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Ageing-Driven Pension Reforms

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Ageing-driven pension reforms

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Abstract

This paper stems from the observation that there are two world-wide trends, pension reform and population ageing, and asks whether the two may be related. Exploring the cases of pension reform in different countries, we find that, although they are very different, the cases share a common characteristic: they shift risks away from workers towards those who are retired. Furthermore, population ageing, by increasing the weight of the elderly relative to working generations, raises the price of intergenerational risk sharing. Combining these findings, we argue and show formally that pension reform can be seen as a welfare-best response to population ageing.

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1 Introduction

This paper stems from two observations. First, many countries have been or are involved with pension reform. OECD (2015), for example, shows that nearly all OECD countries were active in changing their retirement-income provision systems between September 2013 and September 2015. And if reforms are not being implemented, reforms are being discussed. Second, populations in many countries are ageing as a result of low fertility and declining mortality as witnessed by increasing old old-age dependency ratios (see for example UN 2015). This leads us to ask two questions. First, do the pension reforms have something in common? Second, are the pension reforms induced by population ageing?

We answer both questions in the affirmative. Ageing of the population is certainly one of the drivers of pension reform. OECD (2015), Table 1.1 shows that in most cases reforms are implemented that improve the financial sustainability of the pension schemes. We argue that, although implemented in (very) different ways, these pension reforms in many cases imply a shift of risks (demographic risk and/or financial market risk) from workers towards retirees. This holds true for traditional pay-as-you-go (PAYG) schemes and also for partly or fully funded pension schemes. We show that this is a welfare-best response to population ageing. In essence, aging raises the price of intergenerational risk sharing. The obvious effect is that societies therefore want to reduce this type of risk sharing.

We focus on the reform of collective pension plans. These include both nation-wide first-pillar public pension plans and second-pillar employment-based pension plans (Barr and Diamond 2008, Whitehouse 2007). These plans typically include elements of intergenerational redistribution and risk sharing. These elements cannot easily be disentangled: an intergenerational risk-sharing mechanism that is actuarially fair and thus *ex ante* does not lead to redistribution between generations, is likely do so *ex post*, i.e. after the risk has realized. In our analysis, we focus primarily on the intergenerational risk sharing element of pension arrangements and how this changes as a result of demographic developments. The analytical model we develop does allow for ex ante redistribution, but we do not analyse pension reform resulting from changes in the preference for ex ante intergenerational redistribution. Instead, we argue that the observed reforms can be explained as adjustments of the optimal degree of intergenerational risk sharing to changing demographics.

Risk sharing in itself is beneficial as long as people are risk averse. The size of the welfare improvements depends among others on the number of generations included in risk sharing (the length of the smoothing period) and how risk averse people are. Further, it depends on the development of capital markets. The more well-developed are capital markets, the more people can use the market to insure themselves against particular risks and the less welfare improvement that collective risk sharing can bring about. However, there will always be room for welfare-increasing risk sharing as it is impossible to cover all risks through capital markets.

Intergenerational risk sharing may be costly as well. In particular, risk sharing can deteriorate the functioning of labour markets when contribution rates in a collective plan are perceived as taxation. Furthermore, a risk-sharing collective of generations always has to deal with discontinuity risk, as future cohorts may be unwilling to participate in a collective scheme set up by their predecessors. One reason for this unwillingness may be a difference in preferences. Another reason may be an imbalance between what future new participants are required to contribute to the system and what they expect to get out of it. The risk of discontinuity may also arise for well-funded plans as it can be beneficial for its stakeholders to liquidate the plan and to distribute the wealth amongst them.

The analytical model we develop shows that less risk-sharing is a welfare-best response to ageing, where welfare is modeled from a long-run social-planner perspective. The long-run perspective implies that we neglect short-run welfare effects due to redistribution between the generations involved. In general, the elderly will lose whereas the young will gain from the reduction in risk sharing. Often, additional policy measures are taken to (partly) compensate for this short-run reform-induced redistribution. We refer to Auerbach and Lee (2011) for a full analysis of the intergenerational redistribution resulting from various reforms of unfunded pension plans (i.e. in US, Sweden and Germany). Analyses of integenerational redistribution due to reform of funded pension plans can be found in Hoevenaars and Ponds (2009) and Draper et al. (2014) for the Netherlands and Platanakis and Sutcliffe (2016) for the UK. By taking a social planner perspective we abstract form the political economy issues of pension reforms. Countries differ in history, culture and institutional frameworks, and therefore they choose different solutions and reform routes to the ageing problem. We focus on the common trend in these reforms routes and on ageing as the driving force behind this trend. We refer to Sinn (2005), Galasso (2006) and Ebbinghaus (2015) for detailed analyses of how political and social configurations affect the way the driving force of population ageing is translated in actual country-specific reforms of pension arrangements.

The structure of this paper is as follows. We start in Section 2 with a brief discussion of population ageing as a driving force behind pension reform. Section 3 then gives an overview of the main developments in the reform of collective pension plans. This section starts by defining a number of prototype reforms. We distinguish between gradual, parametric reform within existing pension arrangements and fundamental redesign of the pension system. We observe a general trend of moving away from pension plans with defined benefits (DB) towards plans with defined contributions (DC) but adjustable benefits. This trend implies less intergenerational risk sharing and less redistribution from workers to retirees. In Section 4 we develop an analytical 2-OLG model with two types of risk: financial market risk and (macroeconomic) longevity risk. Subsequently, we relate the welfare-maximizing rate of intergenerational sharing of these risks to the demographic situation as measured by the expected old-age dependency ratio. This shows that ageing reduces the optimal rate of intergenerational risk sharing. Section 5 winds up and concludes that the observed reforms are indeed driven by population ageing as they are welfare-best responses to ageing.

2 Ageing and the need for pension reform

The pension plans in most countries have their roots in the thirties or in the period directly after the end of World War II. Many countries started up a first-pillar plan in the form of a traditional *PAYG Defined Benefit (PDB)* plan. In some countries, like the Anglo-Saxon countries and Nordic countries, the base pension plan was supplemented with a funded plan, often a traditional DB plan as well, organized via a pension fund (a *Funded Defined Benefit (FDB)* plan). These plans often contained components of intergenerational risk sharing, as for example in the Netherlands and Switzerland, but not always intentionally. Indeed, the risk sharing itself was rather a by-product of the plan design than something explicitly aimed at.

A main message of this paper is that risk sharing via collective contracts need to be flexible enough to adapt to changing circumstances. A characteristic feature of traditional DB plans is that benefits are paid out as promised and that contribution adjustments are the primary instrument to warrant the financial soundness of those plans. The effectiveness of this strategy has eroded over time.

Regarding traditional PDB, the *ageing of the population* (due to falling fertility rates and the longevity trend) leads to increasing contribution rates, which tend to reach unsustainable levels. Table 1 reports on the dramatic

	Elderly dependency ratio [a]			
	1975	2000	2025	2050
Austria	24	21	32	52
Belgium	22	25	36	43
Denmark	21	22	32	35
Finland	16	22	37	38
France	22	25	37	47
Germany	23	23	34	52
Greece	19	27	37	56
Ireland	19	17	28	43
Italy	19	26	41	69
Luxembourg	20	21	31	40
Netherlands	17	20	36	46
Portugal	16	23	32	49
Spain	16	24	35	66
Sweden	24	26	36	39
United Kingdom	22	24	33	39
Switzerland	19	22	37	50
Turkey	8	9	14	30
Japan	12	24	43	56
United States	16	19	29	35
OECD average	17	21	32	42
Africa average	6	6	7	12
Arab states average	6	5	8	13
Asia and Pacific average	7	9	14	24
Central and Eastern Europe and Central Asia average	14	17	24	34
Latin America and the Caribbean average	8	9	14	26
World Total	10	11	15	23

Figure 1: Old-age dependency ratio 1975-2050 (Source: DICE Report 2003)

increase in the old-age dependency ratio since 1975, doubling or even more in many countries in 2050 compared to 2000. When the replacement ratio (i.e. the pension benefit relative to the wage income before retirement) would remain unchanged, the contribution burden would grow proportional with the dependency ratio, implying unsustainable contribution rates and unsustainable levels of redistribution from the young to the old.

Regarding FDB plans, the main problem is the strong increase of pension liabilities in maturing pension plans in relation to the size of the sponsoring firms and industries. This *increasing maturity* undermines the effectiveness of contribution adjustments to control for the solvency position of the pension funds. Table 2 reports the evolution of pension liabilities and the wage sum as the contribution base for the Dutch pension-fund sector. Pension liabilities in terms of labor income increase from a level of 2.9 in 2009 to a level of 3.7 in 2040. This implies that unanticipated shocks in financial markets and longevity require larger changes in pension contributions in order to shield guaranteed defined pension rights from these shocks (Bonenkamp et al. 2010).

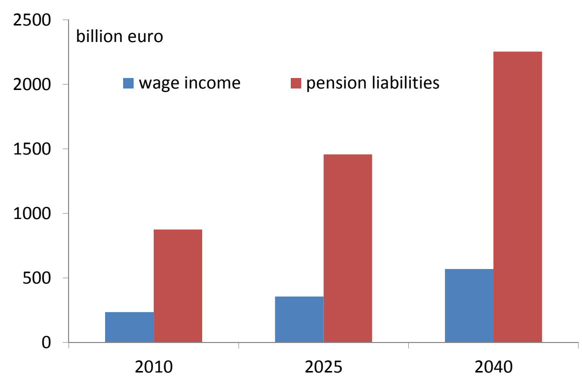

Figure 2: Aggregate pension liabilities and wage income of Dutch pension funds, 2010-2040 (Source: Bonenkamp et al. 2010)

Whereas the lifespan of individuals increases over time, we also observe a general trend of a shortening average lifespan of industries and companies, compare figure 3 (Reeves and Pueschel, 2015). The increasing importance of ICT is an important factor in this respect (Bartelsman 2013). This inverted scissor movement in lifespans undermines any initiative to provide guarantees to employees' pensions.

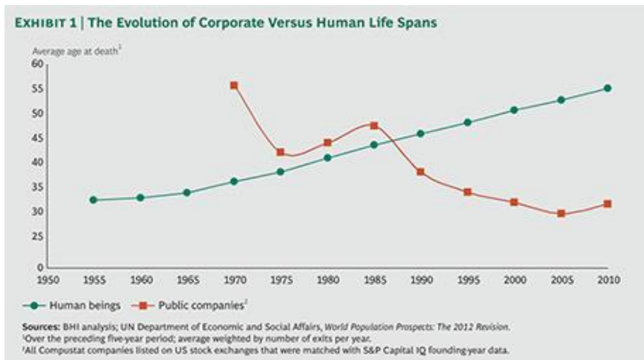

Figure 3: Evolution average lifespans of companies and individuals (Source: Reeves and Pueschel, 2015)

3 An overview of pension reforms

3.1 Prototypes of pension reform

A natural response to the trend towards unsustainable DB plans would be to downsize intergenerational risk sharing in pension plans or even to eliminate it by moving to *Individual Defined Contribution(IDC)* plans. Typical for IDC plans is that only the contribution is defined, no guarantee is given regarding the pension benefit. The pension benefit for the individual with a IDC plan is determined by the sum of contributions plus the (uncertain) return on the investment of these contributions.

Instead of moving from DB plans to IDC plans, an alternative route is to redesign collective arrangements based on intergenerational risk sharing by turning to collective pension plans with adjustable benefits rather than adjustable contributions. In real life, various pension plans indeed have

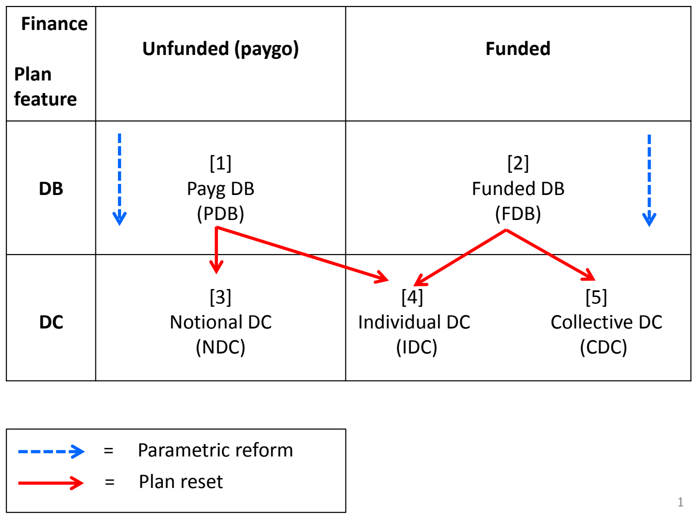

Figure 4: Types of pension reforms

changed to a plan setting with flexible benefits as the primary instrument to keep financial balance. We make a distinction between unfunded collective DC plans, often called *Notional DC (NDC)*, and funded *Collective DC (CDC)* plans.

The matrix in figure 4 shows the five main prototypes of pension plans and the main trends in pension reform. Along the horizontal axis of the diagram, the two main forms in financing pensions are listed: funded plans and unfunded plans (PAYG). The vertical axis illustrates the degree of intergenerational risk sharing. Pensions may be of the DB-type or of the DC-type, the latter typically having a lower degree of intergenerational risk sharing.

Instead of a fundamental redesign, in practice pension reform often is a gradual process involving a series of small parametric reforms that reduce the degree of intergenerational risk sharing in a system that still can be characterized as a DB plan. Such parametric reforms are indicated by the dashed vertical arrows in figure 4. Given the political influence of the elderly in an ageing society, such a gradual, parametric reform often is the only possibility to change the system. However an ongoing process of small parametric reforms may ultimately end up in a fundamental plan change that can be characterized as a redesign as well.

In the rest of this section we will first discuss some prototypes of parametric reforms and then turn to more fundamental reforms, realising that in practice the distinction between parametric and fundamental changes is often not as clear as suggested here. In addition, actual reforms often also involve a combination of different prototype reforms.

3.2 Parametric Reforms

Many countries have pursued a number of gradual adjustments in their first pillar in the form of parametric reforms, implying a lower generosity of pension benefits, for example via a higher retirement age, a lower accrual rate (Bismarckian plan types) or a direct cut in benefit levels. In a way, such parametric reforms imply a break with the practice of defined benefits, as benefits have become flexible to broaden the capacity of bearing and sharing the burden of ageing. Several studies offer detailed overviews of parametric reforms in various countries (European Commission 2012a, 2012b, Schwarz and Arias 2012, Barr and Diamond 2008). The overall assessment of these parametric reforms is that they are not radical enough to warrant sustainable pension arrangements in the longer run. This concerns for example continental European countries in the Bismarckian tradition with large-scale unfunded plans. There are also concerns for public sector workers pension plans - both funded as well as unfunded - which in many countries still provide generous DB promises (Mueller et al. 2009, Ponds et al. 2012).

A number of countries did not follow the route of parametric reforms. Instead, they implemented more radical reforms of their pension plans in the form of a plan reset.

3.3 Individual Defined Contribution

Chile is the most outspoken example of a country that moved away from an unfunded plan to a funded IDC-based first pillar plan. The first steps were set in the early eighties. In 2008 measures were taken in the interest of individual plan holders by new regulations as to competition between providers, asset allocation strategies, and efficiency (Berstein 2010, Impavido et al., 2010).

Anglo-Saxon countries, like the US, the UK, Australia, are also main examples of countries with a general decline of funded DB plans in favor of a move to (voluntary) individual DC plans (Bailey and Kirkegaard, 2009). During the Bush administration, the US witnessed an intense debate on a partial transition to a funded first pillar with individual accounts (Feldstein

2005, Diamond and Orszag, 2005).

Sweden reformed its unfunded public pension plan with DB entitlements in the 1990s into a more sustainable system consisting of two parts. One part is a funded IDC plan with a fixed contribution rate of 2.5%. The other part has a NDC structure which will be discussed below.

3.4 Notional Defined Contribution

The NDC plan structure has recently been introduced in various countries to replace the traditional PDB plan. The most striking feature of the NDC plan is that the contribution rate is fixed and the sustainability of the plan is realized by smoothing the financial burden of a shock (like the ageing of the population) over many generations via structural adjustments in the indexation policy. As the system is steering towards a long-term balance between contributions and benefits, a buffer is needed to absorb deviations between benefits and contributions from year to year.

The transition from a traditional PDB plan to a NDC plan type implies *redesigning intergenerational risk sharing*. A PDB plan captures (demographic) shocks primarily in the contribution rate, which implies different implicit returns for different generations. A NDC plan explicitly aims for equality in the implicit return for the generations by smoothing shocks over a longer time period over many age-cohorts.

The basic idea behind a NDC system is to design a PAYG system that mimics the structure and incentives of an individual DC plan. However, contributions to a NDC system are not invested in financial assets, and hence the returns on these contributions do not depend on stock market or bond returns. The notional returns on these defined contributions are exogenously set, and typically coincide with some long-term average of the per capita growth rate of GDP or of the growth rate of the wage sum. Once a worker retires, the total capitalized value of his/her life-time contributions is transformed into a pension benefit in the form of a real or nominal annuity. This annuity typically will depend, among other things, on life expectancy and retirement age (compare Holzmann et al. 2012).

Sweden provides the most outspoken implementation of NDC in its reformed first-pillar plan. Also Italy, Latvia and Poland are well-known examples of countries with NDC features in their pension system (compare Holzmann et al. 2012).

3.5 Collective Defined Contribution

FDB plans such as second-pillar employer-sponsored plans have been the norm since the 1950's in many countries. This type of plan has been on the retreat, however, since the 1980's. Gradually, private sector FDB plans have been replaced by IDC plans, especially in the UK and the US. Public sector pension plans in these two countries have maintained their DB structure, and funding risks typically have to be borne by (generations of) taxpayers. Other countries, however, have moved into the direction of a CDC plan. The Netherlands probably is the best-known country with a reform of traditional funded DB towards a CDC plan structure. Other CDC plans (compare Blake 2016, Bonenkamp et al. 2014) can be found in Denmark (the ATP plan) and Canada (the Canadian Pension Plan (CPP), the New Brunswick plan, cf. Munnell and Sass 2014, Pension Policy Institute 2014). In the UK the possible introduction of a CDC plan structure is on the political agenda (Blake 2016).

The main difference between a FDB and a CDC plan is the substitution of target benefits for defined benefits. In a CDC plan, each member has the prospect on a target pension (typically related to career average wage path) which increases the longer he or she is a member. The target benefits are flexible and typically will be raised or lowered depending on realised investment performance and the actual longevity experience of retired members. The adjustment of the benefits is typically linked to the funding ratio of the fund, defined as the value of total assets over the value of the target benefits of all members in the scheme. The flexible and periodical adjustments of the target benefits are the primary instrument to correct a situation of overfunding or underfunding compared to a pivot level for the funding ratio.

The contribution rate is typically fixed in these schemes, which necessarily implies that shocks will be absorbed by the pension entitlements. This implies a change in the risk-bearing base, and thus a redesign of the nature of intergenerational risk sharing. Risk sharing via flexible contributions implies that risk is borne by current and future workers. The new form of risk sharing via adjustable benefits implies that risk is borne not only by benefits already accrued by current retirees and current workers, but also (depending on the length of smoothing period) by benefits to-be-accrued by current and future workers in the coming years.

A key aspect of CDC (and also NDC) plans is the length of the smoothing period after a shock. This may range from a relatively short period of 10 years (as in Dutch CDC plans) to a long period of 75 to 100 years (as in the CPP in Canada). The longer the smoothing period, the larger the absorption capacity resulting in relatively small benefit adjustments, but the larger the variation in the funding ratio.

4 The link between population ageing and pension reform

In this section, we develop an analytical model to address the trend in pension plan design from flexible contributions to flexible benefits. We show this trend indeed is an effective response to ageing and increasing maturity, as it implies less risk sharing between the young and the old. The benefits of intergenerational risk sharing can still be maintained but at a reduced scale.

4.1 A 2-period OLG model

We set up a 2-period OLG model. In each period a new generation is born that lives for two periods and in each period two generations coexist. The number of workers, n_y, is constant however, on account of a constant fertility rate. We assume that part of the young generation dies at the end of the first period of life and that the remainder survives into the next period. We define the survival ratio in a period as $\lambda = n_o/n_{y,-1}$, where we use the index -1 to denote that the variable in question relates to the previous period. The number of pensioners, n_o, is a risk factor, it has mean $E(n_o)$ and variance $Var(n_o) > 0$. All workers face the same probability of surviving into the next period, $E(\lambda_{+1}) = E(n_{o,+1}/n_y)$. An increase in this survival probability then implies an increase in longevity and, given that other things remain unchanged, ageing of the population. An alternative interpretation would be that of a homogeneous population that lives longer in retirement in case of a positive longevity shock. The two interpretations can be shown to be equivalent, however. Henceforth, we will stick to the assumption of a heterogeneous population.

We consider both the wage rate and the capital market rate of return as exogenous. This reflects the implicit assumption that changes in the degree of intergenerational risk sharing by the pension fund do not have an effect upon the economywide wage rate and capital market rate of return. The capital market rate of return, denoted r, will be taken stochastic with mean $E(r)$ and variance $Var(r) > 0$.

We allow the two risk factors n_o and r to be correlated. Models of economic growth imply that the interest rate and the rate of population growth are negatively correlated. On the other hand, empirical evidence for a relation between demographic changes and asset returns is rather weak (Poterba 2001). It is a question whether this evidence applies also to long frequencies, however (the unit period of our model covers about thirty years).

A generation receives labour income w only in the first period of his life. This income is distributed over private saving, denoted a, a contribution to the pension scheme, denoted p, and consumption, denoted c_y. The generation's second-period consumption, denoted c_o, equals his saving and any interest earned on it plus a benefit that is paid out by the pension scheme, denoted b. Hence, we have the following expressions for the consumption of young and old cohorts,

$$c_y = w - a - p \tag{1}$$

$$c_o = a_{-1}\left(\frac{1+r}{\lambda}\right) + b \tag{2}$$

The factor $1/\lambda$ can be interpreted as a rate-of-return markup. It is the reciprocal of the survival ratio and reflects the assumption that the private savings left by those who deceased are allocated to the surviving members of the same generation via actuarially fair annuity markets (cf. Yaari, 1965).

For private savings, we assume that they are set at that level that ensures that households will achieve their ambition level if no demographic shocks or capital market shocks occur:

$$a = \frac{E(\lambda_{+1})}{(1 + E(r))}\beta\omega w \tag{3}$$

Here, ω represents the aspired replacement rate, so that ωw is the aspired pension. $0 \leq \beta < 1$ denotes the part of it that is covered through private savings; part $1 - \beta$ is covered through the pension scheme. The index $+1$ denotes that the variable in question is dated one period ahead.

This specification for private savings does not necessarily coincide with the one that would follow from solving an individual's optimization problem. The specification does reflect the idea of life-cycle savings, however, that a large probability to survive into retirement leads workers to save more of their labour income. Next, this specification allows us to derive analytical expressions for the variables of interest.

Substitution of the equation for private savings, (3), into those for consumption, (1) and (2), gives the following two expressions:

$$c_y = w - \frac{E(\lambda_{+1})}{(1 + E(r))}\beta\omega w - p \tag{4}$$

$$c_o = \frac{E(\lambda)}{(1+E(r))\lambda}(1+r)\beta\omega w + b \tag{5}$$

Household preferences are assumed isoelastic. Their utility then combines the two types of consumption in the following way,

$$U = \Big(\frac{c_y^{1-\gamma}}{1-\gamma}\Big) + \frac{\lambda_{+1}}{1+\delta}\Big(\frac{c_{o,+1}^{1-\gamma}}{1-\gamma}\Big) \tag{6}$$

where $\delta > 0$ denotes the individual discount rate.

We are now ready to define our social welfare function. We define it as the expected value of the sum of the utility functions of successive generations, starting with that of the currently old, into the indefinite future:

$$W = E\Big[\sum_{i=-1}^{\infty} n_{y,+i}(1+\Delta)^i U_{+i}\Big] \tag{7}$$

Here, $\Delta \geq 0$ acts as social discount rate.

Two comments apply. First, our economy is stationary: the distributions of n_o and r and the values of non-stochastic variables are constant through time. Given that we explore pension schemes that are time-invariant as well, we can rewrite the expression for W in terms of the felicity functions for the currently old and young in the current period. Second, the individual discount rate δ serves no other role than discounting the felicity function of the old and can be left out without loss of generality. We will do so for the remainder of the analysis. Hence, we obtain the following expression for W,

$$\begin{aligned} W &= E\Big[n_y\Big(\frac{c_y^{1-\gamma}}{1-\gamma}\Big) + \frac{n_o}{1+\Delta}\Big(\frac{c_o^{1-\gamma}}{1-\gamma}\Big)\Big] \\ &= E\Big[n_y\Big(\frac{(w - \Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w - p)^{1-\gamma}}{1-\gamma}\Big) \\ &\quad + \frac{n_o}{1+\Delta}\Big(\frac{(\Big(\frac{E(\mu)}{1+E(r)}\Big)\Big(\frac{1+r}{\mu}\Big)\beta\omega w + b)^{1-\gamma}}{1-\gamma}\Big)\Big] \end{aligned} \tag{8}$$

where the second line follows from substituting the expressions for c_y and c_o in equations (4) and (5) and using the notation μ as a shortcut for the old-age dependency ratio, $\mu = n_o/n_y$. Note that equation (8) does not feature any leads or lags. Indeed, our assumption that n_o is identically and independently distributed means that the expected value of the old-age dependency ratio $E(\mu) = E(n_o/n_y)$ coincides with the survival probability $E(\lambda_{+1}) = E(n_{o,+1}/n_y)$.

The welfare function in equation (8) explores all possible states of nature in one period and weighs the interests of the generations that are young and old in this period. Optimal policies derive from maximizing this ex ante social welfare function with respect to the available policy instrument.

Including only two generations in the risk sharing function is a restriction; first-best policies would include an infinite number of generations (Ball and Mankiw, 2007). However, the two-generation setup is sufficient to demonstrate our claim about the effects of population ageing. Moreover, this restriction may have little relevance for real world pension policies (in the real world, the largest part of shocks seems to be covered within 30 years and the part that is shifted 30 years is quite small).

In the following, we will explore the impact of population ageing upon the optimal degree of intergenerational risk sharing. We do so for the case of a PAYG scheme and that of a funded scheme. We start with the former.

4.2 The case of a PAYG scheme

This section explores the case of a PAYG pension scheme. Both per capita pension contributions and benefits are proportional to the intergenerational transfer per pensioner,

$$p_p = \mu t_p \tag{9}$$

$$b_p = t_p \tag{10}$$

where p_p denotes the amount of pension contributions and b_p that of pension benefits of the PAYG scheme. t_p denotes the transfer from the young to the old generation per pensioner.

For the intergenerational transfer, we propose the following function:

$$t_p = \Big(\frac{\phi_p \mu + (1-\phi_p)E(\mu)}{\mu}\Big)(1-\beta)\omega w \tag{11}$$

The transfer function in equation (11) determines how longevity shocks are shared between the young and the old generation. It covers two polar cases. In the first case, that of zero risk sharing ($\phi_p = 0$), transfers are proportional to the reciprocal of the old-age dependency ratio. In this case, the old generation bears all of any longevity shock. In the second one, the case of maximum risk sharing ($\phi_p = 1$), transfers are unrelated to the old-age dependency ratio. Now it is the young generation that bears all of the longevity risk. All cases in between the two polar cases ($0 < \phi_p < 1$) attribute longevity risk to both the young and the old generation.

We refer to the case of $\phi_p = 1$ as the case of maximum risk sharing, not the case of full risk sharing. To see why, elaborate the equations for the consumption of young and old generations by substituting the expression for transfers in equation (11) into equations (9) and (10) and the results into equations (4) and (5):

$$c_{y,p} = w - \Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w - \Big(\phi_p\mu + (1-\phi_p)E(\mu)\Big)(1-\beta)\omega w \quad (12)$$

$$c_{o,p} = \Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w\Big(\frac{1+r}{\mu}\Big) + \Big(\frac{\phi_p\mu + (1-\phi_p)E(\mu)}{\mu}\Big)(1-\beta)\omega w \quad (13)$$

In the polar case $\phi_p = 1$, the transfer part of the pensions, $1-\beta$, is shielded from longevity risk. Still, the old are subject to longevity risk through their private savings. These private savings are subject to capital market risk as well. Equation (13) shows that the two risks enter multiplicatively. Hence, they can reinforce or weaken each other. In the polar case in which $1+r$ is proportional with μ, they annihilate each other.

Upon substituting the two consumption equations, (12) and (13), into the social welfare equation, (8), an expression for social welfare emerges as a function of ϕ_p:

$$\begin{aligned} W_p & \\ = \quad & E\Big[n_y\Big(\frac{\Big(w - \Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w - \big(\phi_p\mu + (1-\phi_p)E(\mu)\big)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big) \\ & + \frac{n_o}{1+\Delta}\Big(\frac{\Big(\Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w\Big(\frac{1+r}{\mu}\Big) + \Big(\frac{\phi_p\mu+(1-\phi_p)E(\mu)}{\mu}\Big)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big)\Big] \end{aligned} \quad (14)$$

As explained in detail in the appendix, elaborating the first-order condition $\partial W_p/\partial\phi_p = 0$ yields the following closed-form expression for ϕ_p,

$$\phi_p = \frac{\frac{1}{1+\Delta}(\omega)^{-\gamma-1}/(1-\beta)}{E(\mu)(1-\omega E(\mu)\tilde{\beta})^{-\gamma-1} + \frac{1}{1+\Delta}(\omega)^{-\gamma-1}}\Big(1 - \beta\tilde{Cov}(\mu,r)\Big) \quad (15)$$

where $\tilde{\beta}$ is a shorthand notation for $\beta/(1+E(r)) + (1-\beta)$ and $\tilde{Cov}(\mu,r)$ is a shorthand notation for $[Cov(\mu,r)/(1+E(r))]/[Var(\mu)/(E(\mu))]$.

Several features of this expression for ϕ_p are noteworthy. First, if the covariance between demographic and interest rate shocks is zero, the optimal degree of risk sharing, as measured by $(1-\beta)\phi_p$, is generally between 0 and

100 percent. The multiplication of ϕ_p with $(1-\beta)$ reflects that risk sharing applies to the total of private and public saving. In case of non-zero private saving, $\beta > 0$, the optimal value of ϕ_p can exceed one, as it must compensate for the fact that private saving features zero risk sharing. Intuitively, an optimal scheme lets both generations absorb a non-zero part of the risks (Bohn, 2009).

Second, if both the covariance between the two types of shocks and the amount of private savings are very large in absolute terms, $(1-\beta)\phi_p$ may be negative or exceed 100 percent. However, in the absence of private savings, a correlation between longevity shocks and capital market shocks is irrelevant. This is because capital market shocks enter the solution for the optimal degree of risk sharing only through private savings. If shocks in longevity and in the interest rate are negatively correlated as suggested by models of economic growth, $\tilde{Cov}(\mu, r)$ is negative and the optimal degree of risk sharing higher than in case of zero correlation (see equation (15)). This aggravates the impact of population ageing on the optimal degree of risk sharing.

Thirdly, the optimal degree of risk sharing is decreasing in the expected old-age dependency ratio. As we interpret population ageing as an increase in longevity (and thus an increase in the expected old-age dependency ratio), this implies that the optimal degree of intergenerational risk sharing reduces on account of an ageing population (we consider the effect of ageing to be an equiproportional shift in the distribution of n_o, so that the mean and the standard deviation of n_o (and of μ) increase equiproportionally. This implies that $\tilde{Cov}(\mu, r)$ remains unchanged). The fundamental mechanism is that a higher average old-age dependency ratio raises the price of risk sharing: it takes a larger reduction in consumption per capita of workers to increase the consumption per capita of the elderly with a given amount. The optimal reaction to this increase in the price of risk sharing is to demand less of it. In addition, a higher expected longevity increases private savings and pension contributions by the young, thereby reducing their consumption and their capacity to absorb risk. This additional mechanism hinges on the assumption we have made on the link between private savings and expected longevity, however. The same cannot be said about the price effect, which is why we call this the fundamental mechanism.

4.3 The case of a funded pension scheme

The previous section has shown that population ageing reduces the optimal degree of risk sharing in a PAYG scheme. How about a funded scheme?

Does an identical or similar result apply in that case? To explore this, this section models a funded pension scheme. Like the PAYG scheme, the funded scheme allocates demographic risks and financial market risks among the young and the old generations who are alive at the same moment in time. Unlike the PAYG scheme, pension contributions and benefits now contain two parts, a funded part and a PAYG part:

$$p_f = s + \mu t_f \tag{16}$$

$$b_f = s_{-1}\Big(\frac{n_{y,-1}}{n_o}\Big)(1+r) + t_f \tag{17}$$

Here, s denotes the amount of savings through the pension scheme and t_f the transfer between the young and old.

For pension savings, we assume a form that is similar to the one postulated earlier for private savings,

$$s = \frac{E(n_{o,+1})}{(1+E(r))n_y}(1-\beta)\omega w \tag{18}$$

the only difference being that pension savings finance part $1-\beta$ rather than β of the pension benefit.

For transfers, we assume the following form:

$$t_f = -\phi_f\Big(\Big(\frac{E(n_o)}{1+E(r)}\Big)\Big(\frac{1+r}{n_o}\Big) - 1\Big)(1-\beta)\omega w \tag{19}$$

Similar to the transfer function in case of a funded scheme, the transfer scheme in equation (19) allocates the risk that is associated with pension saving to the old generation (in case $\phi_f = 0$) or to the young generation (in case $\phi_f = 1$) or to both of them (in case $0 < \phi_f < 1$). This mimics the case of a DC scheme (for $\phi_f = 0$), a DB scheme (for $\phi_f = 1$) or a hybrid scheme ($0 < \phi_f < 1$). This becomes more clear if we elaborate the two consumption equations (substitute the expression for transfers in equation (19) into equations (16) and (17) and the results into equations (4) and (5)):

$$c_{y,f} = w - \Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w + \mu\phi_f\Big(\Big(\frac{E(n_o)}{1+E(r)}\Big)\Big(\frac{1+r}{n_o}\Big) - 1\Big)(1-\beta)\omega w \tag{20}$$

$$c_{o,f} = \Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w\Big(\frac{1+r}{\mu}\Big) - \phi_f\Big(\Big(\frac{E(n_o)}{1+E(r)}\Big)\Big(\frac{1+r}{n_o}\Big) - 1\Big)(1-\beta)\omega w \tag{21}$$

Note that, unlike the transfer function in the PAYG scheme, transfers are now two-sided. Transfers can be positive or negative, flowing from the young to the old or vice versa.

Before we elaborate the social welfare function, it is convenient to linearize the transfer component of the consumption equations. In particular, $E(n_o)(1+r)/n_o$ can be written as $(1+r)/(n_o/E(n_o))$ or $(1+r)/(1+\hat{n}_o)$, where we use the definition $\hat{n}_o \equiv (n_o - E(n_o))/E(n_o)$. $(1+r)/(1+\hat{n}_o)$ can be linearized as $1+r-\hat{n}_o$. Note that the approximation is good as long as r and $\hat{n}_o$ are not too large.

If we now use this approximation to rewrite the consumption equations, substitution of the results in the social welfare function (8) yields an expression for social welfare in terms of the policy parameter ϕ_f:

$$W_f =$$

$$E\Big[n_y\Big(\frac{\Big(w-\Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w+\phi_f\Big(\frac{\mu}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big)+$$

$$\frac{n_o}{1+\Delta}\Big(\frac{\Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w\Big(\frac{1+r}{\mu}\Big)-\phi_f\Big(\frac{1}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big)\Big] \tag{22}$$

As elaborated in the appendix, this condition implies the following closed-form expression for ϕ_f (we assume $Var(\mu/E(\mu)-r) \neq 0$. This will not hold true only for a very specific value of the covariance between the old-age dependency ratio and the rate of return on savings):

$$\phi_f = \frac{\frac{1}{1+\Delta}\big(\omega\big)^{-\gamma-1}/(1-\beta)}{E(\mu)(1-\frac{E(\mu)\omega}{1+E(r)})^{-\gamma-1}+\frac{1}{1+\Delta}\big(\omega\big)^{-\gamma-1}} \tag{23}$$

This expression tells us three things. First, in case of a funded scheme the optimal degree of risk sharing, as measured by $(1-\beta)\phi_f$, lies strictly between zero and one. The factor $1-\beta$ plays the same role here as in the PAYG scheme that we studied in the previous subsection. Hence, part of the risks associated with life-cycle saving is borne by the old generation and part of it is borne by the young generation.

Second, a large number of elderly, which, given the number of youngsters, translates into a high dependency ratio, reduces the optimal degree of risk sharing (ϕ_f falls). The reason is similar to the case of a PAYG scheme. A higher dependency ratio implies risk sharing is more costly: the young generation has to accept more risk in order to reduce the risk borne by the old generation to a certain extent. In addition, private and pension savings reinforce the mechanism as they imply that the young generation allocates a larger part of his income to saving, thereby reducing his risk absorption capacity.

Thirdly, the covariance between the old-age dependency ratio and the capital market rate of return does not play a role in the expression for the optimal degree of risk sharing. The reason is that transfers in the funded scheme share both longevity and rate of return risks. In contrast, the transfers in the PAYG scheme that was discussed in the previous subsection naturally share only longevity risks.

4.4 Policy implications

The model developed above describes the impact of population ageing on the optimal size of intergenerational risk sharing. The crucial parameter is ϕ which determines to what extent shocks will be born by young, working generations ($\phi = 1$), by old, retired generations ($\phi = 0$) or by both types of generations ($0 \leq \phi \leq 1$). We have concluded that population ageing implies a decrease in ϕ, which boils down to a reduction of intergenerational risk sharing. We have established this result for the case of a PAYG scheme as well as for that of a funded scheme. In practice, a lower value for ϕ can be realized by parametric reform or by one of the moves that we discussed in section 2, i.e. towards an IDC scheme, a NDC scheme or a CDC scheme.

5 Concluding remarks

This paper analyses two questions. First, do the reforms of collective pension plans across the world have something in common? Second, is pension reform induced by population ageing? We answer both questions in the affirmative. The observed reforms of pension arrangements, both gradual parametric reforms and more fundamental reforms, can be classified according to a few protypes of reforms. However, there is a clear common trend in the reform of both nation-wide first-pillar public pension plans and second-pillar employment-based plans, namely a trend of moving away from defined benefits towards plans with defined contributions and more variable benefits. This trend implies less intergenerational risk sharing. By developing a 2-OLG model of optimal intergenerational sharing of financial market risk and demographic risk, we show that this trend can be viewed as a welfare-maximizing response to an increase in the expected old-age dependency ratio. That is, pension reforms indeed seem to be induced by the ageing of populations.

The claim that the observed pension reforms are indeed driven by demographic developments only holds true if the model developed in this paper provides a reasonble description of actual decision making on pension arrangements. This is not straightforward. Our model assumes that pension schemes are used as an instrument to optimize social welfare defined as the expected value of the sum of the utility levels of all current and future generations. Moreover, we only analyse time-invariant pension arrangements in a stationary economy. That is, we focus on the long run and abstract from the welfare effects during periods of transition. Yet, we argue that our model describes the trend in actual decison making on pension arrangements fairly well. Although transitional effects are clearly important for the short run, we believe that the trend in the decisions is mainly determined by what is optimal in the long run. That is, when explaining the trend in the evolution of pension arrangements over a longer period, it is acceptable to ignore transitional effects. Moreover, we believe that, while actual decision making on pension arrangements may deviate from what welfare-maximization would prescribe for many reasons, the trend in these decisions is still reasonably well described by the welfare-maximizing model. The essence of the evolution described by our model is that ageing raises the price of intergenerational risk sharing and thus incites societies to reduce risk sharing. This is a powerful mechanism that will affect decision making, also if this is not purely based on welfare maximization. This leads us to the conclusion that population ageing is a crucial driver of pension reforms around the globe.

References

[1] Auerbach, A. J. and R. Lee (2011). Welfare and generational equity in sustainable unfunded pension systems. *Journal of Public Economics* 95, pp. 16-27.

[2] Baily, M.R. and J. Kirkegaard (2009). *US Pension Reform - Lessons from other Countries*, Washington DC, Peterson Institute for International Economics.

[3] Ball, L., and N.G. Mankiw (2007). Intergenerational Risk Sharing in the Spirit of Arrow, Debreu, and Rawls, with Applications to Social Security Design, *Journal of Political Economy* 115, 523-547.

[4] Barr, N., and P. Diamond (2008). *Reforming pensions: Principles and Policy Choices*, New York and Oxford: Oxford University Press.

[5] Bartelsman, E. (2013). Passend beleid in tijden van welvaart en overvloed, *TPEdigitaal* 7, 7-20.

[6] Berstein, S. (2010). *The Chilean Pension System*. Superintendence of Pensions, Chile.

[7] Blake, D., (2016). *We Need a National Narrative: Building a Consensus around Retirement Income* – The Report of the Independent Review of Retirement, The Pensions Institute.

[8] Bohn, H. (2009). Intergenerational risk sharing and fiscal policy, *Journal of Monetary Economics* 56, 805-816.

[9] Bonenkamp J., L. Meijdam, E. Ponds, and E. Westerhout (2014). *Reinventing Intergenerational Risk Sharing*, Netspar Panel Paper, nr. 40.

[10] Diamond P. and Orszag P. (2005). "Saving Social Security, *Journal of Economic Perspectives*, 19 (2), 11-32.

[11] DICE Report (2003.) *DICE Report 3/2003 (Autumn)* CESifo Institute for Economic Research, Munich.

[12] Draper N., C. van Ewijk, M. Lever and R. Mehlkopf (2014). Stochastic generational accounting applied to reforms of Dutch occupational pensions, *De Economist*, 162 (3), 287-307.

[13] Ebbinghaus B. (2015). The Privatization and Marketization of Pensions in Europe: A Double Transformation Facing the Crisis, *European Policy Analysis* 1, 2015

[14] European Commission (2012a). *The 2012 ageing report. Economic and budgetary projections for the 27 EU Member States (2010-2060)*. European Economy 2/2012.

[15] European Commission (2012b). *White Paper - An Agenda for Adequate, Safe and Sustainable Pensions*, Brussels, 16-2–2012.

[16] Feldstein, M. (2005) "Structural reform of Social Security, *Journal of Economic Perspectives*, 19 (2), pp. 33-55.

[17] Galasso V. (2006). *The political future of social security in aging societies*, MIT Press, Cambride, UK.

[18] Hoevenaars, R., and E.H.M. Ponds (2008). Valuation of intergenerational transfers in collective funded pension schemes, *Insurance : Mathematics and Economics*, 42(2), 578-593.

[19] Holzmann, R., Palmer, E., Robalino, D., eds. (2012). *Nonfinancial Defined Contribution Pension Schemes in a Changing Pension World*, Volume 1. Progress, Lessons, and Implementation. World Bank, Washington, DC.

[20] Impavido, G., Lasagabaster, E., and Garca-Huitrn, M. (2010). *New Policies for Mandatory Defined Contribution Pensions : Industrial Organization Models and Investment Products*. Number 2462 in World Bank Publications. The World Bank.

[21] Mueller C., B. Raffelhueschen, and O. Weddige (2009). *Pension obligations of government employer pension schemes and social security pension schemes estabilished in EU countries*, by order of the European Central Bank, Final Report, Research Center for Generational Contracts, Freiburg.

[22] Munnell A.H. and S.A. Sass (2013). *New Brunswick's New Risk Sharing Plan*, Issue in Brief nr. 33, Center for Retirement Research, August 2013, Boston.

[23] OECD (2015). *Pensions at a Glance 2015; OECD and G20 Indicators.*

[24] Pensions Policy Institute (2014). *Risk Sharing Pension Plans: the Canadian Experience* PPI Briefing Note, no. 60, October 2014, United Kingdom.

[25] Platanakis, E., and Sutcliffe, C. (2016). Pension Scheme Redesign and Wealth Redistribution Between Members and Sponsor: The USS Rule Change in October 2011, *Insurance: Mathematics and Economics*, forthcoming.

[26] Ponds, E.H.M., Severinson, C., and Yermo, J. (2012). Implicit debt in public sector plans: An international comparison. *International Social Security Review*, 65(2), 75-101.

[27] Poterba, J.M. (2001). Demographic Structure and Asset Returns, *Review of Economics and Statictics* 83, 565-584.

[28] Reeves M. and Pueschel L. (2015): Die Another Day: What Leaders Can Do About the Shrinking Life Expectancy of Corporations , in: *BCG Perspectives*, 2 july 2015, Boston Consulting Group.

[29] Sinn, H.- W. 2005. Europes Demographic Deficit A Plea For A Child Pension System, *De Economist*, March 2005, 153 (1), 1-45.

[30] UN, 2015. *World Population Aging 2015 (report)*. United Nations, New York, 2015.

[31] Whitehouse, Edward (2007). *Pensions panorama: retirement-income systems in 53 countries.* World Bank, Washington DC.

[32] Yaari, M.E. (1965). Uncertain Lifetime, Life Insurance and the Theory of the Consumer, *The Review of Economic Studies*, 32(2), 137-150.

A The case of a PAYG scheme

This appendix contains the details of the maximization of the social welfare function in the case of a PAYG scheme.

Equation (14) in subsection 4.3 expresses social welfare in the case of a PAYG scheme, which we repeat here for convenience:

$$W_p =$$

$$E\Big[n_y\Big(\frac{\Big(w - \Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w - \big(\phi_p\mu + (1-\phi_p)E(\mu)\big)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big)$$
$$+\frac{n_o}{1+\Delta}\Big(\frac{\Big(\Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w\Big(\frac{1+r}{\mu}\Big) + \Big(\frac{\phi_p\mu+(1-\phi_p)E(\mu)}{\mu}\Big)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big)\Big] \tag{24}$$

The first derivative with respect to the policy parameter ϕ_p reads as follows:

$$\frac{\partial W_p}{\partial \phi_p} =$$

$$E\Big[-n_y\Big(w - \Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w - \big(\phi_p\mu + (1-\phi_p)E(\mu)\big)(1-\beta)\omega w\Big)^{-\gamma}$$
$$(\mu - E(\mu))(1-\beta)\omega w$$
$$+\frac{n_o}{1+\Delta}\Big(\Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w\Big(\frac{1+r}{\mu}\Big)+\Big(\frac{\phi_p\mu + (1-\phi_p)E(\mu)}{\mu}\Big)(1-\beta)\omega w\Big)^{-\gamma}$$
$$\Big(\frac{\mu - E(\mu)}{\mu}\Big)(1-\beta)\omega w\Big] \tag{25}$$

The expression of the second derivative of W_p with respect to ϕ_p is derived in a similar way:

$$\frac{\partial^2 W_p}{(\partial \phi_p)^2} =$$

$$E\Big[-\gamma n_y\Big(w - \Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w - \big(\phi_p\mu + (1-\phi_p)E(\mu)\big)(1-\beta)\omega w\Big)^{-\gamma-1}$$
$$\Big((\mu - E(\mu))(1-\beta)\omega w\Big)^2$$

$$-\gamma\frac{n_o}{1+\Delta}\Big(\Big(\frac{E(\mu)}{1+E(r)}\Big)\beta\omega w\Big(\frac{1+r}{\mu}\Big)+\Big(\frac{\phi_p\mu+(1-\phi_p)E(\mu)}{\mu}\Big)(1-\beta)\omega w\Big)^{-\gamma-1}$$

$$\Big(\Big(\frac{\mu-E(\mu)}{\mu}\Big)(1-\beta)\omega w\Big)^2\Big] \tag{26}$$

This expression shows that the second derivative of W_p with respect to ϕ_p is unambiguously negative.

Unfortunately, the first-order condition is nonlinear in ϕ_p and it is not possible to directly derive an analytical expression for ϕ_p. We proceed by writing $\partial W_p/\partial\phi_p$ as a function of μ and r:

$$\frac{\partial W_p}{\partial\phi_p}=0 \quad \rightarrow \quad E(\Omega_p(\mu,r))=0 \tag{27}$$

Now, we elaborate the second-order Taylor approximation of Ω_p around the point $(E(\mu),\ E(r))$. This produces the following equation,

$$E\Big(\Omega_p(E(\mu),E(r))+\Omega_{p,\mu}(E(\mu),E(r))(\mu-E(\mu))$$

$$+\Omega_{p,r}(E(\mu),E(r))(r-E(r))$$

$$+\frac{1}{2}\Big[\Omega_{p,\mu\mu}(E(\mu),E(r))(\mu-E(\mu))^2+\Omega_{p,rr}(E(\mu),E(r))(r-E(r))^2$$

$$+2\Omega_{p,\mu r}(E(\mu),E(r))(\mu-E(\mu))(r-E(r))\Big]\Big)=0 \tag{28}$$

where $\Omega_{p,x}$ refers to the derivative of Ω_p to x and $\Omega_{p,xy}$ refers to the derivative of $\Omega_{p,x}$ to y.

It is easy to see that this reduces to the following condition:

$$\Omega_p(E(\mu),E(r))+\frac{1}{2}\Big[\Omega_{p,\mu\mu}(E(\mu),E(r))Var(\mu)$$

$$+\Omega_{p,rr}(E(\mu),E(r))Var(r)+2\Omega_{p,\mu r}(E(\mu),E(r))Cov(\mu,r)\Big]=0 \tag{29}$$

Elaborating this condition gives us the expression for ϕ_p in the main text (equation (15)).

B The case of a funded scheme

This appendix contains the details of the maximization of the social welfare function in the case of a funded scheme.

Equation (22) in subsection 4.3 expresses social welfare in the case of a funded scheme, which we repeat here for convenience:

$$W_f =$$

$$E\Big[n_y\Big(\frac{\Big(w-\Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w+\phi_f\Big(\frac{\mu}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big)+$$

$$\frac{n_o}{1+\Delta}\Big(\frac{\Big(\Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w\Big(\frac{1+r}{\mu}\Big)-\phi_f\Big(\frac{1}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big)^{1-\gamma}}{1-\gamma}\Big)\Big] \tag{30}$$

The derivative of W_f with respect to ϕ_f reads as follows:

$$\frac{\partial W_f}{\partial \phi_f} =$$

$$E\Big[n_y\Big(w-\Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w+\phi_f\Big(\frac{\mu}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big)^{-\gamma}$$
$$\Big(\frac{\mu}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w-$$
$$\frac{n_o}{1+\Delta}\Big(\Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w\Big(\frac{1+r}{\mu}\Big)-\phi_f\Big(\frac{1}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big)^{-\gamma}$$
$$\Big(\frac{1}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big] \tag{31}$$

The expression of the second derivative of W_f with respect to ϕ_f is derived in a similar way:

$$\frac{\partial^2 W_f}{(\partial \phi_f)^2} =$$

$$E\Big[-\gamma n_y\Big(w-\Big(\frac{E(\mu)}{1+E(r)}\Big)\omega w+\phi_f\Big(\frac{\mu}{1+E(r)}\Big)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\Big)^{-\gamma-1}$$

$$\left(\left(\frac{\mu}{1+E(r)}\right)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\right)^2$$

$$\gamma\frac{n_o}{1+\Delta}\left(\frac{E(\mu)}{1+E(r)}\right)\omega w\left(\frac{1+r}{\mu}\right)-\phi_f\left(\frac{1}{1+E(r)}\right)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\right)^{-\gamma-1}$$

$$\left(\left(\frac{1}{1+E(r)}\right)(r-E(r)-\tilde{n}_o)(1-\beta)\omega w\right)^2\Bigg] \tag{32}$$

This expression shows that the second derivative of W_f with respect to ϕ_f is unambiguously negative.

Similar to the case of a PAYG scheme, we proceed by writing $\partial W_f/\partial\phi_f$ as a function of μ and r:

$$\frac{\partial W_f}{\partial \phi_f}=0 \quad \rightarrow \quad E(\Omega_f(\mu,r))=0 \tag{33}$$

The second-order Taylor approximation of the function $\Omega_f(\mu,r)$ around the point $(E(\mu),E(r))$ implies the following expression that is linear in ϕ_f:

$$\Omega_f(E(\mu),E(r))+\frac{1}{2}\Big[\Omega_{f,\mu\mu}(E(\mu),E(r))Var(\mu)+$$

$$\Omega_{f,rr}(E(\mu),E(r))Var(r)+2\Omega_{f,\mu r}(E(\mu),E(r))Cov(\mu,r)\Big]=0 \tag{34}$$

Elaborating this condition yields the closed-form expression for ϕ_f in the main text (see equation (23)).