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Strategic Asset Allocation with Climate Change

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Strategic Asset Allocation with Climate Change ^{*}

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Abstract

We develop a top-down strategic pathway towards green investing and show that eco-investing should not be a puzzle or a sacrifice if investors consider the unknown impact of climate change on different asset classes. We investigate the impact of climate change over a large universe of asset classes including stocks, bonds, alternatives and a list of green assets which have low-carbon emissions, in terms of their time-varying risk-return trade-offs using a factor-based vector autoregression (VAR) model with climate risk as an additional state variable. Estimation results show that green assets are in general resilient against temperature change, while many grey assets are negatively related to the risk of warming. When simulating future investment opportunity set, we disentangle the climate effects from the return processes and introduce two climate scenarios: an optimistic scenario implied by the VAR estimates; and a pessimistic one based on the DICE model. We find a substantial demand for green assets under the pessimistic scenario, which verifies the hypothesis that investors with a decent awareness of climate change should find it profitable to invest in green.

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1 Introduction

William D. Nordhaus won the 2018 Nobel Prize in Economic Sciences for integrating climate change into long-run macroeconomic analysis. His DICE model (Dynamic Integrated model of Climate and the Economy, see [Nordhaus \(1992\)](#), [Nordhaus and Boyer \(2000\)](#) and [Nordhaus \(2008\)](#)) is the first that explores how economic activities such as consumption, production, etc., can affect carbon emissions, how emissions can affect the environment, and how the environment then feeds back to affect the economy. Inspired by his work in climate economics, this paper investigates how institutional investors should react to hedge against the impact of climate change on their investment portfolios.

In this paper, we propose a strategic asset allocation model that allows for hedging climate risk over a large scope of asset classes including stocks, bonds, some alternatives such as corporate bonds, commodities, listed real estates, hedge funds and a list of green assets which have lower exposure to greenhouse gas emissions. We follow the factor-based asset liability management (ALM) approach using a vector autoregression (VAR) model, which is similar to [Van Binsbergen and Brandt \(2007\)](#) and [Hoevenaars et al. \(2008\)](#), to model the time-varying investment opportunity set. In addition to the traditional macro risk factors such as nominal rates, yield spreads, dividend-price ratios, etc., we also include a climate related risk factor into the model to capture the impact of climate change on investment opportunities over various asset classes. Our framework allows us to bring the forecast of the climate related risk factor into the portfolio construction process. However, factor-based model can only help investors understand how each asset class responds to climate change based on the historical data but brings little message about future climate-change pathways. We propose to use a quantitative-scenario-analysis method to incorporate the forecasts of climate change into the portfolio construction process. We consider two different quantitative scenarios: (1) an optimistic scenario implied by the VAR model and; (2) a pessimistic scenario implied by the DICE model. We show a substantial demand for green assets under the pessimistic scenario, indicating that as

long as investors are fully aware of climate change, they should find it beneficial to invest in green assets.

There are two indicators of climate change: (1) physical risk data that collects extreme weather events and natural disasters, such as large-scale coastal flooding and long droughts, and (2) and transferable risk data that collects the information of earth's usual temperature change mainly caused by the increasing greenhouse gases (GHGs) emissions. Among those physical impacts related climate-economics studies, a sizable amount of work has attribute to the role of financial institutions in disaster relief, e.g. [Skoufias \(2003\)](#), [Smolka \(2006\)](#), [Barro \(2006\)](#) and [Kunreuther and Michel-Kerjan \(2011\)](#). While a lot more studies (e.g. [Nordhaus \(1994\)](#), [Econometrics \(2014\)](#) and [Stern \(2008\)](#)) use the latter risk factor as the indicator of climate change. In this paper, we also use the second definition to model climate change. In particular, we follow [Lemoine \(2017\)](#) and [Nordhaus \(2008\)](#) by using the global average temperature relative to its pre-industry level as the indicator of climate change. In a similar fashion, we define climate risk as a risk resulting from abnormal average temperature change affecting natural and human systems and regions.

Fighting climate change is expensive.¹ However, fiduciary duty (see [Board \(2015\)](#) stated in the Task Force on Climate-related Financial Disclosures (TCFD) and [IPCC. \(2014\)](#)) requires institutional investors including pension funds, insurance and reinsurance companies to actively manage their climate risk. Given the non-diversifiable feature of climate risk, the number of existing insurance or financial products that can protect against climate change is zero. A more natural strategic to cope with the challenges of moving towards low-carbon investment activities is to replace a portion of the investment portfolio to a class of assets that can not only reduce the carbon footprint of the total portfolio but also can perform overwhelmingly well when the climate is bad.

As a starting point, institutional investors can hold “green” financial indexes to reduce their carbon exposure. [Andersson et al. \(2016\)](#) categorize two groups of “green” indexes: (1) pure-play indexes that focus on clean technology, renewable energy or environmental services (such as Alternative Energy ETF) and (2) “decarbonized” indexes, initially de-

¹In order to achieve the 2 degree Celsius threshold suggested by the International Panel on Climate Change (IPCC) and the Paris Agreement, leaders of many nations have taken actions to reduce the (GHGs) emissions. Effective GHG abatement actions include shifting energy supply from fossil fuels to low-carbon energy, such as wind, nuclear or hydro power, and transiting towards energy efficient vehicles, buildings and industrial equipment. An estimate from McKinsey ([Nauc ler and Enkvist \(2009\)](#)) shows that the total upfront investment in abatement required to meet the 2 C threshold may be as high as  530 billion in 2020 or  810 per year in 2030.

veloped by AP4, one of Swedens largest pension funds, that reallocate the composition of existing market indexes such as S&P 500 by divesting the high-carbon-footprint stocks. Although it is still a gray area in terms of qualifying a green investment. In this paper, we generally consider all the financial activities that present a positive exposure to both types of the green indexes as green investing.

Over the past decade, the green market enjoys a fast growth rate. According to the latest Bloomberg report, the global green bond market with an annual issuance of less than \$1 billion a decade ago has grown to more than \$170 billion by the end of 2017.² Most of the institutional investors have embraced the concept of ESG (“Environmental, Social, and Governance”) into their investment process. According to the U.S. SIF Foundation Biennial Report 2018³, the ESG assets owned by institutional investors in the U.S. expanded to \$11.6 trillion in 2018, up 44% from \$8.1 trillion in 2016.

However, despite the accessibility to the green market and the awareness of the shift towards more stringent climate mitigation policies, the motivation behind investing in green financial instruments is, at present, more of a moral choice than a speculative opportunity (see [Riedl and Smeets \(2017\)](#)). The myth behind the motivation of green investment is mainly driven by a lack of knowledge about the potential impact of climate change on various asset classes and on the influence on short-term, medium-term and long-term investment performance for institutional investors. Most investors believes that they cannot use their investment portfolio to improve environmental performance without sacrificing investment rewards [Walley and Whitehead \(1994\)](#), especially for short-term investors [Engle et al. \(2018\)](#). On the other hand, some literature such as [Derwall et al. \(2005\)](#) finds a substantial investment opportunity in green investing, periodically.

The first goal of this paper is to help institutional investors seek a strategic pathway to a low-carbon portfolio without giving up their investment rewards. The second goal is to discover the myth behind green investing. We find that uncertainty is the critical barrier to green investing. To eliminate climate risks, investors are confronted with two dimensions of unknowns: (1) unknown knowns (anticipated possibilities) and (2) unknown unknowns⁴ (unforeseeable outcomes). Our model suggests that it is the joint effect of the

²<https://www.bloomberg.com/opinion/articles/2018-07-24/green-bonds-could-become-a-key-part-of-global-capital-markets>.

³<https://www.ussif.org/>.

⁴“Unknown unknown” a phrase from a response of the United States Secretary of Defense Donald Rumsfeld gave to a question at a U.S. Department of Defense news briefing on February 12, 2002 about

two dimensions of unknowns that create the motivation puzzle, and investing in low-carbon assets should not be a myth if taking into account the uncertain impact of climate change on each of the asset classes.

Under the first dimension of unknowns, we address two unknown knowns. First, the impact of climate change on various asset classes is unclear to most of the investors. Despite the amount of existing studies (such as DICE model, Cambridge Econometrics' E3ME model [Econometrics \(2014\)](#) and [Change \(2001\)](#)) that integrated climate risk to the macro-economics model investigating the impact of climate change on regional (Gross Domestic Product GDP), income level and other macro factors; as well as a large amount of studies about climate change on the the commodities' market [Adams et al. \(1990\)](#), [Höök and Tang \(2013\)](#), etc., institutional investors still have very little knowledge about the relationship between climate change and the performance of various financial instruments. Second, returns of invested assets are hard to predict, especially those green assets. It is well known from the literature (such as [Merton \(1980\)](#)) that expected asset returns are notoriously difficult to estimation from historical data. This is driven by the fact the standard error of the historical average return shrinks at the square root of the sample period's time. The longest existing market instrument has a history of no longer than 100 years and the length of low-carbon indexes is even shorter, with a history of fewer than 10 years. Therefore, estimating the expected returns of green instruments using time series data is subject to substantial estimation error.

The second dimension of unknowns worry the investors most. In this paper, we consider two kinds of unknown unknowns. The main source of unknown unknown is an unpredictable climate-change pathway. The projected earth's surface temperature change varies between 1.4°C to 5.8°C by the end of 21st century [Gitay et al. \(2002\)](#). The large dispersion not only reflect the unpredictability of climate-change path, but also the associated impact on capital market. The other challenge is a mismatched and an unknown timing between market awareness of the rewards of green investing and the limited investment horizons. Climate risk is an ultra-long (30 years and beyond) term risk. Thus far, temperature change has had minimal influence on the financial market performance, especially for active managers with short-term investment horizons. Therefore, many fund managers find it risky to replace their portfolio fully by green instruments because of the

the lack of evidence linking the government of Iraq with the supply of weapons of mass destruction to terrorist groups.

short-term risk before the possibility of significant gain from their green investment.

To deal with the two phases of unknowns, we introduce a top-down investment strategy that can guide institutional investors towards green investing strategically. The pathway is demonstrated in Figure 1. The first two steps mean to address the first dimension of unknowns. As a starting point (Step 1), investors can use historical data to determine the probability distribution of their investment opportunity set and the likelihood of climate impact on various asset classes.

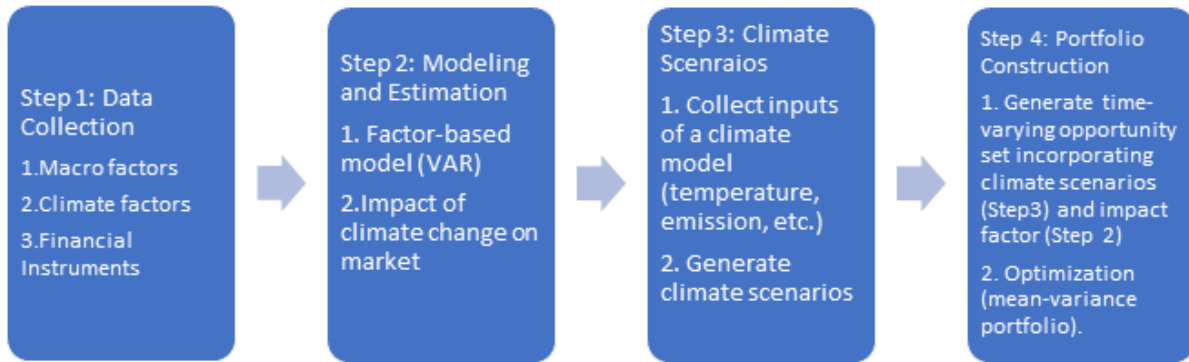
In Step 2, investors need to construct a model to capture the dynamics of asset returns and to quantify the impact of climate risk on asset returns, using the data collected in Step 1. In this paper, we build a factor-based vector autoregression (VAR) model to investigate the impact of climate change on asset returns and to study the time-varying risk-return trade-off over a large universe of asset classes. We model climate risk as the change of land-surface average temperature anomalies relative to the Jan 1951-Dec 1980 pre-industrial level. The temperature-change series plays a role as an external shock quantifying the speed of warming. The estimation results not only prepare investor an opportunity set that carries the information about the time-varying risk return trade-off of the investment portfolio, but also quantify the relationship between climate risk with each asset class. This climate impact is also called the “beta” of climate risk. The beta term is crucial for green investing. It not only address the first phase of unknowns by linking the climate risk and the equity market, but also prepares us for the second phase of unknowns.

Step 3 aims at addressing the first unknown unknown by simulating future temperature trajectories based on climate models. Given the uncertain feature of climate change, we consider two quantitative scenarios to reflect different investors’ view on climate change. For optimistic investors who are either not aware of or do not care about climate change, we simulate climate trajectories using the estimated VAR model since factor models tend to underestimate the speed of warming because the predicting factors barely have any explanatory power on climate change. For pessimistic investors who worry about climate change and its impact, we introduce DICE model to project the future temperature.

Step 4 helps to solve the second unknown unknown. Based on setup of previous three steps, investors can construct an investment horizon-dependent optimal portfolio from the estimated time-varying investment opportunity set. When simulating future return pro-

cesses, we disentangle the climate effect from other financial-market effect. This separated time-varying climate effect is also called “temperature beta”, which incorporates the estimated climate impacts (Step 2) and future climate scenarios (Step 3). Then investors can obtain their optimal portfolio by solving an optimal mean-variance optimization problem. This step The four-step approach not only quantifies the historical climate impact on assets but also brings this impact to the unknown future.

Figure 1: A strategic pathway towards green investing



Our study contributes to the strategic asset allocation literature in two ways. First, we extend the strategic asset allocation framework of [Hoevenaars et al. \(2008\)](#) to allow for climate risk hedging. In particular, we add the change of temperature anomalies as an additional state variable driving expected returns, beyond the traditional macroeconomic variables (i.e. dividend-price ratio, nominal short-term interest rate, credit spread, and yield spread). We show some weakly significant explanatory power of the temperature change on various market instruments. Second, we expand the asset menu to a list of eco-friendly asset classes such as green bonds, socially responsible funds and clean energy funds, in addition to stocks, bonds, and alternatives. These low-carbon assets have caught a great attention from institutional investors but have not been treated with sufficient care among the literature. Low-carbon assets have much shorter time horizons than most stocks or bonds. To deal with the uneven length of historical data, we impose restrictions on the estimation process based on the methodology of [Stambaugh \(1997\)](#) to avoid truncating the data.

We summarize five important findings of the paper. First, most “grey” assets including public equity, corporate bonds, commodities and hedge funds, that have high carbon emissions generally have negative relations with the instantaneous climate risk. However, all green funds are resilient against temperature shocks, as they are either positively

related or not related to the instantaneous temperature change. Second, the short-term investment opportunity set of green assets are better off under the “green” VAR model that incorporates climate risk than under the grey model that only considers macro factors without climate risk. As a result, adding climate risk as an additional risk factor increases the short-term demand for green assets by about 3%. Third, the optimal demand for green assets heavily depends on the underlying climate models, or in other words, the awareness of the climate change from investors. For optimistic investors who do not believe in climate change, green assets are much riskier than other asset classes both in the short-run and in the long-run, despite reasonable returns. Further, with an upward sloping term structure of risks, green assets can hardly attract long term optimistic investors. However, for pessimistic investors, who are fully aware of the trend of warming, we show that the resulting optimal portfolio shifts heavily from high-carbon assets to low-carbon assets. The take away from the two scenarios is that the non-attractiveness of green assets can be driven by the under-estimated and a misspecified climate impact on each asset class. Fourth, green assets can help to hedge liability risk. We find that asset-liability investors who aim to optimize the risk-return trade-off of the funding ratio have more exposure to the “green” assets compared with asset-only investors who do not have a liability constraint. Last, we also find that short-selling constraint largely increases the demand for green assets even under an optimistic climate scenario, but the opportunity cost is much higher than the loss from ignoring green vehicles.

The remainder of the paper is organized as follows. Section 2 introduces the strategic asset allocation strategy that considers the impact of temperature shocks. Section 3 discusses the historical data used for the VAR model and also presents the estimation results. Section 4 introduce the two climate scenarios carried by the temperature beta function and construct a green portfolio. Section 5 analyzes the optimal exposure of green assets for investors with various perspectives of climate change. In Section 6 We use a certainty equivalent for the economic valuation of various sub-optimal portfolios. Section 7 concludes.

2 Strategic Asset Allocation

In this section, we introduce an environment-friendly strategic asset allocation for institutional investors who aim to increase their holdings of green assets without sacrificing long-term rewards. We define green stocks (funds) as the stocks of companies that either belong to environment-friendly industries or operate in a socially responsible manner.

The pathway towards a decarbonized portfolio requires two key components. First, it is important to determine whether the performance of a certain asset class is sensitive to climate change which we proxy by the temperature change relative to its pre-industrial levels. Second, it is necessary to distinguish between well- and poor-performing green assets in terms of their risk and return trade-offs over various investment horizons. Similarly to [Andersson et al. \(2016\)](#) we consider two types of green funds:

- *pure-play* green indexes – include mutual funds that only invest in companies engaged in environmentally supportive businesses, such as alternative energy, clean technology, and/or environmental service;
- *decarbonized* green indexes – financial indexes that underweight or remove the companies with relatively high carbon footprints.⁵

We consider investment strategies for two types of institutional investors: liability-constrained and unconstrained institutions. Institutions with constraints on their liabilities (projected payout obligations) should employ an asset-liability or a liability-driven investment approach that seeks the portfolio that minimizes the risk of capital shortfall. Such liability-constrained institutions include banks, life insurance companies, and defined benefit pension plans due to their solvency capital requirement. The other type of institutions (e.g., defined contribution pension plans) has fewer or no liability constraints and thus, can employ an asset-only approach to construct their optimal portfolio. The asset-only approach follows the mean-variance framework of [Markowitz \(1959\)](#) which maximizes the attainable risk-return trade-off without considering the risk of liabilities.

⁵The carbon footprint of a company refers to its annualized Green House Gas (GHG) emissions relative to its business metric such as revenue, sales or units produced.

2.1 Return Dynamics

We model the dynamics of all relevant variables by a first-order vector autoregressive process (VAR(1)) given by

$$\mathbf{R}_{t+1} = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{R}_t + \mathbf{e}_{t+1} \quad (1)$$

where \mathbf{R}_t is a $h \times 1$ vector of endogenous variables at time t , \mathbf{A}_0 is a $h \times 1$ vector of intercepts, \mathbf{A}_1 is a time-invariant $h \times h$ matrix, and \mathbf{e}_t is a $h \times 1$ vector of normally distributed error terms with zero mean vector and covariance matrix Ω .

The VAR model (1) has been widely used in academic literature. [Campbell et al. \(2003\)](#) developed a stylized setup to describe the dynamics of asset returns and predictive macroeconomic variables using the VAR model. [Hoevenaars et al. \(2008\)](#) extends the model of [Campbell et al. \(2003\)](#) by adding four alternative asset classes (corporate bonds (also called credits), listed real estate, commodities and hedge funds) and additional state variables (risky liabilities, credit spread, real and nominal rate, dividend-price ratio, and the yield spread).⁶ One of the major advantages of the VAR model (1) is that it provides a simple framework to investigate the time-varying feature of the risk-return trade-off of each asset/liability jointly with macro-economic variables.

We extend the VAR of [Hoevenaars et al. \(2008\)](#) in two important ways. First, we introduce temperature change as an additional state variable predicting expected returns. In this respect, some recent empirical studies have shown a significant impact of climate change on capital markets and on economic growth (see [Novy-Marx \(2014\)](#), [Dell et al. \(2012\)](#), [Bansal and Ochoa \(2012\)](#), and [Bansal et al. \(2016\)](#)). In particular, [Bansal et al. \(2016\)](#) find a negative relationship between the temperature change and expected equity returns using the data from the US equity markets. Second, we add several green assets to the investment portfolio.

Next we describe the dynamics of the variables in our model. The time t values of all

⁶Alternative is an investment vehicle that is beyond the conventional investment types, such as stocks, government bonds, and cash. Typical examples of alternatives include corporate bonds, real estates, hedge funds, private equities, commodities, etc.

relevant variables in our model are given by the following vector

$$\mathbf{z}_t \equiv \begin{pmatrix} r_{tb,t} \\ \mathbf{s}_t \\ \mathbf{x}_t \end{pmatrix} \quad (2)$$

where $r_{tb,t}$ represents the real return on 3-month Treasury bill (T-bill), \mathbf{s}_t is a $(k \times 1)$ vector of state variables, and \mathbf{x}_t is an $((l + m + n) \times 1)$ vector of log excess returns on: l conventional assets (stocks and government bonds) and liabilities, m alternative assets, and n green assets.

Now we describe the components \mathbf{s}_t and \mathbf{x}_t of vector \mathbf{z}_t . The state variables \mathbf{s}_t are relevant for predicting asset returns. We use the following five predictive variables:

1. r_{nom} – the nominal 3-month interest rate;
2. dp – the dividend-price ratio;
3. y_{spr} – the yield spread;
4. c_{spr} – the credit spread;
5. ΔT – temperature change.⁷

The first four state variables together with r_{tb} are commonly used in the literature. For instance, [Campbell and Shiller \(1991\)](#) show that yield spreads can predict interest rate dynamics. [Campbell and Shiller \(1988b\)](#) find strong predictive power of dividend-price ratio for stock returns. Nominal rates can also capture the dynamics of the equity market (see [Campbell \(1987\)](#)). [Campbell et al. \(2003\)](#), [Campbell and Viceira \(2005\)](#) include the nominal rate, yield spread, and dividend-price ratio in the VAR and find strong explanatory power of these state variables for the dynamics of future returns. [Brandt and Santa-Clara \(2006\)](#) use the dividend yield, yield spread, credit spread, and the returns on T-bills to capture the dynamics of future returns. Recent studies by [Bansal and Ochoa \(2012\)](#) and [Bansal et al. \(2016\)](#) show that change in temperature can also explain variations in returns. Since we consider green assets in our analysis, we also include the

⁷We use the temperature change series $\Delta T_{t+1} = T_{t+1} - T_t$ for the empirical analysis because the temperature series itself is non-stationary.

temperature change to the vector of state variables. This will allow us to compare the sensitivity of different asset classes to climate risk.

The vector of log returns in excess of the 3-month T-bill return $r_{tb,t}$ is split in three components

$$\mathbf{x}_t = (\mathbf{x}_{1,t}, \mathbf{x}_{2,t}, \mathbf{x}_{3,t})^\top \quad (3)$$

where $\mathbf{x}_{1,t}$ is an $(l \times 1)$ vector containing excess returns on primary asset class and liabilities⁸, $\mathbf{x}_{2,t}$ is an $(m \times 1)$ vector of excess returns on alternative assets, and $\mathbf{x}_{3,t}$ is an $(n \times 1)$ vector of excess returns for both types of green funds, i.e., pure-play and decarbonized green indexes. We split the vector of excess returns in three parts for two reasons. First, the three vectors of excess returns represent three investment categories: $\mathbf{x}_{1,t}$ includes conventional instruments, $\mathbf{x}_{2,t}$ collects alternatives, and $\mathbf{x}_{3,t}$ only focuses on green assets. Second, the available historical data for each category are of different lengths. For the ease of exposition, Table 1 explains all variables that we use including the variables in vector \mathbf{x}_t . Thus, our VAR(1) model contains 21 variables. Recall Figure 1, the collection of macro and climate risk factors as well as the selection of investment vehicles is considered as the first step of our green investment pathway.

An important problem with estimation of our VAR model is the uneven lengths of the available time-series. As should be expected, the time-series for primary assets, such as stocks and bonds, are available over much longer time horizons compared with the other two asset classes. For example, the history of green indexes is fairly short with an average time-series horizon of 10 years. Concurrently, it is not desirable to truncate return histories because the discarded returns may also provide additional information for the analysis. Thus, to optimally use the data to estimate the dynamics of alternatives and green assets, we employ the techniques of [Stambaugh \(1997\)](#) to deal with uneven lengths

⁸We define the return on liabilities as the return on long-term (in our case 20 years constant maturity) real bonds because, for example, a typical defined benefit pension fund has liabilities with a duration of 15 to 20 years in real terms.

⁹Green bonds are fixed income securities in which the proceeds are exclusively applied to businesses that promote environmental sustainability. A green bond index applies an environmental filter to the standard fixed income indexes so as to reduce the carbon footprint of the index.

¹⁰Green-economy funds are also known as low-carbon economy funds whose basic construction principle is to take a standard benchmark, such as the S&P500, and divest companies with relatively high carbon footprints.

¹¹Socially Responsible Funds also take standard stock indexes as their construction principle, while composite stocks are ranked and filtered based on their environmental, social and governance (ESG) rates which not only concern environmental sustainability of the composite stocks but also some social and ethical criteria.

Table 1: Variables in our VAR model

Notation	Variable
$r_{tb,t}$	3-month real interest rate
State variables \mathbf{s}_t	
dp_t	dividend–price ratio
$r_{nom,t}$	3-month nominal rate
$y_{spr,t}$	yield spread
$c_{spr,t}$	credit spread
ΔT_t	temperature change
Primary asset class and liabilities (\mathbf{x}_1)	
$x_{s,t}$	excess return on stocks
$x_{b,t}$	excess return on government bonds
$x_{L,t}$	excess rate of liability change
Alternative asset class (\mathbf{x}_2)	
$x_{cor,t}$	excess return on corporate bonds (credits)
$x_{com,t}$	excess return on commodities
$x_{re,t}$	excess return on listed real estates
$x_{hf,t}$	excess return on hedge funds
Green asset class (\mathbf{x}_3)	
$x_{gb,t}$	excess return on green bonds ⁹
$x_{ge,t}$	excess return on green-economy funds ¹⁰
$x_{sri,t}$	excess return on socially responsible indexes (SRI) ¹¹
$x_{pp1,t}$	excess return on environmental service funds
$x_{pp2,t}$	excess return on alternative energy funds
$x_{pp3,t}$	excess return on clean technology funds
$x_{pp4,t}$	excess return on clean energy funds
$x_{pp5,t}$	excess return on green energy funds

of time series. In this respect, our approach is similar to [Hoevenaars et al. \(2008\)](#) who use these techniques with assets beginning at two different dates. However, in contrast to [Hoevenaars et al. \(2008\)](#), our VAR model has three different starting dates, thereby requiring extra layer of estimation restrictions.

We impose two restrictions on the vectors \mathbf{x}_2 and \mathbf{x}_3 , in response to the large dimension of the VAR(1) model and the uneven historical length of investments. First, the non-classic asset classes (variables in \mathbf{x}_2 and \mathbf{x}_3) are assumed to provide no dynamic feedback to the primary assets \mathbf{x}_1 and state variables \mathbf{s}_t . Second, we assume that green assets \mathbf{x}_3 have zero explanatory power for the dynamics of alternative assets \mathbf{x}_2 . Relaxing the two restrictions leads to the truncated-sample estimators which do not use the early-period observations that are missed in \mathbf{x}_2 or \mathbf{x}_3 . As a consequence, the truncated-sample estimators not only ignore the useful information in the earlier period for estimation, but

also make counter intuitive assumptions that green assets can provide dynamic feedback to state variables and other asset classes.

Now we describe the dynamics of each state variable. The dynamics of the real interest rate $r_{tb,t}$, state variables \mathbf{s}_t , and primary assets with liabilities $\mathbf{x}_{1,t}$, that is,

$$\mathbf{y}_t \equiv (r_{tb,t}, \mathbf{s}_t, \mathbf{x}_{1,t})^\top \quad (4)$$

is specified as an unrestricted VAR with

$$\mathbf{y}_{t+1} = \mathbf{b} + \mathbf{B}\mathbf{y}_t + \epsilon_{1,t+1} \quad (5)$$

where \mathbf{b} is a (9×1) vector, \mathbf{B} is a unrestricted (9×9) matrix, and the error term $\epsilon_{1,t+1}$ is normally distributed with mean zero and a (9×9) covariance matrix $\Sigma_{\epsilon_1 \epsilon_1}$.

The dynamics of excess returns in $\mathbf{x}_{2,t}$, which has a much shorter length of available history than \mathbf{x}_1 , is given by the *restricted* VAR model

$$\mathbf{x}_{2,t+1} = \mathbf{c} + \mathbf{C}_0\mathbf{y}_{t+1} + \mathbf{C}_1\mathbf{y}_t + \mathbf{C}_2\mathbf{x}_{2,t} + \epsilon_{2,t+1} \quad (6)$$

where \mathbf{c} is a (4×1) vector, \mathbf{C}_0 and \mathbf{C}_1 are both unrestricted (4×9) matrices, \mathbf{C}_2 is a diagonal matrix indicating that assets in $\mathbf{x}_{2,t}$ affect only their own returns, but not the returns of the assets in $\mathbf{x}_{2,t}$, $\epsilon_{2,t} \sim \mathbf{N}(0, \Sigma_{\epsilon_2 \epsilon_2})$ with $\Sigma_{\epsilon_2 \epsilon_2}$ being a diagonal covariance matrix.

The *restricted* VAR model for $\mathbf{x}_{3,t}$ is given by

$$\mathbf{x}_{3,t+1} = \mathbf{d} + \mathbf{D}_0\mathbf{y}_{t+1} + \mathbf{D}_1\mathbf{y}_t + \mathbf{D}_2\mathbf{x}_{2,t+1} + \mathbf{D}_3\mathbf{x}_{2,t} + \mathbf{D}_4\mathbf{x}_{3,t} + \epsilon_{3,t+1} \quad (7)$$

where \mathbf{d} is a (8×1) vector, D_0, D_1 are unrestricted (8×9) matrices, D_2, D_3 are unrestricted (8×4) matrices, and D_4 is a (8×8) diagonal matrix, and $\epsilon_{3,t} \sim \mathbf{N}(0, \Sigma_{\epsilon_3 \epsilon_3})$ with $\Sigma_{\epsilon_3 \epsilon_3}$ being a diagonal covariance matrix. The two restrictions on \mathbf{x}_2 and \mathbf{x}_3 implicitly set the covariance of $\epsilon_{2,t}$ and $\epsilon_{3,t}$ equal to zero.

Putting (5), (6), and (7) together, the complete VAR(1) model for the state vector \mathbf{z}_{t+1} is given by

$$\mathbf{z}_{t+1} = \Phi_0 + \Phi_1\mathbf{z}_t + \mathbf{u}_{t+1} \quad (8)$$

where

$$\Phi_0 = \begin{pmatrix} \mathbf{b} \\ \mathbf{c} + \mathbf{C}_0 \mathbf{b} \\ \mathbf{d} + \mathbf{D}_0 \mathbf{b} + \mathbf{D}_2 \mathbf{c} + \mathbf{D}_2 \mathbf{C}_0 \mathbf{b} \end{pmatrix}$$

$$\Phi_1 = \begin{pmatrix} \mathbf{B} & 0 & 0 \\ \mathbf{C}_0 \mathbf{B} + \mathbf{C}_1 & \mathbf{C}_2 & 0 \\ \mathbf{D}_0 \mathbf{B} + \mathbf{D}_1 + \mathbf{D}_2 (\mathbf{C}_0 \mathbf{B} + \mathbf{C}_1) & \mathbf{D}_2 \mathbf{C}_2 + \mathbf{D}_3 & \mathbf{D}_4 \end{pmatrix}$$

representing the 21×1 vector of intercepts and the 21×21 matrix of slope coefficients, respectively. The shocks \mathbf{u}_t satisfy the following distributional assumptions

$$\mathbf{u}_{t+1} \sim \mathbf{N}(0, \Sigma) \quad (9)$$

where the covariance matrix Σ is given by

$$\Sigma \equiv \begin{pmatrix} \Sigma_{\epsilon_1 \epsilon_1} & \Sigma_{\epsilon_1 \epsilon_1} \mathbf{C}'_0 & \Sigma_{\epsilon_1 \epsilon_1} (\mathbf{D}_0 + \mathbf{D}_2 \mathbf{C}_0)' \\ \mathbf{C}_0 \Sigma_{\epsilon_1 \epsilon_1} & \Sigma_{\epsilon_2 \epsilon_2} + \mathbf{C}_0 \Sigma_{\epsilon_1 \epsilon_1} \mathbf{C}'_0 & \Sigma_{\epsilon_2 \epsilon_2} \mathbf{D}'_2 \\ (\mathbf{D}_0 + \mathbf{D}_2 \mathbf{C}_0) \Sigma_{\epsilon_1 \epsilon_1} & \mathbf{D}_2 \Sigma_{\epsilon_2 \epsilon_2} & \Sigma_{\epsilon_3 \epsilon_3} + \mathbf{D}_2 \Sigma_{\epsilon_2 \epsilon_2} \mathbf{D}'_2 + (\mathbf{D}_0 + \mathbf{D}_2 \mathbf{C}_0)' \Sigma_{\epsilon_1 \epsilon_1} (\mathbf{D}_0 + \mathbf{D}_2 \mathbf{C}_0)' \end{pmatrix}.$$

Similarly to [Campbell and Viceira \(2005\)](#) and [Hoevenaars et al. \(2008\)](#), we also assume that shocks can be cross-sectionally correlated but are homoskedastic and independently distributed over time. Our VAR(1) framework not only makes the optimal use of all available data, but it also ensures a semi-definite estimate of Σ .

2.2 Portfolio Choice

We consider two types of asset allocation strategies: *asset-liability* and *asset-only* investment strategies. The former strategy is important for institutional investors that have long-term payment obligations (e.g., defined-benefit pension plans): they need to make sure that their assets are large enough to cover their liabilities. A liability-driven investment approach can help to achieve this goal as it takes into account the correlation between assets and liabilities when determining the optimal portfolio allocation. The more standard asset-only approach to portfolio construction is relevant for institutional investors whose liabilities are not long term (e.g., banks).

We first discuss the asset-liability investment strategy. One common way to deal with asset allocation with liabilities is to formulate a utility criterion that is a function of the funding ratio that is defined as the ratio of the (pension) fund's assets to its liabilities. In this respect, we follow the approach of [Leibowitz and Kogelman \(1994\)](#) by expressing the asset–liability portfolio from a funding ratio return perspective.

The funding ratio return $r_{F,t+j}$ over the period $t + j$, $j \geq 1$, based on the log-linear approximation of [Hoevenaars et al. \(2008\)](#), is given by

$$r_{F,t+j} = \alpha'_t \left(\mathbf{x}_{A,t+j} + \frac{1}{2} \sigma_A^2 \right) - x_{L,t+j} - \frac{1}{2} \alpha'_t \Sigma_{AA} \alpha_t \quad (10)$$

where α'_t is the time- t vector of portfolio weights for the risky assets (the fraction of wealth $1 - \iota' \alpha_t$ is invested in T-bills), $\mathbf{x}_{A,t}$ is a (14×1) vector of logarithmic excess returns on risky assets specified by vector \mathbf{x}_t in (3) excluding the excess rate of liability change $x_{L,t}$, Σ_{AA} is the corresponding (14×14) covariance matrix of returns, σ_A^2 is a (14×1) vector of diagonal elements of Σ_{AA} .

We look for the optimal buy-and-hold investment strategy over the period of length τ . Thus, we let $\alpha_t^{(\tau)}$ be a vector of the τ –period fixed and horizon specific portfolio weights determined at time t . Given the one-period log funding-ratio return (10), the aggregated τ –period logarithmic funding ratio return is given by

$$r_{F,t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} r_{F,t+j} = \alpha_t^{(\tau)'} \left(\mathbf{x}_{A,t+\tau}^{(\tau)} + \frac{\tau}{2} \sigma_A^2 \right) - \frac{\tau}{2} \alpha_t^{(\tau)'} \Sigma_{AA} \alpha_t^{(\tau)} - x_{L,t+\tau}^{(\tau)} \quad (11)$$

where $\mathbf{x}_{A,t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} \mathbf{x}_{A,t+j}$ is the cumulative excess log returns on risky assets over the period of length τ , $x_{L,t+\tau}^{(\tau)}$ is the τ –period cumulative log excess rate of liability change.

In particular, the optimization problem that a τ –period buy-and-hold asset–liability investor aims to solve follows a mean–variance framework which is given by

$$\max_{\alpha_t^{(\tau)}} \mathbb{E}_t \left(r_{F,t+\tau}^{(\tau)} \right) + \frac{1}{2} (1 - \gamma) \text{Var}_t \left(r_{F,t+\tau}^{(\tau)} \right) \quad (12)$$

where $\gamma > 1$ is a risk-aversion parameter. The τ –period expected funding-ratio return $\mathbb{E}_t \left(r_{F,t+\tau}^{(\tau)} \right)$ is given by

$$\mathbb{E}_t \left(r_{F,t+\tau}^{(\tau)} \right) = \tau \left(\alpha_t^{(\tau)'} \left(\mu_{A,t}^{(\tau)} + \frac{1}{2} \sigma_A^2 \right) - \frac{1}{2} \alpha_t^{(\tau)'} \Sigma_{AA} \alpha_t^{(\tau)} - \mu_{L,t}^{(\tau)} \right) \quad (13)$$

where $\mu_{A,t}^{(\tau)} = \frac{1}{\tau} \mathbb{E}_t \left(\mathbf{x}_{A,t+\tau}^{(\tau)} \right)$ and $\mu_{L,t}^{(\tau)} = \frac{1}{\tau} \mathbb{E}_t \left(x_{L,t+\tau}^{(\tau)} \right)$ are the annualized expected τ -period excess log returns on risky assets and liabilities, respectively.

The τ -period variance of the funding ratio returns, $Var_t \left(r_{F,t+\tau}^{(\tau)} \right)$, is given by

$$Var_t \left(r_{F,t+\tau}^{(\tau)} \right) = \tau \left(\left(\sigma_L^{(\tau)} \right)^2 - 2\alpha_t^{(\tau)' } \sigma_{AL}^{(\tau)} + \alpha_t^{(\tau)' } \Sigma_{AA}^{(\tau)} \alpha_t^{(\tau)} \right) \quad (14)$$

where $\left(\sigma_L^{(\tau)} \right)^2$ is the annualized variance of excess rate of liability change over the τ -period horizon, $\sigma_{AL}^{(\tau)}$ is a (14×1) vector of annualized τ -period covariances between excess returns on risky assets and liabilities, $\Sigma_{AA}^{(\tau)}$ is a (14×14) annualized covariance matrix of risky assets (see Appendix A for the derivation of $\Sigma_{AA}^{(\tau)}$).

As derived in [Hoevenaars et al. \(2008\)](#), the optimal τ -period buy-and-hold portfolio for an asset-liability investor is given by

$$\alpha_{AL}^{(\tau)} = \frac{1}{\gamma} \left(\left(1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(\tau)} + \frac{1}{\gamma} \Sigma_{AA} \right)^{-1} \left(\mu_{A,t}^{(\tau)} + \frac{1}{2} \sigma_A^2 - (1 - \gamma) \sigma_{AL}^{(\tau)} \right) \quad (15)$$

which is the solution to (12). As discussed in [Hoevenaars et al. \(2008\)](#), the optimal portfolio (15) can be thought of as having two components: the speculative portfolio

$$\alpha_{S,t}^{(\tau)} = \frac{1}{\gamma} \left(\left(1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(\tau)} + \frac{1}{\gamma} \Sigma_{AA} \right)^{-1} \left(\mu_{A,t}^{(\tau)} + \frac{1}{2} \sigma_A^2 \right) \quad (16)$$

and the hedge portfolio

$$\alpha_H^{(\tau)} = \left(\left(1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(\tau)} + \frac{1}{\gamma} \Sigma_{AA} \right)^{-1} \sigma_{AL}^{(\tau)}. \quad (17)$$

Now we discuss the asset-only approach to portfolio choice. Following [Campbell and Viceira \(2002\)](#), we use the following formulation of portfolio return over the period $t + j$, $j \geq 1$

$$r_{A,t+j} = r_{tb,t+j} + \alpha_t' \left(\mathbf{x}_{A,t+j} + \frac{1}{2} \sigma_A^2 \right) - \frac{1}{2} \alpha_t' \Sigma_{AA} \alpha_t. \quad (18)$$

Similarly to the asset-liability formulation, the optimization problem for asset-only

investors is stated as

$$\max_{\alpha_t^{(\tau)}} \mathbb{E} \left(r_{A,t+\tau}^{(\tau)} \right) + \frac{1}{2} (1 - \gamma) \text{Var}_t \left(r_{A,t+\tau}^{(\tau)} \right) \quad (19)$$

where $r_{A,t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} r_{A,t+j}$ is the aggregated τ -period portfolio return.

Hoevenaars et al. (2008) derive the following optimal portfolio for a τ -period buy-and-hold asset-only investor

$$\alpha_{AO}^{(\tau)} = \frac{1}{\gamma} \left(\left(\left(1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(\tau)} + \frac{1}{\gamma} \Sigma_{AA} \right)^{-1} \left(\mu_{A,t}^{(\tau)} + \frac{1}{2} \sigma_A^2 + (1 - \gamma) \sigma_{Ar}^{(\tau)} \right) \right) \quad (20)$$

where $\sigma_{Ar}^{(\tau)}$ is a (14×1) vector of the annualized covariances between excess asset log returns and the real rate of T-bills over the τ -period horizon.

3 Data

In this section we describe the data that we use in our model. We estimate the VAR model using monthly US data. The data series of unrestricted variables \mathbf{y}_t starts in April 1953 and ends in October 2017. As it was mentioned earlier, the data for variables in \mathbf{x}_2 and \mathbf{x}_3 are only available for much shorter horizons as we describe below (see Table 2).

We obtain the 3-month T-bills data and the seasonally adjusted Consumer Price Index (CPI) from the FRED website.¹² We construct the ex post real rate $r_{tb,t}$ as the difference of the log yield on the 3-month T-bills and log inflation rate (i.e., $\ln \frac{CPI_{t+1}}{CPI_t}$). The nominal yield on T-bills ($r_{nom,t}$) is the log yield on the 3-month T-bills. Data on dividend-price ratio and stock returns are based on the S&P500 data from the ‘‘Irrational Exuberance’’ data of Robert Shiller.¹³ The dividend-price ratio (dp) is the difference of the log dividend payment and log stock price. The yield spread y_{spr} is the difference of log yield on the 10 year zero-coupon bond (denoted by $y_{10,t} = \ln(1 + Y_{10,t})$ where $Y_{10,t}$ is the yield) obtained from FRED website and the nominal yield $r_{nom,t}$. Credit returns are based on the Moody’s Seasoned Baa Corporate Bond Yield obtained from the FRED website. We subtract the log yield on the 10-year zero-coupon bond from the log of 10-year Baa yield to obtain the credit spread $c_{spr,t}$.

¹²See <https://fred.stlouisfed.org/>.

¹³The data from Robert Shiller’s website (<http://www.econ.yale.edu/~shiller/data.html>) are already in real terms.

To evaluate the excess return on government bonds ($x_{b,t} = r_{b,t} - r_{nom,t}$), we construct the gross long-term bond return $r_{b,t+1}$ using the loglinear approximation approach described by [Campbell and Viceira \(2002\)](#) and [Hoevenaars et al. \(2008\)](#),

$$r_{b,t+1} = \frac{1}{12}y_{10,t+1} - D_{10,t}(y_{10,t+1} - y_{10,t}). \quad (21)$$

where $D_{10,t}$ is the duration approximated by

$$D_{10,t} \approx \frac{1 - (1 + Y_{10,t})^{-10}}{1 - (1 + Y_{10,t})^{-1}}$$

Following [Hoevenaars et al. \(2008\)](#), we take the evolution of returns on long-term bonds as a proxy for the liability returns. We construct liability gross return $r_{L,t+1}$ based on the 20-year constant maturity yield rate available from the FRED website.¹⁴

$$r_{L,t+1} = \frac{1}{12}y_{20,t+1} - D_{20,t}(y_{20,t+1} - y_{20,t}) \quad (22)$$

We assume that pension income is fully indexed, implying that the liabilities should be discounted in real terms by subtracting the nominal rate $r_{nom,t}$ from (22). Besides interest rate risk and inflation risk, pension liabilities also exposed to other risks such as the longevity risk and the mortality risk. In the current setup, we assume that the inflows from contributions precisely cancel out the net present value of new liabilities.

We will use monthly temperature data (in degrees Celsius) from Berkeley Earth.¹⁵ The initial data (see Figure 3) carries the land-surface average temperature anomalies (i.e., temperature above the pre-industrial level) relative to the 1951-1980 average. We use the 12-month moving average monthly temperature series T_t to calculate the temperature change series $\Delta T_{t+1} = T_{t+1} - T_t$. The time span used for VAR model matches the length of other state variables.

We obtain the rest of the financial series data from Bloomberg database and convert all asset return series to logarithmic returns. First we describe the data we use to derive the excess returns on assets in alternative class ($x_{cor,t}$, $x_{com,t}$, $x_{re,t}$, and $x_{hf,t}$). We construct the credit returns using the Barclays US aggregate bond index starting from January 1976.

¹⁴The monthly data for the 20-year bond during the period from December 1986 to December 1993 is missing, we therefore convert daily data of that period to monthly based to fulfill the gap.

¹⁵<http://berkeleyearth.org/data/>

The commodity returns are constructed from the S&P 500 GSCI index starting from January 1970. The publicly traded real estate data is obtained from FTSE NAREIT US real estate total index from January 1972. We use the Hedge Fund Research conservative Index (HFRI) to represent hedge fund returns. The data is available from January 1990. Hedge funds often hold illiquid or over-the-counter investment products for which no publicly available trade prices exist. Fund managers may use the last available trade price as a proxy for the current price or intentionally smooth the cash flows by spreading them over periods. Smoothing reduces the volatilities and makes the fund more attractive than it actually is. We therefore use the technique of [Geltner \(1993\)](#) to unsmooth the hedge fund returns.¹⁶

Next, we describe the data used to construct the excess returns on assets in the green asset class ($x_{gb,t}$, $x_{ge,t}$, $x_{sri,t}$, $x_{pp\star,t}$, $\star = 1, \dots, 5$). We use the ICE BofAML green bond index to determine green bond returns starting from January 2011, the longest available green bond index. For the green-economy fund returns we use NASDAQ OMX Green Economy Index and for the socially responsible index we use the S&P 500 Environmental & Social Responsible Index, both starting from October 2010. As an extension of the analysis in [Andersson et al. \(2016\)](#), we also include the following five pure-play green funds: (1) VanEck Vectors Environmental Services ETF starting from October 2006 (x_{pp1}); (2) Van Eck Vectors Global Alternative Energy ETF starting from May 2007 (x_{pp2}); (3) Power Shares Clean-tech Portfolios starting from October 2006 (x_{pp3}); (4) Power Shares Global Clean Energy Portfolio starting from June 2007 (x_{pp4}); (5) First Trust NASDAQ Clean Edge Green Energy Index Fund starting from February 2007 (x_{pp5}).

Table 2 presents the summary statistics of our data. A positive temperature trend of 0.01 (1.03%) indicates that average land-surface temperature increases by 0.1°C for every 10 years. As can see from Figure 3, there is a persistent trend of temperature increase since 1960 and the average temperature increase is around 0.6°C over the past 60 years. The performance of green assets varies. The pure-play assets are much riskier than the other green funds: most of the pure-play funds' annual volatilities are beyond 30% which almost triples the risk of the S&P 500 index. In this respect [Andersson et al. \(2016\)](#) argues that

¹⁶The unsmoothed return series is

$$r_t^{un} = \frac{r_t - \rho r_{t-1}}{1 - \rho}$$

where r_t is the smoothed return series and ρ is the first order autocorrelation coefficient and r_t^{un} is the unsmoothed return series used in the VAR model. The first order autocorrelation $\rho = 0.46$ based on our hedge fund data.

Table 2: Summary statistics.

The table reports summary statistics of all time series. The upper panel reports the statistics of excess returns of each asset class and the lower panel presents the statistics of state variables. The “Mean”, “St dev” and “Sharpe” are annualized (except for the dividend-price ratio). The remaining statistics are on a monthly basis. St dev = standard deviation; Sharpe = Sharpe ratio; Skew = Skewness; Kur = kurtosis.

	Mean	St dev	Sharpe	Min	Max	Skew	Kur	Start
Excess Returns								
Stock (x_s)	6.27	12.09	0.52	-23.72	11.44	-1.45	6.99	1953m04
Bond (x_b)	1.62	6.71	0.24	-9.51	20.96	0.35	4.12	1953m04
Liabilities (x_L)	4.87	8.29	0.59	-11.46	13.97	0.47	4.53	1953m04
Credits (x_{cor})	2.59	5.27	0.49	-7.24	9.64	-0.47	3.95	1976m01
Commodities (x_{com})	4.91	19.84	0.25	-27.77	26.19	-0.36	3.28	1970m01
Real Estate (x_{re})	4.53	17.72	0.26	-36.05	26.34	-0.10	3.54	1972m01
Hedge Funds (x_h)	2.87	5.98	0.48	-10.47	5.01	-0.52	3.49	1990m01
Green Bond (x_{gb})	1.41	7.65	0.18	-9.62	7.20	-0.70	7.32	2011m01
Green Economy (x_{ge})	6.92	14.13	0.49	-13.07	12.15	-0.50	4.55	2010m10
Social Res. (x_{sri})	11.28	10.55	1.07	-6.97	9.58	-0.22	3.58	2010m10
Environmental (x_{pp1})	6.19	17.64	0.35	-22.77	16.76	-0.19	3.42	2006m10
Global Alter. (x_{pp2})	-7.06	32.14	-0.22	-50.30	20.73	-0.74	4.25	2007m05
Cleantech (x_{pp3})	4.19	23.77	0.18	-35.76	18.35	-0.65	4.09	2006m10
Clean Energy (x_{pp4})	-7.21	30.23	-0.24	-43.06	19.60	-0.82	4.82	2007m06
Clean Edge (x_{pp5})	-0.44	31.85	-0.01	-40.87	18.05	-0.29	3.41	2007m02
State Variables								
Real rate (r_{tb})	0.92	0.96		-1.08	1.80	0.51	3.84	1953m04
Dividend-Price (dp)	-3.55	0.40		-4.50	-2.77	-0.23	3.08	1953m04
Nominal Rate (r_{nom})	4.37	0.89		0.00	1.35	2.04	6.12	1953m04
Term Spread (y_{spr})	1.49	0.34		-0.22	0.37	0.14	1.97	1953m04
Credit Spread (c_{spr})	1.89	0.24		0.02	0.50	-0.37	1.99	1953m04
Temperature Trend ΔT	1.03	5.31		-5.75	5.12	-0.02	3.26	1953m04

pure-play indexes are too risky to invest into because they offer no protection against the timing risk of climate change mitigation policy. However, decarbonized indexes can hedge against this timing risk since the decarbonized indexes are generated from a redistribution of the existing benchmark index. Both alternative energy x_{pp2} and clean energy x_{pp4} funds have large negative returns with expected annual losses of more than 7%.

The Green Bonds market has grown rapidly over the past half decade. These instruments are exclusively applied to finance or re-finance climate-related environmental projects aimed at enhancing sustainable development. As it follows from Table 2, our green bond index does not beat the traditional fixed income instruments. The poor performance of the green bond index is driven by insufficient market recognition and a rather limited amount of available data. Decarbonized index is comparable to the S&P

500 index. We would like to note that decarbonized indexes are constructed by eliminating high-carbon-footprint stocks and re-weighting the remaining stocks to meet a certain carbon footprint reduction target.

The SRI fund presents the best performance among all asset classes with an annual expected return of 11.28% and an annual volatility of 10.55% resulting in a Sharpe ratio of 1.07. Besides environmental criterion, SRI funds also concern social impact, such as business ethics, of the composite stocks. The SRI indexes are constructed based on the Environmental, Social and Governance (ESG) rates. [Derwall et al. \(2005\)](#) argues that high ESG-rating equity portfolios can provide substantially higher average returns than low-ranked portfolios. [Statman \(2006\)](#) claim that SRI indexes do not always beat the market and it may vary in subsample periods. Our summary statistics result suggests that the benefit of investing SRI can be substantial.

3.1 Estimation Results

By now, we have finished the first step of the pathway. In this section, we will execute the second step of our green-portfolio pathway (Figure 1) by presenting the parameter estimates of the VAR model. Table 3 reports the parameter estimates of the unrestricted VAR (see the Equation (5)) on the monthly data from April 1953 to October 2017. The correlations and standard deviations of unrestricted variables are presented in Table 4 with the monthly standard deviations on the diagonal.

Dividend-price ratio, nominal rates, yield spreads, credit spreads as well as the temperature change are all highly predictable by their own lag terms and all have very high R^2 indicating the persistency of these variables. The T-bills, dividend-price ratio and nominal rates have significant explanatory power for excess stock returns. Although less significant, bond returns and liabilities also capture some dynamics of stock returns. However, a rather low R^2 of 10% indicates that stock returns are difficult to explain. The negative correlation of shocks in the T-bills, dividend-price ratio, nominal rate, and credit spread with shocks in stocks implies that a positive innovation in these variables has a negative impact on contemporaneous stock returns.

Although bond returns are also hard to predict ($R^2 = 13\%$), the yield spread and stock returns have strong predictive power for their returns. However, [Hoevenaars et al. \(2008\)](#) argue that a low R^2 on a more frequent data basis (such as monthly or quarterly)

implies a higher R^2 on an annual basis. Furthermore, [Campbell and Thompson \(2007\)](#) claim that small R^2 can still be economically meaningful. In general, [Table 3](#) shows that the lag of five macroeconomic factors have significant explanatory power to themselves and to the primary return variables, but not to the temperature change.

Table 3: VAR of core variables.

The table reports parameter estimates of the unrestricted VAR model (see the Equation (5)) with variables: 3-month T-bill, dividend-price ratio, nominal 3-month T-bill, yield spread, credit spread, temperature change, stocks, 10-year government bonds, and liabilities. The t -statistic of each parameter is shown in parenthesis.

	$r_{tb,t}$	dp_t	$r_{nom,t}$	$y_{spr,t}$	$c_{spr,t}$	ΔT_t	$x_{s,t}$	$x_{b,t}$	$x_{L,t}$	b	R^2
$r_{tb,t+1}$	0.42 (13.11)	0.00 (-1.81)	0.27 (6.27)	0.40 (3.61)	-0.44 (-3.02)	0.00 (-0.08)	0.00 (-1.81)	0.01 (1.45)	0.01 (2.34)	0.00 (-2.04)	0.31
dp_{t+1}	-1.73 (-3.64)	0.99 (258.26)	2.07 (3.24)	0.60 (0.37)	-4.73 (-2.19)	0.08 (0.98)	-0.24 (-6.77)	-0.12 (-1.67)	-0.10 (-1.79)	-0.05 (-3.25)	0.99
$r_{nom,t+1}$	0.00 (0.06)	0.00 (-1.87)	1.00 (186.57)	0.08 (5.65)	-0.09 (-4.92)	0.00 (1.13)	0.00 (1.19)	-0.01 (-16.00)	0.00 (-2.88)	0.00 (-1.28)	0.99
$y_{spr,t+1}$	0.00 (-0.07)	0.00 (1.84)	0.00 (-0.38)	0.93 (72.75)	0.09 (5.13)	0.00 (-0.87)	0.00 (-1.58)	0.00 (-4.44)	0.00 (3.12)	0.00 (1.27)	0.92
$c_{spr,t+1}$	-0.01 (-3.70)	0.00 (-2.05)	0.01 (3.47)	-0.02 (-3.52)	0.98 (149.07)	0.00 (-1.13)	0.00 (-8.43)	0.01 (25.78)	0.00 (-4.41)	0.00 (-1.02)	0.98
ΔT_{t+1}	0.09 (0.56)	0.00 (-0.16)	-0.08 (-0.36)	-0.17 (-0.30)	0.59 (0.78)	0.64 (22.84)	-0.02 (-1.73)	-0.03 (-1.05)	0.02 (0.85)	0.00 (-0.17)	0.41
$x_{s,t+1}$	1.16 (2.43)	0.01 (3.27)	-2.10 (-3.32)	-0.95 (-0.60)	2.86 (1.34)	-0.09 (-1.08)	0.25 (7.02)	0.11 (1.56)	0.10 (1.84)	0.05 (3.50)	0.10
$x_{b,t+1}$	-0.26 (-1.02)	0.00 (-1.92)	0.60 (1.75)	3.66 (4.22)	-1.74 (-1.50)	-0.01 (-0.17)	-0.08 (-4.24)	0.28 (7.23)	-0.09 (-3.09)	-0.02 (-2.02)	0.13
$x_{L,t+1}$	-0.37 (-2.53)	0.00 (-0.16)	0.63 (3.23)	0.19 (0.39)	0.27 (0.40)	0.01 (0.21)	0.02 (2.23)	1.22 (54.88)	-0.01 (-0.48)	0.00 (-0.15)	0.82

[Table 5](#) presents the estimation results for the restricted VAR model (see the Equation (6)) which restricts the predictability of alternatives $\mathbf{x}_{2,t}$ on core state variables \mathbf{y}_t . Excess returns on corporate bonds (credits) are most predictable among alternative assets ($R^2 = 68\%$). The nominal rate, yield spread, credit spread and their lags provide strong explanatory power to credits. Credits are also well explained by the long term bonds. It follows from the parameter estimates that excess returns on corporate bonds decrease when yield spread increases or credit spread widens which is due to the fact that credits are exposed to default risks and liquidity shocks which are reflected in both the yield spread and the credit spread. This relationship is consistent with the literature such as [Duffee \(1998\)](#) and [Longstaff et al. \(2005\)](#) and is also in line with the economic notion that default risk induces lower corporate bond returns.

Table 4: VAR of core variables Error correlation matrix.

The table reports the standard deviations (diagonal elements) and correlations (off-diagonal elements) of the unrestricted VAR model (see the Equation (5)) with the variables: 3-month T-bill, dividend-price ratio, nominal 3-month T-bill, yield spread, credit spread, temperature trend, stocks, 10-year government bonds, and liabilities.

	r_{tb}	dp	r_{nom}	y_{spr}	c_{spr}	ΔT	x_s	x_b	x_L
r_{tb}	0.23	0.00	0.09	-0.09	0.06	0.00	-0.08	0.12	-0.13
dp	0.00	0.03	0.04	-0.02	0.30	-0.09	-0.98	0.09	0.00
r_{nom}	0.09	0.04	0.03	-0.99	-0.06	0.00	-0.04	-0.03	0.12
y_{spr}	-0.09	-0.02	-0.99	0.03	0.05	-0.01	0.02	0.02	-0.10
c_{spr}	0.06	0.30	-0.06	0.05	0.01	-0.02	-0.30	0.05	-0.16
ΔT	0.00	-0.09	0.00	-0.01	-0.02	1.18	0.08	-0.02	0.01
x_s	-0.08	-0.98	-0.04	0.02	-0.30	0.08	3.34	-0.10	0.02
x_b	0.12	0.09	-0.03	0.02	0.05	-0.02	-0.10	1.82	-0.05
x_L	-0.13	0.00	0.12	-0.10	-0.16	0.01	0.02	-0.05	1.03

It also follows from Table 5 that commodities are hard to predict ($R^2 = 24\%$). T-bills, credit spreads, and temperature change all have strong and negative predicting power on commodities. The real estate returns are rather predictable ($R^2 = 38\%$) by contemporaneous credit spreads and lagged stock returns. Hedge funds have a strong negative exposure to the news of temperature increase and a wider credit spread. Presumably, alternative assets are not green, hence have a negative exposure to temperature shocks. The negative contemporaneous coefficients on temperature change \mathbf{C}_0 for all alternative assets verify this inverse relationship, although it is insignificant for real estates and credits. In general, we provide empirical evidence showing that warming risk can hurt grey assets.

Table 6 shows the estimation results for the second restricted VAR model (see the Equation (7)). Green assets are more predictable than the alternative assets with their R^2 higher than 70%. Green bonds are well explained by commodities and real estate, but can hardly be explained by primary assets (stocks and bonds). Decarbonized funds x_{ge} are heavily driven by contemporaneous returns on real estate and hedge funds. Although insignificant, decarbonized funds have positive exposure to the news of temperature increase. Social responsible funds are also well explained by contemporaneous changes of real estate and hedge funds, but state variables and primary assets do not contribute significant explanatory power. We also find a weakly significant and positive relationship between social responsible fund returns and temperature change.

It also follows from 6 that pure-play funds are all temperature neutral. We find

Table 5: Excess Return Regressions.

The table reports parameter estimates of the restricted VAR model given by (6) ($\mathbf{x}_{2,t+1} = \mathbf{c} + \mathbf{C}_0\mathbf{y}_{t+1} + \mathbf{C}_1\mathbf{y}_t + \mathbf{C}_2\mathbf{x}_{2,t} + \epsilon_{2,t+1}$) for excess returns of the assets in the subset \mathbf{x}_2 which includes corporate bonds, commodities, listed real estates and hedge funds. The t -statistic of each parameter is shown in parenthesis.

	Contemporaneous \mathbf{C}_0										\mathbf{c}	R^2
	$r_{tb,t+1}$	dp_{t+1}	$r_{nom,t+1}$	$y_{spr,t+1}$	$c_{spr,t+1}$	ΔT_{t+1}	$x_{s,t+1}$	$x_{b,t+1}$	$x_{L,t+1}$			
$x_{cor,t+1}$	-0.04 (-0.18)	-0.05 (-0.71)	-52.37 (-2.01)	-52.13 (-1.99)	-18.87 (-4.68)	-0.03 (-1.21)	-0.02 (-0.32)	0.14 (8.12)	-0.07 (-1.77)	0.00 (-0.55)	0.68	
$x_{com,t+1}$	-8.40 (-4.41)	0.25 (0.44)	210.59 (0.89)	162.20 (0.68)	-122.98 (-3.36)	-0.79 (-3.00)	0.31 (0.52)	0.03 (0.16)	-1.10 (-3.18)	-0.09 (-1.38)	0.24	
$x_{re,t+1}$	2.22 (1.49)	-0.34 (-0.76)	176.82 (0.95)	213.59 (1.14)	-79.86 (-2.79)	-0.18 (-0.88)	0.42 (0.91)	-0.15 (-1.18)	0.37 (1.37)	-0.05 (-1.06)	0.38	
$x_{hf,t+1}$	-0.77 (-1.62)	0.05 (0.37)	-31.43 (-0.53)	-43.24 (-0.72)	-32.23 (-3.50)	-0.14 (-2.17)	0.28 (1.94)	0.02 (0.54)	-0.18 (-2.11)	-0.01 (-0.59)	0.42	
	Lagged \mathbf{C}_1										\mathbf{C}_2	
	$r_{tb,t}$	dp_t	$r_{nom,t}$	$y_{spr,t}$	$c_{spr,t}$	ΔT_t	$x_{s,t}$	$x_{b,t}$	$x_{L,t}$			
$x_{cor,t+1}$	0.00 (0.03)	0.05 (0.72)	52.94 (2.04)	55.30 (2.02)	21.35 (5.50)	0.05 (1.75)	0.00 (-0.12)	0.10 (0.33)	0.04 (2.49)	-0.43 (-7.95)		
$x_{com,t+1}$	3.08 (2.29)	-0.27 (-0.47)	-203.94 (-0.87)	-162.76 (-0.68)	122.32 (3.47)	0.26 (1.00)	-0.12 (-1.04)	3.12 (1.16)	-0.17 (-1.02)	-0.12 (-1.76)		
$x_{re,t+1}$	-0.86 (-0.82)	0.32 (0.72)	-178.96 (-0.97)	-209.82 (-1.13)	72.99 (2.65)	0.06 (0.31)	0.23 (2.45)	2.75 (1.30)	-0.01 (-0.05)	-0.17 (-2.89)		
$x_{hf,t+1}$	0.41 (1.22)	-0.06 (-0.38)	32.99 (0.56)	44.40 (0.74)	33.27 (3.76)	0.09 (1.35)	-0.04 (-1.34)	-0.16 (-0.24)	0.02 (0.38)	-0.32 (-5.65)		

insignificant coefficients on contemporaneous temperature change for all the five pure play funds, indicating that the hypothesis of zero coefficients can not be rejected. Therefore, future warming does not impact the performance of these funds. By definition, green assets are constructed to be resilient against future temperature shocks. This notion is verified in our sample green assets. Environmental service funds x_{pp1} are well explained by the changes of nominal rates and yield spreads. The estimation results go along with the ethical expectation of green assets.

There are two main takeaways from the estimation results. First, green assets are in general resilient against climate risk, since they are either positively related or not related to temperature shocks. Second, most of the grey assets have a negative relationship with the instantaneous temperature shocks, which means warming makes grey funds less attractive. The estimate result of the VAR model enables investors to quantify the historical impact of climate change over various asset classes, as stated in Step 2 of Figure 1.

4 Climate Scenarios and Green Portfolio

The third step of constructing a green portfolio is to generate future temperature pathways. Due to the uncertain feature of climate change, investors may have different perspectives on the future climate change pathways. We consider two climate scenarios based on two quantitative models: an optimistic climate trajectory implied by the estimated VAR model; and a pessimistic scenario based on DICE model.

4.1 Optimistic Climate Scenario

The VAR-model simulated climate scenarios are considered optimistic as it is shown in Table 3 that ΔT_t is mainly driven by its lag term. The rest of the state variables in \mathbf{s}_t barely contribute to the ΔT_t distribution sequence.

In Appendix A, we derive the j -period forecast of the state vector \mathbf{z}_{t+j} . In the same manner, we can obtain the j -period forecast of temperature change from the unrestricted VAR \mathbf{y}_{t+j}

$$\mathbf{y}_{t+j} = \left(\sum_{i=0}^{j-1} \mathbf{B}^i \right) \mathbf{b} + \mathbf{B}^j \mathbf{y}_t + \sum_{i=0}^{j-1} \mathbf{B}^i \boldsymbol{\epsilon}_{t+j-i} \quad (23)$$

with ΔT_{t+j} the last element in vector \mathbf{y}_{t+j} .

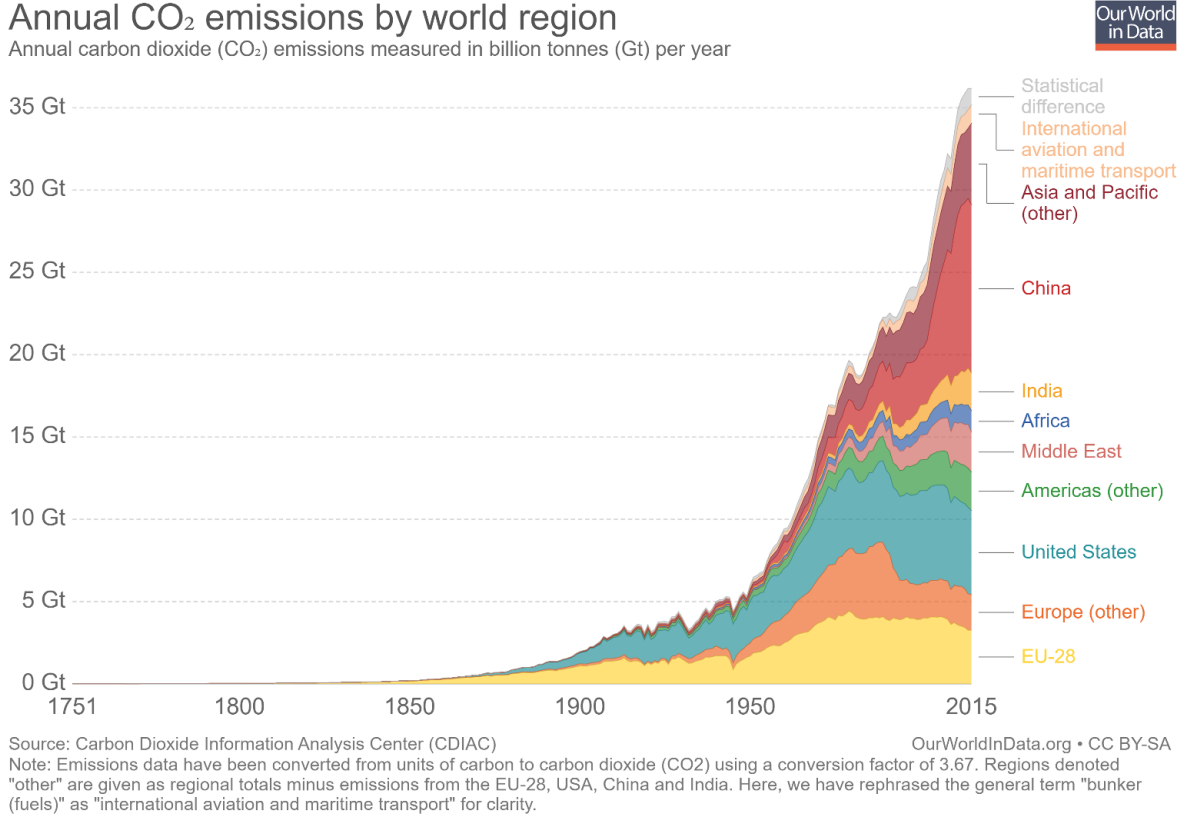
4.2 Pessimistic Climate Scenario

It is now widely accepted that climate change is caused by human activity and is driven by the accumulation of greenhouse gases (GHGs) in the atmosphere (see Stern (2008) and Litterman (2013), among others). Figures 2 and 3 demonstrate the close relationship between global warming and CO₂ emissions. CO₂ is emitted as a byproduct of consumption. We propose to use a consumption-climate model to project future temperature trajectories.

In his seminal work, Nordhaus (1994) introduced a well-known DICE model (Dynamic Integrated model of Climate and the Economy) to explore the correlation between consumption growth and the stock of atmospheric GHGs. As an extension to the DICE assessment model, we impose a forward-looking term to measure the long-term impact of temperature fluctuation towards the risk-return tradeoffs across traditional asset classes,

Figure 2: Historical CO_2 Emissions

Regional and national fossil-fuel CO_2 emissions (1751–2015). Source: Our World in Data <https://ourworldindata.org/>. Original data is published by Carbon Dioxide Information Analysis Center (CDIAC)



alternatives and “green” assets. Employing the insights of [Bansal et al. \(2016\)](#), we treat the climate factor as a penalty term towards “grey” assets while a bonus towards the “green” funds.

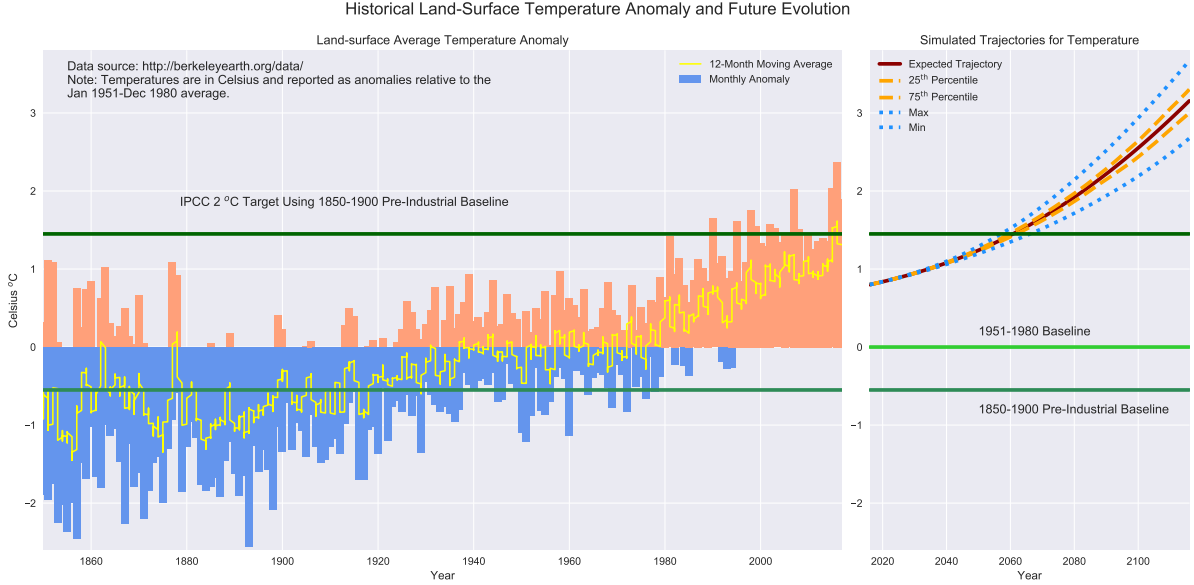
To model the dynamics of global temperature, we follow [Lemoine \(2017\)](#) and [Lemoine and Rudik \(2017\)](#). Let C_t denote the total global consumption (in real terms), then the log growth rate of the consumption is given by

$$r_{C,t+1} = (\mu_C - 0.5\sigma_C) - \alpha T_t + \sigma_C \varepsilon_{C,t+1} \quad (24)$$

where $r_{C,t+1} = \ln(C_{t+1}) - \ln(C_t)$ is the log growth rate, $\mu_C - 0.5\sigma_C$ is the unconditional mean of consumption growth, parameter $\alpha > 0$ represents the expected detrimental effect of global temperature T_t on economic growth, σ_C is the volatility of economic growth, and $\varepsilon_{C,t+1}$ are independent standard normal random variables that reflects the non-climate

Figure 3: Land-Surface Temperature Anomaly and Projections

Left panel: Land-surface average temperature anomaly related to the 1951-1980 average. Temperatures are in Celsius with time span from 1753 to 2017. Source: Berkeley Earth <http://berkeleyearth.org/data/>. Right panel: simulated trajectories for future temperature anomaly (from 2016 to 2116) based on the consumption-climate model of Lemoine (2017) and their calibrations. The solid line depicts the expected trajectory, the dashed lines show the interquartile range, and the dotted line depicts the minimum and maximum values. The 2 ° C warming target is based on the 1850-1900 pre-industrial baseline, as suggested by IPCC Fifth Assessment Report (AR5) <https://www.ipcc.ch/report/ar5/>



factors that make consumption volatile.

Since an increase in consumption is usually associated with larger CO_2 -emissions, we assume that the stock of atmospheric CO_2 emissions M_t evolves according to

$$M_{t+1} = M_t + \gamma_t C_t - \delta (M_t - M_{pre}) \quad (25)$$

where $\gamma_t > 0$ is the rate at which emissions increase with consumption, $\delta \geq 0$ is the parameter of mean-reversion that specifies the speed at which the accumulated emissions revert towards their preindustrial level M_{pre} .

The accumulation of atmospheric GHGs lead to global warming through radiative forcing. Radiative forcing is the difference between energy (sunlight) absorbed by the Earth and energy radiated back to space. In this sense forcing measures the greenhouse effect and higher forcing implies that more energy is trapped by the Earth. Forcing is modelled as the amount of extra heat trapped relative to the heat trapped by preindustrial

level of CO_2 emissions

$$F(M_t) = \nu \ln \left[\frac{M_t}{M_{pre}} \right] \quad (26)$$

where $\nu > 0$ is a forcing factor.

Finally, we link the emissions and global temperature as follows

$$T_{t+1} = T_t + \phi [sF(M_t) - T_t] \quad (27)$$

where the inertia parameter $\phi > 0$ defines the speed at which forcing translates into temperature change, and $s > 0$ is the parameter that specifies how many units of warming are generated by a unit of forcing.

We employ the low emission calibration results of [Lemoine \(2017\)](#) to simulate future temperature scenarios. In particular, we assume that the emission intensity (in Gt C per trillion dollars of output in 2014 dollars) is given by

$$\gamma_t = \gamma_0 \frac{\exp\left(-\gamma_1(t + 2014 - 1960)\right)}{10^{12}}$$

where t is measured in years from 2014. In addition, we assume that

$$s = \frac{S}{5.35 \ln(2)}$$

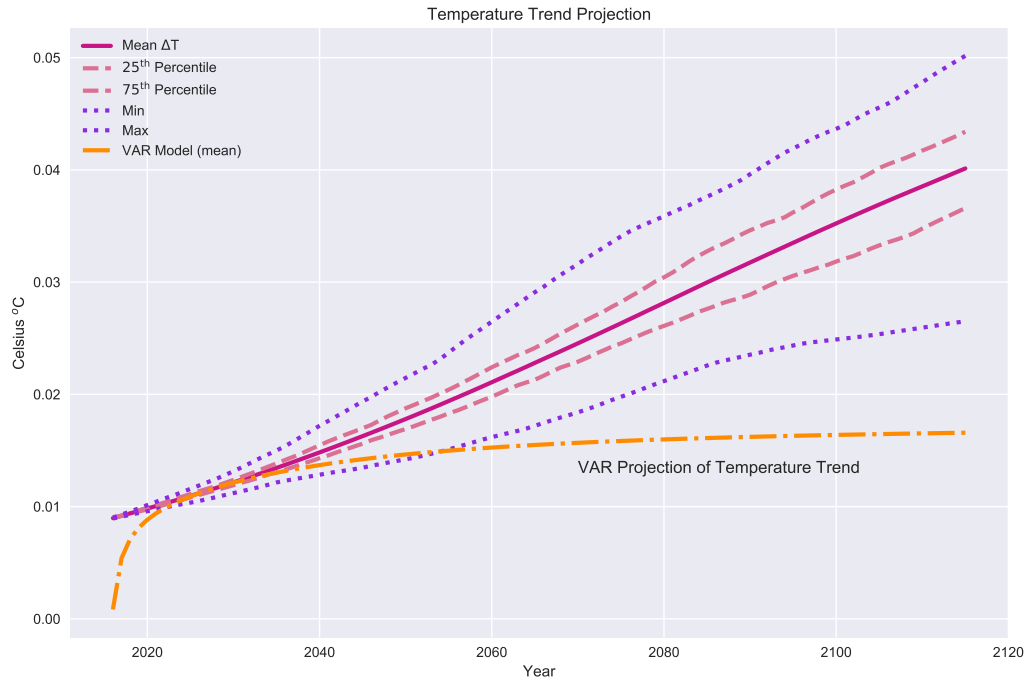
where S is the climate sensitivity parameter (a standard metric that describes the equilibrium warming resulting from doubling the atmospheric concentration of CO_2). The parameter values are given in [Table 7](#).

We simulate 1000 trajectories of pessimistic temperature T_t and their difference ΔT_t over the next 100 years. The right panel of [Figure 3](#) plot the mean, quarterlies, minimum and maximum evolution pathes of temperature. It is almost certain that by the end of the century, total global temperature would grow exceed $2^\circ C$.¹⁷ Temperature trend also increases (see [Figure 4](#)) since the slope of the temperature trajectories is getting steeper

¹⁷The well-known $2^\circ C$ warming limit relies on the baseline of pre-industrial level. The “pre-industrial” society is not clearly defined. It usually refers before the advent of the Industrial Revolution, which occurred from 1750 to 1850. However, regulators frequently update the “pre-industrial” baselines. For instance, IPCC AR5 used several different historical baselines (e.g. 1750, 1850-1900, 1861-1880, etc.) [Schurer et al. \(2017\)](#) show that change the pre-industrial baseline from 1750-1850 to 1850-1900 increases 0.0-0.2 $^\circ C$.

over time.

Figure 4: *The figure plots the mean, quartiles, minimum and maximum value of simulated trajectories for temperature trends using Lemoine (2017) model together with the VAR model implied projection of the temperature trends.*



We compare the two climate scenarios in Figure 4. As can see from the figure, the “green” VAR model’s (optimistic scenario) implied temperature change is much lower than the low-emission version (pessimistic scenario) of the consumption-climate model implied projections. The optimistic ΔT projection is even lower than the minimum simulated path of the pessimistic scenario. In general, factor-based model tends to severely underestimate the warming speed hence also underestimate the impact of temperature increasing on asset returns, especially for long-term investors. We therefore hypothesize that it is a lack of knowledge (or uncertainty) on modelling climate change that causes the eco-investing puzzle.

4.3 Portfolio Choice with Temperature Betas

The sluggish decision on investing in green can be partially attribute to model uncertainty¹⁸. In other words, the optimistic climate scenario implied by the VAR model cannot accurately predict the future impact of temperature change on the green markets.

The uncertainty comes from two sources. First, future temperature change is highly unpredictable. As discussed in Figure 4 the VAR model fails to specify the temperature trend, resulting in an over optimistic temperature trajectory. Investors who employ VAR model to make their asset allocation decision are blind by this misspecified risk factor. Second, given the rather short time series of green assets or even of some alternatives, the impact of climate change on the dynamics of asset returns may not be fully captured by the underlying estimation model. The VAR model only includes macro risk drivers, in addition to the temperature risk. However, many other risks, such as policy risks, non-Gaussian distributed catastrophic risks, unforeseeable news may all help to better understand the risk behaviour of green funds. Unfortunately, these concerns are not addressed in our reference model. In short, investors should incorporate the uncertain forward-looking inputs to analyze the economic impact of investing in low-carbon green assets and in general to any investment vehicles, as also suggested by a recent report from Mercer [Guyatt \(2011\)](#).

In this section, we introduce a climate factor called the “temperature beta” to represent the uncertain climate effect on asset returns. To disentangle the climate effect from other macro effect, we anatomize the portfolio return dynamics as follows:

$$r_{A,t+1} = r_{tb,t+1} + \alpha'_t \left(x_{\tilde{A},t+1} + \beta \Delta T_{t+1} + \frac{1}{2} \sigma_A^2 \right) - \frac{1}{2} \alpha'_t \Sigma_{AA} \alpha_t$$

where $x_{\tilde{A},t+1}$ is the vector of climate-effect clear logarithmic excess returns and $x_{\tilde{A},t+1} + \beta \Delta T_{t+1} = x_{A,t+1}$ if investors only consider optimistic climate scenario. Therefore, we do not have double counting issue.

The separation of temperature beta from the remaining component of excess returns is crucial to answering the green investment puzzle. The temperature beta function, $\beta \Delta T$, contains two components: a climate factor β which according to our estimation (see Section 3.1), has a negative sign for high-carbon assets and a positive sign for some of the

¹⁸Model uncertainty means having fragile beliefs about the probability distribution over a sequence due to limited knowledge or information incompleteness.

low-carbon assets; and the projected temperature trend ΔT which reflects the degree of awareness of climate change from investors. We offer two economic insights of the β in the temperature beta function. First, vector β measures the instantaneous climate impact on asset returns. The estimated β from VAR is generally consistent with the empirical finding of [Bansal et al. \(2016\)](#) which show a negative impact on grey asset returns and a positive impact on green assets if considering climate risk. Second, this β parameter can also be considered as an anticipated climate mitigation policy. With temperature increasing, the β term will penalize grey assets while reward the green assets. The scale of the temperature beta is related to the future climate policies, such as carbon taxation, clean-tech subsidies.

Let $\mathbf{x}_{A^\beta, t+\tau}^{(\tau)} = \mathbf{x}_{A, t+\tau}^{(\tau)} + \beta \Delta T_{t+\tau}^{(\tau)}$, then temperature-beta embedded expression of (11) is given by

$$r_{F, t+\tau}^{(\tau)} = \alpha_t^{(\tau)'} \left(\mathbf{x}_{A^\beta, t+\tau}^{(\tau)} + \frac{\tau}{2} \sigma_A^2 \right) - \frac{\tau}{2} \alpha_t^{(\tau)'} \Sigma_{AA} \alpha_t^{(\tau)} - x_{L, t+\tau}^{(\tau)}$$

The optimal asset-liability portfolio with temperature beta policy takes the same form of (15), except for the expected returns

$$\alpha_{AL^\beta}^{(\tau)} = \frac{1}{\gamma} \left(\left(1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(\gamma)} + \frac{1}{\gamma} \Sigma_{AA} \right)^{-1} \left(\mu_{A^\beta, t}^{(\tau)} + \frac{1}{2} \sigma_A^2 - (1 - \gamma) \sigma_{AL}^{(\tau)} \right) \quad (28)$$

where $\mu_{A^\beta, t}^{(\tau)} = \frac{1}{\tau} \mathbb{E} \left(\mathbf{x}_{A^\beta, t+\tau}^{(\tau)} \right)$.

Similarly, the optimal asset-only portfolio with temperature beta is given by

$$\alpha_{AO^\beta}^{(\tau)} = \frac{1}{\gamma} \left(\left(1 - \frac{1}{\gamma} \Sigma_{AA}^{(\tau)} + \frac{1}{\gamma} \Sigma_{AA} \right) \right) \left(\mu_{A^\beta, t}^{(\tau)} + \frac{1}{2} \sigma_A^2 + (1 - \gamma) \sigma_{Ar}^{(\tau)} \right) \quad (29)$$

The temperature beta changes the scale of temperature exposure on each asset, but the influence is limited to the expected returns. By the end, only speculative portfolios are influenced by the temperature beta and not the hedge demand portfolios. In the following subsections, we consider two scenarios of beta and the temperature trajectories are simulated from the low-emission version of the consumption-climate model.

4.3.1 Scenario 1: Estimated Value of Beta

We first assume beta equals to the estimated coefficient on the instantaneous temperature shocks ΔT_{t+1} for each asset, which describes the size of the climate-change effect on asset returns (see Table 8). Remark, the coefficients shown in Table 8 are in monthly basis, since we use the monthly data to estimate the VAR model. For stocks and bonds, we use the coefficient on ΔT_t instead, for ΔT_{t+1} is also an endogenous variables but is highly correlated with ΔT_t . We choose the threshold significance level at $p \leq 0.15$ which approaches but fails to achieve a customary level of statistical significance. We only take the leaning-towards-significant coefficients (at least 15% significance level) and assume the rest equal to zero. As such, in temperature beta function, $\beta\Delta T$, we sustain the VAR estimation results while only varying the impact of temperature projections when determining the long-term risks and returns of assets. As stated in Table 8, the high-carbon assets, such as stocks and some alternatives, are negatively related to the temperature shocks. The commodities have the most negative climate effect with an estimated monthly beta equal to -0.79 at the 5% significance level. Some green assets, such as socially responsible funds and clean technology portfolios, are positively related to the temperature change. The clean technology portfolio has the most positive beta effect among all green assets. The rest green assets have insignificant betas hence are resilient against climate risk.

4.3.2 Scenario 2: Semi-uniformed Beta

In the second temperature-beta scenario, we split the asset menu into two groups: high-carbon “grey” assets which include stocks, bonds and alternatives have an annualized beta value of -1 (or -0.08 in monthly base); and the green assets have an annualized beta of 1 (or in 0.08 monthly base). Remark, betas shown in Table 8 are in monthly base. Hence betas in Scenario 2 with an annual range between -1 and 1 are reasonable, or even slightly moderate if turning into monthly base $(-1/12, 1/12) \approx (-0.08, 0.08)$. The range of the betas in Scenario 2 is about twice as large as in Bansal et al. (2016),¹⁹ for our model involves a much larger universe of asset class which requires a larger variety of

¹⁹Bansal et al. (2016) calibrate the β based on the sorted stock portfolio. They sort stocks by book-to-market ratio and by size and they find a negative relationship between the average portfolio return and β . In their estimation results, the annualized β for the equity market falls into an interval of -0.04 to 0.02 based on the annual temperature data, which is equivalent to a range of $(-0.48, 0.24)$ if using monthly temperature data.

temperature exposure distributions. The purpose of proposing a semi-uniformed value of betas is to facilitate the climate mitigation policy making.

5 Asset Allocation with Green Assets

In this section, we present the strategic asset allocation for investors with different prospects of climate change. We also demonstrate how uncertainty plays a role in the myth behind green investing.

5.1 With Optimistic Climate Scenario

Do green assets add value for investors who are not aware of climate change or without sufficient diligence? In this section, we investigate the role of green assets on investor’s total portfolio performance under an optimistic view on climate change. In Table 9 we show the optimal asset-liability portfolio $\alpha_{AL}^{(\tau)}$ for investors who employ the “green” model (optimistic climate scenario) and for those who employ the “grey” VAR model (not aware of climate change). The “grey” VAR model takes the same form as (8), but is without the climate factor ΔT_t . The optimal stock-bond portfolio (see Panel (A)) is dominated by long-term bonds over the investment horizon for both VAR models, with a portfolio weight of more than 90% for the short-term ($\tau \leq 5$) investors and more than 70% for the long-term ($\tau \geq 20$) investors. The down-up horizon effect is mainly driven by the term structure of liability risks (see Figure 13). Long-term bonds are attractive in this model due to the mean reversion in bond risks and the negative correlation with liability shocks. Although stocks are also attractive in terms of mean-reverting risks and higher risk premium, they provide less of a liability hedge than the other instruments in the stock-bond portfolio. The “green” model suggests a slightly lower exposure towards stocks because temperature increase induces lower stock returns over the investment horizon (see Figure 10).

In Panel (B), we add alternatives to the stock-bond portfolio and compare the optimal asset allocations implied by the two VAR models over various investment horizons. Corporate bonds and hedge funds dominate the optimal asset-liability portfolio for the short investment horizon, with portfolio weights of more than 130% and more than 80%, respectively. The dominating positions of the two assets are mainly driven by the lower

volatility in the short run. In the long run, however stocks and bonds take over the dominating positions of the optimal portfolio, with portfolio weights more than 100%. T-bills have negative portfolio weight, as the real rate premium is very small (less than 1%). Without the short-selling constraint, real estate also takes a negative position for both models, with an average portfolio weight of -20%. High risk is the main reason behind the unattractiveness of real estate. Commodities are also not recommended in either model, due to their high level of volatility. The “green” model suggests a higher exposure on hedge funds but a lower weight on stocks compared with the “grey” model’s portfolio. This is because temperatures shocks reduce the long-term stock returns but push up the long-term hedge fund returns (see Figure 10 and 11). Awareness of temperature shocks shifts approximately 3% of the portfolio from stocks to hedge funds over various horizons.

Panel (C) includes green assets to the asset mix. The optimal asset-liability portfolio does not suggest much exposure to asset assets for both models, due to their high risk, moderate rewards and the increasing feature risk term structures. Only short-term investors have exposure to the green assets with an aggregate portfolio weight of 10%. Both models suggest negative exposure of green assets for long-term buy-and-hold investors. Similar to the layout in Panel (B), credits and hedge funds dominates the short-term portfolio while stocks and long-term bonds dominate the long-term portfolio. Clean technology funds have the highest exposure among the green asset class, due to the high term structure of returns. However, the mean-reversion structure of clean technology funds diminishes their exposure over time. Although, social responsible funds enjoy the highest Sharpe ratio among all assets (see Table 2), they are not very attractive in the optimal portfolio with negative portfolio weights for most investment horizons, due to their low rewards compared with stocks and increasing term structure of risks. The two models are the same in terms of their allocation of green assets. This is mainly because the “green” model corrects the green funds’ volatilities and returns both upwards, hence the risk-return tradeoff stays the same.

Figure 5 compares the optimal asset allocation with and without liabilities for a long-term buy-and-hold investor with 20 years. We decompose the optimal portfolio into a hedging component and a speculative portfolio. The difference between the asset-liability investment and the asset-only investment is driven by the hedging portfolios. The hedging portfolios aim to hedge liability risks for the asset-liability investor and to minimized the

portfolio variance for the asset-only investor.

At the 20-year horizon, the asset-only hedging portfolio is very different from the asset-liability hedging portfolio. The primary assets and cash dominate the asset-only hedging component, due to their low level of long-term risk. Alternatives and green assets in general are much higher in risk compared with the primary asset classes, hence have negative weights in the asset-only hedging portfolio. The liability hedging portfolio is dominated by long-term bonds and alternatives. Although the T-bill plays a big role in the asset-only portfolio, it does not turn out to be a good liability hedge. Both long-term bonds and credits get substantial weights in the liability hedging portfolio, exactly as in [Hoevenaars et al. \(2008\)](#). Hedge funds present a positive weight in the liability hedging portfolio. Green funds also have small liability hedging potential with a positive aggregate weight of less than 5%. This result sheds some light on the long-term hedging benefit of green assets and selective alternatives for investors with an ALM perspective.

The speculative portfolios are driven by the risk-return trade off of assets, hence are identical to different investors with the same asset menu. The mean reverting characteristics of volatility and reasonably high returns make stocks the dominating asset in the speculation portfolio with a weight of 80%. Neither investors are motivated to speculate from investing in green assets, as the possibility of a big gain is fairly low given the huge uncertainty of the green markets.

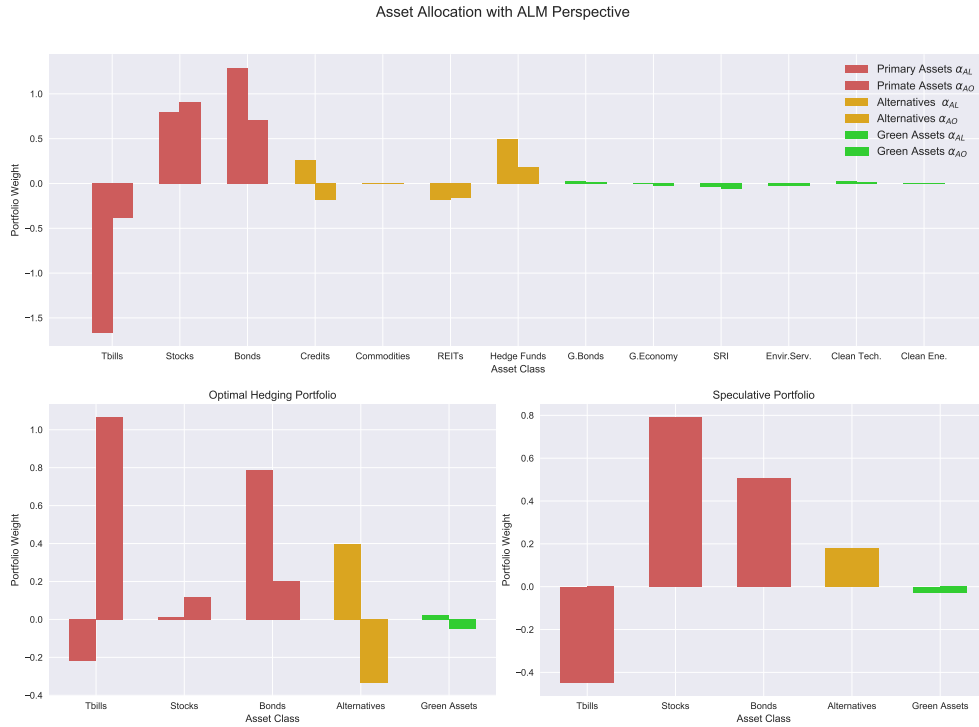
In summary, investors with no awareness or optimistic view of climate change are hardly interested in green assets. Green assets are not attractive to asset-only investors, due to their poor risk-return tradeoff and huge uncertainty compared with those traditional asset classes. However, we show small liability hedging potential from the green asset class. This characteristics is beneficial to long-term investors with an ALM perspective, such as large pension funds. However, in terms of speculative benefits, green funds do not significantly outperform other asset classes. Among all selected green funds, green bonds and clean technology funds have the highest portfolio weights.

5.2 With Pessimistic Climate Scenario

Figure 6 compares the optimal asset allocation strategies with pessimistic and optimistic climate scenarios for both types of investors. Under the pessimistic climate scenario, we also present two beta scenarios: estimated beta, and semi-uniformed beta. We show that,

Figure 5: Asset Allocation with ALM Perspective

Compare the implication of “green” VAR model on optimal asset-liability portfolios and on the asset-only portfolios at an investment horizon of 20 years. The main panel depicts the portfolio weight of each asset class for investors with and without ALM perspective; the left sub-panel presents the hedging portfolio for two types of investors. We aggregate the portfolio weights from alternatives and from green assets. The right sub-panel displays the speculative portfolio for both types of investors. The risk aversion level is set at $\gamma = 10$.



both temperature beta scenarios help promote the social and economic value of green assets, as they enlarge the portfolio weights of green assets substantially. Estimate-beta scenario are more effective than semi-uniformed beta scenario in terms of green assets reallocation. With the estimated-beta scenario, both investors shift more than 50% of their portfolio from the grey assets to the green ones. Uniformed-beta scenario also largely increases the green-asset exposure but is about 10% less than Scenario 1. Investors with an ALM perspective have a slightly higher exposure to the green assets than the asset-only investors. This is because green assets have a liability hedging characteristics, hence are attractive to investors with liability constraints. However, the marginal rewards from holding green assets longer is small due to the mean-diverging feature of green assets’ term structure of risks.

For short-term investors with a 5-year investment horizon, the temperature beta in-

duces a dramatic drop of portfolio weights on alternatives. For asset-liability investors, the alternatives' weight moves from 200% to 40% in Scenario 1; and to 60% in Scenario 2. This is because temperature betas have a much stronger negative impact on alternative especially for credits and hedge funds, as their portfolio weights dominate the demand for alternatives. Stocks and bonds are not very sensitive to the re-scaled temperature exposure, as hedging portfolios play a major role in the optimal portfolios for short-term investors and are not influenced by temperature betas.

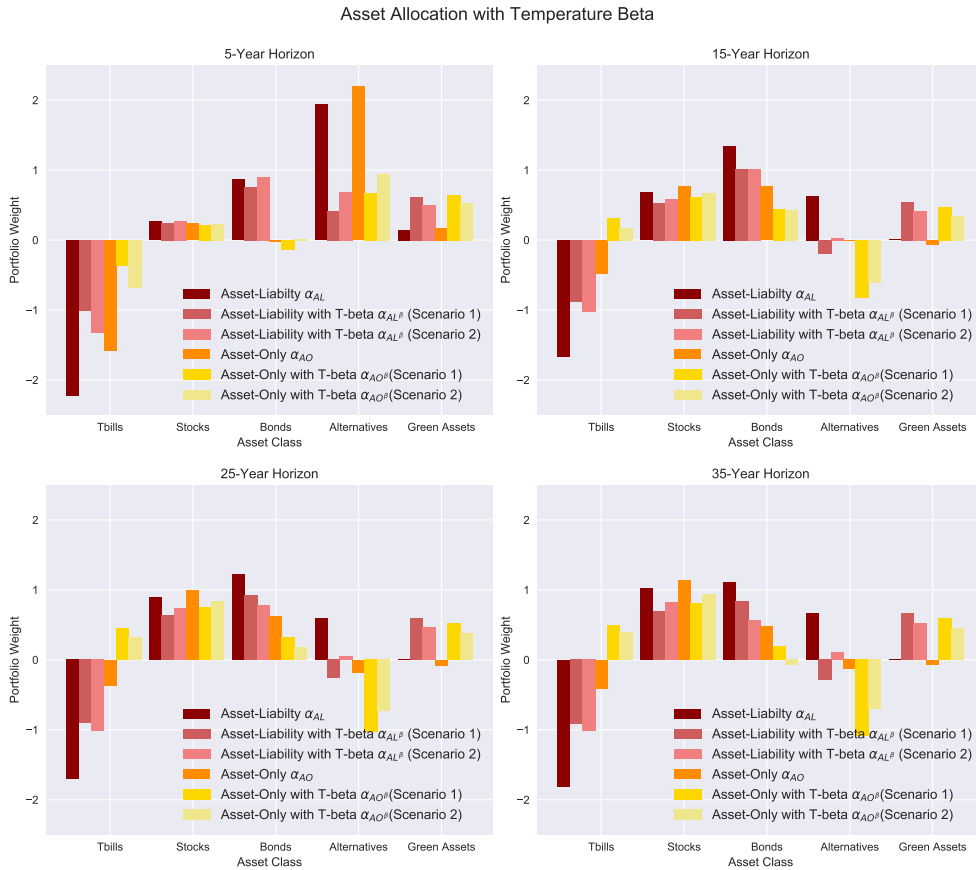
For medium and long-term asset-only investors, their holdings of alternatives drop dramatically to negative weights due to the time-diverging nature of most of the alternatives' term structure of risks. For asset-liability investors, alternatives are also largely depreciated but in a milder manner. Stocks and bonds are less influenced by the depreciation of returns than alternatives, particularly for asset-liability investors. For asset-liability investors, both stocks and bonds keep playing dominating roles in the long-term portfolio, thanks to their mean-reverting and liability-hedging risk characteristics. The temperature beta cuts the stock exposure by about 10% and reduces the bond exposure by about 40% in both beta scenarios. For asset-only investors, the reduction of primary asset's exposure is a bit more dramatic, for the temperature betas largely enhance the speculative opportunities of the green assets hence weaken the attractiveness of long-term bonds.

In general, Figure 6 shows that temperature betas substantially increase the green funds exposure by almost 40% if without short-selling constraint. Investors with liability constraints have a higher preference for green funds, as we find some weakly strong liability-hedging feature of green assets. Stocks are still attractive even suffering from negative temperature betas, thanks to their mean-reversion risk characteristics. The estimated betas in scenario 1 have a wider range than the semi-uniformed betas, hence bring a larger impact to the optimal portfolios.

Figure 7 presents the short-position restricted optimal asset allocation strategies for both types of investors. The demand for green assets increases dramatically when short positions are prohibited, even without imposing temperature betas. We first look at the top figures that do not consider temperature betas. For investors with ALM perspectives (the left figure), their portfolio weight of green assets takes about 10% of their initial investment if the investment horizon is 5 years and goes up to 20% when the investment horizon reaches to 40 years. Primary assets dominate the restricted asset-liability

Figure 6: Optimal Portfolio with Temperature Beta

Implication of temperature beta for asset-liability and asset-only investors with four investment horizons (5, 15, 25 and 35 years) at the risk level of $\gamma = 10$. In temperature beta Scenario 1, we set beta of each asset equal to its annualized estimated coefficient on instantaneous temperature change from the VAR model. In Scenario 2, we set $\beta = -1$ for all “grey” assets which include primary assets and alternatives; and $\beta = 1$ for all “green” assets. Clear-red bars represent the optimal asset-liability portfolios without implementing temperature betas. Red bars with circles show the optimal asset-liability portfolio embedding temperature beta Scenario 1. Red bars with circles present the asset-liability portfolio with temperature beta Scenario 2. The clear orange bars plot the optimal asset-only portfolios without temperature betas. Yellow bars with circles are the results with temperature beta Scenario 1 and yellow bars with stars are with Scenario 2. The x-axis is the unconstrained portfolio weight, and the y-axis represents different asset classes. T-beta: temperature beta.



portfolios over entire investment horizons, due to their relatively low and mean-reverting risk characteristics. Bonds have a portfolio weight of almost 80% when the investment horizon is short, but their exposure vanishes when time span is beyond 30 years. Stock exposure increases with holding period and dominates the asset-liability portfolio in the

long run. Alternatives have an increasing weight over the holding period. For ultra-long term (beyond 30 years) investors, their optimal holding weight on alternatives is more than 30% of their portfolios. For asset-only investors (the right-top figure), alternatives dominates the short-term restricted portfolio, while decreases over time.

Next, we investigate the joint effect of short-selling restrictions and temperature betas on optimal portfolios (middle and bottom rows). In general, both temperature beta scenarios largely promotes the demand for green assets, while asset-liability investors are less influenced compared with asset-only investors. Therefore, even without temperature betas, investing in green funds is rational for asset-liability investors. Under Scenario 1, alternatives are dramatically depreciated by the asset-liability investors, mainly because of the negative beta effects on credits and hedge funds as they dominate the optimal holdings of alternatives. Stocks are also less attractive due to their reasonably large negative beta effect. Compared with Scenario 1, the semi-uniformed temperature beta scenario is much more moderate for asset-liability investors, in the sense that it maintains the time-varying construction of the restricted portfolio while only shrink the exposure of grey assets. As can see, the optimal asset-liability portfolio under Scenario 2 has a similar pattern as the temperature beta free scenario. Even when imposing temperature betas, stocks still have a portfolio weight of more than 50% in the long run. This result is consistent with economic notion that stocks are less risky than bonds in the long run (see [Siegel \(1998\)](#)). Alternatives only attract long-term asset-liability investors and their exposure shrinks by 50 percent when imposing the semi-uniformed temperature beta. Among the four types of alternatives, credits dominates the subclass, due to there low risk and liability hedging feature. In general, lower risk asset classes are more attractive to the restricted asset-liability investors. [Guiso et al. \(1996\)](#) also claims that borrowing constraint can reduce the risk exposure of the optimal portfolio.

Asset-only portfolios are much more sensitive to both temperature beta scenarios, as their hedging portfolio fully depends on the risk-return trade-offs of individual asset class. Without temperature beta, alternatives dominate the restricted portfolio, mainly attribute to hedge funds, which enjoy high rewards with relatively low risk. However, [Stulz \(2007\)](#) argues that it is very difficult to assess hedge fund returns, since it is an unregulated market and reported performances could be biased. Fixed-income assets are not well received in the asset-only portfolio due to their low returns. After adjusting

the temperature exposure of each asset, green exposure dominates the portfolio, with an average portfolio weight of more than 50%. Alternatives shrink dramatically, since their returns are adjusted downwards by the temperature beta. Stocks is not sensitively to the penalty from temperature beta, which is also the case in for the unrestricted portfolios. Tbills are assumed temperature neutral, hence become attractive when returns of long-term bonds and credits are penalized by higher temperature exposure.

In summary, we see significant increase of the green portfolio for both types of investors when imposing short-selling constraint. The temperature beta is not very effective to the asset-liability investor, since the liability hedging component that leads the asset allocation strategy is temperature beta neutral. Asset-only investors shift more than half of their portfolio from the grey assets to the greens if temperature-beta policy is implemented.

6 Economic Valuation

In practice, investors usually have to adopt alternative asset allocation strategies rather than (15) or (20) for reasons of reputation risk, policy constraints, etc. What are the benefits of investing green assets for institutional investors? What are the costs of introducing temperature betas and imposing no-short restrictions? In this section we evaluate the economic benefits or losses from ignoring green assets.

We use the certainty equivalent to evaluate the economic loss of deviating from the optimal strategic asset allocations. We first consider the economics loss of ignoring green assets for optimistic investors with ALM perspective. Define the economic loss of holding some sub-optimal portfolio $\alpha_0^{(\tau)}$ instead of the optimal portfolio (15) by computing the percentage risk-free return the investor requires to be compensated for holding the sub-optimal portfolio $\alpha_0^{(\tau)}$ instead of $\alpha_{AL}^{(\tau)}$. For the “asset-liability ” investor, the certainty equivalent is defined as the difference between the mean-variance utility of the optimal and sub-optimal portfolios,

$$f_t^{(\tau)} = \left(\alpha_{AL,t}^{(\tau)} - \alpha_{0,t}^{(\tau)} \right)' \left(\mu_{A,t}^{(\tau)} + \frac{1}{2} \sigma_A^2 \right) - (1 - \gamma) \left(\alpha_{AL,t}^{(\tau)} - \alpha_{0,t}^{(\tau)} \right)' \sigma_{AL,t}^{(\tau)} \quad (30)$$

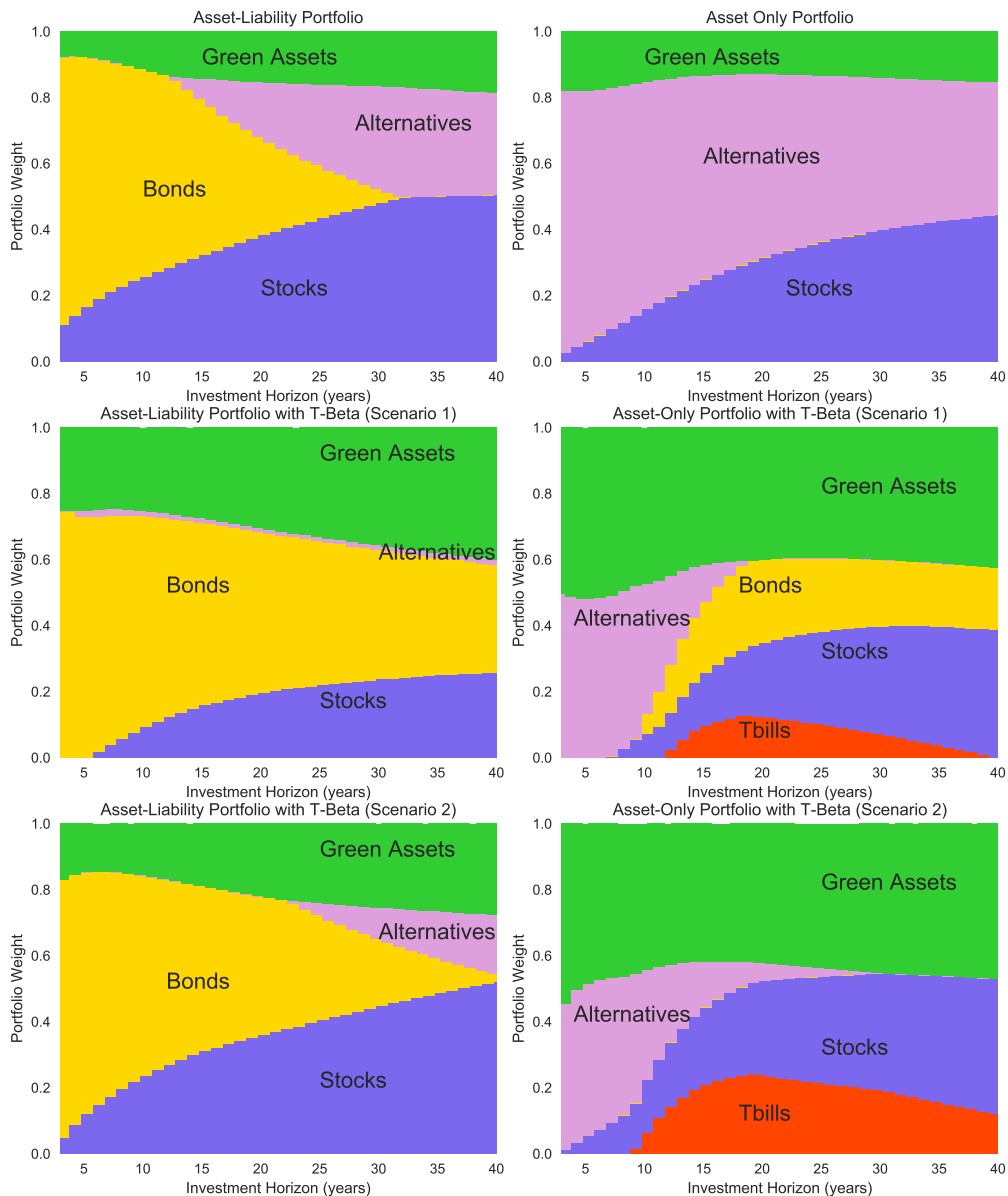
$$- \frac{1}{2} \gamma \left(\alpha_{AL,t}^{(\tau)'} \Sigma_{AA} \alpha_{AL,t}^{(\tau)} - \alpha_{0,t}^{(\tau)'} \Sigma_{AA} \alpha_{0,t}^{(\tau)} \right)$$

The certainty equivalent function consists of three components. The first term of (30)

Figure 7: Optimal Asset Allocation Portfolio with Short-Selling Constraint

Optimal portfolio with borrowing and short-selling constraint over investment horizons up to 40 years. The figure compare the optimal investment strategies with and without imposing temperature beta for both asset-liability and asset-only investors. The left figures plot the restricted asset liability portfolios and the right panels present the restricted asset-only portfolios. The top two figures are without embedding temperature betas. The middle two figures implement the estimated betas (Scenario 1) and the bottom two figures impose the semi-uniformed betas (Scenario 2). T-beta: temperature beta.

Asset Allocation with Short-Selling Constraint



reflects the compensation for the difference in expected returns by investing in the sub-optimal portfolio. The second term represents the compensation for the sub-optimal liability hedge. The third term accounts for diversification of risks among green asset classes.

In Figure 8, we show the certainty equivalent as the percentage the initial funding ratio should rise in order to compensate the investor for sub-optimal investing: $F_t^{(\tau)} = 100 \left(\exp \left(f_t^{(\tau)} \right) - 1 \right)$. It can be interpreted as the monetary compensation the investor requires in dollar terms in order to put 100 dollars in alternative strategies $\alpha_0^{(\tau)}$ rather than $\alpha_{AL}^{(\tau)}$. We consider four different sub-optimal portfolios. The top-left panel of Figure 8 depicts the benefit for an investor with green assets exposure. The certainty equivalent is positive with a hump shaped short-term structure, but decreases to almost zero, due to the time diminishing exposure to greens (see Table 9). The figure tells that the compensation for ignoring green assets is tiny. At the 5-year horizon, a risk averse ($\gamma = 10$) investor with ALM perspective requires a lump sum of only 10 cents for each 100 dollars of initial investment to be compensated for having to ignore green assets. The loss to missed return opportunities is only relevant at shorter horizons. In terms of liability hedge and risk diversification, green assets are worse off. In general, the certainty equivalent of ignoring green funds is mainly attributed the return enhancement.

The compensation for implementing the estimated temperature beta policy is costly (see the top-right figure), which takes about 4% of the initial investment for short-term investors and increases with the hold period of time. The required compensation is attribute to both return enhancement and risk diversification. The semi-uniformed temperature beta policy is much cheaper with a monetary compensation of 1% of the initial investment and is irrelevant to the holding period of time. The semi-uniformed temperature beta enhances the investment returns and is also better off in terms of the liability hedge. However, the benefit from return enhancement and liability hedging is cancelled out by risk diversification.

The bottom-left panel measures the cost of imposing short-selling constraint. The short-term asset-liability investors require a compensation of 2.5 dollars out of 100 dollars for imposing short-selling constraint. The cost increases slowly with investment horizon to 3 dollars at 40-year horizon. Return enhancement dominates the components of certainty equivalent but is largely undone by the better risk diversification of constraint asset-

liability portfolio.

In general, the economic loss of ignoring green assets is negligible, especially for long-term investors. Both the temperature beta and short-selling restriction can help to largely increase the optimal weight of green assets, but the semi-uniformed temperature beta policy is far less costly. However, even with the lowest compensation requirement, the opportunity cost of imposing semi-uniformed temperature beta is still much larger than just ignoring the green assets.

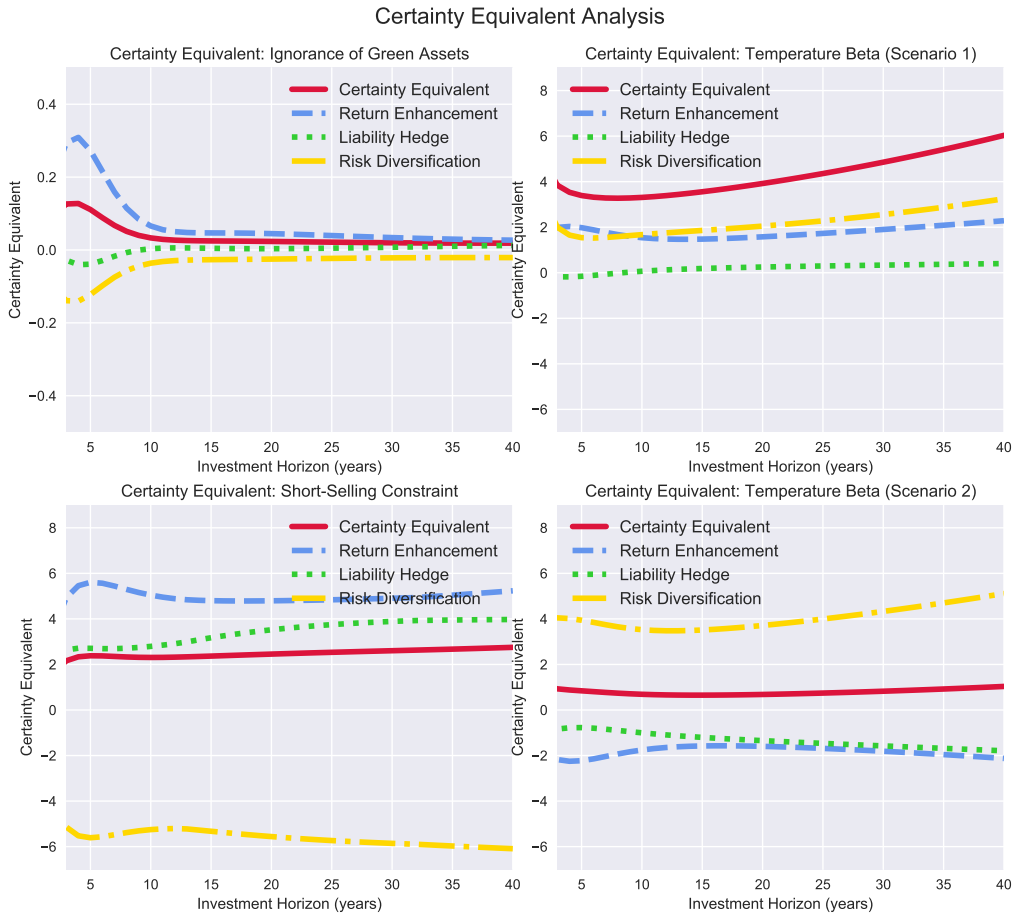
7 Conclusion

We argue that the myth behind the motivation of green investment is mainly driven a lack of knowledge about the potential impact of climate change on various asset classes. To address the uncertain feature of climate effect on various asset returns, we introduce a temperature beta term by disentangling the climate impact on asset returns from the impact from other macro economic factors. Temperature beta is a product of climate sensitivity parameter and the future temperature projection. The climate sensitivity parameter β is estimated using a factor model, with a null hypothesis that grey assets are negatively related with temperature risk and green assets are either positive or not related to climate risk. Our estimated VAR model presents a weak evidence on this hypothesis. The second component is the future temperature trajectory. We employ a quantitative scenario analysis by simulate two climate scenarios: an optimistic scenario based on the VAR estimation; and a pessimistic scenario using DICE model. The optimistic temperature trajectory is much lower than the pessimistic one. The temperature beta term plays an important role on explaining the puzzle behind green investing. It not only connects the financial risk to climate change but also brings the past climate-finance interactions to the future. In general, the temperature beta term tends to penalizes the “grey” assets by perturbing asset returns downwards, while reward the green assets by perturb their returns upwards.

First, we show that investors with optimistic view on climate change (do not believe in global warming) are not motivated to invest in green. Although, our VAR model shows that green funds are more attractive to short-term investors with ALM perspective, due to their short-term rewards and liability hedging characteristics. The optimal portfolio

Figure 8: The Certainty Equivalent

The figure shows the certainty equivalent of alternative asset allocation strategies for a τ -holding period investor. The certainty equivalent is attributed to three components: a return compensation, a hedging compensation and a risk diversification compensation. The analysis is based the optimal asset-liability portfolio (15) with green asset under the risk aversion level of 10. We considers four sub-optimal portfolios: an optimal asset-liability portfolio without green assets; an optimal asset-liability portfolio with estimated temperature beta (Scenario 1); an optimal asset-liability portfolio with short-selling constraint; and an optimal asset-liability portfolio with semi-uniformed temperature beta (Scenario 2). x -axis is the investment horizon and y -axis is the economic compensation of switching to a sub-optimal portfolio for 100 dollars of initial investment.



weight on greens is fairly small compared with other asset classes for optimistic investors. Alternatives dominates the short-term portfolio for both types of investors due to their relatively higher returns. Long-term optimal portfolios are dominated by primary assets, due to their mean-reversion characteristics of term structure of risks. We show that the economic compensation of ignoring green assets for an optimistic investor is tiny.

Second, pessimistic investors want to shift a significant percentage of their portfolio

to green assets. This is because the temperature beta with a pessimistic climate scenario largely promotes the economic value of green assets but downgrades the grey ones. Stocks and bonds are, however less sensitive to the temperature beta policy, as hedging components attribute most to the optimal weights of stocks and bonds

We also find that restricting short-selling can largely increase the demand for green assets even for optimistic investors. An average of more than 15% of the portfolio weight is attributed to green asset for both types of investors. The short-selling restricted asset-only portfolio is very sensitive to the temperature beta policy with a dramatic increase of green assets and a substantial decrease of alternatives. The restricted asset-liability portfolios are much less sensitive to the temperature beta effect.

A Holding-Period Risk and Returns

Given the VAR(1) formulation of \mathbf{z}_t , we can derive the j - period ahead forecast of the state vector \mathbf{z}_{t+j}

$$\mathbf{z}_{t+j} = \left(\sum_{i=0}^{j-1} \Phi_1^i \right) \Phi_0 + \Phi_1^j \mathbf{z}_t + \sum_{i=0}^{j-1} \Phi_1^i \mathbf{u}_{t+j-i}$$

So the j - period ahead forecast (expected value) of \mathbf{z}_{t+j} is

$$\hat{\mathbf{z}}_{t+j|t} = \sum_{i=0}^{j-1} \Phi_1^i \Phi_0 + \Phi_1^j \mathbf{z}_t$$

The cumulative expected return over τ period is given by

$$\mathbf{z}_{t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} \mathbf{z}_{t+j}$$

and the expectation of $\mathbf{z}_{t+\tau}^{(\tau)}$ is given by

$$\hat{\mathbf{z}}_{t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} \left(\sum_{i=0}^{j-1} \Phi_1^i \Phi_0 + \Phi_1^j \mathbf{z}_t \right) \quad (31)$$

with an forecast error of

$$\mathbf{z}_{t+\tau}^{(\tau)} - \hat{\mathbf{z}}_{t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} \sum_{i=0}^{j-1} \Phi_1^i \mathbf{u}_{t+j-i}$$

The covariance matrix of the τ - period error follows as

$$\Sigma^{(\tau)} = \sum_{j=1}^{\tau} \left(\left(\sum_{i=0}^{j-1} \Phi_1^i \right) \Sigma \left(\sum_{i=0}^{j-1} \Phi_1^i \right) \right) \quad (32)$$

B Variables Term Structure

This section explores the long-term dynamics of assets and liabilities implied by the estimated VAR model (see Equation (8) and Section 3). In particular, we discuss the (annualized) forecasts implied by our model over different time horizons, that is,

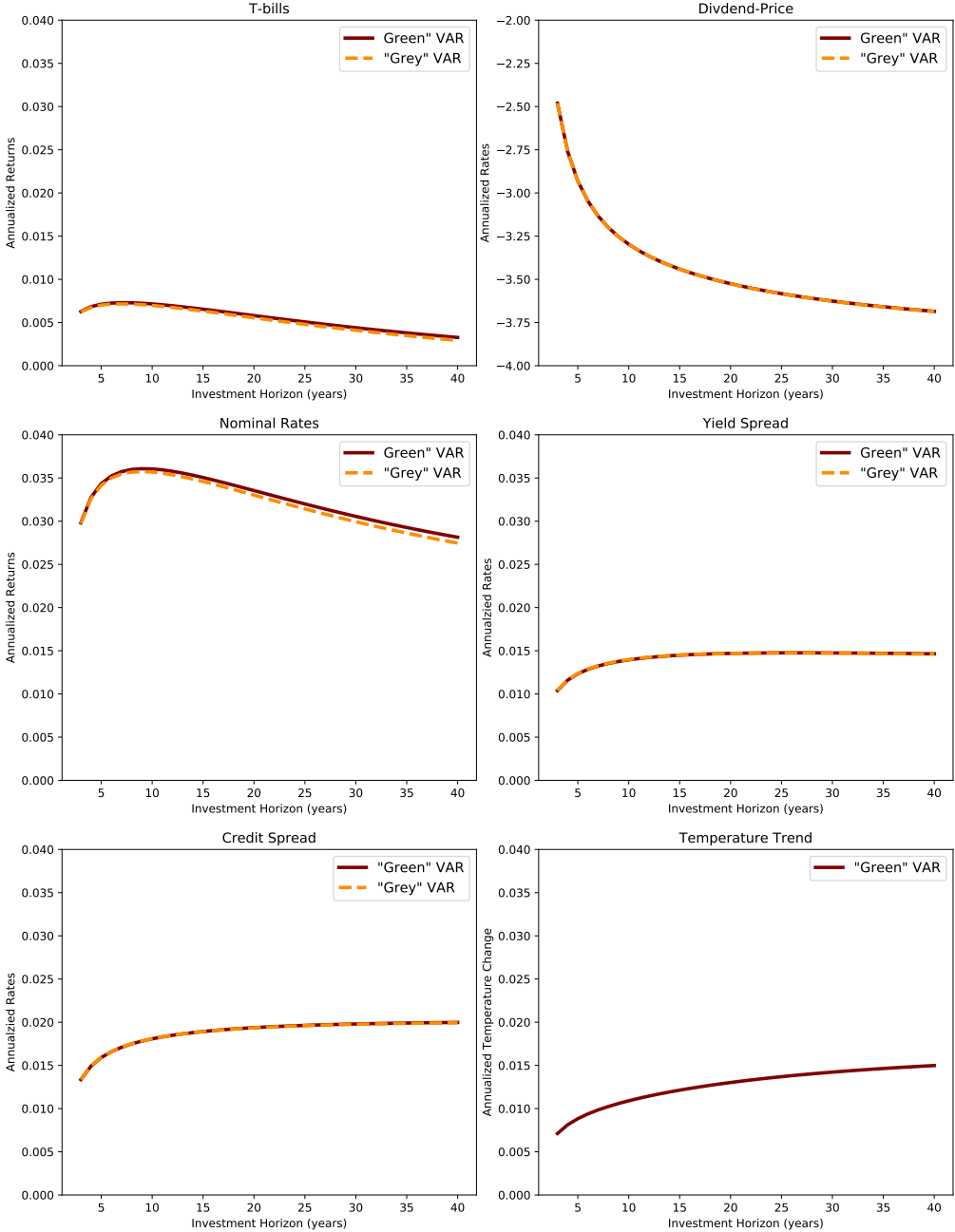
$$\mu_{z,t}^{(\tau)} = \frac{1}{\tau} \mathbb{E}_t \left(\mathbf{z}_{t+\tau}^{(\tau)} \right) \quad (33)$$

where $\mathbf{z}_{t+\tau}^{(\tau)}$ is the vector of all modelled variables (see also Appendix A). As it follows from (33), the analyzed τ -period dynamics take the average of cumulative real holding period return. We compare the “green” model (8) that includes the temperature ΔT_t as a variable in the VAR model with the “grey” model which excludes the temperature dynamics. Figure 9 plots the term structure of some of the state variables over an investment horizon of 40 years. From Figure 9 it follows that the forecasts of the two models coincide for most of the core state variables except for the nominal rates that appear to be somewhat higher in the ‘green’ model.

It is plausible that real interest rates are mean-reverting as the mean-reverting rates reflect per capita economic growth (see Lopez and Reyes (2009)), which is mainly driven by technological innovation. As it follows from Figure 9, the nominal rates also appear to be mean reverting, which goes along with the economic theory since inflation target that meets the gap between real and nominal rates is rather stable. The dividend–price ratio curve predicts dividend growth with a negative sign over time horizon. The downward sloping result supports the notion that expected dividend growth and real rates move at the same direction (Campbell and Shiller (1988a)). Both yield spreads and credit spreads increase with investment horizon at a diminishing rate.

Figure 9: Annualized returns (y-axis) of state variables $r_{tb,t}$ and s_t across different investment horizon (in years on x-axis). Solid lines are implied by the VAR model (8) including temperature shocks. The dashed lines are derived from the traditional VAR model excluding temperature shocks.

Term Structure of Expectations of State Variables



The term structure of the temperature trend also increases at a diminishing rate. The concavity structure indicates a slowing down of warming in the future and the results counter the results from the consumption-climate model (Lemoine and Rudik (2017)). Figure 4 shows that the VAR model implied ΔT projections are even lower than the low-emission projections implied by the consumption-climate model. In general, the VAR model does not offer strong predictive power for temperature change. We therefore use the temperature projections from the consumption-climate model (27) for forward looking analysis.

Figure 10: Term Structure of Primary Asset and Liability Returns

Annualized returns (y-axis) of primary assets and liabilities across different investment horizon (in years on x-axis). Solid lines are implied by the “green” VAR model (8). The dashed lines are derived from the “grey” VAR model excluding temperature shocks.

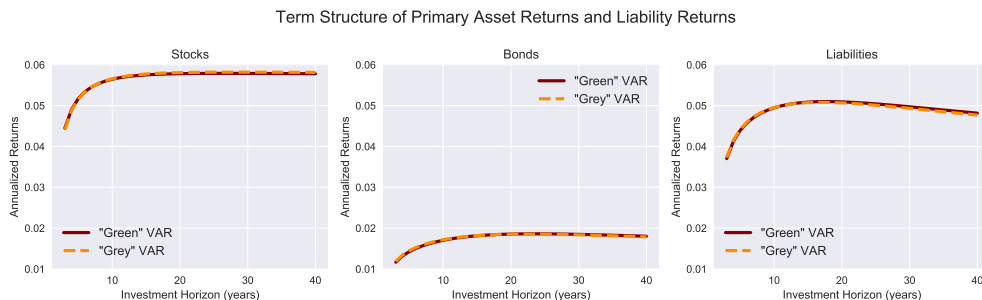
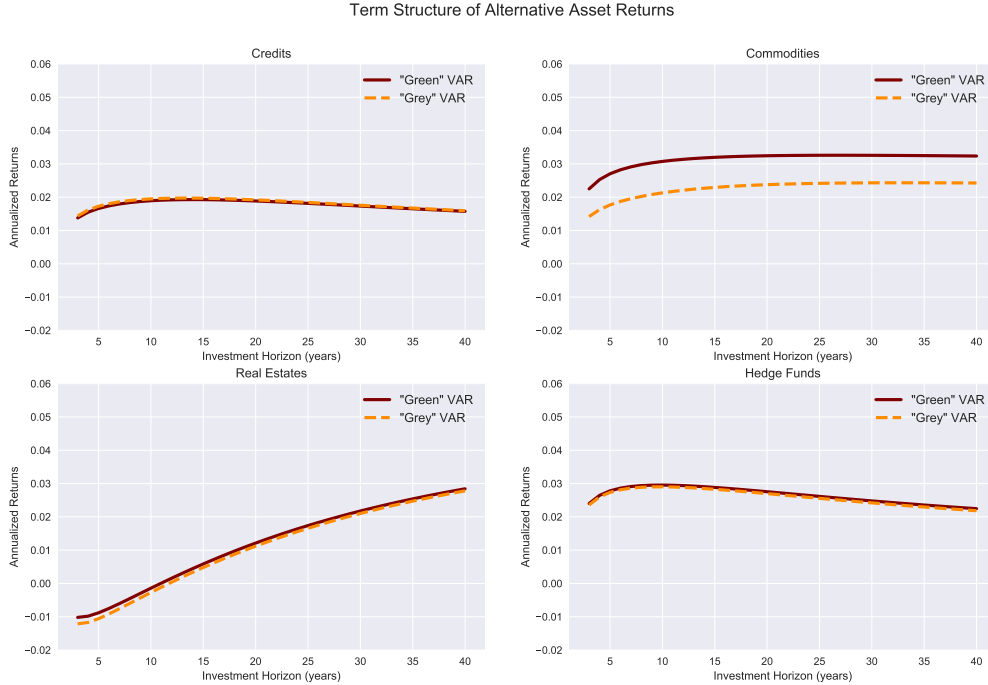


Figure 10 depicts the holding-period’s annual returns for stocks, bonds and liabilities over various investment horizons. The two VAR models imply similar return term structures for the bonds. Hence the trend of warming does not bring much impact to the performance of the long-term bond markets. The upward sloping term structure of stock returns is mainly driven by the credit spreads. The mixed sign of real rates and nominal rates on stock returns weaken the mean-reverting effect of stock returns (see Table 3). The “green” model implies slightly lower stock returns in the long run than the returns from the “grey” model. This means the long-lasting stock markets have been aware of the impact of climate change. Warming can certainly hurt the “grey” markets in the long run, but the impact is too small to be considered. A potential reason for the small impact is due to the underestimated projections for temperature trends, as discussed in Figure 4. The mean-reverting effects of liability returns are mainly driven by the nominal rates. Temperature shocks slightly increase long-term liabilities, which means pension funds would underestimate the value of liabilities if environmental risks are ignored.

Figure 11 presents the expected return structure of alternatives. The annual holding-

Figure 11: Term Structure of Alternative Asset Returns

Annualized returns (*y-axis*) of alternatives across different investment horizon (in years on *x-axis*). Solid lines are implied by the “green” VAR model (8). The dashed lines are derived from the “grey” VAR model.



period return of commodities implied by the “green” VAR model is about 1% higher than the “grey” model across over the investment horizon. Without implementing climate policies, such as carbon taxation, carbon mitigation policy, etc., the awareness of warming corrects the commodity market upwards. This counter-intuitive result is inline with economic notion that warming induces high prices for energy commodities, since controlling supply leads to price increase. A recent study by [McCollum et al. \(2016\)](#) show that climate change may decouple the fuel prices over the next decade. The “green” model implies slightly higher (5 basis points) returns for the real estate, while the difference is small.

Figure 12 plots the hold-period returns of green assets over various investment horizons. Ideally, “green” fund returns implied by the “green” VAR model should have higher returns than the other model’s implied returns. This is because “green” funds by definition are positively related to the temperature increase. This law can be verified by almost all green assets. This result shows that long-term investors tend to underestimate the value of green assets by not adequately considering warming or climate change. Both green

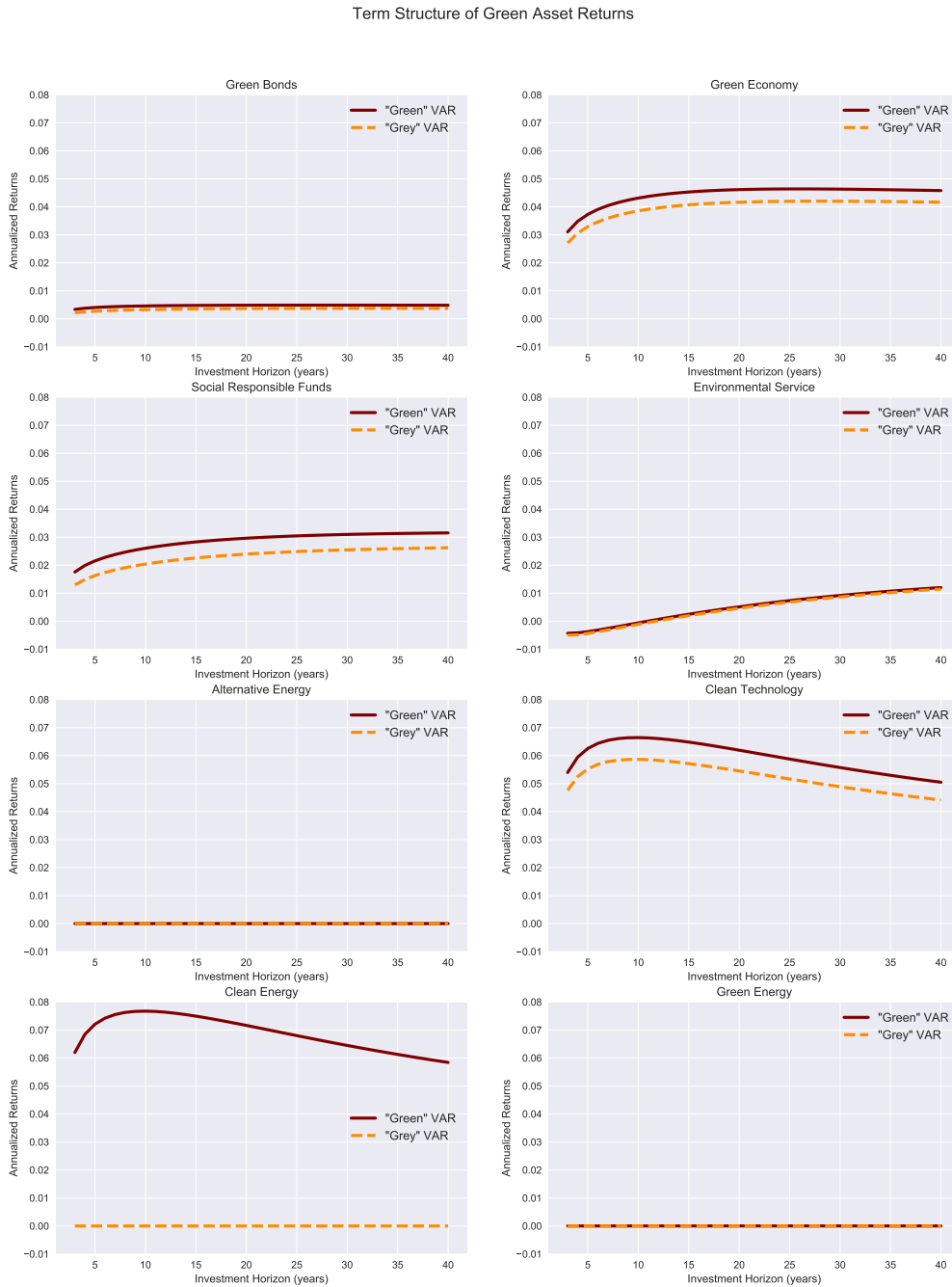
energy funds and alternative energy funds generate no returns in both models. This is mainly because of their negative historical returns as well as their short historical time span. Our VAR model cannot fully capture the long-run relationship between “green” assets and economic drivers of the economy. Second, the dynamics of green funds are likely driven by other factors as well, including social pressure, political uncertainty, news exposure, etc., but are not included in the model. In general, the “green” VAR model implies almost the same returns for primary assets and alternatives. Consideration of temperature change results in additional rewards for most of the green funds except for some “pure-play” funds that have negative annual returns. From the term structure of asset returns, it is fair to conclude that the “green” markets can recognize and benefit from the trend of warming, but the traditional markets and most of the alternatives cannot.

Figure 13 shows the annualized conditional standard deviation of holding period returns for primary assets as well as for liabilities over investment horizons up to 40 years. Figure 14 and 15 depict the term structure of risks for alternatives and green assets. We compare the term structure of risks implied by the two VAR models. The two “pure-play” funds that generate zero returns are no longer included in the investment vehicles for the rest of the analysis. The predictability of dividend yield, nominal rate, yield spread and credit spread, as shown in Table 3, contribute most to the term structure of risks across different asset classes. Both stocks and bonds are found to be less risky in the long run, which agrees with [Campbell and Viceira \(2005\)](#) and [Hoevenaars et al. \(2008\)](#). The mean-reverting pattern of stock volatilities is mostly attributed to dividend price ratio (see Table 4). Bond volatilities also have mean-reverting behavior, which is mainly driven by the negative effect of the nominal rate. Although yield spread induces positive shocks to the bonds, the effect of the nominal rate effect still dominates. Due to the offsetting effect between current and future predictable drivers and their joint positive effect, the term structure of liabilities and credits presents a hump-shaped pattern. For investment horizon up to 10 years, the mean-reverting bond returns drive the time movement of the liability risk. However, positive shocks from nominal rates and spread terms dominate the long run and drive up the long-term liability risk. A similar story applies to the term structure of credits.

The rest of the asset classes all have upward sloping risk term structure, hence losing the attractiveness of time diversification. Both commodities and public real estate are

Figure 12: Term Structure of Green-Asset Returns

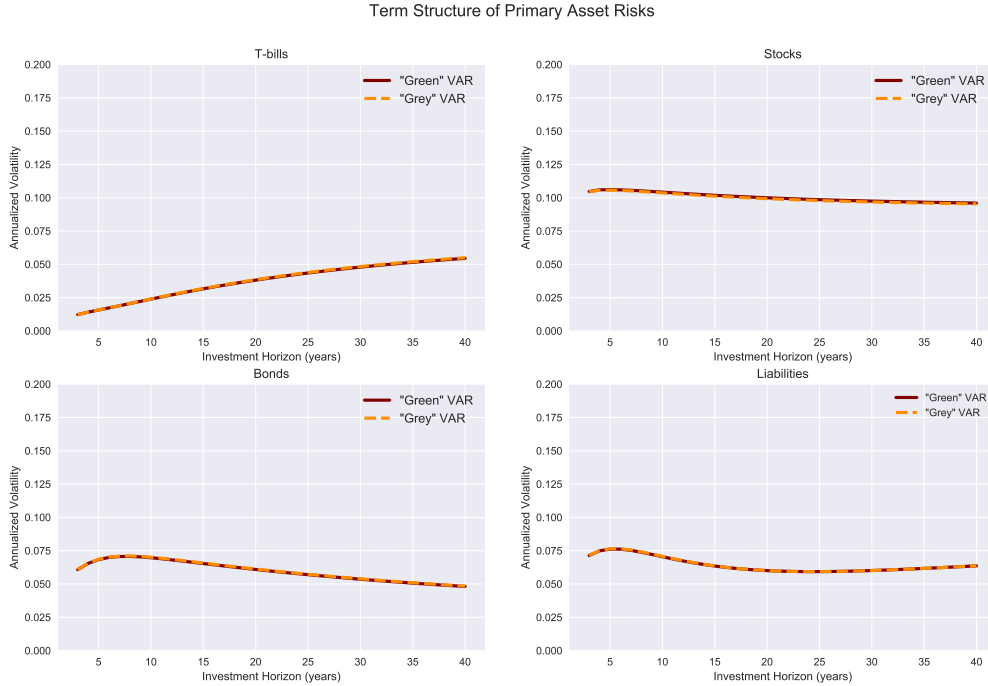
Annualized returns (*y-axis*) of green assets across different investment horizon (in years on *x-axis*). Solid lines are implied by the “green” VAR model (8). The dashed lines are derived from the “grey” VAR model.



high risk for long-term investors. Commodity volatility is mainly attribute to the shocks to the nominal rates, since commodities are frequently used to hedge inflation risks. The

Figure 13: Term Structure of Risks for Primary Assets and Liabilities

Annualized volatility of primary assets and liabilities for a buy and hold investor across different investment horizons (in years). Solid lines represent asset volatilities implied by the “green” VAR model and the dashed lines represent asset volatilities implied by the “grey” VAR model without temperature factor.

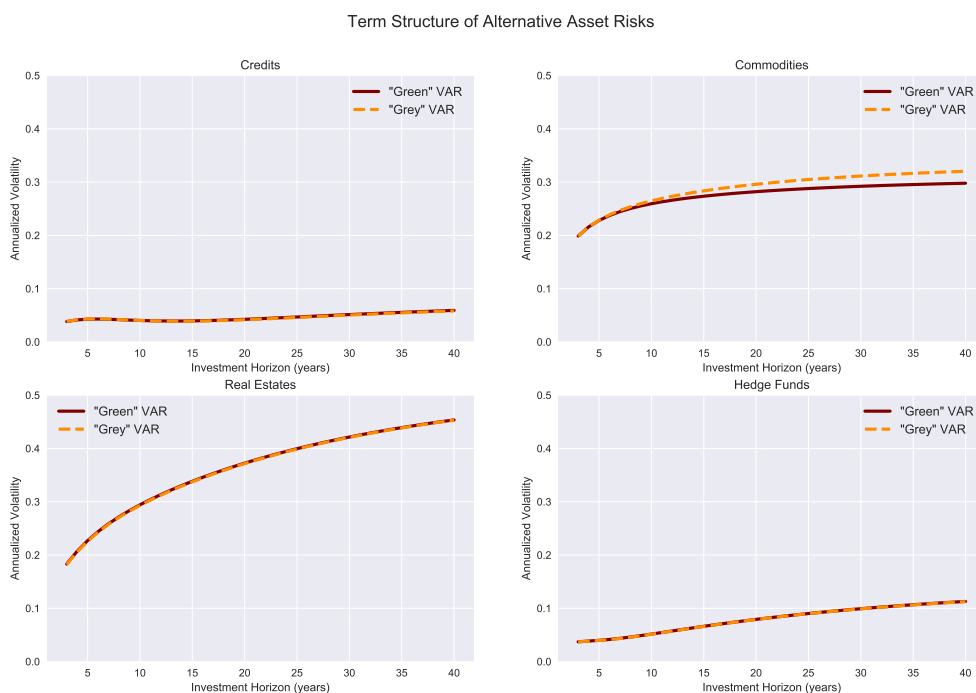


“green” VAR model’s implied commodity risk is about 2% lower than the “grey” model, which still follows the economic theory of [McCollum et al. \(2016\)](#) that says warming induces a sustained decoupling of future energy prices for the next few decades. As also claimed in [Titman and Warga \(1986\)](#), real estate is extremely volatile compared with stocks, especially in the long run. The shape of the real estate risk is driven mainly driven by the yield spreads (see [Table 5](#))

Temperature shocks do not change the term structure of risks for primary assets and alternatives (except for commodities). However, ignoring temperature shocks systematically underestimates the risks of green assets (see [Figure 15](#)). The “Green” model corrects the term structure of green asset risk upwards in response to the higher returns as shown in [Figure 12](#). This result counters the intuition that green assets are resilient against climate change.

Figure 14: Term Structure of Risks for Alternatives

Annualized volatility of alternatives for a buy and hold investor across different investment horizons (in years). Solid lines represent asset volatilities implied by the “green” VAR model and the dashed lines represent asset volatilities implied by the “grey” VAR model without temperature factor.

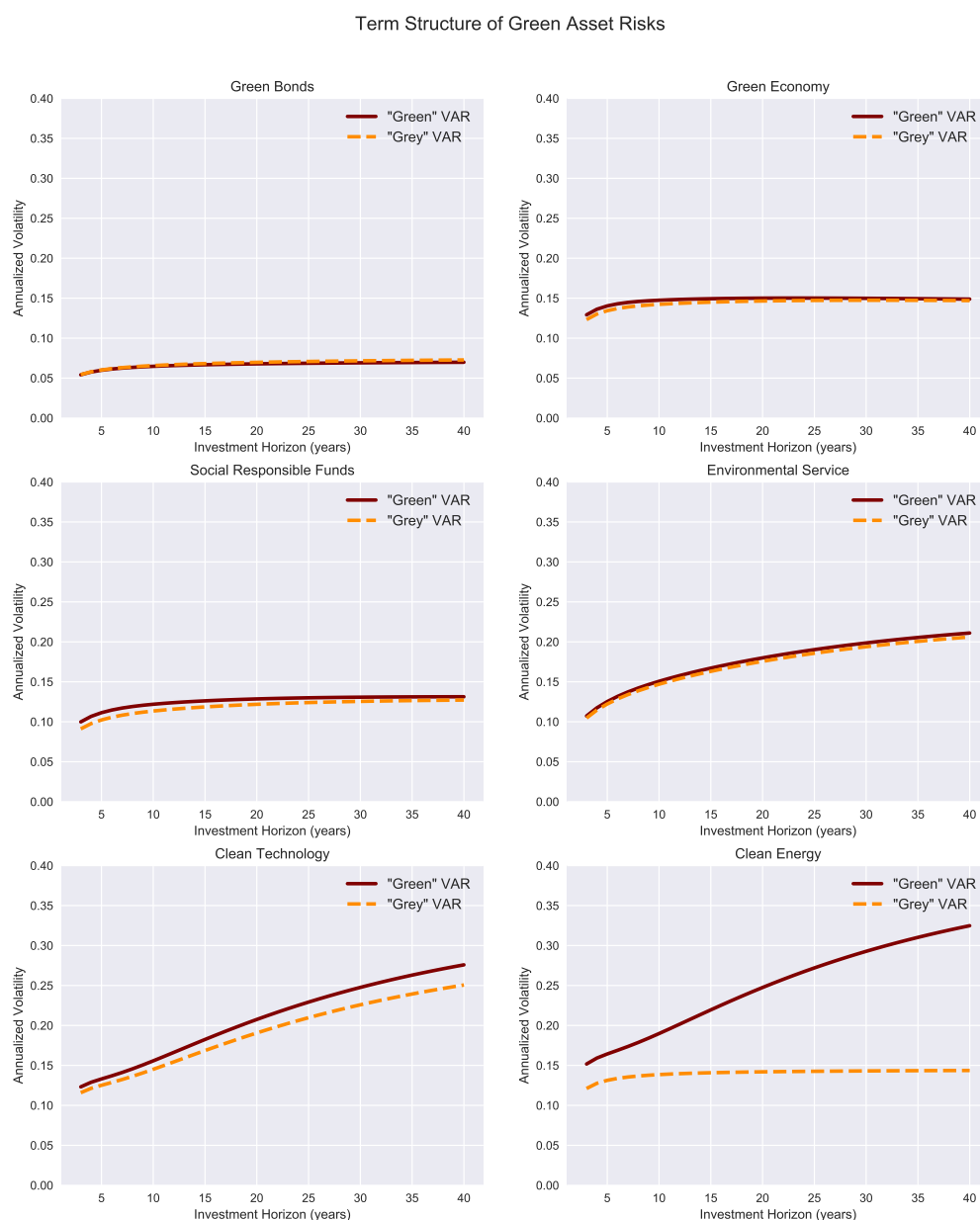


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Figure 15: Term Structure of Risks for Green Assets

Annualized volatility of green assets for a buy and hold investor across different investment horizons (in years). Solid lines represent asset volatilities implied by the “green” VAR model and the dashed lines represent asset volatilities implied by the “grey” VAR model without temperature factor.



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Table 6: Excess Return Regressions.

The table reports parameter estimates for the excess returns of the “green” assets \mathbf{x}_3 with $\mathbf{x}_{3,t+1} = \mathbf{d} + \mathbf{D}_0\mathbf{y}_{t+1} + \mathbf{D}_1\mathbf{y}_t + \mathbf{D}_2\mathbf{x}_{2,t+1} + \mathbf{D}_3\mathbf{x}_{2,t} + \mathbf{D}_4\mathbf{x}_{3,t} + \epsilon_{3,t+1}$. We include green bonds x_{gb} , a decarbonized fund x_{ge} , SRI funds x_{sri} and five “pure-play” clean energy funds x_{ppi} (for $i = 1 \dots 5$) into the “green” portfolio. For the purpose of empirical analysis and portfolio construction, we set all insignificant coefficients equal to zero at the significance level of 10%.

Contemporaneous D ₀										
	$r_{tb,t+1}$	dp_{t+1}	$r_{nom,t+1}$	$y_{spr,t+1}$	$c_{spr,t+1}$	ΔT_{t+1}	$x_{s,t+1}$	$x_{b,t+1}$	$x_{L,t+1}$	\mathbf{d}
$x_{gb,t+1}$	-1.33 (-0.84)	0.54 (0.81)	244.84 (0.35)	184.67 (0.27)	-12.68 (-0.34)	-0.21 (-0.95)	0.30 (0.42)	0.06 (0.68)	-0.15 (-0.33)	0.19 (0.55)
$x_{ge,t+1}$	-0.43 (-0.21)	0.64 (0.76)	-188.94 (-0.21)	-148.40 (-0.17)	-4.68 (-0.10)	0.23 (0.83)	0.76 (0.84)	-0.11 (-0.98)	-0.67 (-1.13)	0.68 (1.50)
$x_{sri,t+1}$	-0.77 (-0.56)	-0.55 (-0.97)	-837.56 (-1.40)	-801.96 (-1.36)	19.57 (0.60)	0.30 (1.56)	-0.22 (-0.37)	0.01 (0.12)	-0.28 (-0.71)	0.37 (1.22)
x_{pp1}	0.13 (0.05)	-1.08 (-1.03)	-2256.72 (-2.02)	-2220.71 (-2.03)	-26.80 (-0.44)	0.00 (0.00)	-1.01 (-0.90)	-0.09 (-0.61)	0.40 (0.53)	0.72 (1.29)
x_{pp2}	6.13 (1.33)	2.23 (1.17)	-1395.22 (-0.69)	-1293.87 (-0.65)	-83.83 (-0.77)	-0.25 (-0.38)	3.32 (1.62)	-0.22 (-0.85)	-2.45 (-1.82)	0.51 (0.50)
x_{pp3}	0.69 (0.25)	0.48 (0.42)	-970.35 (-0.79)	-936.51 (-0.78)	28.80 (0.44)	0.57 (1.47)	0.54 (0.44)	-0.24 (-1.57)	-1.12 (-1.38)	0.39 (0.64)
x_{pp4}	3.43 (0.82)	1.83 (1.05)	-303.19 (-0.16)	-121.18 (-0.07)	-78.51 (-0.79)	-0.38 (-0.65)	2.68 (1.44)	-0.32 (-1.36)	-2.96 (-2.42)	0.54 (0.58)
x_{pp5}	8.75 (1.93)	2.55 (1.36)	-2010.21 (-1.01)	-1901.15 (-0.97)	-18.87 (-0.17)	-0.11 (-0.17)	3.78 (1.87)	-0.32 (-1.28)	-1.50 (-1.13)	0.65 (0.64)
Lagged D ₁										
	$r_{tb,t}$	dp_t	$r_{nom,t}$	$y_{spr,t}$	$c_{spr,t}$	ΔT_t	$x_{s,t}$	$x_{b,t}$	$x_{L,t}$	
$x_{gb,t+1}$	0.83 (0.74)	-0.47 (-0.73)	-217.46 (-0.31)	-172.59 (-0.25)	23.62 (0.61)	-0.07 (-0.30)	0.06 (0.37)	1.67 (0.24)	-0.22 (-1.75)	
$x_{ge,t+1}$	0.19 (0.13)	-0.45 (-0.55)	253.63 (0.28)	175.46 (0.20)	4.78 (0.10)	-0.20 (-0.69)	0.14 (0.68)	-0.03 (0.00)	-0.08 (-0.51)	
$x_{sri,t+1}$	-0.83 (-0.87)	0.64 (1.17)	848.88 (1.41)	814.02 (1.37)	-21.86 (-0.65)	-0.42 (-2.15)	0.09 (0.65)	-6.87 (-1.17)	0.05 (0.46)	
x_{pp1}	-2.21 (-1.23)	1.27 (1.24)	2297.72 (2.05)	2255.81 (2.05)	31.89 (0.51)	-0.14 (-0.39)	-0.11 (-0.44)	-21.71 (-1.98)	0.05 (0.22)	
x_{pp2}	2.44 (0.75)	-2.07 (-1.11)	1531.79 (0.75)	1355.14 (0.68)	84.12 (0.74)	1.19 (1.82)	0.71 (1.56)	-7.49 (-0.38)	-0.27 (-0.74)	
x_{pp3}	-0.68 (-0.35)	-0.34 (-0.31)	1102.49 (0.90)	990.48 (0.82)	-2.80 (-0.04)	-0.12 (-0.30)	0.06 (0.20)	-7.04 (-0.59)	-0.24 (-1.09)	
x_{pp4}	2.35 (0.79)	-1.64 (-0.97)	495.26 (0.27)	197.03 (0.11)	97.29 (0.94)	1.20 (2.00)	0.59 (1.41)	4.12 (0.23)	-0.09 (-0.25)	
x_{pp5}	0.30 (0.09)	-2.33 (-1.27)	2218.90 (1.11)	1990.37 (1.01)	44.92 (0.40)	1.17 (1.81)	0.85 (1.88)	-15.13 (-0.77)	0.12 (0.33)	
Contemporaneous D ₂										
	$r_{cor,t+1}$	$r_{com,t+1}$	$r_{re,t+1}$	$r_{hf,t+1}$	$r_{cor,t}$	$r_{com,t}$	$r_{re,t}$	$r_{hf,t}$	D ₄	R ²
$x_{gb,t+1}$	0.98 (1.90)	0.15 (2.82)	0.23 (2.48)	0.31 (1.03)	1.12 (1.93)	-0.11 (-1.93)	0.00 (-0.03)	0.04 (0.12)	-0.04 (-0.20)	0.73
$x_{ge,t+1}$	-1.06 (-1.60)	0.09 (1.37)	0.44 (3.73)	1.48 (3.82)	0.55 (0.74)	-0.02 (-0.24)	0.06 (0.45)	0.33 (0.69)	0.02 (0.06)	0.86
$x_{sri,t+1}$	-1.02 (-2.30)	0.07 (1.45)	0.40 (5.01)	0.90 (3.47)	0.13 (0.26)	0.02 (0.37)	0.00 (0.01)	-0.09 (-0.28)	0.27 (1.47)	0.89
x_{pp1}	-1.20 (-1.45)	0.15 (1.78)	0.42 (2.86)	0.95 (1.96)	0.09 (0.10)	-0.03 (-0.27)	0.06 (0.37)	0.33 (0.56)	-0.34 (-2.26)	0.73
x_{pp2}	-2.07 (-1.38)	-0.01 (-0.08)	0.11 (0.43)	1.36 (1.55)	-1.31 (-0.78)	0.06 (0.36)	0.54 (1.91)	1.75 (1.64)	0.32 (0.93)	0.71
x_{pp3}	-1.22 (-1.35)	0.05 (0.50)	0.28 (1.73)	2.25 (4.25)	0.64 (0.63)	0.02 (0.16)	0.10 (0.58)	0.92 (1.43)	-0.37 (-1.22)	0.82
x_{pp4}	-1.34 (-0.98)	-0.05 (-0.35)	0.20 (0.84)	1.54 (1.93)	-0.47 (-0.31)	0.10 (0.64)	0.31 (1.20)	2.60 (2.67)	0.21 (0.59)	0.76
x_{pp5}	-2.18 (-1.48)	-0.07 (-0.49)	0.13 (0.48)	1.92 (2.22)	-1.71 (-1.03)	0.12 (0.72)	0.52 (1.90)	2.71 (2.58)	-0.41 (-1.57)	0.76

Table 7: Parameter Values

The table summarize the calibrated parameters for temperature forecasting model.

Parameter	Value	Parameter	Value
α	0.0019	ϕ	0.0091
γ_0	0.29	T_0	0.75
γ_1	0.015	μ_C	0.039
δ	0.0138	σ_C	0.026
M_0	854 Gt C	C_0	\$64.75 trillion
M_{pre}	605 Gt C	S	10° C
ν	5.35 Wm^{-2}		

Table 8: Summary of Estimated Betas

The table summaries the estimated coefficients on instantaneous temperature change ΔT_{t+1} (except for stocks and bonds, which we take the coefficients on ΔT_t , since ΔT_{t+1} is part of the endogenous variables in the core VAR model (5)). The estimated betas shown in the table are in monthly basis. ** = 15% significance level and * = 5% significance level. x_s : stocks, x_b : bonds, x_{cor} : credits, x_{com} : commodities, x_{re} : REITs, x_{hf} : hedge funds, x_{gb} : green bonds, x_{ge} : green economy, x_{sri} : socially responsible index, x_{pp1} : environmental service ETF, x_{pp3} : clean-tech portfolios, x_{pp4} : global clean energy portfolios.

Asset	x_s	x_b	x_{cor}	x_{com}	x_{re}	x_{hf}	x_{gb}	x_{ge}	x_{sri}	x_{pp1}	x_{pp3}	x_{pp4}
β	-0.09**	-0.01	-0.03**	-0.79*	-0.18	-0.14**	-0.21	0.23	0.30**	0.00	0.57**	-0.38

Table 9: Optimal Portfolio Choice

The table shows the fixed-weights optimal portfolio holdings for asset-liability investors implied by the “green” VAR model and the classic VAR model for five investment horizons (5, 10, 20, 30 and 40 years) under the risk aversion level $\gamma = 10$.

Horizon (years)	“Green” ALM					“Grey” ALM				
	5	10	20	30	40	5	10	20	30	40
(A) Stock-Bond Portfolio for Asset-Liability Investors										
Stocks x_s	0.20	0.28	0.24	0.07	-0.08	0.26	0.31	0.27	0.10	-0.06
Bonds x_b	0.99	0.76	0.62	0.75	0.91	1.01	0.77	0.61	0.72	0.87
T-bills r_{tb}	-0.19	-0.05	0.14	0.19	0.17	-0.26	-0.09	0.13	0.19	0.18
(B) Stocks, Bonds and Alternatives for Asset-Liability Investors										
Stocks x_s	0.31	0.53	0.80	0.97	1.08	0.35	0.55	0.83	0.99	1.11
Bonds x_b	0.75	1.25	1.26	1.14	1.05	0.82	1.19	1.16	1.04	0.95
Credits x_{cor}	1.39	0.57	0.31	0.36	0.44	1.30	0.61	0.40	0.46	0.54
Commodity x_{com}	0.00	-0.01	-0.02	-0.01	0.00	0.02	-0.01	-0.03	-0.02	-0.01
REITs x_{re}	-0.23	-0.22	-0.21	-0.19	-0.17	-0.21	-0.22	-0.21	-0.18	-0.17
Hedge Funds x_{hf}	0.87	0.70	0.56	0.52	0.50	0.83	0.66	0.52	0.48	0.47
T-bills r_{tb}	-2.10	-1.81	-1.71	-1.80	-1.91	-2.11	-1.78	-1.67	-1.77	-1.89
(C) Stocks, Bonds, Alternatives and Green Funds for Asset-Liability Investors										
Stocks x_s	0.26	0.53	0.82	0.97	1.09	0.30	0.55	0.84	1.00	1.11
Bonds x_b	0.72	1.27	1.28	1.16	1.06	0.78	1.20	1.17	1.05	0.96
Credits x_{cor}	1.50	0.56	0.30	0.36	0.43	1.40	0.61	0.40	0.46	0.54
Commodity x_{com}	0.01	-0.01	-0.02	-0.01	0.00	0.03	-0.01	-0.02	-0.01	-0.01
REITs x_{re}	-0.20	-0.20	-0.19	-0.17	-0.15	-0.19	-0.20	-0.19	-0.17	-0.15
Hedge Funds x_{hf}	0.80	0.64	0.51	0.47	0.45	0.77	0.60	0.47	0.44	0.42
Green Bonds x_{gb}	0.03	0.03	0.02	0.02	0.02	0.00	-0.02	-0.02	-0.02	-0.02
Green Econ. x_{ge}	0.03	0.00	-0.01	0.00	0.00	0.04	0.00	-0.01	0.00	0.00
SRI x_{sri}	0.00	-0.03	-0.04	-0.03	-0.03	0.00	-0.03	-0.04	-0.03	-0.02
Environmental x_{PP1}	-0.03	-0.03	-0.02	-0.02	-0.02	-0.03	-0.03	-0.02	-0.02	-0.02
Clean Tech. x_{PP3}	0.07	0.04	0.03	0.03	0.03	0.06	0.04	0.03	0.03	0.03
Clean Energy. x_{PP4}	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T-bills r_{tb}	-2.19	-1.80	-1.69	-1.78	-1.89	-2.16	-1.72	-1.62	-1.72	-1.84