such as the Akaike information criterion suggest to include just one lag. However, it turns out that the failure to include more lags for PAYG returns overestimates their downside risk. For single autoregressions it is suggested to include 9 lags, instead, for PAYG returns (but only one or zero for bonds and stocks). Second, the coefficients in the VAR equations for alien lagged variables are always insignificant.

[6] The analysis of this paper abstracts from longevity risk. It considers risk management of old-age consumption at a particular age, given that this age is reached with certainty.

[7] The ith percentile indicates a return level that is undercut with a probability of i percent.

Table 1
Autogressions

PAYG	Constant	0.01 (2.49)
	Lag 1	0.55 (4.63)
	Lag 2	−0.07 (−0.52)
	Lag 3	−0.07 (−0.55)
	Lag 4	−0.15 (−1.15)
	Lag 5	−0.10 (−0.78)
	Lag 6	0.03 (0.25)
	Lag 7	−0.10 (−0.77)
	Lag 8	0.23 (1.86)
	Lag 9	−0.35 (−3.23)
	Variance of disturbance	0.0014
Average bonds	Constant	0.01 (1.18)
	Lag 1	0.26 (2.16)
	Variance of disturbance	0.0032
Long-term bonds	Constant	0.01 (1.12)
	Variance of disturbance	0.0077
Stocks	Constant	0.05 (2.31)
	Variance of disturbance	0.0264

Note: Data source for PAYG returns (growth rates of aggregate wage income) are the National Income and Product Accounts (NIPA Table 1.10). Aggregate wage income is measured by compensation of employees and deflated by the GDP deflator. Data source for bond and stock returns is Shiller (2000) and http://www.irrationalexuberance.com/ShillerSocSec.xls. The sample period ranges from 1929 to 2003. Table 1 reports the results from estimating autoregression equations of the form $\tilde{r}_t^i = \phi_0^i + \sum_{l=1}^{p_i} \phi_l^i \tilde{r}_{t-1}^i + \varepsilon_t^i$ for annual return data. The t-values for the estimated coefficients are indicated in parentheses.

likely that stock returns have been exceptionally high during the twentieth century. From the figures presented above one would have to infer that stocks first-order stochastically dominate bonds, ex ante.[8] This seems hardly compatible with any reasonable notion of equilibrium and, thus, suggests that some adjustments are necessary in order to obtain return distributions that are suitable for our purpose. To be on the safe side I will not assume that bond returns are higher in the future, but rather that stock returns are expected to be lower. Moreover, I will assume that PAYG returns are substantially lower in the future.

Specifically, I shift the historical PAYG and stock return data downwards by an additive constant before estimating the autoregression equations. The amount of this shift is chosen such that the simulated expected T-year returns correspond to some specific target return. For the PAYG system, I set this target return equal to the average of the intermediate and pessimistic (high-cost) projection of the Social Security Administration for the long-run growth rate of aggregate wage income. This yields an annualized value of 1.4%.[9] I take the average of the intermediate and pessimistic scenario, instead of simply the intermediate scenario, as a precautionary estimate of future expected PAYG returns. For stocks I choose a target level for expected returns of 4.8% annually. This corresponds to the international average of stock returns

[8] See also Burtless (2003) for a demonstration of this fact. The large difference between stock and bond returns reflects, of course, the equity-premium and the risk-free-rate puzzle.

[9] For the more distant future, the Social Security Administration projects an annual productivity growth rate of 1.6% and a growth rate of the labor force of .2% for the intermediate scenario. The corresponding values for the high-cost scenario are 1.3% and −.2%, respectively. (See Board of Trustees, 2005, Table V.B1 and V.B2. Short-run projections are higher).

Table 2
Return distributions (returns measured in %)

Expected values	PAYG	58
	Average bonds	49
	Long-term bonds	48
	Stocks	346
Variances	PAYG	15
	Average bonds	46
	Long-term bonds	64
	Stocks	2673
Covariances	PAYG/average bonds	−4
	PAYGO/long-term bonds	−3
	PAYG/stocks	28
	Average bonds/long-term bonds	53
	Average bonds/stocks	113
	Long-term bonds/stocks	130
Percentiles (1st/5th/10th)	PAYG	−11/5/13
	Average bonds	−51/−34/−23
	Long-term bonds	−58/−43/−31
	Stocks	−65/−37/−13

Note: Source for PAYG returns (growth rates of aggregate wage income) is National Income and Product Accounts (NIPA Table 1.10). Aggregate wage income is measured by compensation of employees and deflated by the GDP deflator. Data source for bond and stock returns is Shiller (2000) and http://www.irrationalexuberance.com/ShillerSocSec.xls. The sample period ranges from 1929 to 2003. The reported figures refer to returns cumulated over 32 years. See Section 2 and Appendix A for a description of the derivation of the cumulated return distributions. The underlying data series for PAYG returns has been adjusted by an additive constant, such that expected PAYG returns correspond to the average of the intermediate and pessimistic long-run projection of the Social Security Administration for the growth rate of aggregate wage income. Underlying stock return data have been adjusted such that expected returns correspond to the average of international stock returns during the twentieth century.

during the twentieth century (see Shiller, 2005). It turns out that this adjustment is just sufficient to avoid that stock returns first-order stochastically dominate bonds.

Table 1 contains the coefficients from the autoregressions estimated for the shifted return series. By construction, expected annual returns amount to 1.4% for the PAYG system and to 4.8% for stocks. The corresponding figures for average and long-term bonds are again 1.2%. Cumulated over 32 years, expected returns amount to 58% for the PAYG system, 49% and 48% for average and long-term bonds, respectively, and 346% for stocks. These figures as well as the elements of the covariance matrix for PAYG, bond and stock returns are summarized in the three upper parts of Table 2.

Let us now take a look at the downside risks. For the first (fifth/tenth) percentile, the annualized returns amount to −.4 (.1/.4) % for the PAYG system, to −2.2 (−1.3/−.8) % for average bonds, −2.7 (−1.7/−1.1) % for long-term bonds, and −3.3 (−1.4/−.4) % for stocks. Cumulated over 32 years the corresponding values are −11 (5/13) % for the PAYG system, −51 (−34/−23) % for average bonds, −58 (−43/−31) % for long-term bonds, and −65 (−37/−13) % for stocks. These percentiles are summarized in the bottom part of Table 2. Fig. 1 depicts cumulated 32-year return levels for the thirty lowest percentiles. The solid line corresponds to the PAYG system. The dashed and dash-dotted lines represent average and long-term bonds, respectively. The dotted line corresponds to stocks.

Overall, it turns out that, even if PAYG returns are adjusted in a conservative way, the downside risk exposure of the PAYG system is very limited in comparison to stocks, but still also

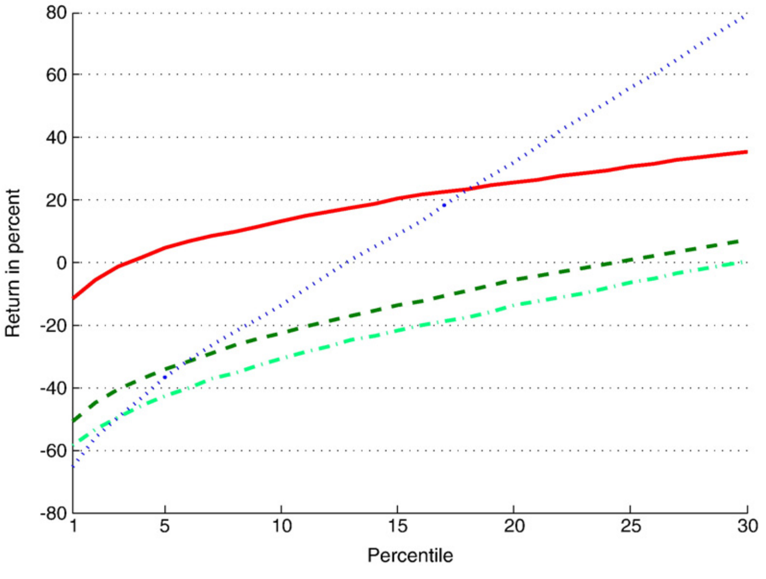

Fig. 1. Bottom percentiles. Note: The solid line represents PAYG returns, the dashed line average bonds, the dash-dotted line long-term bonds and the dotted line stocks. Source for PAYG returns (growth rate of aggregate wage income) is National Income and Product Accounts (NIPA Table 1.10). Aggregate wage income is measured by compensation of employees and deflated by the GDP deflator. Data source for bond and stock returns is Shiller (2000) and http://www.irrationalexuberance.com/ShillerSocSec.xls. The figure shows returns cumulated over 32 years. See Section 2 and Appendix A for a description of the derivation of the cumulated return distributions. The underlying data series for PAYG returns has been adjusted by an additive constant, such that expected PAYG returns correspond to the average of the intermediate and pessimistic long-run projection of the Social Security Administration for the growth rate of aggregate wage income. Underlying stock return data have been adjusted such that expected returns correspond to the average of international stock returns during the twentieth century.

in comparison to bonds. The latter result is rather surprising. Given that expected PAYG returns exceed expected returns for bonds, they dominate, in fact, bonds. Thus, the only potential value of bond investments might stem from a contribution to portfolio diversification.

3. When is a PAYG system desirable for risk management? A theoretical analysis

This section first introduces lexicographic loss aversion (LLA) preferences, originally developed in Binswanger (2005). The main part is then concerned with a theoretical analysis of the conditions under which a PAYG system beneficially contributes to risk management. In particular, it will be explored whether these conditions vary between different income classes. At the end of the section it will be discussed who would be the likely beneficiaries if the Social Security Trust Fund invested in equities.

3.1. Preferences

Lexicographic preferences are represented by a list of arguments or “goals” that individuals seek to achieve. These goals are functions of individuals’ decision variables. The position in the list indicates the hierarchy of a particular goal, where the first goal is the one with the highest priority. Individual behavior is determined by maximization of a goal i in the list, subject to achieving the maximum of all goals with an index smaller than *i*, and subject to budget constraints. The particular goal *i* is determined as the goal with the highest index such that achieving the maximum of all goals with a lower index is still feasible. The higher an individual's income, the higher is the number of goals for which the maximum can be achieved.

Denote by c_1 and $\tilde{c}_2$ working age and retirement consumption, respectively. The tilde indicates that retirement consumption is risky, as it depends on the outcome of risky investments. Let $\tilde{c}_2^{\min}$ denote the minimum of old-age consumption. Preferences are then lexicographic over

$$\Pi = \{I(\tilde{c}_2^{\min} \geq \alpha c_1),\ \min[c_1, \overline{c}],\ E\tilde{c}_2(c_1 - \overline{c})^{\gamma}\}, \tag{1}$$

which constitutes a list of three hierarchically ordered goals. This preference model is characterized by the three parameters $\alpha > 0$, $\overline{c} > 0$ and $\gamma > 0$. I denotes the indicator function. E denotes the mathematical expectation operator.

Consider the first goal. It takes a value of one whenever $\tilde{c}_2^{\min} \geq \alpha c_1$, and of zero, otherwise. Consequently, individuals will never choose a consumption plan for which it is not assured that at least a fraction α of working-age consumption can be consumed during retirement. Thus, they are loss averse with respect to the reference point αc_1. I will refer to this reference point as a habit level. Note that the first goal can always be fully achieved whenever there is an asset with a minimum gross return that strictly exceeds zero (meaning that the net return exceeds minus 100%). Consider next the second goal, which embodies the desire to achieve a particular standard of living $\overline{c}$. Combining the first and the second goal, it follows that an individual maximizes c_1, s.t. $\tilde{c}_2^{\min} \geq \alpha c_1$ and the budget constraint, as long as income is sufficiently low such that the resulting optimal choice of c_1 does not exceed $\overline{c}$.

If the resulting value of c_1 exceeds $\overline{c}$, satiation of the first two goals is feasible. In this case, behavior is determined by the third goal, which is residual in nature. It specifies a desire for "speculative" savings, in the sense of a preference for high expected levels of old-age consumption. Beyond this, it specifies nonsatiation in c_1 and a complementarity between c_1 and $E\tilde{c}_2$. Subtracting $\overline{c}$ from c_1 assures that c_1 is a strictly normal good at all income levels. Note that γ simply represents the inverse of a standard discount factor. To see this take logs of the third goal. If all three goals are "active," individuals maximize $E\tilde{c}_2(c_1 - \overline{c})^{\gamma}$, s.t. $c_1 > \overline{c}$, $\tilde{c}_2^{\min} \geq \alpha c_1$, and the budget constraint.

3.2. Analyzing risk management

Let us start the analysis by introducing some notation. Gross bond returns are denoted by $\tilde{x} \in [\underline{x}, \overline{x}]$.[10] In all the following I will not distinguish any more between different bond returns, but rather assume that there is one homogeneous class of bonds. (For the simulations, I will identify it with average bonds, because their downside risk exposure is smaller, while expected returns coincide with long-term bonds.) Gross stock returns are denoted by $\tilde{y} \in [\underline{y}, \overline{y}]$. Similarly, gross PAYG returns are denoted by $\tilde{z} \in [\underline{z}, \overline{z}]$. To exclude the trivial case where bond returns are first-order stochastically dominated by stocks, I assume that bond returns have a higher minimum return but a lower expected return than stocks. Moreover, I assume that the same is true for the PAYG asset.[11]

[10] Note that gross returns equal net returns plus 1 (or plus 100%).

[11] From a purely theoretical point of view, minimum gross returns are zero (or minimum net returns minus 100%) for any financial asset, as there is some small but positive probability that economies will enter into a disastrous state such as a war. From a more pragmatic point of view, one will identify "minimum" returns with return levels that are exceeded with a high probability, such as 95%.

Assumption 1.

$$\text{(i)}\ \underline{x} > \underline{y}, E\tilde{x} < E\tilde{y}, \quad \text{(ii)}\ \underline{z} > \underline{y}, E\tilde{z} < E\tilde{y}.$$

The position of the PAYG asset with respect to bonds on the risk-return scale is left open at this point. Next, I assume that there is a positive probability that all three assets will take their minimum value.

Assumption 2.

$$\Pr[\tilde{x} = \underline{x}, \tilde{y} = \underline{y}, \tilde{z} = \underline{z}] > 0.$$

This assumption excludes that, for instance, adding stocks to a portfolio only containing bonds and PYG investments will increase the minimum return of this portfolio. As it is plausible to expect that returns of all three assets are particularly low within some “catastrophic” state of the economy, this is a natural assumption.

Denote bond investments by b, stock investments by s and PAYG investments by p. A final assumption rules out the possibility of borrowing and short-selling.

Assumption 3.

$$b \geq 0, \quad s \geq 0, \quad p \geq 0.$$

This is a realistic assumption for all but the highest incomes and is therefore important to be taken into account.

Let us now analyze optimal portfolio plans for lower incomes. Consider an individual with an income w that is small enough, such that only the first two goals in the list (1) are active for this income range. As discussed in the previous subsection, optimal choices are then determined by maximizing c_1, s.t. $\tilde{c}_2^{\min} \geq \alpha c_1$. It follows directly from this program that the individual will only invest in the asset with the highest minimum return. (Note that $\tilde{c}_2^{\min} = b\underline{x} + s\underline{y} + p\underline{z}$.) This allows her to assure the habit consumption αc_1 with the least amount of resources for any given level of c_1. Consequently, she reaches the highest feasible level of c_1. Using this and the budget constraint, it is straightforward to verify that we have $c_1 \leq \overline{c}$ for $w \leq \frac{\alpha + \max[\underline{x}, \underline{z}]}{\max[\underline{x}, \underline{z}]} \overline{c} \equiv w^{\text{crit}}$. Thus, it is for $w \leq w^{\text{crit}}$ that only the first two goals in the list (1) are active.

With respect to the desirability of a PAYG asset, we have the following result.

Proposition 1. *Assume $w \leq \frac{\alpha + \max[\underline{x}, \underline{z}]}{\max[\underline{x}, \underline{z}]} \overline{c} \equiv w^{crit}$. Then $c_1 \leq \overline{c}$, and individuals invest only in the asset with the highest minimum return. The PAYG asset is beneficial if and only if $\underline{z} > \underline{x}$.*

Proof. See the text above. □

Proposition 1 implies that individuals with an income below w^{crit} are best off when having all their private savings replaced by a PAYG system, whenever the PAYG system offers the highest minimum returns in the economy. It is the loss aversion property of preferences (1) that implies this strong result.

I establish next conditions under which a PAYG asset is beneficial for higher incomes where all three preference goals are relevant, i.e. for $w > w^{\text{crit}}$. Consider an individual holding a portfolio that is optimal when having no access to the PAYG asset. Suppose that $b > 0$ holds for this constrained portfolio.[12] Consider now the conditions under which the individual will be better off when given the opportunity to choose positive PAYG investments, instead. This will be the case if there exists a feasible portfolio reallocation (Δb, Δs, Δp), modifying the initial portfolio with $p=0$, such that

$$\begin{aligned} &\Delta p > 0, \Delta b < 0, \Delta s > 0, \\ &\Delta c_1 = \Delta b + \Delta s + \Delta p = 0, \\ &\Delta \tilde{c}_2^{\min} = \Delta b \underline{x} + \Delta s \underline{y} + \Delta p \underline{z} = 0, \\ &\Delta E\tilde{c}_2 = \Delta b E\tilde{x} + \Delta s E\tilde{y} + \Delta p E\tilde{z} > 0. \end{aligned} \tag{2}$$

This can be seen simply by noting that this reallocation will increase the value of the third goal in preferences (1), while leaving the value of the first and second goal unaffected.

Whenever $b=0$ would hold in the absence of the PAYG asset, a pure equity portfolio would already be sufficient to assure the habit consumption level. Thus, the introduction of an asset with a low downside risk, i.e. the PAYG asset, does not have any value. The next proposition summarizes the discussion and provides sufficient conditions under which a PAYG asset is valued if all three preference goals are relevant.

Proposition 2. *Assume* $w > w^{crit}$. *Define* $\overline{\overline{\gamma}} \equiv \dfrac{\underline{y}\Omega}{\alpha(\underline{x}-\underline{y})E\tilde{y}}$, $\overline{\overline{w}} \equiv \dfrac{(\alpha+\underline{y})\Omega}{\underline{y}\Omega - \alpha\gamma(\underline{x}-\underline{y})E\tilde{y}}\overline{c}$, *where* $\Omega \equiv (\alpha+\underline{x})E\tilde{y} - (\alpha+\underline{y})E\tilde{x}$.

i. If $\gamma \geq \overline{\overline{\gamma}}$ or $w < \overline{\overline{w}}$, *then* $b=0$ *and* $p>0$, *i.e. the PAYG asset has positive value, whenever* $\underline{z} > \underline{x}$ *and*

$$\underline{z}(E\tilde{y} - E\tilde{x}) + (\underline{x} - \underline{y})E\tilde{z} > \underline{x}E\tilde{y} - \underline{y}E\tilde{x}. \tag{3}$$

ii. If $\gamma < \overline{\overline{\gamma}}$ *and* $w \geq \overline{\overline{w}}$, *then* $b=0$ *and* $p=0$.

Proof. Proof of *i*. It follows from Propositions 1 to 4 in Binswanger (2005) that, in the absence of the PAYG asset, $b>0$ if $\gamma \geq \overline{\overline{\gamma}}$ or $w < \overline{\overline{w}}$. Thus, $\Delta b<0$ in the system (2) is feasible. Use the second and third line in system (2) to express Δb and Δs in Δp. Note that $\text{sign}\Delta b \neq \text{sign}\Delta p$. Furthermore, $\underline{z}>\underline{x}$ implies $\text{sign}\Delta s = \text{sign}\Delta p$. Insert the expressions obtained for Δb and Δs in the fourth line in system (2) and verify that the inequality holds whenever condition (3) holds. If the reallocation (2) is feasible for a particular value of $b>0$, it is feasible for all values of $b>0$. Therefore we have $b=0$ for the optimal portfolio.

Proof of *ii*. It follows from Proposition 4 in Binswanger (2005) that, in the absence of the PAYG asset, $b=0$ if $\gamma < \overline{\overline{\gamma}}$ and $w \geq \overline{\overline{w}}$. It follows then directly from Assumption 1 and from the fact that all three preference goals are active, that the introduction of a PAYG asset does not have any value. □

Part *i* of Proposition 2 deals with the case in which individuals would not invest purely in stocks in the absence of a PAYG asset. This is the relevant case for basically all reasonable parameter values and all income levels. Under the conditions of part i, the reallocation (2) is feasible. Condition (3), together with $\underline{z}>\underline{x}$, provides an amazingly simple criterion for the desirability of a PAYG system. A particularly nice feature of condition (3) is that it is straightforward to evaluate empirically, which will be done in the next section. It does not even

[12] Some conditions on w or γ, stated in Proposition 2 below, are required to assure that $b>0$ in the absence of a PAYG asset. If γ is very low, and w sufficiently high, the expenditure share of c_1 is low. As a result, the inequality $c_2^{\min} \geq \alpha c_1$ may hold even if all savings are invested in stocks, provided that $\underline{y}$ is sufficiently high. If all preference goals are active and $b=0$ in the absence of the PAYG asset, the introduction of the latter has no value (see below).

require information about preference parameters. Not surprisingly, this condition holds whenever $\underline{z}$ or $E\tilde{z}$ is not too low with respect to $\underline{x}$ or $E\tilde{x}$.[13]

The result that optimal bond holdings are zero may be surprising at first sight. This is explained by the fact that pure loss aversion is the only source of risk aversion in the preference specification (1). There is no motive for portfolio diversification. However, abstracting from diversification allows exploring the implications of loss aversion, a first-order issue for a behaviorally oriented analysis of risk management, in a most transparent way. Section 5 will present an enriched version of the LLA model accounting for a diversification motive. It will turn out that even for these generalized LLA preferences optimal bond holdings are almost negligible.

3.3. Should the social security trust fund invest in equities?

An interesting application of the analysis of the last subsection is the question whether the Social Security Trust Fund should invest in equity. In particular, who are the likely beneficiaries of such equity investments? It is sometimes argued that such equity investments would be particularly valuable for low-income earners, who do not participate in the stock market otherwise, while higher-income earners would be unaffected.[14] Overall, such equity investments would then be beneficial. The analysis here leads to a different conclusion.

If the trust fund only invests in bonds, this implies that Social Security provides its members with a portfolio of PAYG and bond investments, so to speak. If the trust fund additionally invests in stocks, Social Security provides a portfolio of PAYG, bond and equity investments. The LLA model predicts that low-income earners are always better off with a portfolio of PAYG and bond investments than with a portfolio also containing equity. The reason is that bonds are supposed to have a lower downside risk than stocks. The opposite holds with respect to higher-income earners. They do not benefit from bond investments. (As mentioned above, this conclusion remains true under reasonable parameter specifications once a diversification motive is taken into account.) It should be noted, however, that higher-income earners might not be affected by the trust funds' investment policy to the degree that they are able to privately short-sell bonds. In spite of this, it is likely that there would be both, gainers and losers. Thus, there is no simple answer to the question whether the trust fund should invest in equity or not.

4. Calibration of optimal pension plans

Let us start the numerical analysis of optimal pension plans by evaluating condition (3). For the specification of minimum returns I use fifth percentiles. They correspond most closely to return levels that are exceeded with a high probability, while not representing an excessively conservative specification of minimum returns. The values of expected and fifth percentile returns are found in Table 2. In light of these figures, the PAYG system dominates bonds with respect to both, expected values and minimum returns. Thus, $\underline{z} > \underline{x}$ and Eq. (3) holds.[15] It follows then from Propositions 1 and 2, that $b=0$ for all income levels. As a result, optimal plans coincide with the

[13] The simplicity of condition (3) is particularly welcome when noting that no condition of comparable transparency can be expected to be obtained for expected utility models. This conclusion is inferred from the analysis of Eeckhoudt and Gollier (2001), and Gollier (2001, Ch. 10). They show that even the introduction of only a *second* risky asset, in addition to another already available risky asset, is a very intricate issue. In light of their results, the case of *three* risky assets, which is relevant here, appears to be very difficult to characterize under expected utility preferences.

[14] See e.g. Geanakoplos et al. (1998) for a discussion.

[15] Remember from Table 2 and Fig. 1 that this conclusion is robust to changes in percentiles.

plans that are chosen when only two assets, stocks and the PAYG asset, are available. It is thus straightforward to calculate optimal pension plans by applying the formulas in Proposition 1 to 4 in Binswanger (2005). For the convenience of the reader they are summarized in Appendix B.

In the following subsection, I calibrate first optimal pension plans for a baseline parameter specification. A further subsection provides sensitivity analysis with respect to preference parameters, death and retirement age.

4.1. Optimal pension plans under baseline parameter values

A straightforward idea to specify α numerically is to look at Social Security replacement rates (SSRRs). These can be understood as consumption replacement rates, in the case that there would be no private savings. There is substantial heterogeneity in SSRRs across incomes. They amount to 63% for the lowest income levels and decline to 33% for the highest.[16] This decline in SSRRs is motivated by the idea that low-income earners do not save privately. Thus, replacement rates are intended to be more generous for the latter. For this reason, 63% would be too high as a baseline value for α, which represents a *minimal* acceptable consumption replacement rate. Rather, the 63% should be taken as a *normal* replacement rate for low incomes. The opposite is true with respect to the replacement rate for the highest incomes. They are supposed to have private savings and, thus, the minimum acceptable consumption replacement rate is expected to exceed the SSRR. An appropriate estimate for α is obtained by taking the SSRR for an intermediate income quintile. In particular, I choose the SSRR for the middle "average indexed monthly earnings" (AIME) quintile, which amounts to 43.8%. This coincides with the average SSRR over all incomes.

The replacement rate of 43.8% should, however, only be taken as a "raw" value for α and need still to be modified. This raw value is appropriate for comparing annual consumption levels during retirement with annual consumption levels during working age. However, the model in this paper is not specified in annual terms, but time is aggregated into two life-cycle periods, which differ in their length. As described in Section 2, I assume for the baseline specification that working age lasts from age 21 to the beginning of age 65, while retirement lasts from this age until 87, and is thus shorter than working age. In order to adjust the annual replacement rate of 43.8% to this setting, it has to be multiplied by the ratio of the length of retirement to the length of working life.

The parameter $\overline{c}$ represents a "normal" or average standard of living. As such it would be natural to identify it with median consumption in the economy. However, there are no good data available. Instead, I take the mean income of the second income quintile as a conservative proxy for median consumption and thus as a baseline estimate of $\overline{c}$. This amounts to 25,000 year 2001 U.S. dollars.[17]

Concerning the last parameter, γ, inspection of the third goal of preferences (1) shows that it just represents the inverse of a standard discount factor. Take logs of the third goal to see this. In the life-cycle literature a usual specification of the annual discount factor is .96 (see e.g. Campbell et al., 2001). Raising this number to the power of 32, the baseline value for the model's time horizon (see Section 2), and taking the inverse yields a raw estimate of γ. I multiply the resulting number by the ratio of the length of working age to the length of retirement, again, to account for the fact that the retirement period is shorter than working age. This yields a numerical specification for γ of 7.5.

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[16] See Munnell and Soto (2005), Table 10.

[17] Source: U.S. Census Bureau, http://www.census.gov/hhes/income/histinc/h03.html.

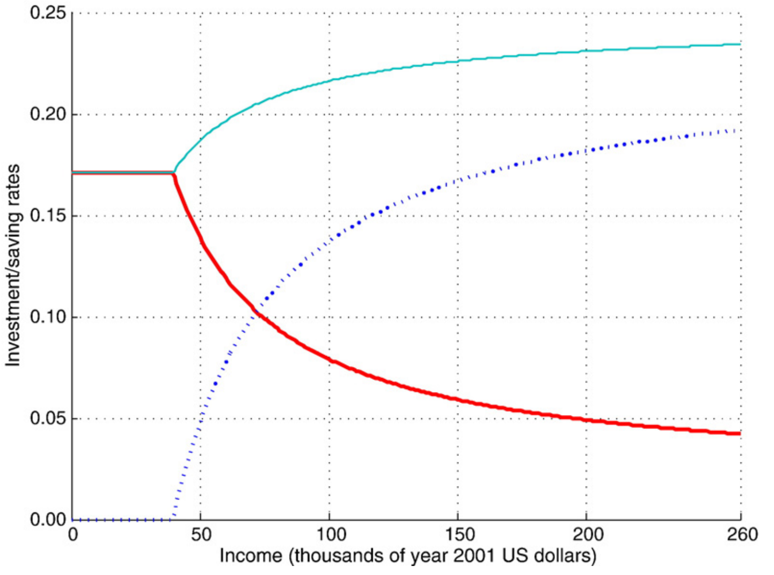

Fig. 2. Optimal pension plans under simple LLA preferences, baseline case. Note: The bold solid line represents PAYG rates, the dotted line stocks. The thin solid line represents the total saving rate. The LLA parameters are $\hat{\alpha}$=0.44, $\bar{c}$=25, $\hat{\gamma}$=0.96, d=87, r=65, T=32. The symbol $\hat{\alpha} = \frac{r-21}{d-r}\alpha$ refers to the annualized value of α. The symbol $\hat{\gamma} = \left(\frac{1}{\gamma}\frac{r-21}{d-r}\right)^{\frac{1}{T}}$ refers to the annualized discount factor underlying γ. Bond holdings are always zero.

Fig. 2 shows optimal expenditure shares for PAYG and stock investments for the baseline parameter values. The bold solid line represents optimal expenditure shares for PAYG investments, i.e. $\frac{p}{w}$. The dotted line shows expenditure shares for equity investments, i.e. $\frac{s}{w}$. The thin solid line represents the sum of both. As discussed previously, optimal bond investments are zero. Therefore, the sum of PAYG and equity expenditure shares represent the total saving rate. It should be remembered that, in order to implement an optimal pension plan, a social planner would set contribution rates of a mandatory PAYG system equal to optimal PAYG expenditure shares. Of course, the latter turn out to differ between different incomes groups. But in a stationary state they would be constant over time/generations.[18]

Overall, the baseline calibration leads to the conclusion that optimal PAYG contribution rates are very substantial for all but very high incomes. For income levels below 40,000 year-2001 dollars, households are best off by having all their savings substituted by a PAYG system.[19] The optimal PAYG contribution rate amounts to 17%. Optimal equity savings are zero. According to the income quintile estimates of the U.S. Census Bureau for 2001, this concerns members of the first two income quintiles and, in addition, lower incomes of the third quintile.[20] At an income of 43,000, representing the mean income of the third quintile,[21] the PAYG rate amounts to 16% and equity investments

[18] A PAYG system with a different "size" for different income groups may, for example, be implemented by crediting each individual's Social Security taxes to a notional account earning a notional interest rate that is equal to the growth rate of the aggregate wage bill. It is interesting to note that Latvia has recently established a PAYG system with notional accounts.

[19] Income levels should be understood as annual permanent income.

[20] The upper limit for the second income quintile amounts to 33,000 dollars for 2001. Source: U.S. Census Bureau, http:/www.census.gov/hhes/income/histinc/h01ar.html.

[21] Source for means of income quintiles: U.S. Census Bureau, http://www.census.gov/hhes/income/histinc/h03.html.

increase to 2%. At the mean income of the fourth income quintile, amounting to 67,000, the PAYG rate reduces to 11%, whereas the equity saving rate increases to 9%. For the mean income of the fifth quintile, amounting to 146,000, the corresponding values are 6% and 16%. Finally, at an income of 260,000, representing the mean income of the top 5% of the income distribution, the PAYG rate takes a value of 4% and the equity saving rate amounts to 19%.

Under the label LLAS1, Table 3a summarizes the values that PAYG, equity investments, their sum as well as equity shares take at the mean income of each income quintile. (The S in LLAS stands for "simple," contrasting with the "augmented" version of the model presented in the next section.) The symbol $\hat{\alpha}$, appearing in the first column, refers to the annualized value of α. Thus, it equals α multiplied by the inverse of the ratio of the length of retirement to working age. The symbol $\hat{\gamma}$ refers to the annual discount factor underlying the numerical specification of γ. Furthermore, d denotes the age of death and r the retirement age.

One means to judge whether these pension plans are reasonable is to compare them to individuals' observed savings and portfolio choices. In particular, total saving rates might be compared to a broad measure of empirical saving rates that would include Social Security savings. Also, prescribed equity shares may be compared to a broad measure of actually observed equity shares, including mutual funds and pension equity. One has, however, to be cautious. Real households, whose decisions are observed in the data, do not face the same choice situation as hypothesized in the setting here. First, individuals are not free to invest an arbitrary amount into a PAYG asset. Rather, Social Security contribution rates are given. Whenever individuals are constrained in their Social Security investments, the LLA model predicts that they would invest additionally in bonds, although they would prefer to invest exclusively in the PAYG asset. Second, returns of the current Social Security system deviate from projected growth rates of the wage bill. Third, and related, actual Social Security includes redistribution, which has been excluded in the analysis here. This implies that actual low-income individuals will save (much) less than prescribed here. In addition to this, the baseline preference parameters have been chosen such that they seem reasonable from a normative point of view. Thus, one should not expect, a priori, that these values would do the best job explaining individuals' observed choices. For instance, the "behavioral" value of α might be lower, and the value of γ higher, than their normative counterpart.

Overall, it is important to emphasize that a comparison of prescribed saving rates and equity shares to the data is not intended to provide a test of LLA preferences.[22] As mentioned above, the idea is that this provides one means to just broadly judge whether the LLA prescriptions are sensible. From a liberal (i.e. non-paternalistic) point of view, it is important to broadly check to what degree prescribed plans correspond to individuals' actual desired levels of old-age consumption and to their amount of risk taking. As a consequence, it is a priori desirable that prescribed saving rates and equity shares do not deviate in an extreme way from their empirical counterparts. A comparison of model prescriptions to the data is particularly helpful when carrying out sensitivity analysis with respect to the models' parameters (which will be done in the next subsection). A model is likely to be judged as less reasonable if parameters are such that the departure of implied saving rates and equity shares from the empirical values is extreme.

A comprehensive measure of empirical saving rates for each income quintile, including Social Security savings but net of redistribution, is found in the second principal row of Table 3a. These saving rates are obtained from Dynan et al. (2004). Specifically, they are calculated by interpolating linearly the "Active+Pensions" saving rates, reported in their Table 3. The interpolation is used to adjust their point estimates to the means of the income quintiles stated in

[22] Such tests are presented in Binswanger (2005), and they are successful.

Table 3a
Old-age provision plans under simple LLA preferences

		Quintiles				
		First	Second	Third	Fourth	Fifth
Income		10.1	25.5	42.6	66.8	146.0
Empirical	Saving rate	0.07	0.10	0.13	0.18	0.27
	Equity share	0	0	0.07	0.37	0.54
LLAS1 (*baseline*):	PAYG rate	0.17	0.17	0.16	0.11	0.06
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Equity rate	0	0	0.02	0.09	0.16
$\hat{\gamma}=0.96$, $d=87$,	Total saving rate	0.17	0.17	0.18	0.20	0.22
$r=65$	Equity share	0	0	0.10	0.46	0.73
LLAS2:	PAYG rate	0.23	0.19	0.16	0.09	0.02
$\hat{\alpha}$ according to SSRR,	Equity rate	0	0	0.02	0.10	0.18
$\bar{c}=25$, $\hat{\gamma}=0.96$, $d=87$,	Total saving rate	0.23	0.19	0.18	0.19	0.20
$r=65$	Equity share	0	0	0.10	0.54	0.92
LLAS3:	PAYG rate	0.14	0.14	0.11	0.06	0.02
$\hat{\alpha}=0.33$, $\bar{c}=25$,	Equity rate	0	0	0.04	0.11	0.18
$\hat{\gamma}=0.96$, $d=87$,	Total saving rate	0.14	0.14	0.15	0.17	0.20
$r=65$	Equity share	0	0	0.27	0.65	0.92
LLAS4:	PAYG rate	0.23	0.23	0.23	0.19	0.13
$\hat{\alpha}=0.63$, $\bar{c}=25$,	Equity rate	0	0	0	0.06	0.14
$\hat{\gamma}=0.96$, $d=87$,	Total saving rate	0.23	0.23	0.23	0.25	0.27
$r=65$	Equity share	0	0	0	0.24	0.50
LLAS5:	PAYG rate	0.17	0.17	0.12	0.08	0.05
$\hat{\alpha}=0.44$, $\bar{c}=18$,	Equity rate	0	0	0.07	0.13	0.18
$\hat{\gamma}=0.96$, $d=87$,	Total saving rate	0.17	0.17	0.20	0.21	0.23
$r=65$	Equity share	0	0	0.38	0.61	0.79
LLAS6:	PAYG rate	0.17	0.17	0.17	0.14	0.07
$\hat{\alpha}=0.44$, $\bar{c}=33$,	Equity rate	0	0	0	0.05	0.15
$\hat{\gamma}=0.96$, $d=87$,	Total saving rate	0.17	0.17	0.17	0.19	0.22
$r=65$	Equity share	0	0	0	0.27	0.66
LLAS7:	PAYG rate	0.17	0.17	0.17	0.16	0.14
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Equity rate	0	0	0	0.01	0.05
$\hat{\gamma}=0.94$, $d=87$,	Total saving rate	0.17	0.17	0.17	0.18	0.19
$r=65$	Equity share	0	0	0	0.08	0.28
LLAS8:	PAYG rate	0.17	0.17	0.11	0.02	0
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Equity rate	0	0	0.09	0.22	0.25
$\hat{\gamma}=0.98$, $d=87$,	Total saving rate	0.17	0.17	0.20	0.24	0.25
$r=65$	Equity share	0	0	0.44	0.90	1
LLAS9:	PAYG rate	0.14	0.14	0.12	0.08	0.04
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Equity rate	0	0	0.02	0.08	0.14
$\hat{\gamma}=0.96$, $d=81$,	Total saving rate	0.14	0.14	0.14	0.16	0.18
$r=65$	Equity share	0	0	0.13	0.50	0.77

Table 3a (*continued*)

		Quintiles				
		First	Second	Third	Fourth	Fifth
LLAS10:	PAYG rate	0.20	0.20	0.19	0.12	0.07
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Equity rate	0	0	0.01	0.10	0.19
$\hat{\gamma}=0.96$, $d=91$,	Total saving rate	0.20	0.20	0.20	0.23	0.26
$r=65$	Equity share	0	0	0.08	0.45	0.74
LLAS11:	PAYG rate	0.15	0.15	0.14	0.09	0.04
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Equity rate	0	0	0.02	0.09	0.16
$\hat{\gamma}=0.96$, $d=87$,	Total saving rate	0.15	0.15	0.16	0.18	0.20
$r=67$	Equity share	0	0	0.15	0.51	0.79

Note: Incomes and $\bar{c}$ are indicated in thousands of 2001 U.S. dollars. Incomes refer to the mean incomes for each income quintile in 2001. Source for income quintiles is the U.S. Census Bureau. Source for empirical saving rates is Dynan et al. (2004). Empirical equity shares are calculated from the Survey of Consumer Finances (SCF) 2001. Saving rates for mean quintile incomes are calculated by linear inter- or extrapolation of the point estimates for income quintiles of the Panel Study of Income Dynamics (PSID), reported under the heading "Active+Pension" in Table 3 of Dynan et al. (2004). Empirical equity shares indicate the median of the ratio of total equity holdings to total financial assets for each income quintile, according to the U.S. Census Bureau. Only observations with age less than 65 are used. Equity holdings include indirect holdings through mutual and pension funds. The symbol $\hat{\alpha} = \frac{r-21}{d-r}\alpha$ refers to the annualized value of α, where d denotes the age of death and r the retirement age. The symbol $\hat{\gamma} = \left(\frac{1}{\gamma}\frac{r-21}{d-r}\right)^{\frac{1}{\gamma}}$ refers to the annualized discount factor underlying γ. Bond holdings are always zero.

the first principal row of Table 3a. Comparing the total saving rates for LLAS1 with the empirical saving rates, it turns out that, for the bottom quintiles, the former are substantially higher.[23] In contrast, for the top quintile, the empirical saving rate exceeds the prescribed value.

As already mentioned, a large part of the distance between empirical and prescribed saving rates is most likely due to the fact that I do not consider redistribution. A potentially remaining part might be due to inadequately low empirical saving rates. To shed light on the question of what should be considered as "adequate" saving rates, consider the probability that annual old-age consumption falls short of 44% of annual working-age consumption. This figure corresponds to the annualized value of α, i.e. $\hat{\alpha}$. Minimum PAYG and stock returns have been identified with fifth percentiles. For optimal choices we always have $\tilde{c}_2^{\min}=\alpha c_1$. Thus, annual old-age consumption is lower than 44% of annual working-age consumption with a probability of roughly 5% for the saving rates obtained. As a consequence, if saving rates are smaller than indicated in Fig. 2 and Table 3a, annual old-age consumption falls short of 44% of annual working-age consumption with a probability that is higher than 5%. Since a replacement rate of 44% is rather low, the downside risk exposure of old-age income should then be taken as substantial. I conclude that, from a normative point of view, and in the absence of redistribution, the saving rates obtained under the baseline LLA model should be taken as reasonable. Independent of the LLA model, saving rates of a similar size are, in fact, always required if, in the absence of redistribution, some minimum consumption replacement around 45% is desirable.

The third principal row in Table 3a contains empirical equity shares that are representative for each income quintile. They are obtained from the Survey of Consumer Finances (SCF) 2001. These equity shares refer to median equity shares for each income quintile.[24] They include equity

[23] The larger distance between actual and prescribed saving rates is not specific to the LLA model, but also occurs for HARA preferences. See Section 6.

[24] See Footnote 20 for the source of quintile bounds.

hold through mutual funds and private pensions. Furthermore, they are based only on observations with age less than or equal to 65. The last row in Table 3a associated to the LLAS1 model shows prescribed equity shares, i.e. $\frac{s}{b+s+p}$. Comparison of empirical and prescribed values shows that they coincide quite closely, except for the top quintile. To shed more light on the adequacy of obtained equity shares, consider, once again, the probability that annual old-age consumption falls short of 44% of annual working-age consumption. This probability equals roughly 5% for the equity shares derived and would increase with higher equity shares. I conclude from this that also equity shares should be taken as reasonable.

4.2. Sensitivity analysis

Table 3a contains a number of pension plans that are obtained under various alternative parameter specifications. The aim of this is twofold. First, it is interesting to check to what degree the prescriptions obtained under baseline parameter values are robust. Second, it is instructive to see how changes of different parameter values affect saving rates and equity shares. This allows to learn something about the "mechanics" of the LLA model. Each time, only one parameter is varied with respect to the baseline specification. The discussion is sketchy. The interested reader will find all relevant information in Table 3a. (Remember that $\hat{\alpha}$ refers to the annualized value of α, and $\hat{\gamma}$ refers to the annual discount factor corresponding to γ. Furthermore, d denotes the age of death and r the retirement age.)

As mentioned previously, there is large heterogeneity associated with SSRRs. It is therefore interesting to see how pension plans would change if the annualized value of α is set to the corresponding SSRRs for each income quintile. The resulting plans are presented in Table 3a under the label LLAS2. Model LLAS3 presents old-age provision plans assuming that the annualized value of α uniformly equals the minimum SSRR, whereas LLAS4 sets the annualized value of α uniformly to the maximum SSRR.

For model LLAS2 total saving rates are not monotonic. They are high and decreasing for low incomes and slightly increasing for higher incomes. I take this pattern as an argument against the adequacy of assuming heterogeneity in α according to the SSRRs. As discussed in the previous subsection, a SSRR of 63%, the value for the lowest quintile, should be considered as a *normal* consumption replacement rate, not a *minimal* replacement rate. The opposite is true for the SSRR for the top quintile, amounting to 33%. Nevertheless, it is still interesting to compare models LLAS3 and LLAS4, allowing observation of how an increase in α increases total saving and PAYG rates and decreases equity shares.

Models LLAS5 and LLAS6 present sensitivity analysis with respect to $\bar{c}$. Under LLAS5 it is set to 18,000, the upper limit of the lowest income quintile. Under LLAS6 it is set to 33,000, the upper limit of the second quintile. An increase in $\bar{c}$ leads to an increase in the threshold income level, above which equity investments are positive. Hence, there is an associated decrease in equity shares (and hence an increase in PAYG shares) for the three upper income quintiles. Next, Models LLAS7 and LLAS8 address the implication of a variation in the annual discount factor $\hat{\gamma}$ (and thus in γ). The annual discount factor amounts to .94 for LLAS7 and to .98 for LLAS8. A higher discount factor implies a higher relative preference weight of $E\tilde{c}_2$ in preferences (1). As a result, equity investments and total saving rates are increased.

Models LLAS9 and LLAS10 explore the implication of a variation in the investment horizon T. Specifically, under LLAS9 the length of retirement is set equal to the life-expectancy for men at age 65 in 2005 under the low-cost scenario of the Social Security Administration. This life expectancy amounts to 16 years. This is the lowest of the projected life-expectancies across all years and scenarios.[25] Under LLAS10, the length of retirement is set equal to the highest projected life-expectancy at age 65 across all years and scenarios. This amounts to 26 years, corresponding to women in 2080 for the high-cost scenario. The resulting values for the time horizon *T*, during which a typical old-age provision dollar is invested, are 29 and 34 for LLAS9 and LLAS10, respectively.[26]

It turns out that the variation in the time horizon has only a minor effect on cumulated minimum returns but leads, of course, to higher cumulated expected returns on PAYG and stock (and bond) investments. More important, a higher life-expectancy directly increases α and decreases γ, holding fixed the baseline values of $\hat{\alpha}$ and $\hat{\gamma}$. The reason is that α is set proportional, and γ inversely proportional, to the length of retirement as discussed at the beginning of the previous subsection. Both, a higher value of α and lower value of γ, lead to higher saving rates. A higher α decreases equity shares while a lower γ leads to higher equity shares. The net effect on equity shares turns out to be small.

A final interesting specification is shown under the label LLAS11. Here, I assume that retirement age is raised to 67, while the age of death is, again, set to the baseline value of 87. The resulting investment horizon equals, again, 32 years. If the retirement age is adjusted upwards, PAYG and total saving rates are lowered by 2 percentage points compared to LLAS1.

For all model specifications, optimal PAYG investments are substantial for at least the three bottom quintiles. For most specifications they are also considerable for the fourth quintile. Optimal equity investments are zero for the two bottom quintiles across all specifications, and for most also quite low for the third quintile. Apart from this, there is considerable variation across specifications with respect to the amount of total saving rates. The same is true with respect to equity shares, provided that they are positive. In light of the discussion of the baseline parameter values in the last subsection, as well as in light of the empirical shape of saving rates and equity shares, I take the baseline specification, or LLAS11, as normatively most appealing.

5. Accounting for "dispersion aversion"

This section explores the implications of augmenting preferences by an aversion to the dispersion of old-age consumption, in addition to loss aversion. It will be explored whether, for reasonable parameter specifications, such dispersion aversion leads to lower equity shares than under the simple version of the model. Furthermore, there might be positive bond investments.

Specifically, augmented preferences are given by

$$\Pi' = \{I(\tilde{c}_2^{\min} \geq \alpha c_1, \sigma(\tilde{c}_2) \leq \hat{\theta}(E\tilde{c}_2)E\tilde{c}_2), min[c_1, \overline{c}], E\tilde{c}_2(c_1 - \overline{c})^{\gamma}\}, \quad (4)$$

where $\sigma(\tilde{c}_2)$ denotes the standard deviation of $\tilde{c}_2$, and $\hat{\theta} > 0$ represents a parameter whose value may be an (increasing) function of $E\tilde{c}_2$. Achieving the first goal now requires not only assuring habit consumption during old-age, but also that the dispersion of old-age consumption, measured by its standard deviation, must not exceed a certain fraction of expected old-age consumption.

Preferences (4) represent the most direct extension of the baseline version (1). However, there might exist some distributions of $\tilde{c}_2$ for which it is not feasible to achieve the inequalities $\tilde{c}_2^{\min} \geq \alpha c_1$ and $\sigma(\tilde{c}_2) \leq \hat{\theta} E\tilde{c}_2$ simultaneously for some values of α and $\hat{\theta}$. This problem is addressed by assuming that behavior is determined by a continuous approximation of preferences (4). In particular, I assume that individuals maximize the function

$$c_1 - \lambda \max(0, \alpha c_1 - \tilde{c}_2^{\min})^2 - \lambda \max(0, \sigma(\tilde{c}_2) - \hat{\theta} E\tilde{c}_2)^2 \quad (5)$$

[25] See Board of Trustees (2005), Table V.A3.

[26] The derivation of *T* is discussed in Section 2.

whenever the resulting value of c_1 falls short of $\overline{c}$, and

$$E\tilde{c}_2(c_1-\overline{c})^\gamma-\lambda\max(0,\alpha c_1-\tilde{c}_2^{\min})^2-\lambda\max(0,\sigma(\tilde{c}_2)-\hat{\theta}E\tilde{c}_2)^2, \qquad (6)$$

otherwise.[27] The parameter λ, a penalty parameter, represents a large but finite number.

It turns out that $\hat{\theta}=\theta_1(E\tilde{c}_2)^{\theta_2}$ is a particularly simple and parsimonious specification for the degree of dispersion aversion. The following reasoning proves helpful for identifying a baseline value for the parameters θ_1, θ_2. Consider an individual with an expected old-age consumption that equals twice the poverty threshold, the latter amounting to 10,700.[28] Ask what level of a standard deviation of $\tilde{c}_2$ the individual would maximally tolerate, expressed as a fraction of $E\tilde{c}_2$. Ask further, by what fraction of $E\tilde{c}_2$ this tolerance would increase if $E\tilde{c}_2$ is increased a further time by the poverty threshold. (It might be helpful for this to know that, for lower consumption levels, the standard deviation of old-age consumption roughly equals the mean absolute deviation of old-age consumption from its mean. The mean absolute deviation seems easier to interpret than the standard deviation.) I take a maximum tolerance of .5 and an increase in this tolerance of .1 as a natural first guess. This figures allow then to determine θ_1 and θ_2, which amount to .15 and .40, respectively.

The income expansion paths for PAYG, bond and equity saving rates are derived numerically, using grid search. The grid width is taken to equal 1% of income. Minimum return rates are again set equal to fifth percentiles. Optimal plans depend on the covariances between different return rates, which are shown in Table 2. Bonds are identified with average bonds. The reason is that they share the same expected return with long-term bonds while having a lower downside risk exposure.

The results for the baseline specification are shown in Table 3b under the label LLAA1 (where the second A stands for "augmented"). All parameters, apart from θ_1, θ_2, are set to the same values than for the baseline specification under simple preferences, i.e. LLAS1. As expected, equity shares are lower than under LLAS1, but only for the two top quintiles. Also, bond investments are positive, but only marginally so and only for the bottom quintile. Roughly speaking, optimal pension plans coincide with the baseline case under simple LLA preferences, except for the two top quintiles.

Under the label LLAA2, Table 3b shows optimal plans for a lower level of dispersion aversion. The maximum tolerated standard deviation of old-age consumption as a fraction of $E\tilde{c}_2$ equals 70% at twice the poverty threshold. The increase in this tolerance due to an increase in $E\tilde{c}_2$ by one time the poverty threshold is set to 20% of $E\tilde{c}_2$. In this case, optimal plans coincide with the baseline case under simple LLA preferences, except that, again, a marginal part of PAYG investments is substituted for bonds for the first quintile. (Apart from this, differences in the numerical values are due to the fact that optimal plans have been derived by grid search in this section. In contrast, in the last section they have been derived using the formulas in Appendix B). Under the label LLAA3, I finally present simulations for a high level of dispersion aversion. Here, the tolerance threshold for the standard deviation is set to 40% and the increase in this tolerance to 6%. In this case, dispersion aversion is so high that the equity share equals only 21% for the top quintile.[29]

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[27] While the functions (5) and (6) resemble mean-variance utility functions, they have the advantage that they do not imply *decreasing* income expansion paths for equity shares, as quadratic utility would because of increasing *absolute* risk aversion.

[28] Source: U.S. Census Bureau. This figure represents a poverty threshold for 2001 for a weighted average of two-person households with age greater or equal to 65 years.

[29] Unreported calibrations show that for some values of θ_1, θ_2 dispersion aversion is relatively high at low levels of $E\tilde{c}_2$, such that high saving rates at low incomes are implied to sufficiently increase the tolerance to a dispersion of old-age consumption. It is interesting to see that a similar pattern occurs under HARA preferences (see Section 6).

Table 3b
Old-age provision plans under augmented LLA preferences

		Quintiles				
		First	Second	Third	Fourth	Fifth
Income		10.1	25.5	42.6	66.8	146.0
Empirical	Saving rate	0.07	0.10	0.13	0.18	0.27
	Equity share	0	0	0.07	0.37	0.54
LLAA1 (baseline):	PAYG rate	0.16	0.18	0.16	0.15	0.10
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Bond rate	0.02	0	0	0	0
$\hat{\gamma}=0.96$, $\theta_1=0.15$,	Equity rate	0	0	0.02	0.04	0.11
$\theta_2=0.40$, $d=87$,	Total saving rate	0.18	0.18	0.18	0.19	0.21
$r=65$	Equity share	0	0	0.11	0.21	0.52
LLAA2:	PAYG rate	0.16	0.18	0.16	0.12	0.06
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Bond rate	0.02	0	0	0	0
$\hat{\gamma}=0.96$, $\theta_1=0.12$,	Equity rate	0	0	0.02	0.08	0.17
$\theta_2=0.57$, $d=87$,	Total saving rate	0.18	0.18	0.18	0.20	0.23
$r=65$	Equity share	0	0	0.11	0.40	0.74
LLAA3:	PAYG rate	0.16	0.18	0.17	0.16	0.15
$\hat{\alpha}=0.44$, $\bar{c}=25$,	Bond rate	0.02	0	0	0	0
$\hat{\gamma}=0.96$, $\theta_1=0.16$,	Equity rate	0	0	0.01	0.02	0.04
$\theta_2=0.30$, $d=87$,	Total saving rate	0.18	0.18	0.18	0.18	0.19
$r=65$	Equity share	0	0	0.06	0.11	0.21

Note: Incomes and $\bar{c}$ are indicated in thousands of 2001 U.S. dollars. Incomes refer to the mean incomes for each income quintile in 2001. Source for income quintiles is the U.S. Census Bureau. Source for empirical saving rates is Dynan et al. (2004). Empirical equity shares are calculated from the Survey of Consumer Finances (SCF) 2001. Saving rates for mean quintile incomes are calculated by linear inter- or extrapolation of the point estimates for income quintiles of the Panel Study of Income Dynamics (PSID), reported under the heading "Active+Pension" in Table 3 of Dynan et al. (2004). Empirical equity shares indicate the median of the ratio of total equity holdings to total financial assets for each income quintile, according to the U.S. Census Bureau. Only observations with age less than 65 are used. Equity holdings include indirect holdings through mutual and pension funds. The symbol $\hat{\alpha}=\frac{r-21}{d-r}\alpha$ refers to the annualized value of α, where d denotes the age of death and r the retirement age. The symbol $\hat{\gamma}=\left(\frac{1}{\gamma}\frac{r-21}{d-r}\right)^{\frac{1}{\gamma}}$ refers to the annualized discount factor underlying γ.

Perhaps the most interesting overall observation to make is that even for high levels of dispersion aversion bond investments are almost negligible. When it comes to policy recommendation, it seems that dispersion aversion is too high under LLAA3. It is harder to discriminate between LLAA1, LLAA2, and finally LLAS1. It is certainly welcome to base policy recommendations on a model accounting for dispersion aversion. This favors the first two models, although the differences between LLAA2 and LLAS1 are marginal. From the point of view of precaution, one might favor LLAA1 when discriminating between LLAA1 and LLAA2.

6. Comparison to HARA preferences

Among standard models, HARA preferences are the natural candidate for exploring how optimal old-age provision varies with income. It is thus interesting to compare optimal pension plans under HARA preferences to the plans for LLA preferences. It will be shown that for reasonable parameter values optimal old-age provision plans have highly implausible properties. In particular, they are characterized by very high and decreasing saving rates. It is only for a range of implausible parameter

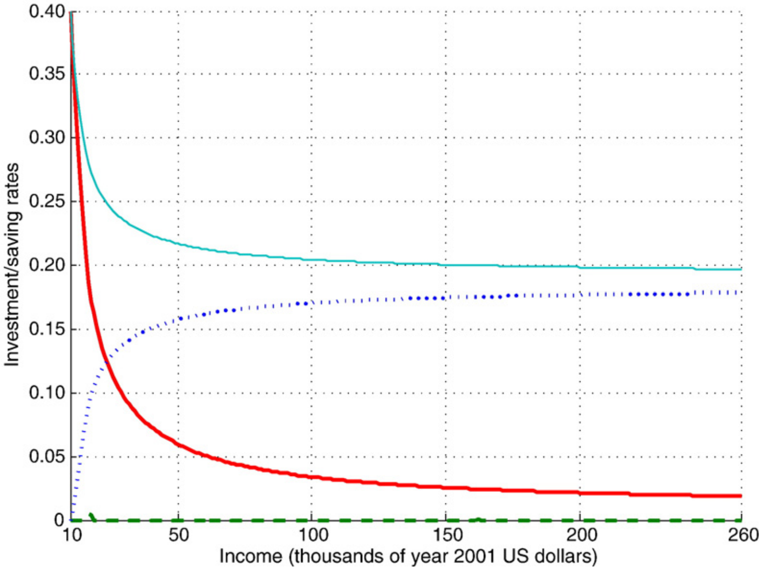

Fig. 3. Optimal pension plans under HARA preferences, baseline case. Note: The bold solid line represents PAYG rates, the dashed line bonds and the dotted line stocks. The thin solid line represents the total saving rate. The HARA parameters are $\eta=2$, $\hat{\beta}=0.96$, $\overline{\overline{c}}_1=5.6$, $\overline{\overline{c}}_2=4.3$, $d=87$, $r=65$, $T=32$. The symbol $\hat{\beta}=\left(\beta\frac{r-21}{d-r}\right)^{\frac{1}{T}}$ refers to the annualized value of β.

values that this problem is mitigated. This suggests that the HARA model is of limited use when thinking about optimal old-age provision.

I consider the HARA specification

$$U(c_1,\tilde{c}_2)=\frac{1}{1-\eta}(c_1-\overline{\overline{c}}_1)^{1-\eta}+\frac{\beta}{1-\eta}E(\tilde{c}-\overline{\overline{c}}_2)^{1-\eta}. \tag{7}$$

The parameters are η, governing relative risk aversion, $\overline{\overline{c}}_1>0$, $\overline{\overline{c}}_2>0$, representing age-specific subsistence levels of consumption, and β, representing the time discount factor. E denotes the mathematical expectation operator. The utility function (7) is well behaved only for the case where $c_1>\overline{\overline{c}}_1, \tilde{c}_2>\overline{\overline{c}}_2$. Note that relative risk aversion is equal to η in the limit case where consumption equals infinity. For finite consumption values relative risk aversion exceeds η. Note further that the limit case where $\eta=1$ corresponds to the usual log-version of Stone–Geary preferences.

For a baseline numerical specification I set $\overline{\overline{c}}_1$ and $\overline{\overline{c}}_2$ equal to a fraction of age-specific poverty threshold levels for the U.S. For the year 2001, these poverty thresholds amount to 14,100 and 10,700 for working-age and retirement, respectively.[30] If $\overline{\overline{c}}_1$ and $\overline{\overline{c}}_2$ were set equal to 100% of these levels, optimal choices would not be defined for incomes below 25,000, because income would not suffice to cover both subsistence consumptions. Therefore, I multiply both poverty thresholds by .4. This is the highest fraction of poverty thresholds such that optimal choices are still defined for the mean income of the first income quintile. Concerning the other parameters, η is set equal to 2 and β equal to $.96^T$ times the ratio of the length of retirement to the length of working age. This accounts for the fact that retirement lasts for fewer years than working age. Retirement age is, again, set to 65, death age to 87. Thus, T equals 32.

[30] Source: U.S. Census Bureau. The figures represent poverty thresholds for a weighted average of three-person households for working age, and a weighted average of two-person households with age greater or equal to 65 years for retirement, respectively.

Table 3c
Old-age provision plans under HARA preferences

		Quintiles				
		First	Second	Third	Fourth	Fifth
Income		10.1	25.5	42.6	66.8	146.0
Empirical	Saving rate	0.07	0.10	0.13	0.18	0.27
	Equity share	0	0	0.07	0.37	0.54
HARA1 (baseline):	PAYG rate	0.41	0.11	0.07	0.05	0.03
$\eta=2$, $\hat{\beta}=0.96$,	Bond rate	0	0	0	0	0
$\bar{\bar{c}}_1 = 5.6, \bar{\bar{c}}_2 = 4.3$,	Equity rate	0	0.13	0.15	0.16	0.17
$d=87$, $r=65$	Total saving rate	0.41	0.24	0.22	0.21	0.20
	Equity share	0	0.53	0.69	0.78	0.87
HARA2:	PAYG rate	0.13	0.10	0.09	0.08	0.08
$\eta=3$, $\hat{\beta}=0.94$,	Bond rate	0.01	0	0	0	0
$\bar{\bar{c}}_1 = 5.6, \bar{\bar{c}}_2 = 1.6$,	Equity rate	0.04	0.10	0.11	0.12	0.12
$d=87$, $r=65$	Total saving rate	0.19	0.19	0.20	0.20	0.20
	Equity share	0.23	0.50	0.55	0.58	0.61

Note: Income and $\bar{\bar{c}}_1, \bar{\bar{c}}_2$ are indicated in thousands of 2001 U.S. dollars. Incomes refer to the mean income for each income quintile in 2001. Source for income quintiles is the U.S. Census Bureau. Source for empirical saving rates is Dynan et al. (2004). Empirical equity shares are calculated from the Survey of Consumer Finances (SCF) 2001. Saving rates for mean quintile incomes are calculated by linear inter- or extrapolation of the point estimates for income quintiles of the Panel Study of Income Dynamics (PSID), reported under the heading "Active+Pension" in Table 3 of Dynan et al. (2004). Empirical equity shares indicate the median of the ratio of total equity holdings to total financial assets for each income quintile, according to the U.S. Census Bureau. Only observations with age less than 65 are used. Equity holdings include indirect holdings through mutual and pension funds. The symbol $\hat{\beta} = \left(\beta\frac{r-21}{d-r}\right)^{\frac{1}{T}}$ refers to the annualized value of β, where d denotes the age of death and r the retirement age.

The results are shown in Fig. 3 and in Table 3c under the label HARA1.[31] The symbol $\hat{\beta}$ appearing in the first column of Table 3c refers to the annual value of the discount factor. The most disturbing feature of the outcome for the baseline specification is that total saving rates are unreasonably high and decreasing in income. Furthermore, equity shares are as high as 53% already for the second quintile. (Remember that the empirical value amounts to zero for this quintile).[32]

I turn next to a parameter constellation where $\bar{\bar{c}}_1$ and $\bar{\bar{c}}_2$ are such that they fulfill the following requirements. First, the ratio $\bar{\bar{c}}_2/\bar{\bar{c}}_1$ is sufficiently reduced, compared to the ratio of age-specific poverty thresholds, such that total saving rates start to increase. (Reducing $\bar{\bar{c}}_2/\bar{\bar{c}}_1$ is the only means to obtain increasing saving rates.) Second, conditional on this, $\bar{\bar{c}}_1$ and $\bar{\bar{c}}_2$ are set as high as possible such that choices are still defined for the first quintile. Specifically, $\bar{\bar{c}}_2$ is set equal to 40% of the poverty threshold for working age, while $\bar{\bar{c}}_2$ is set equal to only 15% of the old-age poverty threshold. Concerning the other parameters, η is increased to 3 to obtain lower equity shares. To avoid a sharp increase in the level of saving rates, the annual value of the discount factor is reduced to .94, in turn. The results are found in Table 3c under the label HARA2. Saving rates turn out to be lower and increasing. Equity shares turn out to be very high for the three bottom quintiles.[33]

[31] For the derivation of optimal pension plans, the return distributions are truncated below at the fifth percentile in order to avoid that saving rates become excessively high due to the fact that feasibility of subsistence consumptions has to be assured with probability one. Additionally, return distributions are truncated at the top in such a way that annualized expected values of the truncated distributions correspond to the target means discussed in Section 2.

[32] Equity shares are lower for higher values of $\bar{\bar{c}}_1$ and $\bar{\bar{c}}_2$. But then choices are not defined any more for the first quintile.

[33] Unreported calibrations show that optimal plans vary only marginally with a change in the length of retirement.

Overall, the following conclusions can be made. For parameter constellations that seem reasonable a priori, in particular a ratio $\overline{\overline{c}}_2/\overline{\overline{c}}_1$ that is proportional to the ratio of poverty thresholds for old age and working age, high and decreasing saving rates are obtained. Given strong empirical evidence for saving rates that increase in income (see Dynan et al., 2004), I take this as a serious argument against the plausibility of the prescriptions of such HARA specifications. Only in the implausible case where the ratio $\overline{\overline{c}}_2/\overline{\overline{c}}_1$ is substantially reduced in comparison to the ratio of poverty thresholds, saving rates increase in income. For such specifications equity shares are quite high, even for the first income quintile, in comparison to the empirical values. Overall, the analysis suggests that the HARA model is of limited use when thinking about old-age provision.

With regard to LLA preferences it is interesting to note that saving rates are much higher under HARA1, and not lower under HARA2. Second, since equity shares are higher under HARA preferences, a PAYG system has substantially higher value under LLA than under HARA preferences. In contrast, both models suggest that optimal bond investments are (almost) negligible.[34]

7. Conclusion

The analysis of this paper suggests that, at all income levels, individuals benefit substantially from the presence of a PAYG system, whose “returns” are given by the growth rate of aggregate wage income. Depending on parameters, PAYG contribution rates range from 15% to 18% for low incomes under the preferred specifications. Furthermore, they vary between 4% and 10% for high incomes. Average contribution rates vary between 11% and 15%. I find that low-income earners are best off by having their entire old-age provision based upon a PAYG system. Only for the two top quintiles old-age provision should also rely substantially on equity investments. It is a robust finding that bonds are not likely do make a valuable contribution to old-age provision, except for the lowest income quintile.

The analysis here abstracts from any form of intragenerational redistribution, whose discussion is beyond scope of this paper. But if redistribution were part of the picture, one might imagine that it would lead to a PAYG system with a uniform contribution rate, where the latter would roughly be equal to the average of derived contribution rates, thus ranging between 11% and 15%. For comparison, the Social Security Administration projects that contribution rates would raise to 15% if benefits were preserved at the current level (see Feldstein and Liebman, 2002).

Clearly, the analysis makes a case against the privatization of Social Security. However, the current Social Security system is characterized by expensive benefit guarantees that endanger its viability. One might consider a partial Social Security reform eliminating any benefit guarantees and making payoffs strictly dependent on the growth rate of the aggregate wage bill. In comparison to privatization and increased prefunding, such a reform seems easier to realize. The reason is that redemption of Social Security’s unfunded liability would be less of an issue. The analysis here suggests that having such a defined contribution Social Security system would be preferable to a privatized and more prefunded system, even when disregarding transition costs. This is the most important implication of the analysis.

Several extensions are worthwhile to be studied. A first-order issue is the consideration of redistributive goals. Second, examining a multiperiod model proves necessary for studying how pension contributions should evolve over working age. A further issue is the study of equilibrium reactions of wages and financial returns.

[34] It seems from this that both, LLA and HARA preferences suggest that in an equilibrium we should observe only very few bond holders. However, this would only hold true in the case where individuals would be free to invest arbitrary amounts in a PAYG asset. Furthermore, bond holdings might be more advantageous for short-term investments, which are not considered here.

Appendix A. Derivation of return distributions

This appendix describes the procedure for deriving the distributions of PAYG, bond and stock returns. The first step consists of estimating autoregression equations for the logs of each historical return series. Before the estimation of the autoregressions, the level of a historical data series $\tilde{x}_t$ is potentially adjusted by a constant shift δ. The particular value of δ is chosen such that the mean of the return distribution, obtained through the procedure described below, corresponds to some prespecified target value (see Section 2). The appropriate value of δ is found by iteration.

I estimate then autoregressions for the *logs* of the adjusted series $\tilde{x}_t - \delta$. The selection of the appropriate number of lags is based on the "general-to-specific sequential *t*-rule," using a significance level of 5%. I start with a prespecified maximal lag length of 10 for all series. This leads to an estimated lag length of nine for PAYG returns, one and zero for average and long-term bonds, and zero for stocks. Based on this, I repeat the model selection procedure with a maximal lag length of nine, leading to the same lag estimations. For each of the estimations I include only the years for which all required lagged observations are available for *all* series. This means, that the sample size is always identical for all four autoregressions, irrespective of the fact, that less lags are required for bonds and stocks than for PAYG returns.

To simulate return distributions over a time horizon of T years I proceed in the following way. I calculate first the residuals from the single autoregressions to estimate the covariance matrix of the disturbance terms. From the variance estimates and the estimated autoregression coefficients, I calculate an estimate for the *unconditional* mean μ_i and variance σ_i for each series i. I assume that log returns are normally distributed with a mean $\mu_i - .5\sigma_i$ and a variance σ_i. Subtracting .5 times the variance for the mean assures that the expected value of level returns equals e^{μ_i} rather than $e^{\mu_i + .5\sigma_i}$. Otherwise, level returns would artificially be boosted up by the assumption of normality of log returns, due to Jensen's inequality. I draw then p_i times a random sample of size N out of the unconditional distributions, where p_i denotes the number of lags for series i.

Based on the estimated covariance matrix for the disturbance terms, I draw next a random sample of disturbance terms of size N for each of T years. I assume that the disturbance terms are jointly normally distributed with a mean vector equal to zero minus .5 times the estimated individual variances of the error terms for each series. The covariance matrix equals the covariance matrix estimated from the residuals of the autoregressions. Similarly to above, the subtraction of .5 times the individual variances assures that the expected values of the levels of the disturbance terms are equal to one for each series.

I assume that in year one of the investment history, there are p_i observations of lagged returns $\tilde{r}^i_0, \ldots, \tilde{r}^i_{-p+1}$ available, that have been drawn from the *unconditional* distributions (see above). Then, returns for each year $t = 1, \ldots, T$ are determined successively according to the equation

$$\tilde{r}^i_t = \phi^i_0 + \sum_{l=1}^{p_i} \phi^i_l \, \tilde{r}^i_{t-l} + \varepsilon^i_t, \tag{8}$$

where ϕ^i_0 denotes the constant and ϕ^i_l the estimated autoregression coefficient for lag l for a series i, and ε^i_t is the disturbance term for time t and series i. Given N random draws of $\tilde{r}^i_t$ for $t = 0, \ldots, -p+1$, and of ε^i_t for $t = 1, \ldots, T$, N histories of log returns can be calculated over T successive years using Eq. (8). Taking the sum over all T years and raising e to the power of this sum yields N level return realizations cumulated over a period of T years. If N is chosen sufficiently large, the resulting sample of random return histories allows to infer the "true" distribution of T-period returns under the assumptions made. It turns out that if N is set to 10,000, the resulting percentiles of the return distributions differ only in a minor way for different sample draws. In particular, the simulations of optimal old-age portfolios remain unaffected by the small differences. This relatively small N allows to save computation time, which is useful because the shift parameter δ that has been mentioned above has to be found by an iteration procedure.

Appendix B. A summary of portfolio choice formulas

This appendix summarizes the formulas for optimal portfolio choice under simple LLA preferences in the presence of two assets, the PAYG asset and stocks. These formulas are used in Section 4 to calibrate optimal pension plans. They have been originally derived in a different context in Binswanger (2005).[35]

Define $\hat{\Omega} \equiv (\alpha+\underline{z})E\tilde{y} - (\alpha+\underline{y})E\tilde{z}$, $\hat{w} \equiv \frac{(\alpha+\underline{z})\hat{\Omega}}{\underline{z}\,\hat{\Omega} - \alpha\gamma(\underline{z}-\underline{y})E\tilde{z}}\bar{c}$, $\hat{\hat{w}} \equiv \frac{(\alpha+\underline{y})\hat{\Omega}}{\underline{y}\,\hat{\Omega} - \alpha\gamma(\underline{z}-\underline{y})E\tilde{y}}\bar{c}$, $\hat{\hat{\hat{w}}} \equiv \frac{\alpha+\underline{y}}{\underline{y}-\alpha\gamma}\bar{c}$, $\hat{\gamma} \equiv \frac{\underline{z}\,\hat{\Omega}}{\alpha(\underline{z}-\underline{y})E\tilde{z}}$, $\hat{\hat{\gamma}} \equiv \frac{\underline{y}\,\hat{\Omega}}{\alpha(\underline{z}-\underline{y})E\tilde{y}}$. Optimal portfolios are then determined according to the following formulas.

i. If $w \le \hat{w}$ or $\gamma \ge \hat{\gamma}$, then $p = \frac{\alpha}{\alpha+\underline{z}}w$, $s=0$.
ii. If $\hat{\hat{\gamma}} \le \gamma < \hat{\gamma}$ and $w > \hat{w}$ or if $\gamma < \hat{\hat{\gamma}}$ and $\hat{w} < w < \hat{w}$, then

$$p = \frac{1}{(1+\gamma)(\underline{z}-\underline{y})\,\hat{\Omega}}\{[\alpha\gamma(\underline{z}-\underline{y})E\tilde{y} - \underline{y}\,\hat{\Omega}]w + (\alpha+\underline{y})\,\hat{\Omega}\,\bar{c}\},$$

$$s = \frac{1}{(1+\gamma)(\underline{z}-\underline{y})\,\hat{\Omega}}\{[\underline{z}\,\hat{\Omega} - \alpha\gamma(\underline{z}-\underline{y})E\tilde{z}]w - (\alpha+\underline{z})\,\hat{\Omega}\,\bar{c}\}.$$

iii. If $\frac{\underline{y}}{\alpha} \le \gamma < \hat{\hat{\gamma}}$ and $w \ge \hat{\hat{w}}$ or if $\gamma < \frac{\underline{y}}{\alpha}$ and $\hat{\hat{w}} \le w \le \hat{\hat{\hat{w}}}$, then $p = 0, s = \frac{\alpha}{\alpha+\underline{y}}w$.
iv. If $\gamma < \frac{\underline{y}}{\alpha}$ and $w > \hat{\hat{\hat{w}}}$, then $p = 0, s = \frac{1}{1+\gamma}(w-\bar{c})$.

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[35] See Propositions 1 to 4 there.

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