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Mandatory pension saving and homeownership

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Abstract

Mandatory pension saving and homeownership

We show that requiring individuals to contribute a constant share of their labor income to a retirement account increases loan-to-value ratios and typically defers homeownership. We investigate three alternative pension systems: (1) early withdrawals to acquire homeownership, (2) age-dependent contributions, and (3) a flexible scheme, which builds on the intuition, that it is not important how individuals build up savings as long as they build up sufficient savings, and only forces individuals to save when they miss the age-dependent savings target. All three systems lead to a similar accumulation of wealth, but lower loan-to-value ratios, usually earlier homeownership, and higher welfare.

Key Words: retirement saving, homeownership, pension system design, loan-to-value ratio, housing market entry

JEL Classification Codes: E21, G11, H23

1 Introduction

Saving for retirement is among the most important financial challenges individuals face throughout their lives. At present, increasing life expectancies combined with decreasing birth rates impose huge challenges on finding suitable ways to sustain a reasonable level of retirement benefits. Most countries around the world impose some form of mandatory retirement savings scheme on their citizens, often financed via a constant share of labor income used to fund the system (1) because forced saving can help individuals overcome lack of retirement planning or self-control problems (Samuelson (1975); Feldstein (1985); Moser and Olea de Souza e Silva (2019); Andersen and Bhattacharya (2021)) and (2) to overcome the Samaritan's dilemma caused by the government's inability to commit to not helping the elderly experiencing poverty.

We set up a realistically calibrated life cycle model which aims to shed light on the side effects of such constant mandatory retirement savings on housing decisions, in particular housing market entries and loan-to-value ratios, and incorporates many important features of models from the literature, including stochastic labor income (e.g., Cocco, Gomes, and Maenhout (2005)), a rent-versus-own decision (e.g., Cocco (2005); Yao and Zhang (2005)), modeling investors' possible irrationality (e.g., Larsen and Munk (2023)) and taxable as well as tax-deferred retirement accounts (e.g., Dammon, Spatt, and Zhang (2004); Fischer and Gallmeyer (2017)). In our model, individuals must make informed decisions about their preferred consumption, their homeownership status, the size of the home they want to live in, the contribution to or, after attaining retirement age, withdrawal from the tax-deferred retirement account, and about the shares of wealth invested into stocks and bonds in the taxable as well as the tax-deferred retirement account.

Our model predicts that mandatory constant minimum contribution requirements are useful for ensuring a sufficient build-up of savings, in particular when explicitly modeling individuals' myopia with respect to financial decision-making. However, individuals subject to constant minimum contribution requirements (1) face substantially higher loan-to-value ratios indicating that they—via the interest rate margin on mortgage debt—pay to borrow their own money that is locked up in the tax-deferred retirement account, (2) typically experience delayed housing market entries, and (3) are more consumption-constrained at younger age, thus rendering consumption-smoothing over the life cycle more difficult.

Existing pension systems of the defined benefit type, in which participants are promised fixed annual benefits for their contributions, face increasing funding problems. We therefore focus on pension systems of the defined contribution type, to which individuals contribute according to a pre-determined schedule, while benefits depend on the amounts of contributions made and their profits earned. Currently, such pension systems already cover large shares of the population's pensions in, e.g., Australia, Denmark, and Sweden, whereas the Netherlands currently undergoes a transition away from a DB system towards a DC system.

Retirement accounts are typically set up as tax-deferred accounts. Contributions to these accounts are (within certain limits) deductible from taxable income, and profits earned from them are generally tax-exempt, while withdrawals during retirement age are subject to income taxation.[1]

Whereas existing funded retirement savings schemes typically provide individuals with some degree of freedom in their asset allocation, they usually impose harsh restrictions on minimum contributions to counteract the empirically documented undersaving for retirement of many individuals that are not forced to save via mandatory retirement schemes (Bernheim, Skinner, and Weinberg (2001); Benartzi and Thaler (2007); Choi, Laibson, and Madrian (2011); Larsen and Munk (2023)). In practice, individuals are often required to contribute a constant minimum share of their labor income to a retirement savings scheme.

We propose three alternative pension systems: First, we explore a setting that allows individuals to withdraw from their retirement savings when acquiring homeownership—a feature found in some tax codes around the world. Second, acknowledging that minimum contribution rates typically bind most for younger individuals who have not been able to build up large savings yet, we investigate minimum contribution rates increasing with age, as, e.g., found in Switzerland. Third, we propose a flexible retirement savings scheme, which builds on the intuition that it is not important through which channels individuals save for retirement, as long as they build up sufficient total wealth. Intuitively, it may make sense to prioritize savings outside the tax-deferred retirement account at a younger age to facilitate making the down payment required for the acquisition of a home. Our flexible scheme only forces individuals to save, if they have not built up sufficient savings in either ordinary taxable accounts, as home equity, or through pre-existing tax-deferred retirement savings. More specifically, our scheme only requires individuals to build up savings, if they have not built up sufficient wealth in the past, but does not require savings to be held in the tax-deferred retirement account.

[1] Few countries, including Denmark and Sweden, impose taxes on profits earned in tax-deferred accounts. However, the tax rate is low compared to that applicable in a taxable account. Withdrawals prior to retirement age are generally not possible or subject to substantial penalty taxes, thus rendering early withdrawals highly undesirable. Hence, an important difference between taxable accounts and tax-deferred retirement accounts is the illiquidity of the latter.

These three alternative pension systems explored in our work all lead to a similar wealth level at retirement age, i.e., enable individuals to attain the same standard of living during retirement as the pension system with mandatory constant minimum contributions. Yet, all three alternative pension systems enable earlier homeownership, lead to lower loan-to-value ratios, more consumption-smoothing over the life-cycle, and higher welfare levels.

Our flexible retirement savings scheme, in particular, has two key advantages. First, it allows individuals to repay their mortgages before building up savings in the tax-deferred retirement account, thus avoiding paying the interest differential between the borrowing rate and the investment rate. Second, it does not impose any savings requirements on individuals that have built up sufficient savings in the past.

As implementing the flexible retirement scheme might be difficult in countries that do not have detailed registry data or impose a wealth tax on their citizens, we also study easily implementable alternatives to constant minimum contribution requirements: (1) the possibility of withdrawals from the tax-deferred retirement account prior to retirement age for the acquisition of an owner-occupied home and (2) minimum contribution requirements that are increasing with age. Both alternatives also alleviate the unintended side effects of constant minimum contribution requirements. However, these alternatives run a higher risk of poverty at retirement age, as there is no frequent assessment of savings.

Our work contributes to a growing strand of literature investigating optimal retirement savings decisions over the life cycle. An important early contribution is the work of Carroll (1997), which studies the optimal life cycle profile of savings when individuals do not have access to tax-deferred retirement accounts. Cocco, Gomes, and Maenhout (2005) extend this work by allowing for a stochastic labor income stream, which is calibrated to socio-demographic data. Studies examining optimal savings decisions with taxable accounts as well as tax-deferred retirement accounts include Amromin (2003), Shoven and Sialm (2003), Dammon, Spatt, and Zhang (2004), Garlappi and Huang (2006), Huang (2008), Fischer and Gallmeyer (2017), and Fischer, Jensen, and Koch (2023). We contribute to this line of literature by highlighting the undesirable side effects of minimum contribution requirements and by proposing three alternative retirement savings schemes, that largely alleviate the unintended side effects, while simultaneously ensuring that individuals build up sufficient savings.

Our work is also related to a strand of literature exploring optimal pension system design (e.g., Dahlquist, Vestman, and Setty (2018)). Larsen and Munk (2023) and Schlafmann, Setty, and Vestman (2024) also investigate optimal pension plan design in a framework with a taxable account as well as a tax-deferred retirement account, but abstract away from explicitly accounting for housing. The former document that individuals who undersave or do not invest into stocks can benefit from mandatory pension saving. The latter advocate age- and retirement wealth-to-labor income dependent contribution rates to retirement savings plans, but do not account for wealth held outside retirement plans in determining their contribution rates. Our work has another focus and differs from those two papers in several regards. First, we explicitly model a rent-versus-own decision and mortgages. Our results show that mandatory constant minimum contributions delay housing market entry and lead to higher loan-to-value ratios. Second, our work explores three alternative pension systems. In particular, we propose a flexible scheme that only requires saving from individuals that have not built up sufficient wealth, and does not require mandatory savings to be located in the tax-deferred retirement account, if, e.g., repaying an expensive mortgage is more desirable.

Other manuscripts investigate empirically the impact of different contribution rules to retirement savings schemes on individuals' financial decision-making (e.g., Beshears, Guo, Laibson, Madrian, and Choi (2023); Larsen, Nielsson, Nutu, and Rangvid (2023)). Our manuscript is, to the best of our knowledge, the first to propose a savings rule that does not solely focus on savings in tax-deferred retirement accounts, but also takes other forms of savings into account. Consistent with economic intuition, our results show that savings outside retirement savings accounts are important for determining how much to save for retirement. Periodically assessing individuals' total savings is further helpful to prevent undersaving, that is among the biggest financial regrets of individuals (e.g., Hurwitz and Mitchell (2022)).

We further contribute to the literature on the interplay between housing and retirement savings choices. In our model, drawdown of financial wealth as well as housing wealth are the exclusive sources of retirement income, whereas existing empirical evidence originates from institutional arrangements with some degree of replacement through public pensions. Existing empirical evidence such as in Poterba, Venti, and Wise (2011) for a US based analysis and Crawford (2018) for a UK based analysis,[2] shows that housing wealth is to a large extent bequeathed rather than consumed and that the average rate of wealth drawdown is much slower than the decline in remaining life expectancy.

In order to assess the possible impact of changes in the design of pension arrangements

[2] The report by Crawford (2018) is closer to our analysis as it is based on defined contribution pensions rather than defined benefit pensions as in Poterba, Venti, and Wise (2011).

and the institutional setup it is important to recognize that a Lucas critique problem is prevalent (Lucas (1976)). The main point of discussing institutional changes and alternative pensions system design is to induce a change the behavior of individuals. Judging suggested changes by an extrapolation of previous experience may well be misleading.[3]

This paper proceeds as follows: section 2 outlines our life cycle model and presents its calibration, while section 3 presents the model predictions. Next, section 4 introduces alternatives to constant minimum contribution requirements, and section 5 concludes.

2 The model

In this section, we outline our life cycle model of optimal consumption, savings, and housing decisions. In our model, individuals work from age 20 to 65 and build up savings for retirement (accumulation phase). At age 66, they are retired and use their accumulated savings during their remaining lifetime. Individuals face a rent-versus-own decision. They can build up savings in both a taxable and a tax-deferred retirement account, and they can invest in a risk-free asset, a risky asset representing a stock market index, and an owner-occupied home.

Individuals are subject to mortality risk and make their decisions in order to achieve the objective of maximizing expected present discounted lifetime utility. The individuals' preference functions are given by recursive Epstein-Zin preferences (Epstein and Zin (1989)) to enable a separation between the individual's degree of relative risk aversion, γ, and its elasticity of intertemporal substitution, ψ.

2.1 Mandatory contributions

Before discussing alternative pension systems, our first goal is to illustrate the implications of mandatory constant minimum contributions to the tax-deferred retirement account for the accumulation of wealth as well as homeownership decisions. For that purpose, we deliberately do not try to replicate a specific retirement savings scheme in a particular country, but aim at contributing to a better understanding of the broader consequences of mandatory constant minimum contributions.

To investigate the impact of mandatory constant minimum contributions, we compare

[3] As stated in Crawford (2018), p. 14: "How individuals behave with respect to their accumulated pension wealth, when it is their main or only source of retirement resource, may be very different from how they use their (typically much smaller) amounts of financial wealth when they have secured income from other sources."

two settings with each other. In our benchmark case, individuals are faced with a minimum contribution requirement to a tax-deferred retirement account, corresponding to a constant share, $\xi \in (0, 1]$, of the individual's labor income, L_t, at time t during working age:

$$Z_t \geq \xi L_t, \quad t < t_{ret}, \tag{1}$$

in which Z_t denotes the individual's contribution to the tax-deferred retirement account, and t_{ret} corresponds to the retirement age, the youngest age at which the individual is retired. Requiring minimum contributions to be a constant share of labor income, is a widespread phenomenon in many pension systems around the world. We explore the implications of age-dependent minimum contributions as an alternative to constant minimum contributions in greater detail in section 4.2.

To understand the implications of constant minimum contributions, we compare the benchmark case with constant minimum contributions to a setting, in which the minimum contribution during working age is zero, i.e.:

$$Z_t \geq 0, \quad t < t_{ret}. \tag{2}$$

We deliberately do not allow for early withdrawals from the tax-deferred retirement account in our base case setting, because such withdrawals are typically either impossible or subject to substantial tax penalties, thus rendering early withdrawals undesirable. Some countries, however, allow for early withdrawals from the tax-deferred retirement account when individuals are acquiring an owner-occupied home. We explore the impact of such a withdrawal opportunity in section 4.1.

Since contributions to the tax-deferred retirement account during working age are typically tax-deductible, they are usually limited by an upper bound, $\phi \in (0, 1]$, of the individual's labor income:

$$Z_t \leq \phi L_t, \quad t < t_{ret}, \tag{3}$$

in which $\xi \leq \phi$.

In our model, individuals are required to fully fund their retirement benefits by drawing down their own savings. Inspired by, for example, the Australian and Dutch pension system, we do not let the individuals choose their withdrawals from the tax-deferred retirement account freely, but require them to annuitize their savings held in the tax-deferred retirement account when attaining retirement age. More specifically, we require individuals to withdraw a certain fraction of their savings in every period of their retirement phase:[4]

$$Z_t = -\frac{W_{R,t-}}{\left(\sum_{i=t}^{100}\left(\frac{1}{1+r}\right)^{i-t}\prod_{j=t+1}^{i} f(j-1)\right)}, \quad t \geq t_{ret}, \tag{4}$$

in which r denotes the risk-free interest rate, $f(j)$ the individual's probability of survival from time j to time $j+1$, and $W_{R,t-}$ the individual's balance in the tax-deferred retirement account before decisions in period t are made.

2.2 Housing

In our model, individuals face a rent-versus-own decision, i.e., they can either live in a rented place or an owner-occupied home. The return on investments in owner-occupied homes, r_t^H, from time t to time $t+1$ is lognormally distributed, with mean μ_H and standard deviation σ_H.

Owner-occupied homes serve a dual role as a durable consumption good and an asset. That is, when owning a home, individuals simultaneously derive utility from living in their home and expose themselves to changes in its value. Following earlier work with owner-occupied housing, such as Cocco (2005), Yao and Zhang (2005), Fischer and Stamos (2013), or Kraft, Munk, and Wagner (2018), we employ a Cobb-Douglas utility function, u, defined over consumption and housing.

Following Kiyotaki, Michaelides, and Nikolov (2011), we account for the preferences of individuals to live in owner-occupied housing, by allowing for a higher utility per home unit when they own it. For that purpose, we multiply the size of the individual's residence, Q, by a factor $1+\zeta h$, in which ζ determines the strength of the preference for living in an owner-occupied home instead of a rented place, and h is an indicator variable, taking the value of one if the individual lives in an owner-occupied home, and zero otherwise. Hence:

$$u(C, Q, h) = C^{1-\theta}\left(Q(1+\zeta h)\right)^{\theta}, \tag{5}$$

in which C is the amount consumed, and θ is the relative preference over housing consumption.

Both owners and renters face costs for housing consumption. These costs can be divided

[4]We solve our model using backward induction. To avoid the numerical difficulties arising from an additional state variable, we do not transform the balance in the tax-deferred retirement account into an annuity stream, but instead impose an age-dependent withdrawal constraint.

into recurring and nonrecurring costs. The former costs depend on whether the individual lives in an owner-occupied home or a rented place. If H_t denotes the price per housing unit at time t, renters pay an annual rent, $m_r Q_t H_t$, that is assumed to be a constant share, m_r, of the value of their residence, $Q_t H_t$. Owners also face recurring costs, such as property taxes and maintenance costs of $m_o Q_t H_t$, in which m_o is a constant share of the value of their home. The rate of recurring housing costs, m, from time t to $t+1$ can be expressed as

$$m\left(h_t\right) = h_t m_o + \left(1 - h_t\right) m_r. \tag{6}$$

Nonrecurring housing expenses, n, occur when the individual acquires a new home. In that case, the individual faces transaction costs of $f_p Q_t H_t$, in which f_p denotes the percentage cost of acquiring homeownership. This cost includes fees to be paid to a real estate agent and taxes. Individuals face these costs when changing homeownership status from renter to owner ($h_t\left(1 - h_{t-1}\right) = 1$) or when remaining a homeowner, but changing home size, i.e., when moving from one owner-occupied home to another ($h_t h_{t-1} \chi_{\{Q_t \neq Q_{t-1}\}} = 1$), in which $\chi_{\{Q_t \neq Q_{t-1}\}}$ is an indicator variable taking the value one, if $Q_t \neq Q_{t-1}$, and zero otherwise.[5] Nonrecurring transaction costs, n, from time t to $t+1$ can then be summarized as follows:

$$n\left(Q_t, h_t, h_{t-1}\right) = f_p Q_t H_t \left(h_t\left(1 - h_{t-1}\right) + h_t h_{t-1} \chi_{\{Q_t \neq Q_{t-1}\}}\right). \tag{7}$$

These transaction costs result in no-trade regions, in which the costs from adjusting the home size outweigh the benefits. That is, trading costs render frequent changes in home size undesirable.

2.3 Capital markets

Apart from investing in owner-occupied homes, individuals can invest into a representative stock market index for which the pre-tax return, r_t^S, from time t to $t+1$ is lognormally distributed with mean μ_S and standard deviation σ_S. Returns on investments into the stock market index and owner-occupied homes may be correlated, reflecting that they may depend on common risk factors, such as the evolution of the economy. When τ_g denotes the tax rate applicable to returns on equity investments in the taxable account, the gross returns on an investment in the stock market index in the taxable and the tax-deferred retirement account from time t to $t+1$ are given by $G_{T,t} = 1 + r_t^S\left(1 - \tau_g\right)$ and $G_{R,t} = 1 + r_t^S$, respectively.

[5] In our model, the buyer covers the costs. Imposing the cost on the seller is economically equivalent and should therefore not affect our findings.

In addition to the stock market, individuals can invest in a risk-free asset paying a constant pre-tax interest rate of r. When τ denotes the income tax rate applicable to labor income, including interest earned in the taxable account, and to withdrawals from the tax-deferred retirement account after attaining retirement age, the gross returns on investments into the risk-free asset in the taxable and the tax-deferred retirement account are given by $R_T = 1 + r(1 - \tau)$ and $R_R = 1 + r$, respectively. Since profits are taxed on a nominal basis, it is important to explicitly account for inflation. We assume a constant inflation rate, which we denote by i.

Homeowners can use their home as a collateral and borrow up to a share, $\delta \in [0, 1]$, of its value, whereas renters are barred from this borrowing opportunity. When B_t denotes the amount invested risk-free ($B_t \geq 0$) or borrowed ($B_t < 0$), it has to hold that:

$$-B_t \leq \delta h_t Q_t H_t. \tag{8}$$

To avoid tax arbitrage opportunities by deducting mortgage interest expenses, while simultaneously earning interest in a tax-deferred retirement account tax free, we assume that borrowers face an interest rate margin, i.e., the after-tax interest burden from borrowing exceeds the after-tax interest earned on investments in the risk-free asset. When R_D denotes the gross after-tax borrowing rate after accounting for the tax deductibility of mortgage interest expenses, it thus has to hold that $R_D > R_R$.

2.4 Optimization problem

In our model, in every period, t, individuals must simultaneously make seven interrelated decisions: 1) how much to spend on nondurable consumption, C_t; 2) how much to contribute to (or, after attaining retirement age, withdraw from) their tax-deferred retirement account, Z_t; 3) the share of retirement savings invested in stocks, $\alpha_{R,t}$; 4) the size of the home they want to live in, Q_t; 5) their homeownership status, h_t; 6) how much of their wealth in the taxable account to invest in stocks, S_t; and, finally, 7) the amount of wealth to invest in bonds or the size of their mortgage, B_t.

In setting up the individual's optimization problem, β denotes the individual's time preference factor. The terminal period in our model is denoted by N. $W_{T,t}$ is the amount of wealth in the taxable account at time t. Retirement savings at time t before (after) contributions/withdrawals are denoted by $W_{R,t-}$ ($W_{R,t+}$). Total wealth, W_t, that effectively belongs to the individual after accounting for the fact that savings in the tax-deferred retirement account are still subject to taxation upon withdrawal, can be expressed as:

$$W_t = W_{R,t-}(1-\tau) + W_{T,t} + h_{t-1}Q_{t-1}H_t. \tag{9}$$

The individual's optimization problem is then:

$$\max_{\{C_t,Z_t,\alpha_{R,t},Q_t,h_t,S_t,B_t\}} V_t(X_t) =$$
$$\left((1-\beta)\left(\left(\frac{C_t}{(1+i)^t}\right)^{1-\theta}\left(Q_t(1+\zeta h_t)\right)^{\theta}\right)^{1-\frac{1}{\psi}} + \beta\left(f(t)\,\mathbb{E}_t\left[V_{t+1}(X_{t+1})^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{10}$$

subject to

$$W_{R,t-} = W_{R,(t-1)+}\left(\alpha_{R,t-1}G_{R,t-1} + (1-\alpha_{R,t-1})R_R\right) \tag{11}$$

$$W_{R,t+} = W_{R,t-} + Z_t \tag{12}$$

$$\begin{aligned} W_{T,t} &= (1-\tau)L_t + S_{t-1}G_{T,t-1} + B_{t-1}\left(\chi_{\{B_{t-1}\geq 0\}}R_T + \chi_{\{B_{t-1}<0\}}R_D\right) \\ &= C_t + S_t + B_t + m(h_t)Q_tH_t + n(Q_t,h_t,h_{t-1}) + h_tQ_tH_t - h_{t-1}Q_{t-1}H_t + Z_t(1-\tau) \end{aligned} \tag{13}$$

$$C_t, Q_t > 0, S_t \geq 0, \alpha_{R,t} \in [0,1], W_{R,t+} \geq 0, \tag{14}$$

and equations (3) to (8), in which $X_t = [t, W_{R,t-}, W_{T,t}, L_t, Q_{t-1}, h_{t-1}]$ is the vector of state variables for this optimization problem. Depending on whether we are investigating the optimal consumption-investment strategy with mandatory contributions to the tax-deferred retirement account or not, the individual's optimization problem is solved subject to the constraint given in equations (1) or (2), respectively. Appendix A explains the more technical details on how we simplify and solve the optimization problem.

2.5 Calibration

This section summarizes the calibration of our model. Individuals enter the life cycle at age 20 ($t = 1$), work until age 65, retire at age 66 ($t_{ret} = 47$), and die at the latest at age 100. Therefore, the accumulation period is 46 years, and the maximum investment horizon is $N = 81$ years. We let one period correspond to one year. Prior to age 100, we use the life table published by the Centers for Disease Control and Prevention (Arias and Xu (2018)) to determine the survival probabilities for male individuals for another year.

To explore the effects of mandatory constant minimum contributions, we compare two different settings: In one setting, individuals do not face any mandatory contribution requirements to a tax-deferred retirement account, i.e., $\xi = 0\%$. In the other setting, they do. The exact level of mandatory minimum contributions imposed differs between countries. Some countries, such as the U.S. and Germany, also split mandatory contributions between employer and employee contributions. While this difference may be important for employees' net salaries, the exact source of funding is mostly an accounting issue and not of primary importance for exploring the economic implications of minimum contribution requirements for housing decisions. If the employer has to bear parts of retirement contributions, it will account for these costs in the wage, it is willing to offer. That is, the employee's net salary after retirement contributions should not rationally depend on whether retirement contributions are legally borne by the employee, the employer, or shared between them. Total mandatory retirement contributions are 12.4% in the U.S., around 15% in Scandinavian countries and as high as 18.6% in Germany. We deliberately stay at the lower range of these values and set mandatory minimum contributions to $\xi = 12.4\%$.[6] The upper bound for contributions to the tax-deferred retirement account is set to $\phi = 100\%$ of labor income in both settings, rendering it essentially non-binding.

The pre-tax risk-free rate is set to $r = 3.9\%$, the average 3–month Treasury Bill rate between 1946 and 2021. The expected annual pre-tax return on the S&P 500 between 1946 and 2021 was 12.6% and its standard deviation $\sigma_S = 16.6\%$. While the historical equity premium was quite high (Mehra and Prescott (1985); Dimson, Marsh, and Staunton (2002)), there are doubts as to whether such a high level will also apply in future periods. As a result, we follow the current consensus, which is around three to four percentage points (Claus and Thomas (2001)) and set the expected return of stocks to $\mu_S = 6.9\%$, implying an equity premium of three percentage points. Inflation is set to its average value between 1946 and 2021, i.e., $i = 3.9\%$, implying a real risk-free rate before tax of zero.

The tax rate on interest income, labor income, and regular withdrawals from the tax-deferred retirement account is set to $\tau = 35\%$ and the tax rate on capital gains is set to $\tau_g = 20\%$, roughly corresponding to the top tax rates in the U.S.

Following Yao and Zhang (2005), we set the housing preference parameter to $\theta = 0.2$, the maximum borrowing rate to $\delta = 80\%$, the annual maintenance rate to $m_o = 1.5\%$, and the transaction costs when acquiring a new owner-occupied home to $f_p = 6\%$. The annual rent

[6] We also computed results for $\xi = 18.6\%$, which lead—as to be expected—to qualitatively identical, though quantitatively larger effects. These results are available from the authors upon request.

rate is $m_r = 6.7\%$, the empirical estimate of Fischer and Stamos (2013). We set the individual's preference for owning over renting to $\zeta = 10\%$, which is in the order of magnitudes chosen in Kiyotaki, Michaelides, and Nikolov (2011) and Fischer and Khorunzhina (2019).

Using the U.S. S&P's CoreLogic Case–Shiller Home Price Index, we estimated the expected nominal return on housing from 1946 to 2021, μ_H, to be 5% and its standard deviation to be 5.8%. Price changes of individual homes are far from perfectly correlated, implying that the House Price Index underestimates the volatility of individual house prices. Case and Shiller (1989) argue that the volatility of individual house prices should be around 15%. Bourassa, Haurin, Haurin, Hoesli, and Sun (2009) provide empirical support. We therefore set the standard deviation of individual homes to $\sigma_H = 15\%$.

To avoid tax arbitrage opportunities as in Marekwica, Schaefer, and Sebastian (2013), the borrowing rate is set to $r_D = 6.4\%$, corresponding to an after-tax borrowing rate of $(1 - \tau)r_D = 4.16\%$, thus exceeding the after-tax risk-free rate in the tax-deferred retirement account of $r = 3.9\%$. Correlation between stock returns and labor income is set to $\rho_{S,L} = 0.047$, correlation between housing return and labor income to $\rho_{H,L} = 0.55$, and correlation between housing and stock returns to $\rho_{H,S} = 0$, which are the empirical estimates from Cocco (2005).

To determine the individual's labor income during working age, we use the labor income process estimated for male high school graduates in Fischer and Koch (2023). The structure of the labor income process is the same as in Cocco, Gomes, and Maenhout (2005) but uses more recent PSID data for estimation. As the goal of our work is to contribute to a better understanding of how contributions to pension systems are best implemented, we require individuals to fully fund their retirement benefits through their own savings. As a result, we do not allow for retirement benefits from other sources and set the replacement ratio to zero, implying that individuals do not receive other income during retirement age besides from their accumulated savings.[7]

Undersaving for retirement is a serious problem for some individuals. To account for this empirical phenomenon, we compute results for both fully rational individuals and individuals that are prone to systematic undersaving. For the latter, we follow Larsen and Munk (2023) and evaluate consumption-investment strategies with a time preference factor of $\beta = 0.96$, but let individuals choose their consumption-investment decisions assuming a time preference

[7]As a robustness check, we also ran our analyses using a replacement ratio of 45%, the value suggested by Larsen and Munk (2023). Setting the replacement ratio greater than zero does not change our results qualitatively but renders it more challenging to isolate the impact of constant minimum contribution requirements on individuals' housing choices.

Table 1: Base case calibration

Description	Symbol	Value	Source
	Preference parameters		
Degree of risk aversion	γ	3	Own choice
Elasticity of intertemporal substitution	ψ	0.15	Own choice
Time preference factor	β	0.96	Own choice
Time preference factor reflecting myopia	β_{irr}	0.85	Larsen and Munk (2023)
	Financial markets		
Expected nominal return on equity	μ_S	6.9%	Historical estimate
Standard deviation of return on equity	σ_S	16.6%	Historical estimate
Inflation	i	3.9%	Historical estimate
Nominal risk-free rate	r	3.9%	Historical estimate
	Housing market		
Housing preference parameter	θ	0.2	Yao and Zhang (2005)
Maximum borrowing rate	δ	80.0%	Yao and Zhang (2005)
Risk-free borrowing rate	r_D	6.4%	Yao and Zhang (2005)
Annual rent rate	m_r	6.7%	Fischer and Stamos (2013)
Annual maintenance rate	m_o	1.5%	Yao and Zhang (2005)
Transaction costs for purchasing home	f_p	6.0%	Yao and Zhang (2005)
Expected nominal return on housing	μ_H	5.0%	Historical estimate
Standard deviation of return on housing	σ_H	15.0%	Case and Shiller (1989)
Preference for owning over renting	ζ	10.0%	Kiyotaki et al. (2011)
	Taxes		
Tax rate on capital gains	τ_g	20%	Own choice
Tax rate on other income	τ	35%	Own choice
	Correlations		
Stock returns and labor income	$\rho_{S,L}$	0.047	Cocco (2005)
Housing return and labor income	$\rho_{H,L}$	0.550	Cocco (2005)
Housing and stock return	$\rho_{H,S}$	0.000	Cocco (2005)

This table summarizes the base case calibration used in our model.

factor of only $\beta_{irr} = 0.85$, leading to systematic undersaving.[8]

While for the fully rational individuals, minimum contribution requirements are an unnecessary constraint, that may hinder them in implementing their otherwise optimal consumption, investment, and housing strategy, for myopic individuals, these contribution requirements may play an important role in securing a sufficient level of savings at retirement age.

The degree of risk aversion is set to $\gamma = 3$. We set the elasticity of intertemporal substitution to $\psi = 0.15$, which is in the range of estimates in Pakoš (2011), Cashin and Unayama (2016), Gayle and Khorunzhina (2018), and Best, Cloyne, Ilzetzki, and Kleven (2020). Table 1 summarizes our base case parameter choice.

3 Results

3.1 Model predictions

It is important to ensure that our simulated model predictions are able to match key features in the data, which is why we begin the presentation of our results by comparing wealth in the Panel Study of Income Dynamics (PSID) data with model-predicted wealth. For our data analyses examining the model fit, we use data from all PSID waves after the financial crisis that we also use for estimating the labor income process, i.e., the PSID waves from 2011 to 2019. A comparison between model predictions and the data is challenging, because Social Security benefits are not part of wealth reported in the data, and there is an extremely high degree of dispersion of wealth in the PSID data. To address these two issues, we proceed as follows.

We add the present value of Social Security benefits to PSID wealth to allow for a meaningful comparison between model predictions and the data. We calculate the present value of Social Security benefits in two steps. First, we calculate the monthly Social Security benefits individuals can expect when retiring at age 65 by following the calculation procedure provided on the U.S. Social Security Administration website.[9] Second, we calculate the present value of the expected benefits with basic actuarial formulas accounting for mortality rates, as provided by the Centers for Disease Control and Prevention (Arias and Xu (2018)) and using our base case interest rate of $r = 3.9\%$. Appendix B outlines the data selection

[8] More technically, we compute optimal consumption-investment decisions using the lower β-value of $\beta_{irr} = 0.85$ and then evaluate welfare using the higher value of $\beta = 0.96$.

[9] https://www.ssa.gov/oact/cola/Benefits.html, last retrieved April 15, 2024.

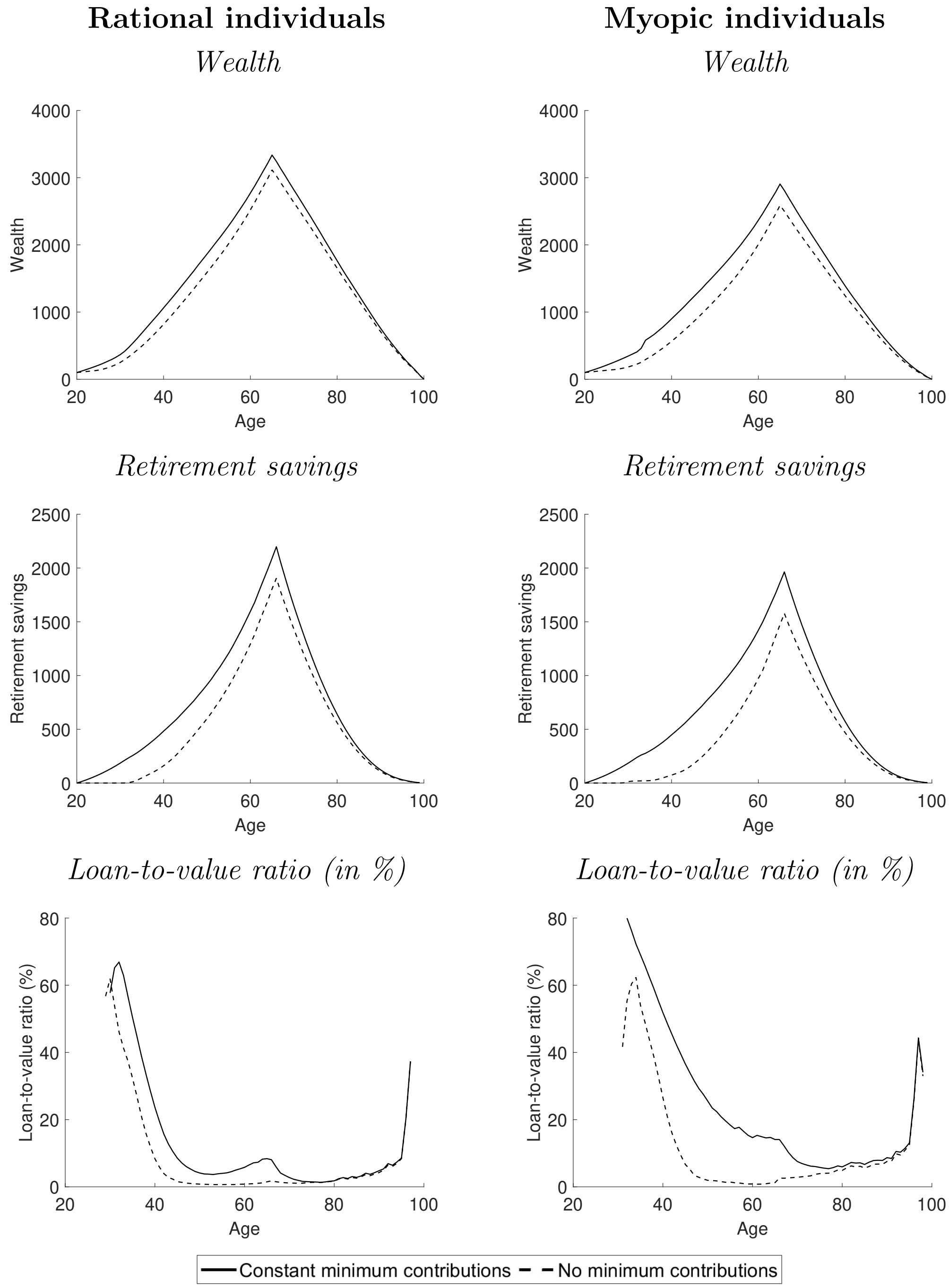
process and the computation of the present value of the Social Security benefits in more detail.

We use median wealth in the data, which reflects that mean wealth is heavily biased upwards by a few outliers. In the data, self-reported median wealth at age 20 is USD 6,781 and grows to USD 31,297 at age 65. The median present value of Social Security benefits at age 65 is USD 191,290, implying a total wealth at age 65 of USD 222,587. Hence, total wealth grows by a factor of about 32.8 from age 20 to 65 in the PSID data. Preference parameters in our model are chosen to achieve a comparable growth of wealth. Specifically, with the choice of parameters introduced in section 2.5, the growth of average wealth in our model from age 20 to 65 is 33.4 for rational individuals and 29.1 for myopic ones, thus matching the growth of wealth in the data very well.

3.2 Implications of minimum contribution requirements

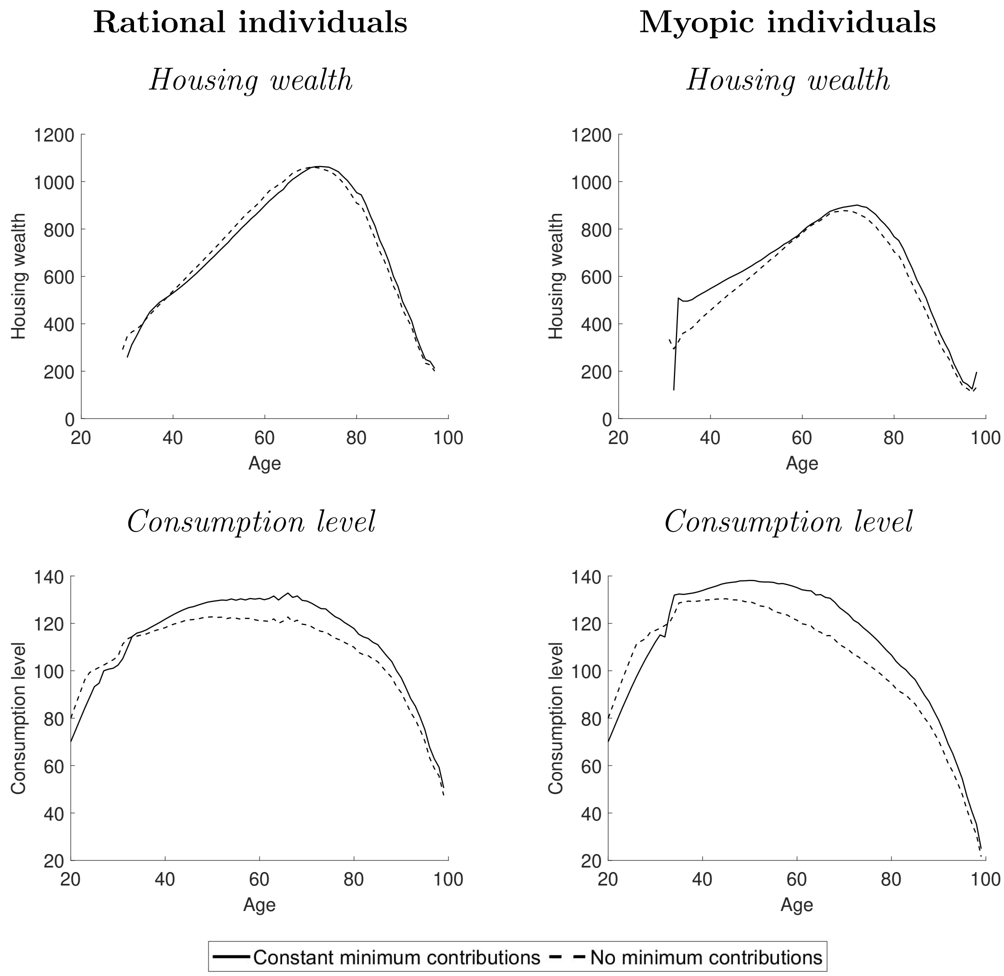
In this section, we explore the implications of mandatory constant minimum contributions for the evolution of the wealth of individuals, their retirement savings, housing market entry, and loan-to-value ratios. Intuitively, mandatory contributions to the tax-deferred retirement account should affect not only the individuals' accumulation of wealth and the location of their savings between the taxable and the tax-deferred retirement account, but they should also affect the rent-versus-own decision, the timing of housing market entry, and the evolution of loan-to-value ratios through two counteracting channels. On the one hand, mandatory contributions reduce the individuals' liquid wealth, thus deferring the age at which individuals have accumulated enough savings for a down payment. Hence, from this liquidity channel, mandatory contributions should defer housing market entry and lead to higher loan-to-value ratios. On the other hand, mandatory contributions should lead to a faster overall accumulation of wealth, enabling individuals to bear possible losses on potential housing investments more easily without running the risk of retirement poverty. Via this wealth channel, mandatory contributions could prepone housing market entry. In this section, we explore, among others, which of these two counteracting channels dominates.

Figure 1: Base case results



This figure depicts the base case results of our model for both, rational (left graphs) and myopic (right graphs) individuals. The upper graphs compare the evolution of wealth over the life cycle, the middle ones of retirement savings, and the lower ones of loan-to-value ratios. The solid lines report results for individuals facing constant minimum contribution requirements to the tax-deferred retirement account, the dashed lines for individuals not facing any contribution requirements. All results are averages from 10,000 simulated paths. In the lower graphs, averages are computed conditional on homeownership and reported for ages with at least 100 observations.

Figure 2: Further base case results



This figure depicts further base case results of our model for both, rational (left graphs) and myopic (right graphs) individuals. The upper graphs compare the evolution of housing wealth over the life cycle and the lower ones of consumption levels. The solid lines report results for individuals facing constant minimum contribution requirements to the tax-deferred retirement account, the dashed lines for individuals not facing any contribution requirements. All results are averages from 10,000 simulated paths. In the upper graphs, averages are computed conditional on homeownership and reported for ages with at least 100 observations.

Figure 1 depicts the evolution of wealth (upper panels), retirement savings (middle panels), and the loan-to-value ratio (lower panels) over the life cycle for both our rational investors (left panels) and our myopic ones (right panels). The solid lines show results for individuals being subject to constant minimum contribution requirements, the dashed lines for individuals that are not. All results reported are averages from 10,000 simulated paths. In the lower graphs, averages are computed conditional on homeownership and reported for ages with at least 100 observations.

From the upper two graphs, mandatory constant minimum contributions increase wealth at retirement age for both rational and myopic individuals. Not surprisingly, myopic individuals reach lower levels of wealth. Yet, the constant minimum contribution requirement leads to an increase of wealth at retirement age of 12.3% relative to the case without minimum contributions, which exceeds its counterpart for rational individuals by about 5.2 percentage points. That is, constant minimum contribution requirements help in bringing wealth at retirement age of myopic individuals closer to the levels reached by rational individuals. As it is important for pension systems to ensure not only a sufficient accumulation of savings for the average individual but also for the poorest individuals, we also calculate increases in wealth due to constant minimum contribution requirements for individuals representing the first percentile of the wealth distribution at retirement age. We find that constant minimum contributions increase total savings by 19.7% for rational individuals and by 23.8% for myopic ones, highlighting that while constant minimum contributions pose unwanted restrictions for rational individuals, they may help the poorest individuals accumulate more savings.

From the middle panels, the increase in wealth is primarily driven by increased retirement savings. When not facing constant contribution requirements, both rational and myopic individuals defer building up retirement savings until their mid-thirties. That is, constant minimum contribution requirements help myopic individuals in building up larger retirement savings at the expense of forcing rational ones to slightly oversave. Again, the increase in retirement savings is higher for myopic individuals with an increase of 24.5% at retirement age compared to 15.4% for rational individuals. Overall, constant minimum contribution requirements reach the goal of increasing myopic individuals' wealth at retirement age—primarily by stipulating a faster build-up of retirement savings.

While higher wealth at retirement age in itself is desirable, ceteris paribus, constant minimum contribution requirements may also come with unintended side effects, particularly when it comes to individuals' housing and mortgage decisions. In the absence of constant minimum contribution requirements, the average age of first-time housing market entry is 30 years for rational investors and 34 years for myopic ones. With constant minimum contribution requirements, the average age of first-time housing market entry is 32 for both types of investors. That is, mandatory constant minimum contribution requirements defer housing market entries of rational investors and prepone housing market entries of myopic ones.

For rational investors, constant minimum contribution requirements render it more difficult to build up sufficient liquid savings for the down payment, thus delaying their housing market entry. Myopic investors on the other hand prepone their housing market entry in the presence of mandatory constant minimum contribution requirements, as they build up wealth at a faster rate in the presence of minimum savings. Overall, constant minimum contribution requirements facilitate earlier housing market entry for myopic individuals, but defer housing market entries of rational individuals.

From the lower two graphs, not only housing market entries, but also loan-to-value ratios of homeowners are heavily affected by constant minimum contribution requirements. For both rational and myopic individuals loan-to-value ratios are substantially higher when they are facing mandatory constant minimum contribution requirements—particularly at younger age. This is consistent with the empirical evidence in Beshears, Choi, Laibson, Madrian, and Skimmyhorn (2022) that automatic enrollment in a retirement plan increases (second) mortgage debt. Without mandatory constant minimum contributions individuals can finance their home with a higher share of equity, thus avoiding the relatively high borrowing rate. Individuals facing minimum contribution requirements, on the other hand, are forced to borrow more for a comparable home. That is, these individuals invest at a relatively low rate in their tax-deferred retirement account and simultaneously borrow at a relatively high rate. In essence, these individuals incur the cost of the interest rate margin by borrowing their own money. From the individual's perspective, it may therefore be more desirable to first repay the relatively expensive mortgage before accumulating savings in the tax-deferred retirement account.

Having explored the impact of mandatory constant minimum contributions on wealth, retirement savings, and loan-to-value ratios, we next ask how mandatory constant minimum contributions affect housing wealth and consumption over the life cycle. For that purpose, Figure 2 depicts the evolution of housing wealth (upper panels) and consumption levels (lower panels) over the life cycle for both our rational individuals (left panels) and our myopic ones (right panels). We find that both groups of individuals accumulate roughly the same level of housing wealth over the life cycle. That is, conditional on homeownership, mandatory constant minimum contribution requirements do not much affect housing wealth. However, constant minimum contributions lead to lower consumption levels in younger years but substantially higher consumption levels in older years. Thus, constant minimum contributions work against the desire for rational individuals to smooth consumption over the life cycle in order to even out the discounted marginal utility. On the other hand, they can be helpful for myopic individuals who lack long-term financial planning skills.

Overall, our results in this section show that constant minimum contribution requirements help myopic individuals in bringing their wealth closer to levels of rational investors—primarily through increased retirement savings. Yet, our results also show that minimum contribution requirements have unintended side effects, namely postponing housing market entries of rational investors, increasing loan-to-value ratios of younger households substantially, and rendering consumption-smoothing over the life cycle more difficult for rational individuals.

4 Alternative retirement savings systems

In this section, we explore alternatives to mandatory constant minimum contribution requirements. In particular, we explore alternatives that should have the potential to alleviate the unintended side effects on housing market entries and loan-to-value ratios while simultaneously ensuring that individuals build up sufficient savings until retirement age. More specifically, we explore three alternative pension systems. First, we explore a system in which individuals still face constant minimum contribution requirements, but are allowed to withdraw from their retirement savings for acquiring homeownership—a feature found in some tax codes around the world. Second, we study a pension system with age-dependent minimum contribution requirements, such as found in Switzerland. Finally, we propose a flexible retirement savings scheme, under which individuals only have to save if they do not live up to an age-dependent total savings target.

4.1 Early withdrawals for home acquisition

Some pension systems around the world, including the German and the Swiss pension system, allow early withdrawals from tax-deferred retirement accounts, if they are used to support the acquisition of a home. Intuitively, the opportunity to withdraw from the tax-deferred retirement savings for acquiring homeownership should help households in meeting the down payment requirement earlier, i.e., enable earlier homeownership and simultaneously allow for a lower loan-to-value ratio.

To explore the implications of such a withdrawal opportunity on individuals' consumption-savings decisions in more detail, we alter the constraint imposed in equation (1). We let $\eta_t \in \{0, 1\}$ be a binary variable that takes value one if the individual acquires a new owner-occupied home at time t and zero otherwise. Then the contribution constraint to the tax-deferred retirement account prior to retirement age, i.e., when $t < t_{ret}$, is

$$Z_t \geq \begin{cases} \xi L_t, & \text{if } \eta_t = 0 \\ \max\left(-W_{R,t-}, -Q_t H_t\right), & \text{if } \eta_t = 1. \end{cases} \tag{15}$$

For our analyses, we solve the optimization problem outlined in section 2.4 now subject to equation (15) instead of equation (1). Allowing individuals to withdraw from their tax-deferred retirement accounts to acquire an owner-occupied home should help them avoid the interest rate margin on mortgage debt.

4.2 Age-dependent minimum contributions

Another way of possibly alleviating the unintended side effects of constant minimum contribution requirements to tax-deferred retirement accounts are age-dependent minimum contributions (see, e.g., Schlafmann, Setty, and Vestman (2024), who advocate age-dependent minimum contributions for Sweden). Intuitively, constant minimum contribution requirements hit younger individuals, that have not been able to build up larger savings hardest and can, as shown in section 3, delay their housing market entry, lead to substantially higher loan-to-value ratios—particularly at younger age—and render consumption-smoothing over the life cycle more difficult. Reducing mandatory minimum contribution requirements at younger age while simultaneously increasing them at older age might dampen these unintended side effects at younger age.

To investigate whether this intuition holds, we alter the constraint imposed in equation (1) as follows:

$$Z_t \geq \xi_{dep,t} L_t, \quad t < t_{ret}, \tag{16}$$

in which $\xi_{dep,t} \in [0\%, 100\%]$ denotes the individual's age-dependent minimum contribution requirement to the tax-deferred retirement account. Following the current contribution requirements of Swiss occupational pension systems (see, e.g., OECD (2021)), we choose[10]

$$\xi_{dep,t} = 0\%\chi_{\{t\in[1,5]\}} + 7\%\chi_{\{t\in[6,15]\}} + 10\%\chi_{\{t\in[16,25]\}} + 15\%\chi_{\{t\in[26,35]\}} + 18\%\chi_{\{t\in[36,46]\}}. \tag{17}$$

That is, prior to attaining age 25, individuals do not have to make any contributions, from

[10] Note that individuals enter the life cycle at age 20, i.e., $t = 1$ corresponds to an individual aged 20 and $t = 46$ corresponds to an individual aged 65.

age 25 to age 34 they have to contribute at least 7% of their labor income, from age 35 to age 44 at least 10%, from age 45 to age 54 at least 15%, and finally from age 55 until retirement age they have to contribute at least 18% to the tax-deferred retirement account.

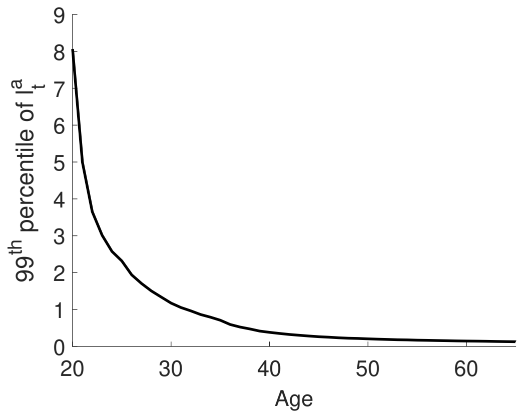
4.3 A flexible retirement savings scheme

In this section, we propose a flexible retirement savings scheme that builds on the intuition that it does not matter whether individuals build up savings for retirement age in the tax-deferred retirement account, specifically labeled as pension savings, or via other means of saving, as long as they build up sufficient total wealth. Under our flexible retirement savings scheme, individuals are therefore only required to save, if they have not built up sufficient total wealth. Individuals that are required to save do not have to build up savings in their tax-deferred retirement accounts, but can also choose to increase their wealth by (partly) repaying a mortgage or by building up wealth in the taxable account. Another positive side effect of our flexible retirement savings scheme is that it provides strong incentives to report all savings correctly, i.e., it renders tax evasion less attractive.

From a legal perspective, it is important to ensure that individuals cannot consume their retirement savings before attaining retirement age to prevent the government from having to sustain their living expenses during retirement. In reality, this could be achieved by requiring individuals to label sufficient savings as "for retirement" each period, which may not be spent. Such a label is traditionally only attached to retirement savings, but can, in the framework of our flexible retirement savings scheme, be moved between different asset classes. For instance, after an appreciation of a home and/or the reduction of mortgage debt, more home equity could be labeled as "for retirement", while wealth in the taxable account could be freed, thus giving the individual more financial flexibility. In practice, this scheme can easily be implemented in countries that have detailed financial information about their citizens, e.g., because of a wealth tax or because they are collecting rich registry data. Examples of such countries include Denmark, Sweden, Switzerland, and the Netherlands. However, the burden of proof of the wealth level could also be imposed on the individuals.

The fundamental idea behind building up savings for retirement is to smooth consumption over the life cycle. To attain this goal, an increase in income should result in an increase in savings, whereas an increase in savings, e.g., due to high realized returns in the past, should decrease the need for saving. Building on this intuition, we advocate an (age-dependent) savings rule that depends on the individual's labor-to-wealth ratio. A high labor-to-wealth ratio indicates that an individual has low savings relative to current income and is more in

Figure 3: Maximum labor-to-wealth ratio in flexible retirement savings scheme



This figure depicts the 99^{th} percentile of the labor-to-wealth ratio after consumption, rents, maintenance, and transaction costs, l_t^a, for the case with mandatory constant minimum contributions, which is used as the age-dependent maximum labor-to-wealth ratio after expenses, $l_{t,MAX}$, in equation (18) in the flexible retirement savings scheme.

danger of not being able to sustain the consumption level at retirement. To nevertheless reach the goal of smoothing consumption over the life cycle, the individual has to save more and consume less.

For every given age, we therefore impose a maximum labor-to-wealth ratio after expenses, $l_{t,MAX}$, to be attained. This maximum labor-to-wealth ratio should decrease with age reflecting that more savings should already have been built up. When W_t is the investor's wealth level at the beginning of period t, already including its earned labor income, L_t, and $E_t = C_t + m\left(h_t\right)Q_tH_t + n\left(Q_t, h_t, h_{t-1}\right)$ are non-durable expenses in period t, i.e., consumption, rents, maintenance, and transaction costs, its labor-to-wealth ratio after expenses in period t is $\frac{L_t}{W_t-E_t}$. Imposing a maximum labor-to-wealth ratio after expenses is then equivalent to imposing a maximum expenses-to-wealth ratio:

$$\frac{L_t}{W_t - E_t} \leq l_{t,MAX} \iff \frac{E_t}{W_t} \leq 1 - \frac{\frac{L_t}{W_t}}{l_{t,MAX}} \tag{18}$$

From equation (18), the maximum expenses-to-wealth ratio decreases in the beginning-of-period labor-to-wealth ratio, $\frac{L_t}{W_t}$, prior to expenses. That is, individuals with high initial labor-to-wealth ratios, $\frac{L_t}{W_t}$, face harsher constraints on their maximum expenses-to-wealth ratios, $\frac{E_t}{W_t}$, than individuals with low initial labor-to-wealth ratios, for whom the expenses constraint is more likely not to become binding.[11] Our flexible retirement savings scheme does, however, not require new savings to be located in a tax-deferred retirement account. If more desirable, additional savings can, for instance, also be used to repay a relatively expensive mortgage before building up other forms of savings.

An important degree of freedom in our flexible retirement savings scheme is the choice and shape of the age-dependent maximum labor-to-wealth ratio, $l_{t,MAX}$, in equation (18). To facilitate a direct comparison with the setting with mandatory constant minimum contributions investigated in section 3, the age-dependent maximum labor-to-wealth ratio, $l_{t,MAX}$, is constructed from the 10,000 simulations on the optimal paths under the constant minimum contribution requirement. Since ensuring the living standards of the poorest individuals is particularly important in pension system design, we compute the age-dependent maximum labor-to-wealth ratio, $l_{t,MAX}$, as the 99^{th} percentile from our 10,000 simulations on the optimal paths under the constant minimum contribution requirement.[12] The evolution of upper bound $l_{t,MAX}$ is depicted in Figure 3.

4.4 Comparison of the three alternative pension systems

Having introduced three alternatives to mandatory constant minimum contributions in sections 4.1 to 4.3, in this section we explore the implications of these alternative pension systems.

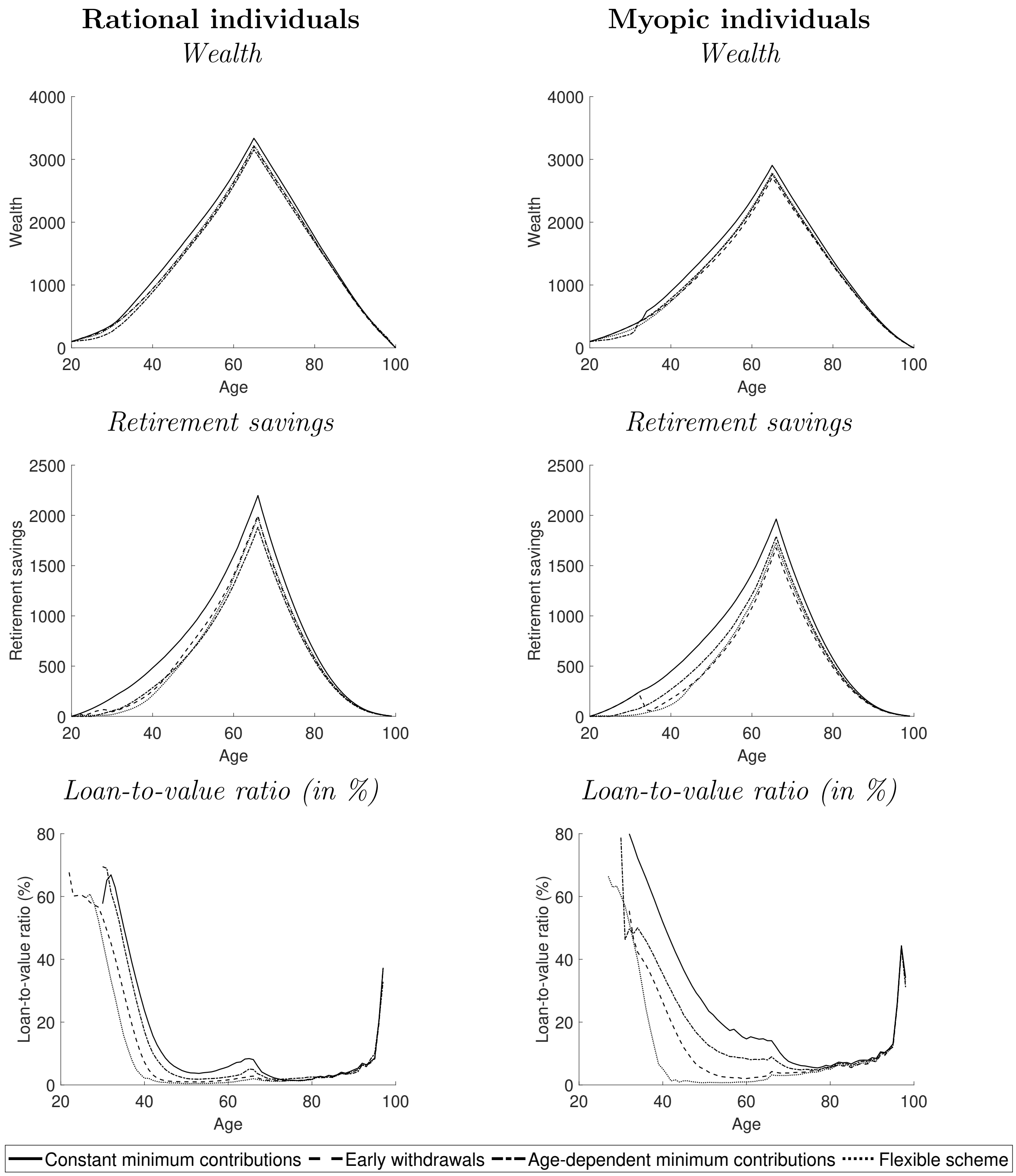
Figure 4 depicts, in a similar fashion to Figure 1, the evolution of wealth (upper graphs), retirement savings (middle graphs), and the loan-to-value ratio of homeowners (lower graphs) over the life cycle for pension systems with a withdrawal opportunity to acquire homeownership from section 4.1 (dashed line), age-dependent minimum contributions from section 4.2 (dash-dotted line), the flexible retirement savings scheme from section 4.3 (dotted line), and the system with mandatory constant minimum contributions (solid line). The left graphs show results for rational investors, the right graphs for myopic ones. All results are averages from 10,000 simulated paths. In the lower graphs, averages are computed conditional on homeownership and only reported for ages with at least 100 observations.

From the upper graphs, the evolution of wealth over the life cycle of the three alternative

[11] Technically, the right hand side of equation (18) can take a negative value—for instance, if an individual faces a dramatic decrease in total wealth. In such (very rare) cases, we set the individual's maximum expenses-to-wealth ratio to 10^{-4}. By restricting the individual's maximum expenses-to-wealth ratio, we are imposing a lower bound on its minimum savings, which may be even negative for individuals with very low labor-to-wealth ratios, i.e., individuals with very low labor-to-wealth ratios may even dissave.

[12] As robustness checks, we also explored settings with other percentiles as upper bound. Our results remain qualitatively the same.

Figure 4: Comparison of alternative pension systems



This figure depicts the evolution of wealth (upper graphs), retirement savings (middle graphs), and the loan-to-value ratio (lower graphs) over the life cycle for the pension systems with a withdrawal opportunity from section 4.1 (dashed line), age-dependent minimum contributions from section 4.2 (dash-dotted line), the flexible retirement savings scheme from section 4.3 (dotted line), and the system with mandatory constant minimum contributions (solid line). The left graphs show results for rational investors, the right graphs for myopic ones. All results are average from 10,000 simulated paths. In the lower graphs, averages are computed conditional on homeownership and reported only for ages with at least 100 observations.

pension systems is very similar to that of the pension system with mandatory constant minimum contributions for both rational and myopic individuals. This is not only true for the average individual as reported in Figure 4 but also for the poorest individuals, i.e., for the individuals whose wealth levels represent the first percentile of our simulated wealth distribution.

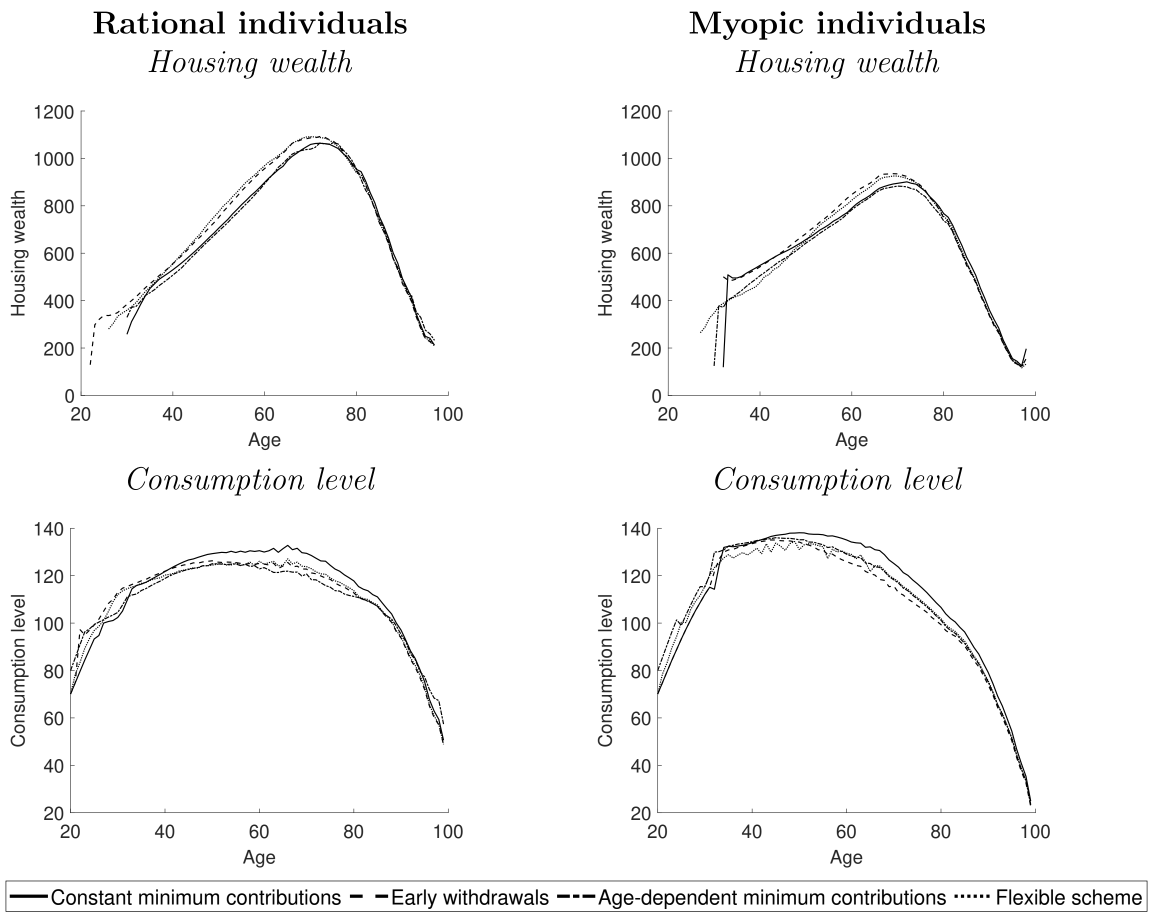
From the middle graphs, it becomes apparent that retirement savings in the tax-deferred retirement account do, however, play a slightly less important role than in the case with mandatory constant minimum contribution requirements. Yet, overall, all three alternative pension systems reach about the same level of total wealth at retirement age as the benchmark with constant minimum contribution requirements, thus allowing for about the same standard of living during retirement age in all four cases. We therefore next ask, whether the three alternative pension systems are potentially associated with advantages over the mandatory constant minimum contribution benchmark.

From the lower two panels of Figure 4, all three alternative pension systems lead to significantly lower loan-to-value ratios at younger age. This effect is particularly pronounced for the flexible retirement savings scheme, but also the pension system with the opportunity for early withdrawals to acquire homeownership still leads to a substantial reduction in the loan-to-value ratio. For instance, at age 40, the average loan-to-value ratio is as high as 24% (52%) under the mandatory constant minimum contribution benchmark, 17% (35%) under age-dependent minimum contributions, 7% (26%) under the withdrawal opportunity, and only 2% (5%) under the flexible scheme for rational (myopic) individuals.

Lower loan-to-value ratios should generally be desirable for the individual as well as on the macro level. On the individual investor's level, the opportunity to operate with a lower loan-to-value ratio is desirable as it relieves the individual from bearing the cost of the interest rate margin it faces when essentially borrowing its own money from the tax-deferred retirement account at an interest margin. On the macro level, lower loan-to-value ratios could increase the stability of the financial system to adverse events, such as a negative shock to house prices.

In addition to reducing loan-to-value ratios, our three alternative pension systems also affect the average age of first-time housing market entry, which is 32 years for both rational and myopic investors under the mandatory constant minimum contribution benchmark. With the opportunity to withdraw early to acquire a home, it decreases to 23 for our rational investors, while it is basically unaffected for the myopic investors for whom the acquisition of a home, i.e., a long-term investment, is less attractive due to their myopic behavior.

Figure 5: Further comparison of alternative pension systems



This figure depicts the evolution of housing wealth (upper graphs) and consumption levels (lower graphs) over the life cycle for the pension systems with a withdrawal opportunity from section 4.1 (dashed line), age-dependent minimum contributions from section 4.2 (dash-dotted line), the flexible retirement savings scheme from section 4.3 (dotted line), and the system with mandatory constant minimum contributions (solid line). The left graphs show results for rational investors, the right graphs for myopic ones. All results are average from 10,000 simulated paths. In the upper graphs, averages are computed conditional on homeownership and reported only for ages with at least 100 observations.

With age-dependent minimum contributions, individuals are facing lower minimum contribution requirements at younger age, thus enabling earlier homeownership, namely at age 31 for rational and age 30 for myopic investors. Unlike the case with early withdrawals, rational investors acquire homeownership slightly later than myopic investors as they account for the increasing minimum contribution requirements in an unbiased fashion in their timing of homeownership.

Under the flexible retirement savings scheme, the average age of first-time housing market entry declines to age 28 for rational and age 30 for myopic individuals. The lower age of first-time housing market entry under the flexible retirement savings scheme makes intuitive economic sense as it allows individuals to build up liquid savings outside the tax-deferred retirement account at a faster rate, thus enabling them to build up sufficient liquid savings for the down payment at an earlier age.

Having explored how the alternative pension systems affect wealth, retirement savings, and loan-to-value-ratios, we next show their impact on housing wealth and consumption levels. For that purpose, Figure 5 depicts the evolution of housing wealth (upper panels) and consumption levels (lower panels) over the life cycle. The left column presents the results for rational individuals and the right column for myopic ones. The upper left panel of Figure 5 depicts the housing wealth of rational individuals over the life cycle and shows that rational individuals with the opportunity to withdraw early invest relatively little in housing in the beginning and only later increase their housing wealth by moving to more valuable houses. We further find that the alternative pension systems with early withdrawals and the flexible scheme lead to a slightly higher accumulation of housing wealth indicating that individuals make use of the flexibility these systems are providing by investing in real estate to accumulate savings.

From the lower panels, consumption-smoothing is most affected by mandatory constant minimum contribution requirements for both rational and myopic individuals. At younger age, consumption levels are lowest and they are highest at older age, again stressing that mandatory constant minimum contribution requirements have the unintended side-effect of constraining younger individuals' consumption opportunities most. All three alternative pension systems allow for more consumption-smoothing over the life cycle.

Given the lower loan-to-value ratios and the opportunity for earlier housing market entry under the three alternative pension systems, it is not surprising that they all lead to welfare gains relative to the benchmark with mandatory constant minimum contributions.

We measure welfare gains in percent of lifetime consumption opportunities and housing services that individuals would have to receive under the pension system with mandatory constant minimum contribution requirements to attain the same level of expected present discounted lifetime utility as an individual trading under one of our alternative pension systems. Welfare gains range from 2.4% for rational and 2.2% for myopic investors under the pension system with the early withdrawal opportunity over 2.6% for rational and 2.1% for myopic investors under the flexible retirement savings scheme to 4.5% and 2.5% for myopic individuals under the pension system with age-dependent minimum contribution requirements. That is, age-dependent minimum contribution requirements lead to the highest welfare gains as the relief from mandatory constant minimum contribution requirements at younger age, when individuals are most liquidity-constrained, is very desirable and harsher contribution requirements at higher age, when individuals have already built up more savings and want to benefit from the tax-exemption of profits in the tax-deferred retirement account are less binding.

While the welfare criterion renders the pension system with age-dependent minimum contributions very desirable, the two other pension systems have their individual advantages as well. Under the pension system with the opportunity to withdraw early for acquiring homeownership, individuals can build up savings in a tax-exempt environment and withdraw them exactly at the point in time when they are needed for acquiring homeownership. The flexible retirement savings scheme leads to the lowest loan-to-value ratios. It is also the only of the alternative pension systems explored that tests for whether individuals have built up sufficient savings before imposing constraints on them.

Overall, all three alternative pension systems introduced in sections 4.1 to 4.3, appear preferable to a pension system with mandatory constant minimum contributions. All of them lead to a similar accumulation of wealth, but lead to lower loan-to-value ratios at younger age, the opportunity of earlier homeownership, and higher welfare. Among the three alternative pension systems, each pension system comes with its own advantages and disadvantages. The pension systems with age-dependent minimum contributions or the early withdrawal on the one hand are easy to implement and already implemented in reality. The flexible retirement savings scheme on the other hand leaves the highest degree of flexibility to individuals that build up sufficient savings, leads to the lowest loan-to-value ratios, could consequently lead to the highest stability of the financial system, and is the only system that tests for whether individuals have built up sufficient savings and thus guarantees that savings goals are reached. However, the flexible retirement savings scheme requires detailed data—not only on individuals' income, but also on their wealth.

5 Conclusion

In this manuscript, we explore the implications of constant minimum contribution requirements to tax-deferred retirement accounts found in many countries around the world in a realistically calibrated life cycle model. Our results show that while constant minimum contribution requirements lead to higher wealth at retirement age, they have unintended side effects of delaying housing market entry, rendering consumption-smoothing over the life cycle more difficult, and leading to substantially higher loan-to-value ratios—particularly at young age. That is, mandatory constant minimum contribution requirements force individuals to pay the higher borrowing rate on their debt, while simultaneously investing at a lower rate in their tax-deferred retirement account. In other words, individuals essentially borrow their own money and face the cost of the interest margin.

We therefore explore three alternatives to constant minimum contribution requirements. First, we explore the implications of withdrawals from the tax-deferred retirement account when acquiring homeownership as found in some countries around the world. Second, we investigate age-dependent minimum contribution requirements, under which contribution rates increase with age. Third, we propose a new flexible retirement savings scheme, which ensures that individuals build up sufficient total savings, but leaves individuals with sufficient savings more flexibility. Our flexible retirement savings scheme therefore only forces individuals to save if they have not built up sufficient total savings. It does, however, not force individuals to build up (illiquid) tax-deferred retirement savings, if, e.g., repaying an expensive mortgage first is more desirable.

All three alternative pension systems lead to similar levels of wealth at retirement age as the pension system with mandatory constant minimum contribution requirements, but enable earlier homeownership and lower loan-to-value ratios, more consumption-smoothing over the life cycle, and, ultimately welfare gains. All three alternative systems have their individual advantages and disadvantages. While the pension system with age-dependent mandatory contribution requirements leads to the highest average welfare gains, the pension system with early withdrawals allows individuals to build up wealth in a tax-exempt environment until these savings are needed for acquiring homeownership. The flexible scheme is the only scheme that does not impose any constraint on individuals that build up sufficient savings, and it also leads to the lowest loan-to-value ratios; thereby also potentially stabilizing the financial system as a whole, at the expense of requiring information about individuals' wealth for its implementation. This may render it more difficult to implement for countries that do not impose a wealth tax or otherwise have detailed information about their inhabitants' wealth.

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Appendix A: Solution of optimization problem

We simplify our optimization by normalizing with aggregate wealth, W_t, that effectively belongs to the individual and does not fall to tax authorities upon withdrawal from the tax-deferred retirement account. Hence, in the normalized optimization problem, $c_t = \frac{C_t}{W_t}$ and $q_t = \frac{Q_t H_t}{W_t}$ are the decision variables that affect utility from immediate consumption of both non-durable and durable consumption. Defining $V_t\left(X_t\right) = v_t\left(x_t\right) \frac{W_t/(1+i)^t}{\left(H_t/(1+i)^t\right)^\theta}$, the individual's optimization problem can be rewritten as:

$$\max_{\{c_t,z_t,s_t,b_t,\alpha_{R,t},q_t,h_t\}} v_t\left(x_t\right) =$$

$$\left(\left(1-\beta\right)\left(c_t^{1-\theta}\left(q_t\left(1+\zeta h_t\right)\right)^{\theta}\right)^{1-\frac{1}{\psi}} + \beta\left(f\left(t\right)\mathbb{E}_t\left[\left(v_{t+1}\left(x_{t+1}\right)\frac{\frac{W_{t+1}}{W_t(1+i)}}{\left(\frac{H_{t+1}}{H_t(1+i)}\right)^{\theta}}\right)^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{A.1}$$

subject to

$$w_{R,t-} = w_{R,(t-1)+}\left(\alpha_{R,t-1}G_{R,t-1} + \left(1-\alpha_{R,t-1}\right)R_R\right) \tag{A.2}$$

$$w_{R,t+} = w_{R,t-} + z_t \tag{A.3}$$

$$\begin{aligned} w_{T,t} &= 1 - w_{R,t-} - (1-\delta)h_{t-1}\frac{Q_{t-1}H_t}{W_t}, \\ &= c_t + s_t + b_t + m\left(h_t\right)q_t + f_p q_t\left(h_t(1-h_t) + h_t h_{t-1}\chi_{\left\{\frac{Q_{t-1}H_t}{W_t}\neq q_t\right\}}\right) + h_t q_t \\ &\quad - h_{t-1}\frac{Q_{t-1}H_t}{W_t} + z_t(1-\tau) \end{aligned} \tag{A.4}$$

$$c_t, q_t > 0, s_t \geq 0, \alpha_{R,t-1} \in [0,1], w_{R,t+} \geq 0, \tag{A.5}$$

in which $w_{R,t-} = \frac{W_{R,t-}}{W_t}$, $w_{R,t+} = \frac{W_{R,t+}}{W_t}$, $w_{T,t} = \frac{W_{T,t}}{W_t}$, $z_t = \frac{Z_t}{W_t}$, $b_t = \frac{B_t}{W_t}$, and $s_t = \frac{S_t}{W_t}$. The new vector of state variables is

$$x_t = \left[t, w_{R,t-}, \frac{L_t}{W_t}, \frac{Q_{t-1}H_t}{W_t}, h_{t-1}\right]. \tag{A.6}$$

We discretize the continuous state variables. The expectation in equation (A.1) is computed using Gaussian quadrature. Parallel computing is used to expedite our computations.

Appendix B: PSID data and present value of Social Security benefits

This appendix explains in more detail, which PSID data we use in our study and how we calculate the present value of Social Security payments (SSP), individuals can expect to receive when working until the age of 65, and being retired at the age of 66.

For our data analyses, we use the biannually available PSID data after the financial crisis that we also use for estimating the labor income process, i.e., PSID data from 2011, 2013, 2015, 2017, and 2019. We only consider heads of households (henceforth, individuals) who are single and male high-school graduates to ensure that we only use the wealth of the same individuals whose labor income process we consider. Furthermore, we adjust total wealth and labor income for inflation using the consumer price index.

To calculate the growth of median wealth, we proceed as follows: First, we restrict the inflation-adjusted total wealth of individuals to be positive. Second, we construct two subsamples, one containing all individuals aged 20 and one containing all individuals aged 65. Third, we calculate the median of the total wealth of individuals for the respective subsamples. We find that the median wealth of individuals aged 20 is USD 6,781 and for those aged 65 it is USD 31,297.

To calculate the present value of Social Security benefits for individuals having the same demographic characteristics and working until the age of 65 and being retired at age 66 we proceed as follows: First, we use inflation-adjusted labor income to calculate the average indexed monthly earnings (AIME). We obtain the AIME by calculating the median earnings for each age from 26 to 60, taking the mean of the median earnings, and dividing this value by 12. Our result is USD 2,250.

Second, we determine the basic Social Security retirement benefit when retiring at age 62, the earliest age at which an individual is generally eligible for Social Security retirement benefits, by calculating the so-called primary insurance account (PIA): For 2019, the PIA is the sum of 90% of the first USD 926 of the AIME, 32% of the AIME exceeding USD 926 but below or equal to USD 5,785, and 15% of the AIME exceeding USD 5,785. Therefore, our PIA is USD 1,257.

Third, as the individuals in our model are retired at age 66 and not already at age 62, we consider cost-of-living adjustments (COLA). We assume the annual COLA to be 1.69%, which is the mean of all COLAs between 2011 and 2019. When taking COLA into account, the PIA for individuals retired at age 66 is USD 1,344. Based on these values, we compute a replacement ratio in the data of: $\frac{1{,}344}{2{,}250} = 59.7\%$.

Fourth, to calculate the present value of SSP, PV_{SSP}, we assume an interest rate of 3.9% (the same as in our model) and use the following formula:

$$PV_{SSP} = \text{average labor income} \cdot \text{replacement ratio} \cdot \sum_{i=1}^{\omega-66} v^{i-1} \cdot_i p_{66}, \tag{A.7}$$

in which ω denotes the technical maximum life time assumed in the life table (100 years), v the discount factor, and ${}_ip_{66}$ the probability that a person aged 66 survives the next i years. Our result for the present value of the annuity is USD 191,290. We do not consider any taxes as Social Security income is (up to certain limits) tax free in the U.S. When adding up the individual's median total wealth right before retirement, i.e., for individuals aged 65, and the present value of SSP, we observe that median total wealth grows by a factor of 32.8 from age 20 to 65 .