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Robustness for asset-liability management of pension funds

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SURVEY 47

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SURVEY PAPER 47

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Colophon
October 2016

Design
B-more Design

Lay-out
Bladvulling, Tilburg

Printing
Prisma Print, Tilburg University

Editors
Frans Kooymans
Netspar

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ROBUSTNESS FOR ASSET-LIABILITY MANAGEMENT OF PENSION FUNDS

Abstract

We survey the literature on robust dynamic asset allocation with an emphasis on the asset-liability management of pension funds. After demonstrating the difference between risk and uncertainty (Section 1), we introduce two levels of uncertainty: parameter uncertainty and model uncertainty (Section 2.1). We describe four of the most widely used approaches in robust dynamic asset allocation problems: the penalty approach, the constraint approach, the Bayesian approach and the approach of smooth recursive preferences (Sections 2.4-2.7). In Section 3 we demonstrate the importance of uncertainty for investors (including pension funds) from both a normative and a positive aspect, then we review the literature on robust asset management and on robust asset-liability management. Section 4 concludes.

Policy recommendations

This paper discusses the effects of uncertainty on optimal investment decisions and on optimal asset-liability management by institutional investors, especially pension funds, by surveying the most recent literature. The implications of robustness for investors are as follows:

- Expected returns are notoriously hard to estimate precisely, thus uncertainty about their value is a primary concern of investors. Uncertainty about expected returns in general induces more conservative investment decisions. Investors who are averse to uncertainty should decrease their myopic (speculative) demand and increase their intertemporal hedging demand for risky securities.

- In simple models uncertainty aversion translates into additional risk aversion. This also suggests that uncertainty-averse investors should behave in a way similar to investors with higher risk aversion.

- By making robust investment decisions, investors can significantly outperform non-robust portfolios and achieve a higher out-of-sample Sharpe ratio and higher out-of-sample expected utility.

- Robustness is especially important for pension funds with a low funding ratio. While robust optimal decisions of financially healthy pension funds are relatively similar to the non-robust optimal decisions of similar pension funds, robustness has a huge effect on the optimal decisions on both the asset and the liability side of underfunded pension funds.

1. Introduction

Pension funds (and investors in general) face both risk and uncertainty during their everyday operation. The importance of distinguishing between risk and uncertainty was first emphasized in the seminal work of Knight (1921), and it has been an active research topic in the finance literature ever since. Risk means that the investor does not know what future returns will be, but she does know the probability distribution of the returns. On the other hand, uncertainty means that the investor does not know precisely the probability distribution that the returns follow. As a simple example, let us assume that the one-year return of a particular stock follows a normal distribution with 9% expected value and 20% standard deviation. A pension fund who knows that the return of the stock follows this particular distribution, faces risk, but it does not face uncertainty. Another pension fund only knows that the return of this stock follows a normal distribution, that its expected value lies between 8% and 12%, and that its standard deviation is 20%. This pension fund faces not only risk, but also uncertainty: not only does it not know the exact return in one year, it also does not know the precise probability distribution that the return follows.

A risk-averse investor is averse of the risk with known distribution, while an uncertainty-averse investor is averse of uncertainty[1]. Decisions which take into account the fact that the

[1] In the behavioral finance/economics literature, *ambiguity* and *uncertainty* have different meanings. Ambiguity refers to missing information that could be known, while uncertainty means that the information does not exist. For a detailed treatment of the difference between ambiguity and uncertainty, we refer to Dequech (2000). In the robust asset allocation literature the two terms are used interchangeably, and we also follow this practice in this paper.

investor faces uncertainty, are called *robust* decisions. These decisions are robust to uncertainty because they protect the investor against uncertain outcomes (“bad events”).

2. Classification of robustness

2.1. Parameter uncertainty and model uncertainty

We can distinguish two levels of uncertainty: parameter uncertainty and model uncertainty. If the investor knows the form of the underlying model but she is uncertain about the exact value of one or several parameters, the investor then faces parameter uncertainty. On the other hand, if the investor does not even know the form of the underlying model, she faces model uncertainty.

For example, if the investor knows that the return of a particular stock follows the geometric Brownian motion

$$\frac{\mathrm{d}S_t}{S_t} = \mu_S \mathrm{d}t + \sigma_S \mathrm{d}W_t^{\mathbb{P}} \tag{1}$$

with constant drift μ_S and constant volatility σ_S, but she does not know the exact value of these two parameters, she faces parameter uncertainty. But if she does not even know whether the stock return follows a geometric Brownian motion or any other type of stochastic process, she faces model uncertainty.

The distinction between parameter uncertainty and model uncertainty is in many cases not clear-cut. The most important example of this from the point-of-view of pension funds is the uncertainty about the drift parameters. If we take the simple example of the stock return in (1), then being uncertain about the drift can be translated into being uncertain about the probability measure $\mathbb{P}$, assuming that the investor considers only equivalent probability measures.[2] Uncertainty about the probability measure is considered model uncertainty according to the vast majority of

[2]Two probability measures are said to be equivalent if and only if each is absolutely continuous with respect to the other. That is, the investor may be uncertain about the exact probability of events, but she is certain about which events

the literature. This is the reason why, e.g., Maenhout (2004) and Munk and Rubtsov (2014) discuss model uncertainty, even though in their model the investor is uncertain only about the drift parameters[3].

The assumption that the investor is uncertain only about the drift, but not about the volatility, is not unrealistic. If constant volatility is assumed, then the volatility parameter can be estimated to any arbitrary level of precision, as long as the investor can increase the observation frequency as much as she wants. Given that in today's world return data are available for every second (or even more frequently), the assumption that the investor is able to observe return data in continuous time is indeed justifiable. For the reasons why expected returns (i.e., the drift parameters) are notoriously hard to estimate, we refer to Merton (1980), Blanchard, Shiller, and Siegel (1993) and Cochrane (1998). Since pension funds' uncertainty mostly concerns uncertainty about the drift, and since it is common practice in the literature to assume that investors only consider equivalent probability measures, whenever we talk about uncertainty in the rest of this paper, we mean uncertainty about the drift term, unless we indicate otherwise.

happen for almost sure (i.e., with probability one) or with probability zero. The assumption that the investor considers only equivalent probability measures is quite common in the robustness literature.

[3]If two probability measures are equivalent, then the standard Wiener processes under the two measures differ only in their drift terms (if expressed under the same probability measure), and their volatility term is the same. For a detailed treatment of this topic we refer to Karatzas and Shreve (1991).

2.2. A generic investment problem

Before discussing robustness in details, we formulate a generic non-robust dynamic asset allocation problem. Later in the paper we extend this model to formulate a robust framework.

Let us assume that the investor derives utility from consumption and terminal wealth. Her goal is to maximize her total expected utility. She has an initial wealth x, and her investment horizon is T. At the end of every period (i.e., at the end of the 0th year, at the end of the 1st year, ..., at the end of the (T-1)th year) she has to make a decision: how much of her wealth to consume and how to allocate her remaining wealth among the assets available on the financial market. For the sake of simplicity, we assume that the financial market consists of a risk-free asset, which pays a constant return r_f, and a stock, which follows the geometric Brownian motion in (1). Then we can formulate the investor's optimization problem as follows.

Problem 1 *Given initial wealth x, find an optimal pair* $\{C_t, \pi_t\}\ \forall t \in [0, ..., T-1]$ *for the utility maximization problem*

$$V_0(x) = \sup_{\{C_t,\pi_t\}\forall t\in[0,...,T-1]} \mathrm{E}^{\mathbb{P}}\left[\sum_{t=1}^{T} U_C(C_t) + U_T(X_T)\right] \quad (2)$$

subject to the budget constraint

$$\frac{\mathrm{d}X_t}{X_t} = \left[r_f + \pi_t(\mu_S - r_f) - \frac{C_t}{X_t}\right]\Delta t + \pi_t\sigma_S\Delta W_t^{\mathbb{P}}. \quad (3)$$

In Problem 1 X_t denotes the investor's wealth at time t, C_t is her consumption at time t, π_t is the ratio of her wealth invested in the stock, and $U_C(\cdot)$ and $U_T(\cdot)$ are her utility functions.

If the investor can consume and reallocate her wealth continuously, the continuous counterpart of Problem 1 can be formulated.

Problem 2 *Given initial wealth x, find an optimal pair* $\{C_t, \pi_t\}$, $t \in [0, T]$ *for the utility maximization problem*

$$V_0(x) = \sup_{\{C_t, \pi_t\} t \in [0, T]} \mathrm{E}^{\mathbb{P}} \left[\int_{t=0}^{T} U_C(C_t)\, \mathrm{d}t + U_T(X_T) \right] \quad (4)$$

subject to the budget constraint

$$\frac{\mathrm{d}X_t}{X_t} = \left[r_f + \pi_t \left(\mu_S^{\mathbb{B}} - r_f \right) - \frac{C_t}{X_t} \right] \mathrm{d}t + \pi_t \sigma_S \mathrm{d}W_t. \quad (5)$$

There are two main methods that can be used to solve optimization problems like Problem (1) and Problem (2): relying on the principle of dynamic programming (which makes use of the Bellman difference equation in discrete time optimization problems and of the Hamilton-Jacobi-Bellman (HJB) differential equation in continuous time optimization problems) and the martingale method of Cox and Huang (1989).[4]

We now briefly explain the intuition behind the principle of dynamic programming. When the investor is making a decision about how much of her wealth to consume and how to allocate the rest, she is working backwards. In the discrete setup of Problem 1 this means that first she solves the optimization problem as if she were at time $T-1$, assuming her wealth before making the decision is X_{T-1}. This way she solves a one-period

[4]For a detailed treatment of the principle of dynamic programming we refer to Bertsekas (2005) and Bertsekas (2012), while the martingale method is treated in detail in Karatzas and Shreve (1998).

optimization problem by maximizing the sum of her immediate utility from consumption and her expected[5] utility from terminal wealth with respect to C_{T-1} and π_{T-1}. This maximized sum is the investor's value function at time $T-1$, and we denote it by V_{T-1}. According to the principle of dynamic programming, the C_{T-1} and π_{T-1} values which the investor has just obtained, are also optimal solutions to the original optimization problem (Problem 1). Then she moves to time $T-2$. She wants to maximize the sum of her utility from immediate consumption C_{T-2}, her expected utility from C_{T-1}, and her expected utility from X_T[6], with respect to C_{T-2}, π_{T-2}, C_{T-1} and π_{T-1}. But according to the principle of dynamic programming, she has already found the optimal values of C_{T-1} and π_{T-1} before. Thus her optimization problem at time $T-2$ eventually boils down to maximizing the sum of her utility from immediate consumption C_{T-2} and the expected value of her value function V_{T-1}, the expectation being conditional on the information available up to time $T-2$. Then she moves to time $T-3$, and continues solving the optimization problem in the same way, until she obtains the optimal C_t and π_t values for all t between 0 and $T-1$. The intuition of solving Problem 2 is the same, but mathematically it means that the investor first obtains the optimal $\{C_t\}$ and $\{\pi_t\}$ processes[7] in terms of the value function, then she solves a partial differential equation (the HJB equation) with terminal condition $V_T = U_T(X_T)$, to obtain the value function. Knowing the value

[5]Conditionally on the information available up to time $T-1$.

[6]Both of these expectations are conditional on the information available up to time $T-2$.

[7]An indexed random variable (the index being t) in brackets denotes a stochastic process, e.g., $\{C_t\}$ is the consumption process.

function, she can substitute it back into the previously obtained optimal $\{C_t\}$ and $\{\pi_t\}$ processes.

Cox and Huang (1989) approached Problem 1 and Problem 2 from a different angle and were the first to use the martingale method to solve dynamic asset allocation problems. The basic idea of the martingale method is that first the investor obtains the optimal terminal wealth as a random variable and the optimal consumption process as a stochastic process. Then, making use of the martingale representation theorem (see, e.g., Karatzas and Shreve (1991), pp. 182, Theorem 3.4.15), she obtains the unique $\{\pi_t\}$ process that enables her to achieve the previously derived optimal terminal wealth and optimal consumption process. In many optimization problems the martingale method has not only mathematical advantages (one does not have to solve higher-order partial differential equations), but it also provides economic intuition and insights into the decision-making of the investor. For such an example, we refer to the optimization problem in Horvath, de Jong, and Werker (2016).

2.3. Robust dynamic asset allocation models

In Problem 1 and Problem 2 we assumed that the investor knows the underlying model properly. In reality, however, investors face uncertainty: they do not know the precise distribution of returns. By incorporating this uncertainty into their optimization problem, they make robust consumption and investment decisions. In the remainder of the paper we assume, for the sake of simplicity, that the investor makes decisions in continuous time, but the intuition can always be carried over to the discrete counterpart of the model. In this subsection we also assume that the financial market consists of a risk-free asset with constant rate of return and a stock, the return of which follows a geometric Brownian motion with constant drift and volatility parameters. The investor knows the volatility parameter of the stock return process, but she is uncertain about the drift parameter. The approaches to robust asset allocation that we introduce in this subsection can straightforwardly be extended to more complex financial markets, e.g., one accommodating several stocks, long-term bonds, a stochastic risk-free rate, etc.

There are several ways to introduce robustness into Problem 2. In this subsection we describe four of the most common approaches in the literature: the penalty approach, the constraint approach, the Bayesian approach and the approach of smooth recursive preferences. The basic idea of all of these approaches is the same: the investor is uncertain about μ_S, thus she considers several μ_S-values that she thinks might be the true one. The differences between these four approaches are twofold: how the investor chooses which μ_S-values she considers possible, and how she incorporates these several possible μ_S-values into her optimization problem (e.g., she selects the worst case scenario, or she takes a weighted average of them, etc.).

2.4. The penalty approach

The penalty approach was introduced into the literature in Anderson, Hansen, and Sargent (2003). The investor has a μ_S-value in mind which she considers to be the most likely. We call this the *base parameter*, and denote it by $\mu_S^{\mathbb{B}}$. She is uncertain about the true μ_S, so she considers other μ_S-values as well. These are called *alternative parameters*, and denoted by $\mu_S^{\mathbb{U}}$. The relationship between $\mu_S^{\mathbb{B}}$ and $\mu_S^{\mathbb{U}}$ is expressed by

$$\mu_S^{\mathbb{B}} = \mu_S^{\mathbb{U}} + u_S \sigma_S \qquad \forall t \in [0, T] \,. \tag{6}$$

u_S is multiplied by the volatility parameter for scaling purposes[8]. Following the penalty approach, the investor adds a penalty term to her goal function, concretely

$$\int_0^T \Upsilon_t \frac{u_S^2}{2} \mathrm{d}t. \tag{7}$$

The parameter Υ_t expresses how uncertainty-averse the investor is, and one might assume that it is a constant, a deterministic function of time, or even a stochastic function of time. $\frac{u_S^2}{2}$ expresses the distance between the base parameter and the alternative parameter[9].

The investor considers all possible $\mu_S^{\mathbb{U}}$ parameters and she chooses the one which results in the lowest possible value

[8]The reason behind u_S being multiplied by σ_S lies in the fact that the investor being uncertain about the drift parameter is equivalent to her being uncertain about the physical probability measure, as long as she only considers probability measures that are equivalent to the base measure that she considers to be the most likely. Changing from her base measure to an alternative measure thus means that the base drift $\mu_S^{\mathbb{B}}$ changes to $\mu_S^{\mathbb{U}} + u_S\sigma_S$, and the stochastic process that under the base measure was a standard Wiener process changes to another stochastic process, namely one which is a standard Wiener process under the alternative measure.

[9]Mathematically, $\frac{u_S^2}{2}$ is the time-derivative of the Kullback-Leibler divergence, also known as the relative entropy. The reason why the Kullback-Leibler divergence is often used in the literature of robustness as the penalty function lies not only in its mathematical tractability, but also in its intuitive interpretation. Actually, the relative entropy of measure $\mathbb{U}$ with respect to measure $\mathbb{B}$ is the amount of information lost if one uses measure $\mathbb{B}$ to approximate measure $\mathbb{U}$. If in the definition of relative entropy the logarithm is of base 2, the amount of information is measured in bits (i.e., how many yes-no questions have to be answered in order to tell $\mathbb{U}$ and $\mathbb{B}$ apart). If the logarithm is of base e, the amount of information is measured in nats. For a detailed treatment of the Kullback-Leibler divergence we refer to Cover and Thomas (2006), Chapter 2).

function. Putting it differently, she considers the worst case scenario. We now formalize the robust counterpart of Problem 2, using the penalty approach.

Problem 3 *Given initial wealth x, find an optimal triplet* $\{u_S, C_t, \pi_t\}$, $t \in [0, T]$ *for the robust utility maximization problem*

$$V_0(x) = \inf_{u_S} \sup_{\{C_t, \pi_t\}} \mathrm{E}\left\{\int_{t=0}^{T}\left[U_C(C_t) + \Upsilon_t \frac{u_S^2}{2}\right] \mathrm{d}t + U_T(X_T)\right\} \tag{8}$$

subject to the budget constraint

$$\frac{\mathrm{d}X_t}{X_t} = \left[r_f + \pi_t\left(\mu_S^{\mathbb{B}} + u_S\sigma_S - r_f\right) - \frac{C_t}{X_t}\right]\mathrm{d}t + \pi_t\sigma_S \mathrm{d}W_t. \tag{9}$$

A robust investor with Problem 3 then solves her optimization problem either by making use of the principle of dynamic programming or by the martingale method, the same way as we described in Section 2.2 for the case of a non-robust investor. The only difference is that instead of only maximizing with respect to $\{C_t, \pi_t\}$ she first maximizes with respect to these two variables, then she minimizes with respect to u_S[10].

[10]In most robust dynamic investment problems the supinf and infsup preferences lead to the same solution, because the order of maximization and minimization can be interchanged due to Sion's maximin theorem (Sion (1958)). So it does not matter whether the investor first maximizes with respect to $\{C_t, \pi_t\}$ and then minimizes with respect to u_S, or if she interchanges the order of maximization and minimization.

The penalty approach is widely used in the literature to study the effects of robustness on dynamic asset allocation and asset prices. Maenhout (2004) finds that if one accounts for uncertainty aversion based on the penalty approach, it is actually possible to explain a substantial part of the “too high” equity risk premium that is termed the “equity premium puzzle” in the literature. Concretely, a robust Duffie-Epstein-Zin representative investor with reasonable risk-aversion and uncertainty-aversion parameters generates a 4% to 6% equity premium. To achieve this result, it is essential that Maenhout (2004) parameterized the uncertainty-aversion parameter Υ_t to be a function of the “value”[11] at the respective time[12]. As he points out, if the uncertainty-aversion parameter is constant (which is actually the case in Anderson, Hansen, and Sargent (2003)), it is not possible to give a closed form solution to the robust version of Merton’s problem. Furthermore, Maenhout (2006) and Horvath, de Jong, and Werker (2016) find that if the investment opportunity set is stochastic, robustness increases the importance of intertemporal hedging compared to the non-robust case.

Trojani and Vanini (2002) examine the asset pricing implications of robustness, comparing their results with those of Merton (1969). The penalty approach is extended by Cagetti, Hansen, Sargent, and Williams (2002) to allow the state variables to follow not only pure diffusion processes but mixed jump processes as well. Uppal and Wang (2003) use the penalty approach and

[11]By “value” we mean the concept known in dynamic optimization theory: the value function at time t shows the highest possible expected value of utility at time t that the investor can achieve by properly allocating her resources among the available assets between time t and the end of her investment horizon.

[12]This approach is criticized by, e.g., Pathak (2002) for its recursive nature.

explore a potential source of underdiversification. They find that if the investor is allowed to have different levels of ambiguity regarding the marginal distribution of any subsets of the return of the investment assets, there are circumstances when the optimal portfolio is significantly underdiversified compared to the usual mean-variance optimal portfolio. Liu, Pan, and Wang (2005) study the asset pricing implications of ambiguity about rare events using the penalty approach. Routledge and Zin (2009) study the connection between uncertainty and liquidity in the penalty framework.

2.5. The constraint approach

The penalty approach determined the set of alternative $\mu_S^{\mathbb{U}}$ parameters by adding a penalty term to the goal function and choosing the least favorable drift parameter. Another way to determine the set of alternative $\mu_S^{\mathbb{U}}$ parameters is to explicitly specify a constraint on u_S. Since the investor considers both positive and negative u_S values[13], it is a straightforward choice to set a higher constraint on u_S^2, concretely

$$\frac{u_S^2}{2} \leq \eta. \tag{10}$$

Using a reasonable value for η, the model assures that the investor considers scenarios which are pessimistic and reasonable at the same time. So, for example, if $\mu_S B = 10\%$, then the investor will not consider $\mu_S^{\mathbb{U}} = -200\%$ as the drift parameter of the stock return, but she might consider $\mu_S^{\mathbb{U}} = 7\%$.

Now we formalize the robust optimization problem, using the constraint approach.

[13]She is allowed to take short positions, so for her only the magnitude of the expected excess return matters, but not the sign.

Problem 4 *Given initial wealth x, find an optimal triplet* $\{u_S, C_t, \pi_t\}$, $t \in [0, T]$ *for the robust utility maximization problem*

$$V_0(x) = \inf_{u_S} \sup_{\{C_t, \pi_t\}} \mathrm{E}\left\{ \int_{t=0}^{T} U_C(C_t)\,\mathrm{d}t + U_T(X_T) \right\} \tag{11}$$

subject to

$$\frac{u_S^2}{2} \leq \eta. \tag{12}$$

and subject to the budget constraint (9).

The form of (12) is very similar to the penalty term in (7). This is not a coincidence: if and only if Υ_t is a nonnegative constant, then there exists such an η, that the solution to Problem 3 and the solution to Problem 4 are the same. For the proof of this statement and a detailed comparison of the penalty approach and the constraint approach we refer to Appendix B in Lei (2001).

The constraint approach is used by, among others, Gagliardini, Porchia, and Trojani (2009) to study the implications of ambiguity-aversion to the yield curve and to characterize the market equilibrium if ambiguity-aversion is also accounted for; and by Leippold, Trojani, and Vanini (2008) to study equilibrium asset prices under ambiguity. Garlappi, Uppal, and Wang (2007) use a closely related approach to build their model for portfolio allocation, but contrary to the majority of literature in this field they examine a one-period (static) setup instead of a dynamic one. Peijnenburg (2014) extends the framework of the constraint-approach and considers, besides the maximin setup, the case of recursive smooth preferences. Moreover, she introduces the concept of learning into the model: the more time elapses, the less uncertain the investor is about the risk premium. We also refer to Cochrane and Saa-Requejo (1996), who use a model similar to the constraint approach to derive bounds on asset pricing in incomplete markets by ruling out "good deals".

2.6. The Bayesian approach

In both the penalty approach and the constraint approach the investor had a set of possible $\mu_S^{\mathbb{U}}$ parameters in mind, and she chose the one which minimized her value function. These two approaches did not make it possible to directly incorporate the investor's view on how likely the different $\mu_S^{\mathbb{U}}$ parameters are. The Bayesian approach builds around this exact idea: the investor has a set of possible $\mu_S^{\mathbb{U}}$ parameters in mind, and she renders likelihoods to all of these values that she considers possible. Putting it differently, she can construct a probability distribution on all $\mu_S^{\mathbb{U}}$ parameters[14]. This probability distribution on all $\mu_S^{\mathbb{U}}$ parameters reflects the view of the investor on how likely the various $\mu_S^{\mathbb{U}}$ values are to be the true parameter value. Now we formulate the optimization problem of a robust investor who uses the Bayesian approach.

Problem 5 *Given initial wealth x, find an optimal pair* $\{C_t, \pi_t\}$, $t \in [0, T]$ *for the robust utility maximization problem*

$$V_0(x) = \sup_{\{C_t, \pi_t\}} \mathrm{E}\left\{\int_{t=0}^{T} U_C(C_t)\,\mathrm{d}t + U_T(X_T)\right\} \tag{13}$$

[14] The wording is important here: she renders a likelihood value to all $\mu_S^{\mathbb{U}}$, regardless of whether she considers it possible or not. If she considers that a particular $\mu_S^{\mathbb{U}}$ cannot be the true parameter value, she renders a likelihood of zero to it.

subject to the budget constraint (9), and assuming that u_s follows a particular probability distribution reflecting the investor's view on how likely different $\mu_S^{\mathbb{U}}$ parameters are.

In the recent literature on robustness, Hoevenaars, Molenaar, Schotman, and Steenkamp (2014) use the Bayesian approach to study the effects of parameter uncertainty on different asset classes, namely stocks, long-term bonds and short-term bonds (bills). They find that uncertainty raises the long-run volatilities of all three asset classes proportionally with the same vector, compared to the volatilities that are obtained using Maximum Likelihood. The consequence of this is that in the optimal asset allocation the horizon effect is much smaller compared to the case of using Maximum Likelihood. Pástor (2000) analyzes the effects of model uncertainty on asset allocation using the Bayesian approach. When calibrating his model to U.S. data, he finds that investors' belief in the domestic CAPM has to be very strong to reconcile the implications of his model with market data.

2.7. Smooth recursive preferences

The approach of smooth recursive preferences makes it possible to separate uncertainty (which reflects the investor's beliefs) and uncertainty-aversion (which reflects the investor's taste). The starting point of the smooth recursive preferences approach is the Bayesian framework in Problem 5. The investor does not know the exact value of the expected excess stock return, but she can construct a probability distribution on it. This probability distribution reflects her beliefs: it shows how likely she considers particular $\mu_S^{\mathbb{U}}$ values.

To incorporate her attitude towards uncertainty, she constructs a "distorted" probability distribution from the original distribution that reflects her beliefs on $\mu_S^{\mathbb{U}}$. Intuitively this means that she gives higher weight to "unfavorable events", i.e., to $\mu_S^{\mathbb{U}}$ values that result in low expected utility, and lower weight to "favorable events". Technically, distorting the probability distribution is achieved by applying a concave function (and later its inverse) on the original probabilities to change their relative importance to the investor. If the uncertainty-aversion of an investor with smooth recursive preferences is infinity, and she has a bounded set of priors for $\mu_S^{\mathbb{U}}$, her optimal solution will be the same as an investor who uses the infsup (minimax) setup in either the constraint approach or the penalty approach with constant Υ_t.

The approach of smooth recursive preferences was developed in Klibanoff, Marinacci, and Mukerji (2005), and it was axiomatized in Klibanoff, Marinacci, and Mukerji (2009). Hayashi and Wada (2010) analyzed the asset pricing implications of this approach. Chen, Ju, and Miao (2014) and Ju and Miao (2012) calibrate the uncertainty-aversion parameter within a framework of smooth recursive preferences.

3. The role of robustness in ALM of pension funds

3.1. Robust Asset Management

Several papers document the relevance of robustness in asset management. Garlappi, Uppal, and Wang (2007) use international equity indices to demonstrate the importance of robustness in portfolio allocation. They assume that the investor is uncertain about the expected return of assets, and she makes robust decisions following the constraint approach described in Section 2.5. Their analysis suggests that robust portfolios deliver higher out-of-sample Sharpe ratios than their non-robust counterparts. Moreover, the robust portfolios are not only more balanced, but they also fluctuate much less over time - which is a desirable property due to attracting less transaction costs in total[15].

Another paper emphasizing the importance of robustness in asset management is Glasserman and Xu (2013). They use daily commodity futures data to extract spot price changes. The investor is assumed to have a mean-variance utility function. The model parameters are estimated based on futures price data of the past 6 months, and they are re-estimated every week. The investor is uncertain about the expected return, and she makes robust decisions following the penalty approach (Section 2.4). The authors find that the portfolio that is based on robust investment decisions significantly outperforms the non-robust portfolio both in terms of the goal-function value and the Sharpe ratio. The difference in performance between the robust and non-robust portfolio is both statistically and economically significant. Moreover, they conclude that the improvement in performance comes mainly from the

[15]Robust portfolios being more balanced and fluctuating less is a "side effect", i.e., they were not designed to have these properties.

reduction of risk, rather than from the increase of return.

Liu (2011) assumes a stochastic investment opportunity set and uncertainty about the expected return within the penalty framework, and demonstrates the superior out-of-sample performance of the robust portfolio. Hedegaard (2014) shows that the robust portfolio outperforms its non-robust counterpart also in the case when the investor knows the expected return, but she is uncertain about the alpha-decay of the predicting factors. Cartea, Donnelly, and Jaimungal (2014) analyze the optimal portfolio of a market maker who is uncertain about the drift of the midprice dynamics, about the arrival rate of market orders, and about the fill probability of limit orders. Using the penalty approach, they demonstrate that the robust strategy delivers a significantly higher out-of-sample Sharpe ratio.

Koziol, Proelss, and Schweizer (2011) show that robust portfolios achieving a significantly higher out-of-sample Sharpe ratio is not only a normative result, but that institutional investors are indeed highly uncertainty-averse and make robust decisions. They find that the average in-sample Sharpe ratio of their asset side is only 60% of the in-sample Sharpe ratio of the corresponding non-robust (i.e. unambiguous) asset portfolio. According to their argument, this result is not due to poor diversification, because institutional investors have the cognitive ability and financial knowledge to optimally diversify their portfolio; moreover, fund managers' compensation is in most of the cases somehow linked to the performance of the managed portfolio. So, a higher in-sample Sharpe ratio means higher compensation for them. The lower in-sample Sharpe ratio thus, as they argue, is a result of uncertainty-aversion. Besides institutional investors being uncertainty averse, they also find that robustness plays a more important role for alternative asset classes (e.g. real estate, private equity, derivatives, etc.) than for stocks and bonds.

As Garlappi, Uppal, and Wang (2007), Glasserman and Xu (2013) and Liu (2011) point out, making robust investment decisions on the asset side ensures that the investment decisions will provide a better out-of-sample Sharpe-ratio on average not only if the pension fund manager knows the exact distribution of asset returns, but also if the model of asset returns that the pension fund manager had in mind turns out to be misspecified. That is, using robust investment decisions helps decrease the investment risk of pension funds.

The main reasons why it is wise for pension funds to make robust investment decisions is the difficulty of obtaining reliable estimates for the risk premiums (the best-known example of which in practice is the equity risk premium), for long term interest rates and for correlations (especially for large portfolios). Maenhout (2004) solves the robust version of the dynamic asset allocation problem of Merton (1971) using the penalty approach: the investor maximizes her expected utility from consumption plus a penalty term, her utility function is of CRRA type, and the financial market consists of a money market account (MMA) with constant risk-free rate and a stock market index. The investment horizon is finite. The investor is uncertain about the expected excess return of the stock market index. The penalty term is quadratic in the difference between the drift term according to the base model and the drift term according to the alternative model. Instead of multiplying the penalty term by a constant (as Anderson, Hansen, and Sargent (2003) did), Maenhout (2004) assumes that the uncertainty-aversion parameter is stochastic, concretely it is linear in the inverse of the value function itself[16]. This particular form of the penalty term makes it possible to obtain a closed form solution for the optimal consumption and investment policy. Moreover, the optimal investment policy is homothetic, i.e., the optimal ratio of wealth to be invested in the stock market index is independent of the wealth itself. Denoting the relative risk-aversion parameter by γ and the uncertainty-aversion parameter by θ, the optimal investment ratio is the same as in the problem of Merton (1971), the only difference being that instead of γ there is $\gamma + \theta$ in the denominator, i.e., $\frac{1}{\gamma+\theta}\frac{\mu_S - r_f}{\sigma_S^2}$. Thus what Maenhout (2004) finds, effectively, is that a robust investor has a lower portion of her wealth invested in the risky asset than a non-robust investor. Or, as sometimes stated in the literature: a robust investor is more conservative in her investment decision.

In another paper Maenhout (2006) analyzes a similar problem, but he assumes that the investment opportunity set is stochastic. To be more precise, the expected excess return of the stock market index follows a mean-reverting Ornstein-Uhlenbeck process. Robustness again decreases the optimal ratio of wealth to be invested in the stock market index, but it increases the intertemporal hedging demand. Thus robustness leads to more conservative decision in two aspects: on one hand the investor invests less in the risky asset by decreasing the myopic (speculative) demand, on the other hand she invests more in the risky asset by increasing the intertemporal hedging demand. The total effect of robustness is thus not straightforward. It can happen, for example, that a robust and a non-robust investor have the same optimal investment ratio, but their motives are different:

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[16]This parameterization has been criticized by some researchers due to its recursive nature (e.g. Pathak (2002)).

a non-robust investor lays more emphasis on the speculative nature of the stock market index than the robust investor, while the robust investor lays more emphasis on its hedging nature than the non-robust investor.

Flor and Larsen (2014) find that it is more important to take uncertainty about stock dynamics into account than uncertainty about long-term bond dynamics. They find that the higher the Sharpe ratio of an asset, the more important the role of uncertainty about the price of that asset. Since historically the stock market has a slightly higher Sharpe ratio than the bond market, uncertainty about stock dynamics plays a more important role than uncertainty about bond dynamics.

Vardas and Xepapadeas (2015) show that robustness does not necessarily induce more conservative investment behavior: if there are two risky assets and a risk-free asset (with constant risk-free rate) in the market and the investor is uncertain about the price processes of the risky assets, it might be the case that the total holding of risky assets is higher than in the case of no uncertainty. Moreover the authors find that if the levels of uncertainty about the price processes of the two risky assets are different, then the investor will decrease her investment in the asset about the price process of which she is more uncertain and she will increase her investment in the asset about the price process of which she is less uncertain. If one of the risky assets represents home equity and the other represents foreign equity, and the investor is more uncertain about the foreign assets, this finding provides an explanation for the home-bias.

Uppal and Wang (2003) also assume a financial market with several risky assets. The investor is uncertain not only about the joint distribution of the asset returns, but she is also uncertain – to various degrees – about the marginal distribution of any subset of the asset returns. The authors find that under specific circumstances[17] the optimal portfolio is significantly underdiversified compared to the optimal mean-variance portfolio.

3.2. Robust liability management and ALM

Introducing robustness into the liability side is less straightforward. If the liability side is given as a one-dimensional stochastic process (in practice this usually means a one-dimensional geometric Brownian Motion), one can add a perturbation term just like one did on the asset side. More sophisticated models allow several state variables to influence the liability side, some of which might influence the asset side as well. Such state variables often used by pension funds include interest rates, wage growth and inflation.

Since inflation can also influence the asset side by, e.g., holding inflation-indexed bonds or by influencing the discount rate used to value bonds, its robust treatment requires a joint ALM framework. The most important papers on robustness about inflation are Ulrich (2013) and Munk and Rubtsov (2014). Ulrich (2013) uses empirical data from the 1970s to the 2010s and concludes that the term premium of U.S. government bonds can be explained by a model with a representative investor with log-utility and uncertainty about the inflation process. Horvath, de Jong, and Werker (2016) find similar results (using a two-factor Vašiček-model, without specifying inflation as a factor): if the investment horizon is assumed to be 30 years, they find that a relative risk-aversion parameter of 1.73 is needed to explain the term premium (the

[17]Concretely: if the uncertainty about the joint return distribution is high and the level of uncertainty about the marginal return distributions is different from each other – even if these differences are small in magnitude.

log-utility case corresponds to a relative risk aversion of 1), assuming model uncertainty. Without model uncertainty a relative risk-aversion of 6.54 is needed to explain market data.

Munk and Rubtsov (2014) also solve a robust dynamic investment problem with stochastic investment opportunity set. But contrary to Maenhout (2006), the stochasticity of the investment opportunity set comes from the short rate being stochastic, and the financial market also includes a long-term nominal bond. Inflation is also explicitly included in the model, and the investor is uncertain not only about the drift of the financial assets (a stock and a long-term nominal bond), but also about the drift of the inflation process. The optimal portfolio weight for both the stock and the bond is the sum of one speculative and three hedging components. The latter three components hedge against adverse changes in the realized inflation, the short rate and the expected inflation. Contrary to Maenhout (2004) and Maenhout (2006), in the optimal solution of the investment problem the uncertainty-aversion parameter is not simply added to the risk-aversion parameter, but it is multiplied by several combinations of the correlation between the inflation process and asset price processes. Intuitively this means that there is a spill-over effect: uncertainty about the inflation process induces uncertainty about the asset price processes. Both the variable nature of the investment opportunity set and the correlation of inflation with asset prices lead to the total effect of uncertainty being not straightforward: whether it increases or decreases the holding of a particular asset depends on which component of the demand (myopic component and three hedging components) of that asset is influenced by a higher degree by uncertainty.

Wage growth can be treated separately as a state variable (where, if inflation is included, wage should be measured in real terms), and the pension fund manager's robustness with respect to this state variable can be expressed by adding an additional penalty term. This is done by Shen (2014). If pensions are indexed to the wage level and/or inflation, higher wage growths and higher inflation leads to higher liabilities. At the same time – assuming that the contribution rate is not changed – the value of the asset side will increase as well. If the pension fund manager makes robust investment decisions and she is ambiguous about the model describing inflation and wage growth, she will effectively base her decision on higher or lower drifts of the wage growth and inflation processes. Whether robustness means higher or lower drifts, depends – among others – on the specification of the financial market (i.e. on other state variables) and on the funding ratio of the pension fund.

Once both the asset and liability sides are described as stochastic processes (which can be functions of several underlying stochastic processes), the objective function can be formulated. The objective function is an expectation of two terms: the utility function and a penalty term. Choosing the exact form of the utility function is a core step in robust optimization for the pension fund, since it determines what exactly the pension fund wants to hedge against. A simple approach, which is used by Shen, Pelsser, and Schotman (2014), is to take the utility function as $-\left[L_T - A_T\right]^+$, where L_T is the value of liabilities at time T and A_T is the value of assets at time T. Then the pension fund manager's goal is to make decisions regarding the state variables (e.g. investment policy, contribution rate, etc.) such that the value function (which contains the above utility function and a penalty term) is maximized (i.e. the manager hedges against the shortfall risk as much as possible) but under the worst case scenario. In mathematical terms this means solving the following optimization problem:

$$\min_{\mathbb{U}} \max_{\Theta} \mathrm{E}^{\mathbb{U}} \left\{ -\left[L_T - A_T\right]^+ + \int_0^T \Upsilon_s \frac{\partial \mathrm{E}^{\mathbb{U}}\left[\log\left(\frac{\mathrm{d}\mathbb{U}}{\mathrm{d}\mathbb{B}}\right)_s\right]}{\partial s} \mathrm{d}s \right\}, \qquad (14)$$

where $\mathbb{B}$ is the base measure, $\mathbb{U}$ is the alternative measure, Υ_s is the (deterministic and time-dependent) uncertainty-aversion parameter and Θ is the set of decision variables (which can be stochastic). The pension fund manager solves the above optimization problem such that the budget constraint holds. The authors find that a robust pension-fund manager follows a more conservative hedging policy. But the effect of robustness heavily depends on the instantaneous funding ratio, precisely: robustness has a significant effect on the hedging policy only if the instantaneous funding ratio is low. Intuitively: if the pension fund is strong enough to hedge against future malevolent events, the robust and the non-robust hedging strategies will be practically identical. This follows from the particular form of the goal function: according to (14), the pension fund manager's goal is to avoid being underfunded, but once there is no significant threat of becoming underfunded (i.e., the funding ratio is high), she does not have any other objectives based on which to optimize. This is reflected in the kink in the goal function. Moreover: the robust hedging policy differs from the non-robust hedging policy only if the drift terms of the state variables are overestimated[18].

[18]According to the model, the pension fund manager is only uncertain about the drift of the state variables.

4. Conclusion

Accounting for uncertainty is of crucial importance for proper asset-liability management of pension funds. As we demonstrated in Section 3, robust investment decisions outperform non-robust investment decisions in terms of both expected utility and the Sharpe ratio. The difference in performance between robust and non-robust portfolios is both statistically and economically significant.

Pension funds can use several approaches to make robust investment decisions. The ones most commonly used are the penalty approach, the constraint approach, the Bayesian approach and the smooth recursive preferences approach. These approaches differ from each other in the assumptions they use and in how they formulate the robust optimization problem. Once this optimization problem is formulated, one can either use the principle of dynamic programming or the martingale method to obtain the optimal investment policy.

Although the vast majority of the robustness literature focuses on the implications of uncertainty on asset management, for the prudent functioning of pension funds it is at least as important to properly account for uncertainty regarding the liability side. As we demonstrated in Section 3.2, there are several factors that influence both the asset and the liability side of pension funds (the most important of which are wage growth and inflation), and accounting for ambiguity about these factors is the basis for robust asset-liability management.

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Robustness for asset-liability management of pension funds

This paper by Ferenc Horvath, Frank de Jong and Bas Werker (all TiU) discusses the effects of uncertainty on optimal investment decisions and on optimal asset-liability management by institutional investors, especially pension funds, by surveying the most recent literature on robust dynamic asset allocation with an emphasis on the asset-liability management of pension funds. The authors also provide several policy recommendations.

This is a publication of:

Netspar

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www.netspar.nl

October 2016