Optimal Inflation Risk Sharing Among Pension Fund Participants

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Abstract

We study inflation hedging from a pension fund perspective. Retired participants in a fund have a demand for inflation protection but face a thin market for inflation-linked bonds. Meanwhile, young participants in a fund have their human capital as a natural inflation hedge. The young members of the fund may be willing to offer some protection to the old, and thus contribute to completing the market, depending on the price at which the inflation risk is internally traded. Our analysis reveals conditions under which a mutually beneficial internal trade can exist. The internal transfer price will, however, generally deviate from the market price of the inflation-linked bond. Using data calibrated from historical inflation rates, interest rates, and stock returns over a long period, we find that the internal real interest rate will likely deviate from the market real interest rate.

Nederlandse samenvatting

Inflatie blijkt al een aantal jaren een hardnekkig fenomeen. Ook in de geschiedenis zijn er meerdere episodes van persistente inflatie. Langdurige inflatie kan de koopkracht van uitkeringen ver uithollen. Een koopkrachtig pensioen behoort tot de doelstellingen van het Nederlandse pensioenstelsel, maar onder de Wtp hebben pensioenfondsen slechts beperkte middelen om deelnemers te beschermen tegen inflatierisico. Beschermingsrendement is voornamelijk ingericht om renterisico af te dekken op basis van een nominale rentetermijnstructuur. Het additioneel expliciet afdekken van het inflatierisico is mogelijk via een direct beschermingsrendement waarbij direct ten behoeve van een cohort in indexleningen wordt belegd.

Theoretisch zijn indexleningen het ideale instrument voor een beschermingsportefeuille. Maar doordat deze producten slechts op kleine schaal worden uitgegeven door Europese overheden, is het aanbod zeer beperkt. De potentieel hoge vraag en het beperkte aanbod heeft waarschijnlijk ook invloed op de prijsvorming. Indexleningen worden als duur beschouwd vanwege het lage verwachte rendement.

We willen een aantal punten maken. Ten eerste, naast de markt voor indexleningen en inflatieswaps, kan ook een interne markt voor inflatiebescherming binnen een pensioenfonds welvaartsverhogend werken, voor zowel jongeren als ouderen. Vanwege hun menselijk kapitaal hebben jongeren een geringere blootstelling aan inflatierisico dan ouderen. Beiden kunnen voordeel hebben wanneer het fonds een deel van de blootstelling van ouderen overhevelt aan jongeren. Ons tweede punt is dat de vergoeding die jongeren daarvoor ontvangen niet noodzakelijkerwijs de marktprijs voor indexleningen is. De marktprijs weerspiegelt niet altijd de evenwichtsprijs voor de uitruil van inflatierisico tussen oud en jong. Belangrijkste reden daarvoor is dat bij interne herverdeling van risico de marktprijs vraag en aanbod binnen de financiële markt weergeeft, maar niet de vraag vanuit de pensioendeelnemers.

We illustreren de meerwaarde van een interne markt, en de omstandigheden waaronder die meerwaarde ontstaat, binnen een gestileerd theoretisch model. In dat model onderscheiden we meerdere typen beleggers die elk een verschillende blootstelling aan inflatierisico hebben. Parameters van het model zijn gecalibreerd op basis van lange historische tijdreeksen voor de VS.

1 Introduction

Pension fund investments are exposed to traded and non-traded risk factors. One major non-traded risk factor that has been of concern since the end of 2021 is inflation, which erodes the purchasing power of a stable nominal pension. For long-term investors saving and investing for a retirement income, the inflation risk exposure varies with age. When they are young, investors have a large share of human capital, which tends to increase in nominal terms with inflation, such that they have limited exposure to inflation shocks in real terms. Hence, young people have a natural hedge against inflation. Human capital decreases with age, while financial wealth increases. With nominal investments, the exposure to inflation risk thus increases over time. This heterogeneity in exposure to inflation is documented, for example, in Doepke and Schneider (2006).

Hedging inflation risk requires real assets such as inflation-linked bonds (ILBs) or inflation swaps. When ILBs are in ample supply, investors can invest in these securities and hedge part of their inflation risk. When the market for ILBs is thin and illiquid, though, investors are unable to implement such a hedge on a large scale. That is why we call inflation a 'non-traded' risk factor.[1] In theory, the human capital of young investors may provide additional inflation hedging. There are two problems with this, however: human capital is not tradable; and young investors cannot issue inflation derivatives. To resolve this market incompleteness, pension funds can bring young and old together and forge a mutually beneficial deal, in which the old get additional inflation protection and the young receive an insurance premium.

We look at the merits and problems of an internal insurance market within a pension fund where the young insure the inflation risk of the old at a premium. Internally, the fund can create an inflation derivative. It pays inflation to the old in return for a fixed premium to the young. We aim to determine what that insurance premium should be and the amount of hedging that would benefit both young and old, and which also takes into account a possible outside option of buying the insurance on the capital market. In short, we study partially segmented markets. Inflation-linked assets are available to all investors in the financial markets, whether through pension funds or by other means. In addition, pension funds can create an internal market where young and old can trade their non-tradable human capital for inflation protection for older participants.

Pension funds aim to provide a stable pension to their participants. In economic terms, stable should be interpreted in real terms. Given a positive average inflation, a fund could define the annuity payout to deliver a cashflow stream that grows at the same rate as expected inflation. This accounts for stable inflation, though, not for inflation risk. It does not

1 For Dutch pension funds, less than 2% of their investments in fixed income are in index-linked bonds. Source: DNB.nl, Statistieken, "Belegd vermogen pensioenfondsen", Table 8.9, "Marktwaarde inclusief derivaten", 2025Q1. In addition, the inflation-linked assets are not tied to inflation in the Netherlands, but usually to inflation in the eurozone.

hedge against unexpected inflation or take into account changes in expected inflation. The fund could also offer a variable annuity based on exposure to equity. It will then adjust the annuity payout if equity excess returns are positive. With sufficient risk exposure, this could lead to a growth in payouts that is equal to expected inflation. But it involves a lot of risk, since equity returns and inflation are not highly correlated, at least not over short horizons.

Our analysis of an internal insurance market is motivated by the new Dutch pension law (Wtp = "Wet toekomst pensioenen" in Dutch). A Dutch pension fund holds a collective portfolio and allocates the returns to all participants. In what is called the "solidarity contract" (SPR = "solidaire premieregeling" in Dutch), the return for every participant is split into a hedge return and an excess return. A small portion of the premium is used to finance a solidarity buffer for exceptional risks. The hedge return has been designed to hedge against interest rate risk in order to deliver nominally stable projected payouts to the participants.[2] Under the current law, this is the nominal yield curve. The real yield curve cannot currently be used as it is perceived as being less reliable due to the illiquidity and thinness of the market for ILBs. After accounting for the hedge return, the remaining excess return is allocated to all participants with age-dependent exposures.[3] Although the pension fund must ensure that the realized hedge return does not deviate too much from the required hedge return, it does not need to hold the assets that generate the hedge return in its collective portfolio outright. The pension fund is allowed to transfer risks between participants. These risks include longevity risk, inflation risk, and default risk due to leveraged positions at young age. In the remainder of the paper we will limit our discussion to inflation risk, but note that the arguments apply to any risk transfer that occurs within a pension fund. The regulation stipulates that the pension fund must use market prices when transferring risks internally. This means that in the event that the fund decides to transfer inflation risk from older participants to younger ones, it must use the prices implied by the traded financial instruments to calculate the premium. An exception is the solidarity reserve itself, for which it is possible to conceive of an arrangement whereby older participants put in more premium in exchange for partial inflation protection.

We investigate the impact of using inflation-linked assets (ILBs or swaps) to maximize the welfare of pension fund participants. It is not clear that the market price would lead to a Pareto improvement, since the market is thin and not all investors participate in this market. This discussion touches upon financial fairness, which dictates that any transfer within a fund must be at market prices. Financially fair transfers provide everyone with the same monetary value, while still allowing, however, for the potential shift of undesirable risk. Our

2 For example, if the fund projects it will pay € 1 in T years from now to a retiree, it can (hypothetically) buy a T-year discount bond that will guarantee that cash flow.

3 It is considered an excess return because initially all participants get 100% of the hedge returns. In the second stage, older participants may have a low exposure to the excess return—such that their pension payout is relatively stable—and younger ones may even have an exposure above 100% to the excess return. One can rewrite the portfolio in the more familiar academic format with x% in a hedge portfolio and $100 - x$% in a speculative portfolio.

analysis focuses on welfare improvements. That criterion is vulnerable to the criticism that it is not objective because we don't know the agent's utility function. But that critique would still apply to financially fair contracts, where the welfare gains still depend on preferences. The pension fund must measure preferences anyhow to determine the risk exposure of its investment portfolio. We analyze the price at which insurance against inflation risk should be set in order to make a fund's investors better off.

To answer the question on the welfare impact of internal insurance markets, we study the following market setups.

1. *Segmented market*: The inflation-linked asset is traded on financial markets by capital market investors, but it is not traded by the pension fund participants. This establishes an equilibrium price for the inflation-linked asset that is determined only by the capital market participants. This is the benchmark setup.

2. *Integrated market*: All assets are freely traded by all investors. This setup establishes the equilibrium price and traded quantities in an unrestricted capital market. To implement this market, the pension fund can trade on the capital market on behalf of the pension fund investors. We assume that there are no information or agency problems.

3. *Policymaker imposed hedging in the internal market:* The pension fund imposes some amount of hedging at the price observed in financial markets. Older pension fund participants buy insurance from younger participants through a sharing rule enforced by the pension fund. Even though the fund is not actually trading the inflation-linked asset, it uses the price from the external financial markets.

4. *Internal market*: The inflation-linked asset is traded independently on the capital market and on the internal pension fund market. We get two equilibrium prices for the asset since cross-trading is not allowed. The internal price may thus differ from the price on financial markets. This scenario deviates from the current Dutch pension rules.

For pension funds, the first three options are all consistent with the new Dutch pension law in the sense that they only involve traded assets with observable prices. The fourth option, *internal market*, involves an allocation of risk and return that is not at market prices but relies instead on preferences of different age cohorts in the pension fund. It may provide partial inflation protection in the solidarity buffer as an arrangement that benefits both old and young participants. We analyze the third option to evaluate the effect of pricing, either internal or external. Even though the price of the ILB in this case is a market price, that price may differ from the *integrated market* scenario when the size of the pension sector in the economy is large enough and pension fund demand differs from the capital sector, for example, because of differences in demographics.

Our main findings are as follows. For the *internal market* scenario, the choice of the internal price at which the mutual insurance between pension fund participants takes place is crucial. Using the price from the capital market in the internal market is not necessarily optimal. At that price, some investor groups may suffer a utility loss as a result of the introduction of the internal market. And even if all investors improve in terms of utility, they might not all be happy with the amount of insurance. This is due to the heterogeneity between the group of capital market investors, which sets the price in the capital market, and the group of pension fund participants, which has to trade at that price. If the two groups are too different, some of the pension fund participants will suffer a utility loss. The trick, then, is to find the equilibrium price in the internal market at which all investors are better off. (Intuitive proof: Going back to their payoff from the completely segmented market, in which they have a zero position in the inflation-linked asset, is possible.) This is the price at which no investor wants to change the amount of insurance. The internal market and its rules are accepted by the pension fund investors. In the *integrated market* scenario, all pension fund investors are better off than in the *segmented market* scenario. The intuitive proof is again that going back to a zero position in inflation-linked assets is an option. Risk sharing now also takes place among pension fund investors and capital market investors. When comparing the *integrated market* and *internal market* setups in terms of equilibrium prices and insurance amounts, there is not necessarily a clear ranking. It might be the case that the *integrated market* Pareto dominates the *internal market*, but it may also be the case that there is no Pareto dominance. In the latter case, one would need a social planner who assigns weights to the pension fund investors to rank the two markets.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model setup. Section 4 discusses the model calibration. The results are discussed in Section 5, and Section 6 presents our conclusion.

2 Literature

Risk sharing within a pension fund has been previously analyzed. For example, Gollier (2008) calculates the welfare gains from intergenerational risk sharing. Our analysis differs, though, in the level of heterogeneity of market participants and in the setup of the financial markets. We have heterogeneity that takes into account the different effects of inflation shocks on the financial wealth of investors. Moreover, our risk sharing is intragenerational, not intergenerational. In our model, generations that are alive at the same time can trade. We do not consider the case of investing on behalf of distant generations that are not yet born.

Chen et al. (2023) explicitly consider the sharing of inflation risk in the absence of market instruments. Their model has two overlapping generations. Human capital is fully indexed by the price level, making it free of inflation risk, just as in our model. With just two generations, they calibrate all parameters such that a period comprises 30 years, which means each generation has 30 years of working age and 30 years of retirement. In our model, there will be a traded instrument to hedge inflation, but its market price need not be equal to an internal transfer price.

Balter et al. (2020) provide a review of optimal risk sharing in pension plans. Intragenerational risk sharing can only add value when some risks are non-traded. Intergenerational risk sharing leads to welfare improvement since a pension fund ("social planner") can pre-invest contributions of future participants before they enter the fund. While the scheme is welfare-improving ex-ante, the risk sharing introduces the political risk that ex-post, the future entrants will refuse to enter if the fund has experienced poor returns. In some cases, this continuity risk can be mitigated (or eliminated) by reducing the size of the "pre-investment". But the smaller the pre-investment, of course, the lower the welfare gains.

Many authors impose the restriction that risk sharing should be financially fair. Bao et al. (2016), for example, define a contract as financially *fair* if the market values of the risk positions before and after risk sharing are equal. This would imply that any risk shifting in a pension fund should be at the observed market prices. One problem for Dutch pension funds is that inflation-linked assets are, at best, tied to eurozone inflation and not Dutch inflation. A certain amount of basis risk, the difference between eurozone and Dutch inflation, will thus always exist. As discussed in the report issued by the independent Dutch government committee (Commissie Parameters, 2022), the two have traditionally been closely related, but that changed just after the start of the war in Ukraine.

Our segmented markets analysis of inflation-linked assets relates to the growing literature on market macrostructure that studies the implications for equilibrium asset prices of the heterogeneity among large players (such as pension funds) and institutional settings in financial markets (Haddad and Muir, 2025). As an example, Greenwood and Vissing-Jorgensen (2018) explore the price impact of the pension and insurance sector on bond markets.

3 Model

3.1 Market Design

We consider three types of investors: *old* pension fund participants (*O*), *young* pension fund participants (*Y*), and *capital market* investors (*C*). Pension fund participants are all in the same pension fund and delegate trading on their behalf to the pension fund. Three different assets are traded on financial markets: stocks (*S*), nominal bonds (*B*), and inflation-linked bonds (*I*). In an integrated financial market, all three investor types can trade with one another in all three assets. Under different pension design settings, the financial markets will be segmented, because the pension fund participants do not trade all assets or are restricted in their demands. In particular, they may not always be able to trade inflation-linked bonds. Our model is highly stylized, with just two periods: now ($t = 0$) and the future ($t = 1$). It ignores many issues in life-cycle investing but is nevertheless sufficient for making our points.[4]

Stocks have an uncertain gross return $\tilde{R}_S$. A tilde indicates a random variable with realization in the future. Gross returns are one plus a rate of return, so an investment of X_S euros in the market generates a payoff $X_S\tilde{R}_S$. The nominal bond has an exogenously given risk-free gross nominal return R_B. As bonds are nominally risk-free, R_B does not need a tilde superscript. By exogenously setting the risk-free rate and the distribution of market returns, we implicitly assume an infinitely elastic supply of both assets.

The payoff of the inflation-linked asset is equal to the next period's price level $\tilde{\Pi}$. With the current price level normalized to one, $\tilde{\Pi}$ is gross inflation—that is, one plus the rate of inflation. The inflation-linked asset will be in zero net supply. Its price is P_I, and its gross nominal return is defined as $\tilde{R}_I = \tilde{\Pi}/P_I$.[5] The equilibrium price P_I is endogenous and will differ across designs for the pension fund investments.

Real returns are denoted by a lowercase letter, $\tilde{r}_S = \tilde{R}_S/\tilde{\Pi}$, $\tilde{r}_B = R_B/\tilde{\Pi}$, and $r_I = \tilde{R}_I/\tilde{\Pi} = 1/P_I$. In the model, r_I is the gross real risk-free rate (without a tilde, as it is certain given the equilibrium price P_I).

Investor endowments for investor n ($n \in \{C, Y, O\}$) are denoted W_n in financial wealth and H_n in real capital ("human capital"). Human capital is a special asset class not traded on financial markets. The payoff of human capital is gross inflation $\tilde{\Pi}$, which is added to next period's financial wealth. Human capital is thus risk-free in real terms. In our two-period

4 We do not, for example, address problems concerning life-cycle asset allocation, such as the valuation of human capital and intermediate consumption and savings. We also sidestep agency issues between pension funds and their participants.

5 Given the zero net supply assumption, it is more natural to think about the inflation-linked asset as a derivative security, such as an inflation swap, but within the model, a swap contract and ILB are equivalent. To see the equivalence, let Z_I be the notional amount invested in the swap contract with Z_I/P_I units that each have a payoff of $\tilde{\Pi} - R_B P_I$. We can interpret the payoff as a swap between receiving the present value of inflation $\tilde{\Pi}/R_B$ and paying the forward price P_I. An ILB requires a slightly different model structure, where we would additionally need a government that issues an exogenously given (small) supply of ILBs.

model, human capital depreciates to zero after the payoff—that is, $\tilde{H}_n = 0$ for all investors. The three investors may have very different real and financial wealth, which will affect their demand for the different assets and their impact on the price for asset *I*. We assume that $H_Y > H_O$ and $W_O > W_Y$. Young pension fund participants have little financial wealth and much human capital. Old participants have high W but nearly zero H.

Investor n allocates an amount $X_{j,n}$ to asset class j ($j \in \{S, B, I\}$). After receiving the investment returns, future nominal wealth is

$$\tilde{W}_n = W_n R_B + X_{S,n}(\tilde{R}_S - R_B) + X_{I,n}(\tilde{R}_I - R_B) + H_n\tilde{\Pi}. \quad (1)$$

The last term is the payoff of the real asset held by the investor, which is assumed to be risk-free in real terms. Dividing by total initial wealth $W_n + H_n$, the nominal return on total wealth is

$$\begin{aligned}\tilde{R}_n &= \frac{W_n}{W_n + H_n} \times \frac{\tilde{W}_n}{W_n} \\ &= y_n\Big(R_B + x_{S,n}(\tilde{R}_S - R_B) + x_{I,n}(\tilde{R}_I - R_B)\Big) + (1 - y_n)\tilde{\Pi}, \end{aligned} \quad (2)$$

where $y_n = W_n/(W_n + H_n)$, and where $x_{j,n} = X_{j,n}/W_n$ are the investment weights in the three assets. The second term on the last line shows the exposure to inflation $\tilde{\Pi}$. Assuming independence between inflation and stock returns, the coefficient on $\tilde{\Pi}$ measures the inflation exposure of nominal returns before hedging–that is, at $x_{I,n} = 0$. The real exposure is then the nominal exposure minus one, such that $-y_n$ is the inflation exposure of the real return.[6] In the remainder of the paper, we will use y_n to indicate the inflation exposure of each investor. A key element of the model is that y_n differs among the three investor types. Young investors have a small y_n and thus only a slightly negative exposure to inflation risk. Old investors have a relatively large financial wealth and their exposure to inflation risk is thus close to one. We treat (2) as a reduced form equation, where the y_n values are exogenously given exposures to inflation, not necessarily to be interpreted as related to human capital.

The capital market investors are sophisticated and always willing to trade the inflation-linked asset. If there is demand from the pension fund, they create the supply, meaning they are the counterparts in the inflation swap. At the equilibrium price P_I, the net supply will be equal to zero; in other words, supply equals demand. If the size of the capital market investors is not infinitely large compared to the pension fund participants, trades by the pension fund will generally increase demand, therefore increasing P_I and lowering the real interest rate.

6 Our definition of real exposure as "nominal minus one" is a linear approximation of the more precise definition that uses the derivative of real returns $\tilde{r}_n = \tilde{R}_n/\tilde{\Pi}$ with respect to inflation, evaluated at $x_{I,n} = 0$,

$$\Delta_n \equiv \frac{\partial \tilde{r}_n}{\partial \tilde{\Pi}} = -y_n \frac{(\tilde{r}_B + x_{S,n}(\tilde{r}_S - \tilde{r}_B))}{\tilde{\Pi}}.$$

Using plausible values—for example, expected inflation of $\bar{\Pi}_I = 1.02$, real equity returns of $\bar{r}_S = 1.04$, and real bond returns of $\bar{r}_B = 1$ and $x_{S,n} = 0.5$—gives us $\Delta_n = -y_n$, as before.

Table 1: Alternative Market Designs

Market design	Price of inflation-linked asset	Pension fund risk sharing
Integrated	Pooled	Fair
Segmented	Market	None
Segmented	Market	Fair
Segmented	Dual: market, internal	Optimal sharing

Table entries show the four cases for the inflation-linked asset analyzed in the paper. In one case, supply and demand from the market and pension fund participants are aggregated (integrated, pooled); in two cases, the price is solely determined in the capital market (market), but pension fund participants internally either do not exchange inflation risk at all or exchange it at market price (fair). In the final case, a separate price exists within the pension fund (internal).

In terms of the market setup, we consider the four cases listed in the introduction, which we summarize in Table 1. The market for the inflation-linked asset is either *integrated* or *segmented*. When it is integrated, all three types of investors are free to trade in the inflation-linked asset market. The equilibrium price is then determined by the aggregate demand. In all other cases, the market for asset *I* is segmented: only the capital market investors can trade the inflation-linked asset, and their demand and supply determine the market price P_I. In the segmented market cases, pension fund participants either do not engage at all in sharing inflation risk, or they share it internally at the externally determined market price, or they share it at an internal transfer price that may be different from the external market price.

We consider two versions for the case where the pension fund engages in internal trades of the inflation asset. In the first version, labeled *fair* in Table 1, the pension fund takes the market price P_I as given and may reallocate some of the inflation risk exposure from the old to the young participants. The positions of both types of pension fund participants are fixed by the pension fund. Unlike the integrated market case, the pension fund does not directly trade with the capital market investors, but it does use their price as the *fair* price for reallocating the risk. In the second version, the pension fund participants obtain their optimal positions $X_{I,n}$ in the inflation asset in the internal market, where old and young trade at an internal equilibrium price. Trading in the internal market is by pension fund participants only. Capital market investors trade asset *I* in the external financial capital market at a price that will differ from the internal price within the pension fund.

3.2 Model Solution

Each investor solves an optimal asset allocation problem to choose the portfolio composition that maximizes their expected utility of real terminal wealth E$[U(\tilde{w}_n)]$. We assume CRRA preferences with relative risk aversion γ. Risk aversion may differ among the different types of investors. Real wealth is defined as

$$\tilde{w}_n \equiv \frac{\tilde{W}_n}{\tilde{\Pi}} = (W_n + H_n)\frac{\tilde{R}_n}{\tilde{\Pi}} = \frac{W_n}{y_n} \times \tilde{r}_n, \tag{3}$$

where $\tilde{r}_n$ is the real investment return. Marginal utility with CRRA preferences is $U'(w) = w^{-\gamma}$, such that

$$U'(\tilde{w}_n) = \left(\frac{W_n}{y_n}\right)^{-\gamma} \times \tilde{r}_n^{-\gamma}. \tag{4}$$

The first term is non-stochastic and can be moved outside of the expectation operator. In taking expectations, we only need to focus on the return distributions. The optimal allocation to the nominal bond follows as one minus the investments in equity and the inflation-linked asset.

3.2.1 *Segmented Market*

The simplest case to solve is when the capital market investors are the only ones who can trade the inflation-linked asset. In this case, they fully determine its price. For all three types of investors, the demand for the inflation-linked asset *I* is zero: for the two types of pension fund investors, because they cannot invest in it; and for the capital market investors, because the inflation-linked asset is in zero net supply.

Equity is in infinite supply, so each investor n can freely choose the optimal fraction $x_{S,n}$. To find the optimal share of equity, each investor solves the first-order condition obtained by differentiating expected utility with respect to $x_{S,n}$,

$$\mathsf{E}\left[U'(\tilde{w}_n)\frac{\partial \tilde{w}_n}{\partial x_{S,n}}\right] = 0, \qquad n \in \{C, Y, O\}. \tag{5}$$

Using the definition of real wealth in (3), and further noting that we can divide through by any positive number, the first-order conditions can be rewritten as

$$\mathsf{E}\left[\tilde{r}_n^{-\gamma}(\tilde{r}_S - \tilde{r}_B)\right] = 0, \tag{6}$$

where now, because of $x_{I,n} = 0$, the portfolio return is $\tilde{r}_n = y_n(\tilde{r}_B + x_{S,n}(\tilde{r}_S - \tilde{r}_B)) + 1 - y_n$, which does not depend on P_I. For each investor, condition (6) therefore only has $x_{S,n}$ as an unknown. Since an analytic solution does not exist, we proceed numerically (see Appendix A).

Next, we find the price P_I such that the demand for asset *I* by the capital market investor is zero. Optimal demand is determined by differentiating expected utility of the capital market investor, $n = C$, with respect to $x_{I,C}$,

$$\mathsf{E}\left[\tilde{r}_C^{-\gamma}(r_I - \tilde{r}_B)\right] = 0. \tag{7}$$

Given the optimal equity demand of the capital market investor, denoted $x^*_{S,C}$, the portfolio return does not contain any unknowns and can be written $\tilde{r}^*_C$, being the real portfolio return evaluated at the optimal equity demand. Since r_I is non-stochastic, we can solve (7) for the equilibrium real interest rate $r_I = 1/P_I$ as

$$r_I = \frac{\mathsf{E}\left[(\tilde{r}^*_C)^{-\gamma}\,\tilde{r}_B\right]}{\mathsf{E}\left[(\tilde{r}^*_C)^{-\gamma}\right]}. \tag{8}$$

Defining $\tilde{M}_C = (\tilde{r}_C^*)^{-\gamma} / \mathsf{E}[(\tilde{r}_C^*)^{-\gamma}]$, we can write this more compactly as

$$r_I = \mathsf{E}[\tilde{M}_C \tilde{r}_B]. \tag{9}$$

The stochastic discount factor $\tilde{M}_C$ prices the real payoffs on the nominal bonds for the capital market investors.[7]

After determining r_I, we can compute the inflation risk premium (IRP) as the difference between the nominal risk-free rate minus the real risk-free rate plus expected inflation using the following equation:

$$\text{IRP} \equiv R_B - (r_I + \mathsf{E}[\tilde{\Pi}] - 1) \tag{10}$$

(the minus 1 because we are using gross returns and gross inflation).

3.2.2 *Integrated Market*

When all investors can freely trade the inflation-linked asset, we find the market clearing price by aggregating demands. To keep the model simple, we do not consider strategic demand, whereby the pension funds and the capital market investors take into account the price impact of each other's demand for asset *I*. For all investors, we have the first order conditions with respect to assets *S* and *I* as:

$$\mathsf{E}\left[U'(\tilde{w}_n)\frac{\partial \tilde{w}_n}{\partial x_{j,n}}\right] = 0 \qquad n \in \{C, Y, O\}, \;\; j \in \{S, I\}. \tag{11}$$

Given P_I, these are six equations in six unknowns. All investors may hold different fractions of their wealth in both equity and the inflation-linked asset. These fractions are driven by their different exposures y_n to inflation risk. When the y_n values differ, there is scope for trade. To the six demand equations, we must add the market-clearing condition that the inflation-linked asset is in zero net supply, implying $\sum_n W_n x_{I,n} = 0$. This adds P_I to the set of unknowns. All seven equations are solved numerically simultaneously (see appendix). Since $x_{I,n} \neq 0$, future real wealth $\tilde{w}_n$ remains dependent on the price P_I.

As before, we can write the price equation for the inflation-linked asset in a more intuitive way. Analogously to (9), it should hold that

$$r_I = \mathsf{E}\left[\tilde{M}_C \tilde{r}_B\right] = \mathsf{E}\left[\tilde{M}_Y \tilde{r}_B\right] = \mathsf{E}\left[\tilde{M}_O \tilde{r}_B\right], \tag{12}$$

where the subscripts denote the stochastic discount factor according to the preferences and optimal positions of each particular investor. In equilibrium, the real interest rate must be the same for all investors. The stochastic discount factors of the three investors are different, as they have different optimal real portfolio returns $\tilde{r}_n$ (because of different inflation exposures y_n and the resulting different optimal demands $x_{I,n}^*$ and $x_{S,n}^*$). Even for the capital market investors, their $\tilde{M}_C$ will differ from the values found in the segmented market case in (9), since they now hold different fractions, $x_{S,C}$ and $x_{I,C}$, than before and thus have different $\tilde{r}_C^*$.

7 Writing equilibrium prices and returns in the form of an expectation involving the product of the payoffs and a stochastic discount factor is a standard notation in asset pricing. See Cochrane (2005) for a textbook introduction. In Appendix A, we further rewrite the price equation as the expected payoff on the nominal bond under a "risk neutral" Q-measure.

3.2.3 *Dual Prices*

In the optimal sharing case shown in Table 1, the two types of pension fund investors can trade with each other but not with the capital market investors. We will now have two different prices for the inflation-linked asset. For the capital market investors, the solution is identical to the segmented market case analyzed in Section 3.2.1. Their demand implies the same real interest as in (9).

For the pension fund investors, we have the same demand equations as in the integrated market case; only the market-clearing condition is different. Since the young and the old participants only trade with each other, the zero net supply condition implies that their aggregate demand is $W_Y x_{I,Y} + W_O x_{I,O} = 0$. To find the optimal demand for both equity and the inflation-linked asset, together with the internal real interest rate, we numerically simultaneously solve the four demand equations and the market-clearing condition for the five unknowns. Solutions for all $x_{j,n}$ and P_I again must be numerical (see appendix).

As before, we can express the equilibrium price—in this case, the internal price—of the inflation-linked asset as

$$r_I^{\text{int}} = \mathsf{E}[\tilde{M}_Y \tilde{r}_B] = \mathsf{E}[\tilde{M}_O \tilde{r}_B], \tag{13}$$

with the superscript to emphasize that this internal price is different from the capital market real interest rate. Obviously, the young and the old participants must agree on the internal real interest rate even though they have different stochastic discount factors.

Given the structure of the solution, it is straightforward to allow for further heterogeneity among the investors, for example, different γ_n. While this will alter the stochastic discount factors for the investors, solving still boils down to finding the $x_{j,n}$ that leads to agreement on the valuation.

4 Calibration of the Tree Model

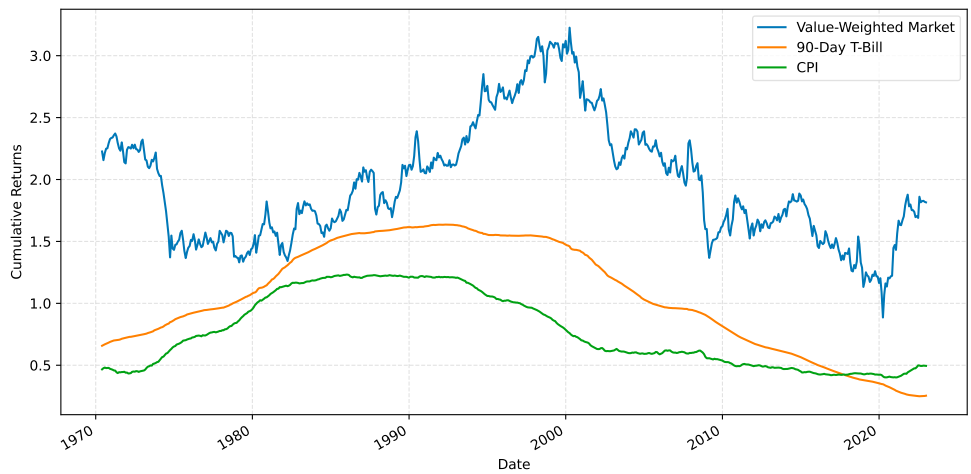
We calibrate the model to empirical data using long historical US data series. For the market return, we rely on the total return of the CRSP Value Weighted Market Index. Inflation is measured using the CPI, and the 90-day T-bill returns are used for the risk-free rate. We collect annual data from the CRSP over the period from 1950 to 2022.

To determine the parameter values, we use annualized values for the realization of cumulative returns over a 20-year period. The main reason for this is to capture possible time series effects that would not be present when just averaging annual data. Inflation is highly persistent; therefore, the annual variance would underestimate the risk that a long-term investor would be exposed to. Similarly, mean reversion in stock returns implies that the annualized variance over a longer horizon is lower than the annual variance (Campbell and Viceira, 2005).

Figure 1 gives the 20-year cumulative log returns for the CRSP value-weighted stock market index, the 90-day T-bill, and the CPI as a function of the point in time when each 20-year period ends. Over this long period, there is quite a lot of variation, particularly in the stock market returns. In our sample period, the stock market has always outperformed the risk-free asset. A comparison of the risk-free asset and the CPI shows that the realized real risk-free rate has been positive most of the time, with the exception of the 20-year periods ending in the last few years. This can be attributed to negative interest rates in the wake of the financial crisis and during the pandemic, as well as to high inflation in 2022.

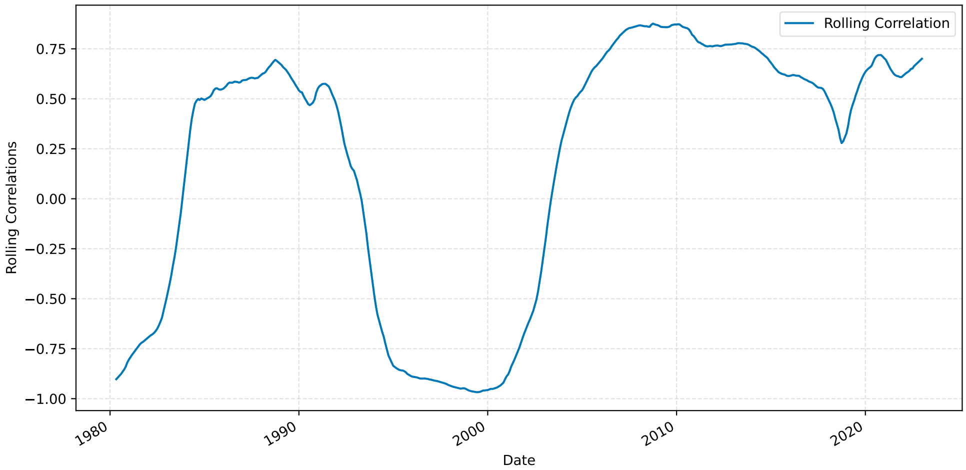
The average cumulative log return for each series is 196% for the stock market, 106% for the T-bill, and 77% for the CPI. Annualizing these data results in 9.8% per year for the stock market, 5.3% for the T-bill, and 3.9% for the CPI. The volatilities of the 20-year returns are

Figure 1: Data for Calibration



The figure shows the 20-year cumulative returns of the stock market, the 90-day T-bill, and the CPI. The x-axis represents the end date of any 20-year period, e.g., 1980 indicates that the cumulative return shown is for the period from 1961 to 1980.

Figure 2: Rolling Correlations Between Stocks and Inflation



This figure shows the rolling correlations between the stock market return and the CPI. Returns are given over a 20-year horizon, and correlations are then computed over a 10-year horizon, which ends at the date given on the x-axis.

48.7% for the stock market, 43.2% for the T-bill, and 29.9% for the CPI. We annualize these numbers, which results in annual volatilities of 10.9%, 9.6%, and 6.9% for each series, respectively.

The correlation between the stock market and the CPI varies over time. This is shown in Figure 2, which shows the rolling correlations between the 20-year stock market returns and the 20-year inflation rate. The correlations are computed over a 10-year period. This finding is in line with Cieslak and Pflueger (2023), who show that around 2000, the sign of the correlation changed from negative to positive. They distinguish between good and bad inflation: good inflation comes with a negative inflation risk premium in bonds, and bad inflation comes with a positive inflation risk premium in bonds.

For the equity market, we set $\mu_S = 0.098$ and $\sigma_S = 0.109$. For the inflation-linked asset, we set $\mu_I = 0.039$ and $\sigma_I = 0.069$. The risk-free rate is $R_B = 0.053$. The correlation between the stock market and inflation in the base case is set to $\rho = 0$; we will consider its impact in a robustness check when discussing the results.

All investors in the market have CRRA utility, with a risk aversion parameter of 10. We set the initial wealth of each of the two types of pension participants equal to one, while the capital market investors have initial wealth equal to two. This implies that the groups of pension fund investors and the capital market investors have the same aggregate initial wealth.

The exposure of the investor to inflation risk is controlled by the parameter y_n. It varies across investors. Old investors whose wealth consists mainly of financial wealth have a negative exposure to inflation risk, which can have the same order as their initial wealth. Young investors, whose wealth is mainly human capital, have an exposure to inflation risk which is only slightly negative. In the benchmark case, we set $y_O = 1.0$, $y_Y = 0.3$, and

$y_C = 0.2$. These exposures get scaled by a constant factor $y_c \in [0, 1]$. The scaling allows for a graphical illustration showing the gradual move away from homogeneous investors toward the heterogeneous investors in the model. In Section 5.5, we explore alternative configurations for the inflation exposure of the capital market investors.

For our analysis, we rely on a 2-period binomial model. While this is admittedly a very stylized setup, it is enough to make our main points. Numerical solution details are provided in Appendix A. We solve the model for the different market setups as explained in Section 3.

5 Results

5.1 Base Case

We start with the base case: the fully segmented market. Capital market investors can trade all assets, while pension fund investors can only trade equity and the nominal risk-free bond. The equilibrium price and real risk-free rate for this setting (and also for all other cases) are reported in Table 2. The price of the inflation-linked asset that is traded on the capital market only is $P_I = 0.9977$, so the real rate of return is 0.0023 and the inflation risk premium is 0.85%. The trading volume in this asset is equal to zero. Pension fund investors cannot trade the inflation-linked asset; capital market investors do not want to trade it because they are all identical and, as such, do not trade with one another.

All investors choose optimal portfolios, given the prices. We calculate their certainty equivalents from the solution to the asset allocation problem in equilibrium. These certainty equivalents will be the benchmark when, later on, we look at the gains and losses of the investors in different market setups relative to the segmented market. Figure 3 shows the certainty equivalents. The factor y_c on the x-axis is a scaling factor for the exposures. For $y_c = 1$, we have the given exposures $y_O = 1.0$, $y_Y = 0.3$, and $y_C = 0.2$. Certainty equivalents are largest for $y_c = 0$—that is, when the exposure to inflation risk is zero for all investors. In this case, the investors are no longer heterogeneous and their certainty equivalents are identical. At a larger y_c, and thus more negative exposures, investors suffer from the additional risk and the average reduction in their real wealth. If y_c were to exceed 1, the certainty equivalents would eventually turn negative.

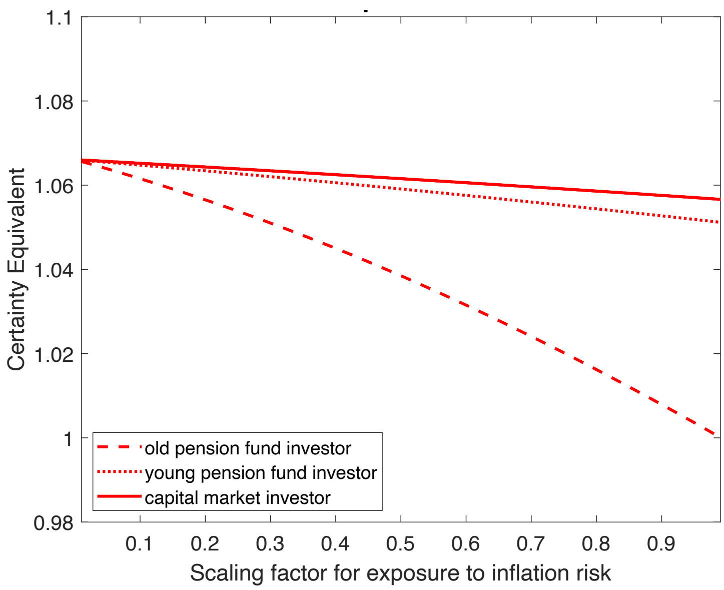
Table 2: Real Return and Inflation Risk Premium

Market structure	Price of inflation-linked asset	Real interest rate	Inflation risk premium
Segmented	0.9977	0.23%	0.85%
Integrated	1.0078	-0.77%	1.85%
Internal	1.0172	-1.69%	2.77%

Table entries show the price of the inflation-linked asset, the real interest rate r_I, and the inflation risk premium defined in (10) for different market settings. In the segmented market, the ILB is priced only on the capital market. The integrated market finds the equilibrium price from the aggregate demand of capital market investors and pension funds. The internal market price uses only the pension fund investors to find the price of the inflation-linked asset; the separate capital market price is identical to the segmented market case.

We can see that the impact of inflation exposure is heterogeneous across investors, being largest for the old pension fund investor and smallest for the capital market investor. This shows that there are gains to be made if the risk becomes tradable.

Figure 3: Certainty Equivalents in a Fully Segmented Market



The figure shows the certainty equivalents for the old pension fund investor (dashed line), the young pension fund investor (dotted line), and the capital market investor (solid line) in a segmented market. The initial exposures to inflation risk are multiplied by the scaling factor on the x-axis.

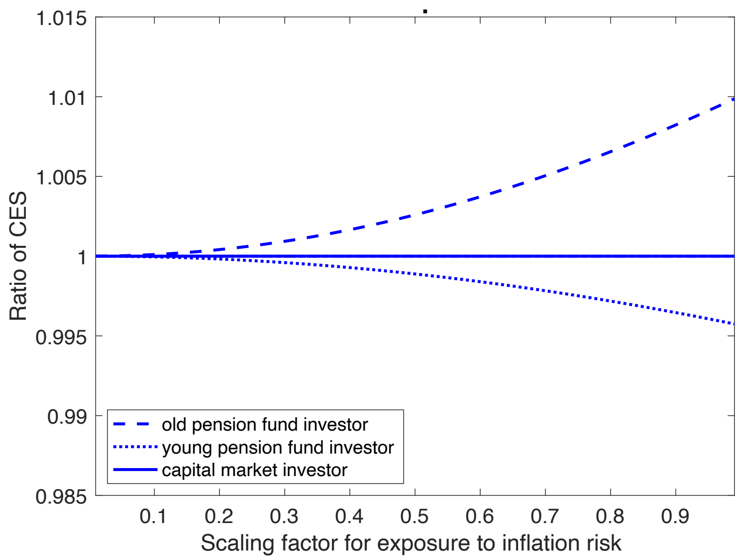
5.2 Internal Risk Sharing at a Fair Price

Next, we look at the case where the pension fund manager sets up an internal market for inflation risk. Technically, the pension fund specifies the sharing rule. This means that the fund defines the shares of the overall fund returns that go to individual pension fund investors. Under this sharing rule, the pension fund imposes a hypothetical trading of an inflation-linked asset. This involves the terminal payoffs of the inflation-linked asset and the initial prices (or premia) that must be paid to obtain insurance. In the following, we will ignore the technical implementation of such a scheme and discuss it as if the pension fund participants trade among themselves.

In our example economy, the initial exposure to inflation risk is more negative for the old than for the young investor. Therefore, young investors should provide some insurance for old investors. In compensation for the additional risk they thus take on, the young investors receive a premium. The pension fund sets the price and the positions for the inflation asset in the internal market. These choices are discretionary; we start with two (sensible) assumptions. First, we set the price in the internal market equal to the price in the capital market. There are several arguments in favor of this. The capital market price is an observable quantity, so there is no subjective component to it. Furthermore, it seems fair. In particular, it is not subject to social or political motivations, such as the redistribution of wealth between groups. This is, in fact, in line with mandatory practice in the Dutch pension system, whereby the internal reallocation of cashflows has to be made using market prices. Second, we set the amount of insurance such that there is full risk sharing. Young and old investors pool their exposures of $y_O = 1.0$ and $y_Y = 0.3$. The resulting overall exposure of 1.3 is then shared equally, and the resulting exposure for each investor is 0.65. To achieve this exposure, the old investor buys $X_{I,O} = 0.35$ inflation-linked assets from the young investor ($X_{I,Y} = -0.35$).

Pension fund participants decide on their optimal position in the market and in the risk-free asset based on the given price and amounts that need to be traded in the internal market. Capital market investors decide on their optimal positions in all three assets.

Figure 4: Certainty Equivalents in the Internal Market When Prices and Amounts Are Given



The figure shows the certainty equivalents for the old pension fund investor (dashed line), the young pension fund investor (dotted line), and the capital market investor (solid line) in an internal market setting when prices and amounts are given. The initial exposures to inflation risk are multiplied by the scaling factor on the x-axis. Certainty equivalents are relative to the CEs in the base case shown in Figure 3.

Figure 4 shows the relative change in the certainty equivalent, where the benchmark is given by the certainty equivalent in the fully segmented market (the market setup analyzed in Section 5.1). For capital market investors, the certainty equivalent does not change. In both market setups, they can trade equity and the nominal bond. Furthermore, they can trade the inflation-linked asset among themselves, while there is no trade between them and pension fund investors. With the same problem to solve, they will come up with the same optimal portfolio, and with the same equilibrium price for the inflation-linked asset in the capital market.

Given our parameters, old pension fund participants benefit from the introduction of the internal market. They gain around 1% p.a. from being able to buy, as it were, insurance against inflation risk. Young pension fund participants lose around 0.45% p.a. (when the scaling factor has a value of one). They profit from the premium they earn from their sale of the inflation-risk insurance. However, that premium is not large enough to compensate for the additional risk they must take on. To understand these results, we look at the initial exposures and the differences between them. The young pension fund participant has an initial inflation risk exposure of $y_Y = 0.3$. He sells inflation-risk insurance to the old pension fund participant, whereupon after that trade, his inflation risk exposure is 0.65. The price at which he has to trade is the equilibrium price set by the capital market investors. These investors have an initial exposure of $y_C = 0.2$. The price for the insurance they set (with zero trading) is thus lower than the price the young pension fund participant would set at his initial exposure of $y_Y = 0.3$. And that higher price hypothetically set by the young pension fund participant would still not induce him to trade; it would just make him happy not to trade. To sell insurance, he would ask for an even higher price. The price the young pension fund participant deems fair, given his initial exposure and his forced trading, is thus higher than the capital market price set by the capital market investors. Being forced to trade at the capital market price makes him worse off.

Figure 4 presents the gains and losses for the different scaling factors y_c of the exogenously given exposures y_n. For a zero scaling factor, inflation risk does not play any role, so gains and losses are zero. In line with intuition, gains and losses then both increase in the scaling factor in absolute terms.

In our example, there are thus pension fund participants (here, young pension fund participants) who are worse off when being forced to trade inflation insurance at the "fair" capital market price. This is the case despite the fact that the equilibrium price of inflation-linked assets from the capital market is an observable market price that is objective and under no political influence. Overall, the price from the capital market may thus not be a good choice for the internal market. It may make some pension fund participants worse off. The reason for this is that capital market investors differ from pension fund participants, so the equilibrium price for the capital market investors will not be the same as the equilibrium price for the pension fund participants.

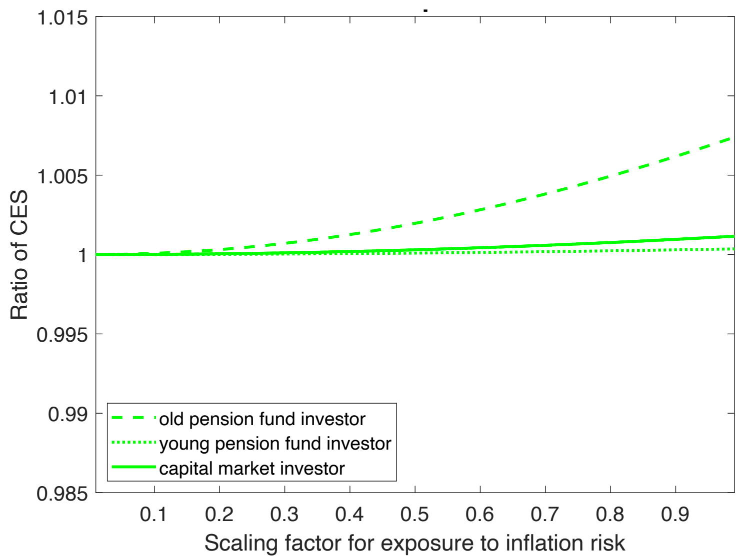
This result depends on the calibrated values for the inflation exposure of the capital market participants versus the pension fund participants. The price observed on the capital market will often not lead to a Pareto improvement resulting from internal risk-shifting. If we modify the exposure for the capital market participants, we can easily construct a case where the losing participants change from the young to the old. We discuss these cases in the next sections.

5.3 Integrated Market

In the integrated market setting, all investors have access to the financial market for the inflation-linked asset and trade with one other. Risk sharing therefore takes place among all investors. The second row of Table 2 shows the results for the inflation-linked asset for the integrated market case. The real interest rate drops to -0.77% and the inflation risk premium increases to 1.85%. The price of the inflation-linked asset increases relative to the segmented market case, since in our calibration, the exposures of the pension fund participants are higher than the exposure of the capital market participants.

Technically, the pension fund trades on behalf of the pension fund participants in the integrated market. It allocates the initial premium payments and terminal payoffs to the pension fund participants via the sharing rule. In the following, we again ignore the technical implementation and assume that the pension fund participants can directly trade in the capital market. In particular, we ignore all problems that may arise with the delegation decision, since that would distract us from focusing on the comparison of the different market setups and the corresponding choices of prices and amounts.

Figure 5: Certainty Equivalent Gains and Losses in the Integrated Market



The figure shows the certainty equivalents for the old pension fund investor (dashed line), the young pension fund investor (dotted line), and the capital market investor (solid line) in the integrated market. The initial exposures to inflation risk are multiplied by the scaling factor on the x-axis. Certainty equivalents are relative to the CEs in the base case shown in Figure 3.

Figure 5 shows the ratios of the certainty equivalents relative to the fully segmented market as a function of the scaling factor y_c for the initial exposures. For the calibrated parameters, the gain of old pension fund participants is around 0.75%, while young pension fund participants only gain some basis points. The gain of capital market investors is also very small. That is because capital market investors are more similar to young pension fund participants than to old pension fund participants. Therefore, old pension fund participants will buy insurance not just from young pension fund participants, but also from capital market investors. Old pension fund participants thus gain a lot from this extended supply of insurance, while the other investors, including young pension fund participants, have to share the premium from selling the insurance.

All pension fund participants gain. This result is not specific to our calibration but holds in general. The reason for this is that the restricted optimal allocation from the segmented market is still accessible to the pension fund participants: an optimal investment in the market (at the exogenously given price) and a zero investment in the inflation-linked asset. If the pension fund participants deviate from this formerly optimal allocation by taking a non-zero position in the inflation-linked asset, they do so because it increases their utility. The gain for the old pension fund participants is smaller compared to the results in Section 5.2, since the equilibrium price for the inflation insurance increases relative to the equilibrium when only capital market investors trade.

Capital market investors are not necessarily better off. For them, the formerly optimal portfolio from the segmented market may no longer be available. That is because the price of the inflation-linked asset in the integrated market will, in general, be different from the price in the segmented market. Their old portfolio, in which they have to trade the inflation-linked asset at the old price, might therefore no longer be accessible.

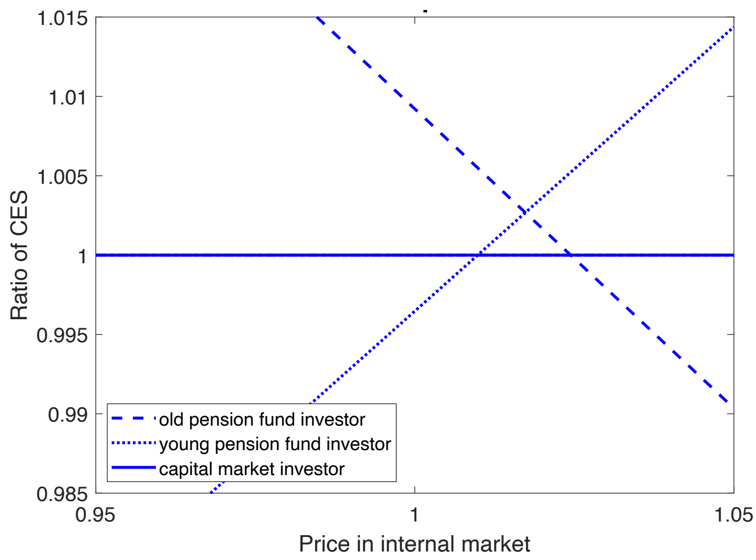
In our setup, we face a special case. Since we assume that capital market investors are identical to one another, they do not want to trade the inflation-linked asset in the segmented market. The old allocation thus involves a zero position in the inflation-linked asset. That portfolio is still available in the integrated market, irrespective of the new equilibrium price of the inflation-linked asset. If capital market investors deviate from this formerly optimal portfolio, the reason is again that they benefit from doing so.

5.4 Internal Market and Equilibrium Price

We now modify the internal market setup in that we do not simply take the capital market price as given. We still assume perfect risk sharing between young and old investors. However, we now vary the price in the internal market. There will thus be two prices for the inflation-linked asset: the price in the capital market, at which capital market investors trade, and the price in the internal market, at which pension fund participants trade. The third row of Table 2 shows the results for the inflation-linked asset as traded in the internal market; the capital market price is identical to the segmented case, since in both cases only the capital market investors determine the price. The internal market price for the inflation-linked asset is 1.0172, with a real interest rate of -1.69% and an inflation risk premium of 2.77%. That price is higher than the capital market price, since the exposures of the pension fund investors are larger than the exposure of the capital market investors.

Figure 6 shows the certainty equivalent gains and losses relative to the segmented market as a function of the price in the internal market. There is a region of intermediate prices for which both young and old pension fund participants gain. At these prices, both types of investors are happy with the introduction of the internal market in the case of perfect risk sharing. Note, however, that perfect risk sharing might not be the optimal choice, and investors might prefer some other amount of trading. In addition, it is only in the event that the capital market price falls into this region that there is the possibility of an internal trade where all pension fund participants gain. However, price is not the only factor in generating utility gains, as the amount of trading is also important.

Figure 6: Certainty Equivalent Gains and Losses in the Internal Market



The figure shows the certainty equivalents for the old pension fund investor (dashed line), the young pension fund investor (dotted line), and the capital market investor (solid line) as a function of the price in the internal market. CEs are relative to the results for the segmented market shown in Figure 3.

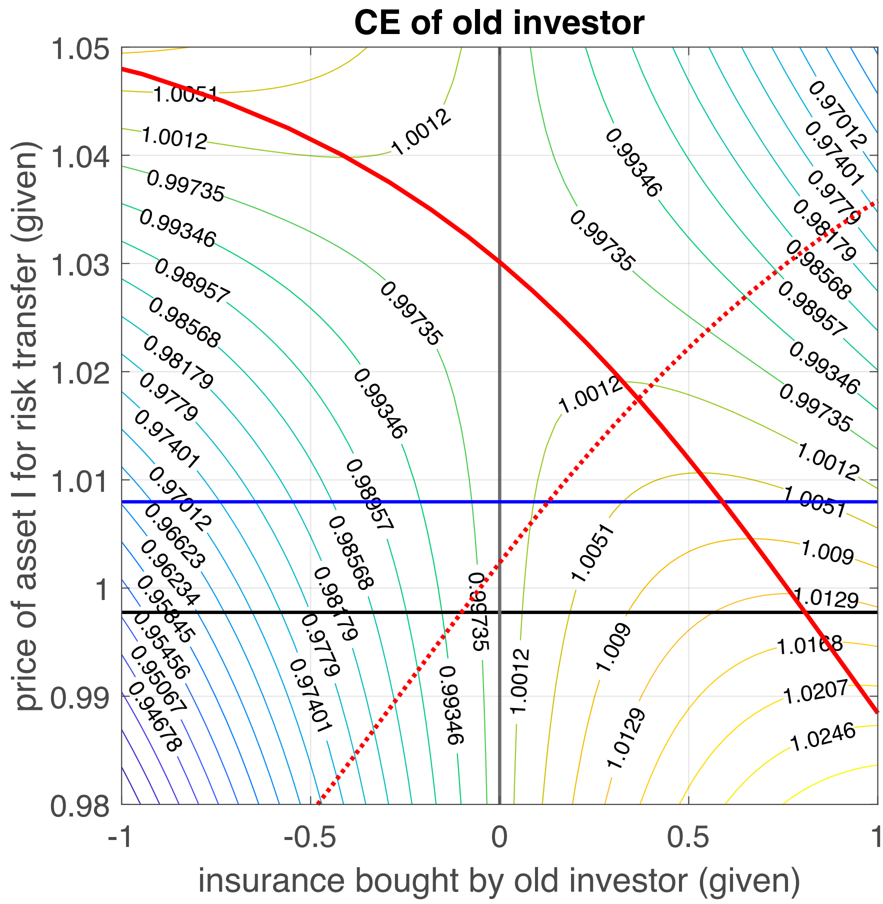
For prices below this region, old pension fund participants gain, since the premium they have to pay for the insurance is small. However, the young pension fund participants, who have to provide insurance at such a low price, lose. For prices above this region, the situation is reversed. Young pension fund participants gain, since they earn a high premium from selling the insurance, while the old pension fund participants, who have to pay a high price for the insurance, lose. In our calibration, the price in the segmented market, where only capital market investors trade the inflation-linked asset, is below this region. That is why old investors gain, while young investors lose in Section 5.2. Note that the results presented are for the case in which there is perfect risk sharing. It is also possible that investors will prefer different trading amounts.

Figure 7 shows the impact of price (y-axis) and position (x-axis) in the internal market. We give the position in the inflation-linked asset of the old investor; he buys insurance for a positive position and sells insurance for a negative position. The position of the young investor is minus the position of the old investor, since the overall net supply of the inflation-linked asset in the internal market is zero.

For each combination of internal price and internal position, we solve the asset allocation problem for the old pension fund participant—that is, we determine his optimal position in the market. We then determine his certainty equivalent and how much he gains as compared to the segmented market. The figure shows the contour plots of the ratios of the certainty equivalents—that is, the indifference curves for the old investor.

For each price level in the internal market, we determine the optimal insurance position for the old investor. These optimal positions (as a function of the internal price) are shown by

Figure 7: Price and Trades



The figure gives the indifference curves for the certainty equivalent gains of the investors as a function of the internal price (y-axis) and the amount of trading of the old investor (x-axis). The red solid line shows the optimal position for the old investor, given the internal price; the red dotted line shows the optimal position for the young investor, given the price. The vertical black line highlights a position of zero. The horizontal black and blue lines show the equilibrium prices in the segmented and integrated markets, respectively.

the solid red line. For a low price, the old investor wants to buy a lot of insurance. His demand for insurance decreases when the price in the internal market rises. If the price becomes too high, he eventually even supplies insurance—that is, he starts to sell insurance.

We do the same calculations for the young investor. The optimal position in insurance (expressed via the opposite position from that of the old investor) as a function of the internal price is shown by the dotted red line. In line with intuition, the young investor wants to sell insurance for a high internal price and starts to demand insurance for a low internal price.

The vertical black line highlights a position of zero. That is the position the pension fund investors have in the segmented market. With identical portfolios, the certainty equivalents in the internal market and in the segmented market coincide and the ratio of certainty equivalents is equal to one.

When we move from the vertical line (zero position in the inflation-linked asset) toward the red solid line (optimal position for old pension fund investors), the certainty equivalent for the old investor increases. When we move beyond the solid red line, however, his certainty equivalent gain starts to decrease again, until he is ultimately even worse off than in the segmented market. When we move from the vertical line away from the solid red line, the old investor loses.

An equivalent result holds when we move from the vertical line toward the dotted red line (the optimal position for young pension fund investors). The certainty equivalent gain of the young investor increases. It decreases when we move beyond the dotted line or when we move in the opposite direction from the vertical line.

The intersection of the solid and the dotted red line gives the equilibrium price and position in the internal market. At this equilibrium price, both types of investors are happy with their equilibrium position. If the pension fund fixes this amount of trading, they will both be satisfied with the amount of risk sharing and also with the introduction of the internal market.

For prices near this equilibrium price, both investors benefit from an internal market as long as the position they are assigned by the pension fund is not too large. For a price slightly below the equilibrium price, the young investor optimally wants to sell less insurance than the old investor wants to buy. If the amount of trading is just slightly above zero, both investors benefit. If the amount exceeds the amount given by the dotted red line (which gives the optimal amount for the young investor), the old investor still gains, while the young investor starts to lose relative to his overall optimal amount. In the beginning, he will still see a utility gain relative to the segmented market, but eventually, his certainty equivalent will drop below its level in the segmented market. If the amount also exceeds the red solid line (which gives the optimal amount for the old investor), both the young and the old investor will be worse off if the amount increases further. For a price slightly above the equilibrium price, the young investor optimally wants to sell more insurance than the old investor optimally wants to buy. Again, both investors benefit from a small amount of trading. If the amount of trading increases by too much, then first the old investor and then the young investor will start to lose again.

For prices above this region, both the young investor and the old investor want to sell insurance. Moving away from a zero position in the direction of a positive amount for the old investor makes the young investor (who cannot sell insurance) better off, but the old investor (who is forced to buy insurance at too high of a price) worse off. Moving away in the direction of a negative amount for the old investor makes the old investor better off and the young investor (who is forced to buy insurance at a high price) worse off.

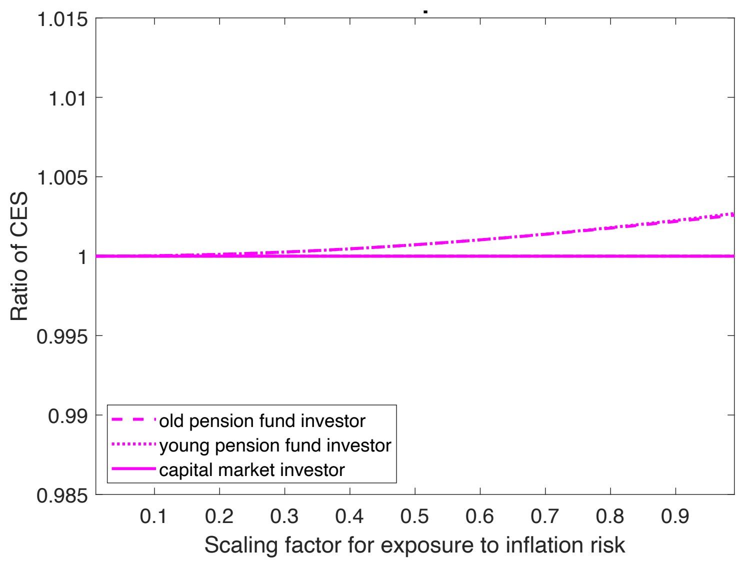
For prices below this region, both the young investor and the old investor want to buy insurance. Moving away from a zero position in the direction of a positive amount for the old investor makes the old investor better off, but the young investor (who is forced to sell insurance at too low of a price) worse off. Moving away in the direction of a negative amount for the old investor makes the young investor better off and the old investor (who is forced to sell insurance) worse off.

The horizontal black line gives the price of the inflation-linked asset in the capital market. The price is below the equilibrium price, and it is also below the region where both investors can benefit. A positive position for the old investor moves the position closer to his optimal position, and he consequently gains, but away from the optimal position for the young investor, who thus loses.

The horizontal blue line represents the price in the integrated market. At this price, too, the old investor gains while the young investor loses. Again, this price is not just suboptimal; it is even harmful for some pension fund investors. The reason is that risk sharing in the internal market takes place between pension fund investors only, while the equilibrium price is determined in the integrated market where risk sharing takes place among all investors. With the capital market investors absent, the price that was determined when they were present is no longer optimal.

Figure 8 gives the ratios of the certainty equivalent gains in the internal market as a function of the scaling factor for the exposures. When the price and the amount of risk sharing are set equal to their equilibrium values, all pension fund investors benefit. For our parameters, the utility gains of the investors are nearly identical. This is because the investors have the same relative risk aversion and differ only in terms of their initial exposures.

Figure 8: Certainty Equivalent Gains and Losses in the Internal Market



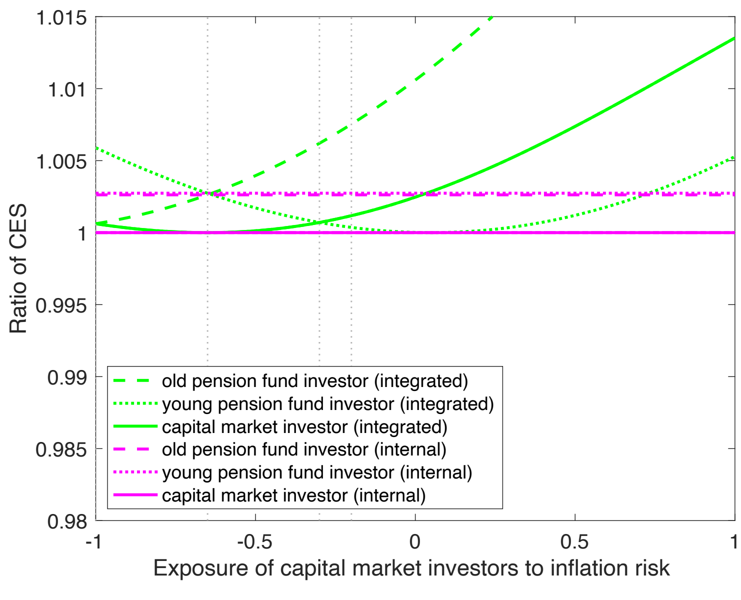
The figure shows the certainty equivalents for the old pension fund investor (dashed line), the young pension fund investor (dotted line), and the capital market investor (solid line) in the internal market. The initial exposures to inflation risk are multiplied by the scaling factor on the x-axis. CEs are relative to the results for the segmented market shown in Figure 3.

The internal market is implemented by the pension fund. Risk sharing takes place according to the appropriate definition of the sharing rule for pension fund participants. The integrated market can be reached by means of trading on the part of the pension fund on behalf of the pension fund investors. In both cases, pension fund investors have some gains in their certainty equivalents. That is because their optimal choice in the segmented market is still accessible, so the certainty equivalent in the segmented market provides a lower bound that can be reached in both setups. For the given calibration, there is no clear ranking of the two markets. Gains are larger for the young investor in the internal market, while old investors gain more in the integrated market. Thus, there is no Pareto dominance relationship between the two market setups. To rank the alternatives, we would need some weighting of the pension fund participants by means of a social planner.

5.5 Role of Initial Exposures and Heterogeneity

Figure 9 analyzes the impact of the initial exposure of the capital market investors. It shows the relative gains in the certainty equivalent for the three groups—old pension fund participants, young pension fund participants, and capital market investors—both in the internal market with the equilibrium prices and positions and in the integrated market.

Figure 9: Capital Market Inflation Exposure and Certainty Equivalent Gains and Losses



The figure shows the certainty equivalents for the old pension fund investor (dashed line), the young pension fund investor (dotted line), and the capital market investor (solid line) in both an internal and an integrated market. CEs are relative to the results for the segmented market shown in Figure 3. The dotted vertical lines represent the original investor exposures to inflation y_n.

For $y_C = y_O = 1$, capital market investors have the same inflation risk exposure as old pension fund participants. In the integrated market, both old pension fund participants and capital market investors want to buy inflation insurance from young pension fund investors. In the internal market, only old pension fund participants want to get insurance from young pension fund participants. Thus, old pension fund participants prefer the internal market, where they do not face an competition for buying insurance against inflation risk. Young pension fund participants, in contrast, benefit from the large demand for insurance in the integrated market and earn high-risk premiums on the insurance they sell. They thus prefer the integrated market to the internal market.

For $y_C = y_Y = 0.3$, capital market investors have the same inflation risk exposure as young pension fund participants. In the integrated market, young pension fund participants and capital market investors thus all sell insurance, while in the internal market, only young pension fund participants sell insurance. Young pension fund participants thus prefer the internal market, where they are the only sellers of inflation-risk insurance and can thus charge higher premiums. Old pension fund participants, in contrast, benefit from the increase in the supply of inflation-risk insurance in the integrated market. They thus prefer the integrated market to the internal market.

For some exposure y_C^* of the capital market investors that is approximately equal to the average exposure of pension fund participants, utility gains in the integrated market and in the internal market coincide. In this case, the equilibrium price in the integrated market is equal to the equilibrium price in the internal market. Additionally, only pension fund investors trade with one another (as in the internal market), while capital market investors have zero demand and do not share any risk with pension fund participants.

For a smaller exposure than y_C^* on the part of capital market investors, the capital market investors want to buy inflation-risk insurance in the integrated market. Due to this competition, old pension fund participants are worse off in the integrated market than in the internal market. On the other hand, young pension fund participants are better off, since they benefit from the demand of both old pension fund participants and capital market investors for inflation-risk insurance.

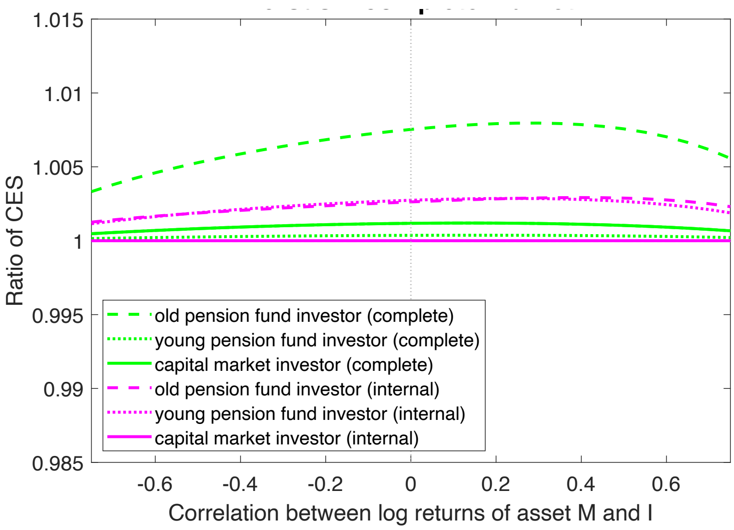
For a somewhat larger exposure than y_C^* on the part of capital market investors, both young pension fund participants and capital market investors offer inflation-risk insurance in the integrated market. Due to this larger supply of inflation-risk insurance, old pension fund participants are better off in the integrated market than in the internal market. Young pension fund participants, in contrast, suffer from the competition from capital market investors in selling insurance and are worse off in the integrated market than in the internal market.

For a much larger exposure than y_C^* on the part of capital market investors, only capital market investors will sell inflation-risk insurance, while both young and old pension fund investors will buy it. An increase in the exposure of capital market investors then makes both young and old investors better off in the integrated market. Ultimately, all pension fund investors will be better off in the integrated market than in the internal market.

5.6 Impact of Correlation

Figure 10 gives the ratios of the certainty equivalents in the integrated and the internal market as a function of the correlation between stock market risk and inflation risk. Gains are largest for a somewhat positive correlation and slightly lower when correlations are very positive or very negative. However, the impact of the correlation is rather small. More particularly, it does not change the overall results.

Figure 10: Correlation and Certainty Equivalent Gains and Losses



The figure shows the certainty equivalents for the old pension fund investor (dashed line), the young pension fund investor (dotted line), and the capital market investor (solid line) as a function of the correlation between inflation and the stock market.

6 Conclusion

Pension fund participants have a heterogeneous exposure to inflation risk. Thus, there is room for risk sharing between them, which can make everyone better off. Risk sharing can be implemented in two ways. First, there can be an internal market within the pension fund in which pension fund participants trade with each other. The pension fund can implement this market by means of an appropriate specification of the sharing rule. Second, there can be an integrated market where pension fund participants trade with each other and with capital market participants. This market can be implemented by the pension fund trading on behalf of its participants.

In the internal market, the choice of the internal price of and positions in the inflation-linked asset are crucial. One alternative is to use the market price of inflation-linked assets from the capital market. This price is objective and easy to verify. However, it is most likely not the optimal choice. It may even make some pension fund participants worse off. This can happen when the group of pension fund participants differs from the group of capital market investors with regard to their initial exposure to inflation risk.

When the price in the internal market is set to the equilibrium price on the internal capital market, all pension fund participants will benefit from the internal market and be happy with their positions. Trading in the integrated market will also make all pension fund investors better off. Concerning a ranking between the integrated and the internal market with equilibrium prices, there is often no Pareto dominance. Therefore, the integrated market with an observable equilibrium market price is not necessarily superior to the internal market with the inter-equilibrium price. The internal market may make any of the types of investors better off, depending on the relation between the average exposures of the pension fund participants and the capital market participants.

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A Numerical Solutions

For the numerical solution, we assume a simple stochastic structure with four possible states of the world defined by two possible states for equity returns and two states for inflation. The random variables $\tilde{R}_S$ and $\tilde{\Pi}$ can take either *up* or *down* values: R_S^d, R_S^u, Π^d and Π^u. Each of the four states, (R_S^k, Π^ℓ), occurs with probability $p^{k\ell}$, with k and ℓ taking values in $\{u, d\}$. In the base case, inflation and equity are independent and all states have probability $\frac{1}{4}$. When $\tilde{\Pi}$ and $\tilde{R}_S$ are independent, equity is not a hedge against inflation. For robustness, we also allow for both positive and negative correlations. With correlation ρ, the probabilities are:

p^{uu}	p^{ud}	p^{du}	p^{dd}
$\frac{1}{4}(1+\rho)$	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1+\rho)$

From the summary statistics in Section 4, we find the means μ_j and standard deviations σ_j of log stock returns and log inflation. With these, we set the up and down states according to:

$\ln R_S^u$	$\ln R_S^d$	$\ln \Pi^u$	$\ln \Pi^d$
$\mu_S + \sigma_S$	$\mu_S - \sigma_S$	$\mu_\Pi + \sigma_\Pi$	$\mu_\Pi - \sigma_\Pi$

Extending the state space to (many) more potential outcomes for equity and inflation will not complicate the analysis and will also not change the qualitative conclusions from the analysis.

With this notation, we can rewrite (6) in the text as

$$\sum_{k\in\{u,d\}} \sum_{\ell\in\{u,d\}} p^{k\ell} \left(y_n(r_B^\ell + x_{S,n}(r_S^{k\ell} - r_B^\ell)) + 1 - y_n \right)^{-\gamma} (r_S^{k\ell} - r_B^\ell) = 0, \tag{A1}$$

where the states for real returns are defined as $r_B^\ell = R_B/\Pi^\ell$ and $r_S^{k\ell} = R_S^k/\Pi^\ell$. For each investor n, this is one non-linear equation in the unknown $x_{S,n}$ and can be easily solved numerically.

For equation (9), we define the elements of the stochastic discount factor $\tilde{M}_C$ as

$$m_C^{k\ell} = \frac{(r_C^{*k\ell})^{-\gamma}}{\sum_{i,j\in\{u,d\}} p^{ij} (r_C^{*ij})^{-\gamma}}, \tag{A2}$$

such that

$$r_I = \sum_{k,\ell\in\{u,d\}} p^{k\ell} m_C^{k\ell} r_B^\ell. \tag{A3}$$

Defining $q_C^{k\ell} \equiv p^{k\ell} m_C^{k\ell}$ and noting that these are all positive and sum to one, we can interpret $q_C^{k\ell}$ as risk-neutral probabilities and rewrite the real interest rate in (A3) as

$$r_I = \sum_{k,\ell\in\{u,d\}} q_C^{k\ell} r_B^\ell \equiv \mathsf{E}_C[\tilde{r}_B], \tag{A4}$$

which states that the real interest rate is the expected real return of the nominal bond under the risk-neutral measure of the capital market investors.

Numerical solutions for the other cases proceed analogously. For example, in the case of an internal market with dual prices, we have, for both the old and the young participants ($n \in \{Y, O\}$), the two implicit demand equations for assets *I* and *S*,

$$\sum_{k,\ell \in \{u,d\}} p^{k\ell} (r_n^{k\ell})^{-\gamma} (r_S^{k\ell} - r_B^{\ell}) = 0 \tag{A5}$$

$$\sum_{k,\ell \in \{u,d\}} p^{k\ell} (r_n^{k\ell})^{-\gamma} (r_I^{\text{int}} - r_B^{\ell}) = 0, \tag{A6}$$

with $r_n^{k\ell} = y_n(r_B^{\ell} + x_{S,n}(r_S^{k\ell} - r_B^{\ell}) + x_{I,n}(r_I^{\text{int}} - r_B^{\ell})) + 1 - y_n$, plus the internal market clearing condition

$$W_Y x_{I,Y} + W_O x_{I,O} = 0. \tag{A7}$$

These are five equations in the five unknowns $x_{S,Y}$, $x_{S,O}$, $x_{I,Y}$, $x_{I,O}$, and r_I^{int}. After solving these equations, we can define the internal stochastic discount factors for the young and the old as

$$m_n^{k\ell} = \frac{(r_n^{*k\ell})^{-\gamma}}{\sum_{i,j \in \{u,d\}} p^{ij} (r_n^{*ij})^{-\gamma}} \qquad (n \in \{Y, O\}), \tag{A8}$$

where, as before, the real portfolio returns $r_n^{*k\ell}$ are evaluated at the optimal demands $x_{j,n}^*$. From the stochastic discount factors, we also obtain the risk-neutral probabilities,

$$q_n^{k\ell} = p^{k\ell} m_n^{k\ell}, \tag{A9}$$

that allow us to rewrite the internal real risk-free rate as

$$r_I^{\text{int}} = \sum_{k,\ell} q_n^{k\ell} r_B^{\ell} = \mathsf{E}_n[\tilde{r}_B]. \tag{A10}$$

From the demand equations, it follows that the risk-neutral probabilities are such that old and young agree on the internal real risk-free rate.

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