Economica (2008) ▮▮, 1–23
doi:10.1111/j.1468-0335.2008.00685.x

Pensions and Intergenerational Risk-sharing in General Equilibrium

By ROEL M. W. J. BEETSMA†‡ and A. LANS BOVENBERG‡

†*University of Amsterdam* ‡*Tinbergen Institute, Netspar, CEPR and CESifo*

Final version received 27 August 2007.

We investigate intergenerational risk-sharing in two-pillar pension systems with a pay-as-you-go pillar and a funded pillar. The funded pension pillar can be either defined contribution or defined benefit. Only a defined-benefit scheme with an appropriate investment policy establishes optimal intergenerational risk-sharing. We show how the pension system affects capital markets in general and the equity premium in particular.

INTRODUCTION

Population ageing is putting retirement systems under financial strain. As a result, social security systems are being reformed around the world. In several countries, public pay-as-you-go (PAYG) pensions are increasingly being supplemented by a funded component. These reforms are occurring in both advanced economies and emerging economies.[1] While in most countries the first pillar continues to dominate the pension system, many countries (such as Germany and Poland) have set up funded tiers in their pension system, in some instances involving personal retirement accounts (e.g. in Ireland and Estonia) and sometimes aimed at new labour market entrants (such as Hungary and Slovakia). Some countries (France and Austria) are limiting the indexation of pension benefits to prices rather than wages. This provides more room for funded private pensions to supplement public PAYG provisions.

Pension funds are becoming more important actors in capital markets. Over the past decade, assets of pension funds in OECD countries have grown at an average annual rate of more than 10%—almost four times faster than the GDP growth rate (see Boeri *et al.* 2006). Although pension funds still face tight limits on their investments in emerging economies (see Roldos 2004), investment restrictions on portfolios of pension funds are gradually being lifted in some countries. This contributes to a shift away from government bonds and towards equity investments and alternative risk-bearing investment categories (such as hedge funds, private equity venture capital and commodities). Aggregate pension fund assets currently represent more than 20% and 10% of capitalization on equity and bonds markets, respectively, in the G10 countries, albeit with great variation across countries (see Visco 2005).

Another important feature of funded pension schemes is the nature of the pension promise that is being offered. Defined-benefit (DB) systems, in which benefits are guaranteed by public or corporate sponsors, are gradually being replaced by defined-contribution (DC) schemes, in which retirement benefits depend directly on the financial returns of the investment portfolio and are thus subject to various risks. The new International Financial Reporting Standards, which require companies to report the market value of their pension guarantees on their balance sheets, has stimulated this move away from DB schemes towards DC schemes. At the same time, in some countries pension funds are developing into collective stand-alone pension funds that share risks between the participants. These pension funds stand alone in the sense that they lack a risk-absorbing sponsor in the form of the government or corporations. Nevertheless, these funds can still have DB features as young participants guarantee the benefits to older, retired generations.

This paper explores how the nature of pension promises and the investment policies of funded pension components can help to share risks optimally over generations in stand-alone pension funds[2] and how pension policies impact on transactions and prices on capital markets in general equilibrium. To that end we formulate a simple, two-period general-equilibrium model, in which a young and an old generation live under the same government and overlap during one period. The economy is subject to productivity shocks and shocks to the depreciation of capital. We consider two-pillar pension systems, with a public PAYG first pillar and a funded second pillar, which may be DC or DB. The second pillar is stand-alone in the sense that it lacks a corporate sponsor. The young participants of the fund absorb the deficits or surpluses that may arise as a consequence of a possible mismatch between assets and liabilities in the second pillar and thus are the residual risk-bearers in that pillar.

The key feature of our setup is that financial markets are incomplete for two reasons. The first is that generations cannot trade risk with each other before the shocks occur because the young generation is born only after these shocks have materialized. The second reason for market incompleteness is that human capital is not traded; accordingly, the old generation cannot acquire a claim on human capital and thus does not share in the wage risk faced by the young generation. In this way, our stylized model captures two realistic and related sources of market incompleteness. In particular, lack of tradability of human capital implies that the young cannot borrow against their human capital to invest in risk-bearing capital (see also Constantinides *et al.* 2002, and Carroll *et al.* 2005). As a result, the young face a disproportionately high wage risk because they hold all claims to wage income and cannot shift this risk on to the older generation. At the same time, older people exhibit an excessive exposure to financial market risk because they cannot share this risk with the young. The associated suboptimal diversification of human capital and financial market risks harms welfare if stock markets (and hence the risk that the old are exposed to) are not perfectly correlated with wages. Indeed, stock markets appear much more volatile than wage incomes. Changes in technology, for example, can give rise to substantial shifts in the distribution of income between capital and labour.

As a result of the market incompleteness in our model, the portfolio choices of the pension fund and the nature of the pension benefits (i.e. DC or DB) do have real effects: the two generations cannot fully offset the transactions of the pension fund. Whereas a funded defined-contribution pension scheme does not add anything to the transaction possibilities in financial markets, and thus leaves allocations unaffected, defined-benefit pension schemes create new opportunities for intergenerational risk-sharing, which private agents do not offset through transactions in financial markets. The reason for these new risk-sharing possibilities offered by defined-benefit pensions is that pension benefits are defined independently from *ex post* returns on the underlying financial assets. As the residual claimant of the assets of the fund, the young generation bears the associated mismatch risk. In this way, the young share in the risks of the financial markets even though they cannot individually borrow to buy risk-bearing financial assets in financial markets (see also Modigliani and Muralidhar 2004). Moreover, by linking retirement benefits to wages, the old acquire an implicit claim on human capital which is not traded on financial markets. In effect, by not matching the risks of its liabilities with the risks of its financial assets, a defined-benefit pension fund allows the young generation to exchange human capital risks for financial risks with the old generation. Accordingly, intergenerational risk-sharing becomes more efficient.

The key question is whether the pension system allows for optimal intergenerational risk-sharing in a decentralized market economy in which incomplete financial markets prevent generations from trading risks. We find that the combination of properly specified first-pillar PAYG systems and second-pillar DB pension funds can establish the twin objectives of optimal intergenerational redistribution and optimal intergenerational risk-sharing. While the PAYG pillar aims at systematic redistribution between generations in accordance with the relative social-welfare weight of the generations, the funded DB pillar allows for optimal sharing of financial-market risks across generations.[3] Both pillars can optimally share wage risks by linking pension benefits to wages. If the funded benefits can be linked to wages, the PAYG pillar can be targeted exclusively at optimal *ex ante* redistribution, while the funded pillar becomes solely responsible for optimal risk-sharing.

The optimal pension arrangement ensures that, for each individual, the exposure to aggregate depreciation risk is the same as the exposure to aggregate productivity risk. Accordingly, for each generation the implicit ownership share of the aggregate capital stock should be equal to that of aggregate human capital. The actual magnitude of this optimal ownership share rises with the social preference weight and falls with risk aversion of the generation concerned.

We develop a general equilibrium framework to analyse intergenerational risk-sharing under a variety of two-pillar pension systems, and describe its implications for capital markets. We extend the literature by investigating how the types of promised benefits in the retirement system and the investment policies of DB pension funds can contribute to optimal intergenerational risk-sharing. We thus formalize some of the arguments that Modigliani and Muralidhar (2004) developed informally. Moreover, our general equilibrium model explores the impact of the pension system on prices and transactions in capital markets, including the equity premium and aggregate saving and investment.

A number of papers have studied how pension systems affect intergenerational risk-sharing, but most of them focus on risk-sharing within PAYG systems only—see e.g. Hassler and Lindbeck (1997); Thogersen (1998); Krueger and Kubler (2002); Wagener (2004). Indeed, the role of funded rather than PAYG pension systems in completing incomplete capital markets has not yet been analysed in the literature. Bohn has extensively investigated optimal intergenerational risk-sharing in a number of papers (see e.g. Bohn 2003), but he has not studied the implications of various types of funded pension system in this regard. Matsen and Thogerson (2004) explore the optimal division between PAYG and funding from a risk-sharing perspective, but they do not include funded systems of the DB type. Teulings and de Vries (2006) investigate a pension arrangement in which each generation builds up (and depletes) its own pension account. Like most other papers in the literature on funded pension systems, they adopt a partial-equilibrium setting with a fixed equity premium and a fixed risk-free interest rate. Moreover, they do not consider wage risk and assume that capital markets are complete so that young agents can invest in equity before they are born. Miles and Černý (2006) simulate the implications of a shift from PAYG to DC funding in a partial-equilibrium framework with borrowing and short-selling constraints and individual risks. They abstract from DB systems. The same holds true for Conesa and Krueger (1999), who also simulate the political feasibility of a transition to a funded DC system. Finally, Smetters (2006) studies how an appropriately chosen capital tax can substitute for social security DC investment in equities.

The main aim of the paper, therefore, is to characterize optimally designed two-pillar pension systems in a market economy, paying specific attention to the respective roles of the two pillars in risk-sharing and redistribution. The structure of the remainder of this paper reflects this objective. Section I lays out the model and Section II solves for the social planner's solution with optimal intergenerational risk-sharing and redistribution. Section III presents the decentralized economy, while Section IV describes general conditions (in particular, how generational accounts should respond to shocks) for a market economy to replicate the social planner solution. Section V shows that a pension arrangement with a fully funded DC second pillar can replicate the social optimum only under special circumstances (i.e. if depreciation shocks are absent). Section VI demonstrates that a properly designed pension arrangement in which the benefits from the funded second pillar are defined in real terms is able to replicate the social optimum. Such a system, however, still suffers from the disadvantage that the PAYG pillar mixes redistribution and risk-sharing. Section VII shows that this drawback can be avoided, while still enabling attainment of the social optimum, if benefits from the fully funded second pillar are indexed to wages. Such a separation of the redistributive and risk allocation roles of the pension system between the two pillars would make the pension system more transparent in terms of its consequences for different generations. Section VIII moves from normative to positive analysis by investigating how a shift from a DC to a DB second pillar impacts capital investment and the equity risk premium, and Section IX concludes the main body of this paper. All technical details are found in an Appendix, which is available on http://www1.fee.uva.nl/toe/content/people/beetsma.shtm.

I. The Model

This section presents a general equilibrium closed-economy model, featuring two periods (0 and 1) and two generations whose lives overlap in period 1. On the one hand, we simplify the standard overlapping-generations (OLG) model by considering only two generations and a two-period horizon. On the other hand, we extend the standard OLG setting by incorporating a richer specification of the retirement system. Our stylized two-period model is the simplest setting for explaining how a two-pillar pension system with a PAYG and a funded component affects intergenerational risk-sharing and intergenerational distribution and impacts capital markets. We need a two-period rather than a one-period model to set up an explicit individual intertemporal optimization problem and explore the consequences of intergenerational risk-sharing for endogenous asset returns, saving and investment. Furthermore, to properly impose the requirement that the second pillar of the pension system be fully funded, we use the first-order conditions from the intertemporal optimization problem facing older individuals.

Since our focus is on *inter*generational rather than *intra*generational risk-sharing and redistribution, we neglect the possible distortionary impacts of intragenerational redistribution and risk-sharing on labour supply. To keep the model as simple as possible without sacrificing the essence of intergenerational risk-sharing and redistribution and its impact on saving and investment, we assume that the labour supply is exogenous.

(a) *Individuals and preferences*

In period 0, a generation of mass $1-\gamma>0$ is born. This generation lives through periods 0 and 1. We call this generation the 'old generation'. The representative agent within this generation features the following utility function:

$$(1) \qquad U(c_{y,0}, c_o) = u(c_{y,0}) + \beta E_0[v(c_o)],$$

where $c_{y,0}$ denotes consumption when this agent is young in period 0, while c_o represents consumption when the agent is old in period 1. In the latter period, a new generation of mass $\gamma > 0$ is born. This generation lives only during period 1 and features the utility function $u(c_y)$. It is termed the 'young generation'. The lives of the two generations thus overlap in period 1. The total population in that period has size unity. The time subscript 0 is reserved for variables that refer to period 0.

(b) Production

The output level of the single good in period 0 is exogenously given at per capita level η_0 (i.e. measured per every person who is alive in that period). Production in period 1, when the two generations coexist, is given by

$$Y = AF(K, L), \tag{2}$$

where K represents the aggregate capital stock and L aggregate employment; A denotes total factor productivity, which is stochastic. The production function exhibits constant returns to scale. In our closed economy the capital stock K is the result of investment in the previous period 0. The old generation is retired in period 1, while each young individual exogenously supplies an amount of labour in that period, which we normalize at unity. Aggregate employment thus amounts to $\bar{L} \equiv \gamma$.

(c) Public expenditures and resource constraints

Exogenous public spending in period 0 is given by $g_0 > 0$. The resource constraints in periods 0 and 1 amount to

$$(1-\gamma)c_{y,0} = (1-\gamma)\eta_0 - K - g_0, \tag{3}$$

$$\gamma c_y + (1-\gamma)c_o = AF(K, \bar{L}) + (1-\delta)K, \tag{4}$$

where $0 \leqslant \delta \leqslant 1$ is the stochastic depreciation rate of the capital shock. The left-hand sides of these expressions denote aggregate consumption in the economy. The right-hand side of (3) represents total endowment income minus the investment in physical capital and public expenditures. The right-hand side of (4) stands for total production plus what is left over of the capital stock after taking depreciation into account.

As in Bohn (1999) and Smetters (2006), we introduce depreciation risk to allow for imperfect correlation between labour and capital income. Depreciation risk may arise for a variety of reasons, e.g. natural disasters, armed conflicts and other violence causing harm to the capital stock. Other sources of depreciation risk are unexpected technological advances and the associated creative destruction that renders capital obsolete. Changes in environmental regulation and various other regulatory standards (such as town planning) also affect the value of the existing capital stock.

II. The Social Planner's Solution

The vector of the stochastic shocks is $\xi \equiv \{A, \delta\}$. It is unknown in period 0, but becomes known before period 1 variables are determined. In period 0, the social planner commits to a state-contingent plan. Hence the consumption levels in period 1 are functions of the shocks: $c_o = c_o(\xi)$ and $c_y = c_y(\xi)$.

The planner weighs all individuals in a generation equally and aims to maximize the sum of the discounted expected utilities of the current and future generations' individuals, where the relative weight of the individuals born in 1 is given by $\chi^p > 0$. By varying χ^p, we can map out all Pareto-optimal solutions.

We can write the planner's problem as

$$£ = \int \begin{bmatrix} (1-\gamma)[u(c_{y,0}) + \beta v(c_o(\xi))] + \chi^p \gamma \beta u(c_y(\xi)) \\ +\beta\lambda_1(\xi) \begin{bmatrix} AF(K,\bar{L}) + (1-\delta)K - \gamma c_y(\xi) \\ -(1-\gamma)c_o(\xi) \end{bmatrix} \end{bmatrix} f(\xi)\mathrm{d}\xi$$

$$+ \lambda_0[(1-\gamma)\eta_0 - K - (1-\gamma)c_{y,0} - g_0].$$

Here, $f(\xi)$ stands for the probability density function for the vector of stochastic shocks ξ. The Lagrange multipliers for the resource constraints in periods 0 and 1 are denoted by λ_0 and $\lambda_1(\xi)$, respectively. Maximization of the planner's program with respect to $c_{y,0}$, K, $c_y(\xi)$ and $c_o(\xi)$ for all ξ yields the following first-order conditions:

$$u_c(c_{y,0}) = \lambda_0,$$

$$\lambda_0 = \int \beta\lambda_1(\xi)(1 + r^{kn})f(\xi)\mathrm{d}\xi,$$

$$\chi^p u_c(c_y(\xi)) = \lambda_1(\xi), \forall\xi,$$

$$v_c(c_o(\xi)) = \lambda_1(\xi), \forall\xi,$$

where the first-order derivative of $u(\cdot)$ is denoted by a subscript c, F_K stands for the marginal product of capital (suppressing the arguments of the production function) and $r^{kn} \equiv AF_K - \delta$, which in the sequel we will refer to as the *net-of-depreciation return on capital*. By eliminating the Lagrange multipliers from these first-order conditions, we establish

(5) $$\chi^p u_c(c_y) = v_c(c_o), \forall\xi,$$

(6) $$u_c(c_{y,0}) = \beta \mathrm{E}_0[(1 + r^{kn})v_c(c_o)].$$

If a decentralized equilibrium is to replicate the planner's solution, these optimality conditions need to be met in addition to the resource constraints (3) and (4).

III. The Decentralized Economy

This section describes the decentralized market economy in which individuals and firms maximize their objective functions under the relevant constraints. A key question will be under what circumstances a market economy can replicate the command optimum. In this connection, we note that we can interpret the optimal risk-sharing condition (5) as the condition for *ex ante* trade in risks between the young and the old in complete financial markets. However, in a decentralized economy the two generations cannot directly trade risk in financial markets, because the young generation is born only after the shocks have materialized. Indeed, in the absence of pension arrangements, the old bear all the depreciation risk δ and cannot shift this risk towards the young. Both generations are exposed to production risk A, but it is unlikely that the allocation of risk across generations is optimal. Hence the generations would like to trade this risk but they cannot do this on financial markets. Other institutions thus have to fill the gap of the missing market for risk-sharing between the old and the young generations. We shall explore the extent to which the pension system can perform that role.

The timing of events in the market economy is as follows.

1. The members of the old generation (who are young in period 0) make their investment decisions, while the government issues debt to cover its period 0 expenditures.
2. The set of shocks ξ materializes.
3. The government levies period 1 taxes. At the same time, firms take hiring and production decisions, while young individuals decide on their savings.

(a) *Individual budget constraints and pensions*

We consider a two-pillar pension system. The first pillar is a pay-as-you-go (PAYG) system composed of a lump-sum part and a wage-indexed part. The second pillar of the pension system consists of a pension fund that collects contributions from old-generation members in period 0, invests these contributions, and pays out benefits to the old generation in period 1.

The exogenous endowment $\eta_0 > 0$ is owned by the old generation, while labour income in period 1 accrues to the young. The budget constraints facing the old generation in period 0 and the young and old in period 1 are given by, respectively,

$$c_{y,0} = \eta_0 - (b + k + \theta^f), \tag{7}$$

$$c_y = (1 - \theta^w)w - \theta^p - \tau + \frac{1-\gamma}{\gamma}(r^a - r^f)\theta^f, \tag{8}$$

$$\begin{aligned} c_o = {} & \frac{\gamma}{1-\gamma}(\theta^w w + \theta^p) + (1 + r^f)\theta^f \\ & + (1 + r)b + (r^k + 1 - \delta)k - \tau, \end{aligned} \tag{9}$$

where b denotes (real) public debt directly held by old households, k the direct claim of old households on the capital stock in period 1, θ^f an exogenous mandatory contribution to a pension fund, w the real wage per unit of labour supplied, τ lump-sum tax payments (levied in equal amounts on both generations) in period 1, θ^p the lump-sum PAYG social security contribution by the period 1 young, θ^w the wage-linked PAYG contribution rate by the period 1 young, r^f the realized real return *to the old* on their contributions to the funded pension scheme, r^a the average net return on the assets held by the pension funds, r the return on debt issued in period 0 (determined at the moment of issuance), and r^k the return per unit of capital in production.

With a first pension pillar that is PAYG, the total contributions by the young, i.e. $\gamma * (\theta^w w + \theta^p)$, equal the aggregate payments to the old, i.e. $(1-\gamma) * [\gamma/(1-\gamma)]$ $(\theta^w w + \theta^p)$(see (8) and (9)). With a fixed parameter $\theta^w > 0$, the PAYG system is of the DC rather than the DB type. Pension benefits are then exposed to wage risk because they vary with wages. Through their PAYG pensions, the old generations thus bear some of the wage risk.

The second-pillar pension fund uses the period 0 contribution θ^f by each old-generation member to invest amounts b^f and k^f in debt and physical capital, respectively:

$$(10) \qquad \theta^f = b^f + k^f.$$

Hence the average net return r^a on the assets held by the pension funds is computed from

$$(11) \qquad (1+r^a)\theta^f = (1+r)b^f + (r^k + 1 - \delta)k^f.$$

We assume the second pension pillar to be fully funded. This requires the old generation to finance its own pension benefits from this pillar: the contributions in period 0 equal the value to the old generation (computed from the *ex ante* perspective of period 0) of the benefits they can expect to collect in period 1. Hence, in contrast to a PAYG system, a fully funded system does not redistribute wealth between generations. From the *ex ante* point of view of period 0, the old put in the same value as they get out of it in the next period.

Even though a fully funded pension system does not redistribute between generations *ex ante*, it may help to share shocks *ex post* between the generations, depending on how the assets and liabilities of the fund respond to shocks. Whereas full funding requires that the assets and liabilities are equal in value *ex ante*, their values may diverge *ex post* if shocks affect them in different ways. Concretely, the term $[(1-\gamma)/\gamma](r^a - r^f)\theta^f$ in (8) arises from the possibility that the total *ex post* payment to the old generation from the pension fund in period 1, i.e $(1-\gamma)(1+r^f)\theta^f$, may differ from the *ex post* value of the fund, $(1-\gamma)(1+r^a)\theta^f$. Such mismatch risk arises if the second pillar is of the DB type. In a stand-alone DB scheme that lacks a corporate sponsor, the young participants are the residual risk-bearers and thus absorb the deficits or surpluses that may arise as a consequence of the mismatch risk.[4] Therefore the young are in fact the owners of the balance sheet of the pension fund. Given that the number of young is γ, each young person receives an amount $[(1-\gamma)/\gamma](r^a - r^f)\theta^f$. Mismatch is thus the channel through which intergenerational risk-sharing can be accomplished by stand-alone DB pension systems featuring young participants as residual risk-bearers.

We shall term an economy without any collective pension arrangements (i.e. $\theta^p = \theta^w = \theta^f = b^f = k^f = 0$) a *laissez-faire* economy.

(b) Individual and firm optimization

As is standard, we solve the model by working backwards. In period 1, a continuum of perfectly competitive representative firms, with mass normalized to unity, produce according to (2) and maximize profits,

$$(12) \qquad AF(K,L) - wL - r^k K,$$

over L and K, taking as given the wage rate and the rental rate of capital. This yields the following first-order conditions:

$$(13) \qquad AF_L = w,$$

$$(14) \qquad AF_K = r^k,$$

where AF_L is the marginal product of labour (suppressing the arguments of the function).

In period 0, the old generation decides on the allocation of its savings over the various assets. They maximize (1) over b and k, where $c_{y,0}$ and c_o are given by (7) and (9), respectively. The first-order conditions are

$$\beta(1+r)\mathrm{E}_0[v_c(c_o)] = u_c(c_{y,0}), \tag{15}$$

$$\beta\mathrm{E}_0[(1+r^{kn})v_c(c_o)] = u_c(c_{y,0}). \tag{16}$$

As we shall see below, differences in pension arrangements affect the responses of c_o to the various types of shock and so also have consequences for investment decisions and the equity risk premium demanded in period 0.

(c) The government budget constraint

The public budget constraint in period 0 reads as

$$d = g_0, \tag{17}$$

where d is the amount public debt issued in period 0. Without any taxation in that period, it equals the exogenous public expenditures (public consumption) in period 0. The public budget constraint in period 1 amounts to

$$\tau = (1+r)d. \tag{18}$$

The term on the right-hand side of (18) represents the redemption of the public debt (including interest payments). The term on the left-hand side represents aggregate lump-sum tax revenue, which adjusts to maintain the government's budget balance.

(d) Market equilibria

The goods market equilibrium conditions in periods 0 and 1 are given by (3) and (4), respectively. Given that k, k^f, b and b^f are amounts *per old-generation person*, the factor market equilibria (for capital in period 0, for labour in period 1) are

$$K = (1-\gamma)(k+k^f), \quad L = \bar{L}, \tag{19}$$

while the debt market equilibrium (in period 0) is

$$(1-\gamma)(b+b^f) = d. \tag{20}$$

(e) Generational accounts

Below, we shall consider various types of pension system. In all these cases, using the optimality conditions, the government budget constraint and the market equilibrium conditions, we can write consumption of the young and old generations in period 1 as

$$c_y = \frac{1}{\gamma}A\bar{L}F_L + G^y, \tag{21}$$

$$c_o = \frac{1}{1-\gamma}(1+r^{kn})K + G^o, \tag{22}$$

where G^y and G^o represent the per capita generational accounts of young and old persons, respectively. These generational accounts measure the net resource flows from the government and the pension system to the individuals in period 1. Hence G^y and G^o are such that the right-hand sides of (21) and (8), respectively (22) and (9), are equal. We use generational accounts because they are a sufficient statistic for the impact of the pension arrangements on consumption and hence the economy. The resource constraint for period 1 (i.e. (4)) implies that the sum of the generational accounts over all individuals is zero:

$$(23) \qquad \gamma G^y + (1-\gamma)G^o = 0.$$

Specifically, in a *laissez-faire* economy, the generational accounts are directly related to the public debt owned by the old generation:[5]

$$(24) \qquad G^o = \frac{\gamma}{1-\gamma}(1+r)d.$$

The benefit to the old of public debt in period 0 depends on the size of the young generation γ. If $\gamma = 0$, all public debt issued in period 0 must be paid off by the members of the old generation through their own tax payments. Hence public debt does not redistribute across generations.

IV. Optimality of Pension Systems

We study how the decentralized market economy can replicate the social optimum with an appropriate choice of the pension system. We thus design the appropriate two pillar pension system (a PAYG first pillar and a funded second pillar) to ensure that the planner's solution derived in Section II is replicated in a decentralized market economy.

A necessary and sufficient condition for a policy to replicate the social optimum is that it implies that $v_c(c_o) = \chi^p u_c(c_y)$ for all possible realizations of the shock vector ξ.[6] In particular, we observe that, by setting $v_c(c_o) = \chi^p u_c(c_y)$, old-generation agents are also induced to choose the same amount of equity investment in period 0 as the planner would select.

Unless we explicitly state otherwise, we assume that both generations are weighted equally in social welfare ($\chi^p = 1$) and that the old and young generation feature the same utility function in period 1 (i.e. $v(\cdot) = u(\cdot)$). This combination of assumptions allows for simple and intuitive analytical solutions. In particular, the necessary and sufficient condition for reproducing the social optimum boils down to $c_y = c_o$. Hence, combining this result with (21) and (22), we observe that the social optimum is reproduced whenever the generational accounts vary such that

$$(25) \qquad \frac{1}{1-\gamma}(1+r^{kn})K - \frac{1}{\gamma}A\bar{L}F_L = -\frac{1}{\gamma}G^o = \frac{1}{1-\gamma}G^y,$$

where we have used (23). Hence, if profit income plus the scrap value of capital per old individual (i.e. $[1/(1-\gamma)]\left(1+r^{kn}\right)K$) exceeds individual wage income per young individual, $(1/\gamma)AF_L\bar{L}$, the old will obtain more per capita resources for consumption in period 1 than the young would get in the absence of intergenerational transfers (so that $G^o = G^y = 0$). Equal consumption levels of the two generations in period 1 then require that the generational accounts offset the differences in incomes in the *laissez-faire* economy. Substituting the expressions for the generational accounts under the various pension fund systems considered below, we can show how the pension system can reproduce the social optimum.

V. A Funded DC System

In a pension scheme of the *defined-contribution* (DC) type, the value of the liabilities always matches that of the assets, not only *ex ante* in period 0 when contributions and pension fund investments are made, but also *ex post* when benefits are paid out. Hence the total pension benefits per old in period 1, i.e. $(1+r^f)\theta^f$, coincide with the actual (gross) returns on the pension contributions in all states of the world. In other words, the benefit per old person equals the per-participant value of the fund $(1+r^a)\theta^f$, implying that $r^f = r^a$. The risks facing the old thus depend directly on the investment policy of the fund.[7] Indeed, under a DC system the old are the residual claimants of the fund. A DC system does not involve the young at all. It thus does not provide intergenerational risk-sharing opportunities in addition to those already provided by the capital market.

The generational account of the old under the *DC* system is given by

$$G^o = \frac{\gamma}{1-\gamma}\left(\theta^p + \theta^w \frac{1}{\gamma} A\bar{L}F_L\right) + \frac{\gamma}{1-\gamma}(1+r)d. \tag{26}$$

Hence not only *explicit* public debt (i.e. d), but also *implicit* public debt, $\theta^p + \theta^w(1/\gamma)A\bar{L}F_L$, features in the generational accounts. The DC funded system (whatever its size or investment policy) does not affect the generational accounts and thus leaves individual consumption and investment decisions unaffected. Indeed, with a DC pension system, any investment b^f or k^f implemented by the pension fund is exactly offset by an equal reduction in individual investments. This is in fact an application of the celebrated Modigliani–Miller theorem to pension financing: individuals see through the veil of the DC pension fund so that the financial policy of the fund is neutral.

Now we want to establish the circumstances under which the *DC* system replicates the social optimum. The potentially available instruments are the investment composition of the pension fund (k^f and b^f, which together define the *ex ante* value of the benefit through the full-funding requirement) and the parameters θ^p and θ^w, characterizing the PAYG system. Since individual investment decisions exactly offset the pension fund investments, only θ^p and θ^w remain as instruments that can affect allocations. These latter parameters are not contingent on the shocks. Although the potential objections to making these parameters dependent on the shocks are not modelled explicitly, frequent changes in the pension parameters inevitably lead to political struggles and introduce additional uncertainty not directly linked to the fundamental economic shocks themselves.

Substitution of (26) into (25) yields the following requirement for reproducing the social optimum:

$$\frac{1}{1-\gamma}\left(\theta^p + \theta^w \frac{1}{\gamma} A\bar{L}F_L\right) + \frac{1}{1-\gamma}(1+r)d = \frac{1}{\gamma}AF_L\bar{L} - \frac{1}{1-\gamma}[AF_K + (1-\delta)]K. \tag{27}$$

Replication of the social optimum requires that this expression holds for all possible realizations of the shock vector ξ. If at all possible, this imposes certain restrictions on the pension system. By inspecting (27), we immediately see the following.

Proposition 1. Assume a PAYG first pillar with a funded DC second pillar. (a) With productivity shocks only, the PAYG pillar can replicate the social optimum by setting $\theta^w = 1 - \gamma F(K,\bar{L})/(\bar{L}F_L)$ and $\theta^p = -(1-\delta)K - (1+r)g_0$. (b) With depreciation shocks in addition to the productivity shocks, the pension system is not able to replicate the social optimum for arbitrary realizations of the shock vector ξ.

The optimal parameter θ^w ensures that the share of the young in overall production, i.e. $[(1-\theta^w)\bar{L}F_L]/F(K,\bar{L})$, corresponds to the share of the young in the population, γ. In this way, each individual in effect has the same per capita claim on production, $AF(K,\bar{L})$. All individuals thus feature the same exposure to productivity shocks—irrespective of age. With θ^w allowing the optimal sharing of productivity shocks, the optimal per capita lump-sum component of the PAYG tax θ^p ensures that the young share equally in the scrap value of capital and the stock of explicit public debt g_0. However, since θ^w is fixed independently of depreciation shocks, it cannot optimally allocate these risks across the two generations. Indeed, with a DC funded system, all depreciation risk accrues to the old generation. This implies suboptimal risk-sharing, because in the optimum the consumption of the two generations should be perfectly correlated. More generally, any source of risk (in addition to depreciation risk, longevity risk would be an example of this) that produces an imperfect correlation of the incomes of our two generations would produce a suboptimal allocation of risk if the pension arrangement could not offset it.

VI. A Funded Defined Real Benefit System

The neutrality of the economy for the introduction of a second pension pillar disappears under DB systems. Intuitively, these pension systems introduce new ways to share risks between the generations that are not offered by financial markets. Hence agents cannot offset the transactions of the pension fund so that funded pension schemes may affect investment and asset returns.

The simplest possible DB system is the defined real benefit (DRB) system. In such a system, the period 1 old receives a safe real benefit B^{drb}. The system is fully funded if it does not redistribute between the two generations *ex ante*. Hence the pension contribution must be equal to the discounted value of the safe benefit, where the appropriate discount rate is the return on debt, r. Indeed, like the payoff on debt, the payoff on the pension contributions is a pre-fixed number of units of the single good. Hence under full funding the return on the contributions to the pension fund, r^f, must be the same as that on a bond:

$$r^f = r. \tag{28}$$

The design of the DRB pension fund in effect requires the choice of four policy parameters: θ^f, B^{drb}, b^f and k^f. These are linked by restriction (10) and the funding requirement that the contribution is actuarially fair: $\theta^f = B^{drb}/(1+r)$. Therefore, only two parameters can be selected independently, which we can take to be b^f and k^f. The amount that the fund invests in debt does not affect the equilibrium because it is not a source of mismatch between assets and liabilities of the pension fund and thus does not provide new possibilities for risk-sharing between the generations. Hence the remaining key choice for the design of the second pillar is k^f.

With the young absorbing the mismatch risk of the pension fund (i.e. what is left in the fund after the payment of the pension benefits to the old), each young person receives $(1-\gamma)/\gamma$ times (where the second equality employs (10) to eliminate θ^f)

$$\begin{aligned}(r^a - r)\theta^f &= (1+r)b^f + (1+r^{kn})k^f - (1+r)\theta^f \\ &= (r^{kn} - r)k^f.\end{aligned}$$

The first line shows that the young *in effect* issue bonds of size θ^f to the old and invest the resources for their own risk according to the portfolio of the pension fund. This way, the young can in fact go short in debt; and, depending on the equity investments of the pension fund k^f, they are effectively participating in the stock market and thus are sharing in equity (productivity and depreciation) risks with the old generation. If the pension fund invests only in bonds ($k^f = 0$ and $b^f = \theta^f$), the DC and DRB systems become identical (given the same contributions θ^f) if the DC scheme invests in public debt only.

The generational account of the old under the *DRB* system is given by

$$G^o = \frac{\gamma}{1-\gamma}\left(\theta^p + \theta^w \frac{1}{\gamma} A\bar{L}F_L\right) + \frac{\gamma}{1-\gamma}(1+r)d + (r - r^{kn})k^f. \tag{29}$$

The final term is new compared with the generational account of a DC system (see (26)). It measures the difference between the non-stochastic real return on the pension fund contributions and the stochastic real investment return on these contributions if invested in equity. This additional term thus captures the mismatch between the pension fund's assets and liabilities that allows the young generation to share in equity market risks. For example, if the net return on equity is unexpectedly low, such that $r^{kn} < r$, the old are hurt through their direct equity holdings k. However, with pension benefits of the DB scheme defined in real terms and the DB fund investing in equity, the young share in the fortunes of the equity market and shield the old from equity market risk. This implicit insurance of the old by the young through the DB scheme explains the positive component $(r - r^{kn})k^f$ of G^o, should $r^{kn} < r$.

Substituting (29) into (25), we find that the social optimum is established in the presence of a DRB funded system if

$$\begin{aligned} &\frac{1}{1-\gamma}\left(\theta^p + \theta^w \frac{1}{\gamma} A\bar{L}F_L\right) + \frac{1}{1-\gamma}(1+r)d + \frac{1}{\gamma}(r - r^{kn})k^f \\ &= \frac{1}{\gamma} AF_L\bar{L} - \frac{1}{1-\gamma}[AF_K + (1-\delta)]K. \end{aligned} \tag{30}$$

The final term on the first line arises from the mismatch between the pension fund's assets and liabilities and provides opportunities to share depreciation risks across the two generations. The following proposition makes this precise.

Proposition 2. With a DRB pension fund in the second pillar, an appropriate combination of the two pension pillars can replicate the social optimum. In particular, the first pension pillar implements optimal *ex ante* redistribution and optimal *ex post* redistribution of wage risk by setting a lump-sum pension premium $\theta^p = -(1+r)(K+g_0)$ and a wage-linked PAYG pension premium $\theta^w = 1-\gamma$. The second pension pillar provides for the optimal *ex post* redistribution of financial market risks with $k^f = [\gamma/(1-\gamma)]K$.

Proof. This follows directly by substitution of the proposed arrangements into (30) and making use of (17). □

The optimal arrangement has young and old agents sharing equally in both types of risk. With a defined-benefit pension fund investing part of its assets in equity capital (i.e. $k^f = [\gamma/(1-\gamma)]K$), the young share in the depreciation risk that would otherwise be born solely by the old. The share of the pension portfolio invested in equity capital is thus determined by the requirement that each agent bears the same depreciation risk. In particular, if the young generation is relatively larger (γ is larger), the proportion of total capital held by the fund rises as the depreciation risk has to be shifted towards a larger young generation and away from a smaller old generation. In fact, using (19) (i.e. $K = (1-\gamma)(k + k^f)$), we can write $k^f = [\gamma/(1-\gamma)]K$ as $k/(k^f + k) = 1 - \gamma$. In other words, the share of capital held by the old is equal to its population share, so that each individual effectively (directly or indirectly as a residual claimant to the pension fund) holds exactly the same amount of capital.[8] The condition $k/k^f = (1-\gamma)/\gamma$ in effect determines the optimal size of the DC pension relative to the DB pension.[9] Hence in an older society (i.e. where γ becomes smaller) the DC part of the second pension pillar should be larger than in a younger society. Intuitively, with a relatively small young generation, the old can shed less capital risk. This result may help to explain the trend away from DB to DC pension systems in ageing societies.

The pension arrangement also requires a wage-linked first PAYG pillar (i.e. $\theta^w \neq 0$). This is because depreciation and productivity risks imply imperfectly correlated wage and capital risks. Accordingly, all generations must have an implicit claim not only on equity capital but also on human capital to share these risks optimally. Without a wage-linked first pillar, the old do not have a claim on human capital and thus do not share in wage risks. By setting $1 - \theta^w = \gamma$, the effective claims of the two generations on aggregate human capital coincide with their population shares. Hence each individual has exactly the same exposure not only to depreciation risk but also to human capital risk. The pension system therefore allows the old to get rid of depreciation risk and the young to shed wage risk.

As regards intergenerational *ex ante* redistribution established by the first pension pillar, optimal intergenerational redistribution requires that $\theta^p = -(1+r)(K + g_0)$ and $\theta^w = 1 - \gamma$. Here θ^w makes sure that the old get the appropriate implicit claim on human capital, whereas θ^p allows the young to obtain the correct implicit claim on economy-wide saving in period 0 so that all resources in the economy are distributed equally. With a fully funded second pillar, the young on average do not have a claim on saving in period 0. To establish equal consumption of all individuals in period 1, the PAYG system therefore has to ensure that the two generations share not only human capital (by setting θ^w appropriately) but also the claims on physical capital and public debt (by setting θ^p appropriately). Indeed, the optimal arrangements ensure that all individuals in society, young or old, hold the same per capita claims on human capital, physical capital and public debt.

VII. Extensions and Further Analysis

This section considers some two extensions to the preceding analysis. First, we explore how a defined wage-indexed benefit-funded pension arrangement can replicate the social optimum. Subsequently, we consider the case in which the planner attaches different weights to the two generations who may also feature different utility functions.

(a) Defined wage benefit

Under a defined wage-indexed benefit (*DWB*) system, the pension benefit is indexed to the wage rate. The DWB system is of particular interest for three reasons. First, funded pension benefits are often at least partially indexed to wages. Second, in contrast to the real promises of the DRB scheme, the wage-indexed promises of DWB schemes are not traded on financial markets. This makes the valuation of the DWB promises more challenging. The final reason why the DWB is of interest is that it can help to share wage risks across generations. With the funded system taking over the role of the PAYG system in sharing wage risks, the PAYG scheme can focus on establishing optimal intergenerational distribution *ex ante* while the funded pillar achieves optimal intergenerational risk sharing *ex post*. Hence the redistributive and risk-allocation roles of the pension system can be completely separated between the two pillars. This would make the pension system more transparent in terms of its consequences for different generations, and might in this way contribute towards the design of better pension systems. For example, reducing the size of the first pillar with the aim of limiting the intergenerational redistribution of wealth towards the old no longer necessarily comes at the cost of less intergenerational risk-sharing.

The benefit to the old in period 1 becomes

$$\theta^{dwb}\frac{1}{\gamma}AF_L\bar{L} = (1 + r^f)\theta^f, \tag{31}$$

where θ^{dwb} captures the fixed (non-stochastic) factor that links the benefit to the wage and r^f represents the real return on each euro of contribution in period 0. The system is fully funded if the old pay at the margin the value they attach to the wage-indexed claim so that $u_c(c_{y,0}) = \beta E_0[u_c(c_o)(1 + r^f)]$,[10] which, by using (31) to eliminate r^f, is equivalent to

$$u_c(c_{y,0}) = \frac{\beta}{\gamma}\left(\frac{\theta^{dwb}}{\theta^f}\right)F_L\bar{L}E_0[u_c(c_o)A]. \tag{32}$$

Substitution of (31) into this expression to eliminate θ^{dwb} yields

$$1 + r^f = \frac{A}{\beta E_0[mA]}, \tag{33}$$

where $m \equiv u_c(c_o)/u_c(c_{y,0})$ denotes the stochastic discount factor of the old.

The optimal design of the DWB pension fund requires the choice of θ^f, θ^{dwb}, b^f and k^f. Again, these are linked by the restriction (10) and the full-funding requirement, which is now given by (32). Accordingly, only two parameters can be chosen independently. For this purpose we select θ^{dwb} and k^f, which determine, respectively, the size of the second pillar and the composition of the pension fund's investment portfolio.

The young receive the residual value of the stand-alone pension fund. Each one of them thus gets $(1 - \gamma/\gamma)$ times

$$\begin{aligned}(r^a - r^f)\theta^f &= (1 + r)b^f + (1 + r^{kn})k^f - (1 + r^f)\theta^f \\ &= (r - r^f)b^f + (r^{kn} - r^f)k^f,\end{aligned}$$

with r^f determined by the full-funding requirement (33). Under the DWB system, the young in effect issue a wage-indexed bond to the old and invest the borrowed resources in bonds and physical capital, in conformity with the portfolio decisions of the pension fund. Wage risk is not traded in financial markets,[11] so the DWB pension fund always suffers from mismatch risk. In this way, the pension fund introduces new possibilities for implicitly trading risk factors. In particular, DWB pension funds allow the young generations not only to participate in the equity market, but also to shed wage risk. If pension liabilities depend on non-traded risk factors, the valuation of pension liabilities becomes problematic and diverges between various agents.[12] By using the stochastic discount factor of the old generation to value liabilities, we are in effect employing the valuation of the old generations in imposing the funding requirement.

The generational account of the old under the DWB system is given by

$$(34)\quad \begin{aligned} G^o &= \frac{\gamma}{1-\gamma}\left[\theta^p + \frac{1}{\gamma}A\bar{L}F_L\left(\frac{1-\gamma}{\gamma}\theta^{dwb}\right)\right] + \frac{1}{1-\gamma}A\bar{L}F_L\theta^w \\ &+ \frac{\gamma}{1-\gamma}(1+r)d - (1+r)b^f - (1+r^{kn})k^f. \end{aligned}$$

If we substitute (34) into (25), we obtain the requirements for the replication of the social optimum under DWB. This gives rise to the following proposition.

Proposition 3. With a DWB scheme in the second pillar, the first pension pillar provides for the appropriate *ex ante* redistribution by setting a lump-sum pension premium $\theta^p = -(1+r)\left[g_0 + K - [(1-\gamma)/\gamma]\theta^f\right]$ and $\theta^w = 0$. The second pension pillar provides for the optimal *ex-post* sharing of all risks with $\theta^{dwb} = \gamma$ and an investment portfolio with $k^f = [\gamma/(1-\gamma)]K$.

In contrast to the DRB scheme, a DWB scheme can completely separate the two roles of the pension system (i.e. intergenerational redistribution and risk-sharing) between the two pillars of the pension system. Whereas the PAYG pillar can now be targeted exclusively at optimal *ex ante* redistribution, the second, funded, pillar is responsible for optimal risk-sharing.

The DWB system does not require systematic redistribution to the old through the wage-indexed component of the PAYG system. Hence the parameter θ^p of the PAYG system has to establish not only the appropriate claim of the young on aggregate saving (this is the term $-(1+r)(g_0+K)$), but also the appropriate claim of the young on human capital (this is the term with $(1+r)[(1-\gamma)/\gamma]\theta^f$). Compared with the situation with the DRB scheme, the lump-sum component of the PAYG system should include an extra systematic transfer from the young to the old. In this way, the old acquire the same per capita claim on the human capital as the young.

(b) Diverging generational weights and different utilities

A planner who attaches equal weights to both generations aims at identical consumption levels for the two generations. In reality, this would usually require such a substantial degree of systematic redistribution between the two groups that it would be considered neither realistic nor desirable. This subsection explores the consequences of a non-unitary welfare weight $\chi^p \neq 1$ as well as heterogeneous utility functions for the two generations. The old, for example, may be more risk averse owing to habit formation. Replication of the planner's solution now requires that $v_c(c_o) = \chi^p u_c(c_y)$. This condition does not generally allow us to find a simple analytical solution for the optimal pension arrangement. Therefore, we log-linearize the model, which enables us to obtain straightforward and intuitive generalizations of the optimal arrangements derived earlier.[13]

We assume that the sources of uncertainty are given by

$$\ln A = \ln \bar{A} + \omega_A, \quad \ln \delta = \ln \bar{\delta} + \omega_\delta,$$

where the shocks ω_A and ω_δ are normally distributed with means zero and standard deviations σ_A and σ_δ, respectively. The condition for optimal risk-sharing follows from the log-linearization of the planner's first-order condition (5) and is given by

(35) $$\widehat{c}_y = \frac{\sigma^o}{\sigma^y}\widehat{c}_o,$$

where a caret above a variable denotes the logarithmic deviation from the point (denoted by a macron) around which the approximation is taken. Further, $\sigma^o \equiv -\bar{c}_o v''(\bar{c}_o)/v'(\bar{c}_o)$ and $\sigma^y \equiv -\bar{c}_y u''(\bar{c}_y)/u'(\bar{c}_y)$ stand for the coefficients of relative risk aversion of the old and the young, respectively. We approximate around the point at which all shocks happen to be at their means—see Bovenberg and Uhlig (2006). This point is solved from a nonlinear system, and hence the solution does not generally correspond to the expected values of the variables involved. Therefore, we label this the *median solution*.[14]

The available instrument sets for the design of a two-pillar pension system with a DRB or DWB second pillar and the respective funding requirements are the same as before when $u(\cdot)$ and $v(\cdot)$ were identical and χ^p was unity. In an appendix (available online—see above) we can show the following.

Proposition 4. (a) Consider a two-pillar pension system with a DRB second pillar. By setting

$$\theta^w = \frac{(1-\gamma)\left(\frac{\bar{c}_o}{\sigma^o}\right)}{\gamma\left(\frac{\bar{c}_y}{\sigma^y}\right) + (1-\gamma)\left(\frac{\bar{c}_o}{\sigma^o}\right)} \text{ and } (1-\gamma)k^f = \frac{\gamma\left(\frac{\bar{c}_y}{\sigma^y}\right)}{\gamma\left(\frac{\bar{c}_y}{\sigma^y}\right) + (1-\gamma)\left(\frac{\bar{c}_o}{\sigma^o}\right)}K$$

$$= (1-\theta^w)K,$$

we simultaneously achieve optimal risk-sharing of ω_A and ω_δ shocks. (b) Consider a two-pillar pension system with a DWB second pillar. By setting

$$\theta^{dwb} = \frac{\gamma\left(\frac{\bar{c}_o}{\sigma^o}\right)}{\gamma\left(\frac{\bar{c}_y}{\sigma^y}\right) + (1-\gamma)\left(\frac{\bar{c}_o}{\sigma^o}\right)}$$

and $\theta^w = 0$, while otherwise keeping the arrangement the same as under (a), the social optimum is replicated.

With $\chi^p = 1$ and $v(\cdot) = u(\cdot)$, $\bar{c}_y = \bar{c}_o$ and $\sigma^y = \sigma^o$, so that the optimal arrangements in Proposition 4 boil down to those in Propositions 2 and 3, respectively.

To obtain some more intuition for arrangement (*a*),[15] we first consider $\sigma^y = \sigma^o$, which is the case in particular if the two generations share the same utility function exhibiting constant relative risk aversion. If the social welfare weight on the young is relatively large (i.e. if $\chi^p > 1$ so that $\bar{c}_y > \bar{c}_o$), we have $1 - \theta^w > \gamma$ and $(1-\gamma)k^f > \gamma K$. The young thus obtain a larger claim on human capital and financial capital than their population share γ and hence bear a larger share of the risks associated with human and financial capital. Intuitively, the young are richer in terms of per capita consumption and therefore absorb a larger share of all risks.[16]

Now, suppose that the old are more risk-averse than the young ($\sigma^o > \sigma^y$) but that $\bar{c}_y = \bar{c}_o$. Again, the young should obtain a larger claim on human and financial capital (i.e. $1 - \theta^w$ and $(1-\gamma)k^f$ should be increased above the young's population share γ), so that they bear a larger share of the aggregate risks. To prevent the old from becoming poorer on average, the government has to set θ^p at a higher level than in the case with equal risk aversion $\sigma^y = \sigma^o$ so that the old collect relatively more safe income.

VIII. Capital Investment and Risk Premia

Policy-makers and the financial sector are at pains to gauge the consequences of pension reforms for returns on financial markets and saving and investment. Our intertemporal general-equilibrium model allows us to explore these effects. In particular, intergenerational risk-sharing conducted by the pension system affects consumption when the old generation is retired, and hence, as suggested by the Euler equations (15) and (16), it also impacts asset returns and investments. This section explores the general-equilibrium effects of replacing a funded DC pension scheme by a funded DB scheme. Hence we depart from the normative analysis of the preceding sections and turn to a positive analysis of the consequences of different arrangements of the second, funded, pension pillar.

We use our earlier log-linearization to compute the equity risk premium (see the appendix, available online as mentioned in the Introduction):

$$(36) \qquad \ln\left[\mathrm{E}_0(1+r^{kn})\right] - \ln(1+r) = \sigma^o \mathrm{cov}(\hat{r}^{kn}, \hat{c}_o),$$

where

$$(37) \qquad \hat{r}^{kn} = \frac{\bar{A}F_K}{\bar{A}F_K + (1-\bar{\delta})}\omega_A - \frac{\bar{\delta}}{\bar{A}F_K + (1-\bar{\delta})}\omega_\delta.$$

If the covariance of the return on equity and old-age consumption is substantial, the equity risk premium needs to be high to persuade the old generation to buy all available equity.

Substitution of the log-linearized generational accounts yields the following expressions for old-generation consumption in period 1 under the DC and DRB schemes, respectively:[17]

$$(38) \qquad \hat{c}_o = \frac{\bar{A}F_K K + \theta^w \bar{A}F_L \bar{L}}{(1-\gamma)\bar{c}_o}\omega_A - \frac{\bar{\delta}K}{(1-\gamma)\bar{c}_o}\omega_\delta,$$

$$(39) \qquad \hat{c}_o = \frac{\bar{A}F_K(K-(1-\gamma)k^f) + \theta^w \bar{A}F_L \bar{L}}{(1-\gamma)\bar{c}_o}\omega_A - \frac{\bar{\delta}(K-(1-\gamma)k^f)}{(1-\gamma)\bar{c}_o}\omega_\delta.$$

The terms involving θ^w represents the beneficial effect of a positive productivity shock, $\omega_A > 0$, on old-generation consumption through the wage-linked PAYG benefit. The terms with $(1-\gamma)k^f$ in the second expression show that productivity and depreciation shocks partly leak away to the young generation through their claim on the equity investment of a DRB scheme.

The equity risk premia under DC and DRB follow from the substitution of (37) and (38), respectively (37) and (39), into (36):

$$\sigma^o\left[\left(\frac{\bar{A}^2 F_K^2 K + \theta^w \bar{A}^2 F_K F_L \bar{L}}{(1-\gamma)\bar{c}_o\left[\bar{A}F_K + (1-\bar{\delta})\right]}\right)\sigma_A^2 + \frac{\bar{\delta}^2 K}{(1-\gamma)\bar{c}_o\left[\bar{A}F_K + (1-\bar{\delta})\right]}\sigma_\delta^2\right],$$

$$\sigma^o\left[\left(\frac{\bar{A}^2 F_K^2\left[K-(1-\gamma)k^f\right] + \theta^w \bar{A}^2 F_K F_L \bar{L}}{(1-\gamma)\bar{c}_o\left[\bar{A}F_K + (1-\bar{\delta})\right]}\right)\sigma_A^2 + \frac{\bar{\delta}^2\left[K-(1-\gamma)k^f\right]}{(1-\gamma)\bar{c}_o\left[\bar{A}F_K + (1-\bar{\delta})\right]}\sigma_\delta^2\right],$$

where we have assumed that the correlation between A and δ is zero. The presence of a wage-linked PAYG pillar (i.e. $\theta^w > 0$), *ceteris paribus*, raises the equity premium because

TABLE 1
EQUITY INVESTMENT AND INTERTEMPORAL PRICES

	Optimal DRB			DC		
	r	K	erp	r	K	erp
Baseline	2.20%	0.297	1.24%	0.51%	0.425	1.90%
$\sigma^o = 10$	1.54%	0.320	1.58%	− 0.14%	0.430	2.25%
$\sigma_A = 0.3$	1.53%	0.323	1.60%	0.24%	0.402	2.12%
$\sigma_A = 0.5$	− 1.88%	0.484	3.07%	− 2.20%	0.428	3.28%
$\sigma_\delta = 1.25$	1.35%	0.298	2.06%	− 0.41%	0.478	2.33%

Notes: Returns are annualized assuming a generation length of 30 years. erp = equity risk premium. The production function is Cobb–Douglas with labour income share 66%. The baseline parameter combination is $\eta_0 = 4$, $g_0 = 0.8$ $\bar{A} = 5$, $\beta = \bar{\delta} = \gamma = 0.5$, $\sigma_A = 0.25$, $\sigma_\delta = 1$, $\theta^w = 0.5$ and $\sigma^o = 7.5$. Other combinations vary one parameter away from the baseline each time, as indicated. The DRB system is always optimal, so that, in conformity with Proposition 2 we have $k^f = K$ and $\theta^p = -(1 + r)(K + g_0)$. The DC system is always combined with a PAYG pillar θ^p (and θ^w) that is identical to the one under the corresponding optimal DRB system.

it raises the covariance between the equity return and old-age consumption that results from the productivity shock. The equity holdings by the young through the DRB pension scheme (i.e. $(1-\gamma)k^f > 0$), *ceteris paribus*, reduce the covariance between the equity return and old-age consumption through two channels, namely not only the productivity shock but also the depreciation shock.

These latter statements about the impact of a wage-indexed PAYG scheme and equity investment by the DRB fund assume that the median solution of the economy is kept constant. However, this solution depends on the parameters of the pension schemes. In order to compute the median solution, we have to resort to numerical simulations.

The baseline parameter combination is selected as follows. We set $\beta = 0.50$, which corresponds to an annual time preference rate of roughly 2.25% if we think of a generation as 30 years, and $\bar{\delta} = 0.50$, which corresponds to a similar (albeit low) annual depreciation rate. First-period resources are fixed at $\eta_0 = 4$, and government consumption is chosen as a share of 20%; i.e. $g_0 = 0.8$. Relative risk aversion σ^o is 7.5—a value that is generally considered reasonable in the finance literature. Further, we select $\sigma_\delta = 1$, which would roughly correspond to an annual standard deviation of stock returns of 20%, assuming that the returns are serially uncorrelated. We set the 30-year standard deviation on productivity lower at $\sigma_A = 0.25$. The other baseline parameter choices are listed in the notes to Table 1. We also perform some sensitivity analysis, whereby each time we vary one parameter away from the baseline. In all cases, we compare an optimal DRB system (cf. Proposition 2) with a DC system that is combined with the same PAYG pension pillar (θ^p, θ^w).

Table 1 shows that the combination of benchmark parameters produces an annual equity premium of 1.2% for the optimal DRB system and of 1.9% for the DC system. These values are of an order of magnitude that is quite realistic. Also, the annual risk-free rate of 2.2% under the optimal DRB system seems reasonable, while that under DC is on the low side. More uncertainty affects investment in risky assets, in two ways. On the one hand, for a given expected return more uncertainty about future resources reduces the desire to invest in risky assets, thereby decreasing investment. On the other hand, more uncertainty raises precautionary saving, which increases the capital stock one-for-one because the supply of public debt is exogenously fixed. In the DC system, the old generation faces more uncertainty in retirement. The larger capital stock under DC indicates that the precautionary savings argument dominates. The DC system also implies a higher equity premium because old-age consumption is more strongly correlated with equity returns than under the DRB system.

An increase in risk aversion ($\sigma^o = 10$) gives rise to more precautionary saving. Both the larger capital stock and the higher price for absorbing risk raise the equity premium. More uncertainty about productivity ($\sigma_A = 0.3$) implies similar qualitative effects. Interestingly enough, with even more uncertainty about productivity ($\sigma_A = 0.5$), the capital stock under optimal DRB is now larger than under DC, while the equity premium is still lower under optimal DRB. In this case, therefore, more efficient risk-sharing under DRB induces the old to invest more in risky assets. Finally, an increase in depreciation risk ($\sigma_\delta = 1.25$) is fully concentrated among the old under DC, producing a relatively strong increase in precautionary savings demand for equity and thus a higher capital stock than under DRB.

IX. Conclusions

This paper has investigated how different pension arrangements affect intergenerational risk-sharing and the capital market in a two-period overlapping-generations model. The two generations in the model overlap only in the second period. As a result, risks cannot be traded directly between the generations, thereby creating a rationale for institutions that take over this role. In particular, the combination of a first PAYG pension pillar and a second, funded, defined-benefit pension pillar can optimally share financial market and wage risks across the generations. An optimal pension system allows each individual to hold the same share of human and physical capital. In this way, the old can get rid of financial market risks, while the young are able to shift labour market risks to the older generation.

Our results shed some light on recent developments in old-age insurance. They suggest that the trend towards more funding does not have to harm intergenerational risk-sharing provided that funded schemes include a DB component. Indeed, the issue of funding can be decoupled from that of intergenerational risk-sharing if some of the retirement benefits in the funded system are linked to wages. In particular, depending on the definition (and thus the stochastic nature) of the retirement benefits and the investments of the pension fund, the associated mismatch risk can create the same risk-sharing as a PAYG scheme. The trend away from DB towards DC schemes can harm intergenerational risk-sharing because DC schemes do not provide intergenerational risk-sharing opportunities in addition to those already provided by the capital market. At the same time, however, the optimal DC share of the funded system should grow if ageing implies that the old can shed less investment risk to a smaller working population. This may help to explain the trend away from DB to DC pension systems in ageing societies.

Our analysis also points to the importance of the investment behaviour of pension funds for intergenerational risk-sharing. Liability-driven investment eliminates mismatch risk, thereby also taking away the opportunities for intergenerational risk-sharing. Indeed, some mismatch can be desirable in order to create new opportunities for risk-sharing and risk-taking. To illustrate, equity investments by stand-alone DB funds allow young participants to take advantage of the equity premium. By bearing the mismatch risk of the DB fund, the young in effect borrow from the old generation to gain exposure to financial market risks and thus take advantage of the associated risk premium. In this connection, the shift towards equity and alternative risk-bearing investment categories that is observed in the portfolios of some pension funds can be welcomed.

This paper has focused on the role of the pension system in conducting intergenerational risk-sharing. An alternative instrument for intergenerational risk-sharing is fiscal policy. In our model, the scope for optimal risk-sharing through fiscal policy is rather limited. With a richer menu of taxes, fiscal policy can play a larger role in optimal risk-sharing. To illustrate, taxes on labour and capital income may help to share productivity and depreciation risks optimally across generations (see e.g. Smetters 2006). At the same time, however, fiscal discretion may give rise to other, political risks. This may in fact further increase the role of the pension system in optimal intergenerational risk-sharing.

Our paper allows for a large number of further extensions. First, the menu of shocks could be extended to include, for example, demographic shocks (such as shocks to fertility and longevity—see e.g. Auerbach and Hassett 2002; Andersen 2005) and health shocks. As a second extension, we can allow for intragenerational heterogeneity in risk preferences. In that case, mandatory pension funds with uniform investments and liabilities cannot be tailored to individual preferences. If individuals do not have access to financial markets to construct their own tailor-made portfolio, mandatory pension funds may give rise to welfare losses compared with a first-best world in which all individuals could buy their own tailor-made portfolio. These losses have to be traded off against the potential benefits of pension funds in allowing young generations to share in financial market risks.

We have assumed that the young cannot participate in capital markets at all to share financial-market risks. One interpretation is that human capital is not tradable and that the young cannot therefore borrow at all against their human capital to invest in financial capital. In practice, however, the young may be able to participate in an equity-market risk that materializes during their working career, either by borrowing or by investing all their savings in the risk-bearing capital. Indeed, capital markets allow in principle for risk-sharing between overlapping generations, especially if the young can borrow. In this regard, therefore, our calculations are likely to overstate the potential risk-sharing benefits from defined-benefit pension plans. At the same time, however, by modelling only two generations, we have underestimated the potential gains of pension funds from risk-sharing between non-overlapping generations. Indeed, with many non-overlapping generations, old generations can benefit from sharing risks not only with the young generations that they overlap but also with future generations that are not yet born when they are alive—see also van Hemert (2006) and Teulings and de Vries (2006). In other words, it may pay to allow not only the young generation but also other future generations to trade with the old through the pension system.

We would like to explore the sensitivity of our results with respect to alternative assumptions about the extent to which the young and the future generations can participate in capital markets and pension institutions. Whereas more scope for the young to participate in capital markets reduces the value-added of pension funds, including more generations in pension arrangements increases the potential of pension funds to create value by opening up new ways to conduct intergenerational risk-sharing.

ACKNOWLEDGMENTS

We thank two anonymous referees and seminar participants at the European Commission and Tilburg University for their helpful comments. Part of this paper was written while Beetsma was visiting DG-ECFIN of the European Commission, which he thanks for its hospitality. He is grateful to Heikki Oksanen for many stimulating discussions. The usual disclaimer applies.

NOTES

1. For an overview of recent reforms in the European Union, see the European Commission (2006).
2. For early work on intergenerational risk-sharing and social security, see Enders and Lapan (1982) and Gordon and Varian (1988). More recent contributions include Demange (2002), De Menil and Sheshinski (2003) and Gottardi and Kubler (2006). For broader recent perspectives, see Shiller (1999) and Lindbeck and Persson (2003).
3. The second pillar is fully funded in the sense that, from an *ex ante* perspective (i.e. in terms of market value), the old generation does not receive any resource transfers from other generations through this pillar (see also Oksanen 2006). Hence this pillar does not systematically redistribute resources across generations.
4. In practice, the young participants can bear this risk in either the pension premium or the pension rights they accumulate.
5. To see this, impose $\theta^p = \theta^w = \theta^f = 0$ on (9); use (18) to eliminate τ, (14) to eliminate r^k (and employ $r^{kn} = AF_k - \delta$), (19) to eliminate k (and use $k^f = 0$), (20) to eliminate b (and use $b^f = 0$), and (22) to eliminate c_o.
6. To see this, one can add γ times equation (21) and $(1-\gamma)$ times equation (22). Using $r^{kn} = AF_K - \delta$, (23), and the constant-returns property of the production function, one can simplify the resulting equation to (4), which coincides with the planner's resource constraint. The combination of the expression $v_c(c_o) = \chi^p u_c(c_y)$, expression (4) and, equations (16) and (7) (combined with expressions (17), (19), (20) and (10)) exactly coincides with the system (3), (4), (5) and (6) to be solved under the planner.
7. The DC system can thus guarantee real benefits *ex ante* by investing in bonds. With the help of the government issuing bonds, a DC system can thus allow for defined benefits.
8. If $\gamma = \frac{1}{2}$ (both generations are of equal size), $k^f = K$. This does not imply that the pension fund holds all equity, but rather that the pension fund holds $(1-\gamma)k^f/K = \frac{1}{2}$ (use (19)) of all equity. The reason is that k^f is defined as capital *per member of the old generation*.
9. Recall that members of the old generation are indifferent between a physical capital investment k via a defined contribution pension fund and just directly investing themselves in physical capital.
10. At the margin, the old should be indifferent about contributing or not contributing one more euro to the pension fund. This requirement implies this last condition.
11. Trade in equity allows only a particular combination of productivity and depreciation risk to be traded.
12. For the valuation of non-traded wage-indexed pension liabilities, see de Jong (2005).
13. For some specific cases, we can find an analytical solution for the optimal pension arrangement without log-linearization. For example, this is the case if the two generations feature the same constant relative risk aversion (CRRA) utility function, since in this case the consumption levels of the young and the old are related in a linear way. If utilities differ across generations, however, no analytical solution can be found even if both utilities are CRRA. In any case, the log-linearization yields intuitive expressions for the optimal pension arrangements. Without log-linearization, we can obtain only expressions that depend in a rather complicated way on the underlying determinants. Moreover, in the next section we exploit log-linearization to derive intuitive expressions that help us to understand the underlying determinants of the equity premium.
14. An appendix (available online as given at the end of the Introduction) shows the complete derivation of the median solution and the log deviations from this solution in response to the shocks.
15. The intuitions for the other arrangement are very similar, and therefore are not explicitly described.
16. Our finding of $\hat{c}_y = \hat{c}_o$ combined with $\bar{c}_y > \bar{c}_o$ implies that the absolute size of the fluctuations in the per capita consumption of the young exceeds the fluctuations in the per capita consumption of the old.
17. To save space, we consider here the alternative only of a DRB system and not that of a DWB system. Real-world pension systems with a funded pillar are on average probably closer to the DRB system because of explicit or implicit guarantees to index benefits to price rises.

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