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# Economic principles in pension design

Annick van Ool

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# **ECONOMIC PRINCIPLES IN PENSION DESIGN**

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# 1

## Introduction

The primary objective of pensions is to maintain the standard of living for individuals during their retirement years (Merton (1983)). Typically, countries organize pensions in a three-pillar structure (World Bank (1994)). The first pillar refers to public pensions that provide a basic safety net for the elderly. These public pensions are administered by the state and are usually financed by social insurance contributions and general tax revenues. The second pillar refers to occupational pensions that supplement the first-pillar pension benefits and are linked to an employment contract. Occupational pensions are administered by a pension fund or an insurance company in a private pension plan and are financed through mandatory contributions that are paid by employers and/or employees. Finally, the third pillar refers to voluntary private pensions that are managed mainly by insurance companies and financed through individual contributions.

This dissertation focuses on second-pillar pensions, which play a key role in providing an adequate retirement income. Globally, second-pillar pension assets amounted to more than 60 trillion US dollars at the end of 2021, which is roughly equal to 60% of global GDP (OECD (2023)). In some countries such as the Netherlands, the amount of pension assets managed by pension funds

is even bigger than the national GDP. As a result, pension funds are among the largest institutional investors and play an important role in financial markets.

At both ends of a continuum, two archetypal types of occupational pension schemes can be distinguished: defined benefit (DB) and defined contribution (DC) pension schemes. In a DB pension scheme, the scheme's sponsor promises a retirement benefit that is determined by a formula based on the employee's wage history, tenure of service, and age. At the other end of the pension scheme spectrum are DC pension schemes. In a DC scheme, the future pension benefit depends on the contributions and the cumulative investment returns on them. Alternative pension schemes have been developed that combine features of both DB and DC pension schemes. Such hybrid pension schemes enjoy increasing popularity due to their attractive features, such as a limited commitment from the sponsor, longevity risk sharing, and economies of scale in administrative and investment costs. An example of a hybrid scheme is a Collective Defined Contribution (CDC) scheme. This dissertation focuses in particular on the design of CDC pension schemes.

Pension schemes offer the ability to pool risks and to smooth financial shocks over time. They can significantly mitigate an individual's risk exposure and increase the probability that individuals receive an adequate level of retirement income.

Risks shared in a pension scheme can be categorized as idiosyncratic risks or systemic risks. Idiosyncratic risks are specific for an individual or investment and can be diversified. Systemic risks impact all participants in a pension scheme and cannot be diversified.

Risks can be shared between individuals of a specific cohort only, shared across cohorts or even shared across generations. Table 1.1 presents the types of risk sharing that are possible for different types of risk in a pension scheme.

**Table 1.1: Types of risk and risk sharing**

*This table presents types of risk sharing for different risks that can be shared in a pension scheme. A cross (black or grey) indicates that a certain risk (row) can be shared in a certain way (column). The black crosses refer to risk and risk sharing combinations that are considered in this dissertation.*

Risk	Intra cohort	Inter cohort (overlapping generations)	Intergenerational (non-overlapping generations)
Micro-longevity risk	X		
Macro-longevity risk		X	X
Inflation risk		X	X
Investment risk		X	X

Sharing idiosyncratic risks within a cohort mitigates risks through the pooling of a large number of individuals. Micro-longevity risk the uncertainty about an individual's time of death, is an idiosyncratic risk because it can be diversified by pooling this risk in a cohort. Individuals who live longer are subsidized by individuals who die earlier. Risk sharing across cohorts allows for inter-temporal smoothing of shocks which can provide retirement income stability. This type of risk sharing can be beneficial for shocks that cannot be diversified through risk pooling, e.g., macro-longevity risk. Macro-longevity risk (the uncertainty about future mortality rates) cannot be diversified through risk pooling but can be shared across cohorts because not all cohorts are affected in the same way by changes in future mortality rates. Inflation risk, which is the risk that the purchasing power of pension benefits erodes over time on the back of increasing prices, can be shared across cohorts in a similar way. Risk sharing across generations can further improve welfare because this is normally not possible due to incomplete markets. For example, intergenerational risk sharing of investment risk allows individuals to already share investment risk before they participate in the pension scheme which extends their investment horizon. Nevertheless, the welfare benefits of intergenerational risk sharing do not always materialise in practice because it exposes a pension scheme to discontinuity risk. Discontinuity risk is the risk that a new cohort is unwilling to participate in the pension scheme, for

instance, because of funding deficits created by previous generations.

The black crosses in Table 1.1 refer to risk and risk sharing combinations that I consider in my dissertation. I focus on sharing investment risk across cohorts and (to some extent) also on sharing investment risk across generations in Chapter 2. Macro-longevity risk sharing between cohorts is discussed in Chapter 3.

To ensure an adequate and sustainable pension at retirement, pension design should satisfy certain economic principles. In this dissertation, I consider the following four economic principles:

- *Fairness*: no ex-ante value transfers between cohorts in a pension scheme.
- *Efficiency*: optimal exposure to market risks for cohorts over their life-cycle and based on their risk preferences.
- *Insurance*: protection against the risk of outliving or not having sufficient pension assets during retirement for cohorts.
- *Accountability*: the responsibility of a pension fund's board of trustees to explain and justify their policy in a transparent way to its stakeholders.

These principles are discussed in more detail in the next subsections.

## 1.1 Fairness

Chapter 2 focuses on sharing market risks in CDC pension schemes, which combine features of DB and DC schemes. From a continuity perspective, an important principle for the design of a CDC scheme is fairness. Fairness requires that no ex-ante value transfer between cohorts in a pension scheme occur. This principle is important because one or more cohorts are better off leaving the CDC pension scheme if this principle is not met. In Chapter 2, I define a general class of fair CDC pension schemes in the presence of equity market risk and interest rate risk through the combination of three key features: the discount rate process, the benefit adjustment process, and the asset allocation. Although a CDC scheme with uniform benefit adjustments intuitively seems a 'fair contract', the results show that such a scheme is generally unfair and that the unfairness, measured as a percentage of annual income, can be substantial. A fair CDC scheme can be designed

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if the scheme is complete and appropriate horizon-dependent benefit adjustments are used to allocate market risks to cohorts.

## 1.2 Efficiency

Another important principle for pension scheme design is efficiency. Efficiency requires that cohorts have an optimal exposure to market risks based on their preferences and life-cycle. The literature on optimal life-cycle investing shows that it is optimal to decrease the equity allocation with age (Bodie et al. (1992), and Bovenberg et al. (2007)). In particular, when the ratio of financial wealth to human capital is low, and human capital is relatively safe, the optimal allocation towards equity is large. This mainly holds for young cohorts, as they have accumulated little financial wealth but have significant human capital with little risk. The optimal interest rate hedge generally increases with age because participants have fewer capabilities to compensate for interest rate decreases. In Chapter 2, I derive the implied exposure to market risks in a CDC scheme and determine to what extent a CDC scheme can provide optimal exposure to market risks for all cohorts. It is not always possible to replicate precisely the optimal individual's exposure to market risks in a CDC scheme, because a CDC scheme itself acts as a constraint on the exposure of cohorts to market risks. Therefore, efficiency may not be reached. I show, however, that the implied exposure to market risks in a fair CDC scheme with benefit smoothing is generally in line with life-cycle theory.

## 1.3 Insurance

Apart from market risks, pension schemes are also exposed to insurance risks such as longevity risk. While micro-longevity risk is an idiosyncratic risk that can be diversified, macro-longevity risk is a systemic risk. While a pension scheme can easily provide insurance for micro-longevity risk to participants by pooling sufficient participants, this is not the case for macro-longevity risk because of its systemic nature. In Chapter 3, I introduce the concept of macro-longevity risk sharing as a risk management tool where workers can provide insurance against the macro-longevity risk of retirees. I

show that macro-longevity risk has a significant impact on pension provisioning, and can affect both retirees and workers, but not all cohorts in the same way or by the same amount. I derive Pareto-improving risk sharing rules for macro-longevity risk and determine a fair risk compensation for cohorts who absorb the macro-longevity risk of other cohorts using a utility-based fairness criterion. I find that the specific features of the retirement age policy have a large impact on the optimal risk-sharing rule and the size of the welfare gains. Due to the lack of sufficiently good market solutions for longevity hedging and increasing longevity, the solutions presented in this chapter, to create an internal market between cohorts in the pensions scheme, are relevant for pension scheme design.

## 1.4 Accountability

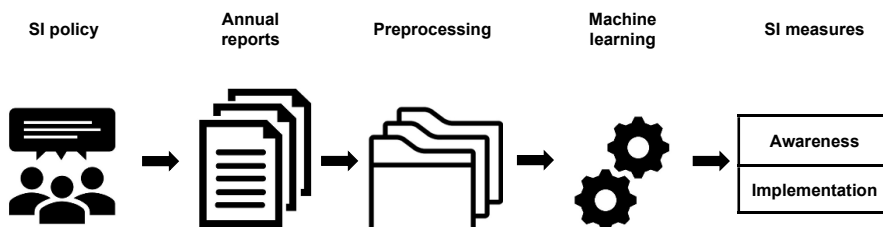
The final principle that I investigate in this dissertation is accountability. A pension fund's board of trustees is responsible for the pension fund's policy and the implementation of this policy, including, e.g., the investment policy, risk management, and benefit administration. In Chapter 4, I investigate a particular aspect of the pension fund policy, namely the sustainable investment (SI) policy. Pension funds are obliged to disclose sustainability-related information: they have to report how they incorporate environmental, social, and governance (ESG) criteria in their investment policy and how they incorporate ESG risks in their risk management. Chapter 4 provides an overview of the disclosures of sustainable investing by a specific group of large institutional investors, Dutch occupational pension funds, by exploiting a unique dataset with a novel tool. I use state-of-the-art natural language processing (NLP) techniques to measure the awareness and implementation of sustainable investing using qualitative data from annual reports. Figure 1.1 visualizes the textual analysis pipeline built to measure sustainable investing. The objective of a textual analysis pipeline is to facilitate the collection and processing of text data. The annual reports, which include disclosures of the SI policy of pension funds, are collected and preprocessed. Subsequently, several machine learning techniques are used to calculate sustainable investment (SI) measures that

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quantify the awareness and implementation of sustainable investing. I combine these SI measures with a proprietary dataset to analyze the relationship between pension fund characteristics and sustainable investing.

**Figure 1.1: Textual analysis pipeline**

*This figure visualizes the textual analysis pipeline built to measure sustainable investing.*



I find that large pension funds have a higher level of awareness and implementation of sustainable investing. Moreover, I analyze the role of signing the International Responsible Business Conduct (IRBC) initiative. The IRBC initiative is a voluntary initiative of Dutch pension funds that aims to bring their investment policy into line with the OECD Guidelines and UN guiding principles.<sup>1,2,3</sup> Large pension funds, pension funds with more female trustees, or pension funds with a positive belief about the risk-return relation of sustainable investing are more likely to sign the IRBC initiative. Although signing this initiative increases the specificity of pension fund statements about sustainable investing, I do not find an effect on the implementation of sustainable investing.

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<sup>1</sup>The IRBC is the 'Convenant Internationaal Maatschappelijk Verantwoord Beleggen Pensioenfondsen' (IMVB) in Dutch.

<sup>2</sup>OECD (2011), *OECD Guidelines for Multinational Enterprises*.

<sup>3</sup>United Nations (2011), *Guiding Principles on business and human rights*.

## 1.5 Integrating economic principles in pension design

Pension design plays a crucial role in ensuring financial security for individuals during both the accumulation and retirement period. To ensure an adequate and sustainable pension at retirement, pension design should satisfy certain economic principles. There is no straightforward way to integrate economic principles in pension design. Legislators, employers and trade unions, and pension fund boards encounter several trade-offs in practice. For example, when determining the level of pension contributions a trade-off has to be made between affordability principle and adequacy principle. Pension contributions have to be sufficiently high to achieve retirement income objectives, but if pension contributions are too high they can be strain on current consumption and financial well-being. In this dissertation, I consider the following four economic principles: fairness, efficiency, insurance and accountability. There are also trade-offs between these four principles. While the first three principles, discussed in Chapter 2 and 3 of this dissertation, focus on financial objectives, i.e., providing an adequate and stable pension, Chapter 4 considers non-financial objectives, i.e., focus on sustainable investing to enjoy retirement in a world worth living. As put forward by Nobel Prize winner in economics Jan Tinbergen (Tinbergen (1952)), accomplishing two objectives (financial and non-financial objectives) with just one instrument (the investment strategy) can be challenging. To find a desirable solution, different objectives need to be prioritized taking into account the preferences of participants. The fact that risk preferences can be strongly heterogeneous among participants within a pension fund makes the design of the investment strategy even more complex (Alserda et al. (2019)). Especially in the Dutch pension sector, where participants are not able to switch their pension provider easily, pension funds have the key responsibility to make sure that they act in the best interest of their beneficiaries.

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## 1.6 Outline

This dissertation consists of three self-contained researches. In Chapter 2, *Fairness and efficiency in CDC pension schemes*, I define a general class of fair and efficient CDC pension schemes in the presence of financial market risks by combining three key design features: the benefit adjustment process, the discount rate process to determine the present discounted value of future benefits, and the asset allocation.

Chapter 3, *The economics of sharing macro-longevity risk*, focuses on sharing macro-longevity risk between cohorts in a pension scheme as a risk management tool. I approach this actuarial topic from an economic perspective. The innovation in this chapter is to derive Pareto-improving risk sharing rules for macro-longevity risk.

Chapter 4, *Walk the green talk? A textual analysis of pension funds' disclosures of sustainable investing*, introduces a novel textual analysis approach using state-of-the-art natural language processing (NLP) techniques to measure the awareness and implementation of sustainable investing by Dutch pension funds in their annual reports.

An overview of the key findings and suggestions for future research are presented in the conclusion in Chapter 5. Chapter 6 contains the research impact and valorization of this dissertation.



# 2

## Fairness and efficiency in Collective Defined Contribution pension schemes<sup>1</sup>

In this paper, we formally analyze a general class of pension schemes: Collective Defined Contribution (CDC) pension schemes. CDC pension schemes combine features of the well-known Defined Benefit (DB) and Defined Contribution (DC) schemes. As in the case of a DB scheme, participants in a CDC scheme accrue benefits expressed in terms of future income. As in the case of a DC scheme, pension contributions are fixed and benefit levels fluctuate as a function of the scheme's funding ratio, i.e., the value of assets to the value of liabilities. In this way, all participants, including those who are working and those who are retired, collectively bear the CDC scheme's exposure to market risks. CDC pension schemes are enjoying increasing popularity. In Canada, Denmark, Sweden, and the Netherlands, CDC schemes already exist, and

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<sup>1</sup>This chapter is based on a working paper co-authored with Ilija Boelaars (ING) and Dirk Broeders (Maastricht University and De Nederlandsche Bank).

CDC schemes will be introduced in the UK in the near future.<sup>2</sup> CDC schemes are attractive to sponsors (employers) because the pension benefits are not guaranteed. The sponsor's commitment is typically limited to paying contributions at a specific level or within a predefined range. CDC schemes also offer advantages to participants, such as longevity risk sharing and economies of scale, because the pooling of retirement assets across a large number of participants leads to lower administrative and investment costs per participant.

The main contribution of this study is to define a general class of fair and efficient CDC schemes in the presence of financial market risks. We distinguish between equity market risk and interest rate risk. We define this class of CDC schemes by combining three key design features: the benefit adjustment process, the discount rate process to determine the present discounted value of future benefits, and the asset allocation. From a continuity perspective, it is preferable that CDC schemes are fair and efficient to prevent new cohorts choosing not to participate in the collective pension scheme. However, in practice, it is a challenge to design a CDC scheme that meets both design criteria.

A pension scheme is fair if all cohorts make an arbitrage-free return on their specific exposure to the market risks at each point in time. As a result, the market value of future pension benefits is equal to the market value of pension contributions at any time. Efficiency requires that cohorts have an optimal exposure to market risks based on their preferences and life-cycle. If these two design criteria are not met, one or more cohorts might be better off leaving the CDC scheme. In this paper, we investigate both criteria. First, we calculate the size of wealth transfers between cohorts in unfair CDC schemes. Second, we design several fair CDC schemes and analyze to what extent these provide an optimal exposure to market risks for all cohorts.

The fairness and efficiency criteria critically depend on how the CDC scheme allocates market risks to cohorts. Market risks impact both assets and liabilities, the latter via the discount rate process. Therefore, more precisely formulated, the scheme allocates the 'mismatch risk' between the value of the

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<sup>2</sup>CDC schemes were introduced in the UK's Pension Schemes Act 2021. Secondary legislation is currently being discussed in Parliament.

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assets and liabilities to its cohorts. This mismatch risk exists if the value of the assets and liabilities in a pension scheme do not develop similarly over time. Depending on the pension scheme's configuration one or more stakeholders have to absorb this mismatch risk. In a pure DB scheme, pension benefits are guaranteed. The sponsor, either the government or a pension guarantee scheme, absorbs the mismatch risk by guaranteeing the benefits (Broeders and Chen (2013)). However, such external benefit guarantees do not feature in a CDC scheme, and cohorts collectively bear the mismatch risk. They do so through benefit adjustments. Benefits are adjusted upwards or downwards depending on the sign and the size of the mismatch risk and the scheme's design features, in particular the benefit adjustment process.

We analyze the fairness and efficiency criteria using a financial market simulation. First, we show that CDC schemes with uniform benefit adjustments across cohorts are generally unfair. For a typical interest rate hedge of 40 percent, this value transfer can be as much as 50 percent of the participant's annual income. Second, we show that a CDC pension scheme can be made fair if the scheme is complete, i.e., all value is explicitly allocated to cohorts, and appropriate horizon-dependent benefit adjustments are used to allocate the market risks to cohorts. Third, we analyze to what extent fair CDC schemes provide an optimal exposure to market risks for all cohorts. We show that the 'return on benefits' in a fair CDC scheme can be replicated with a sum of traded assets and we derive the implied exposure to market risks. Because a CDC scheme pools the assets and liabilities of all cohorts, however, it is not always possible to replicate precisely the optimal individual exposure to market risks in a CDC scheme. However, in a fair CDC scheme with smoothing, the implied exposure to market risks is generally in line with life-cycle theory. Smoothing of benefit adjustments implies that shocks do not translate one-to-one into benefit level adjustments, but benefits are adjusted gradually over time. Under benefit smoothing, the implied exposure to equity decreases with age which is in line with life-cycle theory. In all fair CDC schemes (with and without benefit smoothing), the duration of the bond portfolio decreases with age which is in line with the optimal duration in an individual scheme because the interest rate risk hedging demand decreases with age.

In principle, only the existing cohorts participate in a CDC scheme's risk sharing. However, it is also possible to share risks with future cohorts, so-called intergenerational risk sharing. In the literature, many papers on collective pension schemes focus on the advantages of intergenerational risk sharing in a pension scheme (Ball and Mankiw (2007a), Gollier (2008), Cui et al. (2011), and Chen et al. (2016)). An important disadvantage of intergenerational risk sharing is that it exposes the pension scheme to discontinuity risk, as investigated by, e.g., Chen et al. (2017). Discontinuity risk is the risk that a new cohort is unwilling to participate in the collective pension scheme, for instance, because of funding deficits created by previous generations. We include two CDC schemes with intergenerational risk sharing in our analysis, for comparison purposes.

This paper contributes to the literature of pension scheme design by combining two continuity criteria in the context of CDC pension schemes. First, we use the fairness definition of Boelaars and Broeders (2019), which is in line with the concept of fair contracts in the insurance literature (see, e.g., Grosen and Jørgensen (2002) and Orozco-Garcia and Schmeiser (2019)).<sup>3</sup> This definition implies that if a scheme is not fair, some cohorts could get a better risk-return trade-off by investing directly in financial markets themselves. The concept of fairness is thus an important continuity criterion, because some cohorts can benefit by leaving an unfair scheme, which in turn jeopardizes the collective nature of the pension scheme. This exposes the CDC scheme to discontinuity risk. CDC schemes in practice are generally not fair. In existing CDC schemes, benefit adjustments are usually uniform, i.e., an equal percentage is applied for all cohorts. Although this intuitively seems a 'fair contract', Boelaars and Broeders (2019) show that this is generally not the case, because of the interaction between the benefit adjustment process and the discount rate process. In a financial market model with stochastic interest rates, a uniform benefit adjustment process leads to a return on benefits that cannot be replicated with traded assets. As a result, an arbitrage opportunity exists, implying a continuous wealth transfer from young to old cohorts if the mismatch risk between the value of the assets and liabilities is not fully

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<sup>3</sup>This definition is sometimes called financial fairness (Schumacher (2020)) and differs from fairness in Gollier (2008), who considers fairness from a welfare perspective.

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hedged. Boelaars and Broeders (2019) show that fairness crucially depends on the combined specification of the benefit adjustment process and the discount rate process used in the CDC scheme. Only specific combinations lead to an arbitrage-free, and therefore fair, allocation of market risk.

Second, efficiency is a relevant criterion for the design of CDC pension schemes. There is an extensive literature on optimal life-cycle investing that points out that the optimal equity allocation decreases with age (Bodie et al. (1992), and Bovenberg et al. (2007)). In particular, when the ratio of financial wealth to human capital is low, and the return on human capital is relatively less risky than equity, the optimal allocation towards equity is large. This mainly holds for young cohorts, as they have accumulated little financial wealth but have significant human capital with little risk. Therefore, younger cohorts are better able to absorb shocks than older cohorts. Consequently, it is attractive for young cohorts to invest more of their financial wealth in equity.<sup>4</sup> Besides an optimal equity allocation, adequate interest rate risk management is also essential. For instance, van Bilsen et al. (2018) show that not optimally hedging interest rate risk can lead to significant welfare losses. Young cohorts have a higher interest rate risk exposure than old cohorts because of their longer investment horizon and thus have a higher hedging demand. As a result, the optimal exposure to long-term bonds decreases with age.

The remainder of this paper is organized as follows. Section 2.1 starts with an explanation of the design of a CDC scheme and its most important features. In Section 2.2, we further discuss the concept of fairness. Section 2.3 explains the concept of efficiency and life-cycle theory. Section 2.4 specifies the model. The results are presented in Section 4.4.4 and we conclude in Section 4.5.

## 2.1 Designing CDC pension schemes

As mentioned in the introduction, CDC pension schemes combine features of the well-known DB and DC schemes. In a CDC scheme, the benefit

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<sup>4</sup>In life-cycle models, the optimal asset allocation depends on the amount and the risk level of human capital or the present discounted value of future labor income. For ease of exposition, we will use age in this paper as a proxy of human capital. So, a low age is equivalent to high human capital

levels fluctuate as the mismatch risk between the value of the assets and liabilities is shared with all cohorts, through benefit adjustments. When designing a CDC scheme, policy makers have to decide on the following three key features:

1. Discount rate process
2. Benefit adjustment process
3. Asset allocation

First, a discount rate process is needed to determine the present value of benefits by discounting all future expected payments to derive the total liabilities. Boelaars and Broeders (2019) show that using the default-free term structure of market interest rates as the discount rate process preserves a fair allocation of market risks. Other discount rate processes are also possible, but lead to complicate benefit adjustment processes. Since our paper does not focus on liability discounting and we do not want to add unnecessary complexity, we will only consider one discount rate process, i.e., the risk-free term structure of market interest rates.

Second, the benefit adjustment process prescribes how benefit levels are adjusted in response to changes in the value of assets and liabilities, i.e., the mismatch risk. For example, in the case of uniform benefit adjustments, the benefits of all cohorts are adjusted by the same percentage such that the value of the assets equals the value of the liabilities after the adjustment. An alternative benefit adjustment process is to smooth benefit adjustments. Smoothing of benefit adjustments implies that shocks do not translate one-to-one into benefit level adjustments, but benefits are adjusted gradually over time. As a result, benefits with a short maturity are adjusted by a smaller percentage and benefits by a long maturity are adjusted with a bigger percentage. The benefit adjustment process will be discussed in more detail in Sections 2.4.2 and 2.4.3.

A relevant property in this context is ‘completeness’. A pension scheme is complete if all value in the CDC scheme is explicitly allocated to the cohorts in the scheme at each point in time. This completeness property ensures that directly after the benefit adjustment process has been applied the value of the

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pension liabilities equals the value of the assets. Whether a pension scheme satisfies the completeness property depends on the choice of the discount rate process in combination with the benefit adjustment process.

Third, an asset allocation has to be chosen for the collective pool of assets. We determine the asset allocation using a welfare analysis. In this analysis, total welfare, which is the aggregated expected utility of all cohorts, is maximized. This optimization is explained in more detail in Section 2.4.4. The optimization procedure does not necessarily imply that all cohorts in a CDC scheme have an efficient exposure to market risks. An asset allocation is efficient if it provides an optimal exposure to market risks based on preferences and life-cycle.

## 2.2 Fairness

Because a CDC scheme allocates wealth over time across cohorts, a complex economic problem emerges, namely whether all cohorts are treated fairly. Designing a fair CDC pension contract is therefore not trivial. A pension scheme is fair if all cohorts make an arbitrage-free return on the market risks they bear at any time. Boelaars and Broeders (2019) show that fairness depends on the right combination of the benefit adjustment process and the discount rate process. Fairness in the pension literature has been investigated by, e.g., Teulings and De Vries (2006), who introduce the principle of generational accounting in pension schemes. This principle is sometimes also called financial fairness. Pazdera et al. (2016) provide conditions under which there exists a unique risk sharing scheme that is both Pareto efficient and financially fair. Schumacher (2020) explicitly discusses financial fairness under several changes in assumptions in the collective pension model of Gollier (2008). These papers do not, however, take interest rate risk into account, which is a relevant extension for the fairness criterion in complete pension schemes. We will include interest rate risk in our approach.

In Section 2.2.1, we will discuss unfairness in a simple CDC scheme, and in Section 2.2.2, we will discuss how a simple CDC scheme can be made fair.

### **2.2.1 A simple CDC scheme that is not fair**

In a CDC pension scheme, there is a single pool of assets. We assume that the value of the assets is observed by their latest price in the financial markets. The cohorts accrue pension benefits, expressed in terms of future income, which are adjusted upward or downward based on predefined rules. We require a discount rate to determine the present discounted value of these benefits. We will call this the regulatory value of the benefits, as the regulatory authorities typically prescribe the discount rate. We start by analyzing a fairly simple CDC scheme, in which the pension benefits are adjusted immediately and homogeneously across all cohorts (i.e., the same percentage) in response to changes in the value of the pension fund's assets and the regulatory value of benefits. After the adjustment, the value of total assets equals the value of total liabilities.

Boelaars and Broeders (2019) show that such a CDC scheme with a homogeneous benefit adjustment process is not fair because of the interaction between the benefit adjustment process and the discount rate process. To clarify this interaction, they derive the return on benefits and show it consists of four terms: the return on a zero-coupon bond, the return on the assets, the return on the liability portfolio, and a market-inconsistent return component. The first three terms can be replicated with traded assets, but the market-inconsistent return component cannot. This market-inconsistent return component causes the unfairness in a scheme with homogeneous benefit adjustments.

The market-inconsistent return is a result of the fact that, in general, the interest rate sensitivity of the assets is lower than the interest rate sensitivity of the liabilities, or simply the duration of the assets is lower than the duration of the liabilities. As a result, benefit levels are reduced in scenarios with declining interest rates. The economic intuition is that in these scenarios the present discounted value of benefits increases more than the value of the assets. Hence, all benefits must be reduced to balance assets and liabilities again. The reverse is also true. When interest rates rise, benefit levels are increased. Furthermore, the change in interest rates affects the regulatory value of cohorts' benefits differently based on their investment horizon. When in-

interest rates fall, the young cohorts' benefits become relatively more valuable compared to the old cohorts' benefits.<sup>5</sup> The young absorb a relatively big share of the negative mismatch between assets and liabilities and thus lose relatively valuable benefits. Conversely, when interest rates rise, the young cohorts' benefits become relatively less valuable compared to the old cohorts' benefits. The young absorb a relatively big share of the negative mismatch between assets and liabilities and thus gain relatively cheap benefits.

**Figure 2.1: Market-inconsistent excess return**

*This figure shows the market-inconsistent excess return in a simple CDC scheme for different bond allocations. The bond portfolio consists of long-term bonds with maturity  $tt = 50$ .*

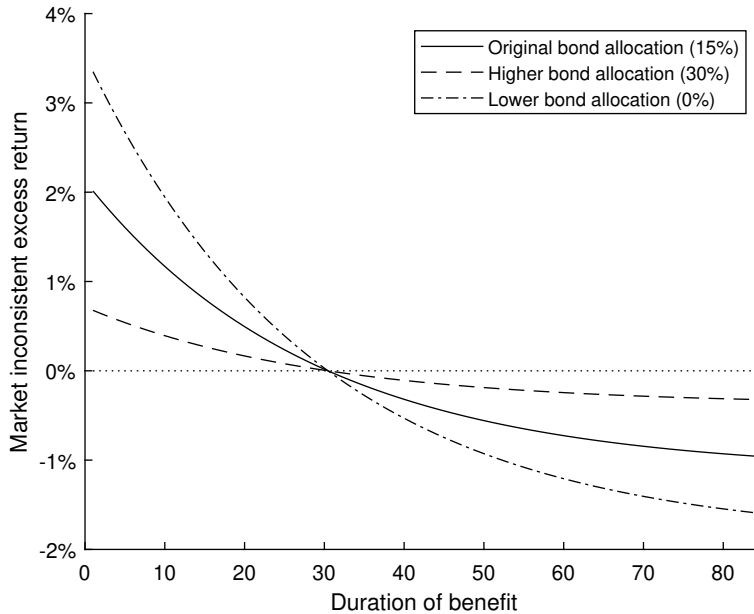


Figure 2.1 visualizes the market-inconsistent return in a simple CDC scheme for three different bond allocations. The graph shows that the size of the market-inconsistent return depends on the duration of the benefit in years.

<sup>5</sup>This is a result of the higher convexity of the young cohorts' benefits compared to old cohorts' benefits.

For old cohorts (i.e., benefits with short maturities), the market-inconsistent return is substantial and positive. A benefit with a maturity of one year, for example, yields an additional return of 2 percentage points on top of the market-consistent return in the case of a 15 percent bond allocation with a duration of 50 years. For young cohorts (i.e., benefits with long maturities), the market-inconsistent return is negative. The size of the market-inconsistent return depends on the size of the asset-liability mismatch: the size of the market-inconsistent return is larger in the case of a bigger asset-liability mismatch. Figure 2.1 shows that the market-inconsistent excess return is bigger in the case of a lower bond allocation (i.e., larger asset-liability mismatch) and smaller in the case of a higher bond allocation (i.e., smaller asset-liability mismatch).

### **2.2.2 A fair CDC scheme**

It is possible to design a fair CDC pension scheme by introducing a horizon-dependent benefit adjustment process instead of a uniform benefit adjustment process. By doing so, we eliminate the market-inconsistent interaction term between the discount rate and the benefit adjustment process as discussed in Section 2.2.1. Because the covariance between the discount rate process and the benefit adjustment process is horizon-dependent, the benefit adjustment process should also be horizon-dependent. As a result of these horizon-dependent benefit adjustments, the market-inconsistent return disappears, and the excess return in Figure 2.1 becomes zero. We derive these horizon-dependent benefit adjustments in Section 2.4.3. First we discuss the second design criterion, efficiency, in the next section.

## **2.3 Efficiency**

Besides fairness, a second key criterion for the design of CDC pension schemes is efficiency. An asset allocation is efficient in the context of a CDC scheme if it provides all cohorts with an optimal exposure to market risks. Section 2.3.1 discusses the theory of optimal life-cycle investing, and Section 2.3.2 discuss smoothing of benefit adjustments in CDC schemes. This feature can bring the exposure to market risks over the life-cycle of all cohorts more

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in line with optimal life-cycle theory.

### **2.3.1 Optimal life-cycle theory**

The literature on optimal life-cycle investing shows that it is optimal to decrease the equity allocation with age (Merton (1971), Merton and Samuelson (1974), Bodie et al. (1992) and Bovenberg et al. (2007)) as long as future labor income is (relatively) risk-free. When the ratio of financial wealth to human capital is low and human capital is relatively less risky than equity, the preferred allocation towards equity is typically larger. This mainly holds for young individuals as they have accumulated little financial wealth but have a lot of human capital. Therefore, young individuals are better able to absorb shocks than older individuals. Consequently, it is optimal for young individuals to invest more in equity. For very young individuals, it is even optimal to lend money to invest in equity such that the equity allocation as a percentage of financial wealth is bigger than 100 percent. After retirement, individuals no longer have human capital; thus, total wealth is equal to financial wealth. Consequently, the optimal exposure to equity is constant after retirement for a given level of risk.

The assumption that human capital is risk-free does not hold in practice generally. If the return on labor income and equity returns are correlated, it is less optimal for young individuals to invest a large fraction of their financial wealth in risky assets. Both Viceira (2001) and Cocco et al. (2005) show that in the presence of labor income risk, it is still optimal to decrease the equity allocation over the life-cycle, although the exposure is lower compared to a setting with risk-free labor income. However, if labor and dividend income are co-integrated, it is sub-optimal to decrease the equity allocation over the life-cycle, according to Benzoni et al. (2007). Instead, a hump-shaped equity allocation over the life-cycle is optimal.

Although the literature on the optimal exposure to equity is considerable, the literature on the optimal exposure to interest rate risk is limited. To protect individuals against changes in pension payments due to interest rate movements, interest rate risk can be hedged by investing in long-term bonds. Both young and old individuals are exposed to interest rate risk. Young in-

dividuals have a higher exposure to interest rate risk because the duration of their pension payments is higher (i.e., pension payments are further in the future). However, young individuals have a lot of (relatively) risk-free human capital, which provides them with a natural interest rate risk hedge. Although the size of interest rate risk is smaller for old individuals, the demand for interest rate risk hedging is larger because they have fewer capabilities to compensate for interest rate decreases. As a result, the optimal interest rate hedge increases with age. This result holds for several model and parameter assumptions (van Bilsen et al. (2020a)). Mehlkopf and Bilsen (2020) show that for old individuals it is optimal to hedge the majority of their interest rate risk. It can even be higher than 100 percent. In this case, the duration of the assets is greater than the duration of the liabilities. As a result, pension payments increase when the interest rate decreases. A large interest rate hedge can be optimal because investing in long-term bonds is beneficial for interest rate hedging and yields a term premium. Investing in long-term bonds yields a positive return in expectation that is usually higher than the return on cash. However, an important reason for not hedging interest rate risk completely is that it increases inflation risk in the event that real bonds are not (sufficiently) available in the market. This effect is not present in our model, since we exclude inflation risk.

Although the age of an individual, as a proxy of human capital, is an important determinant for the asset allocation, a single optimal life-cycle does not exist. The optimal asset allocation also depends on other features such as financial wealth, risk aversion, and future labor income.

### **2.3.2 Smoothing of benefit adjustments in CDC schemes**

To accomplish an exposure to market risks that is in line with optimal life-cycle theory in CDC schemes, we introduce smoothing in the benefit adjustment process. Smoothing of benefit adjustments is already applied in practice in some pension schemes.<sup>6</sup> The two main advantages of benefit smoothing are the following: it reduces the year-to-year volatility of benefit

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<sup>6</sup>For example, smoothing of pension benefit adjustments is permitted in Dutch pension schemes, and smoothing of investment returns is offered by Danish life insurers.

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payments and it creates an age-dependent exposure to market risks (Guillén et al. (2006) and Mehlkopf et al. (2013)).<sup>7</sup> Smoothing of benefit adjustments implies that new shocks do not translate one-to-one into benefit level adjustments, but benefit levels are adjusted gradually over time. The total exposure to market risks does not decrease due to smoothing, but shocks are processed over a longer period. As a result, the exposure to market risks is lower for old cohorts but higher for young cohorts.

In practice, there are many possible types of smoothing policies. The goal of every smoothing policy is to ensure that shocks to the pension schemes' financial position only affect benefit levels gradually, especially for old cohorts. In our setup we apply smoothing to the mismatch between assets and liabilities. For this, the benefit adjustment process is a benefit. In general, policy-makers will choose a low  $\alpha_m$  for benefits with a short maturity and a high  $\alpha_m$  for benefits with a long maturity. As a result, the year-to-year volatility of the short-term pension benefits will decrease. At the same time, the year-to-year volatility of long-term pension benefits will increase. So the smoothing parameter transfers risks from old to young cohorts. The smoothing parameter  $\alpha_m$  is generally linear with the maturity  $m$  but can also have a different distribution (e.g., polynomial or exponential). We consider a linear smoothing parameter only for ease of exposition.

In our paper, we consider the following three smoothing policies based on the completeness property, the benefits that are involved in risk sharing and the funding ratio:

- **Closed smoothing:** under this policy, there is only risk sharing with current pension benefits. The mismatch risk is explicitly allocated to the current pension benefits in the pension scheme and the CDC scheme is complete. The funding ratio always equals 1 after the benefit adjustment.
- **Open complete smoothing:** under this policy, there is risk sharing with current and future pension benefits because the mismatch risk is explicitly allocated to both current and future pension benefits. Because

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<sup>7</sup>Another advantage of benefit smoothing is that it allows for intergenerational risk sharing (Gollier (2008)).

the mismatch risk is completely allocated, the CDC scheme is complete. The funding ratio  $FR_t$  based on current assets and liabilities is not equal to 1 after the benefit adjustment. However, the funding ratio  $FR_t^f$  based on current and future assets and liabilities is. Open complete smoothing allows for intergenerational risk sharing because the mismatch risk is partially allocated to future cohorts. This policy is based on the assumption that future cohorts will be available and willing to participate in the CDC scheme.

- **Open smoothing:** under this policy, there is risk sharing with current and future pension benefits. Because only a fraction of the mismatch between assets and liabilities is allocated to benefits, the CDC scheme is not complete. As a result, the funding ratio is not equal to 1 after the benefit adjustment. Open smoothing allows for intergenerational risk sharing because current shocks are shared with future cohorts. While under open complete smoothing the mismatch risk is explicitly allocated, this is not the case under open smoothing. By definition, a CDC scheme with an open smoothing policy cannot be fair because some value is not explicitly allocated.

Table 2.1 presents some relevant properties of these three smoothing policies. Further details of the different smoothing policies and the benefit adjustments under the different smoothing policies are discussed in Section 2.4.3.

**Table 2.1: Properties of smoothing policies**

*This table presents an overview with properties of the different smoothing policies.*

	Closed smoothing	Open complete smoothing	Open smoothing
Sharing with future benefits	No	Yes	Yes
Complete	Yes	Yes	No
$FR_t = 1$ after benefit adjustment	Yes	No	No
$FR_t^f = 1$ after benefit adjustment	-	No	-

Smoothing of benefit adjustments has an important impact on the implied exposure of cohorts to market risks. Smoothing lowers the exposure to mar-

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ket risks for old cohorts and increases the exposure for young cohorts. This exposure is in line with optimal life-cycle theory. The type of smoothing policy and the length of the smoothing period impact the implied exposure for different cohorts.

An important caveat in CDC pension schemes is that it is not always possible to replicate precisely the optimal individual exposure to equity market risk and interest rate risk over the life-cycle. The benefit adjustments can be made horizon-dependent to mimic the life-cycle, but these adjustments depend on the asset-liability mismatch risk only, i.e., there is no explicit distinction between equity market risk and interest rate risk. Consequently, the number of implicit life-cycles a CDC scheme can offer is limited. The life-cycle is determined by the combination of the benefit adjustment process (i.e., the smoothing policy and length of the smoothing period) and the asset allocation. We will investigate to what extent the optimal individual exposure can be replicated in a CDC scheme by choosing the asset allocation and benefit adjustment process accordingly.

## **2.4 Model specification**

In this section, we build the model to analyze the pension schemes. We start with a description of the financial market in Section 2.4.1. Section 2.4.2 describes the pension schemes' characteristics. We consider two types of pension schemes: Collective Defined Contribution (CDC) schemes and individual defined contribution (IDC) schemes, which act as a benchmark. In Section 2.4.3, we derive the benefit adjustments in the different CDC schemes and discuss the smoothing parameter. To analyze efficiency in CDC schemes, we perform a welfare analysis to determine the optimal asset allocation. This is described in Section 2.4.4. Finally, the optimal exposure for all cohorts to market risks in an IDC scheme is derived in Section 2.4.5.

### **2.4.1 Financial market**

We use a straightforward specification of the financial market with two sources of risk: equity market risk and interest rate risk. The two sources of risk are represented by two standard Brownian motions that we assume to

be uncorrelated:  $dZ_{S,t}$  and  $dZ_{r,t}$ , respectively. The model contains one state variable: the instantaneous interest rate  $r_t$ , which follows a mean-reverting Ornstein-Uhlenbeck process (Vasicek (1977)). We model the development of the price of equity as a geometric Brownian motion. All asset prices are captured by the pricing kernel  $M_t$ .

**Table 2.2: Financial market model**

*This table presents the economic variables in the financial market model, the stochastic differential equations of these economic variables, and parameter values.*

Variable	Parameter	Value
<b>Interest rate</b>	$dr_t = \kappa(\bar{r} - r_t)dt + \sigma_r dZ_{r,t}$	
Mean-reversion parameter	$\kappa$	0.0347
Unconditional mean	$\bar{r}$	0.02
Volatility	$\sigma_r$	0.01
<b>Equity</b>	$\frac{dS_t}{S_t} = (r_t + \lambda_S \sigma_S)dt + \sigma_S dZ_{S,t}$	
Volatility	$\sigma_S$	0.136
<b>Pricing kernel</b>	$\frac{dM_t}{M_t} = -r_t dt + \lambda_r dZ_{r,t} - \lambda_S dZ_{S,t}$	
Price of interest rate risk	$\lambda_r$	-0.1
Price of equity market risk	$\lambda_S$	0.287

We model the economic variables in continuous time to make the simulation as precise as possible. The expressions of the economic variables are derived from the stochastic differential equations in Appendix 2.A. Table 2.2 summarizes the stochastic differential equations and parameter values.

We use the same equity market parameters and mean interest rate as in Gollier (2008). We assume that the equity volatility equals  $\sigma_S = 0.136$  and the price of equity market risk equals  $\lambda_S = 0.287$ . The mean-reversion parameter  $\kappa = 0.0347$  corresponds to a half-life of the interest rate of 20 years. This means that it takes 20 years for a specific interest rate level to revert halfway back to the equilibrium interest rate of  $\bar{r} = 0.02$ . The interest rate volatility equals  $\sigma_r = 0.01$ , and the price of interest rate risk equals  $\lambda_r = -0.1$ . In both the IDC scheme and the CDC schemes, the asset portfolio consists of three investment categories with the following weights: equity ( $x_S$ ), a long-term bond with maturity  $tt = 50$  ( $x_B$ ), and the remainder in cash ( $x_C$ ). We assume

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that the return on cash is equal to the return on a one-year bond. It is not possible to invest in bonds with other maturities. However, this assumption has no impact on the results, because a combination of the long-term bond and cash can always replicate a bond portfolio with a specific maturity.

Section 2.4.4 discusses how the optimal asset allocation in CDC schemes is determined and the optimal asset allocation in an IDC scheme is derived in Section 2.4.5.

## 2.4.2 Pension scheme characteristics

To determine the fairness and efficiency of CDC schemes we consider a pension scheme model with overlapping generations, i.e., a pension scheme in which several generations are alive simultaneously. We use a constant relative risk aversion (CRRA) utility function to describe the preferences of individuals with risk aversion parameter  $\gamma = 5$ . The time preference parameter equals  $\beta = 0.98$ . We assume all participants start working at age 25 and work until age 65, i.e., the length of the working period equals  $T_w = 40$  years. All participants are retired between the ages of 65 and 85, i.e., the length of the retirement period equals  $T_p = 20$  years. For the sake of simplicity, we exclude longevity risk and assume a fixed age of death.<sup>8</sup> Under smoothing, the length of the smoothing period equals  $n = 20$  years. Both the annual individual labor income  $y = 1$  and the individual pension contribution  $p = 0.2$  are fixed. Table 2.3 summarizes the key variables at hand. Next we turn to the description of the pension schemes.

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<sup>8</sup>For an analysis of the implications of longevity risk, see Broeders et al. (2021a).

**Table 2.3: Pension scheme characteristics**

This table presents the pension scheme parameters and values.

Variable	Parameter	Value
Risk aversion	$\gamma$	5
Time preference	$\beta$	0.98
Working period	$T_w$	40
Retirement period	$T_p$	20
Smoothing period	$n$	20
Annual individual labor income	$y$	1
Annual individual contribution	$p$	0.2
Maturity long-term bond portfolio	$tt$	50

### An Individual Defined Contribution (IDC) scheme

For comparison purposes, we start with an Individual Defined Contribution (IDC) scheme. In an IDC scheme, each individual has their own pension wealth  $F_t$ , which is zero at  $t = 0$  when the individual starts working and grows due to yearly pension contribution payments and investment returns before retirement. The wealth dynamics before retirement are given by

$$F_{t+1} = F_t(1 + (x_S r_t^S + x_B r_t^B + (1 - x_S - x_B) r_t^C)) + p, \quad (2.1)$$

where  $r_t^S$  is the return on the equity portfolio,  $r_t^B$  is the return on the long-term bond, and  $r_t^C$  is the return on cash. An IDC pension scheme is fair by construction. Each individual accumulates their own wealth and converts this into a variable annuity at retirement. The market value of the pension contributions is therefore equal to the market value of future pension payments. The pension payments  $C_t$  that the individual receives after retirement are determined using the price of a variable annuity  $pa_t$

$$C_t = \frac{F_t}{pa_t} = \frac{F_t}{\sum_{m=0}^{T_w+T_p+1-t} P(m, r_t)} \quad \forall t \in \{T_w + 1, \dots, T_w + T_p + 1\}, \quad (2.2)$$

where  $P(m, r_t)$  is the price of a zero-coupon bond at time  $t$  that pays one dollar at time  $T$ . Equation (2.2) shows that the pension payments depend on the amount of pension wealth, which typically varies with realized invest-

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ment returns in the case of a variable annuity and the current interest rate. The pension payment can therefore fluctuate over time. After retirement the pension wealth  $F_t$  develops as follows

$$F_{t+1} = (F_t - C_t)(1 + (x_S r_t^S + x_B r_t^B + (1 - x_S - x_B) r_t^C)). \quad (2.3)$$

Because we focus on the optimal exposure to market risks and want to compare the IDC scheme with the CDC schemes in an appropriate way, we assume fixed pension contributions in both the IDC scheme (see Equation (2.1)) and the CDC scheme (see Equation (2.6)). Moreover, we determine the pension payments during retirement in the IDC scheme based on the price of a fixed annuity (see Equation (2.2)). We are aware that these levels of pension contributions and pension payments are not necessarily optimal and one can optimize both the portfolio and consumption choice in an IDC scheme.

### A Collective Defined Contribution (CDC) scheme

In a CDC scheme, there is a single pool of assets  $A_t$ , and the cohorts collectively share market risk via the benefit adjustment process. The regulatory value of the benefits (liabilities)  $L_t$  is determined by discounting all future pension benefits  $b_{j,t}^m$  with a discount rate process. We define  $b_{j,t}^m$  as the pension benefit of cohort  $j$  at time  $t$  that will mature in  $m$  years. For the sake of simplicity, we assume a homogeneous population composition: each cohort consists of one participant.

In this paper, we use the risk-free term structure of market interest rates to discount the benefits.<sup>9</sup> The regulatory value of benefits with time to maturity  $m$  equals

$$L_t^m = \sum_{j=t}^{t+T_w+T_p-1} V_{j,t}^m = \sum_{j=t}^{t+T_w+T_p-1} b_{j,t}^m P(m, r_t) \quad (2.4)$$

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<sup>9</sup>In this case, the pension benefit level can be interpreted as the expected future benefit cashflow under the  $\tau$ -forward measure, i.e. excluding any risk premia earned in expectation. Alternatively, the pension benefit level may be interpreted as a target or expected pension benefit level. In that case one should apply a discount rate that does not ignore risk premia and thus lies above the risk-free interest rate term structure.

where  $V_{j,t}^m$  is the discounted value of the benefit of cohort  $j$  at time  $t$  that matures after  $m$  years. The regulatory value of all pension benefits is equal to

$$L_t = \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t}^m P(m, r_t) \quad (2.5)$$

The accrual of new pension benefits is actuarially fair, i.e., the present value of new benefit accrual is based on the actual risk-free market interest rate

$$\Delta b_{j,t}^m = \begin{cases} \frac{p}{\sum_{i=j-T_p-t+1}^{j-t} P(i, r_t)} & \forall m \in \{j - T_p - t + 1, j - t\}, \\ 0 & \forall m \notin \{j - T_p - t + 1, j - t\}. \end{cases} \quad (2.6)$$

where  $\Delta b_{j,t}^m$  is the new benefit accrual for cohort  $j$  at time  $t$  that matures over  $m$  years and  $p$  is the individual pension contribution.

In a CDC scheme, the pension benefits are not fixed but are adjusted based on the development of the assets and liabilities over time. The benefits are adjusted using an adjustment factor  $\delta_t^m$  in the following way

$$b_{j,t}^m = b_{j,t-1}^{m+1} \delta_t^m \quad \forall m. \quad (2.7)$$

We derive the benefit adjustment factors in the different CDC schemes in the next section.

### 2.4.3 Pension benefit adjustment process

In this section, we derive the benefit adjustment factors in different CDC schemes. We derive the pension benefits and benefit adjustments in discrete time because the benefit payments take place in discrete time. We denote  $\tilde{L}_t$  as the liabilities at time  $t$  after the realized equity and interest rate shocks but before the benefit adjustments

$$\tilde{L}_t = \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} P(m, r_t). \quad (2.8)$$

The assets in the pension scheme grow due to yearly pension contributions ( $p$  for each working cohort) and investment returns while the benefit payments (with maturity  $m = 0$ ) are subtracted

$$A_t = A_{t-1}(1 + (x_S r_{t-1}^S + x_B r_{t-1}^B + (1 - x_S - x_B) r_{t-1}^C)) + T_w p - \sum_{i=t}^{t+T_p-1} b_{i,t}^0. \quad (2.9)$$

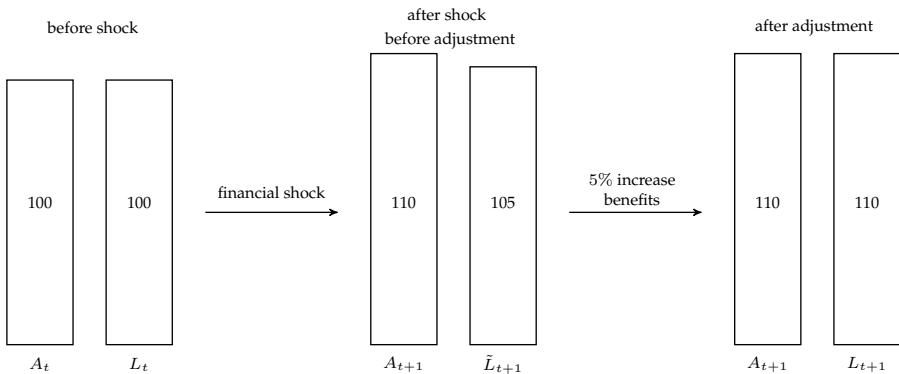
Below we introduce seven different benefit adjustment factors in a CDC scheme.

### 1. No smoothing

In a simple CDC scheme without smoothing, the pension benefits are adjusted immediately and homogeneously such that the value of the pension fund's assets is equal to the liabilities after the benefit adjustment. In the case of homogeneous adjustments, the same adjustment factor is applied for each cohort and each maturity. Figure 2.2 visualizes the balance sheet of such a pension scheme in the event of a financial shock.

**Figure 2.2: Balance sheet no smoothing**

*This figure visualizes the balance sheet of a CDC pension scheme without smoothing in the event of a financial shock.*



Before the financial shock, the assets and liabilities are identical. Directly after the financial shock, a mismatch exists between the assets and liabilities because the financial shock has a different impact on the assets and liabilities. However, a homogeneous pension benefit adjustment of 5 percent ensures that the assets are equal to the liabilities again. In the case of no smoothing, the following equality must hold to absorb shocks

$$\begin{aligned}
 L_t &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t}^m P(m, r_t) \\
 &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} \delta_t^m P(m, r_t) = \tilde{L}_t \delta_t^m = A_t,
 \end{aligned} \tag{2.10}$$

so the adjustment factor equals

$$\delta_t^m = \frac{A_t}{\tilde{L}_t} \quad \forall m \forall j. \tag{2.11}$$

The return on a pension benefit with maturity  $m$  of cohort  $j$  in a CDC pension scheme with this homogeneous benefit adjustment factor equals

$$\begin{aligned}
 \frac{V_{j,t}^m}{V_{j,t-1}^{m+1}} &= \frac{b_{j,t}^m P(m, r_t)}{b_{j,t-1}^{m+1} P(m+1, r_{t-1})} \\
 &= \frac{\delta_t^m P(m, r_t)}{P(m+1, r_{t-1})} \\
 &= \frac{A_t}{\tilde{L}_t} \frac{P(m, r_t)}{P(m+1, r_{t-1})}.
 \end{aligned} \tag{2.12}$$

The return is equal to the return on the asset portfolio multiplied by the return on a zero-coupon bond divided by the return on the liability portfolio. The product in this equation implies a complex non-linear payoff. Because of this non-linearity, it cannot be replicated with traded assets. This market inconsistency causes a CDC with homogeneous benefit adjustments to be unfair. Figure 2.1 plots the size of the market inconsistent return.

---

## 2. No smoothing, fair adjustment factors

The unfairness in the previous section can be corrected with a horizon-dependent adjustment process: adjustments that depend on the maturity of the benefit. Instead of using  $A_t/\tilde{L}_t$  as an adjustment factor, we make the benefit adjustment factor dependent on the asset-liability mismatch and the inverse return on a zero-coupon bond with maturity  $m$ , which acts as the horizon-dependent correction term

$$\delta_t^m = 1 + \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)}. \quad (2.13)$$

This benefit adjustment factor leads to the following return on benefit

$$\begin{aligned} \frac{V_{j,t}^m}{V_{j,t-1}^{m+1}} &= \delta_t^m \frac{P(m, r_t)}{P(m+1, r_{t-1})} \\ &= \left( 1 + \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)} \right) \frac{P(m, r_t)}{P(m+1, r_{t-1})} \\ &= \frac{P(m, r_t)}{P(m+1, r_{t-1})} + \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}}. \end{aligned} \quad (2.14)$$

Equation (2.14) shows that the adjustment factor in Equation (2.13) leads to a return on benefit that is a sum of returns on traded assets. It consists of the return on a zero-coupon bond plus the return on the assets minus the return on the liabilities, which is a portfolio of bonds with different maturities. Appendix 2.B shows that this adjustment factor satisfies the completeness property, i.e., after the benefit adjustment process has been applied the value of the liabilities equals the value of the assets.

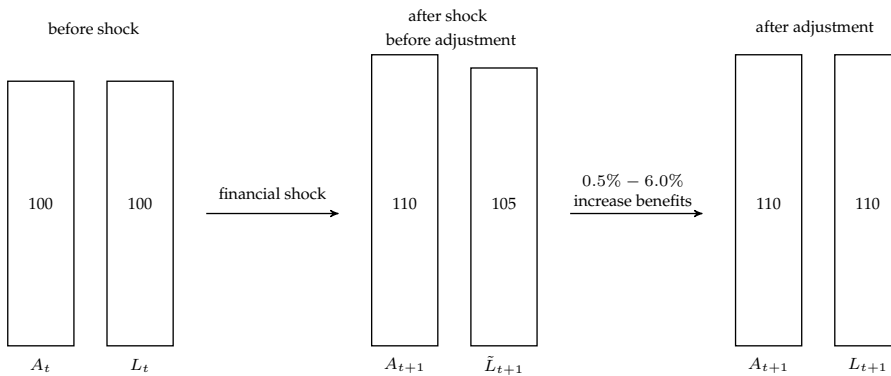
## 3. Closed smoothing

As discussed in Section 2.3.2, the benefit adjustment process includes a smoothing parameter in case of smoothing. The value of the smoothing parameter  $\alpha_m$  determines to what extent benefits with maturity  $m$  are exposed to the asset-liability mismatch. Because  $\alpha_m$  is horizon-dependent and is included in the benefit adjustment process, the benefit adjustment factor  $\delta_t^m$  is also horizon-dependent. The balance sheet under closed

smoothing is visualized in Figure 2.3. The figure shows that the asset-liability mismatch is explicitly allocated to current cohorts and shows that the size of the benefit adjustments differs between 0.5 percent and 6.0 percent depending on the smoothing parameter  $\alpha_m$ . Because the adjustment factor

**Figure 2.3: Balance sheet closed smoothing**

*This figure visualizes the balance sheet of a CDC pension scheme under closed smoothing in the event of a financial shock.*



should satisfy the completeness property, the asset-liability mismatch  $A_t - \tilde{L}_t$  should be completely absorbed by

$$\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m \quad (2.15)$$

We therefore include this term in the horizon-dependent adjustment factor

$$\delta_t^m = 1 + \alpha_m \frac{A_t - \tilde{L}_t}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m}. \quad (2.16)$$

The adjustment factor is equal to the asset-liability mismatch multiplied by the smoothing parameter and a correction term in the denominator to satisfy the completeness property.

---

#### 4. Closed smoothing, fair adjustment factors

In a CDC scheme with closed smoothing and fair adjustment factors, the adjustment factor should depend on the return mismatch  $\frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}}$  (similar to CDC scheme 2 without smoothing and with fair adjustment factors). However, in this case, it is multiplied by the smoothing parameter  $\alpha_m$ . In order to satisfy the completeness property, the adjustment factor has to contain the following correction factor such that the mismatch risk is completely absorbed

$$\frac{L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m}. \quad (2.17)$$

This leads to the following adjustment factor

$$\delta_t^m = 1 + \alpha_m \frac{L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m} \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)}. \quad (2.18)$$

This benefit adjustment factor implies the following benefit

$$\begin{aligned} \frac{V_{j,t}^m}{V_{j,t-1}^{m+1}} &= \delta_t^m \frac{P(m, r_t)}{P(m+1, r_{t-1})} \\ &= \left( 1 + \alpha_m \frac{L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m} \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)} \right) \frac{P(m, r_t)}{P(m+1, r_{t-1})} \\ &= \frac{P(m, r_t)}{P(m+1, r_{t-1})} + \frac{\alpha_m L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m} \cdot \frac{A_t}{A_{t-1}} - \frac{\alpha_m L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m} \cdot \frac{\tilde{L}_t}{L_{t-1}}. \end{aligned} \quad (2.19)$$

Equation (2.19) shows that the adjustment factor leads to a fair benefit return because the benefit return is a sum of returns of traded assets: it includes the return of a zero-coupon bond, the return on the assets, and the return on the liabilities, which act as a portfolio of bonds with different maturities. Appendix 2.B shows that this adjustment factor satisfies the completeness property.

#### 5. Open complete smoothing

As mentioned in Section 2.3.2, shocks are allocated to both current pension benefits and future pension contributions (i.e., contributions for future benefit accrual) in a CDC scheme with open complete smoothing. One can consider infinitely many years of future pension contributions, but this is

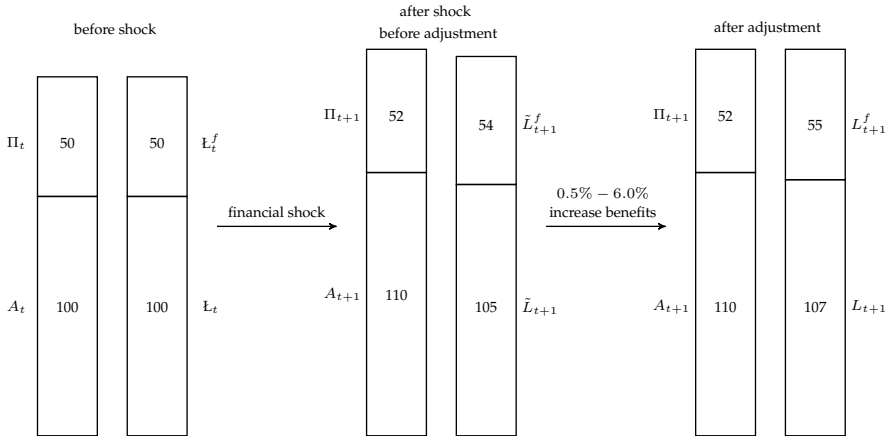
not realistic. We take  $n = 20$  years (length of smoothing period) of future pension contributions into account. We denote the present discounted value of these future pension contributions with  $\Pi_t$  at time  $t$

$$\Pi_t = \sum_{m=1}^n T_w \cdot p \cdot P(m, r_t). \quad (2.20)$$

These future pension contributions will be paid for future benefit accrual  $L_t^f$ . Again, the smoothing parameter  $\alpha_m$  determines to what extent benefits with a specific maturity are exposed to the asset-liability mismatch. The balance sheet of such a pension scheme is visualized in Figure 2.4.

**Figure 2.4: Balance sheet open complete smoothing**

*This figure visualizes the balance sheet of a CDC pension scheme under open complete smoothing in the event of a financial shock.*



The adjustment factor should satisfy the completeness property, so the asset-liability mismatch  $A_t + \Pi_t - \tilde{L}_t - \tilde{L}_t^f$  should be completely absorbed by the current liabilities and future pension accrual

$$\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} \tilde{L}_t^{f,m} \alpha_m. \quad (2.21)$$

---

This leads to the following adjustment factor

$$\delta_t^m = 1 + \alpha_m \frac{A_t + \Pi_t - \tilde{L}_t - \tilde{L}_t^f}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} \tilde{L}_t^{f,m} \alpha_m}. \quad (2.22)$$

Appendix 2.B shows that this adjustment factor satisfies the completeness property.

## 6. Open complete smoothing, fair adjustment factors

In a CDC scheme with open complete smoothing and fair adjustment factors the adjustment factor is dependent on the return mismatch  $\frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}}$  (similar to CDC scheme 2 and 4). However, because of the extended balance sheet, the return mismatch also includes future accrual  $L_t^f$  and future pension contributions  $\Pi_t$  as visualized in Figure 2.4:  $\frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} - \frac{\tilde{L}_t + \tilde{L}_t^f}{L_{t-1}^f + \tilde{L}_{t-1}^f}$ . The adjustment factor should satisfy the completeness property, so the adjustment factor has to contain the following correction factor such that the mismatch risk is completely absorbed by the liabilities and future pension contributions

$$\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} \tilde{L}_t^{f,m} \alpha_m. \quad (2.23)$$

This leads to the following adjustment factor

$$\delta_t^m = 1 + \alpha_m \frac{L_{t-1} + L_{t-1}^f}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \left( \frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} - \frac{\tilde{L}_t + \tilde{L}_t^f}{L_{t-1} + L_{t-1}^f} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)}. \quad (2.24)$$

This benefit adjustment factor implies the following benefit return

$$\begin{aligned}
 \frac{V_{j,t}^m}{V_{j,t-1}^{m+1}} &= \delta_t^m \frac{P(m, r_t)}{P(m+1, r_{t-1})} & (2.25) \\
 &= \left( 1 + \alpha_m \frac{L_{t-1} + L_{t-1}^f}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \right) \\
 &\quad \cdot \left( \frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} - \frac{\tilde{L}_t + \tilde{L}_t^f}{L_{t-1} + L_{t-1}^f} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)} \frac{P(m, r_t)}{P(m+1, r_{t-1})} \\
 &= \frac{P(m, r_t)}{P(m+1, r_{t-1})} + \frac{\alpha_m(L_{t-1} + L_{t-1}^f)}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \cdot \frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} \\
 &\quad - \frac{\alpha_m(L_{t-1} + L_{t-1}^f)}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \cdot \frac{\tilde{L}_t + \tilde{L}_t^f}{L_{t-1} + L_{t-1}^f}.
 \end{aligned}$$

Equation (2.25) shows that the adjustment factor leads to a fair benefit return because the benefit return is a sum of returns of traded assets. It includes the return of a zero-coupon bond, the return of a portfolio consisting of assets and future pension contributions, which act as a portfolio of bonds with different maturities, and the return on the liabilities, which also act as a portfolio of bonds with different maturities. Appendix 2.B shows that this adjustment factor satisfies the completeness property.

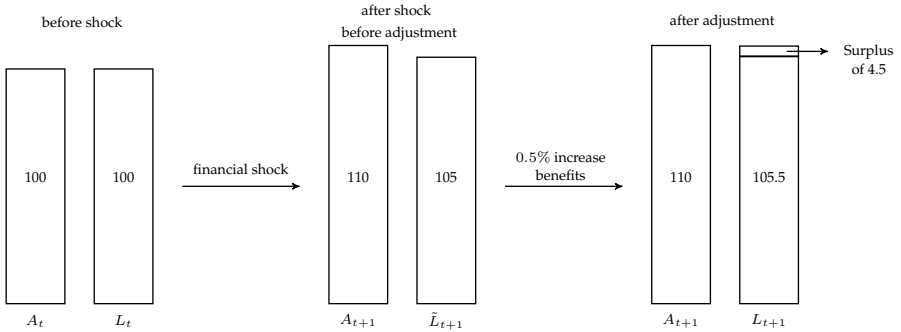
## 7. Open smoothing

Under open smoothing, the benefits are adjusted such that only  $1/n^{th}$  of the asset-liability mismatch  $1 - A_t/\tilde{L}_t$  is absorbed. Figure 2.5 visualizes the balance sheet of such a pension scheme. Because only a fraction of the asset-liability mismatch is absorbed, there is still a surplus or deficit after the benefit adjustment. After the benefit adjustment, the funding ratio equals

$$FR_t^* = \frac{A_t}{\tilde{L}_t} + \frac{1 - \frac{A_t}{\tilde{L}_t}}{n}. \quad (2.26)$$

**Figure 2.5: Balance sheet open smoothing**

This figure visualizes the balance sheet of a CDC pension scheme under open smoothing in the event of a financial shock.



We use this funding ratio to derive the benefit adjustment  $\delta_t^m$

$$FR_t^* = \frac{A_t}{\tilde{L}_t \delta_t^m} \quad (2.27)$$

$$\tilde{L}_t \delta_t^m = \frac{A_t}{FR_t^*}$$

$$\delta_t = \frac{A_t}{\tilde{L}_t FR_t^*}.$$

Table 2.4 presents an overview of the adjustment factors in the different CDC schemes. Note that only the following schemes are fair: 2, 4 and 6. The other pension schemes are not designed fairly and lead to the transfer of value between cohorts.

**Table 2.4: Pension benefit adjustment factors**

This table presents the pension benefit adjustment factor  $\delta_t^m$  in different CDC schemes.

<b>1. No smoothing</b>
$\delta_t^m = \frac{A_t}{\bar{L}_t}$
<b>2. No smoothing fair</b>
$\delta_t^m = 1 + \left( \frac{A_t}{A_{t-1}} - \frac{\bar{L}_t}{\bar{L}_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)}$
<b>3. Closed smoothing</b>
$\delta_t^m = 1 + \alpha_m \frac{A_t - \bar{L}_t}{\sum_{m=0}^{T_w + T_p - 1} \bar{L}_t^m \alpha_m}$
<b>4. Closed smoothing fair</b>
$\delta_t^m = 1 + \alpha_m \frac{L_{t-1}}{\sum_{m=0}^{T_w + T_p - 1} L_{t-1}^m \alpha_m} \left( \frac{A_t}{A_{t-1}} - \frac{\bar{L}_t}{\bar{L}_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)}$
<b>5. Open complete smoothing</b>
$\delta_t^m = 1 + \alpha_m \frac{A_t + \Pi_t - \bar{L}_t - \bar{L}_t^f}{\sum_{m=0}^{T_w + T_p - 1} \bar{L}_t^m \alpha_m + \sum_{m=1}^{T_w + T_p + n - 1} \bar{L}_t^{f, m} \alpha_m}$
<b>6. Open complete smoothing fair</b>
$\delta_t^m = 1 + \alpha_m \frac{L_{t-1} + L_{t-1}^f}{\sum_{m=0}^{T_w + T_p - 1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w + T_p + n - 1} L_{t-1}^{f, m} \alpha_m} \cdot \left( \frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} - \frac{\bar{L}_t + \bar{L}_t^f}{L_{t-1} + L_{t-1}^f} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)}$
<b>7. Open smoothing</b>
$\delta_t^m = \frac{A_t}{FR_t^* \bar{L}_t} \quad \text{with} \quad FR_t^* = \frac{A_t}{\bar{L}_t} + \frac{1 - \frac{A_t}{\bar{L}_t}}{n}$

### Smoothing parameter

The smoothing parameter  $\alpha_m$  is often linear in practice but can also have a different distribution (e.g., polynomial or exponential). In this paper, we consider a linear smoothing parameter only. Under linear smoothing, the smoothing parameter is defined as follows

$$\alpha_m = \begin{cases} \frac{m}{n} & \text{for } m \leq n, \\ 1 & \text{for } m > n. \end{cases} \quad (2.28)$$

---

This definition implies that for a pension benefit with maturity  $m = 1$ , the smoothing parameter is equal to  $1/n$ . The value of the smoothing parameter increases linearly with the maturity  $m$ . For benefits with maturity  $m > n$ , the smoothing parameter equals 1.

The next step is to introduce the welfare analysis that we will perform. We use the welfare analysis to determine the investment policy.

#### 2.4.4 Welfare analysis

To evaluate efficiency in CDC schemes we derive the optimal asset allocation using a welfare analysis in which social welfare is maximized. As mentioned in Section 2.4.2, we use a constant relative risk aversion (CRRA) utility function to describe the preferences of cohorts with respect to consumption  $C$  during retirement

$$U(C_{j,t}) = \frac{C_{j,t}^{1-\gamma}}{1-\gamma}. \quad (2.29)$$

We assume all cohorts have a risk aversion parameter equal to  $\gamma = 5$ . The total expected utility during retirement for cohort  $j$  equals

$$U_j = \mathbb{E} \left[ \sum_{t=\max(1, j-T_p+1)}^j \beta^{t-j+T_p} \frac{C_{j,t}^{1-\gamma}}{1-\gamma} \right]. \quad (2.30)$$

The maximum in the summation index is needed because the oldest cohorts do not go through the whole retirement period in the simulation. The total welfare, or social welfare  $SW$ , is determined by the expected utility of all cohorts

$$SW = \sum_{j=1}^{T_s+T_p-1} \beta^{j-1} U_j. \quad (2.31)$$

We are interested in the certainty equivalent of the total welfare  $SW$ , which is determined by the expected utility of all cohorts in the following way

$$CEC = \left[ \frac{SW(1-\beta^2)(1-\gamma)}{(1-\beta^{T_p})(1-\beta^{T_s+T_p})} \right]^{1/(1-\gamma)}. \quad (2.32)$$

The derivation of this certainty equivalent of total welfare can be found in Appendix 2.C. We maximize this certainty equivalent of total welfare

$$\max_x CEC, \quad (2.33)$$

under the assumption that short selling and borrowing are not allowed

$$\begin{cases} 0 \leq x_i \leq 1 & i = S, B \\ x_S + x_B \leq 1. \end{cases} \quad (2.34)$$

We impose this assumption because short selling and borrowing are generally not allowed in pension schemes in practice.

### 2.4.5 Optimal market risk exposures

We evaluate efficiency in CDC schemes by comparing the market risk exposures in CDC schemes with the optimal market risk exposures in IDC schemes. A CDC scheme potentially restricts some cohorts from having an optimal exposure to market risks. Although the benefit adjustment process and the smoothing policy lead to a differentiated exposure to market risks for cohorts, there are limits to what they can accomplish. To assess whether it is possible to design a CDC scheme with an optimal exposure for all cohorts, we first determine the optimal exposure to market risks in an IDC scheme. In an IDC scheme, the optimal exposure to equity market risk and interest rate risk changes over the life-cycle.

Our financial market model, as formulated in Table 2.2, is similar to Brennan and Xia (2002), although we exclude inflation risk. Moreover, for ease of exposition, we assume that the correlation between equity and interest rate risk equals zero. We start by optimizing the asset allocation assuming financial capital only in a terminal wealth problem. Appendix 2.D shows that the optimal asset allocation  $\mathbf{x}^*$  in the terminal wealth problem equals

$$\begin{bmatrix} x_B^* \\ x_S^* \end{bmatrix} = \begin{bmatrix} -\lambda_r/(\gamma\sigma_r B(tt)) \\ \lambda_S/(\gamma\sigma_S) \end{bmatrix} + \frac{\gamma-1}{\gamma} \begin{bmatrix} B(T)/B(tt) \\ 0 \end{bmatrix}, \quad (2.35)$$

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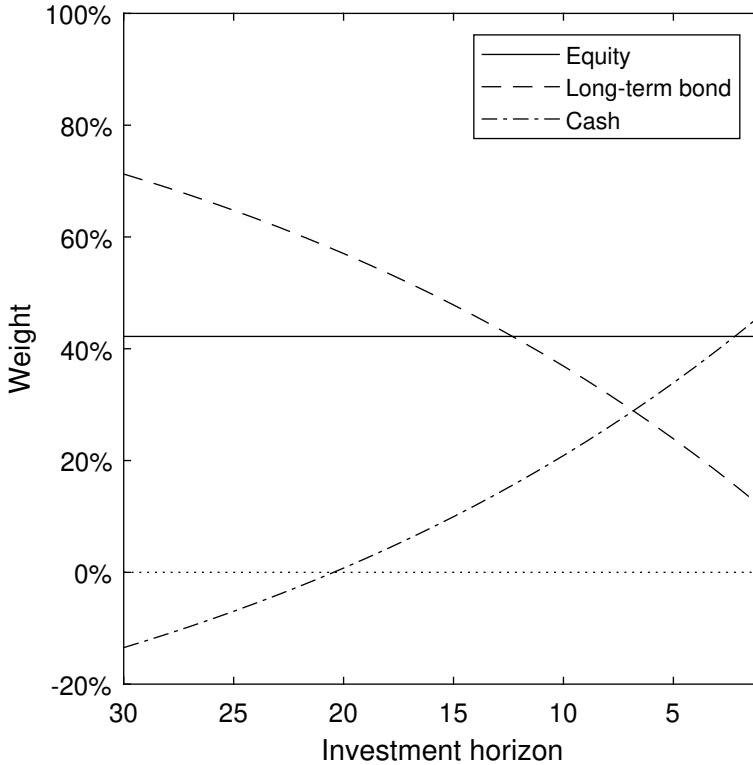
where  $x_B^*$  is the optimal allocation to a long-term bond with a fixed time to maturity  $tt = 50$  and  $x_S^*$  is the optimal allocation to equity. The first term of the right-hand side of Equation (3.25) is the speculative demand. The individual wants to profit from the interest rate risk premium and equity premium. The amount invested in the long-term bond and equity is inversely related to the relative risk aversion  $\gamma$ . The second term is the hedging demand. The individual wants to hedge against a declining interest rate. The hedging demand is higher for individuals with a high risk aversion, and is inversely related to the time to maturity of the long-term bond. Figure 2.6 shows the optimal asset allocation in a terminal wealth problem of Equation (3.25) with parameter values defined in Table 2.2 and 2.3. The optimal allocation to equity does not depend on the investment horizon. This is a well-known result that goes back to Merton (1971). The optimal allocation to the long-term bond increases with the investment horizon due to a higher hedging demand. The figure shows that for long investment horizons borrowing money is optimal, i.e., the optimal asset allocation to equity and the long-term bond exceeds 100 percent.

However, our problem is not a terminal wealth problem but a life-cycle problem, with human wealth. In a life-cycle problem total wealth  $W_t$  consists of financial wealth  $F_t$  and human wealth  $H_t$ , where in our case human wealth (or human capital) is defined as the present discounted value of future pension contributions. In line with Bodie et al. (1992), we assume that future pension contributions are a traded asset, i.e., future pension contributions are equivalent to a risk-free bond. Since the individual can only invest their financial wealth in financial markets, the optimal portfolio allocation of financial wealth  $\hat{x}^*$  considers the human capital's bond-like behavior. The optimal portfolio allocation of financial wealth is derived in Appendix 2.D and equals

$$\begin{bmatrix} \hat{x}_{B,t}^* \\ \hat{x}_{S,t}^* \end{bmatrix} = \begin{bmatrix} \frac{W_t}{F_t} x_B^* - \frac{H_t}{F_t} \frac{D_t^H}{B(tt)} \\ \frac{W_t}{F_t} x_S^* \end{bmatrix}, \quad (2.36)$$

**Figure 2.6: Optimal asset allocation in a terminal wealth problem**

This figure visualizes the optimal asset allocation to equity and a long-term bond with maturity  $t = 50$  in a terminal wealth problem.



where  $D_t^h$  is the duration of human capital

$$D_t^h = \int_0^{T_w + T_p - t} \frac{H_{t,h}}{H_t} B(h) dh. \tag{2.37}$$

Because we define human capital as the discounted value of future pension contributions, the duration of human capital depends on the age of the individual. In line with standard life-cycle theory and as shown by Bodie et al. (1992), the share of financial wealth invested in equity decreases with age.

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Because human capital acts as a risk-free bond, the individual wants to invest a larger part of their financial wealth in equity to obtain the preferred overall exposure to equity market risk. The optimal portfolio allocation of financial wealth to the long-term bond  $\hat{x}_{B,t}^*$  can be rewritten as follows

$$\begin{aligned}
 \hat{x}_{B,t}^* &= \frac{W_t}{F_t} x_B^* - \frac{H_t}{F_t} \frac{D_t^H}{B(tt)} & (2.38) \\
 &= x_B^* + \frac{H_t}{F_t} \left( x_B^* - \frac{D_t^H}{B(tt)} \right) \\
 &= x_B^* + \frac{H_t}{F_t} \left( \frac{D_t^W - D_t^H}{B(tt)} \right),
 \end{aligned}$$

where  $D_t^W \equiv x_B^* B(tt)$  denotes the duration of the optimal long-term bond portfolio.<sup>10</sup> This alternative expression makes it easier to interpret the optimal portfolio allocation of financial wealth to the long-term bond. Since the duration of human capital is usually unequal to the duration of the optimal long-term bond portfolio, the optimal portfolio allocation of financial wealth to the long-term bond is not equal to the allocation of total wealth. When the duration of human capital  $D_t^H$  is smaller than the duration of the optimal bond portfolio  $D_t^W$ , the second term in Equation (2.38) is positive. As a result, the optimal allocation of financial wealth to the long-term bond  $\hat{x}_{B,t}^*$  is bigger than the terminal wealth solution  $x_B^*$ . This makes sense, since the interest rate risk exposure of human capital is insufficient. In contrast, when the duration of human capital is bigger than the duration of the optimal bond portfolio, the second term in Equation (2.38) is negative, implying a smaller portfolio allocation of financial wealth. In this case, the interest rate risk exposure of human capital is higher than required, so the individual should reduce the allocation to the long-term bond to realize the required hedge demand. Equation (2.38) shows that the optimal portfolio allocation to financial wealth is state-dependent, i.e., the portfolio allocation depends on the actual values of financial and human wealth. Figure 2.7 shows the median of the optimal asset allocation in a life-cycle problem of Equation (2.36) and (2.38). The graph shows that the optimal allocation to equity of financial wealth de-

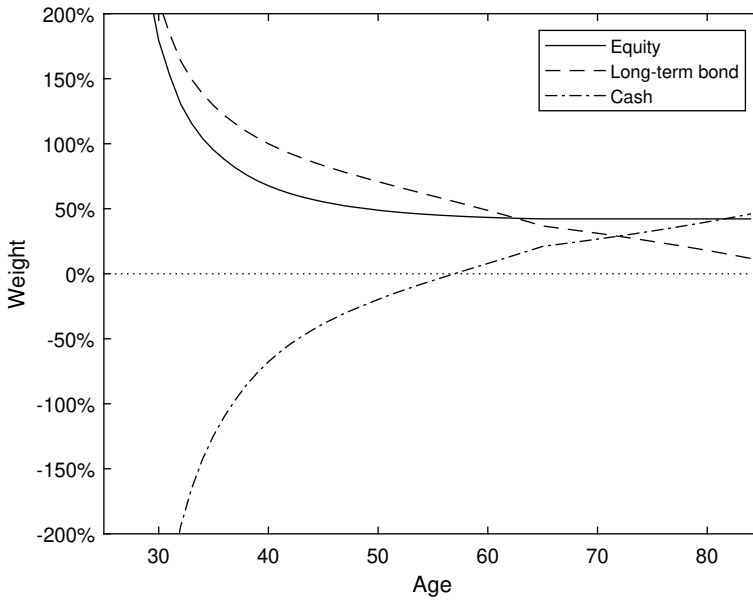
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<sup>10</sup> $B(tt)$  is defined in Appendix 2.A.

creases with age. Moreover, the graph shows that the optimal allocation to the long-term bond also decreases with age because the duration of human capital is smaller than the duration of the optimal bond portfolio in our setting.

**Figure 2.7: Optimal asset allocation in a life-cycle problem**

*This figure visualizes the median of the optimal allocation to equity, a long-term bond with maturity  $tt = 50$ , and cash in a life-cycle problem.*



Besides the optimal portfolio choice there is also an optimal consumption choice (e.g., Merton (1971), Brennan and Xia (2002), Wachter (2002), Munk (2008), van Bilsen et al. (2020a)). The optimal consumption choice determines how accumulated wealth should be consumed during retirement. As discussed in Section 2.4.2, consumption during retirement in an IDC scheme can be determined using the variable annuity price (see Equation (2.2)). However, this consumption profile is not necessarily optimal. Therefore, we also compare the CDC schemes with an IDC Scheme with an optimal consumption choice. The optimal consumption choice is presented in Appendix 2.E.

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## 2.5 Results

We now have all the design features and assumptions in place to study how the CDC schemes perform in terms of fairness and efficiency. Table 2.4 summarized the different pension schemes. The CDC schemes 2 (no smoothing fair), 4 (closed smoothing fair) and 6 (open complete smoothing fair) are all designed in a fair way so there is no need to study their fairness further. However, there are also a number of unfair schemes in the table. We therefore start in Section 2.5.1 by calculating the wealth transfers between cohorts in the following CDC schemes: 1 (no smoothing), 3 (closed smoothing), 5 (open complete smoothing) and 7 (open smoothing). These schemes are not designed fairly. In Section 2.5.2, we assess the efficiency of the different fair CDC schemes of Table 2.4 (schemes 2, 4 and 6). To that end, we compare these schemes' implied exposure to market risks with the optimal exposure to market risks in an IDC scheme. Finally, we compare the welfare under these three schemes with the welfare under an IDC scheme. In the efficiency analysis we focus on the fair CDC schemes because these are most interesting from a design perspective and because we can only derive the implied exposure to market risks for fair CDC schemes.

### 2.5.1 Unfairness in different CDC schemes

In Section 2.2.1 we already saw that uniform benefit adjustments lead to a market-inconsistent return on benefits. This market-inconsistent return results in a wealth transfer from young to old cohorts if the mismatch risk is not fully hedged. We verify this by simulating a CDC pension scheme with an overlapping generation model with a simulation period of  $T_s = 85$  years. To compare the unfairness in different CDC pension schemes, we assume the same asset allocation for each scheme:  $x_S = 50\%$  in equity,  $x_B = 15\%$  in a long-term bond, and the remainder in cash. The average interest rate hedge of e.g., Dutch pension funds equals 40 percent (DNB (2015)). The aggregate duration of the benefits of the pension fund equals 19 years.<sup>11</sup> The measure we use is unfairness, which we define as the difference between the market

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<sup>11</sup>This implies an interest rate hedge of  $(15\% * 50 / 19 =)$  40% in the case of long-term bonds with maturity  $tt = 50$ .

value of the pension benefits for each cohort and the market value of the pension contributions.<sup>12</sup> The distribution of unfairness over different cohorts is a zero-sum game: the sum of the unfairness of all cohorts equals zero. A positive unfairness for one cohort is at the expense of a negative unfairness for another cohort.

### **Unfairness in a CDC scheme without smoothing**

Figure 2.8 plots the unfairness as a percentage of annual income for different cohorts in a CDC scheme without smoothing. The oldest cohort is already retired and only has one year to live at the start of the simulation (age 84). The youngest cohort in Figure 2.8 is born at the start of the simulation (age 0). This cohort starts working 25 years after the start of the simulation. The figure shows that there is a wealth transfer from young to old cohorts. The unfairness of the oldest cohorts (age 48 to 85) is positive because the market-inconsistent excess return on benefits with a short maturity is positive (see Figure 2.1). The maximum positive unfairness equals about 50 percent of annual income. In return, the unfairness of the young cohorts (age below 47) is negative because the market-inconsistent excess return of benefits with a long maturity is negative. The maximum negative unfairness equals about 25 percent. Although the cohorts aged between 5 and 25 at the start of the simulation go through a complete life-cycle during the simulation period, the unfairness is still negative. Apparently, the loss due to a negative market-inconsistent excess return when young is bigger than the benefit of the positive market-inconsistent excess return when old. Because Figure 2.8 does not visualize the unfairness of cohorts who are not yet born at the start of the simulation, the sum of the unfairness of all cohorts in this figure is not equal to zero. However, when all cohorts are included, the sum of the unfairness equals zero.

Of course, the size and distribution of the unfairness depend on the finan-

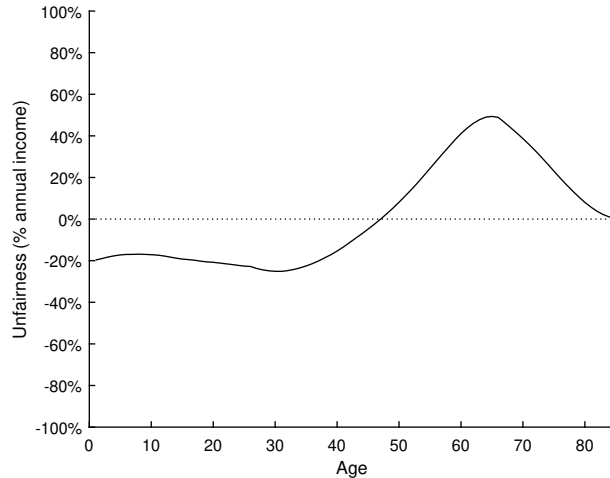
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<sup>12</sup>We have to make some additional assumptions for cohorts that do not go through the whole life-cycle in the simulation. We assume that the market value of the pension contributions paid before the start of the simulation is equal to the market value of the pension benefits at the start of the simulation. Moreover, we assume that the value of the pension benefits that will be paid out after the simulation is equal to the market value of the remaining pension benefits at the end of the simulation.

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**Figure 2.8: Unfairness in CDC scheme without smoothing**

*This figure visualizes the unfairness (as a percentage of annual income) for different cohorts in a simple CDC scheme without smoothing. The horizontal axis presents the age of each cohort at the start of the simulation.*



cial market specification, the asset allocation, and the population composition. Therefore, we perform some sensitivity analyses that are presented in Appendix 2.F. For example, when the allocation to the long-term bond is higher and thus more interest rate risk is hedged, the unfairness is significantly smaller (see Figure 2.F.1 in Appendix 2.F). Furthermore, the interest rate volatility ( $\sigma_r$ ) greatly impacts the size of the unfairness (see Figure 2.F.2 in Appendix 2.F). If we halve the interest volatility from  $\sigma_r = 0.01$  to  $\sigma_r = 0.005$ , for instance, we see a significant drop in the unfairness. Also the population composition has a significant impact on the size of the unfairness. In the main analysis, we assume a homogeneous population composition: each cohort consists of 1 participant. However, pension funds can have different population compositions in practice. Figure 2.F.3 shows the size of the unfairness for different population compositions. Besides the standard population composition we consider a green pension fund (relatively more young participants) and a grey pension fund (relatively more old participants). We assume that

the size of each cohort increases (or decreases) by 2 percent over time for the green (or grey) pension fund. Figure 2.F.3 shows that the positive unfairness for the old cohorts is much bigger in a green pension fund and the negative unfairness for the young cohorts is bigger in a grey pension fund.

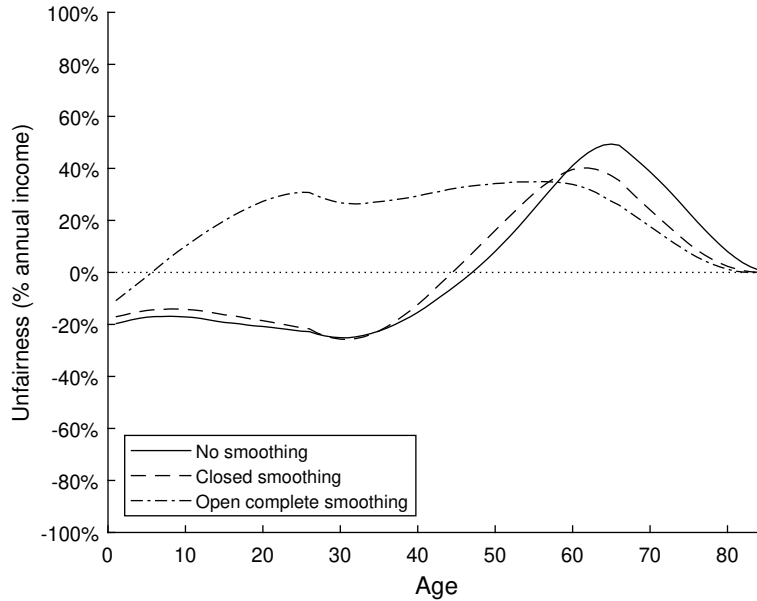
### The impact of smoothing on fairness

We are also interested in the impact of smoothing on fairness. As mentioned in Section 2.4.2, we consider three smoothing policies: closed smoothing, open complete smoothing, and open smoothing. Here we only consider the unfair specifications of these policies, i.e., specifications 3, 5 and 7 from Table 2.4. Figure 2.9 shows the unfairness in the different complete CDC schemes (schemes 3 and 5). We see that the development of unfairness in a CDC scheme with closed smoothing (scheme 3) is very similar to the development of unfairness in a CDC scheme without smoothing (scheme 1). However, the positive fairness for old cohorts (age 60 and higher) is slightly smaller under closed smoothing. Under closed smoothing the benefit adjustments of old cohorts are smaller than under no smoothing because the benefit adjustments include the smoothing parameter  $\alpha_m$ . As a result, the market-inconsistent excess return is also smaller under closed smoothing. When looking at the unfairness in the CDC scheme with open complete smoothing (scheme 5), we notice a positive unfairness for most ages. However, the unfairness is negative for the youngest cohorts, but these cohorts are not shown in Figure 2.9. This result makes sense, because the value transfer from young to old cohorts is bigger in this scheme since future benefits are also taken into account.

Finally, we look at the unfairness in a CDC scheme with open smoothing (scheme 7). By definition, an open smoothing policy cannot be made fair because some value is not explicitly allocated to cohorts in the pension scheme. Moreover, the unfairness depends greatly on the initial funding ratio at the start of the simulation. Figure 2.10 therefore shows the unfairness in a CDC scheme with open smoothing for different initial funding ratios. When looking at the unfairness in Figure 2.10, we first notice that the size of the unfairness is much bigger compared to the complete CDC schemes in Figure 2.9.

**Figure 2.9: Unfairness in different CDC schemes**

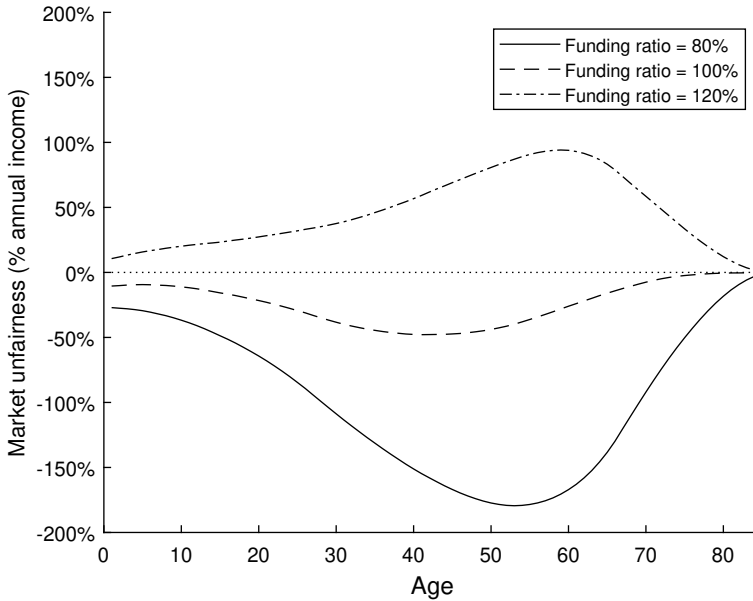
This figure visualizes the unfairness (as a percentage of annual income) for different cohorts in different CDC schemes. The horizontal axis presents the age of each cohort at the start of the simulation.



Moreover, we see that the sign of the unfairness for the oldest cohorts in a CDC scheme with open smoothing and an initial funding ratio of 100 percent is opposite to the complete CDC schemes in Figure 2.9. The unfairness of the oldest cohorts is negative under open smoothing, while it is positive under closed smoothing or without smoothing. The reversed development of the unfairness can be explained by the fact that in a scheme with open smoothing only  $1/n^{th}$  of the asset-liability mismatch is absorbed by the current benefits. Because the asset-liability mismatch is positive in expectation due to the positive risk premium on equity, a positive asset-liability mismatch is passed on to future cohorts. This is at the expense of old cohorts and leads to a negative unfairness for old cohorts.

**Figure 2.10: Unfairness in CDC scheme under open smoothing**

This figure visualizes the unfairness (as a percentage of annual income) for different cohorts in a CDC scheme under open smoothing for different initial funding ratios. The horizontal axis presents the age of each cohort at the start of the simulation.



### 2.5.2 Efficiency

In the previous section we saw that the wealth transfers in unfair CDC schemes can be substantial. In this section we turn our attention to efficiency. We want to assess whether it is possible to design a CDC scheme with an optimal exposure to market risks for all cohorts. We analyze efficiency in a CDC scheme by assessing the implied market exposure and welfare for each cohort. First, we derive the implied exposure to market risks in different CDC schemes and compare this with the optimal exposure to market risks in an IDC scheme (see Section 2.4.5). Although a CDC scheme has only one collective asset allocation, each cohort has a different exposure to market risks due to the benefit adjustment process. The smoothing policy in particular has a significant impact on the implied exposure to market risks. We only consider

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the fair CDC pension schemes (schemes 2, 4 and 6) in this section because we can only derive the implied exposure to market risks for these schemes. The return of the CDC pension benefits in unfair schemes is a complex non-linear payoff that cannot be replicated with traded assets (see Section 2.4.3). Second, we compare the certainty equivalent consumption for each cohort in different fair CDC schemes with the certainty equivalent consumption in an IDC scheme.

### Derivation of implied exposures in CDC scheme

We derive the implied equity allocation and implied duration by determining the replicating portfolio that yields the same return as the return on benefit. Equation (2.14) in Section 2.4.3 shows that the return on a benefit with maturity  $m$  of cohort  $j$  in a fair CDC scheme without smoothing equals

$$\frac{V_{j,t}^m}{V_{j,t-1}^{m+1}} = \frac{P(m, r_t)}{P(m+1, r_{t-1})} + \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}}. \quad (2.39)$$

The return on benefit consists of three returns: the return on a zero-coupon bond with maturity  $m$ , the return on the asset portfolio, minus the return on the liability portfolio of the pension fund, which is a portfolio of bonds with different maturities. We use this equation to determine the implied equity allocation and implied duration.

Because the return on benefit consists of the return on the asset portfolio and the asset portfolio includes equity (see Section 2.4.1), the replicating portfolio of the return on benefit includes equity. As a result, the implied equity allocation  $IE$  in a CDC scheme without smoothing simply equals  $x_S$  for each cohort because  $x_S$  is invested in equity.

The implied exposure to interest rate risk for maturity  $m$  can be found by deriving the implied duration of the portfolio that replicates the return on a benefit with maturity  $m$   $ID_{m,t}$  (see Equation (2.39)). This equals the duration  $DB_m$  of a zero-coupon bond with maturity  $m$  plus the duration  $DA$  of the asset portfolio minus the duration  $DL_t$  of the liability portfolio of the

pension fund

$$ID_{m,t} = DB_m + DA - DL_t = m + x_B \cdot 50 - DL_t. \quad (2.40)$$

The duration  $DB_m$  of a zero-coupon bond with maturity  $m$  equals  $m$  and the duration  $DA$  of the asset portfolio equals the long-term bond allocation  $x_B$  multiplied by the duration of the long-term bond, which is  $tt = 50$ . The duration  $DL_t$  of the liability portfolio of the pension fund equals 19 on average but is time dependent because it depends on the size of the benefits with different maturities. An implied duration of zero means that the discounted value of pension benefits is not sensitive to interest rate changes. A positive implied duration means that the discounted value of benefits is decreased if the interest rate decreases, and a negative implied duration means that the discounted value of benefits is increased if the interest rate decreases.

While Equation (2.40) shows the implied duration of one benefit with maturity  $m$ , we are interested in the implied duration of all benefits of one cohort. Therefore, we have calculated the implied duration of all benefits of a cohort. The implied duration of the benefits of a cohort equals the average implied duration of all benefits of the cohort weighted by the size of these benefits.

In a CDC scheme with smoothing, the implied equity allocation and implied duration can be derived in a similar way using the return of the value of pension benefits in Equation (2.19) for closed smoothing and in Equation (2.25) for open complete smoothing. Table 2.5 presents an overview of the implied equity allocation and implied duration for the three different pension schemes that we consider in this subsection.

### Optimal asset allocation in CDC scheme

Before we can compare the implied market exposures in a CDC scheme with the optimal market exposures in an IDC scheme, we have to determine the optimal asset allocation in a CDC scheme. As discussed in Section 2.4.4, we derive the optimal asset allocation in a CDC scheme using a welfare analysis in which social welfare is maximized. The asset allocation consists of the equity allocation ( $x_S$ ) and the exposure to long-term bonds ( $x_B$ ). The remainder

**Table 2.5: Implied market exposures**

This table shows implied equity allocation and implied duration for all cohorts in different fair CDC schemes.

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<b>No smoothing fair</b>	
$IE = x_S$	
$ID_{m,t} = DB_m + DA - DL_t$	
<b>Closed smoothing fair</b>	
$IE_{m,t} = x_S \cdot \alpha_m \frac{L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m}$	
$ID_{m,t} = DB_m + \alpha_m \cdot \frac{L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m} (DA - DL_t)$	
<b>Open complete smoothing fair</b>	
$IE_{m,t} = x_S \cdot \alpha_m \frac{L_{t-1} + L_{t-1}^f}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \cdot \frac{A_t}{A_t + \Pi_t}$	
$ID_{m,t} = DB_m + \alpha_m \cdot \frac{L_{t-1} + L_{t-1}^f}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \cdot (DA + D\Pi - DL_t - DL_t^f)$	

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( $x_C = 1 - x_S - x_B$ ) is invested in cash. Using the financial market model parameters in Table 2.2, and the pension scheme characteristics in Table 2.3, the optimal asset allocation is calculated for each CDC scheme. Table 2.6 shows the optimal asset allocations. First of all, we notice that the asset allocations are very similar. Moreover, we notice that in each scheme, the allocation to cash equals zero. This is a result of the term premium when investing in long-term bonds. The return on the long-term bond is in expectation higher than the return on cash, so the long-term bond will always be preferred over cash.

### Comparison of implied exposures with an individual DC scheme

In this section we use the equity allocation ( $x_S$ ) and the long-term bond allocation ( $x_B$ ) in Table 2.6 to calculate the implied exposures in the different schemes using the formulas in Table 2.5. Subsequently, we compare these implied market exposures with the optimal market exposures in an IDC

**Table 2.6: Optimal asset allocation in different CDC schemes**

*This table shows the optimal allocation to equity, a long-term bond with maturity  $tt = 50$ , and cash in different fair CDC schemes.*

Smoothing policy	$x_S$	$x_B$	$x_C$
No smoothing fair	47.9%	52.1%	0.0%
Closed smoothing fair	48.2%	51.8%	0.0%
Open complete smoothing fair	46.5%	53.5%	0.0%

scheme. Figure 2.11 shows this comparison for the three fair CDC schemes: no smoothing, closed smoothing and open complete smoothing. The figure shows the implied equity allocation in the CDC schemes and the median equity allocation in the optimal IDC scheme in the left-hand panel.<sup>13</sup> The right-hand panel shows the implied duration in the CDC schemes and the median duration in the optimal IDC scheme. In the CDC schemes with a smoothing policy, a smoothing period of  $n = 20$  years is used.

The equity allocation in an optimal IDC scheme is above 100 percent for the youngest cohorts because human wealth is relatively large and financial wealth is relatively small. The optimal equity allocation in the optimal IDC scheme is fixed after retirement. The left-hand panel shows that the equity allocation is the same for all cohorts in a CDC scheme under no smoothing; it equals  $x_S$ . Under smoothing, the implied equity allocation is higher for young cohorts and lower for older cohorts. This is roughly in line with life-cycle theory. However there are key differences compared to the optimal equity allocation in the IDC scheme. Under the closed smoothing scheme the young and the old cohorts have an excessively low equity allocation, while the middle aged cohorts have an excessively high equity allocation. Under open complete smoothing, the implied equity allocation is even above 100 percent for the youngest cohorts because risk is also allocated to future accrual. We conclude that the implied equity allocation before retirement under open complete smoothing is the closest to the optimal exposure. After retirement, the optimal equity allocation in an IDC scheme is constant, so the CDC scheme without smoothing is most similar to the optimal exposure for retired

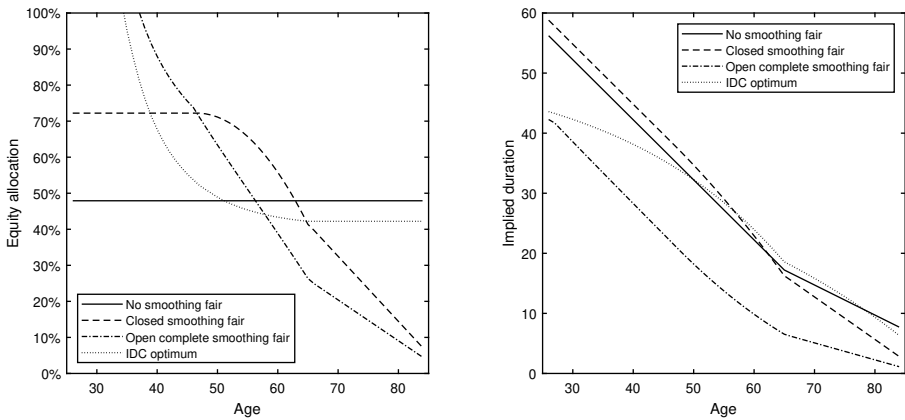
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<sup>13</sup>We determine the median of the equity allocation across all financial market scenarios.

cohorts. However, retirees under the closed smoothing and open complete smoothing end up with an equity allocation that is well below the optimal equity allocation in the IDC scheme.

**Figure 2.11: Comparison of implied market exposures**

*This figure compares the implied equity allocation and implied duration in different CDC schemes and compares it to the median equity allocation and median duration in the optimal portfolio in an IDC scheme for each age over the life-cycle.*



The right-hand graph shows the implied duration. In the optimal IDC scheme the implied duration decreases with age. This decreasing optimal duration is the result of a decreasing hedging demand (see second term in Equation (3.25)) as the investment horizon decreases. Also, in the three CDC schemes the implied duration decreases with age. In a CDC scheme the implied duration of a benefit with maturity  $m$   $ID_{m,t}$  depends on the duration  $DB_m$  of a zero-coupon bond with maturity  $m$  (see Table 2.5). Under no smoothing, the implied duration for the oldest cohort is equal to minus the duration mismatch of the pension scheme  $-(DL_t - DA)$ . Because of the large investment in long-term bonds in the optimal asset allocation, the duration of the asset portfolio is greater than the duration of the liability portfolio. Under closed smoothing, the implied duration converges to almost zero because old cohorts share less in the mismatch risk and thus are less exposed to interest rate changes. Under open complete smoothing, the mismatch risk also includes

future pension contributions and future accrual. The duration of future accrual  $DL_t^f$  is significantly higher than the duration of future pension contributions  $\Pi_t$  because the pension payments lie further in the future compared to the pension contributions. As a result, including future pension contributions and future accrual decreases the positive duration mismatch between the asset and liability portfolio. Due to this smaller duration mismatch the implied duration for each cohort under open complete smoothing is lower. When comparing the duration in the optimal IDC scheme with the implied duration in the different CDC schemes, we conclude that after age 50 the implied duration in a CDC scheme without smoothing is almost identical to the optimal duration.

Although the implied exposures are generally in line with the optimal exposures, it is not always possible to replicate precisely the optimal individual exposure to market risks in a CDC scheme. For example, before retirement, the optimal exposure to market risks in the IDC scheme is state-dependent because the optimal exposure depends on financial and human wealth. Figure 2.11 only presents the median exposure and duration for the IDC scheme. The state dependency in the optimal IDC scheme cannot be replicated in the three CDC schemes.<sup>14</sup>

### **Comparison of certainty equivalent consumption with an individual DC scheme**

Besides comparing the exposure to market risks in CDC schemes with the optimal exposure to market risks in an IDC scheme, we also compare the certainty equivalent consumption (CEC) across the different pension schemes to analyze efficiency. We compare the CEC in an optimal IDC scheme with the CEC in a CDC scheme for all cohorts.

For this comparison we make the following assumptions. The youngest cohort (age 25) starts working at the start of the simulation and goes through the whole life-cycle, in either an IDC or an CDC scheme. Older cohorts, by contrast, have already accrued wealth in the IDC scheme or pension benefits in the CDC scheme before the start of the simulation. For a fair comparison,

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<sup>14</sup>The implied exposure in the CDC schemes depends on the value of the assets and liabilities (see Table 2.5). However, this has a small impact on the implied exposure.

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we assume that the discounted value of the initial benefits in a CDC scheme equals the initial wealth in an IDC scheme for each cohort.

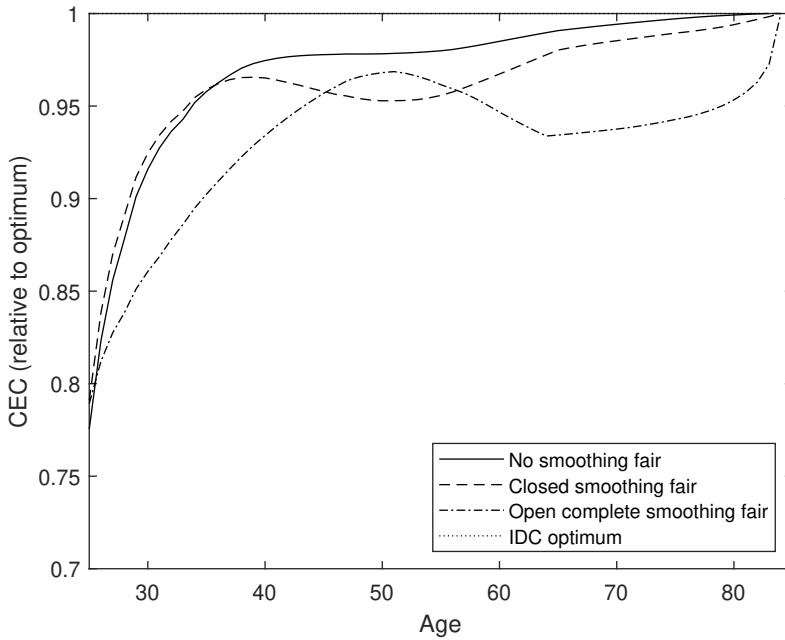
Figure 2.12 shows the CEC in different CDC schemes relative to the CEC in an optimal IDC scheme for all cohorts. The optimum in the IDC is a horizontal line at 1. The development of the relative CEC in Figure 2.12 can be explained to a large extent by the differences in implied exposure in Figure 2.11.

In a CDC scheme with no smoothing, the CEC is approximately equal to the CEC in an optimal IDC scheme for the oldest cohorts. The left-hand panel of Figure 2.11 shows that the equity allocation is constant after retirement in these two schemes and the right-hand graph shows that the implied duration is almost identical in both schemes. However, for the younger cohorts the CDC scheme with no smoothing underperforms the optimal IDC scheme because the equity allocation is too low. In a CDC scheme under closed smoothing, the CEC is below the CEC in an optimal IDC scheme for the oldest cohorts because the implied equity allocation is too low. For young cohorts, the relative CEC decreases significantly, which is a result of the excessively low equity allocation and the excessively high implied duration for young cohorts (see Figure 2.11). The CEC in a CDC scheme under open complete smoothing is significantly lower for retired cohorts compared to the CEC in an optimal IDC scheme. This is a result of the excessively low equity allocation and the excessively low implied duration in the CDC scheme under open complete smoothing for the retired cohorts.

There are several reasons why the CEC in a CDC scheme is lower than the CEC in an optimal IDC scheme. First, the implied exposure to market risks in a CDC scheme deviates from the optimal exposure to market risks in an IDC scheme as discussed in the previous paragraph (see Figure 2.11). Second, the consumption choice during retirement impacts the CEC. The optimal consumption choice during retirement depends, among other things, on the actual value of financial wealth (see Equation (2.110)) and changes in consumption are smoothed during the remaining lifetime. In a CDC scheme consumption is determined by the pension benefits and is not optimized during retirement. Third, the optimal asset allocation in an IDC scheme is state-dependent before retirement, i.e., the optimal asset allocation depends on the

**Figure 2.12: CEC for all cohorts**

This figure shows the CEC in different CDC schemes relative to the CEC in an optimal IDC scheme for different cohorts. The horizontal axis presents the age of each cohort at the start of the simulation.



actual values of financial and human wealth (see Equation (2.36)). Figure 2.11 only presents the median exposure and duration for the optimal IDC scheme. The state dependency in the optimal IDC scheme cannot be replicated in a CDC scheme.

To disentangle these three effects, we compare the CEC in the different CDC schemes with the CEC in an IDC scheme based on different assumptions. First, we consider again the IDC scheme with an optimal asset allocation (see Equation (2.36)) and an optimal consumption choice during retirement (see Equation (2.110)). This IDC scheme is visualized in Figure 2.12 and we refer to this scheme as ‘IDC optimum’. Second, we consider an IDC scheme with an optimal asset allocation and a consumption profile during

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retirement based on the price of a variable annuity (see Equation (2.2)). Such a consumption profile is more in line with the pension payments in a CDC scheme and we refer to this scheme as ‘IDC annuity’. Third, we consider an IDC scheme with the median asset allocation (see Figure 2.6) to exclude state-dependency and with a consumption profile during retirement based on the price of a variable annuity. We refer to this scheme as ‘IDC median’.

Table 2.7 shows the absolute value of the CEC in the different CDC schemes and different IDC schemes and presents the welfare gains of a certain pension scheme relative to the different IDC schemes. In this comparison we focus on the cohort that goes through the whole life-cycle, i.e., the cohort at age 25 in Figure 2.12. When looking at the CEC column in Table 2.7, the IDC optimum has the highest CEC value, which makes sense. When looking at the CDC schemes, the CEC value under smoothing is slightly higher compared to no smoothing. While for older cohorts the CEC is higher in a CDC scheme without smoothing (see Figure 2.12), the CEC is higher under smoothing for the youngest cohort. When looking at the CEC value in the different IDC schemes, we notice that the CEC is significantly lower if the median asset allocation is used (IDC annuity). As a result, the welfare loss of the CDC schemes relative to the IDC median is almost half the welfare loss relative to the IDC optimum.

The implied market exposures and CEC in a CDC scheme depend not only on the type of smoothing policy but also, among other things, on the length of the smoothing period and the population composition. Appendix 2.F contains some sensitivity analyses for the length of the smoothing period and the population composition. Figures 2.F.4 and 2.F.5 show that the length of the smoothing period has a positive impact on the CEC, i.e., the CEC is higher in the case of a longer smoothing period. This holds for both a CDC scheme under closed smoothing (Figure 2.F.4) and under open complete smoothing (Figure 2.F.5). Figure 2.F.6 shows the implied market exposures and CEC in a CDC scheme without smoothing for different population compositions. In the green pension fund the size of each cohort increases by 2 percent each year and in the grey pension fund the size of each cohort decreases by 2 percent each year. Quite surprisingly, the implied equity allocation is higher in the grey pension fund and lower in the green pension fund, which

**Table 2.7: Comparison CEC**

*This table compares the CEC and welfare gain relative to an optimal IDC scheme for different CDC schemes.*

Smoothing policy	CEC	Welfare gain relative to IDC optimum	Welfare gain relative to IDC annuity	Welfare gain relative to IDC median
No smoothing fair	0.95	-22.5%	-20.6%	-14.0%
Closed smoothing fair	0.97	-20.9%	-19.0%	-12.3%
Open complete smoothing fair	0.97	-21.1%	-19.2%	-12.5%
IDC optimum	1.22	-	2.4%	10.9%
IDC annuity	1.20	-2.3%	-	8.3%
IDC median	1.11	-9.8%	-7.7%	-

conflicts with standard life-cycle theory. It turns out that the no-borrowing constraint is binding (see Equation (3.28)), i.e., there is a trade-off in the asset allocation optimization between the equity allocation and the long-term bond allocation. It turns out that in this trade-off the interest rate hedge is more important than the equity allocation. In other words, an implied duration close to the the optimal duration is more important than an implied equity allocation close to the optimal equity allocation. Therefore, the optimal allocation to the long-term bond is higher in a green pension fund and lower in a grey pension fund.

## 2.6 Conclusion

CDC pension schemes pool assets of cohorts that collectively share market risks via benefit adjustments. From a continuity perspective, it is preferable to design CDC schemes in a fair and efficient way. This paper contributes to the literature on pension scheme design by integrating these two continuity criteria within the context of CDC pension schemes. We offer the design features for a general class of fair and efficient CDC schemes through the combination of the benefit adjustment process, discount rate process, and as-

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set allocation in the presence of equity market risk and interest rate risk.

We show that a CDC pension scheme is fair as long as the scheme is complete and it applies an appropriate horizon-dependent benefit adjustment process. This also holds for CDC schemes in which benefit adjustments are smoothed. A pension scheme that is not complete, i.e., shocks are not explicitly allocated to the cohorts in the scheme, cannot be made fair. Moreover, the unfairness is significantly greater in pension schemes that are not complete than in complete pension schemes. In CDC schemes with uniform benefit adjustments, there is a value transfer from young to old cohorts if the mismatch risk between the value of the assets and liabilities is not fully hedged. For a typical interest rate hedge of 40 percent, this value transfer can be as much as 50 percent of annual income. Young cohorts might be better off not participating in such a CDC scheme.

Another important criterion for the design of CDC schemes is efficiency. In fair CDC schemes, the return on benefits can be replicated with traded assets, and the implied exposure to market risks can be derived. Because a CDC scheme acts as a constraint on the exposure of cohorts to market risks, it is not always possible to replicate precisely the optimal individual exposure to market risks in a CDC scheme. Nevertheless, the implied equity allocation and implied duration decrease with age in a CDC scheme with smoothing of benefit adjustments, which is in line with life-cycle theory. The CDC scheme without smoothing outperforms the CDC schemes with smoothing in terms of welfare for the older cohorts. However, for the youngest cohort, the CDC schemes with smoothing slightly outperform the CDC scheme without smoothing. In an optimal IDC scheme the welfare is significantly higher than in the different CDC schemes. This higher welfare is not only the result of a suboptimal exposure to market risks in the CDC schemes but also of the state-dependency of the optimal individual exposure to market risks and an optimal consumption choice.

An interesting extension of the financial market model is to consider inflation risk. If inflation risk is included, the optimal exposure to market risks changes because hedging interest rate risk has to be balanced against hedging inflation risk if real bonds are not (or not sufficiently) available in the financial market. Including inflation risk impacts the optimal exposure to market risks.

Another interesting area for future research is investigating the impact of alternative utility functions on the efficiency of CDC schemes. For example, loss aversion and reference dependence, two pronounced behavioral regularities supported by empirical evidence, significantly impact the optimal asset allocation (see van Bilsen et al. (2020b)).

# Appendix

## 2.A Continuous time derivations

We are interested in the continuous time expression of the short rate  $r_t$ , the stochastic discount factor  $M_t$ , and the price of equity  $S_t$ . We are going to write all three variables as a function of  $\mathbf{Z}_{\Delta t}$ .

$$\mathbf{Z}_{\Delta t} = \begin{bmatrix} Z_{S,\Delta t} \\ Z_{r,\Delta t} \\ \int_t^{t+\Delta t} \exp(-\kappa(t+\Delta t-u)) dZ_{r,u} \end{bmatrix} = \begin{bmatrix} \int_t^{t+\Delta t} dZ_{S,u} \\ \int_t^{t+\Delta t} dZ_{r,u} \\ \int_t^{t+\Delta t} \exp(-\kappa(t+\Delta t-u)) dZ_{r,u} \end{bmatrix} \quad (2.41)$$

$\mathbf{Z}_{\Delta t}$  contains three stochastic integrals that are jointly normally distributed. The mean of  $\mathbf{Z}_{\Delta t}$  is equal to a zero vector since all three integrals are with respect to a Brownian motion. We can derive the variance co-variance matrix of  $\mathbf{Z}_{\Delta t}$ . We assume  $\rho_{r,S} = 0$ . We start by deriving the variance of the first term

$$\begin{aligned} \mathbb{V}_t \left[ \int_t^{t+\Delta t} dZ_{S,u} \right] &= \mathbb{E}_t \left[ \int_t^{t+\Delta t} 1^2 du \right] \\ &= \int_t^{t+\Delta t} du \\ &= \Delta t, \end{aligned} \quad (2.42)$$

In this derivation, we make use of the fact that the expectation of an integral with respect to a Brownian motion is zero and we use the Ito isometry. We

can derive the variance of the second term in the same way. The variance of the third term equals

$$\begin{aligned}
 \mathbb{V}_t \left[ \int_t^{t+\Delta t} \exp(-\kappa(t + \Delta t - u)) dZ_{r,u} \right] &= \mathbb{E}_t \left[ \int_t^{t+\Delta t} \exp(-2\kappa(t + \Delta t - u)) du \right] \quad (2.43) \\
 &= \exp(-2\kappa(t + \Delta t)) \int_t^{t+\Delta t} \exp(2\kappa u) du \\
 &= \exp(-2\kappa(t + \Delta t)) \cdot \frac{1}{2\kappa} \left[ \exp(2\kappa u) \right]_{u=t}^{u=t+\Delta t} \\
 &= \frac{1}{2\kappa} (1 - \exp(-2\kappa\Delta t)) \\
 &= \frac{B(2\Delta t)}{2},
 \end{aligned}$$

where we define  $B(t) \equiv \frac{1}{2\kappa}(1 - \exp(-2\kappa t))$ . Finally, we derive the co-variance between the second and third term. Again we make use of the fact that the expectation of an integral with respect to a Brownian motion is zero.

$$\begin{aligned}
 \text{Cov}_t \left[ \int_t^{t+\Delta t} dZ_{r,s}, \int_t^{t+\Delta t} \exp(-\kappa(t + \Delta t - s)) dZ_{r,s} \right] \quad (2.44) \\
 &= \mathbb{E}_t \left[ \int_t^{t+\Delta t} dZ_{r,s} \int_t^{t+\Delta t} \exp(-\kappa(t + \Delta t - s)) dZ_{r,s} \right] \\
 &= \int_t^{t+\Delta t} \mathbb{E}_t [\exp(-\kappa(t + \Delta t - s))] ds \\
 &= \frac{1}{\kappa} (1 - \exp(-\kappa\Delta t)) \\
 &= B(\Delta t).
 \end{aligned}$$

As a result, the variance co-variance matrix of  $\mathbf{Z}_{\Delta t}$  equals

$$\mathbf{\Sigma}_{\Delta t} = \begin{bmatrix} \Delta t & 0 & 0 \\ 0 & \Delta t & B(\Delta t) \\ 0 & B(\Delta t) & \frac{B(2\Delta t)}{2} \end{bmatrix} \quad (2.45)$$

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## Interest rate

The term structure of interest rates has the same form as in Vasicek (1977)

$$dr_t = \kappa(\bar{r} - r_t)dt + \sigma_r dZ_{r,t}. \quad (2.46)$$

We want to find the continuous time expression of the short rate  $r_t$

$$\begin{aligned} dr_t + \kappa r_t dt &= k\bar{r}dt + \sigma_r dZ_{r,t} & (2.47) \\ \exp(\kappa t)(dr_t + \kappa r_t dt) &= \exp(\kappa t)(k\bar{r}dt + \sigma_r dZ_{r,t}) \\ \exp(\kappa t)dr_t + \exp(\kappa t)\kappa r_t dt &= \exp(\kappa t)(k\bar{r}dt + \sigma_r dZ_{r,t}) \\ d(\exp(\kappa t)r_t) &= \exp(\kappa t)(k\bar{r}dt + \sigma_r dZ_{r,t}), \end{aligned}$$

where in the last line we make use of the product rule. Subsequently, we take the integral from  $t$  to  $t + \Delta t$  on both sides of the equation

1.15

$$\begin{aligned} \int_t^{t+\Delta t} d(\exp(\kappa s)r_s) &= \int_t^{t+\Delta t} \exp(\kappa s)k\bar{r}ds + \int_t^{t+\Delta t} \exp(\kappa s)\sigma_r dZ_{r,s} & (2.48) \\ \exp(\kappa(t+\Delta t))r_{t+\Delta t} - \exp(\kappa t)r_t &= k\bar{r} \int_t^{t+\Delta t} \exp(\kappa s)ds + \int_t^{t+\Delta t} \exp(\kappa s)\sigma_r dZ_{r,s}, \end{aligned}$$

where we make use of the fact that  $\int_a^b dX = X(b) - X(a)$ . Now we can write  $r_{t+\Delta t}$  as

$$\begin{aligned} r_{t+\Delta t} &= \exp(-\kappa\Delta t)r_t + \exp(-\kappa(t+\Delta t))k\bar{r} \int_t^{t+\Delta t} \exp(\kappa s)ds & (2.49) \\ &+ \int_t^{t+\Delta t} \exp(-\kappa(t+\Delta t-s))\sigma_r dZ_{r,s} \\ &= \exp(-\kappa\Delta t)r_t + \exp(-\kappa(t+\Delta t))k\bar{r} \left[ \frac{1}{\kappa} \exp(\kappa s) \right]_{s=t}^{s=t+\Delta t} \\ &+ \int_t^{t+\Delta t} \exp(-\kappa(t+\Delta t-s))\sigma_r dZ_{r,s} \\ &= \exp(-\kappa\Delta t)r_t + \bar{r}(1 - \exp(-k\Delta t)) \\ &+ \sigma_r \int_t^{t+\Delta t} \exp(-\kappa(t+\Delta t-s))dZ_{r,s}. \end{aligned}$$

The expectation of  $r_{t+\Delta t}$  equals

$$\mathbb{E}_t[r_{t+\Delta t}] = \exp(-\kappa\Delta t)r_t + \bar{r}(1 - \exp(-k\Delta t)), \quad (2.50)$$

since the expectation of an integral with respect to a Brownian motion equals zero. The variance is completely determined by the last term in Equation (2.49). We can write  $r_{t+\Delta t}$  as a function of  $\mathbf{Z}_{\Delta t}$

$$r_{t+\Delta t} = \mathbb{E}_t[r_{t+\Delta t}] + \mathbf{A}_r' \mathbf{Z}_{\Delta t}, \quad (2.51)$$

where  $\mathbf{A}_r$  equals

$$\mathbf{A}_r = \begin{bmatrix} 0 \\ 0 \\ \sigma_r \end{bmatrix}. \quad (2.52)$$

### Stochastic discount factor

The stochastic discount factor (or pricing kernel) follows the following process

$$\frac{dM_t}{M_t} = -r_t dt + \lambda_r dZ_{r,t} - \lambda_S dZ_{S,t}. \quad (2.53)$$

We apply Ito calculus to calculate  $d(\ln M_t)$

$$\begin{aligned} d(\ln M_t) &= (\ln M_t)' dM_t + 0.5(\ln M_t)'' dM_t^2 & (2.54) \\ &= \frac{dM_t}{M_t} - 0.5 \frac{1}{M_t^2} dM_t^2 \\ &= \frac{dM_t}{M_t} - 0.5 \frac{1}{M_t^2} \left( r_t^2 M_t^2 dt^2 + \lambda_r^2 M_t^2 dZ_{r,t}^2 + \lambda_S M_t^2 dZ_{S,t}^2 \right. \\ &\quad \left. - 2r_t \lambda_r M_t^2 dt dZ_{r,t} + 2r_t \lambda_S M_t^2 dt dZ_{S,t} - 2\lambda_r \lambda_S M_t^2 dZ_{r,t} dZ_{S,t} \right) \\ &= \frac{dM_t}{M_t} - 0.5 \frac{1}{M_t^2} \left( \lambda_r^2 M_t^2 dt + \lambda_S M_t^2 dt \right) \\ &= -r_t dt + \lambda_r dZ_{r,t} - \lambda_S dZ_{S,t} - 0.5\lambda_r^2 dt - 0.5\lambda_S^2 dt. \end{aligned}$$

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We take the integral from  $t$  to  $t + \Delta t$  on both sides of this equation

$$\begin{aligned}
\int_t^{t+\Delta t} d(\ln M_s) &= - \int_t^{t+\Delta t} r_s ds + \lambda_r(Z_{r,t+\Delta t} - Z_{r,t}) & (2.55) \\
&\quad - \lambda_S(Z_{S,t+\Delta t} - Z_{S,t}) - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t, \\
\ln(M_{t+\Delta t}) - \ln(M_t) &= - \int_t^{t+\Delta t} r_s ds + \lambda_r(Z_{r,t+\Delta t} - Z_{r,t}) \\
&\quad - \lambda_S(Z_{S,t+\Delta t} - Z_{S,t}) - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t, \\
\ln(M_{t+\Delta t}) &= \ln(M_t) - \int_t^{t+\Delta t} r_s ds + \lambda_r(Z_{r,t+\Delta t} - Z_{r,t}) \\
&\quad - \lambda_S(Z_{S,t+\Delta t} - Z_{S,t}) - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t.
\end{aligned}$$

Subsequently, we take the exponent on both sides

$$\begin{aligned}
M_{t+\Delta t} &= M_t \exp\left(- \int_t^{t+\Delta t} r_s ds + \lambda_r(Z_{r,t+\Delta t} - Z_{r,t}) - \lambda_S(Z_{S,t+\Delta t} - Z_{S,t}) \right. & (2.56) \\
&\quad \left. - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t\right) \\
&= M_t \exp\left(- \int_t^{t+\Delta t} r_s ds\right) \exp\left(\lambda_r Z_{r,\Delta t} - \lambda_S Z_{S,\Delta t} - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t\right).
\end{aligned}$$

We eliminate the stochastic  $r_s$  in the integral. Therefore we insert the expression of  $r_s$  in Equation (2.49) into the integral

$$\begin{aligned}
\int_t^{t+\Delta t} r_s ds &= \int_t^{t+\Delta t} \left( r_t \exp(-\kappa(s-t)) + \bar{r}(1 - \exp(-k(s-t))) \right. & (2.57) \\
&\quad \left. + \sigma_r \int_t^s \exp(-\kappa(s-u)) dZ_{r,u} \right) ds \\
&= \int_t^{t+\Delta t} r_t \exp(-\kappa(s-t)) ds \int_t^{t+\Delta t} \bar{r}(1 - \exp(-k(s-t))) ds \\
&\quad \cdot \int_t^{t+\Delta t} \sigma_r \int_t^s \exp(-\kappa(s-u)) dZ_{r,u} ds.
\end{aligned}$$

First, we calculate the integrals separately

$$\begin{aligned}
 \int_t^{t+\Delta t} r_t \exp(-\kappa(s-t)) ds &= r_t \int_t^{t+\Delta t} \exp(-\kappa(s-t)) ds & (2.58) \\
 &= r_t \exp(\kappa t) \cdot -\frac{1}{\kappa} \left[ \exp(-\kappa s) \right]_{s=t}^{s=t+\Delta t} \\
 &= r_t \cdot \frac{-\exp(\kappa t)}{\kappa} \left( \exp(-\kappa(t+\Delta t)) - \exp(-\kappa t) \right) \\
 &= r_t \cdot \frac{1}{\kappa} (1 - \exp(-\kappa \Delta t)) \\
 &= r_t B(\Delta t).
 \end{aligned}$$

The second integral equals

$$\begin{aligned}
 \int_t^{t+\Delta t} \bar{r} (1 - \exp(-\kappa(s-t))) ds &= \int_t^{t+\Delta t} \bar{r} ds - \int_t^{t+\Delta t} \bar{r} \exp(-\kappa(s-t)) ds & (2.59) \\
 &= \bar{r} \Delta t + \bar{r} \exp(\kappa t) \cdot \frac{1}{\kappa} \left[ \exp(-\kappa(t+\Delta t)) - \exp(-\kappa t) \right] \\
 &= \bar{r} \left( \Delta t - \frac{1}{\kappa} (1 - \exp(-\kappa \Delta t)) \right) \\
 &= \bar{r} (\Delta t - B(\Delta t)).
 \end{aligned}$$

We calculate the last integral by changing the order of integration

$$\begin{aligned}
 \int_t^{t+\Delta t} \sigma_r \int_t^s \exp(-\kappa(s-u)) dZ_{r,u} ds & & (2.60) \\
 &= \sigma_r \int_t^{t+\Delta t} \int_u^{t+\Delta t} \exp(-\kappa(s-u)) ds dZ_{r,u} \\
 &= \sigma_r \int_t^{t+\Delta t} \exp(\kappa u) \int_u^{t+\Delta t} \exp(-\kappa s) ds dZ_{r,u} \\
 &= \sigma_r \int_t^{t+\Delta t} \exp(\kappa u) \frac{-1}{\kappa} (\exp(-\kappa(t+\Delta t)) - \exp(-\kappa u)) dZ_{r,u} \\
 &= \sigma_r \int_t^{t+\Delta t} \frac{1}{\kappa} (1 - \exp(-\kappa(t+\Delta t-u))) dZ_{r,u} \\
 &= \sigma_r \int_t^{t+\Delta t} B(t+\Delta t-u) dZ_{r,u}.
 \end{aligned}$$

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We insert these expressions in the original equation

$$\int_t^{t+\Delta t} r_s ds = r_t B(\Delta t) + \bar{r}(\Delta t - B(\Delta t)) + \sigma_r \int_t^{t+\Delta t} B(t + \Delta t - u) dZ_{r,u}. \quad (2.61)$$

We insert this into the expression of  $M_{t+\Delta t}$  in Equation (2.56)

$$\begin{aligned} M_{t+\Delta t} &= M_t \exp(-r_t B(\Delta t) - \bar{r}(\Delta t - B(\Delta t))) \\ &\quad \cdot \exp\left(-\sigma_r \int_t^{t+\Delta t} B(t + \Delta t - u) dZ_{r,u}\right) \\ &\quad \cdot \exp\left(\lambda_r Z_{r,\Delta t} - \lambda_S Z_{S,\Delta t} - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t\right) \\ &= M_t \exp\left(-r_t B(\Delta t) - \bar{r}(\Delta t - B(\Delta t)) - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t\right) \exp\left(-\lambda_S Z_{S,\Delta t}\right) \\ &\quad \cdot \exp\left(\lambda_r Z_{r,\Delta t}\right) \exp\left(-\sigma_r \int_t^{t+\Delta t} B(t + \Delta t - u) dZ_{r,u}\right). \end{aligned} \quad (2.62)$$

To determine the distribution of  $M_{t+\Delta t}$  we first take the natural logarithm

$$\begin{aligned} \ln(M_{t+\Delta t}) &= \ln(M_t) - r_t B(\Delta t) - \bar{r}(\Delta t - B(\Delta t)) - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t \\ &\quad - \lambda_S Z_{S,\Delta t} + \lambda_r Z_{r,\Delta t} - \sigma_r \int_t^{t+\Delta t} B(t + \Delta t - u) dZ_{r,u} \\ &= \ln(M_t) - r_t B(\Delta t) - \bar{r}(\Delta t - B(\Delta t)) - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t \\ &\quad - \lambda_S Z_{S,\Delta t} + \left(\lambda_r - \frac{\sigma_r}{\kappa}\right) Z_{r,\Delta t} + \frac{\sigma_r}{\kappa} \int_t^{t+\Delta t} \exp(-k(t + \Delta t - u)) dZ_{r,u}. \end{aligned} \quad (2.63)$$

We are interested in the distribution of  $\ln(M_{t+\Delta t})$ . The expectation of  $\ln(M_{t+\Delta t})$  equals

$$\mathbb{E}_t[\ln(M_{t+\Delta t})] = \ln(M_t) - r_t B(\Delta t) - \bar{r}(\Delta t - B(\Delta t)) - 0.5(\lambda_r^2 + \lambda_S^2)\Delta t, \quad (2.64)$$

since the expectation of an integral with respect to a Brownian motion equals zero. The variance is determined by the stochastic terms in Equation (2.63). We can write  $\ln(M_{t+\Delta t})$  as a function of  $\mathbf{Z}_{\Delta t}$

$$\ln(M_{t+\Delta t}) = \mathbb{E}_t[\ln(M_{t+\Delta t})] + \mathbf{A}_M' \mathbf{Z}_{\Delta t}, \quad (2.65)$$

where  $\mathbf{A}_M$  equals

$$\mathbf{A}_M = \begin{bmatrix} -\lambda_S \\ \lambda_r - \frac{\sigma_r}{\kappa} \\ \frac{\sigma_r}{\kappa} \end{bmatrix}. \quad (2.66)$$

### Equity price

The price of equity  $S_t$  follows a geometric Brownian motion

$$\frac{dS_t}{S_t} = (r_t + \lambda_S \sigma_S)dt + \sigma_S dZ_{S,t}. \quad (2.67)$$

We apply Ito calculus to calculate  $d \ln(S_t)$

$$\begin{aligned} d \ln(S_t) &= (\ln S_t)' dS_t + 0.5(\ln S_t)'' dS_t^2 \\ &= \frac{dS_t}{S_t} - 0.5 \frac{1}{S_t^2} S_t^2 \sigma_S^2 dt \\ &= (r_t + \lambda_S \sigma_S)dt + \sigma_S dZ_{S,t} - 0.5 \sigma_S^2 dt \\ &= (r_t + \lambda_S \sigma_S - 0.5 \sigma_S^2)dt + \sigma_S dZ_{S,t}. \end{aligned} \quad (2.68)$$

We take the integral from  $t$  to  $t + \Delta t$  on both sides of the equation

$$\begin{aligned} \ln(S_{t+\Delta t}) - \ln(S_t) &= \int_t^{t+\Delta t} (r_s + \lambda_S \sigma_S - 0.5 \sigma_S^2) ds + \sigma_S Z_{S,\Delta t} \\ &= \int_t^{t+\Delta t} r_s ds + (\lambda_S \sigma_S - 0.5 \sigma_S^2) \Delta t + \sigma_S Z_{S,\Delta t}. \end{aligned} \quad (2.69)$$

Subsequently, we plug in Equation (2.61)

$$\begin{aligned} \ln(S_{t+\Delta t}) &= \ln(S_t) + r_t B(\Delta t) + \bar{r}(\Delta t - B(\Delta t)) \\ &\quad + \sigma_r \int_t^{t+\Delta t} B(t + \Delta t - u) dZ_{r,u} + (\lambda_S \sigma_S - 0.5 \sigma_S^2) \Delta t + \sigma_S Z_{S,\Delta t} \\ &= \ln(S_t) + r_t B(\Delta t) + \bar{r}(\Delta t - B(\Delta t)) + \frac{\sigma_r}{\kappa} Z_{r,\Delta t} \\ &\quad - \frac{\sigma_r}{\kappa} \int_t^{t+\Delta t} \exp(-k(t + \Delta t - u)) dZ_{r,u} \\ &\quad + (\lambda_S \sigma_S - 0.5 \sigma_S^2) \Delta t + \sigma_S Z_{S,\Delta t}. \end{aligned} \quad (2.70)$$

We are interested in the distribution of  $\ln(S_{t+\Delta t})$ . The expectation of

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$\ln(S_{t+\Delta t})$  equals

$$\mathbb{E}_t[\ln(S_{t+\Delta t})] = \ln(S_t) + r_t B(\Delta t) + \bar{r}(\Delta t - B(\Delta t)) + (\lambda_S \sigma_S - 0.5\sigma_S^2)\Delta t. \quad (2.71)$$

We can write  $\ln(S_{t+\Delta t})$  as a function of  $\mathbf{Z}_{\Delta t}$

$$\ln(S_{t+\Delta t}) = \mathbb{E}_t[\ln(S_{t+\Delta t})] + \mathbf{A}_S' \mathbf{Z}_{\Delta t}, \quad (2.72)$$

where  $\mathbf{A}_S$  equals

$$\mathbf{A}_S = \begin{bmatrix} \sigma_S \\ \frac{\sigma_r}{\kappa} \\ -\frac{\sigma_r}{\kappa} \end{bmatrix}. \quad (2.73)$$

### Bond price

We denote the price of a zero-coupon bond that pays 1 dollar at time  $T$  by  $P(T-t, r_t)$ . We can write this price as

$$P(T-t, r_t) = \mathbb{E}_t \left[ \frac{M_T}{M_t} \cdot 1 \right]. \quad (2.74)$$

Recall that  $M_T/M_t$  is lognormally distributed and  $\ln(M_T/M_t)$  is normally distributed. To determine the expectation of  $M_T/M_t$  we make use of the expression of the mean of a lognormally distributed variable.

$$\mathbb{E}_t \left[ \frac{M_T}{M_t} \right] = \exp \left( \mathbb{E}_t \left[ \ln \left( \frac{M_T}{M_t} \right) \right] + 0.5 \mathbb{V}_t \left[ \ln \left( \frac{M_T}{M_t} \right) \right] \right). \quad (2.75)$$

We insert Equation (2.64) into the first term

$$\mathbb{E}_t \left[ \ln \left( \frac{M_T}{M_t} \right) \right] = -r_t B(T-t) - \bar{r}(T-t - B(T-t)) - 0.5(\lambda_r^2 + \lambda_S^2)(T-t), \quad (2.76)$$

and derive the second term

$$\begin{aligned}
 \mathbb{V}_t \left[ \ln \left( \frac{M_T}{M_t} \right) \right] &= \mathbb{V}_t \left[ -\lambda_S Z_{S,T-t} + \lambda_r Z_{r,T-t} - \sigma_r \int_t^T B(T-u) dZ_{r,u} \right] \quad (2.77) \\
 &= \mathbb{E}_t \left[ \left( -\lambda_S \int_t^T dZ_{S,u} + \int_t^T (\lambda_r - \sigma_r B(T-u)) dZ_{r,u} \right)^2 \right] \\
 &= \mathbb{E}_t \left[ \lambda_S^2 \left( \int_t^T dZ_{S,u} \right)^2 - 2\lambda_S \int_t^T dZ_{S,u} \int_t^T (\lambda_r - \sigma_r B(T-u)) dZ_{r,u} \right. \\
 &\quad \left. + \left( \int_t^T (\lambda_r - \sigma_r B(T-u)) dZ_{r,u} \right)^2 \right] \\
 &= \lambda_S^2 \int_t^T du - 2\lambda_S \mathbb{E}_t \left[ \int_t^T dZ_{S,u} \right] \mathbb{E}_t \left[ \int_t^T (\lambda_r - \sigma_r B(T-u)) dZ_{r,u} \right] \\
 &\quad + \int_t^T \mathbb{E}_t [(\lambda_r - \sigma_r B(T-u))^2] du \\
 &= \lambda_S^2 (T-t) + 0 + \int_t^T \lambda_r^2 - 2\lambda_r \sigma_r B(T-u) + \sigma_r^2 B(T-u)^2 du \\
 &= (\lambda_S^2 + \lambda_r^2)(T-t) - 2\lambda_r \sigma_r \int_t^T B(T-u) du + \sigma_r^2 \int_t^T B(T-u)^2 du,
 \end{aligned}$$

where we make use of Ito isometry and the independence of  $Z_{r,u}$  and  $Z_{S,u}$  in the fourth line.

We derive both integrals separately. The first integral equals

$$\begin{aligned}
 \int_t^T B(T-u) du &= \int_t^T \frac{1}{\kappa} (1 - \exp(-\kappa(T-u))) du \quad (2.78) \\
 &= \frac{1}{\kappa} \int_t^T du - \frac{1}{\kappa} \int_t^T \exp(-\kappa(T-u)) du \\
 &= \frac{T-t}{\kappa} - \frac{1}{\kappa} \exp(-\kappa T) \left[ \exp(ku) \frac{1}{\kappa} \right]_{u=t}^{u=T} \\
 &= \frac{T-t}{\kappa} - \frac{1}{\kappa^2} \exp(-\kappa T) (\exp(\kappa T) - \exp(\kappa t)) \\
 &= \frac{T-t}{\kappa} - \frac{1}{\kappa^2} (1 - \exp(-\kappa(T-t))) = \frac{T-t}{\kappa} - \frac{1}{\kappa} B(T-t) \\
 &= \frac{1}{\kappa} (T-t - B(T-t)).
 \end{aligned}$$

---

The second integral equals

$$\begin{aligned}
\int_t^T B(T-u)^2 du &= \int_t^T \frac{1}{\kappa^2} (1 - \exp(-\kappa(T-u)))^2 du & (2.79) \\
&= \frac{1}{\kappa^2} \int_t^T du - \frac{2}{\kappa^2} \int_t^T \exp(-\kappa(T-u)) du \\
&\quad + \frac{1}{\kappa^2} \int_t^T \exp(-2\kappa(T-u)) du \\
&= \frac{T-t}{\kappa^2} - \frac{2}{\kappa^3} (1 - \exp(-\kappa(T-t))) + \frac{1}{2\kappa^3} (1 - \exp(-2\kappa(T-t))) \\
&= \frac{T-t}{\kappa^2} - \frac{2}{\kappa^2} B(T-t) + \frac{1}{2\kappa^2} B(2(T-t)).
\end{aligned}$$

We insert both integrals into Equation (2.77)

$$\begin{aligned}
\mathbb{V}_t \left[ \ln \left( \frac{M_T}{M_t} \right) \right] &= \lambda_S^2 (T-t) + \lambda_r^2 (T-t) - 2\lambda_r \sigma_r \int_t^T B(T-u) du & (2.80) \\
&\quad + \sigma_r^2 \int_t^T B(T-u)^2 du \\
&= \lambda_S^2 (T-t) + \lambda_r^2 (T-t) - 2\lambda_r \sigma_r \frac{1}{\kappa} (T-t - B(T-t)) \\
&\quad + \sigma_r^2 \left( \frac{T-t}{\kappa^2} - \frac{2}{\kappa^2} B(T-t) + \frac{1}{2\kappa^2} B(2(T-t)) \right) \\
&= \lambda_S^2 (T-t) + \lambda_r^2 (T-t) - 2\lambda_r \sigma_r \frac{1}{\kappa} (T-t - B(T-t)) \\
&\quad + \frac{\sigma_r^2}{\kappa^2} (T-t - 2B(T-t) + 0.5B(2(T-t))).
\end{aligned}$$

Subsequently, we insert Equation (2.76) and (2.80) into Equation (2.75)

$$\begin{aligned}
 \mathbb{E}_t \left[ \frac{M_T}{M_t} \right] &= \exp \left( \mathbb{E}_t \left[ \ln \left( \frac{M_T}{M_t} \right) \right] + 0.5 \mathbb{V}_t \left[ \ln \left( \frac{M_T}{M_t} \right) \right] \right) \quad (2.81) \\
 &= \exp \left( -r_t B(T-t) - \bar{r}(T-t-B(T-t)) - 0.5(\lambda_r^2 + \lambda_S^2)(T-t) \right. \\
 &\quad \left. + 0.5(T-t)(\lambda_S^2 + \lambda_r^2) - \lambda_r \sigma_r \frac{1}{\kappa}(T-t-B(T-t)) \right. \\
 &\quad \left. + \frac{\sigma_r^2}{2\kappa^2}(T-t-2B(T-t)+0.5B(2(T-t))) \right) \\
 &= \exp \left( - \left( \bar{r} + \frac{\sigma_r \lambda_r}{\kappa} - 0.5 \frac{\sigma_r^2}{\kappa^2} \right) (T-t-B(T-t)) \right. \\
 &\quad \left. - \frac{\sigma_r^2}{2\kappa^2} B(T-t) + \frac{\sigma_r^2}{4\kappa^2} B(2(T-t)) - r_t B(T-t) \right) \\
 &= \exp \left( - \left( \bar{r} + \frac{\sigma_r \lambda_r}{\kappa} - 0.5 \frac{\sigma_r^2}{\kappa^2} \right) (T-t-B(T-t)) \right. \\
 &\quad \left. - \frac{\sigma_r^2}{2\kappa^2} (B(T-t) - 0.5B(2(T-t))) - r_t B(T-t) \right) \\
 &= \exp \left( - \left( \bar{r} + \frac{\sigma_r \lambda_r}{\kappa} - 0.5 \frac{\sigma_r^2}{\kappa^2} \right) (T-t-B(T-t)) \right. \\
 &\quad \left. - \frac{\sigma_r^2}{4\kappa^3} (2 - 2 \exp(-\kappa(T-t)) - 1 + \exp(-2\kappa(T-t))) - r_t B(T-t) \right) \\
 &= \exp \left( - \left( \bar{r} + \frac{\sigma_r \lambda_r}{\kappa} - 0.5 \frac{\sigma_r^2}{\kappa^2} \right) (T-t-B(T-t)) \right. \\
 &\quad \left. - \frac{\sigma_r^2}{4\kappa} B(T-t)^2 - r_t B(T-t) \right).
 \end{aligned}$$

So the price of a zero-coupon bond that pays 1 dollar at time  $T$  equals

$$P(T-t, r_t) = \mathbb{E}_t \left[ \frac{M_T}{M_t} \right] = \exp(-A(T-t) - B(T-t)r_t), \quad (2.82)$$

where

$$A(T-t) = \left( \bar{r} + \frac{\sigma_r \lambda_r}{\kappa} - 0.5 \frac{\sigma_r^2}{\kappa^2} \right) (T-t-B(T-t)) + \frac{\sigma_r^2}{4\kappa} B(T-t)^2 \quad (2.83)$$

$$B(T-t) = \frac{1}{\kappa} (1 - \exp(-\kappa(T-t))). \quad (2.84)$$

This expression is identical to the price of a zero-coupon bond in Sorensen (1999).

---

### Bond return

We are interested in the distribution of the period bond return  $r_t^B$  from time  $t$  until  $t + \Delta t$

$$\begin{aligned}
 r_{t+\Delta t}^B &= \ln \frac{P(T-t-\Delta t, r_{t+\Delta t})}{P(T-t, r_t)} & (2.85) \\
 &= \ln \frac{\exp(-A(T-t-\Delta t) - B(T-t-\Delta t)r_{t+\Delta t})}{\exp(-A(T-t) - B(T-t)r_t)} \\
 &= -A(T-t-\Delta t) - B(T-t-\Delta t)r_{t+\Delta t} + A(T-t) + B(T-t)r_t \\
 &= -A(T-t-\Delta t) + A(T-t) + B(T-t)r_t \\
 &\quad - B(T-t-\Delta t) \left( \exp(-\kappa\Delta t)r_t + \bar{r}(1 - \exp(-k\Delta t)) \right) \\
 &\quad + \sigma_r \int_t^{t+\Delta t} \exp(-\kappa(t+\Delta t-s)) dZ_{r,s}
 \end{aligned}$$

The expectation of  $r_{t+\Delta t}^B$  equals

$$\begin{aligned}
 \mathbb{E}_t[r_{t+\Delta t}^B] &= -A(T-t-\Delta t) + A(T-t) + B(T-t)r_t & (2.86) \\
 &\quad - B(T-t-\Delta t)(\exp(-\kappa\Delta t)r_t + \bar{r}(1 - \exp(-k\Delta t))).
 \end{aligned}$$

We can write  $r_{t+\Delta t}^B$  as a function of  $\mathbf{Z}_{\Delta t}$

$$r_{t+\Delta t}^B = \mathbb{E}_t[r_{t+\Delta t}^B] + (\mathbf{A}_{r^B})' \mathbf{Z}_{\Delta t}. \quad (2.87)$$

where  $\mathbf{A}_{r^B}$  equals

$$\mathbf{A}_{r^B} = \begin{bmatrix} 0 \\ 0 \\ -B(T-t-\Delta t)\sigma_r \end{bmatrix}.$$

## 2.B Proofs of completeness of benefit adjustment factors

All pension benefit adjustments discussed in Section 2.4.3, except for open smoothing, satisfy the completeness property (see also Table 2.1). A pension scheme is complete if all value in the CDC scheme is at each

point in time explicitly allocated to the cohorts in the scheme, i.e., directly after the benefit adjustments have been applied, the value of the pension liabilities equals the value of the assets. This section provides proofs of the completeness property for these benefit adjustment factors.

### 1. No smoothing

$$\begin{aligned}
 L_t &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{m+1} P(m, r_t) & (2.88) \\
 &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} \frac{A_t}{\tilde{L}_t} b_{j,t-1}^{m+1} P(m, r_t) \\
 &= \frac{A_t}{\tilde{L}_t} \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} P(m, r_t) \\
 &= \frac{A_t}{\tilde{L}_t} \tilde{L}_t = A_t.
 \end{aligned}$$

### 2. No smoothing, fair adjustment factors

$$\begin{aligned}
 L_t &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{m+1} P(m, r_t) & (2.89) \\
 &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} P(m, r_t) \left( 1 + \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)} \right) \\
 &= \tilde{L}_t + L_{t-1} \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \\
 &= \tilde{L}_t + A_t - \tilde{L}_t = A_t.
 \end{aligned}$$

### 3. Closed smoothing

$$\begin{aligned}
L_t &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{m+1} P(m, r_t) & (2.90) \\
&= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} P(m, r_t) \left( 1 + \alpha_m \frac{A_t - \tilde{L}_t}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m} \right) \\
&= \tilde{L}_t + \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} P(m, r_t) \alpha_m \frac{A_t - \tilde{L}_t}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m} \\
&= \tilde{L}_t + (A_t - \tilde{L}_t) \frac{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m} \\
&= \tilde{L}_t + A_t - \tilde{L}_t = A_t.
\end{aligned}$$

### 4. Closed smoothing, fair adjustment factors

$$\begin{aligned}
L_t &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{m+1} P(m, r_t) & (2.91) \\
&= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} P(m, r_t) \\
&\quad \cdot \left( 1 + \alpha_m \frac{L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m} \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)} \right) \\
&= \tilde{L}_t + \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} b_{j,t-1}^{m+1} P(m+1, r_{t-1}) \\
&\quad \cdot \alpha_m \frac{L_{t-1}}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m} \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \\
&= \tilde{L}_t + L_{t-1} \left( \frac{A_t}{A_{t-1}} - \frac{\tilde{L}_t}{L_{t-1}} \right) \\
&= \tilde{L}_t + A_t - \tilde{L}_t = A_t.
\end{aligned}$$

### 5. Open complete smoothing

$$\begin{aligned}
 L_t + L_t^f &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{m+1} P(m, r_t) & (2.92) \\
 &+ \sum_{m=1}^{T_w+T_p+n-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{f,m+1} P(m, r_t) \\
 &= \sum_{m=1}^{T_w+T_p+n-1} \sum_{j=t}^{t+T_w+T_p-1} (b_{j,t-1}^{m+1} + b_{j,t-1}^{f,m+1}) P(m, r_t) \\
 &\cdot \left( 1 + \alpha_m \frac{A_t + \Pi_t - \tilde{L}_t - \tilde{L}_t^f}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} \tilde{L}_t^{f,m} \alpha_m} \right) \\
 &= \tilde{L}_t + \tilde{L}_t^f + \sum_{j=t}^{t+T_w+T_p-1} (b_{j,t-1}^{m+1} + b_{j,t-1}^{f,m+1}) P(m, r_t) \\
 &\cdot \alpha_m \frac{A_t + \Pi_t - \tilde{L}_t - \tilde{L}_t^f}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} \tilde{L}_t^{f,m} \alpha_m} \\
 &= \tilde{L}_t + \tilde{L}_t^f + \frac{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} \tilde{L}_t^{f,m} \alpha_m}{\sum_{m=0}^{T_w+T_p-1} \tilde{L}_t^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} \tilde{L}_t^{f,m} \alpha_m} \\
 &\cdot (A_t + \Pi_t - \tilde{L}_t - \tilde{L}_t^f) \\
 &= \tilde{L}_t + \tilde{L}_t^f + A_t + \Pi_t - \tilde{L}_t - \tilde{L}_t^f = A_t + \Pi_t.
 \end{aligned}$$

---

## 6. Open complete smoothing, fair adjustment factors

$$\begin{aligned}
L_t + L_t^f &= \sum_{m=0}^{T_w+T_p-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{m+1} P(m, r_t) \\
&+ \sum_{m=1}^{T_w+T_p+n-1} \sum_{j=t}^{t+T_w+T_p-1} \delta_t^m b_{j,t-1}^{f,m+1} P(m, r_t) \\
&= \sum_{m=1}^{T_w+T_p+n-1} \sum_{j=t}^{t+T_w+T_p-1} (b_{j,t-1}^{m+1} + b_{j,t-1}^{f,m+1}) P(m, r_t) \\
&\cdot \left( 1 + \alpha_m \frac{L_{t-1} + L_{t-1}^f}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \right) \\
&\cdot \left( \frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} - \frac{\tilde{L}_t + \tilde{L}_t^f}{L_{t-1} + L_{t-1}^f} \right) \frac{P(m+1, r_{t-1})}{P(m, r_t)} \\
&= \tilde{L}_t + \tilde{L}_t^f + \sum_{m=0}^{T_w+T_p+n-1} \sum_{j=t}^{t+T_w+T_p-1} (b_{j,t-1}^{m+1} + b_{j,t-1}^{f,m+1}) P(m+1, r_{t-1}) \alpha_m \\
&\cdot \frac{L_{t-1} + L_{t-1}^f}{\sum_{m=0}^{T_w+T_p-1} L_{t-1}^m \alpha_m + \sum_{m=1}^{T_w+T_p+n-1} L_{t-1}^{f,m} \alpha_m} \\
&\cdot \left( \frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} - \frac{\tilde{L}_t + \tilde{L}_t^f}{L_{t-1} + L_{t-1}^f} \right) \\
&= \tilde{L}_t + \tilde{L}_t^f + (L_{t-1} + L_{t-1}^f) \left( \frac{A_t + \Pi_t}{A_{t-1} + \Pi_{t-1}} - \frac{\tilde{L}_t + \tilde{L}_t^f}{L_{t-1} + L_{t-1}^f} \right) \\
&= \tilde{L}_t + \tilde{L}_t^f + A_t + \Pi_t - \tilde{L}_t - \tilde{L}_t^f = A_t + \Pi_t.
\end{aligned} \tag{2.93}$$

## 2.C Certainty equivalent of social welfare

We express welfare in terms of the certainty equivalent of consumption

$$CEC_j = \left( U_j \frac{1 - \gamma}{\sum_{i=0}^{T_p-1} \beta^i} \right)^{1/(1-\gamma)}, \tag{2.94}$$

where  $U_j$  is defined by Equation (2.30). We can rewrite this as follows

$$U_j = \frac{CEC_j^{1-\gamma} \sum_{i=0}^{T_p-1} \beta^i}{1-\gamma}. \quad (2.95)$$

Total welfare, or social welfare  $SW$  is determined by the expected utility of all cohorts. Note that there are  $T_s + T_p - 1$  cohorts in total.

$$\begin{aligned} SW &= \sum_{j=1}^{T_s+T_p-1} \beta^{j-1} U_j \\ &= \sum_{j=1}^{T_s+T_p-1} \beta^{j-1} \frac{CEC_j^{1-\gamma} \sum_{i=0}^{T_p-1} \beta^i}{1-\gamma}. \end{aligned} \quad (2.96)$$

We can derive the certainty equivalent of total welfare as follows

$$\begin{aligned} SW &= \sum_{j=1}^{T_s+T_p-1} \beta^{j-1} \frac{CEC_j^{1-\gamma} \sum_{i=0}^{T_p-1} \beta^i}{1-\gamma} \\ &= \frac{1 - \beta^{T_s+T_p-1}}{1-\beta} U(CEC) \frac{1 - \beta^{T_p}}{1-\beta}. \end{aligned} \quad (2.97)$$

We can rewrite this

$$\begin{aligned} U(CEC) &= \frac{SW(1-\beta)^2}{(1-\beta^{T_s+T_p-1})(1-\beta^{T_p})} \\ CEC &= \left[ \frac{SW(1-\beta)^2(1-\gamma)}{(1-\beta^{T_s+T_p-1})(1-\beta^{T_p})} \right]^{1/(1-\gamma)}. \end{aligned} \quad (2.98)$$

## 2.D Derivation of optimal market risk exposures

Our financial market model as formulated in Table 2.2 consists of two sources of risk: equity market risk and interest rate risk. We can combine the interest rate process in Equation (2.46) and the natural logarithm of the stock price process in Equation (2.68) in the following multivariate equation

$$dY_t = (K_0 + K_1 Y_t)dt + \Sigma dZ_t, \quad (2.99)$$

---

where

$$\mathbf{K}_0 = \begin{bmatrix} \kappa \bar{r} \\ \sigma_S \lambda_S - 0.5 \sigma_S^2 \end{bmatrix}$$

$$\mathbf{K}_1 = \begin{bmatrix} -\kappa & 0 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_r & 0 \\ 0 & \sigma_S \end{bmatrix}.$$

The factor loadings matrix equals

$$\boldsymbol{\sigma} = \begin{bmatrix} -B(tt)\sigma_r & 0 \\ 0 & \sigma_S \end{bmatrix},$$

the risk premium vector equals

$$\boldsymbol{\Lambda} = \boldsymbol{\sigma} \boldsymbol{\lambda} = \begin{bmatrix} -B(tt)\lambda_r \sigma_r \\ \sigma_S \lambda_S \end{bmatrix}.$$

and the variance-covariance matrix equals

$$\boldsymbol{\Omega} = \boldsymbol{\sigma} \boldsymbol{\rho} \boldsymbol{\sigma}' = \begin{bmatrix} B^2(tt)\sigma_r^2 & 0 \\ 0 & \sigma_S^2 \end{bmatrix}.$$

The portfolio allocation  $\mathbf{x}$  determines how much is invested in a long-term bond and in equity. The remaining wealth,  $1 - \mathbf{i}'\mathbf{x}$ , is invested in cash. The wealth process is then given by

$$\frac{dF_t}{F_t} = (r_t + \mathbf{x}'\boldsymbol{\Lambda})dt + \mathbf{x}'\boldsymbol{\sigma}d\mathbf{Z}_t. \quad (2.100)$$

Our financial market model as formulated in Table 2.2 is similar to Brennan and Xia (2002) but without inflation risk ( $\pi = 0$  and as a result  $\boldsymbol{\xi}=0$ ). Moreover, we assume that there is no correlation between both risks. Theorem 3 in Brennan and Xia (2002) shows that the optimal portfolio allocation to a stock

and long-term bond in a terminal wealth problem equals

$$\begin{aligned}
 \mathbf{x}^* &= \frac{1}{\gamma} \Omega^{-1} \Lambda + \frac{(1-\gamma)B(T)}{\gamma} \Omega^{-1} \sigma \rho \mathbf{e}_2 \sigma_r & (2.101) \\
 &= \frac{1}{\gamma} \frac{1}{B^2(tt) \sigma_r^2 \sigma_S^2} \begin{bmatrix} \sigma_S^2 & 0 \\ 0 & B^2(tt) \sigma_r^2 \end{bmatrix} \begin{bmatrix} -B(tt) \lambda_r \sigma_r \\ \sigma_S \lambda_S \end{bmatrix} \\
 &\quad + \frac{(1-\gamma)B(T)}{\gamma} \frac{1}{B^2(tt) \sigma_r^2 \sigma_S^2} \begin{bmatrix} \sigma_S^2 & 0 \\ 0 & B^2(tt) \sigma_r^2 \end{bmatrix} \begin{bmatrix} -B(tt) \sigma_r & 0 \\ 0 & \sigma_S \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sigma_r \\
 &= \frac{1}{\gamma} \frac{1}{B^2(tt) \sigma_r^2 \sigma_S^2} \begin{bmatrix} -\sigma_S^2 B(tt) \lambda_r \sigma_r \\ B^2(tt) \sigma_r^2 \sigma_S \lambda_S \end{bmatrix} \\
 &\quad + \frac{(1-\gamma)B(T)}{\gamma} \frac{1}{B^2(tt) \sigma_r^2 \sigma_S^2} \begin{bmatrix} -\sigma_S^2 B(tt) \sigma_r \\ 0 \end{bmatrix} \sigma_r \\
 &= \begin{bmatrix} -\lambda_r / (\gamma \sigma_r B(tt)) \\ \lambda_S / (\gamma \sigma_S) \end{bmatrix} + \frac{\gamma-1}{\gamma} \begin{bmatrix} B(T)/B(tt) \\ 0 \end{bmatrix}.
 \end{aligned}$$

However, our problem is not a terminal wealth problem but a life-cycle problem with human wealth. In a life-cycle problem, total wealth  $W_t$  consists of financial wealth  $F_t$  and human wealth  $H_t$ , where human wealth (or human capital) is defined as the discounted value of future pension contributions. We assume that future pension contributions are risk-free and thus behave like a bond. Since the individual can only invest financial wealth, the optimal portfolio allocation of financial wealth  $\hat{\mathbf{x}}^*$  takes into account the bond-like behavior of human capital. Human capital is defined as follows

$$H_t = \int_0^{T_w + T_p - t} H_{t,h} dh, \quad (2.102)$$

where

$$H_{t,h} = \mathbb{E}_t \left[ \frac{M_{t+h}}{M_t} p_{t+h} \right], \quad (2.103)$$

where  $p_t = p$  for  $t < T_w$  and  $p_t = 0$  for  $t \geq T_w$ . The human wealth process then equals

$$\frac{dH_t}{H_t} = (r_t + \lambda_r \sigma_r D_t^h) dt - D_t^h \sigma_r dZ_{r,t} - p_t dt, \quad (2.104)$$

where  $D_t^h$  equals the duration of human capital

$$D_t^h = \int_0^{T_w + T_p - t} \frac{H_{t,h}}{H_t} B(h) dh. \quad (2.105)$$

As formulated in Equation (2.D), the financial wealth process equals

$$\begin{aligned} \frac{dF_t}{F_t} &= (r_t + \hat{\mathbf{x}}' \boldsymbol{\Lambda}) dt + \hat{\mathbf{x}}' \boldsymbol{\sigma} d\mathbf{Z}_t \\ &= (\dots) dt + \hat{x}_S \sigma_S dZ_{S,t} - \hat{x}_B \sigma_r B(tt) dZ_{r,t} \end{aligned} \quad (2.106)$$

The total wealth process is equal to

$$\begin{aligned} dW_t &= dH_t + dF_t \\ &= (\dots) dt - D_t^h \sigma_r H_t dZ_{r,t} + \hat{x}_S \sigma_S F_t dZ_{S,t} - \hat{x}_B \sigma_r B(tt) F_t dZ_{r,t} \\ &= (\dots) dt - D_t^h \sigma_r \frac{H_t}{W_t} W_t dZ_{r,t} + \hat{x}_S \sigma_S \frac{F_t}{W_t} W_t dZ_{S,t} - \hat{x}_B \sigma_r B(tt) \frac{F_t}{W_t} W_t dZ_{r,t} \\ &= (\dots) dt - \left( D_t^h \sigma_r \frac{H_t}{W_t} + \hat{x}_B \sigma_r B(tt) \frac{F_t}{W_t} \right) W_t dZ_{r,t} + \hat{x}_S \sigma_S \frac{F_t}{W_t} \cdot W_t dZ_{S,t}. \end{aligned} \quad (2.107)$$

Subsequently, we can derive the portfolio allocation of financial wealth to equity

$$\begin{aligned} \hat{x}_S^* &= \sigma_S \frac{F_t}{W_t} \\ \hat{x}_S^* &= \sigma_S \frac{W_t}{F_t} x_S^*, \end{aligned} \quad (2.108)$$

and derive the portfolio allocation of financial wealth to the long-term bond

$$\begin{aligned} x_B^* &= \frac{D_t^h}{B(tt)} \frac{H_t}{W_t} + \hat{x}_B^* \frac{F_t}{W_t} \\ \hat{x}_B^* \frac{F_t}{W_t} &= -\frac{D_t^h}{B(tt)} \frac{H_t}{W_t} + x_B^* \\ \hat{x}_B^* &= -\frac{D_t^h}{B(tt)} \frac{H_t}{F_t} + x_B^* \frac{W_t}{F_t}. \end{aligned} \quad (2.109)$$

## 2.E Optimal consumption choice

Theorem 2 in Brennan and Xia (2002) shows that the optimal consumption choice at time  $t$  equals<sup>15</sup>

$$C_t^* = Q_1^{-1}(t, T)F_t, \quad (2.110)$$

where  $Q_1(t, T)$  is the fraction of financial wealth  $F_t$  that is consumed and which is chosen to satisfy the budget constraint.  $Q_1(t, T)$  equals

$$Q_1(t, T) = \int_t^T \exp\left(\frac{1-\gamma}{\gamma}(B(s-t)r_t + a(t, s))\right) ds, \quad (2.111)$$

and  $a(t, T)$  equals

$$\begin{aligned} a(t, T) = & \frac{\lambda' \rho \lambda (T-t)}{2\gamma} + \left( \bar{r} + \frac{(1-\gamma)\sigma_r \lambda' \rho e_2}{\gamma \kappa} \right) ((T-t) - B(T-t)) \quad (2.112) \\ & + \frac{(1-\gamma)\sigma_r^2}{4\gamma \kappa^3} \left( 2\kappa(T-t) - 3 + 4\exp(\kappa(t-T)) - \exp(2\kappa(t-T)) \right). \end{aligned}$$

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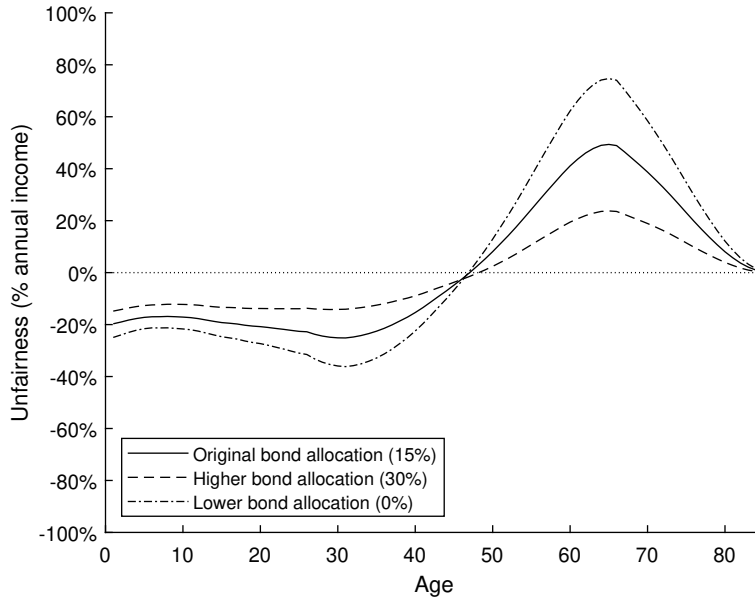
<sup>15</sup>Because our financial market model does not include inflation risk and because we assume that there is no correlation between equity market risk and interest rate risk, several terms disappear.

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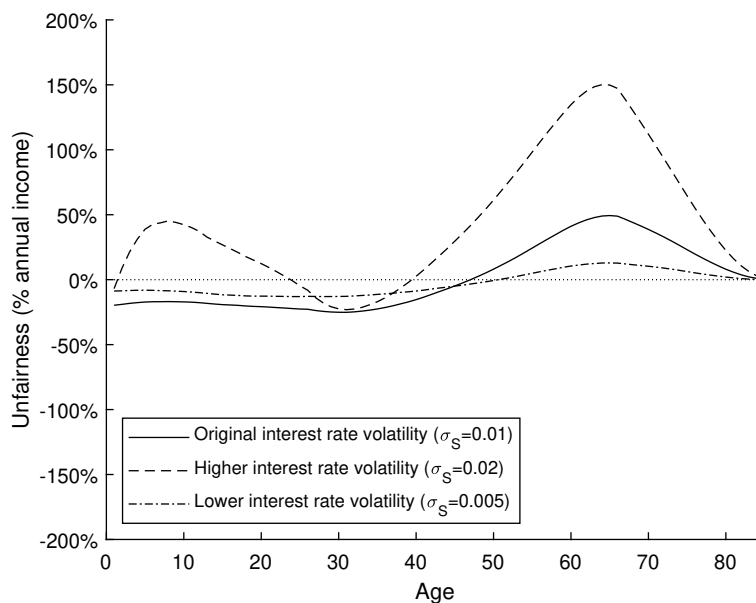
## 2.F Sensitivity analyses

**Figure 2.F.1: Unfairness in a CDC scheme for different bond allocations**

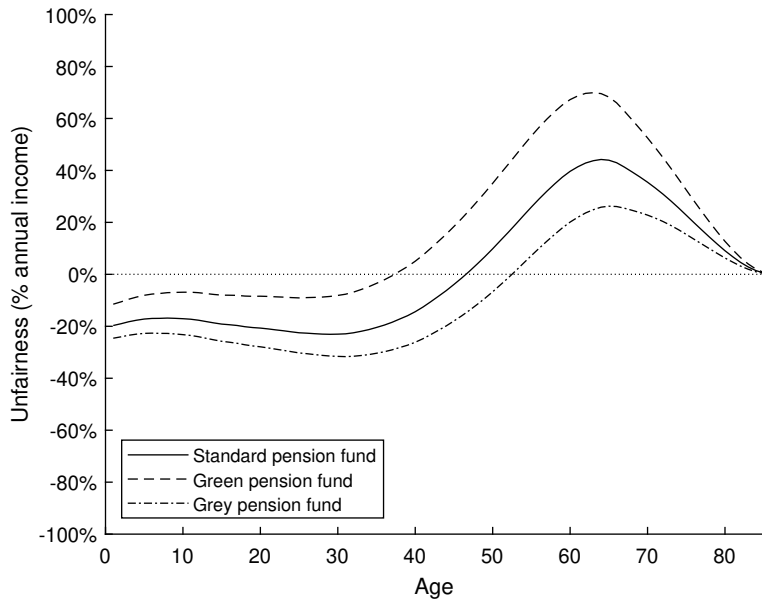
*This figure compares unfairness for different cohorts in a CDC scheme without smoothing for different bond allocations. The horizontal axis presents the age of each cohort at the start of the simulation.*



**Figure 2.F2: Unfairness in a CDC scheme for different interest rate volatilities**  
*This figure compares unfairness for different cohorts in a CDC scheme without smoothing for different interest rate volatilities. The horizontal axis presents the age of each cohort at the start of the simulation.*

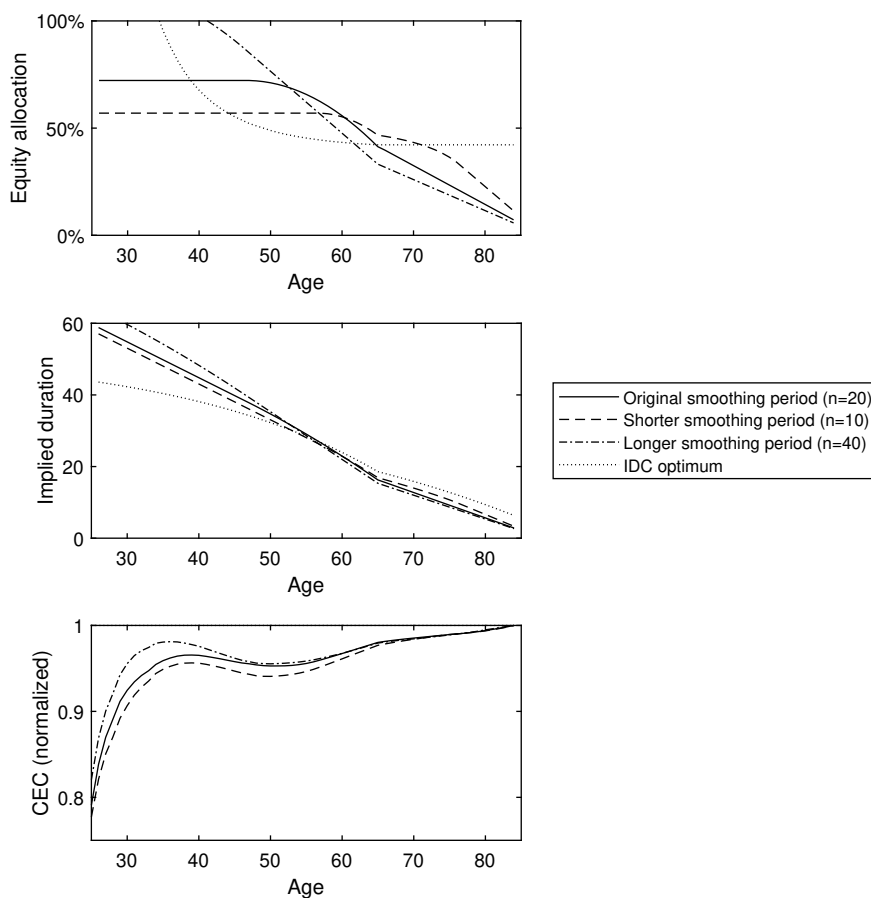


**Figure 2.F.3: Unfairness in a CDC scheme for different population compositions**  
 This figure compares unfairness for different cohorts in a CDC scheme without smoothing for different population compositions. In the standard pension fund each cohort consists of 1 participant. In the green pension fund the size of each cohort increases by 2 percent each year and in the grey pension fund the size of each cohort decreases by 2 percent each year. The horizontal axis presents the age of each cohort at the start of the simulation.

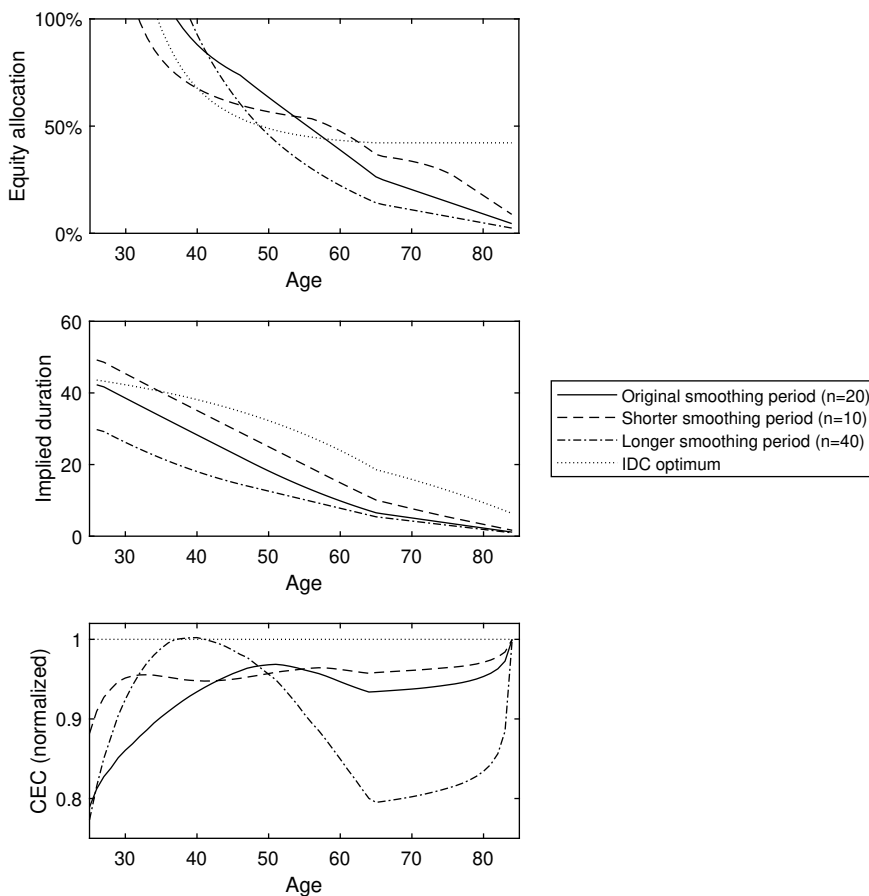


**Figure 2.F.4: Implied market exposures and CEC under closed smoothing**

This figure shows the implied market exposures and CEC in a CDC scheme under closed smoothing for different lengths of the smoothing period. The top graph presents the implied equity allocation and the middle graph the implied duration for each age over the life-cycle. The bottom graph shows the CEC relative to the CEC in the optimal IDC scheme for different cohorts. The horizontal axis in this graph presents the age of each cohort at the start of the simulation.

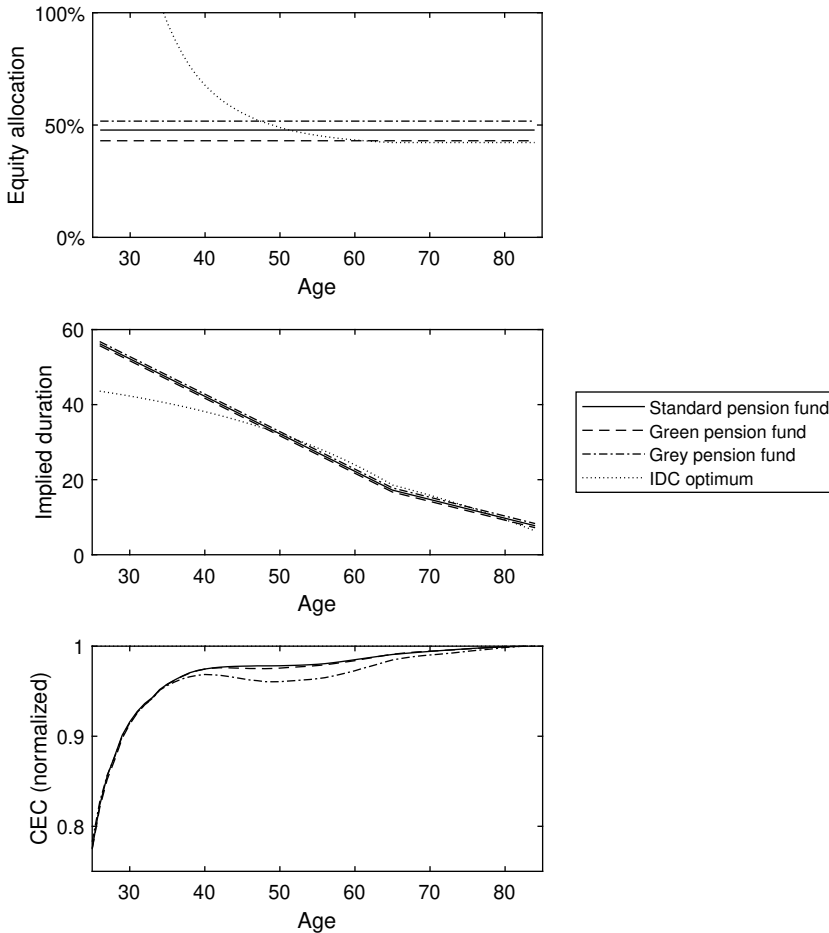


**Figure 2.F.5: Implied market exposures and CEC under open complete smoothing**  
 This figure shows the implied market exposures and CEC in a CDC scheme under open complete smoothing for different lengths of the smoothing period. The top graph presents the implied equity allocation and the middle graph the implied duration for each age over the life-cycle. The bottom graph shows the CEC relative to the CEC in the optimal IDC scheme for different cohorts. The horizontal axis in this graph presents the age of each cohort at the start of the simulation.



**Figure 2.F.6: Implied market exposures and CEC alternative population composition**

This figure shows the implied market exposures and CEC in a CDC scheme under closed smoothing for different population compositions. In the standard pension fund each cohort consists of 1 participant. In the green pension fund the size of each cohort increases by 2 percent each year and in the grey pension fund the size of each cohort decreases by 2 percent each year. The top graph presents the implied equity allocation and the middle graph the implied duration for each age over the life-cycle. The bottom graph shows the CEC relative to the CEC in the optimal IDC scheme for different cohorts. The horizontal axis in this graph presents the age of each cohort at the start of the simulation.



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## 2.G Definitions

Parameter	Definition
$\alpha_m$	smoothing parameter for maturity $m$
$A_t$	value of the assets at time $t$
$b_{j,t}^m$	pension benefit of cohort $j$ at time $t$ that matures after $m$ years
$\Delta b_{j,t}^m$	new benefit accrual of cohort $j$ at time $t$ that matures after $m$ years
$C_t$	consumption of individual at time $t$
$CEC$	certainty equivalent of total welfare
$CEC_j$	certainty equivalent of consumption during retirement for cohort $j$
$\delta_t^m$	pension benefit adjustment for cohort $j$ at time $t$ of benefit that matures after $m$ years
$DA$	duration of the asset portfolio
$DB_m$	duration of zero-coupon bond with maturity $m$
$DL$	duration of liability portfolio
$F_t$	pension wealth of an individual at time $t$
$FR_t$	funding ratio based on current assets and liabilities at time $t$
$FR_t^f$	funding ratio based on current and future assets and liabilities at time $t$
$\gamma$	risk aversion parameter
$ID_m$	implied duration of pension benefit that matures after $m$ years
$IRH_m$	interest rate hedge of a pension benefit that matures after $m$ years
$j$	indicator for cohort
$\kappa$	mean-reversion parameter of interest rate
$L_t^m$	discounted value of benefits that mature after $m$ years at time $t$
$L_t$	discounted value of all benefits at time $t$
$\tilde{L}_t$	discounted value of benefits before benefit adjustment at time $t$
$L_t^f$	discounted value of future benefits at time $t$

*Continued on next page*

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Table 2.G.1 – *continued from previous page*

Parameter	Definition
$\tilde{L}_t^f$	discounted value of future benefits before benefit adjustment at time $t$
$\lambda_r$	price of risk interest rate
$\lambda_S$	price of risk equity
$m$	indicator for maturity
$n$	length of smoothing period
$p$	pension contribution
$P(m, r_t)$	price of a zero-coupon bond at time $t$ that matures after $m$ years
$pa_{j,t}$	price of a variable annuity for cohort $j$ at time $t$
$\Pi_t$	discounted value of future pension contributions
$\bar{r}$	unconditional mean of interest rate
$r_t^S$	return on equity
$r_t^B$	return on long-term bonds
$r_t^C$	return on cash
$\beta$	time preference parameter
$\sigma_r$	volatility of interest rate
$\sigma_S$	volatility of equity
$SW$	total welfare
$t$	indicator for time
$T$	investment horizon
$T_p$	length of retirement period
$T_s$	length of simulation period
$T_w$	length of working period
$tt$	maturity of long-term bond
$U_j$	expected utility during retirement for cohort $j$
$V_{j,t}^m$	discounted value of pension benefit of cohort $j$ at time $t$ that matures after $m$ years
$x_S$	equity allocation
$x_B$	exposure to long-term bonds

*Continued on next page*

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Table 2.G.1 – *continued from previous page*

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Parameter	Definition
$x_C$	exposure to cash
$y$	yearly income
$Z_{r,t}$	standard Brownian motion of interest rate process
$Z_{S,t}$	standard Brownian motion of equity process

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# 3

## The economics of sharing macro-longevity risk<sup>1</sup>

Macro-longevity risk is the uncertainty about future mortality rates. Mortality rates may for example decrease as a consequence of medical improvements, or may increase because of new diseases. Macro-longevity risk is a systemic risk. It affects society at large and consequently the entire population of, e.g., an occupational pension fund. As a consequence, macro-longevity risk does not decrease by pooling a large group of participants. Nonetheless, the key message in this paper is that sharing macro-longevity risk between different age groups of a pension scheme is beneficial because the risk affects cohorts differently. Furthermore, we show that a flexible retirement age is an instrument that enhances the benefits of risk sharing. Macro-longevity risk differs from micro-longevity risk or the individual uncertainty about the time of death. Micro-longevity risk is an idiosyncratic risk that can be diversified by pooling enough participants in a pension scheme.

Macro-longevity risk has a significant impact on pension provisioning, depending on the configuration. A defined benefit (DB) pension scheme pro-

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<sup>1</sup>This chapter is based on Broeders et al. (2021a).

protects its beneficiaries against macro-longevity risk. The risk increases the uncertainty in the funding ratio, or the ratio of assets over liabilities. These changes in the funding ratio are however borne by the employer (i.e. the 'sponsor') and workers that contribute to the pension scheme and not by the retirees. In a defined contribution (DC) pension scheme with a fixed annuity, pension benefits are guaranteed after retirement and macro-longevity risk is borne by the pension provider, for example the shareholders of an insurance company. In a DC pension scheme with a variable annuity, pension benefits are adjusted to changes in future mortality rates. As a consequence, participants bear macro-longevity risk themselves. Retirees are especially vulnerable to macro-longevity risk because they cannot compensate lower pension benefits by working longer or saving more. A decrease in mortality however does not only affect retirees. Also workers might be affected through a decrease in future pension benefits or an increase in contributions to finance a longer retirement period. Hence, macro-longevity risk affects both retirees and workers. However, the risk does not affect all cohorts in the same way or by the same amount. Medical progress or diseases may affect cohorts differently. Furthermore, workers have more risk-absorbing capacity compared to retirees because their labor supply acts as a hedge against changes in mortality rates. These differences between cohorts create a clear case for risk sharing. This case is strengthened further by the fact that a market for macro-longevity risk is close to non-existent.

The central problem we address in this paper is risk sharing between cohorts in a funded occupational pension scheme. Collective risk sharing is a risk management tool that pools risks across cohorts and re-allocates these risks in a different way back to the same cohorts. We approach this actuarial topic from an economic perspective. The innovation in this paper is to derive Pareto-improving risk sharing rules for macro-longevity risk. We find these rules by assuming that a social planner maximizes the welfare gain from risk sharing relative to autarky for all participants in the pension scheme. This social planner makes decisions about risk sharing on behalf of the participants with the aim to smooth their consumption over the life-cycle. Furthermore, we focus on the welfare gains of risk sharing under various exogenous retirement age policies. The retirement age policy has a strong incremental impact

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on the benefits of risk sharing. We confine our analysis to risk sharing and the impact of different retirement age policies on the welfare gains of risk sharing. We do not analyze the suitability of these different retirement age policies from a broader policy perspective. We also do not consider additional private savings by participants or adjustments in state sponsored ‘first pillar’ pensions as ways to absorb longevity risk. This broader policy perspective is a different research question.

We find that the specific features of the retirement age policy have a large impact on the optimal risk-sharing rule and on the size of the welfare gains. If the retirement age is fixed, welfare gains from sharing macro-longevity risk measured on a 10-year horizon are between 0.1 and 0.3 percent of certainty equivalent consumption after retirement. In this case, the impact of macro-longevity risk on consumption after retirement is more or less equal for different cohorts. As a consequence, young cohorts do not absorb macro-longevity risk of other cohorts and the welfare gains from sharing macro-longevity risk are limited. By contrast, if the retirement age is linked to life expectancy the welfare gains from sharing macro-longevity risk are substantially higher, up to 1.8 percent in the Lee–Carter model and up to 2.9 percent in the Cairns–Blake–Dowd model. The risk bearing capacity of workers is large, because their labor supply acts as a hedge against changes in macro-longevity. Human capital of workers increases by working longer. As a result, workers can also absorb the risk from retirees. Moreover, a positive risk compensation is not necessarily required for young cohorts to absorb risk of retirees.

This paper contributes to the knowledge on longevity hedging solutions and the recent literature on managing macro-longevity risk. Related papers are De Waegenare et al. (2017), De Waegenare et al. (2018) and papers that consider group self-annuitisation schemes (GSAs), for example Piggott et al. (2005), Qiao and Sherris (2013) and Boon et al. (2019). These papers investigate sharing micro- and macro-longevity risk. In GSAs longevity risk is shared uniformly among participants in a pool. De Waegenare et al. (2017) and De Waegenare et al. (2018) consider ad hoc risk-sharing rules for micro- and macro-longevity risk. We consider macro-longevity risk only and determine the optimal risk-sharing rule. Moreover, we include a risk compensa-

tion which is not the case in the papers mentioned above.

Optimal sharing of longevity risk has been investigated in other settings by Andersen (2014) and Bommier and Schernberg (2018). Andersen (2014) investigate intergenerational re-distributions and risk sharing in a PAYG scheme with changing longevity and an endogenous retirement age. Bommier and Schernberg (2018) investigate the optimal allocation of longevity risk for a temporally risk averse participant with access to perfect insurance markets.

Our paper also contributes to the literature on risk sharing. Most papers on risk sharing in funded pension schemes focus on financial risks, e.g., Gollier (2008), Cui et al. (2011) and Bovenberg and Mehlkopf (2014). We determine a Pareto-improving risk-sharing rule for a non-financial risk. Finally, we are innovative in investigating the impact of different retirement age policies on risk sharing. Investigating different retirement age policies is relevant, as some countries do and other countries do not link the retirement age to life expectancy. Stevens (2017) investigates the impact of retirement age policies on the individual retirement age, expected remaining lifetime at retirement and value of pension benefits, but does not consider collective risk sharing.

Next to risk sharing across cohorts, there are other ways to manage macro-longevity risk for occupational pension schemes. First, macro-longevity risk can be transferred to institutional investors via financial markets. This process of securitization is described by, for example, Cairns et al. (2006b), Blake et al. (2006b), Ngai and Sherris (2011), Hunt and Blake (2015), and Zhou et al. (2015). Securitization can improve welfare as it achieves a more efficient risk allocation by distributing the risk among market participants who are better able to bear the risk. Second, governments can establish solutions to manage macro-longevity risk by issuing longevity bonds (Brown and Orszag (2006), and Blake et al. (2014)). Government-issued longevity bonds allow for an efficient and fair sharing of longevity risk across generations. Third, buy-ins and buy-outs are ways to insure macro-longevity risk (Lin et al. (2015)). Buy-ins and buy-outs are contracts where the occupational pension scheme pays a single premium for the transfer of macro-longevity risk to an insurer. A buy-in is an asset and the pension scheme maintains the legal responsibility to pay benefits.

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Under a buy-out, the insurer takes over the legal responsibility for paying benefits directly to scheme members. A disadvantage of pension buy-ins and buy-outs is that they are generally considered to be expensive.<sup>2</sup> Fourth, natural hedging is a way to manage macro-longevity risk (Cox and Lin (2007)). Macro-longevity risk in annuity policies can be hedged with mortality risk in life insurance policies.<sup>3</sup> Participants living longer than expected have a negative impact on annuity policies but a positive impact on life insurance products because less participants die at a young age. However, mortality risk only provides a partial hedge to longevity risk due to the different nature of both risks and the different age groups. Moreover, the mortality risk market is more than five times smaller compared to the already small longevity risk market (EIOPA (2011)).

There are several reasons for the lack of solutions and a well-functioning longevity market (Basel Committee on Banking Supervision (2013)). A government is not a natural issuer of longevity bonds because it is already exposed to longevity risk, typically through first pillar pensions. Further, insurance companies are reluctant to underwrite macro-longevity risk because of basis risk.<sup>4</sup> As a case study Blake et al. (2006a) analyse the withdrawal of a longevity bond that was supposed to be issued by the European Investment Bank (EIB) and attribute the lack of demand for the bond to design, pricing and institutional issues. Because there is no well-developed market for longevity risk a, so-called, replicating portfolio does not exist. A replicating portfolio serves as a benchmark for liability valuation and contains standard financial instruments that match the cash flows of the liabilities as good as possible. However, also without the existence of a replicating portfolio, liability valuation is possible. Pelsser (2011) discusses and compares several methods proposed in the literature to price risks in incomplete markets. In these methods one has to define a pricing operator to determine the value

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<sup>2</sup>This is a result of insurance companies being typically subject to more stringent regulation than pension funds and because any initial underfunding requires a lump-sum payment by the sponsor to reach full funding before the pension scheme can be sold to a third party (Basel Committee on Banking Supervision (2013)).

<sup>3</sup>In this context, mortality risk is the risk that people live shorter than expected.

<sup>4</sup>Macro-longevity risk is rather specific to a pension scheme's population. This will create a basis risk compared with generic market solutions that are typically based on a country's population.

of a payoff. The pricing operator is based on the expected value of a payoff minus a penalty term that factors in the risk of the payoff.

Due to the lack of sufficiently good market solutions and increasing longevity, we see that funding risks are increasingly being transferring from pension scheme sponsors to pension participants (Munnell (2006) and Novy-Marx and Rauh (2014)). Because macro-longevity risk lies with participants, it is even more important to take advantage of the differences in the risk exposure of cohorts. The solutions presented in this paper amount to creating an internal market between the participants in the pension scheme. The findings in this paper are therefore relevant for pension scheme design.

The remainder of this paper is organized as follows. Section 3.1 describes the modeling of macro-longevity risk. Section 3.2 explains the concept of collective risk sharing. Section 3.3 describes the different retirement age policies. Section 4.4.4 presents the results. Section 4.5 concludes and gives a policy evaluation.

### **3.1 Macro-longevity risk**

We consider three sources of macro-longevity risk. Figure 3.1.1 visualizes these three sources of risk relative to each other. The first source is stochastic variation. This is the random variation in the aggregate realized number of deaths. A stochastic mortality model captures stochastic variation, assuming that the model and its parameters are known. In a given year more or less people than expected may die.<sup>5</sup> The second source is parameter risk. This is the uncertainty about the true value of the parameters, assuming that the stochastic model is a correct representation of reality. The third source is model risk. This is the uncertainty about the appropriateness of the mortality model. For instance, model risk can occur due to structural breaks that are not captured by the model. Medical innovations or a rapid increase of obesity can cause these structural breaks. All the three sources of uncertainty can

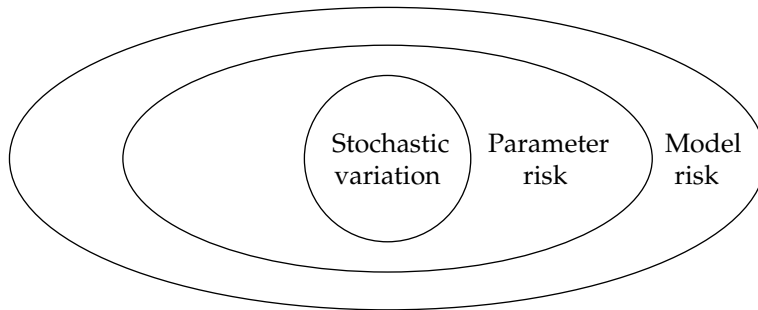
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<sup>5</sup> Stochastic variation in death rates of individuals within cohorts, i.e., individual uncertainty about the time of death, is excluded. We assume that cohorts are large enough so that micro-longevity risk is fully diversified.

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lead to a mis-estimation of mortality rates. In our paper we focus on stochastic variation as source of macro-longevity risk. However, we also consider a type of parameter risk, namely recalibration risk. This will be discussed in more detail in Section 3.1.2. We also address model risk by using an alternative model for macro-longevity risk in Section 3.4.1. We employ the widely

**Figure 3.1.1: Sources of macro-longevity risk.**



used Lee and Carter (1992) model which is a stochastic mortality model that allows for stochastic variation in death rates. It is fitted to historical data to forecast death rates and to quantify macro-longevity risk. Several academics use the Lee-Carter model to model macro-longevity risk, for example Hari et al. (2008), Cocco and Gomes (2012), Stevens (2017) and De Waegenaere et al. (2017). Moreover, the model is the basis of several mortality table forecasts in practice.<sup>6</sup>

Cairns et al. (2011) discuss the suitability of six stochastic mortality models for forecasting mortality and conclude that the Lee-Carter model is both reasonably robust relative to historical data and produces plausible forecasts. Because the Lee-Carter model is a one-factor model with no cohort effects mortality improvements at different ages are perfectly correlated. In reality mortality improvements at different ages are not perfectly correlated and the welfare benefits of sharing macro-longevity risk are higher. Therefore, we

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<sup>6</sup> For example the U.S. Census Bureau and the U.S. Social Security Administration. The Actuarial Society in the Netherlands ('Koninklijk Actuarieel Genootschap') uses an alternative specification of this model.

underestimate the welfare benefits by using the Lee-Carter model. Alternative stochastic mortality models are for example the model of Renshaw and Haberman (2006) that is an extension of the Lee-Carter model including a cohort effect and the two-factor model of Cairns et al. (2006a). We will use the latter model in Section 3.4.1 to perform a sensitivity analysis.

In the remaining of this section we discuss the Lee-Carter model in Section 3.1.1 and elaborate on macro-longevity risk in the Lee-Carter model in Section 3.1.2. In Section 3.1.3 we analyze the impact of macro-longevity risk on the remaining lifetime and on the variable annuity value.

### 3.1.1 Lee-Carter model

The core of the Lee-Carter model is the central death rate  $\mu_{x,t}$  for a cohort of age  $x$  in year  $t$ . This death rate equals

$$\mu_{x,t} = \frac{D_{x,t}}{E_{x,t}}, \quad (3.1)$$

where  $D_{x,t}$  is the number of deaths in year  $t$  among the people in the cohort of age  $x$  and  $E_{x,t}$  is the number of people in the cohort of age  $x$  in year  $t$ .

The Lee-Carter model estimates the log central death rates with the following expression<sup>7</sup>

$$\ln(\mu_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}, \quad (3.2)$$

where  $\alpha_x$  is an age-specific constant,  $\kappa_t$  is a time trend and  $\beta_x$  represents the sensitivity of the log central death rates to the time trend. This sensitivity generally decreases with age. This implies that death rates for high ages are less effected by the time trend compared to death rates for young ages. The time trend reflects the development of death rates over time. This trend is generally downward implying an increasing life expectancy over time. The error term  $\epsilon_{x,t}$  is normally distributed with mean zero and age-dependent variance  $\sigma_{\epsilon,x}^2$ .

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<sup>7</sup> The logarithm of  $\mu_{x,t}$  ensures that death rates cannot be negative. However, death rates can exceed unity but this is not a problem in practice. This can be avoided by modeling  $\ln(\mu_{x,t}/(1 - \mu_{x,t}))$ , but in that case a linear trend in  $k$  does not imply a constant geometric rate of decline for each age-specific death rate (Lee (2000)).

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The Lee Carter model assumes that the central death rates are constant during a year, i.e.,  $\mu_{x+s,t+s} = \mu_{x,t}$  ( $0 \leq s \leq 1$ ). Therefore, we can approximate the one-year death probability  $q_{x,t}$  in the following way

$$q_{x,t} \approx 1 - \exp(-\mu_{x,t}). \quad (3.3)$$

The one-year death probability is the probability that an individual of age  $x$  and alive at the beginning of year  $t$  dies before year  $t + 1$ . The one-year survival probability  $p_{x,t}$  equals

$$p_{x,t} = 1 - q_{x,t} \approx \exp(-\mu_{x,t}). \quad (3.4)$$

One-year survival probabilities can be used to calculate the cumulative survival probability  ${}_i p_{x,t}$ . The Lee-Carter model forecasts survival probabilities by estimating the time trend  $\kappa_t$  in (3.2) with a standard univariate time series model<sup>8</sup>

$$\kappa_t = c + \kappa_{t-1} + \eta_t, \quad (3.5)$$

where  $c$  is the drift and  $\eta_t$  is the error term that is normally distributed with mean zero and variance  $\sigma_\eta^2$ . The Lee-Carter model assumes that the error terms  $\epsilon_{x,t}$  in (3.2) and  $\eta_t$  in (3.5) are independent. This independency implies that for each cohort mortality develops at an own age-specific exponential rate.

### Calibration of the Lee-Carter model

In this paper we use mortality data of Dutch females from 1985 until 2014 from the Human Mortality Database to calibrate the parameters of the Lee-Carter model.<sup>9,10</sup> We apply the method of Kannisto (1994) to extrapolate

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<sup>8</sup>Lee and Carter (1992) conclude after testing several *ARIMA* specifications that the *ARIMA*(0, 1, 0) model, a random walk with drift, is most appropriate to fit the data.

<sup>9</sup>Human Mortality Database (HMD). University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany): <http://www.mortality.org/>.

<sup>10</sup>A calibration period of 30 years is conventional. For statistical reliability, one would prefer a longer calibration period (HMD). However, a shorter calibration period leads to a better estimate of the current trend in mortality improvements.

the central death rates for ages  $x \in \{91, \dots, 110\}$  using the death rates of younger cohorts because death rates for very high ages are not available or not very reliable.<sup>11</sup> We estimate parameters  $\alpha_x, \beta_x$  and  $\kappa_t$  in (3.2) using a singular value decomposition and we impose the standard restrictions to identify the model. We use the standard identification choice of Lee and Carter (1992) that imposes the following constraints. As a result of these restrictions, the age-specific constant  $\alpha_x$  is the average log central death rate of cohort of age  $x$  over time, i.e.,  $\alpha_x = \frac{1}{30} \sum_{t=1985}^{2014} \ln(\mu_{x,t})$ . Subsequently the drift  $c$  and variance  $\sigma_\eta^2$  in (3.5) are estimated using the  $\kappa_t$ 's.

### 3.1.2 Macro-longevity risk in Lee-Carter model

As already mentioned at the beginning of Section 3.1 we focus in our paper on stochastic variation as source of macro-longevity risk. Macro-longevity risk in the Lee-Carter model arises from two random variables:

- *Uncertainty in time trend*: random shock  $\eta_t$  in the time trend  $\kappa_t$  in (3.5). It reflects the uncertainty in the time trend, i.e., development of death rates over time. The impact of this shock on future death rates depends on the size of  $\sigma_\eta$  and  $\beta_x$ .
- *Uncertainty in death rates*: random shock  $\epsilon_{x,t}$  in the log central death rate  $\mu_{x,t}$  in (3.2). It reflects particular age-specific historical influences not captured by the model. The impact of this shock on future death rates depends on the size of  $\sigma_{\epsilon,x}$ .

We model the first source of macro-longevity risk, stochastic variation, as the aggregate effect of those two random variables. We assume that  $\eta_t$  and  $\epsilon_{x,t}$  are independent and normally distributed. The sum of two independent normal random variables is again normally distributed

$$\left. \begin{array}{l} \eta_t \sim N(0, \sigma_\eta^2) \\ \epsilon_{x,t} \sim N(0, \sigma_{\epsilon,x}^2) \end{array} \right\} \Rightarrow \beta_x \eta_t + \epsilon_{x,t} \sim N(0, \beta_x^2 \sigma_\eta^2 + \sigma_{\epsilon,x}^2). \quad (3.6)$$

The trend risk  $\eta_t$  is multiplied with the sensitivity of to the time trend  $\beta_x$

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<sup>11</sup>This method uses a logistic regression based on  $\mu_{x,t}$  for ages  $x \in \{80, 81, \dots, 90\}$ . Death rates above age  $x = 110$  are assumed to be equal to the death rates at age  $x = 110$ .

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because the sensitivity parameter  $\beta_x$  determines the impact of the time trend on death rates. Macro-longevity risk has zero mean because it is the risk that future mortality rates deviate from the best estimate mortality rates.

In this paper we do not consider yearly macro-longevity shocks but consider macro-longevity risk on a 10-year horizon because a pension contract has a long horizon and we want to focus on structural changes in life expectancy only. We determine macro-longevity shocks on a 10-year horizon by aggregating the independent normal random variables in (3.6) over 10 years to get

$$\sum_{i=0}^9 (\beta_{x+i} \eta_{t+i} + \epsilon_{x+i,t+i}) \sim N \left( 0, \sigma_\eta^2 \sum_{i=0}^9 \beta_{x+i}^2 + \sum_{i=0}^9 \sigma_{\epsilon,x+i}^2 \right). \quad (3.7)$$

So far we have focused on stochastic variation as source of macro-longevity risk. Now is a good moment to recall that the second source of macro-longevity risk is parameter risk. When new mortality data become available we can recalibrate the parameters. This recalibration changes the parameter estimates (Cairns (2013)). In this paper we include this as recalibration risk. We use the realized death rates  $\mu_{x,t}$  including the trend shocks  $\eta_t$  and estimation shocks  $\epsilon_{x,t}$  to recalibrate the parameters in (3.2) and (3.5). Subsequently, we use these recalibrated parameters to forecast future death rates. By considering recalibration risk we include the influence of parameter risk.<sup>12</sup> In the main analysis, macro-longevity risk includes both stochastic variation and recalibration risk. We address model risk separately in Section 3.4.1.

### 3.1.3 The impact of macro-longevity risk on remaining lifetime and variable annuity value

Now that we have defined macro-longevity risk in the Lee–Carter model, we are able to assess its impact. We express this impact in two measures: the expected remaining lifetime and the value of a (deferred) variable annuity. Figure 3.1.2 visualizes the impact of macro-longevity risk measured on a 10-

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<sup>12</sup>A more formal way to include parameter risk is to use standard Bayesian methods (Cairns et al. (2006a)).

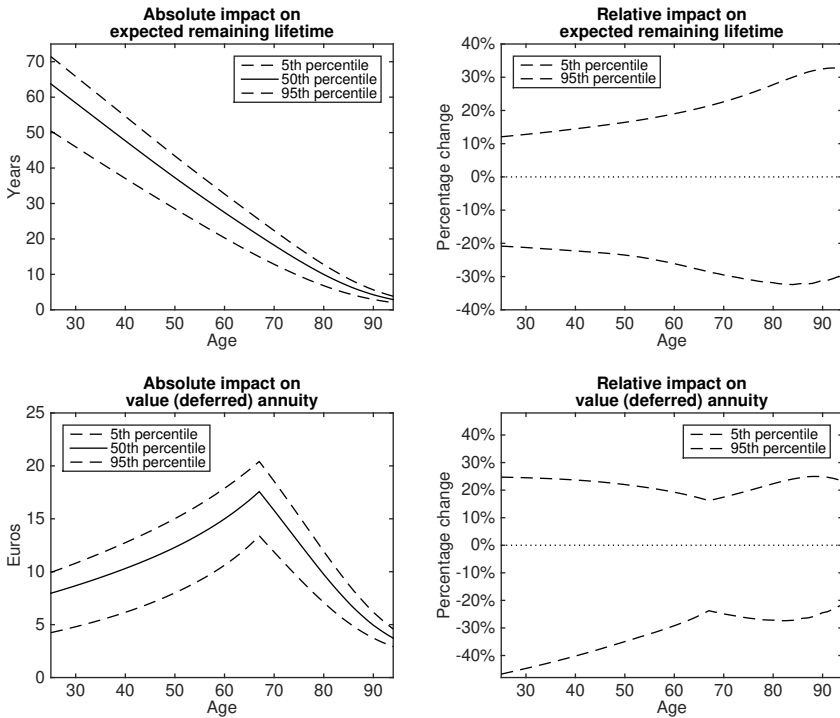
year horizon in the Lee-Carter model on the expected remaining lifetime (top graphs) and the value of a (deferred) variable annuity (bottom graphs) by displaying different percentiles of the distribution.<sup>13</sup> Besides the absolute impact on the expected remaining lifetime and the value of a (deferred) annuity (left-hand graphs), it is also interesting to look at the relative change of these variables (right-hand graphs). We assume that the interest rate - used to determine the value of a (deferred) annuity - equals  $r = 2\%$  and the retirement age equals  $R = 67$ . We observe the following. The top left-hand graph shows that the expected remaining lifetime decreases with age. At the age of 25 the expected remaining lifetime is 64 years while it is 11 years at the age of 80. The intuition behind this decrease is that older people have a higher probability of dying. Moreover, we see that the impact of macro-longevity risk also decreases with age. The difference between the 5th and 95th percentile at age 25 is 21 years while it is 6 years at the age of 80. There are two reasons for this decreasing impact. First, a longevity shock has an impact on all future death probabilities. The expected remaining lifetime of young cohorts depends on more future death probabilities compared to the expected remaining lifetime of old cohorts. Second, the impact of both trend and estimation risk decreases with age. The sensitivity of the death rates  $\beta_x$  decreases with age implying a decreasing impact of the trend risk. The variance of the estimation risk  $\sigma_{\epsilon,x}^2$  generally decreases with age as there is less uncertainty at higher death rates. This implies a decreasing impact of estimation risk.

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<sup>13</sup>Negative (positive) macro-longevity shocks, i.e., negative (positive) random shocks in log central death rates, have a positive (negative) impact on life expectancy and annuity values. To avoid confusion we denote negative (positive) macro-longevity shocks by unexpected increases (decreases) in life expectancy.

**Figure 3.1.2: Impact macro-longevity risk on the expected remaining lifetime and value of a variable annuity.**

This table shows the impact of macro-longevity risk measured on a 10-year horizon in the Lee-Carter model on the expected remaining lifetime and the value of a (deferred) variable annuity for a Dutch female in 2014 in absolute terms (left-hand graphs) and relative change (right-hand graphs) assuming a constant interest rate  $r = 2\%$  and fixed retirement age  $R = 67$ .



The value of a deferred annuity (bottom left-hand graph) increases before retirement because of two reasons:

- The probability that a participant reaches the retirement age increases with age.
- The value of a deferred annuity is lower for young cohorts compared to cohorts just before retirement because of a larger discounting effect.

The relative change of the value of a (deferred) annuity as a result of a macro-longevity shock is in the same order of magnitude for all age cohorts. Later in this paper we will see that this explains the small welfare gains in case of collective risk sharing when the retirement age is fixed.

Another important observation in Figure 3.1.2 is that the impact of macro-longevity risk on the expected remaining lifetime and (deferred) annuity value is asymmetric. Unexpected increases in life expectancy have a smaller impact than unexpected decreases in life expectancy. This follows from the exponential distribution of death rates. A consequence of this asymmetry is that the expectations of a future survival probability is smaller than or equal to its forecasted value. We formally show this in Appendix 3.A. This property also holds for the expected remaining lifetime and expected (deferred) annuity value. Both are smaller than or equal to their forecasted value.

## **3.2 Sharing macro-longevity risk**

After modeling macro-longevity risk, we now turn to collective risk sharing. Pension providers can create an internal market for macro-longevity risk, by reallocating the aggregate risk differently to cohorts. Consequently some cohorts absorb part of the risk of other cohorts in exchange for monetary compensation. We refer to this as collective risk sharing. Collective risk sharing of macro-longevity risk can be welfare enhancing because the risk is not traded on a liquid market and cohorts are affected differently by the risk.<sup>14</sup> In fact, it creates a new asset that can be priced and makes the market more complete.

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<sup>14</sup>Collective risk sharing can also be welfare enhancing if the risk is traded with future cohorts. In this paper we abstract from this dimension.

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We discuss the concept of collective risk sharing in Section 3.2.1. We use a stylized two-agent model in Section 3.2.2 to derive an analytical risk sharing solution. This model gives some economic intuition. Subsequently, we present a full model in Section 3.2.3 that consists of many cohorts representing the population of a pension fund. We use this model to optimally share macro-longevity risk between cohorts in a pension scheme.

### 3.2.1 Risk sharing model

We start with the risk exchange model of Borch (1960). In this model there are  $N$  agents. Agent  $i$  has initial wealth  $W_i$ . The agents are exposed to an exogenous risk factor  $\tilde{y}_i$ . Each individual's preferences are represented by a utility function  $U_i(\cdot)$  with positive and decreasing marginal utility. In autarky consumption consists of wealth and the exposure to the risk factor

$$C_i^a = W_i + \tilde{y}_i \quad i = 1, \dots, N. \quad (3.8)$$

In case of risk sharing the agents aggregate and subsequently redistribute the risk among themselves through the continuous risk sharing function  $T_i(\tilde{y}) = T_i(\tilde{y}_1, \dots, \tilde{y}_N)$ . This leads to the following expression for consumption in case of risk sharing

$$C_i^s = W_i + T_i(\tilde{y}) \quad i = 1, \dots, N, \quad (3.9)$$

under the condition that the aggregate risk is fully distributed over all agents

$$\sum_{i=1}^N T_i(\tilde{y}) = \sum_{i=1}^N \tilde{y}_i. \quad (3.10)$$

Risk sharing is Pareto improving compared to autarky if the welfare of at least one agent improves

$$\mathbb{E}[U(C_i^s)] > \mathbb{E}[U(C_i^a)] \quad \text{for some } i = 1, \dots, N \quad (3.11)$$

and all other agents do not become worse off

$$\mathbb{E}[U(C_i^s)] \geq \mathbb{E}[U(C_i^a)] \quad \forall i = 1, \dots, N. \quad (3.12)$$

A risk sharing rule  $\{T_1, \dots, T_N\}$  is Pareto optimal if no Pareto improvement is possible, i.e. there does not exist an alternative risk-sharing rule  $\{\bar{T}_1, \dots, \bar{T}_N\}$  such that

$$\mathbb{E}[U(W_i + \bar{T}_i(\tilde{y}))] \geq \mathbb{E}[U(W_i + T_i(\tilde{y}))] \quad \forall i = 1, \dots, N, \quad (3.13)$$

and for at least one agent strict inequality holds

$$\mathbb{E}[U(W_i + \bar{T}_i(\tilde{y}))] > \mathbb{E}[U(W_i + T_i(\tilde{y}))] \quad \text{for some } i = 1, \dots, N. \quad (3.14)$$

A Pareto optimal risk-sharing rule yields the highest welfare gain compared to autarky.

The theorem of Borch (1960) provides the following necessary and sufficient conditions for a risk sharing rule  $\{T_1, \dots, T_N\}$  to be Pareto optimal

$$\begin{aligned} U'_i(W_i + T_i(\tilde{y})) &= c_i U'_1(W_1 + T_1(\tilde{y})) \quad \forall i = 1, \dots, N, \\ \sum_{i=1}^N T_i(\tilde{y}) &= \sum_{i=1}^N \tilde{y}_i, \end{aligned} \quad (3.15)$$

where  $c_2, c_3, \dots, c_N > 0$  can be chosen arbitrarily and  $c_1 = 1$ . This theorem shows that in a Pareto-optimal risk-sharing rule the ratio of marginal utilities of two different agents is equal to a constant. Borch (1960) also proofs that in a Pareto-optimal risk-sharing rule  $T_i(\tilde{y})$  is a function of the aggregate risk  $\sum_{i=1}^N \tilde{y}_i$  only. This implies that in a Pareto-optimal risk-sharing rule a pool must be formed of the aggregate risk of all participants.

The conditions for the existence of a Pareto-optimal risk-sharing rule in Borch's theorem in (3.15) are very weak. DuMouchel (1968) shows that if the utility functions are strictly monotonic these conditions are satisfied and thus a Pareto-optimal risk-sharing rule exists.

So far we have not further specified the risk-sharing functions  $\{T_1, \dots, T_N\}$ . A subset of all possible risk-sharing functions  $\{T_1, \dots, T_N\}$  is the

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collection of affine risk sharing rules

$$T_i(\tilde{y}) = t_{0,i} + \eta_i \sum_{i=1}^N \tilde{y}_i. \quad (3.16)$$

In this risk sharing rule the risk transfer  $\eta_i$  is the fraction of the aggregate risk that agent  $i$  absorbs and  $t_{0,i}$  is a constant risk compensation<sup>15</sup> that agent  $i$  receives ex-ante. Some agents receive a positive risk compensation that has to be financed by the other agents. An advantage of affine risk sharing rules is that they are easy to implement. A Pareto-optimal risk-sharing rule is generally affine. Huang and Litzenberger (1985) show that a Pareto optimal risk-sharing rule is affine if the agents have the same cautiousness.<sup>16</sup> This condition is satisfied when the individual utility functions are member of the Hyperbolic Absolute Risk Aversion (HARA) class (Aase (2002)). This is a general class of utility functions that are often used in practice.

The conditions in (3.15) do not imply a unique Pareto-optimal risk-sharing rule since the positive constants  $c_2, c_3, \dots, c_N$  can be chosen arbitrarily. The welfare gain from risk sharing can be distributed over the agents in different ways. However, there must be an upper limit to  $c_i$  since utility decreases with increasing  $c_i$  and agents cannot become worse off in the Pareto-optimal risk-sharing rule. One can find a unique solution within the set of Pareto-optimal risk-sharing rules by looking for an equilibrium. In this approach the agents can trade in a fictitious market. This method is used by, e.g., Krueger and Kubler (2006), Ball and Mankiw (2007b) and Gottardi and Kubler (2011). An alternative way is to consider a Nash bargaining solution that is Pareto optimal (see Aase (2009) and Zhou et al. (2015)). A third way to find a unique Pareto-optimal risk-sharing rule is by making use of a social planner and using a utility-based fairness criterion. The social planner maximizes aggregate welfare and reallocates risk across agents. A social planner is used by, e.g., Gordon and Varian (1988), Gollier (2008), Cui et al. (2011) and Bovenberg and Mehlkopf (2014). The utility-based fairness

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<sup>15</sup>This risk compensation is referred to as side payment in syndicate theory, see Wilson (1968).

<sup>16</sup>Cautiousness is the derivative of the reciprocal of absolute risk aversion. It measures how quickly the coefficient of risk aversion increases as wealth goes down, see, e.g. Wilson (1968).

criterion requires that all agents experience the same welfare gain from risk sharing. This criterion is used by, e.g., Gollier (2008) and Bovenberg and Mehlkopf (2014). We use this criterion in the full model in Section 3.2.3.

### 3.2.2 Stylized two-agent model

To understand how collective risk sharing works and leads to welfare gains we first consider a stylized model in which we can derive the Pareto-optimal risk-sharing rule analytically. The model consists of  $N = 2$  agents which both have exponential utility with risk aversion  $\alpha$ <sup>17</sup>

$$U(C_i) = -\frac{1}{\alpha} \exp(-\alpha C_i). \quad (3.17)$$

We use this utility function because of its analytical convenience and will show that the Pareto-optimal risk-sharing rule is indeed affine in aggregate risk. For the sake of simplicity we assume both agents are exposed to the same risk factor  $\tilde{y}$  but with a different exposure, i.e.,  $\tilde{y}_1 = \beta_1 \tilde{y}$  and  $\tilde{y}_2 = \beta_2 \tilde{y}$ . We assume  $\tilde{y}$  is normally distributed with zero mean and variance  $\sigma^2$ .

We derive the Pareto-optimal risk-sharing rule  $\{T_1(\tilde{y}), T_2(\tilde{y})\}$  by maximizing the expected utility of agent 1 under the condition that agent 2 does not become worse off relative to autarky

$$\begin{aligned} \max_{\eta, t_0} \mathbb{E}[U(W_1 + T_1(\tilde{y}))] \quad \text{such that} \quad & \mathbb{E}[U(W_2 + T_2(\tilde{y}))] \geq \mathbb{E}[U(W_2 + \tilde{y}_2)] \\ \text{and} \quad & T_1(\tilde{y}) + T_2(\tilde{y}) = \tilde{y}_1 + \tilde{y}_2, \end{aligned} \quad (3.18)$$

where we plug in consumption in autarky (3.8) and consumption in case of risk sharing (3.9). The maximization can also be written as follows

$$\begin{aligned} \max_{\eta, t_0} \mathbb{E}[U(W_1 + T_1(\tilde{y}))] \quad \text{such that} \quad & (3.19) \\ \mathbb{E}[U(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y}))] \geq & \mathbb{E}[U(W_2 + \beta_2\tilde{y})]. \end{aligned}$$

This maximization can be solved and yields the following Pareto-optimal

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<sup>17</sup>Identical risk aversions is not a necessary condition for deriving Pareto-optimal risk-sharing rules analytically in case of exponential utility. Gerber and Pafum (1998) derive it for different risk aversions.

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risk-sharing rule  $\{T_1(\tilde{y}), T_2(\tilde{y})\}$

$$\begin{cases} T_1(\tilde{y}) &= t_{0,1} + \eta_1(\beta_1 + \beta_2)\tilde{y} = -\frac{1}{8}\alpha\sigma^2(\beta_1 + 3\beta_2)(\beta_1 - \beta_2) + \frac{1}{2}(\beta_1 + \beta_2)\tilde{y}, \\ T_2(\tilde{y}) &= t_{0,2} + \eta_2(\beta_1 + \beta_2)\tilde{y} = \frac{1}{8}\alpha\sigma^2(\beta_1 + 3\beta_2)(\beta_1 - \beta_2) + \frac{1}{2}(\beta_1 + \beta_2)\tilde{y}. \end{cases} \quad (3.20)$$

A proof is provided in Appendix 3.A.1. We can conclude that the Pareto-optimal risk-sharing rule is indeed affine in aggregate risk. This makes sense since exponential utility belongs to the HARA class. As mentioned in Section 3.2.3 a Pareto-optimal risk-sharing rule is affine for utility functions of the HARA class.

The optimal risk-sharing rule in (3.20) shows that in case of exponential utility the optimal risk transfer  $\eta_i$  is independent of the wealth of the agents and is the same for both agents. They both absorb half of the aggregate shock. Taking the expectation of the Pareto-optimal risk-sharing rule leads to a constant risk compensation  $t_{0,i}$  since risk factor  $\tilde{y}$  has zero mean

$$\begin{cases} \mathbb{E}[T_1(\tilde{y})] &= t_{0,1} = -\frac{1}{8}\alpha\sigma^2(\beta_1 + 3\beta_2)(\beta_1 - \beta_2), \\ \mathbb{E}[T_2(\tilde{y})] &= t_{0,2} = \frac{1}{8}\alpha\sigma^2(\beta_1 + 3\beta_2)(\beta_1 - \beta_2). \end{cases} \quad (3.21)$$

The constant risk compensation  $t_{0,i}$  depends on the risk aversion  $\alpha$ , the exposure of both agents to the risk factor  $\beta_i$  and the variance  $\sigma^2$  of the risk factor  $\tilde{y}$ . The risk compensation  $t_{0,1}$  is negative if  $\beta_1 > \beta_2$ . This implies that agent 1 has to pay a risk compensation to agent 2 if the exposure of agent 1 to the risk in autarky is larger compared to the exposure of agent 2. Because both agents absorb half of the aggregate shock, agent 2 wants to receive a positive risk compensation in return. Agent 2 will require a higher risk compensation if the risk is higher.

The risk compensation  $t_{0,i}$  determines how the welfare gain from risk sharing is distributed among the agents. Because the inequality restriction is binding the welfare gain of risk sharing goes completely to agent 1. The optimal risk-sharing rule in (3.20) is not necessarily the only Pareto-optimal risk-sharing rule. In fact, there is generally a whole set of Pareto-optimal risk-sharing rules. There are also Pareto-optimal risk-sharing rules in which both agents gain from risk sharing. In that case  $t_{0,2}$  is higher and thus  $t_{0,1}$  must be lower as long as agent 1 does not become worse off.

We make the simplifying assumption that both agents are exposed to the same risk factor  $\tilde{y}$ . In case both agents are exposed to a different risk factor the optimal risk sharing rule is still affine in aggregate risk. However, the equation of the risk compensation becomes more complex.

### 3.2.3 Collective risk sharing of macro-longevity risk: full model

We replace the stylized two-agent model of Section 3.2.2 with a full model with many cohorts representing a pension fund's population. Furthermore, we adjust the risk-sharing mechanism to make it suitable for sharing-macro-longevity risk. This is necessary because of the different nature of macro-longevity risk compared to financial risks. Macro-longevity risk does not impact the individual wealth but impacts survival probabilities and therefore also retirement consumption in a non-linear way. For this reason the properties of the stylized two-agent model do not apply to the full model. We will explain this in more detail below.

The full model consists of  $N = 70$  cohorts. These cohorts are all alive when arranging the risk-sharing mechanism. Cohort 1 is aged 25 and cohort 70 is aged 94.<sup>18</sup> We base the number of participants  $n_i$  in cohort  $i$  on the cumulative probability that a participant is still alive at age  $i + 24$ . So old cohorts consist of less participants compared to young cohorts. The left-hand graph in Figure 3.2.1 visualizes this population composition. Participants have identical preferences given by a power utility function with risk aversion  $\gamma = 5$ <sup>19</sup>

$$U(C_i) = \begin{cases} \frac{C_i^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1, \\ \ln(C_i) & \text{if } \gamma = 1. \end{cases} \quad (3.22)$$

---

<sup>18</sup>We exclude cohorts older than age 94 because the number of participants in these cohorts is very small and therefore do not influence the results significantly.

<sup>19</sup>Power utility has become the workhorse of macro-economics and finance and is in line with empirical studies compared to exponential utility. We justify the assumption that agents have the same risk aversion  $\gamma$  because collective risk sharing within a pension fund often occurs within a group of participants with similar characteristics such as education, job, salary, etc.

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We assume power utility because power utility has become the workhorse of macro-economics and finance and is more in line with empirical studies compared to for example exponential utility. Because we use power utility in we implicitly assume risk neutrality with respect to the length of the life-cycle. This means that for any constant consumption profile and any age, a participant is risk neutral with respect to age of death and prefers a fixed pension consumption (Bommier (2006)). This assumption is used in traditional life-cycle models.<sup>20</sup> We measure consumption as the expected yearly consumption and not as the total consumption during retirement. Using expected yearly consumption does not impact the welfare analysis because we implicitly assume risk neutrality with respect to the length of the life-cycle by using power utility.

The total wealth  $W_i$  of a participant in cohort  $i$  depends on his or her age. The right-hand graph in Figure 3.2.1 visualizes the development of wealth over the life-cycle of a participant. Wealth increases during the working period as the participant contributes to the pension fund. Wealth at the start of the working period is positive because wealth consists of financial wealth and human wealth (i.e., future pension contributions).<sup>21,22</sup> Moreover, wealth depends on the retirement age policy which will be discussed in Section 3.3.

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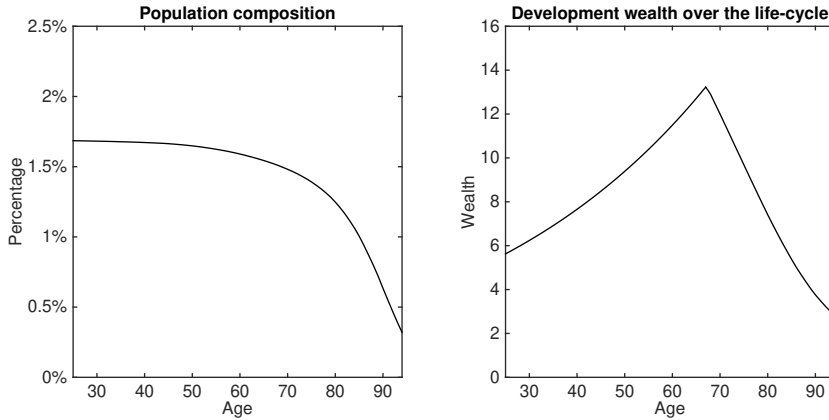
<sup>20</sup>An alternative assumption used by, for example, Bommier and Schernberg (2018) is temporal risk aversion. A temporal risk averse participant prefers a decrease in pension consumption if life expectancy increases, and vice versa.

<sup>21</sup>Human wealth is equal to the present value of future pension contributions and not the present value of future labor income because the pension contributions are fixed in our model.

<sup>22</sup>We assume that labor income is fixed and the same for each cohort and each age. As a result, the impact of macro-longevity risk on replacement rates is similar to the impact of macro-longevity risk on consumption.

**Figure 3.2.1: Population composition and wealth development.**

This figure shows the population composition (left-hand graph) and development of wealth over the life-cycle of a participant assuming a fixed retirement age  $R = 67$  (right-hand graph).



We consider a DC pension scheme in which consumption after retirement depends on the value of a fair annuity. An annuity is fair if the expected present value of the future pension payments matches its current price. We assume that participants buy a variable annuity from the DC scheme. The value of this annuity varies with expected future survival probabilities.<sup>23,24</sup> For example, a negative macro-longevity shock implies an increase of expected future survival probabilities that increase the value of the annuity. This has a negative effect on consumption after retirement. The value of a (deferred) variable annuity  $\ddot{a}_{x,t}$ , that pays 1 dollar annually during retirement, for an individual of age  $x$  in year  $t$  is calculated as follows

$$\ddot{a}_{x,t} = \sum_{j=\max(x,R)}^M \frac{1}{(1+r)^{j-x}} j-x p_{x,t}. \quad (3.23)$$

<sup>23</sup>Variable annuities in practice typically also vary with realized investment returns. Because we exclude investment risk, this is not the case in our paper.

<sup>24</sup>We assume macro-longevity risk is not reinsured by a third party or borne by future cohorts that enter the pension scheme.

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In this formula  $R$  equals the retirement age,  $M$  is the maximum age an individual can reach and  ${}_i p_{x,t}$  is the probability of still being alive after  $i$  years. The retirement age  $R$  depends on the retirement age policy which will be explained in Section 3.3. We assume a constant interest rate  $r = 2\%$ .

For ease of reference we denote the value of a (deferred) annuity in (3.23) for cohort  $i$  by  $\ddot{a}_{x,t} = \ddot{a}_i$ . The value of a (deferred) annuity changes for cohort  $i$  from  $\ddot{a}_i$  to  $\ddot{a}_i^n$  due to a macro-longevity shock. The expected annual consumption after retirement in autarky  $C_i^a$  for cohort  $i$  after a shock is given by

$$C_i^a = \frac{W_i}{\ddot{a}_i^n}. \quad (3.24)$$

So in autarky macro-longevity risk is completely borne by each generation. In practice, retirement consumption also consists of other elements such as first pillar pension and private savings. In our model we exclude those additional elements. Micro-longevity risk is excluded from the risk-sharing mechanism in our paper because it will be fully diversified in case of enough participants in the pension scheme. This implies that if a participant dies her remaining wealth is added to the wealth of the other participants. This is known as a biometric return or mortality credit.

As mentioned in Section 3.1.2 we consider macro-longevity risk on a 10-year horizon. To determine the impact of macro-longevity risk on consumption we calculate for each cohort how much money is needed (or is left) to fully compensate the impact of a macro-longevity shock.<sup>25</sup> We denote this by  $\tilde{y}_i$

$$\begin{aligned} \frac{W_i}{\ddot{a}_i} &= \frac{W_i + \tilde{y}_i}{\ddot{a}_i^n} \\ \tilde{y}_i &= W_i \left( \frac{\ddot{a}_i^n}{\ddot{a}_i} - 1 \right). \end{aligned} \quad (3.25)$$

$\tilde{y}_i$  represents the amount of money to offset the effect of a macro-longevity

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<sup>25</sup>In this paper we assume that consumption before retirement is fixed, i.e., a macro-longevity shock can only be absorbed by changing consumption after retirement. In case a participant can also change consumption before retirement, the impact of a macro-longevity shock on the consumption level after retirement will be smaller for workers.

shock on consumption in autarky. If the annuity value increases (decreases) due to an unexpected increase (decrease) in life expectancy,  $\tilde{y}_i$  is positive (negative) and money is needed (left).  $\tilde{y}_i$  is not the same for each cohort  $i$  because the impact of a macro-longevity shock on future death rates depends on age. We can calculate the total money needed (or left) to fully compensate the impact of a macro-longevity shock for all  $N$  cohorts. We denote this by  $\tilde{y}_T$

$$\tilde{y}_T = \sum_{i=1}^N {}_{10}p_i n_i \tilde{y}_i, \quad (3.26)$$

where  $n_i$  is the number of participants in cohort  $i$  and  ${}_{10}p_i$  the probability that a participant is still alive after 10 years. Macro-longevity risk is shared by distributing the aggregate macro-longevity shock  $\tilde{y}_T$  among cohorts.

Similar to the affine risk-sharing rules in (3.16), each cohort absorbs part of the aggregate macro-longevity shock  $\eta_i$  and receives (or pays) a risk compensation  $t_{0,i}$ . The risk compensation is paid upfront by transferring wealth from cohorts that sell risk to cohorts that underwrite risk. After a macro-longevity shock has occurred another wealth transfer takes place. The size and the direction of this second wealth transfer depends on the nature of the macro-longevity shock, in particular an increase or decrease of mortality rates. Furthermore, we assume that in case a participant dies within the 10-year period she does not benefit from the risk-sharing mechanism. Her wealth is added to the wealth of the other participants. Consumption in case of risk sharing thus equals

$$C_i^s = \frac{W_i + \tilde{y}_i - \eta_i \tilde{y}_T - t_{0,i}}{\ddot{a}_i^r}. \quad (3.27)$$

The numerator contains two adjustments that are needed to process the impact of a macro-longevity shock. First,  $\tilde{y}_i$  is included to compensate the impact of the macro-longevity shock on consumption (see (3.25)). If for example participants live longer yearly consumption will be lowered. The intuition is that a macro-longevity shock does not change the amount of money available to finance retirement consumption. Therefore, consumption itself is adjusted. Second, the total impact of a macro-longevity shock  $\tilde{y}_T$  is redistributed among

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all cohorts according to the risk-sharing rule. Next to the impact of a macro-longevity shock, also the risk compensation  $t_{0,i}$  impacts consumption.

We determine a unique, Pareto-improving risk-sharing rule by assuming the presence of a social planner. The social planner maximizes the welfare gain from risk sharing relative to autarky for all participants in the pension scheme. The social planner does so by simultaneously choosing a risk transfer  $\eta_i$  and risk compensation  $t_{0,i}$  for each participant in the following way

$$\max_{\substack{\eta_1, \eta_2, \dots, \eta_N \\ t_{0,1}, t_{0,2}, \dots, t_{0,N}}} \frac{\mathbb{E}[U(C_1^s)] - \mathbb{E}[U(C_1^a)]}{\mathbb{E}[U(C_1^a)]}, \quad (3.28)$$

under the restriction that the relative welfare improvement is the same for all agents

$$\frac{\mathbb{E}[U(C_i^s)] - \mathbb{E}[U(C_i^a)]}{\mathbb{E}[U(C_i^a)]} = \frac{\mathbb{E}[U(C_1^s)] - \mathbb{E}[U(C_1^a)]}{\mathbb{E}[U(C_1^a)]} \quad \forall i = 1, \dots, N. \quad (3.29)$$

Furthermore, we need the following two additional restrictions

$$\begin{cases} \sum_{i=1}^N 10p_i n_i \eta_i = 1, \\ \sum_{i=1}^N n_i t_{0,i} = 0. \end{cases} \quad (3.30)$$

The first additional restriction makes sure that the aggregate risk is fully distributed over all agents. We multiply with the probability that a participant is still alive after the 10-year period because the risk transfer is settled after the realization of the macro-longevity shock. In case a participant dies within the 10-year period she no longer participates in the risk-sharing mechanism. The second additional restriction guarantees that the total risk compensation that some of the participants receive is paid by the other participants.

To interpret our results later on, it is good to be aware of two limitations of our approach. First, the risk-sharing rule is not necessarily Pareto-optimal. Although the optimization in (3.28) leads to a Pareto-improvement for all participants, it is not necessarily Pareto-optimal because we restrict our analysis to affine rules and because consumption depends on the aggregate

risk in a non-linear way. It is possible that a non-affine risk-sharing rule provides a higher Pareto-improvement. Second, our model does not take macro-longevity risk after the 10-year horizon into account. In practice a new risk-sharing rule will be determined every 10 years between all the participants that are alive at that moment in time. Both limitations imply that the welfare gains in our paper underestimate the true welfare gains. Before we go to the results, we now introduce the different retirement age policies. These retirement age policies impact the welfare gains of risk sharing significantly.

### **3.3 Retirement age policies**

The significant increase in life expectancy during the last decades had a major impact on the sustainability of pension systems. As a response several countries link the state pensionable age to life expectancy developments.<sup>26</sup> In the United Kingdom for example the government plans to link the state pensionable age at future dates to the projected longevity of the population in such a way that people receive state pension during a fixed proportion of their adult life (Hammond et al. (2016)). Under this policy both the working and retirement period increase if life expectancy increases. In the Netherlands the retirement age is linked to life expectancy in a different way. The Dutch government implemented a law that links the retirement age to the remaining life expectancy of the population at age 65. Under this policy the absolute length of the retirement period is fixed and independent of life expectancy while the working period increases if life expectancy increases.

In our paper we focus on occupational pension schemes. The retirement age in occupational pension schemes is often equal to the state pensionable age. As a consequence, the retirement age policy of the government also impacts the retirement age in occupational pension schemes and thus the ability to share macro-longevity risk in occupational pension schemes. We consider three exogenous retirement-age policies:

1. **Fixed retirement age (FRA):** the retirement age is fixed, i.e., the retirement age does not change after macro-longevity shocks. In this policy

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<sup>26</sup>Another option is to increase pension contributions to finance the increase in life expectancy. In this paper we do not consider this alternative option.

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the length of the working period is constant. This policy supports the belief that if people live longer, they extend their retirement period. In most countries, for example in the United States and Australia, the retirement age is not linked to life expectancy.

2. **Partial adjustment of the retirement age (PARA):** the retirement age automatically adjusts to life expectancy developments in a such a way that retirement consumption remains the same.<sup>27</sup> This means, e.g., that if life expectancy increases (decreases) with 12 months, the retirement age should increase (decrease) with roughly 9 months.<sup>28</sup> In this policy consumption after retirement is constant. The adjustment only holds for working participants, since retirees cannot adjust their retirement age anymore. This policy is close to the retirement age policy in the United Kingdom.<sup>29</sup>
3. **Full adjustment of the retirement age (FARA):** the retirement age automatically keeps up fully with life expectancy changes. This means, e.g., that if the remaining life expectancy at retirement increases (decreases) with 12 months, the retirement age also increases (decreases) with 12 months. In this policy the length of the retirement period is constant. The adjustment holds for working participants only, since retirees cannot adjust their retirement age anymore. This policy supports the belief that if people live longer, they increase their labor supply by extending their working period. This policy is similar to the retirement age policy in the Netherlands.<sup>30</sup>

We derive optimal risk-sharing under these three retirement age policies. We

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<sup>27</sup>There are also countries in which the retirement age is not automatically linked to life expectancy but the government decides to increase the retirement age based on life expectancy improvements incidentally. We do not investigate such a policy.

<sup>28</sup>The exact increase (decrease) does not only depend on the size of the longevity shock but also on the impact of the longevity shock on survival probabilities at different ages and the life expectancy before the longevity shock.

<sup>29</sup>The retirement age adjustment in the UK proposal depends on the proportion of adult life that people receive state pension.

<sup>30</sup>The Dutch law states that the retirement age  $R$  is only adjusted in case the remaining life expectancy at age 65 increases but it remains the same if it decreases. In this paper we assume a symmetric rule, i.e., the retirement age is adjusted in case of both positive and negative shocks.

confine ourselves to the benefits of risk sharing. We do not make a statement about the suitability of retirement age policies in general. The exogenous adjustments in retirement ages by the government under PARA and FARA are a consequence of systematic changes in life expectancy. The government could chose to implement changes in the retirement age in a fair way based on expected utility of the cohorts in society. In practice, government decisions on pensions are often politically driven. The suitability of retirement age policies also depends on the healthy life expectancy. Different health status scenarios have been considered in the literature, see, e.g., OECD (2006). A possible scenario is that the share of life spent in bad health increases as life expectancy increases ('expansion of morbidity'). Such a scenario argues in favor of a fixed retirement age policy. A 'compression of morbidity' scenario means that the share of life spent in good health would increase as life expectancy increases. This scenario argues in favor a full adjustment of the retirement age.

Stevens (2017) investigates the effect of different retirement age policies on the distribution of the (forecasted) retirement age. He concludes that if the retirement age is linked to life expectancy macro-longevity risk is effectively hedged. However, such a policy also leads to substantial uncertainty in the retirement age and length of the retirement period.

**Table 3.3.1: Impact increase life expectancy**

*This table shows the impact of an unexpected increase in life expectancy on several variables for working participants in case of different retirement age policies.*

	Working period	Retirement period	Retirement consumption	Value variable annuity	Wealth at retirement
FRA	Constant	++	-	++	Constant
PARA	+	+	Constant	+	+
FARA	++	Constant	+	-	++

Table 3.3.1 presents the impact of an unexpected increase in life expectancy on several variables for the three retirement age policies in autarky. Expected consumption after retirement is determined by the value of a (deferred) annuity and total wealth  $W_i$  (see (3.24)). If the retirement age is linked to life expectancy the development of wealth over the life-cycle is different because the participant accrues more (less) wealth by paying

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pension premia for a longer (shorter) period.<sup>31</sup> The table presents the impact for working participants only because retirees cannot adjust their retirement age as response to longevity shocks. In case of an unexpected decrease in life expectancy, the signs in Table 3.3.1 revert. We focus on the impact of longevity risk on retirement consumption. Longevity shocks not only impact the level of consumption but also the number of years the participants receive pension payments. Since we assume risk neutrality with respect to the length of the life-cycle we focus on the impact on the level of consumption only.

In case of a fixed retirement age the length of the working period is constant. As a result the (expected) length of the retirement period increases in case of an unexpected increase in life expectancy. The annuity value increases as a result of higher survival probabilities. Wealth at retirement remains the same. As a result retirement consumption will decrease.

In case of a partial adjustment of the retirement age both the working and retirement period are extended. The annuity value increases as a result of higher survival probabilities. The wealth at retirement also increases because the participant will work longer. The annuity value and wealth at retirement increase such that consumption after retirement remains the same.

If the retirement age is fully adjusted the length of the retirement period is constant. The (expected) length of the working period increases in case of an unexpected increase in life expectancy. The annuity value is lower than before the longevity shock. Higher survival probabilities have a positive impact on the annuity value, but later retirement has a negative impact on the annuity value. It turns out that the latter effect outweighs. The wealth at retirement increases because the participant will work longer. As a result, retirement consumption will increase.

In this research we measure welfare gains of sharing macro-longevity risk under different retirement age policies. We do not measure welfare gains of different retirement age policies since the retirement age policy is given for both autarky and risk sharing. The welfare gains of a higher average retirement consumption as a result of linking the retirement age to life expectancy

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<sup>31</sup>We assume that the labor market functions perfectly so participants do not experience any difficulties with staying employed.

are excluded on purpose.

Our analysis is based on the assumption that increases in life expectancy also lead to an increase in human capital in case the retirement age is linked to life expectancy. A literature review by Pilipiec et al. (2021) concludes that increases in the retirement age lead to an increase in labor force participation among older workers and the realized retirement age. Moreover, several studies, e.g. Boucekine et al. (2002) and Zhang and Zhang (2005), find a positive correlation between life expectancy and human capital accumulation.

We use exogenous rules in the retirement age policies. An alternative is an endogenous retirement age. This is considered by, e.g., Chang (1991), Cocco and Gomes (2012) and Heijdra and Romp (2009). The participant optimizes his retirement age based on realized life expectancy improvements. In that case it is necessary to include leisure time besides consumption in the utility function to take the labor-leisure trade-off into account. Otherwise a high retirement age would always be optimal because a shorter retirement period implies a higher consumption after retirement. Cocco and Gomes (2012) investigate the impact of macro-longevity risk on the optimal saving and retirement decision in an individual life-cycle model. They conclude that individuals decide to retire later even if this entails a utility cost in terms of foregone utility of (additional) leisure. Although deriving the optimal endogenous retirement age is relevant from an individual point of view, it complicates the risk-sharing mechanism and introduces circularity in the modelling. The optimal risk-sharing rule depends on the risk bearing capacity of generations which depend on the retirement age policy. In turn, the optimal retirement age depends on the risk-sharing mechanism in place. This circularity is beyond the scope of this paper and we consider this circularity not feasible to implement in practice.

Although we do not explicitly model the labor-leisure trade-off in this paper, the retirement age policies represent different preferences regarding consumption and leisure. In case of a fixed retirement age, a life expectancy increase implies a lengthening of the retirement period (leisure) at the expense of the consumption level. In case of a partial adjustment of the retirement age both consumption and leisure (relative to labor) remain approximately equal. A full adjustment of the retirement age implies a higher consumption

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level at the expense of leisure.

### 3.4 Results

In this research we quantify the welfare gains from collective risk sharing in terms of aggregate certainty equivalent consumption after retirement. We use the Lee-Carter model to model macro-longevity risk using mortality data of Dutch females.<sup>32</sup> Table 3.4.1 presents the aggregate welfare gains for the three retirement age policies discussed in Section 3.3.

**Table 3.4.1: Welfare gains sharing macro-longevity risk**

*This table shows the welfare gains in terms of aggregate certainty equivalent consumption after retirement from sharing macro-longevity risk measured on a 10-year horizon.*

<b>Fixed retirement age (FRA)</b>	0.2%
<b>Partial adjustment retirement age (PARA)</b>	0.6%
<b>Full adjustment retirement age (FARA)</b>	1.8%

We observe that for each retirement age policy collective risk sharing of macro-longevity risk is welfare improving compared to autarky. The design of the retirement age policy impacts the size of welfare gains from sharing macro-longevity risk. In case of a fixed retirement age, the welfare gain equals 0.2 percent. This relatively small welfare gain is a result of the fact that in this policy the impact of macro-longevity risk on retirement consumption for different cohorts is more or less equal (Figure 3.1.2). As a result, the welfare gain from risk sharing is limited. When the retirement age is partially adjusted the welfare gain from risk sharing is higher. This is a result of the fact that the expected retirement consumption of workers is not affected by macro-longevity shocks. In case of a full adjustment of the retirement age the aggregate welfare gain increases significantly. This is a result of the large risk bearing capacity of workers. Their labor supply acts

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<sup>32</sup>We focus on risk sharing between different cohorts of the same population. We do not investigate risk sharing between the sexes or between different populations. Sharing macro-longevity risk between sexes or different populations is potentially welfare improving but is out of the scope of this paper.

as a hedge against macro-longevity shocks. This increases the risk absorbing capacity of the workers to provide insurance to retirees.

**Figure 3.4.1: Optimal risk transfer and corresponding risk compensation.**

This figure shows the optimal risk transfer relative to autarky for all participants as percentage of total risk transferred (left-hand graph) and corresponding risk compensation (right-hand graph) in case of sharing macro-longevity risk measured on a 10-year horizon. A positive (negative) risk transfer for cohort  $i$  means that participants in cohort  $i$  absorb risk of (transfer risk to) other cohorts. A positive (negative) risk compensation for cohort  $i$  means that participants receive (pay) a risk compensation.

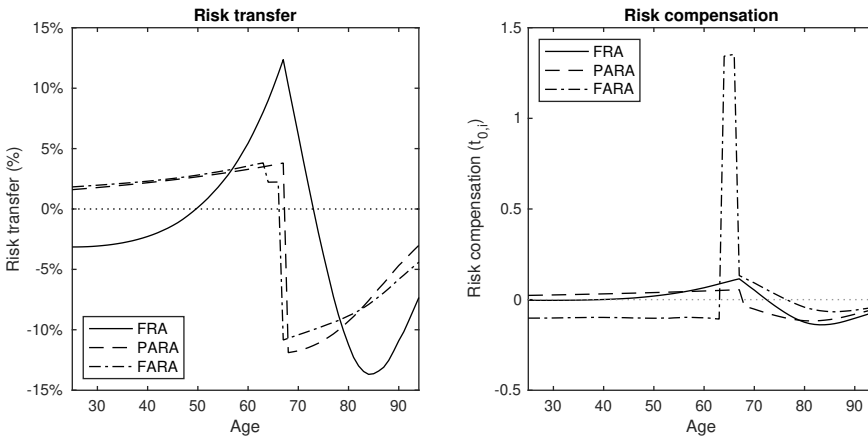


Figure 3.4.1 (left-hand graph) visualizes the optimal risk transfer relative to autarky for a participant in cohort  $i$  as a percentage of total risk. A positive risk transfer for cohort  $i$  means that participants in cohort  $i$  absorb risk of other cohorts. A negative risk transfer means that the exposure to macro-longevity risk of a cohort is (partly) transferred to other cohorts.<sup>33</sup> If thereafter an unexpected increase in life expectancy occurs, cohorts that ‘sold’ risk receive a monetary amount from cohorts that ‘bought’ risk. How much wealth is transferred depends on the size of the longevity shock and the fraction of the exposure that is transferred in the optimal risk transfer. In case of a fixed retirement age the risk transfer increases with

<sup>33</sup>Note that the sum of all the risk transfers in the graph is not exactly equal to zero because each cohort does not consist of an equal number of participants.

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age for the workers until retirement and decreases with age for retirees. Macro-longevity risk of the young workers and old retirees is (partly) absorbed by the other cohorts. The development of wealth over the life-cycle (right-hand graph in Figure 3.2.1) primarily explains this shape. Cohorts who have relatively more wealth can absorb more risk. The risk transfer rule in case of a fixed retirement age significantly differs from the risk transfer rule in case the retirement age is adjusted to macro-longevity shocks. The risk transfer rule in case the retirement age is partially adjusted is very similar to the risk transfer rule in case the retirement age is fully adjusted. The workers absorb risk and the retirees transfer risk. This makes sense because the workers adjust their labor supply to macro-longevity shocks. As a result, they are able to absorb risk of the retirees.

The right-hand graph in Figure 3.4.1 displays the risk compensation  $t_{0,i}$  corresponding to the optimal risk transfer for a participant in cohort  $i$  under the utility-based fairness criterion (left-hand graph).<sup>34</sup> A positive risk compensation for cohort  $i$  means that participants receive a risk compensation. A negative risk compensation for cohort  $i$  means that participants pay a risk compensation. The risk compensation is added to or subtracted from the individual wealth of participants. In general, cohorts who absorb risk from other cohorts receive a risk compensation and cohorts who transfer risk have to pay a risk compensation. However, this does not hold if the retirement age is fully adjusted. Young cohorts absorb risk from other cohorts but do not receive a positive risk compensation; the risk compensation is even negative. Under this policy the labor supply of workers acts as a hedge against macro-longevity shocks. This implies a reverse effect of macro-longevity shocks for workers and retirees (Table 3.3.1). As a result, a positive risk compensation is not required for young cohorts to absorb risk of retirees. A final observation is the peak in the risk compensation around age 66 in case of a fully adjusted retirement age. This peak is due to the fact that cohorts just before retirement cannot fully adjust their retirement age in case of an unexpected decrease in life expectancy, i.e., the retirement age cannot be lower than their current age. As a result, the certainty equivalent consumption of these cohorts is relatively

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<sup>34</sup>The sum of risk compensations in the graph is not exactly equal to zero because each cohort does not consist of an equal number of participants.

high in autarky so risk sharing is less welfare improving for these cohorts. Therefore, these cohorts require a higher risk compensation.

We consider macro-longevity risk on a 10-year horizon. The welfare gains from sharing macro-longevity risk over the whole life-cycle are most likely higher. Moreover, we do not take the inherent dynamics implied by overlapping generations into account because of computational reasons. We maximize aggregate expected utility of all current cohorts but do not take into account that the expected consumption during retirement of young cohorts also depends on risk-sharing with yet unborn cohorts. Although this is a simplifying assumption we expect that this is a second-order effect with limited impact. A final sidenote is that this paper applies a first-best risk-sharing rule as its benchmark for evaluating welfare effects. In practice, however, the first-best risk-sharing rule may not always be feasible. Policymakers might want to limit the maximum risk a participant can absorb to prevent very large wealth transfers in case of extreme macro-longevity shocks.

### 3.4.1 Sensitivity analyses

In this section we verify whether the welfare gains and risk-sharing rules are sensitive to mortality data and model assumptions by performing three types of sensitivity analyses:

1. **Alternative mortality data:** macro-longevity risk in the Lee-Carter model depends on the parameters in (3.2) and (3.5) that are calibrated using historical mortality data. We investigate the impact of alternative mortality data on welfare gains from risk sharing and corresponding risk-sharing rule.
2. **Alternative population compositions:** welfare gains from sharing macro-longevity risk also depend on the population composition. We will investigate the impact of alternative population compositions on welfare gains from risk sharing and corresponding risk-sharing rule.
3. **Alternative models macro-longevity risk:** instead of macro-longevity risk in the Lee-Carter model we assess the impact of alternative shocks in death rates on welfare gains from risk sharing and corresponding risk-sharing rule. We will consider two alternative models, the Cairns-

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Blake-Dowd model and the Solvency-II specification.

### Alternative mortality data

Macro-longevity risk in the Lee-Carter model depends on the parameters in (3.2) and (3.5). In our main analysis we calibrate the parameters using historical mortality data of Dutch females. Using alternative mortality data changes the parameters and therefore also the size and distribution of macro-longevity shocks.

**Table 3.4.2: Welfare gains alternative mortality data.**

*This table shows the welfare gains in terms of aggregate certainty equivalent consumption after retirement from sharing macro-longevity risk measured on a 10-year horizon for alternative mortality data.*

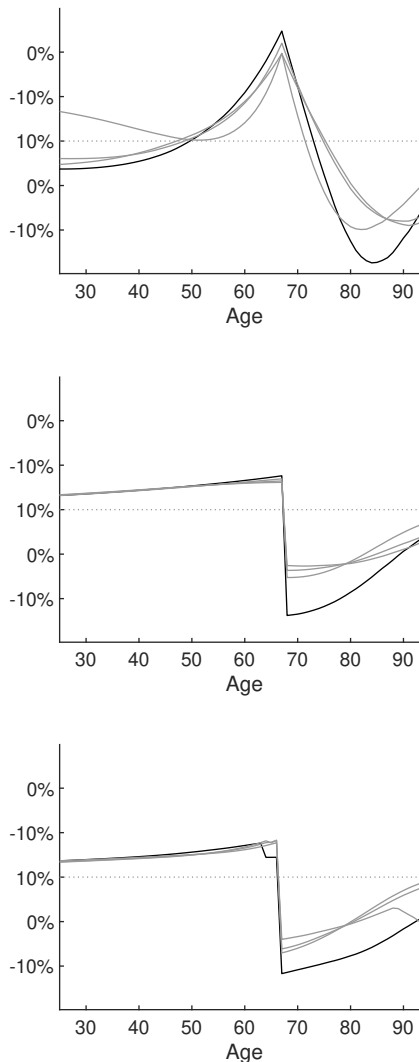
Mortality data	Dutch females	Dutch males	US females	US males
Fixed retirement age (FRA)	0.2%	0.1%	0.1%	0.1%
Partial adjustment retirement age (PARA)	0.6%	0.5%	0.2%	0.4%
Full adjustment retirement age (FARA)	1.8%	1.3%	0.9%	1.1%

The welfare gains from risk sharing using alternative mortality data are presented in Table 3.4.2. We look at Dutch males, US females and US males. In case of a fixed retirement age or partial adjustment of the retirement age, the welfare gains do not change significantly. However, when the retirement age is fully adjusted welfare gains from risk sharing are lower compared to the mortality data of Dutch females. This especially holds for mortality data of US females. This lower welfare gain is caused primarily by lower volatility parameters in (3.6). A lower volatility implies smaller risk and therefore lower welfare gains from risk sharing.

Figure 3.4.2 visualizes the optimal risk transfer relative to autarky as percentage of total risk transferred. The black lines represent the optimal risk transfer rule using the mortality data of Dutch females and the gray lines for the alternative mortality data. We can conclude that for each retirement age policy the optimal risk transfer rule is reasonably robust to the alternative mortality data we consider.

**Figure 3.4.2: Optimal risk transfer alternative mortality data.**

This figure shows the optimal risk transfer relative to autarky as percentage of total risk transferred for the fixed retirement age policy (top graph), partial adjustment retirement age policy (middle graph) and the full adjustment retirement age policy (bottom graph). The black lines represent the risk transfer rules based on Dutch females and the gray lines represent the risk transfer rules using alternative mortality data. The welfare gains in terms of aggregate certainty equivalent consumption after retirement from sharing macro-longevity risk measured on a 10-year horizon for alternative mortality data.



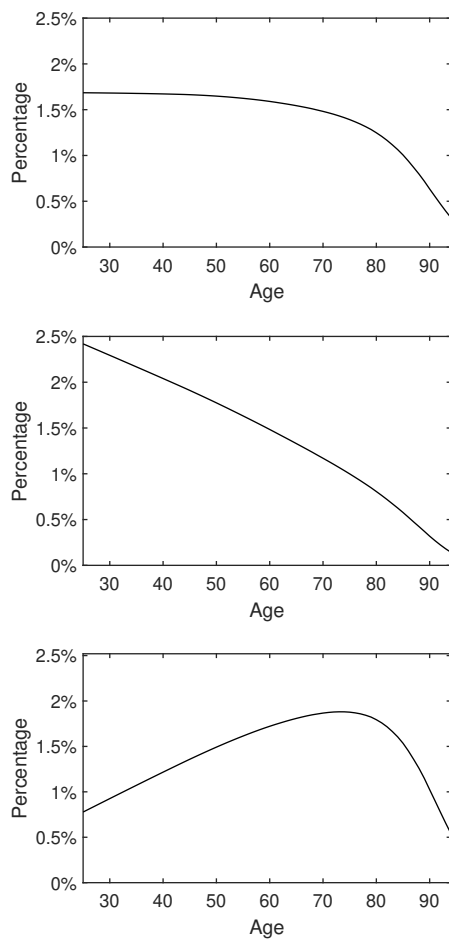
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### **Alternative population compositions**

We determined welfare gains in Table 3.4.1 and risk transfers and risk compensations in Figure 3.4.1 for a population composition of an entire country (left-hand graph in Figure 3.2.1). In practice the population composition of a pension fund is generally not equal to this standard population composition. Therefore, it is interesting to also consider alternative population compositions: a population composition of a green and gray pension fund. We assume that the green pension fund has a relatively young population. We approximate this by assuming that the number of participants in a cohort decreases with 1 percent per age year compared to the standard population composition. In the gray pension fund the number of participants in a cohort increases with 1 percent per age year compared to the standard population composition. The standard and alternative population compositions are displayed in Figure 3.4.3.

**Figure 3.4.3: Different population compositions**

*This figure shows different population compositions: a standard pension fund (top graph), a green pension fund (middle graph) and a gray pension fund (bottom graph).*



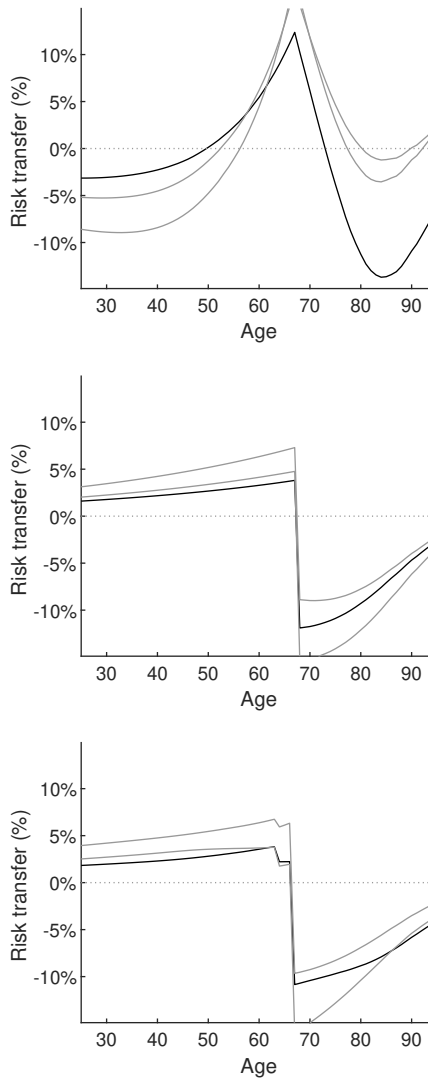
**Table 3.4.3: Welfare gains alternative population compositions**  
*Welfare gains in terms of aggregate equivalent consumption after retirement from sharing macro-longevity risk measured on a 10-year horizon for alternative population compositions.*

<b>Population composition</b>	<b>Standard</b>	<b>Green</b>	<b>Grey</b>
<b>Fixed retirement age (FRA)</b>	0.2%	0.1%	0.1%
<b>Partial adjustment retirement age (PARA)</b>	0.6%	0.4%	0.6%
<b>Full adjustment retirement age (FARA)</b>	1.8%	1.8%	2.3%

Table 3.4.3 presents the welfare gains from risk sharing using alternative population compositions. The welfare gains are not significantly different from the welfare gains for the standard population composition, even if the retirement age is fully adjusted. Figure 3.4.4 visualizes the optimal risk transfer relative to autarky as a percentage of the total risk. The black lines represent the optimal risk transfer rules using the original population composition and the gray lines represent the risk transfer rules using alternative population compositions. The shape of the risk transfer rule is reasonably robust to the population composition but the percentage of total risk an individual participant absorbs or transfers can be different in case of alternative population compositions. A different population composition leads to a different ratio between the individual macro-longevity shock and total macro-longevity shock. This impacts the optimal risk transfer as a percentage of the total risk.

**Figure 3.4.4: Optimal risk transfer alternative population compositions**

Optimal risk transfer relative to autarky as percentage of total risk transferred for the fixed retirement age policy (top graph), partial adjustment retirement age policy (middle graph) and the full adjustment retirement age policy (bottom graph). The black lines represent the original risk transfers and the gray lines represent the optimal risk transfers using alternative population compositions.



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### **Alternative models macro-longevity risk**

Many papers use the Lee-Carter model. Moreover, it is the basis of several mortality table forecasts in practice. Although the Lee-Carter model is reasonably robust relative to historical data and produces plausible forecasts, alternative mortality models might be more suitable under some scenarios. We will consider the Cairns-Blake-Dowd (CBD) model (Cairns et al. (2006a)) as alternative mortality model. Similar to the Lee-Carter model the CBD model is reasonable robust. The most important advantage of the CBD model over the Lee-Carter model is that the CBD model allows different improvements at different ages. This is not possible in the Lee-Carter model in which mortality improvements at different ages are perfectly correlated. Moreover, uncertainty is possibly underestimated in the Lee-Carter model. The CBD model will be explained in more detail below.

Moreover, mortality models are not a perfect representation of reality because there is uncertainty about structural breaks. For example, medical innovations can cause structural breaks that are not captured by a standard mortality model such as the Lee-Carter model and the CBD model. Therefore it is interesting to also look at the impact of alternative shocks in the death rates. We will consider the longevity shocks of the Solvency II framework as alternative shocks in the death rates.

This section is constructed as follows. First, we discuss alternative views about the development of old age survival probabilities and link these views to the different mortality models. Subsequently, we consider the CBD model and Solvency II framework and discuss the results.

### **Alternative views**

There is no scientific consensus on the development of future survival probability at old ages. Buettner (2002) suggests that there are two alternative views about the future survival probability at old ages: compression versus expansion. In case of mortality compression mortality continues to decline over a widening range of adult ages, but meets natural limits for very advanced ages. This development implies that the survival probability approaches a rectangle (Figure 3.4.5). Einmahl et al. (2017) and Dong et

al. (2016) find evidence for the existence of a maximum age. In case of mortality expansion mortality continues to decline for all ages, i.e., there is no maximum age. Wilmoth (2000) and Oeppen and Vaupel (2002) argue that there is indeed no maximum age. Wilmoth (2000) states that, based on available demographic evidence, the human life span shows no sign of approaching a certain limit imposed by biology or other factors. There are even scientists who believe in the possible realization of longevity escape velocity. In this scenario death rates fall so fast that people's remaining life expectancy increases with time because therapies restore health faster than the rate of body deterioration due to biological ageing (De Grey (2004)).

**Figure 3.4.5: Different views future survival probability**

*This figure shows different views of future survival probability: compression (left-hand graph) and expansion (right-hand graph).*

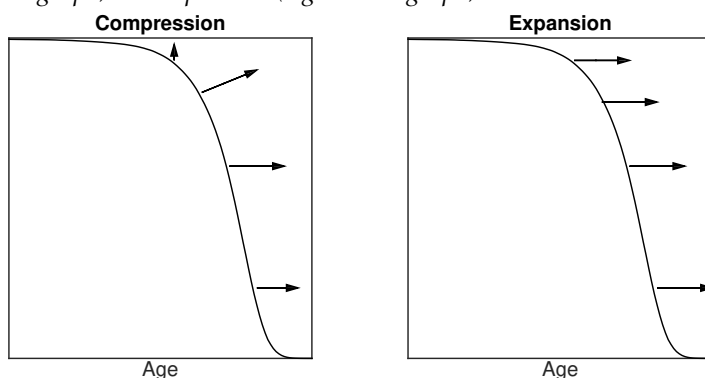


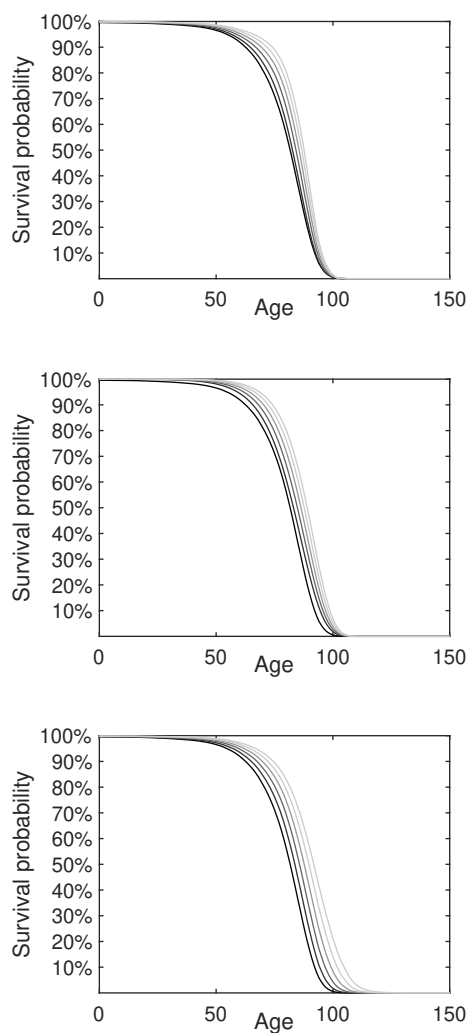
Figure 3.4.6 visualizes the different types of shocks, i.e., macro-longevity shocks in the Lee-Carter model, CBD model and the Solvency II framework. The development of future mortality in the Lee-Carter model is in line with the compression view. The sensitivity of the death rates to the time trend decreases in age  $x$  to almost zero at very high ages. Longevity risk in the Solvency II framework is in line with the expansion view. Survival probabilities improve at all ages at the same rate. The development of future mortality in the CBD model lies in between the compression view and the expansion view. Survival probabilities improve at all ages but at young

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ages survival probabilities improve much more than at old ages. Although both views consider the development of future survival probabilities on a long-term horizon, the effect is - although small - still present at a 10-year horizon.

**Figure 3.4.6: Impact consecutive macro-longevity shocks**

This figure shows the impact of several consecutive macro-longevity shocks in the Lee-Carter model (top graph), in the CBD model (middle graph) and in the Solvency II framework (bottom graph) on the survival probability.



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### Cairns-Blake-Dowd model

The Cairns-Blake-Dowd (CBD) model (Cairns et al. (2006a)) is a two-factor model and is based on the following logistic transformation of the death probability rather than the log central death rates in the one-factor Lee-Carter model:

$$\text{logit}(q_{x,t}) = \ln \frac{q_{x,t}}{1 - q_{x,t}} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}). \quad (3.31)$$

where  $\bar{x}$  is the mean in the range of ages to be estimated and  $(\kappa_t^{(1)}, \kappa_t^{(2)})$  is estimated with a bivariate random walk with drift. Similar to the Lee-Carter model (see Section 3.1) we use mortality data of Dutch females from 1985 until 2014 to calibrate the parameters, apply the method of Kannisto (1994) to estimate the death rates for very high ages, and consider macro-longevity risk on a 10-year horizon and include recalibration risk.

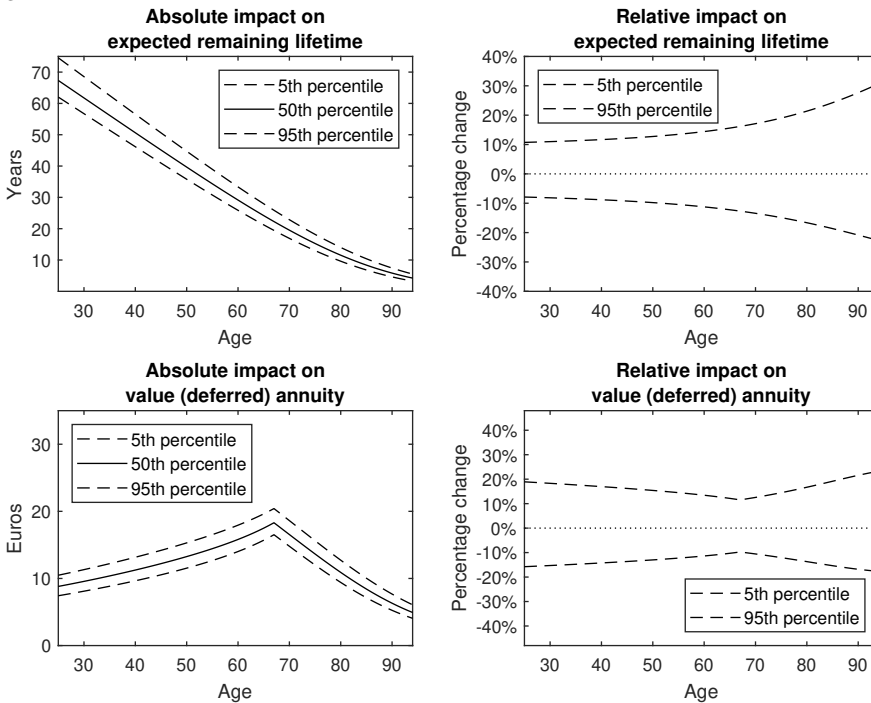
Figure 3.4.7 visualizes the impact of macro-longevity risk in the CBD model on the expected remaining lifetime and the value of a (deferred) variable annuity. When we compare this with the Lee-Carter model in Figure 3.1.2 we observe two important differences. First, we notice that the relative change of the expected remaining lifetime and (deferred) annuity value (right-hand figures) differs significantly. In the Lee-Carter model the impact of macro-longevity risk on death probabilities decreases to almost zero at very high ages while this is not the case in the CBD model. Moreover, Figure 3.4.7 shows that the impact of macro-longevity risk on the expected remaining lifetime and expected (deferred) annuity value is slightly asymmetric. Unexpected increases in life expectancy have a slightly bigger impact than unexpected decreases in life expectancy. This is a result of the recalibration of the parameters in the CBD model. As a result of the recalibration the expectations of future survival probabilities and therefore also the expected remaining lifetime and expected (deferred) annuity value are bigger than its forecasted values.

### Solvency II framework

An alternative shock in death rates is the macro-longevity shock in the Solvency II framework for insurers. The Solvency II capital requirements for longevity risk are determined by applying a uniform shock, i.e., a 20 percent

**Figure 3.4.7: Impact of macro-longevity risk in the CBD model**

This figure shows the impact of macro-longevity risk in the CBD model on the expected remaining lifetime and the value of a (deferred) variable annuity for a Dutch female in 2014 in absolute terms (left-hand graphs) and relative change (right-hand graphs) assuming a constant interest rate of 2% and fixed retirement age  $R = 67$ .



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decrease, to all future death probabilities  $q_{x,t}$ .<sup>35</sup> For mortality risk the capital requirements are determined by applying an increase of 15 percent to all future death probabilities. The longevity shock in the Solvency II framework is in line with the expansion view because all death probabilities decrease at the same rate.

The shocks for longevity and mortality risk in the Solvency II framework are deterministic, i.e., no stochastic mortality model is used to determine the distribution of future death rates. Because we have to make an assumption about the distribution of future death rates when sharing macro-longevity risk, we assume that the shocks for longevity and mortality risk both occur with probability 50%.

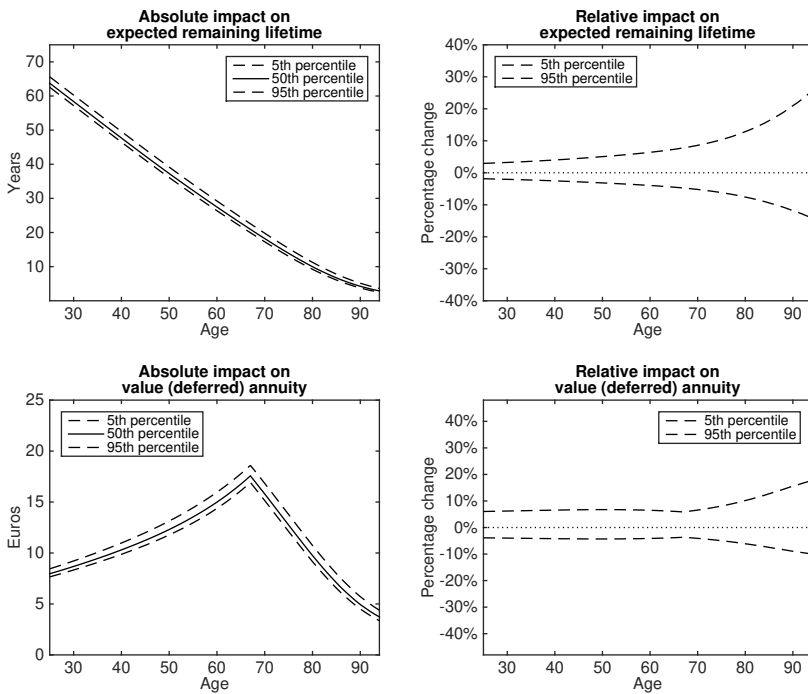
Figure 3.4.8 visualizes the impact of those shocks on the expected remaining lifetime and the value of a (deferred) variable annuity. We cannot compare the size of the impact of macro-longevity risk in the Lee-Carter model (Figure 3.1.2) and Solvency II framework (Figure 3.4.8) directly, because the shocks in the Lee-Carter model are on a 10-year horizon while the shocks in the Solvency II framework are one-off shocks. However, we can still compare the distribution of macro-longevity risk over different cohorts in both models. We notice that the relative change of the expected remaining lifetime and (deferred) annuity value (right-hand figures) differ significantly. In the Solvency II framework the relative change increases significantly after retirement. This is due to the fact that the impact of a uniform improvement of death probabilities on survival probabilities is much higher at high ages compared to low ages because death probabilities are higher at high ages. In the Lee-Carter model the impact of macro-longevity risk on death probabilities decreases with age.

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<sup>35</sup>These capital requirements are based on the 99.5% VaR of the available capital over a one-year horizon.

**Figure 3.4.8: Impact of macro-longevity risk in the SII model**

This figure shows the impact of macro-longevity risk in the Solvency II framework on the expected remaining lifetime and the value of a (deferred) variable annuity for a Dutch female in 2014 in absolute terms (left-hand graphs) and relative change (right-hand graphs) assuming a constant interest rate of 2% and fixed retirement age  $R = 67$ .



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## Results

Table 3.4.4 shows welfare gains from risk sharing in the CBD model and the Solvency II framework for the three retirement age policies. We notice that the welfare gains are higher in the CBD model compared to the Lee-Carter model. This can be explained by the fact that the CBD model contains two correlated factors. The Lee-Carter model contains only one factor which implies that mortality improvements at different ages are perfectly correlated in the Lee-Carter model. Welfare benefits are expected to be higher in case mortality improvements at different ages are not perfectly correlated as in the CBD model. Moreover, the higher welfare gain in case of a fully adjusted

**Table 3.4.4: Welfare gains different models**

*Welfare gains from sharing macro-longevity risk in terms of aggregate certainty equivalent consumption after retirement in the Lee-Carter model and in the Solvency II framework.*

<b>Model</b>	<b>LC</b>	<b>CBD</b>	<b>SII</b>
<b>Fixed retirement age (FRA)</b>	0.2%	0.3%	0.2%
<b>Partial adjustment retirement age (PARA)</b>	0.6%	0.8%	0.5%
<b>Full adjustment retirement age (FARA)</b>	1.8%	2.9%	0.6%

retirement age can also be explained by the fact that the increase (or decrease) of the retirement age is on average bigger in the CBD model compared to the LC model because death probabilities at high ages are more sensitive to macro-longevity shocks in the CBD model. As a result, the hedge effect of the adjusted labor supply to macro-longevity shocks for workers in the CBD model is bigger.

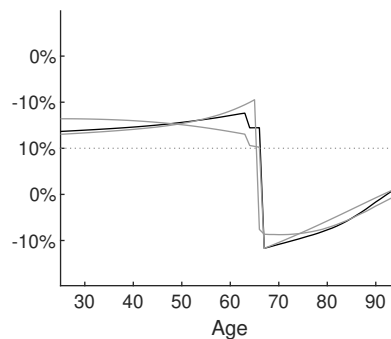
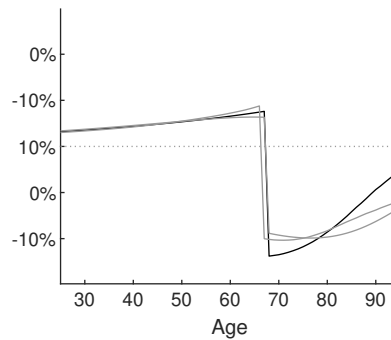
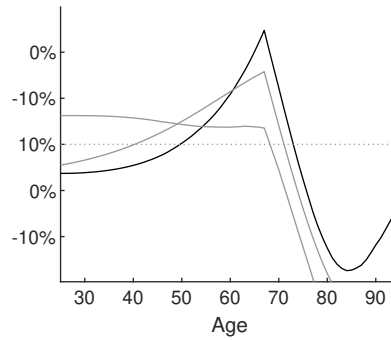
We cannot compare the size of welfare gains in the Lee-Carter model and Solvency II framework directly because both shocks have a different interpretation as mentioned above. In the Solvency II framework the welfare gain does not increase significantly in case of a full adjustment of the retirement age. Recall that the high welfare gain in case the retirement age is fully adjusted in the Lee-Carter model is a result of the hedge effect of the adjusted labor supply to macro-longevity shocks for workers. In the Solvency II framework the impact of macro-longevity risk on the expected remaining life-

time (Figure 3.4.8) is small for workers. As a result, the hedge effect is much smaller in the Solvency II framework compared to the Lee-Carter model.

Figure 3.4.9 plots the optimal risk transfer relative to autarky as a percentage of the total risk that is transferred. The black lines represent the optimal risk transfers in the Lee-Carter model and the gray lines represent the optimal risk transfers in the CBD model and Solvency II framework. We conclude that the optimal risk transfer rule is robust to the alternative mortality models in case of (partial) adjustment of the retirement age but it differs in case of a fixed retirement age. The only significant difference is the difference in risk transfer after retirement in case the retirement age is (partially) adjusted. This can be explained by the fact that the impact of macro-longevity risk decreases after retirement in the Lee-Carter model but not in the CBD model and the Solvency II framework.

**Figure 3.4.9: Optimal risk transfer different models**

Optimal risk transfer relative to autarky as percentage of total risk transferred for the fixed retirement age policy (top graph), partial adjustment retirement age policy (middle graph) and the full adjustment retirement age policy (bottom graph). The black lines represent the original risk transfers in the Lee-Carter model and the gray lines represent the optimal risk transfers in the CBD model and the Solvency II framework.



### **3.5 Conclusion and policy evaluation**

Pension funds face macro-longevity risk or uncertainty about future mortality rates. We analyze macro-longevity risk sharing between cohorts in a pension scheme as a risk management tool. We explore this economic problem as macro-longevity risk is not traded on a liquid market and cohorts are affected differently by macro-longevity risk. We derive Pareto-improving risk-sharing rules that maximize the welfare gain from risk sharing for all participants in the pension scheme for different retirement age policies.

We find that the design of the retirement age policy has a large impact on both the risk-sharing rule and welfare gains from sharing macro-longevity risk. When the retirement age is fixed, welfare gains from sharing macro-longevity risk on a 10-year horizon are between 0.1 percent and 0.3 percent of certainty equivalent consumption after retirement. Under this policy, the impact of macro-longevity risk on retirement consumption for different cohorts is more or less equal. Young cohorts do not absorb macro-longevity risk of old cohorts in the optimal risk transfer rule. As a result, welfare gains from risk sharing are limited.

Some countries link the retirement age to life expectancy developments. If the retirement age is linked to life expectancy, welfare gains from sharing macro-longevity risk measured on a 10-year horizon are substantially higher, up to 1.8 percent in the Lee-Carter model and up to 2.9 percent in the Cairns-Blake-Dowd model. The risk bearing capacity of workers is larger, because their labor supply acts as a hedge against macro-longevity shocks. As a result, workers absorb risk from retirees in the optimal risk transfer rule because the human capital of workers increases if they work longer. As a result, the welfare gain from risk sharing increases. The size of welfare gains from risk sharing is sensitive to the mortality data and model assumptions. This is a result of a different volatility of macro-longevity risk when using different mortality data and a different distribution of macro-longevity risk over cohorts. However, the optimal risk transfer rules are reasonably robust to the alternative mortality data and model assumptions.

The findings in this paper are relevant for pension policy, especially because of the general trend of transferring funding risks from pension scheme

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sponsors to pension participants (Munnell (2006) and Novy-Marx and Rauh (2014)). *First*, we determine the optimal risk-sharing rule for macro-longevity risk in this paper. In practice macro-longevity risk is shared in specific ways. In DB schemes and pooled annuity schemes, e.g., macro-longevity risk is usually shared uniformly. The results in this paper show that uniform risk sharing is suboptimal. Moreover, it is sometimes argued that workers can provide insurance to macro-longevity risk of retirees. The results in this paper show that such a risk distribution is optimal only in case the retirement age is linked to life expectancy. If the retirement age is fixed it is not optimal for young cohorts to absorb risk of retirees. *Second*, we determine a fair risk compensation for cohorts who absorb macro-longevity risk of other cohorts using a utility-based fairness criterion. In practice, there is usually no risk compensation for absorbing macro-longevity risk.

Sharing macro-longevity risk results in higher welfare gains in case of a full adjustment of the retirement age. In this paper we do not make a statement about the suitability of retirement age policies. This is a different research question and involves a broader perspective. Healthy life expectancy and practical implementation are for example relevant but outside the scope of this paper. It is up to policymakers to decide whether it is appropriate to link the retirement age to life expectancy. The goal of this paper is to determine the optimal way to share macro-longevity risk between cohorts *given* a certain retirement age policy.

Sensitivity analyses show that the size of welfare gains depends on the population composition, mortality data and mortality model. For example, welfare gains from sharing macro-longevity risk are smaller for US mortality data compared to Dutch mortality data as a result of a lower volatility. Moreover, welfare gains from sharing macro-longevity risk are higher in a mortality model in which mortality improvements at different ages are not perfectly correlated. An interesting area for future research is to investigate sharing macro-longevity risk between pension funds or even between countries. van Binsbergen et al. (2014) propose sharing risks between heterogeneous pension funds by trading pension guarantees. Bodie and Merton (2002) propose swaps to achieve risk-sharing benefits of broad international diversification. Our framework is useful for further developing such instruments.



# Appendix

## 3.A Expected survival probability

The random shocks in (3.6) in the log central death rates are normally distributed with mean zero, i.e.,  $\mathbb{E}[\beta_x \eta_t + \epsilon_{x,t}] = 0$ . The following holds for the expected survival probability

$$\begin{aligned}\mathbb{E}[p_{x,t}] &\approx \mathbb{E}[\exp(-\mu_{x,t})] = \mathbb{E}[\exp(-\exp(\alpha_x + \beta_x \kappa_t + \epsilon_{x,t}))] & (3.32) \\ &= \mathbb{E}[\exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1} + \beta_x \eta_t + \epsilon_{x,t}))]\end{aligned}$$

$$\begin{aligned}\mathbb{E}[\exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1} + \beta_x \eta_t + \epsilon_{x,t}))] \\ \leq \exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1} + \mathbb{E}[\beta_x \eta_t + \epsilon_{x,t}])),\end{aligned}$$

using Jensen's inequality  $\mathbb{E}[f(x)] \leq f(\mathbb{E}[x])$  with  $f(x) = \exp(-\exp(x))$  being a concave function for  $x \leq 0$ .

$$\begin{aligned}\exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1} + \mathbb{E}[\beta_x \eta_t + \epsilon_{x,t}])) & \\ &= \exp(-\exp(\alpha_x + \beta_x c + \beta_x \kappa_{t-1})) = \hat{p}_{x,t}.\end{aligned} \tag{3.33}$$

So the expected one-year survival probabilities are smaller or equal to the forecasted values.

### 3.A.1 Derivation Pareto optimal risk-sharing rule in stylized two-agent model

The Lagrange function of the maximization problem in (3.19) equals

$$L(T, \lambda) = \mathbb{E}[U(W_1 + T_1(\tilde{y})) + \lambda(U(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y})) - U(W_2 + \beta_2\tilde{y}))]. \quad (3.34)$$

Because  $T_1(\tilde{y})$  is a continuous function we take the Fréchet-derivative

$$D_T L(T, \lambda) \cdot \tau = \mathbb{E}[(U'(W_1 + T_1(\tilde{y})) + \lambda U'(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y})))\tau(\tilde{y})]. \quad (3.35)$$

The first order condition should be zero for each pertubation  $\tau(\tilde{y})$ . This is only possible if the following holds for all values of  $\tilde{y}$

$$U'(W_1 + T_1(\tilde{y})) = \lambda U'(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y})) \quad \forall \tilde{y}. \quad (3.36)$$

This is a non-linear equation which can be solved for  $\{T_1(\tilde{y}), T_2(\tilde{y})\}$ . As mentioned in Section 3.2.2 we assume both agents have exponential utility with risk aversion  $\alpha$ . We plug this utility function into the first order condition

$$\begin{aligned} \exp(-\alpha(W_1 + T_1(\tilde{y}))) &= \lambda \exp(-\alpha(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y}))) & (3.37) \\ -\alpha(W_1 + T_1(\tilde{y})) &= \ln(\lambda) - \alpha(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y})) \\ -2\alpha T_1(\tilde{y}) &= \ln(\lambda) - \alpha(W_2 - W_1 + (\beta_1 + \beta_2)\tilde{y}) \\ T_1(\tilde{y}) &= -\frac{1}{2} \frac{\ln(\lambda)}{\alpha} + \frac{1}{2} (W_2 - W_1 + (\beta_1 + \beta_2)\tilde{y}). \end{aligned}$$

Because the utility function is strictly increasing, the inequality restriction in (3.19) is binding

$$\begin{aligned} \mathbb{E}[U(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y}))] &= \mathbb{E}[U(W_2 + \beta_2\tilde{y})] & (3.38) \\ \mathbb{E}\left[-\frac{1}{\alpha} \exp(-\alpha(W_2 + (\beta_1 + \beta_2)\tilde{y} - T_1(\tilde{y})))\right] &= \mathbb{E}\left[-\frac{1}{\alpha} \exp(-\alpha(W_2 + \beta_2\tilde{y}))\right]. \end{aligned}$$

Plugging in (3.37) yields

$$\begin{aligned}
& \mathbb{E} \left[ -\frac{1}{\alpha} \exp \left( -\alpha \left( W_2 + (\beta_1 + \beta_2) \tilde{y} + \frac{1}{2} \frac{\ln \lambda}{\alpha} - \frac{1}{2} (W_2 - W_1 + (\beta_1 + \beta_2) \tilde{y}) \right) \right) \right] \quad (3.39) \\
&= \mathbb{E} \left[ -\frac{1}{\alpha} \exp(-\alpha(W_2 + \beta_2 \tilde{y})) \right] \\
& \mathbb{E} \left[ -\frac{1}{\alpha} \exp \left( -\frac{1}{2} \ln \lambda - \alpha \left( \frac{1}{2} (W_1 + W_2) + \frac{1}{2} (\beta_1 + \beta_2) \tilde{y} \right) \right) \right] \\
&= \mathbb{E} \left[ -\frac{1}{\alpha} \exp(-\alpha(W_2 + \beta_2 \tilde{y})) \right] \\
& - \frac{1}{\alpha} \mathbb{E} \left[ \exp \left( -\frac{1}{2} \ln \lambda \right) \right] \mathbb{E} \left[ \exp \left( -\alpha \frac{1}{2} (W_1 + W_2) \right) \right] \mathbb{E} \left[ \exp \left( -\alpha \frac{1}{2} (\beta_1 + \beta_2) \tilde{y} \right) \right] \\
&= -\frac{1}{\alpha} \mathbb{E} \left[ \exp(-\alpha W_2) \right] \mathbb{E} \left[ \exp(-\alpha \beta_2 \tilde{y}) \right] \\
& \frac{1}{\sqrt{\lambda}} \exp \left( -\frac{1}{2} \alpha W_1 \right) \exp \left( -\frac{1}{2} \alpha W_2 \right) \exp \left( \frac{1}{8} \alpha^2 (\beta_1 + \beta_2)^2 \sigma^2 \right) \\
&= \exp(-\alpha W_2) \exp \left( \frac{1}{2} \alpha^2 \beta_2^2 \sigma^2 \right) \\
& \exp \left( \frac{1}{2} \alpha (W_2 - W_1) \right) \exp \left( \frac{1}{8} \alpha^2 \beta_1^2 \sigma^2 + \frac{1}{4} \alpha^2 \beta_1 \beta_2 \sigma^2 - \frac{3}{8} \alpha^2 \beta_2^2 \sigma^2 \right) = \sqrt{\lambda} \\
& \exp \left( \alpha (W_2 - W_1) \right) \exp \left( \frac{1}{4} \alpha^2 \beta_1^2 \sigma^2 + \frac{1}{2} \alpha^2 \beta_1 \beta_2 \sigma^2 - \frac{3}{4} \alpha^2 \beta_2^2 \sigma^2 \right) = \lambda
\end{aligned}$$

Plugging  $\lambda$  into (3.37) yields

$$\begin{aligned}
T_1(\tilde{y}) &= -\frac{1}{2} (W_2 - W_1) + \frac{3}{8} \alpha \beta_2^2 \sigma^2 - \frac{1}{8} \alpha \beta_1^2 \sigma^2 - \frac{1}{4} \alpha \beta_1 \beta_2 \sigma^2 \quad (3.40) \\
&+ \frac{1}{2} (W_2 - W_1) + \frac{1}{2} (\beta_1 + \beta_2) \tilde{y} \\
&= -\frac{1}{8} \alpha \sigma^2 (\beta_1 + 3\beta_2) (\beta_1 - \beta_2) + \frac{1}{2} (\beta_1 + \beta_2) \tilde{y},
\end{aligned}$$

and the optimal risk-sharing rule for agent 2 equals

$$T_2(\tilde{y}) = \frac{1}{8} \alpha \sigma^2 (\beta_1 + 3\beta_2) (\beta_1 - \beta_2) + \frac{1}{2} (\beta_1 + \beta_2) \tilde{y}. \quad (3.41)$$

Equation (3.40) and (3.41) show that both risk-sharing rules add up to the total exposure to the risk factor  $(\beta_1 + \beta_2) \tilde{y}$ . Moreover, the constant risk compensation  $t_{0,1}$  is equal to the negative of  $t_{0,2}$  which makes sense because it is a zero-sum game.

### 3.B Definitions

Parameter	Definition
$\alpha_x$	Age-specific constant in log central death rates
Annuity value ( $\ddot{a}_{x,t}$ )	Value of an annuity that pays 1 dollar annually during retirement for an individual of age $x$ in year $t$
Autarky	Situation without risk sharing
$C_i^a$	Consumption after retirement in autarky for a participant in cohort $i$
$C_i^s$	Consumption after retirement in case of risk sharing for a participant in cohort $i$
Certainty equivalent consumption	Guaranteed consumption level that someone would accept rather than a higher uncertain consumption
Central death rate ( $\mu_{x,t}$ )	Average yearly death rate of an individual of age $x$ in year $t$
Cumulative survival probability ( ${}_i p_{x,t}$ )	Probability that an individual of age $x$ in year $t$ is still alive after $i$ years
$c$	Drift in time trend
Uncertainty in death rates ( $\epsilon_{x,t}$ )	Random variation in log central death rates
Fixed retirement age (FRA)	Constant retirement age
Full adjustment retirement age (FARA)	Retirement age keeps up fully with life expectancy
Longevity risk	Risk that people live longer than expected
Macro-longevity risk	Uncertainty about future mortality rates
Micro-longevity risk	Uncertainty about individual time of death
Mortality risk	Risk that people live shorter than expected
One-year death probability ( $q_{x,t}$ )	Probability that an individual of age $x$ and alive in year $t$ dies before year $t + 1$

*Continued on next page*

Table 3.B.1 – *continued from previous page*

Parameter	Definition
One-year survival probability ( $p_{x,t}$ )	Probability that an individual of age $x$ and alive in year $t$ is still alive in year $t + 1$
Parameter risk	Uncertainty in the true value of the parameters
Partial adjustment retirement age (PARA)	Retirement age adjusts to life expectancy such that the value of an annuity remains the same
$\beta_x$	Sensitivity of log central death rates to time trend
Risk compensation ( $t_{0,i}$ )	Financial compensation for absorbing risk for a participant in cohort $i$
Risk sharing	Allocate risks to cohorts via a predetermined rule
Risk-sharing rule ( $t(\tilde{y})$ )	Risk transfer plus risk compensation
Risk transfer ( $\eta_i$ )	Part of total macro-longevity shock a participant in cohort $i$ absorbs
Stochastic variation	Random variation in the aggregate realized number of deaths
Time trend ( $\kappa_t$ )	Development of death rates over time
Uncertainty in trend ( $\eta_t$ )	Random variation in the time trend
$\sigma_\epsilon^2$	Variance death rates
$\sigma_\eta^2$	Variance trend
$W_i$	Wealth of a participant in cohort $i$
Welfare gain	Relative increase certainty equivalent consumption after retirement
$\tilde{y}_i$	Amount of money needed to offset effect of macro-longevity shock for a participant in cohort $i$
$\tilde{y}_T$	Amount of money needed to offset effect of macro-longevity shock for all cohorts



# 4

## Walk the green talk? A textual analysis of pension funds' disclosures of sustainable investing<sup>1</sup>

There is a global trend towards investment policies that take environmental, social, and governance (ESG) information into account. Sustainable investing reached 35.3 trillion dollars in assets under management in 2020 (GSIA (2021)).<sup>2</sup> Pension funds, as long-term investors, in particular may put their capital at work in a way that positively influences the environment and society. There is societal and political pressure on pension funds to do so; several recent examples exist of protesters pushing pension funds to divest from fossil fuels.<sup>3</sup> Moreover, there is growing recognition that climate-related risks are a source of financial risk, impacting the resilience of financial institutions,

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<sup>1</sup>This chapter is based on a working paper co-authored with Rob Bauer (Maastricht University and ICPM) and Dirk Broeders (Maastricht University and De Nederlandsche Bank).

<sup>2</sup>In this paper, we use the term sustainable investing (SI), which is also known as socially responsible investing (SRI), corporate social responsibility (CSR) or ESG investing.

<sup>3</sup>For example, there were protests at the Greater Manchester Pension Fund in July 2019, the Dutch civil service pension fund (ABP) in September 2021, and the Teachers Insurance & Annuity Association of America (TIAA) in October 2022.

including pension funds.<sup>4</sup> Nevertheless, little is known about the design and development of SI policies by pension funds. Pension funds can implement sustainable investing using different strategies, for example, by excluding companies with a negative environmental impact from the investment portfolio (divestment), by voting on shareholder proposals (public engagement), or by directly communicating with companies (private engagement).

Over the years, governments and NGOs launched several initiatives to stimulate the development of a sustainable financial system and to promote the integration of ESG information into investment decisions. For example, the United Nations Environment Programme Finance Initiative (UNEP FI) established and co-created several international programs, including the well-known Principles for Responsible Investment (PRI). This program is a UN-supported initiative founded in 2006 by some of the world's largest institutional investors to stimulate the incorporation of ESG information into investment practices. In this paper we focus on the best-known initiative in the Dutch pension fund sector which is the International Responsible Business Conduct (IRBC) initiative (IRBC (2018)). This initiative is a voluntary effort undertaken of Dutch pension funds that aims to bring their investment policy into line with the OECD Guidelines for Multinational Enterprises (OECD Guidelines) and the United Nations Guiding Principles on Business and Human Rights (UNGPs).<sup>5,6,7</sup> This raises the question of whether pension funds that sign such an initiative enhance their SI policy more than non-signatories. This paper is the first paper investigating the impact of signing the IRBC initiative on a pension fund's SI policy.

This paper contributes to the literature by creating an overview of the disclosures of sustainable investing by a specific group of large institutional investors, Dutch occupational pension funds, by exploiting a unique dataset with a novel tool. Dutch pension funds had more than 1.8 trillion euros worth of assets under management at the end of 2021 (DNB (2022)) and as such the Netherlands have the highest ratio of pension assets to GDP worldwide

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<sup>4</sup>See, e.g., Vermeulen et al. (2018) and FSB (2020).

<sup>5</sup>The IRBC is the 'Convenant Internationaal Maatschappelijk Verantwoord Beleggen Pensioenfondsen' (IMVB) in Dutch.

<sup>6</sup>OECD (2011), *OECD Guidelines for Multinational Enterprises*.

<sup>7</sup>United Nations (2011), *Guiding Principles on business and human rights*.

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(ThinkingAheadInstitute (2022)). We introduce a novel textual analysis approach using state-of-the-art natural language processing (NLP) techniques to measure a pension fund's SI policy using qualitative data from annual reports. The textual analysis approach consists of two steps. To begin, we extract all SI-related sentences from the annual reports by applying a combined rule-based and classification approach using a pre-trained state-of-the-art language model called BERT. Subsequently, we exploit various NLP techniques (rule-based approach, topic modeling, and classification approach) to measure the SI policy of pension funds along two dimensions. First, we measure the awareness of sustainable investing, where we define awareness as the amount of attention paid to sustainable investing in the annual report. We use three measures to quantify awareness: intensity (fraction of SI-related sentences), spectrum (number of SI topics), and specificity (number of specific SI-related paragraphs). Second, we track the implementation of sustainable investing by constructing two measures: variety (number of implemented SI strategies) and scope (fraction of the portfolio included in the SI policy). We combine these SI measures with detailed financial and non-financial information about Dutch occupational pension funds, using a proprietary dataset from the prudential supervisor of pension funds, De Nederlandsche Bank (DNB).

We formulate three hypotheses to analyze the relation between pension fund characteristics and sustainable investing and the role of signing the IRBC initiative. First, we hypothesize that pension fund characteristics impact pension funds' awareness and implementation of sustainable investing. In particular, we expect large pension funds to have higher scores on all five SI measures, because large pension funds have more capacity to implement sustainable investing and might experience more societal pressure. We also expect that beliefs regarding the risk-return relation of sustainable investing impact the SI measures. We expect that board characteristics, such as the average board's age, gender, or stakeholder representation, do not impact the SI measures. Second, we hypothesize that pension fund characteristics also have an impact on the probability of signing the IRBC initiative in line with the first hypothesis. Third, we hypothesize that pension funds that signed the IRBC initiative enhance their SI policy more than pension funds that did

not sign this initiative. We expect that the commitment of signatories to bring the investment policy into line with the OECD Guidelines and UNGPs will increase the awareness and implementation of sustainable investing.

The empirical results show that the pension fund's size increases the pension fund's awareness and implementation of sustainable investing. A positive belief about the risk-return relation of sustainable investing increases the awareness of sustainable investing. In line with our hypothesis, the board of trustees characteristics do not impact the SI measures. Further, we find that large pension funds, pension funds with more female trustees, and pension funds with a positive belief about the risk-return relation of sustainable investing are more likely to sign the IRBC initiative. Signing this SI initiative increases the awareness of sustainable investing, but we do not find a significant effect on the implementation of sustainable investing.

This paper fits into the literature on institutional investors setting up their SI policy. Some papers use survey data to investigate perceptions about and the implementation of sustainable investing by institutional investors (Krueger et al. (2020), Amel-Zadeh and Serafeim (2018), and Ilhan et al. (2021a)). For instance, Krueger et al. (2020) show that institutional investors increasingly account for climate risk in their investment decision-making. Wagemans et al. (2018) investigate engagement at large Dutch pension funds using survey data and interviews.

Another strand of the literature focuses on the impact of institutional ownership on ESG performance. Dyck et al. (2019) and Chen et al. (2020) find a positive relationship between institutional ownership and firms' environmental and social performance. Ceccarelli et al. (2021) find a positive association between responsible institutional investors and ESG scores. The impact of SI initiatives on ESG performance is investigated by Bauckloh et al. (2021), Gibson et al. (2022), and Kim and Yoon (2022), focusing on the PRI. These papers provide mixed evidence. Bauckloh et al. (2021) and Gibson et al. (2022) find that institutional investors who signed the PRI initiative have better ESG scores compared to matched non-signatories. However, this result does not hold for US signatories in the research of Gibson et al. (2022), and Kim and Yoon (2022) also do not observe improved ESG scores for US mutual funds after signing. Bingler et al. (2022) measure the impact of signing differ-

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ent climate initiatives on the quality of corporate climate action disclosures and show that engagement initiatives considerably increase the quality and decision-relevance of corporate disclosures of climate-related commitments and actions.<sup>8</sup>

Finally, this paper relates to literature using textual analysis to measure climate risks in corporate documents. Berkman et al. (2021) follow a rule-based approach to measure climate risk exposure based on 10-K filings. Hail et al. (2021) and Sautner et al. (2021) use a predefined dictionary to measure climate change exposure in earnings conference calls.

The remainder of this paper is structured as follows. In Section 4.1, we describe the institutional setting of Dutch occupational pension funds, relevant legislation, SI initiatives, motives, and strategies. Section 4.2 presents the method for measuring sustainable investing. In Section 4.3, we provide an overview of the data and explain how the different SI measures are constructed. Section 4.4 introduces the empirical design and discusses the results. We conclude in Section 4.5.

## **4.1 Institutional setting**

This study takes place in the Dutch occupational pension sector. We describe the organization of Dutch pension funds in Section 4.1.1 and the legal requirements regarding sustainable investing in Section 4.1.2. Section 4.1.3 gives an overview of SI initiatives that aim to enable and reinforce the development of a financial system. Section 4.1.4 describes why pension funds want to implement sustainable investing and Section 4.1.5 discusses strategies to realize sustainable investing.

### **4.1.1 Dutch occupational pension sector**

Due to the quasi-mandatory status, the participation rate in the Netherlands is high: around 90% of the workforce participates in one or more occupational pension schemes (StvdA (2020)). For some industries, mandatory participation exists, which implies that all companies - and therefore all em-

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<sup>8</sup>Bingler et al. (2022) consider the following climate initiatives: Task Force on Climate-Related Financial Disclosure, the Science-Based Targets Initiative and the Climate Action 100+.

ployees - in such an industry are required to join an industry-wide pension fund. Besides industry-wide pension funds, there are also pension funds for the employees of a specific company (corporate pension funds) or a particular profession (professional group pension funds).

In the Netherlands, pension funds are legally independent, non-profit organizations whose task is to execute the pension scheme that representatives of employers and employees have negotiated as part of labor compensation. The board of trustees is responsible for managing the assets and administering the benefits and consists of employee representatives (labor unions), employer representatives, and external experts. This board formally sets the investment policy and strategic asset allocation, with the help of several advisory councils, consultants, and investment advisors. Most pension funds outsource the implementation of the investment policy to one or more asset management firms. Besides implementing the investment policy, asset management firms can also act as an advisor to the pension fund when developing the SI policy because they often possess more expertise on this topic. Specifically for engagement, pension funds sometimes use ESG service providers who conduct the engagement conversations regarding sustainable investing independently from the asset manager.

### **4.1.2 Legislation**

A number of features in the legislation on Dutch occupational pension funds are relevant to sustainable investing. An important article of the Dutch Pension Act states that pension funds should invest their assets in the sole interest of their beneficiaries. This is the so-called prudent person rule.<sup>9</sup> The prudent person rule is an open norm and does not contain quantitative investment restrictions.<sup>10</sup> To invest the assets in the best interest of beneficiaries, pension funds should take into account the beneficiaries' sustainability preferences. Moreover, Article 135 of the Dutch Pension Act states that pension funds should specify in their annual report how they incorporate ESG criteria in their investment policy.

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<sup>9</sup>Dutch Pension Act, Article 135.

<sup>10</sup>The only restrictions are the prohibition on providing direct loans with a duration of one year or longer and the prohibition on investing more than 5 percent in the sponsoring corporation.

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In addition to Dutch legislation, European legislation is also relevant. IORP II states that pension funds can include ESG criteria in the prudent person rule as long as the application of ESG criteria does not harm the financial interests of the beneficiaries.<sup>11</sup> Another IORP II requirement is the incorporation of ESG risks in risk management.<sup>12</sup> The only hard requirement regarding sustainable investing for Dutch pension funds is the prohibition of cluster munition investments, which has been in place since 2013.<sup>13</sup>

Additional requirements are applicable as of March 2021, resulting from the European Sustainable Finance Disclosure Regulation (SFDR) regarding the provision of information on the sustainability of investments. Pension funds should explain to what extent they integrate ESG risks in their investment process. Moreover, they should indicate whether they take the adverse impacts of investment decisions on ESG factors into account.

### **4.1.3 Sustainable investment initiatives**

Besides the legal requirements, there has also been a rapid increase in voluntary initiatives to stimulate sustainable investing. Such initiatives aim to enable and reinforce the development of a sustainable financial system by promoting ESG integration into investment decisions or transparent disclosures. For example, the United Nations Environment Programme Finance Initiative (UNEP FI) comprised several international programs, including the well-known Principles for Responsible Investment (PRI). This program is a UN-supported initiative founded in 2006 by some of the largest institutional investors to stimulate the incorporation of ESG factors into investment practices. Other programs of the UNEP FI are the Principles for Responsible Banking (PRB), the Collective Commitment to Climate Action (CCCA), the Principles for Sustainable Insurance (PSI), and the Net-Zero Asset Owner Alliance (NZAOA). The best-known initiative in the Dutch pension fund sector is the IRBC initiative (IRBC (2018)) to identify, prioritize, and address ESG-related risks. A group of pension funds signed it at the end of 2018. The initiative aims to bring the investment policy into line with the OECD Guidelines and

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<sup>11</sup>IORP II, Article 19; implemented in Pension Act, Article 135.

<sup>12</sup>IORP II, Article 25; implemented in 'Besluit FTK', Article 18.

<sup>13</sup>'Besluit marktmisbruik Wft', Article 21a.

UNGPs. Another national example is the commitment of a group of financial institutions to the climate goals of the Dutch government in 2019. They agreed to measure the CO<sub>2</sub> emissions of their investments and to publish their CO<sub>2</sub> reduction goals as of 2022.

In this paper, we focus on one particular SI initiative that many Dutch pension funds embraced: the IRBC initiative. The pension funds participating in this initiative made joint arrangements with NGOs, labor unions, and the government regarding integration into the investment policy, outsourcing, monitoring, and reporting. For example, they agreed that the SI policy should include an explanation of how sustainability is integrated into the various asset classes in which the pension fund invests. Moreover, the pension fund should disclose its approach towards voting and engagement and provide its stakeholders with information on which companies are excluded (IRBC (2018)). This raises the question of whether pension funds that sign such an SI initiative enhance their SI policy more than non-signatories. We hypothesize that signatories enhance their SI policy more than non-signatories. The SI measures include the integration of sustainable investing into various asset classes and the implementation of different SI strategies. The measures will be described in more detail in Section 4.3.2.

#### **4.1.4 Sustainable investment motives**

In this section, we discuss the motivation of pension funds to invest sustainably. Pension funds can have financial and moral objectives. Other possible motives are reputational risk and legislation.

The first motive for sustainable investing can be driven by financial objectives. As discussed briefly in the introduction, there is growing recognition that climate-related risks are a source of financial risk. Companies with a positive impact on society may be more likely to attract customers and employees and avoid potential environmental fines or regulatory intervention. These companies generate higher risk-adjusted returns if these benefits are not fully priced (Edmans and Kacperczyk (2022)). As a result, pension funds can decide to invest sustainably based on financial objectives.

Moral objectives can drive the second motive for sustainable investing.

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It can be a result of the perceived moral obligation of a pension fund to contribute to a sustainable world or a reflection of the preferences of the beneficiaries of the pension fund. Research shows that most pension participants have strong preferences for sustainability even at the expense of lower financial performance (Delsen and Lehr (2019) and Bauer et al. (2021)). Especially in the context of the Dutch pension sector, in which beneficiaries are not able to switch their pension provider, pension funds have a strong responsibility to ensure that they act in the best interest of their beneficiaries.

A third motive is a concern about reputational risk. As mentioned in the introduction, there is societal and political pressure on pension funds, and several examples exist of protesters pushing pension funds to divest. Pension funds are aware that the material consequences of their investments cause lasting reputational damage. Peer pressure and benchmarking can also accelerate the activities of a pension fund in the SI domain. An example is the VBDO Benchmark for Responsible Investment by Pension Funds (VBDO (2021)), which compares sustainable investing by the 50 largest pension funds in the Netherlands.

Finally, there are legal requirements regarding sustainable investing, as discussed in Section 4.1.2, which can stimulate (or in the future possibly force) sustainable investing by pension funds.

#### **4.1.5 Sustainable investment strategies**

In this section, we discuss different SI strategies. There are several strategies to realize sustainable investing. We distinguish the following strategies in this paper: divestment, ESG integration, screening, public engagement, and private engagement. It is noteworthy, however, that it is not always possible to distinguish clearly between these five investment strategies. For instance, there is some overlap between the different strategies and it is also possible to distinguish more strategies.

We consider these five SI strategies in this paper. The first strategy is divestment (or exclusion) in which a pension fund excludes companies or projects with a negative (environmental) impact from the investment portfolio. This strategy can have several objectives, for example to shift capital to

positive investments or meet the beneficiaries' demand. Many examples exist of pension funds that publicly declare their divestment from particular industries, such as the tobacco or nuclear weapons industry. In the Netherlands, pension funds have faced considerable pressure from stakeholders recently to divest from fossil fuel producers. Possibly in response to this pressure, some Dutch pension funds recently announced that they would stop investing in fossil fuel producers (IPE (2021a) and IPE (2021b)). There is disagreement in the literature on the effectiveness of a divestment strategy. For instance, Choi et al. (2021) posit that divestment pushes companies to adopt climate-friendly policies and decrease carbon footprints. In contrast to this finding, Berk and Binsbergen (2021) evaluate the quantitative impact of ESG divestitures and conclude that ESG divestiture strategies had little impact on the cost of capital and will likely have little impact in the future. A disadvantage of exclusion is that it takes away the opportunity to directly influence corporate decision-making via engagement (see below).

A second strategy is integrating ESG criteria into the investment process. The key objective of this strategy is to improve the risk-adjusted return of investments. When determining the strategic asset allocation, for instance, financial information is complemented by sustainability information. Since this strategy is quite broad, the exact implementation of this strategy may differ between pension funds. There is no single view in the literature on the impact of, for instance, climate risks on the risk-adjusted return of investments. Some papers provide evidence that carbon risk is starting to be priced in the market (e.g., Boermans and Galema (2020), Bolton and Kacperczyk (2021), and Ilhan et al. (2021b)). Bolton and Kacperczyk (2021) find higher returns for stocks with higher total CO<sub>2</sub> emissions. This evidence indicates that investors demand compensation for carbon emission risk. However, Sautner et al. (2021) do not find a positive risk premium for climate change exposure and Faccini et al. (2021) find that transition and physical risks that take longer to materialize are not yet priced. Integrating sustainability can be done by, for example, tilting portfolios toward certain Sustainable Development Goals or mandates with a small, selected number of highly sustainable companies.

A third strategy is screening. The key objective of screening is to improve the portfolio performance based on specific ESG criteria. Screening is

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the process of selecting investments based on these criteria. There are several screening approaches in practice. For example, under negative (or exclusionary) screening, certain sectors or companies that fail to meet specific ESG criteria are excluded.<sup>14</sup> In the case of positive screening, certain sectors or companies are selected based on their positive (or best-in-class) ESG performance relative to industry peers. With norm-based screening, companies that do not adhere to widely accepted norms of business conduct are excluded. Heinkel et al. (2001) and Gollier and Pouget (2014) predict with an equilibrium model that companies will be incentivized to implement reforms when a significant fraction of investors apply the same screening approach. Opposing conclusions exist in the empirical literature on the impact of screening approaches on asset prices. For example, Hong and Kacperczyk (2009) show that sin stocks have depressed prices relative to otherwise comparable stocks.

The fourth and fifth strategies are two types of engagement. Engagement is the process of shareholders influencing corporate decision-making. The objective of engagement is to encourage companies to adopt more sustainable practices. In this paper, we distinguish two types of engagement: public and private engagement. Investors can engage in active ownership strategies by voting on and sponsoring shareholder proposals (public engagement) or by directly communicating with companies (private engagement) via meetings, calls, or letters. Engagement on ESG topics has become increasingly prevalent in financial markets worldwide. Besides engagement on an individual basis, shareholders regularly join forces and engage in a dialogue with companies as a group of institutional investors (collaborative engagement). By speaking with a unified voice, investors can more effectively communicate their concerns to companies and trigger action. SI initiatives can coordinate these collaborative engagements. Examples are the PRI collaboration platform and the IRBC deep track. Dimson et al. (2015) show that collaboration significantly increases the success rate of environmental and social engagements. Bauer et al. (2022) show that firms targeted by successful material private ESG engagements significantly outperform their peers. Hoepner et al. (2022) provide evidence that investor shareholder engagement can reduce

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<sup>14</sup>There is some overlap between divestment and screening, i.e., negative screening can be classified as both a screening strategy and an exclusion strategy.

downside ESG risks, especially those from climate change.

One of the five SI measures, the variety measure, counts the number of SI strategies each pension fund implements. Section 4.3.2 describes this measure in more detail. Before describing the measures in more detail in Section 4.3, Section 4.2 first explains how sustainable investing can be measured.

## **4.2 Measuring sustainable investing using NLP and self-reported information**

To measure sustainable investing, many papers use ESG ratings of companies to calculate a portfolio's average ESG rating, which acts as a measure of the ESG performance (e.g., Dyck et al. (2019), Gibson et al. (2020), Chen et al. (2020), and Ceccarelli et al. (2021)). A drawback of this approach is that several studies document that ESG ratings can be very different across different ESG rating providers (Chatterji et al. (2016), Gibson et al. (2021), and Berg et al. (2022)). Berg et al. (2022) investigate the ratings of six prominent ESG rating providers and find correlations between ESG ratings from 0.38 to 0.71. Moreover, Bams and Kroft (2022) provide evidence that global ESG ratings are inversely related to sustainable performance. There are also other drawbacks to using ESG ratings to measure the SI efforts of institutional investors. First, engagement activities are not directly visible in the ESG ratings compared to other SI strategies (e.g., divestment). It can take some time before successful engagement activities induce ESG rating adjustments. Second, ESG ratings are not available for all asset classes. While the coverage of ESG ratings for equity and corporate bonds is high, ESG ratings are often not available for alternative asset classes such as private equity or infrastructure.

In this study, we do not rely on ESG ratings and measure sustainable investing in an alternative way by using qualitative data from annual reports. We exploit three different NLP techniques (classification approach, topic modeling, and rule-based approach) to measure the awareness and implementation of sustainable investing by pension funds using five different measures that will be explained in Section 4.3.2. In this section, we discuss

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these three NLP techniques and discuss how the textual analysis pipeline is built.

First, text classification is a supervised machine learning technique that allocates categories to input text. For the classification approach, we use a recent NLP innovation exploiting deep neural network models for text classification called BERT. BERT is a trained transformer-based language model which learns contextual word embeddings (Devlin et al. (2019)).<sup>15</sup> One of the key advantages of using a BERT model for text classification is that it is trained on large amounts of unannotated data. This allows the model to learn more general text patterns and complex non-linear patterns, which significantly improves the model's performance. We use the RobBERT model: a trained Dutch RoBERTa-based language model.<sup>16</sup> This model is trained on large amounts of unlabeled Dutch text, generating powerful semantic representations of words and patterns. To perform text classification, we finetune this model on a supervised task using a labeled dataset. Finetuning is done by adding an output layer to the original model architecture (Devlin et al. (2019)). Finetuning a trained model instead of training a model from scratch is preferred because it is less computationally intensive, faster, and improves generalized performance (Hendrycks et al. (2020)), even for smaller datasets. We finetune the RobBERT model twice for two different classification tasks using labeled datasets. These labeled datasets are created with an annotation approach. Appendix 4.A contains more details on this annotation approach, and Appendix 4.B contains more details on the finetuning and performance of the model.

Some recent studies have used BERT models to measure climate risk in corporate documents. Our classification approach is similar to Kölbel et al. (2022), who use BERT to quantify regulatory disclosure of climate risks in 10-K reports in order to analyze the impact on the spread in the credit default swap market. Friederich et al. (2021) use both BERT and RoBERTa to analyze the development of climate risk disclosures in annual corporate doc-

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<sup>15</sup>Besides the basic BERT model, various model configurations exist, such as RoBERTa, DistilBERT, and ALBERT.

<sup>16</sup>The RoBERTa model is the robustly optimized English BERT model. The RobBERT model uses the RoBERTa architecture and trains it with Dutch data.

uments over the last 20 years. Webersinke et al. (2021) developed ClimateBERT by pretraining the DistilRoBERTa model with climate-related news articles, research abstracts, and corporate climate reports. Bingler et al. (2022) use this ClimateBERT model to measure cheap talk in corporate climate commitments.<sup>17</sup>

Second, topic modeling is an unsupervised machine learning technique that identifies topics in text by detecting patterns and recurring words. We use a topic modelling tool that exploits the same class of language models as BERT, namely BERTopic (Grootendorst (2022)). BERTopic extracts latent topics from a collection of documents by producing topic representations. BERTopic is well suited to the analysis of sentences or paragraphs acting as documents, so coherent and consistent themes can be derived from the text. We use this tool to identify different SI topics and determine which are discussed in each annual report. We discuss this application in more detail in Section 4.3.2.

Third, in a rule-based approach, texts are analyzed using carefully prepared keyword lists. For simple, straightforward tasks, rule-based approaches are suitable because of their transparency and flexibility. In this paper, we use a rule-based approach, for example, to extract sentences with SI-related words. We use a dictionary with SI-related keywords and combinations of keywords to extract all SI-related sentences using a lemmatized keyword search.<sup>18</sup> A well-known example of a rule-based application is the word list of Loughran and McDonald (2011) to measure sentiment in financial texts, which is used in several studies in the finance and accounting literature (for example, Das et al. (2014), Kearney and Liu (2014), and Gandhi et al. (2019)). In the literature on climate-related disclosures, Berkman et al. (2021) use a rule-based approach to measure climate risk exposure in 10-K filings, and Hail et al. (2021) investigate potential greenwashing using a keyword approach by analyzing earning calls. A drawback of a rule-based approach is that such a method falls short of incorporating the language's richness, context dependence, and high dimensionality. Moreover, these approaches

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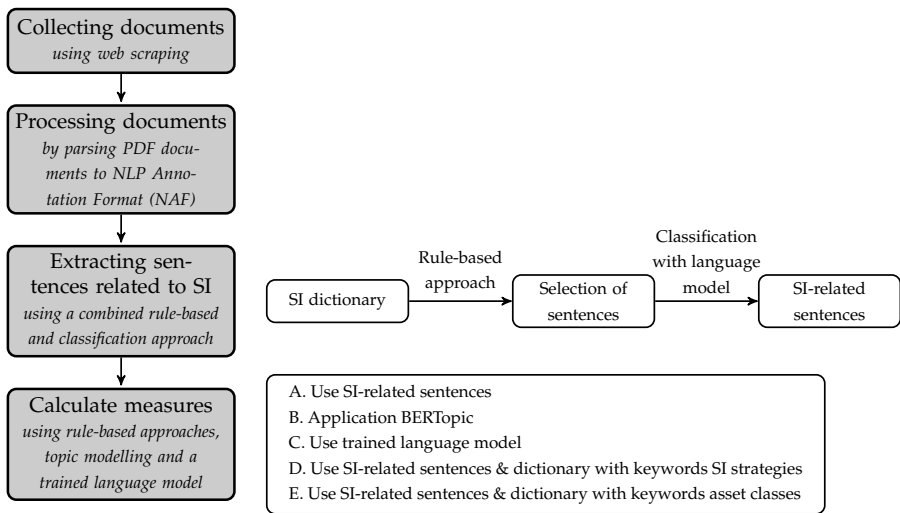
<sup>17</sup>Since the reports of Dutch pension funds are in Dutch, we cannot use a trained climate-related model such as the ClimateBert model.

<sup>18</sup>Negotiations are excluded from the keyword search.

are subjective because they weigh prior information heavily (Gentzkow et al. (2019)). Using a state-of-the-art NLP model such as BERT can overcome these limitations.

**Figure 4.2.1: Textual analysis pipeline**

This figure visualizes the textual analysis pipeline built to collect the documents, process the qualitative data, and calculate the SI measures (left side of figure). The different NLP techniques used to calculate the five SI measures are presented in the box at the bottom right of the figure.



In order to measure sustainable investing by pension funds using qualitative data, we first build a textual analysis pipeline which is visualized in Figure 4.2.1.<sup>19</sup> The objective of a textual analysis pipeline is to facilitate the collection and processing of text data. We start by collecting the annual reports of Dutch pension funds from 2016 to 2021 in an efficient way using web scraping. Subsequently, we process the documents by parsing them to NLP Annotation Format (NAF) files containing all relevant NLP information, such as sentences, headers, parts of speech, and lemmatized words.<sup>20</sup> We extract

<sup>19</sup>The source code of the textual analysis pipeline is published in a Github repository: <https://github.com/AnnickvOol/si-measures>.

<sup>20</sup>The NLP Annotation Format (NAF) is designed to represent linguistic annotations in com-

all SI-related sentences from the documents using a combined rule-based and classification approach. In the rule-based approach, we use an SI dictionary to extract all SI-related paragraphs. Subsequently, the classification approach consists of a language model that is finetuned with a labeled dataset. This model determines whether a sentence is related to sustainable investing or not. Table 4.2.1 presents some examples of labeled sentences and the upper part of Table 4.2.2 presents the performance of the trained language model: the accuracy of the model equals 92 percent in the test set. We use a combined approach because some keywords in the dictionary can have another interpretation unrelated to sustainable investing (see Table 4.2.1). Moreover, the rule-based approach functions as a preselection method, lowering the number of sentences that have to be labeled and classified. After generating a dataset with all sentences related to sustainable investing, we measure the awareness and implementation of sustainable investing with five different measures using various NLP techniques. Figure 4.2.1 presents an overview of the techniques used for each measure. In the next section, we discuss the construction of the measures in more detail.

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plex NLP architectures (Fokkens et al. (2014)). We use the Python package `navigator` to convert the PDF documents to NAF files (<https://github.com/DeNederlandscheBank/navigator>).

**Table 4.2.1: Labeled sentences with regard to sustainable investing**  
*This table presents some examples (translated from Dutch to English) of sentences related to sustainable investing (label 1) and sentences that are not related to sustainable investing (label 0).*

Sentence	Label
The fund also strives to contribute to investments needed to protect people against the impact of <i>climate change</i> .	1
This growth held up in April and May 2016, but was not stable despite a fairly favourable economic <i>climate</i> .	0
With impact investments the pension fund wants to contribute to solutions to worldwide problems, such as poverty and <i>inequality</i> .	1
In this way possible <i>inequality</i> within the board is prevented if the role is assigned to trustees.	0
Portfolio risks resulting from climate risks can be mitigated by implementing an effective and reliable ESG policy, especially regarding <i>transition risks</i> .	1
In 2017 the board paid extra attention to the <i>transition risk</i> of the participant and benefit payment administration.	0

**Table 4.2.2: Performance language models**

*This table shows the performance results of the language models. The upper part shows the results for the classification based on whether sentences are SI-related or not. This classification is used to create a dataset with all SI-related sentences. The lower part shows the results for the classification based on whether paragraphs are specific or not. This classification is used to create the specificity measure. The language models (RobBERT model) are finetuned with labeled datasets. Accuracy equals the overall number of correctly classified sentences (or paragraphs), divided by the total number of sentences in the test set. Precision equals the number of sentences that are correctly classified, divided by the total number of sentences classified as SI-related (or as specific) by the model. Recall equals the number of sentences that are correctly classified as SI-related by the model, divided by the total number of SI-related sentences in the test set. The F1 score is the harmonic mean of precision and recall.*

	Accuracy	Precision	Recall	F1 score
<b>SI-related</b>				
Training set	99.9%	99.9%	99.9%	99.9%
Test set	91.9%	91.9%	91.9%	91.8%
<b>Specificity (measure C)</b>				
Training set	99.7%	99.7%	99.7%	99.7%
Test set	87.1%	86.8%	87.1%	86.9%

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## 4.3 Variable construction and data

In this section we discuss the construction of the SI measures and describe the data. Section 4.3.1 describes the documents we use and Section 4.3.2 describes the construction of the SI measures. The data on the SI initiative are described in Section 4.3.3 and the proprietary dataset containing pension fund and board characteristics is described in Section 4.3.4.

### 4.3.1 Documents

In the analysis, we use qualitative data from annual reports and statements of investment principles of 160 Dutch pension funds from 2016 to 2021. We employ web scraping to collect these documents efficiently (see Figure 4.2.1). However, some annual reports are unavailable online, in which case we collect them via DNB archives. In total, we process more than 1,000 documents. We calculate the five SI measures using the annual reports and by exploiting various NLP techniques. We use the statements of investment principles to extract a pension fund's beliefs regarding the risk-return relation of sustainable investments using a rule-based approach.

We consider the period from 2016 to 2021 because this period allows us to investigate the impact of the IRBC initiative that was initiated and signed by most pension funds at the end of 2018. Moreover, we expect that sustainable investing became a greater priority for pension funds after the Paris Agreement in 2015. Boermans and Galema (2019) find that before 2016 most pension funds did not start measuring or externally disclosing the carbon emissions of their investments, whereas they increasingly started to do so as of 2016.

This paper focuses on annual reports for several reasons. First, all pension funds publish an annual report each year, so we have a balanced panel of pension funds. Second, pension funds are legally required to specify in their annual report how they incorporate ESG criteria in their investment policy and ESG risks in their risk management (see Section 4.1.2). However, since there are no requirements governing how and with how much detail this should be done, there is no guarantee that the relevant statements in the annual report are a complete representation of the SI policy.

Because we focus on disclosures of sustainable investing by pension funds in annual reports, there is concern about potential greenwashing or window-dressing. In the corporate finance literature, there is evidence that companies report mainly positive or general information about sustainable investing and that disclosures therefore suffer from greenwashing (Kim and Lyon (2015), Marquis et al. (2016), Fabrizio and Kim (2019), and Bingler et al. (2022)). Greenwashing or window-dressing incentives could potentially occur in pension funds' annual reports, although the institutional setting of pension funds differs from companies. Dutch pension funds are non-profit organizations but can have other incentives to focus on sustainable investing, e.g., because of beneficiaries' preferences for sustainability (Bauer et al. (2021)).

### 4.3.2 SI measures

As discussed in the previous section, the SI measures are calculated using annual reports. We construct five measures to measure a pension fund's SI policy along two dimensions. First, we measure the awareness of sustainable investing. We use three measures to quantify awareness: intensity, spectrum, and specificity. Second, we track the implementation of sustainable investing by constructing two additional measures: variety and scope. In this section, we describe the construction of these five measures one by one.

#### A. Intensity

Intensity quantifies the attention a pension fund pays to sustainable investing by calculating the proportion of the annual report devoted to sustainable investing. As discussed in Section 4.2, we create a dataset with all SI-related sentences using a combined rule-based and classification approach. Using this dataset, the intensity measure for each pension fund  $i$  in year  $t$  is calculated as follows

$$\text{Intensity}_{i,t} = \frac{\# \text{ SI-related sentences}_{i,t}}{\# \text{ sentences annual report}_{i,t}}. \quad (4.1)$$

---

## B. Spectrum

Spectrum determines how many SI topics are discussed in the annual report in a year. We construct a spectrum of SI topics by applying the BERTopic tool to the dataset of all SI-related sentences. Our dataset of SI-related sentences consists of more than 40,000 sentences. The BERTopic tool generates 27 relevant topics. Table 4.3.1 presents these topics. The topics consist of, amongst others, different SI initiatives (e.g., PRI, IRBC), SI strategies (e.g., exclusions, engagement), and excluded firms (e.g., coal mines, weapons manufacturers). Figure 4.3.1 shows for a selection of topics five related words in order of their c-TF-IDF score.<sup>21</sup> This score represents the importance of a word in the sentence. For example, a sentence on energy transition frequently contains the words energy, renewable, and solar. Figure 4.3.2 shows for a selection of topics how many pension funds discuss this topic over time. It shows that attention paid to the green bond topic has increased significantly over time: in 2016, only four pension funds discussed this topic, whereas forty pension funds discussed it in 2021. Moreover, the graph shows that pension funds discussed the IRBC initiative the most in 2018, which makes sense since the IRBC initiative started in 2018. The spectrum measure is equal to the number of topics pension fund  $i$  discusses in the annual report of year  $t$

$$\text{Spectrum}_{i,t} = \# \text{ SI topics}_{i,t}. \quad (4.2)$$

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<sup>21</sup>c-TF-IDF represents Class-based Term Frequency - Inverse Document Frequency, a procedure that can be used to generate features from textual documents based on their class.

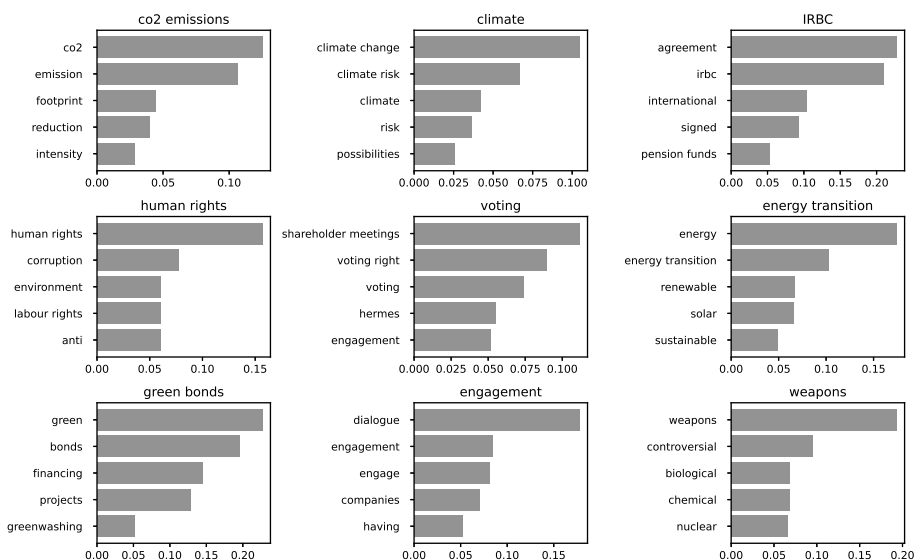
**Table 4.3.1: SI topics**

Overview of the SI topics generated with the BERTopic tool applied to the dataset with all SI-related sentences.

Climate	Exclusions	SFDR
Cluster munitions	Green bonds	Socially responsible investing
CO <sub>2</sub> emissions	GRESB	SRD
Coals	Human rights	Sustainable property
Energy transition	IRBC	Sustainability report
Engagement	OECD guidelines	UN principles
ESG integration	PRI	VBDO
ESG policy	Sanctions	Voting
ESG risk management	SDGs	Weapons

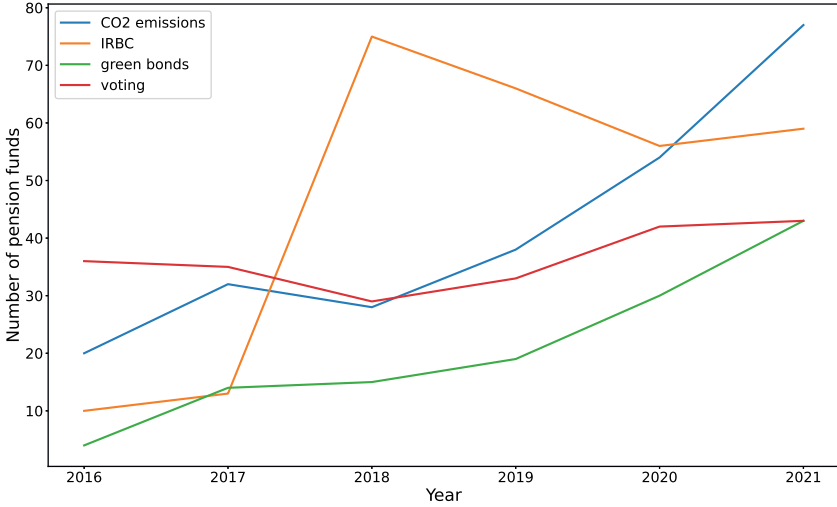
**Figure 4.3.1: Topic word scores**

This figure presents the word scores of the five most relevant words for a selection of topics in the BERTopic model.



**Figure 4.3.2: Development of topics over time**

This graph shows the number of pension funds which discuss a certain topic in a certain year based on the BERTopic output.



### C. Specificity

Specificity quantifies the number of statements regarding sustainable investing that contain details of actions specific to the pension fund, detailed performance information, or tangible and verifiable targets set by the pension fund. A pension fund’s statement is non-specific if it only contains generalized descriptions that can apply to each pension fund or general and non-verifiable goals without explaining how to achieve them. A pension fund’s statement is also non-specific if it contains a description of SI legislation without explaining how the pension fund is implementing it. Our approach is similar to Subramanian et al. (2019), who consider political speeches, and Bingler et al. (2022), who analyze climate-related disclosures of companies. We use a classification approach to determine which SI-related paragraphs are specific and which are not. Table 4.3.2 presents some examples of labeled paragraphs, and Table 4.2.2 shows the performance of the trained language model. The specificity measure for each pension fund  $i$  in year  $t$  equals

$$\text{Specificity}_{i,t} = \# \text{ specific SI-related paragraphs}_{i,t}. \quad (4.3)$$

**Table 4.3.2: Labeled sentences with regard to specificity**

*This table presents some examples (translated from Dutch to English) of specific paragraphs (label 1) and non-specific paragraphs (label 0).*

Paragraph	Label
We continued making the investment portfolio more sustainable, without sacrificing return and risks. The ultimate goal is to have €20 billion in investments that contribute to solving social issues. Moreover, we want to combat climate change by a 50% reduction of CO <sub>2</sub> -emissions in our investment portfolio compared to the baseline measurement in 2014. The recovery contributes to the possibility of achieving our ambition.	1
Besides voting at shareholder meetings, we believe it is important to enter into a dialogue with companies (engagement). In this way the pension fund as an investor makes sure its opinion is heard irrespective of the shareholder meetings. In this continuous dialogue there is a strong focus on ESG issues. In 2021, 1,620 engagements were carried out with 564 companies, of which 122 were closed successfully.	1
It has been decided to implement a best-in-class strategy for the separate allocation to European equity. We invest in companies in the top quartile in terms of ESG scores within the sector. This allocation was implemented at the beginning of 2019.	1
In 2020 we looked at the investment policy for the coming years, the aim of which is that the own portfolio will contribute to a more liveable world. The starting point is a most profitable portfolio which has more positive impact on the living environment and has more relevance for participants.	0
The economy and society face challenges which affect us as an investor. Climate change, the growing demand for renewable energy, and scarcity of natural resources are examples of topics which demand adaptation and innovation.	0
The SFDR contains two key elements for implementation, namely (1) transparency with regard to the inclusion of negative sustainability impact in investment decisions and (2) publication of pre-contractual sustainability information. Each pension provider must also have a description in its pre-contractual information of (1) the way in which sustainability risks are part of the investment decision-making process, or the investment advice or insurance advice, (2) the probable effect of the sustainability impact on the return of the pension fund.	0

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#### D. Variety

The variety measure quantifies the SI implementation by counting the number of SI strategies implemented by each pension fund. As discussed in Section 4.1.5, we distinguish the following five SI strategies: divestment, ESG integration, screening, public engagement, and private engagement. We apply a rule-based approach to the dataset with SI-related sentences using a dictionary with keywords and combinations of keywords for each strategy. In this way, we determine which strategies are implemented by pension fund  $i$  in year  $t$ . The variety measure equals

$$\text{Variety}_{i,t} = \# \text{ implemented SI strategies}_{i,t}. \quad (4.4)$$

#### E. Scope

Finally, scope quantifies the fraction of the asset portfolio that is covered by the pension fund's SI policy. Pension funds invest in various asset classes, but may apply the SI policy only in specific asset classes. We consider the asset classes specified in the OECD guidance for institutional investors (OECD (2017)): equity, corporate bonds, government bonds, real estate, infrastructure, and private equity. We add mortgages as an additional asset class because Dutch pension funds invest a significant fraction of their assets in mortgages.<sup>22</sup> We apply a rule-based approach to the dataset with SI-related sentences using a dictionary with keywords and combinations of keywords for each asset category. In this way, we determine which asset classes are covered by the SI policy of pension fund  $i$  in year  $t$ . We combine this information with asset allocation data of pension funds (see Section 4.3.4). This yields the scope measure

$$\text{Scope}_{i,t} = \frac{\sum_k c_{i,k,t} W_{i,k,t}}{\sum_k W_{i,k,t}}, \quad (4.5)$$

where  $c_{i,k,t}$  takes value one if asset category  $k$  is covered by the SI policy of pension fund  $i$  in year  $t$  and zero otherwise.  $W_{i,k,t}$  is the amount pension fund  $i$  invests in asset category  $k$  in year  $t$ .

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<sup>22</sup>Dutch pension funds invest, on average, 5% of their assets in mortgages.

We exploit a novel textual analysis approach to construct these five SI measures that quantify the awareness and implementation of sustainable investing. By measuring the sustainable investment policy along two dimensions and considering five SI measures, we robustly measure sustainable investing by pension funds, and we reduce the risk of subjectivity. We track the implementation of sustainable investing by constructing two measures that quantify the implementation of sustainable investing at a meta level: the variety measure counts the number of SI strategies implemented and the scope measure is the fraction of the portfolio included in the SI policy. However, these two measures do not quantify the actual quality of the implementation of sustainable investing. While some measures are concrete and objective (e.g., the intensity measure), others are more abstract and somewhat subjective (e.g., the specificity measure). Sometimes, the term ‘measure’ refers to concrete or objective attributes, and the term ‘metric’ refers to abstract or somewhat subjective attributes. For consistency purposes, we use the term ‘measure’ only.

### 4.3.3 Data on the SI initiative

The SI measures, described in the previous section, are used to investigate the impact of signing an SI initiative on the awareness and implementation of sustainable investing. This paper focuses on the IRBC initiative because it is the best-known SI initiative in the Dutch pension sector.<sup>23</sup> The initiative started with a declaration of intent signed by 40 pension funds in March 2017. Subsequently, 73 pension funds signed the initiative at the end of 2018, and several others signed later. The number of pension funds in our sample that signed the initiative in 2018 is 60 instead of 73, because some pension funds left the initiative and some ceased to exist.<sup>24</sup> In our sample, six pension funds signed the initiative in 2019, two in 2020 and two in 2021.<sup>25</sup>

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<sup>23</sup>The IRBC is the ‘Convenant Internationaal Maatschappelijk Verantwoord Beleggen Pensioenfondsen’ (IMVB) in Dutch.

<sup>24</sup>If a pension fund ceases to exist, it transfers its benefits to a different pension fund or insurer.

<sup>25</sup>The pension funds that signed the IRBC initiative owned 92 percent of total pension assets at the end of 2021.

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#### **4.3.4 Pension fund characteristics**

In addition to the public annual reports and SI initiative data, the analysis is based on a proprietary dataset from the prudential supervisor of pension funds, De Nederlandsche Bank (DNB), containing information on occupational pension funds in the Netherlands. All pension funds are obliged to report this information to DNB. This dataset has been used before, by e.g., Bikker et al. (2012), De Haan (2018), Boermans and Galema (2019), and Broeders et al. (2021b). We use a balanced panel of 160 occupational pension funds that reflects almost the entire population of Defined Benefit (DB) pension funds in the Netherlands from 2016 to 2021.<sup>26,27</sup> Pension funds must report general statistics, such as funding ratio, assets under management, liability duration, and the type of pension fund (corporate, industry-wide, or professional group pension fund). Moreover, pension funds report information on their stakeholders. They report information on the board of trustees, including the gender, age, and tenure of each trustee. Finally, pension funds report information on their actual asset allocation, i.e., how much a pension fund invests in each asset category. This information is used to calculate the scope measure.

### **4.4 Empirical design and results**

In this section, we present the empirical design and results. Section 4.4.1 starts with a description of the data. Subsequently, we present the hypotheses in Section 4.4.2 and the models used to test the hypotheses in Section 4.4.3. Finally, the results are presented in Section 4.4.4.

#### **4.4.1 Pension fund sample overview**

Table 4.4.1 shows the statistics of pension fund and board of trustees characteristics for the balanced panel of 160 pension funds. This table shows that the average funding ratio equals 112 percent and the average liability

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<sup>26</sup>The pension funds in our panel owned 98 percent of total pension assets at the end of 2021.

<sup>27</sup>We exclude pension funds that did not exist throughout the whole sample period and general pension funds (pension funds that can execute several pension schemes) from the panel.

duration is 20.3 years.<sup>28</sup> The average size of total assets under management is about 9 billion euros. The sample contains a small number of very large pension funds, hence the skewness in the distribution. Some 66 percent of the pension funds in our panel are corporate pension funds, 28 percent are industry-wide pension funds, and 6 percent are professional group pension funds. Further, 64 percent of the pension funds invest (part of their assets) actively. Only 18 percent of the pension funds have a positive belief regarding the risk-return relation of sustainable investments, i.e., sustainable investing pays off after correcting for risk. The other pension funds either have a more neutral position or do not report their beliefs about the risk-return relation of sustainable investing in their statement of investment principles. The board of trustees' size varies between three and 16 trustees, and the average age of an individual trustee is almost 56. The average fraction of female trustees is 21%, but there are also pension fund boards with no female trustees. There is a wide dispersion in the average tenure of trustees, which varies between one and 20 years. Table 4.4.2 shows some statistics on annual reports. We analyzed 938 annual reports from 160 pension funds from 2016 to 2021.<sup>29</sup> The average report consists of about 2,000 sentences and 840 paragraphs, but there is substantial variation between pension funds.<sup>30</sup> There is also a wide dispersion in the amount of SI-related sentences. In some annual reports, sustainable investing is not discussed at all, whereas one report contains more than 750 sentences related to sustainable investing.

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<sup>28</sup>All pension funds in the sample are Defined Benefit (DB) pension funds.

<sup>29</sup>Unfortunately, 22 annual reports could not be collected via either the pension fund website or DNB archives. As a result, the dataset is not completely balanced.

<sup>30</sup>Note that this ratio of sentences and paragraphs may seem odd. Since, e.g., titles, subheaders, and footnotes count as separate paragraphs, the average number of sentences per paragraph is small.

**Table 4.4.1: Statistics on pension fund and board of trustees characteristics**  
*Panel A presents information on pension funds' characteristics and Panel B on the boards of trustees for the 160 pension funds in our sample. The mean and standard deviation are measured across pension funds and over time for each variable.*

	Obs	Mean	Std dev	Min	25th	75th	Max
<b>A. Pension fund characteristics</b>							
Funding ratio (%)	960	111.7%	13.3%	83.2%	103.3%	117.3%	212.0%
Liability duration	960	20.3	3.9	0.0	17.8	22.5	32.3
Total assets (billion)	960	9.2	41.1	0.0	0.5	4.1	554.4
Log total assets	960	21.1	1.8	12.3	20.0	22.1	27.0
% professional group pension funds	960	5.6%	23.1%	0.0%	0.0%	0.0%	100.0%
% corporate pension funds	960	66.3%	47.3%	0.0%	0.0%	100.0%	100.0%
% industry-wide pension funds	960	28.1%	45.0%	0.0%	0.0%	100.0%	100.0%
% active investing	960	63.6%	48.1%	0.0%	0.0%	100.0%	100.0%
% positive belief risk -return relation SI	960	18.1%	38.5%	0.0%	0.0%	0.0%	100.0%
<b>B. Board of trustees characteristics</b>							
Number of trustees	960	7.6	2.4	3.0	6.0	9.0	16.0
Average age trustees	960	55.9	4.0	43.8	53.5	58.4	67.5
Average tenure trustees	960	5.8	2.9	1.0	3.9	7.2	20.0
% female trustees	960	0.21	0.16	0.00	0.11	0.33	0.83

**Table 4.4.2: Statistics on annual reports**  
*This table presents information on pension funds' annual reports. The mean and standard deviation are measured across pension funds and over time for each variable.*

	Obs	Mean	Std dev	Min	25th	75th	Max
#sentences	938	2014.9	613.4	2.0	1591.5	2376.0	5125.0
#paragraphs	938	842.3	350.9	1.0	627	968.3	5444.0
#SI-related_sentences	938	45.5	52.6	0.0	18.0	57.0	763.0
#SI-related_paragraphs	938	18.0	20.8	0.0	7.0	22.0	267.0

For a first inspection of the SI measures data, we plot the distribution of the SI measures in Figure 4.4.1. In this figure, each plot visualizes the distribution of a particular measure in a specific year. Two stylized facts stand out. First, the figure shows that for each measure the median, visualized by the

dotted vertical black line, increases over time. Second, the value of the scope measure equals zero for a significant number of pension funds. Although this number decreases over time, 40 pension funds still do not report which asset classes are covered by their SI policy in 2021.

**Figure 4.4.1: Distribution of SI measures**

*This figure presents the distribution of the different SI measures for all years. Each plot visualizes the distribution of a particular measure in a specific year. The dotted black vertical line in each plot represents the median.*

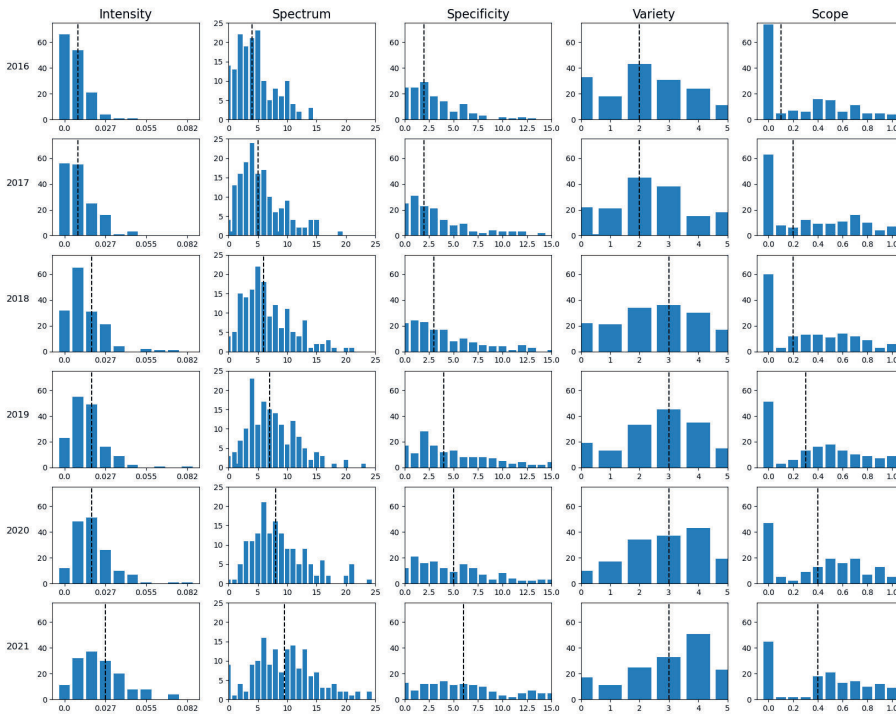


Table 4.4.3 contains statistics on the SI measures (see Figure 4.2.1). The table shows that the average value for all measures increases over time. The intensity and specificity measures show the most significant increase: the average score for these measures increased by 150 percent between 2016 and 2021. Moreover, the standard deviation of these two measures has increased,

i.e., there is more dispersion between pension funds in recent years. As in the case of the intensity measure, Brié et al. (2022) calculate the importance of climate-related information relative to other information in the annual reports of publicly listed European firms. In their sample, the relative importance almost doubled from less than 5.5 percent in 2010 to nearly 10 percent in 2020. In our sample, the average intensity measure increased from 1.2 percent in 2016 to 3.0 percent in 2021.

**Table 4.4.3: Statistics on SI measures**

*This table presents information on the five SI measures over time. The first three measures quantify awareness of sustainable investing and the last two measures track the implementation of sustainable investing. The mean and standard deviation are measured across pension funds for each variable.*

<i>Awareness of sustainable investing</i>							
	Obs	Mean	Std dev	Min	25th	75th	Max
<b>A. Intensity</b>							
2016	151	1.2%	0.8%	0.0%	0.6%	1.7%	4.6%
2017	155	1.5%	1.2%	0.0%	0.6%	2.0%	9.5%
2018	160	1.8%	1.3%	0.0%	1.0%	2.3%	7.6%
2019	159	2.1%	1.7%	0.0%	1.1%	2.5%	16.3%
2020	160	2.6%	2.2%	0.2%	1.5%	3.0%	21.2%
2021	153	3.0%	1.8%	0.0%	1.7%	3.7%	11.4%
<b>B. Spectrum</b>							
2016	151	5.2	3.0	0.0	3.0	7.0	15.0
2017	155	5.9	3.5	0.0	3.0	8.0	21.0
2018	160	7.5	4.0	0.0	5.0	10.0	22.0
2019	159	8.3	3.7	0.0	6.0	10.0	18.0
2020	160	9.6	4.0	0.0	7.0	11.0	22.0
2021	153	11.0	4.4	0.0	8.0	14.0	22.0
<i>Implementation of sustainable investing</i>							
<i>Continued on next page</i>							

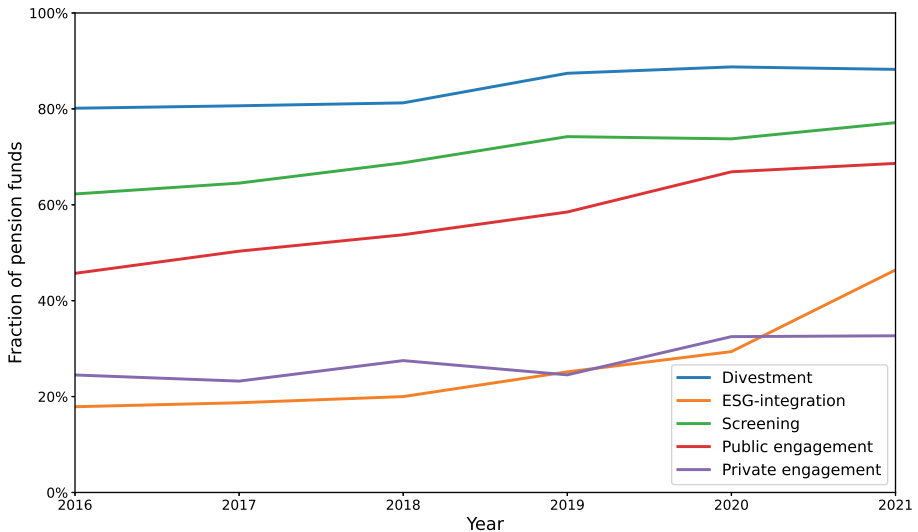
	Obs	Mean	Std dev	Min	25th	75th	Max
<b>C. Specificity</b>							
2016	151	3.2	3.2	0.0	1.0	4.0	18.0
2017	155	4.1	5.9	0.0	1.0	5.0	56.0
2018	160	4.6	5.4	0.0	1.0	6.0	40.0
2019	159	5.9	7.8	0.0	2.0	8.0	76.0
2020	160	7.1	10.7	0.0	2.0	8.0	107.0
2021	153	8.3	9.2	0.0	3.0	12.0	74.0
<b>D. Variety</b>							
2016	151	2.3	1.5	0.0	1.0	3.0	5.0
2017	155	2.4	1.5	0.0	1.0	3.0	5.0
2018	160	2.5	1.5	0.0	1.0	4.0	5.0
2019	159	2.7	1.4	0.0	2.0	4.0	5.0
2020	160	2.9	1.4	0.0	2.0	4.0	5.0
2021	153	3.1	1.4	0.0	2.0	4.0	5.0
<b>E. Scope</b>							
2016	151	25.6%	29.3%	0.0%	0.0%	45.7%	98.7%
2017	155	30.2%	31.5%	0.0%	0.0%	60.0%	97.5%
2018	160	29.0%	29.7%	0.0%	0.0%	55.2%	97.6%
2019	159	34.9%	31.6%	0.0%	0.0%	59.4%	100.0%
2020	160	37.9%	31.6%	0.0%	0.0%	61.2%	100.0%
2021	153	41.1%	32.5%	0.0%	0.0%	65.8%	100.0%

To better understand the implementation of sustainable investing, Figure 4.4.2 and 4.4.3 provide more information on the data underlying the variety and the scope measure. Figure 4.4.2 shows the fraction of pension funds that implemented an SI strategy over time. Divestment is the most popular strategy. This can be explained by the legal requirement introduced in 2013 that pension funds are not allowed to invest in cluster munitions. As a result, most pension funds are forced to implement a divestment strategy for these specific investments. ESG integration shows the biggest relative increase over time, while screening, public engagement, and private engagement have also grown steadily over time.

Similarly, Figure 4.4.3 shows the fraction of pension funds that covered a specific asset class with their SI policy. All asset classes show a significant increase over time. The most popular asset category covered by the SI policy is equity. This observation can be explained by the fact that the SI policy can cover this asset category in various ways. A pension fund can implement exclusion, screening, and ESG integration based on ESG ratings of listed equity. While the coverage of ESG ratings for listed equity is high, ESG ratings are often not available for alternative asset classes. Moreover, the SI policy can cover equity by influencing the decisions of companies in the equity portfolio (engagement). As the green bond market has increased five times in size between 2016 and 2021, it has become easier for pension funds to include fixed income (government bonds and corporate bonds) in their SI strategy.<sup>31</sup>

**Figure 4.4.2: Variety measure over time**

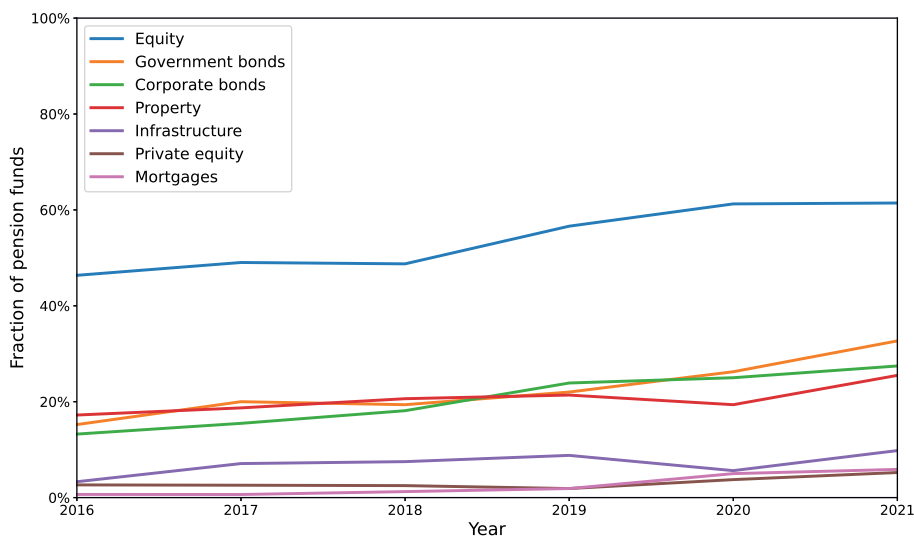
*The percentages in this figure represent the fraction of pension funds that implemented a certain SI strategy over time.*



<sup>31</sup>Source: Bloomberg Finance.

**Figure 4.4.3: Scope measure over time**

The percentages in this figure represent the fraction of pension funds that covered a certain asset category with their SI policy over time.



## 4.4.2 Hypotheses

In this section, we summarize three hypotheses to explain the impact of pension fund characteristics on the awareness and implementation of sustainable investing and the impact of signing an SI initiative.

First, we hypothesize that a pension fund's characteristics impact its SI policy. In particular, we expect that large pension funds will have higher scores for all five SI measures. This hypothesis is in line with the general notion that larger pension funds are more concerned about corporate responsibility (Scholtens (2006)), and are more capable of screening stocks on environmental criteria due to the monitoring cost involved with active management (Kempf and Osthoff (2008), Sievänen et al. (2013), and Egli et al. (2022)). We also expect that pension funds with relatively young beneficiaries, reflected in a higher liability duration, will have higher scores on the SI measures. This hypothesis is in line with empirical findings of Bauer and Smeets (2015) and

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Bauer et al. (2021), who find that young people have stronger preferences for sustainable investing, and Riedl and Smeets (2017), who find that young people are more likely to hold socially responsible mutual funds. Moreover, we hypothesize that board characteristics do not impact the SI measures. Because the board of trustees makes decisions on behalf of the beneficiaries, we do not expect board characteristics to impact the SI policy. In contrast to this hypothesis, Bauer et al. (2020a) find that the average board age and representation of stakeholders impact the asset allocation of corporate pension funds. Finally, we hypothesize that beliefs regarding the return on sustainable investments impact the SI measures. Although no systemic relation can be found between ESG performance and worldwide stock returns during the past two decades (Alves et al. (2022)), pension funds' boards of trustees increasingly express their beliefs regarding the risk-return relation of sustainable investing in the statement of investment principles. We expect that a positive belief about the risk-return relation, i.e., sustainable investing pays off after correcting for risk, has a positive impact on the SI measures.

Second, we hypothesize that pension fund characteristics also have an impact on the probability of signing an SI initiative in line with the first hypothesis. In particular, we expect that large pension funds are more likely to sign the IRBC initiative because they have more capacity to enhance their SI policy. In the same sense, we expect that the liability duration decreases and a positive belief about the risk-return relation of sustainable investments increases the probability of signing the IRBC initiative.

Third, we are interested in the impact of signing the IRBC initiative on the development of the SI policy over time. We hypothesize that signatories of the IRBC initiative enhance their SI policy more than non-signatories. We expect that this holds for both the awareness and implementation of sustainable investing. The goal of the IRBC initiative is to bring the investment policy into line with the OECD Guidelines and UNGPs, and the signatories of the initiative made joint arrangements on how to realize this. We expect that this commitment will increase the awareness and implementation of sustainable investing. This hypothesis is in line with Bauckloh et al. (2021) and Gibson et al. (2022). They find that institutional investors who signed the PRI initiative have better ESG scores than matched non-signatories.

### 4.4.3 Empirical model

In this section, we present empirical models for the three hypotheses of the previous section. To test the first hypothesis, we use the following pooled OLS model

$$ESG_{i,t} = \alpha + \beta' \cdot \mathbf{X}_{i,t} + \theta_t + \epsilon_{i,t}, \quad (4.6)$$

where  $ESG_{i,t}$  is one of the five SI measures of pension fund  $i$  in year  $t$  and  $\mathbf{X}_{i,t}$  contains several explanatory variables.  $\theta_t$  is a set of year dummies to control for year-specific conditions and  $\epsilon_{i,t}$  is the error term.  $\mathbf{X}_{i,t}$  contains both pension fund characteristics and board of trustees characteristics that might impact the SI measures. The pension fund characteristics include the size of the pension fund, represented by the natural logarithm of the total assets under management, the funding ratio, and the liability duration, which is the average time to maturity of the pension liabilities. Further, two dummies for professional group pension funds and corporate pension funds represent the type of pension fund. Industry-wide pension funds are the omitted category. Finally, we include a dummy for active investing. This dummy variable equals one if the pension fund invests (part of its assets) actively and zero otherwise. The board characteristics include the average age of the board of trustees, the fraction of female trustees, and the average tenure of the board of trustees. Moreover, we include a dummy variable that represents the belief regarding the risk-return relation of sustainable investing. This dummy variable equals one if the pension fund expects a positive impact of sustainable investing on the risk-return relation and zero otherwise.

For the second hypothesis, we use a probit model to analyze the effect of pension fund and board characteristics on the probability of signing an SI initiative

$$P[SIGN_i = 1] = \Phi[\alpha + \beta \cdot \mathbf{X}_i + \epsilon_i], \quad (4.7)$$

where  $SIGN_i$  takes value zero for non-signatories and value one for signatories and  $\mathbf{X}_i$  contains several explanatory variables (pension fund and board of trustees characteristics) that explain whether or not a pension fund signs the IRBC initiative.  $\epsilon_i$  is the error term. We do not use the whole panel dataset for this hypothesis because we use a cross-sectional probit model. For pension

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funds that signed the IRBC initiative in 2018 or did not sign the IRBC initiative we use the explanatory variables of year 2018. For pension funds that signed the IRBC initiative in a later year, we use the explanatory variables of the year of signing.

To test the third hypothesis concerning the impact of signing the IRBC initiative on the development of SI measures over time, we use a difference-in-differences (diff-in-diff) specification to estimate the differential effect of signing the IRBC (treatment) on the SI policy measures. We apply the diff-in-diff specification with staggered treatments on the panel of IRBC signatories and non-signatories to evaluate the between-group differences of the change in SI measures over time.<sup>32</sup> The diff-in-diff model is specified as follows:

$$ESG_{i,t} = \alpha + \gamma \cdot SIGN_i + \delta \cdot IRBC_{i,t} + \beta' \cdot \mathbf{X}_{i,t} + \theta_t + \epsilon_{i,t}, \quad (4.8)$$

where  $ESG_{i,t}$  is one of the five SI measures of pension fund  $i$  in year  $t$  and  $SIGN_i$  takes value zero for non-signatories and value one for signatories.  $IRBC_{i,t}$  takes value zero for non-signatories and signatories before signing and value one for signatories after signing, and  $\mathbf{X}_{i,t}$  contains several explanatory variables. Finally,  $\theta_t$  is a set of year dummies to control for year-specific conditions and  $\epsilon_{i,t}$  is the error term of pension fund  $i$  in year  $t$ . We are interested in the coefficient  $\delta$ , which measures the effect of signing the IRBC initiative on the SI measure. A positive coefficient  $\delta$  indicates that, on average, the difference between the SI measure of IRBC signatories and non-signatories has increased after the signatory year.

To rule out spurious correlation we control for the following endogeneity concerns: selection bias, omitted variable bias, reverse causality and measurement error. Below we discuss these endogeneity concerns one by one.

First, selection bias arises in our sample because signing the IRBC is voluntary. Pension funds that have already been enhancing their SI policy in the past or are planning to do so are more likely to sign the IRBC initiative. Therefore, pension funds that signed the IRBC may not be representative and could differ systematically in their main characteristics compared to pension

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<sup>32</sup>Because pension funds can sign the IRBC initiative at different moments in time, the treatment is staggered over time.

funds that did not sign the IRBC. A simple comparison of IRBC signatories and non-signatories is thus not feasible. Since we cannot analyze pension funds in two conditions (signatory and non-signatory) simultaneously, we use a matching methodology. Matching aims to equate the distribution of covariates in the treated (signatories) and control (non-signatories) groups (Stuart (2010)). While several matching methods exist, one of the most common methods is  $r$ :1 nearest neighbor matching (Rubin (1973)). Nearest neighbor matching matches control units to the treated group. For each treated unit  $i$  nearest neighbor matching selects the  $r$  control units with the smallest distance from  $i$ . We conduct a 3:1 nearest neighbor matching with probit regression-based propensity scores using the pension fund and board characteristics as matching variables.<sup>33</sup> The method matches pension funds in the control group (non-signatories) to the treated group (signatories) with the smallest distance, discarding non-matched pension funds. We use nearest neighbor matching with replacement allowing the same control fund to be matched multiple times. The propensity score is used as the similarity measure between pension funds and is defined as the probability of signing the IRBC initiative given the observed pension fund and board characteristics. Subsequently, we weight the regression in Equation (4.8) with these propensity scores.

Second, we address omitted variable bias in two ways. We include explanatory variables ( $\mathbf{X}_{i,t}$  in Equation (4.8)) in the diff-in-diff regression to control for the effect these variables have on the SI measures. Potentially, some variables not included in the model impact the pension fund's SI policy and correlate with the explanatory variables in  $\mathbf{X}_{i,t}$ . As a result, the estimates of the model are potentially biased. Therefore, we include fixed effects as additional explanatory variables in the model to control for omitted variable bias based on the assumption that the omitted variables are constant over time. We add pension fund fixed effects  $\kappa_i$  to the model in Equation (4.8) and analyze this model as a robustness check

$$ESG_{i,t} = \alpha + \delta \cdot IRBC_{i,t} + \beta' \cdot \mathbf{X}_{i,t} + \theta_t + \kappa_i + \epsilon_{i,t}. \quad (4.9)$$

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<sup>33</sup>It is also possible to conduct a 1:1 matching or 2:1 matching, but 3:1 matching yields a better balance between both groups without further reducing the size of the sample.

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In this model,  $X_{i,t}$  contains fewer explanatory variables compared to Equation (4.8) because the time-invariant variables are excluded from  $X_{i,t}$ . The pension fund fixed effects capture the effect of these variables.

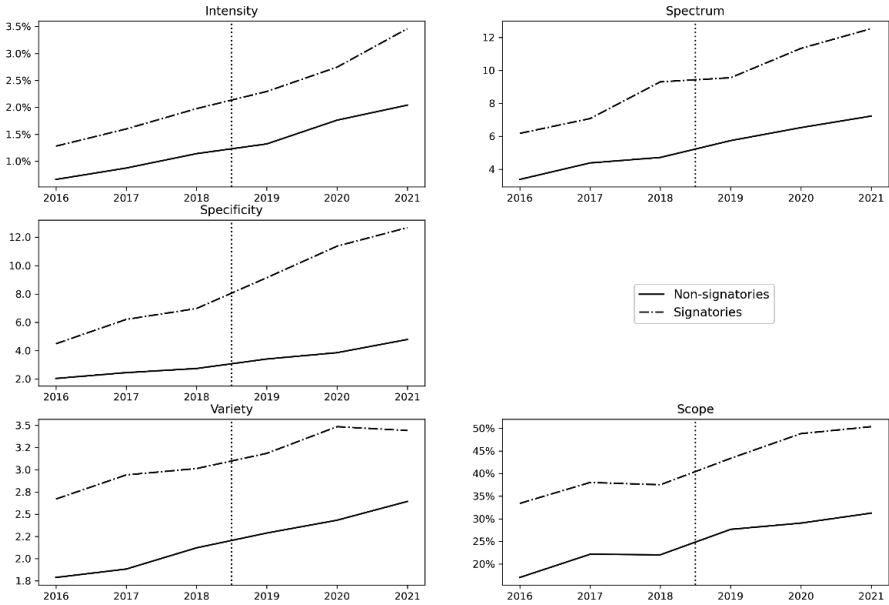
Third, we use the diff-in-diff specification to overcome reverse causality concerns. An essential requirement for a diff-in-diff specification concerns the parallel trend assumption. This assumption requires that the difference between the treatment group (signatories) and control group (non-signatories) is constant before the treatment. Although there is no statistical test for this assumption, visual inspection is useful. Figure 4.4.4 presents for each SI measure the mean value for both the treated group (signatories) and control group (non-signatories). This figure shows that signatories had higher values for all SI measures compared to non-signatories. Generally, the trends of signatories and non-signatories before the IRBC initiative are similar for each SI measure in line with the parallel trend assumption. Note that figure shows the mean values for all signatories and non-signatories. Because we apply matching to reduce selection bias, not all signatories and non-signatories are included in the diff-in-diff model.

Fourth, we reduce measurement error concerns by measuring awareness and implementation of sustainable investing with different SI measures. Moreover, we will construct indices that combine multiple individual SI measures and analyze the impact of signing on these indices as an extra robustness check.

#### **4.4.4 Results**

In this section, we present the key results of the empirical models discussed in the previous section. For our first hypothesis, we run the pooled OLS model of Equation (4.6) for each SI measure. Table 4.4.4 presents the regression results. In line with our hypothesis, we find a statistically significant positive effect for the pension fund's size on the SI measures. This effect is highly significant for each SI measure (i.e., for both awareness and implementation of sustainable investing). However, in contrast to our hypothesis, we do not find an effect for the liability duration. So pension funds with young participants do not have a stronger focus on sustainable investing. In

**Figure 4.4.4: SI measures of signatories and non-signatories over time**  
 This figure presents for each SI measure the mean value for both the treated group (signatories) and control group (non-signatories).



line with our hypothesis, the board of trustees characteristics do not impact the SI policy. The only exception is a statistically significant negative effect of the average tenure of the board of trustees on the spectrum measure and variety measure. However, the size of this effect is rather limited. In line with our hypothesis regarding beliefs about the risk-return relation of sustainable investing, we observe that a positive belief regarding the risk-return relation of sustainable investing increases awareness of sustainable investing. For example, the positive coefficient 0.008 for the intensity measure indicates that pension funds with the belief that sustainable investing pays off devote, on average, 0.8 percent extra of the annual report to sustainable investing. This effect seems small at first glance, but with average attention to sustainable investing of 2 percent of the annual report, this effect is quite substantial. A pos-

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itive belief regarding the risk-return relation of sustainable investing does not have a significant effect on the implementation of sustainable investing. This result could indicate that pension funds with a positive belief about sustainable investing want to enhance their SI policy and also talk about it (reflected by the higher awareness), but are still trying to find out how to integrate sustainable investing in their investment strategy.

For our second hypothesis, we run the probit model of Equation (4.7). Table 4.4.5 presents the results. In line with our hypothesis, large pension funds are more likely to sign the IRBC initiative. This effect is highly significant. A positive belief about the risk-return relation also increases the probability of signing the IRBC initiative. Finally, the fraction of female trustees increases the probability of signing the IRBC initiative. This effect is in line with Harjoto et al. (2015) and Velte (2016), who find that female members on the management board positively impact ESG performance. However, the coefficient of this variable is only significant at the 10 percent level.

For the third hypothesis we conduct a 3:1 nearest neighbor matching with probit regression-based propensity scores using the pension fund and board characteristics. The results are presented in Table 4.4.6. The table shows that for all matching variables the difference in mean value between the treated group (signatories) and control group (non-signatories) is much smaller after matching compared to the original sample. For example, the mean funding ratio of non-signatories is higher than signatories, but after matching the mean funding ratio is approximately equal. A good balance requires statistically insignificant differences between the matched signatories (treated group) and matched non-signatories (control group). For all matching variables the difference between the signatories and matched non-signatories is statistically insignificant.

We run the diff-in-diff model in Equation (4.8) weighted with the propensity scores to investigate the impact of signing the IRBC initiative on the development of the SI measures over time.<sup>34</sup> The results presented in Table 4.4.7 provide evidence that signing the IRBC initiative increases the specificity of

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<sup>34</sup>When running the diff-in-diff model without weighting with propensity scores, the values of the estimates of the IRBC dummy are significantly higher due to the self-selection bias for all SI measures.

**Table 4.4.4: The effect of pension fund characteristics on SI measures**

*This table presents the results of the pooled OLS model in Equation (4.6) for all five SI measures as dependent variable. Pension fund characteristics and board of trustees characteristics are used as explanatory variables. The model includes year fixed effects and the standard errors are clustered at the pension fund level to correct for serial correlation.  $t$  statistics in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .*

	Awareness of sustainable investing			Implementation of sustainable investing	
	Intensity (A)	Spectrum (B)	Specificity (C)	Variety (D)	Scope (E)
Log(assets)	0.004*** (0.001)	1.352*** (0.158)	2.119*** (0.394)	0.309*** (0.055)	0.0506*** (0.011)
Funding ratio	0.000 (0.008)	-1.778 (1.747)	-4.432 (3.821)	-0.287 (0.617)	-0.313 (0.194)
Liability duration	0.000 (0.000)	0.014 (0.063)	-0.020 (0.086)	0.017 (0.022)	0.004 (0.006)
Professional group pension funds	0.001 (0.003)	1.196 (1.187)	-0.231 (1.613)	0.515 (0.383)	-0.032 (0.083)
Corporate pension funds	0.002 (0.002)	0.561 (0.590)	0.466 (0.940)	-0.002 (0.215)	0.017 (0.047)
Positive belief risk-return relation SI	0.008** (0.003)	1.275** (0.633)	3.201* (1.784)	0.151 (0.241)	0.069 (0.042)
Active investing	0.001 (0.002)	0.769* (0.395)	0.957 (0.623)	0.216 (0.159)	-0.007 (0.036)
Fraction female trustees	-0.002 (0.004)	0.278 (1.225)	0.671 (1.772)	0.495 (0.603)	0.096 (0.118)
Average age trustees	0.000 (0.000)	0.044 (0.048)	-0.006 (0.069)	-0.007 (0.023)	-0.003 (0.005)
Average tenure trustees	-0.000 (0.000)	-0.135** (0.063)	0.008 (0.092)	-0.057** (0.028)	0.001 (0.006)
Constant	-0.086*** (0.024)	-25.000*** (5.430)	-37.730*** (9.426)	-3.883* (2.124)	-0.421 (0.514)
$N$	938	938	938	938	938
$R^2$	0.337	0.440	0.301	0.227	0.127

**Table 4.4.5: The effect of pension fund characteristics on the probability of signing the IRBC initiative**

*This table presents the results of the probit model in Equation (4.7) with robust standard errors. The probability of signing the IRBC initiative is the dependent variable and pension fund characteristics and board of trustees characteristics are used as explanatory variables. *t* statistics in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .*

	$P[SIGN = 1]$
Active investing	-0.040 (0.302)
Average age trustees	-0.001 (0.034)
Average tenure trustees	-0.033 (0.051)
Corporate pension funds	-0.340 (0.370)
Fraction female trustees	1.649* (0.905)
Funding ratio	-1.389 (1.675)
Liability duration	-0.004 (0.040)
Log(assets)	0.673*** (0.136)
Positive belief risk-return SI	0.685** (0.348)
Professional group pension funds	-0.254 (0.540)
Constant	-12.629*** (3.717)
<i>N</i>	160

**Table 4.4.6: Evaluation of nearest neighbor matching**

*This table presents the mean values of pension fund and board characteristics for signatories and non-signatories before and after matching using a 3:1 nearest neighbor matching procedure.*

	Signatories	Non-signatories	Matched signatories	Matched non-signatories
Active investing	0.72	0.57	0.71	0.75
Average age trustees	56.06	55.80	55.76	55.12
Average tenure trustees	5.03	6.47	5.43	5.3
Corporate pension funds	0.44	0.83	0.54	0.62
Fraction female trustees	0.25	0.17	0.20	0.22
Funding ratio	1.10	1.13	1.09	1.09
Liability duration	20.52	20.06	20.61	20.58
Log(assets)	22.27	20.18	21.17	21.20
Positive belief risk-return SI	0.29	0.10	0.18	0.16
Professional group pension funds	0.07	0.04	0.10	0.10
N	420	540	220	508

pension fund statements about sustainable investing because the estimate of the IRBC dummy is positive and highly significant for the specificity measure. The positive coefficient 2.803 indicates that IRBC signatories show an average differential increase of almost three specific SI-related paragraphs compared to non-signatories after signing. Since the average value of the specificity measure equals 5.53, this is an increase of more than 50 percent. Surprisingly, we do not find an effect of signing on the other SI measures. As a robustness check, we run the diff-in-diff model in Equation (4.9), which includes pension fund fixed effects. Table 4.4.8 presents these results. Although the values of the estimates are lower, the conclusions stay the same.

**Table 4.4.7: The effect of signing the IRBC initiative on SI measures**

This table shows the results of the pooled OLS model in Equation (4.8) weighted with propensity scores for all five SI measures as dependent variable.  $SIGN_i$  takes value zero for non-signatories and value one for signatories.  $IRBC_{i,t}$  takes value zero for non-signatories and signatories before signing and value one for signatories after signing. Pension fund characteristics and board of trustees characteristics are used as control variables. The model includes year fixed effects and the standard errors are clustered at the pension fund level to correct for serial correlation.  $t$  statistics are in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

	Awareness of sustainable investing			Implementation of sustainable investing	
	Intensity (A)	Spectrum (B)	Specificity (C)	Variety (D)	Scope (E)
IRBC	0.002 (0.002)	0.518 (0.950)	2.803*** (0.736)	0.0350 (0.323)	-0.001 (0.066)
SIGN	0.001 (0.002)	1.287* (0.686)	-1.139 (0.996)	0.248 (0.253)	0.098* (0.045)
Active investing	0.002 (0.001)	0.603 (0.549)	-0.010 (0.574)	0.144 (0.204)	0.001 (0.047)
Average age trustees	0.000 (0.000)	0.044 (0.076)	-0.072 (0.077)	0.016 (0.032)	0.002 (0.005)
Average tenure trustees	0.000 (0.000)	-0.054 (0.077)	-0.070 (0.081)	-0.036 (0.030)	0.003 (0.007)
Corporate pension funds	0.004** (0.002)	1.810* (0.943)	1.421 (1.233)	-0.134 (0.261)	0.106* (0.058)
Fraction female trustees	-0.003 (0.005)	-0.207 (1.829)	0.273 (2.118)	0.769 (0.793)	0.021 (0.139)
Funding ratio	-0.016* (0.009)	-6.031* (3.276)	-4.749 (4.624)	-2.144* (1.110)	-0.668** (0.284)
Liability duration	0.000 (0.000)	-0.027 (0.077)	-0.029 (0.104)	-0.027 (0.028)	0.007 (0.007)
Log(assets)	0.004*** (0.001)	1.744*** (0.545)	2.499*** (0.769)	0.275** (0.107)	0.061** (0.024)
Positive belief risk-return SI	0.003 (0.002)	0.631 (0.754)	0.584 (0.771)	-0.031 (0.345)	0.094 (0.061)
Professional group pension funds	0.001 (0.004)	0.554 (1.618)	-0.909 (1.773)	-0.063 (0.356)	-0.082 (0.098)
Constant	-0.071*** (0.021)	-30.280*** (10.010)	-39.620*** (14.730)	-1.929 (3.487)	-0.777 (0.681)
N	472	472	472	472	472
R <sup>2</sup>	0.369	0.391	0.360	0.143	0.134

**Table 4.4.8: The effect of signing the IRBC initiative on SI measures - robustness check**

This table shows the results of the fixed effects model in Equation (4.9) with pension fund fixed effects and weighted with propensity scores for all five SI measures as dependent variable.  $SIGN_i$  takes value zero for non-signatories and value one for signatories.  $IRBC_{i,t}$  takes value zero for non-signatories and signatories before signing and value one for signatories after signing. Pension fund characteristics and board of trustees characteristics are used as control variables. The model includes year fixed effects and the standard errors are clustered at the pension fund level to correct for serial correlation.  $t$  statistics are in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

	Awareness of sustainable investing			Implementation of sustainable investing	
	Intensity (A)	Spectrum (B)	Specificity (C)	Variety (D)	Scope (E)
IRBC	0.002 (0.003)	0.027 (1.028)	2.391*** (0.697)	-0.186 (0.353)	-0.023 (0.078)
Active investing	-0.001 (0.002)	0.148 (1.131)	1.018 (0.692)	-0.324 (0.280)	-0.0710 (0.055)
Average age trustees	0.000 (0.000)	-0.216 (0.157)	0.072 (0.124)	-0.082** (0.034)	-0.005 (0.011)
Average tenure trustees	0.000 (0.000)	0.042 (0.176)	0.047 (0.088)	0.012 (0.082)	0.009 (0.011)
Fraction female trustees	-0.004 (0.015)	-0.293 (3.364)	2.575 (4.592)	-0.362 (1.278)	-0.380 (0.255)
Funding ratio	0.022 (0.021)	5.886 (9.014)	6.839 (6.583)	0.041 (2.605)	-0.085 (0.752)
Liability duration	-0.001 (0.002)	-0.340 (0.573)	-0.217 (0.443)	-0.097 (0.195)	-0.013 (0.034)
Log(assets)	-0.010 (0.008)	-3.781 (3.038)	-1.666 (3.313)	-1.060 (0.918)	-0.423** (0.202)
Constant	0.24 (0.18)	102.80 (64.98)	33.44 (75.92)	33.60 (21.37)	10.40** (4.60)
N	472	472	472	472	472
$R^2$	0.576	0.643	0.738	0.480	0.428

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In Table 4.4.7 and 4.4.8 we analyze the impact of signing the IRBC initiative on individual SI measures. Because we are interested in the impact of signing on the awareness and implementation of sustainable investing, it is interesting to construct indices that combine multiple individual measures. We construct an awareness index which is a normalized, equally weighted average of the three awareness measures: intensity, spectrum and specificity. We also construct an implementation index which is a normalized, equally weighted average of the two implementation measures: variety and scope. Subsequently, we analyze the models in Equation (4.8) and (4.9) where  $ESG_{i,t}$  is the awareness or implementation index instead of an individual SI measure. Table 4.4.9 shows the results of the diff-in-diff model in Equation (4.8) with awareness and implementation index as dependent variable. The results provide evidence that signing the IRBC initiative increases the awareness of sustainable investing because the estimate of the IRBC dummy is positive and highly significant for the awareness index. Similar to the results in table 4.4.7 on the SI measures, there is no effect of signing on the implementation of sustainable investing. Again, we run the diff-in-diff model in Equation (4.9) as a robustness check, which includes pension fund fixed effects. Table 4.4.10 presents these results. The conclusions stay the same.

**Table 4.4.9: The effect of signing the IRBC initiative on awareness and implementation**

This table shows the results of the pooled OLS model in Equation (4.8) weighted with propensity scores for the awareness and implementation index as dependent variable.  $SIGN_i$  takes value zero for non-signatories and value one for signatories.  $IRBC_{i,t}$  takes value zero for non-signatories and signatories before signing and value one for signatories after signing. Pension fund characteristics and board of trustees characteristics are used as control variables. The model includes year fixed effects and the standard errors are clustered at the pension fund level to correct for serial correlation.  $t$  statistics are in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

	Awareness of sustainable investing (A)	Implementation of sustainable investing (B)
IRBC	0.546*** (0.151)	0.00503 (0.179)
SIGN	-0.101 (0.187)	0.257* (0.135)
Active investing	0.0812 (0.112)	0.0355 (0.109)
Average age trustees	-0.00491 (0.0168)	0.00881 (0.0152)
Average tenure trustees	-0.0139 (0.0181)	-0.00317 (0.0177)
Corporate pension funds	0.413* (0.247)	0.187 (0.136)
Fraction female trustees	-0.231 (0.480)	0.221 (0.401)
Funding ratio	-1.119 (0.983)	-1.858*** (0.679)
Liability duration	0.0152 (0.0207)	0.00743 (0.0164)
Log(assets)	0.555*** (0.159)	0.188*** (0.0531)
Positive belief risk-return SI	0.194 (0.151)	0.185 (0.148)
Professional group pension funds	-0.0610 (0.393)	-0.182 (0.251)
Constant	-9.572*** (2.911)	-2.031 (1.730)
N	472	472
$R^2$	0.428	0.175

**Table 4.4.10: The effect of signing the IRBC initiative on awareness and implementation - robustness check**

*This table shows the results of the pooled OLS model in Equation (4.9) with pension fund fixed effects and weighted with propensity scores for the awareness and implementation index as dependent variable.  $SIGN_i$  takes value zero for non-signatories and value one for signatories.  $IRBC_{i,t}$  takes value zero for non-signatories and signatories before signing and value one for signatories after signing. Pension fund characteristics and board of trustees characteristics are used as control variables. The model includes year fixed effects and the standard errors are clustered at the pension fund level to correct for serial correlation.  $t$  statistics are in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .*

	Awareness of sustainable investing (A)	Implementation of sustainable investing (B)
IRBC	0.477*** (0.145)	-0.0897 (0.204)
Active investing	0.0645 (0.114)	-0.220 (0.139)
Average age trustees	0.00126 (0.0228)	-0.0298 (0.0214)
Average tenure trustees	-0.00243 (0.0193)	0.0201 (0.0337)
Fraction female trustees	0.137 (1.058)	-0.860 (0.700)
Funding ratio	1.887 (1.344)	-0.165 (1.842)
Liability duration	-0.0859 (0.0916)	-0.0492 (0.109)
Log(assets)	-0.830 (0.710)	-1.109** (0.456)
Constant	19.55 (15.99)	28.98*** (10.21)
N	472	472
$R^2$	0.769	0.472

The results of the diff-in-diff models in Equation (4.8) and (4.9) imply that the IRBC initiative improves the awareness of pension fund statements about sustainable investing. However, signing the IRBC initiative does not accelerate the implementation of sustainable investing. This conclusion does not necessarily imply that IRBC signatories do not seriously commit to the IRBC initiative or did not improve the implementation of sustainable investing. First, it could be the case that IRBC signatories want to enhance their SI policy and also talk about it more specifically (reflected by the increased specificity measure), but are still trying to find out how to integrate sustainable investing into their investment strategy. Second, the measures are not a perfect representation of the SI policy of a pension fund. For example, the variety measure counts the number of implemented SI strategies but does not consider to what extent a pension fund uses a specific strategy. A pension fund can, for example, exclude cluster munition investments only, which are prohibited by law. However, a pension fund can also have an extensive exclusion strategy banning all sin and brown stocks. The variety measure does not capture this difference. Third, it could be the case that the IRBC signatories took the lead with the implementation of sustainable investment already before signing the IRBC initiative, and non-signatories are possibly following the forerunners subsequently.

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## 4.5 Conclusion

Pension funds, as long-term institutional investors, play a key role in driving sustainable investing. Nevertheless, little is known about how pension funds implement sustainable investing. This paper creates an overview of the disclosures of sustainable investing by Dutch pension funds in annual reports from 2016 to 2021 by introducing a novel textual analysis approach using state-of-the-art NLP techniques. We measure the awareness and implementation of sustainable investing using five different measures. Further, we analyze the relation between pension fund characteristics and sustainable investing and investigate the impact of signing an SI initiative focusing on the best-known Dutch SI initiative for pension funds: the IRBC initiative.

The empirical results show that the average fraction of SI-related sentences in annual reports and the average specificity of pension fund statements regarding sustainable investing have increased by 150 percent during the past five years, with substantial variation between pension funds. Also, the implementation of sustainable investing has increased significantly over time. We find that the pension fund's size increases both awareness and implementation of sustainable investing. This finding is in line with the general notion that larger pension funds are more concerned about corporate responsibility and are more capable of screening stocks on environmental criteria due to the monitoring cost involved with active management. A positive belief about the risk-return relation of sustainable investing has a positive effect on awareness of sustainable investing but not on the implementation of sustainable investing.

Focusing on the IRBC initiative, we find that large pension funds, pension funds with more female trustees, and pension funds with a positive belief about the risk-return relation of sustainable investing are more likely to sign the IRBC initiative. To analyze the effect of signing the IRBC initiative on sustainable investing, we adopt a diff-in-diff model with propensity score matching to control for possible self-selection bias. Signing the IRBC initiative has a positive and economically significant impact on the awareness of sustainable investing. However, we do not find an effect of signing on the implementation of sustainable investing.

Our findings are subject to some limitations. First, we do not aim to make any causal claims about the effect of signing the IRBC initiative on the SI policy. Moreover, we cannot exclude that an underlying trend towards more sustainable investing drives both the IRBC initiative and the development of SI policies. Second, we are aware that the pension fund statements in the annual report regarding sustainable investing can be an incomplete representation of the SI policy. Although pension funds are legally required to specify in their annual report how they incorporate ESG criteria in their investment policy, there are no requirements about how and with how much detail they should do this.

The results provide important insights for pension funds and the regulatory authority. First, the state-of-the-art textual analysis approach introduced in this paper generates an interesting dataset, including five SI measures exploiting unstructured, qualitative data from annual reports. For example, this approach quantifies the specificity of pension fund statements about sustainable investing, which makes it possible to identify possible vague talk. Second, the results give insights into which pension funds are forerunners in sustainable investing and which pension funds are followers. We show that some pension fund and board characteristics impact the SI policy and the probability of signing an SI initiative. Third, signing an SI initiative seems to go hand in hand with more specific pension fund statements about sustainable investing. However, signing does not accelerate the implementation of sustainable investing. The IRBC initiative does not require that pension funds implement specific SI strategies or cover specific asset classes. However, the initiative requires pension funds to explain their SI strategies and how they integrate sustainable investing in various asset classes. In line with our result, the monitoring committee of the IRBC initiative concluded in 2021 that signatories of the initiative needed to catch up in implementing the agreements of the initiative. Only 13 percent of the pension funds were implementing the agreements thoroughly (IRBC (2021)). This paper does not check whether signatories of the initiative live up to their duties regarding the initiative, but does give insight into the development of the SI policy of signatories compared to non-signatories.

Given that pension fund and board characteristics impact the SI policy

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and the probability of signing an SI initiative, an interesting area for future research is the possible impact of additional stakeholders on the SI policy. For example, it would be interesting to investigate whether advisors (i.e., the asset manager or actuary firm) impact the SI policy. Bauer et al. (2020b) show that asset managers and actuaries impact strategic investment decisions by Dutch pension funds. Moreover, it would be interesting to investigate whether trustees with a seat on multiple boards can explain similarities in the SI policy.

Another interesting research question is whether pension funds ‘walk their talk’ by comparing the SI measures with the ESG performance of the pension fund’s asset portfolio. This question can be answered as better ESG scores become available due to more standardized ESG disclosure frameworks. Including the ESG performance of the pension fund’s asset portfolio makes it possible to investigate the effectiveness of SI policies and to identify potential window-dressing or greenwashing by pension funds. This question is especially relevant since our finding that the pension fund’s size increases both awareness and implementation of sustainable investing is in contrast to the finding of Boermans and Galema (2019) that large pension funds tend to have higher carbon footprints. Therefore, it is interesting to integrate asset portfolio data in the analysis and compare the ESG performance or carbon footprint of the pension fund’s asset portfolio with the pension fund’s SI measures in this paper.



# Appendix

## 4.A Annotation approach

As discussed in 4.2, we finetune the RobBERT model for the following two classification tasks:

- Determining whether a sentence is SI-related or not.
- Determining whether a paragraph is specific or not.

For both tasks, we create a labeled dataset. In the first dataset, a sentence gets label 1 if it is a full sentence and is in any way related to sustainable investing. A sentence gets label 0 if it is not a full sentence (e.g., header) or not related to sustainable investing. Sentences are preselected using a dictionary with SI keywords as discussed in Section 4.2. Table 4.2.1 shows a few examples of labeled sentences. The sentences with label 0 contain a dictionary keyword, but the keyword's interpretation is unrelated to sustainable investing. The labeled dataset is a representative subset containing 2000 sentences of annual reports from 2016 to 2021 and from different pension funds. Table 4.C.1 in Appendix 4.C shows a few examples of the original labeled sentences in Dutch.

Second, we label a dataset with paragraphs in which a paragraph gets label 1 if it is specific and gets label 0 if it is non-specific. A paragraph is specific if it satisfies one of the following conditions:

1. The paragraph contains details of actions that are specific to the pension

fund.

2. The paragraph contains detailed performance information.
3. The paragraph contains a description of tangible and verifiable targets set by the pension fund.

A paragraph is non-specific if it satisfies one of the following conditions:

1. The paragraph contains a general description regarding sustainable investing (e.g., strategies, risks) that can apply to any pension fund.
2. The paragraph contains a description of general and non-verifiable goal(s) regarding sustainable investing without an explanation of how to achieve it.
3. The paragraph contains a description of SI legislation without an explanation of how the pension fund is implementing it or going to implement it.

Table 4.3.2 shows a few examples of labeled paragraphs. The labeled dataset is a representative subset containing 1000 paragraphs of annual reports from 2016 to 2021 and from different pension funds. Table 4.C.2 in Appendix 4.C shows a few examples of the original labeled sentences in Dutch.

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## 4.B Finetuning of the language model

As discussed in Section 4.2, we finetune a trained Dutch RoBERTa-based language model with labeled datasets two times (see Appendix 4.A): we finetune the model to classify whether a sentence is SI-related and whether a paragraph is specific. Both labeled datasets are split up into a training set (80 percent) and a test set (20 percent). The model's tokenizer truncates inputs longer than 256 tokens.<sup>35</sup> The model is finetuned for five epochs and we use AdamW as an optimizer with a learning rate of  $3e-5$ .

Table 4.2.2 contains the performance results after finetuning both models. Both models show good performance results with an accuracy in the test set of 92 percent for the SI-related classification and 87 percent for the specificity classification. The lower performance of the specificity classification can be the result of a smaller labeled dataset or because the specificity classification is more complex. Subsequently, both finetuned language models are applied to the complete datasets. The preselected sentences dataset consists of 82,744 sentences, in which 39,725 sentences get label 0 and 43,019 sentences get label 1. The paragraphs dataset consists of 17,022 paragraphs, in which 11,807 get label 0 and 5,215 get label 1.

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<sup>35</sup>This implies that long paragraphs are truncated. However, increasing the maximum number of tokens to 512 does not improve the performance of the model.

## 4.C Examples of labeled text in original Dutch language

**Table 4.C.1: Labeled Dutch sentences with regard to sustainable investing**  
 This table presents some original examples (Dutch language) of sentences related to sustainable investing (label 1) and sentences that are not related to sustainable investing (label 0).

Sentence	Label
Het fonds streeft ernaar ook een bijdrage te leveren aan investeringen die nodig zijn om de mens te beschermen tegen de impact van <i>klimaatverandering</i> .	1
Die groei hield stand in april en mei 2016, maar was niet stabiel ondanks een vrij gunstig economisch <i>klimaat</i> .	0
Met impactinvesteringen wil het pensioenfonds een bijdrage leveren aan de oplossing van wereldwijde problemen, zoals armoede en <i>ongelijkheid</i> .	1
Hiermee wordt mogelijke <i>ongelijkheid</i> binnen het bestuur indien het houderschap bij bestuurders wordt belegd voorkomen.	0
Portfoliorisico's inzake klimaatrisico's kunnen gemitigeerd worden door effectief en betrouwbaar ESG-beleid te implementeren, met name als het gaat om <i>transitierisico's</i> .	1
Gedurende het jaar 2017 heeft het bestuur extra aandacht besteed aan het <i>transitierisico</i> van de deelnemers- en uitkeringenadministratie.	0

**Table 4.C.2: Labeled Dutch sentences with regard to specificity**  
*This table presents some original examples (Dutch language) of specific paragraphs (label 1) and non-specific paragraphs (label 0).*

Paragraph	Label
In 2017 zijn we verder gegaan met het verduurzamen van onze beleggingsportefeuille, zonder concessies te doen aan rendement en risico's. Met als uiteindelijk doel om in 2020 €20 miljard aan beleggingen te hebben die bijdragen aan het oplossen van maatschappelijke vraagstukken. Daarnaast willen we de klimaatverandering tegengaan door een reductie van de CO <sub>2</sub> -uitstoot met 50% in onze beleggingen vergeleken met de nulmeting in 2014. Het ingezette herstel draagt bij aan de mogelijkheid om onze ambitie te realiseren.	1
Naast het stemmen op aandeelhoudersvergaderingen vinden wij het belangrijk om met bedrijven in dialoog te gaan (engagement). Zo zorgt het fonds er als investeerder voor dat zijn mening ook los van de aandeelhoudersvergaderingen gehoord wordt. In deze voortdurende dialoog ligt sterk de nadruk op ESG-kwesties. In 2021 zijn 1.620 engagements gevoerd met 564 ondernemingen waarvan er 122 succesvol zijn afgesloten.	1
Er is besloten om de afzonderlijke allocatie naar Europese aandelen via een Best-in-class strategie in te vullen. Hiermee wordt belegd in ondernemingen die in het top kwartiel presteren op het gebied van ESG-scores binnen hun sector. Deze allocatie is begin 2019 geïmplementeerd.	1
In 2020 hebben we gekeken naar het beleggingsbeleid voor de komende jaren, dat beoogt dat de eigen portefeuille bijdraagt aan een meer leefbare wereld. Daarbij is het uitgangspunt een zo goed mogelijk renderende portefeuille die meer positieve impact heeft op de leefomgeving en die leidt tot meer relevantie voor de deelnemers.	0
De economie en maatschappij staan voor uitdagingen die ons als belegger raken. Klimaatverandering, de groeiende vraag naar hernieuwbare energie en grondstoffenschaarste zijn voorbeelden van onderwerpen die vragen om aanpassing en innovatie.	0
De SFDR kent twee kernelementen voor de implementatie, namelijk (1) transparantie in het meewegen van negatieve duurzaamheidsimpact bij investeringsbeslissingen en (2) publicatie van precontractuele duurzaamheidsinformatie. Elke pensioenuitvoerder zal daarnaast in de precontractuele informatie een beschrijving moeten hebben van (1) de wijze waarop duurzaamheidsrisico's onderdeel uitmaken van het besluitvormingsproces rondom investeringsbeslissingen, respectievelijk investeringsadvies of verzekeringsadvies, (2) het waarschijnlijke effect van de duurzaamheidsimpact op het rendement van het pensioenfonds.	0



# 5

## Conclusion

Reforms of pension schemes take place around the world. There is a general trend of transferring funding risks from pension scheme sponsors to participants, often driven by the aging of society, lower investment returns. Hybrid pension schemes, such as CDC schemes, that combine features of DB and DC schemes, are enjoying increasing popularity due to their attractive features, such as a limited commitment from the sponsor, longevity risk sharing, and lower administrative and investment costs. This dissertation considers four economic principles that are important for adequate and sustainable pension design, namely fairness, efficiency, insurance, and accountability.

Chapter 2 contributes to the literature by offering design features for a general class of fair and efficient CDC schemes in the presence of equity market risk and interest rate risk. From a continuity perspective, it is preferable to design CDC schemes in a fair and efficient way. I show that a CDC scheme is fair if the scheme is complete and an appropriate horizon-dependent benefit adjustment process is used to allocate the market risks to cohorts. However, a pension scheme that is not complete, i.e., shocks are not explicitly allocated to the cohorts in the scheme, cannot be made fair. Moreover, the value transfers between cohorts are significantly bigger in such schemes compared to

complete pension schemes. This chapter also shows that it is not always possible to replicate the optimal individual's exposure to market risks in a CDC scheme, and therefore efficiency may not be reached. I do show, however, that the implied exposure to market risks in a fair CDC scheme with benefit smoothing is generally in line with life-cycle theory.

Chapter 3 analyzes macro-longevity risk sharing between cohorts in a pension scheme as a risk management tool. It contributes to the knowledge on longevity hedging solutions and the literature on managing macro-longevity risk. I show that workers can provide insurance against the macro-longevity risk of retirees. Macro-longevity risk is a systemic risk that cannot be diversified by pooling enough participants in a pension scheme. This chapter shows that it is nevertheless beneficial to share this risk between different age groups of a pension scheme because the risk affects cohorts differently. I derive Pareto-improving risk-sharing rules that maximize the welfare gain from risk sharing for all participants in the pension scheme for different retirement age policies. I find that the design of the retirement age policy has a large impact on both the risk-sharing rule and welfare gains from sharing macro-longevity risk. When the retirement age is (partially) linked to life expectancy developments, workers provide insurance against the macro-longevity risk of retirees in the optimal risk-sharing rule. I also determine a fair risk compensation for cohorts who absorb macro-longevity risk of other cohorts using a utility-based fairness criterion.

Chapter 4 considers the accountability principle by investigating a particular aspect of the pension fund policy, namely the sustainable investment (SI) policy. This chapter contributes to the literature by providing an overview of the disclosures of sustainable investing by a specific group of large institutional investors: Dutch occupational pension funds. I introduce a novel textual analysis approach using state-of-the-art natural language processing (NLP) techniques to measure the awareness and implementation of sustainable investing. I exploit a proprietary dataset to analyze the relationship between pension fund characteristics and sustainable investing. I find that a pension fund's size increases both the awareness and the implementation of sustainable investing. Moreover, I analyze the role of signing the International Responsible Business Conduct (IRBC) initiative.

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Large pension funds, pension funds with more female trustees, or pension funds with a positive belief about the risk-return relation of sustainable investing are more likely to sign the IRBC initiative. Although signing this initiative increases the awareness of sustainable investing, I do not find an effect on the implementation of sustainable investing.

### **Future research**

The work presented in this dissertation offers opportunities for future work along three dimensions, consistent with the suggestions made at the end of each chapter.

In Chapter 2, I analyze fairness and efficiency in CDC schemes in the presence of equity market risk and interest rate risk. An interesting extension of the financial market model is to also include inflation risk, since pension schemes preferably aim to protect the purchasing power of their participants. If inflation risk is included, the optimal exposure to market risks changes because hedging interest rate risk has to be balanced against hedging inflation risk if inflation-linked bonds are not (sufficiently) available in the financial market. Another interesting extension of Chapter 2 is to investigate the impact of alternative specifications of the utility functions on the efficiency of CDC schemes. For example, loss aversion and reference dependence, two pronounced behavioral regularities supported by empirical evidence, significantly impact the optimal asset allocation (see van Bilsen et al. (2020b)).

Chapter 3 shows that sharing macro-longevity risk between cohorts in a pension scheme yields welfare gains. An interesting area for future research is to investigate sharing macro-longevity risk between pension funds or even between countries to exploit potentially even larger differences in exposures to macro-longevity risk. van Binsbergen et al. (2014) propose sharing risks between heterogeneous pension funds by trading pension guarantees and Bodie and Merton (2002) propose swaps to achieve risk-sharing benefits of broad international diversification. My framework is useful for the further development of such instruments.

In Chapter 4, I exploit a proprietary dataset to analyze the relationship between pension fund characteristics and sustainable investing. Given that pension fund and board characteristics impact the SI policy and the proba-

bility of signing an SI initiative, an interesting area for future research is the possible impact of additional stakeholders on the SI policy. For example, it would be interesting to investigate whether advisors (i.e., the asset management firm, consultants, or the actuary firm) impact the SI policy. Moreover, it would be interesting to investigate whether trustees with a seat on multiple boards can explain similarities in the SI policy. Another interesting research question is whether pension funds 'walk their talk' by comparing the SI measures constructed in Chapter 4 with the ESG characteristics of the pension fund's asset portfolio. This question can be answered as better ESG scores become available due to more standardized ESG disclosure frameworks, including the ESG performance of the pension fund's asset portfolio, which makes it possible to investigate the effectiveness of SI policies and to identify potential window-dressing or greenwashing by pension funds.

# 6

## Research impact and valorization

The findings of this dissertation have several implications for pension design. As discussed in the introduction, this dissertation considers four economic principles that are relevant for adequate and sustainable pension design. These four principles are fairness, efficiency, insurance, and accountability. These principles are relevant for each pension scheme configuration, whether it is DC, CDC, or DB. The research impact of this dissertation presented below relates to these four economic principles.

The conclusions of this dissertation can be used, in particular, by Dutch pension funds and social partners when designing a new pension contract. After more than a decade of negotiations, the Dutch government reached in 2020 an agreement with social partners to reform the second-pillar pension system in the Netherlands. A four-year transition period has started towards this new pension system in 2023 and as of 2027, all pension funds with a DB scheme should have completed this transition. Pension funds can transform their DB scheme into two types of schemes. The first scheme, the ‘new pension contract’ (NPC), is best characterized as a CDC scheme in which risks are shared collectively according to predefined mechanisms specifying how investment returns are allocated to participants. In order to maintain the col-

lective character of pensions some extra collective elements have been added to this scheme such as a solidarity reserve. The solidarity reserve is meant to facilitate intergenerational risk sharing by damping financial and longevity shocks. The other scheme is a so-called 'improved DC contract' which is closer to a traditional DC scheme. In both pension schemes the economic design principles in this thesis are key.

Chapter 2 examines fairness and efficiency in CDC pension schemes. These economic principles support the continuity of a CDC scheme. The results of Chapter 2 show that a CDC scheme is fair under certain conditions and the implied exposure to market risks is generally in line with life-cycle theory in a fair CDC scheme with benefit smoothing. The concept of fairness is an important continuity criterion, because some cohorts can benefit by leaving an unfair scheme, which in turn jeopardizes the collective nature of the pension scheme. Although this argument does not completely hold for participants in the Dutch pension system, because they do not have a choice about which pension fund to join, the support for such a system could be eroded by fairness. Moreover, participants have the option to switch pension funds by changing jobs or sectors. The results of Chapter 2 are also directly applicable to the pension sector in other countries that adopted or are going to adopt CDC schemes. For example, the Pension Schemes Act 2021 introduces CDC schemes in the UK, offering an alternative design for both DB and DC schemes. The secondary legislation for this act is currently being discussed in Parliament. The proposed CDC scheme consists of one collective asset pool and benefit adjustments are based on investment returns similar to the CDC scheme considered in Chapter 2.

Chapter 2 shows that although a CDC scheme with uniform benefit adjustments intuitively seems a 'fair contract', such a scheme is generally unfair and that wealth transfers, measured as a percentage of annual income, can be substantial. Chapter 2 also shows that CDC pension schemes can be made fair if the scheme is complete and an appropriate horizon-dependent benefit adjustment process is used to allocate the market risks to cohorts.

The fairness principle is not only applicable to benefit adjustments in a CDC scheme, but is also relevant for the solidarity reserve, an extra collective element in the new pension contract in the Netherlands. The idea behind

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this solidarity reserve is that it can facilitate intergenerational risk sharing. The solidarity reserve can be filled via a levy on pension contributions or via a levy on positive collective excess returns where both are limited to 10%. Because the solidarity reserve is not allocated to the participants, the pension scheme is not complete and therefore cannot be designed in a fair way. In fact, the specific design of the solidarity reserve causes significant wealth transfers between cohorts. While the idea behind this solidarity reserve is to facilitate intergenerational risk sharing, the legislative constraints on the solidarity reserve prevent pension funds from being able to implement a first-best risk sharing solution.

The other principle in Chapter 2, efficiency, requires that cohorts have an optimal exposure to market risks based on their preferences and life-cycle. The results in Chapter 2 show that although it is not always possible to exactly replicate the optimal individual exposure to market risks in a CDC scheme, the implied exposures are generally in line with life-cycle theory in a CDC scheme with benefit smoothing. These results can be used for the design of the predefined benefit adjustments and allocation mechanisms in the UK and new Dutch pension contract, respectively. In the new Dutch pension contract, predefined allocation mechanisms specify how returns are allocated to participants. This allocation rule replaces the one-size-fits-all approach in the current DB pension schemes. The predefined allocation mechanism allows for heterogeneity between cohorts based on their preferences and life-cycle. This is in line with the efficiency principle.

Chapter 3 shows that workers can provide insurance against the macro-longevity risk of retirees. In this chapter, I analyze macro-longevity risk sharing between cohorts in a pension scheme as a risk management tool. I show that sharing macro-longevity risk is welfare-enhancing it provides insight into how macro-longevity risk should be shared between cohorts given the retirement age policy and what the size of a fair risk compensation should be. These risk-sharing rules can be implemented in the new Dutch pension system, because it provides opportunities to share macro-longevity risk explicitly between different cohorts. In the new pension contract, it is possible to share macro-longevity risk between different cohorts in two ways. First, macro-longevity risk can be shared via the hedge return, which is one of

the components of the allocation mechanism in the new pension contract. The hedge return compensates participants for the realization of annuity risks, like interest rate risk and macro-longevity risk. When sharing macro-longevity risk via the allocation mechanism, a macro-longevity shock is directly absorbed by all cohorts. The allocation mechanism can be based on the risk-sharing rules in Chapter 3. Second, macro-longevity risk can be shared via the solidarity reserve. Because the solidarity reserve implies significant wealth transfers between cohorts, sharing macro-longevity risk via the hedge return is preferable.

While there is a global trend toward investment policies that take ESG information into account, little is known about the design and development of SI policies by pension funds. Chapter 4 provides an overview of the disclosures of sustainable investing. The findings provide insight into which pension funds are forerunners in sustainable investing and which pension funds are followers. I also show that some pension fund and board characteristics impact the SI policy and the probability of signing a SI initiative. I show that both awareness and implementation of sustainable investing by pension funds increase over time, but the increase in awareness is at a much higher rate compared to the implementation. This chapter also shows that although signing a SI initiative increases awareness of sustainable investing, we do not find an effect on the implementation of sustainable investing. In line with my result, the monitoring committee of the IRBC initiative concluded in 2021 that signatories of the initiative needed to catch up in implementing the agreements of the initiative. Only 13 percent of the pension funds were implementing the agreements thoroughly (IRBC (2021)). The fact that the implementation of sustainable investing lags behind the attention for sustainable investing may be due to the fact that it takes some time to develop a policy based on attention and then implement it. However, it could also be a sign of pension funds focusing more on talking about sustainable investing than focusing on the actual implementation of sustainable investing. There are still a lot of examples of greenwashing and cheap talk by companies and investors nowadays. Because participants are not able to choose between pension funds in the Dutch context, it is important that participants should be able to trust that their pension fund is acting in the best interest of their

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participants. This responsibility requires that trustees know what this best interest means and how potential trade-offs should be prioritized.

Chapter 4 also provides insights into applications of NLP innovations in the finance literature. Several NLP techniques are used in this chapter to generate an interesting dataset from qualitative data in annual reports. In particular, recent innovations that exploit deep neural network models expand the possibilities of NLP techniques by allowing the model to learn more general text patterns and complex non-linear patterns making use of large amounts of unannotated data. Such models can not only identify topics of interest in a text but can also be trained to identify how topics are discussed. These NLP techniques can be applied to a wide variety of domains and document types, e.g., annual reports of other financial institutions, corporate disclosures or central bank communication.



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# Nederlandse samenvatting

Wereldwijd worden pensioenstelsels herzien. Er is een algemene trend om financiële risico's over te dragen van de sponsor naar pensioendeelnemers, vaak gedreven door vergrijzing en lagere beleggingsrendementen. Hybride pensioenregelingen, zoals 'Collective Defined Contribution' (CDC) regelingen, nemen in populariteit toe vanwege hun aantrekkelijke kenmerken zoals beperkte betrokkenheid van de sponsor, delen van langlevensrisico en lagere administratieve en beleggingskosten. Dit proefschrift onderzoekt vier economische principes die belangrijk zijn voor adequate en duurzame opzet van een pensioenregeling. Deze vier principes zijn: eerlijkheid, efficiëntie, verzekering en verantwoording.

- **Eerlijkheid:** geen waardeoverdrachten tussen cohorten in een pensioenregeling.
- **Efficiëntie:** optimale blootstelling aan marktrisico's voor cohorten over de levenscyclus en op basis van hun voorkeuren.
- **Verzekering:** bescherming van cohorten tegen het risico om tijdens pensionering langer te leven dan het pensioenvermogen toelaat.
- **Verantwoording:** de verantwoordelijkheid van een pensioenfondsbestuur om hun beleid op een transparante manier uit te leggen en te rechtvaardigen.

Hoofdstuk 2 definieert een algemene set van eerlijke en efficiënte CDC-regelingen in aanwezigheid van aandelenrisico en renterisico. Vanuit een continuïteitsperspectief is het wenselijk om CDC-regelingen op een eerlijke en efficiënte manier te ontwerpen. Ik toon aan dat een CDC-pensioenregeling eerlijk is als de regeling compleet is en adequate horizon-afhankelijke pensioenaanpassingen worden gebruikt om de marktrisico's toe te wijzen aan cohorten. Een pensioenregeling die niet compleet is, d.w.z. schokken worden niet expliciet toegewezen aan de cohorten in de regeling, kan niet eerlijk worden gemaakt. Bovendien is de oneerlijkheid in dergelijke regelingen aanzienlijk groter dan in complete pensioenregelingen. Dit hoofdstuk laat ook zien

dat het niet altijd mogelijk is om de optimale blootstelling van individuen aan marktrisico's te repliceren in een CDC-regeling en dat efficiëntie daarom mogelijk niet wordt bereikt. Ik toon echter aan dat de impliciete blootstelling aan marktrisico's in een eerlijke CDC-regeling waarin schokken worden uitgesmeerd over tijd over het algemeen in overeenstemming is met de theorie van de levenscyclus.

Hoofdstuk 3 analyseert het delen van macro-langlevensrisico tussen verschillende leeftijdsgroepen in een pensioenregeling als risicomangement instrument en laat zien dat werknemers verzekeringen kunnen bieden voor het macro-langlevensrisico van gepensioneerden. Macro-langlevensrisico is een systeemrisico dat niet kan worden gediversifieerd door voldoende deelnemers in een pensioenregeling samen te voegen. Dit hoofdstuk toont echter aan dat het toch voordelig is om dit risico te delen tussen verschillende leeftijdsgroepen van een pensioenregeling omdat het risico leeftijdsgroepen op verschillende manieren beïnvloedt. Ik leid Pareto-verbeterende risicodelingsregels af die het welzijn voor alle deelnemers in de pensioenregeling maximaliseren voor verschillende pensioenleeftijd beleidsmaatregelen. Ik laat zien dat het ontwerp van het pensioenleeftijd beleid een grote impact heeft op zowel de risicodelingsregel als de welvaartswinsten van het delen van macro-langlevensrisico. Wanneer de pensioenleeftijd (gedeeltelijk) gekoppeld is aan de pensioenleeftijd, verzekeren werknemers het macro-langlevensrisico van gepensioneerden in de optimale risicodelingsregel. Ik bepaal ook een eerlijke risicovergoeding voor leeftijdsgroepen die macro-langlevensrisico van andere groepen op zich nemen door gebruik te maken van een op nut gebaseerd eerlijkheidscriterium.

Hoofdstuk 4 onderzoekt het verantwoordingsprincipe door een specifiek aspect van het beleid van pensioenfondsen te onderzoeken, namelijk het duurzaam beleggingsbeleid. Dit hoofdstuk draagt bij aan de literatuur door een overzicht te geven van de openbaarmakingen over duurzaam beleggen door een specifieke groep grote institutionele beleggers, namelijk Nederlandse pensioenfondsen. Ik introduceer een nieuwe aanpak voor tekstuele analyse met behulp van geavanceerde natural language processing (NLP) technieken om de aandacht voor en de implementatie van duurzaam beleggen te meten. Ik maak gebruik van een exclusieve dataset om de relatie

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tussen kenmerken van pensioenfondsen en duurzaam beleggen te analyseren. Ik constateer dat de omvang van een pensioenfonds zowel de aandacht voor als de implementatie van duurzaam beleggen vergroot. Bovendien analyseer ik de rol van ondertekening van het Internationaal Maatschappelijk Verantwoord Beleggen (IMVB) initiatief. Grote pensioenfondsen, pensioenfondsen met meer vrouwelijke bestuursleden of pensioenfondsen die een positieve overtuiging hebben over de risico-rendementsverhouding van duurzaam beleggen, zijn eerder geneigd om het IMVB-initiatief te ondertekenen. Hoewel het ondertekenen van dit initiatief de specificiteit van verklaringen van pensioenfondsen over duurzaam beleggen vergroot, vinden we geen effect op de implementatie van duurzaam beleggen.



## About the author

Annick van Ool was born on July 29, 1993 in Weert, the Netherlands. She obtained her Bachelor's degree in Econometrics and Operations Research in 2015 and her Master's degree in Quantitative Finance and Actuarial Sciences in 2016 at Tilburg University. During an internship at De Nederlandsche Bank, she wrote her master thesis, *Investment strategies for the pre-retirement and retirement phase of IDC pensions*, which was awarded with the Netspar Thesis Award in 2017.

In 2016, she started as an external PhD candidate at the Finance Department of Maastricht University under the supervision of Rob Bauer and Dirk Broeders. She presented her research at several conferences, for example at several Netspar conferences, the Longevity Conference in Washington DC, and International Conference on Pension, Insurance and Savings in Paris, and the Global Finance Conference in Treviso.

In 2016, she also joined the pensions department at the supervision policy division at De Nederlandsche Bank as a policy advisor. In 2018, she switched to the data analytics expertise centre at the insurance supervision division where she worked as a supervisory specialist. In 2023, she joined the macroprudential analysis and policy department at the financial stability division as a senior policy advisor.

