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Model risk in the pricing of reverse mortgage products

Anja De Waegenaere
Bertrand Melenberg
Hans Schumacher
Lei Shu
Lieke Werner

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Netspar

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Affiliations
Anja De Waegenaere – Tilburg University
Bertrand Melenberg – Tilburg University
Hans Schumacher – Tilburg University
Lei Shu – Tianjin University of Finance and Economics,
Lieke Werner – Keylane Actuera

Abstract

Most of the existing reverse mortgage products include a guarantee that the loan value cannot exceed the house value when the contract ends, which is called the No-Negative-Equity-Guarantee (*NNEG*). Although different models have been developed to price the *NNEG*, model risk is typically not investigated. We evaluate the cash flows of different reverse mortgage designs and the implied value of the embedded *NNEG*, taking into account model risk. We compare the values of the *NNEG* and their sensitivities to different parameters generated from two popular models in the literature for the house price process, i.e., the Geometric Brownian Motion model and the VAR model. We calibrate the models using prices of regular mortgages and determine the corresponding price ranges for reverse mortgages. We find quite substantial price ranges for both models, indicating that there is substantial model risk in pricing the *NNEG* in reverse mortgage products. The degree of model risk is larger in the VAR model than in the GBM model. We also find that, among the different parameters that can affect the value of the NNEG, it is most sensitive to the parameter that reflects the probability distribution of the future house price, which is the house price increase rate in the VAR model and the dividend rate in the GBM model.

Samenvatting

Een omkeerhypotheek is een lening die een huiseigenaar in staat stelt om (een deel van) de waarde van zijn huis om te zetten in cash, zonder het te hoeven verkopen. De omkeerhypotheek verschilt van een gewone hypotheek doordat aflossing pas plaatsvindt wanneer de huiseigenaar komt te overlijden of het huis verlaat, en doordat rente in de meest gebruikte constructies niet direct wordt betaald maar wordt bijgeschreven bij de schuld. De omkeerhypotheek bevat meestal een garantie die ervoor zorgt dat het bedrag van de lening op de einddatum van het contract niet hoger is dan de waarde van het huis. Het prijzen van deze ingebedde garantie is complex en onderhevig aan modelrisico. In dit artikel gebruiken we twee verschillende modellen, een Geometric Brownian Motion (GBM) model en een Vector Autoregressive Model (VAR), en vinden dat de prijs van de garantie in beide modellen sterk gevoelig is voor de parameter die de verwachte huisprijsontwikkeling weergeeft. Het kalibreren van deze parameter op basis van prijzen van reguliere hypotheken leidt tot een brede range van mogelijke prijzen voor de omkeerhypotheek. De prijs van een omkeerhypotheek is dus relatief sterk gevoelig voor model-risico, wat mogelijk verklaart waarom de markt voor omkeerhypotheken moeilijk op gang komt.

Management Summary

A reverse mortgage is a loan that allows homeowners to convert (part of) the value of their house into cash. Instead of a fixed end date, as in case of a traditional mortgage, the borrower pays back the loan when (s)he dies, or when the home is sold. Reverse mortgages typically include a guarantee that the loan value cannot exceed the house value when the contract ends. This guarantee is called the *No-Negative-Equity-Guarantee* (*NNEG*). Although different models have been developed to price the *NNEG*, model risk is typically not investigated. In this paper, we determine the value of the *NNEG* and investigate its sensitivity to different parameters, using two popular models for the house price process: a Geometric Brownian Motion (GBM) model and a Vector Autoregressive Model (VAR) model. In both models, we interpret the house as a dividend paying asset, where the dividend is parametrized by the dividend yield. Our analysis shows that the price of the *NNEG* is highly sensitive to the dividend yield. This high sensitivity is a concern because the dividend yield is difficult to estimate or calibrate accurately. We calibrate the dividend yield using prices of regular mortgages and determine the corresponding price ranges for reverse mortgages. We find that the difference between the lower bound and the upper bound of the calibrated price of the *NNEG* can be quite large. For example, for a lump-sum reverse mortgage (the most popular reverse mortgage design) with a loan of 30 percent of the initial house value, the lower bound of the calibrated price of the *NNEG* for a couple consisting of a male aged 67 and a female aged 64 is about 2.7% of the initial house value in the VAR model, while the upper bound is around 19% percent of the initial house value. When the loan-to-value ratio is 50 percent, the price of the *NNEG* ranges from about 19% to almost 40% of the initial house value. The corresponding price intervals for the GBM model are somewhat smaller, but still large. These wide price intervals show that there is considerable model risk when pricing the *NNEG*. This high degree of model risk might imply that reverse mortgage providers are reluctant to offer the product, or that they charge relatively high premiums to compensate for the model risk. In the latter case, reverse mortgages may be unattractive to households. Hence, our findings provide a potential explanation for the fact that the market for reverse mortgages is still relatively small. If an active housing market arises where reverse mortgage providers can use house price derivatives to reduce their exposure to model risk, the reverse mortgage market might expand.

1 Introduction

When people are approaching their retirement age, their human capital decreases, while unexpected expenditures on health care may occur. Moreover, the main source of post-retirement income, the pension payment, is threatened. This occurs because the pension system is under stress due to the financial crisis and its aftermath, the aging population, and retirement of the baby boom generation. With a less stable income from the pension system and longer life expectancy, individuals might face considerable financial problems after retirement. At the same time, a large proportion of their accumulated wealth is typically invested in housing equities, which, if liquidated, can generate substantial post-retirement income. A reverse mortgage is one of the products available on the market that allows the elderly to use home equity to finance retirement income, while staying in their home (until death or permanently moving out of the home due to other reasons).

There is a substantial amount of literature that focuses on the demand and supply of reverse mortgages. In an early empirical study, Venti and Wise (1991) found that very few families chose to increase their post-retirement income by equitizing their house value. They argue that annuity reverse mortgages cannot substantially increase the income of average elderly. However, more recently, the reverse mortgage market began to expand. For example, in the US \$30.21 billion of reverse mortgage loans were made in 2009, while in a survey conducted in 2000 among middle-aged adults in Hong Kong, approximately 11% of the respondents indicated that they would "definitely or probably" apply for a reverse mortgage if it were available (Chou et al. 2006). According to Shan (2011), the main reason that led to the market expansion in the US was the increasing house price. Even though the market seems to have expanded, however, the demand for reverse mortgages is still relatively low. Existing studies point at bequest motives (Davidoff, 2010; Michelangeli, 2008; Chou et al., 2006) and the complexity of the product (Davidoff et al. 2015) as potential explanations for limited demand. In addition, factors such as adverse selection and moral hazard may affect the price of reverse mortgages, and, hence, their attractiveness to potential buyers. Specifically, because some reverse mortgage designs have similar characteristics as annuities, adverse selection problems (i.e., the fact that people who buy annuities tend to have a longer life expectancy) may also exist in the reverse mortgage market. Moral hazard issues arise when a reverse mortgage buyer reduces expenditure on maintaining the condition of the house (Shiller and Weiss, 2000, Miceli and Sirmans, 1994).

There is also a growing amount of literature that focuses on the pricing and risk analysis of reverse mortgage products (Wang, et al., 2008; Li, et al., 2010; Yang, et al., 2011; Chen, et al., 2010; Lee, 2012; Shao, et al., 2015; Ji et al., 2012.). To evaluate different reverse mortgage designs one needs to value the various cash flows involved. This requires the use of quantitative models, including the quantification of interest rates, house prices, mortality, and so on. Typically, in order to get a manageable model, one has to make use of simplifying assumptions. However, the use of a simple model may come at the cost of model risk, i.e., one has to deal with a model that is potentially mis-specified. Although different models have been developed to quantify the risks and price the *NNEG*, model risk is typically not investigated. In this paper, we investigate the impact of model risk on the price of the *NNEG*. We consider two types of models to value the *NNEG* for various reverse mortgage designs. In the first model, the house price follows a Geometric Brownian Motion (GBM) while in the second model, the house price is modeled using a Vector Autoregressive Model (VAR) model. While both models have been used in prior studies (e.g., Szymanoski, 1994; Wang et al., 2008; Ji et al., 2012 use a GBM model, and Shao et al., 2015; Sherris and Sun, 2010; Alai et al., 2014; Cho et al., 2013 use a VAR model), the issue of model risk has, to the best of our knowledge, not been investigated.

In this paper we investigate the sensitivity of the price of the *NNEG* to changes in the (calibrated or estimated) model parameters. In both models we interpret the house as a dividend paying asset, where the dividend is parametrized by the dividend yield. Our analysis shows that the price of the *NNEG* is highly sensitive to the dividend yield. This high sensitivity is a concern because the dividend yield is difficult to estimate or calibrate accurately. This occurs for two reasons. First, due to the recent crisis on the housing market, estimates can be highly sensitive to the period that is chosen to estimate or calibrate the model parameters. Second, the dividend yield cannot be determined using only house price data if the house has a nonzero price of idiosyncratic risk (see, e.g., Shao et al., 2015). Therefore, we use data on prices of regular mortgages to calibrate the dividend yield. Using these calibrated ranges of the dividend yield, we then determine the model implied price ranges of the *NNEG* for different reverse mortgage products. We find quite substantial price ranges, indicating that there is substantial model risk. With considerable model risk, reverse mortgage providers might be reluctant to offer the product or charge high premiums to compensate for the model risk, thus keeping some potential buyers out the market. This suggests that the existence of substantial model risk might be another reason why the market for reverse mortgage products is relatively small.

The organization of the remainder of the paper is as follows. In Section 2, we present three basic reverse mortgage schemes and describe the cash flow pattern for each scheme. In Section 3, we determine the market consistent reverse mortgage rate and quantify the value of the *NNEG*. We price the *NNEG* and analyze its sensitivity to parameter risk with the GBM model in Section 4 and with the VAR model in Section 5. Section 6 studies the model risk. We conclude in Section 7.

2 Reverse Mortgage Schemes

In this section, we present the reverse mortgage contract designs that we investigate in this paper. A summary of the notation used in the paper can be found in Table 11 in the Appendix.

A reverse mortgage is a loan that allows homeowners to convert (part of) the value of their house value into cash. Instead of a fixed end date, as in case of a traditional mortgage, the borrower pays back the loan when (s)he dies, or when the home is sold. This time is denoted as date T. The initial loan amount is typically expressed as a fraction φ of the house value, i.e., the initial loan amount is

$$L_0 = \varphi H_0, \tag{1}$$

where H_0 is the value of the house at the beginning of the contract. The fraction φ is referred to as the *loan-to-value ratio*. This is a contract parameter, whose maximum value is set by the lender, typically depending on characteristics of the house and on the characteristics of the owners, such as, for example, their ages. Instead of receiving a lump sum amount, the loan can also take the form of a whole life annuity of value $L_0 = \varphi \cdot H_0$. On date T, the house is sold and the net revenues of sales are used to pay back the loan. To protect the buyer of the reverse mortgage from negative equity, the contract includes a No-Negative-Equity-Guarantee, *NNEG*. Specifically, the repayment of debt on date T is capped at the net revenue from selling the house. This implies that the outstanding debt is not fully repaid in case the net revenues from the house are lower than the outstanding debt; the lender offers the buyer a guarantee NN_T which is defined as follows:

$$NN_T = \begin{cases} L_T - (1-\delta)\cdot H_T, & \text{if } (1-\delta)\cdot H_T < L_T, \\ 0, & \text{if otherwise,} \end{cases} \tag{2}$$

where L_T is the loan balance on date T, H_T is the house price on date T, and δ is the proportional transaction cost in case of a (forced) sale. On date $t = 0$, the lender charges a premium π_{NN} to cover for the guarantee.

To summarize, the main characteristics of the reverse mortgage are as follows:

- The borrower either receives a lump-sum amount of $L_0 = \varphi H_0$ on date zero, or receives a flat amount per period until the contract terminates.
- On date 0, the borrower pays a premium π_{NN} for the *NNEG* to the lender.
- When the contract terminates on date T, the lender receives the minimum of the loan balance on date T, which is denoted as L_T, and the net revenues from selling the house, i.e., the lender receives $\min\{(1-\delta)\cdot H_T, L_T\} = L_T - NN_T$.

It remains to specify how the loan balance accrues over time. We assume that contract termination occurs at the end of a period, i.e., $T \in \mathcal{T} = \{1, 2, \dots, T_{\max}\}$, and that any intermediate payments occur only on dates $t \in \mathcal{T}$. We consider reverse mortgage schemes in which the lender charges a deterministic interest rate of r per period. Whether the borrower pays interest before date T depends on the type of reverse mortgage, as discussed below.

Lump sum contract—A lump-sum reverse mortgage product pays a lump-sum amount at contract initiation. It is the most common type of reverse mortgage in most of the markets. There are no intermediate interest payments and no principal repayment before contract termination. Thus $L_0 = \varphi H_0$, and the loan balance on date T, denoted by L_T, equals:

$$L_T = L_0(1+r)^T. \tag{3}$$

Hence, the cash flow stream of the reverse mortgage is as follows:

- $t = 0$: the borrower receives $L_0 - \pi_{NN}$ from the lender, where π_{NN} is the premium for the guarantee.
- $t = T$: the lender receives $L_T - NN_T$ from the borrower.

Interest only contract—The contract is the same as the lump-sum contract, except that now each year the borrower pays interest on the outstanding debt. So, the initial loan amount again equals $L_0 = \varphi H_0$, but now, due to payment of interest, the loan balance stays constant. Thus, the loan balance on date T equals:

$$L_T = L_0. \tag{4}$$

The cash flow stream of the reverse mortgage is as follows:

- $t = 0$: borrower receives $L_0 - \pi_{NN}$.
- $t \in \{1, \ldots, T\}$: the borrower pays interest rL_0.
- $t = T$: the lender receives $L_T - NN_T$ from the borrower.

Tenure contract—The tenure reverse mortgage product pays a fixed amount C at the end of each period until the termination of the contract. This means that the loan balance at time T is given by

$$L_T = C \sum_{s=1}^{T} (1 + r)^s. \tag{5}$$

The cash flow stream is as follows:

- $t = 0$: the borrower pays the premium π_{NN}.
- $t \in \{0, \ldots, T - 1\}$: the borrower receives C.
- $t = T$: the lender receives $L_T - NN_T$ from the borrower.

The fixed payment C will be set such that the market value of the payments equals φH_0. This will be determined in the next section.

Table 1 summarizes the net cash flows from the point of view of the lender in each of the three cases.

	$t = 0$	$t = 1$	$\cdots$	$t = T - 1$	$t = T$
Lump-sum	$\pi_{NN} - L_0$	0	$\cdots$	0	$L_T - NN_T$
Interest only	$\pi_{NN} - L_0$	rL_0	$\cdots$	rL_0	$rL_0 + L_0 - NN_T$
Tenure	$\pi_{NN} - C$	$-C$	$\cdots$	$-C$	$L_T - NN_T$

Table 1: This table presents the net cash flows in the three reverse mortgage schemes from the point of view of the lender, with L_T given by (3) for the lump sum contract, (4) for the interest only contract, and (5) for the tenure contract.

3 Pricing Reverse Mortgage Products

To determine the price of the reverse mortgage, we split it into two parts. We first determine the market-consistent interest rate r for the "regular" loan without the *NNEG*. We then calculate the *NNEG* as the present value of the loss of the lender at contract termination. We assume that the conditions under which this loan is offered are market-consistent, i.e., arbitrage opportunities are excluded. This means the existence of a Stochastic Discount Factor (SDF) $M_t > 0$ such that a (random) payoff X_t at time t has price $E(X_t M_t)$ at time 0. In addition, we assume that mortality risk is independent of the other sources of risk (house price risk and financial risk), i.e., T is independent of (H_t, M_t).[1] Thus, we have

$$P_0^{(t)} = \mathbb{E}(M_t),$$

with $P_0^{(t)}$ the time 0 price of a zero coupon bond that pays off 1 at maturity, with time-to-maturity t.

3.1 Termination Date

In this section, we model the probability distribution of the termination date T. We consider the case where the contract terminates upon the decease of the last surviving household member. If the borrower is a single person, $T = T_x$, where T_x is the remaining lifetime of the borrower at time $t = 0$. If the borrower is a couple consisting of a male aged x and a female aged y, then $T = \max\{T_x, T_y\}$, where T_x and T_y are the remaining lifetimes of the spouses at time $t = 0$.

To model the probability distribution of T, we let ${}_s p_{z,t}^{(g)}$ denote the probability that a z-year-old in year t with gender $g \in \{m, f\}$ survives at least s more years, and let $q_{z,t}^{(g)}$ be the probability that a z-year-old in year t with gender g dies within a year. We assume that death always occurs at the end of a year. Moreover, in case of a couple, we assume that T_x and T_y are independent. Then, the probability that the contract terminates at the end of year t, $\mathbb{P}(T = t)$, is given by

$$\mathbb{P}(T = t) = {}_{t-1}p_{x,0}^{(g)} \cdot q_{x+t-1,t-1}^{(g)}$$

in case of a single insured of gender g and aged x at time $t = 0$, and is given by

$$\begin{aligned} \mathbb{P}(T = t) = & \left({}_{t-1}p_{y,0}^{(f)} \cdot q_{y+t-1,t-1}^{(f)} \right) \cdot \left(1 - {}_{t-1}p_{x,0}^{(m)} \right) \\ & + \left(1 - {}_{t-1}p_{y,0}^{(f)} \right) \cdot \left({}_{t-1}p_{x,0}^{(m)} \cdot q_{x+t-1,t-1}^{(m)} \right) \\ & + \left({}_{t-1}p_{y,0}^{(f)} \cdot q_{y+t-1,t-1}^{(f)} \right) \cdot \left({}_{t-1}p_{x,0}^{(m)} \cdot q_{x+t-1,t-1}^{(m)} \right), \end{aligned} \tag{6}$$

for a couple with a male aged x and a female aged y at time $t = 0$.

3.2 Pricing the Loan without the Guarantee

In this section, we determine the market-consistent fixed interest rate for the mortgage without the *NNEG*. For each of the three types of reverse mortgage, the market-consistent interest rate r

[1] The assumption that T is independent of (H_t, M_t) does not exclude the possibility that there is dependence between mortality and other sources of risk under the actual probability distribution: the *actual* SDF $\widetilde{M}_t$ might satisfy $\widetilde{M}_t = M_t \Lambda_t$, where Λ_t represents the change of measure going from the actual probability measure to the probability measure used in pricing (and which might be converted into the risk neutral probability distribution). In the former there might be dependence between mortality and the other risk sources. Only in the latter we assume independence between T and (H_t, M_t).

is determined such that the market value of the net cash flow stream to the lender (as displayed in Table 1) is zero, given $\pi_{NN} = NN_T = 0$.

Lump sum contract—In absence of the *NNEG*, the lender pays L_0 to the borrower on date zero, and receives $L_T = (1+r)^T \cdot L_0$ from the borrower upon contract termination. Therefore, the market-consistent interest rate r solves:

$$\begin{aligned} L_0 &= \mathbb{E}(L_T M_T) = L_0 \cdot \mathbb{E}((1+r)^T \cdot M_T) \\ &= L_0 \sum_{t=1}^{T_{\max}} (1+r)^t \cdot \mathbb{E}(M_t) \cdot \mathbb{P}(T=t), \end{aligned}$$

where $T_{\max}$ denotes the maximum value of T. Rewriting this condition shows that the market-consistent interest rate r solves

$$1 = \sum_{t=1}^{T_{\max}} (1+r)^t \cdot P_0^{(t)} \cdot \mathbb{P}(T=t). \tag{7}$$

Thus, the required interest rate r depends on the term structure of interest rate at contract initiation as well as on the probability distribution of the termination time T.

Interest only contract—The lender offers a loan of L_0 on date zero, receives interest payments rL_0 in any year prior to termination, and receives L_0 upon contract termination. The market-consistent interest rate r in this case satisfies (after canceling L_0 from both sides):

$$1 = \sum_{t=1}^{T_{\max}} \left[r \sum_{s=1}^{t} P_0^{(s)} + P_0^{(t)} \right] \cdot \mathbb{P}(T=t). \tag{8}$$

Tenure contract—The lender pays the constant amount C to the borrower at times $t = 0, \ldots, T-1$, and receives $L_T = C\left(\sum_{s=1}^{T}(1+r)^s\right)$ at time T. We assume that C is set such that the market value of the payments at time $t=0$ equals φH_0, i.e.,

$$\varphi H_0 = C \times \left(\sum_{t=0}^{T_{\max}-1} \mathbb{P}(T>t) \cdot P_0^{(t)} \right). \tag{9}$$

The market-consistent interest rate r in this case follows from equating

$$\varphi H_0 = \mathbb{E}(M_T L_T), \tag{10}$$

with φH_0 given by (9). This yields (after canceling C from both sides):

$$\sum_{t=0}^{T_{\max}-1} \mathbb{P}(T>t) \cdot P_0^{(t)} = \sum_{t=1}^{T_{\max}} \mathbb{P}(T=t) \cdot P_0^{(t)} \cdot \left(\sum_{s=1}^{t} (1+r)^s \right). \tag{11}$$

3.3 Pricing the Guarantee

In this section we discuss the approaches that we consider to determine the price of the *NNEG*, given that r is set equal to the market-consistent rate for the loan without the guarantee ((7), (8), or (11)).

If the lender offers a *NNEG*, she runs the risk that the value of the net revenue from selling the property at contract termination, after transaction costs, is lower than the outstanding debt. The lender then effectively acts as a guarantor who will cover the amount NN_T from (2) on date T. Assuming market-consistent pricing, the date-zero price of the guarantee is:

$$\begin{aligned}\pi_{NN} &= \mathbb{E}\left[\max\left\{L_T-(1-\delta)\cdot H_T,0\right\}\cdot M_T\right]\\ &= \sum_{t=1}^{T_{\max}}\mathbb{E}\left[\max\left\{L_t-(1-\delta)\cdot H_t,0\right\}\cdot M_t|T=t\right]\cdot\mathbb{P}(T=t),\end{aligned}$$

where L_T is the loan balance on date T, given by (3), (4), or (5) depending on the type of reverse mortgage, and where M_T denotes the stochastic discount factor on date T. Because we assume that T is independent of (H_t, M_t), the market price of the guarantee is:

$$\pi_{NN}=\sum_{t=1}^{T_{\max}}\widetilde{\pi}_{NN}(t)\cdot\mathbb{P}(T=t), \tag{12}$$

where $\widetilde{\pi}_{NN}(t)=\mathbb{E}\left[\max\left\{L_t-(1-\delta)\cdot H_t,0\right\}\cdot M_t\right]$ denotes the time $t=0$ price of the guarantee (exactly) ending at date t. For each of the three reverse mortgage designs, the value of $\widetilde{\pi}_{NN}(t)$ depends on the joint distribution of the house price H_t and the stochastic discount factor (SDF) M_t.

In the next section we will consider two approaches to determine the value of $\widetilde{\pi}_{NN}(t)$ by treating it as a put option:

- The GBM model: we price the *NNEG* in a Black-Scholes world.
- The (VAR) model: we derive the joint distribution of the SDF process M_t and the house price process H_t in the context of a Vector AutoRegression (VAR) model. The value of the *NNEG* is then determined via simulation.

4 Pricing with the GBM model

4.1 The NNEG in the GBM Model

In the first model, we interpret the house as a dividend paying asset in continuous time. Assuming the dividend rate is q and the house price and the SDF follow a Geometric Brownian Motion, the *NNEG* can be priced as a Black-Scholes (European) put option on the net house price. Indeed, at contract termination, payments to the lender are capped by the net value of the house. As long as the net house value is above the loan balance, the value of the *NNEG* will be zero. Only when the net house value falls below the loan balance, the guarantee will have material effect. Thus, the *NNEG* is a put option with strike price equal to the loan balance and with the net house price as the underlying asset. The value of the option conditional on $T = t$ is:

$$\begin{aligned} \widetilde{\pi}_{NN}(t) &= \mathbb{E}\left[\max\left\{L_t - (1-\delta)\cdot H_t, 0\right\}\cdot M_t\right] \\ &= \mathsf{BSput}\left((1-\delta)H_0, L_t, r_f, q, t, \sigma\right), \end{aligned} \tag{13}$$

where

- r_f is the risk-free rate, σ is the volatility of the house price, q is the dividend rate, and L_t is the loan balance on date t given, by (3), (4), or (5) depending on the type of reverse mortgage;
- the house price H_t is given by $H_t = H_0 \exp\left(\mu t - \frac{1}{2}\sigma^2 t + \sigma W_t\right)$, with $W_t \sim N(0,t)$;
- the SDF M_t is given by $M_t = \exp\left(-r_f t - \frac{1}{2}\lambda^2 t - \lambda W_t\right)$, with $\lambda = (\mu + q - r_f)/\sigma$;
- $\mathsf{BSput}\left(S_0, K, r_f, q, \tau, \sigma\right)$ is the Black-Scholes price of a put option with initial value S_0 of the underlying, strike K, risk free rate r_f, dividend rate q, maturity τ, and volatility σ.

4.2 Calibration of the GBM Model

The price of the guarantee in the GBM model depends on the dividend rate q, the proportional transaction cost δ, the volatility of the house price σ, and the probability distribution of the contract termination date T. It also depends on the loan balance L_t for all $t \leq T_{\max}$. For the lump-sum contract and the tenure contract, the loan balance depends on the market-consistent mortgage rate r (from (7) and (11)), which in turn depends on the term structure of interest rates and the probability distribution of the contract termination date T. In this section, we discuss the calibration of these parameters.

δ and σ – Since the loan needs to be paid back within a limited period upon contract termination, we conservatively set the transaction cost equal to the one in case of a forced sale, which is $\delta = 30\%$.[2] The volatility of the house price growth rate is calibrated based on the log return of the Dutch House Price Index from the first quarter of 1996 to the second quarter of 2014. This yields $\sigma = 7\%$.

The probability distribution of T – To determine the probability distribution of T, we use the survival rates published by the Dutch Actuarial Society (AG) in their AG2014 cohort life table. Figure 1 displays $\mathbb{P}(T = t)$ for single individuals of different ages, and for a couple with a male aged 67 and a female aged 64.

Risk free rate r_f and mortgage rate r – In the GBM model, the term structure is flat at the risk free rate r_f. To determine this risk free rate r_f (which we shall call the "equivalent risk free

[2] Nederlandse Vereniging van Banken: The Dutch Mortgage Market, 26 May 2014.

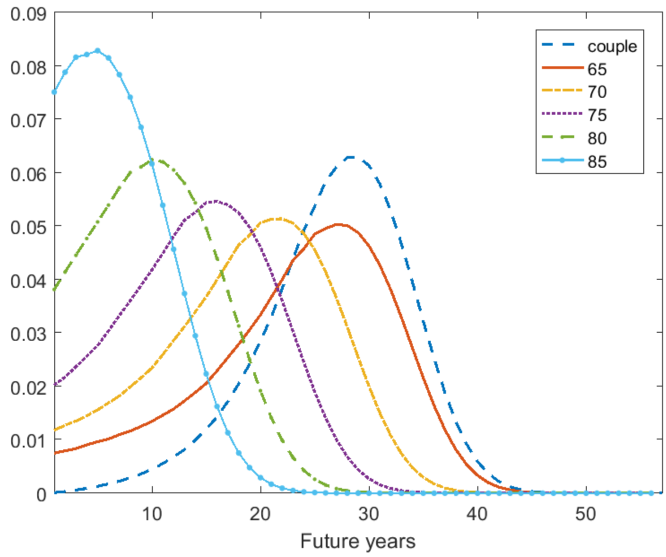


Figure 1: This figure displays $\mathbb{P}(T = t)$, the probability distribution of termination date T, for single buyers of different ages, and for a couple with a male aged 67 and a female aged 64.

rate"), we consider the yield curve determined by the VAR model (see Section 5.1), and we let r_f be equal to the yield corresponding to the expected duration of the reverse mortgage contract in the lump sum and interest only contract, while in the tenure contract, we let r_f be equal to the average yield from the first period to the expected duration of the reverse mortgage contract. The expected duration of the contract is determined as the weighted average of the contract ending times, with the weight of each time equal to $P(T = t)$, the contract termination probability at the corresponding time. The market consistent interest rates then follow from solving (7), (8), and (11), with $P_0^{(t)} = \frac{1}{(1+r_f)^t}$. With a constant equivalent risk-free rate, the market consistent interest rate is equal to this equivalent risk-free rate (i.e., $r = r_f$ solves equations (7), (8) and (11)). The equivalent risk free rate r_f and the market-consistent mortgage rate r depend on the age(s) of the borrower, as these determine the expected duration. The results are displayed in Table 2. The corresponding loan balance as a function of time is displayed in Figure 2 for the three types of reverse mortgages.

	couple	65	70	75	80	85
Lump-sum	0.819%	0.815%	0.805%	0.794%	0.772%	0.744%
Interest-only	0.819%	0.815%	0.805%	0.794%	0.772%	0.744%
Tenure	0.742%	0.732%	0.711%	0.686%	0.649%	0.604%

Table 2: This table displays the market-consistent mortgage rate r, which is equal to the equivalent risk free rate r_f, for single buyers of different ages and for a couple, for the three types of reverse mortgage products.

Dividend rate q – To calibrate the value of the dividend rate q, we use data on interest rates for regular mortgages offered by Florius. Table 3 displays the interest rate charged by Florius, denoted $R^{Florius}(\varphi, T_m)$, for different loan-to-value ratios φ and terms of maturity T_m. The interest rate is not linearly but stair-step increasing with the loan-to-value ratio. In the calibration we use the

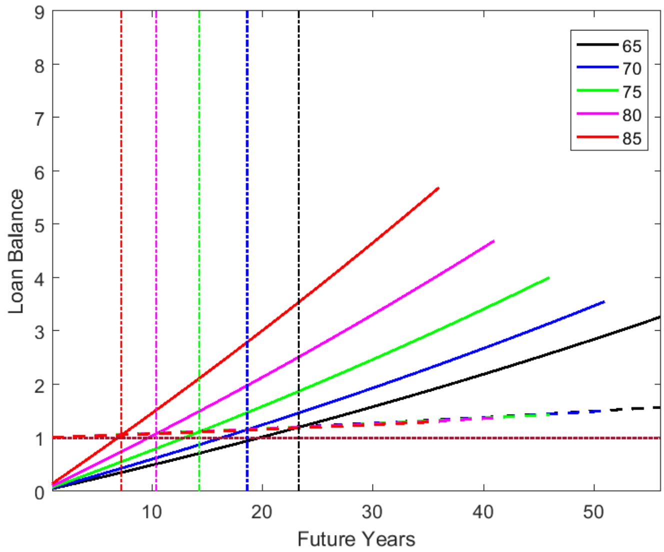


Figure 2: The loan balance L_t as a function of t, with L_0 normalized to 1, for individual borrowers of different ages and for a couple, in the lump-sum scheme (dashed lines), the interest-only scheme (dotted lines), and the tenure scheme (solid lines), respectively. The vertical dashed-dotted lines represent the expected time until decease of the (last surviving) borrower.

φ	1	3	5	6	10	15	20	30
0%-67%	1.25%	1.25%	1.25%	1.8%	1.88%	2.30%	2.7%	3.45%
68%-88%	1.45%	1.45%	1.45%	2.0%	2.08%	2.5%	2.9%	3.65%
89%-98%	1.75%	1.75%	1.75%	2.3%	2.38%	2.80%	3.2%	3.95%
99%+	2.05%	2.05%	2.05%	2.60%	2.68%	3.10%	3.5%	4.25%

Table 3: This table displays the mortgage rate charged by Florius ($R^{Florius}(\varphi, T_m)$) for different values of the term to maturity T_m and loan-to-value ratio φ. The data are obtained from the website of Florius on 8 April 2016.

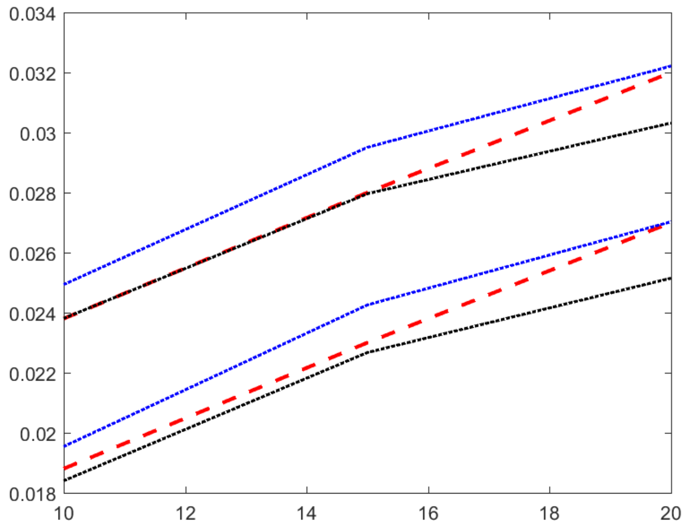


Figure 3: This figure displays the interest rate charged by Florius for regular mortgages, as well as the model-implied interest rate for $q = q_{\min}$ and for $q = q_{\max}$. The upper three lines display $R(q_{\max}, \varphi, T_m)$ (dotted), $R^{\textit{Florius}}(\varphi, T_m)$ (dashed) and $R(q_{\min}, \varphi, T_m)$ (dotted) as a function of the term to maturity for $\varphi = 98\%$. The lower three lines display these rates for $\varphi = 67\%$.

interest rates for loan-to-value ratios $\varphi \in \{67\%, 98\%\}$ and terms to maturity $T_m \in \{10, 15, 20\}$. We first determine the model-implied interest rate for regular mortgages, denoted $R(q, \varphi, T_m)$, for these values of φ and T_m.[3] We then determine the lower bound for q such that the model-implied interest rates are no larger than the Florius interest rates for all $\varphi \in \{67\%, 98\%\}$ and $T_m \in \{10, 15, 20\}$, and the upper bound for q such that the model-implied interest rates are no less than the Florius interest rates for all $\varphi \in \{67\%, 98\%\}$ and $T_m \in \{10, 15, 20\}$, i.e., we let

$$q_{\min} = \max\left\{q : R(q, \varphi, T_m) \leq R^{\textit{Florius}}(\varphi, T_m) \text{ for all } \varphi, T_m\right\},$$
$$q_{\max} = \min\left\{q : R(q, \varphi, T_m) \geq R^{\textit{Florius}}(\varphi, T_m) \text{ for all } \varphi, T_m\right\},$$

The details of the procedure are discussed in the Appendix.[4] We find $q_{\min} = 4.6\%$ and $q_{\max} = 6.6\%$. Figure 3 displays the interest rates charged by Florius, $R^{Florius}(\varphi, T_m)$, as well as the model implied rates, $R(q_{\min}, \varphi, T_m)$ and $R(q_{\max}, \varphi, T_m)$, as a function of the time to maturity T_m, for the two values of φ. We set the calibrated equal to

$$q = (q_{\min} + q_{\max})/2 = 5.6\%.$$

4.3 Sensitivity Analysis

In this section, we investigate the sensitivity of the price of the *NNEG*, π_{NN}, to the various parameter values. The sensitivities of $\widetilde{\pi}_{NN}(t)$ follow straightforwardly from the "Greeks," see,

[3] We take into account that in practice mortgage rates typically include mark-ups for operational costs and Basel requirements.

[4] `https://www.hypotheeklastencalculator.nl/berekenen/executiewaarde/`

for example, Hull (2015). The sensitivity of π_{NN} then follows from (12). We first present the partial derivatives of $\widetilde{\pi}_{NN}(t)$ with respect to the various parameters. Then we illustrate graphically the sensitivities of π_{NN} with respect to these parameters.

To present the partial derivatives of $\widetilde{\pi}_{NN}(t)$, we first notice that $\text{BSput}\,(S_0, K, r_f, q, \tau, \sigma)$, with $S_0 = (1-\delta)H_0$, $K = L_t$, and $\tau = t$, is given by

$$\text{BSput}\,((1-\delta)H_0, L_t, r_f, q, t, \sigma) = e^{-rt} L_t \Phi(-d_2) - (1-\delta) H_0 e^{-qt} \Phi(-d_1),$$

with

$$\begin{aligned} d_1 &= \frac{\log((1-\delta)H_0/L_t) + (r_f - q + \sigma^2/2)t}{\sigma\sqrt{t}}, \\ d_2 &= d_1 - \sigma\sqrt{t}, \end{aligned}$$

where Φ is the cumulative distribution function of the standard normal distribution.

The partial derivatives of $\widetilde{\pi}_{NN}(t)$ with respect to the loan-to-value ratio φ, the dividend rate q, the transaction costs δ, and the house price volatility σ are given by

$$\begin{aligned} \frac{\partial \widetilde{\pi}_{NN}(t)}{\partial \varphi} &= e^{-r_f t} \Phi(-d_2) \times \frac{\partial L_t}{\partial \varphi}, \\ \frac{\partial \widetilde{\pi}_{NN}(t)}{\partial q} &= t(1-\delta) H_0 e^{-qt} \Phi(-d_1), \\ \frac{\partial \widetilde{\pi}_{NN}(t)}{\partial \delta} &= e^{-qt} H_0 \Phi(-d_1), \\ \frac{\partial \widetilde{\pi}_{NN}(t)}{\partial \sigma} &= (1-\delta) H_0 e^{-qt} \phi(d_1) \sqrt{t}, \end{aligned}$$

where $\partial L_t/\partial\varphi$ depends on the reverse mortgage scheme and where ϕ is the density function of the standard normal distribution. These partial derivatives are all positive, implying that $\widetilde{\pi}_{NN}(t)$ and thus also π_{NN} are increasing functions of these parameters.

These positive partial derivatives can easily be understood using the characteristics of the underlying put option. For example, an increase in the loan-to-value ratio φ results in an increase in the strike price of the put option, which leads to a higher put option price. An increase in the dividend rate q results in an increase in the risk premium (i.e., $\lambda = (\mu + q - r_f)/\sigma$). This yields a more widespread distribution of the SDF M_t, resulting in a higher put option price. An increase in the transaction costs δ means a decrease in the underlying value $(1-\delta)H_0$. But the delta (Δ) of a put option (i.e., the sensitivity of the put option price with respect to the underlying) is negative, so that an increase in δ has a positive effect on the put option price. Finally, an increase in the house price volatility σ is positive, following from vega (i.e., the sensitivity of the put option price with respect to the volatility) being positive.

The second order partial derivatives are given by

$$\begin{aligned} \frac{\partial^2 \widetilde{\pi}_{NN}(t)}{\partial \varphi^2} &= e^{-r_f t} \phi(d_2) \frac{\partial \log L_t(\varphi)}{\partial \varphi} \frac{\partial L_t(\varphi)}{\partial \varphi} / \sigma\sqrt{t}, \\ \frac{\partial^2 \widetilde{\pi}_{NN}(t)}{\partial q^2} &= t^2 (1-\delta) H_0 e^{-qt} \left(\phi(d_1)/\sigma\sqrt{t} - \Phi(-d_1) \right), \\ \frac{\partial^2 \widetilde{\pi}_{NN}(t)}{\partial \delta^2} &= e^{-qt} H_0 \phi(d_1)/(1-\delta)\sigma\sqrt{t}, \\ \frac{\partial^2 \widetilde{\pi}_{NN}(t)}{\partial \sigma^2} &= \nu d_1 d_2/\sigma, \end{aligned}$$

with $\nu = \partial\widetilde{\pi}_{NN}(t)/\partial\sigma$, the partial derivative of $\widetilde{\pi}_{NN}(t)$ with respect to σ (known as the "Greek" vega). The second order partial derivatives with respect to φ and δ are positive, implying that $\widetilde{\pi}_{NN}(t)$ and thus also π_{NN} are convex functions of φ and δ. On the other hand, the second order partial derivatives with respect to q and σ can be both positive and negative. This implies that $\widetilde{\pi}_{NN}(t)$ can be both accelerating and decelerating as function of q and σ, depending on the parameter values.

In Figure 4, we illustrate graphically π_{NN} as a function of the loan-to-value ratio (φ, first row), the dividend rate (q, second row), the proportional transaction cost (δ, third row), and the volatility of the house price (σ, last row), with $\varphi \in [0, 1]$, and with the latter three parameters around their calibrated values. The left panels correspond to the the lump sum contract, the middle panels to the interest-only contract, and the right panels to the tenure contract. In each case, results are displayed for single borrowers of different ages. If φ is fixed, we set it equal to $\varphi = 0.50$. If any of the other three parameters is kept fixed, its value is set equal to its calibrated value. The initial house price is set equal to $H_0 = 1$. Figure 5 complements Figure 4 by showing the second order partial derivatives with respect to φ and q as a function of φ and q, respectively, and by showing $\partial^2\widetilde{\pi}_{NN}(t)/\partial\varphi\partial q = \partial^2\widetilde{\pi}_{NN}(t)/\partial q\partial\varphi$ as a function of φ and q. This latter second order derivative is given by

$$\frac{\partial^2\widetilde{\pi}_{NN}(t)}{\partial\varphi\partial q} = \ te^{-r_f t}\phi(d_2)\frac{\partial L_t(\varphi)}{\partial\varphi}/\sigma\sqrt{t}.$$

All curves in Figure 4 are increasing as follows from the positive partial derivatives of $\widetilde{\pi}_{NN}(t)$. Their shapes can be understood by taking into account the second order partial derivatives. For the same parameter values, the curves for older buyers typically correspond to lower values of π_{NN}. Older buyers have a higher probability of shorter remaining lifetimes. This would suggest a higher value, since theta (i.e., the sensitivity of the standard put option price with respect to the time-to-maturity, with symbol Θ) is typically negative, except for deep in-the-money put options. However, there are two opposing effects. First, we have an effect via the strike price. Increasing the time-to-maturity, increases this strike price (except in the interest only contract), which has a positive effect on the put option value. Secondly, we have an effect via the risk free interest rate: older buyers have a lower risk free interest rate (see Table 2). Since rho (i.e., the sensitivity of the standard put option price with respect to the risk free interest rate) is negative, a lower risk free rate implies a higher value of the put option. The graphs show the resulting net age effects. In most cases the two positive effects (via the strike price and the risk free interest rate) dominate the negative time effect (via Θ), except for the tenure contract with very low dividend yields. The age effect is stronger in the lump sum case than in the interest only case. This can be understood as follows. The lump sum and the interest only contract have the same risk free rates, so an important difference in terms of the age effect between these two is the positive strike effect, which is present in the lump sum but absent in the interest only contract. The tenure contract has a lower risk free interest rates than the other two contracts (see Table 2), so that here the magnitude of the net age effect compared to the other two is not immediately clear, but can be observed from the figure (given the considered parameter values).

Among these calibrated parameters (dividend rate (q), proportional transaction costs (δ), and volatility of the house price (σ)), π_{NN} seems to be most sensitive to the dividend rate. For example, π_{NN} for a 65-year-old borrower increases from 1.68% to around 39.8% as the dividend rate increases from 0% to 10% in the lump-sum scheme. In the interest-only scheme, π_{NN} increases from 0.01%% to 17.35% and in the tenure scheme, it increases from 4.21% to 42.28%.

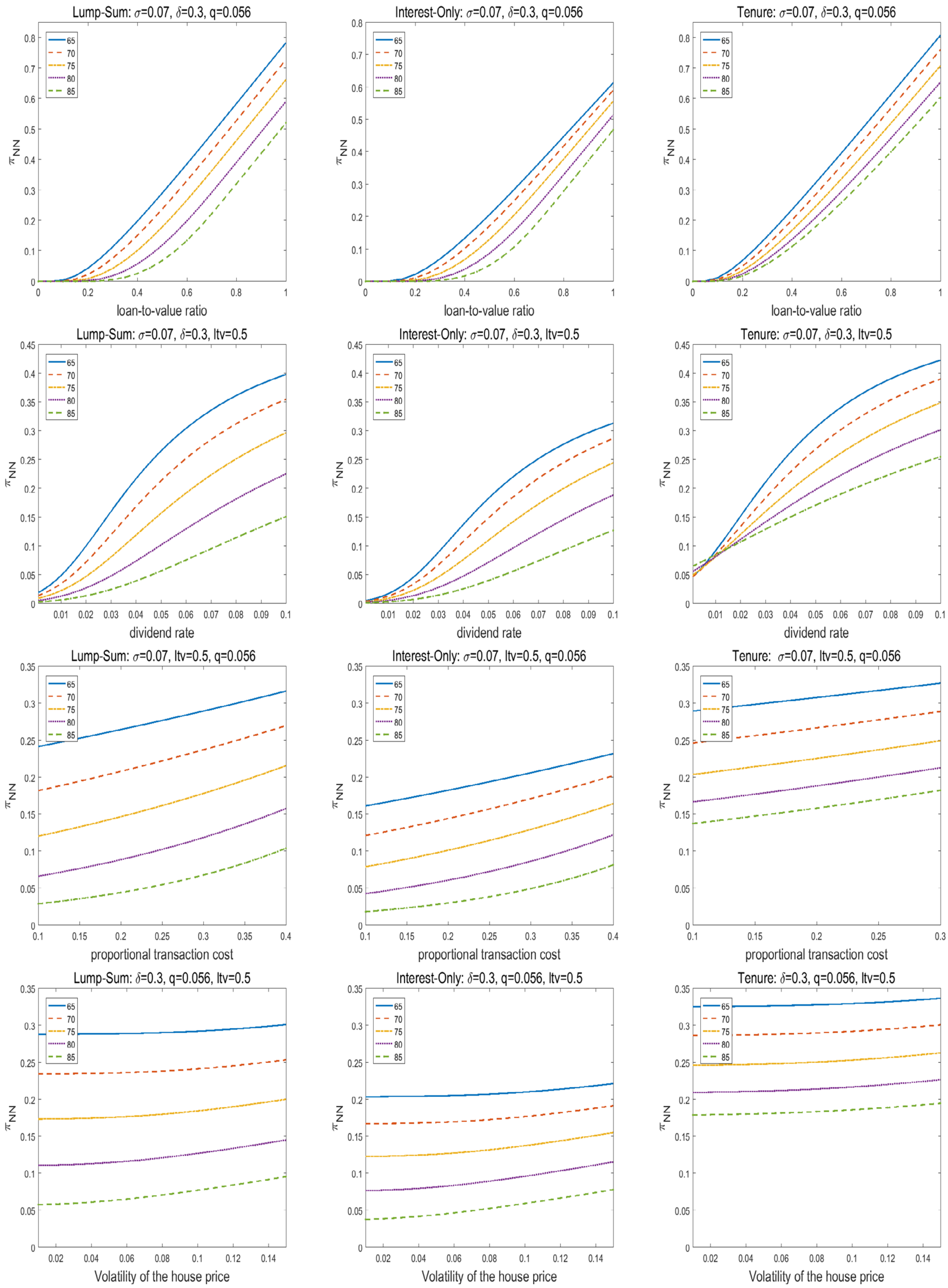


Figure 4: Sensitivity analysis of π_{NN} for the lump-sum scheme (left), the interest only scheme (middle) and the tenure scheme (right). The figure diplays π_{NN} as a function of the loan-to-value ratio (ltv=φ, first row), the dividend rate (q, second row), the proportional cost (δ, third row), and the house price volatility (σ, fourth row).

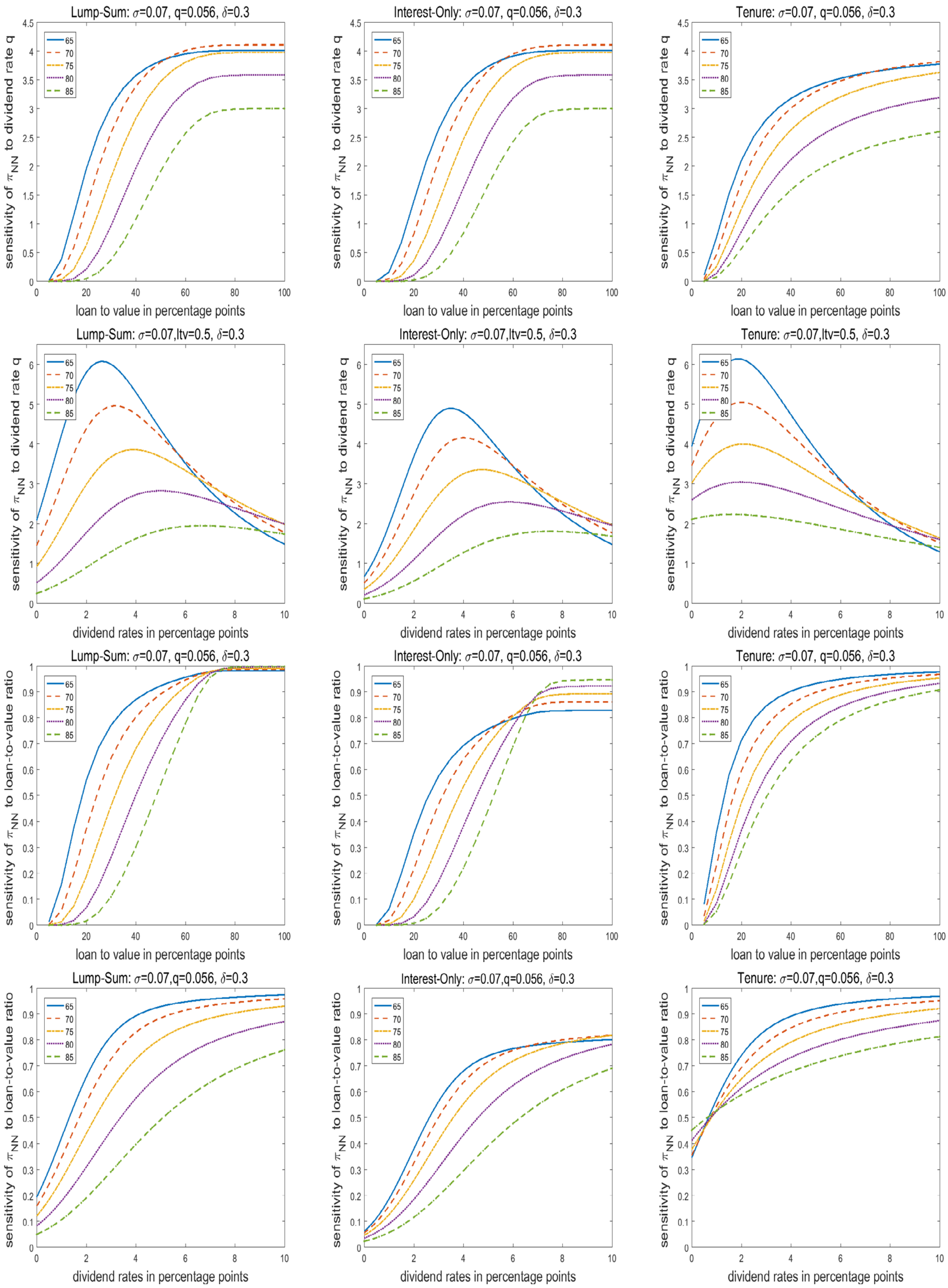


Figure 5: Sensitivity analysis of the derivatives of π_{NN} for the lump-sum scheme (left), the interest only scheme (middle) and the tenure scheme (right). The figure displays $\partial\pi_{NN}/\partial q$ and $\partial\pi_{NN}(t)/\partial\varphi$, respectively, as a function of the loan-to-value ratio (ltv=φ, first and third row) and as a function of the dividend rate (q, second row and fourth row). In each case, the transaction cost is set equal to $\delta = 0.3$, and the volatility of the house price increasing rate is set equal to $\sigma = 0.07$.

5 Pricing with the VAR model

In this section we consider an alternative approach to model the joint distribution of (H_t, M_t). This approach makes use of a Vector Auto Regression (VAR) model that includes the GBM model as a special case. We first present the VAR model, the corresponding SDF, and the implied term structure. Next, we discuss the parameter estimation and calibration. We conclude this section by illustrating the pricing of the *NNEG* (i.e., π_{NN}).

5.1 The VAR Model

Five variables in total will be included in the VAR model: the Dutch GDP log growth rate, the log return on the Dutch House price index (which reflects the growth rate of house prices), inflation (quantified as the log price change in the CPI index), the 3 months Euribor rate, and the spread of the 10 years zero coupon rate over the 3-months Euribor rate. Table 4 presents the variable names and a short description of each of the variables. These choices are motivated by the literature. Brooks and Tsolacos (1999) indicated that interest and inflation are significant factors in explaining house price returns. Abelson et al. (2005) estimated a model using several economic variables. They found that in the long run house prices are affected significantly by disposable income, interest rates, equity prices, consumer price indexes, and the supply of housing. Ang and Piazzesi (2003) describe the joint dynamics of bond yields and macroeconomic variables in a VAR model. They include GDP as a factor in predicting housing prices and the yield curve. Following this study, we also include GDP and inflation in our VAR model. In line with recent studies (Sherris and Sun, 2010; Alai et al., 2013; Shao et al., 2012; and Cho et al., 2013), we will use two factors from the yield curve, namely, the three months Euribor rate and the ten year spread.

variable	definition
hpi	house price growth rate
gdp	GDP growth rate
cpi	inflation
$y^{(1)}$	3-month zero-coupon rate
$y^{(40)} - y^{(1)}$	10-year yield spread

Table 4: This table presents the state variables used in the VAR model.

The dynamics of the five state variables, collected in the n-dimensional vector x_t, with $n = 5$ in our case, is assumed to follow the following VAR:

$$x_{t+1} = \alpha + \Gamma x_t + \Sigma \varepsilon_{t+1}, \tag{14}$$

where α is an n-dimensional vector of parameters, Γ is an $n \times n$-dimensional matrix of parameters, Σ an $n \times n$-dimensional lower triangular matrix, and ε_{t+1} is an n-dimensional vector of error terms representing the shocks to the system, with $\varepsilon_{t+1} \overset{i.i.d}{\sim} N(0_{n\times 1}, I_{n\times n})$, where $0_{n\times 1}$ is an n-dimensional vector of zeros, and $I_{n\times n}$ is the n-dimensional identity matrix.

Before proceeding, we discuss the link with the GBM model. Note that for $\Gamma = 0$, the VAR model implies that

$$H_t = H_0 \exp\left(\alpha_1 t + \sigma_1 \sum_{s=1}^{t} \varepsilon_{1,s}\right), \tag{15}$$

with α_1 the first component of α, σ_1 the $(1,1)$-component of Σ, and $\varepsilon_{1,s}$ the first component of ε_s. Hence, the house price process in the GBM model is a special case of the house price process in the VAR model, with $\Gamma = 0$ and $\alpha_1 = \mu - \frac{1}{2}\sigma_1^2$.

5.2 The Stochastic Discount Factor

The SDF M_t is given by $M_t = \prod_{s=1}^{t} m_s$, where $m_s > 0$ is the SDF between periods $s-1$ and s, i.e., for a payoff X_s at time s the price S_{s-1} at time $s-1$ is given by

$$S_{s-1} = \mathbb{E}_{s-1}(m_s X_s),$$

with $\mathbb{E}_{s-1}$ the conditional expectation operator, conditional upon the information available at time $s-1$. For a payoff X_t at time t, the price at time $t=0$ can be obtained by iterating this equation backward, also using the law of the iterated expectations, resulting in

$$S_0 = \mathbb{E}\left(\prod_{s=1}^{t} m_s X_t\right) = \mathbb{E}(M_t X_t),$$

with $\mathbb{E} = \mathbb{E}_0$.

We model m_{t+1} to be exponentially affine:

$$m_{t+1} = \exp\left(-y_t^{(1)} - \frac{1}{2}\lambda'\lambda - \lambda'\varepsilon_{t+1}\right), \tag{16}$$

with $y_t^{(1)}$ the one-period (continuously compounded) interest rate, λ an n-dimensional vector of "prices of risk", and ε_{t+1} the n-dimensional vector of error terms from (14). Given m_{t+1}, we find the SDF M_t:

$$M_t = \prod_{s=1}^{t} m_s = \exp\left(-\sum_{s=1}^{t} y_s^{(1)} - \frac{1}{2}\lambda'\lambda t - \lambda'\sum_{s=1}^{t}\varepsilon_s\right), \tag{17}$$

where $\sum_{s=1}^{t}\varepsilon_s$ follows an n-dimensional normal distribution with mean vector $0_{n\times 1}$ and covariance matrix $tI_{n\times n}$. We note that the SDF that we use in the GBM model is a special case of the SDF in (17).

5.3 The (Implied) Term Structure

The (nominal) term structure can be derived from the yields of the zero-coupon bonds. Define the yield to maturity of a zero-coupon bond with time-to-maturity $\mathbb{T}$ as $y_t^{(\mathbb{T})}$, then we have

$$P_t^{(\mathbb{T})} = \exp(-\mathbb{T} y_t^{(\mathbb{T})}).$$

A $\mathbb{T}$-year zero coupon bond, like any payoff in the economy, can be priced by the SDF. Therefore, the price of this zero coupon bond at time t satisfies,

$$P_t^{(\mathbb{T})} = \mathbb{E}_t\left(m_{t+1} P_{t+1}^{(\mathbb{T}-1)}\right) \tag{18}$$

For $\mathbb{T} = 1$ we have

$$y_t^{(1)} = -\log(P_t^{(1)}) = -\log\left(\mathbb{E}_t\left(m_{t+1} \times 1\right)\right) = -\log\exp\left(-y_t^{(1)}\right) = y_t^{(1)},$$

showing that the model is self-consistent.

By postulating

$$\log(P_t^{(\mathbb{T})}) = -A(\mathbb{T}) - B(\mathbb{T})' x_t, \tag{19}$$

$A(\mathbb{T})$ and $B(\mathbb{T})$ can be solved recursively using this equation. This results in the following equations:

$$\begin{aligned} A(\mathbb{T}) &= A(\mathbb{T}-1) + B(\mathbb{T}-1)'\alpha - \tfrac{1}{2}B(\mathbb{T}-1)'\Sigma\Sigma' B(\mathbb{T}-1) - B(\mathbb{T}-1)'\Sigma\lambda \\ B(\mathbb{T}) &= \Gamma' B(\mathbb{T}-1) + \delta_1, \end{aligned} \tag{20}$$

with $\delta_1 = (0,0,0,1,0)'$, selecting the fourth component of x_t, which is $y_t^{(1)}$. The starting values for A and B are $A(0) = 0$, $B(0) = 0_n$ (following from $\log P_t^{(0)} = \log(1) = 0$). The zero-coupon yields are thus given by

$$y_t^{(\mathbb{T})} = a(\mathbb{T}) + b(\mathbb{T})' x_t, \tag{21}$$

with $a(\mathbb{T}) = -A(\mathbb{T})/\mathbb{T}$ and $b(\mathbb{T}) = -B(\mathbb{T})/\mathbb{T}$.

5.4 Data

For the five state variables, we use data retrieved from Datastream and the Dutch Central Bank (DNB), from the first quarter of 2009 up to and including the first quarter of 2016. Figure 6 shows the evolution of the five state variables since the second quarter of 1995, when the HPI growth rate became available. Since the GDP growth rate is only available quarterly, we use quarterly data. There was a change in the evolution pattern during the financial crisis, especially for the 3-month rate and the HPI growth rate. Both of them are essential in the pricing of the *NNEG*. To better deal with the impact of the low interest rate and house price growth rate, we use the subsample after the financial crisis in our estimation (i.e., starting from the first quarter of 2009).

In the calibration of the prices of risk (the vector λ) we make use of interest rates, downloaded from the website of the European Central Bank.[5] We use the three-month, nine-month, and one-year to thirty-year interest rates from the third quarter of 2014 to the first quarter of 2016.

5.5 Estimation and Calibration

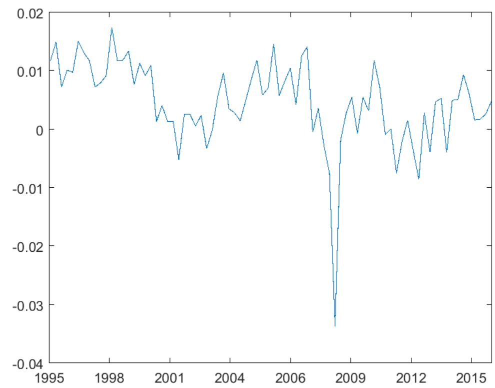
The price of the guarantee in the VAR model depends on the proportional transaction cost δ, the probability distribution of the contract termination date T, and the parameters in the VAR model. In this section, we discuss the estimation/calibration of these parameters.

The proportional transaction cost δ and the probability distribution of T –These parameters are set at the same values as in the GBM model.

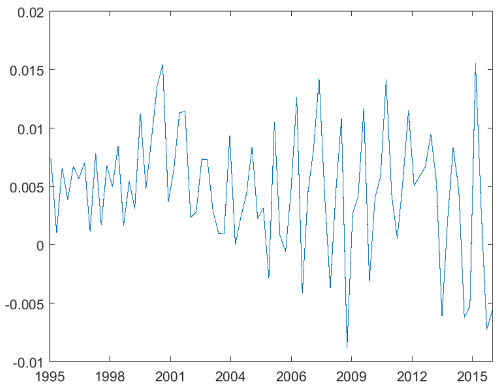
Estimating the parameters of the VAR model – In line with the existing literature (Ang and Piazzesi, 2004; Cochrane and Piazzesi, 2005 and 2008), we conduct a two-step estimation procedure to determine the parameters of the VAR model and the SDF. In the first step, we estimate the VAR model using maximum likelihood. In the second step, we treat the estimated parameters in the VAR model as given, and calibrate λ, the vector of the prices of risk, by minimizing the squared difference between the model-implied term structure of interest rates (as function of λ) to the historical term structure of interest rates. The first step estimation results are shown in Table 5. The calibrated values for the prices of risk are shown in the second and third columns

[5] See `http://www.ecb.europa.eu/stats/money/yc/html/index.en.html`.

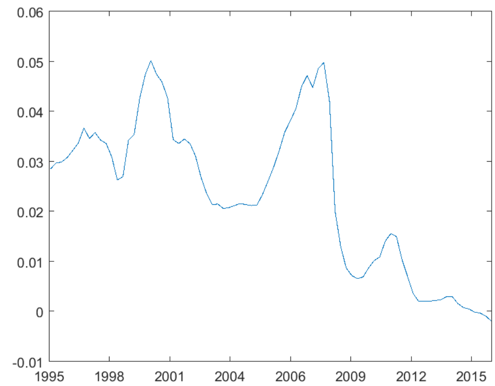
(a) HPI growth rate



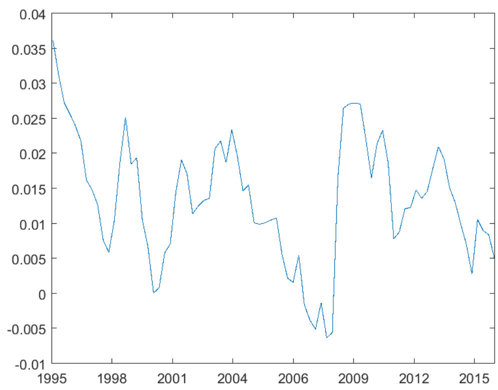
(b) GDP growth rate



(c) Inflation



(d) 3-month rate



(e) 10-year spread

Figure 6: The figure displays the historical movements of the state variables in the VAR model.

(columns “original 1” and “original 2”) of Table 6. In case of “original 1” we do not impose restrictions on λ. In case of “original 2” we assume only the inflation, short rates, and 10-year rate spreads have a non-zero price of risk. In the latter case, calibrated version of the VAR model does not include the GBM model as special case.

	α			Γ		
hpi	-0.00196	0.39646	0.2496	-0.26041	-3.68573	1.37647
	0.00401	*0.14667*	*0.24664*	*0.27633*	*1.52411*	*1.06391*
gdp	-0.00636	0.22988	0.07766	-0.0519	-0.0393	2.25398
	0.003	*0.1097*	*0.18447*	*0.20668*	*1.13992*	*0.79572*
cpi	0.00324	-0.06246	-0.14935	-0.04322	0.55684	-0.16527
	0.00276	*0.10074*	*0.1694*	*0.1898*	*1.04681*	*0.73073*
$y^{(1)}$	0.00067	0.0066	0.03084	0.00348	0.86901	-0.12491
	0.00043	*0.01585*	*0.02665*	*0.02986*	*0.16469*	*0.11497*
$y^{(40)} - y^{(1)}$	0.0013	-0.02394	-0.05615	-0.04688	-0.00171	0.69047
	0.00045	*0.01654*	*0.02782*	*0.03117*	*0.1719*	*0.1200*
	$\mu = (I-\Gamma)^{-1}\alpha$			Σ		
hpi	-0.00491	0.00895	0	0	0	0
gdp	0.00103	0.0001	0.0067	0	0	0
cpi	0.00346	-0.00028	-0.00079	0.00609	0	0
$y^{(1)}$	0.00154	-2.63E-05	-0.0008	5.82E-05	0.00054	0
$y^{(40)} - y^{(1)}$	0.00385	-0.00029	-0.00039	0.00016	0.00027	0.00083

Table 5: Estimation results of the VAR(1) model $x_{t+1} = \alpha + \Gamma x_t + \Sigma\varepsilon_{t+1}$. The first column contains the state variables. The top panel displays the estimated coefficients and the corresponding standard errors. The bottom panel displays the matrix Σ and the vector μ of model implied quarterly equilibrium values of the state variables.

	Original 1	Original 2
hpi	-0.055	0
gdp	0.114	0
cpi	-0.003	-0.0060
$y^{(1)}$	-0.126	-0.2525
$y^{(40)} - y^{(1)}$	-0.008	-0.0162

Table 6: This table presents the calibrated λ (the prices of risk) for the case where we do not impose restrictions on λ (“Original 1”), and for the case where we assume that only the inflation, short rates, and 10-year rate spreads have a non-zero price of risk (“Original 2”).

For the log return on the house price index we find that it depends positively on its own lag (with coefficient around 0.40) and negatively on the one-year interest rate (with coefficient around -3.7). These effects are statistically significant (at the 5% significance level). The implied long run average of the log return on the house price index (shown in the bottom panel of Table 5) turns out to be -0.491%, i.e., around $\mu_{\text{hpi}} = -1.96\%$ on an annual basis. The estimated volatility of the house price index, equal to around 1.8% on an annual basis (i.e., ca. $\sqrt{4} \times 0.895\%$), is lower than the calibrated value of the volatility used in the GBM model. This occurs because the estimated volatility is based on a shorter sample, starting from the first quarter of 2009.

Re-calibrating the parameters of the VAR model – As discussed in the context of the GBM model, the price of the *NNEG* is rather sensitive to the dividend yield. In the VAR model, the price depends on the long run average of the log return on the house price index, which is related to the dividend yield in the GBM model. Specifically, suppose that in the VAR model the house-cum-dividend value $\widetilde{H}_{t+1}$ at time $t+1$ is given by

$$\widetilde{H}_{t+1} = H_{t+1}\exp(q_t), \tag{22}$$

with q_t the dividend yield. Then $H_t = \mathbb{E}_t(M_{t+1}\widetilde{H}_{t+1})$ yields,

$$q_t = y_t^{(1)} - \alpha_1 - \frac{1}{2}\sigma_1^2 - \gamma_1' x_t + \lambda_1\sigma_1, \tag{23}$$

with (as before) α_1 the first component of α, σ_1 the $(1,1)$-component of Σ, λ_1 the first component of λ, and with γ_1 the first row of Γ. Like in the GBM model, we shall calibrate q_t, based on observed prices of regular mortgages. Due to the idiosyncratic risk component (see e.g., Shao et al., 2015), we expect that the calibrated value based on regular mortgage data better reflects the risk premium in the house price increase rate than the estimated value based on house price data. The dividend yield q_t can be calibrated in different ways, for example, by (re-)calibrating one or more of the parameters of the right hand side of (23). In this paper we choose to calibrate q_t by (re)calibrating μ_{hpi}, the first component of $\mu = (I_n - \Gamma)^{-1}\alpha$. To re-calibrate μ_{hpi}, we use the same procedure as described for the GBM model, taking into account that a re-calibrated value of μ_{hpi} results in (re)calibrated values of the whole vector $\alpha = (I_n - \Gamma)\mu$. Details are available in the Appendix. To distinguish this re-calibrated value based on (regular) mortgage data from the estimated value based on house price data, we denote the former by μ_{hpi}^m and the latter by μ_{hpi}^h. The resulting upper and lower bound for the re-calibrated (annualized) μ_{hpi} are given by:

$$\mu_{\mathsf{hpi},\min}^m = -4.99\% \text{ and } \mu_{\mathsf{hpi},\max}^m = -1.14\%.$$

Figure 7 displays the interest rates charged by Florius, $R^{\mathsf{Florius}}(\varphi, T_m)$, as well as the model implied rates, $R(\mu_{\mathsf{hpi},\min}^m, \varphi, T_m)$ and $R(\mu_{\mathsf{hpi},\min}^m, \varphi, T_m)$, as a function of the time to maturity T_m, for the two values of φ.
We set the re-calibrated value equal to the average of the upper and lower bound:

$$\mu_{\mathsf{hpi}}^m = (\mu_{\mathsf{hpi},\min}^m + \mu_{\mathsf{hpi},\max}^m)/2 = -3.065\%.$$

For each of the three values $\mu_{\mathsf{hpi},\min}^m$, $\mu_{\mathsf{hpi},\max}^m$, and μ_{hpi}^m, we re-calibrate the prices of risk λ to derive the corresponding SDF. In this way, by not re-calibrating just a single (*ad hoc*) parameter, we aim to avoid disturbing the links between the different parts of the model. Table 7 displays the resulting values of λ.
We conclude with a few observations. First, the re-calibrated drift based on mortgage data is substantially lower than the estimated value based on house price data, i.e., $\mu_{\mathsf{hpi}}^m = -3.065\% < \mu_{\mathsf{hpi}}^h = -1.96\%$. However, the estimated value μ_{hpi}^h lies between the lower and upper bound of the re-calibrated value. Second, when translated in terms of dividend rate (using $q_t = -\mu_{\mathsf{hpi}} - \frac{1}{2}\sigma_1^2 + y_t^{(1)}$, with $y_t^{(1)} = 0$, using annualized parameter values), the lower and upper bounds $\mu_{\mathsf{hpi},\min}^m$ and $\mu_{\mathsf{hpi},\max}^m$ correspond to dividend rates of $q_{\max} = 4.97\%$ and $q_{\min} = 1.12\%$, respectively. Thus, there is some (minor) overlap with the range found in the GBM model.

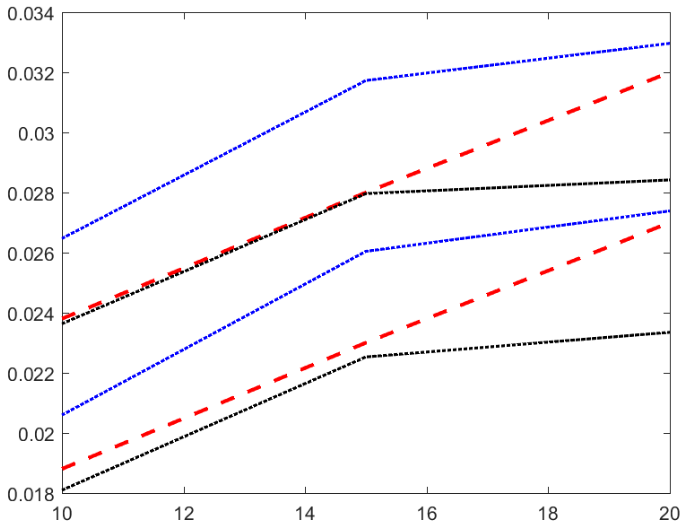


Figure 7: This figure displays the mortgage rate charged by Florius as well as the model-implied mortgage rate for $\mu_{\text{hpi}} = \mu_{\text{hpi,min}}$ and for $\mu_{\text{hpi}} = \mu_{\text{hpi,max}}$. The upper three lines display $R(\mu_{\text{hpi,min}}, \varphi, T_m)$ (dotted), $R^{\textit{Florius}}(\varphi, T_m)$ (dashed) and $R(\mu_{\text{hpi,max}}, \varphi, T_m)$ (dotted) as a function of the term to maturity for $\varphi = 98\%$. The lower three lines display these rates for $\varphi = 67\%$.

	μ^m_{hpi}	$\mu^m_{\text{hpi,min}}$	$\mu^m_{\text{hpi,max}}$
hpi	0	0	0
gdp	0	0	0
cpi	-0.0058	-0.0055	-0.0061
$y^{(1)}$	-0.2505	-0.2470	-0.2540
$y^{(40)} - y^{(1)}$	-0.0163	-0.0166	-0.0161

Table 7: This table presents the re-calibrated λ corresponding to μ^m_{hpi} (second column), $\mu^m_{\text{hpi,min}}$ (third column), and $\mu^m_{\text{hpi,max}}$ (last column).

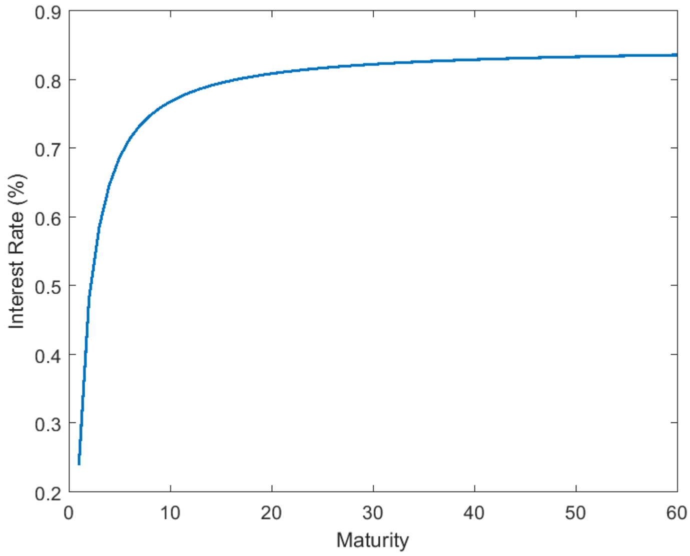


Figure 8: This figure displays the term structure of interest rates in the VAR model.

5.6 Pricing the NNEG

We can now price the NNEG with the VAR model. We use the end-of-sample term structure of interest rates according to the VAR model, given by equation (21) with x_t equal to the state variables in the last period of the sample. The resulting term structure is displayed in Figure 8. The corresponding market-consistent interest rates follow from solving (7), (8), and (11), with $P_0^{(t)} = \exp(-ty^{(t)})^t$. The results are displayed in Table 8.

	couple	65	70	75	80	85
Lump-sum	0.819%	0.815%	0.806%	0.793%	0.773%	0.741%
Interest-only	0.816%	0.811%	0.803%	0.790%	0.769%	0.737%
Tenure	0.847%	0.846%	0.844%	0.841%	0.835%	0.823%

Table 8: This table displays the market-consistent mortgage rate r in the VAR model, for single buyers of different ages and for a couple, for the three reverse mortgage schemes.

We determine the price of the guarantee conditional at time t, i.e.

$$\widetilde{\pi}_{NN}(t) = \mathbb{E}\left[\max\left\{L_t - (1-\delta)\cdot H_t, 0\right\}\cdot M_t\right],$$

via simulation. In our simulation we generate 5000 scenarios for (H_t, M_t). For each scenario, we determine the corresponding value of $\max\left\{L_t(\varphi) - (1-\delta)\cdot H_t, 0\right\}\cdot M_t$, and we set $\widetilde{\pi}_{NN}(t)$ equal to the average of these simulated values. Combined with the probability distribution of T, this yields the value of π_{NN} following from (12).

In Figure 9, we display π_{NN} for the three reverse mortgage products as a function of the two input parameters φ (the loan-to-value ratio) and δ (the forced sale proportional transaction cost), with the other parameters set to their estimated or calibrated values, and with $H_0 = 1$.

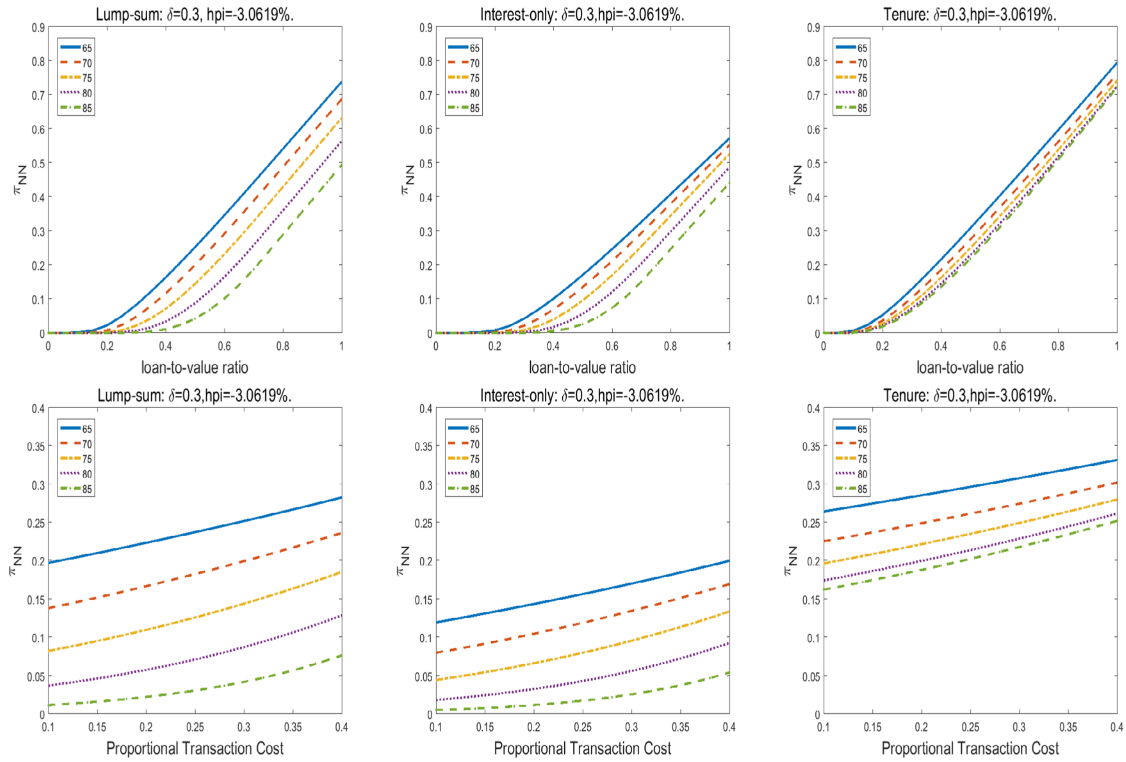


Figure 9: Sensitivity analysis of π_{NN} for the lump-sum scheme (left), the interest only scheme (middle) and the tenure scheme (right) in the VAR model. π_{NN} is plotted as a function of the loan-to-value ratio (φ) in the first row and as a function of the proportional transaction cost (δ) in the second row.

The interest-only contract yields the lowest π_{NN} value, and the tenure contract yields the highest π_{NN} value. With $\mu^m_{\mathsf{hpi}} = -3.065\%$, π_{NN} for a 65-year-old borrower is 56.97% for the interest-only scheme when the initial loan-to-value ratio is 100%. The corresponding π_{NN} is around 73.55% for the lump-sum scheme and 79.16% for the tenure scheme.

6 Model risk

In the previous sections we discussed two approaches to price the *NNEG*. The results show that among the calibrated/estimated parameters, the value of the *NNEG* seems to be quite sensitive to the dividend rate q in the GBM model, and to the house price growth rate μ_{hpi} in the VAR model. For both these parameters, we have calibrated the range of plausible values using data on regular mortgages. This led to (in annual terms)

$$\begin{aligned} q &\in [4.6\%, 6.6\%], \text{ for the GBM model,} \\ \mu^m_{\mathsf{hpi}} &\in [-4.99\%, -1.14\%], \text{ for the VAR model.} \end{aligned}$$

In this section, we compare the ranges of the value of π_{NN} for the GBM model and for the VAR model, resulting from the calibrated ranges of q and μ^m_{hpi}, respectively.

Figure 10 and Table 9 display the upper bound and the lower bound for π_{NN} for both models, as a function of the loan-to-value ratio. All other parameters are set equal to their calibrated/estimated values, as discussed in Sections 4.2 and 5.5, respectively. We present results for the case where the borrower is a couple, consisting of a male and a female, with the male aged 67 and the female aged 64. As before, the initial house value is normalized to 1. Figure 10 also includes the results using the calibrated value of q in the GBM model (i.e., $q = 5.6\%$) and using the estimated value of μ_{hpi} in the VAR model (i.e., $\mu^h_{\mathsf{hpi}} = -1.96\%$).

The results show that the difference in the values of π_{NN} between the lower and upper bounds can be quite substantial. For example, for the lump sum contract, we find using the VAR model that the interval of π_{NN} is around $[0\%, 5.4\%]$ for the lowest loan-to-value ratio considered ($\varphi = 15.5\%$), changing to $[24\%, 45\%]$ for the largest loan-to-value ratio considered ($\varphi = 55\%$). The corresponding intervals for the GBM model are somewhat smaller, but still large, and always included in the intervals of the VAR model. These wide intervals, based on a regular mortgage based calibration, show that there is considerable model risk when pricing the *NNEG* of reverse mortgage products. This might be another reason why the reverse mortgage market is so small. If there is an active housing market, where reverse mortgage providers can use derivatives to reduce the exposure to model risk, the reverse mortgage market might expand.

Table 10 shows the maximum allowed loan-to-value ratio by the Dutch mortgage provider Florius for lump sum reverse mortgages, as a function of the age(s) of the buyer(s).[6] For the couple that we consider, the maximum loan-to-value ratio is 15.5%. The reverse mortgage interest rate charged by Florius is fixed at 3.9% regardless of the age and loan-to-value ratio.[7] So, we cannot directly make a comparison with our model based calculations of π_{NN}. Based on a simple "back-of-the-envelope" calculation, assuming that the NNEG corresponds more or less to the difference between the reverse mortgage interest rate and a regular thirty-year mortgage interest rate (3.45%, see Table 3), we find a value for the NNEG around 3.9%, which is inside the intervals of both the GBM and VAR models.

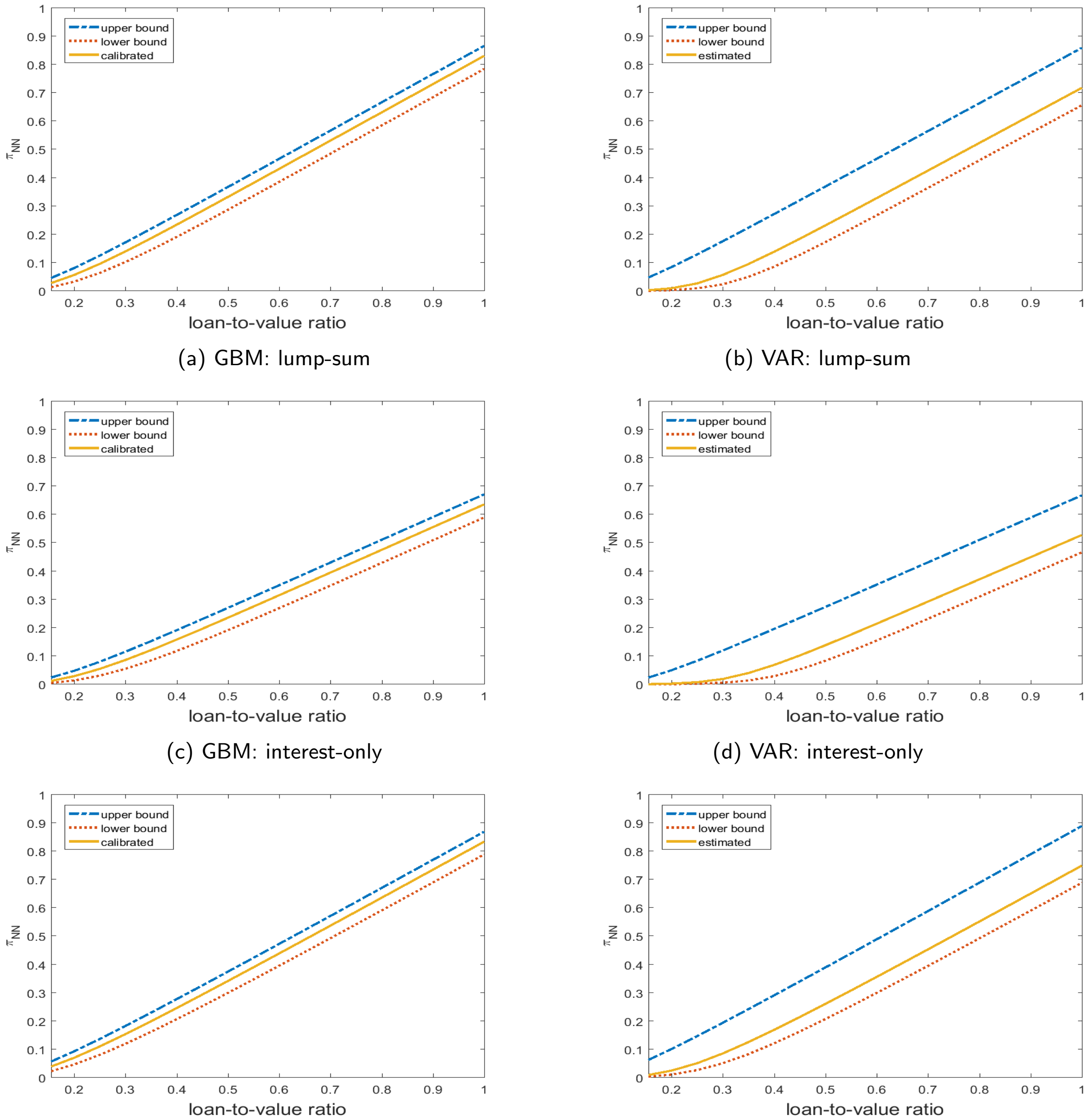
In Figure 11 we display the upper and the lower bound of π_{NN} for the GBM model and for the VAR model, for single buyers as a function of the age of the buyer. The loan-to-value ratio is set equal to the maximum allowed value by Florius, as displayed in Table 10. Thus, for the 65-year-olds, $\varphi = 18.5\%$, for the 70-year-olds, $\varphi = 23.9\%$, and so on. We also include as reference the

[6] These data are obtained from the Florius website on 8 April 2016, see `https://www.florius.nl/Pages/handig/bereken-verzilver-hypotheek.aspx`. After we fill in the date of birth and the house value of the buyer, the maximum loan-to-value ratio will be obtained with the existing mortgage on the property equal to zero.

[7] The mortgage rate is also obtained from the Florius website on 8 April 2016, see previous footnote.

Lump-sum Scheme				
φ	GBM		VAR	
15.5%	1.320%	4.560%	0.000%	5.370%
20%	3.270%	8.090%	0.070%	9.360%
30%	10.210%	17.160%	2.710%	19.200%
40%	19.090%	26.860%	9.720%	29.500%
50%	28.700%	36.760%	19.130%	39.930%
55%	33.620%	41.740%	24.180%	45.170%
Interest-only Scheme				
φ	GBM		VAR	
15.5%	0.430%	2.390%	0.000%	2.760%
20%	1.330%	4.750%	0.000%	5.560%
30%	5.470%	11.510%	0.330%	13.080%
40%	11.760%	19.120%	3.320%	21.230%
50%	19.120%	27.020%	9.350%	29.560%
55%	22.990%	31.010%	13.080%	33.760%
Tenure Scheme				
φ	GBM		VAR	
15.5%	2.260%	5.680%	0.170%	7.100%
20%	4.650%	9.290%	0.960%	11.320%
30%	11.950%	18.250%	5.890%	21.440%
40%	20.650%	27.760%	13.880%	31.980%
50%	29.960%	37.470%	23.340%	42.670%
55%	34.740%	42.370%	28.340%	48.060%

Table 9: The boundaries of π_{NN} in the two models. The first column displays the loan-to-value ratio; the second and third columns display the lower bound and the upper bound of π_{NN} in the GBM model; the fourth and fifth columns display the lower bound and the upper bound of π_{NN} in the VAR model.



(a) GBM: lump-sum

(b) VAR: lump-sum

(c) GBM: interest-only

(d) VAR: interest-only

(e) GBM: tenure

(f) VAR: tenure

Figure 10: The figure displays π_{NN} as a function of the loan-to-value ratio in the three reverse mortgage schemes. The left panels display π_{NN} in the VAR model for the estimated value of the house price increase rate ($\mu^h_{\text{hpi}} = -1.96\%$) and for the lower bound and the upper bound of the calibrated range ($\mu^m_{\text{hpi},\min}$ and $\mu^m_{\text{hpi},\max}$). The right panels display π_{NN} in the GBM model for the calibrated value of the dividend yield ($q = 5.6\%$) and for the lower bound and the upper bound of the calibrated range ($q = q_{\min}$ and $q = q_{\max}$). All other parameters are set equal to their calibrated value.

Age	65	70	75	80	85	couple (64+67)
$\max\varphi$	18.5%	23.9%	29.6%	35.3%	40.1%	15.5%

Table 10: The maximum loan-to-value ratio allowed by Florius.

simple "back-of-the-envelope" based calculation of the NNEG using the reverse mortgage and regular mortgage rates of Florius. Similar to the results for the couples, we find a wide range for π_{NN}, ranging from close to 0% to almost 7% for a 70-year-old to a range from close to 0% to around 3.5% for an 85-year-old. Again the interval in case of the GBM model is narrower than in case of the VAR model, but for higher ages the GBM range is no longer fully contained in the VAR range.

In our analysis, we have assumed that the termination rates are mortality rates given by the AG2014 life table. However, there are two opposite forces that may drive the termination rates away from the mortality rates in the AG2014 life table. On the one hand, there might be adverse selection in buying a reverse mortgage product. Those who have a longer subjective life expectancy may be more willing to buy a reverse mortgage product. Thus, the mortality rates might be overestimated. On the other hand, the termination probability may be underestimated because we ignore other reasons for termination, for instance moving permanently to a nursing home. The fact that in our analysis we did not account for adverse selection and have assumed away the possibility of termination due to causes other than decease implies that the degree of model risk in the price of reverse mortgages is likely underestimated. We therefore performed a sensitivity analysis in which we investigate the effects of alternative assumptions regarding the termination rate on the price of reverse mortgages. We look at both cases where termination rates increase and cases where termination rates decrease as compared to mortality rates from AG2014. Chen, et al. (2011) and the HUD assume that the mobility rate is about 30% of the mortality rate. Therefore, as an upper bound we consider the case where the termination rate is 130% of the mortality rate given by the AG2014 table. We also consider cases where the termination rate is 110% or 120% of the mortality rate. To investigate the potential impact of adverse selection, we consider the case where the termination rate is 10%, 20% or 30% lower than the mortality rate according to the AG2014 table. We find that increasing the termination rate reduces the value of the *NNEG* and the sensitivity of π_{NN} to the main parameters while decreasing the termination rates has the opposite effect. But qualitatively, the results are largely consistent with the findings when using the AG2014 life table. The main difference is that the rates charged by Florius fall below the lower bound of the rates for the GBM model when termination rates decrease by at least 20%. However, this is a quite extreme case. If the mobility rate is 30% of the mortality rate, then a decrease of 20% in the termination rate as compared to AG2014 implies that the adverse selection reduces the mortality rate by 38.5%.

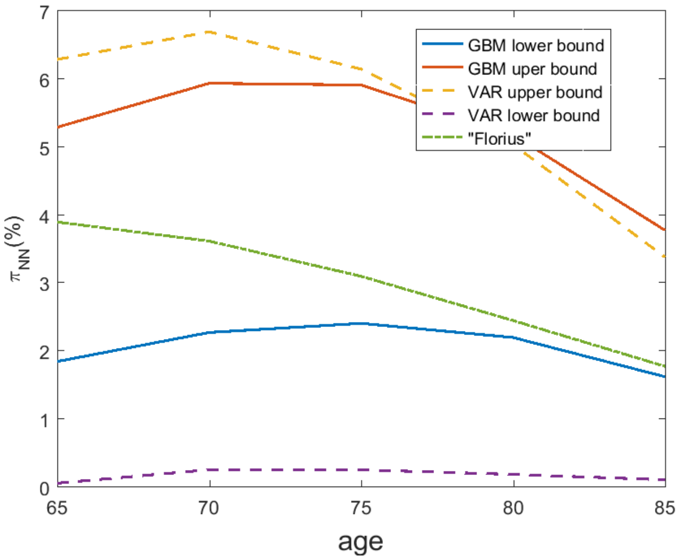


Figure 11: This figure displays the model implied price of the *NNEG* for single buyers, as a function of the age of the buyer. The loan-to-value ratio is set equal to the maximum allowed value by Florius, as displayed in Table 10. All other parameters are set equal to their calibrated value. The solid lines display the lower bound and the upper bound of π_{NN} in the GBM model. The dashed lines display the lower bound and the upper bound of π_{NN} in the VAR model. As reference, the dashed-dotted line displays the value of π_{NN}, assuming that it corresponds to the difference between the reverse mortgage interest rate charged by Florius (3.9%) and the interest rate charged by Florius for a regular thirty-year mortgage (3.45%, see Table 3).

7 Conclusion

We have analyzed the effect of model risk on the price of the No-Negative-Equity-Guarantee (*NNEG*) which is typically imbedded in reverse mortgage products. We considered two different models to determine the price, a GBM model and a VAR model, and investigated the sensitivity of the price of the *NNEG* to the parameter values in each model. We find that in both models, the price of the *NNEG* is particularly sensitive to the parameter that reflects the expected future house price development, which is the house price increase rate in the VAR model and the dividend rate in the GBM model. Calibration of these parameters using data on prices of regular mortgages yields a dividend rate varying from 4.6% to 6.6% in the GBM model, and a house price increase rate varying from -4.99% to -1.14% in the VAR model. Because the price of the *NNEG* is highly sensitive to the value of the house price increase rate in the VAR model, and to the value of the dividend rate in the GBM model, these wide ranges imply that there is considerable model risk when pricing the *NNEG*. For example, for a lump-sum reverse mortgage with loan-to-value-ratio of 30 percent issued to a couple with a male and a female aged 67 and 64, respectively, the lower bound of the calibrated price of the *NNEG* is about 2.7% of the initial house value in the VAR model, while the upper bound is around 19% percent of the initial house value. The model risk for other reverse mortgage designs and lenders of different ages is similar in magnitude.

This substantial model risk means that pricing the *NNEG* in reverse mortgage products is quite a challenging task. In our approach we made a number of possibly restrictive assumptions, which might have impact on our findings. For example, in practice regular and reverse mortgage interest rates might be determined in a different way than we model. This might affect our calibration results, in particular, it might affect the degree of model risk that we find. In the GBM model we only allow for one source of risk. In the VAR model we allow for multiple sources of risk, but we assume that the house price risk factor is equal to zero. Other ways of calibrating the prices of risk and the dividend yield might yield different outcomes. When calibrating the ranges of the dividend yield (in the GBM model) and the house price increase rate (in the VAR model), presented in the Appendix, we also impose strong assumptions. For example, we assume independence between financial risk, house price risk, and default risk. Relaxing these and other assumptions might affect the degree of model risk. Finally, we present results for both the GBM and a version of the VAR model, but without making a clear choice between them. These are topics of future research.

References

[1] Abelson, Peter, Roselyne Joyeux, George Milunovich, and Demi Chung, 2005, Explaining house prices in Australia: 1970-2003, *The Economic Record* 81, Special Issue, August, 2005. S96-S103, The Economic Society of Australia.

[2] Alai, Daniel H., Hua Chen, Daniel Cho, Katja Hanewald, and Michael Sherris, 2014, Developing equity release markets: risk analysis for reverse mortgages and home reversions. *North American Actuarial Journal* 18, 217-241.

[3] Ang, Andrew, and Monika Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and Latent Variables. *Journal of Monetary Economics* 50(4), 745-787.

[4] Brooks, Chris, and Sotiris Tsolacos, 1999, The impact of economic and financial factors on UK property performance. *Journal of Property Research* 16(2) 139-152.

[5] Chen, Hua, Samuel H. Cox, and Shaun S. Wang, 2010, Is the Home Equity Conversion Mortgage in the United States sustainable? Evidence from pricing mortgage insurance premiums and non-recourse provisions using the conditional Esscher transform. *Insurance: Mathematics and Economics* 46(2), 371-384.

[6] Cho, Daniel Wanhee, Katja Hanewald, and Michael Sherris, 2013, The risk management and payout design of reverse mortgages. *Australian School of Business Research Paper* No. 2013ACTL01.

[7] Chou, Kee-Lee, Nelson W.S. Chow, and Iris Chi, 2006, Willingness to consider applying for reverse mortgage in Hong Kong Chinese middle-aged homeowners. *Habitat International* 30(3), 716-727.

[8] Cochrane, John H., and Monika Piazzesi, 2005, Bond risk premia. *The American Economic Review* 95(1), 138-160.

[9] Cochrane, John H., and Monika Piazzesi, 2009, Decomposing the yield curve, AFA 2010 Atlanta Meetings Paper.

[10] Davidoff, Thomas, 2010, Home equity commitment and long-term care insurance demand. *Journal of Public Economics* 94(1), 44-49.

[11] Davidoff, Thomas, Patrick Gerhard, and Thomas Post, 2015, Reverse mortgages: What homeowners (don't) know and how it matters. Available at SSRN 2528944.

[12] Finkelstein, Amy, and James Poterba, 2002, Selection effects in the United Kingdom individual annuities market. *The Economic Journal* 112(476), 28-50.

[13] Ji, Min, Mary Hardy, and Johnny Siu-Hang Li, 2012, A semi-Markov multiple state model for reverse mortgage terminations, *Journal Annals of Actuarial Science* 6, 235-257.

[14] Lee, Yung-Tsung, Chou-Wen Wang, and Hong-Chih Huang, 2012, On the valuation of reverse mortgages with regular tenure payments. *Insurance: Mathematics and Economics* 51(2), 430-441.

[15] McCarthy, David, and Olivia S. Mitchell, 2010, International adverse selection in life insurance and annuities, in *Ageing in Advanced Industrial States*, edited by S. Tuljapurkar, A. H. Gauthier, and N. Ogawa, New York: Springer-Verlag, 119-135.

[16] Siu?Hang Li, Johnny, Mary R. Hardy, and Ken Seng Tan, 2010, On pricing and hedging the No-Negative-Equity Guarantee in equity release mechanisms. *Journal of Risk and Insurance* 77(2), 499-522.

[17] Miceli, Thomas J., and C. F. Sirmans, 1994, Reverse mortgages and borrower maintenance risk. *Real Estate Economics* 22(2), 433-450.

[18] Michelangeli, Valentina, 2008, Does it pay to get a reverse mortgage. Boston University. Manuscript.

[19] Nederlandse Vereniging van Banken, 2014, The Dutch mortgage market, available at `www.nvb.nl/media/document/001406_the-dutch-mortgage-market.pdf`.

[20] Shan, Hui, 2011, Reversing the trend: The recent expansion of the reverse mortgage market. *Real Estate Economics* 39(4), 743-768.

[21] Shao, Adam W., Katja Hanewald, and Michael Sherris, 2015, Reverse mortgage pricing and risk analysis allowing for idiosyncratic house price risk and longevity risk. *Insurance: Mathematics and Economics* 63, 76-90.

[22] Sherris, Michael, and David Sun, 2010, Risk based capital and pricing for reverse mortgages revisited. *UNSW Australian School of Business Research Paper* 2010ACTL04.

[23] Shiller, Robert J., and Allan N. Weiss, 2000, Moral hazard in home equity conversion. Real Estate Economics 28(1), 1-31.

[24] Szymanoski, Edward J., 1994, Risk and the home equity conversion mortgage. *Real Estate Economics* 22 (2), 347-366.

[25] Tigchelaar, Roel, 2014, Evaluation of methods for determining the credit risk premium for mortgages. *University of Twente*.

[26] Venti, Steven F., and David A. Wise, 1991, Aging and the income value of housing wealth. *Journal of Public Economics* 44(3), 371-397.

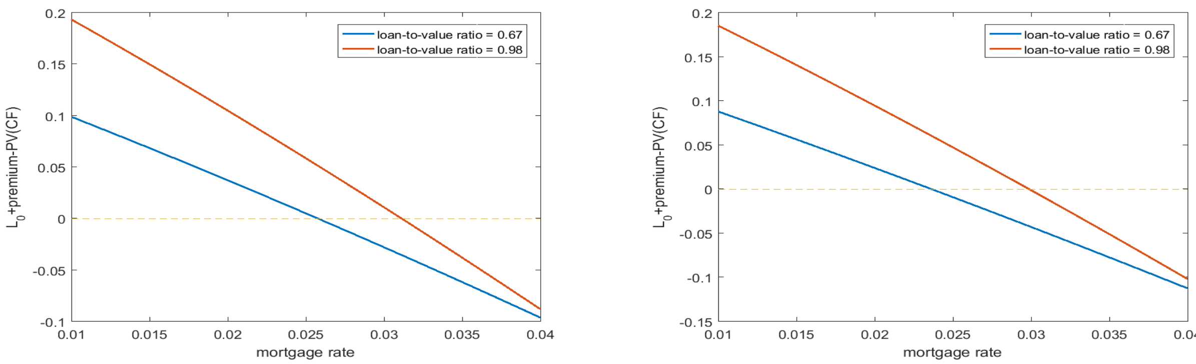
[27] Wang, Liang, Emiliano A. Valdez, and John Piggott, 2008, Securitization of longevity risk in reverse mortgages. *North American Actuarial Journal* 12(4), 345-371.

[28] Yang, Sharon S., 2011, Securitisation and tranching longevity and house price risk for reverse mortgage products. *The Geneva Papers on Risk and Insurance-Issues and Practice* 36(4), 648-674.

Appendix

We use ordinary mortgage products to calibrate the dividend rate q in the Geometric Brownian Motion Model and the expected house price growth rate μ_{hpi} in the VAR Model. First, we describe the mortgage products used in the calibration. Next, we discuss the calibration of the two aforementioned parameters in the two models. Table 11 summarizes the notation used in the main text and in this appendix.

Mortgage Products—The cash flow pattern associated with a regular mortgage is very straightforward. A cash outflow equal to the initial loan amount is lent to the borrower at the beginning of the contract and in the following years cash inflows will be generated from the interest payments and the repayments of the loans. From the lenders' perspective, risks involved in a normal mortgage are due to the default of the borrowers in combination with a house value lower than the loan balance. To compensate the default risk, the bank charges an interest-rate that is above the market-consistent rate without default. We can use the default probability and the recovery rate in case of a default to assess the present value of the expected cash inflow at the contract initiation, which can be compared to the cash outflow– the initial loan amount. In addition to the default premium, the bank needs to reserve some capital that is at least 4% of the loan balance as required by the Basel 1. The required return on the reserved capital, the hurdle rate, is set to be 9.33%.[8] Besides, we assume there is a 1% operation cost proportional to the loan balance. By setting the net present value of all cash flows to the sum of the present value of the operation cost and required profit on the reserved capital, we can derive the model-implied mortgage rate, which depends on the loan-to-value ratio φ. We then compare the interest rates for loan-to-value ratios $\varphi = 98\%$ and $\varphi = 67\%$ to the market rates, which are displayed in Table 3.



(a) GBM (b) VAR

Figure 12: These graphs plot the difference between the sum of the cash outflow and the required premium (the LHS of equations (26) and (27)) and the present value of all the future cash flows (the RHS of equations (26) and (27)) against the interest rate R for mortgage with $T_m = 15$ years. We set the dividend rate q in the GBM model and the house price growth rate μ_{hpi} in the VAR model equal to their calibrated values.

The Geometric Brownian Motion (GBM) Model—The mortgage contract that we consider is an annuity mortgage, which means that the lender pays a fixed amount per year during the entire lifetime of the mortgage. In line with the continuous-time nature of the GBM model, we

[8] R. Tigchelaar, March 2014, Evaluation of methods for determining the credit risk premium for mortgages.

Variable	Definition
(x, y)	The ages of the two spouses of the household (x: husband; y: wife)
T_x, T_y	The times of death of the two spouses of the household
${}_sp_{z,t}^{(g)}$	The probability that a z-year-old in year t with gender g survives at least s more years
$q_{z,t}^{(g)}$	The probability that a z-year-old in year t with gender g dies within a year
$p_0^{(t)}$	The date-zero price of a zero-coupon bond with a unit payoff in date t
L_t	The value of the loan balance at date t
H_t	The value of the property at date t
φ	The loan-to-value ratio
δ	The proportional transaction cost related to selling the house
T	The time at which the reverse mortgage contract terminates, i.e., $T = \max\{T_x, T_y\}$
T_m	The term to maturity of a regular mortgage
$T_{\max}$	The maximum value of T
m_t	The stochastic discount factor linking year $t-1$ to year t
M_t	The stochastic discount factor linking year 0 to year t
C	The fixed payments made each period in the tenure contracts
NN_T	The value of the *No-Negative-Equity-Guarantee* (*NNEG*) at contract termination
π_{NN}	The premium for the *No-Negative-Equity-Guarantee* (*NNEG*)
$\tilde{\pi}_{NN}(t)$	The premium for the *No-Negative-Equity-Guarantee* (*NNEG*) ending at date t
q	Fixed house net dividend rate
q_{min}	The lower bound of the net dividend rate
q_{max}	The upper bound of the net dividend rate
Φ	The cumulative distribution function of the standard normal distribution
ϕ	The probability distribution function of the standard normal distribution
S	The initial value of the underlying asset
K	Strike price of the put option
r	Reverse mortgage rate
r_f	The risk-free rate in the GBM model
R	Mortgage rate
$R^{Florius}$	Mortgage rate charged by Florius
σ	Annualized house price volatility
κ	Markup for π_{NN}
A	Fixed rate of payments of the regular mortgage
T_m	Term to maturity, $T_m \in \{10, 15, 20\}$
T_f	Horizon for redemption of the loan, which is 30 year
β	The default rate for the mortgage
τ	Time of default
hpi	The house price growth rate
gdp	The GDP growth rate
cpi	The inflation rate
$y^{(1)}$	The 3-month zero-coupon rate
$y^{(40)}$	The 10-year rate
λ	The price of risk
μ_{hpi}^{h}	The mean of the house price increasing rate estimated using historical house price data
μ_{hpi}^{m}	The mortgage based drift
$\mu_{hpi,min}^{m}$	The lower bound of the mortgage based drift
$\mu_{hpi,max}^{m}$	The lower bound of the mortgage based drift

Table 11: This table contains the notations used in this paper.

assume that the lender pays a continuous cash flow at an annual rate denoted by A. In the GBM model, with R denoting the mortgage rate, the remaining liability L_t of the mortgage then satisfies

$$\frac{dL_t}{dt} = RL_t - A,$$

which implies

$$L_t = e^{Rt} L_0 - A\,\frac{e^{Rt} - 1}{R}\,. \tag{24}$$

Let T_f be the horizon for redemption of the loan ($T_f = 30$ years). The constant A is chosen such that $L_{T_f} = 0$. In other words,

$$A = \frac{RL_0}{1 - e^{-RT_f}}. \tag{25}$$

The value of such a cash flow during the period from 0 to t is:

$$\int_0^t Ae^{-r_f t} dt = A\frac{1 - e^{-r_f t}}{r}.$$

The time-to-default is modeled as an exponential distribution, with default rate β set equal to 2% in the calibration. In the calculations, we assume the time-to-default to be independent of (H_t, M_t).[9]

The constant payment should satisfy that the present value of all the future cash flows equals to the initial loan plus the required premium:

$$\begin{aligned}
&L_0 + (\rho + \alpha h)\int_0^{T_m} L_t \exp(-t \cdot r_f) dt \\
=&\int_0^{T_m} \left(A\frac{1 - e^{-r_f t}}{r_f} + \mathbb{E}\left(M_t \min\left\{L_t, (1-\delta)H_t\right\}\right)\right)\beta e^{-\beta t} dt \\
&+ \left(A\frac{1 - e^{-r_f T_m}}{r_f} + e^{-r_f T_m} L_t\right) e^{-\beta T_m} \\
=&A\frac{1 - e^{-(r_f+\beta)T_m}}{r_f + \beta} + \int_0^{T_m} \mathbb{E}\left(M_t \min\left\{L_t, (1-\delta)H_t\right\}\right)\beta e^{-\beta t} dt + e^{-r_f T_m} L_t e^{-\beta T_m} \\
=&A\frac{1 - e^{-(r_f+\beta)T_m}}{r_f + \beta} + \int_0^{T_m} e^{-r_f t} L_t \beta e^{-\beta t} dt - \int_0^{T_m} \mathbb{E}\left(M_t \max\left\{L_t - (1-\delta)H_t, 0\right\}\right) \\
&\beta e^{-\beta t} dt + e^{-r_f T_m} L_t e^{-\beta T_m} \\
=&A\frac{1 - e^{-(r_f+\beta)T_m}}{r_f + \beta} + \int_0^{T_m} e^{-r_f t} L_t \beta e^{-\beta t} dt - \int_0^{T_m} BSput\left[(1-\delta)H_0, L_t, r_f, q, t, \sigma\right] \\
&\beta e^{-\beta t} dt + e^{-r_f T_m} L_t e^{-\beta T_m}
\end{aligned} \tag{26}$$

where $h = 9.33\%$ is the hurdle rate, $\alpha = 4\%$ the reserve rate, and $\rho = 1\%$ the operation cost.

The interest rate R is determined by solving (26) using (24) and (25). Figure 12 plots the difference between $L_0 +$ Premium (the LHS of (26)) and the present value of all the future cash flows (the RHS of (26)) against the interest rate R. The R that leads to an intersection with the horizontal line solves (26).

[9] See footnote 1.

VAR Model—In the VAR model, we adopt a similar approach to calibrate the parameters by fitting the model implied mortgage rate to the observed mortgage rate of Florius. We assume payments occur at the end of every period. The maximum maturity is 30 years. Mortgage with maturity shorter than 30 years needs to refinance at the end of the contract period.

With default risk, the annual payment A is set such that the expected present value of all the future cash flows equal to the initial loan plus the required premium:

$$\begin{aligned} &L_0 + (\rho + \alpha h) \sum_{t=1}^{T_m} L_{t-1} \mathbb{E} M_t \\ &= \sum_{t=1}^{T_m} \left(\sum_{s=1}^{t-1} \mathbb{E}\left[A \cdot M_s\right] + \mathbb{E}\left[\min\{L_t, (1-\delta) H_t\} M_t\right] \right) \mathbb{P}(\tau = t) \\ &+ \left(\sum_{s=1}^{T_m - 1} \mathbb{E}\left[A \cdot M_s\right] + \mathbb{E}\left[L_{T_m} \cdot M_{T_m}\right] \right) \mathbb{P}(\tau > T_m) \end{aligned} \tag{27}$$

The time-to-default is modeled as a geometric distribution, with default probability β set equal to 2% in the calibration. In the calculations, we assume the time-to-default to be independent of (H_t, M_t).[10] The time-to-default τ is assumed to follow a geometric distribution,

The corresponding interest rate R solves

$$A \cdot \sum_{t=1}^{30} \left(\frac{1}{1+R} \right)^t = L_0, \tag{28}$$

and the corresponding loan balance (before period t payment of A) is given by:

$$L_t = L_0 \cdot (1+R)^t - A \cdot \sum_{s=1}^{t-1} (1+R)^s \tag{29}$$

for $t > 1$ and $L_1 = L_0 \cdot (1+R)$

The interest rate R follows from solving (27), using (28) and (29). Similar to the GBM model, the R in Figure 12 that leads to an intersection with the horizontal line solves (27).

[10] See footnote 1.

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