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The Recovery Potential for Underfunded Pension Plans

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Summary

In this paper, we consider defined benefit schemes which are designed to provide participants with a specific "ambition", which we model as a stochastic benchmark, where its market value is higher than the market value of the contributions. We consider strategies based on preferences which become more risk-averse in disadvantageous scenarios (CRRA utilities) and preferences which become less risk-averse in such scenarios (SAHARA utilities). We also examine the influence of constraints that force the replacement ratio to exceed a certain lower bound. We find that by a suitable choice of the parameters and of SAHARA utility functions, the average replacement ratio can be improved, with only a slightly higher probability of worse replacement ratios than in the CRRA case. We also investigate the effect of downward jumps in asset prices for both types of strategies. When the extra risk due to such jumps is compensated for by a higher value of the average rate of return for those assets, outcomes actually improve a bit in the sense that the probability of ending up above the minimally guaranteed level of the funding ratio increases for both CRRA and SAHARA-based investment portfolios. This suggests that optimization based on SAHARA preferences with lower bound constraints can be a useful tool to generate good investment strategies whenever the market value of the "ambition" is higher than the market value of the contributions.

Samenvatting

In dit paper beschouwen we defined-benefit (DB) pensioenregelingen die tot doel hebben hun deelnemers een specifieke 'ambitie' te bieden die we modelleren als een stochastische benchmark, waarbij de marktwaarde van de stochastische benchmark hoger is dan de marktwaarde van de premiebijdragen. We beschouwen strategie¨en op basis van voorkeuren die meer risico-avers worden in nadelige scenario's (CRRA nutsfuncties) en voorkeuren die in dergelijke scenario's minder risico-avers worden (SAHARA nutsfuncties). Ook kijken we naar de invloed van restricties om de vervangingsratio boven een bepaalde ondergrens uit te laten komen. We vinden dat door een geschikte keuze van de parameters en van SAHARA-nutsfuncties, de gemiddelde vervangingsratio kan worden verbeterd, met slechts een iets grotere kans op slechtere vervangingsratio's dan in het geval van CRRA. We onderzoeken ook het effect van neerwaartse sprongen in activaprijzen voor beide soorten strategie¨en. Wanneer het extra risico als gevolg van dergelijke sprongen wordt gecompenseerd door een hogere waarde van het gemiddelde rendement van die activa, verbeteren de uitkomsten zelfs een beetje in de zin dat de kans om boven het minimaal gegarandeerde niveau van de dekkingsgraad uit te komen voor zowel op CRRA en SAHARA gebaseerde beleggingsportefeuilles toeneemt. Dit suggereert dat optimalisatie op basis van SAHARA-voorkeuren met ondergrensbeperkingen een nuttig instrument kan zijn om goede investeringsstrategie¨en te genereren wanneer de marktwaarde van de "ambitie" hoger is dan de marktwaarde van de bijdragen.

1 Motivation

Pension funds have been challenged by low interest rates since 2008. In addition, the recent COVID-19 outbreak has led to a significant drop in equity prices in the first months of 2020. As a result, the Dutch government temporarily reduced the minimal required funding ratio for pension funds from 104% to 90%. Several large pension funds announced in the autumn of 2020 that their coverage ratios were still below that level and, if this were still the case at the end 2020, pensions entitlements for participants in these funds would probably have been cut at the beginning of 2021. This has been avoided by an improvement in funding ratios in the last months of 2020, but many have not yet reached the desired level of 100%.

The importance of reaching a certain required funding ratio after a limited time period raises whether such aims can be included in the optimization problems that are solved by pension funds for their asset and liability management. In classical life cycle problems for defined contribution pension schemes, the optimization criteria which determine the investment strategy are often stated in terms of nominal wealth at a future time, typically the point of retirement. Risk preferences are then expressed in terms of a utility function of an individual agent and an investment policy is chosen which maximizes the expected utility of wealth at retirement. On the other hand, for pension funds which implement defined benefit schemes, communication to a fund's participants, and decisions by policymakers, are not based on the total wealth of funds or individuals but on funding ratios, i.e., on a ratio of terminal wealth and the terminal value of a liability which represents the pension entitlements. Low funding ratios may have a detrimental impact on pension outcomes when they lead to pension cuts, i.e., when the value of the entitlements is reduced. Optimization criteria which are based on funding ratios instead of terminal wealth, may therefore be more natural in some circumstances, especially for funds which find themselves in a position of underfunding.

In this paper, we consider defined benefit schemes with a *defined ambition*. These types of schemes attempt to combine the best of both DC and the DB schemes for their participants. They are designed to provide their participants with a specific "ambition" (e.g. a fully price-indexed annuity at retirement), without providing any hard guarantee that the ambition is achieved under all economic circumstances. We model the ambition as a stochastic benchmark, where its market value is higher than the market value of the contributions[1] In the remainder of this paper, we will use the term "underfunded" DC scheme to emphasize the fact that the benchmark is too expensive relative to the contributions. We will also consider the replacement ratio as the ratio between the pension

[1] Otherwise, the ambition is not very "ambitious".

capital at retirement relative to the stochastic benchmark.

The usual stochastic optimization problem, which considers the expected utility of terminal wealth under CRRA preferences, was first studied in [Merton(1969)] and [Merton(1971)]. Such problems can be solved by dynamic programming techniques when a Markovian assumption about the state process is satisfied. Optimal strategies can then be derived after solving a non-linear partial differential equation, the Hamilton-Jacobi-Bellman equation, which characterizes the best possible expected utility of terminal wealth. Later, [Cox & Huang(1989)] showed that in complete markets this dynamic optimization problem can be reduced to a static problem. This leads to explicit solutions based on a dual approach and even allows the Markov assumption on state variables to be dropped.

An extension in which the agent optimizes expected utility but must, under all circumstances, outperform a certain stochastic benchmark, was covered in [Basak(1995)] and [Tepla(2001)]. In the latter paper, it was shown that the optimal investment policy under such a constraint can be interpreted as an investment in the benchmark plus an investment of the remaining wealth in a contingent claim which has a positive value. This shows that in this setting the agent's initial wealth must be larger than the initial value of the benchmark portfolio and that if these two initial values are equal, the solution to the maximizing problem is simply to invest in the benchmark for the entire investment period. Other research has shown how the ratio of terminal wealth and the terminal value of a benchmark can be included in the utility function. In [Brennan & Xia(2002)], for example, optimal strategies are found for the expected utility of wealth in real terms under investment in nominal assets. The optimization criterion is therefore based on nominal wealth divided by a stochastic price index. A recent Netspar paper by [Balter et al(2020)] on optimal investment based on a stochastic benchmark shows that by using double power utility functions, the probability of attaining the desired benchmark can be improved, even with an underfunded starting position.

In this paper, we assume that a participant converts his or her pension wealth into an annuity. The stochastic benchmark that we define depends on the value of the annuity at retirement but also on equity returns over the period under consideration. How well a certain standard of living can be maintained after retirement is measured in terms of the ratio of the terminal wealth and this benchmark, the so-called replacement ratio. We use the Black-Scholes model for the economy, but we also investigate the effect of negative jumps in asset prices on the outcomes for the replacement ratio. The desire to overcome the underfunding means that we look beyond the classical choice of preferences which exhibit constant relative risk aversion (CRRA). We make use of the class of Symmetric Asymptotically Hyperbolic Absolute Risk Aversion (SAHARA) utility functions to implement preferences that are always risk-averse but which, in contrast to what holds in the CRRA case that is usually considered, show decreasing risk aversion as a function of wealth when the level of wealth is low enough. This implies that investment strategies will prescribe more rather than less risk taking once funding ratios fall below a certain threshold. This *risk-taking for resurrection*[2] prevents excessive de-risking leading to an almost risk-less investment strategy, which makes it impossible to leave the situation of underfunding.

The rest of the paper is organized as follows. Section 2 describes the basic financial model and other assumptions. Section 3 introduces SAHARA preferences. Section 4 shows the impact of different preferences on the distribution of the optimal final replacement ratio outcomes. Section 5 studies the results when a lower bound is introduced for these ratios. Section 6 describes the Merton jump diffusion model and studies the impact of downward jumps in asset prices on the distribution of optimized pension outcomes. We present our conclusions in Section 7.

[2]We avoid the term *gambling for resurrection* here, since gambling typically assumes risk-seeking behavior. As mentioned before, when we find ourselves in the least favorable scenarios we become less risk averse but never risk-seeking: our risk aversion coefficient becomes smaller but never crosses zero.

2 Financial Market Model and Pension Annuity

Our financial model is the Black-Scholes model with a stock or stock-market index S and a risk-free asset B and prices that satisfy:

$$dS_t = \mu S_t dt + \sigma S_t dW_t^{\mathbb{P}}, \qquad dB_t = rB_t dt, \tag{1}$$

where $W^{\mathbb{P}}$ denotes a standard Brownian motion under a probability measure $\mathbb{P}$. The expected return $\mu > r$ and the volatility σ of the stock-market are assumed to be constant. The optimizing agent can only invest in these two assets so an optimal strategy is uniquely characterized by the amount of money θ_t that is invested in the risky asset S at time t. The process θ must be adapted to the filtration $\{\mathcal{F}_t,\ t \in [0, T]\}$ that is generated by the Brownian motion $W^{\mathbb{P}}$.

To model the replacement ratio at the fixed retirement date $T > 0$, we need to make some assumptions about the terminal value of the real pension annuity, which we denote by L_T. For simplicity, we assume it equals the product of a fixed amount $\tilde{A}$ and some positive power $d > 0$ of the terminal stock price S_T. It thus satisfies the following equation:

$$L_T = (AS_T)^d, \tag{2}$$

for given constants $A > 0$ and $d > 0$, where we have rewritten $\tilde{A} = A^d$ to ease later notation. The specification (2) incorporates the fact that the value of the liabilities will show some dependence on the behaviour of financial markets, while remaining in the setting of complete markets. For example, when the real annuity is tied to the wage inflation, then it is reasonable to tie L_T to the stock-market index S_T and this removes the need to specify an exogenous model for inflation. We note that other choices for the liability are possible, see for example the paper [van Binsbergen & Brandt(2016)] in which the optimization criterion also involves the ratio of the values of assets and liabilities but the latter is represented by a fixed payment after 15 years while explicit Value-at-risk constraints are imposed.

To capture the underfunded feature of the starting position, the initial value of the assets X_0 is assumed to be a fraction $\phi \in (0, 1)$ of L_0, the price at time 0 of the real annuity which pays L_T at time T, so

$$X_0 = \phi \mathbb{E}\left[\frac{M_T}{M_0} L_T\right] = \phi e^{-rT} \mathbb{E}^{\mathbb{Q}}[L_T], \tag{3}$$

where $\mathbb{Q}$ represents the risk-neutral measure and M is the pricing kernel process for this economy which satisfies

$$M_T = M_0 e^{-(r+\frac{1}{2}\nu^2)T - \nu W_T^{\mathbb{P}}} := \eta\, S_T^{-\frac{\nu}{\sigma}} \tag{4}$$

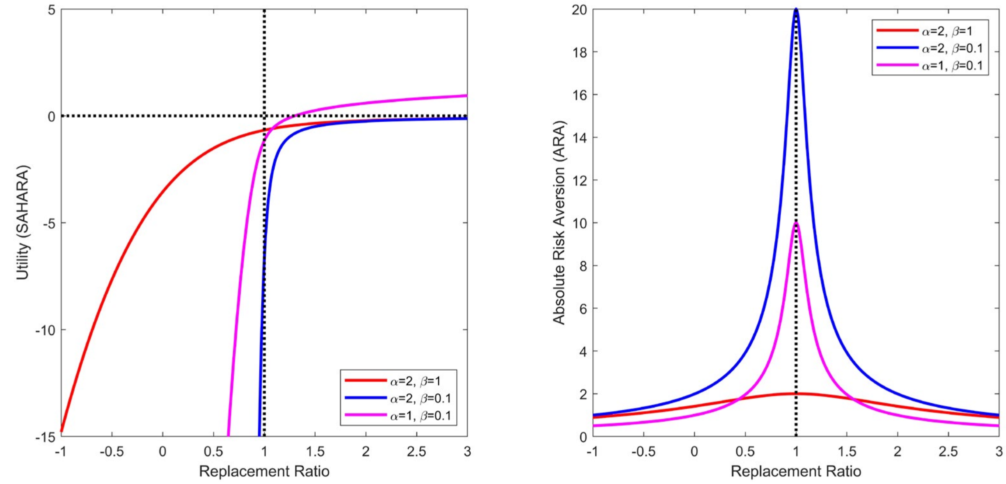
for a constant $\eta > 0$ which follows from this definition and where $\nu = (\mu - r)/\sigma$ is the Sharpe ratio for equity. A direct calculation shows that

$$\mathbb{E}^{\mathbb{Q}}[S_T^d|\mathcal{F}_t] = S_t^d e^{\zeta(T-t)}, \tag{5}$$

for $\zeta = d(r - \frac{1}{2}\sigma^2) + \frac{1}{2}d^2\sigma^2$, so we can characterize the initial wealth explicitly as

$$X_0 = \phi e^{-rT}\mathbb{E}^{\mathbb{Q}}[(AS_T)^d] = \phi(AS_0)^d e^{(\zeta - r)T}. \tag{6}$$

Performance criteria for the agent's optimization problem are formulated in terms of the replacement ratio $C_T := X_T/L_T$, which thus involves terminal wealth X_T and the benchmark L_T.



(a) SAHARA utility function

(b) Absolute Risk Aversion (ARA)

Figure 1: Plot of SAHARA utility function and absolute risk aversion (ARA) for $w_0 = 1$.

3 Modelling Preferences

A utility function U with domain $\Re$ is a SAHARA utility function if its absolute risk aversion function A satisfies

$$A(x) := \frac{-U''(x)}{U'(x)} = \frac{\alpha}{\sqrt{\beta^2 + (x - w_0)^2}} \tag{7}$$

for a certain scale parameter $\beta > 0$, risk aversion parameter $\alpha > 0$ and threshold wealth $w_0 \in \Re$. Clearly, $A(x)$ is strictly positive for all values of $x \in \Re$ which shows that U represent preferences which are risk-averse for all wealth levels.

The utility functions U can be written in an explicit form[3] by solving the differential equation (7). Figure 1 shows three examples of the resulting SAHARA utility functions U and the corresponding absolute risk aversion functions A. The SAHARA utility functions are defined on the whole real line and are concave everywhere, which indicates that it has a higher dis-utility for the same loss at low wealth than at high wealth.

To give some more intuition for the risk aversion parameters α, β in the SAHARA utility function, we can compare the absolute risk aversion γ / C_T for the CRRA utility function (with C_T the replacement ratio that we introduced earlier), with the risk aversion $\alpha / \sqrt{\beta^2 + (C_T - 1)^2}$ for the SAHARA utility function. We see two important qualitative differences between the two utility functions:

- A CRRA utility function will always ensure that $C_T > 0$, as the risk aversion γ / C_T

[3] See [Chen, Pelsser & Vellekoop(2011)] for these closed-form expressions and a more extensive discussion of the properties of SAHARA utility functions.

approaches infinity for $C_T \downarrow 0$. However, for SAHARA utility the risk aversion stays finite, even for $C_T < 0$. This means that negative wealth is possible for SAHARA utility, and we may have to control for this.

- The CRRA utility function attaches no specific significance to the replacement ratio $C_T = 1$. On the other hand, the SAHARA risk aversion peaks at $C_T = 1$, and decreases to zero for $C_T > 1$ or $C_T < 1$. The height of the peak in risk aversion is controlled by the parameter β: for smaller βwe get a higher the peak. The peak in risk aversion at $C_T = 1$ implies that the optimal investment strategy will de-risk near $C_T = 1$ and will involve more risk-taking for $C_T > 1$ or $C_T < 1$. For replacement ratios $C_T \ll 1$, we can explain this behaviour as "risk-raking for resurrection"; for replacement ratios $C_T \gg 1$, we can explain this as pure return-seeking behaviour. This feature of positive but diminishing risk aversion when we reach low levels of wealth (or, in our case, replacement ratios) helps to overcome an initial position of underfunding. We remark that one can also distinguish explicitly between the cases $C_T < 1$ and $C_T > 1$ when a mean-shortfall criterion is used; in [Siegmann(2007)] the resulting optimal investment strategies for defined benefit pension funds are derived and analysed.

We remark that the SAHARA class of preferences contains the exponential and power utility functions as limiting cases. When $\beta \searrow 0$ and $w_0 = 0$ we find for $x > 0$ that, $A(x) \rightarrow \frac{\alpha}{x}$, which corresponds to the absolute risk aversion function of the power utility function. On the other hand, if $\alpha \rightarrow \infty$ and $\beta \rightarrow \infty$ while $\frac{\alpha}{\beta}$ stays constant, we find that $A(x) \rightarrow \frac{\alpha}{\beta}$ which corresponds to the absolute risk aversion function of exponential utility.

4 Optimizing the Replacement Ratio at Retirement

If we consider a Black-Scholes economy, every[4] stochastic payoff at a future time T can be generated by an appropriate self-financing trading strategy, if there are sufficient initial funds. In our case, an agent tries to maximize the expected utility of his or her replacement ratio at time T which equals $C_T = X_T/L_T$. The objective function can now be specified as follows:

$$\max_{X\in\mathcal{A}(X_0)} \mathbb{E}\left[U\left(\frac{X_T}{L_T}\right)\right], \tag{8}$$

where $\mathcal{A}(X_0)$ denotes the class of all possible wealth processes that can be generated by self-financing strategies θ in this market with an initial capital X_0. Following the solution method of [Cox & Huang(1989)], we thus see that the dynamic stochastic optimal control problem (8) can be formulated as a static optimization problem for which the objective function can be specified as follows:

$$\begin{aligned} &\max_{X_T} \mathbb{E}\left[U\left(\frac{X_T}{L_T}\right)\right] \\ &\text{s.t. } \mathbb{E}\left[\frac{M_T}{M_0}X_T\right] = X_0. \end{aligned} \tag{9}$$

Note that the choice of the initial value M_0 will ensure that the budget $\mathbb{E}[(M_T/M_0)\cdot X_T] = X_0$ is satisfied. The optimal terminal wealth X_T^* follows from the Lagrange optimization:

$$X_T^* = I(M_T L_T)\, L_T, \tag{10}$$

with $I = (U')^{-1}$ the inverse of the marginal utility function.

4.1 SAHARA preferences

For SAHARA utility functions, we have that

$$U'(x) = \left((x-w_0) + \sqrt{\beta^2 + (x-w_0)^2}\right)^{-\alpha} = \beta^{-\alpha} e^{-\alpha \operatorname{arcsinh}((x-w_0)/\beta)}, \tag{11}$$

hence the inverse marginal utility I equals

$$I(y) = \left(U'\right)^{-1}(y) = \beta \sinh\left(-\frac{\ln y}{\alpha} - \ln\beta\right) + w_0 = \frac{1}{2}\left(y^{-1/\alpha} - \beta^2 y^{1/\alpha}\right) + w_0, \tag{12}$$

with domain $\Re^+$. The optimal final wealth is:

$$\begin{aligned} X_T^* &= I(M_T L_T) L_T = \left(\beta \sinh\left(-\tfrac{1}{\alpha}\ln(M_T L_T) - \ln\beta\right) + w_0\right) L_T \\ &= \tfrac{1}{2} A^d \left((\eta A^d)^{-\frac{1}{\alpha}} S_T^{d-\frac{d}{\alpha}+\frac{\nu}{\sigma\alpha}} - \beta^2 (\eta A^d)^{\frac{1}{\alpha}} S_T^{d+\frac{d}{\alpha}-\frac{\nu}{\sigma\alpha}}\right) + w_0 A^d S_T^d. \end{aligned} \tag{13}$$

[4] To be precise: every stochastic payoff which is measurable with respect to the filtration generated by the Brownian Motion which drives the stock price process S.

so the optimal replacement ratio at retirement is given by

$$C_T^* = \frac{X_T^*}{(AS_T)^d} = \frac{1}{2}\left((\eta A^d)^{-\frac{1}{\alpha}} S_T^{-\frac{d}{\alpha}+\frac{\nu}{\alpha\sigma}} - \beta^2(\eta A^d)^{\frac{1}{\alpha}} S_T^{\frac{d}{\alpha}-\frac{\nu}{\alpha\sigma}}\right) + w_0. \tag{14}$$

The parameter η can then be solved in explicit form by setting $X_0^* = X_0$ with X_0 as in (6) and X_0^* following from (13) for $T = 0$. The value of the investment portfolio value X at earlier times t can then be found using $X_t^* = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[X_T^* \mid \mathcal{F}_t]$.

4.2 CRRA preferences

For power utility functions with risk aversion parameter $0 < \gamma \neq 1$ we have

$$U(x) = \frac{x^{1-\gamma} - 1}{1-\gamma} \tag{15}$$

while $U(x) = \ln x$ when $\gamma = 1$, with domain $\Re^+ \cup \{0\}$ if $\gamma < 1$ and domain $\Re^+$ otherwise. Since $U'(x) = x^{-\gamma}$ we find that $I(y) = y^{-\frac{1}{\gamma}}$ for all $y > 0$ and $A(x) = \frac{\gamma}{x}$ for this class of preferences.

For an agent applying power utility to the replacement ratio, the optimal final wealth is:

$$X_T^* = (M_T L_T)^{-\frac{1}{\gamma}} L_T = \eta^{-\frac{1}{\gamma}} A^{d-\frac{d}{\gamma}} S_T^{d+\frac{\nu-d\sigma}{\sigma\gamma}}, \tag{16}$$

and the terminal replacement ratio then follows:

$$C_T^* = \frac{X_T^*}{(AS_T)^d} = \eta^{-\frac{1}{\gamma}} A^{-\frac{d}{\gamma}} S_T^{\frac{\nu-d\sigma}{\sigma\gamma}}. \tag{17}$$

As before, η can be found by setting $X_0^* = X_0$ and using both (6) and (16), while $X_t^* = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[X_T^* \mid \mathcal{F}_t]$ determines the portfolio value at all times $t \in [0, T]$.

As a special case, note that when $\nu = d\sigma$, we have $C_T^* = \eta^{-\frac{1}{\gamma}} A^{-\frac{d}{\gamma}} = \phi$. Moreover, when $\nu > d\sigma$, C_T^* is positively correlated to S_T; on the other hand, when $\nu < d\sigma$, C_T^* is negatively correlated to S_T.

4.3 Numerical Results

This section explores the effect of the parameters α and β of the SAHARA utility function on the optimal replacement ratio outcomes C_T^* in a numerical case study. All other parameters are kept fixed. We assume a Black-Scholes market with equity rate of return $\mu = 4\%$, volatility $\sigma = 16\%$ and risk-free interest rate $r = 1\%$. The initial underfunding is 20% so $\phi = 0.8$. For SAHARA utility functions, the threshold w_0 is set to 1, which implies that the optimizing agent is the most risk-averse around a replacement ratio of $C_T = 1$ and is willing to take more risk if the replacement ratio moves away from one.

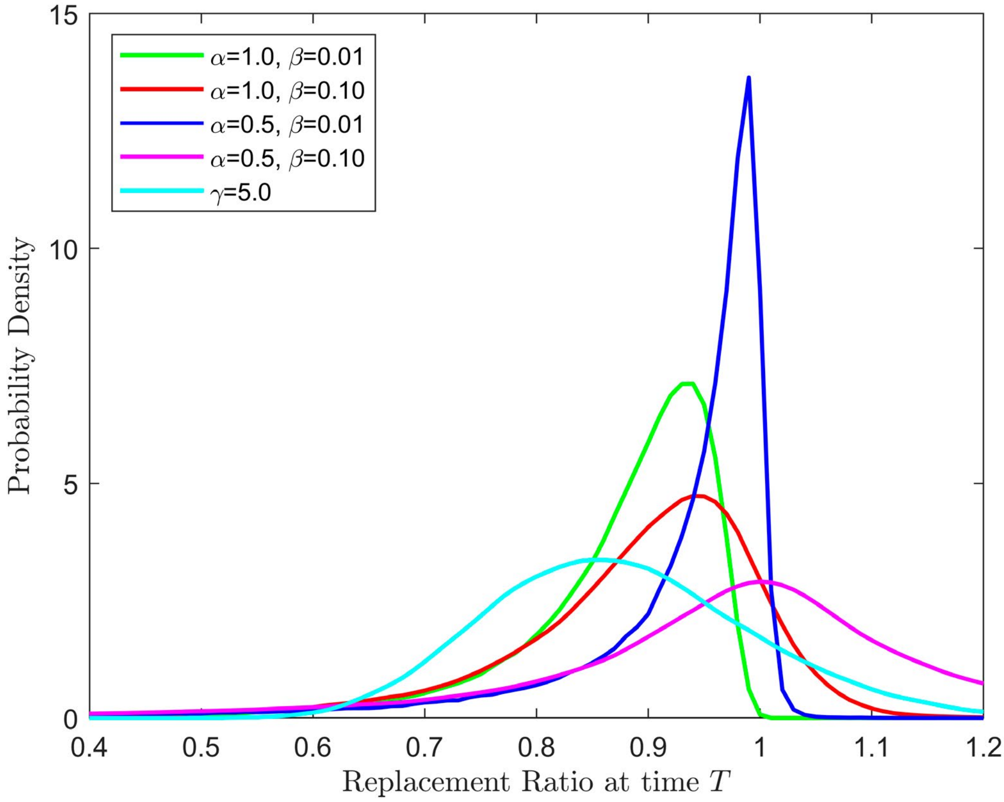


Figure 2: Plot of the probability density for the replacement ratio $C_T^* = X_T^*/L_T$.

For α and β, we take $\alpha \in \{1, 0.5\}$ and $\beta \in \{0.01, 0.1\}$ and explore all four possible combinations. To illustrate the effects on C_T^*, its probability distribution under the real-world probability measure $\mathbb{P}$ is shown. Results for CRRA preferences with $\gamma = 5$ are included for comparison.

Table 1: Overview of model parameters.

μ =0.04	r = 0.01
σ=0.16	T =40.0
S_0 = 1.00	d = 0.50
A = 1.00	w_0 = 1.00
ϕ = 0.80	

Figure 2 shows the probability density plots. Keeping α unchanged and increasing β shifts the mode of the distribution to a lower value while the right tail becomes fatter. Keeping β unchanged and decreasing α makes the mode shift slightly to the right. Table 2 displays a number of statistics to assess the optimal replacement ratio at retirement C_T^*: the mean, variance and a number of relevant quantiles. We also show the absolute

Table 2: Overview C_T^* statistics with d = 0.5, w_0 = 1.

	CRRA	SAHARA			
Statistics	γ = 5	α = 1.00, β = 0.01	α = 1.00, β = 0.10	α = 0.50, β = 0.01	α = 0.50, β = 0.10
ARA (at X_0)	8.6762	2.3539	2.2970	1.1797	1.1485
mean	0.8775	0.8742	0.8914	0.9223	1.0500
variance	0.0144	0.0093	0.0145	0.0323	0.2016
$P(C_T^* \geq 1)$	15.16%	0%	12.55%	8.74%	55.64%
$P(C_T^* \geq 0.90)$	39.87%	50.01%	55.80%	80.15%	78.55%
$P(C_T^* \geq 0.50)$	100%	99.12%	98.80%	97.90%	96.77%
$P(C_T^* < 0)$	0%	0.03%	0.05%	0.55%	0.92%

risk aversion (ARA) at the (underfunded) starting position, since this is the most important feature which distinguishes CRRA and SAHARA preferences.

If we keep α fixed and increase β from 0.01 to 0.1 , the average replacement ratio improves slightly, from 87.42% to 89.14% when α = 1 and from 92.23% to 105.00% when α = 0.5. We note that the probability of a replacement ratio above 100%, i.e., the probability of recovering from an underfunded position, increases substantially, from 0% to 12.55% when α = 1, and from 8.74% to 55.64% when α = 0.5. The probability of attaining optimal replacement ratios below zero increases slightly, from 0.03% to 0.05% when α = 1 and from 0.55% to 0.92% when α = 0.5. If we keep β unchanged, reducing α from 1 to 0.5, the average replacement ratio increases from 87.42% to 92.23% when β = 0.01, and from 89.14% to 105.00% when β = 0.1. The probability of C_T^* being above 100% increases substantially, from 0% to 8.74% when β = 0.01, and from 12.55% to 55.64% when β = 0.1, while the probability of attaining negative optimal replacement ratios increases only slightly.

Three strategies based on SAHARA utility functions outperform strategies based on CRRA utility functions with γ = 5 in terms of the expectation of the outcomes, with the case when α = 1 and β = 0.01 the only exception. The probability of overcoming the underfunding problem is higher when α = 0.5 and β = 0.1 but the price for this (and a higher average replacement ratio) is a small but positive probability (almost 1%) of a negative replacement ratio at retirement and a probability of almost 3% that the replacement ratio will be below 50%. Moreover, the distribution of the replacement ratio has a relatively high variance, as can also be deduced from Figure 2.

All this suggests that the probability of overcoming underfunding and the average replacement ratio could both be improved by reducing the value of absolute risk aversion, if one is willing to allow a small probability of ending up with negative wealth at retirement. In the next section we design adjusted investment strategies that are forced to respect an *a priori* chosen lower bound for the replacement ratio at retirement, so that negative wealth can then be avoided.

5 Introducing Lower Bounds

To ensure that the final coverage ratio does not fall below a predetermined floor, a lower bound can be imposed on the level of the coverage ratio as long as the present value of the lower bound is smaller than the initial wealth. This section illustrates how such a lower bound changes the probability density of replacement ratios at retirement when we use power or SAHARA utility functions to define our performance criteria.

Denote the lower bound by K. In general, the objective function can then be stated as follows:

$$\begin{aligned} \max_{X_T} \ & \mathbb{E}\left[U\left(\frac{X_T}{L_T}\right)\right] \\ \text{s.t. } & \mathbb{E}\left[\frac{M_T}{M_0}X_T\right] = X_0, \\ & X_T \geq KL_T \end{aligned} \tag{18}$$

for a constant K satisfying $0 \leq K < \phi$. We know that for $K = 0$ the optimal final wealth $X_T^* = I(M_T L_T)L_T$.

The paper by [Grossman & Zhou(1996)] shows that the optimal final wealth in this case can be expressed as

$$\begin{aligned} X_T^{**} &= \max\{I(M_T L_T)L_T, KL_T\} \\ &= (I(M_T L_T)L_T - KL_T)^+ + KL_T, \end{aligned} \tag{19}$$

and $X_t^{**} = e^{-r(T-t)}\,\mathbb{E}^{\mathbb{Q}}[X_T^{**} \mid \mathcal{F}_t]$ then follows.

5.1 SAHARA and CRRA preferences

To arrive at more explicit expressions, we need to determine M_0 (or, alternatively, η) in such a way that the budget constraint is satisfied when we evaluate this expression for $t = 0$. For SAHARA utility functions this tedious but straightforward calculation shows that

$$\begin{aligned} X_t^{**} = {} & c_1 A^d S_t^{d-\frac{d}{\alpha}+\frac{\nu}{\sigma\alpha}} e^{h_1\cdot(T-t)}\Phi(g_1(t)) - c_2\beta^2 A^d S_t^{d+\frac{d}{\alpha}-\frac{\nu}{\sigma\alpha}} e^{h_2\cdot(T-t)}\Phi(g_2(t)) \\ & + w_0 A^d S_t^d e^{h_3\cdot(T-t)}\Phi(g_3(t)) + KA^d S_t^d e^{h_4\cdot(T-t)}\Phi(g_4(t)), \end{aligned} \tag{20}$$

where Φ denotes the standard-normal cumulative distribution function. For the value of the strictly positive constants c_i and h_i and a specification of the functions g_i, see the Appendix. The optimal replacement ratio at retirement becomes

$$C_T^{**} = \frac{X_T^{**}}{L_T} = \left(w_0 + c_1 S_T^{-\frac{d}{\alpha}+\frac{\nu}{\sigma\alpha}} - c_2\beta^2 S_T^{\frac{d}{\alpha}-\frac{\nu}{\sigma\alpha}} - K\right)^+ + K. \tag{21}$$

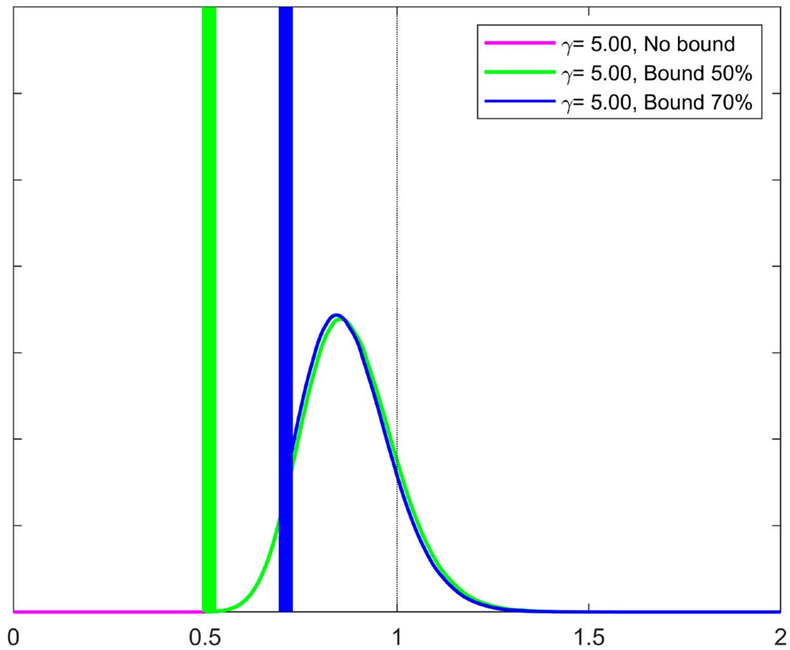


Figure 3: Probability density plots for Power utility functions with and without bounds.

The optimal outcomes for power utility functions follow from the preceding calculations if we set $\beta = w_0 = 0$ and $\alpha = \gamma$, because of (7). We thus find

$$X_t^{**} = c_1 A^d S_t^{d-\frac{d}{\alpha}+\frac{\nu}{\sigma\alpha}} e^{h_1\cdot(T-t)}\Phi(g_1(t)) + KA^d S_t^d e^{h_4\cdot(T-t)}\Phi(g_4(t)) \tag{22}$$

and

$$C_T^{**} = \left(c_1 S_T^{-\frac{d}{\gamma}+\frac{\nu}{\sigma\gamma}} - K\right)^+ + K. \tag{23}$$

5.2 Numerical Results

In this section we make a comparison between the optimization with and without lower bounds on the replacement ratio at retirement. Figures 3 and 4 show probability densities for the replacement ratios at retirement for a SAHARA utility function (with $\alpha = 0.5$ and $\beta = 0.1$) and a power utility function (with $\gamma = 5$) in the absence of a lower bound, for a lower bound of $K = 0.5$ and for a lower bound of $K = 0.7$.

When lower bounds are introduced, the probability mass for the most disadvantageous scenarios ends up in the lower bounds. This turns out to have an an effect on the mean, which decreases, and the variance, which decreases as well since the uncertainty for the worst outcomes is reduced. The probability of the optimal replacement ratio ending up

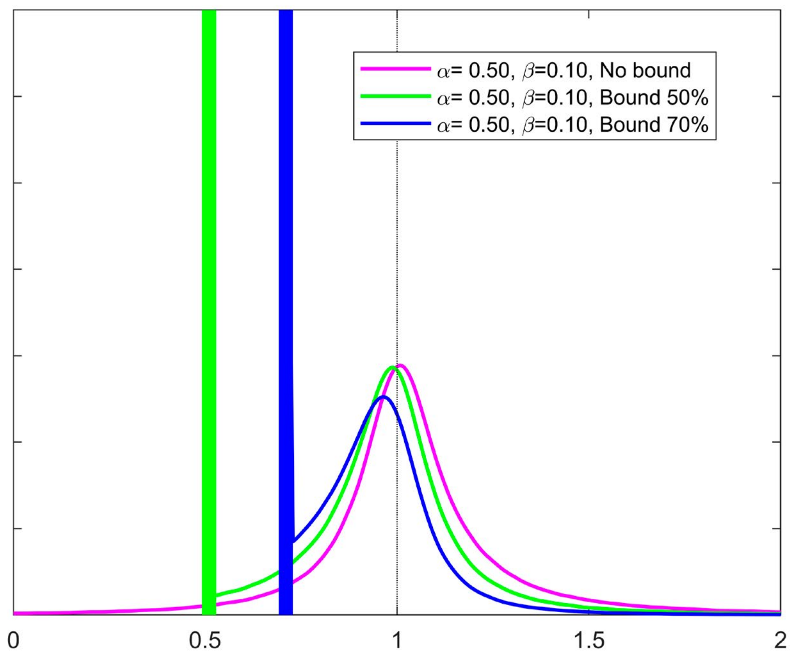


Figure 4: Probability density plots for SAHARA utility functions with and without bounds.

below 50% is almost zero for the power utility case, and we therefore see that enforcing that lower bound has hardly any effect on those preferences.

This can also be seen in Table 3, which displays a number of statistics for the optimal distribution C_T^{**}. For a lower bound of 70%, using SAHARA instead of CRRA preferences almost doubles the probability of overcoming underfunding (25% instead of 13%) but also increases the probability of ending up on the lower bound (25% instead of 7%).

5.3 Decomposition of the Optimal Strategy

The general expressions (20)-(21) allow a straightforward interpretation of the optimal investment strategies in the presence of lower constraints. The optimal strategy consists of four parts:

- **Guaranteeing a minimal funding ratio.**
  The last term in the expression, corresponding to

$$C_t^{**} = Ke^{h_4(T-t)}\Phi\left(g_4(t)\right),$$

  becomes

$$C_T^{**} = K$$

Table 3: Overview C_T^* statistics for different preferences and bounds K.

	Power			SAHARA		
lower bound K	none	0.5	0.7	none	0.5	0.7
ARA (at X_0)	8.6762	8.6762	8.6762	1.1485	1.1485	1.1485
mean	0.8775	0.8775	0.8681	1.0500	0.9564	0.8969
variance	0.0144	0.0144	0.0129	0.2016	0.0866	0.0384
$P(C_T^{**} \geq 1)$	15.16%	15.17%	12.93%	55.64%	39.94%	25.18%
$P(C_T^{**} \geq 0.90)$	39.87%	39.95%	36.11%	78.55%	65.12%	48.94%
$P(C_T^{**} \geq 0.50)$	100%	100%	100%	96.73%	100%	100%
$P(C_T^{**} = K)$		0.20%	6.78%		7.44%	25.34%

at maturity, which implies that the minimal funding ratio of K will always be reached, since this term in the optimal strategy replicates the required terminal wealth $X_T^{**} = K(AS_T)^d$.

- **Reaching the target.**
  The third term in the expression
  $$C_t^{**} = w_0 e^{h_3(T-t)}\Phi(g_3(t))$$
  becomes
  $$C_T^{**} = w_0 \mathbb{1}_{\{S_T \geq S^*\}},$$
  so the target funding ratio of $w_0 = 1$ is reached when stock prices are high enough at maturity[5]. This part of the strategy is zero if we choose the target $w_0 = 0$.

- **Risk taking.**
  The first term in the expression is
  $$C_t^{**} = c_1(S_t)^{-\frac{d}{\alpha}+\frac{\nu}{\sigma\alpha}} e^{h_1(T-t)}\Phi(g_1(t))$$
  which becomes
  $$C_T^{**} = c_1(S_T)^{-\frac{d}{\alpha}+\frac{\nu}{\sigma\alpha}} \mathbb{1}_{\{S_T \geq S^*\}}$$
  at maturity which implements our risk preferences and thus depends on the Sharpe ratio ν and the risk aversion parameter. Notice that this term would disappear if $\alpha \to \infty$ i.e. if our risk aversion were infinitely large.

[5] Here and in the rest of the paper we always assume that parameter values are chosen in such a way that S_T and C_T^{**} are positively correlated.

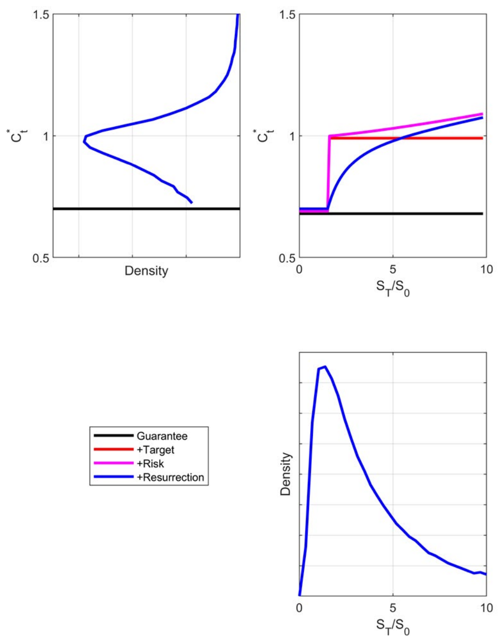
- **Extra risk taking for resurrection.**
  The second term in (20) represents

$$C_t^{**} = -c_2\beta^2(S_t)^{\frac{d}{\alpha}-\frac{\nu}{\sigma\alpha}}e^{h_1(T-t)}\Phi(g_2(t))$$

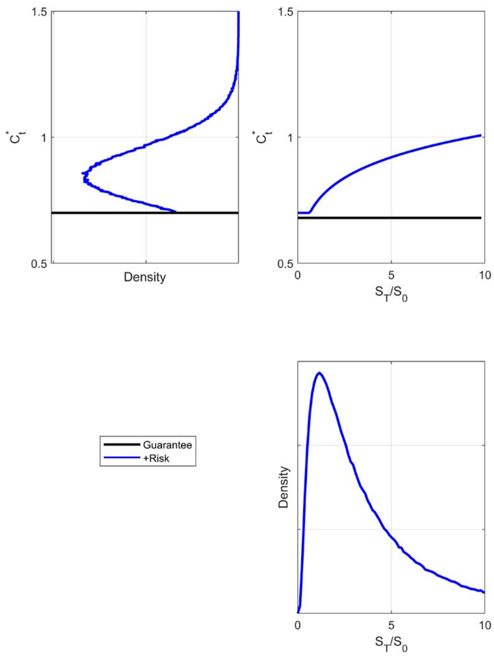
  and becomes

$$C_T^{**} = -c_2\beta^2(S_T)^{\frac{d}{\alpha}-\frac{\nu}{\sigma\alpha}}\mathbb{1}_{\{S_T\geq S^*\}}$$

  at maturity which means that this term would disappear if $\beta = 0$ i.e., if our utility function is of the CRRA class. This payoff is negative but the the sum of this component and the previous two will always be positive, ensuring that the guaranteed funding ratio K is always reached.



(a) SAHARA case



(b) CRRA case

Figure 5: Decompositions for the Optimal Strategies.

In Figure 5(a) we illustrate the different components which make up the optimal strategy for the case where $\alpha = 0.50$ and $\beta = 0.10$ when the guaranteed minimal level equals $K = 70\%$. Shown are the payoff of the portfolio replicating the guarantee (in black), the guarantee + target value (in red), the guarantee + target + optimized equity risk portfolio under CRRA (in pink) and the total portfolio (in blue), which also includes our diminished risk aversion when we are in an underfunded position, i.e., our willingness to take extra risk "for resurrection".

In the lower right corner we plot the distribution of final stock returns S_T/S_0, which is translated by the different transformations which describe optimal strategies' components in the upper right corner into the distribution of the funding ratio C_T^* in the upper left corner. The mean of S_T is 4.95 which corresponds roughly to the target value $C_T^* = w_0 = 1$ and the median of S_T is almost 2.97 which corresponds to $C_T^* = 90\%$. The value of S^* is 1.51 so whenever the terminal stock price S_T is below that value, the terminal funding ratio equals the minimal guaranteed funding ratio K.

For comparison, Figure 5(b) shows the results when we follow a CRRA strategy for the same guaranteed terminal funding ratio (70%) as well. We can clearly see that for this strategy less probability mass ends up at funding ratio levels $C_T \geq 1$ or $C_T = K$, so we have a smaller probability to evade underfunding, but we also have a smaller probability of ending up with the minimal guaranteed level K.

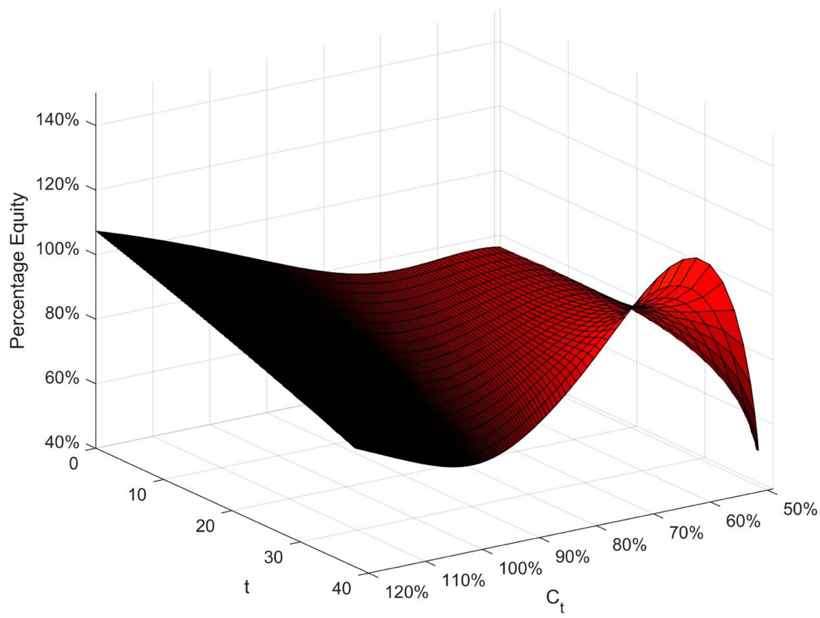


Figure 6: Equity percentage as a function of time and target value.

In Figure 6 we show for the SAHARA case with $K = 50\%$ what percentage of equity will be invested in stock at different points in time and for different values of the replacement ratio $C_t = X_t/L_t$ at that time. At the final time $T = 40$, investment in equity is minimal around $C_T = 1$ and maximal at values just above the lower bound. In the underfunded position at the start, where $t = 0$ and $C_0 = 80\%$ in this example, the percentage invested in equity is just below 77%.

6 Effect of Downward Jumps in Asset Prices

The Black-Scholes model served as a basis for describing the dynamics of asset prices but it fails to capture features such as jumps in stock prices. We now analyse the influence of such jumps on the outcomes of our investment strategy. The asset price process is modelled by a Merton jump diffusion process and we denote it by S^M to distinguish it from the earlier case. In the paper by [Merton(1976)] it is assumed to satisfy:

$$dS_t^M = \mu S_t^M dt + \sigma S_t^M dW_t^{\mathbb{P}} + S_{t-}^M (dQ_t - \mu_{Poi}\kappa dt). \tag{24}$$

The process $\{Q_t\}_{0\leq t\leq T}$ is a compound Poisson process of the form

$$Q_t = \sum_{i=1}^{N_t} (Y_i - 1), \tag{25}$$

where the stochastic variables $Y_i \in (0,1]$ represent the relative price shocks due to the i-th jump of the stock price. These are assumed to be i.i.d. and independent of all other stochastic variables and processes. The process $\{N_t\}_{0<t\leq T}$ is a Poisson process with intensity $\mu_{Poi} > 0$, which implies that on average, once per every $1/\mu_{Poi}$ units of time, the stock price will exhibit an unexpected jump and κ is the expectation (under the original measure P) of the change in the price of S as a fraction of the current price, so $\kappa = \mathbb{E}[Y_i] - 1 \leq 0$. This makes the compensated compound Poisson process $Q_t - \mu_{Poi}\kappa t$ a martingale under the original measure P.

Standard results from stochastic calculus show that

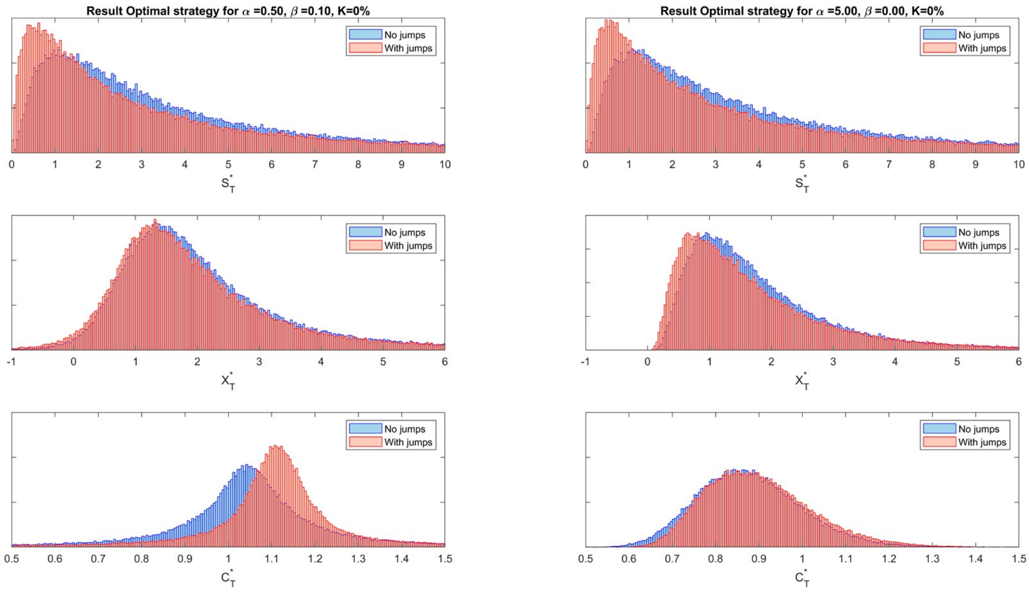
$$\begin{aligned} d\ln(S_t^M) &= (\mu - \mu_{Poi}\kappa - \tfrac{1}{2}\sigma^2)dt + \sigma dW_t^{\mathbb{P}} + \ln(Y_i)dN_t \\ S_t^M &= S_0^M e^{(\mu-\mu_{Poi}\kappa-\frac{1}{2}\sigma^2)t+\sigma W_t^P} \prod_{i=1}^{N_t} Y_i. \end{aligned} \tag{26}$$

Since the market is incomplete, there is no unique risk neutral measure and perfect replication of all stochastic payoffs that depend on the path of S on $[0, T]$ is no longer possible. The simulation study in the next section is therefore based on the implementation of the strategy for the case without jumps in stock prices. We then study whether and how the performance of our strategies deteriorates in the presence of such jumps.

6.1 Numerical Results

We take the intensity of the Poisson distribution equal to $\mu_{Poi} = 10\%$ and set $Y_i = 70\%$ so on average a crash occurs every ten years and the stock price drops 30% each time this happens[6]. The graphs show what the probability density plot for our two classes of

[6] We investigated what happens when the jump sizes have a lognormal distribution with similar characteristics, but we found that the results are almost exactly the same in that case.



(a) SAHARA case

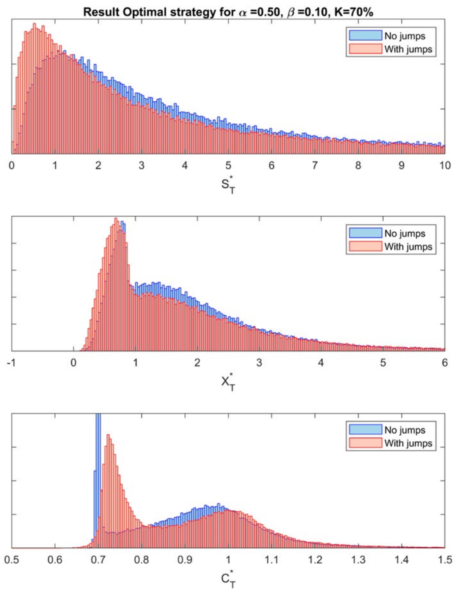
(b) CRRA case

Figure 7: Results when $K = 0$.

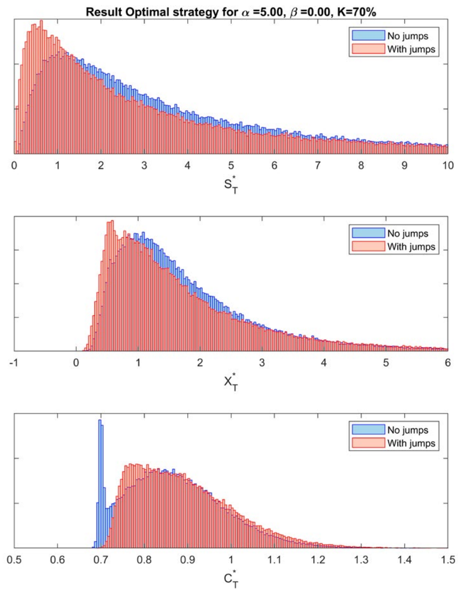
preferences will become when the dynamics of the stock follows Merton's jump diffusion process, but our investment strategy is based on the standard Black-Scholes model.

Figures 7(a) and 7(b) show that the introduction of jumps skews the distribution of stock prices to the left, but we kept its expected value the same. Even though the dynamics are quite different now, the replication strategy for the SAHARA preferences reaches roughly the same distribution as before and the funding ratio actually improves since the terminal value of assets remains the same while the terminal value of liabilities is skewed to the left, since these are proportional to S^d_T. For the CRRA case we see that terminal wealth levels are more skewed to the left and as a result the distribution of the funding ratio hardly changes at all.

When we apply the optimal strategies with lower bound constraints to the funding ratio, we obtain the results in Figures 8(a) and 8(b). For CRRA preferences, on the right, we see that the lower bound pushes some probability mass below K to the level K when there are no jumps, but when there are jumps the higher mean rate of return before jumps has the effect that the lower bound is not reached anyway in almost all cases. We see the same effect for SAHARA preferences and again notice that much more probability mass ends up around the target funding ratio value of one, both for the case with and without jumps.



(a) SAHARA case



(b) CRRA case

Figure 8: Results when $K = 70\%$.

7 Conclusion

In this paper, we consider defined benefit schemes with a *defined ambition* which aim to provide their participants with a specific “ambition”, which we model as a stochastic benchmark, where its market value is higher than the market value of the contributions.

Hence, we compare stochastic outcomes for the replacement ratio at retirement for different investment strategies when starting from an underfunded position. We consider strategies based on preferences which become more risk-averse in disadvantageous scenarios (CRRA utilities) and preferences which become less risk-averse in such scenarios (SAHARA utilities). We also examine the influence of constraints that force the replacement ratio to end up above a certain lower bound.

We find that by a making suitable choice of the parameters α and β of SAHARA utility functions, the average replacement ratio can be improved, with only a slightly higher probability of worse replacement ratios than in the CRRA case. By introducing a lower bound constraint, the probability of ending with a replacement ratio above 100% increases, but there may also be a substantial probability of ending up on the lower bound. The SAHARA utility function emphasizes the defined ambition level for the investor and the position of underfunding forces him or her to invest more in equities (compared to CRRA) in order to optimize the expected utility of terminal wealth. The long investment horizon and the positive equity risk premium ensure that the risk properties of the replacement ratio do not deteriorate much compared to the CRRA case.

We also investigated the effect of downward jumps in asset prices for both types of strategies. When the extra risk due to such jumps is compensated for by a higher value of the average rate of return for those assets, outcomes actually improve slightly in the sense that the probability of ending up above the minimally guaranteed level of the funding ratio increases for both CRRA and SAHARA-based investment portfolios.

This suggests that optimization based on SAHARA preferences with lower bound constraints is a useful tool which can be used to generate good investment strategies whenever the market value of the “ambition” is higher than the market value of the contributions.

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Appendix: Outcomes under Constraints

Optimal Terminal Wealth

Optimizing $\mathbb{E}[U(X_T/L_T)$ under the constraint $\mathbb{E}[X_T M_T] = x_0$ for a given x_0 gives that $X_T^* = L_T I(\nu L_T M_T)$ for a certain (Lagrange) parameter ν. Introducing an extra constraint $X_T \geq KL_T$ for given (x_0-feasible) scalar value $K \geq 0$ gives

$$X_T^* = \max\{KL_T, L_T I(\nu L_T M_T)\} = KL_T + L_T(I(\nu L_T M_T) - K)^+.$$

The Lagrange parameter ν is chosen to satisfy the constraint, i.e. by making

$$\mathbb{E}[M_T \max\{KL_T, L_T I(\nu L_T M_T)\}] = x_0.$$

If $M_T = \eta(S_T)^m$ and $L_T = \xi(S_T)^d$ and $I(y) = c_1 y^{p_1} + c_2 y^{p_2} + w_0$ we can rewrite this as

$$\begin{aligned} X_T^* &= \mathbb{1}_{\{S_T \geq S^*\}}(\xi S_T^d) c_1 \nu^{p_1} (\eta S_T^m)^{p_1} (\xi S_T^d)^{p_1} + \mathbb{1}_{\{S_T \geq S^*\}} w_0 (\xi S_T^d) \\ &\quad + \mathbb{1}_{\{S_T \geq S^*\}}(\xi S_T^d) c_2 \nu^{p_2} (\eta S_T^m)^{p_2} (\xi S_T^d)^{p_2} + \mathbb{1}_{\{S_T \leq S^*\}} K (\xi S_T^d) \\ &= (a_1 S_T^{b_1} + a_2 S_T^{b_2} + a_3 S_T^{b_3}) \mathbb{1}_{\{S_T \geq S^*\}} + a_4 S_T^{b_4} \end{aligned}$$

where S^* solves $K = I(\nu\eta\xi(S^*)^{m+d})$ while $a_3 = (w_0 - K)\xi$, $a_4 = K\xi$ and $b_3 = b_4 = d$ and for $i = 1, 2$ we have $a_i = \xi c_i (\nu\eta\xi)^{p_i}$ and $b_i = d + p_i(m + d)$.

Replication of Power Digitals

The value at time $t \in [0, T[$ of a contingent claim paying $(S_T)^b \mathbb{1}_{\{S_T \geq S^*\}}$ at time T equals

$$f_{b,S^*}(S_t, t) = e^{-rT} \mathbb{E}^{\mathbb{Q}}[(S_T)^b \mathbb{1}_{\{S_T \geq S^*\}} | S_t] = (S_t)^b e^{(b-1)(r + \frac{1}{2} b\sigma^2)(T-t)} N(d_{1,b,S^*}(S_t, t))$$

with

$$d_{1,b,S^*}(S, t) = \frac{\ln(S/S^*) + (r + \sigma^2(b - \frac{1}{2}))(T-t)}{\sigma\sqrt{T-t}}.$$

The number of stocks to hold at time t to replicate this payoff then follows, since

$$\Theta_{b,S^*}(S_t, t) = \partial_S f_{b,S^*}(S_t, t) = \frac{f_{b,S^*}(S_t, t)}{S_t}\left(b + \frac{N'(d_{1,b,S^*}(S_t, t)))}{N(d_{1,b,S^*}(S_t, t)\sigma\sqrt{T-t}}\right).$$

Replication of Optimal Terminal Wealth

We thus find that the optimal wealth process and optimal number of stocks to invest in are given by

$$\begin{aligned} X_t^* &= a_1 f_{b_1,S^*}(S_t, t) + a_2 f_{b_2,S^*}(S_t, t) + a_3 f_{b_3,S^*}(S_t, t) + a_4 f_{b_4,0}(S_t, t), \\ \theta_t^* &= a_1 \Theta_{b_1,S^*}(S_t, t) + a_2 \Theta_{b_2,S^*}(S_t, t) + a_3 \Theta_{b_3,S^*}(S_t, t) + a_4 \Theta_{b_4,0}(S_t, t). \end{aligned}$$

with

$$
\begin{aligned}
a_1 &= \tfrac{1}{2}A^d(\nu\eta A^d)^{-1/\alpha}, & b_1 &= \tfrac{\mu-r}{\alpha\sigma^2} + d(1-\tfrac{1}{\alpha}),\\
a_2 &= -\tfrac{\beta^2}{2}A^d(\nu\eta A^d)^{1/\alpha}, & b_2 &= -\tfrac{\mu-r}{\alpha\sigma^2} + d(1+\tfrac{1}{\alpha}),\\
a_3 &= (w_0 - K)A^d, & b_3 &= d,\\
a_4 &= KA^d, & b_4 &= d,\\
S^* &= (U'(K)/(\nu\eta A^d))^{1/(d-\sigma^{-2}(\mu-r))}.
\end{aligned}
$$

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