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Longevity Risk in Solvency II

Standard Formula and Internal Model Compared

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Longevity Risk in Solvency II: Standard Formula and Internal Model Compared

by
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Abstract

Two methods to determine the Solvency Capital Requirement (SCR) for longevity risk as described by the Solvency II Directive are compared: the Solvency II standard approach and an internal model based on the stochastic Lee-Carter mortality model. The Lee-Carter (1992) model is used to estimate and forecast future mortality for Dutch gender-specific data. Forecasts from this model are used to simulate portfolio participants' lifetimes to determine the SCR with the 99.5% VaR. The standard approach uses a one-off shock for the best estimate one-year mortality rates as a simplification for the 99.5% Value-at-Risk of the internal model, both in excess of the best estimate value of the liabilities for sample pension portfolios.

The results indicate that, in case investment risk is hedged and interest rate risk has been accounted for, the internal model proposed in this thesis consistently gives lower SCRs than the standard approach. Although the SCRs' size depends on portfolio composition and factors such as the estimation period, this may be a reason for insurers to choose to develop an internal model rather than using the standard approach. However, one should be reminded that the Lee-Carter model used here only takes into account trend risk, while disregarding level, model and parameter risk. The uncertainty with respect to portfolio-specific mortality is found to be of significant influence to the results, although its impact still seems smaller than the shock proposed by Solvency II. This result falls in line with the insurance field arguing that the 20% shock size is too large.

KEYWORDS: longevity risk, pensions, insurance, forecasting, (portfolio) mortality risk, Solvency II, stochastic mortality modeling, Solvency Capital Requirement, SCR, life expectancy.

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1 Introduction

Insurance companies are generally exposed to risks which can influence their financial position: e.g., interest rate risk, catastrophe risk, mortality risk. Insurers are usually considered solvent when they are able to meet their long-term financial obligations, while taking into account the aforementioned risks. In order to ensure that insurance companies across Europe limit their risk of insolvency to a desired level[1], the European Insurance and Occupational Pensions Authority (EIOPA) has proposed the Solvency II Directive, both to harmonize EU insurance regulation, as well as introduce risk-based regulation more tailored to the risk profile of the respective insurance company. One aspect of an annuity provider and pension insurer's risk profile is the age its participants are expected to obtain, the life expectancy.

	Men				Women			
	1950	1975	2000	2050	1950	1975	2000	2050
Austria	62.2	67.7	75.0	82.8	67.3	74.7	81.1	87.2
Belgium	63.8	68.8	74.6	82.1	68.9	75.2	80.9	87.5
Czech Republic	62.0	67.0	71.6	79.7	66.8	74.0	78.3	84.1
Denmark	69.1	71.3	74.4	81.4	71.5	77.0	79.1	85.2
UK	66.5*	69.7*	75.7*	82.4	71.3*	75.9*	80.4*	86.7
Finland	60.4	67.4	74.2	81.9	67.9	76.1	81.0	86.6
France	63.4	69.0	75.3	82.3	69.2	76.9	82.8	87.9
Germany	n/a	68.1**	75.3**	82.0	n/a	74.7**	81.2**	86.8
Hungary	59.9	66.3	67.4	78.1	64.3	72.4	76.0	83.4
Italy	64.0	69.5	76.6	82.8	67.5	75.9	82.5	87.8
Netherlands	70.3	71.5	75.7	81.1	72.6	77.7	80.8	85.2
Spain	59.4	70.5	75.8	81.7	64.2	76.3	82.7	87.3
Sweden	69.8	72.2	77.4	82.6	72.4	77.9	82.0	86.6

* England and Wales
** West Germany

Table 1.1: Historical and forecasted gender-specific period life expectancy at birth for selected EU countries, in years. Source: Hári (2007).

Table 1.1 shows the historical and forecasted period life expectancies (see Section 2.1 for elaboration on this term) at birth, for selected EU countries, where "the historical life expectancies ... are taken from the Human Mortality Database[2], [whereas] the forecasted ones are based on the report by the Economic Policy Committee and European Commission (2005)" (Hári, 2007). When looking at the development of the life expectancy of populations of European countries in Table 1.1, it becomes clear that life expectancy for both genders has increased considerably over the past 50 years and is also expected to improve even further in the future, as a result of an expected decrease in mortality rates.

[1]Solvency II requires a 99.5% probability of remaining solvent during one year, i.e., being able to meet their financial obligations.

[2]Human Mortality Database, University of Californa, Berkely (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or humanmortality.de (data downloaded on 4 November 2005).

One should bear in mind, however, that the forecasted life expectancies are dependent on the method used to forecast mortality rates and that there is uncertainty in these predictions caused by longevity risk. Academics define longevity risk as "[the] uncertainty ... regarding the future development of mortality" (De Waegenaere, Melenberg, & Stevens, 2010). Institutions offering pension products or life insurance have to take this expected increase in life expectancy and the uncertainty in the development of mortality into account when assessing the future liabilities of their portfolios, since longevity risk cannot be diversified by pooling and it is hardly hedgeable since only a small and relatively illiquid market exists to securitize this risk. This is especially relevant since increases in life expectancy being underestimated are a worldwide phenomenon, often caused by setting a maximum age based on biological reasons (Oeppen & Vaupel, 2002; Wilmoth, 2000). If life expectancies are underestimated, the premiums paid or the life insurance offered are either too low or too high respectively, which could lead to problems for an insurer's solvency.

For insurers there are two types of risk related to mortality: the longevity risk, i.e., the risk of decreasing mortality rates, and mortality risk, the risk of increasing mortality rates.[3] Note that longevity risk is a long-term risk, i.e., this is a risk concerning changes in the trend, whereas mortality risk is a short-term risk, comparable to a natural catastrophe risk. In order to account for these risks, Solvency II requires the insurance company to hold a so-called buffer in terms of capital: the Solvency Capital Requirement (SCR).

Solvency II offers two methods to calculate these SCRs: a standard formula as formulated by Solvency II based on a one-off shock on all mortality rates and a stochastic mortality model (referred to in Solvency II as an internal model) which is able to estimate and forecasts future mortality rates and the uncertainty in the forecasts. The most well-known stochastic mortality model was proposed by Lee & Carter (1992), which provides a relatively simple model to extrapolate the time trend in mortality rates. After this Lee-Carter model, many extensions and adaptions have been proposed. Dutch institutions are also performing research on future mortality: Statistics Netherlands (Centraal Bureau voor de Statistiek, CBS), the Royal Actuarial Association (Koninklijk Actuarieel Genootschap, AG) and the Pension and Annuity Tables group (Pensioen- en Lijfrentetafels, PLT) of the Dutch Association of Insurers (Verbond van Verzekeraars, VvV) have provided a forecast for Dutch mortality.

These stochastic models can be used to forecast mortality and quantify the uncertainty with respect to the death rates. Using the forecasted mortality rates, it is then possible to also calculate SCRs to be held, according to Solvency II, as described in Section 4. Past studies (Börger, 2010; Börger et al., 2014; Coppola & D'Amato, 2014) have shown

[3]Note that these differ from the scientific definitions of longevity and mortality risk: longevity risk is defined as the uncertainty in future mortality rates, whereas (individual) mortality risk is the risk stemming from the fact that, given the death probabilities, the remaining lifetime of an individual is a random variable.

that the size of these SCRs can differ significantly from the standard formula SCRs. If that is indeed the case, insurers may find it more convenient to develop their own internal model, rather than the standard formula proposed by Solvency II.

The aim of this thesis is to compare the longevity risk SCRs found from the Solvency II shock to the SCRs obtained after modeling and forecasting mortality rates using a well-known stochastic mortality model. First, the Lee-Carter (1992) stochastic mortality model will be estimated for the entire Dutch population based on population data, to extract the general time trend, as well as discern age-related influence. After estimation, a mortality model is available with which future mortality rates can be forecasted and uncertainty can be measured via confidence intervals. This model is then used as an internal model to calculate the longevity risk SCRs and compare the results to the SCRs based on Solvency II's standard formula. The results indicate that the internal model gives lower SCRs than the Solvency II standard approach, and it could be concluded that the shock size set by Solvency II regulation is too prudent. The exact size of the SCRs depend mainly on the portfolio composition (average age of the portfolio) and whether or not the uncertainty surrounding experience mortality is taken into account, but for all cases the Solvency II standard approach yields higher SCRs. This already indicates that an insurer seeking to decrease its buffers should perhaps opt for an internal model rather than using the Solvency II standard formula. Insurers could wish to do so to be able to pay more dividends to their shareholders. A significant impact follows from the uncertainty in the adverse selection experienced by insurers: namely that portfolio participants have considerably lower mortality rates than the general population. However, because of limited data the factors for experience mortality are not so straightforward to estimate. Nevertheless, their influence on the SCR calculations are relevant.

This thesis is structured as follows: Section 2 introduces the notation used throughout the scope of the thesis. Section 3.1 and Section 4.1 appraises relevant literature about both mortality models as well as articles concerning Solvency (II) requirements with respect to longevity and mortality risk. In Section 3.2 and 4.2 the data is described. Section 3.3 and 4.3 follow by first describing the estimation method of the mortality models of Lee & Carter (1992) and afterwards showing how the SCR is determined for longevity risk under certain assumptions. In Section 3.4 and 4.4 the results of the mortality model estimation and the SCRs are presented. Section 5 concludes and proposes material for future research, following from the results found in this thesis.

2 Notation

The notation introduced and explained below is used throughout this thesis and is commonly used in studies about mortality modeling. More elaborate explanations can be found in Pitacco, Denuit, Haberman, & Olivieri (2009), for example. Note that, throughout this thesis, I take t and x as follows: a person is aged x on January 1 of the year t, also assuming his or her birthday is on January 1.

- $p_{x,t}$ denotes the probability that at time t, a person born on January 1 of year $t-x$ will survive at least the year t, so live up to January 1 of year $t+1$. Similarly, a k-year survival probability is defined as ${}_kp_{x,t}$ for someone aged x in year t (if $k=0$, then ${}_0p_{x,t}=1$), which means the chance of surviving until January 1 of year $t+k$;

- $q_{x,t}$ denotes the probability that at time t, a person with age x will pass away during the year t before reaching January 1 of year $t+1$, so $q_{x,t}=1-p_{x,t}$. Similarly, a k-year death probability is defined as ${}_kq_{x,t}$ for someone aged x in year t, so ${}_kq_{x,t}=1-{}_kp_{x,t}$, and ${}_0q_{x,t}=0$, and this is defined as the probability of someone dying before reaching January 1 of year $t+k$. In applications, k is typically integer, but need not be. Throughout the rest of the thesis, the terms mortality rates and death rates/probabilities typically refer to one-year death probabilities;

- $\tilde{T}_{x,t}$ denotes the remaining lifetime of a person aged x at time t: this is a random variable with the probability distribution $P(\tilde{T}_{x,t}\leq h)=P(\tilde{T}_{0,t-x}\leq x+h \mid \tilde{T}_{0,t-x}\geq x)$, $h\geq 0$;

- $T_{x,t}=[\tilde{T}_{x,t}]$ is the curtate remaining lifetime of a person aged x in year t, with $P(T_{x,t}=k)=P(k\leq \tilde{T}_{x,t}<k+1)$, see also Gerber (1997);

- $e_{x,t}$ denotes the remaining life expectancy of a person with age x at time t, so $e_{x,t}=\mathbb{E}_t(T_{x,t})$.

- $\mu_{x,t}$ denotes the force of mortality of a person with age x at time t, where the force of mortality is defined as follows:

$$\mu_{x,t}=\lim_{\Delta k\to 0}\frac{P_t(0\leq T_{x,t}\leq \Delta k)}{\Delta k},$$

  which is often referred to as the instantaneous rate of mortality;

- $D_{x,t}$ denotes the observed number of deaths during year t in a cohort aged x at the beginning of year t;

- $E_{x,t}$ denotes the number of person years lived during year t in a cohort aged x at the beginning of year t, which is also called the exposure-to-risk. Assuming that the people dying during a year have lived, on average, the first half of that year, the exposure-to-risk can be approximated by the number of survivors plus half the number of deaths in this cohort.

Furthermore, we assume that for any integer age x, and any time t, the following holds:

$$\mu_{x+u,t+\tau} = \mu_{x,t}, \qquad \text{for } 0 \leq u, \tau < 1, \tag{2.1}$$

In doing so, it follows from (2.1), which is also denoted as $\mu_{x,t}$ being piecewise-constant, that the force of mortality is equal to the central death rate, i.e., $\mu_{x,t} = D_{x,t}/E_{x,t}$. Then the following is obtained for the survival probabilities:

$$p_{x,t} = \exp(-\mu_{x,t}). \tag{2.2}$$

For the exact derivation, the reader is referred to Pitacco et al. (2009).

2.1 Calculating Period and Cohort Life Expectancy

The remaining life expectancy can be calculated in the following two ways:

- Period life expectancy: the (remaining) life expectancy for a person aged x in a certain year t, based on the death rates that hold for the year t;
- Cohort life expectancy: the (remaining) life expectancy for a person aged x in a certain year t, based on the death rates from year t onwards (cohort mortality rates). For this person the life expectancy is based on the probability of reaching age $x+1$ in year $t+1$, age $x+2$ in the year $t+2$, etc. However, for example for $t = 2014$ the death probabilities after t are not known, but random, because of longevity risk. Therefore cohort life expectancy is a random variable, but we can calculate, e.g., mean, median and quantiles by using a stochastic mortality model. If death rates are not assumed constant over time the cohort life expectancy differs from the period life expectancy: if death rates decrease over time, the cohort life expectancy is higher than the period life expectancy, the difference between these two life expectancies is illustrated in Section 3.4.

Period life expectancy	**Cohort life expectancy**
${}_kp_{x,t} = \Pi_{i=1}^{k} p_{x+i-1,t}$	${}_kp_{x,t} = \Pi_{i=1}^{k} p_{x+i-1,t+i-1}$
$P(T_{x,t} = k) = {}_kp_{x,t} \cdot q_{x+k,t}$	$P(T_{x,t} = k) = {}_kp_{x,t} \cdot q_{x+k,t+k}$
$e_{x,t}^{period} = \frac{1}{2} + \sum_{k=1}^{\infty} k \cdot P(T_{x,t} = k)$	$e_{x,t}^{cohort} = \frac{1}{2} + \sum_{k=1}^{\infty} k \cdot P(T_{x,t} = k)$
$= \frac{1}{2} + \sum_{k=1}^{\infty} k \cdot {}_kp_{x,t} \cdot q_{x+k,t}$	$= \frac{1}{2} + \sum_{k=1}^{\infty} k \cdot {}_kp_{x,t} \cdot q_{x+k,t+k}$
$= \frac{1}{2} + \sum_{k=1}^{\infty} {}_kp_{x,t}$	$= \frac{1}{2} + \sum_{k=1}^{\infty} {}_kp_{x,t}$

3 Modeling Mortality

This section concerns the mortality modeling part of this thesis. Firstly, the literature review in Section 3.1 examines the most relevant (stochastic) mortality models in the academic literature. Afterwards, the data used for the mortality modeling is described in Section 3.2. Section 3.3 explains the motivation for and estimation of the stochastic mortality model used. Finally, in Section 3.4 the results of the mortality model are presented and discussed. A sensitivity analysis to certain input parameters is done in Section 3.4.1 and Section 3.4.2 concludes with a comparison of the findings to mortality forecasts of well-known Dutch institutions.

3.1 Literature Review

This section presents an overview of the most relevant (stochastic) mortality models that are used to forecast future mortality. A general overview of mortality modeling is given first and the models developed in the academic literature are covered afterwards.

In general, one can distinguish broadly three approaches to determine and forecast future mortality (Booth & Tickle, 2008):

- Expert judgment: Forecasts are based on subjective expert opinions, e.g., assuming a certain value for life expectancy at a certain point in future time with a specified path (Olshansky, 1988; Pollard, 1987) or using expert judgment to adjust trends in age- or cause-of-death-specific trends (Waldron, 2005);
- Explanation: a method where mortality is forecasted using known relations between mortality and exogenous explanatory variables. An example is the dependence between real per capita income and mortality (Preston, 1975, 2007);
- Extrapolation: This method uses the regularity found over time as well as across ages, in mortality measures such as life expectancy or death rates, to extrapolate such quantities. The assumption is made that future trends are a continuation of past trends.

Although all three approaches described above have been previously employed in the literature, there is general consensus that the third method, extrapolation, gives the most plausible results (Booth & Tickle, 2008), especially when the purpose of the model is long-term forecasting. Therefore, the focus of this literature review will be on extrapolative models. Do note, however, that the extrapolation approach can be invalidated by (future) medical advances whose impact differs in magnitude from past medical advances, that may change the time trend of mortality.

An extensive range of literature is available with respect to modeling stochastic mortality rates using extrapolation, and over time many researchers have reviewed the models that have been proposed in the past decades, e.g., Booth & Tickle (2008); Cairns et al. (2009);

Dowd et al. (2010); Haberman & Renshaw (2011) and Cairns et al. (2011). Likely the most well-known of these stochastic mortality models is the one proposed by Lee & Carter (1992), followed in popularity by the Cairns et al. (2006) model and the class of P-spline models[4]. The P-spline models are not discussed in detail in this thesis because the model's estimation is not so straightforward. In short, the Lee-Carter (LC) model estimates the log force of mortality as follows: $\ln \mu_{x,t} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$. Defining the log mortality this way assumes there is a time trend in the log mortality rates, measured by κ_t, which affects individuals of different ages in its own way through β_x, in addition to the average level of mortality at a certain age, measured with α_x, and the error term is $\epsilon_{x,t}$. Besides giving point estimates of future mortality rates, the model also allows for the computation of confidence intervals. A more elaborate explanation of the Lee-Carter model is given in Section 3.3. The literature finds, surprisingly, that κ_t is usually linear, which implies that gains in life expectancy are relatively constant each year in most populations.

The model was found by Lee & Carter (1992) to give a good in-sample fit for age-specific death rates and also showed stability with respect to the data interval used. However, after its initial publication in 1992, many extensions have been proposed, to solve certain disadvantages and unrealistic assumptions of the original model, such as the model's homoskedasticity assumption of the errors and its disregard for the integer-valued character of the number of deaths needed for the estimation (Brouhns et al., 2002). Several papers have also suggested alterations to improve the model's fit to the data. For example, Brouhns et al. (2002) propose an alternative to the original Lee-Carter model by modeling the number of deaths $D_{x,t}$, conditional on the exposure-to-risk, as a Poisson random variable, rather than taking the central death rates as a random variable. In other words, Brouhns et al. (2002) assume $D_{x,t} \sim \text{Poisson}(E_{x,t} m_{x,t})$, where $m_{x,t} = \alpha_x + \beta_x \kappa_t$ as in the Lee-Carter model. Then using Maximum Likelihood estimation also gives parameter uncertainty, whereas more effort is needed in the Lee-Carter method to quantify this (for example, by using a bootstrapping method). This reformulation allows for the estimation process even for data cells where no deaths are observed, which is impossible in the original specification.

Another well-known stochastic mortality model, particularly in the U.K., is the Cairns-Blake-Dowd (CBD) model (2006), based on the fact that $\log \frac{q_{x,t}}{1-q_{x,t}}$ is approximately linear across age, for fixed t. The CBD-model is defined by $\log \frac{q_{x,t}}{1-q_{x,t}} = \kappa_t^{(1)} + \kappa_t^{(2)} x$, where $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are stochastic processes and by $\log(\cdot)$ we mean the natural logarithm. The advantage of the CBD-model over the LC-model is that it does not impose perfect correlation in mortality across ages, because it is a two-factor model instead of a one-factor model (Pitacco et al., 2009). A notable disadvantage is, however, that the CBD-model only seems to work well for higher ages, namely 40 and older (Peters et al., 2012). Plat (2009) introduces a model which combines the strong points of both the LC and the CBD-model, and seems to give better fits, especially for Dutch data. The Plat model is

[4]See, e.g., Currie et al. (2004); Eilers & Marx (1996) and Currie et al. (2006) for examples of P-spline models.

more complex, however (Haberman & Renshaw, 2011). Other well-known models that integrate the two general families of LC-models and CBD-models are found in O'Hare & Li (2012) and Börger et al. (2014).

In addition to the age and the time factor, a third factor one could include is the cohort factor, for example, as proposed by Renshaw & Haberman (2006) in the LC-model. The authors define log-mortality as follows: $\log m_{x,t} = \alpha_x + \beta_x^{(0)} i_{i-x} + \beta_x^{(1)} \kappa_t$, where $\beta_x^{(0)} i_{i-x}$ captures the cohort effect. Estimation of this model is more complex than the original LC-model, but including the cohort effect also increases flexibility. Another proposal for including the cohort effect, but then in the CBD-model, is described in Cairns et al. (2009). The cohort effect means that there is a significant change in mortality rates for one generation, for example those born between 1940-1950, compared to the rest of the population. This differs from a time effect, since the time effect holds for the entire population. If such a cohort effect is present in a population's data, adding a cohort factor can give more accurate projections (Haberman & Renshaw, 2011), for example in the United Kingdom. For the Netherlands no conclusive evidence for a cohort effect has been found yet.

Hunt & Blake (2014) note, however, that there is a proliferation of new mortality models, some of which are "standard" algorithms. As an example, Hunt & Blake (2014) give the principal components analysis (PCA), which is deployed mechanically on various datasets without careful consideration of the underlying causes driving the population's mortality, and which uses terms that seem demographically insignificant. In addition, other models only add terms to "fix" certain problems in existing models. The authors therefore suggest a general procedure to provide a structured approach to building a stochastic mortality model, the interested reader is referred to Hunt & Blake (2014).

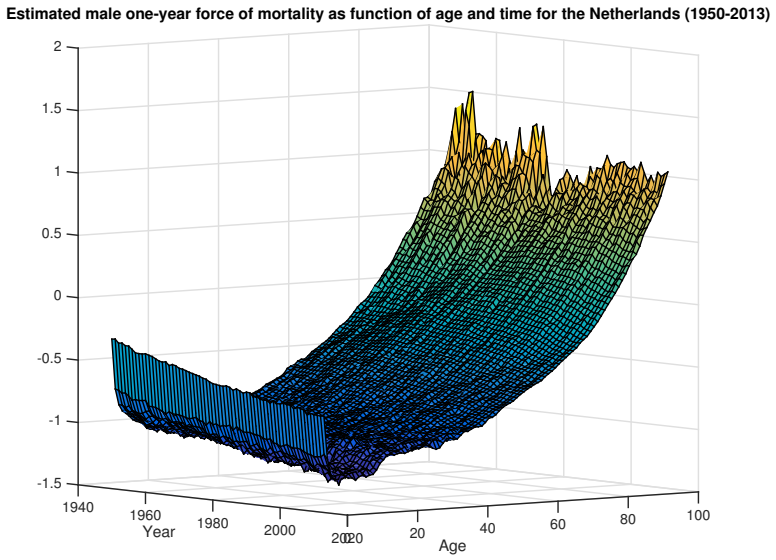
Noteworthy is that the models described above do not only provide point estimates, but also give confidence intervals for future mortality projections. Since longevity risk concerns the uncertainty in future mortality rates, the construction of confidence intervals is of essence when assessing longevity risk, also with respect to the Solvency II regulation concerning longevity risk.

3.2 Data

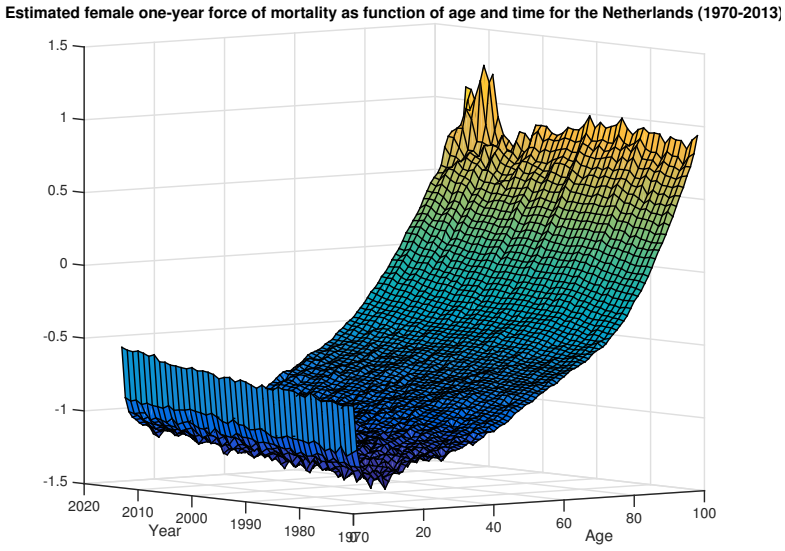
The mortality data used consist of the one-year death probabilities ($q_{x,t}$) as made available by Statistics Netherlands. The Dutch one-year death probabilities are available from the year 1950 up to 2013. These death probabilities are given in the form of gender-specific period life tables. The maximum attainable age in these period life tables is set at 99. However, it should be noted that these death rates are half-year averages, i.e., they are the death rates for ages $0, 0.5, 1.5, ..., 98.5$. To convert the rates to whole-year ages, so for ages $0, 1, 2, ..., 98$, the following transformation is applied to all death rates with the exception of $q_{0,t}$:

$$q_{x,t} = 1 - \sqrt{(1 - q_{x-0.5,t})(1 - q_{x+0.5,t})}. \tag{3.1}$$

See Figure 3.1 for a graphical representation of the empirical death rates as force of mortality for Dutch men and women from 1950 to 2013. Note the linearity in the force of mortality



(a) Males



(b) Females

Figure 3.1: The one-year death probabilities for males (left) and females (right) for ages 0 to 98 and time period 1950-2013

3.3 Methodology

After having appraised different stochastic mortality models in Section 3.1, this thesis will use the Lee-Carter model, with some alterations as proposed by other researchers throughout the years, to model and forecast population mortality. The motivation for using this model is as follows: the model is relatively simple, but does not perform significantly less accurate than other models, in terms of backtests and goodness-of-fit (Cairns et al., 2009; Dowd et al., 2010).

Since the literature does not provide evidence for a significant cohort effect in the Netherlands, it is not necessarily of added value to include a cohort effect, as in the APC variant of the Lee-Carter model: doing so would likely only increase the complexity of the model without significantly improving the forecast.

Lee & Carter (1992) use SVD estimation for the model's parameters. An alternative is to use the Poisson variant as described in Brouhns et al. (2002). If the data contains empty cells the original Lee-Carter estimation would fail whereas the Poisson variant does provide estimates, but in the data used for this thesis there are more than zero deaths for all ages and years and therefore SVD estimation will suffice.

Moreover, the Lee-Carter method is seldom systematically outperformed by other models in the academic literature, see for instance Cairns et al. (2009) and Dowd et al. (2010). Nevertheless, the model is still subject to certain drawbacks, for which mitigating adjustments are considered in this thesis, which are explained below. However, first a more complete description of the original Lee-Carter model is given.

Lee & Carter (1992) introduced the following dynamic mortality model to forecast future values of the force of mortality:

$$\ln \mu_{x,t} = \ln \frac{D_{x,t}}{E_{x,t}} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t} \tag{3.2}$$

However, without constraints the parameters[5] of this model are not identified, so the authors introduced two standard constraints that ensure identification, where t_1 and t_n are the first and last year of observation respectively, and with x_1 and x_m being the lowest and highest age in the data:

$$\sum_{t=t_1}^{t_n} \kappa_t = 0, \qquad \sum_{x=x_1}^{x_m} \beta_x = 1, \tag{3.3}$$

With these constraints the parameters can be interpreted in the following way:

- α_x is considered as the base shape of the mortality, or the average $\ln \mu_{x,t}$ over the time period;
- β_x is considered the sensitivity of age x to κ_t;

[5]Note that the 'parameter' κ_t becomes a time series process in order to estimate future death rates, which is explained on the next page.

- κ_t captures the changes in the mortality pattern over time;
- $\epsilon_{x,t}$ is the error term, independent over x and t, while having mean zero and homoskedastic variance σ_ϵ^2.

Estimation of the model is done in three steps. First $\hat{\alpha}_x$, $\hat{\beta}_x$ and $\hat{\kappa}_t$ are calculated with least squares, using Singular Value Decomposition (SVD). This method is explained in more detail in Appendix A. After estimating these parameters, Lee & Carter (1992) propose adjusting the $\hat{\kappa}_t$ such that they reproduce the observed number of deaths for each year t to avoid large differences, i.e., $\hat{\hat{\kappa}}_t$ solves:

$$\sum_x D_{x,t} = \sum_x E_{x,t} \exp(\hat{\alpha}_x + \hat{\beta}_x \hat{\hat{\kappa}}_t). \tag{3.4}$$

However, the above step has been criticized by Girosi & King (2007) because it is possible for the re-estimation to give zero or multiple solutions, and it is therefore not implemented in this thesis. Instead of the SVD estimation, other estimation methods are also possible, for example Maximum Likelihood or Weighted Least Squares. More detailed information on these estimation procedures can be found in Wilmoth (1993).

After the estimation of the parameters, the authors use the Box-Jenkins method to identify and estimate $\hat{\kappa}_t$. The Box-Jenkins method selects an appropriate fit of a time series by means of ARIMA models, and then uses the fitted model to make forecasts. Lee & Carter (1992) find the following model for κ_t, an ARIMA(0,1,0) time series:

$$\kappa_t = C + \kappa_{t-1} + \epsilon_t, \tag{3.5}$$

with ϵ_t i.i.d. normal with mean zero and variance σ_κ^2. Consequently the difference $\delta_t = \kappa_t - \kappa_{t-1}$ is modeled as a stationary time series. This result for κ_t is in line with the results found by Tuljapurkar et al. (2000), who investigated the patterns of mortality decline across the G7 countries, and found that κ_t follows an ARIMA(0,1,0) process for most of the countries inspected.

In case past events occurred that are not representative of the development of mortality rates over time (think of the Spanish flu in 1918 or the hunger winter in the Netherlands in 1944/1945), dummies can be included to filter out these events. On the other hand, it can also be argued that such catastrophic events might occur in the future as well, and are therefore in some sense representative for the development of mortality rates. However, Lee & Carter (1992) used a dummy for the influenza epidemic of 1918 as their aim is to estimate the long term trend of mortality, excluding casualties. The authors find that exclusion of the dummy hardly changes the parameter estimates and the forecasts, although the confidence interval for κ_t becomes wider. The data period used for the estimation does not contain such extraordinary events though, and therefore it is not necessary to decide whether to use a dummy variable or not.

The Lee-Carter model is an extrapolative model using historic data. A salient drawback of this model is its inability to take into account sudden and large improvements (decreases) in future mortality rates, for example because of medical or technological

advancement (deterioration), e.g., a cure for cancer, solution for the pollution problem (vice versa epidemics or grave pollution), which are not immediately seen in the mortality rates themselves. These developments are not captured properly in the estimation of the parameters, since the model only uses past death rates as input, without any other information. One cannot incorporate any already known information about future death patterns. Furthermore, there is no guarantee that observed trends from the past will continue as they have. However, no other forecasting methods have as of yet been found to perform more accurately[6] than extrapolative methods (Wilmoth, 2000).

The estimates for C and σ_κ^2 are the empirical mean and empirical variance of κ_t respectively. Mortality projections are then calculated by predicting values $\tilde{\kappa}_{T+t}$, where T is the final year of the sample. Afterwards the future central death rates can be predicted as follows:

$$\hat{\mu}_{x,T+t} = \exp(\hat{\alpha}_x + \hat{\beta}_x \hat{\tilde{\kappa}}_{T+t}),$$

and then the corresponding projected future one-year survival and death probabilities $p_{x,t}$ and $q_{x,t}$ are calculated using (2.2). Instead of using the estimated values $\hat{\mu}_{x,t_n}$ with t_n the final data year used for estimation, this thesis uses the observed μ_{x,t_n} for all x in order to avoid a jump-off bias, as proposed in Lee & Miller (2001).

The method outlined above is applied to the one-year death probabilities as described in Section 3.2, for the time period of 1970 - 2013. The year 1970 is chosen as the starting year because the development, i.e., the increase in mortality rates before 1970 seems a less reasonable indicator for the future trend in mortality rates, which shows steady decrease after 1970. However, since the Lee-Carter estimations are quite dependent on the starting year of the data used, a robustness analysis is done on the results using different starting years, i.e., 1960 up to 1980 with 5-year intervals respectively, in Section 3.4.1. The forecasts are done for a time horizon of 100 years after the last observed data point (2013, so the starting year for the forecasts is 2014), which is necessary for the SCR calculations since a number of young participants are present in the portfolio. The best estimate values of the future mortality rates are obtained by setting $\epsilon_t = 0$ for the κ process in Equation (3.5). These are the mortality rates used when computing the best estimate value of the liabilities. A confidence interval for the death rates is obtained by estimating future values of κ_t using Equation (3.5) for both genders, with ϵ_t denoting a draw from a Normal distribution with mean zero and standard deviation σ_κ. This procedure is repeated 1,000 times to give 1,000 different future developments of κ_t. More simulations do not significantly change the results, as discussed in Section 3.4.1.1. Future death rates can then be predicted using these κ_t's as described in the paragraph above, and confidence intervals can be constructed based on the empirical distribution. In the original Lee-Carter method, the confidence interval would be based on assumed normality of the error terms in κ_t. This latter confidence interval construction is described in Appendix B.

Three types of risks can be distinguished in the model described in this section: process risk, parameter risk and model risk. The uncertainty in the development of

[6] In terms of backtesting and in-sample fit.

future mortality rates, caused by the uncertainty in the development of κ_t is the process risk. The quantification of this risk is found in the error terms ϵ_t in Equation (3.5). Parameter risk stems from the fact that the parameter estimates are incorrect, which can be quantified by bootstrapping. Finally, whether or not the chosen model, in this case Lee-Carter, is the correct model, is the crux of model risk. As only one model is considered in this thesis, model risk is not taken into account. One could account for model risk by also estimating and forecasting mortality rates using other models, for example the CBD-model or the Plat model. In addition, as mentioned above, the Lee-Carter model is sensitive to the data period used, i.e., the start and end year for estimation. To overcome this latter sensitivity, multiple data periods are used for estimation to check the robustness of the results.

Furthermore, Lee-Carter does not recognize the integer-valued character of $D_{x,t}$, and the model requires a calibration step for the $\hat{\kappa}_t$'s in order to match the predicted death rates with the realized in-sample death rates.

3.4 Results

The results of the methodology to forecast future mortality of the Dutch population, as presented in the previous Section 3.3, are given and discussed below. The sensitivity of these results to certain input parameters is appraised in Section 3.4.1. In Section 3.4.2 these findings are also compared to the mortality forecasts of relevant institutions in the Netherlands, namely Statistics Netherlands (Centraal Bureau voor de Statistiek, CBS) and the Royal Actuarial Association (Koninklijk Actuarieel Genootschap, AG).

Graphs depicting the estimates of the Lee-Carter parameters α_x, β_x and κ_t for both males and females are given below, based on the estimation period from 1970 to 2013. The parameter estimates displayed are consistent with results found in other papers concerning Dutch mortality, e.g., De Waegenaere et al. (2010).

α_x is estimated for the ages 0-98 for both genders and displayed in Figure 3.2. Recall that α_x can be interpreted as the average ln $\mu_{x,t}$ over the time period. At age 0 the average mortality is relatively high because of infant mortality. Afterwards it decreases up to age 10, and increases approximately linearly after this age, except for the well-known 'accident hump' for adolescents, especially pronounced for males. The α_x are higher for males than for females, which makes sense since males have a slightly higher probability of death than females for all ages.

In Figure 3.3 the estimates for β_x for males and females are displayed for ages 0-98. Recall the interpretation of β_x as the sensitivity of age x to κ_t. It is observed that the younger ages profit more from the decrease in mortality rates (high values of β_x) than the older ages. The oldest males even see a slight increase in mortality (negative β_x). This pattern again shows consistency with previous results of Lee-Carter estimations, for example Stevens et al. (2010). Noteworthy is the higher variation in the values of β_x, implying that the mortality rates for the younger Dutch population has varied more in the past.

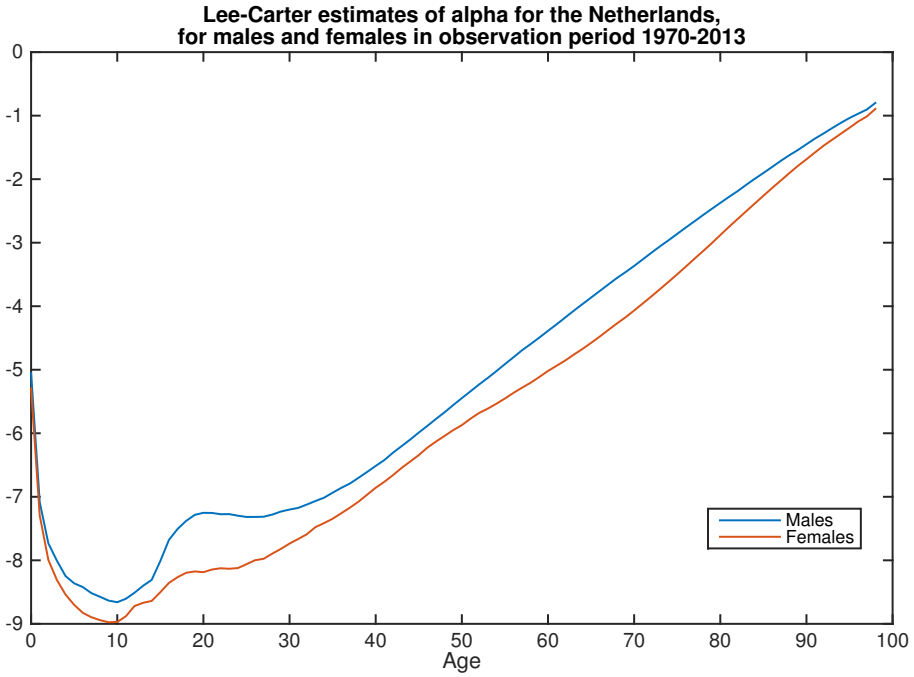


Figure 3.2: Estimates of α_x for both males (blue) and females (red) using Dutch gender-specific data from time period 1970-2013.

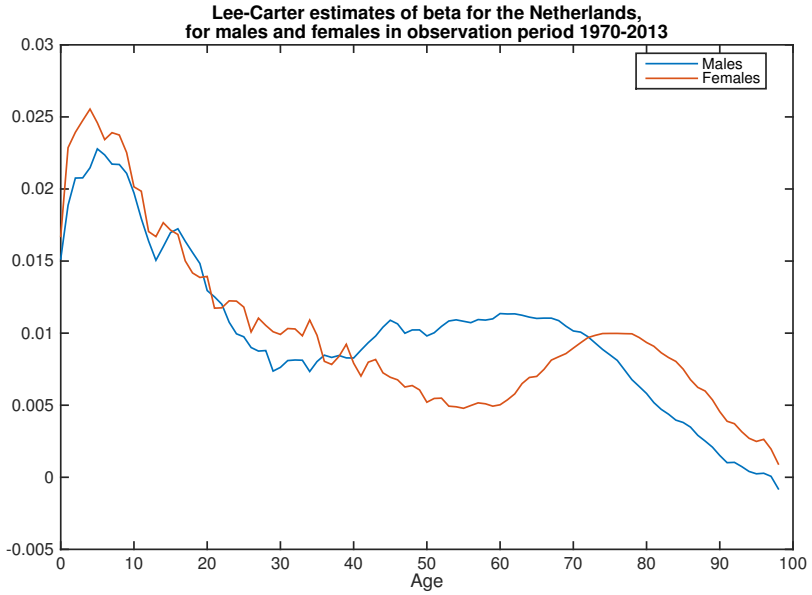


Figure 3.3: Estimates of β_x for both males (blue) and females (red) using Dutch gender-specific data from time period 1970-2013.

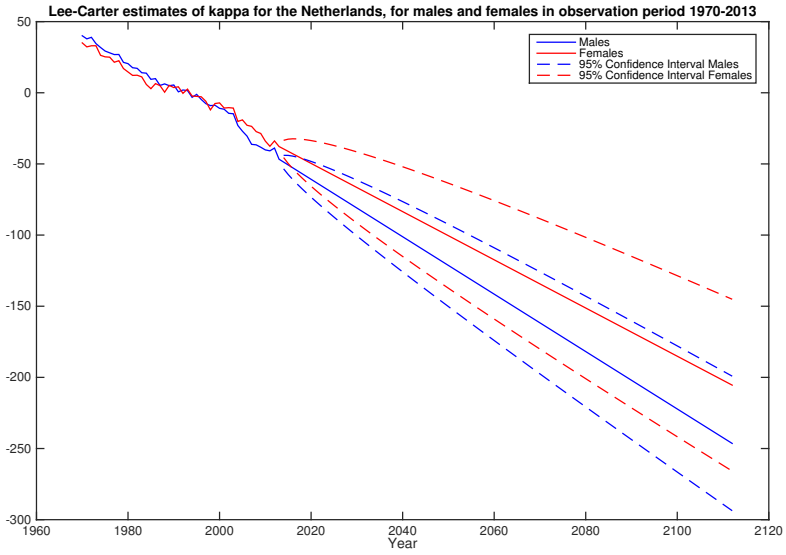


Figure 3.4: Best estimates of κ_t (solid) and the 95% confidence interval (dashed) for both males (blue) and females (red) using Dutch gender-specific data from time period 1970-2013.

Finally, the estimates for κ_t and the 95%[7] confidence intervals are given in Figure 3.4. κ_t gives the changes in the mortality pattern over time. The decline in mortality rates over the past 40 years was somewhat more rapid for males than for females, although one should keep in mind that females had lower mortality rates than males already. There has also been more variation in the decreasing trend for females than for males, which gives a higher variance of the process κ_t for females (9.6 for females versus 5.9 for males) and as a result, a larger confidence interval for females.

The percentage of variance explained by the Lee-Carter model is 90.11% for males and 83.66% for females. These are lower than in Lee & Carter (1992) and other literature, but the difference is mainly caused by the wider age interval in this thesis. To illustrate: Brouhns et al. (2002) finds percentages close to 90%, but only consider ages 65-98. If the same age group is considered here, the model gives comparable figures.

The cohort and period[8] life expectancy for both males and females with their respective 2.5% and 97.5% quantiles are given for calendar years 2014, 2034 and 2054 and the ages 0, 25 and 65 in Table 3.1. By best estimate (BE) are meant the forecasted life tables using the Lee-Carter model with the κ_t process errors set at 0.

A number of observations follow from the tables:

[7] 95% is the typical percentage used for confidence intervals in the literature.

[8] See Section 2.1 for the difference between period and cohort life expectancy.

	0			25			65		
	2.5%	BE	97.5%	2.5%	BE	97.5%	2.5%	BE	97.5%
2014	86.06	86.36	86.53	59.37	59.65	59.83	18.25	18.40	18.54
	(78.69)	(79.07)	(79.46)	(54.27)	(54.62)	(54.96)	(17.28)	(17.51)	(17.75)
2034	87.66	87.94	88.09	61.17	61.46	61.62	19.89	20.13	20.33
	(80.53)	(81.91)	(83.20)	(55.94)	(57.22)	(58.42)	(18.41)	(19.29)	(20.13)
2054	88.88	89.13	89.27	62.62	62.89	63.05	21.31	21.59	21.79
	(82.57)	(84.19)	(85.68)	(57.83)	(59.36)	(60.79)	(19.71)	(20.81)	(21.86)

(a) Males.

	0			25			65		
	2.5%	BE	97.5%	2.5%	BE	97.5%	2.5%	BE	97.5%
2014	88.67	89.13	89.02	62.15	62.57	62.80	21.48	21.73	21.92
	(82.20)	(82.62)	(83.01)	(57.65)	(58.03)	(58.38)	(20.23)	(20.55)	(20.84)
2034	90.14	90.59	90.77	63.69	64.15	64.40	23.00	23.36	23.61
	(83.16)	(84.84)	(86.42)	(58.52)	(60.06)	(61.58)	(20.96)	(22.24)	(23.49)
2054	91.36	91.79	91.99	65.04	65.61	65.69	24.37	24.78	25.00
	(84.6)	(86.75)	(88.65)	(59.85)	(61.88)	(63.72)	(22.06)	(23.74)	(25.27)

(b) Females.

Table 3.1: Cohort and period (between brackets) expected remaining lifetimes for males (top) and females (bottom) of ages 0, 25 and 65 and for calendar years 2014, 2034 and 2054 with their respective quantiles. The best estimate (BE) calculations are based on the Lee-Carter model with κ_t process errors all 0.

First of all, the difference between male and female remaining life expectancy is sizable, with the females having at least a 3 year higher remaining life expectancy for all ages and calendar years displayed.

Secondly, the difference between period and cohort expected remaining lifetimes is also sizable for both males and females, for all ages and calendar years shown, ranging from approximately 2 to 10% of the period life expectancy. The difference in calculating a period versus a cohort life expectancy was explained in Section 2.1, and as mentioned in that section, cohort life expectancies are higher because of expected decreases in future mortality rates.

Thirdly, life expectancy will increase in the future according to the Lee-Carter model: in 40 years time newborn males and females are expected to have a 2.83 and 2.66 years increase in total life expectancy. For the 25-year olds 40 years from now the remaining life expectancy increases with 3.24 and 3.04 for males and females respectively. For the elderly the increase is of even greater size given the lower remaining lifetime: 3.19 for males and 3.05 for females. The size of the increase is smaller between 2054 and 2034 than between 2034 and 2014 though, which may indicate that, at least according to the Lee-Carter model, increases in life expectancy will slow down in the (far) future.

Furthermore, the width of the confidence interval is distinctly narrower for cohort life

expectancies (0.29-0.71 years) than for period life expectancies (0.58-5.38 years). The narrow confidence intervals are a well-known characteristic of the Lee-Carter model, caused by the limited influence of κ_t's uncertainty on the life expectancy in case of initially low mortality rates, according to Lee (2000). Since cohort life expectancies take into account the future decrease in mortality rates whereas the period life expectancies do not and thus use higher mortality rates, it follows that cohort life expectancies have narrower confidence intervals than the confidence intervals for expected period remaining lifetimes.

In addition, the difference between the 2.5% quantile and the best estimate is larger than the difference between the 97.5% quantile and the best estimate, i.e., the distribution of the expected remaining lifetimes is slightly skewed to the left. Although the width of the confidence interval of κ_t is not skewed, the skewness in the life expectancy confidence interval stems from the transformation of the (relatively linear) log mortality rates to the mortality rates $q_{x,t}$ using the exponential function.

Of special interest for the research question of this thesis are the 99.5% quantile of the expected remaining lifetimes compared to the Solvency II standard formula. The latter will be explained in more detail in the next section, but in short it comes down to the following: the standard formula is formulated as decreasing all one-year death rates by 20%, so $q_{x,t}^{SII} = 0.8 \cdot q_{x,t}^{BE}$, $\forall x, t$. These shocked death rates are then used to calculate the remaining life expectancy, of which the calculations were described in Section 2.1. The 99.5% quantile and the Solvency II shocked life expectancies for ages 0, 25 and 65 and years 2014, 2034 and 2054 are given in Table 3.2.

	0			25			65		
	BE	99.5%	SII	BE	99.5%	SII	BE	99.5%	SII
2014	86.36	86.58	87.94	59.65	59.93	61.37	18.40	18.58	20.03
2034	87.94	88.17	89.32	61.46	61.69	62.99	20.13	20.42	21.64
2054	89.13	89.33	90.38	62.89	63.15	64.26	21.59	21.84	22.99

(a) Males.

	0			25			65		
	BE	99.5%	SII	BE	99.5%	SII	BE	99.5%	SII
2014	89.63	89.91	91.12	63.07	63.39	64.71	22.23	22.46	23.75
2034	91.09	91.36	92.39	64.65	64.97	66.11	23.86	24.20	25.25
2054	92.29	92.61	93.43	66.01	66.34	67.30	25.28	25.60	26.55

(b) Females.

Table 3.2: Cohort expected remaining lifetimes for males (top) and females (bottom) of ages 0, 25 and 65 and for calendar years 2014, 2034 and 2054 for the best estimate forecast, the 99.5% quantile and the Solvency II standard approach. The best estimate (BE) calculations are based on the Lee-Carter model with κ_t process errors all 0, the Solvency II calculations using the standard formula.

It is promptly seen from Table 3.2 that all remaining life expectancies calculated using the Solvency II (SII) standard method are higher than the 99.5% quantile of the Lee-Carter model. In addition, the difference between the 99.5% quantile and the SII standard formula are quite large, at least a year or more for both genders, all ages and calendar years, with the exception of 2054 for females (although the smallest difference here is also 0.82 years). These observations may already indicate that the Solvency II standard formula will also lead to higher buffers (Solvency Capital Requirement) for longevity risk than the Lee-Carter model. The calculation of the SCR and its results will be discussed further on in Section 4. In Section 4.4.1 a sensitivity analysis will be conducted partly based on the results found below.

3.4.1 Sensitivity Analysis

To check the robustness of the results above, a sensitivity analysis is done for certain input parameters. A sensitivity analysis checks whether the uncertainty in the input parameters of the model impacts uncertainty in the model's output. The following inputs are taken into account: the estimation period for the Lee-Carter model, the volatility of the κ_t process and the number of simulations. If some of these indicate a relevant impact on the remaining life expectancy, consequently these input parameters are also likely to influence the size of the SCR, which will be calculated in Section 4.

3.4.1.1 Estimation Period

To check the robustness of the results for the estimation period used, estimations of the Lee-Carter model are done using different estimation periods, with starting years 1960 up to 1980 with 5-year intervals, while the end year remains at 2013 (the latest year with data available).

For males the remaining life expectancy steadily increases when the estimation period is shortened to more recent years: for newborns the expected lifetime increases by 1 - 2% for almost all years, with the exception of the increase from starting period 1970 to 1975 and estimation year 2054. For 25-year olds the expected remaining lifetime also increases with 1-2% for all estimation windows. The increase in remaining life expectancy, from starting period 1960 to starting period 1980, is the largest for 65-year olds, namely ranging from 3-5%. These increases also hold for the 99.5% quantile and the Solvency II standard formula calculations.

On the other hand, taking a more recent (earlier) starting year for the observation window does not significantly change the life expectancy of females: the increases are limited to 1-2% changes at most, although most life expectancies change with less than 1%. For the starting year change from 1970 to 1975, the life expectancies even decrease by 1-2%. These changes in life expectancies hold for all life expectancies, i.e., the best estimate, the 99.5% quantile and the Solvency II standard formula.

The changes above may imply that a change of observation window changes the size of the SCR significantly for (older) males, whereas the impact on the SCR for a portfolio of females will likely remain limited.

3.4.1.2 Volatility

In the original Lee-Carter estimation the empirically found variation for the estimated κ_t is used as the volatility to generate future κ_t's. It may be well possible, however, that the empirical volatility is not the real volatility, so it is useful to check whether changing the value of κ_t's volatility will affect the life expectancies calculated above.

Since the best estimate value for the life expectancies are calculated using a κ_t process without uncertainty and because the best estimate is also the basis for the Solvency II standard formula, changing the volatility of the κ_t process will only influence the 99.5% quantile. Note that the best estimate is not the same as the mean of the life expectancies, as the best estimate is calculated while setting all $\epsilon_{x,t}$ to 0. One expects that a lower volatility will decrease the 99.5% quantile and that a higher volatility will increase the value of the 99.5% quantile.

To check the above statement, the empirical value of the volatility (5.88 and 9.58 for males and females respectively) is doubled and halved to investigate the effect on the 99.5% quantile of the remaining life expectancy. Halving the volatility hardly decreases the 99.5% quantile: the largest decrease is 0.5% at most, with most decreases around 0.1-0.2%. Doubling the volatility increases the 99.5% quantile, although the increase is larger for older ages and for females, because the κ_t volatility for females was larger to begin with. Nevertheless, the increases are still negligible at a maximum of 0.6%, so it is unlikely changing the κ_t volatility will change the size of the SCR, unless it is changed to the extreme. This will, however, further be discussed in Section 4.4.1.

3.4.1.3 Simulation Size

Increasing the number of simulations to 5,000 or more hardly changes the results, i.e., the life expectancy 99.5% quantiles remain almost exactly the same, as can be seen in Table 3.3. Thus, to limit the computational load, 1,000 simulations of κ_t are deemed a reasonable number of repetitions to give robust results.

3.4.2 Comparison

In this section the results are compared to findings about Dutch mortality by two institutions, namely the Royal Actuarial Association (Koninklijk Actuarieel Genootschap, AG) and Statistics Netherlands (Centraal Bureau voor de Statistiek, CBS). These two institutions have also created models to forecast future mortality which include confidence intervals. The methods of these two institutions differ and thus yield slightly different results.

3.4.2.1 AG

The Royal Actuarial Association, from here onwards referred to as AG, has in the past presented two (deterministic) forecast tables that use a short and a long-term trend to predict future mortality. The trends used were based on Dutch mortality data. These Dutch forecast tables were published in 2010 and 2012 respectively. AG's most recent forecast table, presented in 2014, is based on a stochastic mortality model, namely the

	0		25		65	
	1,000	5,000	1,000	5,000	1,000	5,000
2014	86.58	86.61	59.93	59.89	18.58	18.58
2034	88.17	88.15	61.69	61.72	20.42	20.39
2054	89.33	89.35	63.15	63.12	21.84	21.85

(a) Males.

	0		25		65	
	1,000	5,000	1,000	5,000	1,000	5,000
2014	89.91	89.94	63.39	63.41	22.46	22.50
2034	91.36	91.38	64.97	64.97	24.20	24.17
2054	92.61	92.68	66.34	66.31	25.60	25.62

(b) Females.

Table 3.3: 99.5% quantiles of the cohort remaining life expectancy for males (top) and females (bottom) for ages 0, 25 and 65 and calendar years 2014, 2034 and 2054, using 1,000 and 5,000 simulations.

Li-Lee methodology as described in Li & Lee (2005). To model the population mortality the mortality data of Western European countries with an above average GDP per capita[9] is used, whereas the subpopulation mortality data is the Dutch data provided by Statistics Netherlands, the same as used in this thesis. The future trend of mortality is forecasted based on the past trend development of the prosperous Western-European countries, which shows a stable linear development. The Dutch mortality forecast is then based on this trend development, while taking the past Dutch mortality deviation from this trend into account. So while this thesis focuses on Dutch data only, AG's model bases the Dutch future trend on the past Western-European trend, in a method similar to the original Lee-Carter model. Technical details can be found in Li & Lee (2005) and Actuarieel Genootschap (2014). AG has also published period and cohort life expectancies for both sexes for certain calendar years, which are compared to the results based on the model in this thesis in Table 3.4.

One immediately deduces from the table that the Lee-Carter model as proposed in this thesis underestimates life expectancy compared to the AG model, which means that the AG model expects mortality rates to decrease more than the Lee-Carter model. However, both models expect the increase in life expectancy to become smaller in the far future (compare the increase from 2014 to 2039 to the increase from 2039 to 2064). Most noteworthy is, however, that none of the life expectancies reported by AG fall in the 99.5% confidence interval of the L-C model, which implies that, according to the Lee-Carter model, the likelihood of future life expectancy becoming as high as the AG model predicts is almost negligible. So the AG model is relatively optimistic about

[9]The countries Austria, Belgium, Denmark, England & Wales, Finland, France, Germany, Iceland, Ireland, Luxembourg, The Netherlands, Norway, Sweden and Switzerland.

	Males				Females			
	Age 0		Age 65		Age 0		Age 65	
	AG	L-C	AG	L-C	AG	L-C	AG	L-C
2014	89.9	86.36	19.7	18.4	92.2	89.63	22.8	22.23
	(79.7)	(79.07)	(18.2)	(17.51)	(83.2)	(83.12)	(21.1)	(21.05)
2039	92.4	88.27	22.9	20.52	94.5	91.42	25.6	24.23
2064	94.1	89.63	25.5	22.22	96.1	92.81	27.8	25.92

Table 3.4: Cohort and period (between brackets) life expectancies for males (left) and females (right) of several starting years and ages, as calculated by AG and in this thesis.

future mortality, whereas the Lee-Carter model based on Dutch data is significantly more pessimistic. This is hardly surprising though, since the increase in Dutch life expectancy has been lagging behind the Western-European trend and only recently started catching up (Stoeldraijer et al., 2013; Van Duin & Stoeldraijer, 2012).

3.4.2.2 CBS

Statistics Netherlands, from here onwards referred to as CBS, has also come up with a mortality model, but rather than predicting future one-year death rates based on the general past trend of mortality, CBS differentiates between the trend in smoking (which causes decrease in mortality rates) and a general trend. This general trend is partly based on the mortality data of Western-European countries[10]. Details concerning the model and its assumptions can be found in Stoeldraijer et al. (2013).

The main results are given in terms of period life expectancy, so for comparison the period life expectancies for newborns and 65-year olds as computed by CBS as well as using the method described in this thesis are given in Table 3.5, with their 95% confidence intervals. These data are obtained from CBS' database website StatLine[11].

The tables show that the CBS model expects a higher increase in life expectancy over the time period 2012-2054 than the Lee-Carter model. As indicated in Section 3.4.2.1, the Western European life expectancy has been growing at a faster pace than the Dutch life expectancy, so the CBS model will also predict a higher increase for the future than the L-C model. The 95% confidence intervals computed by the CBS model are also much wider than the L-C confidence intervals. This is likely caused by the past fluctuations in the Western-European data, which have been sizable.

[10]The Western-European countries used in the CBS model are Denmark, England & Wales, Finland, France, Germany, Italy, Norway, Spain, Sweden and Switzerland.

[11]http://statline.cbs.nl/Statweb/, found on the page 'Prognose bevolking kerncijfers, 2014-2060' (Forecast population key figures, 2014-2060)

	Age 0						Age 65					
	CBS			L-C			CBS			L-C		
	2.5%	BE	97.5%	2.5%	BE	97.5%	2.5%	BE	97.5%	2.5%	BE	97.5%
2014	78.28	79.61	80.94	78.69	79.07	79.46	17.21	18.22	19.23	17.28	17.51	17.75
2034	79.47	83.16	86.85	80.53	81.91	83.20	17.78	20.78	23.78	18.41	19.29	20.13
2054	81.18	86.23	91.28	82.57	84.19	85.68	18.86	23.15	27.44	19.71	20.81	21.86

(a) Males.

	Age 0						Age 65					
	CBS			L-C			CBS			L-C		
	2.5%	BE	97.5%	2.5%	BE	97.5%	2.5%	BE	97.5%	2.5%	BE	97.5%
2014	81.74	83.07	84.40	82.7	83.12	83.51	20.02	21.03	22.04	20.73	21.05	21.34
2034	82.13	85.82	89.51	83.66	85.34	86.92	19.72	22.72	25.72	21.46	22.74	23.99
2054	83.99	89.04	94.09	85.1	87.25	89.15	21.01	25.3	29.59	22.56	24.24	25.77

(b) Females.

Table 3.5: Period life expectancies for males (top) and females (bottom) of several starting years and ages, as calculated by CBS and in this thesis, and their 95% confidence intervals. The CBS data has been obtained from CBS' StatLine database under the name 'Prognose bevolking kerncijfers, 2014-2060' (Forecast population key figures, 2014-2060).

Compared to both the AG and the CBS model, the L-C model expects the smallest increase in life expectancy for the future with the least uncertainty in terms of confidence interval width, and will therefore probably also give smaller buffer sizes than if one of these two alternatives were used. This might already indicate that it is of added value to consider another mortality model in future analyses.

4 Longevity Risk in Solvency II

In the previous section a number of mortality tables were simulated. With these mortality tables one can calculate the buffer needed to set an insurer's probability of defaulting on its portfolio of annuities to 0.5%, in both ways as described in Solvency II. Investment and interest rate risk are not taken into account here, in order to focus on longevity risk. However, first an overview of the Solvency II regulation as well as the academic literature concerning solvency buffers in general is discussed in Section 4.1. Afterwards, the data and methodology are described in Section 4.2 and 4.3 respectively. This section concludes with the baseline results and a sensitivity analysis with respect to certain input parameters in Section 4.4.

4.1 Literature Review

The regulation with respect to Solvency II buffer calculations is described below in Section 4.1.1 and 4.1.2. In addition, Section 4.1.3 appraises the literature relevant for the research question of this thesis: literature which deals with calculating the Solvency Capital Requirement (SCR) for longevity risk under Solvency II regulation specifically, and papers about solvency requirements for longevity risk in general are also evaluated. Note that the time subscript t from here on is considered as the starting year for the calculations, so future years are denoted with $t + s$, $s \geq 0$.

4.1.1 Solvency II: Standard Formula

As described in the introduction, the Solvency II Directive was announced to harmonize insurer regulation across the European Union, and to introduce a more risk-based approach compared to Solvency I. The Solvency II regulation encourages insurers to implement internal models to give another assessment of their risk. This method, referred to as the internal model, is further elaborated on in Section 4.1.2 and 4.3.3. However, especially for smaller insurers, developing an internal model is costly. As an alternative, Solvency II gives insurers the possibility to use a so-called standard formula instead to calculate an insurer's capital requirements. This standard model distinguishes seven different risk modules for which buffers, or rather SCRs, have to be held by the insurer. Some of these modules are divided further into submodules. The modular approach is illustrated in Figure 4.1. For insurers that issue lifelong (pension) annuities, longevity risk, as submodule under the life module, is essential in their total risk assessment and is, therefore, discussed in detail in this thesis.

For each of these submodules, Solvency II requires an insurance company to be able to cover their liabilities during one year with respect to these risks with 99.5% certainty. That is, the insurer has to calculate the 99.5% Value-at-Risk of the available capital over a one-year time horizon (Börger, 2010). Equivalently, the insurer's funding ratio $FR_{t+1} = \frac{A_{t+1}}{L_{t+1}}$ has at time t a probability of 0.5% or less to fall below one[12], where

[12]This is often referred to as being underfunded, the situation in which the value of the assets is lower than the value of the liabilities.

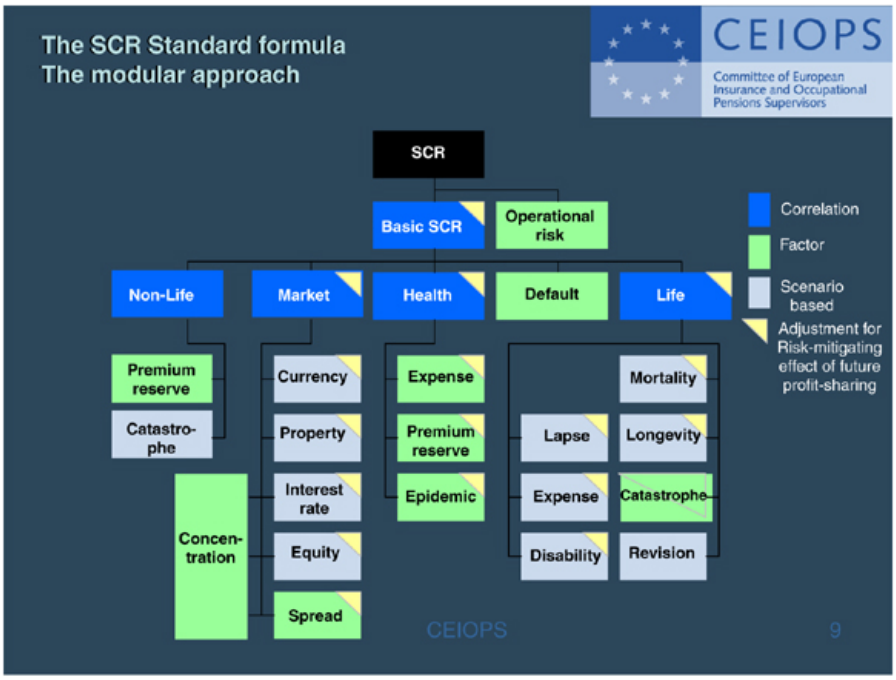


Figure 4.1: The seven Solvency II modules and their submodules, used for the SCR calculation. Source: Steffen (2008).

A_{t+1} is the value of the assets at time $t+1$ and L_{t+1} is the value of the liabilities at time $t+1$. The definition for the value of the liabilities will be discussed later on in this section. Then the insurer should hold at time t a minimum level of assets $A_t^* = \min\{A_t \mid \mathbb{P}_t\left(\frac{A_{t+1}}{L_{t+1}} < 1\right) \leq 0.005\}$, with $\mathbb{P}_t$ the objective probability measure at time t. The SCR to be held for a certain submodule by the insurer at time t is $SCR_t \equiv A_t^* - L_t$.

Supposing assets pay out a return of r_t during year t and liabilities paid at the end of year t are given by $\tilde{L}_t$, then A_{t+1} is defined as $A_{t+1} = A_t^*(1+r_t) - \tilde{L}_t$. Combining the above we find that $\mathbb{P}_t\left(\frac{A_{t+1}}{L_{t+1}} < 1\right) = \mathbb{P}\left((1+r_t)\cdot A_t - \tilde{L}_t < L_{t+1}\right)$ and thus the following expression for the SCR:

$$SCR_t = Q_{0.995,t}\left(\frac{\tilde{L}_t + L_{t+1}}{1+r_t}\right) - L_t, \tag{4.1}$$

with $Q_{0.995,t}$ denoting the function that returns the 99.5% quantile of the distribution of $\frac{\tilde{L}_t+L_{t+1}}{1+r_t}$, conditional on the information available at time t. Therefore, one could say that the SCR is the level of assets needed on top of the expected present value of the liabilities in order to ensure with 99.5% certainty coverage for the insurer's liabilities during year t. To determine the SCR, one should have an expression for the value of the liabilities, as no market price is observed for the liabilities. Solvency II proposes an

approximation using so-called Technical Provisions (Börger, 2010), which decomposes the value of the liabilities into a best-estimate value of the liabilities BEL_t including a risk margin RM_t: $L_t = BEL_t + RM_t$, where BEL_t is the expected present value of all future payments defined by

$$BEL_t = \sum_{s \geq 0} \mathbb{E}_t[\tilde{L}_{t+s}] \cdot P_t^{(s+1)}, \tag{4.2}$$

with $P_t^{(s+1)}$ the discount factor (or equivalently, the price at time t of a zero-coupon bond maturing at time $t + s + 1$)[13].

The so-called cost of capital approach is used to compute the risk margin. This approach takes the concept of selling the insurer's liabilities to another insurance company with an empty portfolio (European Commission, 2010). The price for taking over the liabilities must then be the sum of the liabilities' value and a risk premium for taking over the risks associated with the liabilities, based on all future SCRs. This risk premium is afterwards multiplied with a Cost of Capital (CoC) rate, the rate on top of the risk-free rate that reflects the dividends which should be paid to attract capital for the purchase of the liabilities. In Solvency II the CoC rate is set at 6%. The risk margin is then determined as follows:

$$RM_t = CoC \sum_{s \geq 0} SCR_{t+s} \cdot P_t^{(s+1)}. \tag{4.3}$$

One sees, however, that defining the risk margin like this will lead to a loop in calculating the SCR and the RM: the SCR depends on the risk margins to be held from t onwards and vice versa. Moreover, it is impossible to determine the future SCRs because these are contingent on the run-off behavior of the portfolio, which is uncertain at time t. Therefore assumptions are needed to solve these issues. CEIOPS (2009b) proposes discarding the risk margins in the liabilities computation for the SCRs, as these are supposedly relatively small compared to the total liabilities. This solves the first issue and makes it possible to calculate both the SCRs and the RMs, but the process remains cumbersome because one needs to simulate the development of the portfolio over the entire future time span or approximate the distribution of the underlying variables in closed form. QIS5 (CEIOPS, 2010), the fifth Quantitative Impact Study on the calibration of Solvency II, therefore proposes further computational simplifications for the SCR computation.

The first simplification concerns the uncertainty surrounding the mortality rates: although the original formulation of Solvency II demands the 99.5% Value-at-Risk, many insurers use best estimate and thus deterministic death rates only. Hence in QIS5 a 20%[14]

[13] Note that the liability payments are due ultimo year t, hence the $s + 1$ rather than s.

[14] 20% as the percentage for a one-off permanent decrease in the one-year mortality rates is based on two analyses: one of historic improvements in unisex mortality tables in nine European countries and one about the shocks of future improvement in mortality rates. The former found a median decrease in mortality rates between 19% and 25% between 1992-2006, the latter used a stochastic model to find the 99.5% percentile, based on simulations with the assumption that annual mortality improvements follow a Normal distribution. The mean and standard deviation are calculated using the annual unisex mortality improvements for all ages for the life tables of the earlier mentioned nine countries between 1992-2006. See CEIOPS (2009a) for more details.

immediate shock to all one-year death probabilities (so for all ages x and for every year t) is introduced as corresponding to the 99.5% quantile, while the best estimate value of the liabilities is computed with the deterministic probabilities. The SCR is then the difference between these two values, after discounting:

$$SCR_t = \sum_{s \geq 0} (\tilde{L}^{shock}_{t+s} - \tilde{L}^{BE}_{t+s}) \cdot P_t^{(s+1)}, \tag{4.4}$$

with the superscript *shock* referring to the shock scenario and the superscript BE denoting the best estimate. For future years $t + \tau$ the expectation of SCR_{t+s} can be calculated as follows:

$$\mathbb{E}_t[SCR_{t+\tau}] \approx \sum_{\tau \geq 0} \mathbb{E}_t[(\tilde{L}^{shock}_{t+\tau+s} - \tilde{L}^{BE}_{t+\tau+s}) \cdot P_{t+\tau}^{(s+1)}]. \tag{4.5}$$

Computation of the risk margin at time t is then as follows:

$$RM_t \approx CoC \sum_{s \geq 0} \mathbb{E}_t[SCR_{t+s}] \cdot P_t^{(s+1)}. \tag{4.6}$$

Although it is now possible to calculate the SCR_{t+s} using equation (4.5), the outcome is still contingent on how the portfolio develops between time t and time $t + s$, so it remains impossible to determine the expectation. Therefore QIS5 proposes another simplification: future values of the SCR are determined using a constant fraction of the best estimate:

$$\frac{\mathbb{E}_t[SCR_{t+s}]}{SCR_t} \approx \frac{\mathbb{E}_t[BEL_{t+s}]}{BEL_t}, \tag{4.7}$$

with BEL_t as defined in (4.9). Equation (4.7) implicitly assumes that the SCR is proportional to the best estimate. However, one should note this assumption does not hold when, for example, the underlying risks change over time. Börger (2010) claims that the above method gives rather inaccurate approximations of future SCRs, as he finds that for a closed portfolio, the fraction of the SCR compared to the best estimate value of the liabilities tends to be increasing over time in general.

The final simplification allowed by QIS5 also concerns the future SCR computations. QIS5 allows insurers to assume that run-offs in the portfolio occur according to the best estimate, between time t and time $t + s$. As a result, the SCR_{t+s} is only dependent on deviations from the best estimate after time $t + s$, which can be written as follows:

$$\mathbb{E}_t[SCR_{t+s}] \approx SCR_{t+s}(\xi_t^s), \tag{4.8}$$

where the ξ_t^s stands for the scenario in which death probabilities develop as in the best estimate between time t and time $t+s$, so the right-hand side is the realization of SCR_{t+s} for the scenario ξ_t^s.

Börger (2010) argues this SCR and risk margin computation method is seen as giving the "exact" risk margin in general. However, through an analytical approximation Stevens et al. (2010) claim that this simplification is not always exact. Nevertheless, it seems to give more accurate results than the second simplification proposed, so from here on the standard approach is defined as the method which uses the 20% longevity shock and this final simplification to calculate future SCRs.

4.1.2 Solvency II: Internal model

The standard formula, and specifically the longevity shock scenario proposed in QIS5 as described above is deemed too conservative by many insurers (CEIOPS, 2010). In addition, such a mortality shock produces an unusual shape for the capital requirements (Richards, 2012). Furthermore, the effect of the insureds' age on the uncertainty with respect to the present value of the liabilities is ignored in the standard approach, as the standard formula assumes a portfolio with average age over 60. Hence there is general consensus that the SCR computed via an internal model might be more accurate. The research question of this thesis is whether the sizes of the SCRs calculated using the standard formula and the internal model differ.

Rather than calculating best estimate liabilities and the value of the liabilities after applying a one-off shock to the death probabilities, as the standard approach suggests, the internal model approach uses a stochastic mortality model to determine the 99.5% Value-at-Risk (VaR). The procedure for the best estimate liabilities remains the same as before, but for the shock scenario, the following is done: the stochastic internal model simulates future mortality rates $q_{x,t}$. Using these simulated $q_{x,t}$'s it is possible to find the 99.5% Value-at-Risk for longevity risk. A more detailed explanation concerning this approach is given in Section 4.3.3.

4.1.3 Solvency (II) in the Literature

There are several academic papers which have explicitly discussed solvency requirements for pension annuities with respect to longevity risk[15], e.g., Börger (2010); Hári (2007); Hári et al. (2008); Olivieri & Pitacco (2003); Stevens (2011) and Börger et al. (2014). The methods used to calculate solvency differ across these articles in several terms, e.g., which stochastic mortality model to use; whether financial market risk and/or parameter risk are taken into account as well as the method proposed to determine solvency buffers. For instance, Olivieri & Pitacco (2003) does not use a stochastic mortality model, but incorporates uncertainty through the use of a small, medium and high decrease in mortality rates that can occur with a certain probability. This probability measure is then used to calculate solvency requirements via two methods: the first are so-called reserve-based solvency requirements, the second concerns obligations-based solvency requirements, where obligations are both with respect to policyholders as well as with respect to shareholders. Olivieri & Pitacco (2003) also take into account investment risk, which originates from the uncertainty caused by the insurer or pension fund investing in the financial market. The authors calculate the buffers for portfolios of immediate annuities. It is found that significant buffers need to be held, a solvency margin of approximately 13% in case of no investment risk (the authors only implement interest rate risk for investment risk) and approximately 15% with interest rate risk, although the results are strongly dependent on the choices made in the model, in particular on the mortality model used.

[15]These papers all define longevity risk as the uncertainty in future mortality rates, rather than the Solvency II definition of longevity risk: the risk of decreasing future mortality rates.

Hári et al. (2008) considers portfolios of pension annuities, and appraises the impact of longevity risk as well as mortality risk on the probability of underfunding. This latter risk is found to be diversified in portfolios with a large number of participants, the so-called "pooling effect". This effect was also found in Olivieri & Pitacco (2003). Hári et al. (2008) uses a variant of the Lee-Carter model and forecast mortality rates using Dutch gender-specific data. Additionally, parameter risk is taken into account. The authors find that, in the absence of financial market risk, longevity and parameter risk are of substantial size. When financial market risk is increased, so when the fund invests a larger fraction in risky assets, the other two risks have a relatively smaller impact. The solvency measures considered are the Value-at-Risk (VaR) and Expected Shortfall (ES) based on a funding ratio approach. The required solvency buffers range from 7% to 8.5% for a 2.5% insolvency probability, dependent on the horizon and the solvency requirement. In their analysis, Expected Shortfall gives higher buffers than the Value-at-Risk. However, when financial market risk is included and a substantial percentage is invested in risky assets, the uncertainty in these assets decreases the relative significance of uncertainty in the mortality rates, so the buffers needed are lower.

Yet other methods are used in Olivieri & Pitacco (2008) and Olivieri & Pitacco (2009). Olivieri & Pitacco (2008) focuses on mortality risk and longevity risk specifically, disregarding all other risks. Again, immediate life annuities are considered, for one cohort specifically. The buffers for such a portfolio are calculated based on two different solvency rules, and compared to the Solvency II standard formula as described in Section 4.1.1, using Italian mortality data. Olivieri & Pitacco (2009) utilizes the same method as Olivieri & Pitacco (2008), with the difference in the mortality model used. The authors find it is possible to gain efficiency in capital allocation when a properly designed internal model is used, and that the use of the two defined solvency requirements do not give fundamentally differing results. Adopting the standard formula could lead to unnecessarily high or undesirably low capital requirements, dependent on the age profile of the portfolio considered (Olivieri & Pitacco, 2009).

Olivieri (2011) uses the same approach as Olivieri & Pitacco (2009) but extends certain classical results with respect to the modeling of parameter risk, joint to the number of deaths. The author find that it is recommendable to reference the insurer's portfolio mortality to a reference population when updating the parameters of the mortality model, so that the underlying trend can be traced more accurately.

Richards et al. (2014) introduces yet another way of calculating solvency requirements for annuity portfolios and defined-benefit pension schemes, in light of the Solvency II regulation: although longevity risk is a long-term accumulation of small changes, Solvency II demands insurers to look at longevity risk from a one-year Value-at-Risk point of view. Richards et al. (2014) therefore introduces a method where the mortality rates in the upcoming year are simulated with the use of stochastic mortality models, and hence to deduce the required solvency buffers using these simulations to project new mortality tables into the future. The authors find that such a 'stressed trend' method gives a lower required buffer than the shock approach as defined in Solvency II, while taking into account the one-year view.

The Lee-Carter model is used as the stochastic mortality model in Stevens et al. (2010), to derive a closed form approximation for the capital requirements in an internal model that fits with the Solvency II requirements with respect to longevity risk, while also taking into account financial risk. In addition, the effect of the simplifications proposed in Solvency II, as also described in Section 4.1.1 are quantified. Again a funding ratio approach is used for the capital requirements. The results indicate that the Solvency II shock will lead to an overestimation of the solvency buffer needed compared to using an internal model, since the long-term trend is not taken into account in the most logical manner. The simplifications proposed by the regulation of Solvency II do therefore not lead to an accurate estimation of the necessary buffer's size, assuming the Lee-Carter model gives an accurate forecast of future mortality rates.

Börger (2010) finds, in addition, that different risk margin approximations can give significantly different values. The authors also find, in line with above results by other researchers, that the one-off shock from Solvency II has structural shortcomings when compared to an internal model. They therefore propose defining a different shock size for each age and maturity to improve the risk perception. Noteworthy compared to the earlier papers is the fact that Börger (2010) uses a so-called forward model as described in Bauer et al. (2008). Börger et al. (2014) again uses another stochastic mortality model to compare capital requirements to those computed using the Solvency II standard formula. The difference is including a stochastic trend component such that the model does not have to be fully re-estimated after one year. Again the results indicate that the Solvency II standard formula seems relatively prudent and gives higher capital requirements than the internal model proposed in Börger et al. (2014).

To summarize, the majority of the literature finds that capital requirements are estimated more accurately if an insurance company develops its own internal model in line with the mortality profile of its policy holders, rather than using the standard formula proposed in Solvency II.

4.2 Data

Certain assumptions are made with respect to the portfolios considered in this thesis. The literature usually considers simple, closed[16] portfolios of only 65-year olds, which typically only receive an old age pension or both an old age and a partner pension. See for example Börger (2010); Hári et al. (2008); Stevens et al. (2010) and Börger et al. (2014). The aforementioned papers find that it is indeed favorable to use an internal model over the standard approach. However, the same may not necessarily be the case for a different portfolio composition, for example, a younger and more diversified portfolio. The assumption of a 65-year old only portfolio is unrealistic for the average Dutch insurer. Therefore, the age composition of the base portfolio considered in this thesis is chosen to show more resemblance to the Dutch population pyramid, although the robustness of the results with respect to this portfolio composition is analyzed as

[16] A closed portfolio means that there are no new entrants to the portfolio.

well. It is noteworthy to mention though, that the standard formula in Solvency II assumes the average age of the participants in the portfolio is at least 60, which is not the case for the majority of the Dutch insurers.

Portfolios with the participants' age normally distributed around different means (35, 45, 55 and 65) and variance 12 are considered. Such age compositions are deemed representative for most Dutch insurers' portfolio participants.

To determine the present value of future liabilities, the nominal term structure of zero-coupon bonds for insurers as published by the Dutch Central Bank (De Nederlandsche Bank, DNB) is used, dating from November 30, 2014. This term structure of interest rates is based on the price of zero-coupon bonds. This and older term structures of interest rates are available at the website of DNB[17]. The exact figures of the term structure used are given in Appendix C.

4.3 Methodology

This section describes the methods used to determine the Solvency Capital Requirement (SCR) for longevity risk. Firstly, in Section 4.3.1, the necessary portfolio assumptions are explained. Afterwards, the methodology to determine the Solvency II buffers using the standard formula and the internal model are discussed in Sections 4.3.2 and 4.3.3 respectively.

4.3.1 Portfolio Assumptions

Although the basics for the SCR computation have been outlined in Section 4.1 to a certain extent, assumptions about the portfolio and the rights built up by the participants are needed to be able to do the actual computations. The portfolio compositions were already shortly described in Section 4.2. The means and variances of the normally distributed portfolio age composition are as follows: average ages 35, 45, 55 and 65 respectively with variances set at 12. The ages of the people in the portfolio range from 25 to 85.

Furthermore, the retirement age is set at 65 and participants build up an equal amount of pension rights throughout their (working) life each year from age 25 to 65, i.e., an x-year-old participant has built up $\frac{\min\{40,x-25\}}{40}\%$ of their total pension. To ease the computational load, the totals per age group and gender are considered for the portfolio development, rather than each policy individually. Moreover, the career patterns of the participants are assumed to be flat, i.e., the insured amounts remain unchanged in the future.[18] Indexation is also not taken into account. As a result, entitlements for each age group only change because of portfolio mortality, assuming the fund is closed to new participants.

[17] www.statistics.dnb.nl/financiele-markten/rentes/index.jsp

[18] Note that in the Netherlands the size of the pension entitlements is nowadays based on the average pay of a participant, and that a participant's wage usually increases as the participant ages. This is not taken into account here for the sake of simplicity and without loss of generality assumed.

For the discounting of the liabilities the nominal term structure of zero coupon bonds for insurers as published on November 30, 2014 by the Dutch Central Bank (De Nederlandsche Bank, DNB) are used. This term structure provides discount rates $P_t^{(s+1)}$, based on the price of zero coupon bonds with duration up to 100 years: for durations of more than 20 years the Ultimate Forward Rate (UFR) extrapolation[19] is used to determine the discount factor. In order to focus on longevity risk only, interest rate risk is neglected by considering the term structure constant over time: i.e., the price of a τ-year zero coupon bond in year t is equal to the price of a τ-year zero coupon bond in year $t+s$, for $s > 0$. Therefore the subindex t in $P_t^{(s+1)}$ is dropped from here onwards. The discount factors used are displayed in Appendix C.

Finally, in order to focus on longevity risk specifically, no financial market risk is taken into account, i.e., interest rates are taken to be deterministic and constant over time and it is assumed that the assets held by the insurer are risk-free zero-coupon bonds only.

4.3.2 Standard Approach

The standard approach for the longevity risk SCR computation was already explained quite specifically in Section 4.1.1, so to summarize the analytics from Section 4.1.1, calculating the SCR using the standard approach comes down to the following:

- The value of the liabilities is defined as a best estimate value and a risk margin: $L_t = BEL_t + RM_t$, with

$$BEL_t = \sum_{s \geq 0} \mathbb{E}_t[\tilde{L}_{t+s}] \cdot P^{(s+1)}. \tag{4.9}$$

  $P^{(s+1)}$ is the discount factor, also see Section 4.3.1. RM_t is a risk margin determined as

$$RM_t = CoC \sum_{s \geq 0} SCR_{t+s} \cdot P^{(s+1)},$$

  where CoC is the Cost-of-Capital rate, set at 6% in Solvency II. However, because calculation of the SCR and the RM leads to a loop, CEIOPS (2009b) allows to discard the risk margins in the liabilities computation for the SCRs.

- The standard approach then states that the SCR is the difference between the best estimate value of the liabilities and the 20% mortality rates shocked value of the liabilities. The 20% decrease is applied to the best estimate forecast of the one-year death rates as calculated from the stochastic mortality model described

[19] The UFR extrapolation method is described in detail at the website of DNB: http://www.toezicht.dnb.nl/en/binaries/51-226788.pdf.

in Section 3.3, with all errors ϵ_t in the κ_t process set at 0.

$$SCR_t = \sum_{s\geq 0} (\tilde{L}^{shock}_{t+s} - \tilde{L}^{BE}_{t+s}) \cdot P^{(s+1)} \tag{4.10}$$

$$\mathbb{E}_t[SCR_{t+\tau}] \approx \sum_{\tau\geq 0} \mathbb{E}_t[(\tilde{L}^{shock}_{t+\tau+s} - \tilde{L}^{BE}_{t+\tau+s}) \cdot P^{(s+1)}]. \tag{4.11}$$

The risk margin at time t is then $RM_t \approx CoC \sum_{s\geq 0} \mathbb{E}_t[SCR_{t+s} \ \cdot P^{(s+1)}]$.

- Although it is now possible to calculate the SCR, the outcome is still dependent on the portfolio development. To simplify SCR calculations QIS5 allows insurers to assume that run-offs in the portfolio occur according to the best estimate between time t and time $t+s$, thus only the deviations from the best estimate after time $t+s$ are needed for the SCR_{t+s} calculation.
- It is now possible to calculate both the SCR needed at time t as well as for all future years $t+s$, with $s > 0$.

Note that this approach neglects individual mortality risk[20], although past literature (De Waegenaere et al., 2010; Olivieri & Pitacco, 2003) has found that individual mortality risk can be pooled away in a large insurance portfolio, as the observed number of deaths will then converge to the expected value.

4.3.3 Internal Model

The main research question of this thesis is whether the shock scenario of 20% decrease in death rates as proposed in QIS5 is comparable to an internal model's results. The 20% shock is too prudent according to many insurers(CEIOPS, 2010), and it is also argued that the standard approach neglects the effect of the insureds' age on the uncertainty of the liabilities' present value (Richards, 2012).

Rather than the standard approach described in Section 4.3.2, it is an option for insurers to build an internal model and determine the 99.5% Value-at-Risk (VaR) of the liabilities. This 99.5% VaR in excess of the best estimate value of the liabilities is the SCR for longevity risk. It may well be possible that this internal model approach yields different SCRs for longevity risk than the standard approach.

The best estimate value of the liabilities is computed in exactly the same way as for the standard approach (see Equation (4.9)): by using the best estimate death probabilities. The 99.5% quantile calculation requires a more detailed explanation:

In Section 3.3 the method of forecasting mortality rates for the SCR calculation was explained. 1,000 sets of different mortality scenarios for all ages x, running from age 0 to age 98 for the years 2014 to 2113 are simulated using the Lee-Carter model. These

[20]Individual mortality risk has earlier been defined as the risk that, given a certain mortality distribution, the remaining lifetime of an individual is still a random variable, i.e., one can live shorter or longer than expected.

simulations are drawn using the parameter estimates obtained from the SVD estimation, as described in Section 3.3 and in Appendix A.

As stated before, the old age pension of 1 ultimo each year of survival is paid out to a participant aged 65 or older. So at the end of each year t, an amount of $\tilde{L}_t = \sum_{x=65}^{98} \pi_{x,t}$ is paid to the living retired participants, with $\pi_{x,t} = \pi_{x-1,t-1} \cdot (1 - q_{x,t})$, and $\pi_{x,2013} = 1, \forall x$. This method is employed for the 1,000 simulated mortality scenarios for $t = 2014, \ldots, 2113$ to determine the yearly payments to the participants above 65 in all mortality scenarios, discounted to the present time. The 99.5% quantile of these 1,000 summed present values is then taken and subtracted by the best estimate value of the total liabilities for each year, to obtain the SCR for each year. In formula this is written as:

$$SCR_t = Q_{0.995,t}(L_t) - L_t^{BE} \tag{4.12}$$

with $L_t = \sum_{s\geq 0} \tilde{L}_{t+s} \cdot P^{(s+1)}$ and $L_t^{BE} = \sum_{s\geq 0} \tilde{L}_{t+s} \cdot P^{(s+1)}$.

However, recall that in order to calculate future SCRs, a simplification has been put forward in QIS5 that allows insurers to assume the run-off in the portfolio occur according to the best estimate between time t and time $t+s$. Although this risk margin is not always exact (Stevens et al., 2010), Börger (2010) claims this computation method suffers only a small loss in accuracy. So for each year $t+s, s = 1, \ldots, 99$, it is assumed that the portfolio run-off follows the best estimate death probabilities between time t and $t+s$, denoted by ξ_t^s. Then 1,000 simulations are run from year $t+s$ as described in Section 3.3. Then the 99.5% quantile of the present value of the total liabilities can again be calculated and the SCR is determined as follows:

$$\mathbb{E}_t[SCR_{t+s}] = Q_{0.995,t}(L_{t+s}) - L_{t+s}^{BE} \tag{4.13}$$

with $L_{t+s} = \sum_{\tau\geq 0} \tilde{L}_{t+s+\tau} \cdot P^{(\tau+1)}$ and $L_{t+s}^{BE} = \sum_{\tau\geq 0} \mathbb{E}_t[\tilde{L}_{t+s+\tau}|\xi_t^s] \cdot P^{(\tau+1)}$, with $\tilde{L}_{t+s+\tau}|\xi_t^s]$ denoting the realization of $\tilde{L}_{t+s+\tau}$ if the run-off of the portfolio was according to the scenario ξ_t^s between year t and year $t+s$.

After all these simulations and calculations have been done for every year, one can determine the risk margin for the first year of the calculations as

$$RM_t \approx CoC \cdot \sum_{s\geq 0} \mathbb{E}_t[SCR_{t+s}] \cdot P^{(s+1)}. \tag{4.14}$$

The internal model should give more plausible results with respect to the Solvency Capital Requirements than the standard approach described in QIS5, since this latter approach actually calculates the 99.5% quantile of the liabilities. The standard approach neglects the age composition of the portfolio, as it assumes the same deterministic shock over all ages. Intuitively however, it seems more logical that a portfolio with younger participants will suffer from a larger longevity risk than a portfolio with old participants, since there is more time for the younger participants' death rates to deviate from current estimations in the long run. Therefore it is quite likely that the assumption of a 20% shock being a decent estimation for a portfolio of 60-year olds will give severely overestimated results for younger portfolios and underestimated SCR buffers for older

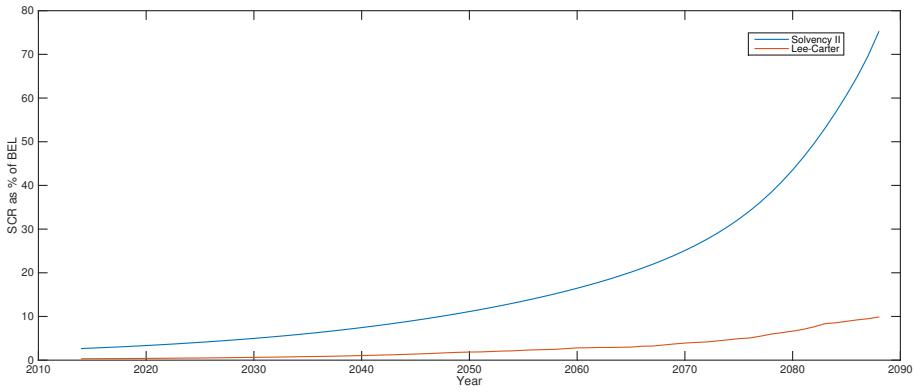


Figure 4.2: The SCR as a percentage of BEL_t for each year t for both the Solvency II standard approach (blue) and the Lee-Carter model (red), with a portfolio consisting of an equal number of males and females with average age 45 and the age composition normally distributed around 45 with standard deviation 12.

portfolios. However, already it seems that even for a portfolio of 65-year olds only the deterministic shock of 20% in the standard approach is too prudent (Börger, 2010; Stevens et al., 2010). In the next section the results of this thesis are discussed and compared for both approaches, for the model portfolios described earlier.

4.4 Results

The baseline results following from the methodology described above are given below. Furthermore, in Section 4.4.1, a sensitivity analysis is done with respect to certain input parameters such as volatility and taking into account experience mortality, also known as portfolio-specific mortality.
The SCR for each year from 2014 onwards is displayed in Figure 4.2 as a percentage of the best estimate liabilities to be held by the insurer, for both the Solvency II standard approach and the internal model approach, denoted by Lee-Carter. One immediately sees that the SCR increases evidently over time for both approaches, which can be explained by the fact that, as time passes, a decreasing number of participants remain in the portfolio and therefore the influence of micro-longevity risk, i.e., individual mortality risk[21] as defined in Section 1 increases. However, what is more evident is the far larger increase in the SCR needed for the Solvency II method compared to the Lee-Carter approach. We already ascertained that the Lee-Carter model, albeit a stochastic model, gives very tight confidence bands, so it is not very surprising that the SCR needed for the internal model is of moderate size. Considering the fact that every one-year death rate is decreased by 20% in the Solvency II standard approach and the fact that the elderly

[21]The fact that uncertainty remains even if the death probabilities are known, because one's lifetime is a random variable.

have relatively high death rates, it is derived that the effect of a 20% shock decreases mortality rates significantly in absolute value, especially for the elderly, which are also the people to which liabilities have to be paid and thus the dominating age group in the portfolio concerning the results, and therefore the SCR grows at a much faster pace over time and is larger to begin with. However, keep in mind that in absolute value the influence is neglectably small, because in the last few years of the portfolio's run-off there are only few units to be paid. If one were to estimate the 'shock' imposed by the Lee-Carter 99.5% quantile death rates, shock sizes of less than 10% are found, which are significantly lower than the 20% imposed by Solvency II. One might wonder whether 20% is the proper percentage for a deterministic shock imposed on the one-year death rates.

Thus, from this figure the tentative conclusion can be drawn that the internal model based on the Lee-Carter extrapolation gives lower buffers than the Solvency II standard formula, in case of a Dutch portfolio with the characteristics as described above. However, one should keep in mind that the Lee-Carter model only accounts for a possible change in trend, i.e., it considers deviations from the trend estimated through κ_t. The Solvency II standard approach, on the other hand, also considers the risk of having used the 'wrong' model and deviations in the level of the mortality rates, i.e., the chances that the mortality rates of the people considered are different from those of the entire population. In this case, one generally finds that the insured have lower mortality rates than the uninsured, which may give different results if this deviation is taken into account. This phenomenon will be discussed in more detail in Section 4.4.1.4.

Furthermore, the approach in this thesis is based on a number of assumptions and input parameters, which do not necessarily hold for all insurers, such as the estimation period for the Lee-Carter model and the average age of the participants. The impact of these parameters is discussed in the next section.

4.4.1 Sensitivity Analysis

The results in Section 4.4 depend on a number of input parameters. It is possible that a change in some of these parameters influences the results found significantly, so a sensitivity analysis is done for the following input: the portfolio composition, the observation window for the Lee-Carter model estimation, the volatility of the κ_t process in the Lee-Carter model, and the influence of experience mortality, also known as portfolio-specific mortality.

4.4.1.1 Average Portfolio Age

The baseline results are based on a portfolio with average age 45 and the age composition distributed normally around this average of 45 with a standard deviation of 12. However, there are many insurers with either younger or older portfolios, so the same calculations are done for average ages 35, 55 and 65, the standard deviation still held at 12. One would expect that an older portfolio will have lower SCRs since there is less uncertainty concerning the future development of mortality rates, and vice versa higher SCRs are expected for younger portfolios.

The graphs with SCRs for the respective portfolios are given in Appendix D. For the standard approach we surprisingly find that the SCRs are higher for a portfolio with average age 45 rather than 35. Apparently the 20% shock in the one-year death rates adds up to a larger direct risk in terms of money than the larger long-term uncertainty for the young participants. For the internal model the difference between these two portfolio average ages is negligible. Vice versa, we find higher SCRs for older portfolios (average 55 and average 65) for both methods, although again the difference between the SCRs are relatively small for the internal model results. For the standard approach, the differences are considerable, especially for 65-year olds, where a steeper increase in the SCR takes place compared to 45-year olds.

4.4.1.2 Observation Window

We saw in Section 3.4.1.1 that the mortality rate estimation showed some sensitivity to the starting year of the estimation period used, especially for males. The baseline analysis used estimation period 1970 - 2013. To test for the observation window sensitivity the SCR is recalculated using starting years 1960 up to 1980 with 5-year intervals, the end year remaining the same at 2013.

Graphs for these SCRs with different starting years in the observation period are given in Appendix E. For the starting years 1960 and 1965 the change in the SCRs over the years is negligible. For starting year 1975, however, the standard approach shows a steeper increase near the end of the portfolio's duration than for starting year 1970. For the internal model the SCRs are also slightly higher near the end. This is likely caused by the steeper decrease in mortality rates from more recent years. This effect of steeper increasing SCRs near the end of the timeline is even larger when starting year 1980 is used.

Thus, although the starting years 1960 - 1970 show little difference in results, using less data points and more importantly, more recent years can influence the SCRs near the end of the portfolio's run-off.

4.4.1.3 Volatility κ_t

In Section 3.4.1.2 it was found that changing the estimated volatility of the κ_t process did not affect the 99.5% quantile of one's life expectancy significantly. However, it is still of value to find out by how much the volatility should increase for the internal model's SCR to become similar in size to the standard approach's SCR.

If the estimated volatility of κ_t is increased by a factor 30, the SCRs using both methods are quite similar in shape and size, as displayed in Figure 4.3. This solidifies the belief that the Lee-Carter confidence intervals are quite narrow, and that the SCR determined from the internal model is somewhat sensitive to the size of κ_t's estimated volatility.

4.4.1.4 Experience Mortality / Portfolio-specific Mortality

The experience mortality[22] is a mortality factor used by insurers to take into account the

[22]Dutch: ervaringssterfte.

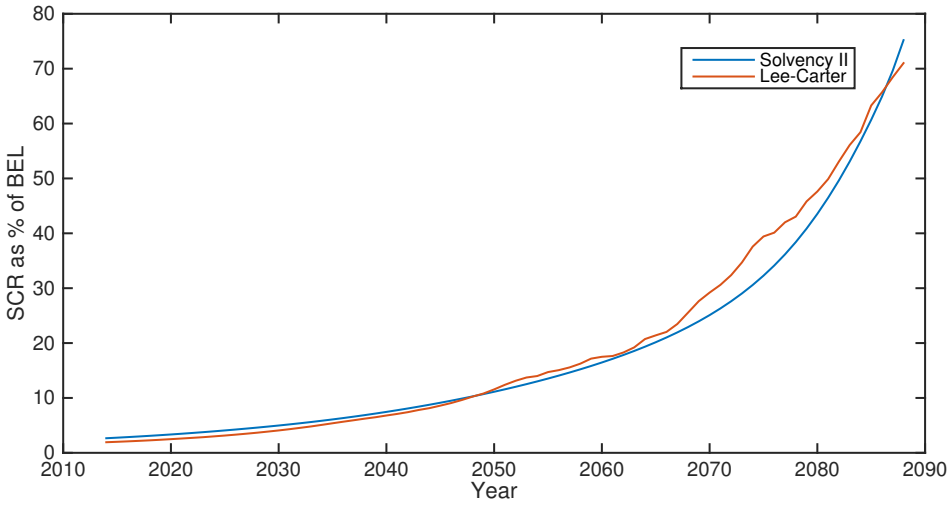


Figure 4.3: The SCRs for both internal model and standard approach under the baseline portfolio assumptions, with the volatility of κ_t increased by a factor 30.

fact that insured people generally have lower mortality rates than the uninsured. The general consensus though, is that this effect decreases in size when a person is older. Work group PLT (2010) have published a set of mortality experience factors which insurers can apply to the population mortality rates to account for this phenomenon. The factors are calculated using data obtained from the largest insurers in the Netherlands. These factors are displayed in Figure 4.4.
To see how the results change because of this experience mortality, in this thesis the mortality factors from Work group PLT (2010) are applied to the forecasted one-year death rates. Afterwards the SCRs are calculated using these portfolio-specific mortality rates. The resulting SCRs are displayed in Figure 4.5.

We find that the SCRs have decreased for both methods, especially at the end of the portfolio's run-off. The size of the SCR in the first 40 years is comparable to the results found in Section 4.4. The decrease in later years makes sense because even though the death rates have decreased through experience mortality, this gain in life expectancy diminishes significantly for the highest ages.

However, despite the fact that insurers are aware of the lower mortality rates of their participants, there is still uncertainty with respect to the level of an insurer's portfolio-specific mortality. The factors from Figure 4.4 are not necessarily the true factors, as these are estimates by Work group PLT. To see how much a difference in level can influence the size of the SCRs, the factors from Figure 4.4 are all decreased by 0.2 and applied to the population mortality forecast. The base SCRs from Figure 4.5 are subtracted from the SCRs calculated using these 'shocked' mortality rates. The resulting figures can be considered the SCR for the level risk of experience portfolio,

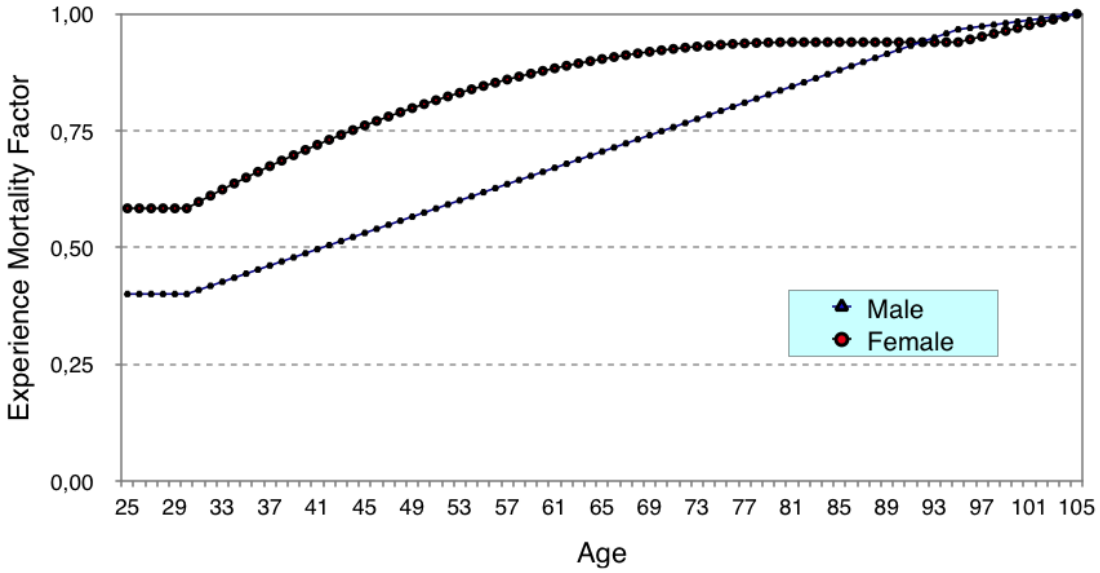


Figure 4.4: Portfolio-specific Mortality factors for men (blue triangles) and women (red circles), for ages 25 - 105, as published in Work group PLT (2010).

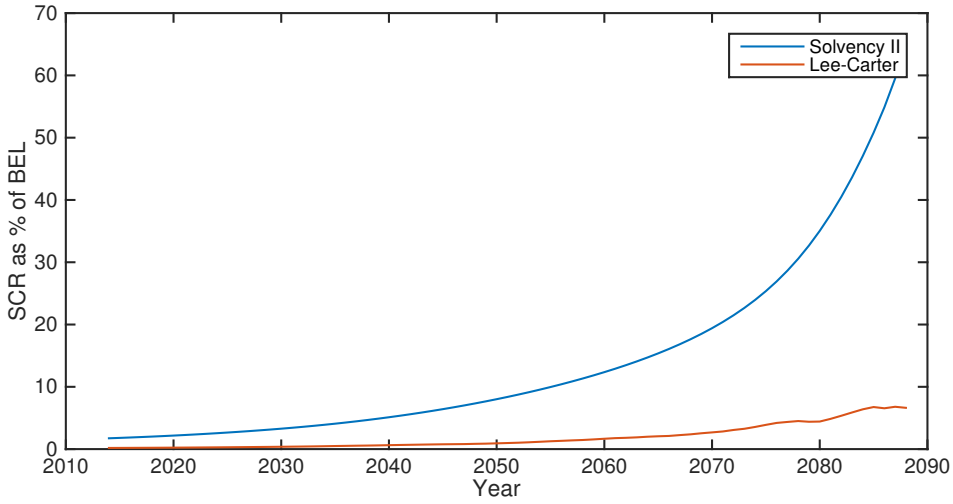


Figure 4.5: The SCRs for both the internal model and the Solvency II standard approach for the baseline portfolio, after experience mortality factors have been applied to the forecasted one-year mortality rates.

and are added to the base SCRs to find the 'total' SCR for longevity risk[23]. However,

[23]Keep in mind parameter and model risk are not accounted for in this thesis.

because these two shocks are related, since both concern a decrease in mortality rates, a correlation factor of 0.5 is used. The resulting SCR as percentage of the best estimate liabilities (including the base experience mortality) are given in Figure 4.6.

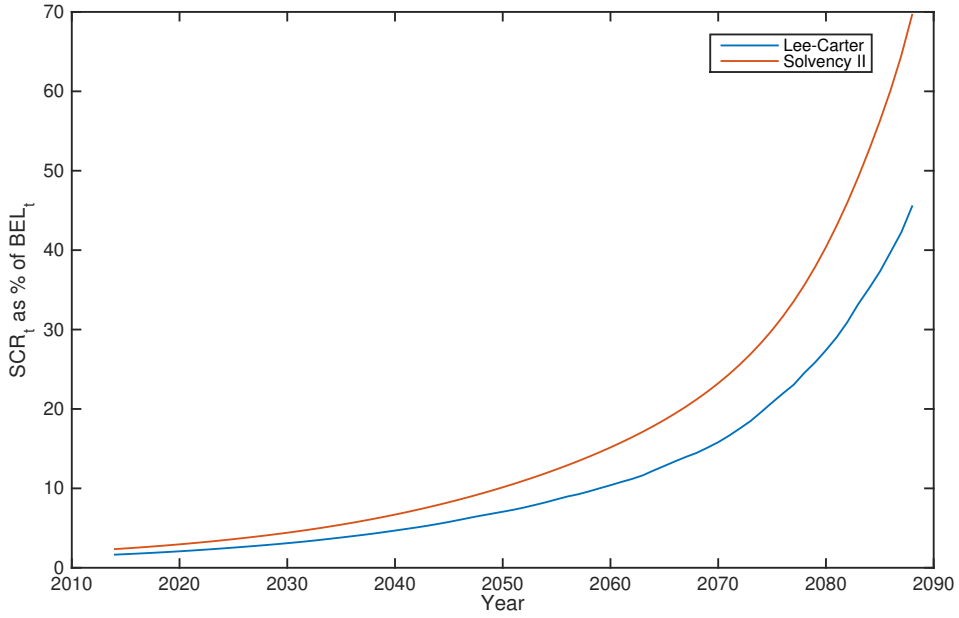


Figure 4.6: The SCRs for both the internal model and the Solvency II standard approach for the baseline portfolio, after experience mortality factors have been applied to the forecasted one-year mortality rates and including the SCR for the experience portfolio level risk.

The impact of adding an SCR for level risk are considerable, as the SCR in the Lee-Carter model has now become quite similar to the Solvency II standard formula SCRs, especially compared to the baseline results displayed in Figure 4.2. However, the shock of 0.2 on the experience mortality factors seems implausible, although its impact is still smaller than a 20% shock on the one-year death rates, as proposed by Solvency II. This result might indicate that, even after accounting for level risk, the 20% shock in Solvency II is perhaps too conservative in size. Nonetheless, Figure 4.6 does show that it is relevant for an insurer to consider the uncertainty surrounding the experience mortality of its participants, as the impact on the SCR can be sizable.

Overall, it would seem that the method used in this thesis is relatively robust to most of the input parameters. The most salient of the input parameters are the portfolio composition and the experience mortality, based on the sensitivity analysis done above.

5 Conclusions and Recommendations

This thesis tried to find an answer to the question: how do an internal model and the Solvency II standard formula compare in terms of the SCR that needs to be held by an insurer? For the internal model, the Lee-Carter (1992) model with some adaptations was used, and the data provided by Statistics Netherlands, on Dutch mortality rates were employed. Certain commonly used assumptions were made concerning the insurance and the portfolio used for the calculations.

One could conclude from the initial results that an insurance company has two very different options: using an internal model, in this thesis the Lee-Carter method, gives lower SCRs than the standard formula, which means that the insurer has to hold less capital which can then be used for other purposes. However, it is important to keep in mind that this method, first of all, only considers the uncertainty in the mortality's trend development, whereas the Solvency II standard formula also takes into account deviations in the mortality level, e.g., insureds are generally found to have lower mortality rates than the general populace of the Netherlands, in this specific case. This latter phenomenon can influence the results significantly, as was found in the sensitivity analysis on the baseline results. However, the shock used to determine the level risk SCR for experience mortality was sizable. Combining the results from the baseline analysis and the sensitivity analysis one could tentatively conclude that, if mortality were to develop according to the Lee-Carter method, the Solvency II formulas SCRs can be quite strict, and an insurer seeking to decrease its SCR buffers could consider investing in the development of an internal model. Other factors influence the size of the SCR for both methods as well: the portfolio composition also increases the SCR if the average age is lower than the baseline assumptions done in this thesis and vice versa for higher average ages. Again, the effect is larger for the Solvency II standard approach than for the internal model. Also, this thesis assumed that the number of male and female participants were equal in the portfolio, which may not necessarily be the case.

Salient for the research question is how the 20% deterministic shock was established: since the Lee-Carter model 99.5% quantile of the death rates showed a 'shock' of less than 10% for all ages and years, one could wonder whether 20% is too prudent. Many insurers have argued that this is the case and the model used here seems to confirm their suspicions. Even adding a 0.2 shock to the experience mortality factors determined in Work group PLT (2010) gives lower SCRs than the Solvency II standard formula, although the influence of experience mortality uncertainty is considerable. Nevertheless, this certainly argues in favor of the insurers claiming that the 20% shock is too prudent.

The considerable influence of the experience mortality level risk with respect to the size of the SCRs also indicates it would be wise for insurers to scrutinize the development of their portfolio mortality. If insurers have a reasonable amount of data available about portfolio-specific mortality, they could also estimate and forecast these portfolio-specific mortality rates with a stochastic mortality model. However, small insurers will probably find this investment too costly, and judging from the results, for these smaller insurance companies the Solvency II standard formula seems a strict but acceptable method.

To conclude, the results from this thesis are indicative of the prudence of the Solvency II standard formula: although the impact of longevity risk should not be underestimated, especially in the long run, there are indications that the method proposed by the Solvency II Directive are perhaps too harsh for the average insurer. Insurers might therefore benefit from having an internal model developed. In addition, if possible, developing a model for portfolio-specific mortality might further increase the accuracy of insurers' buffer requirements and also give these financial institutions more insight in the risks they are exposed to.

5.1 Recommendations

Although a certain number of topics of importance for this research question have already been addressed in this paper, there are always more relevant matters to investigate, with respect to the main topic of this thesis. One of these is, for example, the use of multiple mortality models: the majority of the papers concerning the longevity risk in Solvency II have only considered one mortality model versus the standard formula. It would be of interest to find out whether another mortality model will give significantly different results from the mortality model used here or from the mortality models in the other papers, while employing the same methods otherwise.

Another salient impact that has only been shortly brought to attention in this thesis is the portfolio-specific mortality. The stand alone influence of the experience mortality uncertainty as well as its impact combined with a population stochastic mortality model would be of interest to investigate, to see how and if this portfolio-specific mortality differs much across insurers and to shed more light on the uncertainty surrounding this phenomenon.

It is also of importance to take into account the combined effects of mortality and longevity risk. Whereas longevity risk concerns the risk of living longer than the insurer had expected, mortality risk deals with the uncertainty of an insurer's participants living shorter than expected. Solvency II allows insurers to mitigate part of the longevity risk via mortality risk and vice versa, in case the insurer holds portfolios with products to which mortality risk applies, e.g., widower or orphan pensions. Since most larger insurers do hold these types of insurance products as well, this mitigating effect is of interest.

Other possible topics for future research are the influence of the investment portfolio's asset mix and including interest rate risk, as other papers have already found that portfolios with larger investment risk generally needed a smaller buffer for longevity risk than safer investment strategies.

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A Estimating the Lee-Carter model

As stated in Section 3.3 the parameters of the Lee-Carter method are in this thesis estimated using the so-called Singular Value Decomposition method, which is described below.

The following minimization is to be solved:

$$\min_{\hat{\alpha},\hat{\beta},\hat{\kappa}} \sum_x \sum_t \left(\log m_{x,t} - \hat{\alpha}_x - \hat{\beta}_x \hat{\kappa}_t\right)^2, \tag{A.1}$$

while subject to the constraints

$$\sum_{t=t_1}^{t_n} \kappa_t = 0, \qquad \sum_{x=x_1}^{x_m} \beta_x = 1. \tag{A.2}$$

However, note that the quantity on the right hand side of Equation (A.1) is unobserved, so it is not possible to use OLS (Ordinary Least Squares). Therefore Lee & Carter (1992) propose estimating α_x by

$$\hat{\alpha}_x = \frac{1}{n} \sum_{t=t_1}^{t_n} \log m_{x,t}. \tag{A.3}$$

Equation (A.3) follows from the constraints in Equation (A.2) and setting the derivative of (A.1) with respect to α_x equal to zero. As a result, it becomes possible to estimate β_x and κ_t by performing Singular Value Decomposition (SVD) on the matrix A, defined as

$$A_{x,t} = \log m_{x,t} - \hat{\alpha}_x \quad \forall x, t, \tag{A.4}$$

into $A = USV$, with U an orthogonal $m \times m$ matrix, S a diagonal $m \times n$ matrix and V an orthogonal $n \times n$ matrix. The following estimates are then obtained for β_x and κ_t:

$$\hat{\beta}_x = \frac{1}{c} s_{11} u_1 \tag{A.5}$$

$$\hat{\kappa}_t = c v_1, \tag{A.6}$$

where s_{11} is the first element in the matrix S, the first column vector of U is denoted by u_1, v_1 denotes the first column vector of V, and c is the sum of all elements of the vector $s_{11}u_1$, to ensure that all $\hat{\beta}_x$'s sum up to 1, see the constraint in (A.2). Combining this constraint with the estimate $\hat{\alpha}_x$ ensures that the κ constraint in (A.2) is also obeyed. Note that, in case of 0 deaths for a certain age and time, $m_{x,t}$ becomes 0 and then log $m_{x,t}$ is undefined. There are, however, no 0's in the data used in this thesis.

B Derivations in Lee-Carter model

Recall from Section 3.3 that the difference in κ_t's is modeled as a stationary time series by $\delta_t = \kappa_t - \kappa_{t-1} = C + \epsilon_t$. Then it follows that, for $k \geq 1$ and t_n the final year of observation:

$$\kappa_{t_n+k} = \kappa_{t_n} + \sum_{j=1}^{k}(\kappa_{t_n+j} - \kappa_{t_n+j-1}) = \kappa_{t_n} + \sum_{j=1}^{k}\delta_{t_n+j},$$

Forecasting κ_{t_n+k} and μ_{x,t_n+k} is done as follows:

$$\tilde{\kappa}_{t_n+k} = \hat{\kappa}_{t_n} + \sum_{j=1}^{k}\mathbb{E}[\delta_{t_n+j}] = \hat{\kappa}_{t_n} + k \cdot \hat{C}$$

$$\tilde{\mu}_{x,t_n+k} = exp(\hat{\alpha}_x + \hat{\beta}_x\tilde{\kappa}_{t_n+k}) = \tilde{\mu}_{x,t_n}exp(k\hat{\beta}_x\hat{C}) \tag{B.1}$$

Since we have

$$\kappa_{t_n+k} = \kappa_{t_n} + \sum_{j=1}^{k}\delta_{t_n+j} = \kappa_{t_n} + kC + \sum_{j=1}^{k}\epsilon_{t_n+j},$$

The $p \cdot 100\%$ confidence intervals for κ_{t_n+k} and for μ_{x,t_n+k} are then given by

$$(\hat{\kappa}_{t_n} + k\hat{C} - z\sqrt{k\hat{\sigma}_\kappa^2}\ ,\ \hat{\kappa}_{t_n} + k\hat{C} + z\sqrt{k\hat{\sigma}_\kappa^2}),$$

$$(\tilde{\mu}_{x,t_n}exp(k\hat{\beta}_xC - z\hat{\beta}_x\hat{\sigma}_\kappa\sqrt{k})\ ,\ \tilde{\mu}_{x,t_n}exp(k\hat{\beta}_xC + z\hat{\beta}_x\hat{\sigma}_\kappa\sqrt{k})), \tag{B.2}$$

with $\mathbb{P}[-z \leq Z \leq z] = p$ for Z a standard normal random variable.

Also note that in this thesis, jump-off bias is avoided by taking μ_{x,t_n} instead of $\tilde{\mu}_{x,t_n}$ in (B.1) and (B.2).

C Discount Factors DNB

In Figure C.1 and Table C.1 below the discount factors for the yield on zero-coupon bonds are displayed, as published by the Dutch Central Bank (De Nederlandsche Bank, DNB) for November 30, 2014. Note that this term structure uses the Ultimate Forward Rate (UFR) extrapolation for durations of more than 20 years.

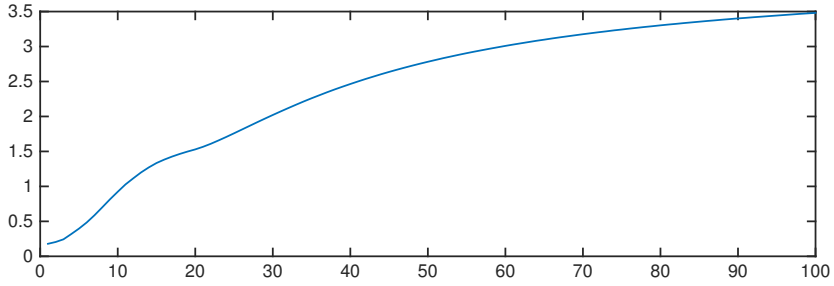

Figure C.1: The yield curve for zero-coupon bonds in percentage points as published by the Dutch Central Bank for November 30, 2014.

Term	Factor	Term	Factor	Term	Factor	Term	Factor	Term	Factor
1	0.9982	21	0.7216	41	0.3632	61	0.1621	81	0.0713
2	0.9959	22	0.7039	42	0.3492	62	0.1556	82	0.0685
3	0.9927	23	0.6852	43	0.3357	63	0.1494	83	0.0657
4	0.9874	24	0.6662	44	0.3225	64	0.1433	84	0.0630
5	0.9804	25	0.6467	45	0.3100	65	0.1376	85	0.0605
6	0.9714	26	0.6269	46	0.2978	66	0.1321	86	0.0581
7	0.9599	27	0.6072	47	0.2861	67	0.1267	87	0.0557
8	0.9457	28	0.5875	48	0.2749	68	0.1217	88	0.0535
9	0.9296	29	0.5679	49	0.2640	69	0.1168	89	0.0513
10	0.9122	30	0.5487	50	0.2536	70	0.1121	90	0.0492
11	0.8936	31	0.5297	51	0.2436	71	0.1076	91	0.0473
12	0.8754	32	0.5110	52	0.2338	72	0.1032	92	0.0454
13	0.8565	33	0.4927	53	0.2246	73	0.0991	93	0.0436
14	0.8379	34	0.4748	54	0.2156	74	0.0951	94	0.0418
15	0.8197	35	0.4574	55	0.2070	75	0.0913	95	0.0401
16	0.8027	36	0.4406	56	0.1988	76	0.0876	96	0.0385
17	0.7861	37	0.4240	57	0.1909	77	0.0841	97	0.0369
18	0.7698	38	0.4080	58	0.1832	78	0.0807	98	0.0354
19	0.7539	39	0.3926	59	0.1759	79	0.0775	99	0.0340
20	0.7382	40	0.3777	60	0.1688	80	0.0743	100	0.0327

Table C.1: The discount factors for zero-coupon bonds used in the Solvency Capital Requirement calculations, as published by DNB for November 30, 2014.

D Sensitivity Analysis Portfolio Composition

In the graphs below the SCRs for a portfolio with average age 45 is compared to portfolios with average ages 35, 55 and 65 respectively. See Section 4.4.1.1.

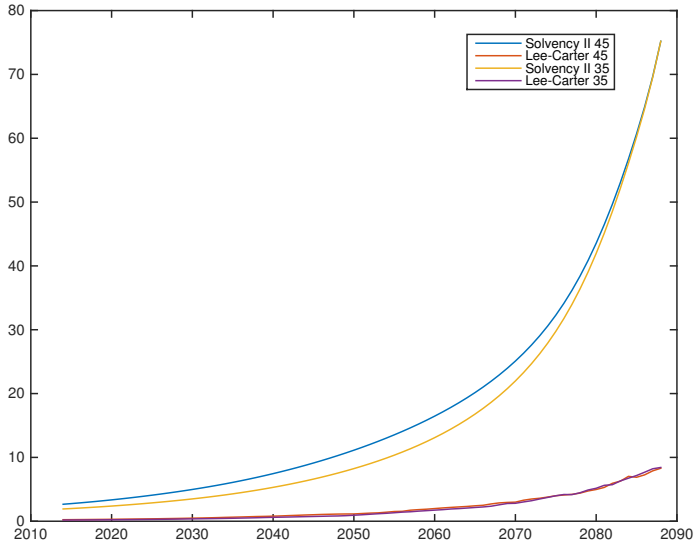

Figure D.1: SCRs as a percentage of BEL_t for portfolios with average age 35 and average age 45 for both the Solvency II standard formula and the Lee-Carter model.

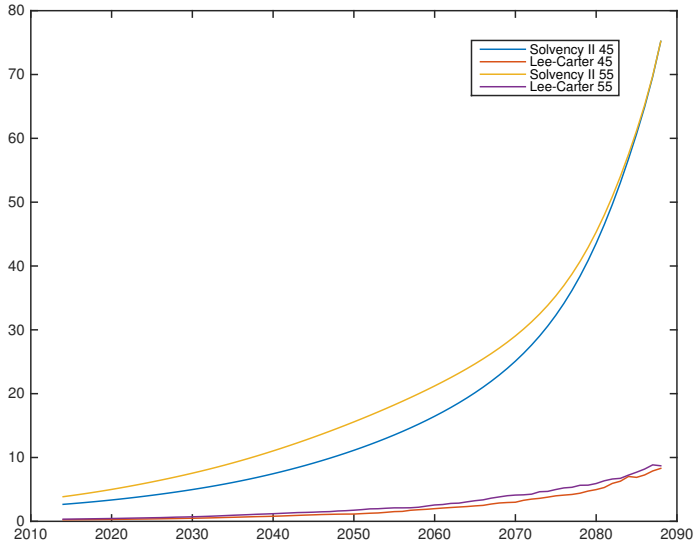

Figure D.2: SCRs as a percentage of BEL_t for portfolios with average age 45 and average age 55 for both the Solvency II standard formula and the Lee-Carter model.

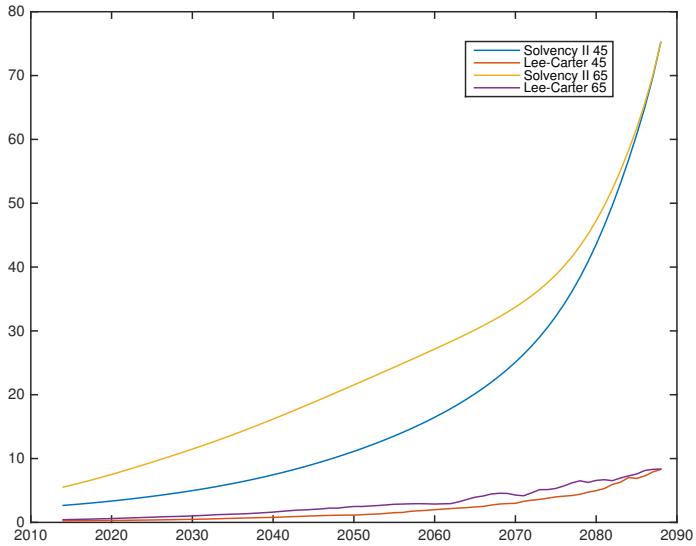

Figure D.3: SCRs as a percentage of BEL_t for portfolios with average age 45 and average age 65 for both the Solvency II standard formula and the Lee-Carter model.

E Sensitivity Analysis Observation Window

In the graphs below the SCRs for a portfolio with observation window 1970 - 2013 is compared to portfolios with starting years 1960, 1965, 1975 and 1980 respectively, with the end year set at 2013. See Section 4.4.1.2.

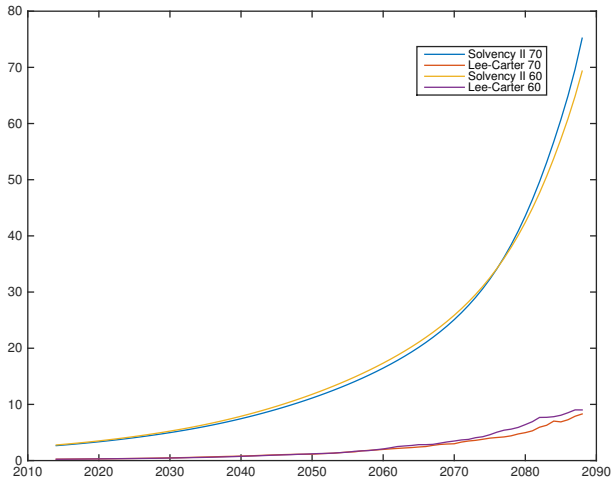

Figure E.1: SCRs as a percentage of BEL_t for portfolios with starting year 1960 and starting year 1970 for both the Solvency II standard formula and the Lee-Carter model.

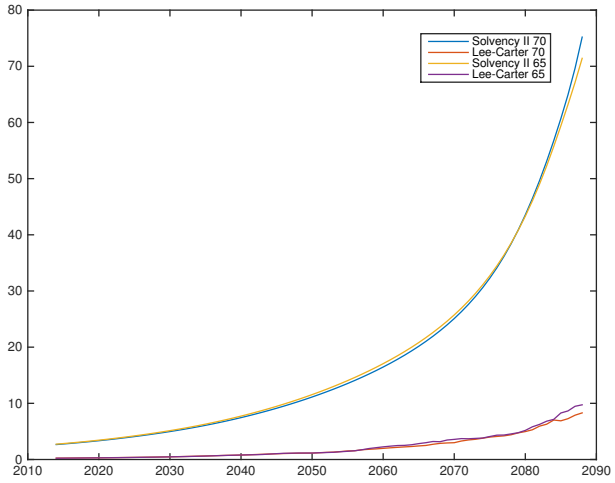

Figure E.2: SCRs as a percentage of BEL_t for portfolios with starting year 1965 and starting year 1970 for both the Solvency II standard formula and the Lee-Carter model.

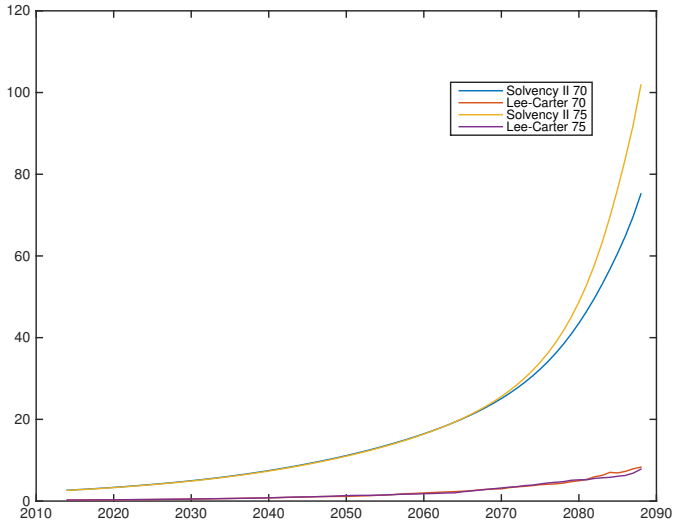

Figure E.3: SCRs as a percentage of BEL_t for portfolios with starting year 1970 and starting year 1975 for both the Solvency II standard formula and the Lee-Carter model.

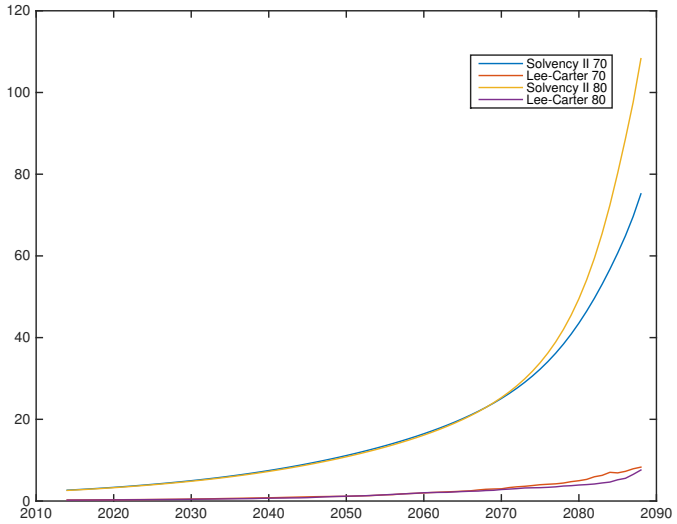

Figure E.4: SCRs as a percentage of BEL_t for portfolios with starting year 1970 and starting year 1980 for both the Solvency II standard formula and the Lee-Carter model.