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Formalizing the New Dutch Pension Contract

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Abstract

Dutch employers and labor unions agreed in 2010 to support a major pension reform. The newly proposed contract replaces suggested nominal guarantees by "soft real rights". The soft real rights are adjusted annually to unexpected real investment returns. In this paper we provide a formalization of this new Dutch pension contract with special emphasis on the market-consistent valuation of the entitlements and the investment strategy for the collective that is consistent with the promise made to the participants. For simplicity we restrict ourselves to the case of a closed fund and a single risk factor in financial markets.

1. Introduction

Dutch employers and labor unions agreed in June 2010 to support a major reform of the Defined Benefit contracts. In June 2011 a more detailed agreement was reached where "soft real rights" were introduced as the main new element in the pension contract. Soft real rights are adjusted annually to unexpected real investment returns. Moreover social partners have proposed to create an automatic link between the eligibility age for retirement benefits and life expectancy a few years before retirement.

A key feature of the new contract is that shocks in financial markets are smoothed over a ten year period before they hit actual pension income. We show that this smoothing generates a life-cycle feature in that younger participants are more exposed to shocks in the financial market than older participants. For this, we provide some numerical illustrations. An individual that desires the risk exposure that is generated by this contract can do so by reducing his or her equity exposure while aging (assuming that inflation is either constant or traded). We also derive the investment strategy for a collective fund that is consistent with the intended exposures of the individual participants. Finally we derive the market-consistent value of such promises.

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Market-consistent valuation of pension entitlements is vital in DB schemes for a number of reasons. First, market-consistent valuation specifies the minimum amount of money that is needed to implement the contracts that have been offered. Secondly, it determines the required new contributions for accrual of new pension rights without any negative consequences for current participants. Likewise it determines whether or not sufficiently many assets are available relative to the liabilities to increase current projected pension outcomes or that, on the contrary, existing rights have to be cut. As such, thirdly, it leads to a market-consistent funding ratio whose interpretation is that, at 100%, the fund has exactly enough assets available to meet its promises.

We provide the market-consistent value of pension entitlements by means of an appropriate discount curve. It is important to note that, in line with Dutch practice, the pension entitlements to be discounted are the so-called rights which are identified as the annual pension income to be paid in case of no further future adjustments, i.e., no new entitlements, no indexation, no right cuts, etc.

In this paper we provide a formalization of this new Dutch pension contract with special emphasis on the market-consistent valuation of the entitlements and the investment strategy for the collective that is consistent with the promise to the participants. For simplicity we restrict ourselves to the case of a closed fund and a single risk factor in financial markets (which we take as equity risk). Fluctuations in interest rates, inflation rates, or life expectancy are thereby ignored for simplicity of the argument. In order to generate transparent results we even assume in the numerical examples that the date of entry to the labor market, of retirement, and of death are all known in advance and equal 25, 65, and 85 respectively. The second important ingredient of the new Dutch pension contract, adjustment of pension income to increases in life expectancy, is thus ignored for simplicity.

2. Risk profiles

Our analysis for individual participants of the new Dutch pension contract starts with a specification of the risk profile each individual in the pension fund has. In this paper we restrict ourselves to one financial market risk factor, which is identified as a stock market index. In Section 5 we discuss additional risk factors, such as interest rate, longevity, and inflation risk. We also impose some simplifying assumptions on the individuals.

2.1. Characteristics of individuals

Individuals are characterized as follows:

- All individuals enter the pension fund at age 25, retire at $a_r = 65$, and die at age $a_{max} = 85$.
- Individual's wages are assumed to be normalized to 1. In particular there is no wage profile.

In the remainder we will index individuals with a, their current age.

2.2. Characteristics of the financial market

We assume a financial market with a single risk factor, namely a stock-market index, whose stochastic behavior is governed by the Black and Scholes (1973) model. Thus, with S_t denoting the value of the stock index at time t, we have

$$dS_t = \mu S_t dt + \sigma_S S_t dW_t,$$

where μ represents the constant drift (expected return), σ_S a constant volatility, and W a standard Brownian Motion/Wiener process.[1] Returns on the stock market index are denoted by r_t.

Throughout the paper we use the following parameter values. The constant nominal risk-free interest rate r^f is assumed to be 4%. The constant inflation π is assumed to be 2%. The volatility σ_S is taken to be 20% on an annual basis and the (nominal) financial market risk premium, $\lambda = \mu - r_f$ is constant and equals 4%.

2.3. Risk profiles for individuals

The key ingredient of the new Dutch pension contract is that future pension payments depend on shocks in investment returns and that both positive and negative shocks are smoothed using a linear mechanism. This is implemented by adjusting pension benefits in accordance with the shocks occurring on the financial market. More precisely, if $B_{a,t}$ denotes the accumulated pension benefits of an individual that is a years old at time t, then, without the accumulation of new entitlements due to premium payments, we have

$$\begin{aligned} B_{a,t+h} &= B_{a,t}[1 + \alpha_1 w(r_{t+h} - r^f - \lambda) + \ldots + \alpha_h w(r_{t+1} - r^f - \lambda)] \\ &= B_{a,t}[1 + w \sum_{j=1}^{h} \alpha_j (r_{t+h-(j-1)} - (r^f + \lambda))]. \end{aligned} \quad (1)$$

Here $w\alpha_h$ is the (desired) exposure to unexpected shocks in the financial market at horizon h. Note that this way of representing pension promises, in particular that α_j does not depend on a, leads to a restriction in the possible risk profiles. It is made because of the customary practice in the Netherlands that entitlements are increased or decreased uniformly for all age groups in funds in case of indexation or right cuts.

An individual only receives an actual pension payment when retired and alive, thus, in our simple model if his or her age is between a_r and a_{max}. In mathematical terms, individual a receives a pension payment in case $\max(1, a_r - a) \leq h < a_{max} - a$. As a result, actual pension payments to individual a at time $t + h$ satisfy

$$\begin{aligned} P_{a,t+h} &= B_{a,t+h} I_{\max(1,a_r-a) \leq h < a_{max}-a} \\ &= B_{a,t}[1 + w \sum_{j=1}^{h} \alpha_j (r_{t+h-(j-1)} - (r^f + \lambda))] I_{\max(1,a_r-a) \leq h < a_{max}-a}. \end{aligned}$$

[1] A process is called a Wiener process, if $W_0 = 0$, increments are independent, and $W_t - W_s \sim N(0, t-s)$ for $t > s \geq 0$.

Note that throughout the paper it is assumed that pensions are paid at the beginning of the year. This means that an 85 year old will receive a pension payment before he dies.

Provided that the actual investment strategy coincides with the desired strategy, which we will assume throughout, the expected pension benefits at $t+h$ given the information at time t is equal to:

$$E_t\left\{P_{a,t+h}\right\} = B_{a,t} I_{\max(1,a_r-a)\leq h<a_{max}-a},$$

since $E_t\left\{r_{t+j} - r^f - \lambda\right\} = 0$ for $j = 1, .., h$.

This result underpins the term expected pension benefits for $B_{a,t}$. The expectation of the pension payment at time $t+h$ given the information at time $t+1$ and $t+2$ is obtained as

$$\begin{aligned}
E_{t+1}\left\{P_{a,t+h}\right\} &= B_{a,t}[1+\alpha_h w(r_{t+1} - r^f - \lambda)] I_{\max(1,a_r-a)\leq h<a_{max}-a}, \\
E_{t+2}\left\{P_{a,t+h}\right\} &= B_{a,t}[1+\alpha_h w(r_{t+1} - r^f - \lambda) + \alpha_{h-1} w(r_{t+2} - r^f - \lambda)] I_{\max(1,a_r-a)\leq h<a_{max}-a},
\end{aligned}$$

and so on. These equations show how shocks affect future expected pension income. When an individual ages, thus a increases, the indicator will be non-zero for smaller values of h, because it takes less years before the retirement age will be reached. In the typical case where $\alpha_h \geq \alpha_{h-1}$ this shows that the risk exposure of an individual remains the same or decreases if he or she ages.

2.4. Two concrete implementations of exposure to unexpected shocks

The pension benefit of an employee participating in a risk-taking pension fund is exposed to financial market risks. Hence, a pension payment h years from now will not necessarily be equal to the current pension benefit. This is the nature of "soft" contracts. The exposure to the unexpected portfolio shock on a h horizon is defined by α_h (see (1)). We discuss two concrete choices for these exposures α_h.

RAM mechanism, given N. In case the new Dutch pension deal is interpreted strictly, shocks have to be absorbed completely by the current pension fund participants within $N = 10$ years. For given N this smoothing mechanism is formalized as

$$\alpha_h = \begin{cases} h/N & \text{if } h < N, \\ 1 & \text{if } h \geq N. \end{cases}$$

Note that in case of no amortization period ($N = 1$), we have $\alpha_h = 1$. In that case the exposure to the market risk factor for every horizon h equals w: the equity exposure you desire to give to the youngest participants.

Policy Ladder or Autoregressive (AR) mechanism, given ρ. An alternative mechanism, that is more closely related to the existing policy ladders, is to annually absorb a fixed percentage of a shock. In Dutch this mechanism is called "staffel".

It leads to mathematically convenient first-order autoregressive (AR) behavior of the relevant state variables in a pension fund. Formally,

$$\alpha_h = 1 - \rho^h.$$

Again it holds that, in case of no amortization period ($\rho = 0$), the exposure to the risk factor on every horizon h equals w.

For both specifications it holds that $\alpha_h \to 1$ for $h \to \infty$. Hence, the fraction w can also be interpreted as the exposure to the financial market risk factor of the youngest participants in the fund.

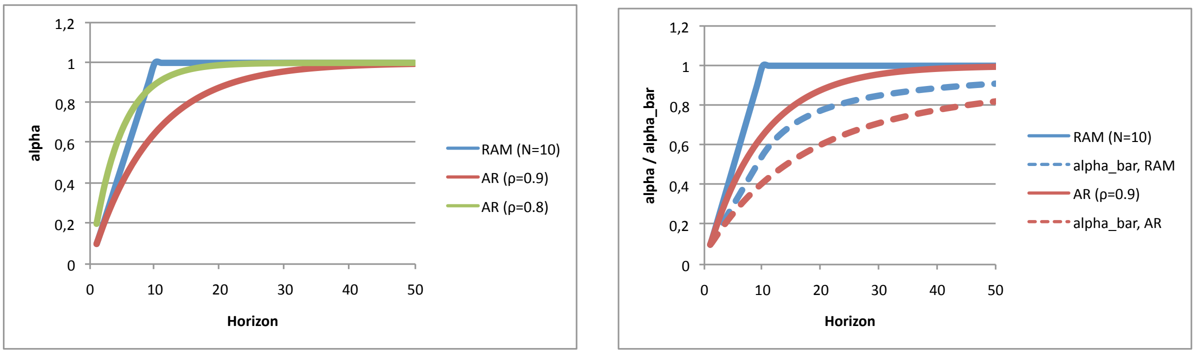

Figure 1: The left figure illustrates the behavior of α_h as a function of the horizon h for the RAM mechanism with $N = 10$ and for the AR mechanism with $\rho = 0.8$ and $\rho = 0.9$. The right figure shows how $\overline{\alpha}_h$ (which will play a role in Section 3) and α_h behave.

Figure 1 graphically illustrates the behavior of α_h as a function of the horizon h for the RAM and AR mechanisms. The short-term pension payments absorb a lower fraction of the unexpected shocks than the longer-term pension payments. In case of the RAM mechanism with $N = 10$ α_h increases the first ten years linearly and remains equal to one afterwards. For the AR mechanism it holds that the smaller ρ is the bigger α_h on every horizon h. Also note that, in terms of exposure, the RAM mechanism with $N = 10$ and the AR mechanism with $\rho = 0.8$ do not differ enormously.

2.5. Induced risk profiles

The risk profile a participant has, depends on the fund's asset allocation and the smoothing mechanism that is applied. As explained in the previous section, w can be interpreted as the equity exposure you want to give to the youngest participants. The level w is identified by the condition that $\alpha_h \to 1$ as $h \to \infty$. Standard calculations show that this implies that the cumulative risk an individual faces over an horizon h is described by the volatility[2]

$$w\sigma_S\sqrt{\sum_{l=1}^{h}\alpha_l^2}.$$

This volatility readily leads to prediction intervals for pension payments.

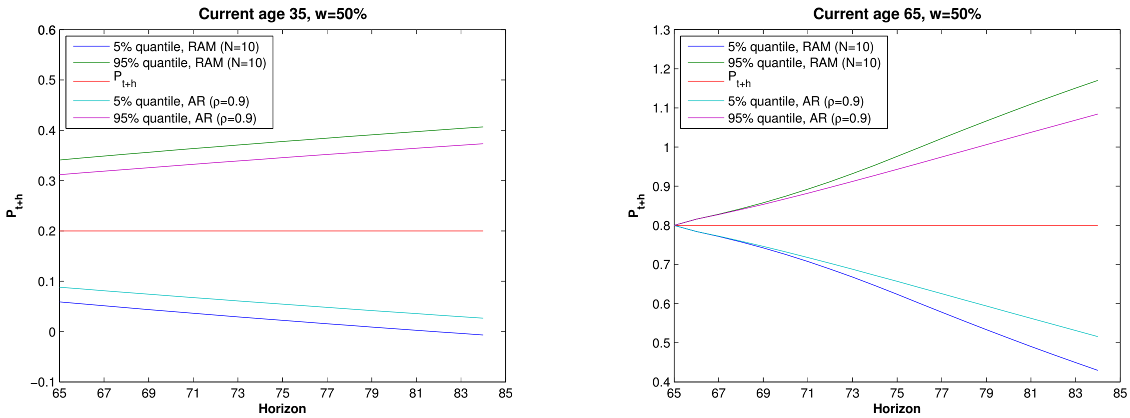

Figure 2: This figure displays the risk in actual pension payments (pension entitlements), $P_{a,t+h}$ ($B_{a,t+h}$) for a 35 year old (left) and a 65 year old participant (right), where $w = 50\%$ and with the RAM ($N = 10$) mechanism and AR ($\rho = 0.9$) mechanism. The figures are based on a hypothetical 2% accrual per year, i.e., 0.20 for the 35 year old and 0.80 for the 65 year old.

Consider two individual participants with age 35 and 65. Figure 2 shows that the pension entitlements of a 35 year old are much more uncertain than those of a 65 year old, due to the larger exposure to financial market shocks. Note that the smoothing mechanism, either RAM or AR, reduces the risk of the retirees while young participants are exposed to more risk, which is in line with the standard Merton life-cycle consumption model with labor income. Both the left and right panel of Figure 2 show that the exposure to unexpected portfolio shocks is somewhat higher for the RAM mechanism with $N = 10$ than for the AR mechanism with $\rho = 0.9$.

Besides the horizon of the pension payments and the mechanism that is used, w and the amortization period (N and ρ) affect the riskiness of the pension benefits as well. Figure 3 illustrates that a smaller value of w implies that pension benefits become less risky at all horizons h.

[2]This assumes that the shocks are independently and identically distributed with volatility σ_S. This holds for the assumed Black-Scholes model.

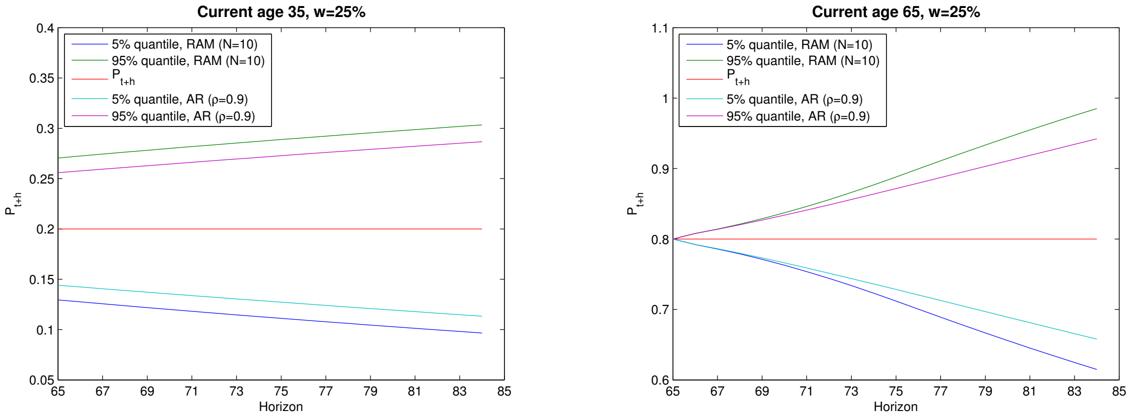

Figure 3: This figure displays the risk in actual pension payments (pension entitlements), $P_{a,t+h}$ ($B_{a,t+h}$) for a 35 year old (left) and a 65 year old participant (right), where $w = 25\%$ and with the RAM ($N = 10$) mechanism and AR ($\rho = 0.9$) mechanism. The figures are based on a hypothetical 2% accrual per year, i.e., 0.20 for the 35 year old and 0.80 for the 65 year old.

Finally, the amortization period influences the exposure to unexpected shocks.

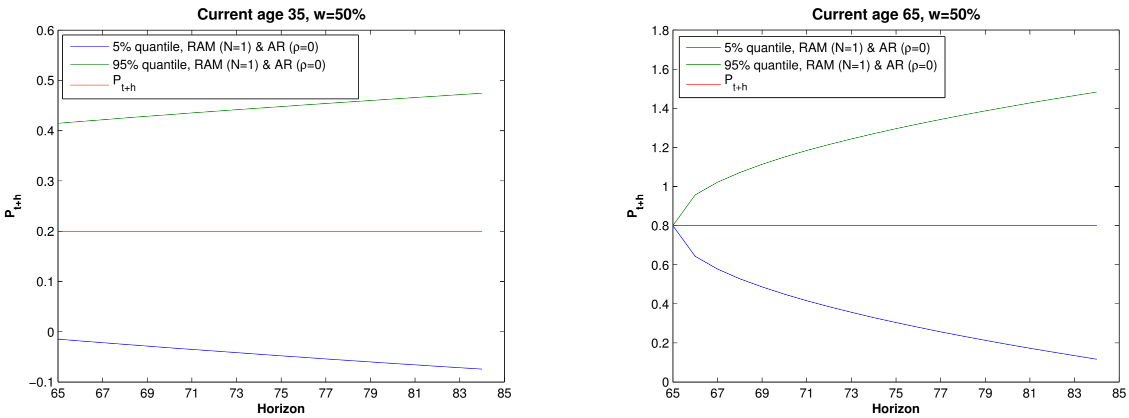

Figure 4: This figure displays the risk in actual pension payments (pension entitlements), $P_{a,t+h}$ ($B_{a,t+h}$) for a 35 year old (left) and a 65 year old participant (right), where $w = 50\%$ and with the RAM ($N = 1$) mechanism and AR ($\rho = 0.0$) mechanism. The figures are based on a hypothetical 2% accrual per year, i.e., 0.20 for the 35 year old and 0.80 for the 65 year old.

The effect of smoothing is that pension payments on shorter horizons are exposed to only a fraction of the shocks. In the extreme case of no smoothing ($N = 1/\rho = 0$), we have $\alpha_h = 1$, for all h and for both mechanisms, which results in the risk profiles depicted in Figure 4.

Note that the above figures can also be used to find the level of risk exposure that optimally suits the preferences of the participants. While the smoothing mechanism is likely to be prescribed in the regulation for Dutch supplementary pensions, the value of w can probably be chosen freely. In DC contracts also the adjustment parameters α_h can be set to accommodate the preferences of participants.

3. Market-consistent value of pension benefits

Market-consistent valuation of pension benefits determines the required sum of money that is needed to be able to honor the actual pension promises. In other words this equals the amount required to set up a replicating portfolio, i.e., an investment strategy with exactly the same cash flows. This amount plays an important role in the valuation of pension promises and in the Solvency and FTK legislations. In the context of collective pension funds it is moreover used to determine whether or not sufficient funds are available to increase benefits or to cut rights.

The market-consistent value at time t of a pension payment paid h periods from now can be derived in the following way.[3] We use the standard pricing result that market-consistent values can be obtained as risk-neutral ("under $\mathbb{Q}$") expected discounted pay-offs. Thus, the market-consistent value of the pension payment at horizon h of an individual indexed by his age a, is given by, for $\max(1, a_r - a) \leq h < a_{max} - a$,

$$\begin{aligned} V_{a,t}^{(h)} &= E_t^{\mathbb{Q}}(e^{-r^f h} P_{a,t+h}) \\ &= e^{-r^f h} B_{a,t} E_t^{\mathbb{Q}}(1 + w \sum_{j=1}^{h} \alpha_j [r_{t+h-(j-1)} - (r^f + \lambda)]) \\ &= e^{-r^f h} B_{a,t}(1 - w\lambda \sum_{j=1}^{h} \alpha_j) \\ &\approx B_{a,t}[1 + r^f + w\lambda h^{-1} \sum_{j=1}^{h} \alpha_j]^{-h} \\ &= \frac{E_t(P_{a,t+h})}{(1 + r^f + w\lambda\overline{\alpha}_h)^h}, \end{aligned} \tag{2}$$

where $\overline{\alpha}_h = h^{-1} \sum_{j=1}^{h} \alpha_j$ equals the average exposure over horizon h. Here we used $E_t^{\mathbb{Q}} r_{t+h} = r^f$, which holds by definition under $\mathbb{Q}$.

Thus, $V_{a,t}^{(h)}$ denotes the market-consistent value at time t of a pension payment at time $t+h$, P_{t+h}, if the participant is indeed entitled to a pension payment at time $t+h$, i.e., if $\max(1, a_r - a) \leq h < a_{max} - a$. Moreover, we denote by $V_{a,t} = \sum_{h>0} V_{a,t}^{(h)}$ the market-consistent value of all pension promises at time t to the individual of age a.

[3] In Section 5, we mention some possible extensions of this work, in particular, the consequences of inflation risk.

In Section 2.5, we demonstrated that the exposure to unexpected shocks in the pension portfolio depends, among others, on the horizon of the pension payment, the investment mix, and the smoothing parameter (N or ρ). We now discuss how each of these affect the current market-consistent value of a future pension payment. This also illustrates (2).

First, consider a payment one year from now ($h = 1$). This payment is only exposed to a fraction, $w\alpha_1$, of the equity shock. Said differently, the expected return on a pension benefit paid one year from now equals $\mu(1) = r^f + w\lambda\alpha_1$. Therefore, since the payment is not exposed to all the risk, only a part of the financial market risk premium can be incorporated in the market-consistent discount factor. Thus, the market value of a pension payment one year from now is given by

$$V_{a,t}^{(1)} = \frac{B_{a,t}}{1 + r^f + w\lambda\alpha_1}.$$

The expected pension benefit over two periods from now absorbs α_2 of the shock in the first period and α_1 of the shock in the second period. Thus, the expected return of a pension benefit on a two-year horizon equals the average expected return over a two-year horizon for the replicating portfolio, i.e., $\mu(2) = r^f + w\lambda(\alpha_1 + \alpha_2)/2$. Given the fact that the pension benefit absorbs risk in period one and two, the corresponding discount factor is based on the average of α_1 and α_2 times $w\lambda$. Thus, the market-consistent value of the pension benefit paid two years from now is

$$V_{a,t}^{(2)} = \frac{B_{a,t}}{(1 + r^f + w\lambda\overline{\alpha}_2)^2}.$$

Note that since α_h is increasing in the horizon of the payment (h), $\overline{\alpha}_h$ also increases over time but less steeply, see Figure 1.

We consider two simple, but important, special cases. In case of no smoothing period ($N = 1$, $\rho = 0$, or, in general, $\alpha_h = 1$ for all $h > 0$) the relevant discount rate is $1 + r^f + w\lambda$, which shows that the full equity premium can be used in the discount factor, as all pension payments are fully exposed to financial market risk. Secondly, in case of a riskless portfolio ($w = 0$), the discount rate is the riskless rate for all horizons.

Equation (2) can be interpreted both in terms of a nominal and a real pension. This is merely a matter of defining the unit in which pension payments are measured (nominal Euros or a real price index). When switching from a nominal to a real contract, the nominal interest rate must be corrected for inflation. In formulas, this would mean that $r^f_{nominal} = r^f_{real} + \pi$ or $r^f_{real} = r^f_{nominal} - \pi$, where π denotes the inflation and where we ignore inflation risk. These relations become (much) more involved in case of interest rate risk.

In Figure 5 the flat nominal and real interest rate are depicted along with the market-consistent discount curves for the RAM and AR mechanism.

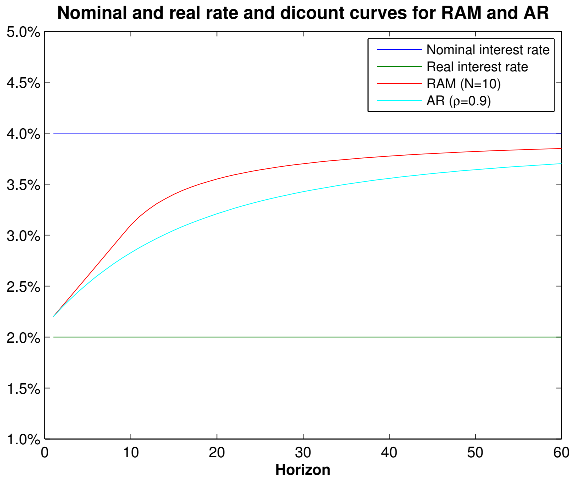

Figure 5: The constant nominal and real term structure and the market-consistent discount rate for the RAM ($N = 10$) and AR ($\rho = 0.9$) mechanism. In this specific example $r^f_{nominal} = 4.0\%$, $\pi = 2.0\%$, $w = 50\%$, and $\lambda = 4.0\%$.

The red line lies above the light blue one since $\overline{\alpha}_h$ is bigger for all h for the RAM ($N = 10$) mechanism than for the AR ($\rho = 0.9$) mechanism (see Figure 1). For longer horizons the red and light blue line approach 4.0% as the horizon dependent premium approaches $w\lambda = 2.0\%$.

4. The investment strategy of one-person and collective pension funds

Up to now individual pension benefits, the associated risks, and corresponding market-consistent value have been considered. In this chapter the investment strategy of a pension fund will be discussed. In Section 4.1 two pension funds are considered each consisting of only one individual. This analysis will then be extended in Section 4.2 to a situation of pension funds consisting of several individuals.
Throughout this chapter we assume that at $t = 0$ there are exactly enough assets to match the liabilities. In other words the initial funding ratio is equal to 100%.

The most important variables that have been and will be employed are summarized as follows:

- $B_{a,t}$: the pension entitlement of individuals of age a at time t. Note that the sum is taken over all individuals of age a.
- $P_{a,t+h}$: the actual pension payment to individuals of age a at time $t + h$. Again, the sum is taken over all individuals of age a.
- $V^{(h)}_{a,t}$: the market-consistent value at time t of the pension payments at time $t + h$ to individuals of age a ($P_{a,t+h}$).

- $V_{a,t}$: the market-consistent value of all future pension payments to individuals of age a at time t.

Given $V_{a,t}^{(h)}$, the total market-consistent value at time t of the pension payments at horizon h summed over all participants, is given by:

$$V_t^{(h)} = \sum_a V_{a,t}^{(h)}.$$

The market-consistent value at time t of the pension payments summed over all individuals a and horizons h, which is equal to the value of the liabilities, is given by:

$$V_t = \sum_a \sum_{l=1}^{h} V_{a,t}^{(l)} = \sum_{l=1}^{h} \sum_a V_{a,t}^{(l)} = \sum_{l=1}^{h} V_t^{(l)}.$$

Now consider a pension fund that implements an investment strategy such that the induced exposure to each of the age groups, equals the exposure that has been promised, i.e., equals $w\alpha_h$ at horizon h. We also assume that we take a closed fund perspective in the sense that the fund is not allowed to fulfill its promise to the current age groups by exposing future participants to risk. Then, as $w\alpha_h$ is the promised exposure to the next period's financial market shock of pension entitlements at horizon h, which have a total market-consistent value of $V_t^{(h)}$, the collective exposure must equal

$$w \sum_h \alpha_h V_t^{(h)}.$$

Thus, in order to honor the pension promise, the pension fund should not invest w of the portfolio in the equity index, but less, namely $w\omega_t$, such that

$$w \sum_h \alpha_h V_t^{(h)} = w\omega_t V_t,$$

i.e.,

$$\omega_t = \frac{\sum_h \alpha_h V_t^{(h)}}{V_t}. \tag{3}$$

We consider two special cases. In the absence of smoothing, i.e., $\alpha_h = 1$, for $h > 0$, we find $\omega_t = 1$. In such case the effective exposures of all age groups correspond to those of individual Defined Contribution contracts with financial market exposure w.
For a very young fund ω_t is (close to) unity if $V_t(h)$, for which $\alpha_h < 1$, is negligible compared to the total liabilities V_t. On the other hand in case of an extremely old fund, for which $V_t^{(h)} = 0$ for $h \geq 2$, we have $V_t = V_t^{(1)}$ and thus $\omega_t = \alpha_1$: the exposure promised at horizon $h = 1$.

4.1. The investment strategy of a one-person pension fund

Before more general pension fund compositions will be treated, two pension funds that consist of only one participant will be considered: the first with a participant of age 35 and the second with a participant of age 65 at time $t = 0$. We consider the RAM and the AR smoothing mechanism, which specifies the pattern for α_h. Figure 6 displays the investment mix, $w\omega_t$, that the pension fund must apply such that the ability of the individual to absorb shocks, defined by α_h, is taken into account. An older participant is exposed to less financial market risk than a younger one since α_h is increasing in h.

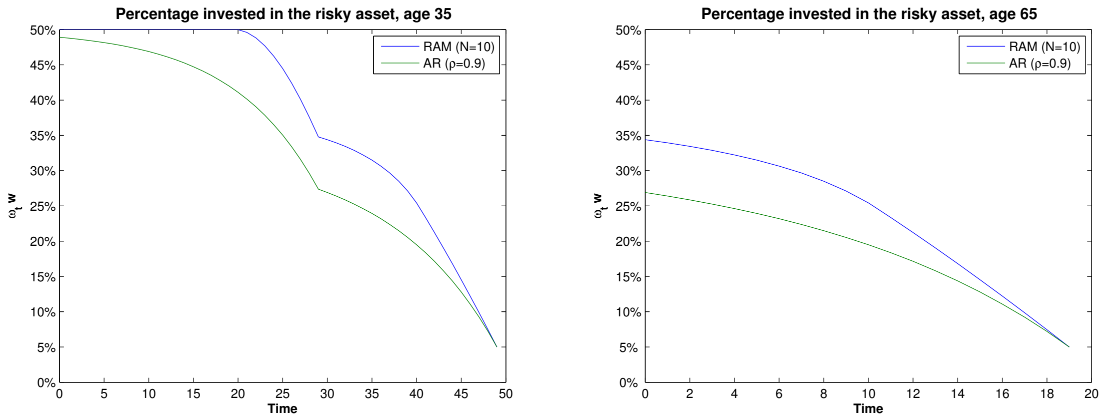

Figure 6: The left figure describes the investment mix of the pension fund, $w\omega_t$, over time for the RAM ($N = 10$) and AR ($\rho = 0.9$) mechanism in case the fund only has a 35 year old participant. The right figure represents the same, but now for a fund having only a 65 year old participant.

First of all we note that the right figure is embedded in the left one, because at time $t = 30$ a 35 year old has become 65 years old, which is the situation of the right figure at $t = 0$. Because of this feature only the left figure will be discussed.

Up to time $t = 20$ $w\omega_t = 50\%$ in case of the RAM mechanism, as all α_h's are equal to one and consequently (3) is equal to one as well. At $t = 21$ the individual is 56 years old, which implies that the pension payment at 65 is within 10 years from that moment. Therefore, not all α_h's are equal to one, more specifically $\alpha_9 = 0.9$, which lowers the value of ω_{21} under one (see (3)). In the following nine years this process continues and the amount invested in stocks decreases further.

In order to generate intuition for the required equity exposure as of $t = 21$ note the similarity between (3) and the definition of the duration of the liabilities in which case $\alpha(h) = h$. Therefore, for $h < N$, ω_t and the duration only differ a factor N. In more general cases we will refer to ω_t as the reweighted duration. The weight for longer maturities will be smaller than in standard duration computations. From ages 55 to 64 the duration of the pension payments to be made drops by one year every year (ignoring discounting effects). The reweighted duration drops slower because long maturities are capped weights. As of age 65 the duration of the remaining payments drops by half a year every year (again ignoring discounting). Likewise the reweighted exposure drops at a slower rate than before retirement which explains the kink in Figure 6. The last positive number is equal to $w \cdot \omega_{49} = w \cdot \frac{\alpha_1 V_{49}^{(1)}}{V_{49}^{(1)}} = w \cdot \alpha_1 = 5.0\%$ (see (3)).

The same line of reasoning applies to the AR mechanism. The main difference is that the kink at $t = 20$ is absent, because the development of α_h for the AR mechanism is smooth, whereas the RAM mechanism has a kink at $h = 10$ (see Figure 1).

4.2. The investment strategy of a collective pension fund

The case of a pension fund consisting of only one individual can easily be extended to pension funds representing several ages. In Figure 7 the percentage of the portfolio that has to be invested in the risky asset to assure that the risk exposure of the assets coincides with the risk exposure of all benefits according to (3) is depicted over time for a homogeneous fund where the ages of 25 up to 85 are represented. We define a fund to be homogeneous if the number of participants in each group is equal and all participants have had pension accruals of a fixed percentage per year as of age 25.

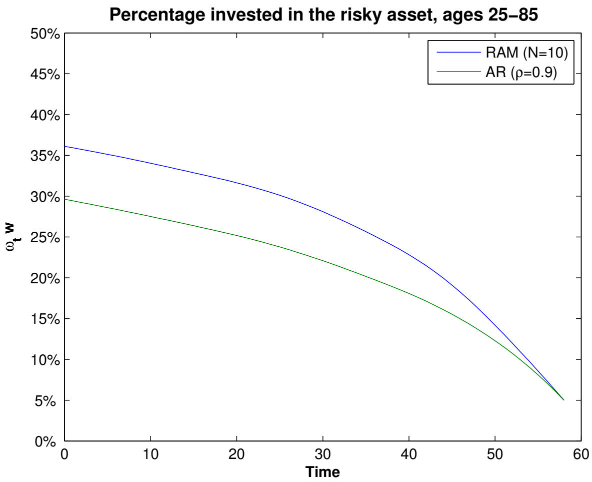

Figure 7: This figure describes the fraction that a pension fund must invest in the risky asset, $w\omega_t$, over time for a homogeneous pension fund where the ages of 25 up to 85 are represented once. The blue line corresponds to the RAM ($N = 10$) mechanism and the green line to the AR ($\rho = 0.9$) mechanism.

Figure 7 shows a decreasing pattern, which is due to the fact that as time proceeds, participants age (and no new participants enter since a closed pension fund is considered) and the risk absorption capacity of the pension fund as a whole decreases. In order to align this decreasing capacity with the investment policy of the fund, a smaller part of the assets has to be invested in the risky asset.

At time $t = 58$ the risk absorption capacity reaches its minimum as the only participant left in the pension fund is the 84 year old, which was 26 year old at $t = 0$. This participant has to absorb the full equity shock in the pension benefit at 85 and therefore the equity exposure of the fund as a whole is equal to $\omega_t \cdot w = \alpha_1 \cdot w = 10\% \cdot 50\% = 5\%$.

The fact that the blue line lies above the green one on every horizon is due to the fact that α_h is bigger for all h in case of the RAM mechanism with $N = 10$ compared to the case of the AR mechanism with $\rho = 0.9$ (see Figure 1).

In Figure 8 the development of the percentage of the portfolio invested in the risky asset for a green fund, ages 25 up to 65 represented once, and for a grey fund, ages 65 up to 85 represented once, are displayed.

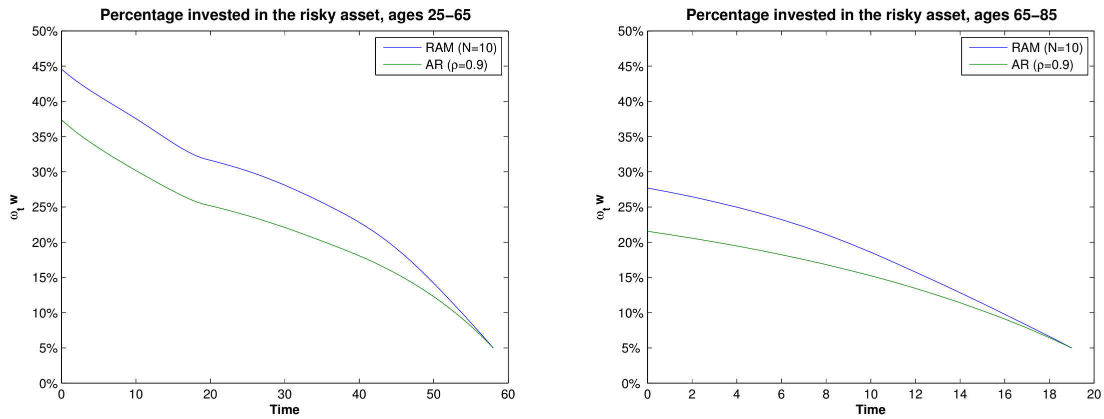

Figure 8: These figures describe the fraction that a pension fund must invest in the risky asset, $w\omega_t$, over time for a green fund, ages 25 up to 65 represented once (left figure) and for a grey fund, ages 65 up to 85 represented once (right figure). The blue line corresponds to the RAM ($N = 10$) mechanism and the green line to the AR ($\rho = 0.9$) mechanism.

In case the fund consists of workers only (left figure) at time $t = 0$ and hence the fund is younger compared to the fund in Figure 7, the amount invested in the risky asset is bigger for the first 20 years. After 20 years the pension fund composition of both funds coincides and the blue and green lines are the same in Figure 6 and 7.

In case the initial fund consists of retirees only the initial risk exposure is lower compared to Figure 7, since the risk absorption capacity of elderly is lower. The course of the lines as well as the end points are comparable to Figure 7. The maximum horizon is now equal to $t = 19$, since the 65 year old at $t = 0$ has then reached the age of 84.

5. Concluding remarks

In this paper we have formalized and analyzed a simple version of the new Dutch "soft real rights" contract. We have shown that the contract contains attractive life-cycle features and implies decumulation of pension wealth using variable annuities with horizon-dependent risk exposure. We have also derived market-consistent discount rates for the entitlements. Risk-free discounting does not yield market-consistent valuation because the soft rights are quite different from guarantees. Likewise, discounting against an expected portfolio return is misleading because the shocks on the financial markets are smoothed over a 10 year period. We show that the market-consistent discount curve for the new Dutch contract can be written as the nominal risk-free curve minus expected inflation and possibly an inflation risk premium, plus a horizon-dependent risk premium. The horizon dependent risk premium reflects the fact that the exposure to investment risk for nearby pension income is much smaller than for pension income far in the future.

We conclude this paper by discussing a few possible extensions.

5.1. Interest rate risk

The present paper assumes the absence of interest rate risk. Extending the analysis is obviously relevant in a world with interest rate risk, where a fund could also possibly benefit from taking that risk. This would on the one-hand induce a second financial market state variable, and on the other hand affect the hedging strategies of funds.

5.2. Longevity risk

In this paper we have also abstracted from longevity risk. Micro longevity risk, which is by definition supposed to be diversifiable and not priced, is easily taken into account. This is not the case for macro longevity risk, certainly not if this risk is partly hedgeable in the financial market. Like interest rate risk, such an extension would on the one hand introduce a second risk factor in the analysis and on the other hand affect the value of the liabilities in a systematic way.

5.3. Inflation risk

By allowing for inflation risk the real term structure is obtained by subtracting the expected inflation and the inflation risk premium, which is typically considered to be positive, from the nominal term structure. Furthermore, as individual or pension fund ambitions are likely to be stated in real terms, an assessment of inflation-hedge properties of the available investment assets is required.

5.4. Open fund/continuity perspective

In this paper a closed fund perspective has been adopted, which implies that the fund cannot share risks with future generations, a government, and/or a sponsor. Consequently, it has to honor the promises to its participants, implying that upon liquidation of the fund it has to have a funding ratio of exactly 100%. This in turn has important implications for the investment policy of the fund over time.

By abandoning this point of view and considering an open fund one has to take, among others, pension premiums, corresponding accumulation of pension rights, and the risk absorption capacity of future participants into account.

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