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Pension Contributions and the Economic Value of the Accrued Pension Entitlements in Dutch Second Pillar Pension Schemes

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Pension contributions and the economic value of the accrued pension entitlements in Dutch second pillar pension schemes

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Management summary

The economic value of pension entitlements takes an important role in the Dutch policy debate. If the economic value of accrued pension entitlements is not equal to the contribution that is paid, value transfers between participants can materialize.

This thesis is written at PGGM and makes use of their ALM model in case stochasticity of the economy is modeled.

Dutch regulations for second pillar pension schemes leave several opportunities that can cause value transfer to materialize. Of these opportunities three relevant settings are analyzed in which value transfers occur:

1. Discounting method of the contribution differs from discounting method of funding ratio

2. Participating in a fund with a surplus or a deficit

3. The uniform accrual and contribution system

This thesis first introduces the basics about valuation and generational accounting. It is also explained that some important research on Dutch pension topics use deterministic calculations in linear pension contracts. This is because deterministic calculations are understandable and fast. Besides that, literature suggests that a change of the equity allocation in a fund with a linear pension contract, does not cause value transfers between participants. This last statement is widely known in the pension sector.

This thesis proceeds with an analysis of the three mentioned settings. This is done for pension funds with a linear pension contract in a deterministic setting. Dependent on the specific setting, value transfers can range from +60% of an annual income for young, to a loss of a full annual income for the retired in setting **1**. Setting **2** shows value transfers that range from +40% for the retired to -20% of an annual income for the young and future generations when the pension fund has a 10% deficit. Setting **3** which is nowadays an important point of debate in the Netherlands leaves several sub-situations under which new value transfers can occur or existing value transfers can change. The value transfers range from +80% for the elderly to a permanent deficit for the future generations caused by the implicit pension debt.

Next, this thesis gives some elaboration on risk neutral valuation and Monte Carlo simulation. This allows to also study non linear pension contracts because from now on also stochasticity is incorporated in the calculations. The statement that a change of the equity allocation in a linear pension contract does not cause value transfers between participants is studied. It is found that this statement does not hold exact in a pension fund model that is not strictly bounded by mathematical assumptions. This could serve as an "eye-opener" for the Dutch pension sector.

The thesis proceeds with an analysis of the three mentioned settings in the pension fund model of PGGM. A link is made between the outcomes from the PGGM model and the results obtained from the earlier calculations in a linear pension contract under a deterministic setting. The results from the linear pension contracts show, as expected, some relevant similarities. It is also concluded that on a high level, the generational effects under the two nonlinear pension contracts, used in this thesis, do not deviate much from the generational effects found under the linear pension contract.

One of the main conclusions in this thesis is that Dutch pension regulations allow situations in which sizable value transfers can occur between participants of second pillar pension schemes. This can be considered as important point because literature suggests that value transfers can make the whole pension system vulnerable due to a decline in confidence of the participants. It is also found that deterministic calculations in linear pension contracts show to give a good view of the more complex pension problems in reality. To gain insight in the dynamics of a realistic problem, deterministic calculations in linear contracts offer a way to obtain a clear view of the size and direction of the value transfers. Last but not least, this thesis provides some awareness on the frequently debated linear contracts that are well known in the Dutch pension sector.

Abstract

This thesis finds that Dutch pension regulations leave several opportunities in Dutch second pillar pension schemes that can cause sizable value transfers between groups of participants. Three realistic situations under which value transfers occur are studied in both a deterministic setting for a linear pension contract and stochastic setting that allows an extension to nonlinear pension contracts. It is found in this thesis that in general, simple deterministic calculations in linear pension contracts give a clear view of the size and direction of value transfers as they can occur in a realistic setting. Also value transfers in linear pension contracts are studied, that are caused by a change of the equity allocation of the pension fund. It is concluded that in a more realistic setting these transfers are not guaranteed to be exact zero, as is assumed throughout the pension sector.

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Contents

1 Introduction

This thesis will focus on the economic value of accrued pension entitlements and the pension contribution made for accrual in Dutch second pillar pension schemes. The economic value of pension entitlements has an important role in the policy debate. When the economic value of accrual is not equal to the contribution, value transfers between participant originate. This can contribute to the discontent of participants and a declining support for the collective pensions in the Netherlands.

The Dutch pension system The Dutch pension system is internationally known for its magnitude. The funded part of the pension system in the Netherlands owns assets worth 1,130 billion Euros in 2013 according to Lever et al. (2013). The Dutch pension system can be divided in to three pillars.

1. The first pillar is a government provided pay as you go (PAYG) scheme, named AOW. This unfunded first pillar alleviates old age poverty and gives a minimum amount of money to citizens of the Netherlands.

2. The second pillar pensions are mandatory for all workers in the Netherlands that work under a collective labor agreement. Second pillar pension plans complement the first pillar and are aimed on remaining welfare standards for employees. Most industries in the Netherlands have their own sector wide pension fund where all workers in that industry are a member of. Some big companies have their own company pensions fund of which the scheme is specified by the company wide labor agreement.

3. The third pillar consists of voluntary participation of mainly self employed in the Netherlands. It can also be used in addition to first and second pillar pension provisions. This third pillar provides individual products that are sold by banks and insurance companies.

Legal value and Economic value Dutch second pillar pension entitlements are most often valued as if they are guaranteed nominal benefits that would not change in the future. If entitlements where indeed guaranteed nominal agreements that would not change in future, valuing them is straight forward. This can be done by discounting the future cash flows against the nominal risk-free term structure. The value that results from this way of considering pension entitlements is called the legal value. The legal value thus uses the assumption that pension entitlements are guaranteed and nominal and will not be raised or cut in future. For many purposes, pension entitlements are valued in this way. Examples are: people joining or leaving the fund, decisions about raising or cutting entitlements or in the past for the communication to participants. The term legal value is used because from legal point of view this is the value a participant is entitled to.

In reality the benefit that a participant receives when he or she is retired will depend on the solvency position of the pension fund. The funding ratio is used as the solvency parameter for pension funds. It is given by the ratio of the value of assets and the value liabilities. It becomes clear that when a pension fund invests in risky, volatile, assets the funding ratio and thus the amount of pension benefit becomes dependent on the performance of those risky investments. When a pension fund has completely specified how entitlements are raised or cut conditionally on the solvency of the fund, the fund is said to have a complete contract. When a fund offers a complete contract the economic value of the conditional entitlements can be calculated with the use of mathematical pricing techniques. The economic value gives the price for which these conditional entitlements could be sold or bought on the financial market, it thus leads to a market-consistent value. The price depends on how volatile the entitlements are and thus how certain it is that particular payouts will materialize in the future. The financial market demands a risk premium for holding assets or claims that are risky. The fund's pension contract (a mechanism for raising and cutting entitlements) and the future state of the funding ratio will there for be important determinants for the market value of pension entitlements.

Often risk bearing pension entitlements are valued as risk-free because the legal value is used. The pension industry tries to reconcile the legal value and the economic value in future. In this way, the legal value will mirror the actual economic value of the entitlements and not the value the entitlements have under the assumption that there are no future cuts or raises.

The importance of economic valuation Today the Dutch second pillar pension funds face several challenges that affect the confidence of participants in the system. Groups of participants think they contribute too much for what they will receive back. They think that they are subsidizing other groups of participants. In order to investigate these statements, economic valuation of pension entitlements provides a useful tool. According to Bonenkamp (2009) differences between contribution and the economic value of pension entitlements imply value transfers between groups of participants. So when a mismatch arises between the economic value of new accrual and contribution, economic value is transferred. Economic valuation also provides a useful tool for the board of a pension fund and the Dutch government. With this tool it is possible to consider the effects of intended adjustments to the policy of the pension fund.

The landscape in which pension funds operate changed during the past decade. Changes may have caused or increased value transfers and caused them to flow more in one direction. This resulted in a more critical evaluation by the participants of the Dutch pension system. Some of the most important factors that changed the pension landscape are: a more flexible labor market, demographic changes (aging society), political reforms (changes to accrual rate) and also the financial crisis which resulted in low interest rates and poor investment returns.

According to Boeijen et al. (2006) value transfers between groups of participants can make the Dutch pension system vulnerable. Boender et al. (2013) finds that the whole pension system could become more transparent when economic valuation of pension entitlements is used. The paper argues that future cuts or rises should be incorporated in the valuation of pension entitlements. This will help to restore confidence in the Dutch pension system.

Problem statement This thesis will investigate situations in which the economic value of pension entitlements deviates from the value of the contribution. Three main settings will be analyzed in which value transfers are known to occur. These settings are chosen because they are topical and can occur under Dutch pension regulations. The three settings are:

1. Discounting method of the contribution differs from discounting method of funding ratio
2. Participating in a fund with a surplus or a deficit
3. The uniform accrual and contribution system

The three problems will first be studied in a linear pension contract under a deterministic world. After that the same problems will be analyzed in a stochastic setting. This time the analysis is expanded with nonlinear pension contracts. This approach is chosen because this thesis will also study a well known statement in the pension sector. This statements says that "a change of equity allocation in a pension fund with a linear pension contract should not cause value transfers between participants". As a consequence, deterministic calculations on pension funds with a linear pension contract should lead to similar results as stochastic calculations of linear pension contracts. This is because the equity allocation of the pension fund can be put to zero, which eliminates all stochasticity. According to the statement this will not influence the economic value of entitlements. The statement will be verified and the results from the linear contracts in a deterministic setting will be compared to the results of the linear contract under a stochastic setting.

Contribution differs from the economic value of the entitlement The first setting in which value transfers are studied is the setting in which a dissimilarity is present between the discounting methods used for the calculation of the premium rate and the calculation of the funding ratio. The funding ratio is calculated by discounting liabilities against the risk-free nominal term structure. The premium that is needed to cover the new accrual is calculated by discounting liabilities against the expected return of a portfolio (nominal or real). This portfolio can also contain risky assets. If the expected return of the portfolio deviates from the risk-free rate, too little or too much contribution is flowing in to the fund. As a result the funding ratio will depart from a stable position. The mechanism that adjusts entitlements in case of under- or overfunding (in this thesis referred to as the pension contract) will start cutting or raising entitlements in order to resolve the problem of under- or overfunding. In this way entitlements are adjusted, also entitlements of the retired who did not pay too much or too little. This will result in a shift of the economic value between groups of participants. First the value transfers that occur by this mismatch between discounting methods are analyzed in a linear pension contract under a deterministic (risk-free) world. Subsequently the analysis is done in a stochastic setting in which the extension is made to nonlinear pension contracts.

Participating in a fund with a surplus or deficit The second setting in which a mismatch between contribution and the economic value of accrual is studied is a setting in which people participate in a fund with a surplus or a deficit. The participant will pay premium to the fund independent of the deficits or surpluses of the fund. By this a new participant will become the partial owner of the deficit or the surplus. In this way the economic value of his or her newly accrued entitlements will not be equal to the value in case the funding ratio was at the equilibrium level. Future cuts or raises can be expected. By this the economic value will not be equal to the contribution, a value transfer between participants can materialize. This problem is also first addressed in a deterministic setting with a pension fund that has a linear contract. A second analysis is done in a stochastic setting that also incorporates the effects of nonlinear pension contracts.

Uniform accrual and contribution The third setting in which a deviation between contribution and the economic value of entitlements is studied is the setting in which a uniform accrual and contribution system is used. This kind of scheme is mandatory for Dutch second pillar pension plans. The scheme raises the same percentage of premium among all participants[1] and gives for this the same percentage of accrual to all participants. This results in a mismatch between contribution and the economic value of accrued pension entitlements. The departure is mainly caused by two reasons. First, premium payments of younger persons can yield an investment return for a longer period than premium payments of elder people. For this reason contribution of young persons is worth more at retirement than the contribution of older persons at retirement. Nevertheless in this scheme everyone receives the same accrual for their contribution. As a consequence the young pay too much and the elderly pay too little for their accrual. The second reason that contributions do not match the economic value of accrual is the implicit debt that originated from giving the first generation of elderly their benefit. This debt causes a lower return on contribution of participants because partly these contributions are used to finance the debt. The CPB (Central Plan Bureau, Netherlands) made a report on the uniform accrual and contribution system. The report (Lever et al., 2013) estimates that the size of the implicit debt is around 100 billion Euro in the whole Dutch second pillar pension system. This amount is almost 10% of the total pension assets. Earlier Boeijen et al. (2006) finds that replacing the system with an actuarial fair system would cost around 20% of the funding ratio.

PGGM This thesis is written at PGGM. The second largest pension provider that executes the pensions of the second largest pension fund in the Netherlands, PfZW ("Pensioenfonds Zorg en Welzijn"). PGGM wants to advise the board of PfZW on the basis of economic valuation. In order to investigate the economic value of pension entitlements PGGM started a broad project with three Netspar[2] students of the Tilburg University. The project in general is focused on economic valuation and has as three sub topics: longevity risk, parameter sensitivity & shocks on the financial market and the third sub topic is, economic value determination of new accrual. This thesis addresses the last topic, the determination of economic value of new accrual. The main research question of this thesis is: under what circumstances and scheme types do pension contributions depart from the economic value of newly accrued entitlements? Of particular interest is the answer to this question in the three aforementioned settings. First, the discounting method of the premium is different from the discounting method of the funding ratio. Second, participating in a fund with a surplus or a deficit. And third, the uniform accrual and contribution system.

The outline of this thesis is as follows. Chapter 2 will introduce concepts like valuation and generational accounting. Chapter 3 will study the three settings in which the economic value deviates from the contribution in linear pension contracts under a deterministic (riskless) world. Chaper 4 will explain concepts like risk neutral valuation and simulation, it will also study the well known statement about changing the equity allocation in a fund with a linear pension contract. The three settings will be analyzed again in chapter 5, this time also nonlinear pension contracts will be analyzed under a stochastic setting. Chapter 6 concludes.

[1] In reality the premium is a percentage over income minus the franchise. This franchise is set to zero for the remainder of this thesis.

[2] `http://www.netspar.nl/`

2 Valuation and generational accounting

This chapter will introduce some concepts and techniques used in the remainder of this thesis. The basics about valuation will be explained in section 2.1. Next some elaboration is given about generational accounting in section 2.2. After that, remarks are made about a common way to study pension funds. This is done in section 2.3.

2.1 Valuation

Assigning the right value to something is very important because you do not want to pay too much, or receive to less for an asset. If an asset is not volatile and its price in future is certain the value of that asset can be obtained by discounting the future price against the risk-free rate, in this way an investor only gets compensated for the time value of money. If an asset is volatile, investors demand a risk premium to compensate them for the possibility that the future price of the asset is different from what they expect it to be. In general it holds that market prices are not equal to the expected payoff of an asset because of this additional risk premium. Commonly, assets that move with the market are priced at a discount, assets that are expected to move against the market are priced at a premium. For now we will assume that the volatile assets in this example will have a positive risk premium, and are therefor priced at discount.

The very basics If a person owns money worth X_0 and invests this money in a portfolio that will expect to have a value of X_1 one period from now. The expected return per period is given by

$$\mathbb{E}_0\left(\frac{X_1 - X_0}{X_0}\right) = \frac{\mathbb{E}_0(X_1) - X_0}{X_0} = \mu. \tag{2.1}$$

This result can also be applied to pension payments. If a pension fund promises to pay in expectation a benefit Y_1 in one year. The fund can do this by investing the received contribution in a portfolio of risky assets that has expected return μ_A, the value of the contribution needed in period 0 can be derived by using equation 2.1. The value of the contribution needed (V_0^A) is given by

$$V_0^A = \frac{Y_1}{(1+\mu_A)}. \tag{2.2}$$

In case the pension fund agrees to give a certain benefit Y_1, the fund should invest the contribution V_0 in a portfolio of risk-free bonds that have a non-volatile return of r^f, the risk-free rate. The amount of contribution V_0^B needed to fulfill this certain payment in one year is given by

$$V_0^B = \frac{Y_1}{(1+r^f)}. \tag{2.3}$$

In case μ_A of equation 2.2 is greater than r^f of equation 2.3, the following will hold: $V_0^A < V_0^B$. This means that the expected return on the risky asset is higher due to the presence of risk premium. Risk premium is a compensation for bearing risk.

A world in which only the risk-free asset is traded In a world in which only the risk-free asset is traded, valuation is strait forward. The only asset available for trade has no risk and thus also no risk premium. In order to attach a value to a future cash flow in this setting, one only has to know when the cash flow will take place and what the risk-free rate r^f is. Here it will be assumed that the risk-free term structure is flat. The value of the future cash flow can than be calculated by discounting it against the risk-free rate like

$$V_t = \frac{C_T}{(1+r^f)^{T-t}}. \tag{2.4}$$

Here V_t is the value at t of a cash flow C_T that occurs at time $T \geq t$. Note that equation 2.4 is discrete time. Later on in this thesis continuous time finance will be used. The continuous time version of equation 2.4 is

$$V_t = C_T \cdot e^{-r^f(T-t)}. \tag{2.5}$$

2.2 Generational accounting when only the risk-free asset is traded

A pension fund cannot create wealth. Its function is to enable participants in risk sharing, accessing the financial market and guide participants in making adequate decisions for the future. A pension fund can only invest money of participants and redistribute between participants. This is the reason why a pension fund often is referred to as a "zero-sum game". If one generations receives more economic value than the economic value of their contribution, an other generation has to receive less than the economic value of their contribution.

According to Boeijen et al. (2006) solidarity is a deeply embedded feature of the Dutch second pillar pension schemes. The paper makes a distinction between *risk solidarity* and *subsidizing solidarity*. In *risk solidarity*, ex ante it is not known who will benefit because unforeseeable risks are shared. In expectation everyone gains welfare from this kind of solidarity. This is different for *subsidizing solidarity*, in this kind of solidarity ex ante it is known who will benefit and who will lose.

To study the financial benefit or burden of participating in a pension scheme or policy, generational accounting is commonly used. Generational accounting looks at the net benefit or deficit that a particular generation experiences when it takes part in a policy or scheme such as a pension scheme. It can thus be used to investigate *risk solidarity* and *subsidizing solidarity*.

An intuitive description is given by the following example. A generational account can be seen as a bank balance to which all contribution of a generation are paid. When the particular generation is retired all benefits of the generation should be deducted from this bank balance. When finally the bank balance is negative the generation received more than it paid in. When the bank balance is positive, the generation paid in more than it received.

In order to compare the generational accounts of different generations the value of all the balances should be expressed at one particular point in time. This can be done by discounting the final amount in the generational account to the time at which the comparison is made.

Mathematical illustration of generational accounting Here it is assumed that the risk-free asset is the only asset available for trade. The risk-free rate r^f again is flat. For this explanation the payments to the fund by generation x, will be split from the receivables of generation x. The value of this contribution will be discounted to time $t = 1$. The account value at time $t = 1$ of premium payments (contribution) to the fund are given by

$$\text{Contr}(x, 1) = \frac{P_1^x}{(1 + r^f)^0} + \frac{P_2^x}{(1 + r^f)^1} + \frac{P_3^x}{(1 + r^f)^2} + \dots. \tag{2.6}$$

Here P_1^x is the premium payment of generation x to the scheme at time $t = 1$. This payment is not discounted because it takes place at the same time as the generational account will be studied ($t = 1$). In a similar way also the pension benefits from the fund to the participants can be calculated. For the benefits this results in

$$\text{Ben}(x, 1) = \frac{B_1^x}{(1 + r^f)^0} + \frac{B_2^x}{(1 + r^f)^1} + \frac{B_3^x}{(1 + r^f)^2} + \dots, \tag{2.7}$$

with B_1^x the received benefit of generation x at time $t = 1$. The generational account of generation x is given by subtracting the sum of contribution (equation 2.6) from the benefits (equation 2.7). This comes down to

$$\text{GA}(x, 1) = \text{Ben}(x, 1) - \text{Contr}(x, 1). \tag{2.8}$$

More general, to obtain a generational account of a generation x, at time $t = \tau$ the positive and negative cash flows should be added. This results in

$$\text{GA}(x, \tau) = \sum_{t=1}^{\infty} \frac{\text{CF}_t^x}{(1 + r^f)^{t-\tau}}, \tag{2.9}$$

which thus adds up the cash flows CF_t^x of generation x at time t and discounts all those cash flows back to time τ. Note that in a risk-free world, thus a world without uncertainty, discounting with r^f leads to the market consistent value of the cash flow.

If the generational account of a generation is positive, the generation is said to have a net benefit of joining or participating in a scheme or policy. If the generational account is negative, the generation has a net loss of participating. A pension fund itself cannot create value, it can only redistribute. For this reason the sum of all generational accounts should be zero.

Generational accounting is not only a good tool to study the net benefit or burden for a generation when it starts to participate in a policy or scheme. Generational accounting can also be used to evaluate changes in the policy of a pension fund. This can be done, and will be done in chapter 3, 4 and 5. There generational accounts in case of no policy change can be compared with the generational accounts that result from the change in policy. In this way the effects of a change in policy can be studied as financial gains or losses that are divided among the generations in a pension fund. The change that is observed in the generational accounts is called a wealth transfer that originates due to the change in policy. Too large wealth transfers can lead to discontent among groups of participants (generations) in the fund.

In chapter 4 the generational account will be derived in case the world is not riskless but also incorporates stochastic processes that model the financial markets and other economic parameters over time.

2.3 Linear pension contracts used to study the risky reality

Important literature on topics like the uniform accrual and contribution system is based on deterministic calculations. Some examples are Boeijen et al. (2006) and the CPB report of Lever et al. (2013). These deterministic calculations do not incorporate risky assets that are present in the real world. The reason why these studies do not incorporate stochastic processes that describe the behavior of volatile stocks, is that the deterministic models are better understandable and easier to use. Next to this, an other reason is known which will be explained below.

Linear pension contracts Pension funds have a mechanism that cuts or raises the value of pension entitlements. If the funding ratio of a pension fund is above the desired level, there are more assets than needed to cover liabilities. A pension fund can increase the value of entitlements. In Dutch this is known as "indexeren", giving indexation in English. In this way the surplus that the fund has is divided among the participants. If the fund has a lower funding ratio than desired, the pension fund is also able to cut entitlements. In Dutch this is known as "korten". As a result of cutting entitlements the value of the liabilities goes down and the funding ratio increases.

If the amount of cutting and raising of pension entitlements is a linear function of the funding ratio, the pension fund is said to have a linear pension contract. In figure 1 three linear contracts can be found. Such a linear contract is specified by the desired funding ratio F_D and a smoothing parameter α that determines which proportion of the deviation from the desired funding rate is dissolved per period (a year). For example, if $\alpha = 1$, 100% of the difference between the actual funding ratio and the desired funding ratio is dissolved within one period.

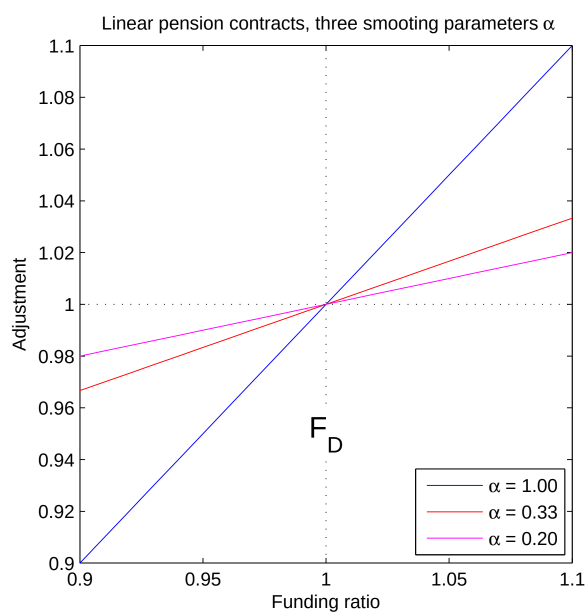


Figure 1: Three different linear pension contracts. Desired funding ratio $F_D = 1.00$. $\alpha = 1.00$ (Smoothing period: one), $\alpha = 0.33$ (Smoothing period: three), $\alpha = 0.20$ (Smoothing period: five)

In section 3.1.3 equation 3.13, a formula of the smoothing mechanism is given. The lines plotted in figure 1 are a visualization of the function in equation 3.13.

Equity allocation w of a pension fund In the pension sector a well known statement argues that "within a pension fund with a linear contract, a change of the equity allocation w of the fund does not cause value transfers between participants". This is an important result because it implies that valuation can take place as if all investments of a pension fund are riskless. It also explains why the literature is based on riskless discounting: implicitly linear contracts are assumed.

Bovenberg et al. (2012) argues that changing the equity allocation hardly causes generational transfers in linear pension contracts. Bilsen and Bovenberg (2014) derives an expression for the economic value of pension entitlements in which the equity allocation w is absent. This expression only holds exact under strict mathematical assumptions. Nevertheless changing the equity allocation in the pension funds, with a linear contract, that are used in this thesis should not cause large value transfers. In other words, the value transfers should be small compared to the value transfers of the three main settings in this thesis, which are studied in chapter 3 and chapter 5.

The statements implies the following. If the equity allocation is such that the fund only invests in risk-free bonds ($w = 0$) there should be no difference in economic value of entitlements compared to the case in which the fund does invest in equity ($w > 0$). From this argument it is adequate to study pension funds with a linear pension contract in a risk-free setting. Because incorporating volatile behavior of stocks would lead to the same results.

Mathematical illustration that equity allocation w is absent in value of entitlements This simple illustration will show some intuition behind why equity allocation w is not present in the expression for the economic value of a benefit flow.

In this example μ is the expected return on equity. The risk-free rate r^f is the flat return of the bond. In this economy the risk premium is $\lambda = \mu - r^f$.

A pension fund that invests fraction w of its wealth in risky equity and fraction $(1 - w)$ in risk-free bonds, has an expected return μ_p on its portfolio of

$$\mu_p(w) = r + w\lambda = (1 - w)r^f + w\mu.$$

Now suppose a person, at time 0, in that pension fund has accrued entitlements that will give him amount Y_0 every year for the next T years. These entitlements can increase or decrease in value according to the stochastic return on the portfolio. The adjustment of the entitlements is symmetrically over the upside and the downside of the returns. It will be shown that in this case the risk premium appears both in the numerator and the denominator of the calculations. The value of the entitlements can be calculated by the use of equation 2.2 for T periods in future,

$$V_0 = \frac{E_0^{\mathbb{P}}(Y_1)}{(1 + r + w\lambda)} + \frac{E_0^{\mathbb{P}}(Y_2)}{(1 + r + w\lambda)^2} + \cdots + \frac{E_0^{\mathbb{P}}(Y_T)}{(1 + r + w\lambda)^T}. \tag{2.10}$$

Because of the assumption that the cuts and raises are linearly related to the funding ratio and therefor to the investment return, the expectations can be replaced by

$$V_0 = \frac{Y_0(1 + w\lambda)}{(1 + r^f + w\lambda)} + \frac{Y_0(1 + w\lambda)^2}{(1 + r^f + w\lambda)^2} + \cdots + \frac{Y_0(1 + w\lambda)^T}{(1 + r^f + w\lambda)^T} \tag{2.11}$$

In expectation the risk premium would let the entitlements grow, but the risk premium also appears in the denominator. If now the following mathematical approximation is used

$$\frac{(1+w\lambda)}{(1+r^f+w\lambda)} \approx \frac{1}{(1+r^f)}, \tag{2.12}$$

equation 2.11 reduces to

$$\begin{aligned} V_0 &= \frac{Y_0}{(1+r^f)} + \frac{Y_0}{(1+r^f)^2} + \cdots + \frac{Y_0}{(1+r^f)^T}, \\ &= \sum_{t=1}^{T} \frac{Y_0}{(1+r^f)^t}. \end{aligned} \tag{2.13}$$

From this it can be seen that the equity allocation w has no effect on the current value of future pension payments. These pension payments are dependent on volatile stock returns but their current value is not.

The example above is very simplistic and does not state details on investment returns and the link between investment returns and the funding ratio and the mechanism that cuts or raises entitlements. When these factors are incorporated in the calculations the result would be recursive functions of the funding ratio and the mechanism. In that case specific assumptions will be required to show that equity allocation w drops out of the equations. For these more comprehensive derivations one should study Bilsen and Bovenberg (2014). This paper gives an derives an expression for the individual value of an annuity which is not influenced by the choice of the equity allocation.

In section 4.2, the statement that a change in the equity allocation w should not lead to value transfers between generations in a pension fund with a linear contract, will be evaluated. It is expected that only under certain strict mathematical assumptions the value transfers are exact 0. In a pension fund that is not bounded by these strict mathematical assumptions the differences in economic value of entitlements should still be minor in case the equity allocation w is changed.

Why does this statement only apply to linear contracts and not to nonlinear contracts? A nonlinear pension contract is a pension contract that adjusts entitlements (raises or cuts entitlements) as a nonlinear function of the funding ratio. For example a contract that has a smoothing period of 3 years ($\alpha = 0.33$) below the desired funding ratio F_D and a smoothing period of 10 years ($\alpha = 0.10$) above the desired funding ratio F_D. A visualization of a nonlinear contract can be found in figure 19b and figure 19c in chapter 4.

If in a volatile world, a pension fund has a nonlinear contract, the entitlements of some generations within the fund can be viewed as option like contracts (see Hoevenaars and Ponds (2007)). For example, some generations could profit from the upward potential of the investments, but are not harmed if the solvency position of the fund gets worse. The particular generation will benefit if the fund takes on more risk (increases w). The generation is said to hold a call option. It is known that the value of options goes up if volatility increases. This is the reason why the aforementioned statement can only be applied to linear contracts. In those contracts all generations benefit as much from good investment performance as they suffer from poor investment performance.

3 Economic value in linear pension contracts

This chapter will study the three settings in which pension contributions do not match the economic value of entitlements that are bought with that contribution. The results in this chapter are obtained from deterministic calculations in linear pension contracts. The first setting is the setting in which the discounting method used for the calculation of the premium is different from the discounting method used to calculate the value of the liabilities in the funding ratio. The second setting is about participating in a fund with a surplus or a deficit. The third setting is the uniform accrual and contribution scheme.

Section 3.1 will study the first setting in which the premium is calculated in a different way as the liabilities in the funding ratio. Section 3.2 will look at the generational effects of underfunding. Last, section 3.3 will study value transfers in the uniform accrual and contribution system.

Assumptions of a risk-free model For the analysis that will be performed in this chapter (chapter 3) some simplifying assumptions are made. As said earlier, there will be no uncertainty in this chapter. The only asset available for investment will be a risk-free bond that yields a flat term structure with interest rate r^f. There will be no inflation so $\pi = 0$. There are also no transactions costs. The participants are assumed to live until an age of a_D and then pass away with given survival probability s. Initial wage of the participants is w_0 at age a_W at which they start working. Participants could have wage growth g (set to zero for now) per year until they retire at age a_R.

3.1 Discounting method of the contribution differs from the discounting method of the funding ratio

In the Netherlands the financial assessment framework (FTK, Financieel Toetsingskader) prescribes how pension liabilities have to be valued. The financial assessment framework is part of the Dutch pension law, which specifies the financial requirements for Dutch pension funds. The old FTK (a new FTK was adopted during the writing of this thesis) allows pension funds to choose between several kinds of discounting methods in order to determine the contribution that is needed to cover pension accrual. The two most important methods that can be chosen in the old FTK to calculate the premium are:

> *Nominal expected return of portfolio* The required premium may be based on discounting liabilities against the nominal expected return of a portfolio, possibly also containing risky assets.
>
> *Nominal interest rate plus premium* The second method that can be chosen is to discount liabilities against a the nominal interest rate plus a premium for equity capital.

According to the new FTK (nFTK) the premium needs to be determined on the basis of:

> *Real expected return of portfolio* This is the expected return of a portfolio possibly also containing risky assets but now corrected for inflation.
>
> *10 year average of interest rate plus premium* A ten year average of the interest rate plus a premium for equity capital may be used to calculate the premium level.

The funding ratio on the other hand must be determined by discounting pension liabilities against the risk-free term structure. By calculating the liabilities in the funding ratio in this way, it is done as if liabilities are risk-free guarantees that are paid out with 100% certainty.

The funding ratio is an important parameter in a pension fund. When the funding ratio is below 1 (or 100%) the fund does not have enough assets to back the value of the liabilities. In case the funding ratio is higher than 1, the fund has more assets than needed to back liabilities. For this reason the funding ratio is a very commonly used solvency parameter for pension funds. Figure 1 in chapter 2 gave a graphical illustration of a linear pension contract. In such a contract the adjustment of entitlements is performed in a linear relationship with the funding ratio. Raising and cutting entitlements is also done proportional over all participants.

With the funding ratio used as input parameter for a mechanism that can cut or raise pension entitlements it is important that the method to calculate the funding ratio is in line with the method used to calculate the contribution rate. If this is not the case, too little or too much contribution is paid in terms of the funding ratio. As a consequence the funding ratio will depart from the equilibrium situation and entitlements are adjusted. As described above, the FTK allows the liabilities in the funding ratio to be calculated in a different way than the premium rate.

In this way the economic value of newly accrued pension entitlements can depart from the value of the contribution made to cover those newly accrued pension entitlements. Usually the expected return on a portfolio of risky assets is higher than the expected return on a portfolio only containing risk-free bonds. In this way, the contribution made to the fund will be too low to keep the funding ratio at the 100% level. This will trigger the mechanism that cuts or raises pension entitlements. To restore the funding ratio to the desired level the value of the entitlements will be cut in such a way that the funding ratio will reach the desired level again after a specified period. All in all this means that insufficient contribution to a scheme, because of a dissimilarity in the methods of discounting liabilities, will result in a cut of entitlements owned by all current participants.

For young participants this will be attractive, they pay less due to a low premium rate. Everyone participating in the fund will see that his or her entitlements are cut. Because the young do not own a lot of entitlements that can be cut, they will win. The older generations of participants have accrued a lot of pension entitlements. If the entitlements of them are cut, they could lose a lot of wealth. Next to that, the retired do not even profit from paying in too little premium.

3.1.1 Illustrative mathematics

In this subsection mathematical results on a degressive pension scheme will be derived, these results are used in the stylized pension fund that is used for the analysis in section 3.1.3. First discrete time mathematics are presented about pension wealth and the derivation of the premium rate. After that the formulas for a degressive accrual scheme are given, these will be continuous time finance which is also used in the stylized pension fund.

Pension wealth As an example, the value of all entitlements of a participant at the time of retiring is denoted with X_R. The formula for the economic value (EV) of pension provision at the time of retirement $i = R$ is given by

$$EV(X_R)_{i=R} = w_0 \cdot \gamma \cdot \sum_{j=0}^{\infty} \frac{s_j}{(1+r)^j}, \tag{3.1}$$

with γ the fraction of wage (wage is flat, wage growth $g = 0$) someone receives as a benefit every year he or she is retired. s_j is the survival probability in the jth year of retirement. Note that $EV(\cdot)_i$ is the economic value at i years in the fund (R years in this particular case). For convenience j is used as index to indicate the number of years spent in retirement.

If a pension fund uses a fixed premium rate p for all ages, the appropriate level of p could be solved from other parameters like wage and interest. This can be done for a participant that spends his or her full career in the fund. This participant will start in year $i = 0$ and will retire at $i = R$ years in the fund. Information is needed about the return μ_p that is expected on the portfolio where the contribution is invested in. The premium rate needed to cover pension wealth X_R can be solved by equating the return of the portfolio to X_R. In discrete time finance, for a participant that pays R years of premium and has a fixed wage w_0[3] this result in

$$\begin{aligned} X_R &= \sum_{i=0}^{R-1} p \cdot w_0 \cdot (1+\mu_p)^{R-i}, \\ p &= \frac{X_R}{\sum_{i=0}^{R-1} w_0 \cdot (1+\mu_p)^{R-i}}. \end{aligned} \tag{3.2}$$

It is clear from equation 3.2 that the premium level p is highly dependent on the expected return μ_p of the portfolio. For higher values of the expected return μ_p the premium rate p, necessary to reach pension wealth X_R, has a lower value.

Pension fund with degressive accrual Now a pension fund with a degressive accrual system is considered. Simplifying assumptions are made on the risk-free rate, wage growth and mortality of a participant. In this fund, participants pay in premium rate p (contribution, this is uniform over all ages) at the beginning of each year i, to receive pension accrual $\theta(i)$. Finally when retired the total accrual of pension entitlements adds up to level γ, the replacement income. In order to determine the premium rate and the accrual per year the following steps need to be followed:

- **Step 1.** Determine the replacement income γ for a participant that will spent his whole career participating in the fund. (Equation 3.3 and illustration)
- **Step 2.** Determine the premium rate p (age independent and fixed) needed to cover a benefit that is fraction γ of average wage, from retirement until death. (Equation 3.8 and illustration)
- **Step 3.** Determine the yearly accrual rate $\theta(i)$ (degressive with age), from the premium percentage p, such that the economic value of the premium payment is always equal to the economic value of new accrual. (Equation 3.9 and illustration)

[3] In reality a person pays premium over wage minus franchise. In this thesis the franchise is assumed to be 0.

Step 1, replacement income From step 1 the replacement income is chosen, this faction of nominal annualized income is the benefit a participant will receive until death. Equation 3.3 gives the total accrual at the time of retiring, when a participant participated his or her whole career (R years) in the fund. From the equation it can be seen that the accrual rate $\theta(i)$ is dependent on i,

$$\gamma = \sum_{i=0}^{R-1} \theta(i). \tag{3.3}$$

This is because over time a constant contribution is made to the fund, therefore the accrual is degressive with the age of the participant. Here the economic value of the new accrual is always equal to the value of the contribution.

Step 2, the premium rate Then step 2 follows. Now, not an amount of pension wealth at retirement is considered, like in equation 3.2, but a yearly payment from retirement until death. Equation 3.2 need to be adapted further.

Suppose the pension fund collects premium p with the participants at the beginning of each year when the participants works that year. The age at which participants start to work is $a = a_W$ ($i = 0$). Participants retire at age $a = a_R$ ($i = R$) and after retirement age have survival probabilities s_j. Thus participants survive for sure upto age a_R. If the pension fund wants to calculate what premium percentage is necessary to cover a replacement income γ from retirement until death, they can do that in a similar way as equation 3.2. For X_R the economic value at retirement of the total pension entitlements can be substituted. The total value of pension entitlements will be given below in equation 3.5.

Continuous time versions of equations will be considered for the remainder of this section because this will also be used in section 3.1.3. The continuous time version of equation 3.2 is given by equation 3.4:

$$p = \frac{X_R}{\sum_{i=0}^{R-1} w_0 \cdot e^{\mu_p(R-i)}} \tag{3.4}$$

The economic value of pension entitlements at the time of retirement ($i = R$, $a = a_R$), for a participant that accrued a replacement income of γ and receives this replacement income until death, is given by the next equation. It is assumed from now that the pension fund only invests in the risk-free bond with return r^f. Note that equation 3.5 is the continuous time equivalent of equation 3.1,

$$EV(X_R)_{i=R} = w_0 \cdot \gamma \cdot \sum_{j=0}^{\infty} s_j \cdot e^{-r \cdot j}. \tag{3.5}$$

The economic value at the time of joining the fund ($i = 0$, $a = a_W$), of all benefits received by a participant during his whole retirement are given by:

$$\begin{aligned} EV(\text{benefit})_{i=0} &= e^{-r^f(a_R - a_W)} \cdot EV(X_R)_{i=R} \\ &= e^{-r^f(a_R - a_W)} \cdot w_0 \cdot \gamma \cdot \sum_{j=0}^{\infty} s_j \cdot e^{-r^f \cdot j}. \end{aligned} \tag{3.6}$$

The economic value, at the time of joining the fund ($i = 0$, $a = a_W$), of the total premium paid during a whole career is given by

$$EV(\text{premium})_{i=0} = p \sum_{i=0}^{R-1} w_0 \cdot e^{-r^f \cdot i}. \tag{3.7}$$

The contribution rate needed to cover pension the pension benefits is given by equating equation 3.6 to equation 3.7. This leads to

$$e^{-r^f(a_R - a_W)} \cdot w_0 \cdot \gamma \cdot \sum_{j=0}^{\infty} s_j \cdot e^{-r^f \cdot j} = p \sum_{i=0}^{R-1} w_0 \cdot e^{-r^f \cdot i}$$

$$p = \frac{e^{-r^f(a_R - a_W)} \cdot w_0 \cdot \gamma \sum_{j=0}^{\infty} s_j \cdot e^{-r^f \cdot j}}{\sum_{i=0}^{R-1} w_0 \cdot e^{-r^f \cdot i}}. \tag{3.8}$$

Step 3, accrual rate per year Then the last step (step 3) is used to determine the accrual rate $\theta(i)$ per year i in the fund (with the first year in the fund $i = 0$) is given by

$$EV(\text{premium in year } i)_{i=0} = EV(\text{accrual year } i)_{i=0}$$

$$p \cdot w_0 \cdot e^{-r^f \cdot i} = e^{-r^f \cdot R} \cdot \sum_{j=0}^{D-1} \theta(i) \cdot e^{-r^f \cdot j}$$

$$\theta(i) = \frac{p \cdot w_0 \cdot e^{-r^f \cdot i}}{e^{-r^f \cdot R} \cdot \sum_{j=0}^{D-1} e^{-r^f \cdot j}}. \tag{3.9}$$

If in step 1 $\gamma = 0.8$, is chosen and it is assumed that participants live for sure until age $a = a_D = 85$ and die at that age with probability 1. This implies a participant spend $i = 60$ years in the fund. If the risk-free interest rate is assumed to be $r^f = 0.03$, then the yearly accrual rate is given by figure 2.

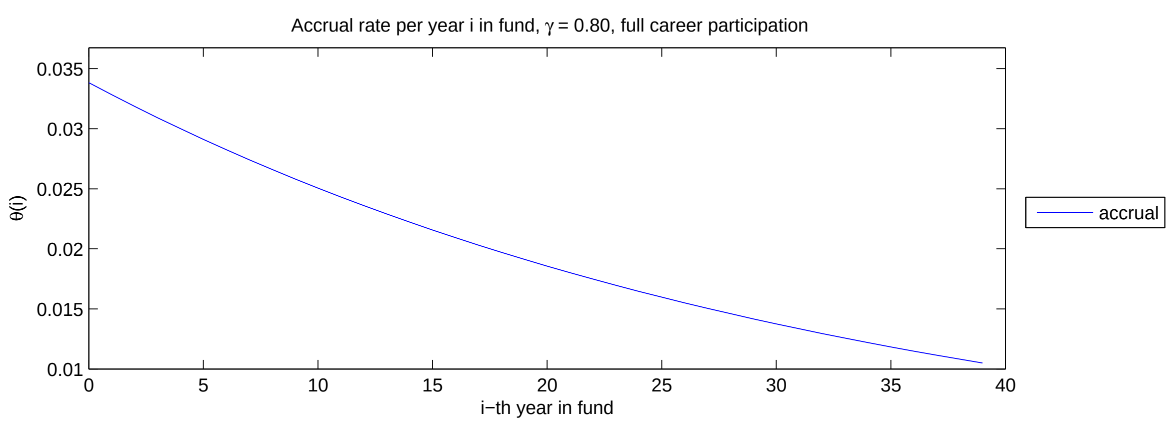


Figure 2: Accrual rate $\theta(i)$ per year i, $r^f = 0.03$, $\gamma = 0.80$, flat annualized wage, a full career is assumed to be 40 years. The retirement period is assumed to be 20 years.

The expected return of a portfolio In reality a pension fund does not only invest in a risk-free bond but also in risky assets. A small illustration will make clear what the effect is of using portfolio return as discount factor for a liability that has a maturity of T years. The expected return of a portfolio depends on the chosen asset mix. The assets mix determines how much risk premium in expectation is collected. The risk premium λ is given by:

$$\lambda = (\mu - r^f) \tag{3.10}$$

For a portfolio that allocates fraction w to risky equity investments and $(1-w)$ to risk-free bonds the nominal expected return is given by the following equation

$$\mu_N(w) = (r^f + w\lambda) = (1-w)r^f + w\mu. \tag{3.11}$$

The real expected return of a portfolio is given by an equation similar to equation 3.11 only now corrected for inflation, this results in

$$\mu_R(w) = (r^f + w\lambda) - \pi = (1-w)r^f + w\mu - \pi \tag{3.12}$$

with π the inflation.

The nominal expected return can be used to discount pension entitlements with in order to determine the premium rate that is in line with the old Dutch FTK regulation. The same holds for real expected return in the new FTK.

The Dutch government prescribes the parameters values advised by commission parameters (Langejan et al., 2014). This commission has set the expected return on equity to 7%[4]. For the risk-free rate, the term structure should be used. For now again a flat term structure is assumed with return $r^f = 3\% = 0.03$. Also the inflation is assumed to be $\pi = 2\% = 0.02$. As an example these values can be used in equation 3.11 for the nominal expected return or in equation 3.12 in order to calculate the real expected return of a portfolio. The expected portfolio return implies a discount factor that can be used to discount liabilities with different maturities T.

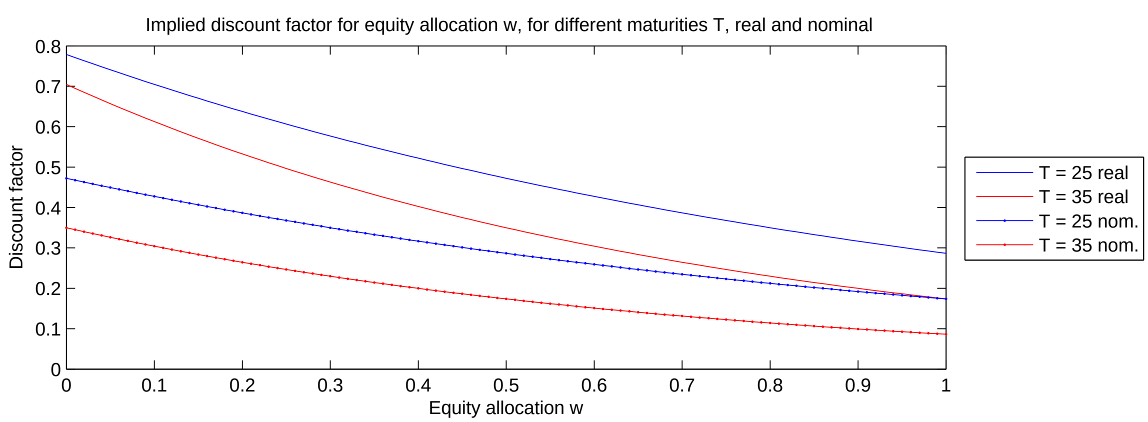


Figure 3: Implied discount factor for different values of equity allocation w plotted for two maturities: $T = 25$ and $T = 35$ for both a nominal liability and a real liability. Inflation $\pi = 2\%$.

[4]Listed shares, geometric mean.

The implied discount factors are given by figure 3. As it can be seen, the discount factor for a nominal liability is below the discount factor of a real liability. This is because in real terms the investment return on the portfolio is less compared to a nominal return. Further it can be seen that for values of w around 1, the discount factors for liabilities with maturity $T = 35$ tends to values around 0.1. Long maturities are common for pension liabilities. In reality a portfolio with equity allocation of $w = 0$ is used to determine the funding ratio because this is equivalent with discounting liabilities against the risk-free rate r^f. On the other hand, a portfolio with an equity allocation of $0 \leq w \leq 1$ is used to determine the contribution that is actually paid by participants.

Expected portfolio return used to determine premium level As already explained earlier, by the old FTK a pension fund is allowed to calculate the contribution rate on the basis of the nominal expected portfolio return $\mu_N(w)$. To do this $\mu_N(w)$ is used in equation 3.8 instead of r^f. By the new FTK (nFTK) a pension fund is allowed to use the expected real portfolio return $\mu_R(w)$. In this case expected real return needs to be used in equation 3.8.

If additional assumptions are made, the contribution rate of a stylized pension fund can be calculated. These assumptions are that a participant will live for sure until age $a = a_D = 85$, and dies with probability 1 at that age. He or she will reach a replacement income of $\gamma = 0.8$ after working a full career (40 years). The premium rate p, required when premium is invested in a portfolio with equity allocation w and thus has nominal expected return $\mu_N(w)$ or real expected return $\mu_R(w)$ is given in figure 4.

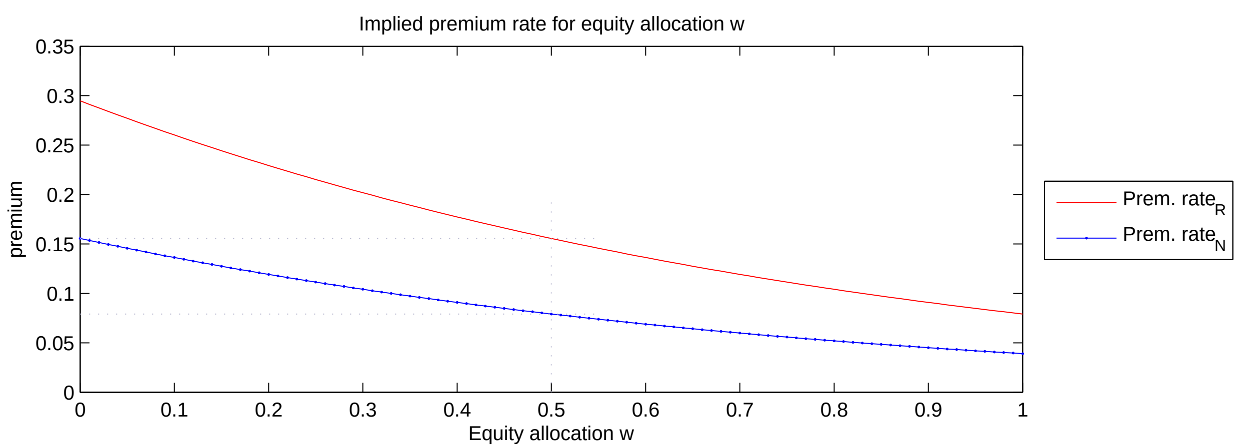


Figure 4: The premium rate is given as function of the expected real and nominal portfolio return μ_w. μ_w as given by equation 3.11 and equation 3.12. Participants join, retire and die respectively at $a_W = 25$, $a_R = 65$, $a_D = 85$. The replacement income $\gamma = 0.80$.

It can be seen from figure 4 that the implied premium rate p differs heavily whit the equity allocation w of the portfolio. It is shown in figure 4 that an equity allocation of $w = 50\%$ results in roughly 50% lower premium rate (discounted nominal: from 15% to around 8%). When the premium is determined on the basis of expected real portfolio return the premium rate is around the same level as the funding ratio demands. This is because the real return of a portfolio with $w = 50\%$ is around the same level as the risk-free rate $r^f = 3\%$. Note that this could vary if the risk-free rate or the inflation would be assumed different.

An equity allocation of $w = 50\%$ is very common for a Dutch pension fund. From this the problem studied in this section becomes clear: under the old FTK, if $w > 0$ the premium that is paid by the participants is too little to keep the funding ratio from decreasing. Below this is used when a fictive scheme is analyzed in section 3.1.3. Note that when real expected return is used to determine the premium (new FTK), this almost corresponds with the amount needed to keep the funding ratio at $F = 1$ if $w = 50\%$.

3.1.2 Insights from literature

The Dutch CPB (Central Planning Bureau) has made a small report (Lever and Bonenkamp, 2013) on the use of expected return in the pension premium. The paper mentions that the amount of premium is important for employers and employees, but also for income of the Dutch government because of tax regulations. The amount of pension premium paid in the Netherlands is around 40 billion a year and for this reason of macro economic importance according to the paper.

Lever and Bonenkamp (2013) also find that the method used to discount a liability is dependent on the nature of the liability (certain or uncertain nominal or real). Next to that the paper finds that a too low premium percentage can lead to cutting entitlements. This is something that is also argued above but also confirmed in the analysis of section 3.1.3 below.

3.1.3 Analysis

In this section a stylized pension fund will be analyzed. In figure 4 it can be seen that, in the nominal case, when a pension fund chooses for an equity allocation of $w = 0.5$ the premium that participants have to pay is around 50% too little compared with what is needed to keep the funding ratio stable . The effects of contribution too little premium to a pension fund will be analyzed in this section. Points of interests are which groups of participants benefit or lose and how much and the generational effects of the smoothing period in the adjustment mechanism (see equation 3.13 and explanation).

Model description: The economy and pension fund participants The economy is assumed the same as as above, only a risk-free bonds is used to invest in, this bond yields a flat risk-free return r^f. Everything about the participants of the fund is known (e.g. entrance, mortality, wage, etc.).

Model description: The pension scheme A degressive accrual system is still used, because this scheme type is actuarial fair in the basic situation. The analysis will be done such that the fund will depart from the basic situation, and value transfers will occur.

As previously mentioned a pension fund also has an entitlement adjustment mechanism, a linear pension contract in this case. This mechanism can cut or raise entitlements of all participants in the fund. It uses the funding ratio F as input parameter to adapt all liabilities L. The mechanism is given by

$$\delta(F) = 1 + \alpha(\frac{F}{F_D} - 1), \tag{3.13}$$

$\delta(F)$ is the correction that will be multiplied with the liabilities L. F_D is the desired level of the funding ratio. The smoothing parameter α determines what fraction of the difference between F_D and F is solved per period. In section 5 also nonlinear mechanisms will be considered.

Model description: Parameterization The basic parameterization of the economy, the participants and the scheme can be found in table 1. In the following analysis some of the parameters may be changed in order to deviate from the basic situation, this will be clearly stated. Note that t is used as time index for the years the pension fund itself is active. With $t = 1$ the starting year. Also observe that in the basic parameterization $\alpha = 1$ and thus there is no smoothing because any deviations of the funding ratio from $F_D = 1$ will be totally dissolved in that particular year.

Parameter	Symbol	Value
Risk-free rate	r^f	3.0%
Start working age	a_w	25
Retirement age	a_R	65
Death age	a_D	85
Replacement income	γ	80.0%
Wage growth	g	0.00
Start wage	w_0	1
Premium rate (equation 3.8)	p_0	15.6 %
Participants at $t = 1$		none
Participants for $1 < t \leq 120$		one every year, age $a = a_W$
Desired funding rate	F_D	1.00
Smoothing parameter	α	1.00

Table 1: Basic parameterization of the deterministic economy, the population and the pension scheme.

Departure from equilibrium situation In order to mimic the real situation in which discounting against expected nominal portfolio return leads to a too low premium rate, a fund is started and kept in the equilibrium state ($F = 1$). Insights from figure 4 are used to lower the premium rate, causing a situation of underfunding. From figure 4 it can be seen how the premium rate varies for different equity allocations w. If w is changed from 0 to 0.5 the premium percentage changes to a value around 50% of the premium that would be needed if $w = 0$ (this corresponds with discounting against r^f as the funding ratio does). All in all the real world situation in which a fund could invest in risky assets, leading to a premium percentage that is too low is replicated here by just levying a premium percentage that is a fraction of the original premium percentage. The period over which this too low premium percentage is levied can also vary from 1 year to a longer period.

Different fractions of original premium rate are levied The parameterization of the economy, the scheme and the population can be found in table 1. At $t = 1$ the fund is empty, for $t > 1$ every year a participant will join the fund and will spend his whole career as a member of the fund. This will go on for the next 120 years.

Fractions of the original premium rate p_0 are chosen to be $0.50, 0.60, 0.70$ and $0, 80$ this is approximately the difference in premium rate that occurs when liabilities are discounted with μ_w if equity weights w are chosen to be $0.4, 0.3, 0.2$ and 0.1 respectively.

At $t = 70$ (the oldest participant is then dead for 9 years) the original premium rate p_0 is cut by a certain percentage. This will only be the case for one year. At $t = 71$ the original premium p_0 is applied once again. Because the funding ratio drops below 1 ($F < 1$), the mechanism that adjusts entitlements cuts the entitlements of all participants proportionally. Here $\alpha = 1$ is used, which means that the shortfall will be resolved in one year.

For the year in which the premium deviates from p_0, $t = 70$ is chosen. At that time the fund is full and there are no side effects caused by starting the fund from an empty situation.

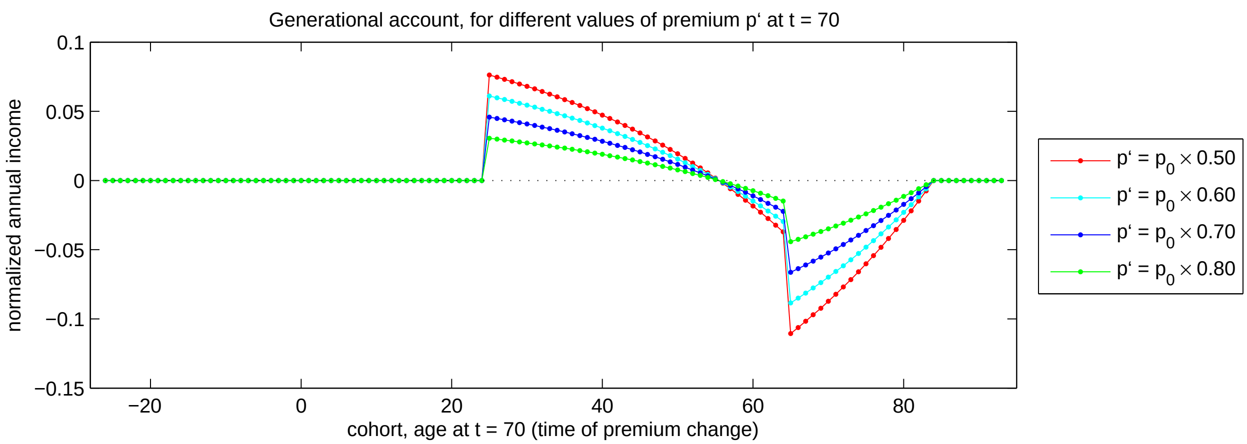


Figure 5: Generational account for different fractions of the original premium rate p_0. Only in year $t = 70$ the premium rate deviates from p_0. Generational account calculated to time $t = 70$. On the horizontal axis the age of participants is given at $t = 70$. Settings of the economy, the scheme and the population given in table 1.

The effect of the underfunding, caused by insufficient premium payments to the fund, over the generations can be found in figure 5. On the horizontal axis the age of participants is given at time $t = 70$. On the vertical axis the generational account is given measured in normalized annual income (income is normalized to 1). The following can be observed:

- **The unaffected** It can be seen that early generation (right side of the graph, age $a > 83$ at $t = 70$) experience no effect at all, they already received their last benefit or are already dead by the time entitlements are cut.
- **The retired** The age cohort that has age $a = 83$ at $t = 70$ is the oldest cohort that experiences an effect of the too low inflow of money in the fund. The generations that have age $65 \leq a \leq 83$ at $t = 70$ experience that their entitlements are cut. They do not profit from paying in a too low premium, because they are retired and for this reason do not pay in premium. The generations that have age $a = 65$ at $t = 70$ and slightly older ($a = 65, 66, 67, \dots$) face the largest burden. Those generations own a lot of entitlements that are cut.

- **Close to retirement** The generations that have age $a = 64, 63, \ldots, 56$ at $t = 70$ profit from paying in to less premium but also experience a cut of their right. They are close to retirement at $t = 70$ and for this reason own a lot of entitlements that can be cut, therefore their generational account is still negative.
- **The young** Looking at younger generation (age $a = 25, 26, \ldots, 55$ at $t = 70$) one can observe positive generational accounts. This is due to the fact that those generation profit from contributing less to the scheme and do not own that much entitlements that are cut. The participant that has age $a = 25$ at $t = 70$ experiences the greatest profit. By looking at the red line for instance, one can see that this person (age $a = 25$) pays in 50% too little contribution and has only accrued a small amount of rights that can be cut. For this reason his or her benefit results in approximate $w_0 \cdot p_0 \cdot 0.50 = 1 \cdot 0.156 \cdot 0.50 \approx 0.08$.
- **Future generations** Future generations are not harmed at all, they pay in p_0 which is exactly equal to the economic value of the benefits they will receive.

From figure 5 it can also be concluded that a greater departure from the original premium rate p_0 results to more redistribution between generations. If the deviation of p_0 is large, the shortage is also greater, as a result the funding ratio will also be lower and the mechanism has to cut entitlements more.

If in reality the board of a pension fund, has decided to hold a large proportion of equity in the portfolio (higher w), the expected return on this portfolio is also high. The difference between the premium that is paid by the participants (p' in the simplified model used here) and the premium needed to keep the funding ratio at 100% (p_0 here) will for this reason also be larger. Which results in a greater difference between the contribution and the economic value of newly accrued entitlements.

Different settings of the entitlements adjustment mechanism The effect of the smoothing period (α in equation 3.13) is analyzed here. The setting of this smoothing period (also known as recovery period) is something that is determined by the FTK and the board of a pension fund.

Once again insufficient contribution will flow in to the scheme because in year $t = 70$, $p' = 0.50 \times p_0$ is contributed. The underfunding is dissolved by cutting entitlements. If $\alpha = 1$ the entitlements of all participants are cut in such a way that the funding ratio returns to $F_D = 1$ within one year. If $\alpha = 0.2$ every year only one-fifth of the shortcoming is eliminated, F_D is not reached in one year, but gradually. If $\alpha = 0.1$ only one-tenth of the shortcoming is dissolved. In figure 6 the generational account for different generations can be found. The economy stated in table 1 is still used, but the parameter α is varied.

From figure 6 it can be seen that α has an effect on how gains and losses are divided over generations. The red line corresponds with the red line from figure 5, those settings are the same.

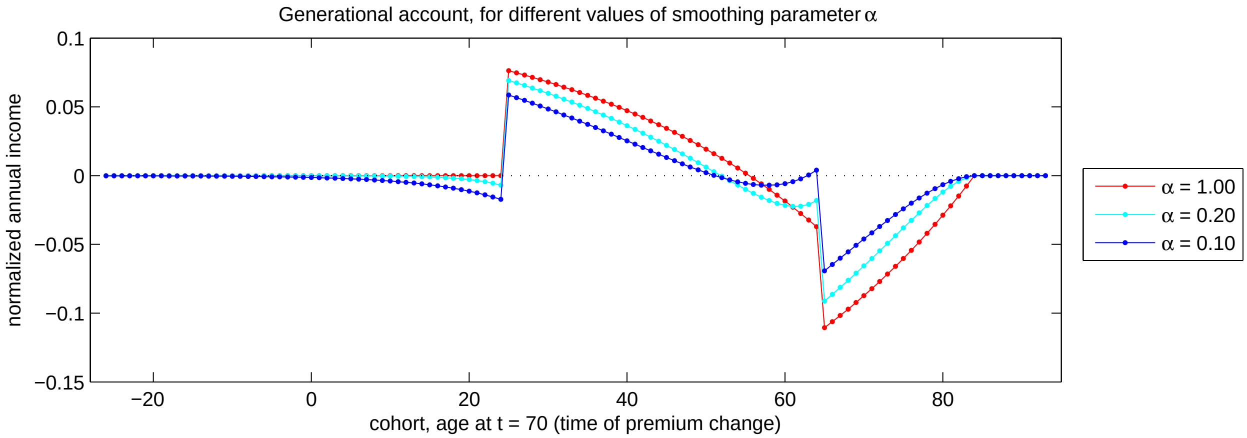


Figure 6: Generational account for different values of smoothing parameter α. In year $t = 70$ the premium rate deviates from p_0 by cutting it 50%. On the horizontal axis the age of participants is given at $t = 70$. Settings of the economy, the scheme and the population given in table 1, except α.

The following can be observed for some of the participants groups:

- **The retired** Generations that are retired at time $t = 70$ (age $65 \leq a \leq 84$) again feel the biggest disadvantage. They own the most entitlements that can be cut, and do not profit from lower premium. Although when the deficit is dissolved in more than one year ($\alpha < 1$), the retired gain relative to the situation in which the burden is taken in one year ($\alpha = 1$). There are two reasons for this. First, years shortly after $t = 70$, they receive a higher benefit, compared with $\alpha = 1$, because it takes a few years to completely dissolve the underfunding and thus before their entitlements are fully cut. Second, the working participants will help to dissolve the problem, this is in favor of the retired. If younger generations take a larger share in the burden, the retired are partly spared.

- **Close to retirement** Participants close to retirement will gain when $\alpha < 1$ compared to the situation in which $\alpha = 1$. They profit from paying in too little premium but they do not lose as much by the cut in their entitlements. Because when they retire,they still receive a relative large part of their original accrued entitlement, because the entitlements are only partly cut every year.

- **The young** The young (age $25 \leq a \leq 51$) do not gain as much from spreading the deficit over a longer period ($\alpha < 1$). They are subjected to cuts for a longer period, a period in which their amount of entitlements grows because they participate in the fund. Bottom line they thus own more rights that are cut, when $\alpha < 1$.

- **Future generations** In case $\alpha = 0.2$ or 0.1 the defect is not eliminated in one year. The participants joining the fund after $t = 70$ will experience cuts in the accrued entitlements. They do not profit from paying in too little premium. For $\alpha = 0.1$ it can be seen that 15 to 20 additional generations feel the negative consequences of the too low contribution to the fund. Note that this effect is pure theoretical because the size is minor.

The situation of underfunding that arises when the contribution is determined by discounting entitlements against the expected portfolio return μ_w while the funding ratio is based on discounting against the risk-free rate r^f, can be solved in different periods. The choice of this period is something the board of a pension fund can decide on. This decision will have an impact on value transfers between groups of participants, even on future participants.

A longer period of premium payment that is too low Here the effects of the smoothing parameter are studied in case insufficient premium payments to the fund will persist for a longer period. From $t = 70$ and further 50% of the premium rate necessary to keep the funding ratio at 100% is collected by the fund.

In figure 7 the generational accounts can be found for different levels of the smoothing parameter α. The gains and losses now take larger values compared with the situation in which the insufficient inflow only lasted one year (note the difference of the vertical axis with figure 6). Losses can grow up to the size of annual income, gains to more than half a annual income.

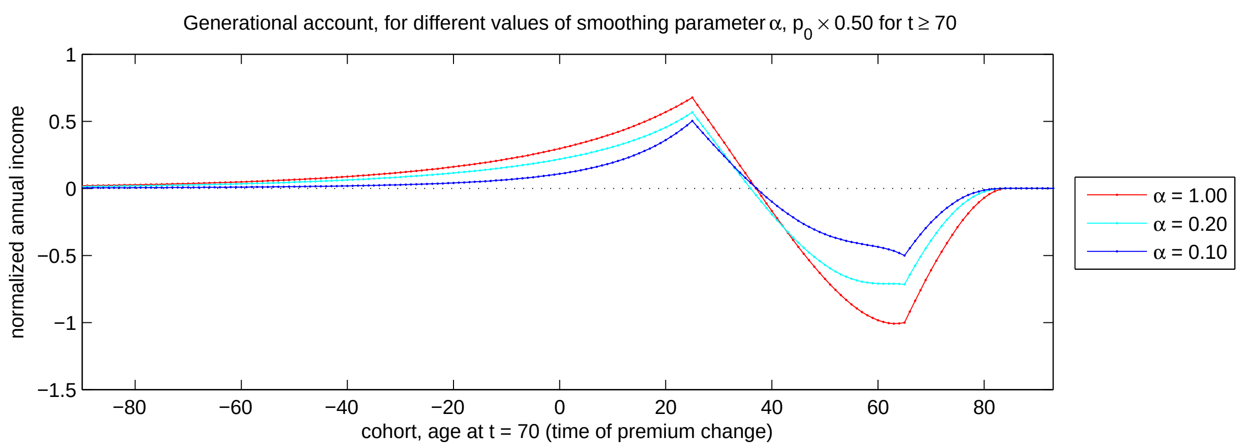


Figure 7: Generational accounts for different values of smoothing parameter α. For $t \geq 70$ the premium is $p' = 0.5 \times p_0$ (it will not be changed back to p_0). On the horizontal axis the age of participants is given at $t = 70$. Settings of the economy, the scheme and the population given in table 1, except α, also more future generations are used.

The generational accounts given in figure 7 show an adsorbing effect in case $\alpha < 1$. For different groups the following effects are observed:

- **The retired** In case insufficient premium payments will persist, the burden on the retired is larger compared with only one year of too low premium. Lower values of the smoothing parameter α have a cushioning effect on the size of the burden for the retired. As mentioned earlier in the illustration around figure 6, the retired are partly spared if $\alpha < 1$. This corresponds to the findings of figure 7.
- **Close to retirement** The people close to retirement and half way trough their career will now lose wealth compared to the situation in which insufficient premium was used for only one year. They profit from too low premium payments, but future generations that pay in too little imply cuts for more years, which is negative for the people close to retirement

and halfway trough their career. Their generational account drops below 0 in the case the insufficient contribution persists.

- **The young** The young people participating in the fund at $t = 70$ will benefit less in case the premium rate will permanently be too low. In contrast to the earlier analysis above, the young will also see their entitlements been cut at the time they already have accrued a lot (for instance half way their career).
- **Future generations** Earlier (figure 6 and illustration) the future generations only felt the negative consequences of the one year in which too little premium was paid to the fund if $\alpha < 1$. Now they also benefit from paying in too little themselves. That is why their generational account is positive. Lower values of α (which thus mean more smoothing) for this reason result in lower values of their generational account. Because then, more of the deficit is solved with them. This is also why the generational account of the retired is less negative in case of a low α.

Again different smoothing periods induce different wealth transfers between groups of participants. For lower values of α the generational effects (wealth transfers) seem to be smaller, than if $\alpha = 1$.

3.1.4 Conclusions

Although the previous analysis is conducted in a world that is heavily restricted by simplifying assumptions, the results give a good view of the causes and directions of value transfers.

If the method used for discounting entitlements is not the same for the funding ratio and the contribution rate, value transfers between generations can occur. The economic value of accrual will differ from the value of contribution. The smoothing parameter α and the equity allocation w have the following effects.

The choice of equity allocation w The choice of equity allocation influences the nominal expected return on a portfolio. From figure 4 it was found that the equity allocation w has an important effect on the amount of premium that has to be paid. If w goes from 0 to 1, expected portfolio return grows, which reduces the premium rate. The funding ratio is based on discounting against the risk-free rate r^f which is equivalent with a portfolio with equity allocation $w = 0$. If w is raised, indirectly the funding ratio is decreased because new accrual is bought to cheap in terms of the funding ratio.

The choice of smoothing parameter α If the fund has $\alpha = 1$ the deficit will be solved with all participants, also the retired that did not profit from paying too little premium. If α is decreased smoothing will occur and the underfunding will be solved among more (also future) generations.

Restrictions and the new FTK Given that an equity allocation of $w = 50\%$ is used, value transfers under the use of real expected return are expected to be small compared to the use of nominal expected return (in this particular setting). In chapter 5 section 5.1 the use of real expected return will be studied. There it will be found that value transfers that originate from the use of real expected return can also flow into the direction of the elderly instead of the young.

3.2 Participating in a fund with a surplus or deficit

The second setting in which a deviation between contribution and the economic value of new accrual is studied is a setting in which a participant joins a pension fund that has a deficit or a surplus. If a participant joins a pension fund with a surplus or a deficit, and pays a premium independent of the solvency position, this participant will become the partial owner of the deficit or surplus. Ex ante the new participant knows that it is very likely that his entitlements need to be cut or will be raised. This will affect the economic value of new accrual. Nowadays this is relevant, currently in the Netherlands allot of pension funds are at risk to cut entitlements the upcoming years and cannot compensate pensions entitlements for inflation. This in order to dissolve a deficit or a too low funding ratio. These deficits or too low funding ratios resulted from the financial crises, which cropped high interest rates and lowered equity returns. In the past the opposite was observable, buffers of pension funds where high. Entitlements were expected to be indexed with a large probability which resulted in a higher economic value of new accrual than the contribution made.

What happens is the following. A pension fund has a deficit and a person joins that fund. He or she has to pay the ordinary contribution level to obtain new accrual. This contribution level is for now assumed independent of the solvency position of the fund. In case of a deficit it is very likely that in the near future entitlements need to be cut. Ideally, if the pension fund was in equilibrium, the contribution of the new participants would match the economic value of newly accrued entitlements. But now, because of underfunding, cuts are expected with high probability. This will reduce the economic value of the new accrual and will result in a difference between contribution and the economic value of new accrual. Note that in reality the contribution level could be raised to dissolve the deficit, making the difference between the contribution and the economic value of new accrual even larger. This last effect will not be studied here, only the effect of an unchanged contribution rate when the fund has a deficit will be analyzed here.

How a deficit is solved with the participants of the fund and whether the deficit is solved with the help of future participants is a decision partially made by the board of the fund (also regulations prescribe in what period an underfunding should be dissolved). By definition decisions made by the board of a pension fund are a zero-sum game. The board (or regulation) cannot create or erase wealth. A board's decision thus can only redistribute wealth. The focus of this section is on effects that participants, mainly new participants, experience caused by choice of a smoothing period in case of underfunding.

3.2.1 Analysis

In this section again a stylized pension fund will be analyzed. The focus is on how a choice for a smoothing period affects the economic value of new accrual. If the economic value of accrual is affected and the contribution stays unchanged, value transfers between groups of participants are expected.

Model description: The economy, pension fund and participants The same simplifying assumptions that where made in the earlier section (section 3.1) still hold. Again a degressive accrual system is used again. The premium rate is now determined in correspondence with the calculation of the funding ratio, on the basis of the risk-free rate r^f.

Model description: Parameterization The parameterization of the economy, the scheme and population as used here can be found in table 2.

Parameter	Symbol	Value
Risk-free rate	r^f	3.0%
Start working age	a_w	25
Retirement age	a_R	65
Death age	a_D	85
Replacement income	γ	80.0%
Wage growth	g	0.00
Start wage	w_0	1
Premium rate (equation 3.8)	p_0	15.6%
Participants at $t = 1$		none
Participants for $1 < t \leq 120$		one every year, age $a = a_W$
Desired funding rate	F_D	1.00
Smoothing parameter	α	1.00

Table 2: Basic parameterization of the deterministic economy, the population and the pension scheme for section 3.2.1

Departure from equilibrium situation In order to mimic the real world situation in which a deficit will be present in the fund, again a fund is started in the equilibrium state. In this deterministic setting underfunding cannot happen in a stable degressive accrual scheme because there are no unexpected shocks. The fund only invests in the risk-free bond with deterministic return r^f. In order to attain a situation of underfunding, at a certain time ($t = 70$ is chosen for this) a fraction of assets is taken out of the fund (this could be seen as a big downwards shock). Hereby the funding ratio that was $F_{69} = 1$ will become smaller at $t = 70$ ($F_{70} < 1$). The underfunding is then solved by a particular setting of the linear pension contract.

The generational accounts of the participants can once again be used to assess the effect of the smoothing period. Because a fraction of the assets is taken out to the fund, the generational accounts add up to the size of the shock at $t = 70$. The shock to the assets of the fund at $t = 70$ is something that is not influenced by the board, the way the shock is divided between participants is partially influenced by the board. That decision is indeed zero-sum. The decision how to dissolve the deficit does not create or erase wealth, it only redistributes.

For this reason the effect of α on generational accounts of age cohorts can be analyzed from two viewpoints.

1 By looking at the absolute, unconditional, effects on the generational accounts of cohorts. In this way, the generational accounts will add up to the total amount of assets taken out of the fund at $t = 70$. The generational accounts will develop differently regarding the choice of parameter α.

2 By looking at the difference between two possible solutions chosen (two different levels of α). In this way, the generational accounts will add up to zero, because either way, the generational accounts represent the fraction of the total assets that was taken out at $t = 70$. This second method reflects also the zero-sum property of decisions made by the board, as mentioned above.

A kind of benchmark solution could be chosen for comparison in case the analysis is conducted as described in the second viewpoint (**2**). The benchmark situation will be chosen $\alpha = 1$, in this situation all participants immediately lose a proportional amount of their entitlements in case of underfunding. It could be argued that setting ($\alpha = 1$) uses minimum interference of the board and therefore could be used as the benchmark solution.

Asset shock solved for different settings of adjustment mechanism The parameterization of the economy, the scheme and the population can be found in table 2. At $t = 0$ the fund is empty, for $t \geq 1$ every year a participant will join the fund and will spend his or her whole career in the fund.

At $t = 70$ an asset shock of -10% is applied: $A'_{70} = A_{70} \times 0.90$. Shortly after $t = 70$ but before new premium is received by the fund and new accrual is earned, the funding ratio drops to $F = 0.90$. Then the entitlement adjustment mechanism starts cutting entitlements of all participants. How much of the deficit is solved within one year depends on α. After the mechanism applied a cut, the new premiums are collected and new entitlements are accrued.

In figure 8 the losses for different cohorts can be seen from viewpoint one (see above). As said, the generational accounts now will not add up to zero, but to $0.10 \times A_{70}$.

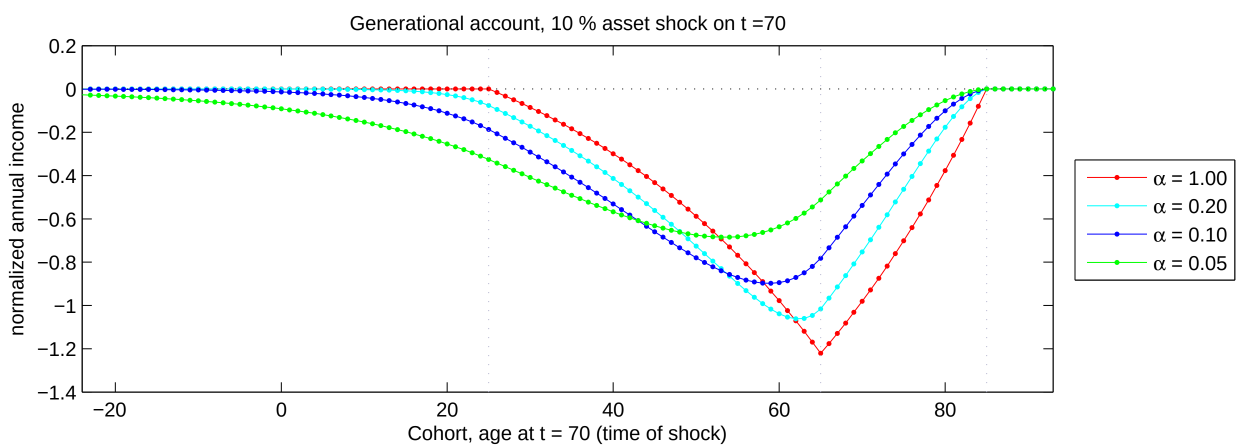


Figure 8: Generational effects for different values of α when at $t = 70$ the fund is subjected to a negative asset shock of 10% (first viewpoint). The economy, scheme, and participants as in table 2.

The red line shows the solution for dissolving the underfunding without smoothing. Looking at the red line, it can be seen that the deficit is spread over the participants proportional to the amount of entitlements they own. It can be seen that in this case (red line) only participating generations are hurt, no future generations.

- **At retirement age** The participants with most entitlements feel the hardest burden. This concerns participants with age $a = 65$ at time $t = 70$. They have accrued their whole career and at the maximum accrual, just before they receive their first benefit, all their entitlements are cut.

- **The retired** Moving to older generations for the red line (age $a > 65$ at $t = 70$) it can be seen that the loss decreases with age. This can be explained by the fact that people closer to mortality own less entitlements that are cut.
- **The working** For younger generations (age $a < 65$) also the burden decreases, these generation also do not own allot of entitlements because they are still accumulating them.

Differences between methods to solve the underfunding are also present as can be seen from the different lines in figure 9. In figure 9 the viewpoint of the second method (described above, **2**) is used. This figure thus gives the differences with the benchmark (not smoothing, $\alpha = 1$) solution. The generational account, which represent differences now does add up to zero.

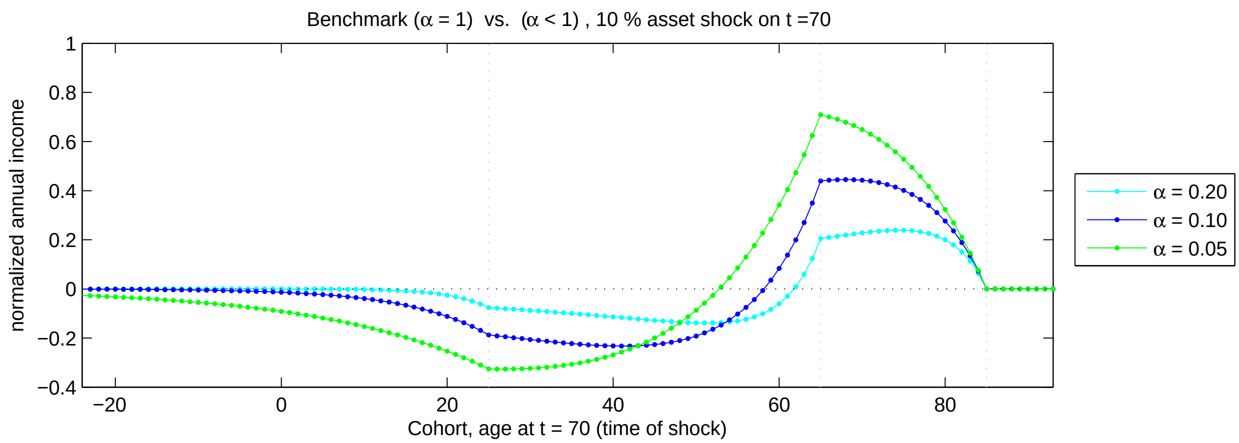


Figure 9: Differences in generational effects when several values of α are compared with the benchmark solution $\alpha = 1$ (second viewpoint). At $t = 70$ the fund is subjected to a negative asset shock of 10%. The economy, scheme, and participants as in table 2.

From the figure 9 it becomes clear, that when the underfunding is not solved immediately but gradually during a certain period the older participants gain. The younger and future participants lose by the decision to smooth the asset shock over a period. The main intuition behind the differences of $\alpha < 1$ compared with $\alpha = 1$ is given by:

- **The retired** The lower the value of α the longer the period over which the defect is smoothed. For the retired a lower value of α is desirable. They still receive a reasonable part of their benefit, because in case $\alpha < 1$ their entitlements are not fully cut immediately. By cutting entitlements at a later time the retired also own less entitlements that can be cut because they received a part of their benefit for some years. Next, there are also some new generations entering, which get engaged in solving the deficit. This will decrease the size of the cuts which is also something retired profit from.
- **Just before retirement age** For small α the deficit is smoothed over a longer period. For people just before retirement (age around $a = 60$ to $a = 65$ at $t = 70$) ,these are participants

who also own a reasonable amount of entitlements, this is beneficial. Their entitlements are not cut that hard as in the case of $\alpha = 1$ and when they reach retirement they receive a benefit on the basis of their entitlements that are not fully cut yet. This will make their benefit received in the early years of retirement rather high, which is beneficial for them.

- **The young working** For the younger participants, working at $t = 70$, a smaller value of α is not desirable. By dissolving underfunding over a period, the rights which the young accrue during that period will grow, therefor more of their rights are subjected to cuts if α decreases. Because the degressive accrual system entitlements of the young grow harder than that of older participants in comparison with a uniform accrual system. Making their share in the rights cut even larger.
- **Future generations** In case $\alpha = 1$ future generations enter the fund at a time the asset shock is totally dissolved. When $\alpha < 1$ generations that enter in the years after $t = 70$, enter a fund with a deficit. This deficit solved in a period in which new participants join the fund, which means that their rights are also subjected to cuts. Because the deficit is not solved immediately, the benefits that are received by the retired are too high for the level of the funding ratio, hereby the outflow of money will be too high, making the deficit even harder to solve. This also influences the generational account of the future generations negatively. This last part is also an effect that the working generations experience.

If $\alpha = 1, 0.2, 0.1$ or 0.05 it means that every year, the whole, one-fifth, one-tenth of on-twentieths of the deficit is solved. In figure 9 asymptotic behavior of the generational account for future generations can be seen if $\alpha < 1$. The amount of extra generations that experience negative consequences exceeds five, ten and twenty. By solving every year a certain fraction of the deficit, the retired receive a too high benefit for the level of the funding ratio. This too high benefit worsens the deficit, because it is an additional factor that brings the funding ratio under 100%. This increases as mentioned the burden on the working and future generations.

Premium rate dependent on the funding ratio for new participants A new participant that joins a pension fund which has a deficit will know that he or she will pay in too much contribution for the economic value of the accrual that he or she will receive back. This is studied above. As a solution to this mismatch a funding ratio dependent premium rate could be argued. These funding ratio dependent premium rates are such that they already incorporate the fact that new accrual will be subjected to cuts in future and is for this reason lower than the normal premium rate. As a result generations that join the fund after a situation of underfunding will not contribute to the elimination of the deficit. As a result the participants of the fund at the time of the shock have to solve the deficit on their own.

The effects of this premium rate dependent on the funding rate, that only holds for new participants, can be studied. This situation in which new participants do not contribute to the elimination of the deficit, is practically the same as a situation in which after the shock ($t = 70$ in this case) no new participants would join the fund. In that case the problem needs to be entirely solved by the participants of the fund at the time of the shock.

In figure 10 the generational effects are given relative to the benchmark solution in which $\alpha = 1$ (the generational account curve with $\alpha < 1$ is subtracted with generational account curve with $\alpha = 1$). The results can be interpreted as follows:

- **The retired** The retried gain from this solution relative to the benchmark ($\alpha = 1$), because they are, if $\alpha < 1$, not fully subjected to the cut but partially. If $\alpha < 1$ they still receive too much benefits the first years after the shock. Besides that they also leave the fund before the deficit is totally solved, making the effects on them also less. This reasoning was made before in figure 9.

- **The working** The working are hit harder if $\alpha < 1$. This is because the elderly leave the fund because they pass away at age $a = a_D = 85$, making the number of people that have to eliminate the deficit smaller, thus the burden per person higher. It can also be observed that the generational account of the working if $\alpha = 0.2$ and $\alpha = 0.1$ is lower in the case of no new inflow of participants than in the case of figure 9 in which there are new participants joining who help to solve the underfunding.

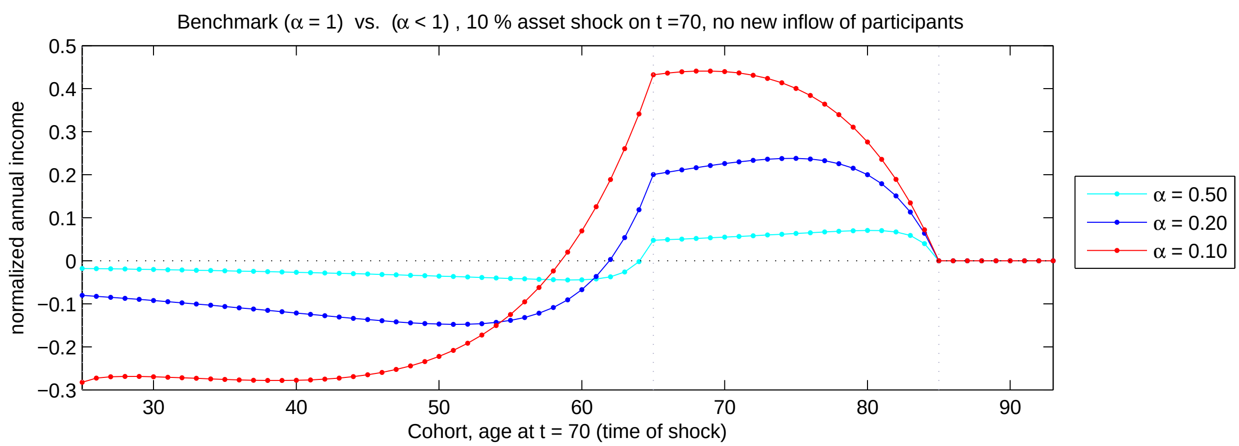


Figure 10: Differences in generational accounts when several values of α are compared with the benchmark solution $\alpha = 1$ (second viewpoint). After $t = 70$ there is no new inflow of participants. At $t = 70$ the fund is subjected to a negative asset shock of 10%. The economy and scheme, as in table 2.

A premium rate that is dependent on the funding ratio makes it more fair for new participants because their contribution is in line with the economic value of their accrual. For the participants in the fund that keep paying the original premium rate, this situation could be undesirable. The elderly benefit at the cost of the working.

3.2.2 Conclusions

Concluding from this section it can be said that new participants that would keep paying a premium rate independent of the funding ratio would lose because of the underfunded pension fund. On the other hand, new participants could gain if they are old themselves and a new future generation would help them to recover a shock. If $\alpha = 1$ future participants are not involved in any elimination of the deficit, on the other hand this puts a greater burden on the participants of the fund at the time of the shock. Certainly for the old who have no human wealth to compensate the financial loss this could pose a big problem.

Life cycle theory suggests that exposure to volatile stocks should be decreased near retirement. For this reason a 10% shock that is taken here as an example is quite big in reality as a shock on retired people. Nevertheless the analysis gives a good view of in which way value transfers flow if shocks are smoothed ($\alpha < 1$).

In the Netherlands second pillar pension funds could also raise the premium rate in order to solve a deficit. This case is not studied here, but it can be imagined that this would increase the burden on new participants even more. Next to that their entitlements will be cut in future they also have to pay more for the entitlements.

In chapter 5, section 5.2 also the generational effects of underfunding will be studied. This time also nonlinear pension contracts will be used to solve the underfunding.

3.3 Uniform accrual and contribution system

The last setting that is studied because it is known to have a mismatch between contribution and the economic value of new accrual is the setting in which a fund has an uniform accrual and contribution scheme.

The Dutch second pillar pension funds use the system of uniform accrual and contribution, this scheme type is compulsory for the industry wide pension funds. This pension scheme gives all participants the same uniform accrual, independent of personal characteristics like age and gender. The premium percentage is also uniform among all working participants. When retired, a participant receives a benefit until death.

This section will first give an overview of the literature on the uniform accrual and contribution system. Next some stylized mathematical derivations will show what the main points are that cause value transfers. Finally a stylized pension fund with a uniform accrual and contribution scheme is studied.

3.3.1 Insights from literature

In the uniform accrual and contribution system the value of the contribution is under most circumstances not equal to the economic value of new accrual. The recent CPB report (Lever et al., 2013) on uniform accrual and contribution argues that value transfers occur due to *differences in life expectancy* and *time value of premium payments*. According to the paper the public debate focuses on the latter one. This thesis will only study the effects around *time value of premium payments.*

> *Time value of premium payments* By differences in time at which accrual is earned and will be paid out as a pension benefit, the value of accrual is dependent on time to retirement (age of participant). Although all participants pay the same contribution and receive the same accrual.
>
> *Differences in life expectancy* By differences in life expectancy, the period over which a benefit is received (retirement) differs per person. Hereby the total benefit is dependent on life expectancy although all participants pay the same contribution.

Time value of premium payments Premium payments of young participants can yield a return for a longer period then premium payments of elderly. The young simply have to wait longer before they retire. For this reason contribution of the young is worth more at retirement than premium payments made by elderly, who retire within a relative short period of time. By the use of uniform accrual and uniform contribution, the young pay more for the same accrual than the old. The premium in this kind of scheme is determined such that for the total fund, the total costs of the accrual made in a particular year is divided uniform over all working participants that year. By means of this the young always pay more than the economic value of their accrual, the old always pay less than the economic value of their accrual. Subsidies from young to old take place. When the young become older themselves, they will in their turn receive a subsidy of the young at that time.

If someone participates his or her whole career in a uniform accrual and contribution scheme, first he or she will pay a subsidy and later he or she will receive one. It can be expected that to some extent the negative effects of paying a subsidy when young cancel out the positive effects of receiving a subsidy when old. There are several reasons why full cancellation does not take place.

Implicit pension dept In the past the first generation of elderly received a subsidy on their accrual, this resulted in an implicit dept within the system. The first generation elderly never paid the subsidy when young, they only received one when old. This implicit debt (implicit promise) can be seen as a PAYG element in the scheme. This implicit debt (PAYG element) is financed by premium payments that for this reason cannot yield the full investment return. As a result, if an implicit debt is present, the total contribution over a full career is higher than the economic value of all accrued entitlements because partly the debt is financed.

Lever et al. (2013) in their paper on uniform accrual and contribution estimate that the adverse consequences caused by the implicit pension debt have a negative effect of around 8% of pension premium. In other words, the CPB estimates that with 8% less premium the same benefit can be reached in a pension scheme without an implicit debt. This would thus mean that in the uniform accrual and contribution scheme the premium is around 8.7% ($\frac{1}{0.92} - 1$) higher than in an actuarial fair scheme. On the long run, the CPB estimates that participating in the uniform accrual and contribution system will result a burden of around 18% to 22%[5] of gross annual wage. The paper also states that generational effect of this implicit debt are dependent on the economic parameters and the average age in the fund. In an aging society the burden of this implicit debt is placed with fewer people making it heavier for those people. The total value of the implicit debt in the Dutch second pillar pensions is estimated around 100 billion Euro, 10% of total pension wealth. Boeijen et al. (2006) estimates that abolishment of the uniform accrual and contribution system will cost around 20% of pension wealth. The difference of the estimate between the two papers can be explained by different assumptions on economic parameters and also by the fact that the Dutch government has implemented some austerity measures. For instance the uniform accrual rate was lowered by government decision. According to Boeijen (2014) these austerity measures have also decreased the size of the implicit debt. On page 39 of this thesis, the implicit debt of the stylized pension fund used for the analysis in this section will be studied.

[5]Lever et al. (2013) page 40, estimate on basis of weighted average men and women.

Heterogeneous wage growth The pension benefit from retirement until death is in terms of a replacement income, this is a fraction of average wage (in the past final wage). Participants who have an above average wage growth later in their career will see that their accrual is more in the period when it is subsidized, resulting in a advantage for them and a burden for the rest of the fund who pay the subsidy. Boeijen et al. (2006) finds that a participant that has a steep income growth in the second half of his or her career will receive a subsidy around 5% of total pension wealth at retirement, in the particular setting of the paper. The opposite is also true for people with a below average wage growth.

Incomplete careers If participants do not spend their whole career in the fund, the value of contribution can heavily deviate from the economic value of accrual. Participating only the first part of a career will mean accruing pension rights in a period in which the participant pays too much for his or her accrual. Participating only the last part of a career will mean that entitlements are accrued in a period in which they are subsidized by the young, this will mean that the value of contribution is smaller than the economic value of the accrual.

Both Boeijen et al. (2006) and Lever et al. (2013) mention that in the past there where good reasons for the presence of value transfers. Some generations where impoverished by the second world war and trough the baby boom the value transfers to the old were paid by allot of young people.

Also in the past more people worked for the same company their whole life. Nowadays labor mobility increased and more heterogeneous salary paths arose. Bovenberg and Boon (2010) mentions wider career paths as source of increased wealth transfers between participants. Lever et al. (2013) mentions that the number of self employed in the Netherlands increased from 6.4 % in 1996 to 10.2 % of the labor force in 2012. This paper also finds that if someone only participates the first half of his career in a uniform accrual and contribution system, he or she would pay 35% of pension wealth at retirement too much. Someone only participating the second half will have a benefit of around 20% of pension wealth at retirement. Boeijen et al. (2006) also studies incomplete careers and finds that a person who works from age 25 until 35 and than leaves the fund pays almost 40% too much premium. A person that only participates the second half of his career will pay too little, around 20%.

Population of the pension fund For a pension fund with more young participants (a green fund) the premium will be relative low because allot the benefits have to be paid out over a relative long period. For a relative old fund (grey fund) the premium rate will be relative high because benefits have to be paid in a relative short period, making the total investment return on them low. If a young (old) person participates in a grey (green) fund he or she will pay a higher (lower) contribution compared to an averaged aged fund, seen over his or her full life.

Boeijen et al. (2006) mentions that indeed, an aging fund induces a higher premium rate for participants while the benefit remains the same. Currently the Dutch society is aging, which means higher premium rates and a relative high burden for the young. Lever et al. (2013) finds that in an aging society, subsidies of young to the old are carried by fewer people. Next to that, if people have the retirement age and decide to keep working, the period over which they accrue entitlements that are subsidized is extended.

Differences in life expectancy All participants in the uniform accrual and contribution scheme pay the same premium percentage. Participants receive a benefit from the age that they retire until death. The length of retirement is one of the most important determinants for the total cost of retirement. Life expectancy differs between groups within the fund. For instance higher educated people are expected to live longer than low educated people. And women are expected to live longer than man. By the fact that everyone pays in the same contribution (based on average life expectancy) but does not receive the same benefit (because in expectation certain groups of participants live longer or shorter than the collective expectation), the economic value of entitlements deviates from the contribution.

In this thesis, the value transfers that occur trough *differences in life expectancy* will not be analyzed. The CPB report of Lever et al. (2013) argues that the public discussion focuses on transfers between generations, not within[6].

Criticism on the uniform accrual and contribution scheme According to Boeijen et al. (2006) the Dutch second pillar pensions experience increasing criticism of pension experts. Also the confidence of participants is declining, because over the past years pension funds where not able to correct entitlements for inflation and even entitlements had to be cut. The CPB report of Lever et al. (2013) mentions that the support for the uniform accrual and contribution system could decrease because of subsidizing solidarity. Of subsidizing solidarity it is ex ante known who will benefit, in expectation not everyone gains welfare by this kind of solidarity. Subsidizing solidarity through *time value of premium payments* is the main source of dissatisfaction in the Dutch second pillar pension plans. On forehand it is known who will contribute more or less than the economic value of pension accrual. This mismatch between contribution value and economic value of accrual will result in value transfers between participants.

3.3.2 Illustrative mathematics

Here some mathematical results will be shown that can help to gain insight in the dynamics of the premium rate in uniform accrual and contribution scheme. Next to that, an equation for the implicit pension debt will be derived.

The premium rate in an uniform accrual and contribution scheme The uniform accrual and contribution scheme gives participants a fixed accrual rate $\theta(i) = \theta$ per year worked, this accrual rate is uniform over all ages. Participating a full career will result in an pension benefit that is a percentage (γ) of average wage, $\sum_{i=0}^{R-1} \theta = \gamma$ (similar as equation 3.3). For simplicity the wage growth $g = 0$.

The pension fund will determine the amount of contribution in a particular year by taking the total costs of accrual for the whole fund in a particular year and divide those costs uniformly over all participants. The total costs of new accrual on fund level can written as a function of the age of

[6] Currently at PGGM there is also a Netspar student of the Tilburg University that is studying the effects of longevity shocks on the value of pension payments. For the results of this study see the master thesis of Veeneman (2014).

the participants in the fund, this results in

$$EV(\text{New accrual})_t = \sum_{k=1}^{N} e^{-r^f(R-i_{k,t})} \cdot \sum_{j=0}^{\infty} w_0 \cdot \theta \cdot s_j \cdot e^{-r^f \cdot j}. \tag{3.14}$$

With the k'th participant having spend i years in the fund from age 25 at time t. The amount of years in the fund at which a participant retires is again denoted with R. w_0 is the flat wage and s_j the survival probabilities during retirement (it is assumed that participants do not die before retirement). The total costs of accrual, or in this case the economic value[7] of all new accrual in the fund, as given in equation 3.14 will result in a contribution for each participant according to

$$p_t = \frac{EV(\text{New accrual})_t}{N} = \frac{\sum_{k=1}^{N} e^{-r^f(R-i_{k,t})} \cdot \sum_{j=0}^{\infty} w_0 \cdot \theta \cdot s_j \cdot e^{-r^f \cdot j}}{N} \tag{3.15}$$

with p_t the premium rate in year t and N the total amount of participants. Rewriting leads to

$$p_t = \frac{\sum_{k=1}^{N} e^{r^f \cdot i_{k,t}} \left[e^{-r^f R} \cdot \sum_{j=0}^{\infty} w_0 \cdot \theta \cdot s_j \cdot e^{-r^f \cdot j} \right]}{N}. \tag{3.16}$$

From equation 3.16 it already becomes clear that the premium rate p_t rises in the variable $i_{k,t}$ (the time spent in the fund from age 25 at time t). It can also be concluded that the premium rate p_t thus rises with average age of the fund, although not linearly.

Implicit pension debt The size of the implicit pension debt within a uniform accrual and contribution scheme can be calculated by taking the difference between the premium in a uniform accrual and contribution scheme and the premium in an actuarial fair scheme, like the scheme used in section 3.1.3 and section 3.2.1. The pension benefit γ and population and all other settings need to be taken equal in both funds. The extra premium that is needed in a uniform accrual and contribution scheme is necessary to pay the interest on the implicit debt.

A formula for the premium in both schemes is at hand. The premium in a degressive accrual scheme is given by equation 3.8. The premium in a pension fund with an uniform accrual and contribution scheme is given by equation 3.16. By taking the difference between the two premium rates the interest payment on the implicit debt can be calculated. Similar as in equation 3.8 the premium rate in a degressive accrual scheme is given by

$$p = \frac{e^{-rR}\gamma \cdot \sum_{j=0}^{\infty} s_j \cdot e^{-r \cdot j}}{\sum_{i=0}^{R-1} w_0 \cdot e^{-r \cdot i}}. \tag{3.17}$$

[7]This world is still risk-free, only flat known interest rate r^f.

This premium (equation 3.17) can be subtracted from equation 3.16. This can be equated to the total interest payment on the implicit debt like

$$\mathrm{IPD} \cdot (e^{r^f} - 1) = N \cdot \left(\frac{\sum_{k=1}^{N} e^{r^f \cdot i_{k,t}} \left[e^{-r^f R} \cdot \sum_{j=0}^{\infty} w_0 \cdot \theta \cdot s_j \cdot e^{-r^f \cdot j} \right]}{N} - \frac{e^{-rR} \cdot \gamma \cdot \sum_{j=0}^{\infty} s_j \cdot e^{-r^f \cdot j}}{\sum_{i=0}^{R-1} w_0 \cdot e^{-r^f \cdot i}} \right). \tag{3.18}$$

With N the total amount of participants. This can be rearranged by taking out $e^{-r \cdot R}$ on both sides, this leads to

$$\mathrm{IPD} \cdot (e^{r^f} - 1) = N \cdot \left(\frac{\sum_{k=1}^{N} e^{r^f \cdot i_{k,t}} \sum_{j=0}^{\infty} w_0 \cdot \theta \cdot s_j \cdot e^{-r^f \cdot j}}{N} - \frac{\sum_{j=0}^{\infty} s_j \cdot \gamma \cdot e^{-r^f \cdot j}}{\sum_{i=0}^{R-1} w_0 \cdot e^{-r^f \cdot i}} \right) e^{-r^f \cdot R}. \tag{3.19}$$

Also the term $\sum_{j=0}^{\infty} s_j \cdot \gamma \cdot e^{-r^f \cdot j}$ is present on both sides, and can be taken out. This yields

$$\mathrm{IPD} \cdot (e^{r^f} - 1) = N \cdot \left(\frac{\sum_{k=1}^{N} w_0 \cdot \theta \cdot e^{r^f \cdot i_{k,t}}}{N} - \frac{\gamma}{\sum_{i=0}^{R-1} w_0 \cdot e^{-r^f \cdot i}} \right) \cdot e^{-r^f \cdot R} \cdot \sum_{j=0}^{\infty} s_j \cdot e^{-r^f \cdot j}, \tag{3.20}$$

which can no longer be simplified. The implicit debt can now be calculated by dividing the whole right side by $(e^r - 1)$.

From equation 3.20 it can be seen that both the interest rate r^f and the age of the participants $i_{k,t}$ are important determinants of the size of the implicit debt. If the participants are not changed but the interest rate r^f grows, the left side term between the big brackets in equation 3.20 grows harder than the right hand side between the brackets. This will lead to a larger difference between the premium of an actuarial fair scheme, like a degressive accrual scheme, and the premium in a uniform accrual and contribution scheme. This difference is needed to cover the implicit debt. Thus a larger premium percentage, leads ceteris paribus to an higher implicit debt.

3.3.3 Analysis

Model description: The economy and participants The economy is again assumed risk-free (see section 3.1.3 and section 3.2.1 above). There is only a risk-free bond with flat return r^f. Also everything is known about the participants (e.g. age of entrance, mortality wage, when they leave the fund).

Model description: The pension scheme This time the pension scheme that is used is the uniform accrual and contribution scheme. Each year a participant works he or she will accrue a fixed percentage of his or her wage. The contribution rate is determined by taking the total cost of new accrual on fund level and dividing this uniformly over all working participants.

Model description: Parameterization In table 3 the basic parameterization of the economy, the scheme and population as used here can be found. In the analysis of the stylized fund below, some of these parameter values could be adapted in order to study a specific setting, this will be clearly stated.

Parameter	Symbol	Value
Risk-free rate	r^f	3.0%
Start working age	a_w	25
Retirement age	a_R	65
Death age	a_D	85
Replacement income	γ	80.0%
Wage growth	g	0.00
Start wage	w_0	1
Premium rate (equation 3.8)	p_0	15.6%
Participants at $t = 1$		none
Participants for $1 < t \leq 120$		one every year, age $a = a_W$
Desired funding rate	F_D	1.00
Smoothing parameter	α	1.00

Table 3: Basic parameterization of the deterministic economy, the population and the pension scheme for section 3.3.3

Implicit pension debt As stated above the CPB study on the uniform accrual and contribution system of Lever et al. (2013) finds that the an average Dutch second pillar pension scheme has an implicit debt such that around 8.7% more premium is needed to cover this implicit debt compared to an actuarial fair scheme. The implicit pension debt itself is estimated to be 10% of total pension wealth. Boeijen et al. (2006) estimates the implicit debt to be around 20% of total pension wealth.

On page 15 of CPB report (Lever et al., 2013) a graph is presented about the effects of implicit debt on the pension premium. Figure 11 shows the same graph but now for the assumptions made in this section of the thesis (table 3). The CPB report does not state the scale of the axes, figure 11 does.

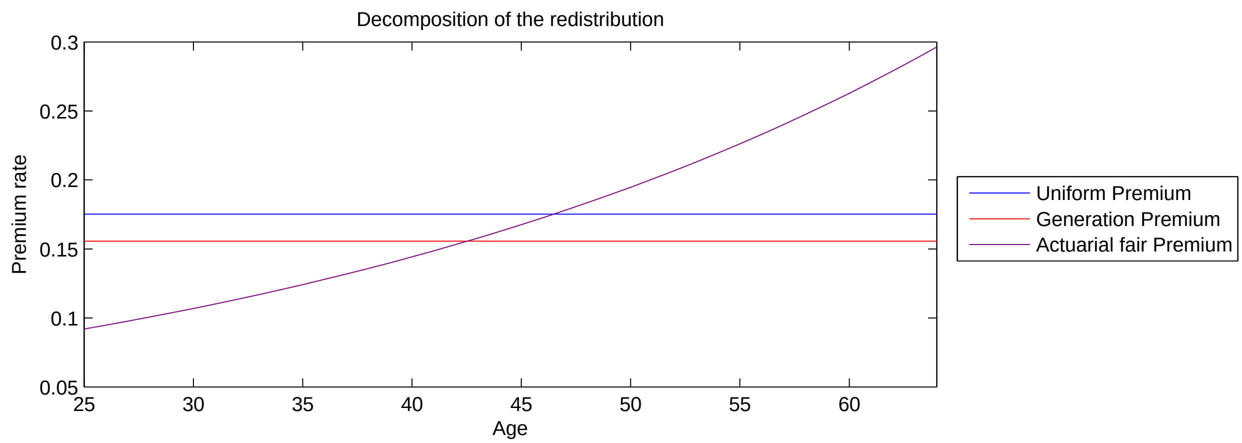


Figure 11: Decomposition of the redistribution, as on page 15 of the CPB report by (Lever et al., 2013) but now for the economy as assumed in this section. Interest rate $r = 3\%$, age start working $a_w = 25$, retirement age $a_R = 65$, death age $a_D = 85$. Pension benefit is $\gamma = 0.80$ of wage (flat normalized at 1).

From the stylized model used in this section the following can be observed from figure 11:

- **Blue line** The blue line gives the uniform premium rate within the uniform accrual and contribution scheme, 17,5%. This is the premium rate if the population is kept constant with equal amount of participants per age cohort. A part of this premium rate is used to finance the implicit debt.

- **Red line** The red line would be the actuarial fair premium rate that is constant over age. This constant premium rate is 15.6%, it is already seen in the previous sections (degressive accrual scheme). The difference between the blue and the red line is the pay as you go (PAYG) element. This actuarial fair premium rate would be sufficient if there was no implicit debt. The premium in the uniform accrual and contribution scheme is thus around 12% higher than under an actuarial fair scheme.

- **Purple line** The purple line is the actuarial fair age dependent premium rate for an uniform accrual. In the calculation of this age dependent premium rate the time value of money is taken into account.

The total value of the implicit pension debt can be obtained by comparing the total asset value of two different pension funds. One fund is the stylized fund with a uniform accrual and contribution scheme as in this section. The other fund has an actuarial fair scheme with degressive accrual (fund from section 3.2). The total asset values of the two stylized funds can be compared when they both are in the same equilibrium state (e.g. same amount of participants). This moment is taken when both funds have participants in all age groups, thus 40 working members and 20 retired members.

Asset value of a fund with an actuarial fair scheme:	330.82
Asset value of a fund with an uniform accrual and contribution scheme:	304.30

From the difference in asset value the implicit debt is around 9% of the total asset value of the pension fund with a uniform accrual and contribution scheme. This value is also found if equation

3.20 is used. If the interest rate was higher than the assumed $r^f = 3\%$, the surface between the blue and red line in figure 11 would be greater and the purple line would be steeper.

The first generation in a uniform accrual and contribution scheme Now a stylized pension fund with a uniform accrual and contribution scheme that is started, will be studied. At the start the implicit debt originates in the system, that is why this setting is studied. In figure 12 the magenta colored line shows the generational account of the basic population (pop_0). In this basic population, every generation has the same size. The following could be observed for the magenta colored line:

- **Age 32 and older** Al generations in the fund of age $a \geq 30$ will feel a benefit from the uniform accrual and contribution scheme. They spend fewer years in a situation in which they subsidize others, and they spend more years in a situation in which they are subsided by others.
- **Age between 26 and 31** The generations with age $26 \leq a \leq 31$ at $t = 1$ will feel a burden from the uniform accrual and contribution scheme. Although they spend fewer years than a 25 year old in a situation in which they subsidize the elderly, the payments for the implicit debt make their generational accounts less than 0.
- **Age 24 and younger** All generations that participate a full life in the scheme will pay too much premium because of the implicit debt. The net loss of participating in the scheme will be the same for all future participants. In terms of money at time $t = 1$ it will be less because of the time value of money. This is the reason the line goes asymptotically to 0.

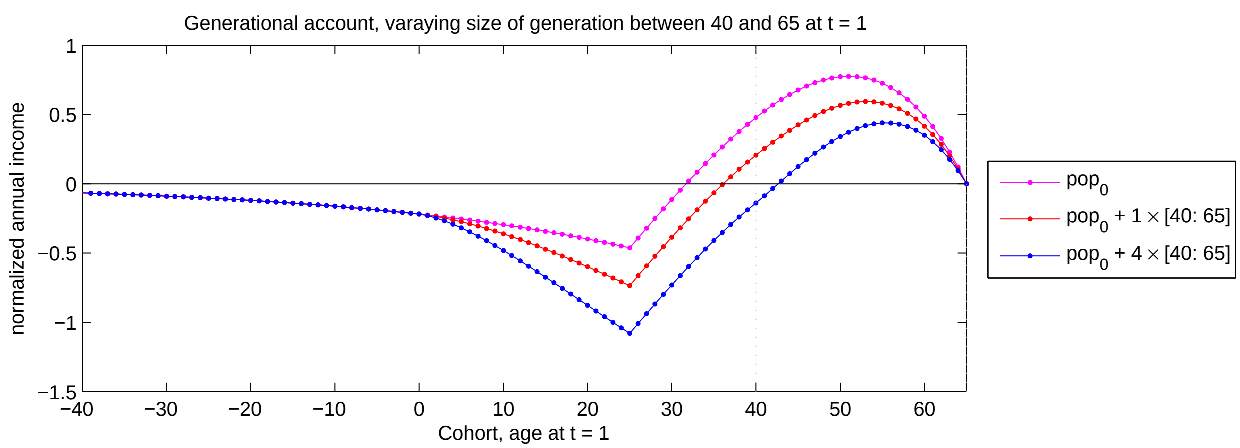


Figure 12: Generational account for different compositions of the first generations in a pension fund with an uniform accrual and contribution scheme. Size of generation with age between 40 and 60 at $t = 1$ can be varied. The settings of the economy, the fund and the basic population (pop_0) can be found in table 3.

Uniform accrual and contribution scheme with varying age distribution at start Next also the size of the first generations that is present in the fund could be varied. The first 25 generations of the basic population (magenta colored line, figure 12) in the fund as described in table 3 will be supplemented. The oldest 25 generations, people that have age $a = 40$ to $a = 65$ at $t = 1$, will be with more people. Normally there is one person in every generation (basic case, magenta line), now this amount could be 2 (red line) or 5 (blue line).

The effects of the varying size of oldest 25 generations can also be found in figure 12. The following effects can be observed:

- **The larger generations itself** The larger generations itself will make up a larger part of the fund compared to an basic population (every generation the same size). In this case their age is always closer to the average age in the fund. For this reason they pay a premium rate that is closer to the actuarial fair premium rate for their age (see equation 3.16 and illustration). This is the reason that the red line (2 persons in the larger generation) and the blue line (5 persons in the larger generation) are below the magenta line.

- **The generations that have the ordinary size** The generations that have the ordinary size (one per generation) will feel a burden. Their age is always below that of the larger generation. And when the larger generation is old, the premium rate will go up compared with a uniform population. This will hurt the younger, normal sized, populations because they now need to subsidize more compared to a uniform population.

Uniform accrual and contribution scheme with varying age distribution after start The effect of a varying population size can also be studied after the start of a fund. Figure 13 again shows the generational account of three different populations that flow trough a pension fund with an uniform accrual and contribution scheme. The basic population (pop_0) has the same amount of participant in each generation (magenta colored line). The red line in figure 13 shows the generational account of a population with two times more people of the age $a = 20$ to $a = 30$ at $t = 1$. The blue line represents the generational account if the generations with an age from $a = 20$ to $a = 30$ is five times larger than the rest of the generations.

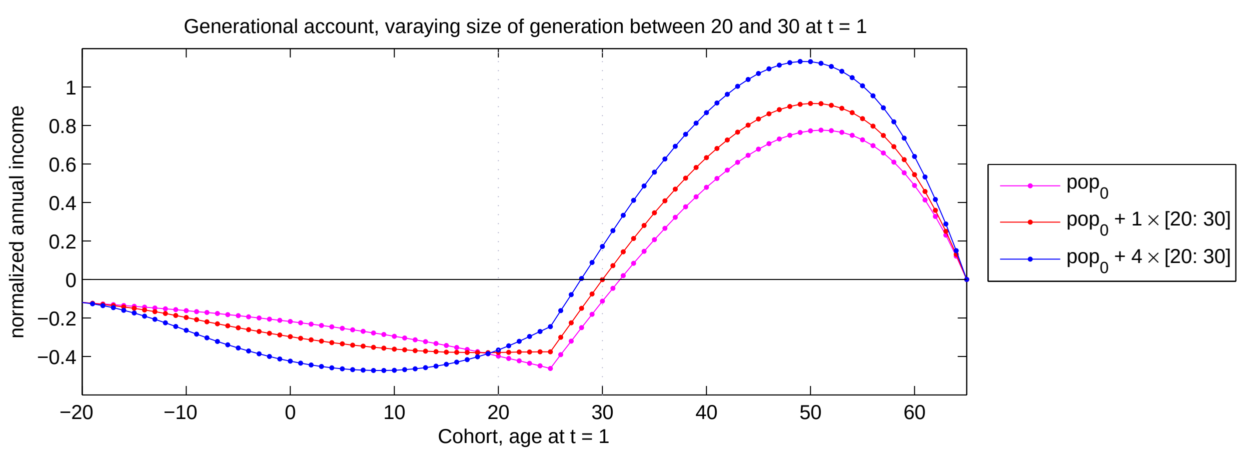


Figure 13: Generational account for different compositions of the generations in a pension fund with a uniform accrual and contribution scheme. The settings of the economy, the fund and the basic population (pop_0) can be found in table 3.

From the generational accounts in figure 13 the following can be observed:

- **Generations older than the large generations** As said earlier, the average age in the fund will in the case of a larger generation always be closer to the age of the larger generation itself, compared to the case of a uniform population. Generations older than the large generation profit from this. They experience a lower average age as long as they are working, and thus lower premium payments. Said short, there are now more people that subsidize them.

- **The large generations** The larger generation itself will experience that they always pay a premium rate that is relative closer to their actuarial fair premium rate, compared to the case in which the population was uniform. From this the larger generation itself benefits. This can also be found from the fact that the blue and red line lie above the magenta colored line, for the generations that have age $a = 20$ to $a = 30$.

- **Generations younger than the large generations** The generations younger than $a = 20$ at $t = 1$, are smaller in size than the generations that have age $a = 20$ to $a = 30$ at $t = 1$. Those younger generations will experience the opposite of the effect experienced by the older generations. The younger (normal sized) generations will see that the average age in the fund is most of the time higher compared to what the average age would be if there was a uniform population (pop_0). This results also in a higher premium rate for those younger generations. Which makes their generational account more negative (blue and red line lie below the magenta line)

The situation as described in figure 13 is in large extent comparable with the Dutch situation during the past 60 years. A large group of workers, the baby boom generation, subsidized the by war impoverished elderly. The baby boom generation itself was large and could carry this subsidy. Now the baby boom generation itself becomes old and the premium rates go up because the average age in the fund raises.

A person only participates first or second half of his or her career As can be seen from figure 11 a participant in a pension fund with a uniform accrual and contributions scheme will pay more than the actuarial fair premium until an age of approximate 45. A person pays less than the actuarial fair premium rate after the age of proximate 45.

For this reason it is interesting to study the effects of an incomplete career on the generational account. In figure 14 the magenta line again gives the generational accounts of the basic population. There is also an additional person in the generation that has age $a = 24$ at $t = 1$. That additional person starts working at an age of $a = 44$, his or her generational account is indicated by the red dot. So the generation that has age $a = 24$ at $t = 1$ consists of twice as much persons as a generation in the basic population (pop_0) and half of these persons starts participating the second half of his or her career.

The effects on the generational account of participating the second half of a career are sizable. The person that starts working at age 44 has a benefit of around 35% of annual income. This is a lot if compared to the rest of his generation (magenta line, age 24) that has a negative effect of around 40% of annual income. For the person that starts working late, this thus means that the economic value of the entitlements he receives is much more than the value of the premium payments he makes. These result are in line with findings in Lever et al. (2013).

Figure 14: Generational accounts in a uniform accrual and contribution scheme that is started. There is one participant of the generation that has age $a = 24$ at $t = 1$ that starts to participate at age $a = 44$.

Figure 15 shows the same as figure 14, the person that previously starts working at an age of 44 now will stop working at that age. The magenta colored line again shows the generational accounts of the basic population. The red dot indicates the generational account of the preson that stops working at an age of $a = 44$.

Figure 15 shows that the person that stops working early will experience sizable negative consequences. Normally his generation (age 24 at $t = 1$), if active in the fund for the full career will feel a burden of around 40%. The person that stops working prematurely will feel an additional burden of around 40% leading to a total burden of 80% of annual income. The economic value of the entitlements accrued by this person are thus obviously lower than the contribution he pays to the pension fund. Later in section 5.3 of chapter 5 we will see that this result is approximately similar to the result obtained when the calculations are made 25 years ahead.

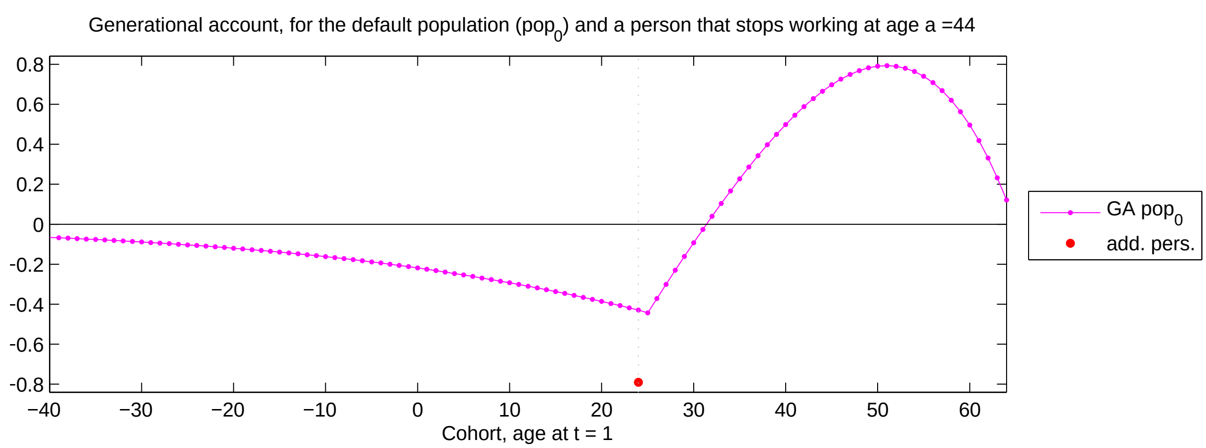


Figure 15: Generational accounts in a uniform accrual and contribution scheme that is started. There is one participant of the generation that has age $a = 24$ at $t = 1$ that stops participating at age $a = 44$.

The participants that have an incomplete career themselves are not the only ones who encounter a change in generational accounts caused by their incomplete career. The other generations that are working at the time someone else has an incomplete participation also see their premium rates change. For the other working participants the economic value of new accrual will deviate from the contribution also. The benefit for the person in figure 14 that only participates the second half of his career is paid by the rest of the participants. These participants see their generational account decrease by a bit. The other way around is also true. The person in figure 15 that felt a burden from participating only the first half of his career in the fund provided the rest of the participants a gift by paying too much contribution in comparison with the economic value of accrual he or she received. This increased the generational account of the others active in the fund.

A participants has a steep carrier growth in the second half of his or her career If participants do have an above average career growth, for instance by making a big promotion in the second half of a career, they accrue more entitlements in the period in which accrual is subsidized by the young. In that way the economic value of new accrual is above the contribution paid for this person. This situation is much alike working only the second half of a career.

The generational accounts of generations in the fund can be found in figure 16. In this figure the magenta colored line again gives the generational account of the basic population (pop_0). The red dot gives the account of the particular person that has a higher wage the second half of his or her career. The wage of that person is chosen to be 50% higher than a normal wage. In this way the effect becomes clear. It can be seen that the person with a higher wage profits from its wage growth. Normally his generation would have a loss by participating in the uniform accrual and contribution system of around 45% of an annual income. Now the additional person has a higher wage in the period in which his or her accrual is subsidized this loss reduces to around 27% of an annual wage which thus is sizable effect.

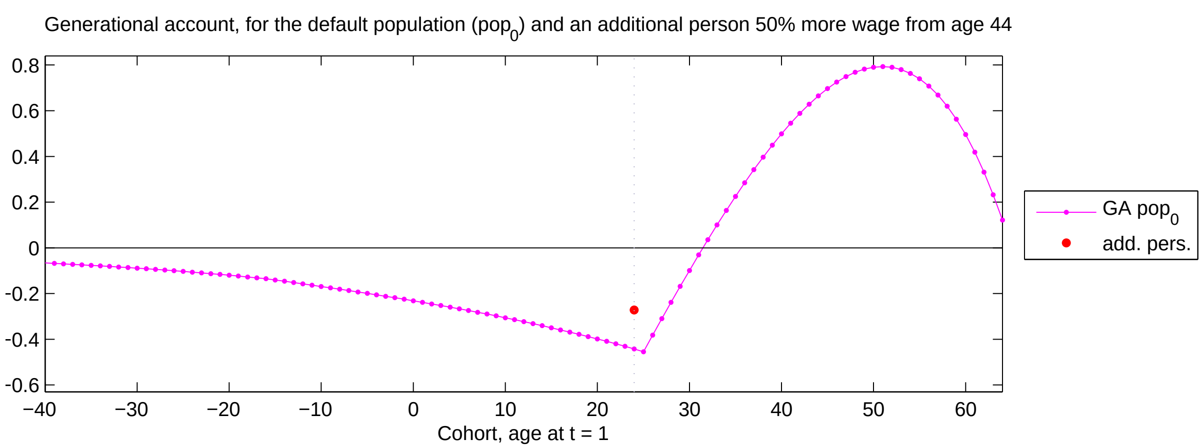


Figure 16: Generational accounts of an uniform accrual and contribution scheme that is started. There is one participant of the generation that has age $a = 24$ at $t = 1$ that has a 50% higher wage for age $a \geq 44$ (the second half of a career).

Uniform accrual and contribution scheme of which the uniform accrual rate will be lowered In the past the Dutch government decided to lower accrual rate for second pillar pensions.

This kind of measure lowers the absolute value of the implicit debt within a pension fund. The implicit debt is a part of a future promise that is not funded. By lowering the future promise, also the implicit debt is lowered.

In the Netherlands pension experts and pension funds are thinking about ways to abolish the uniform accrual and contribution scheme. The paper of the CPB on the uniform accrual and contribution scheme (Lever et al., 2013) finds that abolishing the system can cause allot of transition issues. Also Boeijen et al. (2006) finds that the transition to an other kind of system is problematic and costly. By lowering the accrual rate, the Dutch government in fact solves a small part of the transitional problems by putting the burden by the working participants.

In figure 17 the impact on the generational account can be found if the accrual rate is lowered. In this figure the lines indicate the difference between not lowering the accrual rate θ and the generational accounts of lowering θ. Observe that the lines do have exactly the opposite shape of the magenta colored lines in previous figures (e.g. figure 12). From this it can be concluded that by lowering the accrual rate, the opposite effects take place of installing an uniform accrual and contribution system.

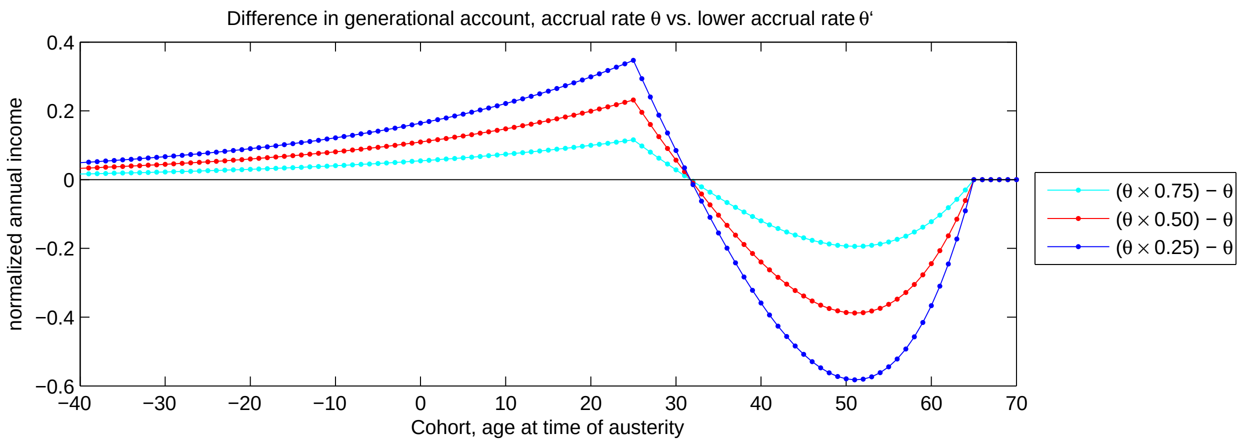


Figure 17: Differences in generational account (GA) between a reduction of the annual accrual rate and the original accrual rate. GA(lower accrual) − GA(original accrual). Effects calculated to time of austerity (reduction).

The impacts of lowering the accrual rate on generations within the fund can be found in figure 17. The effects are as follows:

- **The retired** The retired people age $a \geq 65$ at the time of the reduction of the accrual rate do not feel anything. They have already accrued their rights and those accrued rights are not changed. They just receive the ordinary benefit as was present in the base situation in which no lowering of the accrual rate takes place.
- **The working, older than age 30** The working participants older than 30 years, experience a loss. Normally, those people had paid too much premium when young. When they would grow older, they would accrue rights which are subsidized by the young at that time. When the accrual rate is reduced they will see that they will accrue less in a period in which their accrual is subsidized. This causes the negative consequences.

- **Future generations** The future generations gain by the fact that the payment to the implicit debt is less after the reduction of the accrual rate. They experience a permanent advantage from this.

Combination of uniform accrual and contribution scheme and too little contribution It is also interesting to study what happens when the situation of section 3.1, too little inflow of contribution, is combined with the situation of section 3.3, the uniform accrual and contribution scheme. In reality this could be the case because Dutch pension funds use the system of uniform accrual and contribution and by the old FTK they were allowed to determine the premium by discounting against nominal expected return. It is expected that the combination of those two settings would reduce the value transfers because both settings create value transfers in the opposite direction.

To just study the effects of this combination, a pension fund with an uniform accrual and contribution scheme is started. Right from the start the premium for the whole fund is calculated by discounting the liabilities against $\mu(w = 0.5) = 5\%$ (equation 3.11). The funding ratio is still determined on the basis of $r^f = 0.03$. Return the fund actually makes on its investments is still kept at r^f (in section 5.1 also the change of this return will be taken into account).

Figure 18 gives the generational accounts that results from the combination. The magenta colored line gives the ordinary generational accounts that result from the uniform accrual and contribution scheme in case the premium is calculated with r^f. This line is thus in accordance with the lines seen earlier (e.g. figure 12). The blue line indicates what would happen if premium is calculated with $\mu(w = 0.5) = 5\%$ and the smoothing parameter is $\alpha = 0.10$. The cyan colored line gives the generational account for the same premium calculation but now in case $\alpha = 0.20$.

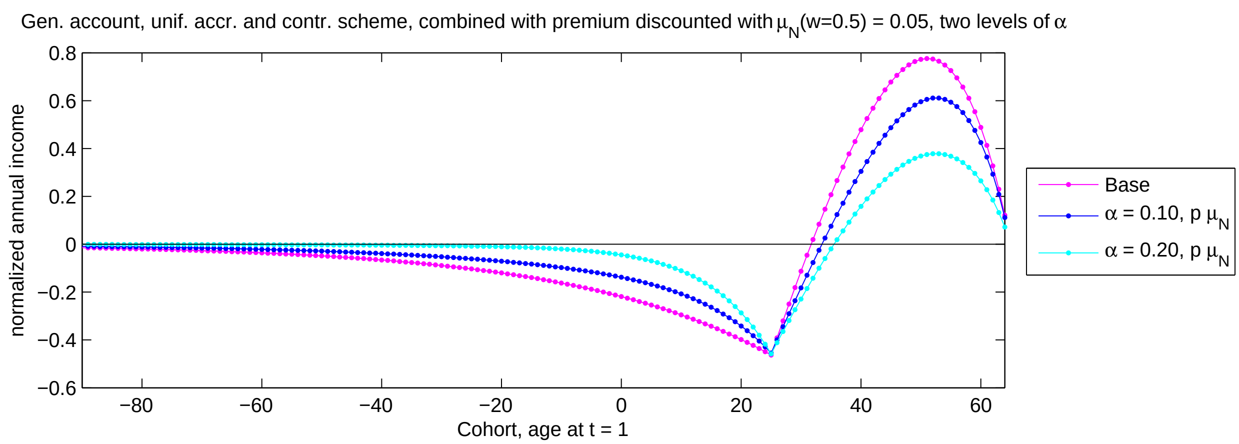


Figure 18: Generational accounts in a uniform accrual and contribution scheme that is started, but too little premium is paid to the scheme.

From figure 18 the following can be observed. In case $\alpha = 0.1$ the difference between the actual funding ratio and the desired funding ratio is dissolved by one-tenth every year. For the young this brings a benefit because they pay in too little premium and the older participants share in the deficit. This is also the reason why the generational account of the elderly is below that of the standard situation (magenta line) and that of the young is above the standard situation.

In case $\alpha = 0.2$ the difference between the blue line and the magenta line is even larger. This is because with a smoothing period of five years, a larger part of the underfunding is dissolved per year. This has the result that the older participants take a larger share in the solution of the underfunding. As a result the 80% benefit that a person of age 50 at $t = 1$ would have reduces to around 40% of annual income. The generational accounts of the young also increase on the long run.

From figure 18 it can indeed be confirmed that combining two settings that create opposite value transfers results in lower value transfers between generations in general. The use of this setting is limited by the fact that investment return is not properly adapted to $\mu(w = 0.5)$ (this will be done in section 5.1). However it shows that the aggregation of two situations in which opposite value transfers appear, can result in a partial cancellation of the transfers in general.

3.3.4 Conclusions uniform accrual and contribution system

We have seen several situations within the uniform accrual and contribution system in which sizable value transfers between generations occurred. The analysis studied value transfers that could be caused by demographic reasons (varying population). The transfers can also be caused by an individual participant (e.g. incomplete career). But transfers can also be caused by government decision to adapt the rules of the scheme (e.g. reducing the accrual rate).

Deterministic calculations about the uniform accrual and contribution system that has a linear pension contracts showed to give a clear view and interpretation of the generational effects. In general the same kind of results where found as in Lever et al. (2013) and Boeijen et al. (2006), the differences that are observed can be explained by the difference in assumptions.

Chapter 5, section 5.3 will study the basics about the uniform accrual and contribution system again. This time also nonlinear pension contracts will be analyzed.

4 Risk-neutral valuation and linear pension contracts

In this chapter the concept of risk neutral valuation will be introduced, also the statement about value transfers in linear contracts when the equity allocation is changed is studied. Section 4.1 will first show how to obtain a risk neutral price process for a stock. Next an analytical expression will be given for the price of a call option. Thereafter it will be shown how the same price of a call option can be obtained by simulation. Section 4.2 will analyze the statement ”varying the equity allocation in a linear pension contract does not cause value transfers between generations”.

4.1 Risk-neutral valuation

In chapter 2 section 2.1 the basics of valuation were explained. It was also said that in a world in which risk is absent valuation was not complex. Discounting against the flat risk-free interest rate r^f was sufficient. In chapter 3 the three problems were analyzed in a riskless world with a linear contract. Chapter 5 will study the three settings again. This time the world in which the pension fund models operate will also contain risky assets. Nevertheless the results from chapter 3 should be largely similar to the results from chapter 5 in case of a linear pension contract.

As explained in chapter 2, investors demand a risk premium for investing in risky assets. This risk premium compensates investors for the possibility that their assets could also turn out in a loss. If pension benefits depend on the funding ratio, these benefits also depend on the price processes of stocks where the fund invests in. Also the economic value of pension entitlements will depend on the the financial market. A pension entitlement is thus a contingent claim. To determine the economic value (market consistent value) of pension entitlements the same difficulties as pricing a call option or an other financial derivative are experienced.

How investors (or financial markets) value negative shocks or positive shocks and diverse sizes of these shocks is hard to determine.

The First Fundamental Theorem of Asset Pricing The first Fundamental Theorem of Asset Pricing (FTAP) in (Schumacher, 2013, Theorem 4.2.1) states that the market specified by objective measure $\mathbb{P}$ is free of arbitrage if and only if given any numéraire N there is a measure $\mathbb{Q}_N$ which is equivalent to the objective measure $\mathbb{P}$ and which is such that all relative price processes are martingales under $\mathbb{Q}_N$.

If a risk-neutral probability measure $\mathbb{Q}_N$ is found, the numéraire dependent pricing formula, stated in (Schumacher, 2013, Formula 4.22) and given by equation 4.1, can be used to obtain a market-consistent price for future cash flow C_T. The numéraire dependent pricing formula or risk-neutral pricing formula is given by

$$\frac{C_t}{N_t} = E_t^{\mathbb{Q}_N}\left[\frac{C_T}{N_T}\right], \tag{4.1}$$

with $t < T$. In this way any cash flow, whether it is premium payments to the fund or benefits received from the fund, can en be valued such that the price would correspond to the price the financial market would pay for such a cash flow.

An intuitive explanation of the risk neutral measure is given by: "Under the risk neutral measure there are no risk premia, but price processes under this measure are tweaked, such that they would lead to correct prices in markets that do incorporate risk premia"[8].

Illustration with Black Scholes Here an illustration is given of how to obtain a price process of the stock under the risk neutral measure, in a Black Scholes economy. This illustration is given with the use of the course notes of financial models (Schumacher, 2013) (page 68 - 71).

The price process of a stock S_t and a bond B_t in a Black and Scholes (1973) economy are given by

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{4.2}$$

$$dB_t = r^f B_t dt, \tag{4.3}$$

with μ the drift term of the stock, $\sigma > 0$ the volatility, r^f the risk-free rate of the bond and W_t an standard Brownian motion or Wiener process. Equation 4.2 and 4.3 are the price processes under the objective measure $\mathbb{P}$. To calculate market consistent prices a change of measure is applied (a change to the risk neutral measure). This change of measure is equivalent with a change of the drift term of the stock.

In an arbitrage free market the following holds (Schumacher, 2013, Theorem 4.2.3):

$$\mu - \sigma\lambda = r^f. \tag{4.4}$$

λ is often called the market price of risk. Girsanov's theorem (Girsanov, 1960) can be used to apply the change of measure (course notes of financial models (Schumacher, 2013) (page 49). This begins with

$$d\tilde{W}_t = \lambda dt + dW_t, \tag{4.5}$$

where $\tilde{W}_0 = 0$. Under the new measure $\tilde{W}_t$ is a standard Brownian motion. Equation 4.5 can be substituted in equation 4.2. This will give the price process of the stock S_t under the risk neutral measure $\mathbb{Q}_B$

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{4.6}$$

$$dS_t = (\mu - \lambda\sigma) S_t dt + \sigma S_t d\tilde{W}_t \tag{4.7}$$

$$dS_t = r^f S_t dt + \sigma S_t d\tilde{W}_t. \tag{4.8}$$

The relative price process under the numéraire

$$d\frac{S}{B} = \sigma \frac{S}{B} d\tilde{W}_t, \tag{4.9}$$

is indeed a martingale because

$$E_t^{\mathbb{Q}}[\frac{S_T}{B_T}] = \frac{S_t}{B_t}. \tag{4.10}$$

[8]Explanation as given by prof. dr. B.J.M. Werker during Asset Liability Management lecture, UvT spring 2014.

So equation 4.8 gives the price process of the stock under the risk neutral measure $\mathbb{Q}_B$ which can be used for pricing purposes.

In some cases it is possible to obtain the explicit price of a financial derivative. In more complex cases this is not possible. In section 4.1.1 an explicit solution for the price of a call option will be derived in a Black Scholes world. A call option is a financial derivative which gives the holder the right to buy a stock against a certain price K at a certain time in future T. In section 4.1.2 the price of the call option will be estimated by taking the mean of many risk neutral scenario's of the stock. This is called simulation, also known as the Monte Carlo method.

4.1.1 Analytical expression for the price of a call option method

Here an illustration will be given of how one can obtain the explicit formula for the price of a call option in a Black Scholes economy. This will also be done along the lines of the course notes of Schumacher (2013) (see page 79 and 80 of the course notes).

The payoff function of a call option with strike K at maturity T is given by

$$F(S_T) = \max(S_T - K, 0), \tag{4.11}$$

with S_T the value of the stock at time T. With the use of the numéraire dependent pricing formula the call option can be priced according to

$$\frac{C_t}{B_t} = E_t^{\mathbb{Q}_B}\left[\frac{F(S_T)}{B_T}\right] \tag{4.12}$$

in which $B_T = 1$ because it is the bond maturating at time T[9]. Because the expectation is taken under the risk neutral measure $\mathbb{Q}_B$, with the bond as numérair, the relative price process of S_t/B_t is as derived in equation 4.9 need to be used. If to equation 4.9 the explicit solution for a Geometric Brownian motion is applied the following results

$$\frac{S_T}{B_T} = \frac{S_t}{B_t}\exp(-\frac{1}{2}\sigma^2(T-t) + \sigma(\tilde{W}_T - \tilde{W}_t)) \tag{4.13}$$

with $\tilde{W}$ a Brownian motion under $\mathbb{Q}_B$ and with $(\tilde{W}_T - \tilde{W}_t)$ normally distributed with variance $T-t$. Note that $B_T = 1$, the explicit formula for S_T under $\mathbb{Q}_B$ is given by

$$S_T = e^{r(T-t)}S_t\exp(-\frac{1}{2}\sigma^2(T-t) + \sigma\sqrt{T-t}Z), \tag{4.14}$$

with $Z \sim N(0,1)$. Now the derived expression of S_T can be plugged in to equation 4.12 which yields the following integral

$$C_t = e^{-r(T-t)}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\max(e^{r(T-t)}S_t\exp(-\frac{1}{2}\sigma^2(T-t) + \sigma\sqrt{T-t}\cdot z) - K, 0)\exp(-\frac{1}{2}z^2)dz \tag{4.15}$$

which can be solved. The solution results in the famous Black Scholes option pricing formula. This is thus an explicit formula for the price of a call option in a Black Scholes economy. In the next section the use of simulation is illustrated.

[9] Zero coupon bond with interest rate r^f, maturity T and face value 1. Value at t is given by $e^{-r^f(T-t)}$

4.1.2 Monte Carlo method for the price of a call option

Earlier the risk neutral price process of the stock was derived. This was the process under the equivalent martingale measure associated with taking the bond as numéraire, the $\mathbb{Q}_B$ measure. As stated earlier it is not always possible to derive an explicit solution for the price of a financial derivative (or cash flow) like in section 4.1.1. Often this is a very demanding job or even not possible. In that case, a lot of trajectories of the underlying (the stock S_t in the example of the call option here) can be simulated under the risk neutral measure. The expectation of equation 4.12 is estimated by $\hat{C}_t$, the mean of the simulation results.

Thus the numéraire dependent pricing formula is estimated by taking M simulated trajectories of the stock price process. These trajectories are started at time t that end at time of maturity T, which result in

$$\hat{C}_t = e^{-r(T-t)} \frac{1}{M} \sum_{i=1}^{M} F(S_T^i) \tag{4.16}$$

$$= e^{-r(T-t)} \frac{1}{M} \sum_{i=1}^{M} \max(S_T^i - K, 0) \tag{4.17}$$

with S_T^i the price under the risk neutral measure of S at time T for the i-th simulation trajectory. Note that $B_t = e^{-r(T-t)}$. $\hat{C}_t$ will be a random variable and for large M, $\hat{C}_t$ will converge to its expectation,

$$\hat{C}_t \xrightarrow{M \to \infty} C_t.$$

Because $\hat{C}_t$ is a random variable it will have standard deviation $\hat{s}$,

$$\hat{s} = \sqrt{\frac{1}{M-1} \sum_{i=1}^{M} \left(C_t^i - \hat{C}_t \right)^2} \tag{4.18}$$

with C_t^i the price at t in the i-th simulation trajectory. The standard deviation $\hat{s}$ can be used to construct confidence intervals. A 95% confidence interval around the simulation estimate $\hat{C}_t$ is given by

$$\mathrm{CI}_{0.025,0.975} : \left[\hat{C}_t - \Phi^{-1}(0.975) \cdot \frac{\hat{s}}{\sqrt{M}}, \qquad \hat{C}_t + \Phi^{-1}(0.975) \cdot \frac{\hat{s}}{\sqrt{M}} \right]. \tag{4.19}$$

By thus obtaining risk neutral trajectories of the financial market, complex financial derivatives like pension entitlements can be priced. This is done by replacing the expectation in equation 4.1 by the mean of the payoffs in the risk neutral world. From this the economic value of pension entitlements in stochastic models for the financial market can be obtained.

The two illustrations above are on the basis of the economy described by Black and Scholes (1973) and given in equations 4.2 and 4.3. Also for more complicated models risk neutral trajectories can be obtained. When in chapter 5 the KNW-capital market model is used, also risk neutral trajectories will be simulated in order to obtain the market price of pension entitlements.

Monte Carlo vs. explicit solution Both methods of obtaining a price have their advantages and disadvantages. If it is possible to derive an explicit formula for the price of a financial derivative the computer program works very fast and accurate. As said earlier this is not always possible because these formulas are very complex. The Monte Carlo method only needs risk-neutral trajectories to make an estimation of the market price. The disadvantage is that this methods calculates a mean, which also has variance. In order to reduce the variance of the mean, more trajectories M could be simulated. This requires a lot of computing power, which is a disadvantage of the Monte Carlo method.

4.2 Value transfers due to varaying the equity allocation of the pension fund

In section 2.3 it was argued that within the pension industry the statement that the equity allocation w of a pension fund does not cause value transfers between generations as long as the pension contract is linear. This statement is very well known in the pension sector and also applied in research. The statement will be studied in this section with the use of a simplified version of the ALM model of PGGM.

A linear contract was illustrated before in chapter 3. The linear mechanism was given by equation 3.13. A visualization of such a linear pension contract can also be found in figure 19a. Next to the linear contracts there are also nonlinear contracts. Figure 19b gives a graphical interpretation of the "single kink" contract, this contract is named after its single kink. Figure 19c visualizes the "Dutch staffel" contract, this is a very common kind of contract for Dutch pension funds.

A mathematical notation of the linear pension contract also used in chapter 3 was already given in equation 3.13 for clarity it is given again

$$\delta(F) = 1 + \alpha(\frac{F}{F_D} - 1), \tag{4.20}$$

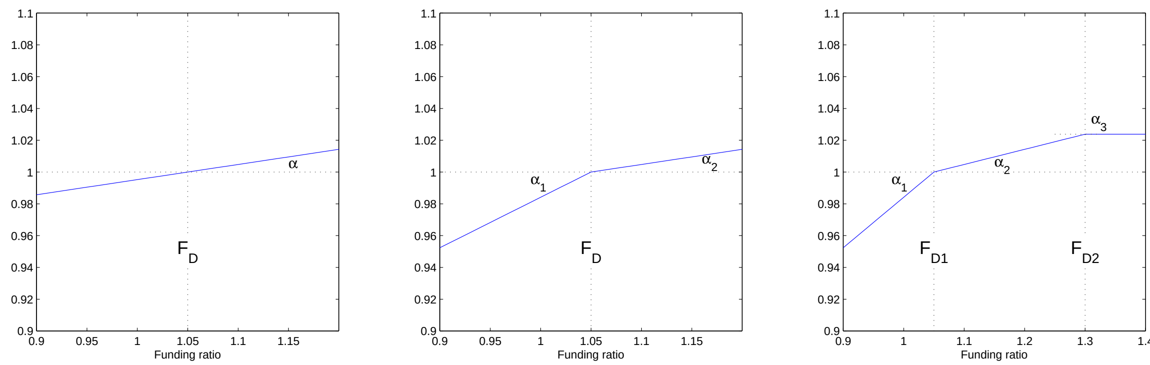
with $\delta(F)$ the correction factor that is multiplied with the entitlements to obtain the new entitlements after adjustment. The "single kink" pension contract visualized by figure 19b has the following mathematical notation:

$$\delta(F) = \begin{cases} 1 + \alpha_1(\frac{F}{F_D} - 1) & F \leq F_D \\ 1 + \alpha_2(\frac{F}{F_D} - 1) & F > F_D \end{cases}. \tag{4.21}$$

The Dutch staffel contract visualized in figure 19c has two breakpoints. The mathematical notation for this contract is given by

$$\delta(F) = \begin{cases} 1 + \alpha_1(\frac{F}{F_{D1}} - 1) & F \leq F_{D1} \\ 1 + \alpha_2(\frac{F}{F_{D1}} - 1) & F_{D1} < F \leq F_{D2} \\ 1 + \alpha_2(\frac{F_{D2}}{F_{D1}} - 1) + \alpha_3(\frac{F}{F_D} - 1) & F > F_{D2} \end{cases}. \tag{4.22}$$

(a) Linear contract | (b) Single kink contract | (c) Dutch Staffel contract



(a) $\alpha = 0.1,\ F_D = 1.05$

(b) $\alpha_1 = 0.33333,\ \alpha_2 = 0.1,$ $F_D = 1.05$

(c) $\alpha_1 = 0.33333,\ \alpha_2 = 0.1,$ $\alpha_3 = 0.0,$ $F_D = 1.05\ ,\ F_D = 1.30$

Figure 19: Three different types of pension contracts: linear, single kink and Dutch staffel

4.2.1 Value transfers in a linear contract

Here an analysis will be made about the statement that "in a linear pension contract, a change of the equity allocation w should not lead to value transfers between generations". Bilsen and Bovenberg (2014) derives an expression for the economic value of pension entitlements in which the equity allocation w does not appear. This result is derived under strict mathematical assumptions. In this section the statement will be studied in a setting in which these strict mathematical assumptions are weakened. A more realistic pension fund model is used, namely the ALM model of PGGM. This setup will be very relevant because people working in the pension industry or studying pensions in general think that the statement holds under all conditions.

The ALM model of PGGM will also be used throughout chapter 5. A more detailed overview of the pension fund model can be found in appendix B. The analysis here will be performed under a Black Scholes economy, this economy has only one risk factor. When the equity allocation w is put to 0, the model is identical to the model used in chapter 3, where risk was absent. The parameters of the Black Scholes economy are chosen such that the risk-free rate $r^f = 3\%$ and the volatility of the stock is $\sigma = 20\%$. Inflation π is set to 0.

For now the population of the pension fund is taken the same as in chapter 3. This means that people start working at an age of 25 and die at an age of 85. The retirement age is 65. Participants receive a degressive accrual and earn a flat wage which is the same for all generations. When a participant spend his or her full career (40 years) in the fund he or she has accrued an replacement income of $\gamma = 0.8\%$ of (average) wage.

The simulation is done with $M = 10.000$ trajectories, 25 years in to the future. At the end of the simulation horizon the surplus or deficit is divided among all participants in the fund, proportionally to the amount of entitlements they own. This is referred to as the closing rule.

The effect of changing the equity allocation on economic value In figure 20, 21 and 22 the generational account of the default equity allocation $w = 50\%$ is compared to the generational account of the participants in case the equity allocation is increased to $w = 70\%$ or decreased to $w = 30\%$. Thus the generational account under the changed equity allocation is subtracted by the default equity allocation ($w = 50\%$).

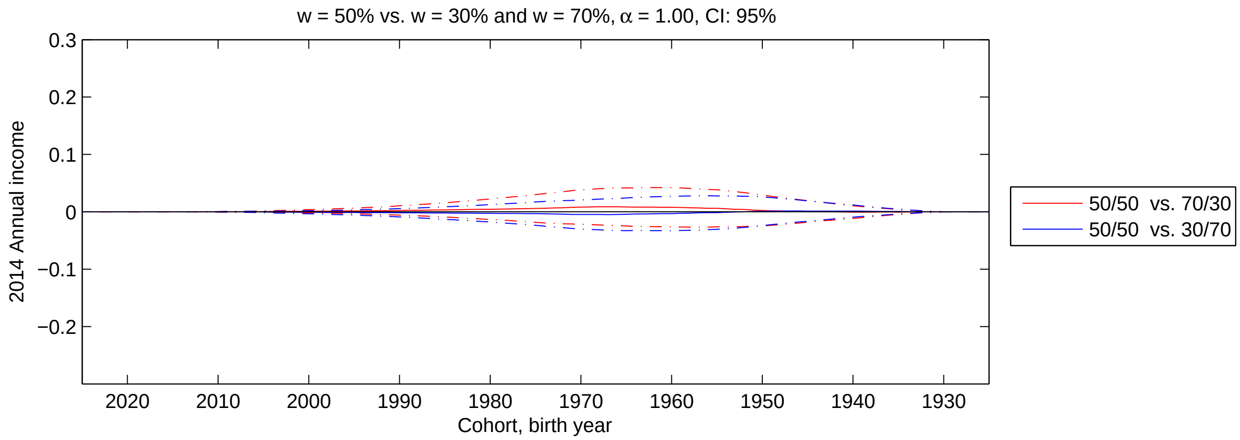


Figure 20: Difference in generational account (GA) if the equity allocation w of the pension fund is changed. The lines indicate the difference compared to the default equity allocation of $w = 50\%$. $\text{GA}(w = 70\%) - \text{GA}(w = 50\%)$ & $\text{GA}(w = 30\%) - \text{GA}(w = 50\%)$. The linear contract is specified by $\alpha = 1$, $F_D = 1$.

Figure 20 shows the differences of the generational accounts in a linear contract in which $\alpha = 1$, the desired funding ratio is also one ($F_D = 1$). The red line gives the change in economic value for all generations if the equity allocation is increased from 50% stocks to 70% stocks. The blue line gives the change in the economic value for all participants if the equity allocation is lowered from 50% stocks to 30% stocks. Both the red and the blue line indicate that there is a negligible value transfer of at maximum less than 1% of an annual income. So far the statement seems to hold, even in a model that is not extensively bounded by mathematical assumptions.

Figure 21 shows a linear contract in which $\alpha = 0.33$, the desired funding ratio is still kept at one ($F_D = 1$). The red line gives the change in generational accounts if the equity allocation is increased. Again, the blue line gives the change in the generational accounts if the equity allocation is decreased to 30% stocks. The red line shows a small value transfer to older participants. The value transfer is 3% at its maximum. The young lose some, at maximum this loss is around 2% of an annual income. For the blue line, which indicates a change to less equity and more investment in bonds, the gain for the young is around 1% and the loss for the elderly around 1.5%.

The value transfers in case the smoothing period is 3, $\alpha = 0.33$ are not 0 but are still very small. If the value transfers are compared to the value transfers seen in chapter 3 they can be considered irrelevant.

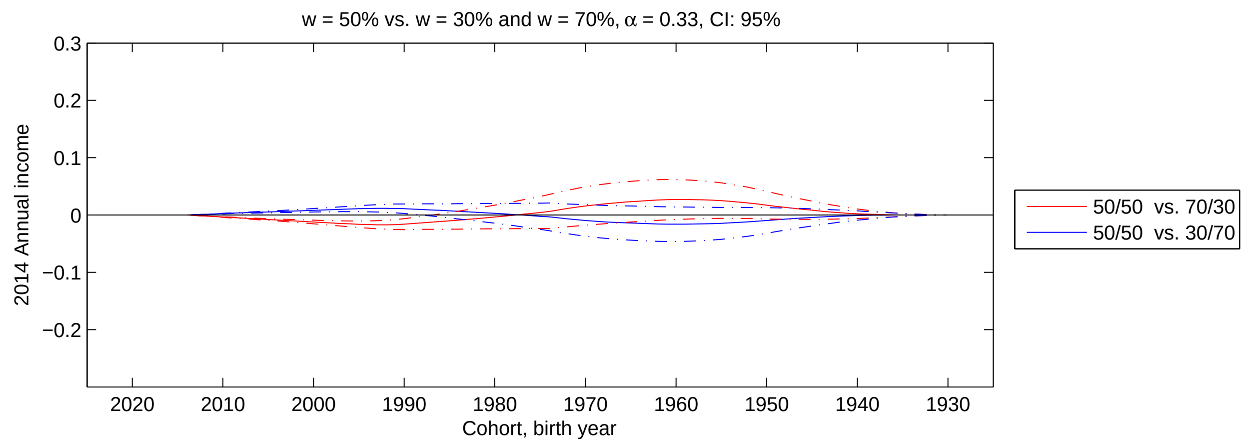


Figure 21: Difference in generational account (GA) if the equity allocation w of the pension fund is changed. The lines indicate the difference compared to the default equity allocation of $w = 50\%$. GA($w = 70\%$) - GA($w = 50\%$) & GA($w = 30\%$) - GA($w = 50\%$). The linear contract is specified by $\alpha = 0.33$, $F_D = 1$.

Figure 22 again gives the change in the generational account relative to the default situation in which the pension fund invests $w = 50\%$ in equity. In this linear pension contract the smoothing period is 5 years, which means $\alpha = 0.20$. The red line indicates the difference if the equity allocation is increased from $w = 50\%$ to $w = 70\%$. The same direction of the transfers are observed as in figure 21.

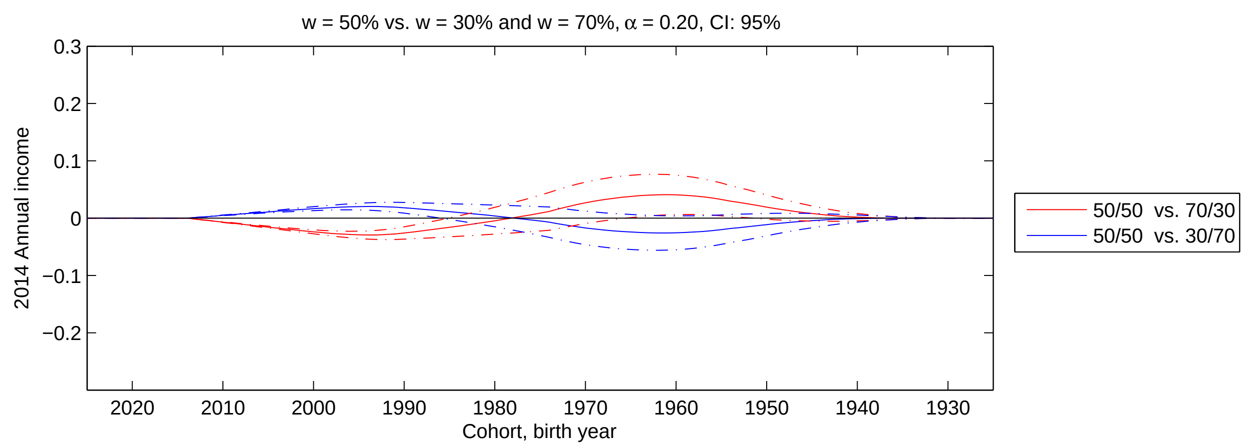


Figure 22: Difference in generational account (GA) if the equity allocation w of the pension fund is changed. The lines indicate the difference compared to the default equity allocation of $w = 50\%$. GA($w = 70\%$) - GA($w = 50\%$) & GA($w = 30\%$) - GA($w = 50\%$). The linear contract is specified by $\alpha = 0.20$, $F_D = 1$.

This time they are somewhat larger in size. The red line indicates an advantage for the elderly of max 4% of an annual income. The young lose at most 3% when the equity allocation is increased. The blue line also shows similar directions of the value transfers as figure 21. The gain for the younger in the fund is around 2%, the loss for the elderly less than 3%. Again the differences are small compared to the value transfers seen in chapter 3. When α becomes smaller the value transfers seems to grow in size, a reason for this will be given at the end of this chapter.

It has to be noted that when a pension contract is taken which has a smoothing period that is longer than five years (e.g. 10 years) and the equity allocation w is varied more extreme than from $w = 50\%$ to $w = 70\%$ or $w = 30\%$, much larger value transfers can be observed, even in a linear pension contract.

Conclusions, value transfers in linear contracts From the analysis above it is clear that value transfers are not exact zero when the equity allocation is changed in a pension fund with a linear pension contract. The pension fund model used here is a simplified version of a pension model that is actually used at PGGM. It is not built according to strict mathematical assumptions but built based on how pension funds truly work. Considering this and the fact that the value transfers are very small compared to the value transfers found in chapter 3, it seems very reasonable to conclude that the statement may still be stated. Nevertheless, people working in the pension industry or studying pensions have to be aware of the fact that formally the statement requires some strict mathematical nuances.

4.2.2 Value transfers in a nonlinear contract

It is also interesting to show the results when the equity allocation w is changed as before but now if the pension contract is nonlinear. In order to do this a "single kink" pension contract is constructed that is a mixture of the contract in figure 20 ($\alpha = 1$) and figure 22 ($\alpha = 0.20$). The first contract setting that will be studied has smoothing parameters $\alpha_1 = 1$ and $\alpha_2 = 0.20$ and desired funding ratio $F_D = 1$. The second contract setting is exactly opposite of the first. It has smoothing parameters $\alpha_1 = 0.2$ and $\alpha_2 = 1$ and the same desired funding ratio.

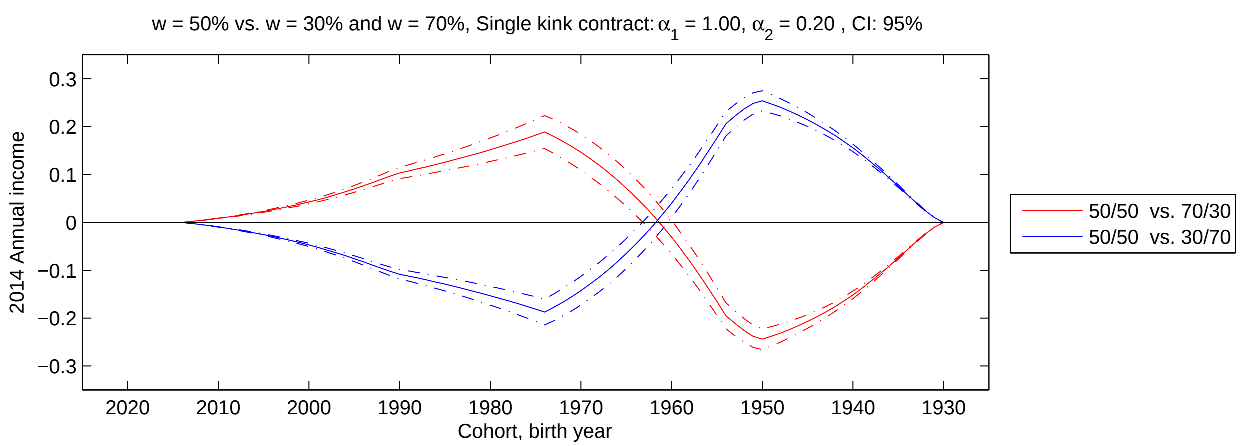


Figure 23: Difference in generational account if the equity allocation w of the pension fund is changed for a nonlinear pension contract: "single kink'. The lines indicate the difference compared to the default equity allocation of $w = 50\%$. GA($w = 70\%$) - GA($w = 50\%$) & GA($w = 30\%$) - GA($w = 50\%$). The single kink contract is specified by $\alpha_1 = 1$, $\alpha_2 = 0.20$, $F_D = 1$.

Figure 23 shows difference in the generational accounts if the equity allocation is changed. The red and blue line have the same as meaning as before. First of all it can be seen that there are sizable value transfers when the equity allocation is changed in a nonlinear pension contract. This can be explained from the fact that now an option structure is present within the pension fund.

If the funding ratio is below the desired level of 100%, cuts are implemented right away such that after the adjustment of entitlements the funding ratio is restored to the desired level. If the funding ratio is above this desired level indexation is not given that fast. This will mean that the elderly in the fund do not profit as much from the upside of the financial market. They do lose in case of bad returns. By this the chances for positive indexation for the young increase if the equity allocation is raised because on the long run the fund has a more healthy funding ratio in expectation. The young benefit at the cost of the elderly.

The blue line in figure 23 show the opposite effect of the red line. The call option that the young are said to hold drops in value if the equity allocation is lowered. This is due to the fact that volatility decreases as the equity allocation is lowered. The old gain from lowering the equity allocation from 50% to 30%, the bad states of the world are on average less bad.

Figure 24 shows the results if the pension fund has the opposite kind of contract. This time smoothing parameters are $\alpha_1 = 0.2$ and $\alpha_2 = 1$. This implies that when the funding ratio is below the desired level, cuts are not made immediately. In the good state of the economy indexation is given right away. If the equity allocation is raised the older participants of the fund profit. For the young the opposite holds, if the fund takes on more risk their chances on indexation decrease because all the surpluses are given to the elderly right away.

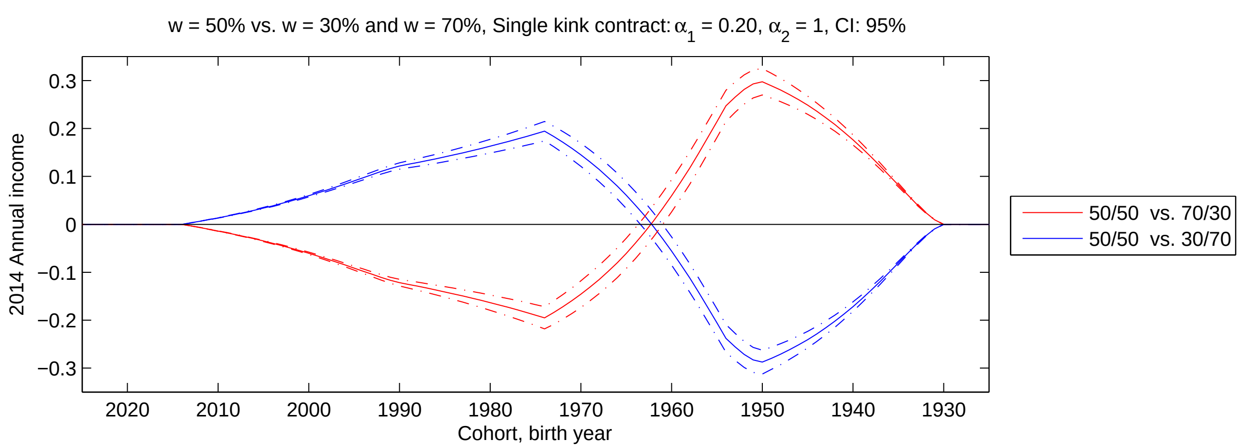


Figure 24: Difference in generational account if the equity allocation w of the pension fund is changed for a nonlinear pension contract: "single kink'. The lines indicate the difference compared to the default equity allocation of $w = 50\%$. GA($w = 70\%$) - GA($w = 50\%$) & GA($w = 30\%$) - GA($w = 50\%$). The single kink contract is specified by $\alpha_1 = 0.20$, $\alpha_2 = 1$, $F_D = 1$.

The blue line in figure 24 indicates opposite value transfers as the red line. The reasoning is the same as before. The young lose less in the case of a lower equity allocation this results in a profit compared to 50% equity. The older participants hold a call which drops in value if less volatility is taken on by the fund.

Conclusions, value transfers in nonlinear contracts We find sizable value transfers between generations when the equity allocation is changed in a pension fund with a nonlinear pension contract. The direction of the transfers that occur is very well explainable.

4.2.3 Discussion and conclusions

As explained above, the statement that "a change of the equity allocation in a pension fund with a linear contract should not cause value transfers between generations", still seems valid. Small value transfers are observed which can be explained by the fact that the used pension fund model does not meet the necessary assumptions, because the pension model is built for practical purposes. In a nonlinear pension contract sizable value transfers between generations do occur.

It is interesting to come up with the possible causes in the pension fund model, that are responsible for the matter that the statement is not exactly met. These possible causes could serve as a basis for future research on this statement.

A continuous time model The pension fund model used is programmed in discrete time. The expression in (Bilsen and Bovenberg, 2014) is stated in continuous time. This discrepancy is expected to be responsible for a sizable part of the transfers that occur. Said short, here an error arises.

Future research about this topic should preferably take place in a model that is totally specified in continuous time, such that it meets the requirements of Bilsen and Bovenberg (2014). Parts of the model can than be adapted to match a more realistic fund, such as used in this section, to see what the isolated effects are of weakening assumptions.

Adjustment mechanism suited for lognormal evolution of the funding ratio In the Black Scholes economy assets prices follow a log normal distribution. A pension fund invests in those assets. For this reason the funding ratio also evolves according to a lognormal distribution. The linear adjustment mechanism given in equation 4.20 is not suited well for adapting a funding ratio that evolves lognormal. As a result the linear mechanism adjusts the upper quantile of the funding ratios (over the scenarios) more than the lower quantile. Hereby the mean of the funding ratio over all scenarios drops. The result is that on average (mean over the scenarios) a pension fund is underfunded. From section 3.2 we know that this causes a value transfer from young to old.

A loglinear adjustment mechanism, like

$$\delta(F) = \alpha(\ln(F) - \ln(F_D)) \tag{4.23}$$

would be better suitable to steer a funding ratio that evolves according to a lognormal distribution. Unfortunately this provided no solution in the pension fund model of PGGM used here, the rest of the model could not be adapted to this. Next to that in reality Dutch pension funds do not adapt their entitlements with a logarithmic adjustment mechanism.

Closing rule Because the method of simulation requires intensive calculation power of a computer, the time horizon is finite. Because it is practically impossible to simulate far into the future (25 years ahead with 10.000 scenarios already takes around 25 minutes) a closing rule is required.

This closing rule divides the deficits and surpluses among the participants at the end of the simulation horizon. As a result of this, the participants that join the fund in the last year of the simulation are not able to fully smooth a deviation of the funding ratio from the desired funding ratio. So for certain groups of participants the smoothing period set by the parameter α does not apply. This also brings in an error.

As a solution to this, several alternative closing rules could be studied to see what the effects are of a closing rule. This was not done here due to a lack of time and the ability to perform radical changes to the PGGM ALM model. This would also be an interesting subject for future research.

Smaller values of alpha indicate larger transfers In the result above about the linear contracts it was observed that the value transfers gained size for smaller values of α. This can be explained by the following reasoning. Given the fact that an error is made, a smaller value of α brings in, ceteris paribus, a greater deviation of the funding ratio from the desired funding ratio F_D. The absolute value of the error that is made soars. It explains why greater generational transfers are observed for contracts that have a smaller value of α (a longer smoothing period).

Overall conclusion, varying the equity allocation in a linear pension contract As stated earlier, the value transfers due to varying the equity allocation in linear pension contracts are small compared to value transfers that naturally occur in the Dutch second pillar pension schemes (e.g. situations studied in chapter 3). As a consequence deterministic calculations in linear pension contracts would sure be able to give approximate similar results as the results that can be obtained from a linear contract under a stochastic world.

An important part of the conclusion is that often people working in or analyzing the pension industry have heard about the statement and see it as applicable to all situations without the need to nuance the mathematical assumptions for the statement. The results above indicate that the statement approximately holds in a general pension fund model, but a strong framework is needed to let the statement holds exact. Even when the pension fund model is not bounded by strict assumptions.

Future research on this topic will preferably take place in a pension fund model that is designed to study value transfers due to varying the equity allocation. It is recommended to be able to begin simple and extend the model further to a more realistic pension fund model, and evaluate the statement along the way. This proved to be a limitation of the PGGM ALM model.

5 Economic value in nonlinear pension contracts

In this section, again the three settings in which the pension contributions do not match the economic value of the new accrual will be studied. The first is the situation in which the discounting method of the contribution differs from the discounting method of the funding ratio. The second setting is when people participate in a fund with a surplus or a deficit. Third and last is the uniform accrual and contribution system.

Here the ALM model of PGGM is used. This time the study that is performed will differ from the study in chapter 3 on three parts. The three differences can be grouped as:

The economic model The model for the economy will not be deterministic as in chapter 3 but stochastic, introducing volatile stocks and bonds (term structures) for the pension fund to invest in. The specific model that will be used is the KNW-capital market model by Koijen et al. (2010). As a more simple illustration in section 5.1.1 also the Black Scholes model will be used.

The pension fund Chapter 3 only studied a linear pension contract because a deterministic economic model was assumed. In the current chapter, a richer class of adjustment mechanisms can be used to adapt entitlements of participants according to the solvency position of the fund, the funding ratio. This is because now also the stochastic behavior of the economy is modeled.

The participants of the fund Mortality will not be bounded by extensive assumptions like a fixed age at which participants die for sure as in chapter 3.

The economic model The economic model used in this section will be the KNW-capital market model developed by Koijen et al. (2010). This model is advised by the commission parameters of the Dutch government (Langejan et al., 2014) and has a wide variety of applications within the pension industry. It can for instance be used to make estimations about risk and the feasibility of ambitions but also for ALM studies. The KNW model describes the stock and bond market (from the bond market a term structure is derived) as well as inflation. The for Dutch pension funds mandatory parameterization of the KNW model is also prescribed by the commission parameters (Langejan et al., 2014).

The scenarios used in the simulations are obtained from Tilburg Finance Tool[10] (TFT). In this software scenario's of a KNW-capital market model can be simulated with a chosen parameterization[11]. TFT allows a user to export risk neutral scenarios such that those scenarios can be used for the market valuation of pension entitlements (in section 4.1 and section 4.1.2 risk neutral valuation and simulation where explained). The default KNW scenario set has a term structure that corresponds to an economy that is in equilibrium. This means that the state variables X_1 and X_2 have a value of zero at start (2014).

The Central Planning Bureau of the Netherlands studied the KNW model[12] and calibrated it on Dutch data. The study (Draper, 2014) found that the model is appropriate to evaluate derivative

[10]Free download: `https://www.tilburguniversity.edu/webwijs/files/center/werker/TilburgFinanceTool/`

[11]The scenario's do not incorporate the UFR correction as proposed by Langejan et al. (2014)

[12]A third Netspar student of the Tilburg University is studying model and parameters sensitivities at PGGM, the findings can be read in the master thesis Teeuwen (2014).

products. Draper (2014) gives a summary of the model, along to his findings a brief summary is given here. The KNW model has four sources of uncertainty. The first is uncertainty about the real interest rate. Second, uncertainty of the instantaneous expected inflation. Third, unexpected inflation. Last and fourth, stochastic stock returns. Note that the Black Scholes model that was illustrated in section 4.1 only has one source of uncertainty, the stock. The KNW model also has a nominal discount factor which can be used as one divided by the numéraire $\frac{1}{MMA}$.

The pension fund model The model for the pension fund used in this section is a simplified version of the actual model that PGGM uses to study the pension fund of their clients, like PfZW. It is simplified in the sense of costs for the fund and the premium that is collected with the participants. All costs (like operation and administration costs) are set to zero. Normally the premium is rounded and can be fixed for a period of time. In the version of the fund used in this thesis the premium that the participants pay is set equal to cost covering premium on fund level. This cost covering premium can be calculated in many ways. In the default situation the nominal risk-free term structure is used to determine the value of new accrual. In section 5.1 real expected return is also used for this. In the model there is no additional premium on the contribution for solvency requirements.

Normally a pension fund has a complex mechanism that can cut or raise pension entitlements. These complex mechanisms take into account catch-up adjustments (”inhaalindexatie” in Dutch) and are not only a function of the funding ratio but also other parameters of the fund. In the fund used in this section, these more complex mechanisms are replaced by one of the three contracts given in section 4.2 and visualized in figure 19. Thus there is one linear contract (figure 19a), and two nonlinear contracts, the ”single kink” contract (figure 19b) and the ”Dutch staffel” contract (figure 19c). In appendix B a chronological description is given of how the fund exactly works.

The participants of the fund The ALM model of PGGM, can handle diverse settings of the member base (population of the fund). The characteristics of the populations that can be varied are for instance size (also per birth year), survival probability, age at which one starts to work and age at which retirement begins. But also wage, wage inflation and accrual needed to be specified.

In this chapter a default participant population is used. This default population will consist of 100 participants per cohort (generation, same birth year). All cohorts will have the same survival probabilities, which are equal to estimate of the real Dutch 2014 survival probability of men made by the Dutch Actuarial Society (Actuarieel Genootschap, 2012). The age at which participants start working is set to $a_W = 25$, the age at which they retire is set to $a_R = 65$. Wage inflation is also taken into account. From the past until 2014 a fixed percentage per year is assumed, after 2014 the economic model used (e.g. KNW-capital market model) models wage inflation further.

Below the three settings will be analyzed under which the contribution to the pension fund is not equal to the economic value of new accrual.

5.1 Discounting method of the contribution differs from the discounting method of the funding ratio

In this section again a situation is studied in which the contribution to a pension fund deviates from the economic value of new accrual. This will, as before, be analyzed by looking at the generational accounts of the cohorts in the pension fund.

As explained in section 3.1, during the writing of this thesis a new FTK (Financial Assessment Framework) was adopted. This nFTK allows pension funds to calculate the contribution by choosing out of two options.

> *Real Expected return* Pension fund are allowed use real expected return in order to determine the premium rate. This method was explained in section 3.1 but not studied because there the use of nominal expected return was studied.
>
> *10 year average of term structure plus premium* Pension fund are also allowed to chose a 10 year average of the risk-free term structure plus a premium for equity capital in order to determine the premium rate for their participants.

The Central Planning Bureau (CPB) of the Netherlands has studied the effects of both methods (Lever and Bonenkamp, 2014). They find that an average Dutch pension fund that uses one of the two above mentioned options, will have a discount factor that is on average 1% below the risk-free term structure. Due to this, the premium rate could rise in comparison with the premium rate that is calculated by using the risk-free term structure. They find that if the term structure is matched with the current observed low interest rates, the use of real expected return could lead to somewhat lower premium rates. The report (Lever and Bonenkamp, 2014) also notes that because of the recent reduction of the maximal annual accrual rate (Witteveen kader) the premium rates do not rise.

The remainder of this section will focus on the effects of calculating the premium by discounting against real expected return μ_R. The results are applicable to a situation in which the use of real expected return is not reversed in future. Next to that also the calculations do not incorporate the fact that in future a new commission parameters can adapt parameters such that value transfers will eventually change because of this.

Real expected return Equation 3.12 gives the formula for the real expected return of a portfolio with an asset allocation w and bond allocation $(1-w)$. μ will be taken as the maximum allowed expected return on stocks as set by the commission parameters 2014 (Langejan et al., 2014), $\mu = 7\%$. Inflation π will taken 2%. What the expected return on bonds should be and how this must be estimated is still not decided by the Dutch government. In equation 3.12 the bond return is given by r^f but in reality, and also in the KNW model, there is no flat term structure. Next to that the term structure varies per year. The formula for real expected return is

$$\mu_R(w) = (r^f + w\lambda) - \pi = (1-w)r^f + w\mu - \pi. \tag{3.12}$$

After some inquiries at PGGM and at the Dutch ministry of Social affairs and employment it became clear that a temporary solution is used until the law (details of nFTK) is finished in detail. It was argued that for new accrual a pension fund would approximately invest in bonds that have a duration that matches the duration of the new accrual. For this reason the bond return is estimated by the interest rate that belongs to the maturity of the new accrual. Equation 3.12 will be written as

$$\mu_R(w) = (r^f_{D_\theta} + w\lambda) - \pi = (1-w)r^f_{D_\theta} + w\mu - \pi, \tag{5.1}$$

with $r^f_{D_\theta}$ the risk-free rate that holds for a maturity D_θ in the future. D_θ is an estimate of the duration of the new accrual in the pension fund.

As it can be seen in figure 4 (section 3.1) by looking at the red line (premium discounted by real return) the premium rate does not always have to be less than the premium rate that is calculated with the use of the risk-free rate. With an equity allocation of $w = 25\%$ the premium rate will be higher than a premium rate calculated with the use of the risk-free term structure. In this case too much premium flows into the fund, which will increase the funding ratio, and from this the mechanism that cuts or raises entitlements as a function of the funding ratio will start to allocate the surplus of the fund to all the participants.

First the analysis is done in a Black and Scholes (1973) economy. Next the analysis is done in the KNW-capital market model by Koijen et al. (2010). In both economic worlds the generational accounts of discounting against expected real return will be compared with the generational accounts of discounting against the risk-free term structure. In this case the calculation method of the premium rate and the liabilities in the funding ratio are the same. The difference that is obtained by the subtraction $(\text{GA}(\mu_R) - \text{GA}(r^f))$ gives the pure effects that appear by different methods of discounting, per pension contract.

5.1.1 Impact of different options to set the discount rate, Black Scholes with inflation

The Black Scholes model has one source of uncertainty which is the return of the stock. Next to that it has a flat term structure with interest rate r^f and it can have flat inflation π. For the analysis here the risk-free rate is set to $r^f = 3\%$. The inflation is set to $\pi = 2\%$. The volatility of the stock is set to $\sigma = 20\%$. The default population of the fund (see earlier description) will be given a 2% wage inflation per year until 2014, from then on the 2% inflation of the Black Scholes model will be applied to the wages. The participants of the fund will receive a degressive accrual.

The balance case: a fund with $w = 50\%$ equity allocation In case a pension fund operates in the Black Scholes economy described above ($r^f = 3\%$, $\sigma = 20\%$ and $\pi = 2\%$) and the return on stocks is taken $\mu = 7\%$ (commission parameters 2014) the risk premium is given by

$$\lambda = \mu - r^f = 0.07 - 0.03 = 0.04. \tag{5.2}$$

If the fund has an equity allocation of $w = 50\%$ the real expected return on the portfolio is given by

$$\mu_R(w) = r^f + w\lambda - \pi \tag{5.3}$$
$$\mu_R(0.5) = 0.03 + 0.5 \cdot 0.04 - 0.02 = 0.03. \tag{5.4}$$

In this case discounting against the real expected portfolio return $\mu_R(0.5) = 0.03$ is the same as discounting against the flat risk-free term structure $r^f = 0.03$. The generational effects are zero for all generations, because both discounting methods leave the same calculation of the premium rate, no differences arise.

The case of a too high discount factor: a fund with $w = 75\%$ equity allocation If a pension fund chooses an equity allocation of $w = 75\%$ in a Black Scholes world the real expected return will deviate from the risk-free rate. This will mean that the real expected return now will be

$$\mu_R(0.75) = 0.03 + 0.75 \cdot 0.04 - 0.02 = 0.04. \tag{5.5}$$

In this case $\mu_R(0.75) > r^f$ which will imply that the premium that is flowing in to the pension fund is too low (around 25% too low, when output of simulation is analyzed) in order to keep the funding ratio stable. The new accrual is sold to participants too cheap, which will result in underfunding. The mechanisms that can cut or raise pension entitlements (pension contracts) will start cutting entitlements in order to dissolve the deficit. Figure 25 shows the differences in the generational account between discounting with the risk-free term structure, as is used to calculate the funding ratio, and discounting against real expected portfolio return.

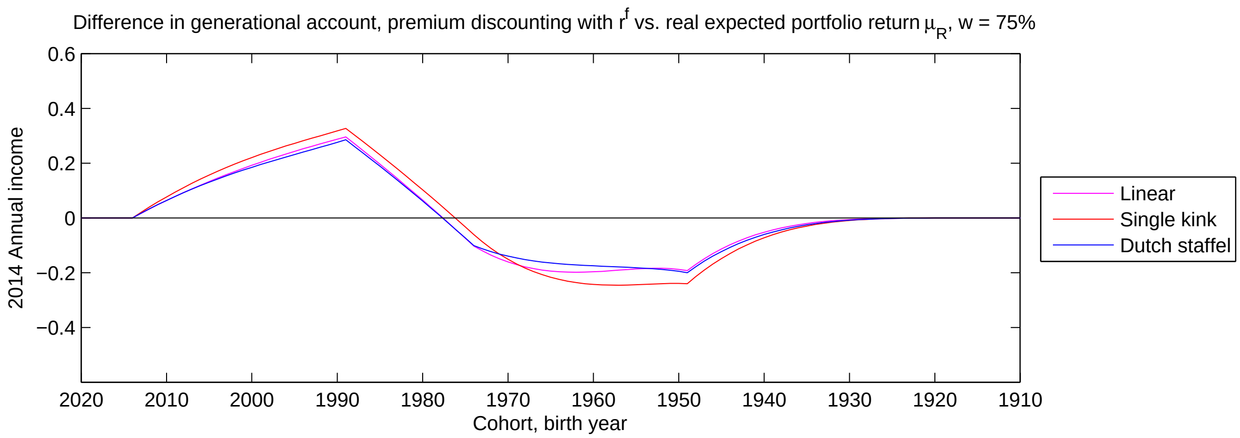


Figure 25: Difference in generational accounts, premium calculated by discounted with $\mu_R(w = 75\%)$ minus discounted with r^f. Black Scholes model with flat inflation. Three different pension contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

From figure 25 the following can be concluded:

- **The joiners** During the simulation horizon of 25 years people born between 1989 and 2014 will turn 25 years old (a_W), they thus join the fund. Clearly they benefit from paying in too little contribution. They do not own a lot of entitlements that can be cut. At most the benefit for a person born in 1989 is around 35% of an 2014 annual wage which is sizable.
- **The working** The people born in year 1990 and a few years earlier also benefit from the fact that they pay less contribution than the economic value of their accrual. They do not own allot of entitlements that can be cut, so the negative consequences are minor for them.
- **The older working** The older people in the fund that are sill working working, benefit from the fact that they can accrue cheap, but lose from the fact that they own a lot of entitlements that are cut. In general they lose more from the cutting of entitlements than they gain from a low premium rate. In all three contracts they lose around 20% of a 2014 annual income.

- **The retired** The retired at the beginning of the simulation in 2014 lose because they do not profit from paying in too little premium because they do not pay premium anymore. They own a lot of entitlements which are cut. People born earlier than 1949 lose less because they have already received a part of their benefit and thus own less entitlements that can be cut.

Next to the effects of inappropriate discounting on the generations, it is also interesting to look at the differences between the three types of pension contracts that are used. By looking at figure 25 it can be seen that the differences between the three lines are very small. The maximum difference is around 5% of a 2014 annual income. This can be explained by the fact that especially the nonlinear contracts (single kink and Dutch staffel) behave similar in case of an funding ratio $F < F_{D2}$.

The case of a too low discount factor: a fund with $w = 25\%$ equity allocation If a pension fund that operates in the Black Scholes world as described above, allocates 25% of its equity to stocks the real expected return will become:

$$\mu_R(0.25) = 0.03 + 0.25 \cdot 0.04 - 0.02 = 0.02, \tag{5.6}$$

which is less than the risk free rate. This will result in exact the opposite situation as $w = 75\%$ (above). Now too much contribution will flow in to the pension fund. As a result the funding ratio will grow and the surplus will be divided among the participants. Figure 26 gives the difference of the generational accounts between discounting against the risk-free term structure and discounting with expected portfolio return $\mu_R(0.25) = 0.02$.

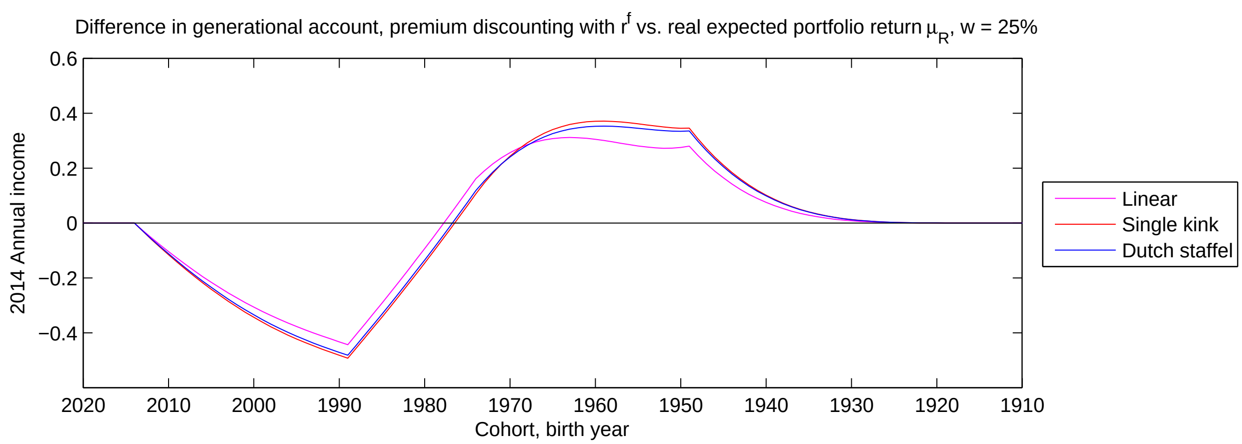


Figure 26: Difference in generational accounts, premium calculated by discounted with $\mu_R(w = 25\%)$ minus discounted with r^f. Black Scholes model with flat inflation. Three different contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

The effects of discounting against the real expected return of a portfolio that has an asset allocation of $w = 25\%$ can be summarized as follows:

- **The joiners** The participants that join the fund during the simulation horizon pay too much for their new accrual. This causes a surplus in the fund, which is divided among all participants. The entitlements of the joiners are raised but not enough to compensate the too high contribution. The more years premium paid, the higher the loss.

- **The working** The generations that just started working (born in 1998 and earlier) also experience that they pay too much contribution for the economic value of their accrual. Entitlements are raised because the fund has a surplus, by this the more entitlements someone owns, the more indexation he or she accrues in absolute value. For this reason the loss for older generations will decline and turn in to a benefit for generations born after 1975.

- **The older working** The generations born after 1975 lose from paying in too much premium, but they own a lot of entitlements that are raised. The absolute value they gain from indexation (raising entitlements) is larger than the loss of paying too much premium.

- **The retired** The retired at the beginning of the simulation in 2014 (generation born in 1949) benefit from the surplus in the fund. They never paid too much for their accrual and only experience that their entitlements are raised. For retired persons born even before 1949 the benefit is less, they own less entitlements because a part of their pension is already received by them.

In figure 26 a small difference is visible in the generational effects between the linear and two nonlinear contracts. The linear contract deviates by almost 8% of an 2014 annual income from the two other nonlinear contracts. By looking to what happens into more detail, it becomes clear that in all three the funds the funding ratio grows from the start funding ratio of 1.05 to around 1.15. In that region of the funding ratio all three the contracts are the same, they all smooth the surplus over 10 years. Only for the scenarios with higher funding ratios a difference in indexation is observed. The Dutch staffel caps the amount of positive indexation in that case. This explains the small differences.

5.1.2 Impact of different options to set the discount rate, the KNW model

The same situations as above will now be studied in the KNW-capital market model. In this model there is no flat term structure and the term structure has volatility. The default parametrization of the KNW model is taken such as prescribed by the commission parameters (Langejan et al., 2014). This also means that the state variables X_1 and X_2 are zero at the start. The participants of the fund still accrue a degressive accrual[13] and start working (participating) at an age of $a_W = 25$. They will retire at an age of $a_R = 65$. This time the assumed wage inflation until 2014 is set slightly higher to 2.6% because this matches the average inflation over simulation horizon better. From 2014 the KNW model, models the wage inflation.

As explained earlier, there is no prescribed method to set the real expected return. This is why the interest rate that holds for the duration of the new accrual (D_θ) is used. The duration of new accrual is determined by using the following formula

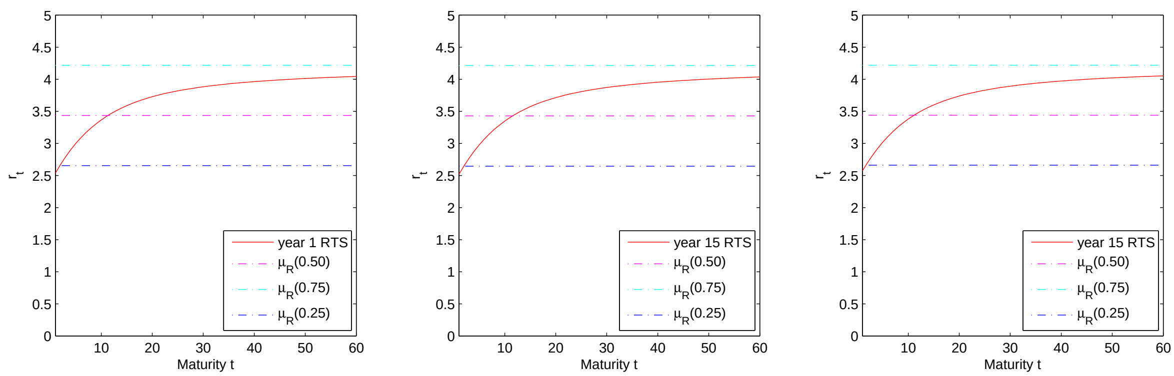
$$D_\theta = \frac{\sum_{t=1}^{n} t \cdot \frac{B_t}{(1+r_t^f)^t}}{\sum_{t=1}^{n} \frac{B_t}{(1+r_t^f)^t}}, \tag{5.7}$$

[13]This degressive accrual is determined on the basis of a flat interest rate. This is done because after some talks at PGGM it was concluded that this would match with how degressive accrual would work in reality if it were used.

with D_θ the duration of new accrual, n the number of years into the future, B_t the benefit at time t and r_t^f the interest rate that holds for maturity t. The calculation is made for the new entitlements accrued in 2014 for the default population and with r_t^f the average therm structure from KNW in 2014. The duration of new accrual D_θ is around 29 years.

It is important to gain some insight in the dynamics of the term structure as it appears in the KNW model. It is very insightful to see what the relationship is between real expected return and the term structure. Figure 27 gives the mean (over all simulations) of the risk free term structure in the 1st, 15th and 25th year of the simulation horizon. Next to the mean, also the mean real expected return is plotted as it is implied by the term structure. Equation 5.1 is used to calculate the real expected return. It can be seen that for short maturities real expected return ($w = 50\%$) is larger than the interest rate for short maturities. For higher maturities real expected return ($w = 50\%$) is less than the term structure. This will imply a higher premium rate in case new accrual is discounted against real expected return ($w = 50\%$).

(a) 1st year term structure | (b) 15th year term structure | (c) 25th year term structure



Mean term structure and mean $\mu_R(w)$ at 1 year. | Mean term structure and mean $\mu_R(w)$ at 15 year. | Mean term structure and mean $\mu_R(w)$ at 25 year.

Figure 27: Mean term structure and mean expected real portfolio return according to equation 5.1 under the objective measure $\mathbb{P}$, with duration $D_\theta = 29$. KNW capital market model with paramerization of commission parameters 2014 with state variables in equilibrium, $X_1 = 0$ and $X_2 = 0$.

A fund with $w = 50\%$ equity allocation An average pension fund in the Netherlands invests around 50% of its wealth in stocks, this is the reason why this equity allocation is studied first. In figure 28 the difference in the generational accounts is given. It is the difference in generational accounts between discounting the liabilities for the premium calculation with the risk-free term structure (similar as the funding ratio) and discounting the liabilities with real expected return.

Over time the premium rate that is calculated on the basis of discounting against the real expected return (equity allocation 50%) turns out to be higher than the premium rate when it is discounted against the risk-free term structure. In the first year on average it is around 14% higher than under risk-free term structure discounting. The effects on the generational accounts are as follows.

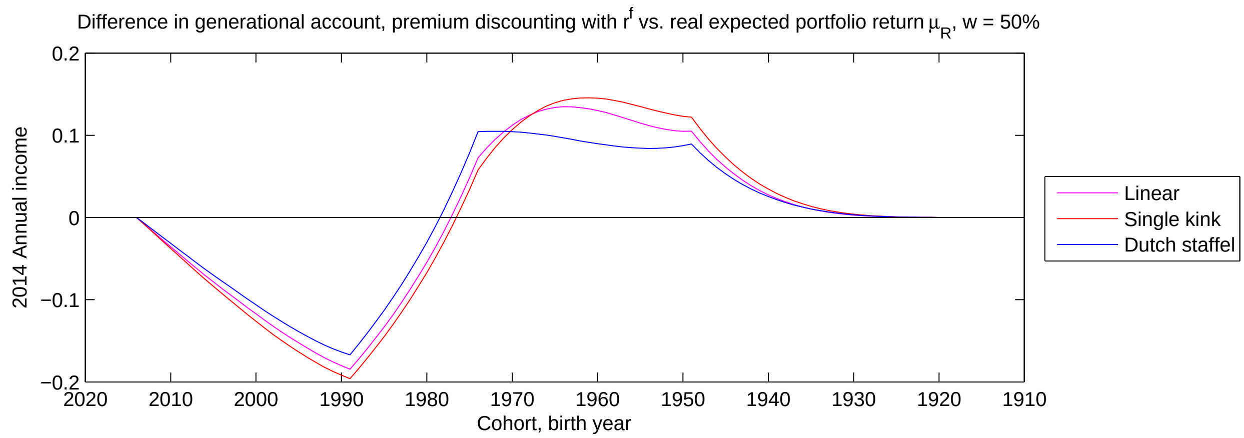


Figure 28: Difference in generational accounts, premium calculated by discounting with $\mu_R(w = 50\%)$ minus discounting with r^f. KNW capital market model. Three different pension contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

The joiners which join the fund during the simulation horizon, born between 1998 and 2014, feel a burden. They pay too much for the economic value of accrual they receive. At most they lose around 17% of a 2014 annual income. For *the young working*, born between 1975 and 1988, also loss is experienced. They pay too much for their accrual. Their entitlements are raised because a surplus in the fund originates, but they do not own that much entitlements. Overall, the raised value of entitlements is not enough to compensate the too high premium for this group. Looking at *the older working*, born before 1975, it can be seen that they feel a benefit. They own more entitlements that are raised. And this positive effect overrules the negative effects of paying too much premium. *The retired* own a lot of entitlements that can be raised. Next to that they do not pay premium anymore, for this reason they do not feel the negative consequences at all. At maximum they benefit 12% of an 2014 annual income.

This time the differences between the three types of contracts is at max around 7% of a 2014 annual income. The Dutch staffel clearly is less beneficial for the elderly. This is explained by the fact that in the last 10 - 8 years before the end of the simulation horizon there is an increasing probability of a funding ratio beyond the level of 130. In that region of the funding rate, the Dutch staffel contract is very different from the other two contracts because above a funding ratio of 130% the indexation is capped. Which would mean less raising of entitlements for the retired.

In comparison with the value transfers seen in chapter 3 the value transfers from figure 28 look small. One has to keep in mind that the simulation horizon is 25 years. If the calculation had a longer horizon the maximum gains and losses could grow somewhat bigger. Next to that, this calculation implicitly assumes that the parameters set by the commission parameters are not changed in future. In reality it is imaginable that the interest rate can change in future and that the value transfers as displayed here will change. Nevertheless, this situation, an equity allocation of $w = 50\%$, is very common in the Netherlands. This setting thus already causes value transfers between participants that can grow upto 15% to 20% at max.

A fund with $w = 25\%$ equity allocation A pension fund that has an equity allocation of $w = 25\%$ is studied. By knowing the outcome of the previous analysis ($w = 50\%$) one can already expect that the premium rate in this example deviates even more from the premium rate that would result from using the risk-free term structure. By analyzing the premium rates in the simulation output it became clear that with an equity allocation of 25%, the premium rate that was obtained by discounting against the real expected return was around 45% higher than the premium rate obtained by using the risk-free term structure (same as funding ratio). From this it is clear that the people who pay premium, pay too much compared with the benchmark case in which premium is discounted against the risk-free term structure.

Figure 29 again gives the differences in the generational accounts between discounting against the risk-free term structure and discounting against real expected return. Note that the scale of the axis is different now, three times larger than in the previous figure.

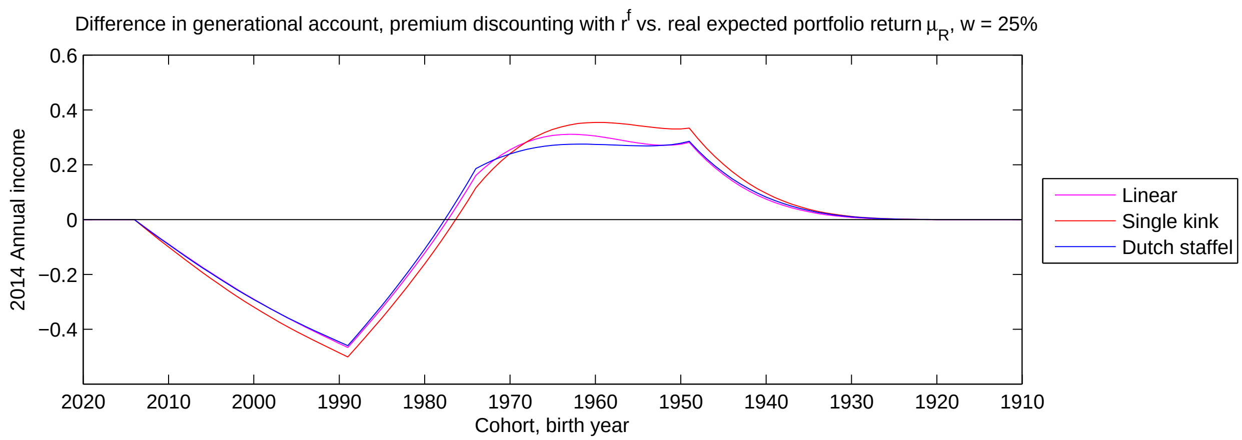


Figure 29: Difference in generational accounts, premium calculated by discounting with $\mu_R(w = 25\%)$ minus discounting with r^f. KNW capital market model. Three different contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

By looking at figure 29 the same direction of value transfers is observed as in the analysis of figure 28 only now the transfers are larger. *The joiners* lose from paying too much contribution, they create a surplus that is divided among everyone. For *the working* the same reasoning as in the earlier case applies, this time the values are larger. *The older working* own a substantial amount of entitlements that are raised due to the surplus in the fund. The gain from this outweighs the loss of paying in too much premium. *The retired* gain from the fact that their entitlements are indexed. They do not lose from a higher premium because they do not pay premiums anymore. All in all, when the equity allocation is $w = 25\%$, value transfers are sizable. This is due to a large difference between the premium and the economic value of new accrual.

This time the generational effects under the different pension contracts show some deviations. Between the three contracts the differences are maximum 8% for generations born between 1949 and 1960. The Dutch staffel contract results in the lowest benefit for the older and retired participants.

This contract limits indexation above a funding ratio of 130% which hurts the older and retired. The small difference between the linear contact and the single kink contract can be explained from the fact that in bad states of the world (low funding ratio) the single kink contracts implements cuts earlier.

A fund with $w = 75\%$ equity allocation Here the generational effects of discounting with real expected return will be studied in a fund that has an equity allocation of $w = 75\%$. By looking at figure 27 it can be expected that in this setting ($w = 75\%$) the difference between real expected return and the risk-free term structure is smaller. By looking at the premium rates in the simulation output, it can be found that the differences between the premium calculated by real expected return, do indeed not differ much from the premium rates calculated by using the risk-free term structure.

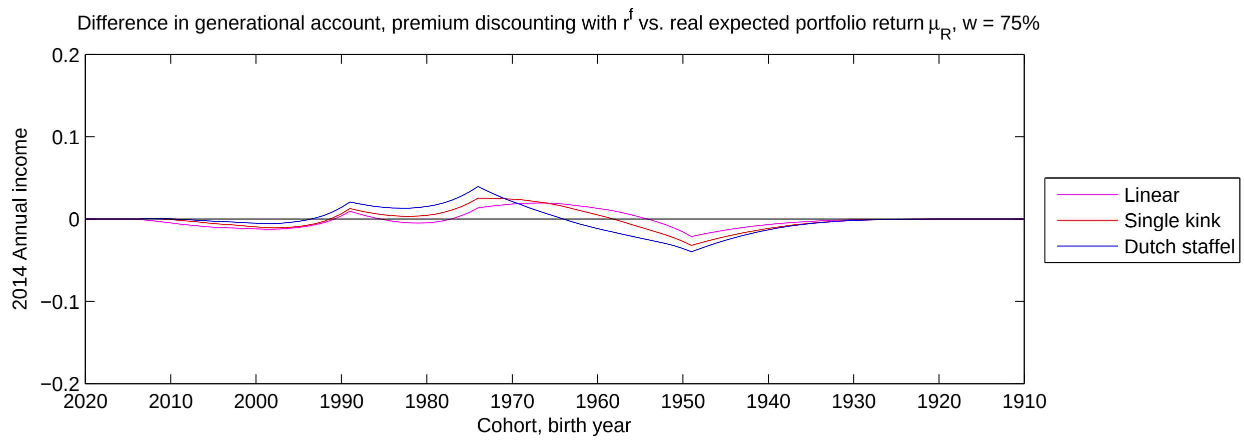


Figure 30: Difference in generational accounts, premium calculated by discounting with $\mu_R(w = 75\%)$ minus discounting with r^f. KNW capital market model. Three different pension contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

Figure 30 gives the differences in generational accounts for the participants of the fund. Again the difference is taken between the situation in which the premium rate is based on discounting against the risk-free return and determining the premium on the basis of real expected return. Note that now the scale of the axis is again smaller. The overall value transfers are very small. The positive value transfers are at max 4% for the generation born around 1974. The max loss is around 5.5% for the generation born in 1949. This generation is retired at the start of the simulation horizon, it thus never gains from paying a too little premium, it only suffers from cuts in entitlements.

Also the differences between the three contract types are minor. At max the differences are 2.5% of an annual income. Thus the generational effects of calculating the premium with real expected return ($w = 75\%$) is approximately similar for all the three pension contracts, regardless of the fact that there is a linear or a nonlinear contract used.

A fund with $w = 100\%$ equity allocation The for Dutch pension funds, extreme setting of 100% equity allocation will also be studied. Because this setting is not very realistic, the figure of the generational effects can be found in appendix A.1. This setting is only studied because here the full risk premium is incorporated and value transfers could flow into the other direction compared to previous settings (above, $w = 50\%$ and $w = 25\%$).

By looking to the output file of the simulation it became clear that the premium implied by discounting against the real expected return ($w = 100\%$) is lower than the premium implied by discounting against the risk-free term structure. Figure 36 in appendix A.1 shows the differences of the generational accounts between using the real expected return and the risk-free term structure to base the premium on. Because the premium rate implied by the real expected return is less than the premium rate implied by the risk-free term structure the value transfers flow from old to young. New accrual is purchased too cheap by the participants. The opposite reasoning of figure 28 applies to figure 36. More illustration can also be found in appendix A.1.

The differences between the three contract types are present but minor in case $w = 100\%$. The differences have the greatest size for the participants born between 1949 and 1974. For those participants the difference is at max 5%.

5.1.3 A term structure that matches the current DNB term structure better

Within the KNW capital market model it is also possible to adapt the value of state variables X_1 and X_2 at the start. In this way it is possible to let the nominal term structure of the KNW model look more like the term structure of "De Nederlandse Bank" (the Dutch national bank, DNB). This can be done by solving a set of two the equations for two randomly chosen points on the DNB term structure, for instance the interest rate for a maturity of 11 years r^f_{11} and the interest rate for a maturity of 50 years r^f_{50}. The the formula for the affine term structure in year 1 of the simulation horizon is given by

$$\begin{bmatrix} r^f_{11} \\ r^f_{50} \end{bmatrix} = \begin{bmatrix} -0.002880 & -0.003612 \\ -0.006906 & -0.005705 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \tag{5.8}$$

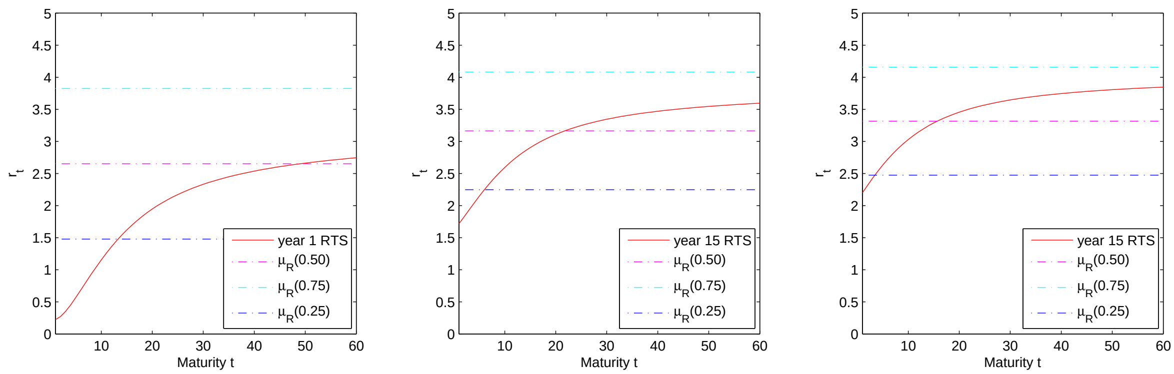
With $r^f_{11} = 1.210\%$ and $r^f_{50} = 2.841\%$ known from the DNB[14], this system can be solved which leads to

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2.019 \\ 1.432 \end{bmatrix}.$$

The term structure in the KNW model is now more adapted to the DNB term structure in the first year. In figure 31 the term structure is given again just as the real expected return in the 1st, 15th and 25th year of the simulation horizon.

[14]DNB term structure for pension funds 31-8-2014, without averaging the interest rate ("rentemiddeling"), with UFR.

(a) 1st year term structure (b) 15th year term structure (c) 25th year term structure



Mean term structure and mean $\mu_R(w)$ at 1 year. Mean term structure and mean $\mu_R(w)$ at 15 year. Mean term structure and mean $\mu_R(w)$ at 25 year.

Figure 31: Mean term structure and mean expected real portfolio return according to equation 5.1, under the objective measure $\mathbb{P}$, with duration $D_\theta = 29$. KNW capital market model with paramerization of commission parameters 2014 but with state variables $X_1 = 2.019$ and $X_2 = 1.432$.

With the new obtained term structure the previous analysis of section 5.1.2 can be performed again. Because an average Dutch pension fund has an equity allocation of 50% only this situation will be analyzed now.

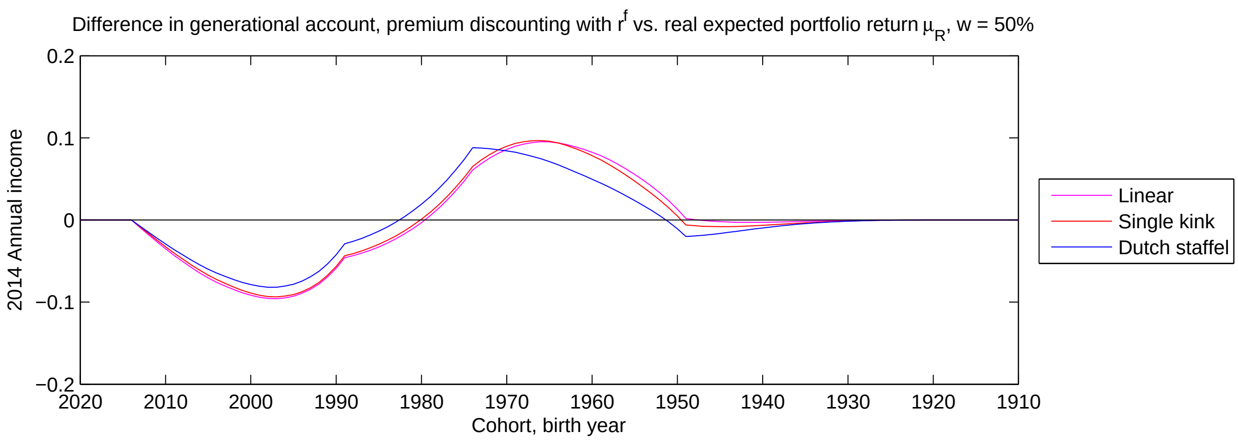


Figure 32: Difference in generational accounts, premium calculated by discounted with r^f vs. $\mu_R(w = 50\%)$. KNW capital market model. Three different pension contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

Figure 32 gives the differences in the generational accounts in case real expected return is used instead of the risk-free term structure when the premium is calculated. Again the same kind of value transfers are observed as in the previous case when the fund also had an equity allocation of 50% (see figure 28). This time the size of the value transfers is smaller. This can be explained by the fact that real expected return is now closer the the risk-free term structure.

This can also be found when figure 31 is compared to figure 27. In the first years of the simulation horizon the term structure is lower and for later simulation years on average closer to real expected return in case $w = 50\%$.

Figure 32 shows a maximum benefit of 10% of an 1014 annual income, for people born around 1965. A maximum loss is observed for generations born around 1989, the loss is also around 10% of an 2014 annual income. The KNW scenarios that were obtained from state variables that were in equilibrium showed value transfers for these generations of 14% and 19% respectively (figure 28).

Even under a KNW economy that has a term structure that is closer to the DNB term structure, we still find value transfers when pension premium is calculated on the basis of real expected return instead of the risk free term structure. Nevertheless these value transfers are smaller than the same setting with an KNW economy that starts in equilibrium. A 10% value transfer over a 25 year horizon is minor compared to the transfers seen earlier in this thesis. The currently low inflation rates that are observed in the Netherlands are not taken into account. A fixed inflation rate $\pi = 2\%$ was used in the calculation of real expected return. If low inflation levels were incorporated it is expected to observe a further reduction of the value transfers, possibly even a change of direction.

5.1.4 Conclusions: Inappropriate discounting of premium

The new FTK (still) allows pension funds to choose between options to set the discount rate for determining the required premium. These options are different from the method used to calculate the funding ratio. Sizable value transfers can occur between groups of participants when the discounting method of the premium and the funding ratio are not in line. An average Dutch pension fund has an equity allocation of 50%. When in such a fund real expected return is used as a discount rate for the calculation of premium, value transfers can range from 15% to almost 20% of a 2014 annual income for some of the groups. These value transfers flow from young to old. Later, in section 5.3 the generational effects of the uniform accrual and contribution system are analyzed. There it is also concluded that value transfers in this system flow from young to old which can result in an even larger value transfers in the same direction if the two problems are combined.

The current observed low interest rates in the Netherlands have a reducing effect on the size of the value transfers. This is explained by the fact that a low term structure yields a discount rate that is closer to real expected return when an equity allocation of 50% is in place. In this way the difference between the two methods of discounting becomes smaller. If interest rates would be higher, the equity premium λ becomes smaller if still 7% return on equity from commission parameters 2014 is used. When a 2% inflation level is used, the discount rate from real expected return will always be smaller than the discount rate from the (higher) term structure. This will result in higher premiums when real expected return is used .

Linear contracts compared with nonlinear contracts The settings studied in this section indicate that discounting with real expected return instead of the risk-free term structure has similar generational effects in all three contract types. If the probability grows that the funding ratio reaches levels for which the pension contracts are very different, more sizable differences can be observed. For the analysis above this was only the case when the equity allocation was 25%, still then the observed differences in the generational effects between the three contracts was maximal 8% of an annual income while value transfers itself could grow up to 35%.

A comparison with the results found in chapter 3, where a linear pension contract was studied in a deterministic setting, is hard to make. The analysis there was about nominal expected return. Nevertheless, the value transfers due to a different discounting method indicate a similar kind of value transfer in the case too little premium is paid (section 3.1). There are no indications that the approach used in chapter 3 (deterministic linear pension contracts) will give other results than found in this section for linear pension contracts.

Next to that, the isolated effects of a different discount method for the calculation of the premium rate, was approximately similar among all three the pension contracts. Which would mean that for this specific setting of the problem (different discounting methods) and the specification of the pension contracts, the approach of chapter 3 would also be able to give a reasonable intuition of the size and direction of the value transfers that occur in nonlinear contracts.

5.2 Participating in a fund with a surplus or deficit

In this section the situation will be studied in which participants join a fund that is underfunded. The value of the assets is thus less than the value of the liabilities at the start (2014). This situation was also studied in section 3.2. Now, next to the linear mechanism that can adjust entitlements, two nonlinear mechanisms are also available, the single kink contract and the Dutch staffel contract. The calculations are once again made in the simplified PGGM ALM model of which a more detailed description can be found in appendix B.

Again the KNW-capital market model (Koijen et al., 2010) will be used with the paramterization as prescribed by the Dutch commission parameters (Langejan et al., 2014), the state variables X_1 and X_2 are again set to zero. Just as before risk neutral scenarios are used because the analysis here is about the economic value of pension entitlements.

The participants of the fund are taken the same as in the previous analysis (section 5.1). They again have a degressive accrual and the size of all generations is the same. All generations have the same survival probabilities, namely the male 2014 estimate made by the Actuarieel Genootschap (2012). The pension fund in this section will have an asset allocation of 50% thus $w = 50\%$. The contribution that is needed is calculated by discounting the new accrual against the risk-free term structure. This is similar to the calculation of the liabilities in the funding ratio.

At the start of the 25 year simulation horizon the funding ratio will be $F_{2014} = 0.90$. The level of the funding ratio from which indexation is given (entitlements are raised) is 1.05 for all three the contracts. In order to reach a funding ratio of $F \geq 1.05$ entitlements have to be cut.

The generational effects of the particular pension contract that dissolves the deficit is of main interest here. This is done by comparing the generational account of a not underfunded fund with the generational accounts that result from a fund that is underfunded at the start. These generational effects are obtained by subtracting the generational account of the the equilibrium setting from the generational accounts of the underfunded setting. This will give the isolated generational effects of underfunding at the start.

Three pension contracts to dissolve underfunding Figure 33 gives the differences in the generational accounts (GA) between a fund that is started with a funding ratio of 105% and a fund that is started with a funding ratio of 90% (GA($F_{2014} = 90\%$) − GA($F_{2014} = 105\%$)).

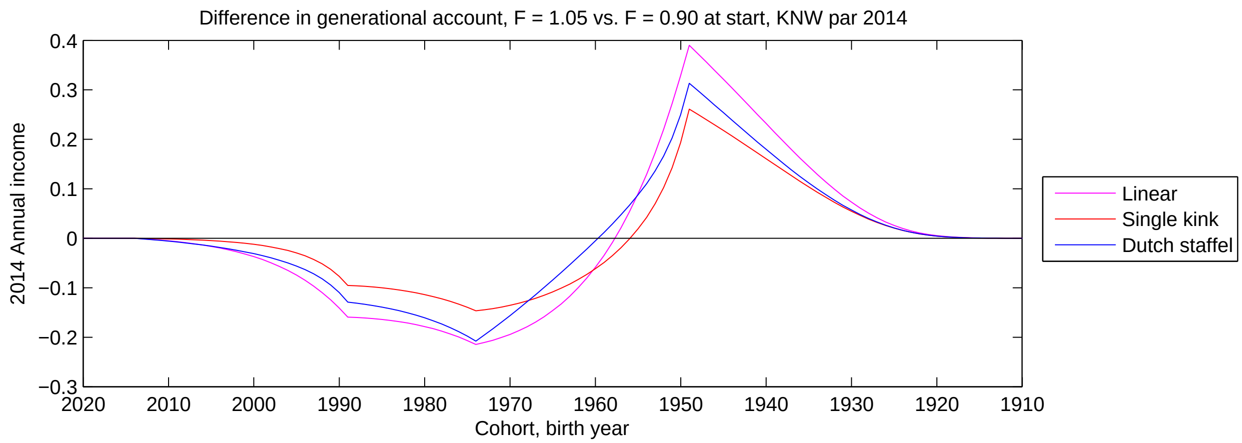


Figure 33: Difference in generational accounts, GA($F_{2014} = 0.90$) − GA($F_{2014} = 1.05$). Economy: KNW-capital market model. Three different pension contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

The effects of underfunding on the generational accounts of the different groups can be summarized as follows.

- **The new joiners** In 2014 the funding ratio of the pension fund is $F_{2014} = 0.90$, the youngest generation in the fund at that time is born in 1989. The generations born after 1989 thus join a fund that is recovering from an underfunded position. The generational effect on the new joiners is negative, their entitlements are cut while they pay in premiums for uncut entitlements. New participants that join in the beginning of the simulation horizon (around 2014, 2015...) lose 10% of 2014 annual income in the single kink contract and around 15% in the linear contract.

  An intuitive way to verify this result of the linear contract is to look at the expected cuts over time for a person born in 1989 and look at how much this reduces the premium payments. The first year premium is thus 25 year subjected to cuts, the second year premium is subjected to cuts for 24 years. By adding the expected value of all these cuts together one comes around an 12% of an annual income. The time value of money would make this amount less. On the other side the fact that older people in the fund receive too much pension benefit makes the problem worse and would result in higher cuts. A loss of 15% for a person born in 1989 very plausible, also from a simple calculation.

- **The young working** The young working (born between 1974 and 1998) do not profit from smoothing (in either of the three pension contracts). They do not own a lot of entitlements that are spared from cutting due to smoothing. In time they accrue more entitlements that are cut in order to dissolve the deficit. This makes their situation much alike that of the *new* *joiners.* At max they lose around 15% (single kink contract) to 22% (linear and Dutch staffel contract) of a 2014 annual income.

- **The middle aged working** The generations born between 1955 and 1974 also feel a loss from the underfunding. They own a reasonable amount of entitlements which are partial spared from cutting due to smoothing. The middle aged working also accrue entitlements over time that will be subjected to cuts. This last effect overrules the gains from smoothing, that is why the middle aged working finally end up with a loss on their generational account.

- **The older working** The generations born before 1955 feel a benefit from the smoothing. This generation has spend a long period in the fund and has accrued a lot of entitlements. In case the start funding ratio is 90% they could lose around 15% of their entitlements. By dissolving the underfunding over a period of time (smoothing), they do not get the full 15% cut in one year. This allows them to share the deficit with younger, new, generations. Next to that if they enter retirement they receive a benefit on the basis of not fully cut entitlements. In that case they receive a pension benefit while still a part of the deficit in the fund needs to be solved.

- **The retired** The retired also gain from smoothing. They own a lot of entitlements that normally would be worth around 15% less. By dissolving the underfunding over a period of time, the retired receive pension benefit that is not fully cut. This results in pension payments that are too high seen in the light of the funding ratio. In this way the retired are using the working and future working to solve the problems later in time.

Figure 33 also shows differences between the way how the three pension contracts solve the underfunding among the generations. The following reasoning can explain theses differences.

Linear contract The linear contract has the longest smoothing period of all three contracts, 10 years. The other two have a 3 year smoothing period in case of underfunding ($F < 1.05$). This is the reason why the retired and almost retired in 2014, gain around 12% of an annual income more in the linear contract. In the extreme case the difference between the linear contract and the Dutch staffel contract is around 15% for the generation born in 1950. Compared to the other two contracts, the entitlements of the elderly are cut less in the linear contract. This also implies that a relative larger part of the deficit has to be solved by other (younger/ future) generations. This explains why the generational accounts for the people born after 1960 is the lowest for the linear contract.

The linear contract needs around 15 years, from the start, to reach a funding ratio of 105%. The other two contracts (both nonlinear) reach 105% after already 5 years. This explains the lower generational accounts in the linear contract, for participants born after 1960, they can expect less positive indexation.

The nonlinear contracts The single kink contract is similar to the Dutch staffel contract for funding ratios below 105%. Though there are differences between the generational account in both contracts. These differences are caused by an increasing probability of high funding ratios, where the Dutch staffel caps the amount of indexation and the single kink contract does not. In the Dutch staffel contract the surplus of the fund grows but it is only limited divided among the participants because indexation is limited. In the single kink contract the indexation is on average higher because indexation is not limited. This explains why people born after 1968 have a less negative generational account under a single kink contract than under the Dutch staffel.

5.2.1 Conclusions: Underfunding

In case of underfunding it can be concluded that value transfers from future and younger generations are used to dissolve the underfunding if a smoothing period is used. Generations that own relative much entitlements (older) gain. They gain because without smoothing their entitlements were cut harder. This gain is paid by the future and the young generations.

The setting of underfunding is studied because funds in the Netherlands experience relative low funding ratios nowadays, compared to the funding ratios of the past. If the same situation was studied but now for a pension fund that has a surplus, approximately the opposite direction of value transfers would be observed. Then wealth would flow from old to young.

In chapter 3 it was also mentioned that an asset shock of 10% to 15% would be large for someone close to or in retirement. In reality risk towards retirement is reduced. Next to that it can also be argued that this kind of risk sharing is (smoothing shocks with future generations) is beneficial in some way. If the young are old themselves they also like to share those risks at a time when their own human capital is depreciated. These aspects are not taken into account in the calculations here but can be considered as very important.

Linear contracts compared with nonlinear contracts There are some differences in the generational effects between how the different pension contracts solve the deficit. Differences of almost 15% can be observed in the extreme case. For the rest the differences stay well under 7% of an annual income. Overall it can be concluded that the way how the generational accounts are affected by the underfunding, results in the same direction of value transfers for all three the pension contracts. The size of the value transfers are somewhat different but this is explained by the different smoothing periods that the individual contracts have when the funding ratio is below F_D or F_{D1}, 10 years and 3 years respectively.

If the results are compared with the results from section 3.2 it can be argued that at least the linear contract of the deterministic setting shows similar results with the linear contract of the section here. Due to the absence of large differences in generational effects between the contract types, calculations under a deterministic setting, can already give an intuitive view of the direction and size of the value transfers that occur under nonlinear contracts.

5.3 Uniform accrual and contribution system

Now the third setting in this thesis, under which the economic value of accrual deviates from the pension contribution will be studied. This setting is the setting in which a pension fund uses the uniform accrual and contribution system. Chapter 3 gave a clear view of the sizable value transfers that occur in such a system under a linear pension contract. The main reasons that cause value transfers where found to be:

Young pay too much contribution compared to the economic value of accrual they earn.

Old pay too little contribution for the economic value of the accrual they earn.

The implicit debt in general made the pension premium higher than under an actuarial fair scheme.

In chapter 3 the uniform accrual and contribution scheme was studied under a linear pension contract. In this section the scheme will also be studied under nonlinear contracts namely, the single kink contract and the Dutch staffel contract.

The analysis is again made in the simplified version of the ALM model of PGGM of which a more detailed description can be found in appendix B. The economic scenarios originate once again from the KNW-capital market model by Koijen et al. (2010). The scenarios represent an economy in equilibrium, thus the state variables X_1 and X_2 were zero at the start.

Participants and their accrual The main difference with the previous sections is that now, the participants do not accrue a degressive percentage of average wage, but a uniform percentage of average wage, namely 2% per year worked. If a person participates in the scheme for 40 years, he or she will accrue $\gamma = 80\%$ of average wage.

The default member base of the fund is again the same as previous. Thus every generation has the same amount of participants and the same survival probabilities, that are taken according to the 2014 male estimate (Actuarieel Genootschap, 2012).

The analysis Figure 34 gives the generational accounts as they result from a simulation 25 years into the future. This figure does not show the differences between two settings it just shows the generational accounts under the three contracts in a uniform accrual and contribution system.
From figure 34 a transfer is observed from the participants born around 1950 and earlier and the participants born around 1982 and later to the participants that born between 1950 and 1982. The explanation of these transfers is given by:

- **Older participants** The older participants born before 1950 experience a loss. This loss gives a somewhat distorted view. The loss namely originates from the fact that the fund is started at a funding ratio of 105%. In the model for the pension fund this surplus will be accounted such that the generations all profit proportionally from this surplus at start. But this surplus will not be handed out to participants until the fund is closed by the closing rule. As a result the older participants receive benefits that do not include a part of the surplus of 5%. This is accounted as a loss for them, this explains the negative generational accounts for one part. Next to that the older do not accrue any entitlements because they are retired in 2014, therefor their generational accounts has nothing to do with the uniform accrual and contribution system such as it is analyzed in this section.

  The second part is found in the fact that the two nonlinear contracts solve underfunding (funding ratio below $F_D = 1.05$) in 3 years while they handout the overshoot (funding ratio above $F_D = 1.05$) in 10 years. This makes the pension contracts more durable on the long run, which implies that elderly will profit less of the upward en lose more from the downward scenarios in the nonlinear contracts. This additionally brings down their generational accounts.

The linear contract thus only shows the effects of the 5% of surplus the elderly miss from the start. This linear contract does not create an option like structure as we know, so the second part (reasoning above) does not apply to the linear contract.

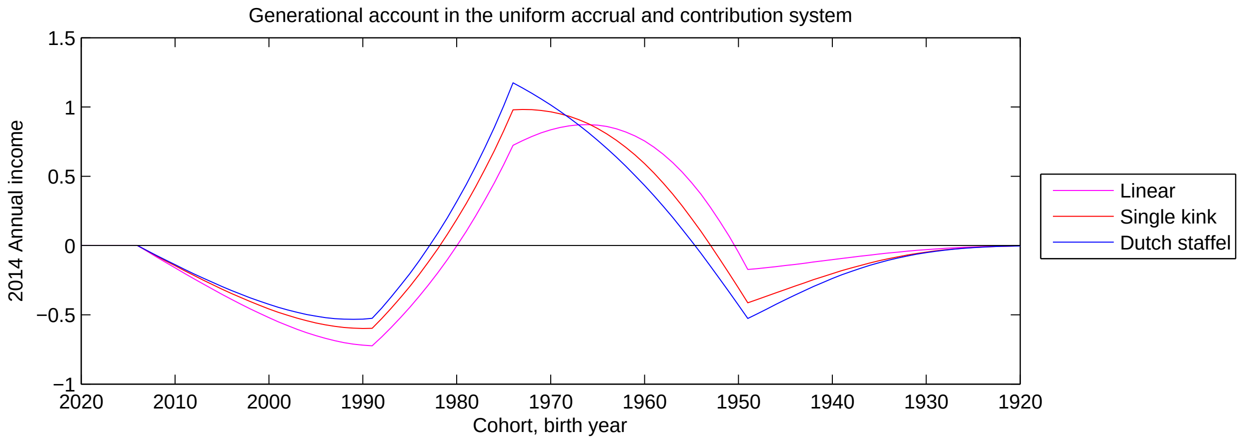


Figure 34: Generational accounts in a uniform accrual and contribution system. Three different mechanisms to adjust entitlements:
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch Staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

- **Older working participants** The participants born between 1950 and 1982 gain. There are three reasons for this. First, they gain from accruing entitlements in the uniform accrual and contribution scheme. They are old and thus receive a subsidy from the young. The second reason is found in the fact that the two nonlinear contracts will have a more durable solvency position (high funding ratio) on the long run which increases the probability on positive indexation.The third reason is that a decent part (mainly people born around 1982) of these generations is still alive after 25 years. Which means that they do profit from the closing rule that hands out the surplus.

- **The young** Participants born after 1982 experience an overall loss. This loss is caused by the uniform accrual and contribution system in which they pay too much premium compared to the economic value of their accrual. They thus subsidize the elderly. The loss is somewhat softened because these young generation gain from the surplus that is handed out after 25 years. At that time almost everyone of these generations is still alive. Next to that, in the two nonlinear contracts, they gain because those contracts yield higher funding ratios on the long run. This increases the chances on positive indexation and make the surplus at the end higher.

More interesting and insightful is, just like the previous two sections, to analyze the isolated effects of the uniform accrual and contribution system. This means comparing the findings of figure 34 with the alternative of degressive accrual per contract. This results in figure 35.

Figure 35 shows the difference between the generational accounts that occurs when the generational account, calculated under the uniform accrual and contribution scheme, is subtracted by the generational account calculated under the degressive accrual system. This thus gives the isolated generational effects of the uniform accrual and contribution system over a time span of 25 years per contract type.

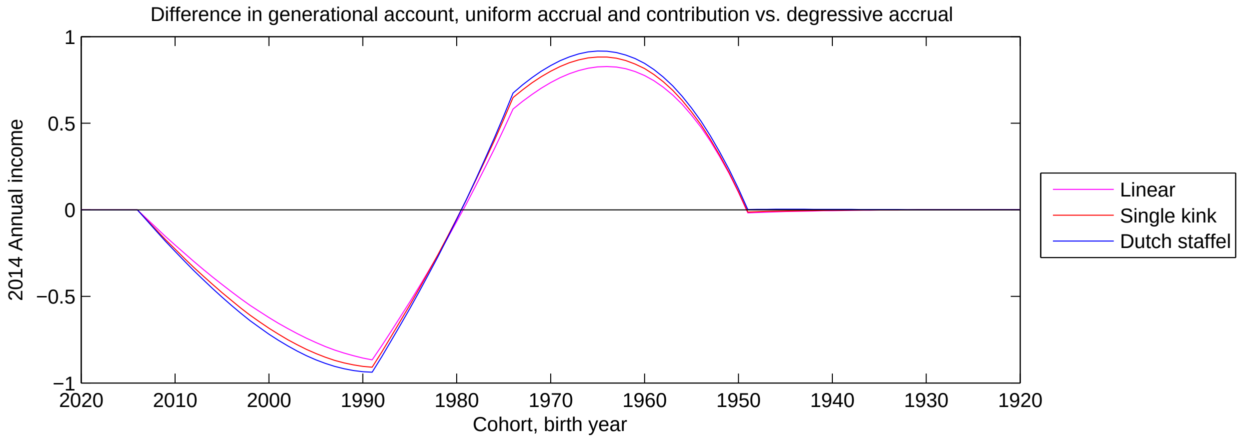


Figure 35: Difference in generational accounts, uniform accrual and contribution minus degressive accrual. Three different mechanisms to adjust entitlements:
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch Staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

Figure 35 indicates that the isolated effects are about similar for all three the contract types. Thus regardless the contract type, the move from degressive accrual to uniform accrual results in the same value transfer.

It can be seen from figure 35 that the direction of the value transfers goes from young to old. The net gain for the elderly is at max around 80% to 90% of an 2014 annual income for the Dutch staffel contract and the linear contract respectively. The net loss for the young is around 85% tot 95% of an 2014 annual income for the generation born in 1989, in the linear contract and the Dutch staffel contract respectively.

It is also interesting to compare the results from figure 35 with the results from figure 13 (section 3.3, linear contract that was calculated deterministic). It can be seen that the gain for the elderly is in line with what was found in section 3.3. There are differences observable between the outcomes of the younger, born between 1984 and 2014. Figure 35 indicates huge losses of max around 80% to 90% of an 2014 annual income while figure 13 (section 3.3) indicates losses around at max 45% for this generation. This has to do with the fact that the calculations of this section are bounded by a time horizon. In this way, the subsidy that a young participant would receive when he or she is old is not accounted for, because this is not covered by the 25 year horizon. By this the people born between 1984 and 2005 have a loss that is too much, compared with their actual loss on a longer horizon. The people born between 2005 and 2014 have a too small loss compared with what the loss would be on a longer horizon.

Because the interpretation and explanation of the value transfers and their direction was already extensively given in section 3.3 and partial in the illustration of figure 34, this will not be done here again. Here it will be tried to connect the findings of the CPB report on the uniform accrual and contribution system (Lever et al., 2013) to the results found here. As stated in chapter 2 the CPB study was performed in a deterministic setting. This raises two points of interest. First, is there a relevant difference between the outcomes under a linear contract and nonlinear contracts and how much do these results differ? Second, is it possible to find the same key results from the model in this section as the CPB study (Lever et al., 2013) found?

5.3.1 Linear contracts compared with nonlinear contracts

It was already concluded that the differences in the generational accounts, given in figure 35, show little deviations between the three contract types. One can find the largest differences appear between the Dutch staffel contract and the linear contract. At max this difference is 10% for someone born in 1970 and someone born in 1996 or 1997. All in all the isolated effects between the uniform accrual and contribution system and a degressive accrual system are almost similar for all three contract types.

The standalone results (no comparison with an alternative scheme) on the uniform accrual an contribution system, shown in figure 34, indicate larger differences between the three pension contracts. This makes sense because bottom line the three contracts represent three different ways to adapt entitlements. At most the difference between the linear contract and the Dutch staffel contract is around 50% of an 2014 annual income. This is explained by the fact that the Dutch staffel distributes more wealth to future generations (young) compared with a linear contract. What has to be noted is that this also happens under the alternative of degressive accrual.

To study the effect of the uniform accrual and contribution system it makes sense to compare the result with an alternative (degressive accrual system). By this way the pure generational effects of the uniform accrual and contribution system become visible. Those effects show to be the same for all three the pension contracts.

A suitable conclusion is that the generational effects of the uniform accrual and contribution scheme shows no relevant differences between a linear, single kink and Dutch staffel contract in the situation as analyzed here. Once again it is observed that a deterministic calculation of a linear pension contract (chapter 3) gives core results about the isolated generational effects in the linear contract. These isolated effects under linear contracts are very similar to the isolated effect observed under the nonlinear pension contracts. This would imply that deterministic calculations on linear pension contracts are able to give a good view of isolated generational effects that appear under nonlinear contracts in the uniform accrual and contribution system as they are studied here.

5.3.2 The key findings relative to the findings of the CPB

It is tried to link the findings of the CPB report on uniform accrual and contribution (Lever et al., 2013) to the findings found in the analysis performed for this section. The comparison will be made on findings about: long run negative effects, the implicit debt and green and grey funds.

Negative generational effect on the long run Lever et al. (2013) (CPB report) find that participating in an uniform accrual and contribution scheme will result in a burden between 18% and 22% of gross wage. Which would result in a loss of around 28% to 35% if the franchise was set to 0, like in the analysis here[15].

[15] Gross wage in Lever et al. (2013) assumed EUR. 35.000 and franchise EUR. 13.000.

Because a 25 year horizon is used, the negative effects over a full participation in the fund cannot be confirmed with certainty. The long run estimate of the CPB on the generational effects is somewhat lower than found in chapter 3. What can be reasoned is that losses around 25% to 40% of an annual income over a full career are not unlikely to be observed. The deviations can be explained by the differences in assumptions that are believed to easily move this number 5 to 10 percentage points.

Higher premium rate A commonly used benchmark for the implicit debt is the amount that the premium in a uniform accrual and contribution scheme is higher than the premium rate in an actuarial fair scheme. In more complex settings, like in the KNW economy, an explicit formula for the implicit debt (like in section 3.3) cannot be derived easily. The CPB study on the uniform accrual and contribution system (Lever et al., 2013) finds that the premium in the uniform accrual and contribution system is around 8.7% higher. This higher premium is needed to finance the implicit debt and is not invested to obtain a pension benefit over an amount of years. Table 4 summarizes the percentages of premium that are needed to finance the implicit debt in the three studies (CPB, section 5.3 and section 3.3).

Analysis	IPD of premium
CPB (Lever et al., 2013)	8.7%
Chapter 5 (linear / nonlinear stochastic)	8.6%
Chapter 3 (linear deterministic)	12%

Table 4: Overview of percentage premium in a uniform accrual and contribution scheme that is paid to finance the implicit pension debt (IPD). Calculated by $(\frac{P_{\text{UNIF}}}{P_{\text{DEG}}} - 1) \times 100\%$

From table 4 is can be seen that all three studies indicate that an amount higher than 8% of the premium is paid too much. In section 3.3 is was concluded that this proportion was heavily dependent on the interest rate and the population of the fund. This can explain the differences between the results of chapter 3 and the other two results. Next to that, a larger difference was expected between the findings of the CPB and the results from this section (section 5.3). This was expected because of differences in for instance: population, interest rate and survival probabilities. Thus the conclusion that the calculation of this current section (section 5.3) exactly mimic the CPB setting is certainly not drawn. Nevertheless the results are close and do indicate some relevant similarities.

Green or Grey fund In appendix A.2 the generational accounts of a pension funds with a uniform accrual and contribution system are calculated once again. This time the size of the young and old generations is varied to obtain result for a green fund (more younger) and for a grey fund (more elderly). The results are in line with the findings of Lever et al. (2013). Although the CPB used a more realistic setting for the green and the grey fund than increasing the size of a generation with just 100% as in appendix A.2. The direction of the value transfers are corresponding to what the CBP study finds and are well interpretable (see illustration in appendix A.2).

When the simulation output is studied in more detail, it is found that the premium paid by the participants of the larger group (the old in case of a grey fund, the young in case of a green fund) deviates less from the actual economic value of their accrual (over a 25 year horizon). So the larger generation pays a more fair premium in relation to the economic value of their accrual. This was also concluded in section 3.3. The other generations see on average a larger deviation between the economic value of their accrual and the actual amount they pay as premium. In case of the green fund this is beneficial for the other generations, the large group of young keeps the premium low. In case of the grey fund, this results in a disadvantage for the other generations than the old generation. In this situation, the old cause the premium rate to increase.

The analysis about the grey and the green fund shows the same kind of differences in generational accounts between the three contract types as seen earlier from the analysis in figure 34 and figure 35. This thus indicates that the effects of a varying population are not influenced much by the kind of pension contract in place.

5.3.3 Conclusions uniform accrual and contribution scheme

When nonlinear pension contracts are studied, sizable value transfers are observed in the uniform accrual and contribution system. The transfers flow from young to old and can reach values of up to 90% of an annual income. The results found, indicate that on a 25 year horizon the difference between premium payments and the economic value of accrual is large. An estimation of the generational effects on the long run was not possible because a horizon of 25 years was used due to the lack of calculation power and time.

It is expected that value transfers even grow larger when the premium in a uniform accrual and contribution scheme would be calculated on the basis of real expected return. This would be a combination between section 5.1 and 5.3 where value transfers in the same directions take place (equity allocation of $w = 50\%$).

Linear contracts compared to nonlinear contracts Generational effects of the uniform accrual and contribution system are similar for all three pension contracts if compared with a degressive accrual system.

From this it can be concluded that, with the degree of nonlinearity that is present in the studied pension contracts, deterministic calculations in a linear contract already give a good view of the core results. Next to that, the findings of this section do indicate that the deterministic approach of the CPB (Lever et al., 2013) is able to give sufficient insight in the complex reality.

If one is interested in the generational effects over a full life cycle, which is desirable in a study about the uniform accrual an contribution scheme, the use of simulation could cost a lot of calculation power and time. Given that deterministic calculations in a linear pension contract are fast and accurate enough to make an estimate of the isolated effects, they are well suited to gain a sufficient level of insight.

6 Conclusions, discussion & future research

This thesis used a two-step approach to study the economic value of accrued pension entitlements in relation with pension contribution. The analysis began with a linear pension contract in a deterministic setting. This yielded a clear view about size and direction of value transfers that originate in Dutch second pillar pension schemes. Later, next to the linear contract, nonlinear pension contracts were studied in a stochastic setting. Conclusions of this thesis can be placed in three categories: policy conclusions, methodological conclusions and "a point of attention" for the Dutch pension sector.

Policy conclusions Dutch regulations leave second pillar pensions funds several situations in which the economic value of accrual differs from the pension contribution. This creates a possibility for sizable value transfers to occur, which can harm the sustainability of and the confidence in the Dutch second pillar pension schemes.

1. Discounting method of the contributions differs from discounting method of funding ratio. If the premium rate is based on discounting against real expected return and not on the risk-free term structure, which is used in the funding ratio, value transfers can originate. In an average Dutch fund that has an equity allocation of 50% too much premium is paid by the participants. A surplus arises that is divided among all (also non premium payers) members of the fund. The results of this over a 25 year horizon range from a gain for the elderly of around 10% to 15% of an 2014 annual income to a loss of 10% to 20% of an 2014 annual income for the younger. If the current low interest rates are incorporated in the model the size of the value transfers decreases. The limitations of the results can also be discussed. The currently observed low inflation levels are not included in the calculations, next the Dutch government has not yet decided in detail how real expected return should be estimated. Implicitly the calculations performed in this thesis assume that the parameters as prescribed by the commission parameters and the regulation that allows this dissimilarity between discounting methods will never change in future, which is doubtful.

2. Participating in a fund with a surplus or a deficit. In this topic we have seen value transfers to the participants that own a lot of entitlements (that are spared from cutting) from participants that still have to accrue most of their pension wealth. The specific setting used in this thesis, a funding ratio of $F = 0.9$ instead of $F = 1.05$, resulted in value transfers that at max ranged from a benefit of 25% to 40% of an annual income for the elderly and losses of 10% to 20% for the young working and future generations. The results do not incorporate the gain that the current young would have if in future a younger generations helps them to recover from a negative shock. Also the calculations did not incorporate recovery premium, which is expected to make the burden for the future generations and working even higher. The opposite situation in which a fund has a surplus is also not studied. This is expected to show an opposite value transfer, from old to young.

3. The uniform accrual and contribution system. The uniform accrual and contribution system results in the largest value transfers of all three the settings that were analyzed. The gain of the elderly who receive a subsidy on their accrual can grow upto 90% of a 2014 annual income. The young lose because they pay that subsidy. The 25 year horizon in chapter 5 gave a somewhat distorted view because the calculations do not take into account that a young person receives a subsidy when old. The calculations of chapter 3 were able to identify this. Analysis also showed that young lose more in an aging (grey) fund and elderly gain in a young (green) fund. Some limitations on this topic were that value transfers within generations were not addressed (life expectancy men/women, low/high educated). In reality the population of a pension fund is not as uniform as assumed which could influence value transfers.

There are also some overall limitations to the results of this thesis, mainly because assumptions had to be made. Assumptions are made about the population of the fund, we know that the population of a fund can have impact on value transfers. In reality a pension fund has a more complex mechanism that cuts or raises entitlements than the two nonlinear pension contracts used in this thesis, although the contracts provide a realistic degree of nonlinearity. Another limitation is that the calculations of chapter 5 only reach 25 years into the future, even a 35 or 55 year horizon would still not be satisfactory because it does not represent the full situation. The calculations did also not incorporate longevity risk.

Methodological conclusions An underlying subject of this thesis was, to what extent are deterministic (simple) models able to provide answers to question for which normally stochastic (complex) models are used. This thesis studied both methods and concludes that on a high level deterministic models give a lot of intuition and can provide fast results which help to gain insight in generational transfers when parameters or policies are changed. Advantages of simple models are that, if calculations go far into the future (which is possible by the short calculation time) the results become less dependent on the closing rule. For more specific results on a specific setting, stochastic models are still preferable, this approach does not downplay small details in the contract or other characteristics of the specific setting.

Advances can be made when details of the real world are better incorporated in deterministic calculations. For instance in chapter 3 more simplifying assumptions are made than necessary to keep the calculations fast and simple. If in future more effort would be put in a better modeling of the details, results are expected to become even closer to reality.

Point of attention for the Dutch pension sector Value transfers between participant showed to be small in case the equity allocation of a fund with a linear pension contract was changed. Literature suggests that these value transfers should be zero under the right mathematical framework. People active in the pension sector are not aware of the fact that there are some assumptions needed for these value transfers to be zero. This is an important finding because linear pension contracts are debated frequently without people knowing that there are some practical restrictions.

From practical point of view, linear pension contracts are frequently used because they are simple. If one likes to state all the necessary framework needed to keep the value transfers at zero exact, the simplistic application of linear pension contracts wears out.

Suggestions for future research An interesting point could be to focus a study more on a combination of the three problems that were central in this thesis. Next it is also interesting to incorporate facts of the past in the analysis. For example in the past generations received accrual in terms of final wage instead of average wage, or had not to pay premiums for several years because of high funding ratios. Also the transitional issues that arise when the uniform accrual and contribution is abolished provide a relevant subject in which economic valuation plays an important role.

In chapter 4 when value transfers due to a change of the equity allocation in linear pension contracts were analyzed, some suggestions for future research were already made. The main suggestion was to study the economic value of entitlements in a pension fund model that meets the mathematical assumptions. The question arises how far is a realistic pension fund off from these strict assumptions and what exactly is the size of the error? Preferably a pension fund model is built that can alleviate the restrictions one by one, the ALM model of PGGM was not suited for this.

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A Additional figures

A.1 Section 5.1.2, Equity allocation w = 100%, KNW

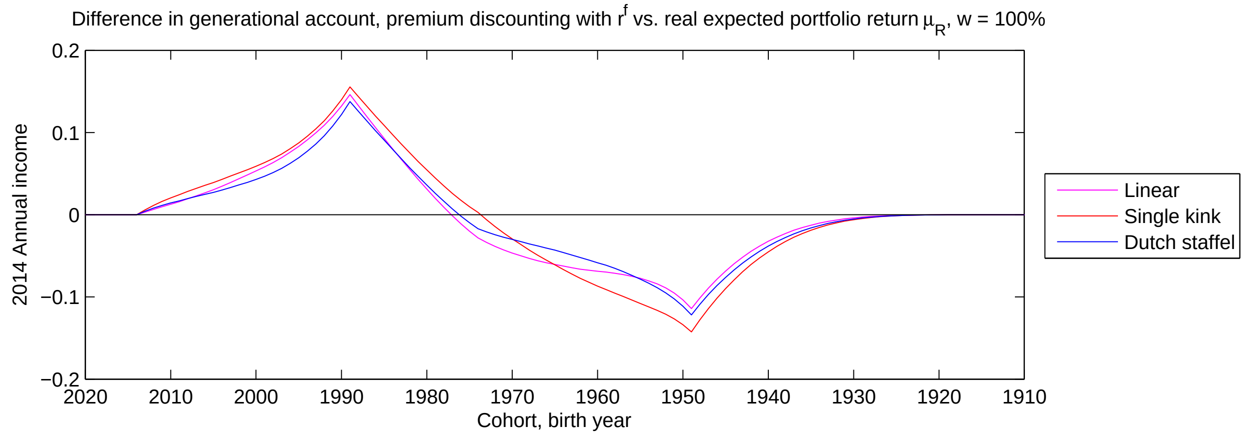

Figure 36: Difference in generational accounts, premium calculated by discounted with r^f vs. $\mu_R(w = 75\%)$. KNW capital market model. Three different pension contracts to adjust entitlements.
Linear: $F_D = 1.05$, $\alpha = 0.1$. Single kink: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$. Dutch staffel: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

The difference in the generational accounts that appear when the premium calculation is based on real expected return and when the risk-free term structure is used can be found in figure 36. The effects can be explained as follows.

- **The joiners** The new joiners during the simulation horizon (born between 1989 and 2014) gain from paying to little premium. The funding ratio skews down because of a too low inflow of contribution. The deficit is solved among all participants not only the participants that create the underfunding.
- **The working** The working generations during the simulation horizon (born in 1998 and earlier) also experience a gain. They benefit from paying too little premium and are not hurt that hard by the cut because they do not own many entitlements.
- **The older working** The older working own a lot of entitlements from the start of the simulation. They suffer from the fact that their entitlements are cut in order to dissolve the deficit of the fund. Compared to this burden, the gain from paying to less premium is small, that is way over all they lose.
- **The retired** The retired do not pay premium anymore. They only experience a cut of their entitlements which thus has a negative effect on their generational account.

A.2 Section 5.3, green and grey fund, KNW

Here the generational account are visualized of participants in a pension fund that has a uniform accrual and contribution system with a non uniform population. A uniform population has the same amount of persons per generations. This time a "green" and a "grey" fund are also studied. A green fund has more younger participants. A grey fund has more older participants. The generational account give the values per person. Thus the wealth of the greater generations are calculated per person.

In the green fund the participants that are born between 1998 and 2013 has twice the amount of persons as the default population (uniform population). The grey fund has twice as much participants born between 1950 and 1974 as the default population.

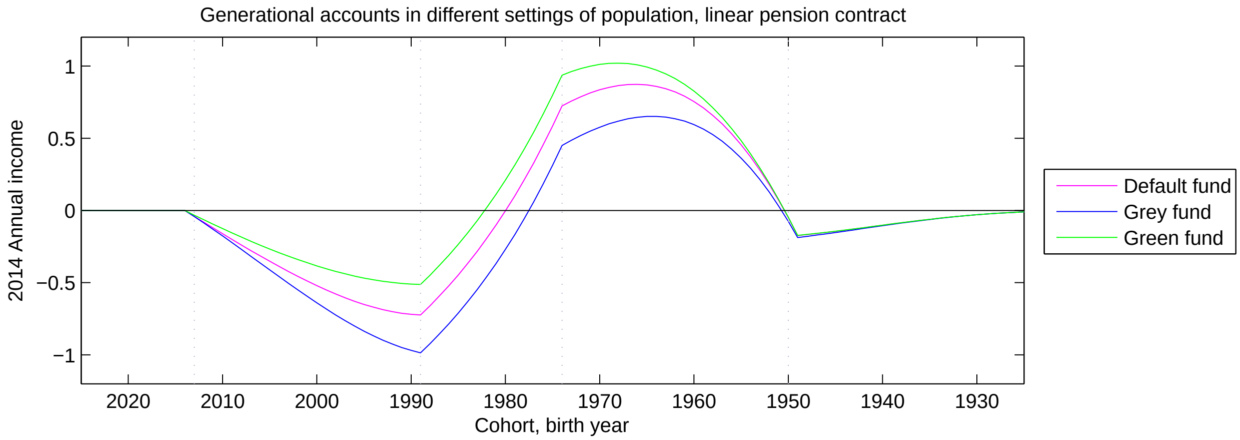

Figure 37: Generational accounts (per person), KNW capital market model. Green and grey pension fund with a linear pension contract: $F_D = 1.05$, $\alpha = 0.1$.

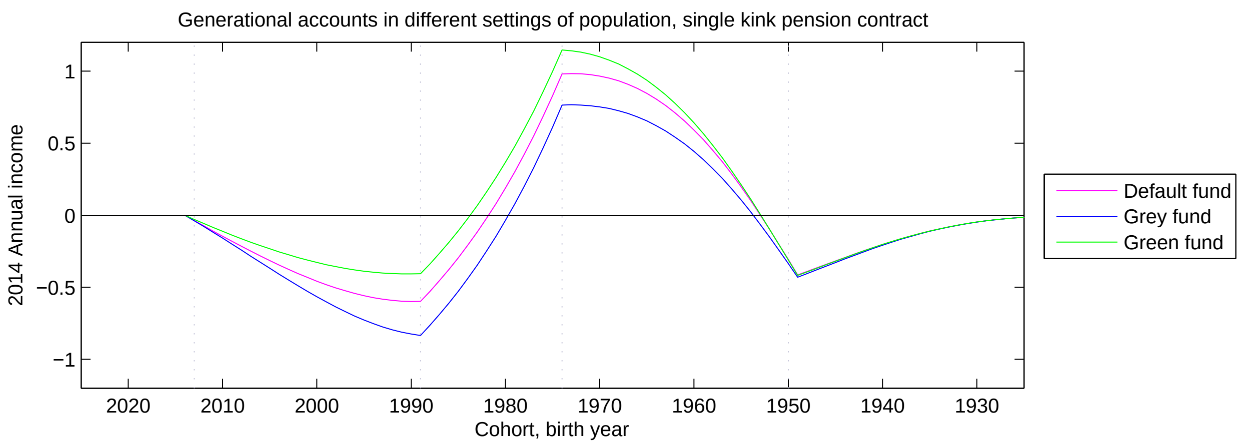

Figure 38: Generational accounts (per person), KNW capital market model. Green and grey pension fund with a single kink pension contract: $F_D = 1.05$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$.

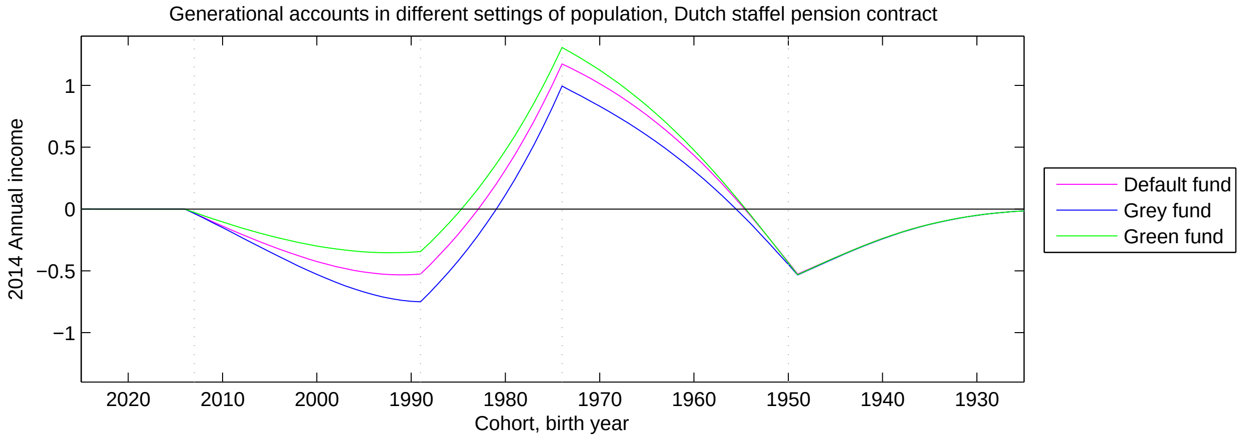

Figure 39: Generational accounts (per person), KNW capital market model.
Green and grey pension fund with a Dutch staffel pension contract: $F_{D1} = 1.05$, $F_{D2} = 1.30$, $\alpha_1 = 0.333$, $\alpha_2 = 0.1$, $\alpha_3 = 0.0$

In figure 37, figure 38 and figure 39 the generational accounts are given in a pension fund with a default, green and grey population in a pension fund with a linear, single kink and Dutch staffel contract respectively.

It can be seen from each of the three figures that the generational account in a green fund are located above the generational account of the default fund. This is because the larger amount of young people pay a more actuarial fair premium for themselves, but because they are young, they also keep premium levels low for the rest of the fund.

When looking to the blue line (grey fund) in relation with the magenta colored line (default fund) it can be seen that the generational account of the grey fund is below the default fund. This is because the elder pay a more fair premium for their age, this means they are subsidized less. The larger amount of elderly increases the average age in the fund which results in higher premiums also for the other generations who also lose in a grey fund.

B Chronological description of the PGGM pension fund model

The pension fund model used to make the calculations for chapter 4 and chapter 5 is in basis the same model as the model that PGGM uses to make pension calculations for PfZW. The main adaptions that are made are about yearly operating and administration costs. Next to that also the premium that participants pay is not rounded. In reality the model has a complex mechanism that can cut or raise entitlements. This complex mechanism is replaced by one of the three mechanisms (pension contracts) introduced in section 4.2.

Initialization, start values The pension fund model needs a cash flow matrix. This cash flow matrix specifies in which year entitlements are accrued and in which year that accrual will actually result in a pension payment. This cash flow matrix needs input like: the amount of participants, their survival probabilities, retirement age, wage growth until 2014, etc. Next the model also needs economic scenarios, in this thesis the KNW scenarios from TFT or Black Scholes scenarios are used.

The setting in which the model will start is specified in an Excel input sheet. Here the start funding ratio which sets the amount of assets to the proper level can be chosen. The discounting methods for the funding ratio (nominal term structure in this thesis) and the premium (real expected return or nominal term structure) can be chosen. Also the contract can be specified: linear, single kink and Dutch staffel. An important input variable is also the equity allocation (used in section 4.2 and section 5.1). The pension fund can hedge interest rate risk on a percentage of the liabilities, for the calculations in the KNW model 50% is hedged, for calculations in the Black Scholes 0% is hedged.

Year 0 In year 0 all information is loaded (settings, cash flow matrix and scenarios). Memory space needed for the calculations is reserved. All term structures are calculated with the use of Nelson and Siegel (1987) parameters for all scenarios and simulation years. The begin values of the assets are determined for all scenarios on the basis of the chosen funding ratio and the liabilities that are determined from the cash flow matrix and the term structure.

Year on year calculations The year on year calculations are repeated for every year of the simulation horizon (25 years in this thesis). The following steps are preformed for all scenarios individually:

- Information of the specific year is loaded from the cash flow matrix. From this the pension payments, the liabilities and the new accrual is determined. Also the total salary is used to base the premium percentage on.
- The cost of new accrual is determined on fund level. This is done by discounting the total new accrual in the current year by either the nominal term structure or the real expected return (section 5.1)
- The costs of new accrual is actually set as the premium that is paid by the participants. In the original version of the ALM model there were additional factors that influences premium rate, these are switched off.
- The liabilities are calculated. These are all entitlements as obtained from the cash flow matrix and discounted against the the nominal term structure.
- The return on the investments is calculated. This step uses the returns on stocks and bonds as provided by the scenario set and the equity allocation of the fund to calculated the overall return per scenario.
- Adjustment of the liabilities. Here the funding ratio is used as an input parameter in one of the three pension contracts (linear, singe kink or Dutch staffel). This step yields an adjustment parameter per scenarios where the liabilities are corrected with.
- Funding ratio is reported. In this step the funding ratio is calculated once again. Now it is the funding ratio after the mechanism has cut or raised the pension entitlements.

The model keeps track of all payments to individual generations and jumps to the beginning of the year on year calculations to perform the same steps until the simulation horizon is reached. Note that when liabilities are used that these liabilities include adjustments of previous years. The same holds for wage, inflation from the KNW model is used as a source for wage inflation. Every year the model corrects the wages for the cumulative wage inflation.

Final year When the simulation horizon is reached the surplus or deficit of the pension fund is nominally divided among the participants proportional too the amount of liabilities each generation owns relative to the total amount of liabilities. This is also referred to as the closing rule.

Output file At the end of the calculations an output Excel file is created. The output file contains information about: the settings that were chosen, premium rates, funding ratios over time, returns, etc. The most important information for this thesis is provided by the generational account overview within the output file. This overview gives market consistent prices of the different options that each generation holds. In this overview, option prices for the following aspects are given: indexation (raise of entitlements), cuts (decrease of entitlements), shortage at start, surplus at start, shortage at end, surplus at end. Also the start entitlements and the necessary amount of assets of each generation is given just as the amount of premium they paid and what the economic value was of the accrual bought with this premium.