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A Cohort-Specific Collar Approach to Retirement Security

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A cohort-specific collar approach to retirement security

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Abstract

This thesis aims at developing an innovative approach towards retirement security with reference to the recommendations of the committee Goudswaard. The focus is on the relation between contribution, ambition and guarantees. Specifically, the implemented strategy aims at realizing an ex-ante defined target benefit based on financial market insurance. The target benefit analyzed, is defined by a guaranteed level of pension income necessary to safeguard a minimum standard of living, while it allows for reaching an aspired level of pension income in case favorable economic scenarios realize. At retirement, the payoff corresponding to the target benefit is defined by a collar option contingent on the state of the economy. Assuming a complete market for equity risk, the payoff of the contingent claim is constructed by executing a cohort-specific and dynamic delta replication strategy. In addition, the proposed approach is compared to life cycle strategies more commonly executed in the pension industry. The results indicate that the collar approach potentially outperforms the evaluated life cycle strategies in terms of the aspired replacement rate that can be offered given a pessimistic replacement rate. Moreover, the collar approach may offer a larger probability of realizing an aspired benefit, given a predefined level of downside risk.

Contents

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1 Introduction

1.1 Review

Worldwide, traditional defined benefit (DB) plans experienced a solvency crisis at the beginning of the twenty-first century. The crisis was triggered by a combination of adverse shocks at global financial markets and the introduction of market-based accounting standards. The international move towards fair value accounting principles compelled pension funds to operate on a mark-to-market basis, using fixed-income instruments to provide discount rates for measuring the liabilities. As a consequence, the financial position of pension funds became explicitly related to the dynamics of the capital markets. However, at the same time of the move towards fair value accounting, markets moved in unfavorable directions for pension funds. Falling stock market returns and historic low long-dated fixed-income yields caused a severe drop in asset values and a sharp increase in the fair value of the pension liabilities. The joint impact of both developments led to substantial funding pressure on DB plans.

Moreover, longevity and aging increased the cost of guaranteeing pension benefits. The maturing of DB plans reduced the funds' ability to absorb unanticipated shocks in financial markets and longevity. As pension liabilities expanded relative to the premium base, larger volatility of the pension contributions was required to protect the pension rights. With the financial and actuarial risks of pension liabilities starting to dominate those of their core business, sponsoring companies no longer wanted to underwrite the risks of their pension funds (Boeri, Bovenberg, Cœuré and Roberts, 2006). As a result, many countries experienced a shift from DB plans towards defined contribution (DC) plans. Particularly, in the Anglo-Saxon countries traditional DB plans have largely been replaced by individual DC schemes (Munnell, 2006). The shift towards individual DC plans involved a shift of risk from the employer to the employee. In individual DC schemes, participants themselves are responsible for the saving and investment decisions. At retirement, the pension wealth directly depends upon the contributions and returns realized. Moreover, the individual bears the risk of longevity before annuitization.

In The Netherlands, the reconsideration of traditional DB plans led to the development of hybrid DB-DC pension schemes with collective risk sharing among participants according to explicit rules (Boeri et al., 2006). Similar to DB plans, the benefit entitlement is based on years of service and a reference wage. The reference wage used to be a participant's final wage but in recent years most funds moved to career-average wage schemes with conditional indexation. In conditional indexation schemes, the pension contract guarantees a minimum pension benefit which is updated every year by a solvency-contingent factor of indexation. Therefore, similar to DC plans, the indexation factor depends on the funds' financial status and hence on the realized returns on investment. Since indexation is conditional rather than guaranteed, the system may be characterized as a hybrid system of guarantees and ambitions. Conditional indexation schemes distinguish between hard pension rights, which are typically guaranteed in nominal terms, and soft pension rights, which typically involve an ambition to index pension rights to prices or wages (Boeri et al., 2006). By shifting away from final wage schemes to career-average wage schemes with conditional indexation, pension funds made the indexation of the pension rights of retired participants as well as rights of active members conditional on the financial performance of the fund. As a result of the reform, the ability of pension funds to control solvency risk strengthened.

However, recent years have seen a growing divergence between the risks pension funds are exposed to and their ability to manage these risks. As the number of retired participants is steadily increasing in relation to the number of active participants due to aging, the contribution instrument is growing less effective. Moreover, increasing contribution rates can have adverse effects, such as declining international competitiveness of domestic firms and reduced labor market participation. Hence, more emphasis is placed on the solvency controlling ability of the condition indexation instrument to restore the funds' financial position after a financial or demographic shock. If solvency is insufficient, this implies pension funds grant less indexation and therefore require fewer financial resources to index pension benefits. Instead, these resources can be used to improve the funding ratio. As a result, however, pension funds increasingly fail to fulfill the ambition to index accrued pension benefits and hence the purchasing power of the participants' entitlements diminishes.

In particular, the latest global financial crisis had a profound impact on many pension systems worldwide. Declining equity prices combined with a lower yield curve rapidly resulted in the deterioration of pension funds' funding ratios. As a consequence, pension funds were unable to index accrued benefits and experienced significant difficulty in guaranteeing participants' hard pension rights. Goudswaard, Beetsma, Nijman and Schnabel (2010) conclude that the Dutch supplementary pension system is insufficiently shock-resistant and may be unsustainable in the future. In their report, Goudswaard et al. (2010) indicate the costs of guaranteeing pension benefits are expected to increase in the future due to lower expected interest rates and equity returns. Moreover, asset returns display larger volatility which leads to sharp fluctuations in funding ratios and in turn leads to larger volatility in participants' real benefits. The sustainability of the system, however, is not exclusively affected by financial factors. In addition, Goudswaard et al. (2010) conclude that the costs increase due to demographic trends such as aging and longevity.

Goudswaard et al. (2010) point out a new balance between costs, guarantees and ambition is inevitable for the future. Given the increasing costs of guaranteeing pensions, current contribution rates will be insufficient to maintain today's ambition level[1]. Consequently, new choices regarding the parameters of the pension contract are expected to be taken. Possible directions for the future, involve a combination of increasing contribution rates, reduced ambition levels or reduced guaranteed levels. Additionally, Goudswaard et al. (2010) indicate innovative risk allocation is desirable. Promising proposals involve incorporating developments in life expectancy and financial markets into the benefit formula. After demographic or financial shocks, either the targeted retirement date or the the targeted income as of a fixed date may be adjusted. Both approaches require more risks are transferred directly to the pension fund participants. However, risk exposure may be differentiated according to participants' ability and willingness to absorb risk. Accordingly, heterogeneous risk preferences among different groups may be served and conflicts of interest between groups could be averted. A prerequisite for differentiated risk allocation is an explicitly defined pension contract which clearly specifies to what extent participants are taking part in risk bearing.

This thesis aims at developing an innovative approach towards retirement security with reference to the recommendations of Goudswaard et al. (2010). The focus is on the relation between contribution, ambition and guarantees. We discuss a cohort-specific approach to pension income accrual over the life cycle based on a collar strategy. The strategy allows for differentiated risk exposure among cohorts and protects each generation's property rights.

[1] Currently, the contribution rate is approximately 13% of the average wage in the Dutch supplementary pension system. Given today's level of ambition and guarantees, the Dutch Bureau for Economic Policy Analysis expects the contribution rate to reach 17% of the average wage in 2025 (see Goudswaard et al., 2010).

Moreover, the accrual of pension income is based on an explicit and financially fair pension contract. The next section introduces the subject of research in detail.

1.2 Research description

At financial markets, the no-arbitrage principle guarantees that the market-based compensation for taking risk is financially fair. Within pension funds, the rules of the pension contract define the risk and reward allocation among participants. Unlike derivative holders in financial markets, it is often ambiguous whether pension fund participants are fairly compensated for taking risk. Embedded value transfers may occur when risk bearing parties are not compensated properly. In current ageing society, this endangers the long-term sustainability of pension schemes. Given current low funding ratios, young participants could become reluctant to participate as they may expect to contribute in an increasing degree to fund the present shortfalls. While at the same time accepting the funds' residual risk, young participants face substantial risk to end up with poor benefits (Kocken, 2006). Pension fund redesign aimed at creating a sustainable system for the future, therefore asks for explicitly defined contracts which properly specify the property rights of participants.

Moreover, the introduction of conditional indexation aggravated conflicts of interest between young and old pension fund participants. Elderly participants are primarily concerned with the protection of the purchasing power of the pension entitlements and it is therefore in their interest to minimize risk exposure corresponding to the fund's asset allocation. However, little equity exposure implies low expected returns and high contribution rates given the ambition to index pension rights. In particular for young participants this policy is disadvantageous. It is in their interest to take substantial investment risk to benefit from expected excess returns and low contribution rates. Although conflicts of interests between young and old participants have always been present in collective pension schemes with uniform policies, they are expected to become more prevalent in the future as risks are increasingly transferred to participants (Goudswaard et al., 2010). Recent research, however, explored the reform of uniform policy rules to meet heterogeneous preferences of different groups of pension fund participants. Molenaar and Ponds (2009) discuss age-dependent indexation rates by adding return-related indexation policy to the current widespread practice of wage-related indexation policy. To acknowledge different preferences of participants in the future, innovative pension contracts are also expected to allow for differentiated risk exposure among participants.

This thesis aims at developing an approach to retirement security which clearly defines each generation's property rights on the return on investment. Similarly to Bodie, Merton and Samuelson (1992) and Teulings and De Vries (2006) each generation's pension entitlements are administered separately. The system is financially fair as, a priori, the value of the pension benefit is equal to the value of the contributions. In addition, the approach allows for differentiated risk exposure among cohorts to realize an ex-ante defined target benefit. The strategy implemented to attain the required pension benefit is based on financial market insurance. Assuming a complete market for equity risk, a target benefit can be realized at the financial markets by executing a cohort-specific and dynamic investment strategy. The target benefit analyzed in this thesis is specified in terms of a minimum and maximum pension benefit. Whereas the minimum benefit is guaranteed, the maximum benefit represents an aspired level of pension income, the realization of which depends on the returns realized. Specifically, the pension product aims at securing a level of income necessary to safeguard a minimum standard of living, while it allows for reaching an aspired level of pension income in case of favorable economic scenarios.

Rather than maximizing the expected pension benefit, the investment strategy aims at realizing the aspired pension benefit with an ex-ante defined probability. Equivalently, the risk of ending up with the guaranteed level is also specified in terms of a probability. Whereas the probability of realizing the aspired pension benefit would preferably be large, the probability of ending up with the guarantee is expected to be small. However, the set of choices regarding the minimum and maximum pension benefit, together with the corresponding probabilities, determines the price of the pension product. Evidently, appealing choices regarding the product's parameters require large contribution payments over a participant's life cycle. Since pension fund boards often predefine the level of an acceptable contribution rate, boards face a trade-off between the product's parameters. Given a contribution rate, a feasible combination of ambition, guarantees and the likelihoods of realizing the chosen minimum and maximum benefit has to be chosen. To present insight in the specific choices to make, the trade-offs between the option parameters are illustrated in this thesis. Once choices regarding the product's parameters are taken, the investment strategy is defined. Hence, rather than taking investment decisions, the pension fund board is expected to define the desired product. The dynamic investments strategy associated with the chosen pension product follows from a specific algorithm.

Essentially, the pension product guarantees a minimum benefit in case adverse scenarios materialize and provides a maximum benefit in case prosperous economic scenarios occur. Hence, the product could be framed in terms of owning a protective put option and writing a covered call option to obtain downside protection at the expense of accepting a limited upside potential. In financial markets, contingent claims called collar options are traded which offer downside protection by guaranteeing a minimum payoff but simultaneously provide limited upside potential to suppress the cost of downside insurance. The pension product proposed in this thesis, is defined by such a collar option but is specifically designed for retirement security. To tailor the collar option for retirement security, the floor of the option is expressed in terms of a minimum pension benefit a participant receives, whereas the cap represents a maximum, or aspired pension benefit. More specifically, the collar option aims to provide a real pension benefit in the form of an annuity. The floor and the cap of the collar option represent the price of real annuities corresponding to a minimum and maximum replacement rate, which is defined as the fraction of pension income relative to the real wage earned by a participant during the active career. Since the collar option proposed requires specific strike prices and an exercise date different from those available in exchange-traded option markets, a dynamic investment strategy is carried out to replicate the payoff of a collar option synthetically. Depending on a cohort's time to retirement and experienced realized returns, a cohort's equity exposure is dynamically adjusted to construct the minimum pension income, while maintaining the prospect of realizing the aspired pension benefit as returns turn out to be favorable.

Given a set of option parameters, the contribution rate is determined by the price of the collar option that is to be replicated. Using risk-neutral pricing techniques, the cost-based contribution rate is set such that the guaranteed pension benefit can be realized and the ambition may be fulfilled. Moreover, the accrual of pension income depends on the contribution rate in a financially fair way. Given a constant contribution rate, pension income is accrued based on a decreasing accrual system. That is, given a uniform contribution rate each participant receives age-related accrual of pension entitlements. The accrual percentage is determined such that the contribution exactly covers the fair value of new pension entitlements. Young participants therefore accrue more than old participants (see Boeijen, Jansen, Kortleve and Tamerus, 2006). In addition to accrual of pension income from contribution payments, a participant's benefit entitlement is conditionally indexed. The indexation policy of the pension scheme is return-driven and cohort-specific. Given the replication strategy, a cohort's factor of indexation depends on the realized excess return on investment and hence may be smaller than one or even negative. On the other hand, however, indexation may be well in excess of inflation. Alternatively to typical conditional indexation schemes, annual indexation is not restricted by the limits of a policy ladder. In the pension scheme discussed in this thesis, a target benefit is constructed over the total active period of a participant's life cycle.

Although the point of departure is based on an individual pension product, retirement security corresponding to the collar approach is organized collectively. It is assumed the board of the pension fund decides on the product that is to be replicated. Academic studies show that individuals typically lack the basic financial knowledge and computational ability to implement financial life cycle planning to secure adequate pension income and even lack the ambition to gain more in-depth knowledge about retirement planning (Lusardi and Mitchell, 2007; Van Rooij, Lusardi and Alessie, 2006; and Van Rooij, Kool and Prast, 2007). Moreover, empirical research points out individuals tend to be prone to myopia and procrastination. Literature shows individuals delay saving for retirement and tend to save too little (Thaler and Benartzi, 2004; Kooreman and Prast, 2007; and Van Rooij, Kool and Prast, 2007). In addition to paternalistic reasons for organizing retirement security collectively, scale advantages present an important argument. The pension scheme discussed in this thesis, manages the assets with a single asset allocation in order to perform the replication strategy of the different cohorts in the scheme. Accordingly, the scheme benefits from internal transactions and reduced transaction costs compared to individual trades. Moreover, the scheme exploits scale economies which tends to reduce operational costs (see Bikker and De Dreu, 2007).

Another crucial advantage of collectively organized pension schemes is that participants benefit from risk sharing of non-traded risks. As a consequence of non-traded risks, individuals are inevitably exposed to mismatch risk in realizing a target benefit. For example, markets fail to provide adequate protection of real interest rate risk, wage risk and macro longevity risk. Collective pension funds may complete markets for risk that cannot be insured against by creating an internal market within the fund to trade these risks among participants. Cui and Ponds (2010) propose an internal swap market where cohorts trade financial market risk and wage risk. A similar approach may be used to price other incomplete market risks and hence improve participants' risk-return trade-off. In this thesis, however, we focus on the relation between contribution, ambition and guarantees from the perspective of complete markets. We do not include in the analysis several features that are often considered in life cycle investment literature, such as stochastic interest rates, stochastic inflation, macro longevity risk, and asset return predictability. In a more comprehensive investigation, such factors should be taken into account; here the aim is to present a first analysis. Nevertheless, we do present an extensive analysis of the pension income accrual over the life cycle and compare the proposed approach to life cycle strategies more commonly executed in the pension industry.

This thesis is organized as follows. Chapter 2 specifies the financial market and labor market settings. Chapter 3 introduces the pension scheme and the collar option is illustrated in chapter 4. In addition, chapter 4 discusses the trade-offs associated with the collar approach and the pension income accrual over the life cycle. An alternative approach based on maximizing expected utility given a collar constraint is presented in chapter 5. Chapter 6 compares the collar approaches to life cycle models and chapter 7 concludes.

2 Market settings

2.1 Financial market

The investment opportunities are represented by the instantaneous riskless bond and the risky stock of a standard Black-Scholes market (see Black and Scholes, 1973). The only risk factor is stock market risk, and it is traded through a stock market index S_t driven by Brownian motion. The riskless cash bond B_t is subject to deterministic exponential growth at a constant rate r. Specifically, the asset price processes are given by

$$\begin{aligned} dS_t &= \mu S_t dt + \sigma S_t dZ_t \\ dB_t &= r B_t dt \end{aligned} \tag{1}$$

where μ and σ are the constant drift and volatility parameters, and Z_t is a standard Brownian Motion. The unique pricing kernel η_t of the Black-Scholes market is defined by

$$d\eta_t = -r\eta_t dt - \lambda\eta_t dZ_t \tag{2}$$

where $\lambda = \frac{\mu - r}{\sigma}$ is the market price of risk.

Note that the Black-Scholes market is a highly simplified representation of the real financial market. The underlying assumptions of the Black-Scholes model are constant volatility, a constant riskless interest rate, efficient and perfectly liquid markets, no dividend payoffs, and identically and independently log-normally distributed stock returns.

In subsequent sections, results are illustrated by numerical examples. For this purpose, it is assumed that the economy is characterized by a real riskless interest rate of 2%, expected real stock market return of 5% and stock market volatility equal to 18%. Hence, the market price of risk is equal to 0.167. The financial parameter values are summarized in table 1.

r	μ	σ	λ
0.02	0.05	0.18	0.167

Table 1: Financial parameter values.

2.2 Labor market

The economy considered is populated by sixty overlapping generations which range from the age of 25 to 85. The population is assumed to be a homogenous group of individuals who start working at age 25, retire at age 65 and have an expected remaining life time of twenty years. We assume all idiosyncratic mortality risk to be fully diversified and the expected improvement in life expectancy is included in the twenty-year retirement period. Moreover, it is assumed individuals participate in the pension scheme during both the active period and the retirement period. There are no individuals who enter late or leave prematurely. Let the parameters T_R and T_D denote the length of the active period and the period individuals participate in the pension scheme, respectively.

During the active period individuals work and earn a flat real labor income Y.[2] Part of the labor income is saved in the pension scheme for consumption after retirement. During the retirement period, individuals receive no income other than their pension benefit. Furthermore, it is assumed participants save only through the pension scheme. Apart from pension savings, there are no savings. The labor market parameters are summarized in table 2.

T_R	T_D
40	60

Table 2: Labor market parameter values.

3 Pension scheme settings

The pension scheme analyzed in this thesis is represented by a fund with conditionally indexed liabilities. Pension fund participants pay periodic contributions and correspondingly accrue pension income. The indexation policy is return-driven and cohort-specific according to ex-ante defined rules. Subsequent subsections introduce the pension scheme settings in detail. We discuss the contribution rate, the accrual of pension income, the indexation policy, and the dynamics of the assets and the liabilities.

3.1 Contribution rate

In a complete market, such as the Black-Scholes market, there is a unique price for any contingent claim, namely the initial wealth needed to finance the replicating portfolio. The contribution rate of the pension scheme is based on the amount of initial wealth needed to replicate the payoff of the collar option. Since participants pay periodic contributions, rather than one single contribution, the sum of the discounted pension contributions has to be equal to the price of the option in order to execute the option replication strategy. If it is assumed that participants contribute a constant fraction of labor income over the active period of the life cycle, then the contribution rate c is solved form

$$V_0 = cY \int_0^{T_R} e^{-rs} ds \tag{3}$$

where V_0 denotes the unique price of the option upon entrance into the scheme. As a participant's wage Y is assumed to be constant in real terms, the discount rate equals the real riskless return r.

3.2 Accrual of pension income

The pension scheme is framed in terms of accrued pension income instead of accrued pension wealth. Rather than monitoring participants' accumulated savings, the pension scheme keeps account of the amount of pension income participants receive after retirement. In fact, the amount of pension income is expressed in terms of a participant's real wage Y and resembles a deferred real annuity due for retirement.

The accrual of pension income is assumed to depend on the contribution rate in a financially fair way. Given a constant contribution rate, the financially fair accrual of pension income

[2] Wage inflation is assumed to be identical to price inflation.

decreases over the life cycle as a consequence of the time value of money. After all, the contribution of a young participant can yield for many more years than the contribution paid by a participant only a few years away from retirement. Given the contribution rate c, the newly accrued pension income ΔP^x of an x-year-old participant is defined by

$$\Delta P^x = \varepsilon^x Y \tag{4}$$

where the pension accrual rate ε^x depends on the price of a deferred real annuity and is to be solved form

$$c = \varepsilon^x \int_{T_R}^{T_D} e^{-r(s-x)} ds. \tag{5}$$

Figure 1 displays the decreasing pension accrual rate ε^x over the life cycle of a participant in case the contribution rate is equal to 17.5% of a participant's real wage Y.[3] Note that the accrual rate is expressed in real terms.

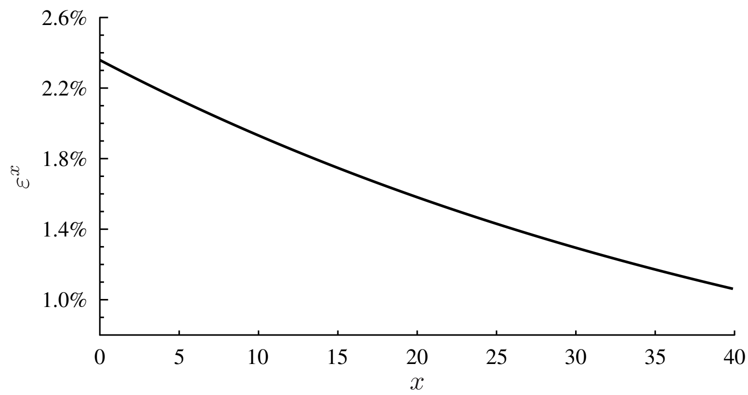

Figure 1: Decreasing accrual of pension income.

3.3 Indexation policy

In addition to the accrual of pension income from contribution payments, a participant's pension income is conditionally indexed. The indexation policy of the pension scheme is return-driven and cohort-specific. Depending on the cohort-specific replication strategy α_t^x of a cohort aged x at time t, the cohort receives return-driven indexation. Let P_t^x denote the accumulated pension income of age cohort x at time t, then the dynamics of the accrual of pension income are given by

$$dP_t^x = \Delta P^x \cdot 1_{\{x<T_R\}} dt + P_t^x di_t^x \tag{6}$$

where di_t^x represents the dynamics of the indexation policy of age cohort x, and the indicator function $1_{\{x<T_R\}}$ implies participants accrual pension income only during the active period. The indexation process di_t^x is adapted to Brownian motion Z_t and is defined by

$$di_t^x = \alpha_t^x(\mu - r)dt + \alpha_t^x \sigma dZ_t. \tag{7}$$

[3]The contribution rate corresponding to the benchmark collar option discussed in section 4.4 is equal to 17.5% of a participant's real wage.

Since the growth rate of the pension liabilities is equal to the riskless interest rate r, the indexation factor depends on the realized excess return. Depending on the investment strategy and the realized excess return on stocks, the indexation factor can be either positive or negative. Given this return-driven indexation policy, the accumulation of pension income of the age cohorts exactly corresponds to the accumulation of the underlying pension assets according to the replication strategy.

3.4 Liabilities

The liabilities of the pension scheme are equal to the market value of the accrued pension income of the participants. Since the indexation policy is cohort-specific, the liabilities of the cohorts have to be accounted for separately. At time t, the liabilities corresponding to the accrued pension income of age cohort x are equal to

$$L_t^x = P_t^x \int_{\max\{T_R,x\}}^{T_D} e^{-r(s-x)} ds. \tag{8}$$

The discount rate is equal to r, as pension income is expressed in real terms. The total liabilities of the pension scheme are equal to the aggregated value of the cohort-specific liabilities. Total pension liabilities are given by

$$L_t = \int_0^{T_R} L_t^x dx. \tag{9}$$

3.5 Assets

The value of the pension assets is equal to the contributions made into the fund and the returns realized on the contributions. During retirement, pension assets are liquidated to pay the pension benefits of the participants. Let A_t denote the value of the pension fund's assets and let α_t denote the fund's investment strategy, then the dynamics of the assets are given by

$$dA_t = \left[A_t(r + \alpha_t(\mu - r)) + C - P_t\right] dt + \alpha_t \sigma A_t dZ_t \tag{10}$$

where C denotes the amount of total contributions received and P_t denotes the value of the pension benefits paid at time t. More specifically, the definitions are

$$\begin{aligned} C &= cY \cdot T_R \\ P_t &= \int_{T_R}^{T_D} P_t^x dx. \end{aligned} \tag{11}$$

Note that the amount of total contributions C is independent of time, because the contribution rate is constant and generations of pension fund participants are assumed to be homogeneous.

3.6 Asset allocation

The pension scheme manages the assets with a single asset allocation in order to serve the risk preferences of the different age cohorts in the scheme. Given the replication strategies of the age cohorts, the asset allocation of the pension fund is determined such that all investment risk is absorbed by the different cohorts. The investment risk is completely attributed to the cohorts in case the asset allocation of the pension fund is equal to the weighted average of the cohort-specific investment strategies according to the cohorts' stake in the liabilities. Therefore, the investment strategy of the pension fund is determined by

$$\alpha_t = \int_0^{T_D} \alpha_t^x \frac{L_t^x}{L_t} dx. \tag{12}$$

Given the indexation policy introduced in section 3.3, the ex-post realized return on the assets is assigned to the age cohorts according to each cohort's replication strategy. Note that the fund's asset allocation also depends on the investment strategy of the retired cohorts. To finance the annuity payments, the retirees' assets are fully invested in the riskless asset.

3.7 Funding ratio

The funding ratio is defined as the ratio of the market value of the assets over the market value of the liabilities, i.e.

$$FR_t = \frac{A_t}{L_t}. \tag{13}$$

The funding ratio is commonly used to monitor the solvency position of pension funds. In the pension scheme employed in this thesis, the market value of the liabilities always exactly matches the value of the pension assets. Hence, the funding ratio equals one at every point in time.

4 Collar strategy

In this chapter, the collar option replication strategy is discussed. First, the collar option is specified and the option parameters are defined. In addition, an analytical expression for the price of the option is derived and accordingly the contribution rate is determined. Subsequently, the trade-off between the option parameters is illustrated and the investment strategy presented to create the payoff of a collar option synthetically. Finally, the return-driven indexation policy, the accumulation of pension income and the asset allocation of the pension fund are discussed.

4.1 Option specification

A standard collar option on a stock market index can be constructed by a long position in the stock index, a long put option at strike K_1, and a short call option at strike K_2.[4] The effect of a collar option is to place a floor and a cap on the value of the underlying index at maturity of the option. The strategy offers protection of a put in return for accepting a limited upside potential on the underlying asset as a result of writing a call. The premium received from writing the call partly offsets the costs of the put. Thus by accepting a limited upside, the investor is able to obtain downside put protection at smaller cost than the cost of the put alone. Figure 2 depicts the option composition of a standard collar option with strike prices K_1 and K_2. Moreover, the figure illustrates the dependence of the option payoff on the value of the underlying stock market index S_T at maturity T. Depending on the value of the stock market at time T, the payoff of the option is equal to the minimum payoff K_1, the maximum payoff K_2, or a value in between. For stock market values in between the two strike prices, the payoff of the option is linearly increasing with a slope equal to unity.

[4] Since the focus is on insuring bad case scenarios in combination with a capped upside in good scenarios, the strike price of the long put is assumed to be smaller or equal than the strike price of the short call, i.e. $K_1 \leq K_2$.

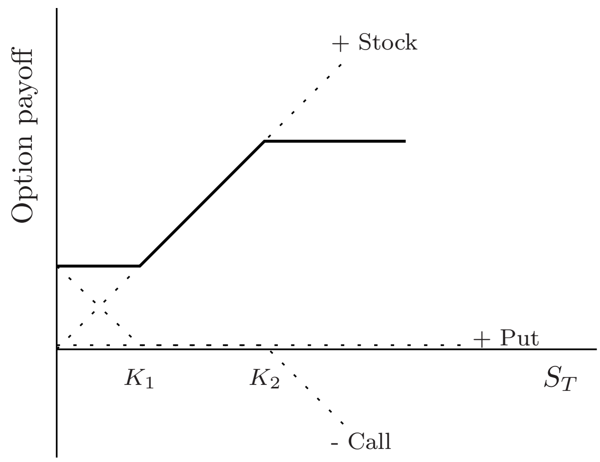

Figure 2: Option composition of a collar option.

An alternative to buying and selling options is creating the payoff of a collar option synthetically. There are at least two reasons why it may be more attractive, or even necessary, for a portfolio manager to create a required option synthetically compared to buying the option in the market. First, options markets do not always have the liquidity to absorb the trades required by managers of large funds. Second, portfolio managers may require strike prices and exercise dates that are different from those available in exchange-traded option markets (Hull, 2008). The pension product proposed in this thesis is tailored for retirement security and therefore requires an unusually large time to maturity. In addition, the product is specified more generally than a standard collar option. The collar payoff replicated to secure a pension benefit at retirement represents a linear combination of stocks and multiple long puts and short calls. However, the weights, and hence the slope of the option, need not necessarily be equal to unity. To define the collar option, we introduce two additional contract parameters. Let V_T be the value of the collar option at maturity T, and let θ_1 and θ_2 be the floor and the cap, respectively. Given the strike prices K_1 and K_2, the collar option is defined as

$$V_T = \max\left\{\theta_1, \min\left\{\theta_1 + \frac{\theta_2 - \theta_1}{K_2 - K_1}(S_T - K_1), \theta_2\right\}\right\}. \tag{14}$$

Figure 3 illustrates the value of the collar option at maturity. The payoff of the option is equal to the floor θ_1 if the value of stock market index at time T is smaller than the lower strike K_1. Equivalently, the payoff is equal to the cap θ_2 if the value of the stock market index at time T is larger than the upper strike K_2. For stock market values in between the two strike prices, the payoff of the option is linearly increasing with a slope equal to $\frac{\theta_2 - \theta_1}{K_2 - K_1}$.

4.2 Option parameters

The collar option replication strategy aims to provide pension fund participants with a retirement income level bound by a minimum and maximum pension benefit. The floor of the collar option is given by the price of a real annuity that pays participants a benefit corresponding to a minimum, or guaranteed replacement rate. A replacement rate is defined as the fraction of pension income relative to the real wage level earned by a participant

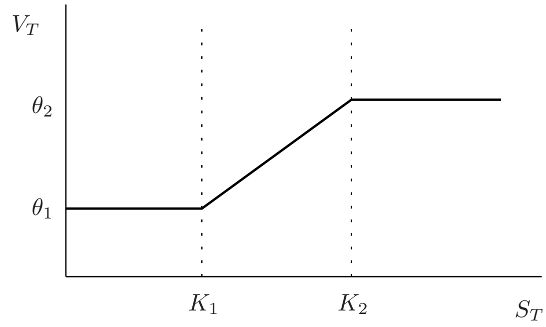

Figure 3: Option value at maturity.

during the active career (i.e. P^{40}/Y). Equivalently, the cap of the collar option is given by the price of a real annuity that pays a benefit corresponding to a maximum, or aspired replacement rate. Let the *guarantee* (κ_1) be defined as the minimum replacement rate and let the *ambition* (κ_2) be defined as the maximum replacement rate. Then, the floor and the cap of the collar option are given by

$$\theta_i = \kappa_i Y \int_0^{T_D - T_R} e^{-rs} ds, \qquad i \in \{1, 2\}. \tag{15}$$

The strike prices K_1 and K_2 determine for which values of the stock market index at maturity, the option pays the guaranteed or the aspired level of pension income. To obtain an intuitive interpretation of the parameter values, the strikes are expressed in terms of the probabilities of reaching the ambition level and ending up with the guaranteed level. Hence, the strike prices reflect the likelihood of realizing the ambition and the risk of ending up with the guarantee. Respectively, the lower and upper strike are solved from

$$\begin{aligned} p_1 = \mathbb{P}(V_T = \theta_1) &= \Phi\left(\frac{\log\frac{K_1}{S_0} - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \\ p_2 = \mathbb{P}(V_T = \theta_2) &= 1 - \Phi\left(\frac{\log\frac{K_2}{S_0} - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \end{aligned} \tag{16}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, and p_1 and p_2 represent the probabilities of realizing the guarantee and the ambition, respectively. Figure 4 displays the lognormal distribution of the stock market index value at time T. The spikes in the distribution indicate how much probability mass is assigned to reaching the ambition level and ending up with the guaranteed level. Note that the probability of ending up with the guarantee is increasing in K_1, whereas the probability of realizing the ambition is decreasing in K_2.

4.3 Option price

The value of the option and hence the contribution rate are determined by the choices regarding the floor, the cap and the strike prices. In this section, we present an explicit formula for the value of the collar option before maturity T. In the complete Black-Scholes market, risk-neutral pricing techniques can be used to determine the unique price for a contingent claim. Subsequently, the risk-neutral pricing formula is used to price the collar option.

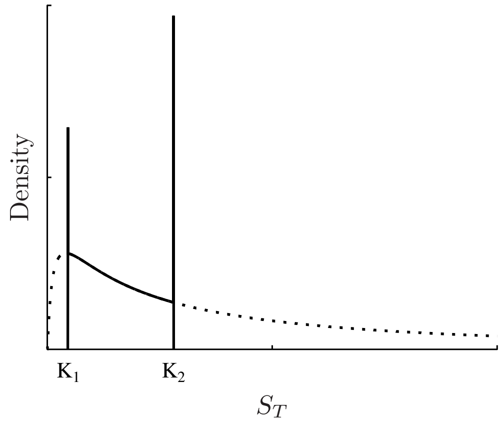

Figure 4: Lognormally distributed stock market value.

Let $\mathbb{Q}$ be the unique equivalent martingale measure corresponding to the bond as numéraire. According to the fundamental theorem of asset pricing, the process defined by $\{\frac{V_t}{B_t}\}$ is a martingale under the equivalent measure $\mathbb{Q}$ and therefore the value of the option at time $t \leq T$ satisfies

$$V_t = E_t^{\mathbb{Q}}\left[\frac{B_T}{B_t}V_T\right] = e^{-r(T-t)}E_t^{\mathbb{Q}}V_T. \tag{17}$$

Solving for the option value at time t yields a function depending on time t and of the value of the underlying stock market index S_t at time t. The value of the collar option at time t is given by

$$\begin{aligned} V_t &= e^{-r(T-t)}\Big(\theta_2 + a(K_1\Phi(d_1) - K_2\Phi(d_2))\Big) \\ &\quad + aS_t\Big(\Phi(d_2 - \sigma\sqrt{T-t}) - \Phi(d_1 - \sigma\sqrt{T-t})\Big) \end{aligned} \tag{18}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function,

$$d_i = \frac{\log(\frac{K_i}{S_t}) - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \qquad i \in \{1, 2\} \tag{19}$$

and $a = \frac{\theta_2 - \theta_1}{K_2 - K_1}$ represents the slope of the option as displayed in figure 3. Appendix A.1 contains the derivation of the value of the collar option. Upon entrance into the pension scheme, the value of the collar option, i.e. V_0, is equal for every cohort. Section 3.1 describes the dependence of the contribution rate on the initial option price.

4.4 Trade-off between option parameters

Given a contribution rate, the option parameters have to be chosen. Conversely, given the option parameters, the contribution rate is determined. In this section, the trade-off between the ambition, the guarantee and the strike prices is discussed for a range of contribution rates. The relationship between the option parameters and the contribution rate is illustrated by numerical examples. The market parameters used are mentioned in tables 1 and 2.

The benchmark collar option considered is defined by the parameter values presented in table 3. The guarantee and the ambition are equal to 50% and 80% of a participant's real wage, and the probabilities of realizing the guarantee and the ambition are equal to 2.5%

and 70%, respectively. The contribution rate corresponding to the option parameters equals 17.5% of a participant's wage. Appendix C.1 relates the benchmark option parameters to the parameters of the pension contract currently enforced in The Netherlands.

κ_1	κ_2	p_1	p_2	c
0.5	0.8	0.025	0.7	0.175

Table 3: Benchmark option parameter values.

Subsequent subsections illustrate the trade-offs for a range of contribution rates. The focus is on two types of trade-offs:

i *The strike prices are fixed.*
Assume that the board of the pension fund intends to offer a pension product which realizes the ambition with 70% probability and incorporates a risk of ending up with the guarantee given by 2.5% probability. Given a contribution rate, the board faces a trade-off between the ambition level and guaranteed level that can be offered.

ii *The ambition and guarantee are fixed.*
Assume that the board of the pension fund intends to offer a pension product whose ambition level is equal to 80% and whose guaranteed level is equal to 50%. Given a contribution rate, the board faces a trade-off between the probability of realizing the ambition and the probability of ending up with the guarantee.

Note that different trade-offs between option parameters are possible. For the remaining trade-offs, we refer to appendix C.2.

4.4.1 Floor and cap

The strike prices are fixed such that the ambition level and guaranteed level are realized with 70% and 2.5% probability, respectively. Figure 5 illustrates the trade-off between the ambition level and the guaranteed level for a given contribution rate in terms of the option payoff scheme. In addition, the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves. The curves show that as the floor increases,

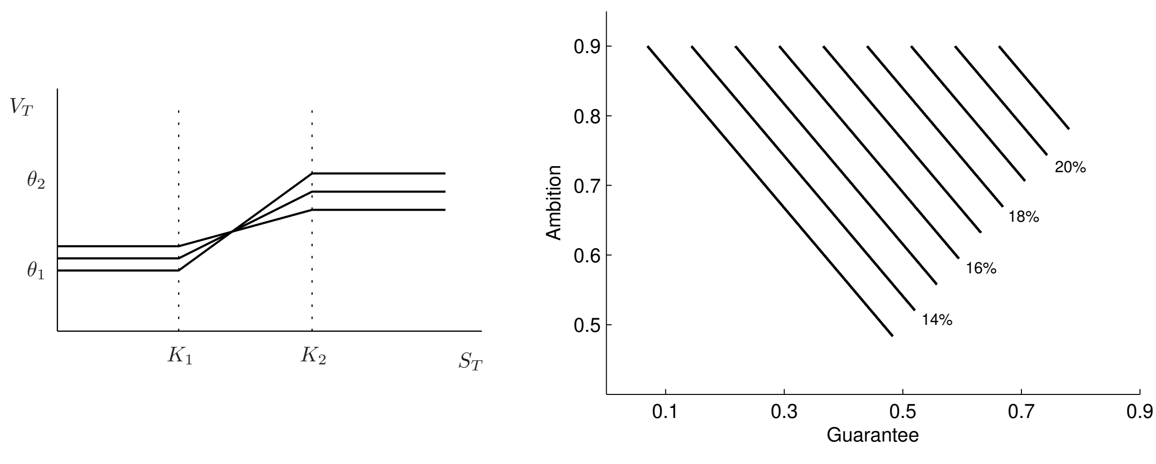

Figure 5: Trade-off between ambition and guarantee.

more insurance is need in the bad-state region, and accordingly the cap in the good-state region decreases to satisfy the budget constraint imposed by the constant contribution level. Depending on the assumed strike prices, the slope of the curves shows that the ambition level and the guaranteed level are approximately equal substitutes. Hence, increasing the ambition is approximately equally expensive in terms of giving up guarantees as the other way around. For the benchmark collar option, the guarantee increases from 50% to 60% in case the ambition decreases from 80% to 70%.

Yet, note that the slope of the trade-off curves depends on the chosen strike prices. Ceteris paribus, the slope is decreasing in K_2 and increasing in K_1. For example, if the probability of realizing the aspired pension benefit increases, the upper strike K_2 decreases and therefore the slope of the trade-off curves increases. Accordingly, increasing the ambition becomes more expensive in terms of giving up guarantees. Further, figure 5 displays the additional contribution required to increase the guarantee or the ambition. Increasing the guarantee or the ambition by 10%-points costs approximately 1.35%-points extra contribution.

4.4.2 Strike prices

The guarantee and the ambition are fixed at replacement rates equal to 50% and 80%, respectively. Figure 6 illustrates the trade-off between the strike prices for a given contribution rate in terms of the option payoff scheme. In addition, the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves. The curves show that as the upper strike price K_2 decreases, more states pay the ambition level and therefore the probability of realizing the ambition increases. As a consequence, the amount of bad states paying the guaranteed level has to increase in order to maintain a constant contribution level. This implies the lower strike price K_1 increases and thus the probability of ending up with the guarantee increases. The slopes of the trade-off curves show that increasing the probability of realizing the ambition is slightly more expensive in terms of the probability of realizing the guarantee. For the benchmark option, the probability of realizing the ambition increases from 70% to 80%, in case the probability of realizing the guarantee increases from 2.5% to 10%.

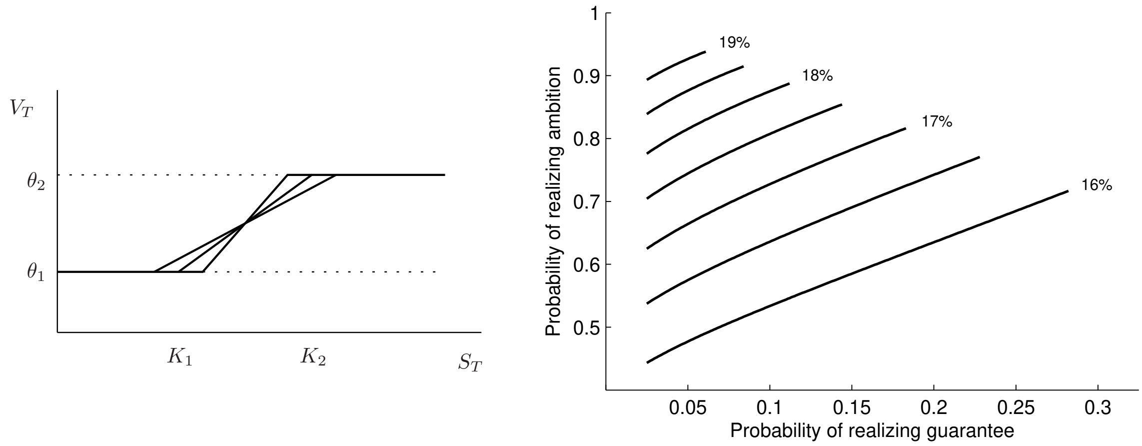

Figure 6: Trade-off between strike prices.

Further, the figure shows that the upward shift of the trade-off curve decreases as the contribution increases. This implies that the additional costs of increasing the probability of realizing the ambition rise as the contribution rate increases. A similar conclusion holds for decreasing the probability of realizing the guarantee. Hence, the marginal improvement of the probabilities, as a result of extra contribution, decreases.

4.5 Investment strategy

In a complete market, such as the Black-Scholes market, every claim can be replicated perfectly by suitable trading in the underlying asset. Black and Scholes (1973) show that a stock option at any instant behaves like a weighted portfolio of stocks and riskless bonds. Accordingly, it is possible to construct a portfolio that exactly replicates the payoff of an option. The strategy to create a replicating portfolio is referred to as delta-hedging and involves continuously rebalancing the portfolio weights as time passes and the stock price moves. It is called delta-hedging because the strategy involves maintaining a position in the underlying asset such that the delta of the replicating portfolio is equal to the delta of the required option. The delta of an option is defined as the rate of change of the option price with respect to the price of the underlying asset. Mathematically, the delta δ_t is the first derivative of the option value with respect to the underlying stock value, i.e.

$$\delta_t = \frac{\partial V_t}{\partial S_t}. \tag{20}$$

In case the replicating portfolio maintains a delta equal to the delta of the required option, the change in replicating portfolio value is equal to the change in option value for any move in the price of the underlying asset. Hence, the replicating portfolio value mimics the value of the option and therefore the delta-hedge strategy exactly replicates the option payoff. To maintain the required portfolio delta, at any instant the replicating portfolio is constructed out of δ_t stocks and a remaining investment in the riskless bond. The delta of the collar option is given by

$$\delta_t = a\left(\Phi(d_2 - \sigma\sqrt{T-t}) - \Phi(d_1 - \sigma\sqrt{T-t})\right) \tag{21}$$

where a represents the slope of the option. Since the replicating portfolio value is equal to V_t, the fraction of portfolio value invested in stocks is given by

$$\phi_t = \frac{\delta_t S_t}{V_t}. \tag{22}$$

Appendix A.2 contains the derivation of the delta of the collar option.

To present the intuition behind the delta-hedge strategy, the strategy is discussed by displaying the dynamics of the stock value, the option delta and the fraction of replicating portfolio value invested in stocks. Figure 7 illustrates the dynamics of S_t, δ_t and ϕ_t for three scenarios.[5] For the ease of viewing, the dynamics are shown for the last twenty years only. The intuition is as follows.

- Scenario 1 displays an increasing stock value. As the stock value increases, the replicating portfolio value tends to the amount of wealth needed to realize the ambition without being exposed to stock market risk. Therefore, the volume of stocks in the portfolio, given by the delta, tends to zero and the fraction invested in stocks decreases. As the time to maturity decreases, the fraction invested in stocks tends to zero.

[5]The scenarios presented in figure 7 correspond to an initial stock value $S_0 = 1$. For the ease of viewing, the replicating portfolio is rebalanced at an annual basis.

- Scenario 2 displays the dynamics of a relatively constant, though low, real stock value. Returns are disappointing and the replicating portfolio value tends to the amount of wealth at least required to realize the ambition without being exposed to stock market risk. To be able to guarantee the minimum payoff, the volume of stocks in the portfolio decreases. Also the fraction of portfolio value invested in stocks decreases.

- Scenario 3 displays the evolution a stock value, which corresponding option value does not approximate the discounted value of either the cap or the floor. As a consequence, the option value varies more strongly with financial market shocks. As time increases and the put and the call remain out of the money, the option delta increases.[6] As a consequence, the fraction invested in stocks increases.

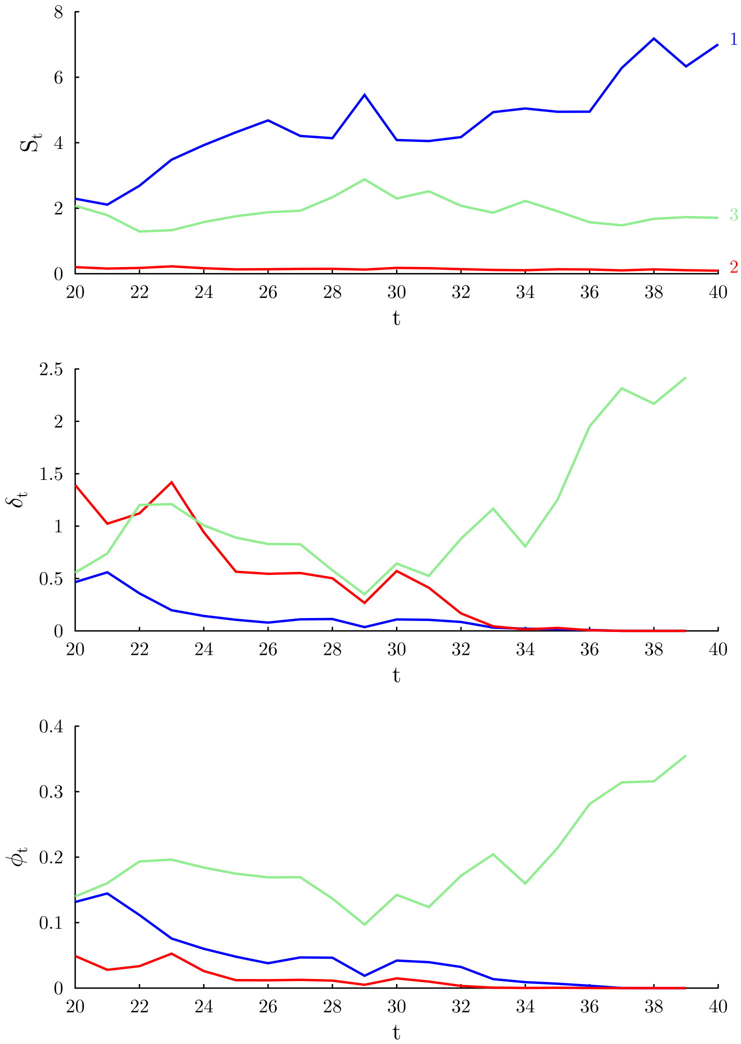

Figure 7: Three scenarios of S_t and corresponding δ_t and ϕ_t.

In general, the option delta can be expressed as a function of time and stock market return. Figure 8 illustrates the investment strategy by representing the delta as a function of time and

[6]In the subsequent paragraph, this relation is explained.

annualized real return. The relation between the delta and the return on the stock market is explained similarly to the explanation corresponding to figure 7. In case the return on the stock market is such that the replicating portfolio value approximates the discounted value of the cap or the floor, little stock market exposure is needed to replicate the required option value. Hence, the delta decreases and eventually tends to zero. However, if the portfolio value does not approximate the discounted value of either the cap or the floor, the option value varies more strongly with financial market shocks. As the time to maturity decreases, and the put and the call remain out of the money, the option delta increases. This is explained as follows. When the time to maturity tends to zero, the option delta displays an increasingly discrete path. The delta either tends to zero, or to a in case the put and the call remain out of the money. Variable a represents the slope of the collar option as displayed in figure 3.

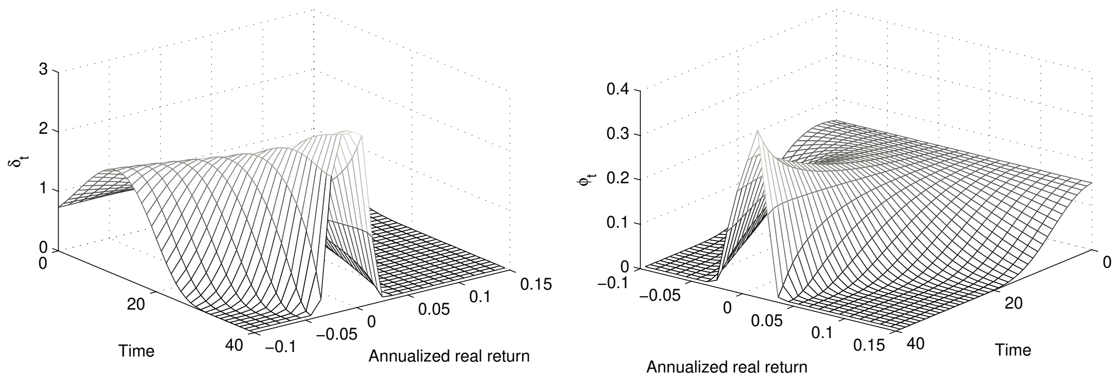

Figure 8: Option delta and replication strategy.

In addition, figure 8 displays the fraction of replicating portfolio value invested in the stock, i.e. ϕ_t, as a function of time and annualized stock return. The figure shows that the stock weight at the beginning of the investment period is approximately equal to 15%. Depending on the realized returns and the time to maturity, the stock weight is continuously adjusted to replicate the collar option payoff. The relation is explained similarly to the relation between the option delta and the realized return. In case returns are either very strong or very weak, the fraction of portfolio value invested in stocks tends to reduce to zero. In case the payoff is likely to realize between the cap and the floor, the stock weight tends to increase and may approximate 40% equity risk.

The delta of the collar option determines the investment strategy the pension scheme has to execute on behalf of the age cohorts to replicate the option payoff. However, the initial wealth V_0 necessary to execute the replication strategy is not available upon entrance of a cohort into the scheme. Since participants pay periodic contributions, rather than one single contribution equal to V_0, the risk exposure of the cohorts' pension assets has to be altered to perform the option replication strategy. The delta-hedge strategy requires $\delta_t^x S_t$ to be invested in the stock market together with a remaining investment equal to $V_t^x - \delta_t^x S_t$ in riskless bonds.[7] The value of the replicating portfolio V_t^x may now be interpreted as the value of the accumulated pension wealth plus the discounted value of future contribution payments. Since the accumulated pension wealth, and hence the value of the liabilities corresponding to the pension entitlements of age cohort x are equal to L_t^x, the cohort-specific investment

[7]The superior x is added to indicate that the replication strategy is executed on a cohort-specific basis.

strategy is given by

$$\alpha_t^x = \frac{\delta_t^x S_t}{L_t^x}. \tag{23}$$

Also the cohort-specific investment strategy can be expressed as a function of time and stock market return. Figure 9 illustrates the investment strategy by representing the stock weight as a function of time and annualized real return. The left plot shows that age cohorts initially borrow against the risk-free rate to carry out the replication strategy. Taking the benchmark collar option parameters gives $\delta_t^0 S_t = 0.73 \cdot Y$, which implies that upon entrance into the pension scheme, a participant borrows 73% of its annual wage to invest in the stock market. As time increases, the faction invested in the stock market rapidly decreases. After five years, the stock weight is likely to be less than 100%. The right plot displays the stock weight for the last twenty years only. Depending on the realized returns, the fraction invested in the stock markets varies between 0% and approximately 40% during the final years of the active period.

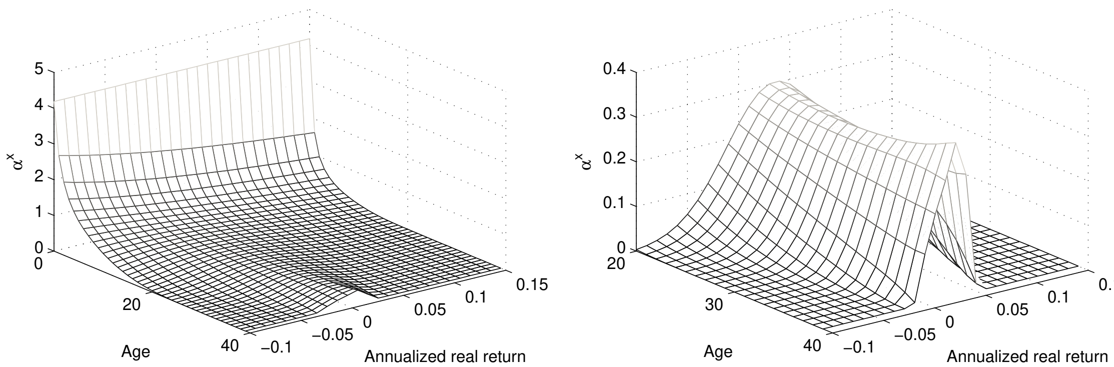

Figure 9: Cohort-specific investment strategy.

Similarly to the life cycle models discussed by Bodie et al. (1992) and Teulings and De Vries (2006), the investment in equity is initially financed by borrowing against the risk-free rate. However, these life cycle models find a much larger amount participants may want to borrow to acquire the optimal asset allocation. Their research shows that young participants may want to borrow as much as six times their annual wage to invest in the equity market. In practice, borrowing against the riskless assets may be hard to realize since young participants face credit constraints due to a lack of collateral. On the other hand, one rationale of pension funds is that borrowing constraints for young participants may be alleviated (see e.g. Bovenberg, Koijen, Nijman and Teulings, 2007). Within a collective pension scheme, participating cohorts may ex-ante decide on the fund's aggregated asset allocation and accordingly specify the division of the realized return on investment. As such, an internal market for equity risk could serve to overcome borrowing constraints.

4.6 Return-driven indexation

The indexation policy of the pension scheme is return-driven and cohort-specific. The pension entitlements of the cohorts are conditionally indexed depending on stock market return and the cohort-specific investment strategy α_t^x. The indexation policy, defined in section 3.3, is such that the accumulation of pension income exactly corresponds to the accumulation of the underlying pension assets. Since the return on the pension liabilities is equal to the riskless interest rate r, the indexation factor depends on the cohort-specific realized excess return on investment.

Figure 10 describes the indexation policy over the active period of the life cycle. The black solid line represents the expected factor of indexation of a cohort, and the dotted lines provide the 1% and 99% quantiles. The red line represents the annual indexation for a randomly selected scenario. If indexation is equal to one, this implies no excess return on investment is realized and therefore the entitlements of a cohort remain constant in real terms. The quantiles indicate that indexation can deviate from the a priori expected factor of indexation. Particularly early in life, indexation may be volatile due to equity exposure in excess of 100%. Whereas the upper quantile shows that the pension entitlements of cohorts could be doubled or even tripled, the lower quantile depicts indexation might be negative. As time increases, the indexation policy becomes less risky because stock market risk is reduced.

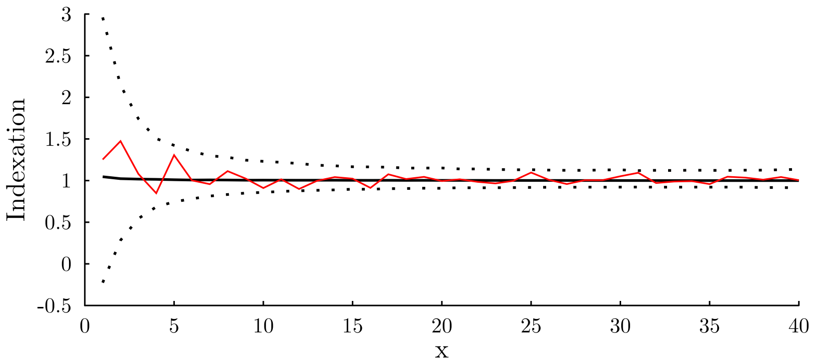

Figure 10: Return-driven indexation.

As a consequence of negative indexation, participants accumulate debt rather than wealth. Young pension fund participants could therefore face negative pension entitlements. The possibility of negative pension entitlements presents a potential problem of the collar-based pension product. In current labor markets, employees are becoming increasingly mobile as they more often shift between employers. In case a young employee who experienced unfortunate equity returns, decides to accept a job at another firm, he faces the obligation to pay off the debt corresponding to the pension product offered by the pension fund he used participate in. In practice, this would be an undesirable outcome which needs to be resolved. Based on accurate data about the amount of participants leaving the fund within the first couple of years, the fund could introduce collective insurance against leaving in debt. In this thesis, however, the problem is omitted since it is assumed no participants leave prematurely.

4.7 Pension income accrual

In this section, the accrual of pension income over the active period of the life cycle is illustrated. Participants accrue pension income from contribution payments and return-driven indexation. Figure 11 presents the income accrual over the investment period associated with the collar strategy. The black solid line presents the expected pension accrual, and the dotted lines provide the 1% and 99% quantiles. Upon entrance into the scheme, the expected replacement rate is equal to 75% of a participant's real wage. Moreover, the quantiles show that the distribution of accrued pension income does not fan out as time increases. As a result of the collar strategy, the pension benefit is bound by replacement rates equal to 50% and 80%. In addition, figure 11 displays the probability distribution of the pension benefit at retirement. It is displayed that the probability mass in between the ambition and the guarantee is approximately equally divided.

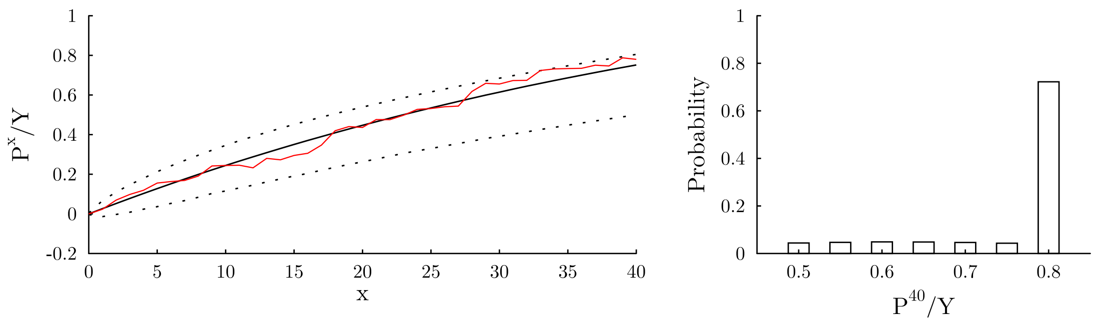

Figure 11: Distribution of the pension benefit.

In general, the pension accrual can also be expressed as a function of time and stock market return. Figure 12 illustrates the accrued income as a function of time and annualized real stock return. For the ease of viewing, the pension accrual is displayed over the last twenty years only. The red line depicts the payoff scheme of the option in terms of the benefit at retirement. The figure shows that the surface tends to display an increasingly discrete dependence on the realized return as the time to retirement decreases. This is a direct consequence of the development of the option value over time. As the time to maturity decreases, the value function increasingly tends to the discrete payoff function.

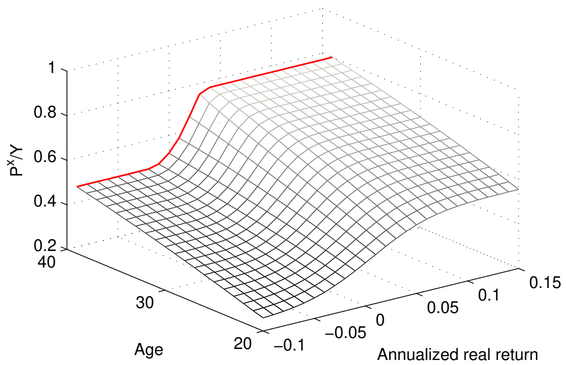

Figure 12: Pension income accrual.

4.7.1 Scenario analysis

So far, the focus has been on the accrual of pension income from the perspective of a cohort at the start of its career. Similarly, the income accrual can be analyzed for cohorts already participating in the scheme. Depending on the realized returns on investment, these cohorts face different distributions of the pension benefit at retirement. From the perspective of retirement planning, it may be interesting to investigate how the accrual of already participating generations may evolve over the remaining investment period, depending on their position at a specific moment in time.

Figure 13 presents a scenario analysis at three points during the life cycle, which are respectively, after ten, twenty and thirty years of saving. At each point in life, the evolution of pension income is displayed based on different initial positions. The origin of the blue lines, in the plots on the left, corresponds to the position of a cohort which experienced fortunate returns up to that point in time. Equivalently, the origins of the green and red lines correspond to average and bad positions. More specifically, after ten, twenty and thirty years, the accrual of pension income is analyzed for cohorts situated at the mean (green), and the 10% (red) and 90% (blue) quantiles. From these initial positions three lines start, which display a good, bad and average evolution.[8] In addition, the distribution of the pension benefit at retirement is given for the different initial positions, in the plots on the right.

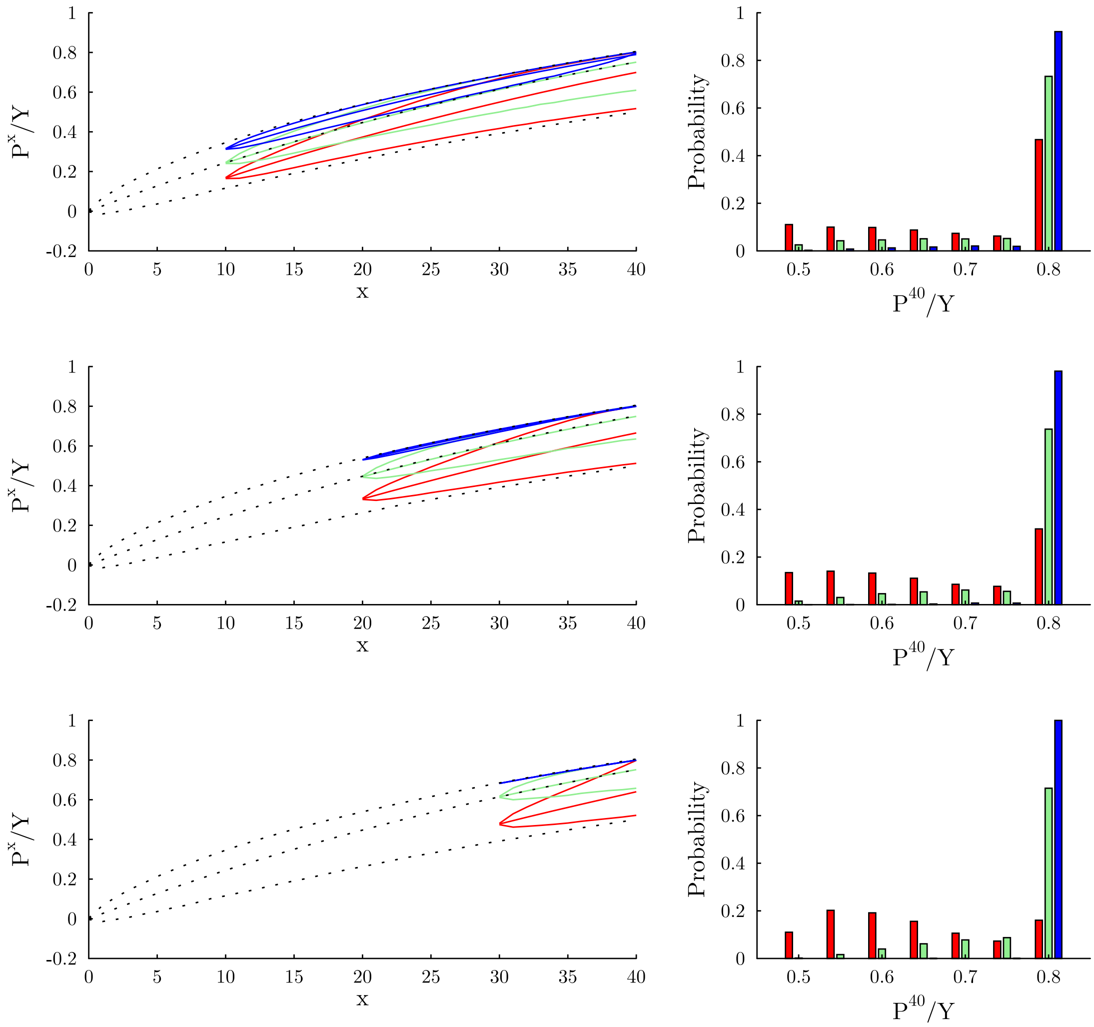

Figure 13: Scenario analysis over the life cycle.

The results of the scenario analysis are summarized as follows. For all three points in time, the figure shows that cohorts experiencing good returns, are expected to realize the ambition at retirement. Furthermore, it is shown that the downside risk is strongly reduced. For cohorts at the path given by the expected accrual, the probability of realizing the ambition

[8] Again, the terms average, bad and good correspond to the mean and 10% and 90% quantiles, respectively.

remains approximately constant at 70%. Yet, the distribution of the benefit slightly skews to the right as time increases. In case cohorts experience bad returns, the probability of realizing the ambition falls. Whereas unfortunate generations still have 40% probability of realizing the ambition after 10 years, the probability falls to 15% after 30 years. Nevertheless, unfortunate cohorts are expected to realize a pension benefit significantly better than the minimum guarantee. The expected replacement rate after twenty years, and even after thirty years of unfortunate returns is approximately equal to 65%.

4.7.2 Overlapping generations

In this subsection, the pension accrual associated with the collar strategy is discussed from a social point of view. Specifically, the differences in realized replacement rates are analyzed for overlapping generations. Since the pension savings of the participating generations are invested on a cohort-specific basis, the generations experience different rates of returns and therefore also realize different pension benefits at retirement. From a social perspective it may be undesirable if two cohorts, retiring relatively close to each other, obtain benefits which vary significantly.

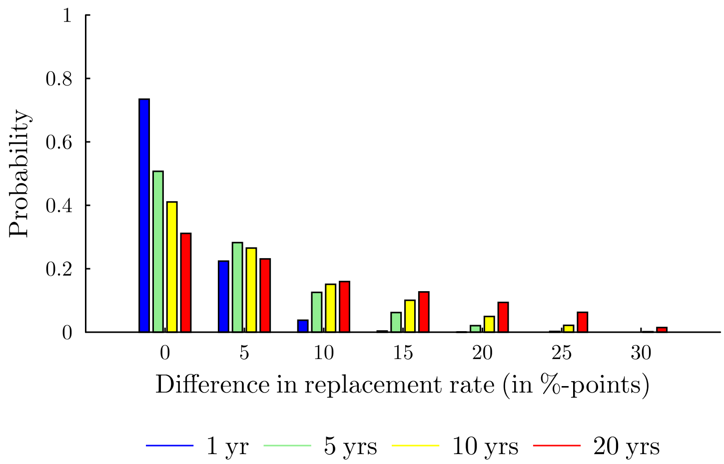

Figure 14: Difference in benefit between overlapping generations.

Figure 14 presents a probability distribution of the absolute difference in replacement rate for overlapping generations which retire within, respectively, one, five, ten and twenty years from each other. It is shown that cohorts which are one year apart, have approximately 75% probability of realizing replacement rates which differ less than 5%-points from each other. For cohorts five, ten and twenty years apart, this probability reduces to 50%, 40% and 30%, respectively.

4.8 Asset allocation pension fund

The pension scheme manages the pension assets with a single asset allocation to execute the replication strategies of the cohorts in the scheme. Figure 15 illustrates the asset allocation of the fund together with the corresponding stock value for three scenarios over a 40-year horizon. The dependence of the asset allocation of the fund on the stock value is explained similarly to the relation between the stock and the replication strategy. In case the fund experiences either favorable or adverse returns, the aggregated asset allocation in stocks is reduced in favor of bonds. Furthermore, figure 15 presents the expected asset allocation and the 10% and 90% quantiles, which are given by the dotted lines. The figure shows that the fraction of pensions assets the fund is expected to invest in stocks, is approximately equal to 12%. The quantiles indicate that the fraction may vary between 4% and 20%.

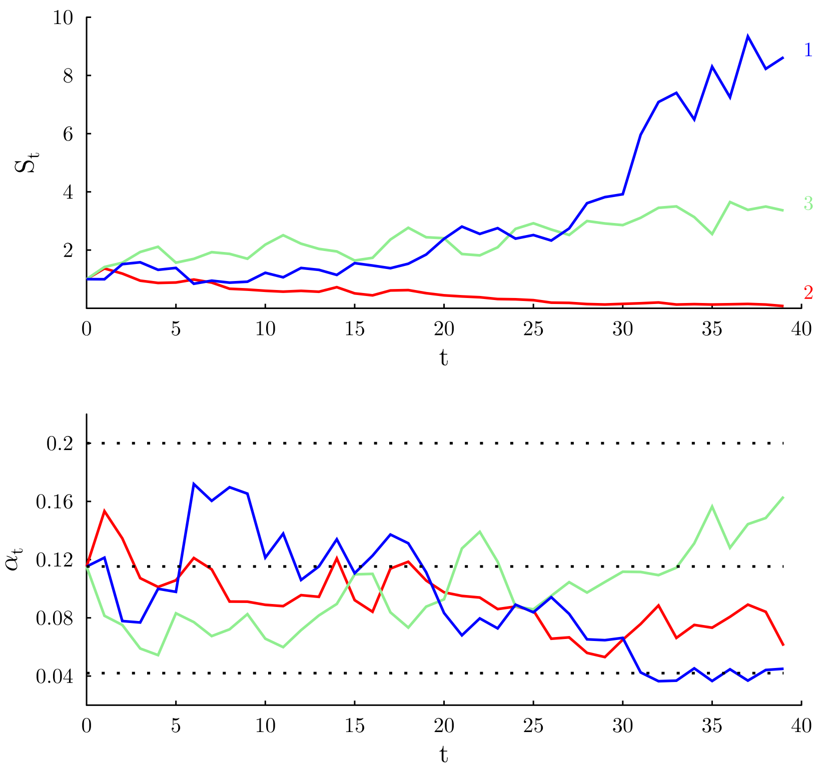

Figure 15: Asset allocation of the pension fund.

On aggregated fund level, the asset allocation corresponding to the replication strategy, may be characterized as conservative compared to the current asset allocations of Dutch pension funds. Before the latest financial crisis, Dutch pension funds invested approximately 50% of their assets in stocks (see Broeders and Rijsbergen, 2010). In fact, the asset allocation of the scheme analyzed in this thesis, shows a stronger resemblance to the investment strategies more commonly implemented twenty years ago, when Dutch funds invested 20% of the pension assets in equities. In view of the characteristics of the pension product offered, it is not remarkable that the asset allocation towards stocks is low. The collar strategy guarantees a minimum payoff which requires a significant investment in fixed-income securities to make sure at least the guarantee is realized. Moreover, the assets of the retirees in the fund are fully financed by investments in the riskless cash bond. Figure 16 illustrates the aggregated asset allocation of the active cohorts only. Again, the average allocation and quantiles are shown together with a randomly chosen scenario.

Yet, a conservative asset allocation presents a disadvantage from a macroeconomic perspective. Low risk-taking due to extensive liability-driven investment aimed at matching pension promises, may endanger macroeconomic stability and growth. In case pension funds mainly

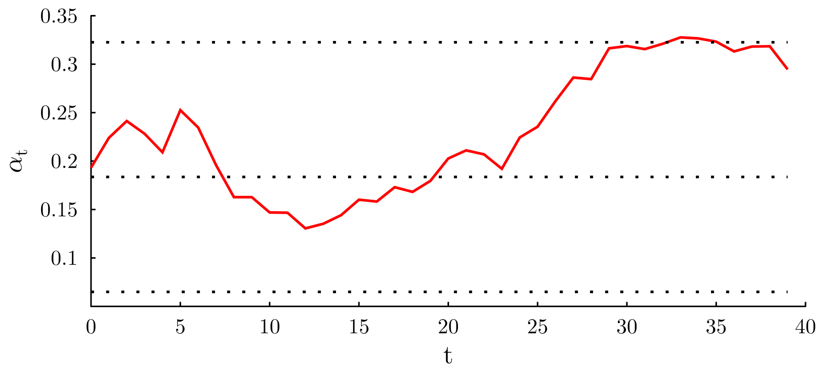

Figure 16: Aggregate asset allocation of active cohorts only.

invest in low-risk assets and government bonds, guaranteed pension benefits become more expensive as the funds' demand for bonds drives down long-term interest rates. Consequently, additional saving is required to maintain the prospect of realizing the guaranteed benefit. This process may further distort asset prices and could set in motion a deflationary spiral. At the same time, the supply of risk-taking capital may dry up, which crowds out productive investments and lowers long-term growth, as less innovation and R&D are financed (Boeri et al., 2006).

The asset allocation of the fund could also be viewed in relation to the economic conditions of the market. In times of recovery after economic downturn, the fund benefits from increasing asset prices as equity exposure is expected to be increased. Since the long put options tend to move out of the money, the cohort-specific investment strategies require the risk profile of the assets to be raised by increasing the allocation to equity. In case market recovery carries through, and the economy enters a boom, it becomes more likely that cohorts realize the ambition level. Therefore, a lock-in strategy is performed by increasing the allocation to bonds at the expense of equities. Hence, in an up going market, the pension fund is able to buy low and sell high. When the bull market subsequently turns into a bear market, the fund increases its stock exposure again, as now the short call options tend to move out of the money. If, in turn, the recession moves into a depression, the fund has to start selling stocks to make sure the guaranteed benefits can be realized. Essentially, a stop-loss strategy is executed to prevent the portfolio value from falling through the crucial minimum level. Concluding, in a downwards moving market, the fund is required to buy high and sell low.

5 Utility-based collar strategy

In this chapter, an alternative approach is discussed based on maximizing expected utility given a collar constraint. Contrary to the collar approach presented in chapter 4, the utility-based strategy takes into account that participants are likely to prefer pension benefits close to the ambition rather than benefits approximating the guarantee. Whereas the collar approach describes a linearly increasing function to connect the floor and the cap, the utility-based approach proposes a concave function based on the participants' preferences. In subsequent sections, the approach is developed. First, the cohorts' presumed preferences are specified and the optimization problem is defined. Next, the investment strategy is derived and finally the distribution of the pension benefit is illustrated.

5.1 Preferences

Preferences of the cohorts are assumed to be described by the power utility function. Power utility reflects constant relative risk aversion (CRRA) and is defined by

$$u(V) = \begin{cases} \frac{V^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma > 0,\ \gamma \neq 1 \\ \log(V) & \text{if } \gamma = 1 \end{cases} \tag{24}$$

where γ represents the level of risk aversion. The power utility function is an often used evaluation measure in the literate on dynamic asset allocation.

5.2 Optimization problem

The objective of the utility-based approach is to maximize expected utility given the assumption that participants want to limit downside risk exposure at the expense of upside potential. Formally, the optimization problem to maximize expected utility given a collar constraint is defined by

$$\begin{aligned} & \max_{V_T} E\left[u(V_T)\right] \\ \text{subject to} \quad & E\left[\eta_T V_T\right] = \eta_0 V_0 \\ & V_T \geq \theta_1 \\ & V_T \leq \theta_2 \end{aligned} \tag{25}$$

where the floor and the cap are given by θ_1 and θ_2, respectively. In the complete Black-Scholes market, the optimal payoff can be obtained as a function of the state of the economy by using the equivalent martingale method (Cox and Huang, 1989). Taking into account the collar constraint, the optimal value of the pension assets at maturity T is given by

$$V_T = \max\{\theta_1, \min\{I(y\eta_T), \theta_2\}\} \tag{26}$$

where $I(\cdot)$ denotes the inverse of the marginal utility function, and y is a Lagrange multiplier which is determined by the budget constraint $E\left[\eta_T V_T\right] = \eta_0 V_0$. In the context of a pension fund, the budget constraint can be interpreted as imposing financial fairness between the participating cohorts.

5.3 Risk aversion parameter

Given the one-to-one correspondence between the pricing kernel and the stock market index in the Black-Scholes economy, the optimal payoff V_T can be expressed as a function of the stock market value S_T at maturity. Figure 17 illustrates the payoff scheme of the utility-based collar option for different values of the risk aversion parameter. As a result of decreasing marginal utility, reflected by the power utility function, the slope of the option becomes more concave as risk aversion increases. Moreover, the figure displays that as risk aversion becomes larger, the implied strike prices shift further apart. Hence, the bad-state region decreases at the expense of the good-state region.

Instead of choosing probabilities of realizing the ambition and ending up with the guarantee, the collar-based approach requires the input of a risk aversion parameter to define the pension product. Given the risk aversion parameter, implied strike prices and the concavity of the slope are defined. Hence, the risk aversion parameter may be related to the probabilities of realizing the ambition and the guarantee. For this purpose, it is assumed that

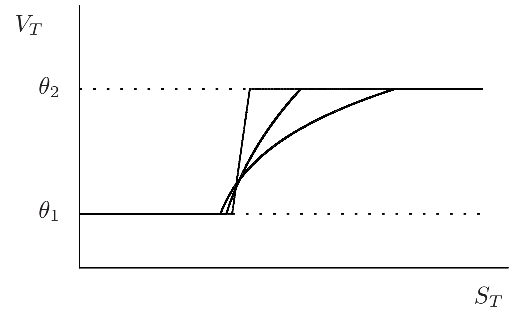

Figure 17: Utility-based collar option.

the parameters of the utility-based collar option are equal to the corresponding parameters of the benchmark collar option discussed in chapter 4. Accordingly, the cap and the floor correspond to replacement rates equal to 50% and 80%, respectively. The contribution rate is equal to 17.5% of a participant's real wage.

Figure 18 displays the probabilities of realizing the ambition and the guarantee for risk aversion parameters $1 \leq \gamma \leq 5$. The curve shows that as the risk aversion parameter increases, the probability of realizing the guarantee decreases at the expense of the probability of realizing the ambition. In fact, the effect of the risk aversion parameter is shown to be substantial. Whereas the probability of ending up with the guarantee drops from 4% to 1.2%, the probability of realizing the ambition decreases from approximately 90% to 50%. Hence, a slight decrease of the bad-state region requires a strong decrease of the good-state region. Conversely, increasing the probability of realizing the ambition is cheap in terms of the probability of ending up with the guarantee.

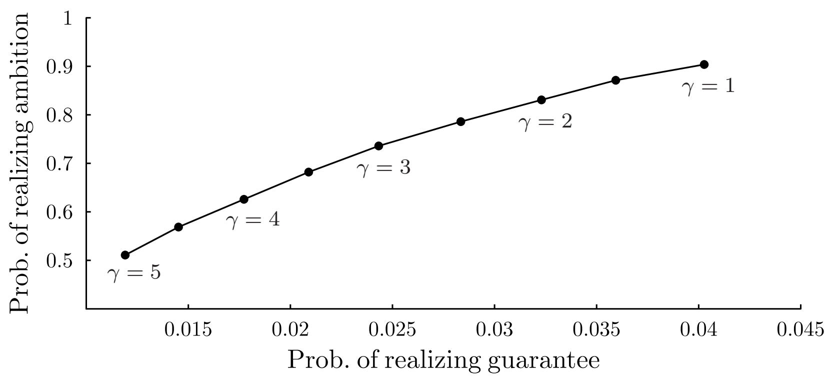

Figure 18: Risk aversion in relation to strike prices.

Subsequent section presents the investment strategy associated with the utility-based approach. To illustrate the strategy, the risk aversion parameter γ is assumed to be equal to 3.3. Given this parameter value, the probability of realizing the ambition is equal to 70%. As such, the utility-based collar option offers the same probability of realizing the ambition as the benchmark collar option discussed in chapter 4. The associated probability of ending up with the guarantee is equal to 2.2%, and therefore slightly smaller than the corresponding probability of the benchmark collar option. The utility-based collar option parameters values are summarized in table 4.

κ_1	κ_2	γ	c
0.5	0.8	3.3	0.175

Table 4: Utility-based collar option parameter values.

5.4 Investment strategy

The value of the utility-based collar option is determined using the equivalent martingale method. The optimal value of the pension assets at maturity is given by a contingent claim and therefore the payoff can be replicated by a delta-hedge strategy. According to the fundamental theorem of asset pricing, the process defined by $\{\eta_t V_t\}$ is a martingale. This implies that the value of the utility-based option at time $t \leq T$ satisfies

$$V_t = E_t\left[\frac{\eta_T}{\eta_t} V_T\right]. \tag{27}$$

The option value V_t can be solved as a function of time t and of the value of the pricing kernel η_t. At time t, the value of the utility-based collar option is given by

$$\begin{aligned} V_t &= e^{-r(T-t)}\left(\theta_1\Phi(d_1+\lambda\sqrt{T-t})+\theta_2\Phi(-d_2-\lambda\sqrt{T-t})\right) \\ &+ \frac{e^{\Gamma(t)}}{(y\eta_t)^{\frac{1}{\gamma}}}\left(\Phi(d_2+\lambda\sqrt{T-t}(1-\frac{1}{\gamma}))-\Phi(d_1+\lambda\sqrt{T-t}(1-\frac{1}{\gamma}))\right) \end{aligned} \tag{28}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function, and

$$\Gamma(t) = (-r-\frac{1}{2}\lambda^2\frac{1}{\gamma})(T-t)(1-\frac{1}{\gamma}) \tag{29}$$

$$d_i = \frac{\log(y\eta_t)+\gamma\log(\theta_i)-(r+\frac{1}{2}\lambda^2)(T-t)}{\lambda\sqrt{T-t}} \tag{30}$$

for $i \in \{1,2\}$. Appendix B.1 contains the derivation of the value of the utility-based collar option.

The one-to-one correspondence between the pricing kernel and the stock market index can be used to express the option value as a function of time t and the stock market value S_t. In turn, the holdings in the risky asset (i.e. the delta) can be determined by the partial derivative of the option value at time t with respect to the stock market value at time t. However, as an alternative to computing the delta, we specify the optimal investment strategy by presenting the fraction of replicating portfolio value to be invested in stocks. The replication strategy to create the payoff of the utility-based collar option is given by

$$\phi_t = \frac{\mu-r}{\gamma\sigma^2}\cdot\frac{e^{\Gamma(t)}}{V_t(y\eta_t)^{\frac{1}{\gamma}}}\left(\Phi(d_2+\lambda\sqrt{T-t}(1-\frac{1}{\gamma}))-\Phi(d_1+\lambda\sqrt{T-t}(1-\frac{1}{\gamma}))\right). \tag{31}$$

Appendix B.2 contains the derivation of the optimal investment strategy. The analytical expression for the fraction of replicating portfolio value to be invested in stocks depends on the weight invested in risky assets required by a constant proportion strategy, i.e. $\frac{\mu-r}{\gamma\sigma^2}$. The replication strategy tends to the constant proportion strategy in case the return on the stock is such that the utility-based option is likely to realize a payoff between the floor and the cap.

Based on the replication strategy, the pension scheme determines the cohort-specific investment strategies which are executed on behalf of the cohorts. The risk exposure over the cohorts' pension entitlements is different from the replication strategy because participants pay periodic contributions rather than a single contribution equal to the equal initial wealth required to replicate the payoff. The replication strategy requires $\phi_t^x V_t^x$ to be invested in the stock market.[9] Since the value of the pension entitlements of age cohort x are equal to L_t^x, the cohort-specific investment strategy is given by

$$\alpha_t^x = \frac{\phi_t^x V_t^x}{L_t^x}. \tag{32}$$

Figure 19 illustrates the replication strategy and the cohort-specific investment strategy by representing the stock weight as a function of time and annualized real stock return. The plots show a strong similarity to the plots representing the replication strategy and cohort-specific investment strategy of the collar option analyzed in chapter 4. For the explanation of the figure, we therefore refer to section 4.7.

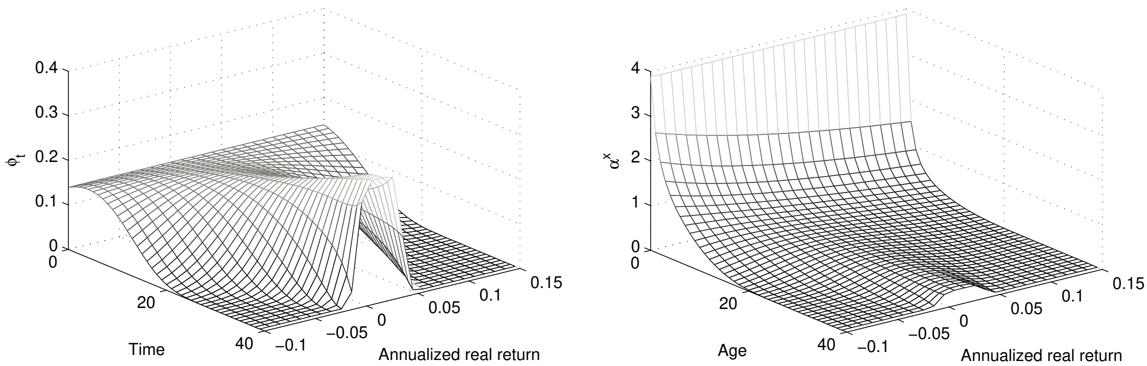

Figure 19: Replicating strategy and cohort-specific investment strategy.

5.5 Pension benefit

In this section, the distribution of the pension benefit is compared for the two different collar approaches. Figure 20 displays the distribution of the utility-based approach as well as the distribution of the standard collar approach. For the utility-based approach, it is shown that the probability mass in between the guaranteed replacement rate and the aspired replacement rate is more skewed to the right compared to the standard collar approach. More probability mass is assigned to payoffs near the cap than to payoffs near the floor because of the concavity of the slope. Nevertheless, the effect is not very strong. Since only little probability mass is assigned to payoffs in between the guaranteed replacement rate and the aspired replacement rate, the utility-based approach does not present a strong advantage over the standard collar option. For collar options which assign more probability mass to payoffs in between the cap and the floor, the effect of the utility-based approach is likely to become stronger.

In the following chapter, life cycle strategies are compared to the standard collar approach. In comparing different strategies, we choose the standard collar option as benchmark approach

[9] The superior x is added to indicate that the replication strategy is executed on a cohort-specific basis.

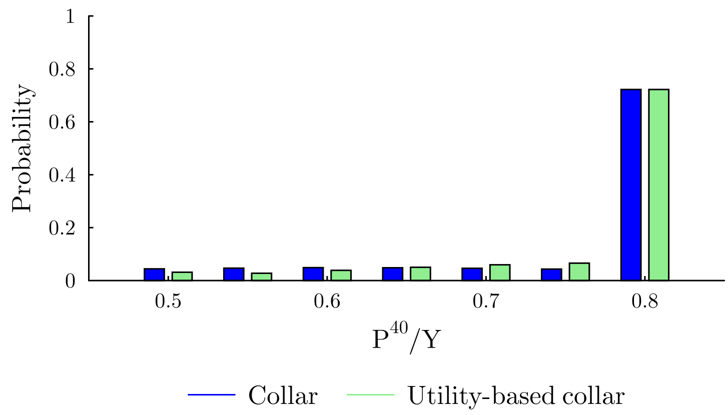

Figure 20: Distribution of the pension benefit.

because the interpretation of the parameters more closely fits the notions currently considered in the pension sector. Nevertheless, note that the utility-based approach could present an improvement in terms of the distribution of the benefit.

6 Life cycle strategies

The board of the pension fund has to decide on the characteristics of the targeted pension benefit it intends to offer to its participants. Subsequently, the contribution rate and the investment strategy have to be determined such that the fund lives up to the offered product. The collar approach requires the preferred product to be formulated by a feasible combination of the option parameters and a contribution rate. Once choices are taken, the dynamic investment strategy is defined. The replication strategy requires continuously rebalancing the portfolio weights as time passes and asset prices move. At retirement, the payoff of the strategy corresponds to the promised retirement product.

In practice, collective pension funds often determine a strategic asset allocation corresponding to the long-term characteristics of the liabilities. The strategic asset allocation refers to the optimal base mix of stocks, bonds and potentially other asset classes, based on expected returns, volatility and correlations. In the effort to realize the long term objectives, the fund's asset allocation is rebalanced to the optimal allocation at a regular basis. In case the fund implements a uniform policy regarding the benefit allocation (e.g. uniform indexation), participants are implicitly exposed to approximately constant equity exposure over the life cycle. As an alternative, target date funds offer investment strategies which reduce the allocation towards stocks as the time to retirement decreases. The portfolios of target date funds typically start out with significant stock exposure and shift to more conservative fixed-income holdings as individuals near retirement. Target date funds rarely provide a minimum guaranteed return on participants' pension assets.

In this chapter, the performance of life cycle strategies is compared to the collar approach. The life cycle strategies discussed are classified as target date schemes or myopic schemes. All strategies rebalance the asset allocation according to predefined schemes which either provide decreasing or constant equity exposure over the life cycle. The criteria to evaluate the performance of the life cycle strategies are related to the targets set by the board of the pension fund. It is assumed that the targets are formulated in terms of the option parameters of the collar option. Corresponding to the parameter values of the benchmark collar option, the focus is on two types of performance measurement:

i *Fixed probabilities*
It is investigated which aspired and pessimistic pension benefit a strategy offers, given the prerequisites that the aspired benefit is realized with 70% probability and the risk of falling short of the pessimistic benefit does not exceed 2.5% probability.

ii *Fixed levels*
The aspired benefit and the minimum desired benefit are assumed to be equal to 80% and 50% of a participant's real wage. It is investigated how much probability a strategy assigns to realizing the aspired benefit and to falling short of the minimum desired benefit.

Subsequently, the performance of the life cycle strategies is related to the performance of the collar approach. The following sections discuss the life cycle strategies and the performance measures in detail.

6.1 Schemes

6.1.1 Target date schemes

The target date schemes start out with 100% equity exposure and shift to the riskless portfolio at retirement. The speed with which the different schemes reduce the equity exposure may vary over the life cycle. Figure 21 depicts the age-related stock exposure α^x corresponding to the schemes evaluated. Three strategies are emphasized and referred to as *risk averse*, *linear* and *risk seeking*. Respectively, these schemes reduce the equity exposure quickly early in life, on a straight-line basis, or not until later in life. In the next section, the performance of these schemes is illustrated explicitly.

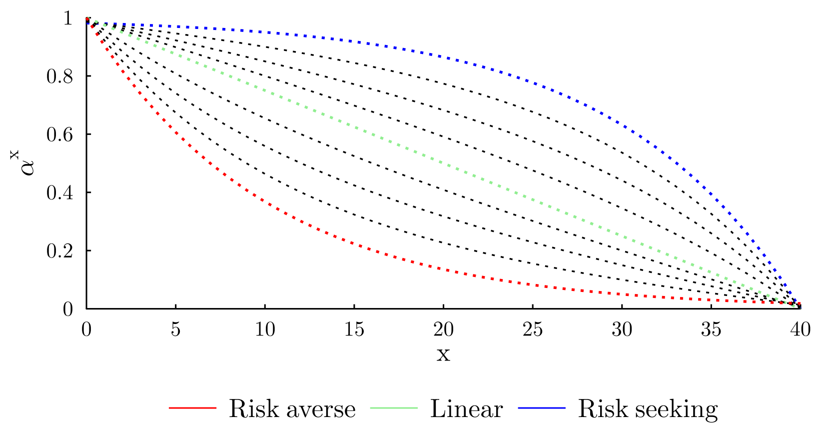

Figure 21: Target date schemes.

6.1.2 Myopic schemes

Myopic schemes have constant equity exposure over the life cycle. Figure 22 displays the myopic schemes given by 10%, 30%, 50% and 70% equity exposure. In the next section, the performance of these schemes is illustrated explicitly.

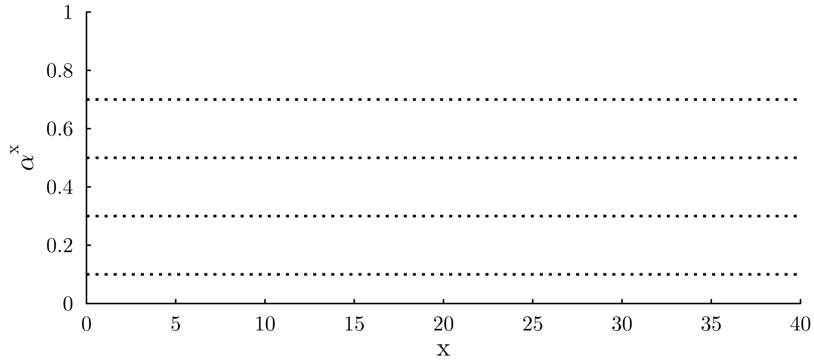

Figure 22: Myopic schemes.

6.2 Performance measurement

This section relates the performance of the life cycle strategies to the performance of the collar option. The contribution rate equals 17.5% of a participant's wage, similarly to the contribution rate corresponding to the benchmark collar option analyzed in chapter 4.

6.2.1 Fixed probabilities

Assume a pension scheme intends to offer an aspired pension benefit which is realized with at least 70% probability. Moreover, the scheme requires that the probability of falling short of a pessimistic benefit does not exceed 2.5% probability. Given these prerequisites, it is investigated which aspired and pessimistic replacement rate the pension scheme can offer its participants in case a life cycle investment strategy is executed.[10]

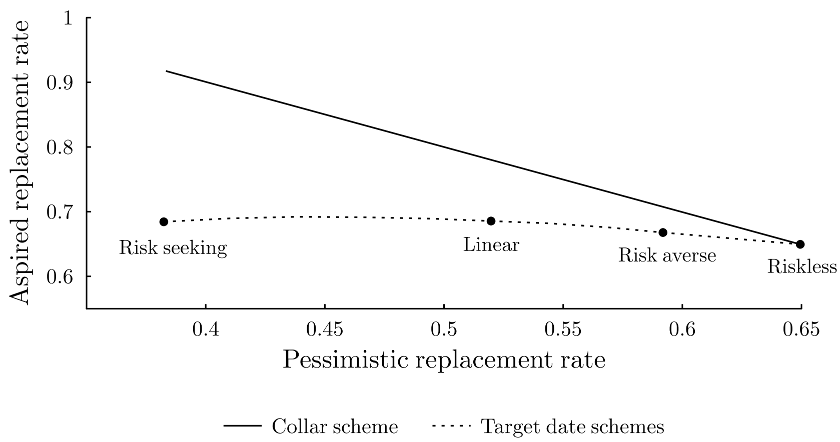

Figure 23: Trade-off between aspired and pessimistic replacement rate.

Given the definitions, every target date scheme provides a pessimistic and aspired replacement rate. However, as a class of strategies, the set of target date schemes yields a trade-off curve. The curve is displayed in figure 23 and shows that as the schemes become riskier,

[10]Formally, the aspired replacement rate is defined by $\max\{\nu \geq 0 \mid \mathbf{P}(P^{40}/Y \geq \nu) \geq 70\%\}$ and the pessimistic replacement rate by $\max\{\nu \geq 0 \mid \mathbf{P}(P^{40}/Y \leq \nu) \leq 2.5\%\}$.

the pessimistic replacement rate increases, whereas the aspired replacement rate remains approximately constant. Hence, accepting more investment risk does not provide a proportional increase of the aspired pension benefit compared to the increase of the downside risk. In addition, figure 23 displays the trade-off curve for the collar approach given that the ambition level and guaranteed level are realized with 70% and 2.5% probability, respectively. In case a substantial level of certainty (i.e. 70%) is required, it is shown that given a pessimistic replacement rate, the collar approach offers a higher aspired replacement rate than the target date schemes. Moreover, it is displayed that given the assumed contribution rate, the collar approach may offer aspired replacement rates which are unattainable in case a target date scheme is implemented. Figure 24 displays the trade-off curve for the myopic schemes. It is shown that the curve shows a strong similarity to the curve associated with the target date schemes.

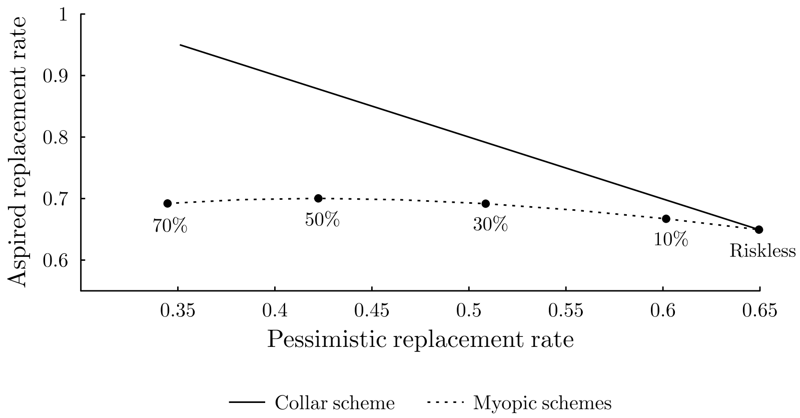

Figure 24: Trade-off between aspired and pessimistic replacement rate.

Note that the collar approach provides a maximum and a minimum pension benefit. The return on life cycle strategies is not bound by a floor and a cap. Participants face the risk of ending up with a benefit lower than the pessimistic replacement rate, but also have the upside potential to realize pension benefits well in excess of the wage earned in the active period. However, if participants require a substantial level of certainty in realizing the aspired benefit, it is shown that given a pessimistic replacement rate, the collar approach outperforms the life cycle strategies in terms of the offered aspired replacement rate. Conversely, given an aspired replacement rate, a larger pessimistic replacement rate may be offered.

6.2.2 Fixed levels

Assume a pension scheme intends to offer an aspired benefit and a minimum desired benefit equal to 80% and 50% of a participant's wage, respectively. Given these targets, it is investigated how much probability a life cycle strategy assigns to realizing the aspired benefit and to falling short of the minimum desired benefit. The life cycle strategies analyzed, all provide a probability of realizing the aspired benefit and a probability of falling short of the minimum desired benefit. Similarly to the previous section, trade-off curves are constructed for the target date and myopic schemes.

Figure 25 displays the trade-off curve for the set of target date schemes. It is shown that the risk averse scheme has zero probability of falling short of the minimum desired benefit. However, as a consequence of conservative equity exposure the scheme offers only 10% probability of realizing the aspired level. As the target date schemes become riskier, the probability of realizing the aspired replacement rate increases at the expense of the probability of falling short of the minimum desired replacement rate. In addition, figure 25 displays the trade-off curve for the collar approach. The floor and the cap correspond to replacement rates equal to 50% and 80%, respectively. The figure shows that given a probability of shortfall (c.q. realizing the guarantee), the collar approach offers a larger probability of realizing the aspired replacement rate than the target date schemes.[11]

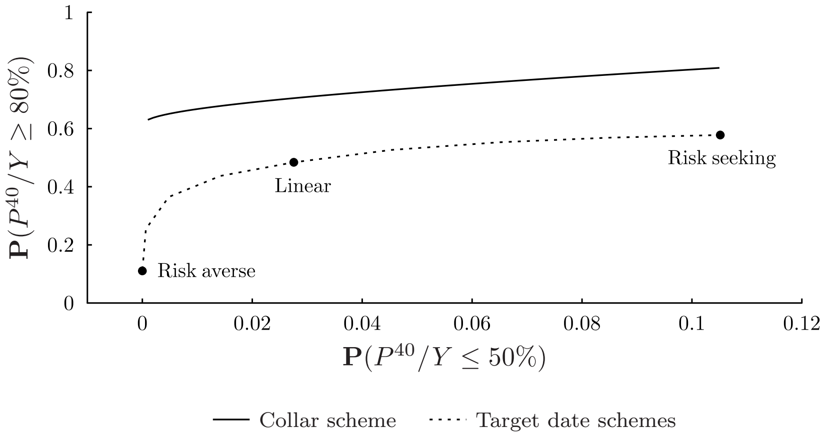

Figure 25: Probabilities of realizing aspired and minimum desired replacement rate.

Figure 26 displays the trade-off curve for the myopic schemes. Similarly to the curve associated with the target date schemes, the trade-off curve implied by the myopic schemes is displayed below the trade-off curve of the collar approach. Hence, given a probability of falling short of the minimum desired benefit (c.q. realizing the guarantee), the collar approach outperforms the life cycle strategies in terms of the offered probability of realizing the aspired benefit.

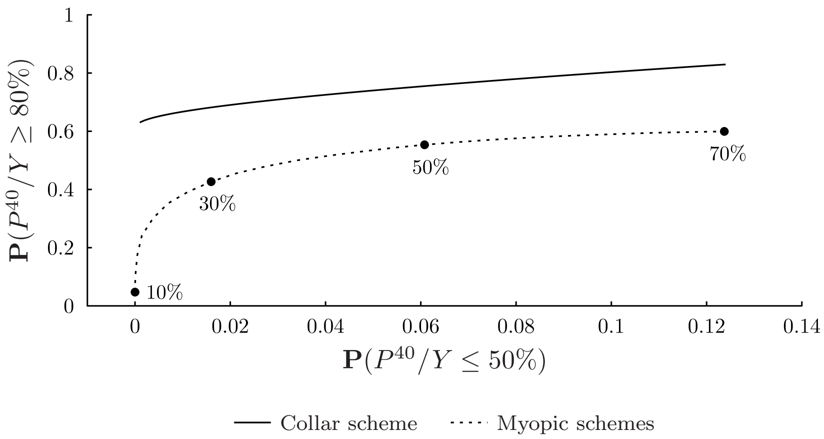

Figure 26: Probabilities of realizing aspired and minimum desired replacement rate.

[11]In fact, for the collar approach the axis are given by $\mathbf{P}(P^{40}/Y = 50\%)$ and $\mathbf{P}(P^{40}/Y = 80\%)$.

7 Conclusion

In this thesis, an approach to retirement security is developed that focusses on realizing an ex-ante defined target benefit. The target benefit analyzed, aims at securing a level of pension income necessary to safeguard a minimum standard of living, while it allows for reaching an aspired benefit in case favorable economic scenarios materialize. The strategy implemented to attain the required pension benefit is based on financial market insurance. At retirement, the payoff corresponding to the target benefit is defined by a claim contingent on the state of the economy. Assuming a complete market for equity risk, the payoff of the contingent claim is constructed by executing a cohort-specific and dynamic replication strategy. Depending on a cohort's time to retirement and realized returns on investment, the equity exposure is dynamically adjusted to create the guaranteed benefit, while maintaining the prospect of realizing the aspired benefit.

More specifically, the proposed pension product is defined by a collar option on a stock market index. The option offers downside protection by guaranteeing a minimum benefit but simultaneously provides limited upside potential to suppress the cost of downside insurance. The value of the option and hence the contribution rate directly depend on the preferred set of option parameters. Using risk-neutral pricing techniques, the cost-based contribution rate is set such that the guaranteed benefit can be realized and the ambition may be fulfilled. Given fixed strike prices such that the ambition and guarantee are realized with 70% and 2.5% probability, it is illustrated that the ambition level and the guaranteed level are approximately equal substitutes. In case the contribution rate is required to remain constant, increasing the ambition is approximately equally expensive in terms of giving up guarantees as the other way around. Moreover, the additional costs of increasing the guarantee or the ambition are displayed. It is shown that for 1%-point additional contribution, the guarantee, the ambition or a combination of both may be increased by 7.4%-points in total.

Furthermore, the proposed system is financially fair because the payoff is constructed on a self-financing basis. As a result of the decreasing accrual system in combination with the return-driven and cohort-specific indexation policy, the value of pension entitlements exactly matches the value of the underlying pension assets. The property of financial fairness ensures that it is justified to compare the performance of the collar approach to life cycle strategies more commonly executed in the pension industry. Given the prerequisite that an aspired benefit is realized with 70% probability, the collar approach outperforms the evaluated life cycle strategies as a higher aspired benefit can be offered given a pessimistic benefit (i.e. 2.5% quantile). Moreover, given a probability of falling short of the minimum desired benefit (c.q. realizing a guarantee), the collar approach outperforms the life cycle strategies in terms of the offered probability or realizing the aspired benefit.

Although the collar approach is based on an individual pension product, retirement security is assumed to be organized collectively. The pension scheme manages the pension assets with a single asset allocation to execute the replication strategies of the cohorts in the scheme. On aggregated fund level, the asset allocation corresponding to the collar approach is characterized by a conservative risk profile compared to the asset allocation of Dutch pension funds. Whereas Dutch funds invest approximately 50% of their assets in equities, the stylized scheme is expected to invest only 12% of the assets in equities. Taking into account the asset of the active cohorts only, the allocation towards stocks approximates 20%. The aggregate asset allocation of the fund indicates that the guarantee to realize a minimum payoff requires a significant investment in fixed-income securities. Yet, the conservative asset allocation may present a disadvantage form a macroeconomic perspective. Low risk taking at the expense of risk-bearing capital may be unfavorable as it could harm innovation and economic growth (Boeri et al., 2006).

To conclude, a few limitations of the scheme are addressed and some points are put forward for further research. First of all, the implementation of the collar approach may be challenged by the initial requirement to finance risky equity investments by borrowing against the risk-free rate. Although the collective organization of the pension scheme could serve to overcome restricted access of young individuals to capital markets, young cohorts are exposed to the risk of losing wealth in excess of their accumulated savings as a consequence of the leveraged equity exposure. The possibility of pension debt at young ages presents a potential problem for the approach even though a guaranteed benefit is ensured at retirement. In case a young employee who experienced unfortunate returns, decides to become self-employed or switches jobs, he faces the obligation to pay off the debt associated with the pension product offered by the fund he used to participate in. One way to resolve the problem, could be to introduce collective insurance against leaving in debt.

Second, the collar approach is developed in a highly simplified representation of the real financial market. The Black-Scholes economy incorporates only stock market risk and assumes the presence of a cash bond subjected to constant interest accrual. In real life however, long-dated pension claims are exposed to large interest rate risk as a result of time-varying interest rates. Moreover, inflation presents a source of risk that has to be addressed, as inflation may strongly erode the real value of pension savings in the long-run. For further research, it is recommended that the replication of a target benefit is analyzed in a more comprehensive model which takes into account stochastic interest rates and stochastic inflation. Since more factors representing the state of the economy have to be hedged, it is expected that incorporating additional sources of risk increases the cost of realizing a target benefit. Hence, either higher contributions would be required or a new balance between ambition and guarantees has to be determined.

Finally, a reflection is cast upon the delta replication strategy. As discussed, the investment strategy to replicate the payoff of a contingent claim requires a complete market setting and involves continuously rebalancing portfolio weights as time passes and the underlying assets move. In real financial markets, however, transaction costs and non-traded risks may prevent portfolio managers from performing the perfect hedge. As a consequence, financial market insurance may fail to realize the ex-ante defined target benefit a fund intends to offer. For the future, research is recommended to be aimed at exploiting the potential of collective pension funds to complete markets and their ability to benefit from scale advantages. As a result, the funds' risk-return trade-off may improve and mismatch risk in realizing a target benefit could be reduced.

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Appendices

A Collar option

A.1 Option value

The payoff of the collar option is defined by

$$V_T = \max\left\{\theta_1, \min\left\{\theta_1 + \frac{\theta_2 - \theta_1}{K_2 - K_1}(S_T - K_1), \theta_2\right\}\right\}.$$

Let $\mathbb{Q}$ be the unique equivalent martingale measure corresponding to the bond as numéraire, then the value of the collar option at time $t \leq T$ satisfies

$$\begin{aligned}
V_t &= E_t^{\mathbb{Q}}\left[\frac{B_T}{B_t} V_T\right] = e^{-r(T-t)} E_t^{\mathbb{Q}} V_T \\
&= e^{-r(T-t)} E_t^{\mathbb{Q}} \max\{\theta_1, \min\{\theta_1 + a(S_T - K_1), \theta_2\}\}
\end{aligned}$$

where $S_T = S_t e^{(r-\frac{1}{2}\sigma^2)(T-t)+\sigma(Z_T - Z_t)}$ and $a = \frac{\theta_2-\theta_1}{K_2-K_1}$. Let $z \sim N(0,1)$ and d_i denote the value of z such that $K_i = S_t e^{(r-\frac{1}{2}\sigma^2)(T-t)+\sigma\sqrt{T-t}z}$ for $i \in \{1,2\}$. Then,

$$\begin{aligned}
V_t &= \frac{e^{-r(T-t)}}{\sqrt{2\pi}}\left(\int_{-\infty}^{d_1} \theta_1 e^{-\frac{1}{2}z^2} dz + \int_{d_2}^{\infty} \theta_2 e^{-\frac{1}{2}z^2} dz\right) \\
&\quad + \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{d_1}^{d_2} \left(\theta_1 + a(S_t e^{(r-\frac{1}{2}\sigma^2)(T-t)+\sigma\sqrt{T-t}z} - K_1)\right) e^{-\frac{1}{2}z^2} dz \\
&= e^{-r(T-t)}\left(\theta_1 \Phi(d_1) + \theta_2 \Phi(-d_2)\right) \\
&\quad + \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{d_1}^{d_2} (\theta_1 - aK_1) e^{-\frac{1}{2}z^2} dz \\
&\quad + \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{d_1}^{d_2} aS_t e^{(r-\frac{1}{2}\sigma^2)(T-t)+\sigma\sqrt{T-t}z} e^{-\frac{1}{2}z^2} dz \\
&= e^{-r(T-t)}\Big(\theta_1 \Phi(d_1) + \theta_2 \Phi(-d_2) + (\theta_1 - aK_1)(\Phi(d_2) - \Phi(d_1)\Big) \\
&\quad + \frac{aS_t e^{-\frac{1}{2}\sigma^2(T-t)}}{\sqrt{2\pi}} \int_{d_1}^{d_2} e^{\sigma\sqrt{T-t}z} e^{-\frac{1}{2}z^2} dz \\
&= e^{-r(T-t)}\Big(\theta_2 + a(K_1\Phi(d_1) - K_2\Phi(d_2))\Big) \\
&\quad + \frac{aS_t e^{-\frac{1}{2}\sigma^2(T-t)}}{\sqrt{2\pi}} \int_{d_1}^{d_2} e^{-\frac{1}{2}(z-\sigma\sqrt{T-t})^2} e^{\frac{1}{2}\sigma^2(T-t)} dz \\
&= e^{-r(T-t)}\Big(\theta_2 + a(K_1\Phi(d_1) - K_2\Phi(d_2))\Big) \\
&\quad + aS_t\left(\Phi(d_2 - \sigma\sqrt{T-t}) - \Phi(d_1 - \sigma\sqrt{T-t})\right).
\end{aligned}$$

A.2 Option delta

The delta of a stock option is the first derivative of the option value with respect to the underlying stock market value, i.e.

$$\delta_t = \frac{\partial V_t}{\partial S_t}.$$

Specifically, the delta of the collar option is given by

$$\begin{aligned}\delta_t = &-\frac{ae^{r(T-t)}}{S_t\sigma\sqrt{T-t}}\left(K_1\Phi'(d_1) - K_2\Phi'(d_2)\right)\\ &+ a\left(\Phi(d_2-\sigma\sqrt{T-t}) - \Phi(d_1-\sigma\sqrt{T-t})\right)\\ &+ \frac{a}{\sigma\sqrt{T-t}}\left(\Phi'(d_1-\sigma\sqrt{T-t}) - \Phi'(d_2-\sigma\sqrt{T-t})\right).\end{aligned}$$

Subsequently, we use that $S_t\Phi'(d_i - \sigma\sqrt{T-t}) = e^{-r(T-t)}K_i\Phi'(d_i)$ for $i \in \{1,2\}$ and therefore it holds that the delta of the collar option is equal to

$$\delta_t = a\left(\Phi(d_2-\sigma\sqrt{T-t}) - \Phi(d_1-\sigma\sqrt{T-t})\right).$$

Finally, we prove the equality used above. First note that[12]

$$\begin{aligned}d_i^2 - (d_i - \sigma\sqrt{T-t})^2 &= (\sigma\sqrt{T-t})(2d_i - \sigma\sqrt{T-t})\\ &= 2\left(\log(\frac{K_i}{S_t}) - r(T-t)\right).\end{aligned}$$

Second, we use $\Phi'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$ to complete the proof. The final steps are given by

$$\begin{aligned}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(d_i^2-(d_i-\sigma\sqrt{T-t})^2\right)} &= \frac{1}{\sqrt{2\pi}}\frac{\Phi'(d_i)}{\Phi'(d_i-\sigma\sqrt{T-t})}\\ &\Leftrightarrow\\ e^{-\log\left(\frac{K_i}{S_t}\right)+r(T-t)} &= \frac{\Phi'(d_i)}{\Phi'(d_i-\sigma\sqrt{T-t})}\\ &\Leftrightarrow\\ S_t\Phi'(d_i-\sigma\sqrt{T-t}) &= e^{-r(T-t)}K_i\Phi'(d_i).\end{aligned}$$

B Utility-based collar option

B.1 Option value

The optimization problem is given by

$$\begin{aligned}&\max_{V_T} E_t\left[u(V_T)\right]\\ s.t.\quad &E_t\left[\eta_T V_T\right] \le \eta_0 V_0\\ &V_T \ge \theta_1\\ &V_T \le \theta_2\end{aligned}$$

[12] Note: $x^2 - y^2 = (x-y)(x+y)$.

where $u(V) = \frac{V^{1-\gamma}}{1-\gamma}$. Using the equivalent martingale method (Cox and Huang, 1989), optimal wealth at time T is given by

$$V_T = \max\{\theta_1, \min\{I(y\eta_T), \theta_2\}\}$$

where $I(x) = x^{-\frac{1}{\gamma}}$ is the inverse of $u'(\cdot)$ and $y \geq 0$ solves $E[\eta_T V_T] = \eta_0 V_0$. According to the pricing kernel formulation, the value of the utility-based collar option at time $t \leq T$ satisfies

$$\begin{aligned} V_t &= E_t\left[\frac{\eta_T}{\eta_t} V_T\right] \\ &= E_t\left[e^{(-r-\frac{1}{2}\lambda^2)(T-t)-\lambda\sqrt{T-t}z} \max\{\theta_1, \min\{I(y\eta_T), \theta_2\}\}\right] \end{aligned}$$

where $\eta_T = \eta_t e^{(r-\frac{1}{2}\lambda^2)(T-t)-\lambda(Z_T - Z_t)}$. Let $z \sim N(0,1)$ and d_i denote the value of z such that $\theta_i = I(y\eta_t e^{(r-\frac{1}{2}\lambda^2)(T-t)-\lambda\sqrt{T-t}z})$ for $i \in \{1,2\}$. Then,

$$\begin{aligned} V_t &= \frac{e^{(-r-\frac{1}{2}\lambda^2)(T-t)}}{\sqrt{2\pi}} \int_{-\infty}^{d_1} \theta_1 e^{\lambda\sqrt{T-t}z} e^{-\frac{1}{2}z^2} dz \\ &+ \frac{e^{(-r-\frac{1}{2}\lambda^2)(T-t)}}{\sqrt{2\pi}} \int_{d_2}^{\infty} \theta_2 e^{\lambda\sqrt{T-t}z} e^{-\frac{1}{2}z^2} dz \\ &+ \frac{1}{\sqrt{2\pi}(y\eta_t)^{\frac{1}{\gamma}}} \int_{d_1}^{d_2} e^{((-r-\frac{1}{2}\lambda^2)(T-t)-\lambda\sqrt{T-t})(1-\frac{1}{\gamma})} e^{-\frac{1}{2}z^2} dz \end{aligned}$$

$$\begin{aligned} &= \frac{e^{(-r-\frac{1}{2}\lambda^2)(T-t)}}{\sqrt{2\pi}} \int_{-\infty}^{d_1} \theta_1 e^{-\frac{1}{2}(z+\lambda\sqrt{T-t})^2} e^{\frac{1}{2}\lambda^2(T-t)} dz \\ &+ \frac{e^{(-r-\frac{1}{2}\lambda^2)(T-t)}}{\sqrt{2\pi}} \int_{d_2}^{\infty} \theta_2 e^{-\frac{1}{2}(z+\lambda\sqrt{T-t})^2} e^{\frac{1}{2}\lambda^2(T-t)} dz \\ &+ \frac{e^{(-r-\frac{1}{2}\lambda^2)(T-t)(1-\frac{1}{\gamma})}}{\sqrt{2\pi}(y\eta_t)^{\frac{1}{\gamma}}} \int_{d_1}^{d_2} e^{-\lambda\sqrt{T-t}(1-\frac{1}{\gamma})z} e^{-\frac{1}{2}z^2} dz \end{aligned}$$

$$\begin{aligned} &= e^{-r(T-t)} \left(\theta_1 \Phi(d_1 + \lambda\sqrt{T-t}) + \theta_2 \Phi(-d_2 - \lambda\sqrt{T-t})\right) \\ &+ \frac{e^{(-r-\frac{1}{2}\lambda^2)(T-t)(1-\frac{1}{\gamma})}}{\sqrt{2\pi}(y\eta_t)^{\frac{1}{\gamma}}} \int_{d_1}^{d_2} e^{-\frac{1}{2}(z+\lambda\sqrt{T-t}(1-\frac{1}{\gamma}))^2} e^{\frac{1}{2}\lambda^2(T-t)(1-\frac{1}{\gamma})^2} dz \end{aligned}$$

$$\begin{aligned} &= e^{-r(T-t)} \left(\theta_1 \Phi(d_1 + \lambda\sqrt{T-t}) + \theta_2 \Phi(-d_2 - \lambda\sqrt{T-t})\right) \\ &+ \frac{e^{(-r-\frac{1}{2}\lambda^2\frac{1}{\gamma})(T-t)(1-\frac{1}{\gamma})}}{\sqrt{2\pi}(y\eta_t)^{\frac{1}{\gamma}}} \int_{d_1}^{d_2} e^{-\frac{1}{2}(z+\lambda\sqrt{T-t}(1-\frac{1}{\gamma}))^2} dz. \end{aligned}$$

Let $\Gamma(t) = e^{(-r-\frac{1}{2}\lambda^2\frac{1}{\gamma})(T-t)(1-\frac{1}{\gamma})}$, then the utility-based collar option value is given by

$$\begin{aligned} V_t &= e^{-r(T-t)} \left(\theta_1 \Phi(d_1 + \lambda\sqrt{T-t}) + \theta_2 \Phi(-d_2 - \lambda\sqrt{T-t})\right) \\ &+ \frac{e^{\Gamma(t)}}{(y\eta_t)^{\frac{1}{\gamma}}} \left(\Phi(d_2 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) - \Phi(d_1 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma}))\right). \end{aligned}$$

B.2 Optimal investment

The wealth process of the replicating portfolio is given by

$$dV_t = (r + \phi_t(\mu - r))V_t dt + \sigma\alpha_t V_t dZ_t.$$

where ϕ_t denotes the portfolio process. Applying Itô's lemma to the option value derived in appendix B.1 yields

$$dV_t = \frac{\partial V_t}{\partial t}dt + \frac{\partial V_t}{\partial \eta_t}d\eta_t + \frac{1}{2}\frac{\partial^2 V_t}{\partial \eta_t^2}d[\eta_t, \eta_t].$$

Hence, it holds that $\sigma\phi_t V_t = -\lambda\eta_t \frac{\partial V_t}{\partial \eta_t}$. To determine the optimal portfolio process ϕ_t, we first determine

$$\begin{aligned}\frac{\partial V_t}{\partial \eta_t} &= \frac{e^{-r(T-t)}}{\eta_t\lambda\sqrt{T-t}}\left(\theta_1\Phi'(d_1 + \lambda\sqrt{T-t}) - \theta_2\Phi'(d_2 + \lambda\sqrt{T-t})\right) \\ &\quad - \frac{\frac{1}{\gamma}ye^{\Gamma(t)}}{(y\eta_t)^{\frac{\gamma+1}{\gamma}}}\left(\Phi(d_2 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) - \Phi(d_1 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma}))\right) \\ &\quad + \frac{e^{\Gamma(t)}}{\eta_t\lambda\sqrt{T-t}(y\eta_t)^{\frac{1}{\gamma}}}\left(\Phi'(d_2 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) - \Phi'(d_1 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma}))\right).\end{aligned}$$

Subsequently, we use that $\frac{e^{\Gamma(t)}}{(y\eta_t)^{\frac{1}{\gamma}}}\Phi'(d_i + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) = \theta_i e^{-r(T-t)}\Phi'(d_i + \lambda\sqrt{T-t})$ for $i \in \{1, 2\}$ and therefore it holds that

$$\frac{\partial V_t}{\partial \eta_t} = -\frac{\frac{1}{\gamma}ye^{\Gamma(t)}}{(y\eta_t)^{\frac{\gamma+1}{\gamma}}}\left(\Phi(d_2 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) - \Phi(d_1 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma}))\right).$$

Concluding, the portfolio process is defined by

$$\phi_t = \frac{\mu - r}{\gamma\sigma^2} \cdot \frac{e^{\Gamma(t)}}{V_t(y\eta_t)^{\frac{1}{\gamma}}}\left(\Phi(d_2 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) - \Phi(d_1 + \lambda\sqrt{T-t}(1-\frac{1}{\gamma}))\right).$$

Finally, we prove the equation used above. First, note that[13]

$$\begin{aligned}&\left(d_i + \lambda\sqrt{T-t}\right)^2 - \left(d_i + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})\right)^2 \\ &\qquad = \left(\frac{1}{\gamma}\lambda\sqrt{T-t}\right)\left(2d_i + 2\lambda\sqrt{T-t} - \frac{1}{\gamma}\lambda\sqrt{T-t}\right) \\ &\qquad = \frac{2}{\gamma}\log(y\eta_t) + 2\log(\theta_i) - \frac{2}{\gamma}(r - \frac{1}{2}\lambda^2)(T-t) - \frac{1}{\gamma^2}\lambda^2(T-t).\end{aligned}$$

Second, we use $\Phi'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$ to complete the proof. The final steps are given by

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\left(d_i+\lambda\sqrt{T-t}\right)^2 - \left(d_i+\lambda\sqrt{T-t}(1-\frac{1}{\gamma})\right)^2\right)} = \frac{1}{\sqrt{2\pi}}\frac{\Phi'(d_i + \lambda\sqrt{T-t})}{\Phi'(d_i + \lambda\sqrt{T-t}(1-\frac{1}{\gamma}))}$$

$$\Leftrightarrow$$

$$e^{-\frac{1}{\gamma}\log(y\eta_t) - \log(\theta_i) + \frac{1}{\gamma}(r - \frac{1}{2}\lambda^2)(T-t) + \frac{1}{2}\frac{1}{\gamma^2}\lambda^2(T-t)} = \frac{\Phi'(d_i + \lambda\sqrt{T-t})}{\Phi'(d_i + \lambda\sqrt{T-t}(1-\frac{1}{\gamma}))}$$

$$\Leftrightarrow$$

[13] Note: $x^2 - y^2 = (x-y)(x+y)$.

$$\frac{1}{(y\eta_t)^{\frac{1}{\gamma}}} e^{\frac{1}{\gamma}(r-\frac{1}{2}\lambda^2)(T-t)+\frac{1}{2}\frac{1}{\gamma^2}\lambda^2(T-t)} \Phi'(d_i + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) = \theta_i \Phi'(d_i + \lambda\sqrt{T-t})$$

$$\Leftrightarrow$$

$$\frac{1}{(y\eta_t)^{\frac{1}{\gamma}}} e^{(-r-\frac{1}{2}\lambda^2\frac{1}{\gamma})(T-t)(1-\frac{1}{\gamma})} \Phi'(d_i + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) = \theta_i e^{-r(T-t)} \Phi'(d_i + \lambda\sqrt{T-t})$$

$$\Leftrightarrow$$

$$\frac{e^{\Gamma(t)}}{(y\eta_t)^{\frac{1}{\gamma}}} \Phi'(d_i + \lambda\sqrt{T-t}(1-\frac{1}{\gamma})) = \theta_i e^{-r(T-t)} \Phi'(d_i + \lambda\sqrt{T-t}).$$

C Benchmark collar option

C.1 Parameters

Guarantee

Assume that the guarantee in the Dutch system of supplementary pensions is defined by nominal accrual only. Indexation is not incorporated. Further, assume a constant nominal pension accrual rate of 2% per year and annual inflation equal to 2%. In this case, the price of a nominal annuity paying the guaranteed benefit is equal to the price of a real annuity that pays 46% of a participant's wage after retirement. Again, wages are assumed to be constant in real terms. Rounded upwards, the guarantee of the benchmark collar option is chosen equal to 50%.

Ambition

Assume that the ambition in the Dutch system is defined by a fully indexed pension benefit. Given a working period of 40 years, a fully indexed pension benefit corresponds to 80% of a participant's wage. Therefore, the ambition of the benchmark collar option is chosen equal to 80%.

Probability of realizing the guarantee

There is no direct link with the Dutch system. The 2.5% quantile is assumed to be an acceptable quantile to describe downside risk. Therefore, the lower strike is determined such that the probability of realizing the guarantee is equal to 2.5%.

Probability of realizing the ambition

The report Evaluation Financial Assessment Framework specifies the consistency requirement that the indexation quality has to larger or equal to 70% of the ambition.[14] Therefore, the upper strike price is determined such that the probability of realizing the ambition is equal to 70%.

C.2 Additional trade-offs

Cap and upper strike

Figure 27 illustrates the trade-off between the ambition level and the probability of realizing the ambition in terms of the option payoff scheme. In addition, the trade-off curves are depicted for a range of contribution rates, which are displayed next to the trade-off curves.

[14] The report Evaluation Financial Assessment Framework is attached to Dutch policy file 30413 nr. 142, released 9 April 2010.

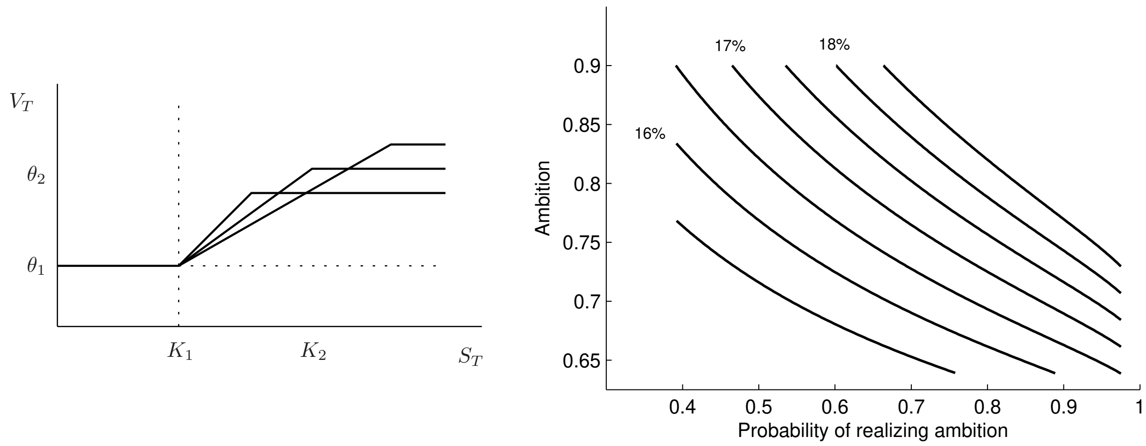

Figure 27: Trade-off between ambition and upper strike.

Floor and lower strike

Figure 28 illustrates the trade-off between the ambition level and the probability of realizing the ambition in terms of the option payoff scheme. In addition, the trade-off curves are depicted for a range of contribution rates, which are displayed next to the trade-off curves.

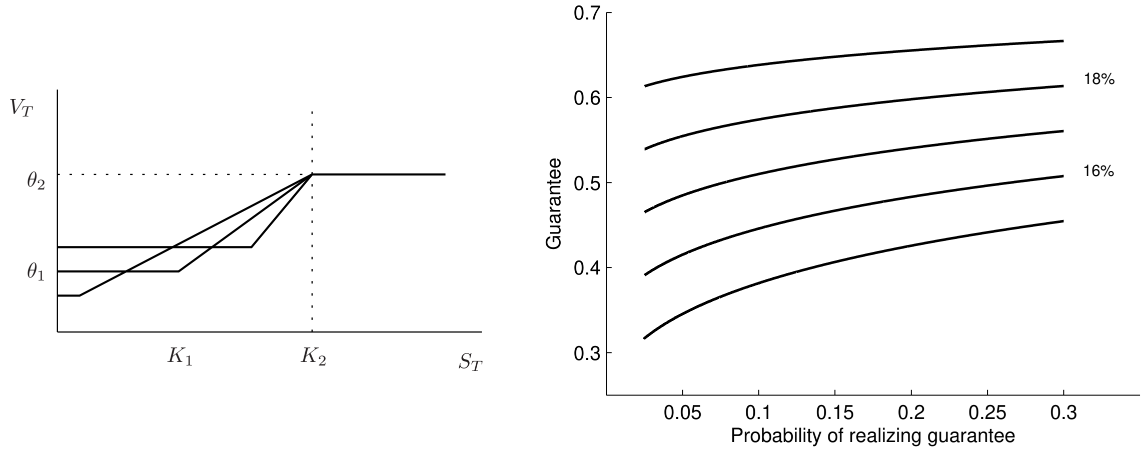

Figure 28: Trade-off between guarantee and lower strike.

Cap and lower strike

Figure 29 illustrates the trade-off between the ambition and the probability of realizing the guarantee in terms of the option payoff scheme. In addition, the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves.

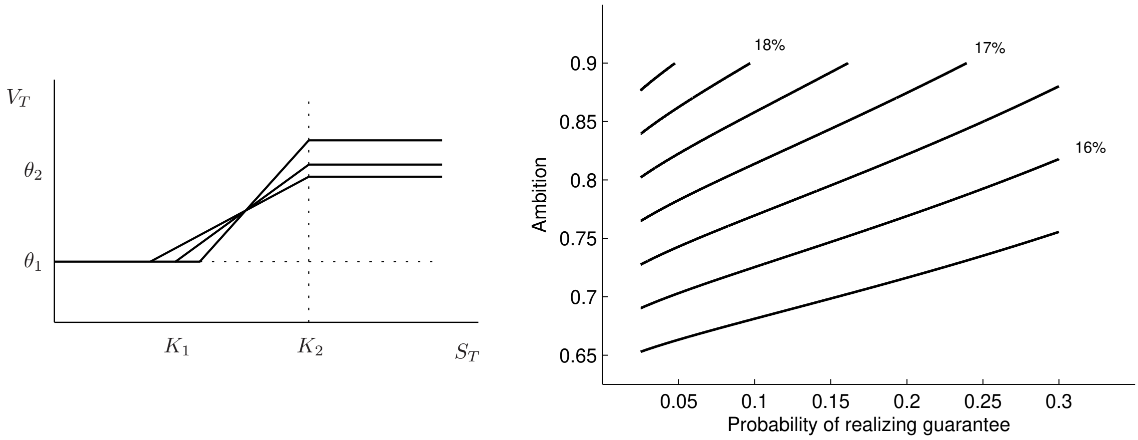

Figure 29: Trade-off between ambition and lower strike.

Floor and upper strike

Figure 30 illustrates the trade-off between the guarantee and the probability of realizing the ambition in terms of the option payoff scheme. In addition, the trade-off is depicted for a range of contribution rates, which are displayed next to the trade-off curves.

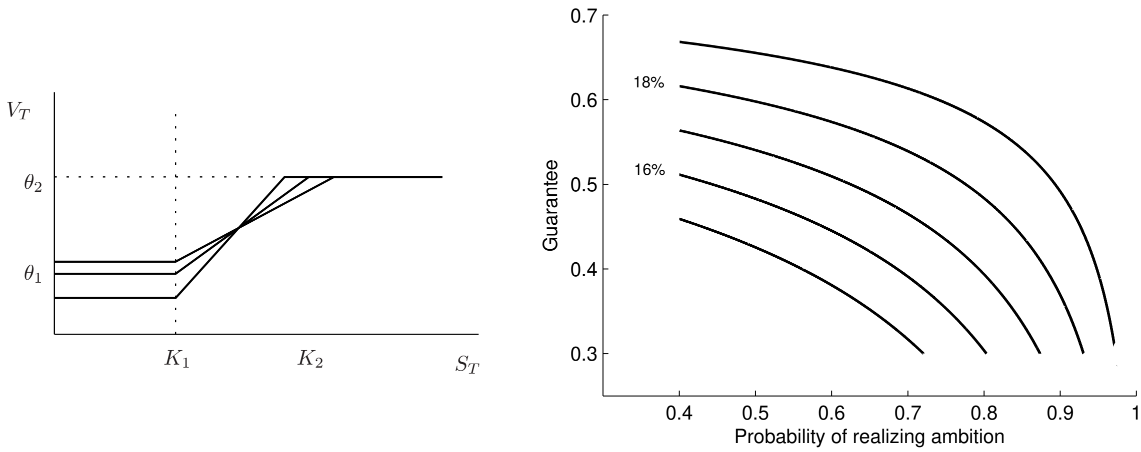

Figure 30: Trade-off between guarantee and upper strike.