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Abstract We study inflation hedging from the perspective of a pension fund. Retired participants in the fund have a demand for inflation protection but face a thin market for inflation-linked bonds. Young participants in the fund have their human capital as a natural inflation hedge. The young may be willing to offer some protection to the old, and thus contribute to complete the market, depending on the price at which the inflation risk is internally traded. Our analysis shows conditions such that a mutually beneficial internal trade exists. The price of the internal transfer will, however, generally deviate from the market price of the inflation-linked bond. Using data calibrated to a long history of inflation, interest rates and stock returns, we find that the internal real interest rate is likely to deviate from the market real interest rate.

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1. INTRODUCTION

Pension fund investors are exposed to traded and non-traded risk factors. One major non-traded risk factor that has been in focus since the end of 2021 is inflation since it erodes the purchasing power of a stable nominal pension. For a long-term investor, who is saving and investing for a retirement income, inflation risk exposure of wealth varies with age. When they are young, investors have a large share of human capital, which has a real value that has a low correlation with inflation. Hence the young have a natural hedge against inflation. When they get older, human capital decreases, while financial wealth, largely invested in nominal bonds, increases. With nominal investments, the exposure to inflation risk thus increases over time. This heterogeneity in exposure to inflation is documented, e.g., in Doepke and Schneider (2006).

Hedging inflation risk requires real assets such as inflation-linked bonds (ILB) or inflation swaps. When ILBs would exist in ample supply, investors could invest in these securities and hedge part of their inflation risk. When the market for ILBs is thin and illiquid, investors cannot implement such a hedge on a large scale. That is why we call inflation a 'non-traded' risk factor. In theory, the human capital of young investors may provide additional inflation hedging. However, neither is human capital tradeable nor can young investors write inflation derivatives. To resolve this market incompleteness, pension funds can bring young and old together and forge a mutually beneficial deal, where the old get additional inflation protection and the young receive the insurance premium. Many investors save for retirement within a pension fund that invests on behalf of participants of all ages, from young to old. The pension fund holds a collective portfolio and allocates the returns among its participants.

We look at the merits and problems of an internal insurance market within the pension fund, where the young insure the inflation risk of the old at a premium. Internally the fund can create an inflation derivative. It pays inflation to the old in return for a fixed premium to the young. We aim to determine an insurance premium and amount of hedging that benefits both young and old, and that takes into account a possible outside option of buying the insurance in the capital market. In short, we study partially segmented markets. Inflation-linked assets are available to all investors, in or out of pension funds, in the financial markets. In addition, pension funds can create an internal market where young and old can trade their non-tradable human capital for inflation protection of the old.

The pension fund aims to provide a stable pension to its participants. In economic terms stable should be interpreted in real terms. Given positive average inflation the fund could define the annuity payout to deliver a cashflow stream that grows at the same rate as expected inflation. This accounts for stable inflation, not for inflation risk. It is neither a hedge against unexpected inflation nor changes in expected inflation. The fund could also offer a variable

annuity based on exposure to equity. It will then adjust the annuity payout if equity excess returns are positive. With sufficient risk exposure, this could lead to a growth in payouts that is equal to expected inflation. But it involves a lot of risk, since equity and inflation are not highly correlated, at least not over short horizons.

Our analysis of an internal insurance market is motivated by the new Dutch pension contract. A Dutch pension fund holds a collective portfolio and allocates the returns over all participants. In what is called the solidarity contract (SPR = 'solidaire premieregeling' in Dutch), the return for every participant is split into a hedge return and an excess return. The hedge return has been designed to hedge against interest rate risk. The hedge return should provide the returns that can deliver the nominally projected payouts to the participants.[1] In the current law, this is the nominal yield curve. The real yield curve is currently not allowed as it is perceived as less reliable due to the illiquidity and thinness of the market for Inflation Linked Bonds (ILB's). After providing the hedge returns, the remaining excess return is allocated to all participants with age-dependent exposures.[2] The pension fund does not need to hold the assets that generate the hedge return in its collective portfolio. The pension fund is allowed to transfer risks between participants but must use market prices when doing so. This means that in case the fund decides to transfer inflation risk from old participants to the young it must use the prices implied by the traded ILB's to calculate the premium.

Our analysis investigates the impact of the last rules, i.e., using the ILB's to compute transfer prices, on the welfare of the pension fund participants. It is not clear that the market price would lead to a Pareto improvement since the market is thin and not all investors participate in this market. This discussion touches upon financial fairness, which dictates that any transfer within a fund has to be at market prices. Financially fair transfers leave everyone with the same monetary value but still allow potentially undesirable risk shifting. Our analysis focuses on welfare improvements. That criterion is vulnerable to the criticism that it is not objective because we don't know the agent's utility function. But that critique would still apply to financially fair contracts, where the welfare gains still depend on preferences. We analyze the setup in which the price at which insurance against inflation risk should be done to make the investors better off.

To answer the question on the welfare impact of internal insurance markets, we study the following market setups

[1] For example, if the fund projects it will pay €1 in n years from now to a retiree, it can (hypothetically) buy an n-year discount bond that will guarantee this cash flow.

[2] It is an excess return since initially, all participants get 100% hedge returns. In the second stage the old may have a low exposure to the excess return — such that their pension payout is relatively stable —, and the young may even have an exposure above 100% to the excess return. This is the official description of the system, but one can rewrite the portfolio in the more familiar academic format with x% in a hedge portfolio and $100 - x\%$ in a speculative portfolio.

1. Incomplete market: The inflation asset is traded on the general capital market but it is not traded by the pension fund participants. That is the current setup. It establishes an equilibrium price for the inflation-linked asset that is determined only by the capital market participants. This is the benchmark setup.

2. Complete market: All assets are freely traded by all investors. This setup establishes the equilibrium price and traded quantities in an unrestricted capital market. To implement this market, the pension fund can trade in the capital market on behalf of the pension fund investors. We assume that there are no information or agency problems.

3. Policymaker imposed hedging in the internal market: The pension fund manager imposes some amount of hedging at some price. At this price, the old pension fund participants buy insurance from the young participants, which is implemented via a sharing rule enforced by the person fund. The internal market has no relation to the capital market in which the capital market participants trade the inflation-linked asset. When the internal price is set equal to the price from the capital market, this setup is the policy proposal by the Dutch Central Bank. However, the price might also be set at a different level. In the latter case, we deviate from the current Dutch pension rule.

4. The inflation asset is traded independently on the capital market and on the internal pension fund market. We get two equilibrium prices for the asset since there is no cross-trading allowed.

Our main findings are as follows. For the internal market, the choice of the internal price at which the mutual insurance between pension fund participants takes place is crucial. Using the price from the capital market in the internal market is not necessarily optimal. At this price, some investor groups may suffer a utility loss due to the introduction of the internal market. And even if all investors improve in terms of utility, they might not all be happy with the amount of insurance. The reason is that the group of capital market investors, which set the price in the capital market, and the group of pension fund investors, which have to trade at this price, are heterogeneous. If they are "too different", part of the pension fund investors will suffer a utility loss. There is an equilibrium price in the internal market. This price makes all investors better off (intuitive proof: going back to their payoff from the incomplete market, in which they have a zero position in the inflation-linked asset, is possible). At this price, no investor wants to change the amount of insurance. The internal market and its rules are accepted by the pension fund investors. In the complete market, all pension fund investors are better off than in the incomplete market. The intuitive proof is again that going back to a position of zero in inflation-linked assets is viable. Risk sharing now also takes place among pension fund investors and capital market investors. When comparing the complete market and the internal market with equilibrium prices and amounts, there is not necessarily

a clear ranking. It might be the case that the complete market Pareto dominates the internal market, but it may also be the case that there is no Pareto dominance. In the latter case, one would need a social planner who assigns weights to the pension fund investors to rank the two markets.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model setup. The results are discussed in Section 5. Section 6 provides the conclusion.

2. LITERATURE

Risk sharing within a pension fund has been analysed before. For example, Gollier (2008) calculates the welfare gains from intergenerational risk-sharing. Our analysis differs in the level of heterogeneity of market participants and in the setup of the financial markets. We have heterogeneity that goes beyond age and takes into account the different effects of inflation shocks on the financial wealth of investors. Our risk sharing is intra-generational, not inter-generational. In our model generations that are alive at the same time can trade. We do not consider the case of investing on behalf of distant generations that are not yet born.

Chen, Beetsma, and Van Wijnbergen (2023) explicitly consider sharing inflation risks in the absence of market instruments. Their model has two overlapping generations. Human capital is fully indexed by the price level, so free of inflation risk just as in our model. With just two generations they calibrate all parameters such that that a period comprises 30 years, which means each generation has 30 years of working age and 30 years of retirement.

Balter, De Jong, and Pelsser (2020) provide a review of optimal risk sharing in pension plans. Intra-generational risk sharing can only add value when some risks are non-traded. Inter-generational risk sharing leads to welfare improvement since a pension fund ('social planner') can pre-invest contributions of future participants before they enter the funds. While the scheme is welfare-improving ex-ante, the risk-sharing introduces the political risk that ex-post the future entrants will refuse to enter if the fund has experienced poor returns. In some cases, this continuity risk can be mitigated (or eliminated) by reducing the size of the 'pre-investment'. But the smaller the pre-investment, of course, the lower the welfare gains.

Many authors impose that risk-sharing should be financially fair. Bao, Ponds, and Schumacher (2016), for example, define a contract as *financially fair* if the market values of the risk positions before and after risk sharing are equal. This would imply that any risk shifting in the pension fund should be at the observed market prices.

3. MODEL

We consider a discrete-time model with one period and two points in time: now ($t = 0$) and the future ($t = 1$). The financial market contains three assets, M, I and B. First, the market asset M is always traded. It has an uncertain payoff $\tilde{D}_M$. The price of the market asset is set to $P_M = 1$, so $\tilde{D}_M$ is also its gross return. By setting the price instead of determining the equilibrium price, we assume an infinitely elastic supply of the market asset. Second, asset I is an inflation-linked bond (ILB), which may be traded or non-traded. It has payoffs $\tilde{D}_I$ equal to the uncertain price level next period. The price of the ILB is P_I, which needs to be determined in equilibrium. This then also gives the real risk-free rate as $1/P_I$. The return of the ILB has two components, realised inflation and real risk-free rate. In log terms, it holds that

$$\log \frac{\tilde{D}_I}{P_I} = \log \tilde{D}_I + \log \frac{1}{P_I}.$$

Later on, we will consider different choices for the price P_I. If markets are complete, there is a unique price. Otherwise, markets are segmented and there are two equilibria: the equilibrium price in the capital market (where capital market investors trade) and one maybe different price in the internal market (where pension fund investors have to trade). The final asset is a nominal bond B, which has a risk-free exogenous gross return R_f and is always traded. By exogenously setting the risk-free rate, we assume an infinitely elastic supply of the risk-free asset.

In the base case, the payoffs of the two risky assets are independent of each other, and the probability for each of the four states (each of the two assets can see an up- or a down-state) is 0.25. A tilde indicates a random variable with realisation in the second period ($t = 1$).

Investor i has initial wealth W_i. Future real wealth $\tilde{W}_i$ is a function of the asset holdings for given prices P_M and P_I as well as inflation. We denote the number of shares in assets M and I as $x_{i,M}$ and $x_{i,I}$. Future real wealth is then given by

$$\tilde{W}_i = W_i R_f + x_{i,M}\left(\tilde{D}_M - P_M R_f\right) + x_{i,I}\left(\tilde{D}_I - P_I R_f\right) + y_i(\tilde{D}_I - 1), \tag{1}$$

There may be restrictions on the position in asset I, including zero positions in I for some investors (when pension fund investors do not participate in the market for inflation risk insurance) or exogenously given positions in asset I (when pension funds choose the positions on behalf of their participants in an internal market).

We use a reduced form setup to capture inflation risk. The overall exposure of investor i to inflation risk is described by y_i; it is multiplied by the net inflation rate $\tilde{D}_I - 1$. Usually, y_i is negative. To motivate Equation (1), we can start with the real wealth of the investor after

he has decided on his position in the market. Regressing his real wealth $\tilde{W}$ at time $t = 1$ on $\tilde{D}_M$ and $\tilde{D}_I$ gives

$$\tilde{W} = \alpha_0 + \beta_M \tilde{D}_M + \beta_I \tilde{D}_I + \tilde{\epsilon}_i \tag{2}$$

Rewriting give

$$\tilde{W} = \underbrace{\alpha_0 + \beta_M R_f + \beta_I}_{=:WR_f} + \beta_M \left(\tilde{D}_M - R_f\right) + \beta_I \left(\tilde{D}_I - 1\right) + \tilde{\epsilon}_i \tag{3}$$

We can interpret the real wealth at time $t = 1$ as resulting from investing the (implicitly defined) initial wealth W into the risk-free asset with nominal return R_f and buying β_I units of the market with nominal payoff $\tilde{D}_M$; β_I gives the exposure to inflation risk which we denote by y_I. The error term is the idiosyncratic component of the investor's wealth, which we will ignore in the following.

For a more economic explanation, assume that the financial wealth of the investor is $W_F \tilde{R}_F$ where $\tilde{R}_F$ is the return on financial wealth. With 1 as the consumer price index in $t = 0$ and $\tilde{D}_I$ as the consumer price index in $t = 1$, the inflation rate is given by $\tilde{D}_I - 1$. The real wealth is then

$$W_F \tilde{R}_F \cdot \frac{1}{\tilde{D}_I} \approx W_F \tilde{R}_F - W_F \left(\tilde{D}_I - 1\right) \tag{4}$$

Financial wealth thus gives the investor an exposure to the inflation rate which equals the negative of the initial wealth. Now assume that aggregate wealth can be decomposed into human capital and financial wealth. Furthermore, assume that human capital has a zero exposure to inflation risk. The exposure of aggregate wealth to inflation risk is then given by the initial financial wealth, which is now smaller than aggregate initial wealth W_i.

Each investor solves an optimal asset allocation problem. Given the prices P_M and P_I, the investor chooses the portfolio composition such as to maximize his expected utility of terminal wealth $\tilde{W}$. We assume CRRA-preferences with relative risk aversion γ. Risk aversion may differ among the different types of investors. While the position in the market is not restricted at all, the position in the inflation-linked asset may be restricted to zero (for pension fund investors in the incomplete market) or to some exogenously given amount (for pension fund investors in the internal market).

There are four different investors. Investors 1 and 2 are participants of the pension fund; we call them *Old* and *Young* pension fund participants in the following. They differ in their exposure to inflation risk. Young investors have small financial wealth and large human capital. Their exposure to inflation risk is thus slightly negative. Old investors have large financial wealth and small or even zero human capital. Their exposure to inflation risk is

thus equal to their initial wealth. Investors 3 and 4 are more sophisticated capital market participants. Besides the market, which is traded by all investors, the capital market investors can also trade the inflation-linked asset I.

The next step consists of finding the equilibrium price P_I to clear the market for ILB's. The price P_I (at the capital market) has to be chosen such that

$$\sum_i x_{i,I} = 0 \tag{5}$$

where the summation is done over all investors who participate in the capital market for the ILB. If there is an internal market, the (given or optimally chosen) internal exposures $\hat{x}_{i,I}$ also have to add up to zero, *i.e.* $x_{young,I} + x_{old,I} = 0$.

In terms of the market setup, we consider several cases described in Section 1, namely the incomplete market, the complete market, and two versions of an internal market. In the incomplete market case, only the capital market investors can trade the ILB and their demand and supply determines the price P_I. In the complete market, all investors trade the ILB with the equilibrium price being determined by the aggregate demand and supply of all investors, young, old, and capital market investors. If the size of the capital market investors is not infinitely large compared to the pension participants, this will generally increase demand, therefore increasing P_I and lowering the real interest rates. For the internal market, we have two versions. In the first version, the positions of the pension fund investors and the price in the internal market are fixed by the pension fund. In principle, the pension fund is completely free in its choice. It might use the price at which the capital market investors trade the ILB in the capital market, but it is by no means forced to do so. It might set the positions such that the resulting exposures of the pension fund investors to inflation risk are equal in the end, but again, it is by no means forced to do so. In the second case, the pension fund investors choose their positions in the internal market optimally, and the price in the internal market follows from market clearing. Note that trading in the internal market involves the pension fund participants only. Capital market participants trade the ILB in the capital market at a price that differs from the one in the internal market.

We now turn to the solution method. All investors can freely choose their position in the market portfolio M. Their position $x_{i,M}$ follows from the first order condition

$$0 = \mathrm{E}\left[\tilde{W}_i^{-\gamma}\left(\tilde{D}_M - P_M R_f\right)\right]. \tag{6}$$

The expectation is taken under the physical probability measure. The supply of M is infinitely elastic, and all demand will be absorbed at the exogenous price $P_M = 1$. We can therefore prespecify the expected rate of return of the market as well as the risk-free rate.

For those investors who can freely trade the inflation-linked asset I, the position $x_{i,I}$ follows from the first order condition

$$0 = \mathrm{E}\left[\tilde{W}_i^{-\gamma}\left(\tilde{D}_I - P_I R_f\right)\right]. \tag{7}$$

In the incomplete market, the position in asset I is fixed at zero for pension fund investors. If there is an internal capital market within the pension fund, the position is exogenously fixed at $x_{1,I}$ and $x_{2,I}$ by the pension fund manager (case 1) or optimally chosen by the pension fund investors (case 2).

The price of asset I depends on the market we consider. For the capital market investors, the price is always equal to the equilibrium price in the capital market. For the pension fund investors, the price is equal to the equilibrium price in the capital market (complete market), to an exogenously fixed price (internal market with price given by pension fund manager), or to the equilibrium price in the internal market where only the pension fund investors trade with each other (internal market with equilibrium price).

Given the asset demand of all investors, we equal demand and supply to determine the equilibrium price of asset I. The inflation-linked asset is in zero net supply.

We rely on a stylized model to concentrate on our main point, the impact of how pension fund investors can trade inflation risk insurance. We thus e.g. ignore problems from life-cycle asset allocation as the pricing of human capital and intermediate consumption and savings, but also agency issues when pension fund investors delegate trading on their behalf to the pension fund. Furthermore, we rely on a one-period model, which is nevertheless sufficient to make all our points.

4. CALIBRATION OF THE TREE MODEL

We use data from CRSP to calibrate the model to the data. For the market return we rely on the total return of the CRSP all-stock-index. Inflation is measured using the CPI, and the 90-day t-bill returns are used for the risk-free rate. We collect annual data from CRSP, the sample period is from 1950 to 2022.

Since we deal with a pension savings problem, we consider a time horizon of 20 years in the empirical analysis. In the model, we will work later on with a time horizon of one year, since annualized numbers are easier to put into perspective. We will thus annualize the empirical moments and also discuss the results in annual terms.

Figure 1 gives the 20-year cumulative log returns for the stock market ("vwretdLog20"), the 90-day t-bill ("t90retLog20"), and the CPI ("cpiretLog20") as a function of the point in

time when the twenty year period ends. Over this long period, there is quite some variation in particular in the stock market returns. In our sample period, the stock market has always outperformed the risk-free asset. A comparison of the risk-free asset and the CPI shows that the realized real risk-free rate has been positive most of the time with the exception of the 20-year periods ending in the last few years. This can be attributed to negative interest rates from after the financial crisis and during the pandemic as well as to high inflation in 2022.

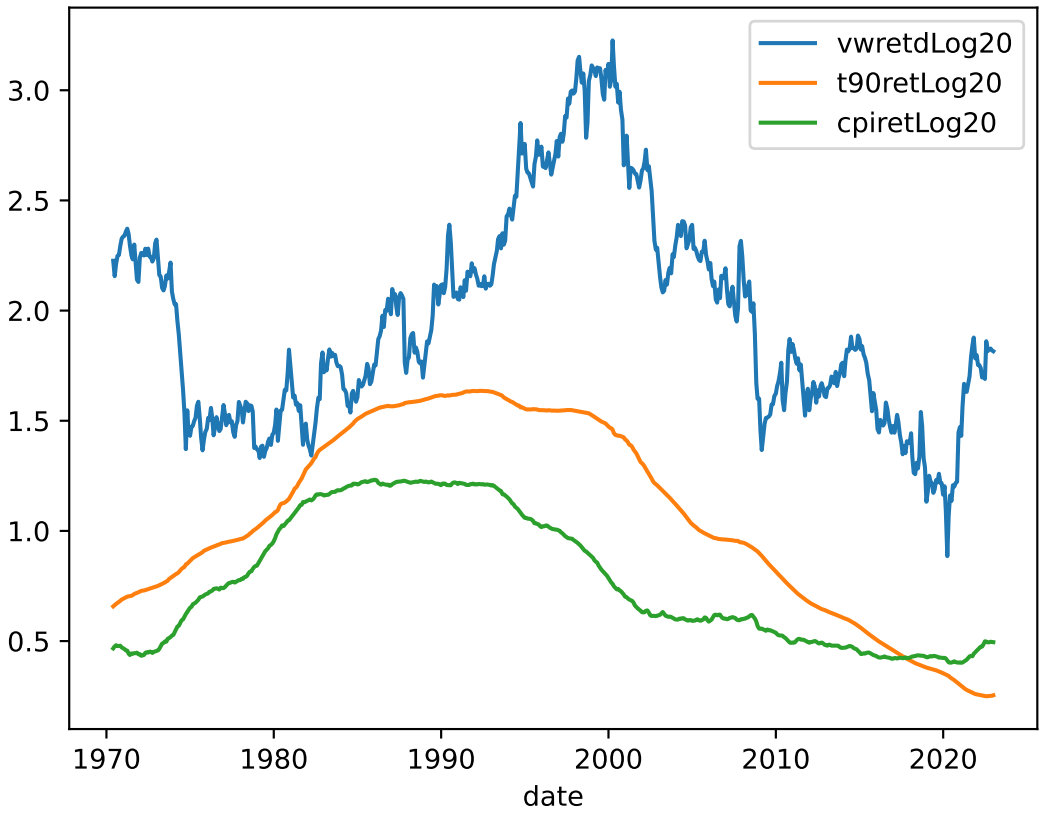

Figure 1. Data for Calibration
This figure shows the returns of the CRSP stock market index, the 90-day t-bill, and the CPI. Returns are given over a twenty-year horizon which ends at the date given on the x-axis.

The average cumulative log return for each series is 196% for the stock market, 106% for the t-bill, and 77% for the CPI. Annualizing these data results in 9.8% per year for the stock market, 5.3% for the t-bill, and 3.9% for the CPI. The volatilities of the 20 year returns are 48.7% for the stock market, 43.2% for the t-bill, and 29.9% for the CPPI. We annualize these numbers, setting the autocorrelation to zero, which results in annual volatilities of 9.8%, 5.3%, and 3.9% for each series, respectively.

The correlation between the stock market and the CPI varies over time. This is shown in Figue 2, which shows the rolling correlations between the 20-year stock market return

and the 20-year inflation rate. The correlations are computed using a 10 year period. This finding is in line with Cieslak and Pflueger (2023), who show that the sign of the correlation has changed around 2000 from negative to positive. They distinguish between good and bad inflation: good inflation comes with a negative inflation risk premium in bonds, and bad inflation comes with a positive inflation risk premium in bonds.

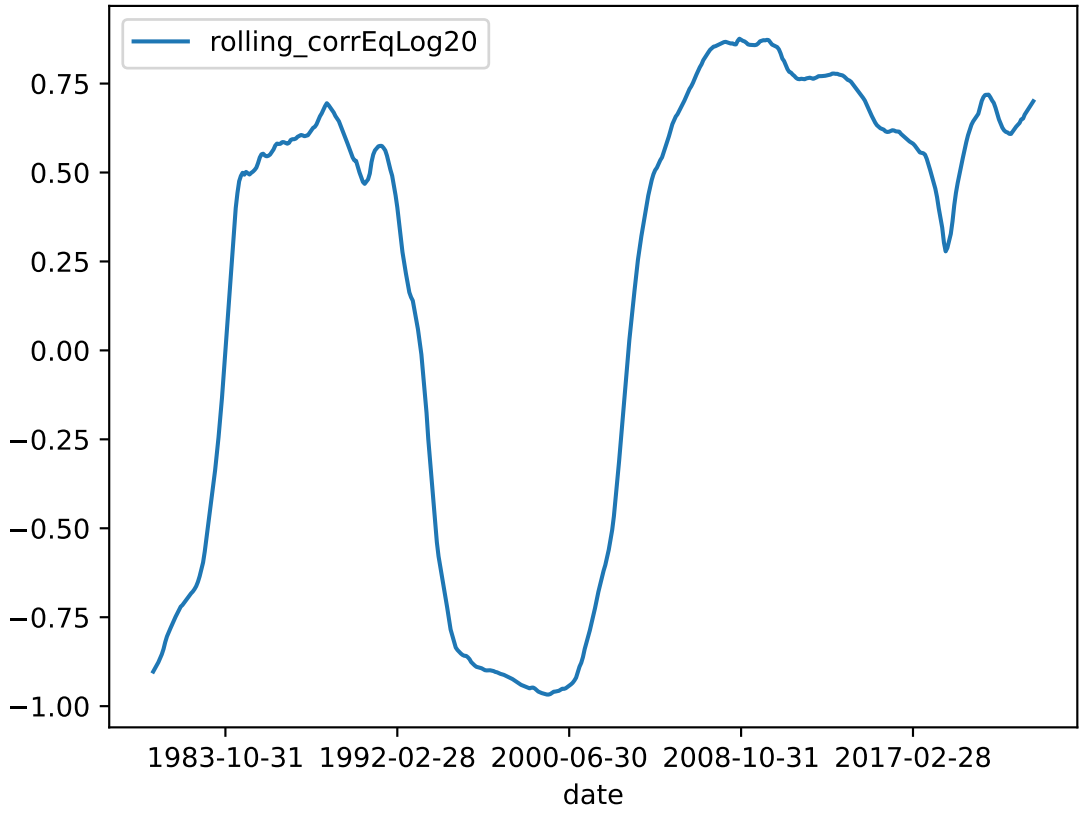

Figure 2. Rolling Correlations Between Stocks and Inflation
This figure shows the rolling correlations between the CRSP stock market index return and the CPI. Returns are given over a twenty-year horizon and correlations are then computed over a 10-year horizon which ends at the date given on the x-axis

For the market, we set $\mu_M = 0.098$ and $\sigma_M = 0.109$. For the inflation-linked asset, we set $\mu_I = 0.039$ and $\sigma_I = 0.069$. The risk-free rate is $r_f = 0.053$. The correlation between the stock market and inflation in the base case is set to $\rho = 0$; we will consider its impact in a robustness check when discussing the results.

All investors in the market have CRRA utility with a risk aversion parameter of 10. We set the initial wealth of each of the four investors equal to one. This implies that the groups of pension fund investors and capital market investors have the same aggregate initial wealth, and it furthermore implies that we can interpret the exposures $y_{i,I}$ to inflation risk

as exposures per unit of wealth.

The exposure of the investor to inflation risk is controlled by the parameter $y_{i,I}$. It varies across investors. Old investors whose wealth consists mainly of financial wealth have a negative exposure to inflation risk which can have the same order as their initial wealth. Young investors, whose wealth is mainly human capital, have an exposure to inflation risk which is only slightly negative. In the benchmark case, we set $y_{old,I} = y_{1,I} = -1.0$, $y_{young,I} = y_{2,I} = -0.3$, and $y_{CapMarkInv,I} = y_{3,I} = y_{4,I} = -0.2$. These exposures get scaled by a constant $y_c \in [0, 1]$.

For our analysis, we rely on a one-period binomial model. While this is admittedly a very stylized setup, it is enough to make our main points. The probabilities of the four states are thus given by 0.25. We solve the model for the different market setups as explained in Section 3. Given the asset demand of all investors we set demand equal to supply to determine the equilibrium price of the inflation-linked asset I. Remember that the market and the risk-free asset are in infinitely elastic supply; we fix their prices and returns at the values given above.

5. RESULTS

5.1 Base Case

We start with the base case, i.e., the incomplete market. In this setup, capital market investors can trade all assets, and pension fund investors can trade the market and the risk-free asset only. The price of the market and the risk-free rate are given exogenously. We solve for the equilibrium in this market. The price of the inflation-linked asset which is traded in the capital market only is $P_I = 0.9977$, so the real rate of return is 0.0023. The trading volume in this asset is equal to zero. Pension fund investors can not trade the inflation-linked asset; capital market investors do not want to trade the inflation-linked asset, since they are all identical and do thus not trade with each other.

All investors chose the optimal portfolios, given the prices. We calculate their certainty equivalents from the solution to the asset allocation problems in equilibrium. These certainty equivalents will be the benchmark when we look at the gains and losses of the investors in different market setups relative to the incomplete market later on. Figure 3 shows the certainty equivalents. The factor y_c on the x-axis is a scaling factor for the exposures. For $y_c = 1$, we have the given exposures $y_{old,I} = -1.0$, $y_{young,I} = -0.3$ and $y_{CapMarkInv,I} = -0.2$. Certainty equivalents are largest for $y_c = 0$, i.e. if the exposure to inflation risk is zero for all investors. In this case, the investors are no longer heterogeneous, and their certainty equivalents are identical. For a larger y_c and thus more negative exposures, investors suffer from the additional risk and the average reduction in their real wealth. If y_c were above 1,

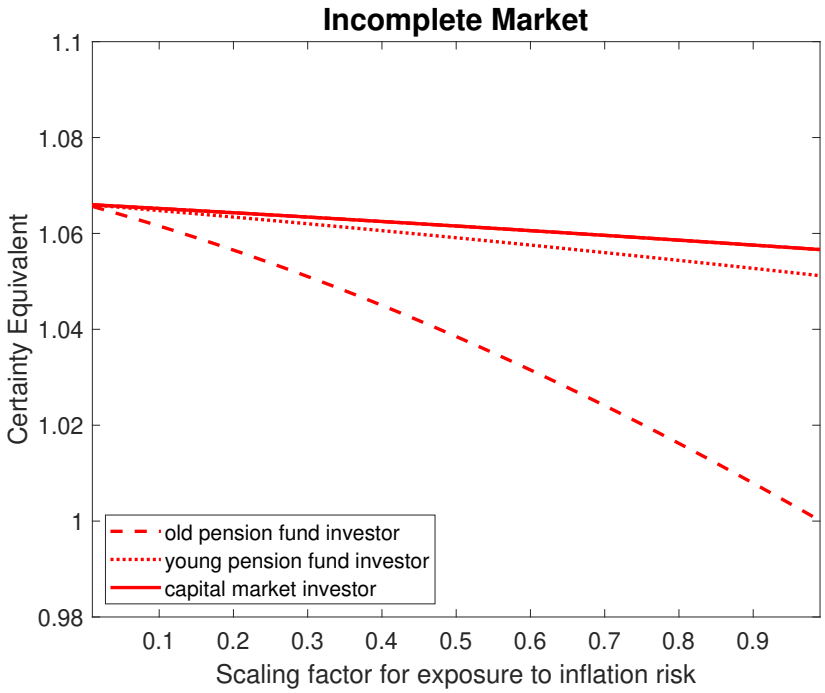

Figure 3. Certainty equivalents in incomplete market

The figure shows the certainty equivalents of the old pension fund investor (dashed line), the young pension fund investor (dotted line) and the capital market investor (solid line). The initial exposures to inflation risk are multiplied by the scaling factor on the x-axis.

the certainty equivalents would eventually turn negative.

We can see that the impact of the inflation exposure is heterogeneous across investors, being largest for the old-person fund investor and smallest for the capital market investor. This shows that there are gains to be made if the risk becomes tradable.

5.2 Internal Market

We next look at the case where the pension fund manager sets up an internal market for inflation risk. Technically, the pension fund specifies the sharing rule, i.e. it defines the shares of the overall fund value that go to the individual pension fund investors. In this sharing rule, the pension fund can integrate a hypothetical trading of an inflation-linked asset. This involves the terminal payoffs of the inflation-linked asset and the initial prices (or premia) that have to be paid for getting insurance. In the following, we will ignore the technical implementation and speak of trading by the pension fund participants.

In our example economy, the initial exposure to inflation risk is more negative for the old than for the young investor. Therefore, young investors should insure old investors. To compensate the young investors for the additional risk they have to take on, they get a premium. The pension fund sets the price and the positions for the inflation asset in the internal market. These choices are discretionary; we start with two (sensible) assumptions. First, we set the price in the internal market equal to the price in the capital market. There are several arguments in favour of this price. It can be observed so that there is no subjective component to it. Furthermore, this price seems fair. In particular, there are no political arguments involved, like a redistribution of wealth between different groups. In fact, this is mandatory in the Dutch pension system, i.e., the internal re-allocation of cash flows has to be made using market prices. Second, we set the amount of insuring such that there is full risk sharing. Young and old investors pool their exposures of $y_{old,I} = -1.0$ and $y_{young,I} = -0.3$. The resulting overall exposure of -1.3 is then shared equally, and the resulting exposure for each investor is -0.65. To achieve this exposure, the old investor buys $x_{old,I} = 0.35$ inflation-linked assets from the young investor ($x_{young,I} = -0.35$).

Given the price and the amounts that have to be traded by pension fund participants in the internal market, pension fund participants decide on their optimal position in the market and the risk-free asset. Capital market investors decide on their optimal positions in all three assets.

Figure 4 gives the relative change in the certainty equivalent, where the benchmark is given by the certainty equivalent in the incomplete market, i.e., the current market setup analyzed in Section 5.1.

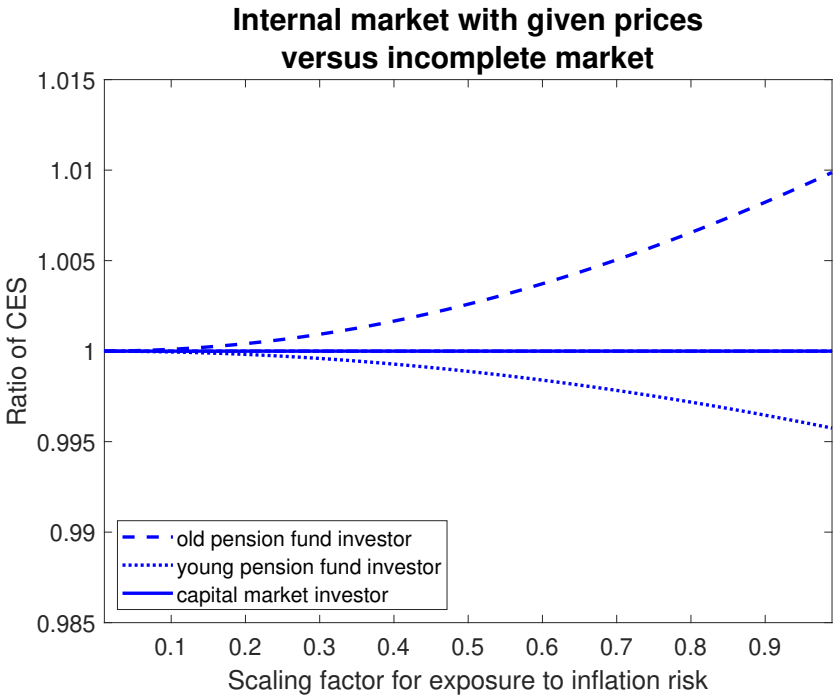

Figure 4. Certainty equivalents in internal market when pries and amounts are given
The figure shows the certainty equivalents of the old pension fund investor (dashed line), the young pension fund investor (dotted line) and the capital market investor (solid line). The initial exposures to inflation risk are multiplied by the scaling factor on the x-axis.

For capital market investors, the certainty equivalent does not change compared to the incomplete market. In both market setups, they can trade the market and the risk-free asset. Furthermore, they can trade the inflation-linked asset between them, while there is no trade between them and pension fund investors. With the same problem to solve, they will come up with the same optimal portfolio, and with the same equilibrium price for the inflation-linked asset in the capital market.

Given our parameters, old pension fund participants profit from the introduction of the internal market. They gain around 1% p.a. from being able to buy inflation-risk insurance. Young pension fund participants lose around 0.45% p.a.. They profit from the premium they earn on the sold inflation-risk insurance. However, the premium is not large enough to compensate them for the additional risk they have to take on. To get the intuition, we look at (the differences of) the initial exposures. The young pension fund participant has an initial inflation risk exposure of $y_{young,I} = -0.3$. He sells inflation risk insurance to the old pension fund participant, and his inflation risk exposure after trading will be -0.65. The price at which he has to trade is the equilibrium price set by the capital market investors. These investors have an initial exposure of $y_{CapMarkInv,I} = -0.2$. The price for the insurance they set (with zero trading) is thus lower than the price the young pension fund participant

would set if he was the one not to trade at his initial exposure of $y_{young,I} = -0.3$. The higher price hypothetically set by the young pension fund participant would just make him happy not to trade. To sell insurance, he would ask for an even higher price. The price the young pension fund participant deems fair given his initial exposure and his forced trading is thus larger than the capital market price set by the capital market investors. Being forced to trade at the capital market price makes him worse off.

The figure gives the gains and losses for different scaling factors y_c of the exogeneously given exposures y_{iI}. For a zero scaling factor, inflation risk does not play any role, so gains and losses are zero. In line with intuition, gains and losses then both increase in the scaling factor in absolute terms.

In our example, there are thus pension fund participants (here: young pension fund participants) who are worse off when being forced to trade inflation insurance at the capital market price. This is the case although the equilibrium price of inflation-linked assets from the capital market is an observable market price, it is objective, and there is no political influence on this price.

Overall, the price from the capital market may thus not be a good choice for the internal market. It may make some pension fund participants worse off. The reason is that the capital market investors differ from the pension fund participants so the equilibrium price of the capital market investors will not be the equilibrium price of the pension fund participants.

5.3 Complete Market

Next, we consider the complete market. In this market, all investors have access to the capital market for the inflation-linked asset and trade with each other. Risk sharing therefore takes place between all investors.

Technically, the pension fund trades on behalf of the pension fund participants in the complete market. It allocates the initial premium payments and terminal payoffs to the pension fund participants via the sharing rule. In the following, we again ignore the technical implementation and assume that the pension fund participants can directly trade in the capital market. In particular, we ignore all problems that may arise with the delegation decision since that would distract us from focusing on the comparison of the different market setups and the corresponding choices of prices and amounts.

Figure 5 gives the ratios of the certainty equivalents relative to the incomplete market as a function of the scaling factor y_c for the initial exposures.

For the assumed parameters, the gain of old pension fund participants is around 0.75%, while young pension fund participants only gain some basis points. The gain of capital market

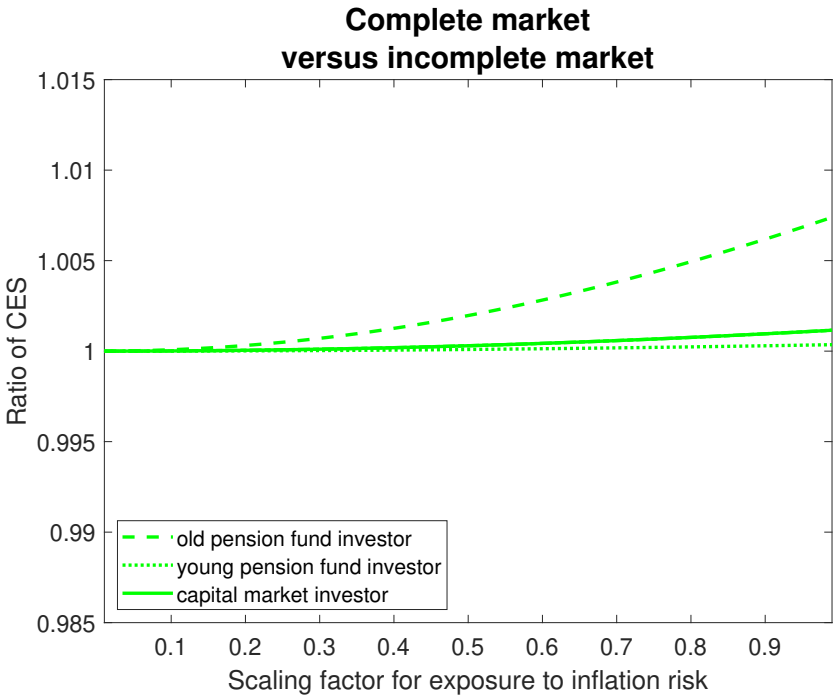

Figure 5. Base case: certainty equivalent gains and losses in complete market
The figure shows the certainty equivalents of the old pension fund investor (dashed line), the young pension fund investor (dotted line) and the capital market investor (solid line). The initial exposures to inflation risk are multiplied by the scaling factor on the x-axis.

investors is very small, too. The reason is that capital market investors are more similar to young pension fund participants than to old pension fund participants. Therefore, old pension fund participants will buy insurance not just from young pension fund participants, but also from capital market investors. Old pension fund participants thus gain a lot from this extended supply of insurance, while the other investors, including young pension fund participants, have to share the premium from selling insurance.

All pension fund participants gain. This result is not specific to our calibration but holds true in general. The reason is that the restricted optimal allocation from the incomplete market is still accessible to the pension fund participants: an optimal investment in the market (at the exogenously given price) and a zero investment in the inflation-linked asset. If the pension fund participants deviate from this formerly optimal allocation by taking a non-zero position in the inflation-linked asset, then the reason is that their utility increases when doing so.

Capital market investors are not necessarily better off. For them, the formerly optimal portfolio from the incomplete market may no longer be available. The reason is that the price of the inflation-linked asset in the complete market will in general be different from the price in the incomplete market. Their old portfolio, for which they have to trade the

inflation-linked asset at the old price, might therefore no longer be accessible.

In our setup, we face a special case. Since we assume that capital market investors are identical to each other, they do not want to trade the inflation-linked asset in the incomplete market. The old allocation thus involves a zero position in the inflation-linked asset. This portfolio is still available in the complete market, irrespective of the new equilibrium price of the inflation-linked asset. If capital market investors deviate from this formerly optimal portfolio, the reason is again that they profit from doing so.

5.4 Internal Market and Choice of Price

We now modify the internal market setup in that we do not simply take the capital market price as a premium. We still assume perfect risk sharing between young and old investors. However, we now vary the price in the internal market. There will thus be two prices for the inflation-linked asset: the price in the capital market, at which capital market investors trade, and the price in the internal market, at which pension fund participants trade.

Figure 6 shows the certainty equivalent gains and losses relative to the incomplete market as a function of the price in the internal market.

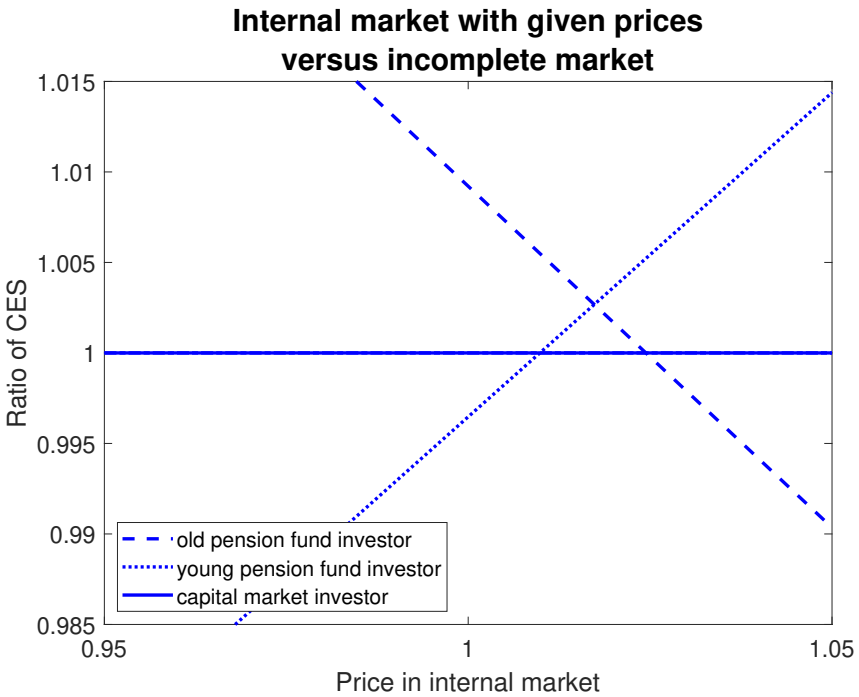

Figure 6. Base case: certainty equivalent gains and losses in internal market
The figure shows the certainty equivalents of the old pension fund investor (dashed line), the young pension fund investor (dotted line) and the capital market investor (solid line) as a function of the price in the internal market.

There is some region of intermediate prices for which both young and old pension fund participants gain. For these prices, both investors are happy with the introduction of the internal market in case of perfect risk sharing. Note, however, that perfect risk sharing might not be the optimal choice, but that they might prefer some other amount of trading.

For prices below this region, old pension fund participants gain since they have to pay a small premium for the insurance. Young pension fund participants, who have to provide insurance for this low price, however, lose. For prices above this region, the argument is vice versa. Young pension fund participants gain since they earn a high premium when selling insurance. Old pension fund participants, who have to pay a high price for the insurance, however, lose. For our calibration, the price in the incomplete market where only capital market investors trade the inflation-linked asset, is below this region. This is the reason why old investors gain, while young investors lose in Section 5.2. Note that we give the results for the case of perfect risk sharing. The investors might well prefer some different amounts of trading.

Figure 7 shows the impact of the price (y-axis) and the position (x-axis) in the internal market. We give the position in the inflation-linked asset of the old investor; he buys insurance for a positive position and sells insurance for a negative position. The position of the young investor is minus the position of the old investor since the overall net supply of the inflation-linked asset in the internal market is zero.

For each combination of internal price and internal position, we solve the asset allocation problem of the old pension fund participant, i.e. we determine his optimal position in the market. We then determine his certainty equivalent and how much he gains as compared to the incomplete market. The figure shows the contour plots of the ratios of the certainty equivalents, i.e. the indifference curves for the old investor.

For each price level in the internal market, we determine the optimal insurance position of the old investor. These optimal positions (as a function of the internal price) are shown by the solid red line. For a low price, the old investor wants to buy a lot of insurance. His demand for insurance decreases when the price in the internal market rises. If the price becomes too high, he eventually even supplies insurance, i.e. he thus starts to sell insurance.

We do the same calculations for the young investor. The optimal position in insurance (expressed via the opposite position of the old investor) as a function of the internal price is shown via the dotted red line. In line with intuition, the young investor wants to sell insurance for a high internal price and starts to demand insurance for a low internal price.

The vertical black line highlights a position of zero. That is the position the pension fund investors have in the incomplete market. With identical portfolios, the certainty equivalents in the internal market and in the incomplete market coincide, and the ratio of certainty

equivalents is equal to one.

When we move from the vertical line (zero position in the inflation-linked asset) towards the red solid line (optimal position of old pension fund investors), the certainty equivalent of the old investor increases. When we move beyond the solid red line, however, his certainty equivalent gain starts to decrease again, until he is ultimately even worse off than in the incomplete market. When we move from the vertical line away from the solid red line, the old investor loses.

An equivalent result holds true when we move from the vertical line towards the dotted red line (the optimal position of young pension fund investors). The certainty equivalent gain of the young investor increases. It decreases when we move beyond the dotted line, or when we move from the vertical line into the opposite direction.

The intersection of the solid and the dotted red line gives the equilibrium price and position in the internal market. For this equilibrium price, both investors are happy with their equilibrium position. If the pension fund fixes this amount of trading, both investors will be happy with the amount of risk sharing, and they will also be happy with the introduction of the internal market.

For prices around this equilibrium price, both investors profit from an internal market as long as the position they are assigned by the pension fund is not too large. For a price slightly below the equilibrium price, the young investor optimally wants to sell less insurance than the old investor wants to buy. If the amount of trading is just slightly above zero, both investors profit. If the amount exceeds the amount given by the dotted red line (which gives the optimal amount for the young investor), the old investor still gains, while the young investor starts to lose relative to his overall optimal amount. In the beginning, he will still see a utility gain relative to the incomplete market, but eventually, his certainty equivalent will drop below its level in the incomplete market. If the amount also exceeds the red solid line (which gives the optimal amount for the old investor), both the young and the old investor will be made worse off when the amount increases further. For a price slightly above the equilibrium price, the young investor optimally wants to sell more insurance than the old investor optimally wants to buy. Again, both investors profit from a small amount of trading. If the amount of trading increases by too much, then first the old investor and then the young investor will start to lose again.

For prices above this region, the young investor and also the old investor want to sell insurance. Moving away from a zero position in the direction of a positive amount for the old investor makes the young investor (who can not sell insurance) better off, but the old investor (who is forced to buy insurance at a too-high price) worse off. Moving away in the direction of a negative amount for the old investor makes the old investor better off and the young investor (who is forced to buy insurance at a high price) worse off.

For prices below this region, the young investor and also the old investor want to buy insurance. Moving away from a zero position in the direction of a positive amount for the old investor makes the old investor better off, but the young investor (who is forced to sell insurance at a too-low price) worse off. Moving away in the direction of a negative amount for the old investor makes the young investor better off and the old investor (who is forced to sell insurance) worse off.

The horizontal black line gives the price of the inflation-linked asset in the capital market. The price is below the equilibrium price, and it is also below the region where both investors can profit. A positive position of the old investor moves the position closer to the optimal position of the old investor, who consequentially gains, but away from the optimal position of the young investor, who thus loses.

The horizontal blue line is the price from the complete market. At this price, too, the old investor gains while the young investor loses. Again, this price is not just not optimal, but it is even harmful for some pension fund investors. The reason is that risk sharing in the internal market takes place between pension fund investors only, while the equilibrium price is determined in the complete market where risk sharing takes place among all investors. With the capital market investors missing, the price that was determined when they were present is no longer optimal.

Figure 8 gives the ratios of the certainty equivalent gains in the internal market as a function of the scaling factor for the exposures. When the price and the amount of risk sharing are set equal to their equilibrium values, all pension fund investors profit. For our parameters, the utility gains of the investors are nearly identical. The reason is that the investors have the same relative risk aversion and differ in their initial exposures only.

Figure 9 compares the gains in the certainty equivalents in the complete market and the gains in the internal market when the price and the positions in the internal market are equal to their equilibrium values. The x-axis gives the scaling factor y_c of the initial exposures.

The internal market is implemented by the pension fund. Risk sharing takes place by the appropriate definition of the sharing rule for the pension fund participants. The complete market can be reached via trading of the pension fund on behalf of the pension fund investors. In both cases, pension fund investors have some gains in their certainty equivalents. The reason is that their optimal choice in the incomplete market is still accessible so the certainty equivalent in the incomplete market provides a lower bound that can be reached in both setups. For the given calibration, there is no clear ranking of the two markets. Gains are larger for the young investor in the internal market, while old investors gain more in the complete market. There is thus no Pareto dominance of one of the two market setups. To rank the alternatives, we would need some weighting of the pension fund participants via a social planner.

5.5 Role of Initial Exposures and Heterogeneity

Figure 10 analyzes the impact of the initial exposure of the capital market investors. It shows the relative gains in the certainty equivalent for the three groups – old pension fund participants, young pension fund participants, and capital market investors – both in the internal market with the equilibrium prices and positions and in the complete market.

For $y_{CapMarkInv,I} = y_{old,I} = -1$, capital market investors have the same inflation risk exposure as old pension fund participants. In the complete market, both old pension fund participants and capital market investors want to buy inflation insurance from young pension fund investors. In the internal market, only old pension fund participants want to get insurance from young pension fund participants. Thus, old pension fund participants prefer the internal market without any competition in buying inflation risk insurance. Young pension fund participants, in contrast, profit from the large demand for insurance in the complete market and earn high-risk premium on the insurance they sell. They thus prefer the complete market to the internal market.

For $y_{CapMarkInv,I} = y_{young,I} = -0.3$, capital market investors have the same inflation risk exposure as young pension fund participants. In the complete market, young pension fund participants and capital market investors thus all sell insurance, while in the internal market, only young pension fund participants sell insurance. Young pension fund participants thus prefer the internal market in which they are the only sellers of inflation risk insurance and can thus charge higher premia. Old pension fund participants, in contrast, profit from the increase in the supply of inflation risk insurance in the complete market. They thus prefer the complete market to the internal market.

For some exposure $y^*_{CapMarkInv,I}$ of the capital market investors which is approximately equal to the average exposure of pension fund participants, utility gains in the complete market and the internal market coincide. In this case, the equilibrium price in the complete market is equal to the equilibrium price in the internal market. In this case, only pension fund investors trade with each other (as in the internal market), while capital market investors have zero demand and do not share any risks with pension fund participants.

For a smaller exposure than $y^*_{CapMarkInv,I}$ of capital market investors, also the capital market investors want to buy inflation risk insurance in the complete market. Due to this competition, old pension fund participants are thus worse off in the complete market than in the internal market. On the other hand, young pension fund participants are better off in the complete market than in the internal market, since they profit from the demand of both old pension fund participants and capital market investors for inflation risk insurance.

For a somehow larger exposure than $y^*_{CapMarkInv,I}$ of capital market investors, both young pension fund participants and capital market investors offer inflation risk insurance in the

complete market. Due to this larger supply of inflation risk insurance, old pension fund participants are better off in the complete market than in the internal market. Young pension fund participants, in contrast, suffer from the competition of capital market investors in selling insurance and are worse off in the complete market than in the internal market.

For a much larger exposure than $y^*_{CapMarkInv,I}$ of capital market investors, only capital market investors will sell insurance, while both young and old pension fund investors will buy inflation risk insurance. An increase in the exposure of capital market investors then makes both young and old investors better off in the complete market. Ultimately, all pension fund investors will be better off in the complete market than in the internal market.

5.6 Impact of Correlation

Figure 11 gives the ratios of the certainty equivalents in the complete and in the internal market as a function of the correlation between stock market risk and inflation risk. Gains are largest for a somehow positive correlation and slightly lower when correlations are very positive or very negative. However, the impact of the correlation is rather small. In particular, it does not change the overall results.

6. CONCLUSION

Pension fund participants have a heterogeneous exposure to inflation risk. There is thus room for risk sharing between them which can make everyone better off if implemented correctly.

Currently, the market is incomplete. While inflation-linked assets are traded in the capital market, the market is thin and its volume is small. Pension fund participants usually do not participate in this market.

Risk sharing can be implemented in two ways. First, there can be an internal market within the pension fund where pension fund participants trade with each other. The pension fund can implement this market via an appropriate specification of the sharing rule. Second, there can be a complete market where pension fund participants trade with each other and with capital market participants. This market can be implemented by the pension fund trading on behalf of its participants.

In the internal market, the choice of the internal price and the positions in the inflation-linked asset are crucial. One alternative is to use the market price of inflation-linked assets from the capital market. This price is objective and easy to verify.[3] However, it is most likely not the optimal choice. It may even make parts of the pension fund participants worse

[3]In this case, it also follows the rules of the Dutch pension system.

off. This can happen when the group of pension fund participants differs from the group of capital market investors concerning the initial exposure to inflation risk.

When the price in the internal market is set to the equilibrium price in the internal capital market, all pension fund participants will profit from the internal market and be happy with their positions.

Trading in the complete market will also make all pension fund investors better off. Concerning a ranking between the complete and the internal market with equilibrium prices, there is often no Pareto dominance. Therefore, the complete market with an observable equilibrium market price is not necessarily superior to the internal market with the inter-equilibrium price. The internal market may make any group of investors better off, depending on the relation between the average exposures of the pension fund participants and the capital market participants.

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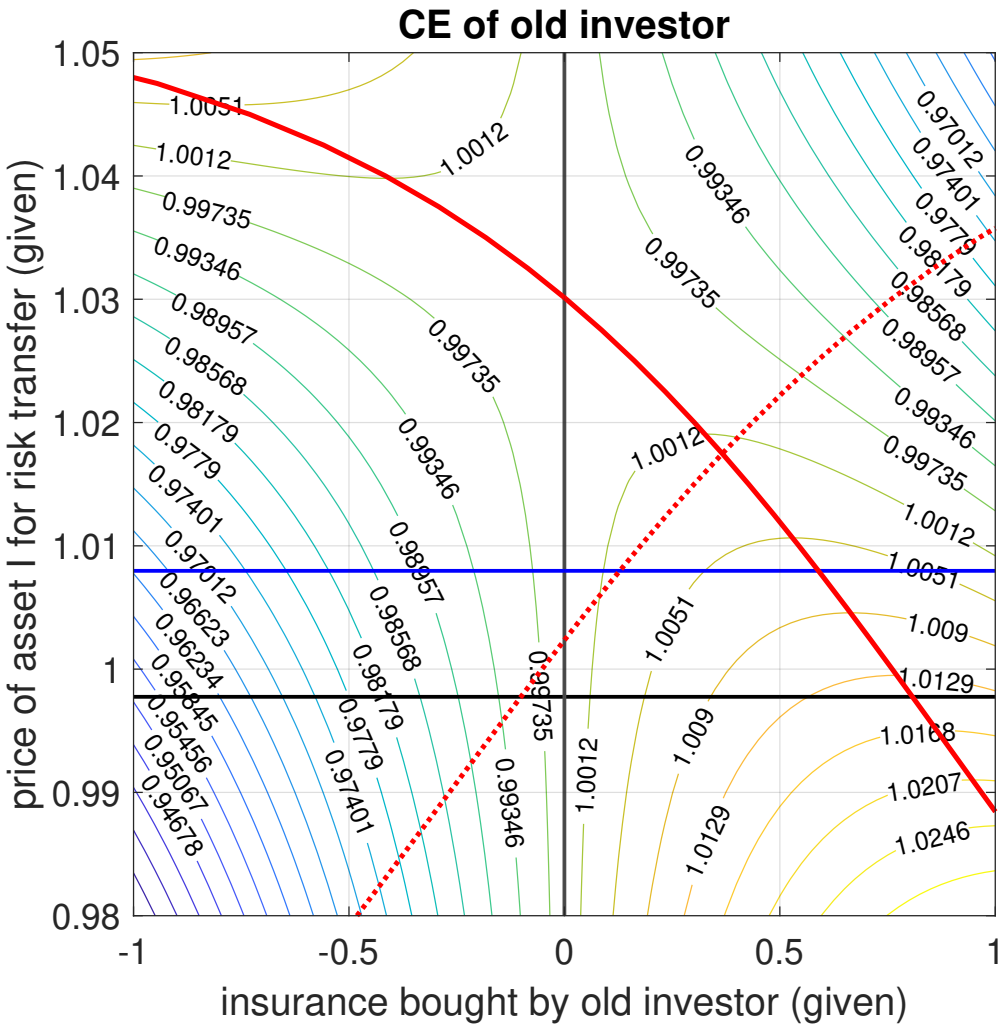


Figure 7. Impact of internal price and internal amount in internal market

The figures give the indifference curves for the certainty equivalent gains of the investors as a function of the internal price (y-axsis) and the amount of trading of the old investor (x-axis). The red solid line shows the optimal position of the old investor, given the internal price; the red dotted line shows the optimal position of the young investor, given the price. The vertical black line highlights a position of zero. The horizontal black (blue) line gives the equilibrium price in the incomplete (complete) market.

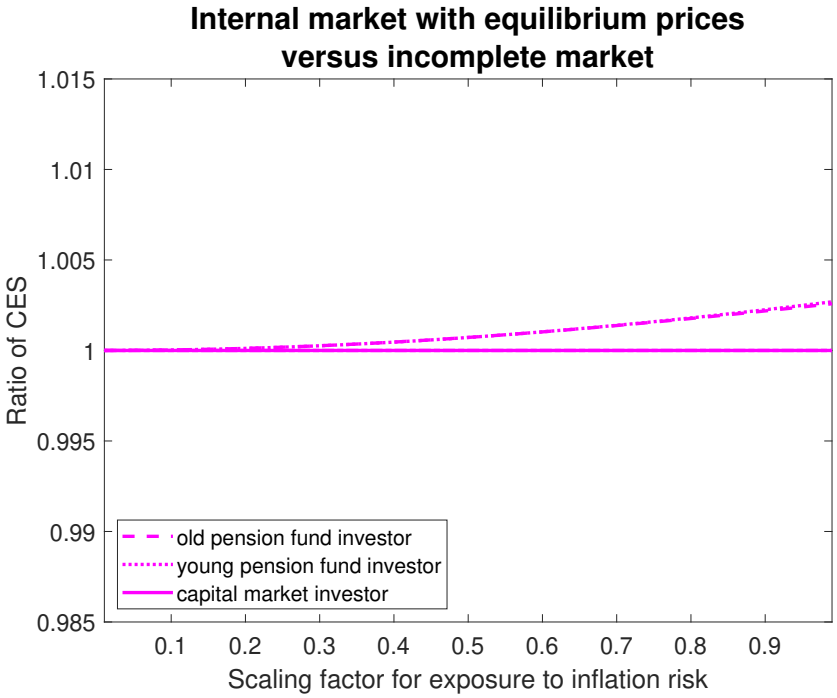
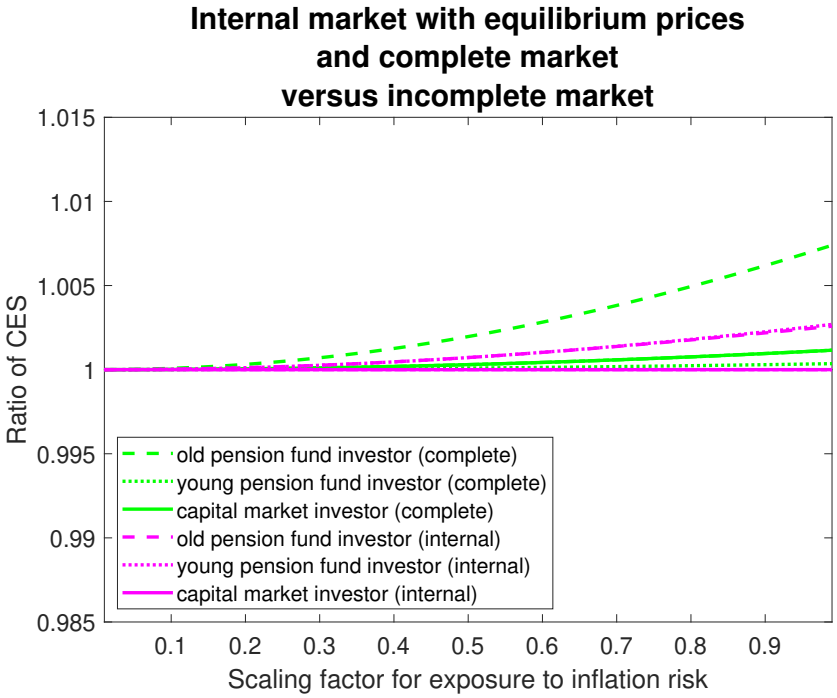


Figure 8. Base case: certainty equivalent gains and losses in internal market
The figure shows the certainty equivalents of the old pension fund investor (dashed line), the young pension fund investor (dotted line) and the capital market investor (solid line) as a function of the price in the internal market.



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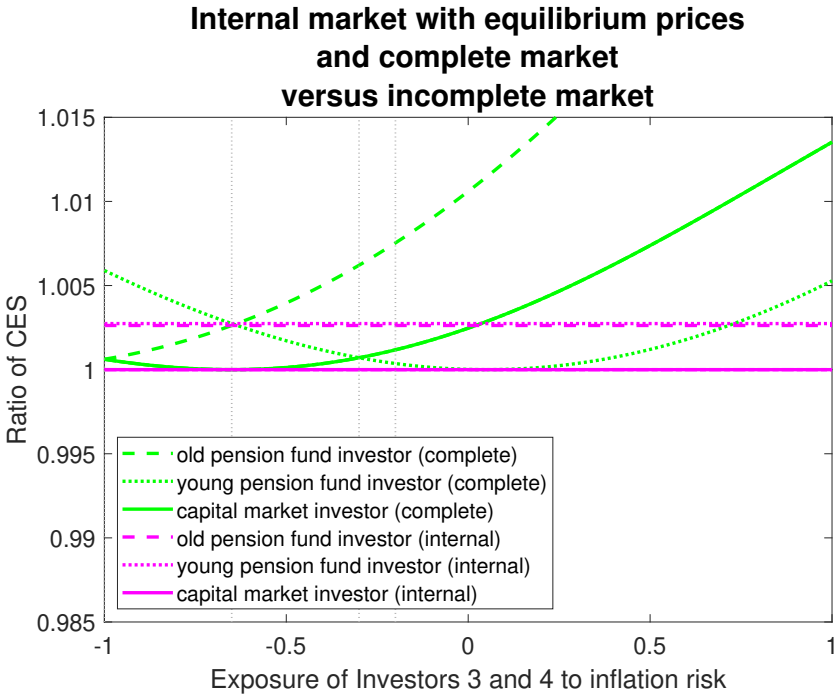

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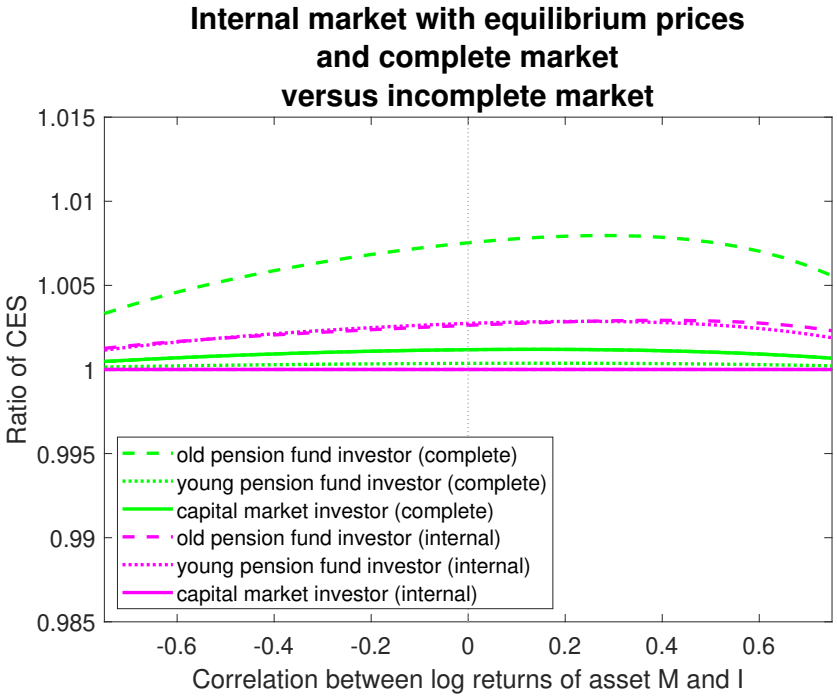

Figure 11. Base case: certainty equivalent gains and losses in internal market
The figure shows the certainty equivalents of the old pension fund investor (dashed line), the young pension fund investor (dotted line) and the capital market investor (solid line) as a function of the correlation between inflation and the stock market.