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Abstract

We analyze a collective defined contribution pension fund which aims at intergenerational risk sharing among different age cohorts using a return smoothing mechanism. Using a utility based framework, we find that approximately one third of unexpected return shocks should be directly passed on to all the cohorts in the year the shock occurs by means of the smoothing mechanism. We demonstrate that risk sharing implemented in this way is welfare improving compared to a plan with no risk sharing and more sustainable compared to defined benefit pension fund plans. Additionally, we show that the asset allocation of such a pension fund automatically corresponds to the life-cycle portfolio choice theory.

Keywords: Collective Defined Contribution, Funded pension system, Overlapping generations, Intergenerational risk sharing

JEL codes: H55, G23, H80

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1 Introduction

Despite the crucial dependence of current working generations and retirees on the pension system for their retirement security, pension systems around the world have not lived up to their expectations. There are well-known problems associated with traditional plans, in particular with Defined Benefit (DB) plans. Novy-Marx and Rauh (2011) have documented large unfunded liabilities in US public pension plans. For corporate pension funds, sponsors are increasingly withdrawing their guarantees as they do not want spillover effects of underfunding on the company's balance sheet and therefore they prefer the workers to be the risk bearers for their own pensions. This usually involves freezing DB pension plans (Rauh, Stefanescu and Zeldes (2013)) and replacing them with Defined Contribution (DC) plans for new and old employees.[1] However, in this case the responsibility for making complex financial decisions falls on individual households, who can make sub-optimal decisions. This has been documented by, for example, Choi, Laibson and Madrian (2011), who demonstrate that individual households make poor choices in investing for their retirement, and by Lusardi and Mitchelli (2007), who show that households lack financial skills for retirement planning.

We analyze a Collective Defined Contribution (CDC) pension system, which is one promising solution to the DB problem discussed in the literature. In this type of pension contract, the assets of the fund are pooled and institutionally managed, and the benefits depend on the financial situation of the fund. This keeps the pension system collective, as needed because individuals make poor financial decisions when it comes to retirement saving and investing.[2] Another important benefit of collective pension funds is risk-sharing. Gollier (2008) finds potentially large benefits of intergenerational risk-sharing; a 25% increase in the certainty equivalent of the pension benefits paid to all current and future generations. In this chapter, we provide an estimate of the benefits of intergenerational risk sharing in the absence of any form of sponsor guarantee and with

[1] Freezes can either be mandatory or voluntary, see Brown and Weisbenner (2014). Another common solution implemented is reducing the generosity of pensions, for example by withholding inflation protection or cost of living adjustments (COLA) for pension payments. Again this solution is far from ideal and erodes trust in the pension system as it amounts to an ex-post reduction in benefits.

[2] It is also possible to share portfolio risk and take advantage of economies of scale in these pension systems.

no option of increasing contributions. We find that a collective pension fund is able to reap the benefits of intergenerational risk sharing purely by managing pension rights.

Intergenerational risk sharing is generally welfare-increasing.[3] However, efficiency of risk-sharing can depend on the risk-sharing mechanism in place. For example, Cui, Jong and Ponds (2011) find a decrease in welfare compared to optimal individual welfare in the cases of for defined-benefit plans with benefit adjustments only and of defined-benefit plans with contributions adjustments only.[4] In this chapter, we analyze a collective defined contribution pension system that allows for intergenerational risk sharing through return smoothing.[5] We model intergenerational risk sharing with an explicit age independent return smoothing rule to provide full transparency in risk-sharing. Only a fraction of any financial shock is transferred to the individual cohorts' pension wealth, while the rest is smoothed through a buffer. Our first contribution is that we find that intergenerational risk-sharing implemented in this way increases the certainty equivalent of pension benefits paid by approximately 6% ($\gamma = 3$) for all current and future generations, compared to a pension fund with no risk-sharing.

Our second contribution is that we demonstrate that there is a tradeoff between low risk-sharing which is beneficial for current working generations and high risk-sharing which is beneficial for current retirees and generations close to retirement. If there is high smoothing (more risk-sharing) then this is beneficial to current retirees and generations closer to retirement as they will not fully experience a shock. However, this is not preferable for the current working population and future generations entering the

[3] For a recent treatment of welfare benefits of intergenerational risk-sharing see Cui, Jong and Ponds (2011); Beetsma and Bovenberg (2009); Beetsma, Romp and Vos (2012); Beetsma, Romp and Vos (2013); Chen et al. (2014);van Bilsen and Bovenberg (2014) etc. Risk sharing can be amongst all generations currently in the scheme or amongst all current and future generations, the latter is welfare enhancing with market-traded risks. Moreover, limiting risk sharing to participants only in accumulation or decumulation phase will limit the benefits.

[4] See also Beetsma and Bucciol (2015)

[5] Some pension funds in the Netherlands have implemented reforms to switch from DB to CDC plans. One specific plan where this change happened in January, 2014 is Rabobank Pension Fund (www.rabobankpensioenfonds.nl). There is no sponsor guarantee, and benefits are flexible and depend on the financial situation of the fund. The UK government is considering similar 'Defined Ambition' pension schemes for the Pension Schemes Bill 2014-15 see for example Thurley (2014). In the US, cash balance plans and the Wisconsin Retirement System (WRS) are in a similar spirit of risk sharing, see Novy-Marx and Rauh (2014) who find substantial benefits in reducing unfunded liabilities by making pensions a function of returns

pension system, as they may enter into an underfunded fund. Due to this tradeoff, it is possible to calculate the optimal level of risk-sharing or smoothing in our model. The board of a pension fund needs to decide on optimal risk allocation, as it needs to balance the interests of current and future participants. Depending on the social discount factor considered, optimal risk-sharing implies that only about one-fourth to one-third of underfunding should be passed on to all the future generations in the year underfunding occurs.

The pension funds portfolio allocation to risky assets remains constant in our model. Our third contribution is that we demonstrate that the individuals exposure to risky returns decreases as the individual approaches retirement. This is consistent with life-cycle portfolio choice theory. It also extends the findings of Guillén, Jørgensen and Nielsen (2006) to collective pension systems. Furthermore, we look at the long-term steady state of pension funds and find that CDC pension funds are more sustainable compared to our proxy of DB fund. Lastly, we calculate the minimum funding ratio that is required to ensure that the benefits of risk sharing are not outweighed by the detrimental effect of underfunding for the generation that is about to enter the fund. It may still be welfare-improving to enter into a underfunded pension fund when intergenerational risk sharing provides a welfare improvement compared to a system with no risk sharing. We find that at about 11% ($\gamma = 3$) underfunding, it is still welfare improving for a new generation to join the fund.[6]

Our model is based on Gollier (2008). We first extend his model by departing from the assumption that the present value of future contributions and present value of future payouts are part of the pension funds balance sheet. Secondly, our risk-sharing rule resembles that of Goecke (2013). Like him, we specify the smoothing rule as a function of the asset liability ratio, but we extend his model by explicitly modeling overlapping generations and cash flows due to premiums and pension payments.[7] Third, we develop a utility framework that can take into account intra-generational fairness. We model an

[6] Discontinuity risk for pension fund can also arise from a very high funding ratio (Van Bommel, 2007), something that we do not consider here. See also Siegmann (2011) for a discussion of minimum fundings ratios for defined-benefit pension funds.

[7] See Grosen and Lochte Jorgensen (2000) for a treatment of return smoothing mechanisms in life insurance products.

open fund in which a new cohort enters each year and shares in the risk with the existing cohorts. This aids in understanding the behavior of the fund in the long run, and enables the sustainability of the pension system to be checked. In our model, the accumulation of the assets through the investment returns deviates from the accumulation of the pension rights at a rate which is a smoothed function of investment returns. We refer to the latter as the pension return. The assets are a function of market returns, whereas the pension rights are dependent on the funding ratio of the fund. Since the pension rights are adjusted based on the funding level, a smoothing parameter is used for risk sharing across cohorts. This parameter determines the extent of intergenerational risk sharing.

In our model, the benefits are defined implicitly through a pension indexation rule. The ultimate objective of the fund is to design an optimal benefits policy. For the special case of power utility and lump-sum pension payments, Gollier (2008) shows that the optimal benefit policy is a linear function of current assets. Another example of a benefit rule is given by Cui, Jong and Ponds (2011), who suggest a cohort-specific benefit rule, where each generation receives a fixed benefit plus a fraction of the returns earned on their contributions. Our benefit rule is similar to Gollier (2008) and Cui, Jong and Ponds (2011) but differs in the way that risk sharing is implemented.

2 Risk sharing by return smoothing

2.1 Model

We consider a pension fund with N working and K retired overlapping generations. Each year the oldest generation dies and a young generation enters the system. Each generation lives $N + K$ years. The pension scheme has mandatory enrollment. Participants accumulate pension rights when working and decumulate the rights during retirement. We assume that the entire cohort τ dies at time $\tau + K$. Generations are indexed by their retirement date τ; therefore at time t working generations are in $\mathcal{W}_t = \{\tau : t < \tau \leq t+N\}$ and retired generations are in $\mathcal{R}_t = \{\tau : t - K < \tau \leq t\}$.

The pension rights at time t of participants retiring at time τ are denoted by $Z_t(\tau)$. Working generations pay annual contributions equal to $C_t(\tau)$. Their pension rights accumulate as

$$Z_{t+1}(\tau) = \left(Z_t(\tau) + C_t(\tau)\right) I_t, \tag{1}$$

where $I_t = e^{i_t}$ is a pension return factor set by the fund. The pension rights of all generations are adjusted by the same I_t. Total contributions to the fund are defined as $C_t = \sum_{\tau \in \mathcal{W}_t} C_t(\tau)$. Retired generations receive pension payments $X_t(\tau)$ and decumulate their pension rights according to

$$Z_{t+1}(\tau) = \left(Z_t(\tau) - X_t(\tau)\right) I_t \tag{2}$$

After benefit payments, the remaining rights accumulate with the same pension return in a similar manner as the rights of the working generations. Thus, risk is not only shared during the working life but also during the retirement years, with benefit payments determined in part by the investment returns. While risk taking during retirement is not essential to the return smoothing mechanism described below, this assumption is in line with Koijen, Nijman and Werker (2011), who find that individuals should optimally convert a sizable proportion of their retirement wealth to variable annuities.

Benefit payments are assumed to come in the form of a variable annuity with i_t as the "assumed interest rate" (cf. Brown and Poterba (2006)),

$$X_t(\tau) = a_t(\tau) Z_t(\tau), \qquad \tau \in \mathcal{R}_t, \tag{3}$$

with annuity factor $a_t(\tau) = (1 - I_t^{-1})(1 - I_t^{t-K-\tau})^{-1}$. The payout would be a level annuity if the pension index I_t is constant. With a time-varying I_t the payout is a variable annuity. The total fund payout is $X_t = \sum_{\tau \in \mathcal{R}_t} X_t(\tau)$. The sum of all rights of both working and retired generations is

$$Z_t = \sum_{\tau \in \mathcal{W}_t \cup \mathcal{R}_t} Z_t(\tau) \tag{4}$$

The accrued pension rights can be considered as the liability of the pension fund. However, since Z_t is not a guaranteed claim on the fund, it is not a formal liability. It does not include the present value of the future expected increase in the pension rights of both working and retired generations. In that sense Z_t is similar to the actuarial concept of Accumulated Benefit Obligations (ABO).

The fund has assets A_t. After paying out the benefits to the retired generations and collecting the contributions from the working generations, assets are invested in the financial market where they earn a risky return $R_t = \mathrm{e}^{r_t}$. Therefore the assets of the fund develop according to

$$A_{t+1} = (A_t - X_t + C_t)\, R_{t+1} \tag{5}$$

The assets of the fund should be balanced against the accrued pension rights Z_t. In principle the fund has two policy variables: the pension indexation I_t, and the investment portfolio which determines the risk/return trade-off of the returns R_t. We consider I_t as the policy instrument of the fund, and assume that it sets I_t to maintain a long-term balance.

The pension rights are adjusted annually based on the returns earned by the fund portfolio by using a return smoothing mechanism. The pension rights are indexed depending on the funding position of the fund through

$$i_t = \mu_p + \alpha \ln \left(\frac{A_t}{Z_t} \right) \tag{6}$$

where μ_p is the expected return of the pension fund portfolio. The pension return i_t in year t is the expected portfolio return with an adjustment. This adjustment depends on the funding mismatch between assets and liabilities, where the parameter α is used for smoothing this mismatch. It determines the extent to which the accumulation of asset returns is passed on to accumulation of pension rights. This has a dampening effect on liabilities, resulting in a smooth development of the liabilities. Since only part of the mismatch is passed on to the current generations, the parameter α determines the extent to which intergenerational risk sharing is allowed.

The adjustment is symmetrical: pension rights increase faster than average if the funds assets yield above average returns. Pension rights can also decrease after bad returns. Since the fund does not offer a minimum return guarantee, all investment risk is borne by current and future generations. Linearity of the adjustment mechanisms also means that there is no limit to pension wealth.[8] The return smoothing makes risk sharing possible both over all cohorts in a given year and over time. It provides a relatively stable accumulation of pension rights compared to the more volatile asset return. The pension return rule provides a means of automatic adjustment of pension rights to enable the pension fund to maintain a healthy solvency level. However, since a shock is not immediately passed on to participants, new entrants may be faced with large unrealized bad returns. The pension return rule generates mean reversion in the funding ratio of the fund.[9]

To complete the model, we need to specify the contributions and portfolio choice. In the stylized model, we keep both fairly simple. Many assumptions of the stylized model can be relaxed in an empirical application. All the generations are assumed to have an equal number of workers. We assume that for each period t

$$C_t(\tau) = 1, \qquad \tau \in \mathcal{W}_t \tag{7}$$

Contributions are constant in real terms. With the fixed population size, they can be normalized to one. In a more elaborate realistic setting, it is possible to add income risk and demographics. If y_t is real per capita income and $n(\tau)$ the size of the entering cohort, contributions could be $C_t(\tau) = \kappa n(\tau) y_t$.

The pension fund invests in a portfolio of risk-free and risky assets. The return on the risky assets at time t is denoted by R_t^S. The risk-free interest rate is assumed constant and denoted by R^f. The share in the risky asset is denoted by ω, which is assumed to be constant. Hence the portfolio return at time t equals $R_t = \omega R_t^S + (1-\omega)R^f$. For the risky asset we assume that the log return $r_t^S = \ln R_t^S$ is independently normally distributed with mean μ and variance σ^2. Both the existence of a real risk-free rate

[8] Other possible alternatives for the pension return rule could be a S-shape function which would avoid extreme values in assets, but also slow down the recovery after a bad shock. However, since we want to illustrate the simple return smoothing for pension funds, we refrain from such extended rules.

[9] Since the fund does not have liabilities in a strict sense, the term funding ratio is not entirely accurate. However, it is used here and elsewhere in the chapter for lack of a better term.

and the normality of the returns are clearly simplifications to explain the main ideas of the model. Under the assumption of constant investment opportunities and a constant population size, the optimal portfolio weight ω will also be constant.

2.2 Preferences and the fund policy

This section considers the optimal return smoothing parameter α in (6). We assume that the pension fund designs the adjustment rule with the aim of maximizing the welfare of all the participants of the fund. Since the pension fund board represents both current working and retired cohorts as well as future generations, it faces an intertemporal trade-off in the benefits paid to current and future retirees. Since there are also K overlapping generations receiving benefits at each time t, the fund should also consider the intratemporal distribution of benefits over different age groups. Several utility functions express both the intratemporal and intertemporal effects of the benefits policies. We use the following simple parametric specification

$$Q_t = \mathrm{E}_t \left[\sum_{s=0}^{\infty} \delta^s U_{t+s} \right], \tag{8}$$

and where

$$U_t = \frac{V_t^{1-\gamma}}{1-\gamma} \tag{9}$$

$$V_t = \left(\sum_{\tau \in \mathcal{R}_t} X_t(\tau)^\rho \right)^{1/\rho} \tag{10}$$

The intertemporal choice is determined by the pension fund's social discount factor δ and the constant relative risk aversion parameter γ. The intratemporal choice is specified by using a CES specification as an aggregator of the benefits to the different overlapping generations. The parameter ρ defines the preference for equality over cohorts. If $\rho = 1$, the fund is only concerned with the total payout and $V_t = X_t$. For smaller ρ the pension fund views a large dispersion in payments made in the same year to different cohorts as an undesirable feature. Such preferences induce a degree of fairness in the distribution of payments. In the limit as $\rho \to -\infty$ the intratemporal utility reduces to $V_t = \min_{\tau \in \mathcal{R}_t} X_t(\tau)$, which is the 'Rawlsian social welfare functional' and appears standard in the literature (c.f. Kreps (1990), p 160). Since the CES function is first degree homogeneous, an overall proportional increase in payments $X_t(\tau)$ to all cohorts raises the utility V_t by the same proportion. The CES specification is related to a multiplicative habit model, in which individuals view their utility as a function of their benefits relative to the average payments to all retirees.

The pension fund's utility function is still time-separable in V_s. The utility function is not necessarily separable in the benefit payments of the individual cohorts. With an objective function as in (8), the pension board makes decisions for the fund such that higher payouts are better, smoothing over time is encouraged (γ) and intratemporal equality has value (ρ).

The optimal risk-sharing parameter α is chosen such that it maximizes the discounted sum of expected utility generated by pension wealth. The fund faces a budget constraint in its optimization, since contributions are fixed by design as we are considering a collective defined contribution pension system. Changing α generally has two effects. First, a smaller α provides more smoothing and will reduce risk for older generations. Secondly, a smaller α increases the volatility of the funding ratio, which increases the risk for young generations of entering the fund when its funding ratio is very low.

To obtain a transparent adjustment rule with a fixed smoothing parameter we solve for the optimal α that maximizes the average of Q_0 when we start the system in a steady state. From there, we simulate a series of paths of length T and compute the realized fund utilities

$$Q_0 = \sum_{t=0}^{T} \delta^t \frac{1}{1-\gamma} \left(\sum_{\tau \in \mathcal{R}_t} X_t(\tau)^\rho \right)^{(1-\gamma)/\rho}, \tag{11}$$

and find the value of α that maximizes the average $\bar{Q}_0$.

For the optimal smoothing parameter, we look at the utility from the perspective of the pension fund, not from an individual's perspective. For an individual cohort retiring at time τ, the life-time utility is $V_{\tau-N}(\tau) = \mathrm{E}_{\tau-N} \sum_{s=0}^{K-1} \delta_\tau^s U(X_{\tau+s}(\tau))$, where U is a utility function and δ is pension fund's social discount factor. The pension fund's objective is not simply the sum of the individual utilities, but rather at each time t it calculates the utility by including all cohorts that are alive, both working and retired, at that time. The fund needs to aggregate individual cohorts' utilities, without being able to assess whether individual utilities are preference independent. In defining preferences for a pension scheme, inter-cohort comparisons may be an important consideration, since generations may view their benefits from the system relative to what other cohorts receive at the same time. A reduction in pension seems less painful when everyone is facing a reduction. Additionally, an extensive literature on habit formation (cf. Abel (1990)) questions the assumption of preference independence. Luttmer (2005) provides empirical evidence for the effect of the neighbor's consumption on the individual's own well-being. For this reason, we have chosen the objective function (8).

3 Results

We analyse the pension system using simulation. As in Cui, Jong and Ponds (2011) we take $N = 40$ and $K = 15$ and thus have 55 overlapping generations. We set the return parameters for the risky asset at $\mu = 5\%$ and $\sigma = 15\%$, and its portfolio share at $\omega = 0.6$. The risk-free rate is $R^f = 2\%$. We start the simulations at $t = 0$, assuming that returns before $t = 0$ have always been equal to the expected return $\mathrm{E}[R_t]$. Section 5.1 provides the details for the initial pension rights $Z_0(\tau)$. The initial conditions for the pension rights are set independently of the adjustment parameter α.

3.1 Stochastic steady state

[Insert Table 1 here]

Using 100,000 simulated paths, we evaluate the cross-sectional distribution of the outcomes at time $t = 200$, assuming that after such a long period we have reached the stationary distribution. Table 1 presents the value of various pension fund indicators for different values of the policy parameter α. First of all, with $\alpha < 1$, the distribution of the pension index I_t is much more concentrated than that of the actual portfolio returns R_t, showing that the smoothing of risk across generations implies less uncertainty in pension returns. The standard deviation (and 95% interval) shrinks with α. Another aspect of the smoothing is that the pension return is highly autocorrelated, even though returns themselves are not autocorrelated. The autocorrelation increases as we decrease α.

[Insert Figure 1 here]

To characterize the pension returns further, we consider the regression of the pension return on current and lagged asset returns

$$i_t = c + \sum_{j=0}^{60} b_j r_{t-j} + e_t \tag{12}$$

Figure 1 plots the regression coefficients for different values of α. For $\alpha < 1$ the coefficients show a slowly (geometrically) declining pattern. The coefficient b_0 for the current return is approximately the weight in the risky asset (ω) times the risk-sharing parameter (α). The coefficients decline faster for the larger α; for small α, the pension return is simply a long weighted moving average of market returns. The log-linear relationship between the pension return and the market returns is almost perfect, with R^2 of the regression above 0.993.

[Insert Figure 2 here]

Table 1 also shows the downside of return smoothing: a lower α implies a more volatile funding ratio. This is because the fund does not adjust quickly to the mismatch in funding ratio with a lower value of α, but delays the shock over a long horizon. This has implications for generations entering the fund during times of underfunding. For them, the return smoothing may be sub-optimal. When entering at a low funding ratio, the expected return will be below the market average. New entrants will thus face a low expected return, although they will later benefit from the lower risk at retirement. This shows the tradeoff between uncertainty in pension returns and uncertainty in funding ratio. The distributions of pension return and funding ratio are shown in figures 2 and 3 respectively. It is clear from the figures that the model with highest risk sharing

($\alpha = 0.25$) has the highest standard deviation of funding ratio but lowest standard deviation of pension return.

[Insert Figure 3 here]

Table 1 also shows that the average benefit level and the average pension return are independent of α. Given the specification of the return smoothing rule (6), the average funding ratio is equal to one, and assets are on average equal to the total liabilities. The average level of the assets is also independent of the adjustment parameter α. In the stylized model, the effect of α is concentrated on the volatility of the funding ratio and the stability of the pension return and benefits.

[Insert Figure 4 here]

To learn more about the stability and adjustment speed of the system, we rerun the simulations with different initial conditions for the assets at time $t = 0$. We experiment with different values for the funding ratio $f_0 = A_0/Z_0$ ranging between 80% and 120%, Given the funding ratio, the initial assets are taken as the funding ratio times the pension rights, $A_0 = f_0 Z_0$. Figure 4 presents the result of the shock to the assets on the evolution of the system. The fund reaches the steady-state ratio of approximately one after some years. Convergence to the steady state is faster in case of a higher value for α.

3.2 Optimal risk sharing

To compute the optimal adjustment parameter, we need to choose the pension fund preference parameters. To compare the welfare of different generations over time, we require a discount factor (δ) that reflects the rate of social time preference. This social discount factor is used to discount future utility from the point of view of the social planner. There is considerable debate on the choice of this long-term discount rate. Studies that look at intergenerational risk-sharing in the pension context like Gollier (2008) use a discount factor of 0.9728, whereas Cui, Jong and Ponds (2011) use 0.9615 and in climate change literature, Nordhaus (2007) uses 0.9852. We compare results for different values of δ between 0.96 and 0.98 as the values for the discount factor. We also use alternative values for the preference parameters γ and ρ. For each set of preference parameters, we determine numerically the value of α that maximises the pension fund objective (11).

[Insert Table 2 here]

Table 2 reports the results. The optimal value of the risk-sharing parameter generally falls between 1/4 and 1/3. Thus it is optimal to have the risk sharing such that about one third or one fourth of the funding mismatch is passed on to all participants and retirees in the next year. The optimal value for α decreases as the discount factor decreases. A lower discount factor implies that the pension fund cares less about future generations. Thus lower α is preferred, which means more risk sharing, less uncertainty in pension returns and high auto-correlation but also high uncertainty on the funding ratio (see table 1). High uncertainty on the funding ratio is not preferable for cohorts just entering the fund, as they may be joining an underfunded pension system. The parameter ρ is used to emphasize equality across generations at a particular point in time. If $\rho = 1$, the fund is only concerned with the total payout. For smaller ρ ,the pension fund views a large dispersion in payments made in the same year to different cohorts as an undesirable feature. Thus it is optimal to have more risk sharing if intra-generational equality is valued.

As an economically interpretable measure of the cost of a suboptimal α, we calculate a certainty equivalent. For an arbitrary value of α, we compute the objective $\bar{Q}_0(\alpha)$. By construction this will be less than the maximized value $\bar{Q}_0$ at the optimal α. We then find a constant c such that if we multiply all the payouts $X_t(\tau)$ of the suboptimal policy with that constant, these benefits would yield the same expected utility as the optimal policy. This is equivalent to $c = \left(\bar{Q}_0/\bar{Q}_0(\alpha)\right)^{\frac{1}{1-\gamma}}$.

[Insert Figure 5 here]

Figure 5 presents the value of c against the value of given α for the preference parameters $\gamma = 3$, $\delta = 0.97$ and $\rho = 1$, which corresponds to an optimal α of 0.31 (see table 2). At $\alpha = 1$ the pension payments should be increased by as much as 7% per annum as compared to the benefits at the optimal α.[10] The certainty equivalents are U-shaped, showing the clear optimum for α. Too much risk sharing is not optimal. The limiting case of $\alpha = 0$, a defined benefit system where the pension payments are fixed, is not sustainable in this model without sponsor guarantees.

Another way to measure the value of optimal risk sharing is by computing the equivalent initial funding ratio at different values of α. The equivalent funding ratio is such that the utility generated by a underfunded pension system with risk sharing α is the same as the utility generated by a pension system that is fully funded ($f_0 = 1$) and has no risk smoothing ($\alpha = 1$). Without risk smoothing, all return shocks in the system are immediately passed on to the current generations. How much lower can the initial funding ratio f_0 be at different values of α without a loss in expected utility? We find f_0 by numerically solving

$$\bar{Q}_0(f_0, \alpha) = \bar{Q}_0(f_0 = 1, \alpha = 1) \tag{13}$$

The funding ratio of f_0 can be interpreted as the minimum funding ratio that needs to be maintained so that it is still welfare improving for future generations to enter this pension fund. The collective defined contribution pension system in our model goes through cycles of over and under-funding depending on the returns of the fund portfolio. Some generations therefore enter the system when it is in a state of underfunding. The generations that enter the fund at a funding ratio below the minimum level may refuse to enter into the mandatory pension contract and choose to renegotiate, thus rendering the pension system unsustainable. If the funding ratio drops below this level, newly entering cohorts would be better off in welfare terms to save and invest individually. Even for them, however, it could still be worthwhile to enter the pension fund for reasons outside the model, for example to avoid welfare loss resulting from financial mistakes by households and to benefit from scale economies of pension funds.

[Insert Figure 6 here]

[10] See Bonenkamp et al. (2014) for a comparison of welfare gains of intergeneration risk sharing in the literature which range from 1 to 25%.

Figure 6 shows the minimum funding ratio for a given level of risk sharing. For the optimal level of risk sharing, the minimum required funding ratio is approximately 89% percent. Therefore about 11% underfunding is sustainable without creating incentives to abandon the risk smoothing rule. Table 3 shows probabilities of underfunding (funding ratio less than one) of the pension fund with various values for α. Note that although for $\alpha = 0.5$, the probability of underfunding is about 53% the probability of the funding ratio falling below the sustainability threshold is relatively low at approximately 19%.

[Insert Table 3 here]

3.3 Implicit exposure to risky asset

Optimal life-cycle theory (Merton (1969), Merton (1971), Bodie, Merton and Samuelson (1992) and Bovenberg et al. (2007)) states that a young person should hold a larger component of financial wealth in risky assets than an older person. Some models advise borrowing to invest in risky investments at a young age when financial capital is small and human capital is large, in order to obtain optimal exposure to the risky assets.[11] However, young people might be reluctant to do so, which may explain the emergence of institutionally managed life-cycle or target date funds which invest more conservatively over time. Individuals may also lack the capability to do so, due to limited financial literacy (Lusardi and Mitchelli, 2007). Pension wealth is a significant component of an individual's financial wealth and therefore there are arguments in favor of age-dependent exposure to risky assets by a pension fund on behalf of the members of the fund (Bikker et al., 2012). We evaluate how the benefits provided by a collective defined contribution system with risk sharing implemented via return-smoothing are dependent on past returns on the risky asset. To keep the analysis simple, we assume here that $N = 40$ and $K = 1$, thus considering lump-sum pension wealth. We regress this lump-sum pension wealth at retirement date on the past returns on equity, using the regression

$$\log(X_t) = c + \sum_{j=1}^{60} b_j r_{t-j} + e_t, \tag{14}$$

[11] This also depends on the riskiness of human capital including covariance with stock returns.

similar to (12). Figure 7 plots the coefficients for different values of α. For $\alpha = 1$ the payouts only depend on the returns during the working life and payouts are most sensitive to the most recent returns close to retirement. For smaller α the payout also depends on returns before a generation enters the fund. This illustrates the benefits of intergenerational risk sharing. When a young worker enters the pension system, he has a higher exposure to risky asset than in a system with no risk sharing. This is in line with the optimal life-cycle theory. Initially the exposure still increases with age due to annual contributions, but the most important element of the graph is the decreasing exposure closer to retirement. Because of the return smoothing, the exposure to the risky asset decreases automatically, again in line with life cycle theory.

[Insert Figure 7 here]

4 Concluding remarks

We have analyzed a stylized model of a collective pension system that allows for intergenerational risk sharing based on return smoothing. The pension fund's board sets the optimal amount of return smoothing taking into account the payouts to current and future generations as well as intra-generational fairness. Since a tradeoff exists between uncertainty in pension returns and uncertainty in the funding ratio, there is an optimal level of return smoothing. Given our specification of the risk and return tradeoff in the financial market, and given the preference parameters of the pension fund, the optimal amount of return smoothing implies that only less than one third of a shock should be passed on immediately. As a result, the pension return for participants becomes a long weighted moving average of past returns. The implied age-related exposure to the risky asset return automatically attains a shape that is broadly in line with life-cycle portfolio choice. Certainty equivalent calculations show that the welfare improvements provided by the intergenerational risk sharing are economically significant in the order of a 6% increase in annual benefits compared to the case without return smoothing.

The current stylized model can be extended in several directions. First, instead of focusing solely on financial risks, an empirical model should also include income and demographic risks. In the collective defined contribution system these risks affect both the contributions and the retirement benefits. Secondly, a more realistic return model would deviate from a normal distribution with independent returns and constant mean and variance. Thirdly, we assumed the existence of a real risk-free rate. With only nominal financial instruments, inflation should also be included as an additional risk factor. Fourthly, given these additional risks, the portfolio allocation problem can be treated more seriously by including additional financial instruments and optimizing over the portfolio weights.

A separate aspect is the robustness of the pension system. In our stylized model, the log-linear adjustment rule contains the expected portfolio return as a constant that determines the average pension return. This parameter will be hard to estimate from the data. Misspecification of the expected return parameter may have consequences for the dynamic behavior of the system. For example, setting the expected return too low will give current generations a low indexation of their pension rights, leading to the creation of a persistent financial buffer in the fund that will benefit future generations. To avoid an explosion or collapse of the system the simple log-linear rule needs amendment at the extremes when the assets are either exceptionally high or low.

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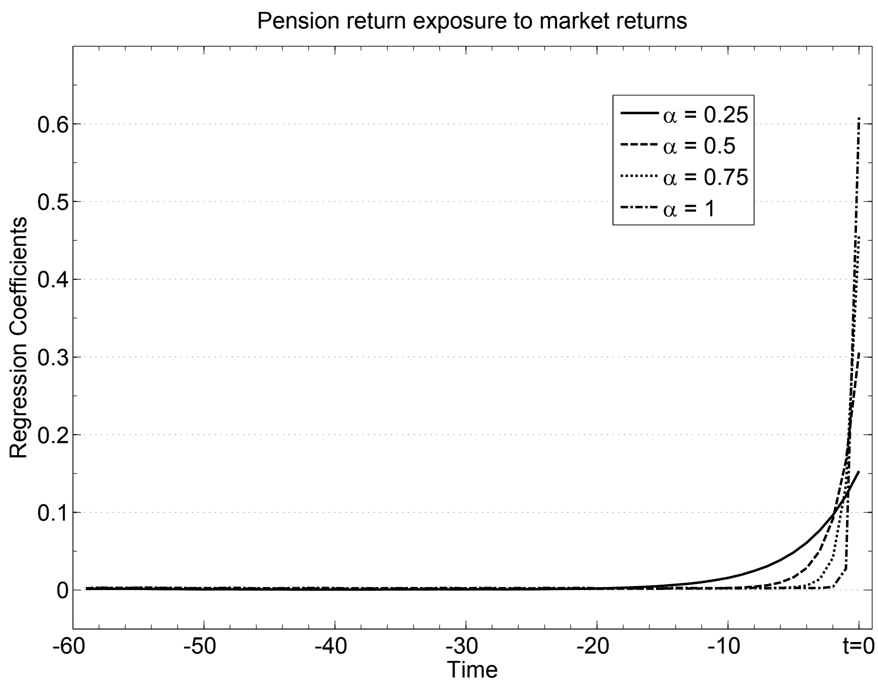

Figure 1: Smoothing effect of the parameter α. This figure plots the coefficients of regression (12) for different values of the return smoothing parameter α.

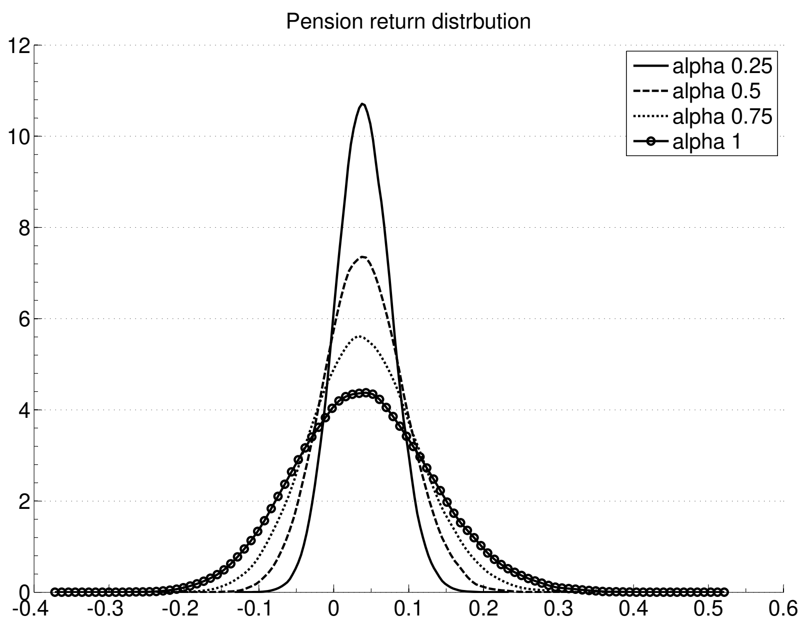

Figure 2: Distribution of pension return in steady state for different values of the return smoothing parameter α.

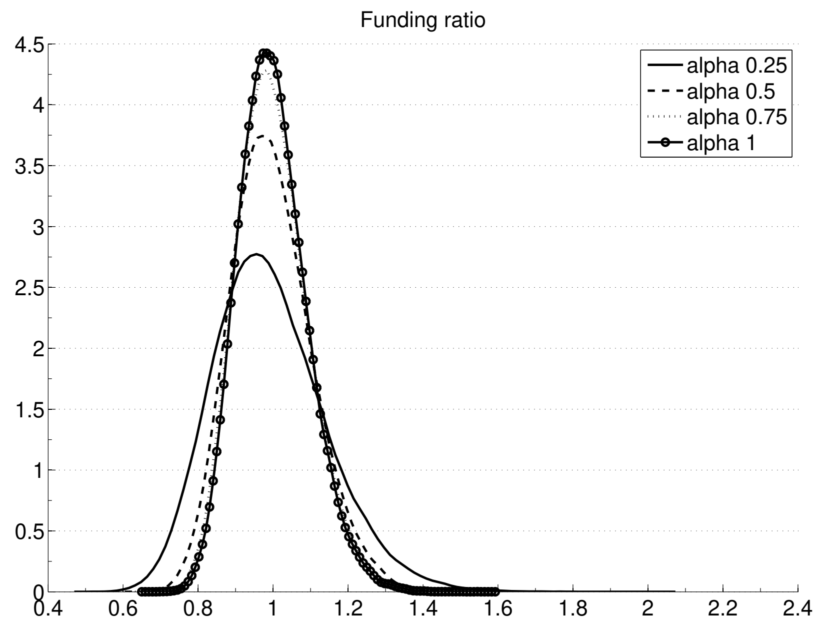

Figure 3: Distribution of funding ratio in steady state for different values of the return smoothing parameter α.

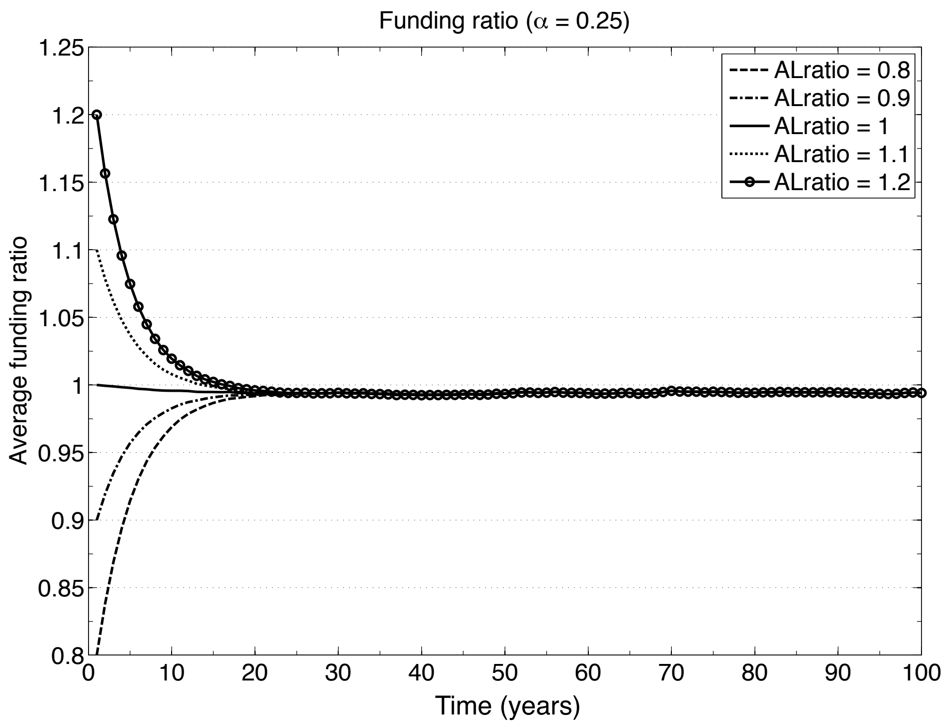

Figure 4: Average funding ratio of the pension fund following different initial funding ratios.

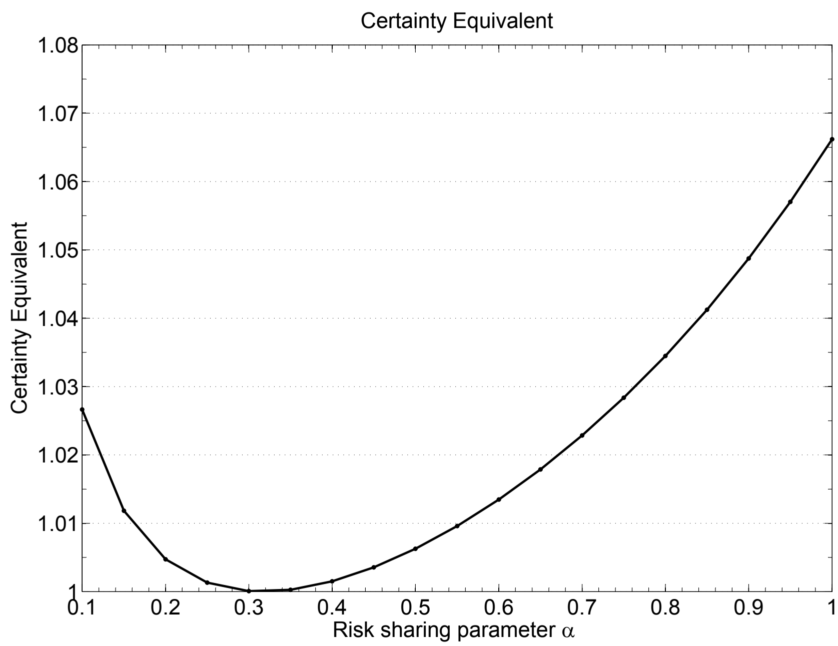

Figure 5: This figure shows certainty equivalent per annum. Preference parameters are $\gamma = 3$, $\delta = 0.97$ and $\rho = 1$. Return parameters are as in table 1.

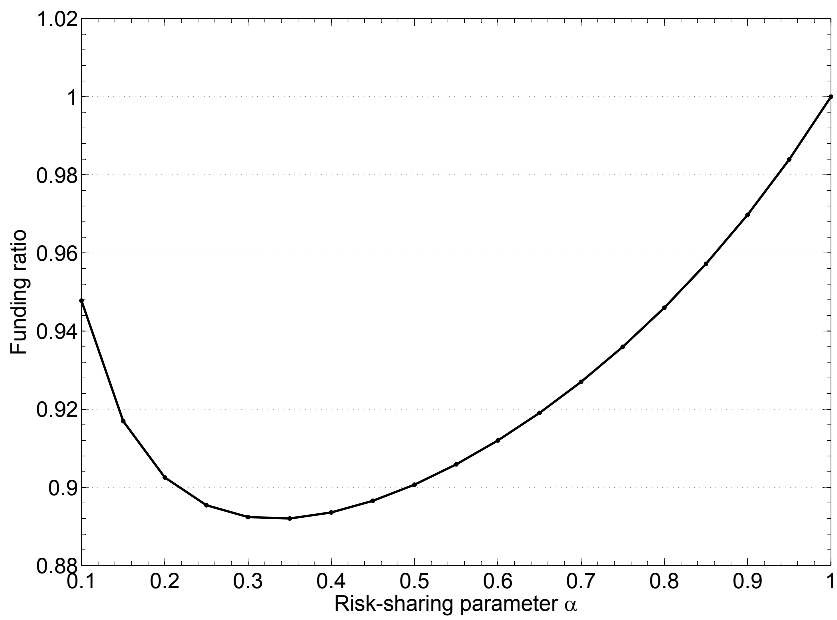

Figure 6: This figure shows the equivalent funding ratio (EFR) for different levels of risk-sharing. It is calculated using (13).

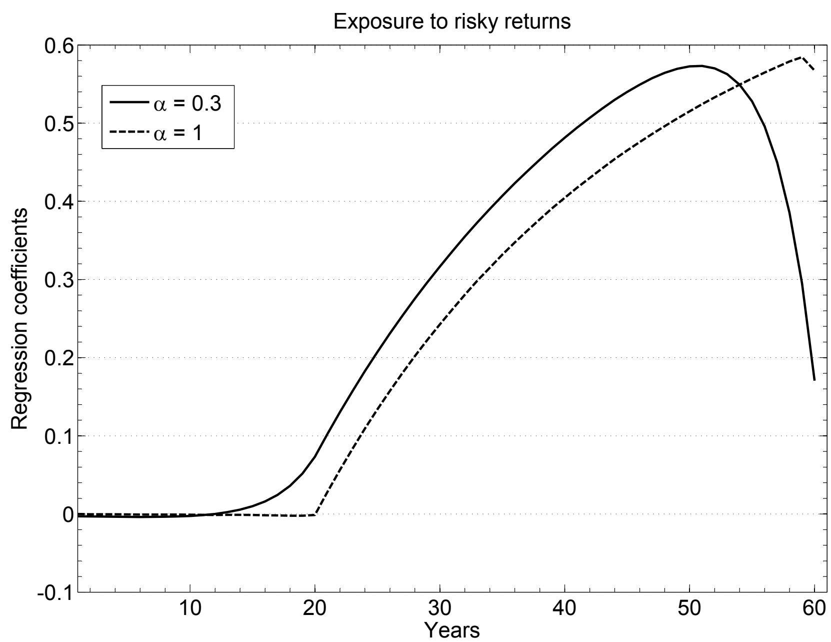

Figure 7: Sensitivity of the pension wealth to past returns. This figure plots the coefficients of regression (14) for two different values of the return smoothing parameter α.

Table 1: Pension fund indicators in steady state

	Mean	95 percentile	5 percentile	Std. error	Autocorrelation
Panel A. α = 0.25					
Pension return	1.042	1.109	0.981	0.0001	0.7063
Payouts	157.0	326.0	63.0	0.2913	0.8804
Assets	2660	5190	1210	4.2920	0.8754
Liabilities	2630	4730	1370	3.5483	0.9454
Funding ratio	0.995	1.264	0.774	0.0005	0.7027
Panel A. α = 0.5					
Pension return	1.044	1.143	0.956	0.0002	0.4808
Payouts	157.0	315.0	64.0	0.2684	0.7424
Assets	2656	4810	1332	3.6136	0.8626
Liabilities	2640	4610	1410	3.3012	0.9332
Funding ratio	0.999	1.193	0.835	0.0003	0.4781
Panel A. α = 0.75					
Pension return	1.046	1.177	0.930	0.0002	0.2524
Payouts	157.0	317.0	63.0	0.2704	0.5612
Assets	2660	4700	1370	3.4216	0.8554
Liabilities	2650	4570	1420	3.2331	0.9140
Funding ratio	1.001	1.170	0.856	0.0003	0.2510
Panel A. α = 1					
Pension return	1.048	1.217	0.902	0.0003	0.0236
Payouts	157.0	324.0	60.0	0.2814	0.3506
Assets	2660	4660	1400	3.3391	0.8506
Liabilities	2660	4560	1430	3.2134	0.8859
Funding ratio	1.002	1.164	0.863	0.0003	0.0241

Note: This table shows the average, standard deviation, 95 and 5 percentiles, and first order autocorrelation for various outcomes of the pension fund in steady state (here at time point 200 after start) for different values of the smoothing parameter α. The number of simulated paths is 100,000. The mean return for each of the simulations is 1.046, with 95th percentile = 1.216 and 5th percentile = 0.901. The parameters of the log-normal distribution are $\mu = 5\%$ and $\sigma = 15\%$; the share in the risky asset is fixed at $\omega = 0.6$. The risk-free rate is $R^f = 2\%$

Table 2: Optimal risk sharing parameter α

ρ	Discount factor (δ)		
	0.96	0.97	0.98
Panel A. γ = *2*			
0.50	0.22	0.29	0.35
0.75	0.23	0.30	0.35
1.00	0.24	0.31	0.36
Panel B. γ = *3*			
0.50	0.23	0.30	0.35
0.75	0.24	0.30	0.36
1.00	0.25	0.31	0.36

Note: This table shows the optimal value of α obtained by numerically maximizing the pension fund objective (11). Return parameters are as in table 1.

Table 3: Sustainability of the pension fund

α	P(FR<0.7)	P(FR<1)	P(FR>1.3)	EFR ($\gamma = 3$)	P(FR<EFR)
0.10	0.128	0.592	0.127	0.948	0.515
0.25	0.010	0.555	0.034	0.895	0.268
0.50	0.000	0.532	0.009	0.901	0.187
0.75	0.000	0.523	0.005	0.936	0.260
1.00	0.000	0.519	0.004	1.000	0.519

Note: This table reports the probability of underfunding, significant shortfall and significant overfunding. The table also reports equivalent funding ratio (EFR) for different levels of risk-sharing and the probability that the funding ratio falls below this level. It is calculated using (13). Return parameters are as in table 1.

5 Appendix

5.1 Initial conditions

We start our simulations from a deterministic steady state. At time $t = 0$ the fund starts as if returns have always been equal to $R = \mathrm{E}[\mathrm{e}^r] = \exp(\mu_p + \frac{1}{2}\omega\sigma^2)$ and pension indexation equal to $I = R$. With contributions normalised to one, the initial conditions for pension rights become

$$Z_0(\tau) = \begin{cases} R\frac{R^{N-\tau}-1}{R-1} & \tau \in \mathcal{W}_0 \\ R\frac{R^{N-\tau}-1}{R-1} \times \frac{1-R^{-(K+\tau)}}{1-R^{-K}} & \tau \in \mathcal{R}_0 \end{cases} \tag{15}$$

and $Z_0(\tau) = 0$ for all other τ. The initial conditional can be interpreted as the value of individual accounts, where each cohort has been contributing one unit per year during the working life and where retired generations have been extracting annuity benefits. Implicitly the payouts to the retired cohorts are then equal to

$$X_0(\tau) = \frac{R^N - 1}{1 - R^{-K}} \tag{16}$$

To fix the inital funding ratio at one, the starting value for the assets is equal to the sum of the pension rights

$$A_0 = \sum_{\tau \in \mathcal{W}_0 \cup R_0} Z_0(\tau) \tag{17}$$

These initial conditions are independent of the smoothing parameter α.