Network for Studies on Pensions, Aging and Retirement

NETSPAR ACADEMIC SERIES

A Multi-population approach to forecasting all-cause mortality using cause-specific mortality data

Pintao Lyu, Bertrand Melenberg, Anja De Waegenaere

DP 02/2018-006

A Multi-Population Approach to Forecasting All-Cause Mortality Using Causes-of-Death Mortality Data

Pintao Lyu[1,2], Bertrand Melenberg [1], and Anja De Waegenaere[1]

[1]Department of Econometrics and OR, Tilburg University, PO Box 90153, 5000LE Tilburg, The Netherlands

[2]Pricing & Hedging department, Nationale-Nederlanden, Weena 505

ARTICLE HISTORY
Compiled January 10, 2018

ABSTRACT
All-cause mortality is driven by various types of cause-specific mortality. Projecting all-cause mortality based on cause-of-death mortality allows one to understand the drivers of the recent all-cause mortality improvements. However, the existing literature has argued that all-cause mortality projections based on cause-specific mortality experience have a number of serious drawbacks, including the inferior cause of death mortality data and the complex dependence structure between causes of death. In this paper, we use the recent WHO access version causes of death data to address this issue in a multi-population context. We construct a new model in the spirit of Li and Lee (2005) but in terms of cause-specific mortality. A new two-step beta convergence test is used to capture the cause-specific mortality dynamics between different countries and between different causes. We show that the all-cause mortality projections produced by the new model perform similarly in-sample as the projections by the Lee-Carter and the Li-Lee all-cause mortality models. However, in contrast to results from earlier studies, we find that the all-cause mortality projections of the new model have a better out-of-sample performance in a long forecast horizon. Moreover, for the case of the Netherlands, about one year higher remaining life expectancy projections of a 67-year-old Dutch male in a 30-year forecast horizon is obtained by this new model, compared to the all-cause Li-Lee mortality model.

Corresponding email: p.lyu@uvt.nl. The authors are grateful to Richard MacMinn, Hong Li, Yang Lu, David Schrager, Jan Dob, and the seminar participants at Longevity 13 for their useful comments. All errors are our responsibility.

1. INTRODUCTION

Ongoing increases in life expectancy have attracted substantial attention in recent years. The uncertainty around these increases is a concern to social security authorities and pension industries. Several stochastic mortality models have been developed to quantify the so-called longevity risk. Among these models, the Lee-Carter (all-cause mortality) model (henceforth the LC model) is regarded as the benchmark method (Booth et al. 2006). Since its appearance in 1992, the literature has expanded in various directions, such as, for example, including cohort effects (Cairns et al. 2006; Renshaw and Haberman 2006), and developing multi-population approaches aimed at avoiding diverging projected mortality rates in similar populations (Tuljapurkar et al. 2000; Li and Lee 2005), to name just a few. These approaches have not yet taken into account the causes of death information. An alternative approach that has been considered in the literature is to construct all-cause mortality projections based on cause-specific mortality experience. While some literature states merits of mortality forecasts based on cause-of-death mortality data (Tabeau et al. 1999), several studies have discuss potential drawbacks of this approach (Wilmoth 1995, Gutterman and Vanderhoof 1998, and Booth and Tickle 2008). The main concerns are inferior data quality and availability, and the complex dependence structure between causes. This paper addresses these two issues by modeling the cause-specific mortality in a multi-population context, where the dependence structure will be quantified based on a two-step beta convergence test.

The multi-population mortality experience has yet not been incorporated in cause-specific mortality modeling to the same extent as in all-cause mortality modeling. Similar decreasing patterns of all-cause mortality among the developed countries are illustrated by a large literature (Wilmoth 1998; Wilson 2001; White 2002; Janssen et al. 2005). To accommodate this feature in the all-cause mortality modeling, Li and Lee (2005) extend the LC model to an augmented common factor model that maintains the long-run common trend of the mortality pattern in a group of populations, while allowing for short-run variations in each member of the group. The resulting Li-Lee model produces coherent all-cause mortality forecasts for a group of populations, where coherence means that the all-cause mortality forecasts of this group of populations improve at the same speed, maintaining constant inter-population mortality differences in the long run. Li and Lee (2005) attribute this coherence to close links between the group members of the different populations in terms of communications, medical technologies, and lifestyles. "Generating coherent all-cause mortality forecasts" is then used by Millossovich et al. (2014) as one of the important criteria to evaluate multi-population mortality models. Moreover, they shed light on the usage of multi-population mortality models in the assessment of basis risks.

Compared to the coherence of all-cause mortality, the coherence of cause-specific mortality might be easier to identify because the cause-specific driving factors are less confounding than the all-cause ones. For example, all-cause mortality improvements are buttressed by many cause-specific medical advances. Also, fast-growing information and communication technologies help cause-specific medical advances to exchange quickly across borders, potentially leading to a similar improvement speed of cause-specific mortality of developed countries. It has been well documented that mortality rates for most cancers (Jemal et al. 2010), cardiovascular diseases (Vallin and Meslé 2004), and infectious diseases (Omran 1998), are declining in the US and other western countries simultaneously. Therefore, it seems a natural follow-up to model cause-specific mortality in a multi-population context. Furthermore, to better test cause-specific coherence assumptions, we propose a new “two-step beta convergence test” that is borrowed from the beta convergence approach in the growth literature (Barro 1991) and in the European financial integration studies (Adam et al. 2002). Briefly speaking, the first-step test is to verify whether the cause-specific mortality of different countries are tending to a same level and the second-step test is to investigate whether these cause-specific mortality are tending to improve at a same speed. Using cause-specific multi-population mortality modeling allows one to avoid undesired divergence between similar countries in terms of cause-specific mortality forecasts. Also, for the countries with poor causes of death mortality records, multi-population forecasts seem to be more reasonable than the one based on the single-population mortality histories, because of averaging out country-specific disturbances by incorporating extra information from other similar countries.

Causes of death are usually correlated with each other. The dependence structure between causes plays a pivotal role in modeling cause-specific mortality. Several dependence structures of duration are well-studied in the competing risk framework (Carriere 1994). Yet, the true (net) dependence structure is impossible to be identified (Tsiatis 1975). Honoré and Lleras-Muney (2006) give a famous example of the non-identifiability problem, i.e., the failure in the war on cancer. The well-observed stagnation in the cancer mortality improvement could be partially explained by a positive dependence between cancer and cardiovascular disease. Both cardiovascular diseases and cancer are positively correlated to some unobserved common risk factors. In the last three decades, the innovations of vascular medicines and emergency medical care have reduced the sensitivity of cardiovascular mortality to these common risk factors. As such, the relative risk exposure of cancer has increased because the individuals that are exposed to these common factors will eventually die of one cause, either cancer or cardiovascular disease (supposing these are only two causes of death). Moreover, the decrease in cardiovascular mortality is more beneficial to individuals with a higher common risk exposure, who are more likely to die from cancer. The individual's exposure to cardiovascular is substituted by the exposure to cancer. Consequently, the mortality improvement of cancer is partially masked by such a substitution effect. More recently, copulas have been used to recover the latent cancer mortality improvements from the observed one, net of such substitution effects (see Zheng and Klein 1995; Carrière 1995; Kaishev et al. 2007; Dimitrova et al. 2013). However, further assumptions, such as the functional form of copula and the choices of the dependence parameter(s), are needed when taking these copula-related models into practice.

Additional assumptions on the dependence structure between causes of death have to be made to be able to utilize cause-specific mortality in all-cause mortality projections. The independence assumption between causes of death is most widely used (see Wilmoth (1995) and Putter et al. (2007)). Although the independence assumption might not be realistic, it brings tractability to model the cause-specific mortality and still maintains some interesting features of cause-specific mortality patterns. Alternatively, assuming Hierarchical Archimedean Copulas for the latent cause-specific survival functions, Li and Lu (2017) derive the implied latent cause-specific mortality from the observed cause-specific mortality and use the LC model to extract the latent mortality improvement for each cause. However, the dependence structures between causes of death that they assume, i.e., the functional form of the copulas, cannot be identified from the observed cause-specific mortality data, resulting in a substantial amount of the model uncertainty. The true development of some cause-specific mortality could possibly be masked by these assumptions. Instead, Honoré and Lleras-Muney (2006) derive semi-parametric bounds for cancer mortality so as to evaluate the true cancer mortality development with fewer assumptions. They discover that the semi-parametric bounds for cancer mortality are fairly tight and suggest much faster improvements in cancer than the ones that are previously estimated under the independence assumptions.

Unlike the aforementioned competing risk literature that accounts for the dependency between causes by recovering the true, but unobserved cause-specific mortality from the observed one, Arnold and Sherris (2013) focus directly on the dependency between the observed cause-specific mortality. They apply Vector Error Correction Models (VECM) to identify long-run stationary relationships between the observed cause-specific mortality and use these long-run stationary relationships in their forecasts. This results in better performance of the observed cause-specific mortality forecasts. But Arnold and Sherris (2013) provide cause-specific mortality projections in a single population context only, which does not exclude possible divergences between the projections of a group of similar countries. In their spirit, we study the observed cause-specific mortality in a multi-population context. Instead of using VECM to identify the dependence structure between causes of death, we use the outcomes of the two-step beta convergence tests to model the co-movements between causes of death both in mortality level and in mortality improvement, embedding the inter-cause co-movements in a cause-specific multi-population model, and deriving the mortality implications of the inter-cause dynamics on all-cause mortality. We incorporate both international dynamics and inter-cause dynamics in our multi-population cause-specific mortality models. We introduce two new models, namely, a model that only accounts for cause-specific international co-movements, (which we shall refer to as the CoDLi-Lee model) and a model that accounts for both international dynamics and inter-cause dynamics, based on the outcomes of the two-step beta convergence test (which we shall refer to as the nestedCoDLi-Lee model). We compare these two new models to the traditional all-cause mortality models, namely, the Lee-Carter and the Li-Lee model. We find that the nestedCoDLi-Lee model has a comparable in-sample fit but a better out-of-sample performance in a long (i.e., eight to fourteen years) forecast horizon than the conventional all-cause mortality models.

The rest of our paper is organized as follows. Section 2 introduces the data and the two-step beta convergence test. Section 3 presents the new cause-specific models. Section 4 discusses the comparison of the in-sample fit and the out-of-sample performance between the aforementioned two models and the conventional all-cause mortality models. Section 5 concludes. Large graphs are presented in the Appendix.

2. DATA

According to the International Classification of Diseases (ICD), the definition of a cause of death is: "*the disease or injury which initiated the train of morbid events leading directly to death, or the circumstances of the accident or violence which produced the fatal injury*." A compilation of mortality data[1] by age, sex, and causes of death from 1979-2015 is provided by the World Health Organization (WHO). The Access version of this database is used here, which features the consistent causes of death data collected officially by WHO from the countries that are using the newest ICD 9 and ICD 10 and the data on population and live births. Through these features, we ensure there are fewer discontinuities caused by the changing of ICD versions as indicated by Gaille and Sherris (2011). The cause-specific mortality trends estimated and projected via this database are more robust to the evolution of ICD in different countries and different time periods. The disadvantages of this database, however, are also obvious. First, the range of age groups is relatively large, i.e., 0-1, 1-4, 5-14, 15-24, 25-34, 35-54, 55-74, 75+. Second, to cope with the new ICD versions, few developed countries have full records of long enough mortality histories for all important causes of death. From the aforementioned data, we use the causes of death mortality and population numbers for male, age groups (25-34, 35-54, 55-74, 75+), and time period (1979-2013) in

[1] `http://www.who.int/healthinfo/mortality_data/en/`

the Netherlands, Belgium, and France. We abstract from the infant and junior age groups because the cause-specific data in those age group is sparse. In addition, data availability restricts us to these three countries at the current stage.[2] Moreover, the causes of death mortality statistics in France and the Netherlands are credited by Tabeau et al. (1999) as the exceptions robust to the reclassification of diseases in ICD.

For the male populations of the Netherlands, Belgium, and France, the cause-specific mortality is defined as follows. First, let $D^{s,v}_{x,t}$ be the number of deaths of cause s in country v for age group x at time t, and let $E^{v}_{x,t}$ be the all-cause exposure in country v for the age group x at time t. Then, the (raw) cause-specific central death rate of cause s in country v for age group x at time t, $m^{s,v}_{x,t}$, is given by

$$m^{s,v}_{x,t} = \frac{D^{s,v}_{x,t}}{E^{v}_{x,t}} \tag{1}$$

The all-cause mortality rate in country v for age group x at time t, $m^{v}_{x,t}$, is then given by

$$m^{v}_{x,t} = \frac{\sum_s D^{s,v}_{x,t}}{E^{v}_{x,t}} \tag{2}$$

In our case $v \in \{Netherlands, France, Belgium\}$, x refers to the age groups 25-34 years, 35-54 years, 55-74 years, and 75+ years, and (in-sample) $t \in \{1979, \cdots, 2013\}$. The all-cause mortality is dis-aggregated into five main causes of death, i.e., cancer (CM), vascular diseases (VM), other diseases (DM), accidents and murders (AMM), and the unexplained (UEM). Thus, $s \in \{CM, DM, VM, AMM, UEM\}$. Table 1 shows the coding rules. We choose the most common causes in the developed countries, more or less in line with earlier studies. For instance, Arnold et al. (2015) consider cancers, diseases of the circulatory system, and other diseases as the three main causes in their study. Also, similar categorizations of the causes of death are used in more previous studies (Oeppen et al. 2008; Alai et al. 2015).

We show the past trends of the causes of death mortality in Figure 1 for the different age groups and the three countries in terms of Mortality Component Graphs (MCGs). For the age group 25-34, there is a jump in the central mortality rate of other diseases (DM) in the 1990s in France. This is the result of a relatively big epidemic of influenza-like illness[3] at that time, see Carrat et al. (1998). The attack rate was the highest for young people and decreased by increasing age. Therefore, we barely see any signs of such an epidemic in the older age groups in France. We do not see a similar jump in the Netherlands and Belgium, implying that a common mortality trend of other diseases (DM) in these three countries might be more or less insensitive to this country-specific epidemic. Among the age groups 25-34 and 35-54 (the first and second row), accident and murders (AMM), including external injuries, is also a major cause of death in the three countries. This stylized fact is in line with the findings of European countries in Mathers et al. (2009). For the elderly age groups 55-74 and 75+ in these three countries (the third and forth row), we observe somewhat smoother patterns of causes of death mortality trends than the ones of the younger age groups. The cardiovascular revolution since the 1970s can be seen as the decreasing proportions

[2]Other countries, e.g., United Kingdoms, United States, and Nordic countries, have the similar length of data only for some causes of death but not for all causes. Given that we would like to provide implications to all-cause mortality, we require full records for all causes of death mortality data. In the WHO access version database, only Netherlands, Belgium, and France have full records of all causes of death mortality in the longest historical period from 1979-2013.

[3]More specifically ICD chapter I (Certain infectious and parasitic diseases)

Table 1.: Coding rules of causes of death

Cause of Death(male)	Abbreviation(male)	ICD Chapter(male)
Cancer	CM	2(Neoplasms)
Vascular Disease	VM	9(Diseases of the circulatory system)
Other Diseases	DM	1(Certain infectious and parasitic disease)
		4(Endocrine, nutritional and metabolic diseases)
		5(Mental and behavioural disorders)
		6(Diseases of the nervous system)
		7(Diseases of the eye and adnexa)
		8(Diseases of the ear and mastoid process)
		10(Diseases of the respiratory system)
		11(Diseases of the digestive system)
		12(Diseases of the skin and subcutaneous tissue)
		13(Diseases of the musculoskeletal system and connective tissue)
		14(Diseases of the genitourinary system)
		17(Congenital malformations, deformations and abnormalities)
Accidents and Murders	AMM	20(External causes of morbidity and mortality)
Unexplained	UEM	15(Pregnancy, childbirth and the puerperium)
		16(Conditions originating in the perinatal period)
		18(Symptoms, signs and abnormal clinical and laboratory findings)

of VM from 1979 to 2013 in the Mortality Component Graphs. On the other hand, according to Bongaarts (2014), the cancer phase mortality transition of high-technology societies has just begun in the developed countries, implying that cancer mortality might decrease at a slower pace initially but might possibly improve faster in case the transition is close to finishing in the future.

2.1. International comparison of cause-specific mortality

Similar to Arnold and Sherris (2013), we construct age-adjusted cause-specific mortality rates (level) of the three countries for all five causes of death.

Mortality level

In Figure 2 we present the the mortality rates for cancer (CM, left panel) and cardiovascular diseases (VM, right panel). The left panel shows that for most of the periods, cancer mortality in the three countries behaves similarly, except for the period 1995-2005. We observe a catch-up effect among these three countries during this period, i.e., the cancer mortality of Belgium (solid line) initially starts to decline at a significantly faster pace at around 1995, followed later by France (dash line) and the Netherlands (dash-dotted line). After 2005, cancer mortality of these three countries behaves similarly again. The right panel of Figure 2 indicates that the vascular mortality rates of the Netherlands, Belgium, and France improve at a similar pace. Visual inspection of Figure 2 thus suggests international coherence in cancer and vascular mortality, i.e., similar countries' mortality improve at the same pace in the long run (after catching-up), defining coherence similarly as Li and Lee (2005)). [4] The phenomena observed in Figure 2 are in line with the find-

[4]Figure 15 indicates the mortality of the other diseases and the accident and murders improve at a slow and similar pace in all three countries, meaning the international coherence also holds for the mortality of minor causes. But for the unexplained, we see a more

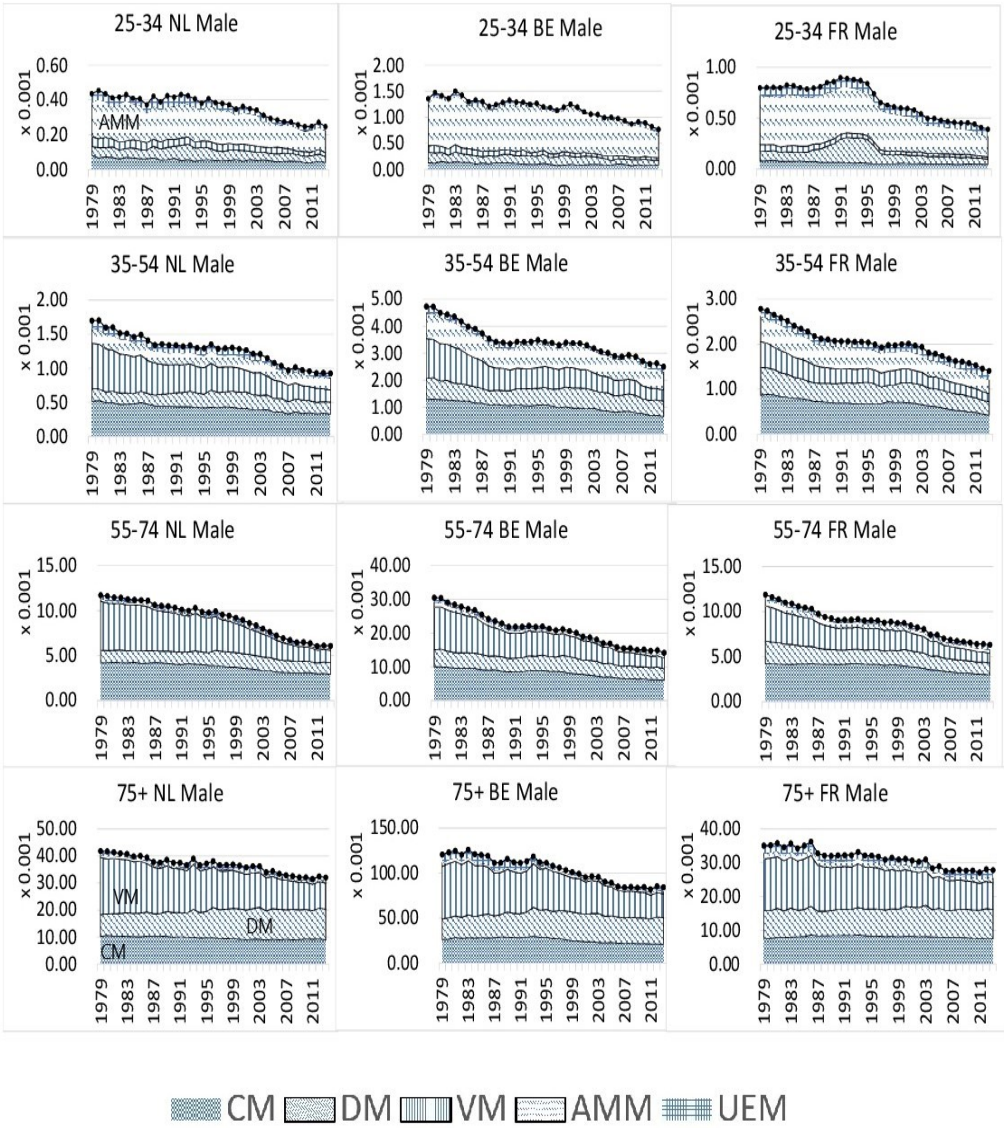

Figure 1.: Mortality component graph(MCG) of NL, BE and FR

ings of Bremberg (2017). He shows that there exists a significant catch-up effect in the all-cause mortality rates of the 55-75 year-old in the OECD countries, i.e., the gap between the rates of improvement in all-cause mortality in the OECD countries are becoming smaller since the 1990s. Faster dissemination of medical innovations during the 1990s is believed to be one of the main drivers.

Mortality improvement

Figure 3 shows the mortality improvements corresponding to the mortality rates presented

volatile mortality pattern, from which we could hardly draw a meaningful conclusion

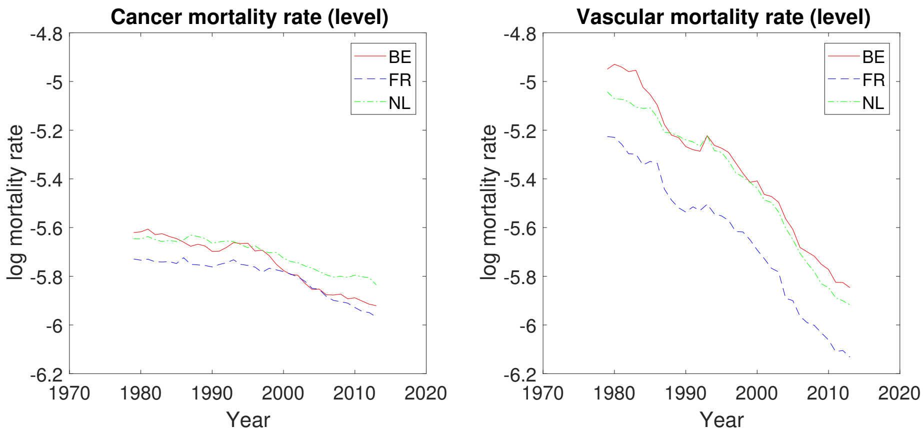

Figure 2.: Cancer and vascular mortality rate

in Figure 2. The mortality improvements are estimated via a 5-year rolling window or a 10-year rolling window.[5] We observe that cancer and vascular mortality in Belgium, the Netherlands, and France are improving roughly at a same speed, We observe converging cancer and vascular mortality improvements in the three countries using both the 5-year rolling window and the 10-year rolling window.[6]

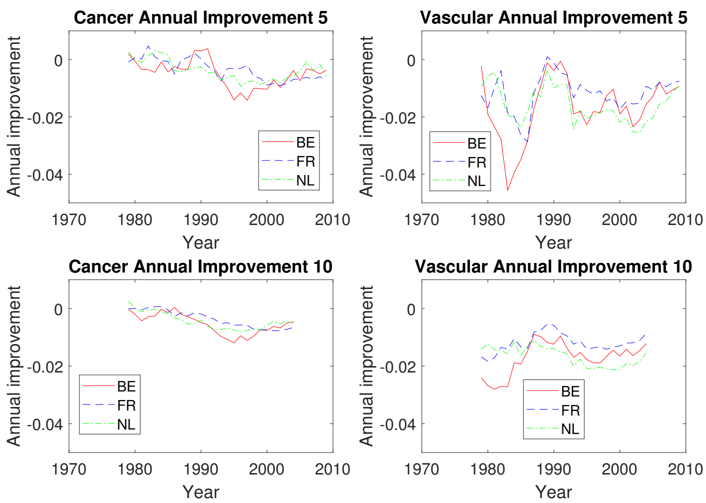

Figure 3.: Cancer and vascular mortality improvement in different rolling windows

2.2. Two-step beta convergence test

Section 2.1 shows some empirical evidence in favor of international coherence of cause-specific mortality. In order to model these cause-specific mortality patterns in a Li-Lee frame-

[5]These rolling window improvements are defined as $dm^{s,v}_{i,t} = \frac{log(m^{s,v}_{i,t+k})-log(m^{s,v}_{i,t})}{k-1}$, for $k = 5$ and $k = 10$.
[6]For the minor causes in Figure 16, the mortality improvements are mild, i.e., fluctuating around zero.

work, we need to determine which countries to include to construct a coherent group for the Li-Lee model. There are several ways to determine the coherence membership. One possibility is the original way proposed by Li and Lee (2005) who test whether the member-specific period effect in the second stage (after taking out the common period effect in a first stage) is stationary or not[7]. The other one is proposed by Hatzopoulos and Haberman (2013) who determine coherence memberships using a generalized linear model (GLM) framework.

As pointed out by Li and Lee (2005) themselves, the definition of a coherent group is intentionally set to be vague, because some countries that are going through a transitory period of mortality, like, for example, the East EU countries, could also be included in a group with the West EU countries, even though the country-specific period effects might not satisfy the stationarity assumption. However, the definition in terms of stationarity of member-specific period effect might also rule out some countries whose inclusion in a coherent group seems plausible. For example, Hatzopoulos and Haberman (2013) show that Scotland is excluded from the coherent group of male mortality of Western Europe, when using the Li-Lee method to determine the group membership. Apparently, Scotland is not a country that is in a transitory period and is arguably similar to England and Wales in socioeconomic status. To deal with such a dilemma a more precise way to define the membership of a coherent group is needed.

Unlike Hatzopoulos and Haberman (2013) who determine the coherent membership of a country via the coefficients of a dynamic linear regression (DLR) in a generalized linear model (GLM), we propose to determine the coherence membership within the Li-Lee model with a two-step beta convergence test. We first describe this two-step beta convergence test, after which we establish the link between this test and the Li-Lee model.

Two-step beta convergence test and its link to the Li-Lee model

The two-step beta convergence test is built on the one-step beta convergence test proposed by Barro (1991). d'Albis et al. (2012) introduce a single-step beta convergence test in studies on mortality. The beta convergence test is originally used to check whether there exists a negative relationship between the GDP growth rate and its initial GDP level. In our case, we replace GDP growth with the mortality improvement rate. More specifically, we want to investigate whether the mortality rate would improve faster (slower) when the mortality rate is at a relatively low (high) level, or intuitively whether there exists a catch-up effect in mortality rates between different countries. This means that different countries or different causes of death are heading towards the same mortality level in the long run. Moreover, we not only test for convergence in the mortality levels, but we also test for convergence in mortality improvements. Convergence in mortality improvements means that the mortality improvements of different countries tend to a constant in the long run, i.e., the considered countries share the same mortality improvement.

First step: beta convergence test in mortality level

Let $dm^v_{x,t} = log(m^v_{x,t+k}) - log(m^v_{x,t})$ represent the k rolling window mortality improvement for age x, time t, and country v. Consider the following regression equation,

$$dm^v_{x,t} = \alpha_v + \beta_{level} log(m^v_{x,t}) + \sum_{l=1}^{L} \gamma_l dm^v_{x,t-1} + \varepsilon^v_{x,t}, \quad v \in \{Netherlands, France, Belgium\} \tag{3}$$

[7] we also provide the related results from the Li-Lee method in the section 6.1 in the appendix

In this equation α_v is the country-specific effect, β_{level} quantifies the relation between the mortality improvement and its initial mortality level, and γ_l are the coefficients of the lagged terms, which control for past information. The error term $\varepsilon^v_{x,t}$ is assumed to satisfy the usual regression assumptions.

The key coefficient is β_{level}. A negative value of β_{level} corresponds to convergence in the mortality level (type 1 co-movement in Figure 4); a positive value of β_{level} corresponds to a divergence in the mortality level (type 2 co-movement in Figure 4). Thus, a significantly negative β_{level} (for example, at a two-sided 5 % significance level) indicates convergence in the mortality level, while a significantly positive β_{level} indicates divergence in the mortality level. If β_{level} is not significantly different from 0, there is no clear link between the mortality improvement of a country and its initial mortality level.

The results of the beta convergence test will be sensitive to the choice of the length k of the rolling-window in $dm^v_{x,t}$: likely, there will be more noise in the estimated mortality improvement with a shorter rolling window, which might potentially bias the test, but there will be fewer degrees of freedom in the estimation of the fixed effect model with a longer rolling window, which might reduce the efficiency. To address these issues, we present the p-values of the two-step beta convergence test with different rolling-window choices, so as to check the robustness of the test.

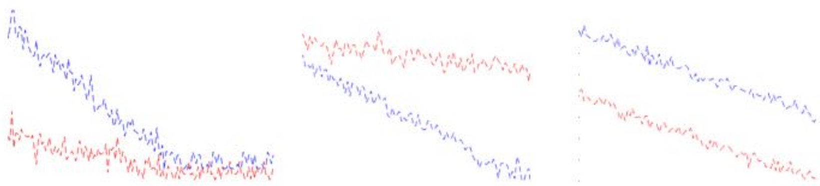

Figure 4.: Co-movement types

Notes: Left to right: Type 1 Convergence, Type 2 Divergence, Type 3 Coherence

To guarantee coherent mortality forecasts, i.e., the members of a coherent group would not have a diverging mortality forecast in the long run, either Type 1 convergence or Type 3 coherence (see Figure 4) should be present. As Type 1 convergence will already be pointed out by the negative β_{level}, we need an extra indicator for Type 3 coherence. Therefore, we propose an additional step to be discussed next.

Second step: beta convergence test in mortality improvement

Let $\Delta dm^v_{x,t} = log(dm^v_{x,t+1}) - log(dm^v_{x,t})$ represent the change of the mortality improvement for age x, time t and country v. Consider the following regression equation, which is the same as equation (3), but with $dm^v_{x,t}$ replaced by $\Delta dm^v_{x,t}$

$$\Delta dm^v_{x,t} = \alpha_v + \beta_{trend} dm^v_{x,t} + \sum_{l=1}^{L} \gamma_l \Delta dm^v_{x,t} + \varepsilon^v_{x,t}, \quad v \in \{Netherlands, France, Belgium\}. \quad (4)$$

We apply the beta convergence test in Equation (3) again, but now in terms of mortality improvements. The interpretation of the parameters of the models stays the same. The key coefficient is now β_{trend}. A significantly (two-sided 5 % significant level) negative β_{trend} would indicate a convergence in mortality improvement, i.e., a coherence in mortality levels (type 3 co-movement, coherence see Figure 4).

Link β_{level} and β_{trend} to Li-Lee Model

The two-step beta convergence test aims at determining the co-movement types (convergence, divergence, or coherence) of a group of time series processes. For a country to be included in a coherent group of the Li-Lee model, it is required that the mortality pattern of this country should not diverge from the ones of the other members in the coherent group. We use the two-step beta convergence test to determine whether there exist convergence, divergence, or coherence of the cause-specific mortality trajectories of a group of countries, so as to decide which country should be included in the Li-Lee model. In addition, we translate the coefficients of the two-step beta convergence test β_{level} and β_{trend} to the model specifications of the Li-Lee model.

Let us first briefly review the Li-Lee model:

$$ln(m^v_{x,t}) = \alpha^v_x + B_x K_t + b^v_x k^v_t + \varepsilon^v_{x,t} \tag{5}$$

Here, a^v_x is the age effect. To prevent sub-populations of the group to diverge from each other in the long run, $B_x K_t$ is the common factor for each country in the group. K_t is the common period effect and B_x is the common sensitivity of age x to this common period effect. To allow for short-run variations of sub-populations of the group, $b^v_x k^v_t$ is the individual factor of country v that is supposed to vanish in the long run. k^v_t is the individual period effect and b^v_x is the sensitivity of age x to the individual period effect. The typical normalizations are: $\sum_x B_x = 1$, $\sum_t K_t = 0$, $\sum_x b^v_x = 1$, and $\sum_x k^v_t = 0$. We estimate the Li-Lee model via singular value decomposition (SVD) in two steps (Li and Lee 2005). For the common period effect K_t, a random walk with drift (RWD) process is commonly selected, i.e., $K_t = B_0 + K_{t-1} + e_t$. Specifically, B_0 is the drift term of the RWD process and e_t is the random variable with an i.i.d normal distribution across time. For the country-specific period effect k^v_t, Li and Lee (2005) suggest to keep the population v in the coherent group if the explanation ratio[8] of the augmented common factor model is large enough and k^v_t could be well described by a stationary process, such as an AR(1) process, i.e., $k^v_t = \beta^v_0 + \beta^v_1 k^v_{t-1} + u^v_t$, in which β^v_1 is required to be less than one.

In our approach, whether a country should be included in a coherent group or not will be determined by the value of β_{level} and β_{trend} of the two-step beta convergence test. First, we utilize the results of the two-step beta convergence test on $m^v_{x,t}$ to profile the characteristics of the common period effect K_t and the country-specific period effect k^v_t. Second, we select the countries whose k^v_t is stationary into a coherent group. Third, as suggested by the Li and Lee (2005), we re-estimate K_t and k^v_t based on a refined coherent group of countries. Subsequently, we use the re-estimated K_t and k^v_t to generate the mortality forecasts.

---

[8]The explanation ratio is: $R^v_{AC} = 1 - \frac{\sum_t \sum_x \left[ln(m^v_{x,t}) - a^v_x - B_x K_t - b^v_x k^v_t \right]^2}{\sum_t \sum_x \left[ln(m^v_{x,t}) - a^v_x \right]^2}$.

Table 2.: Two-step beta convergence test and its link to Li-Lee model

First Step	Second Step	Li-Lee Model	Co-Movement Type at Steady
$\beta_{level} < 0$	$\beta_{trend} < 0$	Li-Lee RWD(1)+ARIMA(1,0,0)	Convergence
$\beta_{level} > 0$	$\beta_{trend} = 0$	Li-Lee RWD(1)+ARIMA(0,1,0)	Divergence
$\beta_{level} > 0$	$\beta_{trend} > 0$	Li-Lee RWD(1)+ non-stationary ARIMA(1,1,0)	Divergence
$\beta_{level} = 0$	$\beta_{trend} < 0$	Li-Lee RWD(1)+AR(1) or Li-Lee RWD(1)+ stationary ARIMA(1,1,0)	Coherence

Notes: Table 2 summarize the link between the results two-step beta convergence test and the Li-Lee model. The test captures the dynamics between mortality patterns of different countries in the sample data.

Table 2 summarizes the link between the two-step beta convergence test and the Li-Lee model. If $\beta_{level} < 0$ and $\beta_{trend} < 0$ this indicates that the mortality levels of different countries are tending to a same value, and so does the mortality improvement. If this tendency were to continue in the projections, the mortality levels and mortality improvements of the different countries will be the same in the long run. This long-run interplay between the different countries in mortality levels and mortality improvements could be characterized by the Li-Lee model with a K_t that is modeled as a random walk with drift (RWD) and k_t^v that are modeled (for example) as AR(1)-processes.

If $\beta_{level} > 0$ and $\beta_{trend} = 0$ this shows that the mortality levels of different countries are tending to different values and there is no clear tendency in the mortality improvements. Were this tendency to continue in the projections, the mortality levels of the different countries would diverge in the long run. This long-run interplay could be characterized by the Li-lee model with K_t as random walk with drift (RWD) and k_t^v as ARIMA(0,1,0).

If $\beta_{level} > 0$ and $\beta_{trend} > 0$ this suggests that the mortality levels of the different countries are tending to different values and so are the mortality improvements. Were this tendency to continue in the projection, the mortality levels and the mortality improvements of the different countries would diverge in the long run. This long-run co-movements between different countries could be captured by the Li-Lee model with a K_t that is modeled as a random walk with drift(RWD) and the k_t^v that are modeled as non-stationary processes, such as ARIMA(1,1,0) (see Equation 33 in the appendix).

If $\beta_{level} = 0$ and $\beta_{trend} < 0$, this suggests that the mortality improvements of the different countries tend to a same value. Were this tendency to continue in the projections, the mortality improvements of the different countries would converge in the long run. Such long-run co-movements between different countries could be captured by the Li-Lee model with a K_t that is modeled as a random walk with drift (RWD) and the k_t^v that are modeled like an AR(1)-process or an ARIMA(1,1,0).

We only present the case that all countries in a coherent group share the same type of time series processes, for example, all the cause-specific period effects are AR(1)-processes. In the appendix, we discuss the case where one country could have an AR(1)-process and the other country could have an ARIMA(1,1,0)-process. But the main results do not change much if we have a stationary AR component in the country-specific period effect k_t^v, no matter whether k_t^v follows an ARIMA(1,1,0)- or an AR(1)-process. We rule out the impossible scenarios at the steady state, for example, $\beta_{level} < 0$ and $\beta_{trend} > 0$, because if the mortality levels of different countries converge to the same value in the steady state, their mortality improvements should also be equal. Also, we rule out the cases that there exist two identical country-specific factors of two different countries, i.e., we impose $b_x^v \beta_0^v \neq b_x^{v'} \beta_0^{v'}, v \neq v', v, v' \in \{NL, BE, FR\}$.

We utilize the two-step beta convergence tests in subsection 2.3 to determine whether there exists international coherence and inter-cause coherence. Subsequently, we construct the optimal group memberships of international coherence.

2.3. Results of the two-step beta convergence test: international comparison

In this section, we apply the two-step beta convergence test to cause-specific mortality across the three countries. We present the results for age-adjusted cause-specific mortality. The results for the individual age classes are quite similar. Figure 5 shows β_{level} (left panels) and β_{trend} (right panels) for cancer (upper panels) and cardiovascular mortality (lower panels). Figure 6 shows the corresponding *p*-values, using the same format. The left panels of Figure 5 show that β_{level} of cancer mortality is negative for most rolling window choices while the one of vascular mortality is fluctuating around zero. However, both of them are insignificant at 5% double-sided level for all the rolling-window choices, according to the left panels of Figure 6. This means that the mortality improvement is not significantly related to the initial mortality level, i.e., $H_0 : \beta_{level} = 0$ cannot be rejected, meaning no sign of convergence or divergence in mortality level. On the contrary, the right panels of Figure 5 suggest that β_{trend} of both cancer and vascular mortality are negative. Moreover, the right panels of Figure 6 show that the negative β_{trend} of cancer and vascular mortality are significant at the double-sided 5% level. This indicates that the change of mortality improvement is negatively related to the initial state of the mortality improvement, i.e., $\beta_{trend} < 0$, meaning convergence in mortality improvement.

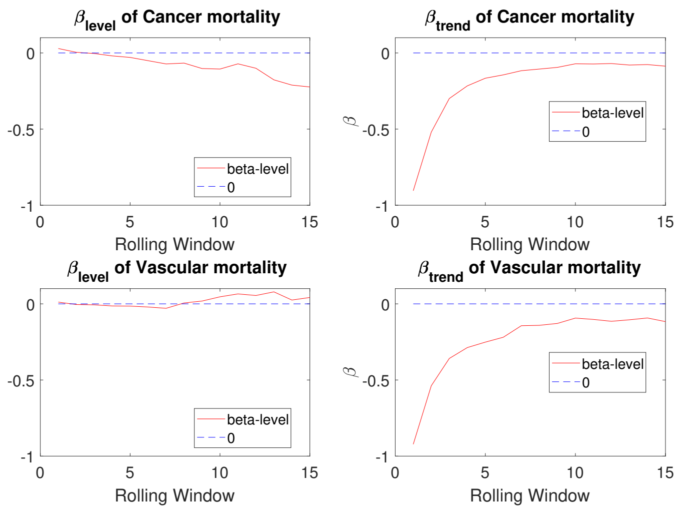

Figure 5.: International comparison: β_{level} and $\beta_{trend} of cancer and vascular mortality$

Referring to Table 2, the combination $\beta_{level} = 0$ and $\beta_{trend} < 0$ indicates that the co-movements between the Netherlands, Belgium, and France are coherent [9]. Figure 17 and Figure 18 (in the appendix) suggest international coherence for the other diseases mortality in the three

[9]The Li-Lee method of determining the coherent group (see Table 8) suggests the similar results as our two-step beta convergence test, except for the Accident & Murder (AMM). The Li-Lee method suggests France should be excluded from the coherent group of Belgium and Netherlands in the AMM mortality because the cause-specific mortality AMM of France is diverging away from the ones of Belgium and Netherlands. On the other hand, our test shows they are actually converging.

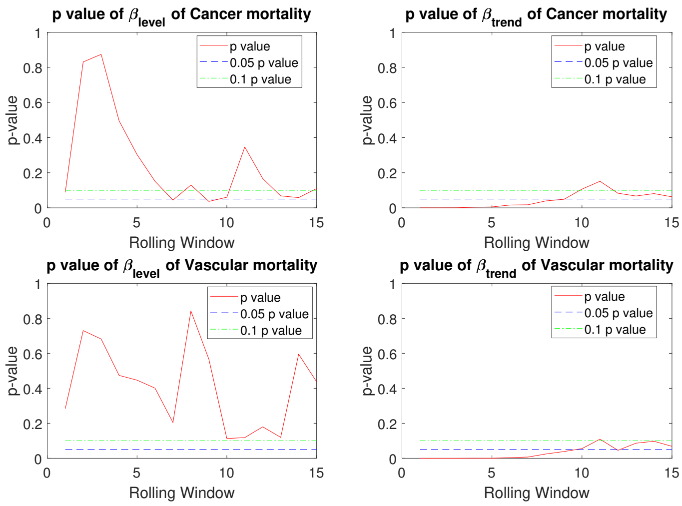

Figure 6.: International comparison: p-value of β_{level} and β_{trend} of cancer and vascular mortality

countries and international convergence (Type 1) for accident and murders and the unexplained. Although the international coherence does not hold for the latter two causes of death, we also impose the international coherent assumption in this case.

2.4. Inter-cause comparison

In addition to the international comparison in cause-specific mortality across the three countries, in this section we also provide an inter-cause comparison. This inter-cause comparison will shed lights on possible catch-up effects between the different causes, which could improve the forecasts of a cause that initially improves at a slower speed but catches up with other causes at a faster speed. Take cancer and vascular diseases as an example. Given that vascular mortality is decreasing much faster than cancer mortality, reaching a historically low level, more focus of media coverage and more efforts of medical researches might shift from preventing and curing the former to the latter, resulting in the catch-up of cancer mortality improvement. Also, some medical innovations that apply to vascular mortality could be transplanted to the cure of cancer. The catch-up effect between the two causes is expected to be more significant if these two causes are highly correlated, because the higher the correlation the easier the dissemination of medical innovations from one cause to the other cause.

Given that the international coherence in cancer and vascular mortality have been supported by the results of subsection 2.1, let the common cause-specific mortality in the Netherlands, Belgium, and France be given as follows [10]

[10] Alternatively, we could also use $m^s_{x,t} = \frac{\sum_v \frac{D^{s,v}_{x,t}}{E^v_{x,t}}}{3}$, given we have 3 countries. If the international coherence does exist, these two indicators should be similar

$$m^s_{x,t} = \frac{\sum_v D^{s,v}_{x,t}}{\sum_v E^v_{x,t}} \quad (6)$$

The common cause-specific mortality will be used to examine the inter-cause comparison between the five main causes of death.

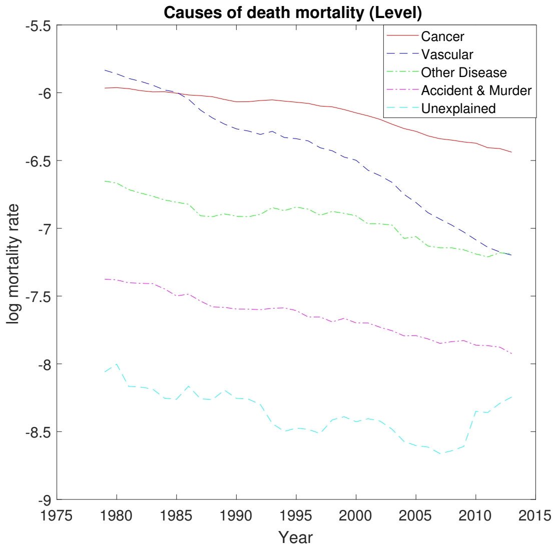

Figure 7.: Cause-specific common mortality Rate

Figure 7 presents the common cause-specific mortality levels. The figure shows that the fastest-improvement cause of death is vascular mortality. Vascular disease was the most important cause of death in the Netherlands, Belgium, and France but become second to the cancer mortality after 1985. The figure also suggests that cancer mortality decreases faster since the 1990s, while the other causes only decrease slightly. The turning point of cancer mortality improvement is roughly 10 years after cancer mortality surpassed vascular mortality and became the most deadly disease. This indicates that there might be a catch-up effect between cancer and vascular mortality, which might be informative for the forecasts of cancer mortality.

Figure 8 presents the common cause-specific mortality improvements for a rolling window of 5 years (left panel) and a rolling window of 10 years (right panel). From this figure, we can observe that the mortality improvement of cancer and vascular converge to a similar level since the 1990s, suggesting a possible cancer-vascular coherence, while other causes are fluctuating around zero, which is in line with the results in mortality level (see Figure 7).

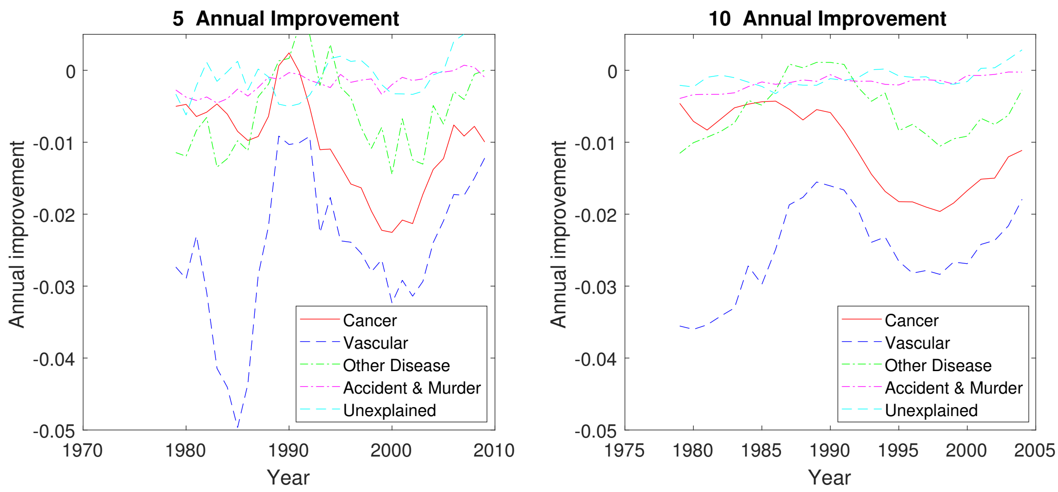

Figure 8.: Cause-specific mortality improvement in different rolling windows

2.5. Results of two-step beta convergence test: inter-cause comparison

Following a similar logic as in the subsection 2.2, in this subsection, we apply the two-step beta convergence test to the mortality levels of inter-cause comparison

$$dm^s_{x,t} = \alpha_s + \beta_{level} m^s_{x,t} + \sum_{l=1}^{L} \gamma_l dm^s_{x,t-1} + \varepsilon^s_{x,t}, \quad s \in \{CM, DM, VM, AMM, UEM\}, \tag{7}$$

and to the mortality improvements of inter-cause comparison

$$\Delta dm^s_{x,t} = \alpha_s + \beta_{slope} dm^s_{x,t} + \sum_{l=1}^{L} \gamma_l \Delta dm^s_{x,t} + \varepsilon^s_{x,t}, \quad s \in \{CM, DM, VM, AMM, UEM\}. \tag{8}$$

so as to test inter-cause coherence. Via the two-step beta convergence test, we obtain the results presented in Table 3. This table shows the found co-movement types between the causes of death. One pitfall in the results of inter-cause coherence is that other diseases might not be coherent to the accident & murders because their correlation is weak. However, the coherence suggested by the test outcome might result from the fact that both of these two causes are relatively more stationary (see Figure 8), compared to vascular mortality that decreases dramatically.

Based on the two-step beta convergence test, we find that there only exists cancer-vascular coherence and other disease-accident& murder coherence, which is consistent with the data visualization of Figure 7 and Figure 8 [11]. In the following sections, we incorporate these inter-cause coherence and the aforementioned international coherence in a new nestedCoDLi-Lee model.

[11] The Li-Lee method of determining the coherent group member also supports these two coherence, as Table 9 and Table 10

Table 3.: Inter-cause two-step beta convergence test results

Inter-cause	Co-movement type	Figure
Cancer-Vascular	Coherence	Figure 19
Cancer-Other Diseases	No Coherence	Figure 20
Cancer-Accident & Murders	No Coherence	Figure 21
Cancer-Unexplained	No Coherence	Figure 22
Vascular-Other Diseases	No Coherence	Figure 23
Vascular-Accident & Murders	No Coherence	Figure 24
Vascular-Unexplained	No Coherence	Figure 25
Other Diseases-Accident& Murders	Coherence	Figure 26
Other Diseases-Unexplained	No Coherence	Figure 27
Accident& Murders- Unexplained	No Coherence	Figure 28

3. MODELS

In this section, we introduce two types of mortality models. One type is all-cause mortality models that are based on all-cause mortality, including the all-cause Lee-Carter model and the already introduced all-cause Li-Lee model. The other type is cause-specific mortality models that are based on cause-specific mortality, including a causes of death Lee-Carter model (referred to as CoDLC, not presented[12]), a causes of death Li-Lee model (referred to as CoDLi-Lee), that only accounts for cause-specific international coherence, and a nested causes of death Li-Lee model (referred to as nestedCoDLi-Lee) that accounts for both international coherence and inter-cause coherence as suggested by the two-step beta convergence test.

3.1. All-cause mortality models

In this subsection we only present the the all-cause Lee-Carter model, since we already introduced the all-cause Li-Lee model. The Lee-Carter model Lee and Carter (1992) is applied to all-cause mortality and is defined as follows

$$ln(m^v_{x,t}) = \alpha^v_x + B^v_x K^v_t + \varepsilon^v_{x,t} \tag{9}$$

where a^v_x is the age effect of country v, K^v_t is the period effect of country v, B^v_x is the sensitivity of age x to the period effect in country v, and $\varepsilon^v_{x,t}$ is the error term of age x at year t in country v. We use the following normalizations: $\sum_x B^v_x = 1$ and $\sum_t K^v_t = 0$. The typical estimation procedure of the LC model is the singular value decomposition (SVD). A random walk with drift (RWD) is assumed for the period effect K^v_t. More details are given in Lee and Carter (1992).

[12]The cause-specific Lee-Carter model consists of applying the Lee-Carter modeling approach directly to each cause-specific mortality in each country without imposing international coherence or inter-cause coherence.

3.2. Cause-specific mortality models

In this section, we introduce two cause-specific mortality models, namely, the causes of death Li-Lee model (CoDLi-Lee) and the nested causes of death Li-Lee model (nestedCoDLi-Lee). Deriving all-cause mortality projections using a cause-specific mortality model requires an assumption on the dependence structure between the causes of death, which we base on the inter-cause coherence as suggested by the two-step beta convergence test in subsection 2.2.

In the literature on causes of death studies, typically independence between causes is assumed (see Wilmoth 1996; Caselli et al. 2006), because the independence assumption brings more tractability to cause-specific mortality modeling. In order to compare our results with the previous studies, we also make this inter-cause independence assumption in the causes of death Li-Lee model (CoDLi-Lee: Equation (10) to (12)) but still maintain international coherence. As an extension, we also incorporate the inter-cause dependence structure and international coherence as suggested by the two-step beta convergence test in the nested causes of death model (nestedCoDLi-Lee: Equation (14) to (20)). Through these settings, we will be able to evaluate whether incorporating international coherence and inter-cause coherence will enhance the performance of the all-cause mortality projections, when compared to the existing models.

Causes of death Li-Lee model (CoDLi-Lee)

We apply the Li-Lee model to the cause-specific central mortality rate for cause s and a group of countries, assuming that the countries of this group are internationally coherent.

$$ln(m_{x,t}^{s,v}) = \alpha_x^{s,v} + B_x^s K_t^s + b_x^{s,v} k_t^{s,v} + \varepsilon_{x,t}^{s,v}. \tag{10}$$

Here, $a_x^{s,v}$ is the age effect. To prevent sub-populations of the group to diverge from each other in the long run, $B_x^s K_t^s$ is the cause-specific common factor for each country in the group. K_t^s is a common period effect of cause s at year t in the group. B_x^s is the sensitivity of age x to the common period effect of cause s. To allow for short-run variations in sub-populations of the group, $b_x^{s,v} k_t^{s,v}$ is the individual factor that vanishes in the long run. $k_t^{s,v}$ is the individual period effect at year t in country v for cause s. $b_x^{s,v}$ is the sensitivity of age x to the individual period in the country v for cause s. We estimate the causes of death Li-Lee model via singular value decomposition (SVD) in two steps.

The common period effect K_t^s of cause-specific mortality of a coherent group for cause s is assumed to follow the random walk with drift (RWD) process:

$$K_t^s = B_0^s + K_{t-1}^s + e_t^s \tag{11}$$

B_0^s is the drift term of the RWD process and e_t^s is the random variable with i.i.d standard normal distribution.

The individual-specific period effect $k_t^{s,v}$ of cause s and country v is assumed to follow an AR(1) process:

$$k_t^{s,v} = \beta_0^{s,v} + \beta_1^{s,v} k_{t-1}^{s,v} + u_t^{s,v} \tag{12}$$

For the codLi-Lee model to work, $\beta_1^{s,v}$ is required to be smaller than 1.

The normalizations of the codLi-Lee model are $\sum_x B_x^s = 1$, $\sum_t K_t^s = 0$, $\sum_x b_x^{s,v} = 1$, and $\sum_t k_t^{s,v} = 0$. We assume independence between causes of death in the codLi-Lee model, which serves as a comparison to the nestedCoDLi-Lee model in which we allow for dependence between causes. Motivated by the independence assumption between causes of death, we obtain the all-cause mortality by adding up the cause-specific mortality generated by the CoDLi-Lee model in Equation 13 as follows:

$$m_{x,t}^v = \sum_s m_{x,t}^{s,v} = \frac{\sum_s D_{x,t}^{s,v}}{E_{x,t}^v}. \tag{13}$$

Nested causes of death Li-Lee model (nestedCoDLi-Lee)

Next, we introduce a new nestedCoDLi-Lee model to not only account for cause-specific international coherence, but to account for inter-cause coherence as suggested by the two-step beta convergence test as well. The co-movement between cancer (CM) and vascular (VM) and the co-movement between other disease (DM) and accident & murder (AMM) are suggested to be coherent. We choose to model the coherent group CM & VM, the coherent group DM & AMM, and the UEM separately, as Table 3 of two-step beta convergence test suggests. We present the nestedCoDLi-Lee model for CM-VM coherence. The other DM-AMM coherence could be obtained by simply switching the index of CM and VM to the index of DM and AMM. As for the unexplained (UEM), we keep it as independent from all other causes. In sum, the dependence structure between causes in the nestedCoDLi-Lee is illustrated in Figure 9.

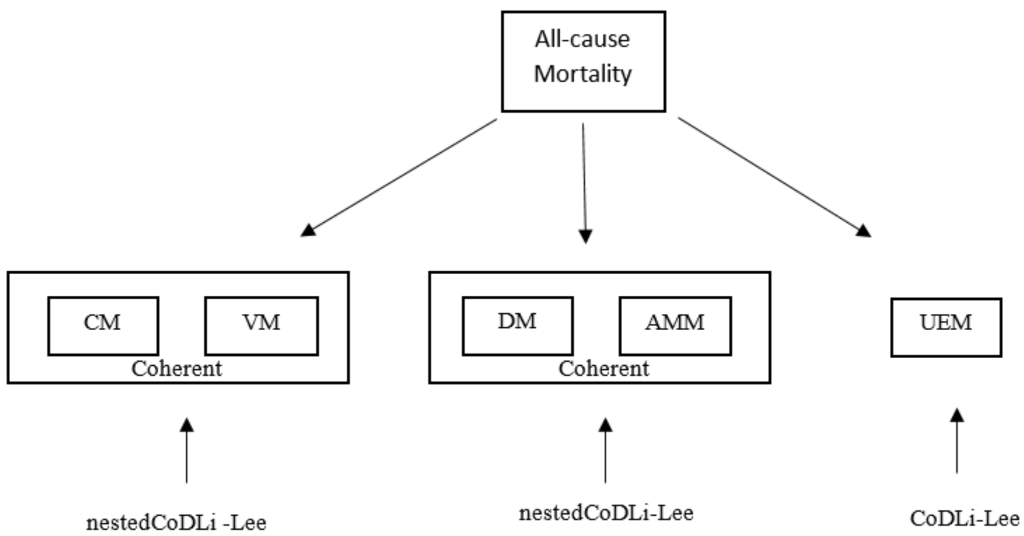

Figure 9.: Inter-cause dependent structure in nestedCoDLi-Lee model

To incorporate international coherence to the cancer-vascular coherence, we extract the common trend of the nestedCoDLi-Lee model from the central mortality rate as follow:

$$m_{x,t}^{CM\&VM} = \frac{\sum_v (D_{x,t}^{CM,v} + D_{x,t}^{VM,v})}{\sum_v E_{x,t}^v} \tag{14}$$

Here, $m_{x,t}^{CM\&VM}$ represents the sum of cancer and vascular mortality of all countries. The sum of cancer and vascular mortality presents the common trend of these two causes of death. In addition, this sum is net of the affect of cancer-vascular substitution effect brought by the competing nature between these two causes. Given the projections of nestedCoDLi-Lee model in the long run rely heavily on this this common trend, it alleviates the effects of competing nature between these two causes on their mortality projections. $D_{x,t}^{CM,v}$ represents the death counts at age x, year t, in country v caused by the cancer, and $D_{x,t}^{VM,v}$ represents the death counts at age x, year t, in country v caused by vascular diseases. The nestedCoDLi-Lee model is summarized as follows:

$$\begin{aligned} ln(m_{x,t}^{CM,v}) &= \alpha_x^{CM,v} + B_x^{CM\&VM} K_t^{CM\&VM} + b_x^{CM,v} k_t^{CM,v} + \varepsilon_{x,t}^{CM,v} \\ ln(m_{x,t}^{VM,v}) &= \alpha_x^{VM,v} + B_x^{CM\&VM} K_t^{CM\&VM} + b_x^{VM,v} k_t^{CM,v} + \varepsilon_{x,t}^{VM,v} \end{aligned} \tag{15}$$

The estimation and forecast procedure of the nestedCoDLi-Lee model takes the following steps. First, following Lee and Carter (1992), we apply single value decomposition (SVD) to $ln(m_{x,t}^{CM\&VM})$ in order to obtain the common factor $B_x^{CM\&VM} K_t^{CM\&VM}$ for cancer and cardiovascular mortality in each country.

$$ln(m_{x,t}^{CM\&VM}) = \alpha_x^{CM\&VM} + B_x^{CM\&VM} K_t^{CM\&VM} + \varepsilon_{x,t}^{CM\&VM} \tag{16}$$

Second, we subtract the cancer-vascular common factor $B_x^{CM\&VM} K_t^{CM\&VM}$ from the log cancer mortality and log vascular mortality of each country, and we apply single value decomposition (SVD) again to the remains to obtain the individual components for cancer and vascular disease in each country.

$$\begin{aligned} ln(m_{x,t}^{CM,v}) - B_x^{CM\&VM} K_t^{CM\&VM} &= \alpha_x^{CM,v} + b_x^{CM,v} k_t^{CM,v} + \varepsilon_{x,t}^{CM,v} \\ ln(m_{x,t}^{VM,v}) - B_x^{CM\&VM} K_t^{CM\&VM} &= \alpha_x^{VM,v} + b_x^{VM,v} k_t^{VM,v} + \varepsilon_{x,t}^{VM,v} \end{aligned} \tag{17}$$

Third, we extrapolate the common trend and individual trends of the nestedCoDLi-Lee model to generate the cancer and vascular mortality projections.The nestedCoDLi-Lee common period effect $K_t^{CM\&VM}$ is modeled as a random walk with drift. The individual trends $k_t^{CM,v}$ and $k_t^{CM,v}$ are modeled as AR(1)-processes. The error terms are assumed to be normally distributed with mean 0 and a constant variance.

$$K_t^{CM\&VM} = B_0^{CM\&VM} + K_{t-1}^{CM\&VM} + e_t^{CM\&VM} \tag{18}$$

$$\begin{aligned} k_t^{CM,v} &= \beta_0^{CM,v} + \beta_1^{CM,v} k_{t-1}^{CM,v} + u_t^{CM,v} \\ k_t^{VM,v} &= \beta_0^{VM,v} + \beta_1^{VM,v} k_{t-1}^{VM,v} + u_t^{VM,v} \end{aligned} \tag{19}$$

Forth, similarly for the other disease (DM) and accident and murders (AMM), we redo the aforementioned procedures to obtain the nestedCoDLi-Lee model estimation for $m_{x,t}^{DM,v}$ and $m_{x,t}^{AMM,v}$. For unexplained (UEM), we apply the CoDLi-Lee model under the independence assumption, following equation (10) - (12). We obtain the all-cause mortality of the nested CoDLi-Lee model by adding up the cause-specific mortalities. The normalizations of the nestedCoDLi-Lee model are: $\sum_x B_x^{CM\&VM} = 1$, $\sum_t K_t^{CM\&VM} = 0$, $\sum_x b_x^{CM,v} = 1$, $\sum_t k_t^{CM,v} = 0$, and similarly for vascular mortality.

$$m_{x,t}^{v} = (m_{x,t}^{CM,v} + m_{x,t}^{VM,v}) + (m_{x,t}^{DM,v} + m_{x,t}^{AMM,v}) + m_{x,t}^{UEM,v} \tag{20}$$

4. Model Performance

We present model performance of the various models in two parts. Part 1 presents the cause-specific mortality performance and Part 2 presents the all-cause mortality performance

Part 1 Cause-specific mortality performance

In this section, we compare the performance of the two new cause-specific mortality models, namely, the CoDLi-Lee model (see Equation (10)) and the nestedCoDLi-Lee model (see Equation (15)), with the performance of a conventional model, namely a cause-specific Lee-Carter model, which consists of applying the Lee-Carter modeling approach directly to each cause-specific mortality in each country without imposing international coherence or inter-cause coherence (referred to as CoDLC, as Wilmoth (1995) and Lee and Miller (2001)). All cause-specific models are fitted over the period 1979 to 2005 and their forecasts are calculated until 2013. Both the in-sample fit and the out-of-sample performance of these cause-specific mortality models are presented.

We compare the fit of the models in terms of the explanation ratios (Li and Lee 2005). For example, for the Li-Lee model applied to specific cause of death and specific country, the explanation ratio would be given by

$$R_{AC}^{s,v} = 1 - \frac{\sum_t \sum_x \left[ln(m_{x,t}^{s,v}) - a_x^{s,v} - B_x^s K_t^s - b_x^{s,v} k_t^{s,v} \right]^2}{\sum_t \sum_x \left[ln(m_{x,t}^{s,v}) - a_x^{s,v} \right]^2}.$$

We present these explanation ratios for different models, causes of death, and countries in Tables 4 and Table 5. The results in these tables show that both the CoDLi-Lee model and the nestedCoDLi-Lee model perform slightly better than the CoDLC model. For accident and murders, all models have a similar performance. Given that the other diseases, accidents & murders, and the unexplained typically contain more turbulence in the mortality rates than cancer and the vascular diseases, the explanation ratios of the nestedCoDLi-Lee and CoDLi-Lee are as low as 56% in some cases. For the CoDLC model, it is even worse. The explanation ratio of CoDLC for other diseases in Belgium is as low as 40%. In a nutshell, for cause-specific mortality, the CoDLi-Lee model and the nestedCoDLi-Lee model have comparable in-sample performance as the CoDLC model in the causes with less volatile histories, such as cancer and vascular diseases, but have better performance than the CoDLC model in causes with more volatile histories, namely other diseases, accident & murders, and the unexplained.

Table 4.: In-sample fit of CoDLC, CoDLi-Lee, and nestedCoDLi-Lee

Country	Cancer			Vascular		
	CoDLC	CoDLi-Lee	nestedCoDLi-Lee	CoDLC	CoDLi-Lee	nestedCoDLi-Lee
Belgium	0.83	0.89	0.86	0.95	0.97	0.95
France	0.87	0.88	0.79	0.95	0.98	0.95
Netherlands	0.78	0.91	0.80	0.94	0.96	0.95

Table 5.: In-sample fit of CoDLC, CoDLi-Lee, and nestedCoDLi-Lee

Country	Other Disease			Accident & Murders			Unexplained	
	CoDLC	CoDLi-Lee	nestedCoDLi-Lee	CoDLC	CoDLi-Lee	nestedCoDLi-Lee	CoDLC	CoDLi-Lee
Belgium	0.40	0.56	0.65	0.83	0.80	0.71	0.94	0.94
France	0.77	0.89	0.97	0.95	0.96	0.93	0.64	0.69
Netherlands	0.70	0.79	0.64	0.86	0.90	0.78	0.57	0.68

Cause-specific mortality projections

In previous studies (see Wilmoth 1995, Tabeau et al. 2001, and Arnold and Sherris 2013), cause-specific mortality projections are usually extrapolated based on cause-specific mortality data within one country, which are usually volatile. Hence, it is more difficult to identify the long-run behavior of cause-specific mortality from such volatile data. Also, single population models might fail to capture the international coherence and inter-cause coherence suggested by the data.

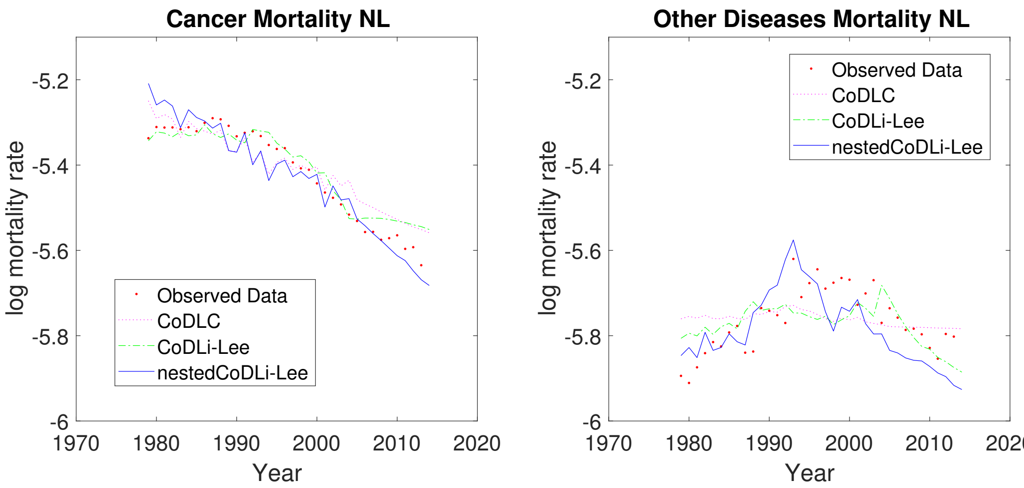

Figure 10.: Observed, fitted and forecasted cause-specific mortality, males in Netherlands

We focus on the case of the Dutch population in Figure 10, in which we show cause-specific mortality, namely, cancer (left panel) and other diseases (right panel), fitted over the period 1979 to 2005 and forecasted until 2013. (The results for Belgium and France are presented in the appendix). The results are twofold. First, for the cause-specific mortality with a smooth historical pattern, e.g., cancer (on the left panel), only the projections of nestedCoDLi-Lee model capture the recent improvements of cancer mortality [13]. Second, for the cause-specific mortality with a volatile historical pattern, for example, other diseases that contains influenza, the projections of the codLi-LEE nestedCoDLi-Lee seem to be much more reasonable than the codLC, because international coherence can drive out most of the country-specific turbulence.

The nestedCoDLi-Lee model incorporates international coherence and inter-cause coherence. In the long run, the international coherence drives out the short-term shocks in one country and presents a better-behaved cause-specific mortality profile. More importantly, the inter-cause coherence between cancer and vascular mortality captures the catch-up effect between these two diseases so as to improve the forecasts of cancer mortality.

Out-of-sample performance

We compare the out-of-sample performance of all three cause-specific mortality models, in terms of the root mean square error (RMSE) of the cause-specific mortality averaged over all the age groups. To be concise, we present the results of the RMSEs averaged over the three countries. Figure 11 shows these averaged RMSEs for a forecast horizon from 1 to 15 years. In case of a forecast horizon of one year, we use the data period 1979-2012 to estimate and forecast 2013, in case of a forecast horizon of two years, we use the data period 1979-2011 to estimate and forecast until 2013, and so on.

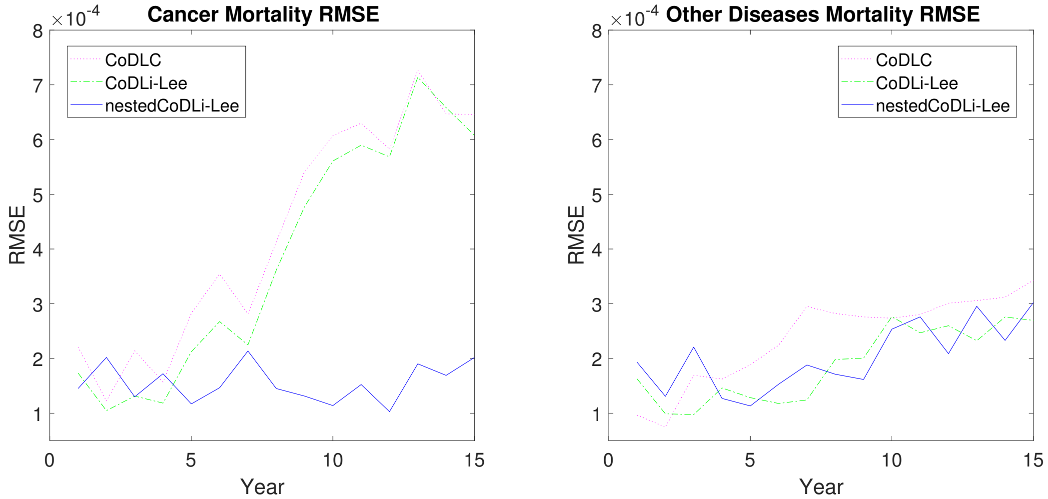

Figure 11.: Cause-specific mortality RMSE average over three countries

From Figure 11, we see that the out-of-sample performance of the nestedCoDLi-Lee model is better than the one of the CoDLC model and the one of the CoDLi-Lee model in cancer mortality (CM), especially for the long-horizon forecasts, which is consistent with Figure 10. This is because it takes several periods for the cancer mortality improvement to catch up with one of cardiovascular mortality. For the cause of death with a relatively volatile historical pattern within one country (like DM), the nestedCoDLi-Lee model and CoDLi-Lee model, which take into account international coherence, could filter out the short-term turbulence within one country resulting in slightly better out-of-sample performance in the long run.

Incorporating international coherence and inter-cause coherence improves the forecasts of

[13]This is even more obvious for the case of Belgium (Figure 29) and France (Figure 30)

cause-specific mortality in two ways. First, as the cause-specific mortality of one country is not likely to diverge from the common trend of a coherent group in the long run, the nestedCoDLi-Lee model could capture an identifiable pattern of cause-specific mortality by summarizing the common trend from a coherent group with richer mortality data. Especially for a country with volatile historical patterns, its cause-specific mortality projections could be corrected by a similar country with more stable historical patterns. Secondly, the nestedCoDLi-Lee model that accounts for inter-cause coherence, is able to capture the recent improvement of cancer mortality, which might be driven by the dissemination of medical innovations from the related vascular diseases. In the nestedCoDLi-Lee model, both cancer mortality and vascular mortality are expected to improve at the same speed in the long run, which leads the cancer mortality to improve faster while forces the vascular mortality to improve slower. This catch-up of cancer mortality towards vascular mortality is captured by the nestedCoDLiLee model, contrary to the CoDLi-Lee and the CoDLee-Carter models.

Part 2 All-cause mortality performance

In this part, we compare the all-cause Lee-Carter model (equation 9), the all-cause Li-Lee model (equation 5), the codLC model, the codLi-Lee model (equation 10), and the nestedcodLi-Lee model (equation 15), in term of their in-sample fit and out-of-sample performance when considering all-cause mortality.

	Cause of Death Info	International Coherence	Inter-cause Coherence
Lee-Carter	NO	NO	NO
Li-Lee	NO	YES	NO
CoDLC	YES	NO	NO
CoDLi-Lee	YES	YES	NO
NestedCoDLi-Lee	YES	YES	YES

Table 6.: All-Cause Mortality Models Summary

Table 6 summarizes the core features of the all-cause mortality models, which will be discussed in the following subsections. Containing causes of death information means that the cause-specific mortality data is used by the model to generate all-cause mortality. International coherence means that the model imposes that the all-cause mortality of a country converges to the common trend of the countries in a coherent group. Inter-cause coherence means that the mortality model accounts for the inter-cause co-movement as suggested by the two-step beta convergence test.

All-cause mortality projections

In Figure 12 we use the cause-specific mortality data of Netherlands from 1979-2005 to construct all-cause mortality projections until 2013 for all models, as Part 1 cause-specific mortality performance. The figure shows that the nestedCoDLi-Lee model produces the fastest-improving all-cause mortality projections, almost capturing the recent mortality improvements of the Netherlands. The main driver of this is the cancer-vascular coherence. Incorporating this coherence helps the nestedCoDLi-Lee model captures the catch-up effect of the cancer mortality in the recent years.

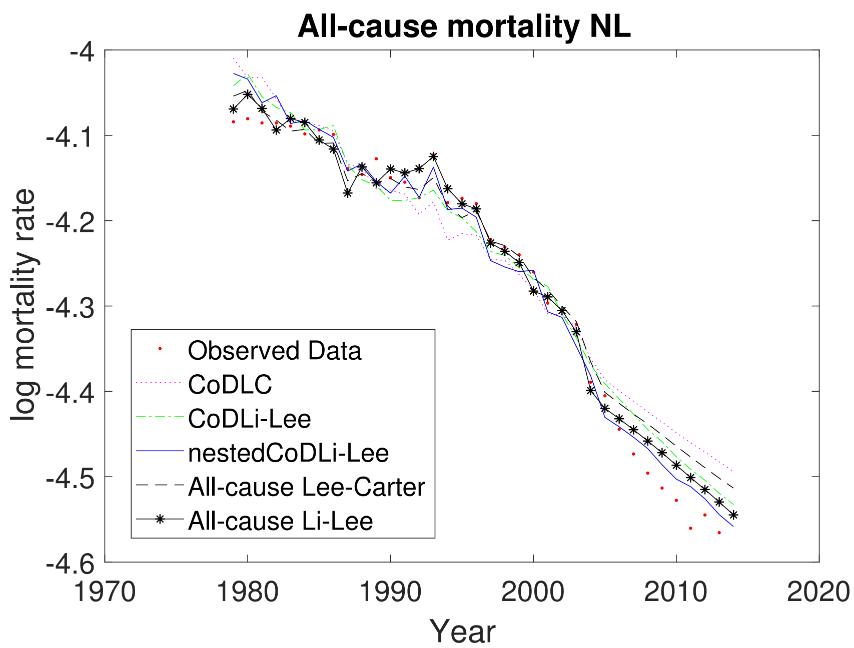

Figure 12.: Observed, fitted and forecasted all-cause mortality, males in Netherlands

In addition, the international coherence helps both all-cause Li-Lee model and codLi-Lee model perform better than their single-population competitors, namely all-cause Lee-Carter model and codLC model respectively. This is because the international coherence captures the catch-up effect and avoids the divergence between different countries in a coherent group. [14]

In-sample fit of the all-cause mortality models

We compare the fit of the LC model, the standard Li-Lee model, the codLC model, the codLi-Lee model and the nestedcodLi-Lee model in terms of the explanation ratios (Li and Lee 2005).

Explanation Ratios	all-cause Lee-Carter model	all-cause Li-Lee model	CoDLee-Carter model	CoDLi-Lee model	nestedCoDLi-Lee model
Belgium	0.94	0.96	0.91	0.93	0.94
France	0.91	0.98	0.90	0.97	0.95
Netherlands	0.94	0.96	0.91	0.94	0.92

Table 7.: Explanation ratios of the different models 1979-2013 male population

Table 7 shows the explanation ratios for the different models and different countries. Among the single-population models, the all-cause LC model performs slightly better than the codLC model. This is because codLC model needs to fit for five cause-specific mortality to construct the all-cause mortality, which includes more noise than the all-cause LC model that directly applies to the all-cause mortality. This is also the case in the multi-population models. Comparing all-cause Li-Lee model to codLi-Lee model and nestedCoDLi-Lee, the all-cause Li-Lee model performs slightly better than the latter two models that based on cause-specific information. Comparing the multi-population models to the single-population models, we discover incorporating either

[14]See appendix Figure 31 and Figure 32 for the similar results of Belgium and France.

all-cause mortality information or cause-specific mortality information from similar countries in a coherent group helps to improve the in-sample performance of the multi-population models. To sum up, all the models perform more or less similarly with the explanation ratios over 90%.

Out-of-sample performance of all-cause mortality models

Figure 13 presents the all-cause mortality Root Mean Squared Errors (RMSE) of the forecasts, following the same method as Figure 11.

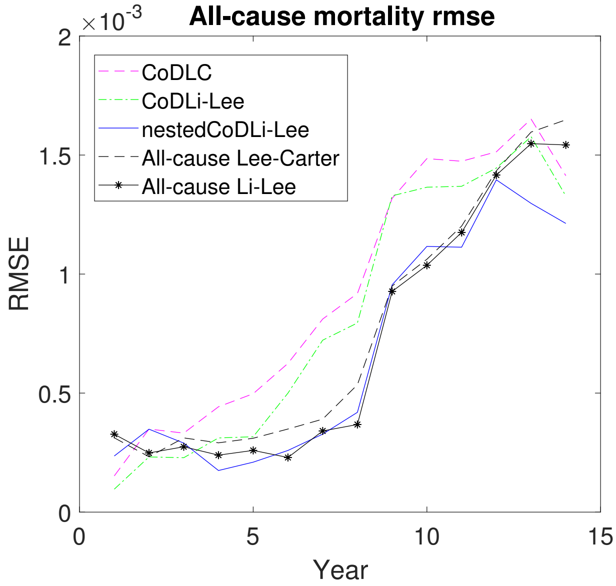

Figure 13.: All-Cause mortality RMSE average over three countries

The results of the out-of-sample performance of the models in all-cause mortality projections can be summarized as follows. First, among the cause-specific mortality models, namely CoDLC model (light dash line), CoDLi-Lee model (dash-dotted line), and the nestedCoDLi-Lee (solid line)), the nestedCoDLi-Lee model that incorporates both international coherence and the inter-cause coherence has the best out-of-sample performance in all forecast horizons. Second, among the all-cause mortality models, namely all-cause LC model (heavy dash line) and all-cause Li-Lee model (line with star markers), the all-cause Li-Lee model that accounts for international coherence perform better in all forecast horizons. Third, comparing the winner of the cause-specific mortality models, i.e., nestedCoDLi-Lee model and the winner of the all-cause mortality models, i.e., all-cause Li-Lee model, these two models have a similar out-of-sample performance in the forecast horizon from 1 to 10 years. After the forecast horizon become longer than 10 years, the nestedCoDLi-Lee model has a slightly better out-of-sample performance than the all-cause Li-Lee model, and subsequently the other models.

There are two main reasons why nestedCoDLi-Lee model has comparable or even better out-of-sample performance than the all-cause mortality models. On one hand, international coherence allows the nestedCoDLi-Lee model to identify a clear long-run trend of cause-specific mortality from richer mortality data of a coherent group, resulting in a lower country-average RMSE.On the other hand, the main inter-cause coherence, i.e., cancer-vascular coherence, helps the nestedCoDLi-Lee model to capture the changing improvement of cancer mortality in recent decades. In details, cancer-vascular coherence makes the cancer mortality projections of nestedCoDLi-Lee model to improve faster as the cancer-vascular common trend in the long run and maintains its own improvement rate in the short run, thus capturing the catch-up effects of cancer mortality toward vascular mortality, which is observed in the data and confirmed by the two-step beta convergence test.

Mortality implications of all-cause mortality models

In Figure 14, we present the remaining period life expectancy of a 67-years-old [15] Dutch male as projected by all the aforementioned models. The nestedCoDLi-Lee model (the solid), that takes into account both international coherence and inter-cause coherence, leads to higher longevity gains than all other models in the Dutch case. More specifically, in a 30-year forecast horizon, the remaining life expectancy of a 67-years-old Dutch male is projected by the nestedCoDLi-Lee model to be approximately one year more than the one that is projected by the all-cause Li-Lee model. In short, such a longevity gain is believed to come from the faster improvement of cancer mortality in the mortality projections of the nestedCoDLi-Lee model, which is a consequence of the cancer-vascular coherence. [16]

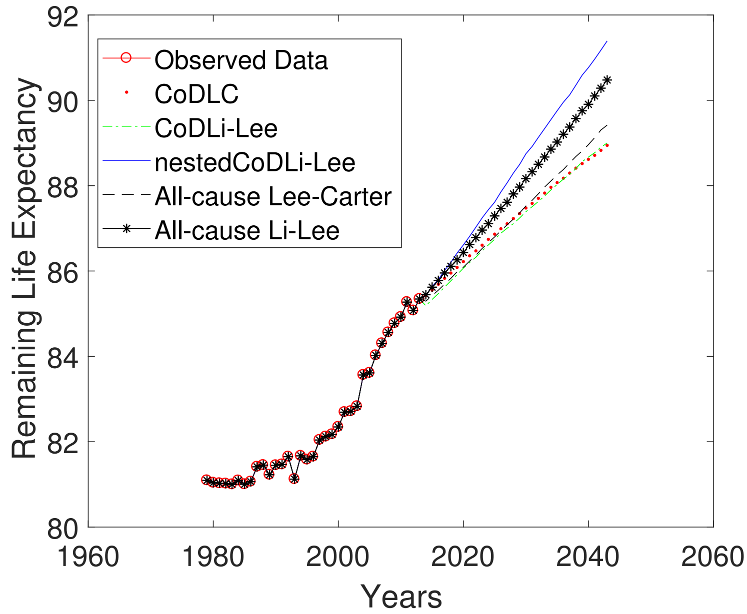

Figure 14.: Observed and forecasted 67-year-old male remaining life expectancy, males in Netherlands

5. Conclusions

Using the Access Version of the WHO Causes of Death Mortality Data from 1979-2013 in the Netherlands, Belgium, and France, our paper aims at providing all-cause mortality projections that incorporate international coherence and causes of death information. Based on international

[15]Recently, the compulsory retirement age of the Netherlands has increased to age 67 years old.
[16]See Figure 33 and Figure 34 for the case of Belgium and France.

coherence and inter-cause coherence as suggested by the two-step beta convergence test, we propose a multi-population model, referred to as nestedCoDLi-Lee, to forecast cause-specific mortality. We show that the nestedCoDLi-Lee model has a similar in-sample fit as the benchmark models used in the literature, but shows a much better out-of-sample performance than the benchmark models. We attribute the merits of the nestedCoDLi-Lee model to the fact that it can prevent the cause-specific turbulence within one country (for example, because of an influenza-like outbreak in a certain period in a specific country), and preserve a well-behaved long-term forecast by incorporating international coherence, and it can capture up-to-date inter-cause dynamics by allowing for inter-cause coherence in the cause-specific mortality projections. Furthermore, we derive mortality implications in terms of all-cause mortality projections from the nestedCoDLi-Lee model and demonstrate that additional one year of life expectancy could be gained by 67 years-old Dutch male in a 30-year forecast horizon if the international coherence and inter-cause coherence were to continue as implied in the projections. Such longevity gains might result from more efficient inter-government collaborations in the countries that share similar socioeconomic characteristics (international Coherence), and from the dissemination of medical innovations between correlated causes of death (inter-cause coherence).

On the other hand, there are several limitations to the approach followed in our paper. The international coherence is restricted to three countries due to a limited availability of WHO Access Version causes of death data; the dependence structure between causes of death is highly abstracted to three types (convergence, divergence and coherence) while there might be more possible dependence structures that are indicated by the data but are overlooked by the two-step beta convergence test. The results of the cause-specific mortality models are sensitive to the definitions of causes of death in Table 1. For future research, it would be interesting to work on constructing an approach to identify more possible dependence structures between causes of death and incorporating the dependence structure in the mortality projections. Also, future research could focus on building an all-cause mortality model based on the cause-specific mortality that allows for more than one type of dependence structure between different causes of death.

REFERENCES

Adam, Klaus, Tullio Jappelli, Annamaria Menichini, Mario Padula, and Marco Pagano (2002), "Analyse, compare, and apply alternative indicators and monitoring methodologies to measure the evolution of capital market integration in the european union." *Report to the European Commission*, 1–95.

Alai, Daniel H, Séverine Arnold, and Michael Sherris (2015), "Modelling cause-of-death mortality and the impact of cause-elimination." *Annals of Actuarial Science*, 9, 167–186.

Arnold, Séverine, Alexandre Boumezoued, Héloïse Labit Hardy, and Nicole El Karoui (2015), "Cause-of-death mortality: What can be learned from population dynamics?"

Arnold, Séverine and Michael Sherris (2013), "Forecasting mortality trends allowing for cause-of-death mortality dependence." *North American Actuarial Journal*, 17, 273–282.

Barro, Robert J (1991), "Economic growth in a cross section of countries." *The quarterly journal of economics*, 106, 407–443.

Bongaarts, John (2014), "Trends in causes of death in low-mortality countries: implications for mortality projections." *Population and Development Review*, 40, 189–212.

Booth, Heather, Rob Hyndman, Leonie Tickle, and Piet De Jong (2006), "Lee-carter mortality forecasting: a multi-country comparison of variants and extensions." *Demographic Research*, 15, 289–310.

Booth, Heather and Leonie Tickle (2008), "Mortality modelling and forecasting: A review of methods." *Annals of actuarial science*, 3, 3–43.

Bremberg, Sven G (2017), "Mortality rates in oecd countries converged during the period 1990–2010." *Scandinavian journal of public health*, 45, 436–443.

Cairns, Andrew JG, David Blake, and Kevin Dowd (2006), "A two-factor model for stochastic mortality with parameter uncertainty: theory and calibration." *Journal of Risk and Insurance*, 73, 687–718.

Carrat, Fabrice, Antoine Flahault, Eric Boussard, Nathalie Farran, Laure Dangoumau, and Alain-Jacques Valleron (1998), "Surveillance of influenza-like illness in france. the example of the 1995/1996 epidemic." *Journal of Epidemiology and Community Health (1979-)*, 32S–38S.

Carriere, Jacques F (1994), "Dependent decrement theory." *Transactions of the Society of Actuaries*, 46, 45–74.

Carrière, Jacques F (1995), "Removing cancer when it is correlated with other causes of death." *Biometrical Journal*, 37, 339–350.

Caselli, Graziella, Jacques Vallin, and Marco Marsili (2006), *How useful are the causes of death when extrapolating mortality trends: an update*. na.

d'Albis, Hippolyte, Loesse Jacques Esso, and Héctor Pifarré I Arolas (2012), "Mortality convergence across high-income countries: An econometric approach."

Dimitrova, Dimitrina S, Steven Haberman, and Vladimir K Kaishev (2013), "Dependent competing risks: Cause elimination and its impact on survival." *Insurance: Mathematics and Economics*, 53, 464–477.

Gaille, Séverine and Michael Sherris (2011), "Modelling mortality with common stochastic long-run trends." *The Geneva Papers on Risk and Insurance Issues and Practice*, 36, 595–621.

Gutterman, Sam and Irwin T Vanderhoof (1998), "Forecasting changes in mortality: a search for a law of causes and effects." *North American Actuarial Journal*, 2, 135–138.

Hatzopoulos, Petros and Steven Haberman (2013), "Common mortality modeling and coherent forecasts. an empirical analysis of worldwide mortality data." *Insurance: Mathematics and Economics*, 52, 320–337.

Honoré, Bo E and Adriana Lleras-Muney (2006), "Bounds in competing risks models and the war on cancer." *Econometrica*, 74, 1675–1698.

Janssen, Fanny et al. (2005), "Cohort patterns in mortality trends among the elderly in seven european countries, 1950–99." *International Journal of Epidemiology*, 34, 1149–1159.

Jemal, Ahmedin, Melissa M Center, Carol DeSantis, and Elizabeth M Ward (2010), "Global patterns of cancer incidence and mortality rates and trends." *Cancer Epidemiology and Prevention Biomarkers*, 19, 1893–1907.

Kaishev, Vladimir K, Dimitrina S Dimitrova, and Steven Haberman (2007), "Modelling the joint distribution of competing risks survival times using copula functions." *Insurance: Mathematics and Economics*, 41, 339–361.

Lee, Ronald and Timothy Miller (2001), "Evaluating the performance of the lee-carter method for forecasting mortality." *Demography*, 38, 537–549.

Lee, Ronald D and Lawrence R Carter (1992), "Modeling and forecasting us mortality." *Journal of the American statistical association*, 87, 659–671.

Li, Hong and Yang Lu (2017), "Modelling competing risks using hierarchical archimedean copula with application to longevity forecast." *Working Paper*.

Li, Nan and Ronald Lee (2005), "Coherent mortality forecasts for a group of populations: An extension of the lee-carter method." *Demography*, 42, 575–594.

Mathers, Colin D, Ties Boerma, and Doris Ma Fat (2009), "Global and regional causes of death." *British medical bulletin*, ldp028.

Millossovich, P, S Haberman, VK Kaishev, S Baxter, A Gaches, S Gunnlaugsson, and M Sison (2014), "Longevity basis risk a methodology for assessing basis risk."

Oeppen, Jim et al. (2008), "Coherent forecasting of multiple-decrement life tables: a test using japanese cause of death data."

Omran, Abdel R (1998), "The epidemiologic transition theory revisited thirty years later." *World Health Statistics Quarterly*, 51, 99–119.

Putter, Hein, Marta Fiocco, and Ronald B Geskus (2007), "Tutorial in biostatistics: competing risks and multi-state models." *Statistics in medicine*, 26, 2389–2430.

Renshaw, Arthur E and Steven Haberman (2006), "A cohort-based extension to the lee–carter model for mortality reduction factors." *Insurance: Mathematics and Economics*, 38, 556–570.

Tabeau, Ewa, Peter Ekamper, Corina Huisman, and Alinda Bosch (1999), "Improving overall mortality forecasts by analysing cause-of-death, period and cohort effects in trends." *European Journal of Population/Revue Européenne de Démographie*, 15, 153–183.

Tabeau, Ewa, Anneke van den Berg Jeths, and Christopher Heathcote (2001), *Forecasting mortality in developed countries: Insights from a statistical, demographic and epidemiological perspective*, volume 9. Springer Science & Business Media.

Tsiatis, Anastasios (1975), "A nonidentifiability aspect of the problem of competing risks." *Proceedings of the National Academy of Sciences*, 72, 20–22.

Tuljapurkar, Shripad, Nan Li, and Carl Boe (2000), "A universal pattern of mortality decline in the g7 countries." *Nature*, 405, 789–792.

Vallin, Jacques and France Meslé (2004), "Convergences and divergences in mortality: A new approach of health transition." *Demographic research*, 2, 11–44.

White, Kevin M (2002), "Longevity advances in high-income countries, 1955–96." *Population and Development Review*, 28, 59–76.

Wilmoth, John R (1995), "Are mortality projections always more pessimistic when disaggregated by cause of death?" *Mathematical population studies*, 5, 293–319.

Wilmoth, John R (1998), "Is the pace of japanese mortality decline converging toward international trends?" *Population and Development Review*, 593–600.

Wilmoth, JR (1996), "Mortality projections for japan: a comparison of four methods."

Wilson, Chris (2001), "On the scale of global demographic convergence 1950–2000." *Population and Development Review*, 27, 155–171.

Zheng, Ming and John P Klein (1995), "Estimates of marginal survival for dependent competing risks based on an assumed copula." *Biometrika*, 82, 127–138.

6. Appendix: Alternatives and Proofs

6.1. The method of determining the coherent group in Li and Lee (2005)

6.1.1. International comparison

Applying Li-Lee model on the cause-specific mortality for five causes of death (s) across three countries (v).

$$ln(m_{x,t}^{s,v}) = \alpha_x^{s,v} + B_x^s K_t^s + b_x^{s,v} k_t^{s,v} + \varepsilon_{x,t}^{s,v} \tag{21}$$

We estimate this model using two-step SVD as described in the Li and Lee (2005). The method used by their paper on determining the membership of coherent groups of this cause-specific mortality is as follows.

First, the country-specific period effect $k_t^{s,v}$ for country v and cause s, is fitted to a AR(1) process.

$$k_t^{s,v} = \beta_0^{s,v} + \beta_1^{s,v} k_{t-1}^{s,v} + u_t^{s,v} \tag{22}$$

Second, a coherent group membership of cause-specific mortality s could be granted to a country v, if $\beta_1^{s,v} < 1$. In other words, the country v could only be included in this cause-specific coherent group if its country-specific period effect is stationary process.

The results of this method are as follows. The estimation is based on the data period from 1979 to 2013.

Table 8.: Li-Lee method of determining coherent membership

	$\beta_1^{s,v}$				
	CM	VM	DM	AMM	UEM
BE	0.33	0.12	0.84	0.87	0.83
FR	0.82	0.74	0.89	1.03	0.98
NL	0.16	0.43	0.81	0.94	0.88

From the Table 8, the Li-Lee method suggest that the international coherence exists in cancer mortality, vascular mortality, other diseases mortality, and the unexplained mortality. However, for the accident & murder, the France should be excluded from the coherent group that consists of Netherlands and Belgium.

6.1.2. Inter-cause comparison

Cancer-vascular coherence

Follow the similar logic as above and, we apply the Li-Lee method to determine the inter-cause coherence between cancer and vascular and obtain the β_1^{CM} and the β_1^{VM} for the cancer-specific period effect and the vascular-specific period effect. The results are as follows.

Table 9.: Li-Lee method on cancer-vascular coherence

	β_1^s
CM	0.97
VM	0.98

Other Diseaes & Accident Murder coherence

Follow the similar logic as above and, we apply the Li-Lee method to determine the inter-cause coherence between other-diseases and accident&murders, and obtain the β_1^{DM} and the β_1^{AMM} as follows.

Table 10.: Li-Lee method on Other Diseases-Accident & Murder coherence

	β_1^s
DM	0.91
AMM	0.88

6.2. Two-step beta convergence and its link to the Li-Lee model

According to Figure 4, different types of co-movements could be summarized as follows. The convergence means the difference between two time series process tends to zero at the steady state. The divergences means such a difference tends to infinity. The coherence means such a difference tends to a constant in the steady state. On the other hand, a negative β_{level} of the two-step beta convergence test indicates the innovations of the time series process is negatively related to the initial level, meaning a higher initial level leads to a lower improvement while a lower initial level leads to a higher improvement. This suggests that the difference between two time series process with different initial level is tending to zeros in the steady state. By a similar logic, a negative β_{trend} of the test reveals that the difference between the innovations of two time series process tends to zero in the steady state. In other words, it means that these two time series process would increase or decrease at a same speed.

To sum up in a context of multi-population mortality, a negative β_{level} means type 1 co-movements in the Figure 4, i.e., the difference between the mortality levels of two countries tends to zero in the steady state. A negative β_{trend} suggests type 3 co-movements, i.e., the difference between the mortality improvements of two countries tends to zeros in the steady state.

Mathematically, the steady-state difference between the mortality levels for age x of the country v and the one of the country v' can be expressed as follows. $v, v' \in \{NL, BE, FR\}$

$$\lim_{T_L\to\infty}\Big[\mathbb{E}\big(log(m^{v}_{x,t+T_L})\big)\mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big)\Big] \tag{23}$$

The steady-state difference between the mortality improvements for age x of the country v and the one of the country v' can be expressed as follows.

$$\lim_{T_L\to\infty}\Big[\mathbb{E}\big(\frac{log(m^{v}_{x,t+T_L+\Delta})-log(m^{v}_{x,t+T_L})}{\Delta}\big)-\mathbb{E}\big(\frac{log(m^{v'}_{x,t+T_L+\Delta})-log(m^{v'}_{x,t+T_L})}{\Delta}\big)\Big] \tag{24}$$

Expressing these two differences in the Li-Lee model, we have the following equations. One is the difference between mortality levels of two countries.

$$\begin{aligned}&\lim_{T_L\to\infty}\Big[\mathbb{E}\big(log(m^{v}_{x,t+T_L})\big)-\mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big)\Big]\\&=\lim_{T_L\to\infty}\Big[\alpha^{v}_{x}+B_xK_{t+T_L}+b^{v}_{x}k^{v}_{t+T_L}-\alpha^{v'}_{x}-B_xK_{t+T_L}-b^{v'}_{x}k^{v'}_{t+T_L}\Big]\\&=\lim_{T_L\to\infty}\Big[\alpha^{v}_{x}-\alpha^{v'}_{x}+b^{v}_{x}\mathbb{E}k^{v}_{t+T_L}-b^{v'}_{x}\mathbb{E}k^{v'}_{t+T_L}\Big]\end{aligned} \tag{25}$$

The other is the difference between mortality improvements of two countries.

$$\begin{aligned}&\lim_{T_L\to\infty}\Big[\mathbb{E}\big(\frac{log(m^{v}_{x,t+T_L+\Delta})-log(m^{v}_{x,t+T_L})}{\Delta}\big)-\mathbb{E}\big(\frac{log(m^{v'}_{x,t+T_L+\Delta})-log(m^{v'}_{x,t+T_L})}{\Delta}\big)\Big]\\&=\lim_{T_L\to\infty}\Big[\Big(B_x\mathbb{E}K_{t+T_L+\Delta}+b^{v}_{x}\mathbb{E}k^{v}_{t+T_L+\Delta}-B_x\mathbb{E}K_{t+T_L}-b^{v}_{x}\mathbb{E}k^{v}_{t+T_L}\Big)\\&-\Big(B_x\mathbb{E}K_{t+T_L+\Delta}+b^{v'}_{x}\mathbb{E}k^{v}_{t+T_L+\Delta}-B_x\mathbb{E}K_{t+T_L}-b^{v'}_{x}\mathbb{E}k^{v'}_{t+T_L}\Big)\Big]\end{aligned} \tag{26}$$

The long-run behavior of the difference in the mortality levels and the mortality improvements are advised by the two-step beta convergence test. To include a country in a coherent group, we need to ensure there is no divergence in the difference the mortality level and the mortality improvements between this country and the other group members. To make sure that the Li-Lee model captures the long-run behavior of these two difference, specific types of country-specific period effect k^{v}_{t} should be selected. We abstract from the discussion of common period effect K_t because these two differences between countries could not be reflected in the K_t.

In a nutshell, we leverage the two-step beta convergence to determine whether to include a country in a coherent group. Then, we select the proper country-specific period effect of the Li-Lee model to capture the international coherence suggested by the tests. We separate the selection procedure into two parts. The first part is assume all countries share the same country-specific period effect. The second part is assume different countries could have different country-specific period effects. Both parts lead to a similar conclusion, i.e., we require country-specific period effect to be stationary so as to include such country in a coherent group. For country-specific period effect, we consider three type of time series process, i.e. ARIMA(1,0,0) (AR(1)), ARIMA(0,1,0) (random walk with drift), and ARIMA(1,1,0). We do not consider moving average components because all the discussions in the main text focus on the best estimates without accounting for the error term.

For the discussion below, the common period effects K_t follows the random walk with drift process (RWD).

$$K_t = B_0 + K_{t-1} + e_t \tag{27}$$

B_0 is the drift term of RWD process and e_t is random variable with i.i.d normal distribution .

6.2.1. All countries use the same type of the time series process for the country-specific period effect

Suppose $k^v_{t+T_L}$ follows an AR(1) process, i.e., $k^v_{t+1} = \beta^v_0 + \beta^v_1 k^v_t + u^v_t$

Note that we impose a normalization on k^v_t such that $\beta^v_0 = 0$. Given a stationary AR(1) process (β^v_1), the difference between mortality levels of two countries is as follows.

$$\lim_{T_L\to\infty} \mathbb{E}\big(log(m^v_{x,t+T_L})\big) - \mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big) = \alpha^v_x - \alpha^{v'}_x + b^v_x \frac{\beta^v_0}{1-\beta^v_1} - b^{v'}_x \frac{\beta^{v'}_0}{1-\beta^{v'}_1} = \alpha^v_x - \alpha^{v'}_x \tag{28}$$

This difference in the mortality levels is constant, i.e., $\alpha^v_x - \alpha^{v'}_x$ in the Equation 28. Next, the difference of the mortality improvements between two countries is as follows.

$$\begin{aligned}&\lim_{T_L\to\infty} \left[\mathbb{E}\Big(\frac{log(m^v_{x,t+T_L+\Delta}) - log(m^v_{x,t+T_L})}{\Delta}\Big) - \mathbb{E}\Big(\frac{log(m^{v'}_{x,t+T_L+\Delta}) - log(m^{v'}_{x,t+T_L})}{\Delta}\Big)\right] \\ &= B_x B_0 - B_x B_0 = 0\end{aligned} \tag{29}$$

To sum up, the Li-Lee model with AR(1) country-specific period effect present the steady-state behavior: (1) the difference of the mortality levels between two countries is constant as $\alpha^v_x - \alpha^{v'}_x$ (2) the difference of the mortality improvements between two countries is zero. This country-specific period effect can link to the two possible two-step beta convergence outcome: (1) $\beta_{level} < 0$ and $\beta_{trend} < 0$ with the special condition $\alpha^v_x - \alpha^{v'}_x = 0$, convergence(2) $\beta_{level} = 0$ and $\beta_{trend} < 0$, coherence.

Suppose $k^v_{t+T_L}$ follows an ARIMA(0,1,0) process, i.e., $k^v_{t+1} = \beta^v_0 + k^v_t + u^v_t$

The difference in the mortality levels between two countries:

$$\begin{aligned}\lim_{T_L\to\infty} \mathbb{E}\big(log(m^v_{x,t+T_L})\big) - \mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big) &=_{(b^v_x\beta^v_0 \neq b^{v'}_x \beta^{v'}_0)} \pm\infty \\ &=_{(b^v_x\beta^v_0 = b^{v'}_x \beta^{v'}_0)} \alpha^v_x - \alpha^{v'}_x + b^v_x k^v_t - b^{v'}_x k^{v'}_t\end{aligned} \tag{30}$$

The differences between the mortality levels of two countries cound be positive infinite or negative infinite under the condition that $(b_x^v\beta_0^v \neq b_x^{v'}\beta_0^{v'})$, i.e., divergence. The differences could also be constant, as long as $(b_x^v\beta_0^v = b_x^{v'}\beta_0^{v'})$.

The difference in mortality improvements between two countries:

$$\lim_{T_L\to\infty}\Big[\mathbb{E}\big(\frac{log(m^v_{x,t+T_L+\Delta}) - log(m^v_{x,t+T_L})}{\Delta}\big) - \mathbb{E}\big(\frac{log(m^{v'}_{x,t+T_L+\Delta}) - log(m^{v'}_{x,t+T_L})}{\Delta}\big)\Big] \\ = b_x^v\beta_0^v - b_x^{v'}\beta_0^{v'} \tag{31}$$

The difference in mortality improvements between countries is the same if $b_x^v\beta_0^v = b_x^{v'}\beta_0^{v'}$. If not, this difference would also stay constantly different. To sum up, this country-specific period effect links to three possible outcomes: (1) if $b_x^v\beta_0^v = b_x^{v'}\beta_0^{v'}$, $\beta_{level} = 0$ and $\beta_{trend} < 0$, constant difference in mortality levels and zeros difference in mortality improvements, coherence (2) if not, $\beta_{level} > 0$ and $\beta_{trend} = 0$, divergence. (3) if $\alpha_x^v = \alpha_x^{v'}$, $b_x^v k_t^v = b_x^{v'}k_t^{v'}$ but $b_x^v\beta_0^v \neq b_x^{v'}\beta_0^{v'}$, $\beta_{level} < 0$ and $\beta_{trend} = 0$, convergence.

Suppose k_t^v follows ARIMA(1,1,0) process, $k_{t+1}^v - k_t^v = \beta_0^v + \beta_1^v(k_t^v - k_{t-1}^v) + u_t^v$

Note that we impose a normalization on k_t^v such that $\beta_0^v = 0$.

For k_t^v,

$$\mathbb{E}(k^v_{t+T_L}) = \frac{1(1-(\beta_1^v)^{T_L+1})}{1-\beta_1^v}k_t^v - \frac{\beta_1(1-(\beta_1^v)^{T_L})}{1-\beta_1^v}k_{t-1}^v \tag{32}$$

When $-1 < \beta_1^v, \beta_1^{v'} < 0$

The limit of $k^v_{t+T_L}$ and $k^{v'}_{t+T_L}$ become stationary, e.g.,$\lim_{T_L\to\infty}\mathbb{E}(k^v_{t+T_L}) = \frac{1}{1-\beta_1^v}k_t^v - \frac{\beta_1}{1-\beta_1^v}k_{t-1}^v$. The differences between mortality levels of two countries are constants.

$$\lim_{T_L\to\infty}\mathbb{E}\big(log(m^v_{x,t+T_L})\big) - \mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big) = \alpha_x^v - \alpha_x^{v'} + [\frac{1}{1-\beta_1^v}k_t^v - \frac{\beta_1^v}{1-\beta_1^v}k_{t-1}^v] \\ - [\frac{1}{1-\beta_1^{v'}}k_t^{v'} - \frac{\beta_1^v}{1-\beta_1^{v'}}k_{t-1}^{v'}] \tag{33}$$

The difference in the mortality improvements between two countries are the same when $b_x^v\beta_0^v = b_x^{v'}\beta_0^{v'}$. If not, the mortality improvements would also be different in the long run.

$$\lim_{T_L\to\infty}\Big[\mathbb{E}\big(\frac{log(m^v_{x,t+T_L+\Delta}) - log(m^v_{x,t+T_L})}{\Delta}\big) - \mathbb{E}\big(\frac{log(m^{v'}_{x,t+T_L+\Delta}) - log(m^{v'}_{x,t+T_L})}{\Delta}\big)\Big] \\ =_{-1<\beta_1^v,\beta_1^{v'}<0} B_xB_0 - B_xB_0 = 0. \tag{34}$$

When $\beta_1^v, \beta_1^{v'} \notin (-1, 1)$

$$\lim_{T_L \to \infty} \mathbb{E}\big(log(m^v_{x,t+T_L})\big) - \mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big) = \pm\infty \tag{35}$$

$$\lim_{T_L \to \infty} \Bigg[\mathbb{E}\Big(\frac{log(m^v_{x,t+T_L+\Delta}) - log(m^v_{x,t+T_L})}{\Delta}\Big) - \mathbb{E}\Big(\frac{log(m^{v'}_{x,t+T_L+\Delta}) - log(m^{v'}_{x,t+T_L})}{\Delta}\Big)\Bigg] = \pm\infty \tag{36}$$

In sum, this time series process of countries specific period effect links to two possible outcomes: (1) if $-1 < \beta^v_1, \beta^{v'}_1 < 0$, $\beta_{level} = 0$ and $\beta_{trend} < 0$, constant difference in mortality levels and zeros difference in mortality improvements, coherence (2) if not, $\beta_{level} > 0$ and $\beta_{trend} > 0$, divergence.

6.2.2. All countries use the different types of the time series process for the country-specific period effect

Suppose k^v_t follows ARIMA(1,1,0) process and $k^{v'}_t$ follows ARIMA(1,0,0) process

Considering $-1 < \beta^v_1, \beta^{v'}_1 < 0$ and $0 < \beta^{v'}_1 < 1, \beta^{v'}_1 < 0$

$$\begin{aligned}\lim_{T_L \to \infty} \mathbb{E}\big(log(m^v_{x,t+T_L})\big) - \mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big) &= \alpha^v_x - \alpha^{v'}_x + [\frac{1}{1-\beta^v_1}k^v_t - \frac{\beta^v_1}{1-\beta^v_1}k^v_{t-1}] - b^{v'}_x \frac{\beta^{v'}_0}{1-\beta^{v'}_1} \\ &=_{\beta^v_0 = 0} = \alpha^v_x - \alpha^{v'}_x + [\frac{1}{1-\beta^v_1}k^v_t - \frac{\beta^v_1}{1-\beta^v_1}k^v_{t-1}]\end{aligned} \tag{37}$$

Equation 37 shows that in the long run even if one country is AR(1) and the other country is AR(1,1,0), the difference between mortality levels of two countries is still constant. From Equation 29 and Equation 31 we also know that both country v and country v' share the same mortality improvement, i.e., $B_x B_0$. Given we have constant mortality level difference and same mortality improvement ($\beta_{level} = 0$ and $\beta_{trend} < 0$), we can conclude that both country v and country v' are in the coherent group, even though they don't share the same AR processes.

Suppose k^v_t follows ARIMA(0,1,0) process and $k^{v'}_t$ follows ARIMA(1,0,0) process

Considering $0 < \beta^{v'}_1 < 1$ and only impose $\beta^{v'}_0 = 0$ because in ARIMA(0,1,0) the drift term is set to non-zero.

$$\lim_{T_L \to \infty} \mathbb{E}\big(log(m^v_{x,t+T_L})\big) - \mathbb{E}\big(log(m^{v'}_{x,t+T_L})\big) = \alpha^v_x - \alpha^{v'}_x + b^v_x \beta^v_0 T_L - b^{v'}_x \frac{\beta^{v'}_0}{1-\beta^{v'}_1} = \pm\infty \tag{38}$$

Equation 38 shows that in the long run even if one country is AR(1) and the other country is AR(0,1,0), the difference between mortality pattern of one country and the one of other country is exploding. Given we have exploding mortality level difference and same mortality improvement($\beta_{level} > 0$ and $\beta_{trend} = 0$), we can conclude that both country v and country v' are not in the same coherent group, which is similar to the cases where all the country-specific period effects share the same ARIMA(0,1,0) process.

7. Appendix: Tables and Figures

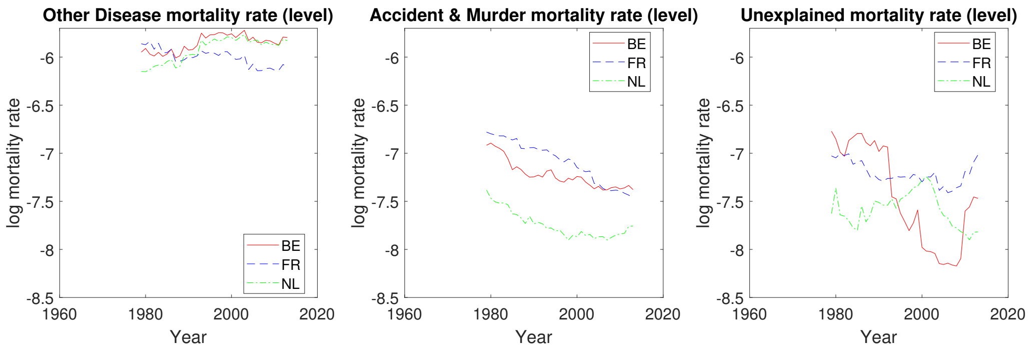

Figure 15.: International comparison: DM, AMM, and UEM mortality rate (Level)
Notes: For DM and AMM, the mortality rates of Belgium, France, and Netherlands are relatively stationary with mild improvements at some periods. The figure shows that these cause-specific mortality across three countries improves at a similar pace but does not converge to a same value , implying a potential international coherence. For UEM, the mortality rates are volatile in three countries, making it hard to draw any conclusion.

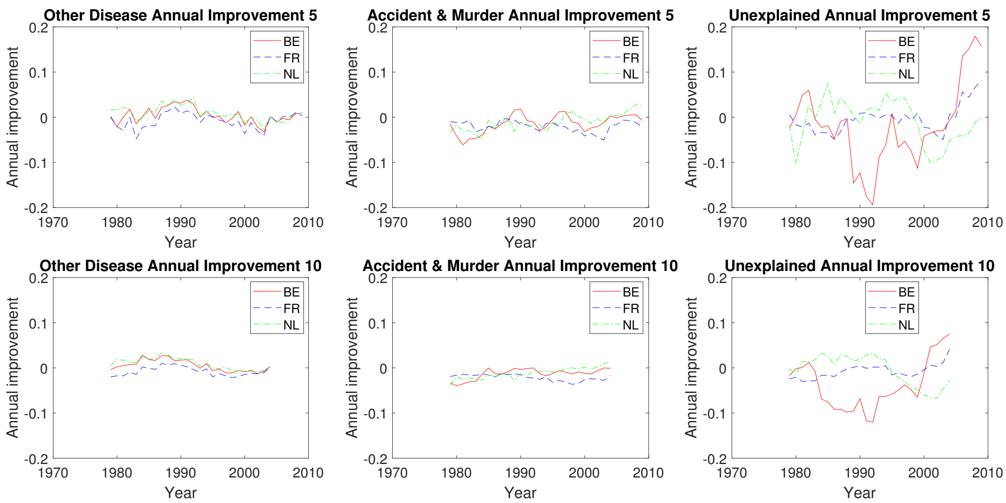

Figure 16.: International comparison: cancer and vascular mortality improvements
Notes: For DM and AMM, the mortality improvements of Belgium, France, and Netherlands are converging to a similar value, which is fluctuating around zeros. This is consistent with Figure 15. The figure again implies a potential international coherence. For UEM, the results are unclear.

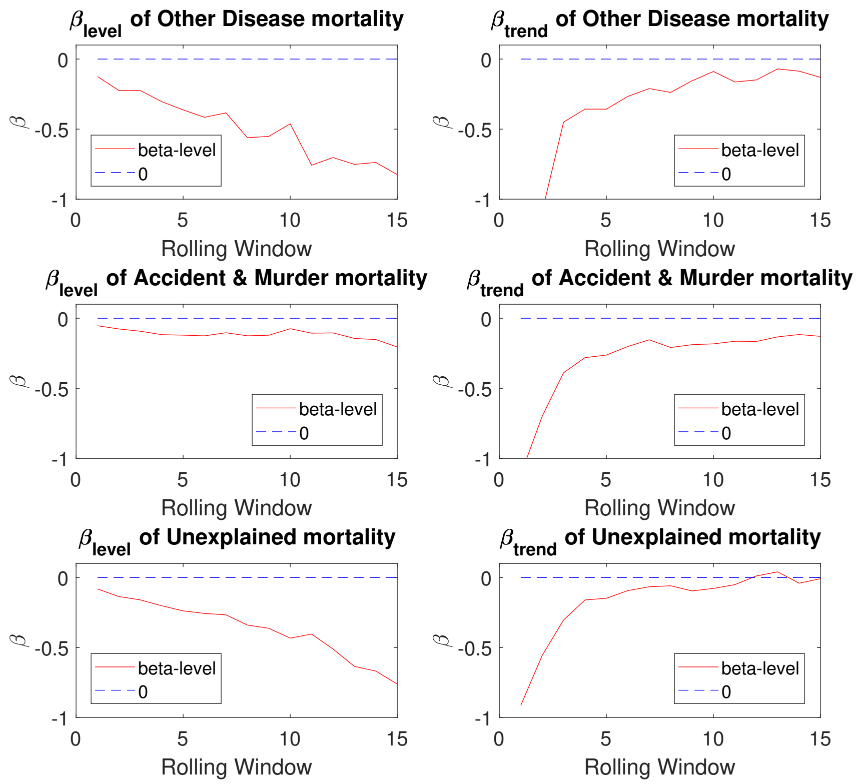

Figure 17.: International comparison: β_{slope} and β_{trend} of DM, AMM, and UEM

Notes: The left panels of the figure shows that β_{level} of DM, AMM, and UEM are negative for all rolling window choices. The right panels shows that β_{trend} of DM and AMM are negative for all rolling window choices while the one of UEM is negative for most of rolling window choices

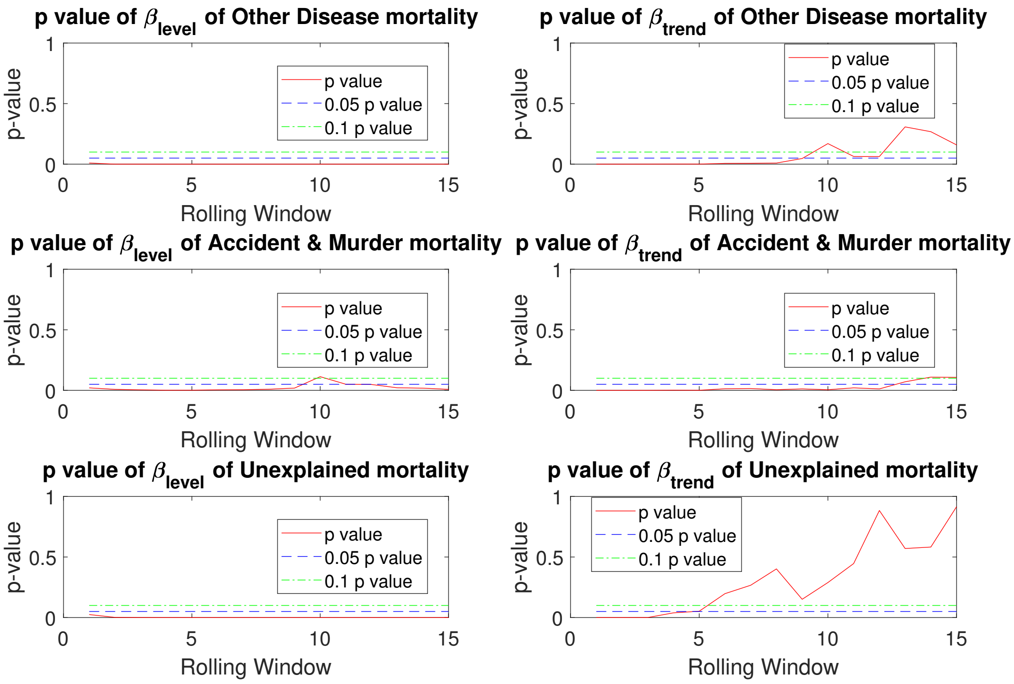

Figure 18.: International comparison: p-value of β_{slope} and β_{trend} of DM, AMM, and UEM

Notes: Combing with the left panels of Figure 17 and the left panels of this figure, we discover that the β_{level} of DM, AMM, and UEM are negative at the significant level of 5 % level. With the right panels of Figure 17 and this figure, the β_{trend} of DM, AMM are suggested to be significantly negative for most rolling windows, while the one of AMM is suggested to be significantly negative for some rolling windows.

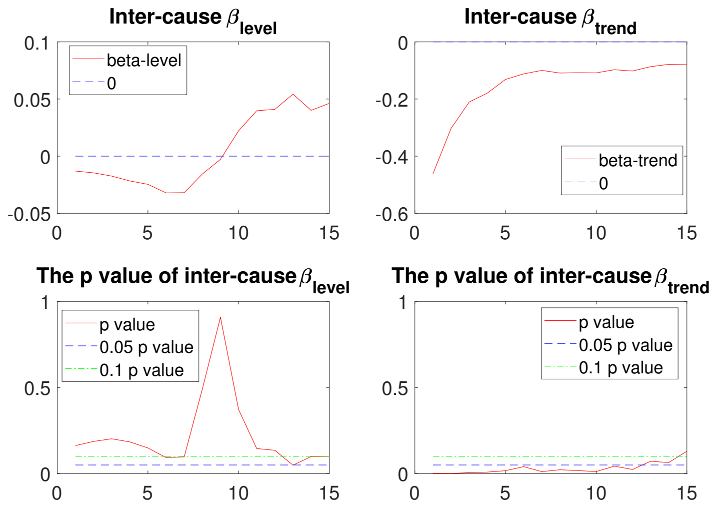

Figure 19.: Cancer-vascular coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for all rolling-window choice, meaning no sign of convergence or divergence in mortality levels of cancer (CM) and vascular mortality (VM). The right panels suggest a significantly negative β_{trend} for most of the rolling-window choice, presenting a strong evidence of cancer-vascular coherence.

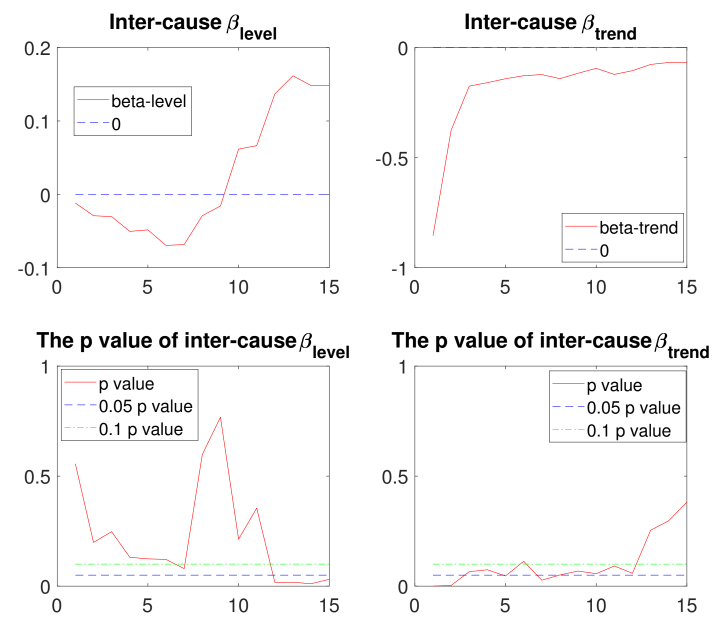

Figure 20.: No cancer-other diseases coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for all rolling-window choice, meaning no sign of convergence or divergence in mortality levels of cancer (CM) and other diseases (DM). The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting no evidence of CM-DM coherence.

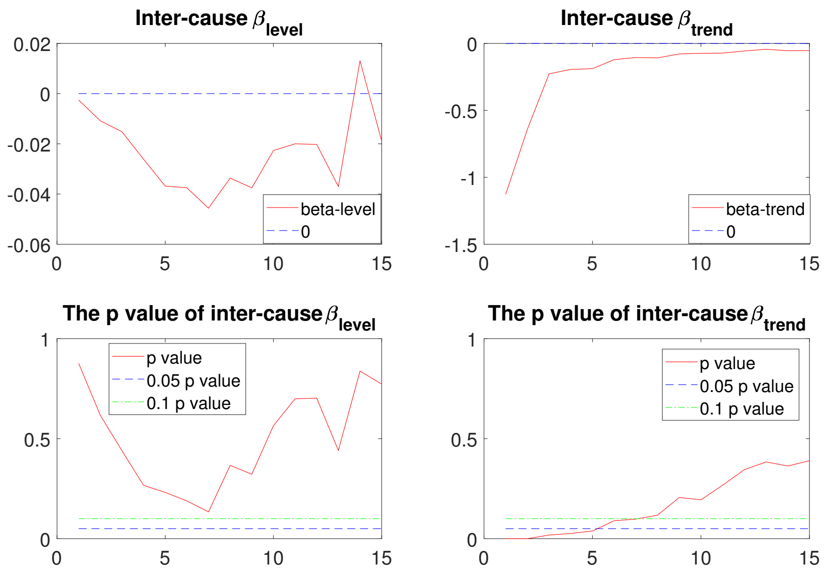

Figure 21.: No cancer-accident&Murders coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for all rolling-window choice, meaning no sign of convergence or divergence in mortality levels of cancer (CM) and accidents & murders (AMM). The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting no evidence of CM-AMM coherence.

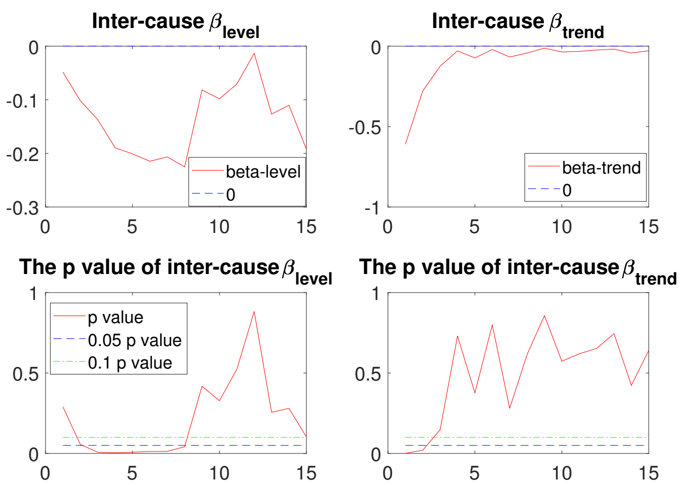

Figure 22.: No cancer-unexplained coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for most rolling-window choice, meaning weak sign of convergence or divergence in mortality levels of cancer (CM) and the unexplained (UEM). The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting no evidence of CM-UEM coherence.

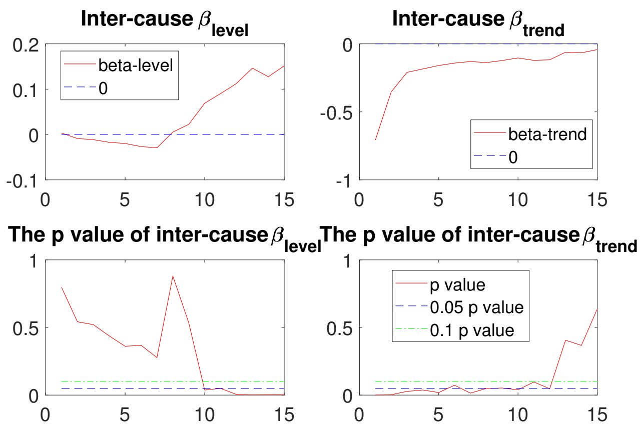

Figure 23.: No vascular-other disease coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for the rolling-window choice with a negative β_{level}, meaning no sign of convergence or divergence in mortality levels of vascular (VM) and the other disease (DM). But for the rolling window choices with positive β_{level}, it shows a sign of significance in β_{level}, meaning potential divergence. The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices. To sum up, it presents the evidence of divergence for some rolling-window choices but no evidence of VM-DM coherence.

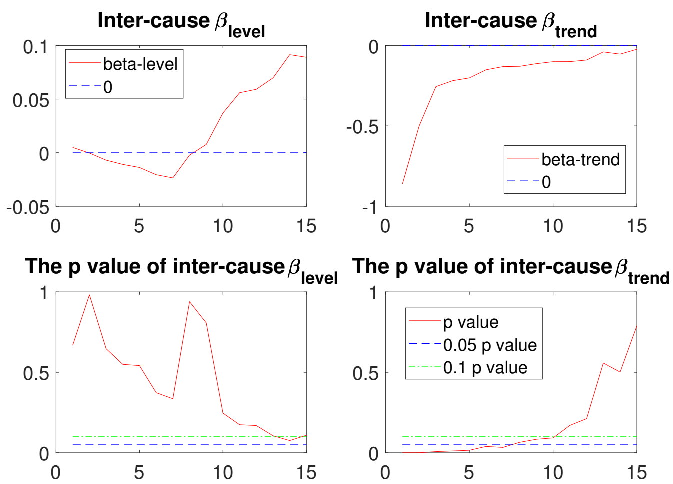

Figure 24.: No vascular-accident&murder coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for most rolling-window choice, meaning weak sign of convergence or divergence in mortality levels of vascular (VM) and accident & murder (AMM). The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting no evidence of VM-AMM coherence.

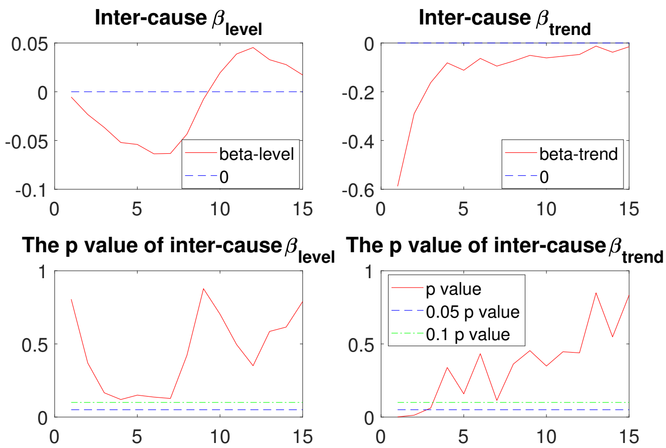

Figure 25.: No vascular-unexplained coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for most rolling-window choice, meaning weak sign of convergence or divergence in mortality levels of vascular (VM) and the unexplained (UEM). The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting no evidence of VM-UEM coherence.

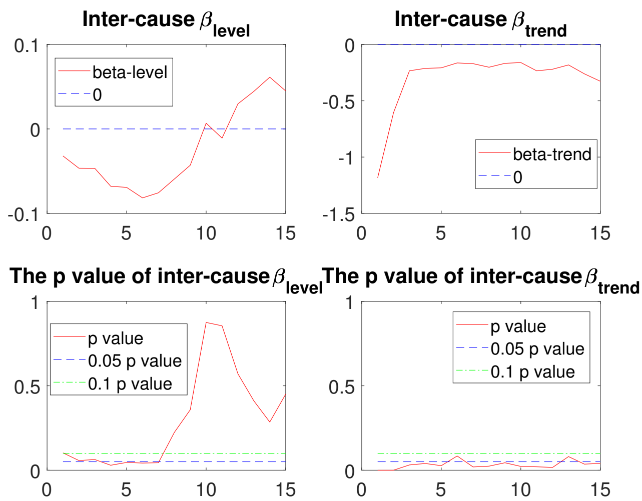

Figure 26.: Other disease-accident&murder coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for most rolling-window choice, meaning weak sign of convergence or divergence in mortality levels of other diseases (DM) and accidents & murder (AMM). The right panels suggest an significant negative β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting the evidence of DM-AMM coherence.

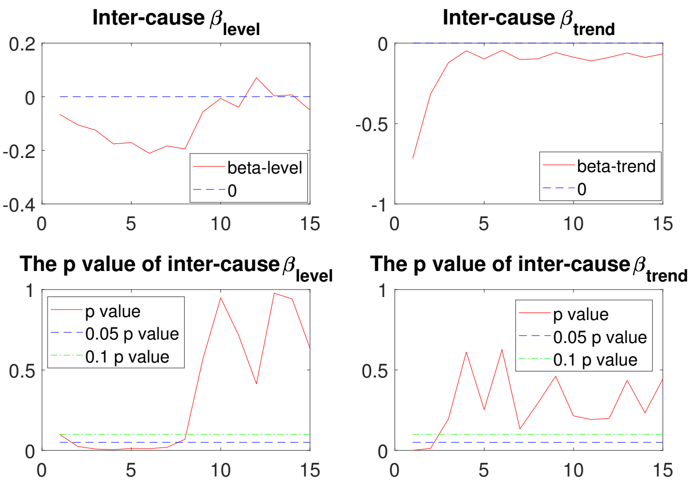

Figure 27.: No other disease-unexplained coherence

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for most rolling-window choice, meaning weak sign of convergence or divergence in mortality levels of other diseases (DM) and the unexplained (UEM). The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting no evidence of DM-UEM coherence.

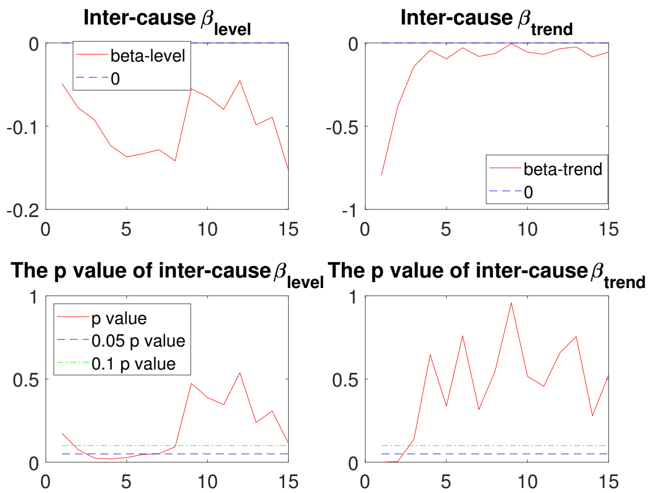

Figure 28.: Accident&Murder-Unexplained

Notes: The left panels of this figure indicate that $H_0 : \beta_{level} = 0$ cannot be rejected at a significant level of 5% for most rolling-window choice, meaning weak sign of convergence or divergence in mortality levels of accidents & murders (AMM) and the unexplained (UEM). The right panels suggest an insignificantly β_{trend} at 5 % level (dotted line) for most of the rolling-window choices, presenting no evidence of AMM-UEM coherence.

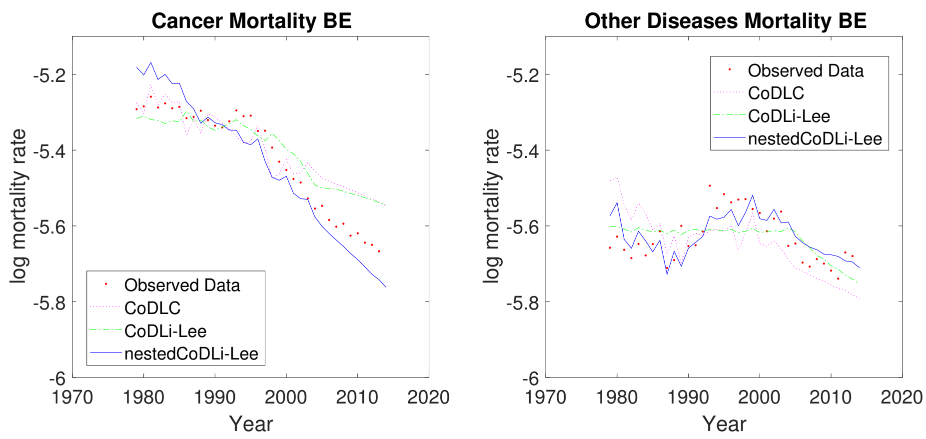

Figure 29.: Observed, fitted and forcasted cause-specific mortality, males in Belgium

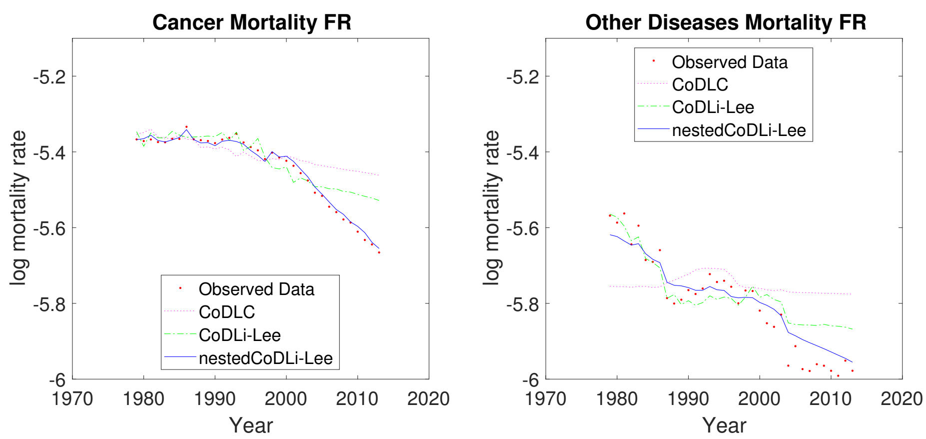

Figure 30.: Observed, fitted, and forcasted cause-specific mortality, males in France

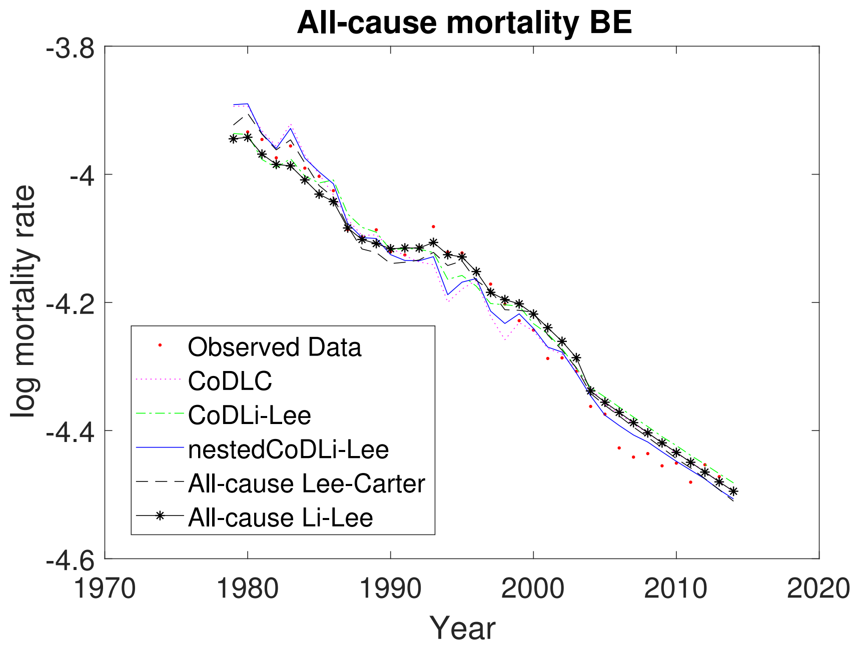

Figure 31.: Observed, fitted, and forcasted all-cause mortality, males in Belgium

Notes: For the case of Belgium, all models perform similarly. But the nestedCoDLi-Lee model produces the all-cause mortality projections that are closest to the observed data.

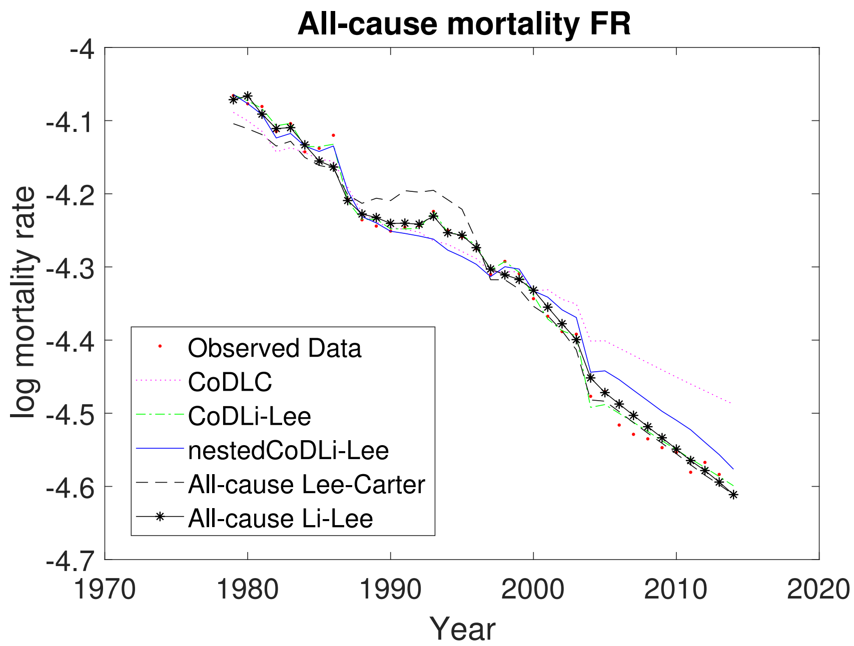

Figure 32.: Observed, fitted, and forcasted all-cause mortality, males in France

Notes: For the case of France, the models that do not account for the inter-cause coherence produce the closest projections to the observed data. This might result from the fact that the vascular mortality of France (blue dash line) is improving at the slowest pace across three countries as Figure 3. Inter-cause coherence between cancer and vascular mortality drag the mortality improvement of France slower, which is in line with the instinct feature of Li-Lee model (e.g., Japan life expectancy are expected to improved slower than the single population case as Li and Lee (2005)). Also, this might result from the mortality histories that chosen to estimate the nestedCoDLi-Lee model. Changing the estimation period might help to improve the out-of-sample performance of nestedCoDLi-Lee, as shown in the Figure 13.

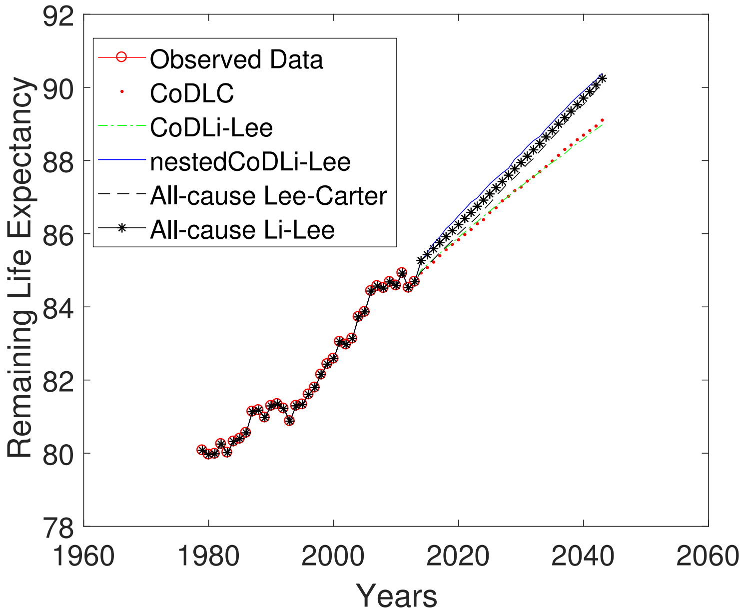

Figure 33.: Observed and forecasted 67-year-old male remaining life expectancy, males in Belgium
Notes: For the case of Belgium, the remaining life expectancy of a 67-years-old male from nestedCoDLi-Lee model is similar to the one from the all-cause mortality model.

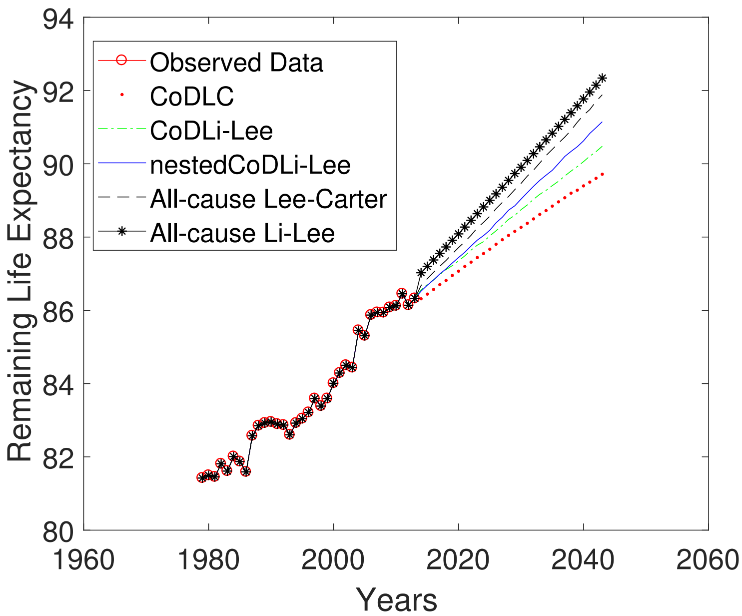

Figure 34.: Observed and forecasted 67-year-old male remaining life expectancy, males in France
Notes: For the case of Frace, the remaining life expectancy of a 67-years-old male from nestedCoDLi-Lee model is approximately one year less than the one from the all-cause Li-Lee model. This is because of the international coherence that the mortality level of France could not be forever lower than the one of the Belgium and the one of the Netherlands.