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Longevity risk in portfolios of pension annuities☆

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Abstract

We analyze the importance of longevity risk for the solvency of portfolios of pension annuities. We distinguish two types of mortality risk. Micro-longevity risk quantifies the risk related to uncertainty in the time of death if survival probabilities are known with certainty, while macro-longevity risk is due to uncertain future survival probabilities. We use a generalized two-factor Lee–Carter mortality model to produce forecasts of future mortality rates, and to assess the relative importance of micro- and macro-longevity risk for funding ratio uncertainty. The results show that if financial market risk is fully hedged so that uncertainty in future lifetime is the only source of uncertainty, pension funds are exposed to a substantial amount of risk. Systematic and non-systematic deviations from expected survival imply that, depending on the size of the portfolio, buffers that reduce the probability of underfunding to 2.5% at a 5-year horizon have to be of the order of magnitude of 7% to 8% of the initial value of the liabilities.

JEL classification: G23; J11; E47

Keywords: Longevity risk; Mortality projection; Pension fund; Funding ratio; Solvency buffer

1. Introduction

The current EU solvency margin requires holding 4% of life insurance ‘mathematical’ reserves as solvency capital.[1] This requirement puts some insurers at a competitive disadvantage as they have more capital locked in than the risk profile of the company would imply. This situation will change with the new solvency regulation. Risk-based solvency requirements will be introduced within Pillar 2 of Solvency II. The new supervisory principles suggested by the authorities allow more room for internal models in assessing the financial situation of insurance companies. Companies either use the capital requirements laid down by the supervisory authorities, or they are replaced by capital requirements (e.g. target capital) resulting from their own risk modeling. Target capital is risk-based and grounded in a market consistent assessment.

Since the nature of the old-age pension is very close to the life insurance business, the same phenomenon can be observed in pension fund regulation as well. Some regulators are beginning to take a more sophisticated approach to evaluate the risk profile of pension funds. The Netherlands, UK and Switzerland (and probably other countries as well) already took steps to introduce risk-based capital or funding requirements to the pension system, which is in line with the Solvency II of the EU. According to the proposal of the Dutch regulator, a capital adequacy test must be performed for three different time horizons. The minimum test ensures that the accrued benefits are covered/funded by sufficient assets in the case of immediate discontinuance. The solvency test checks the funding requirement in a 1-year horizon. The continuity test assesses the fund’s long term prospects.

Performing a capital adequacy test for a maturity of 1 year or for a longer horizon requires aggregate risk for the portfolio

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[1] For more details, see FOPI (2004).

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of the insurance company or pension fund to be evaluated. Several sources of risk constitute to the overall risk. Stock market risk typically influences the asset side of the portfolio, while interest rate risk influences both assets and liabilities through discounting future cash flows. The focus in this paper, however, is on the risk that results from uncertainty in the annuitant's remaining lifetime. We distinguish *micro-longevity risk*, which results from non-systematic deviations from an individual's expected remaining lifetime, and *macro-longevity risk*, which results from the fact that survival probabilities change over time.[2] In the past century survival probabilities have indeed increased for each age group, accompanied by the following three phenomena: (i) increasing concentration of deaths around the mode at adult ages, which is denoted as the "rectangularization" of the survival function, (ii) increasing mode at adult ages of the death curve, which implies the "expansion" of the survival function, and, (iii) higher level and a larger dispersion of death at young ages (for more details see e.g. Pitacco (2004), Olivieri (2001)). The change of mortality experience clearly has a direct effect on the expected lifetime of people. Expected lifetime[3] at birth for men in the Netherlands increased from 47 years to almost 76 years in the last 100 years. For women the life expectancy at birth was 50 years at the beginning of the 20th century, while it increased above 80 years in 2000. Life expectancies for other than newly born age groups also increased. The increase in the past 100 years for men with the age of 65 was almost 4.5 years, while for the 65-year-old women it was nearly 8 years.

The goal of this paper is twofold. First, as discussed above, solvency and continuity regulations typically require that the probability of underfunding in a given time horizon is sufficiently low. A pension fund would be underfunded if its funding ratio (the ratio of the market value of the assets to the market value of the liabilities) is below one. Therefore, we first investigate the extent to which micro- and macro-longevity risk affect the probability distribution of the future funding ratio of a portfolio of annuities.

Studies by Olivieri (2001), Coppola et al. (2000, 2002, 2003), Di Lorenzo and Sibillo (2002) assume a finite number of scenarios for the evolution of future survival probabilities. They find that the micro-longevity risk for an annuity portfolio (measured by the variance of the payoff) becomes unimportant when the size of the portfolio becomes large. In contrast, the relative size of macro-longevity risk is independent of portfolio size. Cossette et al. (2005) use a Poisson log-bilinear model estimated on Canadian data to forecast future death rates, and use a relational model to produce a dynamic mortality table for a specific pension fund. They then use these forecasts to study the impact of mortality changes on the expected remaining lifetime, the price of an annuity, and the probability of ruin for a pension fund.

Our goal is to quantify the uncertainty in a pension fund's funding ratio caused by longevity risk, distinguishing explicitly micro- and macro-longevity risk, and incorporating parameter risk. We use a generalized two-factor Lee–Carter model estimated on Dutch data to forecast the probability distribution of future mortality. Specifically, the 1-year difference in the age-specific log mortality rate is modeled as the sum of two age-specific coefficients multiplied by latent time-dependent factors. The latent factors capture the common movements among mortality rates over time. In order to capture particular age-specific influences that are not properly accounted for by the model, an additional error term, which is time- and age-specific, is added. Several extensions and modifications to the methodology originally developed by Lee and Carter (1992) have been proposed, e.g. Renshaw and Haberman (2003) and Brouhns et al. (2002, 2005).

In many empirical applications the Lee–Carter approach results in a model that describes the log central death rate by means of a linear trend, where different age groups have different trends. However, due to the volatility in mortality data, the estimation of these trends, and, thus, the forecasts based on them, are rather sensitive to the sample period employed. Hári et al. (in press) allows for time-varying trends, depending on a few underlying factors, to make the estimates of the future trends less sensitive to the sampling period. They formulate the model in a state-space framework, and use the Kalman filtering technique to estimate it.

We use the model estimated in Hári et al. (in press) to simulate the distribution of future mortality rates, including micro-longevity, macro-longevity and parameter risk, and determine characteristics of the probability distribution of the funding ratio in the future. In order to quantify the effect of portfolio size, we consider portfolios of different sizes with identical annuitants. In addition, we consider portfolios of different sizes for which the age and gender composition reflects that of the Dutch population beginning of 2004. We first investigate the case where financial market risk is fully hedged, so that funding ratio uncertainty is exclusively due to longevity risk. We show that uncertainty in future survival can significantly affect the probability distribution of the funding ratio. While micro-longevity risk becomes negligible for large portfolios, macro-longevity and parameter risk remain substantial. Next, we allow for financial market risk. We find that the relative importance of longevity risk for funding ratio uncertainty becomes smaller when financial market risk becomes more important, e.g. when the fund invests a larger fraction in risky assets.

Our second goal is to investigate possibilities to enhance solvency in the presence of micro- and macro-longevity risk. A common method to enhance solvency of a pension fund or insurance company is to keep a fraction of its assets in a buffer. Olivieri and Pitacco (2003) calculate solvency requirements for life annuity portfolios and funded pension funds. Their analysis is based on a mortality model with a finite number of postulated scenarios for the evolution of death rates in the future. We use

[2] Whereas part of the literature uses the term *longevity* to refer exclusively to long duration of human life, we use the term to refer to its second definition according to Merriam–Webster (http://www.m-w.com), which is *length of life*. Thus, *longevity risk* relates to any uncertainty in future survival, regardless of whether it leads to longer or shorter than expected lifetime.

[3] The numbers in this section are based on period life tables and reflect the estimates in the Human Mortality Database, University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at http://www.mortality.org or http://www.humanmortality.de.

the probability distribution of future survival rates, estimated on Dutch data with a two-factor generalized Lee–Carter model, to determine the size of the buffer required to meet solvency requirements based on Value at Risk (VaR) and Expected Shortfall (ES), respectively. We distinguish the effects of micro-, macro-longevity, and parameter risk. If all these sources of uncertainty in the annuitant's remaining lifetime are taken into account and financial market risk is perfectly hedged, the initial funding ratio of a large pension fund has to be as high as 107.2% to reduce the probability of underfunding in a 5-year horizon to 2.5% (i.e. to obtain $\text{VaR}_{0.975} = 1$). In order for the Expected Shortfall to equal one in a 5-year horizon, the initial funding ratio has to be as high as 108.4%.

The paper is organized as follows. In Section 2, we describe the model that is used to forecast future mortality. In Section 3, we determine the effect of changes in mortality rates on the expected remaining lifetime of an individual, including uncertainty generated by parameter risk. In Section 4, we determine the effect of longevity risk on the market value of pension annuities. In Section 5, we assess the relative importance of micro-longevity, macro-longevity risk and parameter risk for funding ratio uncertainty. In order to focus on longevity risk only, the expected liabilities are considered to be cash flow matched, so that financial market risk is eliminated. In Section 6, we discuss several possibilities to enhance the solvency of a fund. Specifically, we determine the size of the buffer required to meet solvency requirements based on Value at Risk and Expected Shortfall, respectively. In Section 7, we allow financial market risk, and quantify the uncertainty in the funding ratio due to the combination of longevity risk and financial market risk.

2. Forecasting future mortality

Due to macro-longevity risk, future survival probabilities are uncertain. In this section we present a model to estimate and forecast time-dependent survival probabilities. The model can be seen as a generalization of the widely used mortality forecast model introduced by Lee and Carter (1992). Let us first introduce some notation.

Notation.
- $p_{x,t}$ denotes the probability at time t that a person with age x will survive at least one more year;
- $\mu_{x,t}$ denotes the *force of mortality*[4] of a person with age x at time t;
- $D_{x,t}$ denotes the observed number of deaths in year t in a cohort aged x at the beginning of year t;
- $E_{x,t}$ denotes the number of person years during year t in a cohort aged x at the beginning of year t, the so-called exposure.

We assume that for any integer age x, and any time t, it holds that:

$$\mu_{x+u,t} = \mu_{x,t}, \quad \text{for all } u \in [0, 1), \tag{1}$$

Then, one can verify that

$$p_{x,t} = \exp(-\mu_{x,t}). \tag{2}$$

A maximum likelihood estimator for the force of mortality is given by[5]:

$$\widehat{\mu}_{x,t} = \frac{D_{x,t}}{E_{x,t}}. \tag{3}$$

We use the model developed in Hári et al. (in press) to produce forecasts of $\frac{D_{x,t}}{E_{x,t}}$. Specifically, we assume that the differentiated time series $\log \frac{D_{x,t+1}}{E_{x,t+1}} - \log \frac{D_{x,t}}{E_{x,t}}$ is driven by nf latent factors, and modeled as the sum of an age-specific constant and the product of an age-specific coefficient vector and a vector containing the latent time-varying factors. In order to capture particular age-specific influences that are not properly accounted for by the model, an additional error term, which is time- and age-specific, is added.

In order to reduce the number of parameters to be estimated, we introduce na age groups. Let us denote $m_t = \left(\ln\left(\frac{D_{1,t}}{E_{1,t}}\right) \ \cdots \ \ln\left(\frac{D_{na,t}}{E_{na,t}}\right)\right)'$ for the vector of the log force of mortality for age groups $x \in \{1, \ldots, na\}$ at time t. Then the time-series evolution of m_t is modeled as

$$m_t - m_{t-1} = a + B'u_t + \xi_t + \Theta\xi_{t-1}, \tag{4}$$

$$u_t = \mu_u + \psi_t + \Xi\psi_{t-1}, \tag{5}$$

with u_t an nf-dimensional vector of underlying latent factors, driving the change in the force of mortality, where $a \in \mathbb{R}^{na}$ is the long-run trend, $B \in \mathbb{R}^{na\times nf}$ are the factor loadings, $\mu_u = E(u_t) \in \mathbb{R}^{nf}$, with $\Theta \in \mathbb{R}^{na\times na}$ a matrix with unknown parameters capturing the MA-effects in the force of mortalities, and $\Xi \in \mathbb{R}^{nf\times nf}$ a matrix with unknown parameters capturing the MA-effects in the underlying latent factors. The vectors of error terms ψ_t and ξ_t are white noise, satisfying the distributional assumption

$$\begin{pmatrix}\psi_t \\ \xi_t\end{pmatrix} | \mathcal{F}_{t-1} \sim \left(\begin{pmatrix}0 \\ 0\end{pmatrix}, \begin{pmatrix}\Sigma_\psi & 0 \\ 0 & \Sigma_\xi\end{pmatrix}\right) \tag{6}$$

with $\Sigma_\psi \in \mathbb{R}^{nf\times nf}$ and $\Sigma_\xi \in \mathbb{R}^{na\times na}$ the unknown covariance matrices of ψ_t and ξ_t, respectively, and $\mathcal{F}_{t-1}$ the information available up to time $t-1$. This model is the result of a selection procedure from a broader class of models.

We use yearly survival data for the Netherlands (NL) for men and women separately from 1850 to 2003, provided by the Human Mortality Database.[6] We create the following 18 age groups: 1–4, 5–9, 10–14, ... 80–84 and 85+. Since the database provides data starting at the middle of the 19th century, and the number of people in age groups above 85 (e.g. 85–89, or 90–94 etc.) is relatively low in that period, we merge all the age

[4] The force of mortality, at time t, of an individual with age x is defined as: $\mu_{x,t} = \lim_{\triangle t\to 0} \frac{P(0\leqslant T_{x,t}\leqslant \triangle t)}{\triangle t}$, where $T_{x,t}$ denotes the remaining lifetime of an individual with age x at time t; see also Gerber (1997).

[5] For more details on estimating the force of mortality by the exposure and the death number, see Gerber (1997).

[6] Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at http://www.mortality.org or http://www.humanmortality.de (data downloaded on 04.11.2005).

groups above 85, resulting in the 85+ category.[7] Moreover, the maximum attainable age is assumed to be 110.

The estimation results are presented in Appendix A. For a more detailed exposition of the model, the estimation technique, and the estimation results we refer the reader to Hári et al. (in press).

3. Uncertainty in expected remaining lifetime

Let us denote $T_{x,t}$ for the curtate remaining lifetime at time t of an individual with age x at time t, and ${}_\tau p_{x,t}$ for the probability that an x-year-old at time t will survive at least another τ years, i.e.

$$ {}_\tau p_{x,t} = p_{x,t} \cdot p_{x+1,t+1} \cdot \dots \cdot p_{x+\tau-1,t+\tau-1}. \tag{7}$$

Moreover, we denote $1_{(T_{x,t} \geqslant \tau)}$ for the indicator random variable that indicates whether an individual with age x at time t will survive at least τ more years, i.e.

$$\begin{aligned} 1_{(T_{x,t} \geqslant \tau)} &= 1 \quad \text{if } T_{x,t} \geqslant \tau, \\ &= 0 \quad \text{if } T_{x,t} < \tau. \end{aligned}$$

Then, conditional on survival rates up to period t, the expected curtate remaining lifetime at time t of an individual aged x at time t equals:

$$\begin{aligned} \mathbb{E}_t\left[T_{x,t}\right] &= \sum_{\tau=1}^{110-x} \mathbb{E}_t\left[1_{(T_{x,t} \geqslant \tau)}\right] && (8)\\ &= \sum_{\tau=1}^{110-x} \mathbb{E}_t\left[{}_\tau p_{x,t}\right] && (9)\\ &= \sum_{\tau=1}^{110-x} \mathbb{E}_t\left[\exp\left(-\sum_{s=0}^{\tau-1} \mu_{x+s,t+s}\right)\right] && (10)\\ &\cong \sum_{\tau=1}^{110-x} \mathbb{E}_t\left[\exp\left(-\sum_{s=0}^{\tau-1} \frac{D_{x+s,t+s}}{E_{x+s,t+s}}\right)\right], && (11) \end{aligned}$$

where (9) follows from the law of iterated expectations, and (11) results from the fact the force of mortality is approximated by the ratio of death numbers and exposures.

In order to illustrate the size of improvements in life expectancy, in Table 1 we first determine the expected remaining lifetime at the age of 25, 45 and 65 for men and women, for selected historical years under the assumption that there is no further improvement in mortality, i.e. $\mu_{x+s,t+s} = \mu_{x+s,t}$, so that for $t \leq 2004$,

$$\mathbb{E}_t[{}_\tau p_{x,t}] = \exp\left(-\sum_{s=0}^{\tau-1} \mu_{x+s,t}\right). \tag{12}$$

Next, we calculate life expectancy when improvements in survival rates are taken into account. For certain age cohorts, e.g. the 25-year-old in 1900, all relevant death numbers and exposures have been observed, so that there is no randomness with respect to death rates, and life expectancy can be readily calculated. For most cohorts, however, the maximum attainable age has not been reached in 2004,[8] so that death numbers and exposures for time periods beyond 2003 need to be forecasted. We use (4) and (5) to forecast the ratios of death numbers and exposures, and incorporate macro-longevity risk by allowing for randomness in the error terms ξ_t and ψ_t. Moreover, uncertainty in the estimated parameters in (4) and (5) creates an additional source of risk, which we refer to as *parameter risk*.

Table 1
Expected remaining lifetime based on period life tables

Gender	Year	Age		
		25	45	65
Men	1900	39.6	24.0	10.5
	1925	45.3	27.6	12.3
	1950	48.4	29.7	13.6
	1975	47.7	28.7	13.0
	2000	51.2	32.0	14.9
Women	1900	40.6	25.3	11.0
	1925	45.4	28.2	12.8
	1950	49.8	30.9	14.1
	1975	53.5	34.2	16.7
	2000	56.0	36.6	18.9

The table shows the expected remaining lifetime at the age of 25, 45 and 65 for men and women, for selected historical years under the assumption that there is no further improvement in mortality.

Table 2 presents the expected remaining lifetime with the 95% confidence interval for parameter risk for historical time periods from 1900 to 2000. Since expected remaining lifetime in 2025 is currently a random variable,[9] we present its expectation as well as two prediction intervals. The narrower represents the 95% prediction interval in absence of parameter risk, while the wider includes parameter risk.

From Table 1 we see that if there had been no improvement in life expectancy after 1975, the expected remaining lifetime of a 25-year-old man would have been 47.7 years. However, if improvement is taken into account the same cohort has a life expectancy of 51.9 years, with an upper bound of 53.3 years for parameter risk (Table 2). The forecasted expected remaining lifetime for a 25-year-old man in 2025 equals 54.6, with an upper bound of 57.5 without parameter risk and 64.3 with parameter risk. These results show that methods based on period life tables seriously underestimate life expectancy. Moreover, parameter risk creates substantial uncertainty.

4. Effect of longevity on market value of annuities

We consider the market value of an annuity that guarantees a nominal yearly payment of 1, starting at the end of the year in which the annuitant reaches the age of 65, with a last payment in the year he dies. We assume that mortality risk and financial

[7] Alternatively, assumptions on old-age mortality could be imposed (see e.g. Coale and Guo (1989)).

[8] The highest attainable age is assumed to be 110.

[9] Note that for given values of the estimated parameters in (4) and (5), i.e. in absence of parameter risk, the expected remaining lifetime conditional on death rates observed up to time t as defined in (8) is a deterministic number for periods $t \leq 2004$, but a random variable for time periods beyond 2004.

Table 2
Expected remaining lifetime based on cohort life tables

Gender	Year	Age 25	Age 45	Age 65
Men	1900	44.3	25.9	11.2
	1925	46.4 (46.4;46.4)	28.8	12.1
	1950	49.2 (49.1;49.3)	29.0 (29.0;29.0)	13.9
	1975	51.9 (50.7;53.3)	31.0 (30.8;31.3)	13.4 (13.4;13.4)
	2000	53.3 (50.0;57.2)	33.1 (31.5;35.1)	15.4 (14.9;15.8)
	2025	54.6 (51.3;57.5) (39.2;64.3)	34.3 (31.6;37.0) (25.8;41.8)	16.1 (14.1;18.1) (11.1;21.2)
Women	1900	44.7	27.0	11.8
	1925	51.1 (51.1;51.1)	29.7	12.4
	1950	55.3 (55.1;55.7)	34.8 (34.8;34.8)	15.2
	1975	57.2 (55.6;59.5)	36.6 (36.1;37.2)	18.2 (18.2;18.2)
	2000	58.9 (55.4;63.7)	38.2 (36.2;41.0)	19.4 (18.7;20.4)
	2025	60.6 (57.8;63.3) (52.2;70.1)	39.8 (37.2;42.4) (32.7;47.4)	20.6 (18.5;22.8) (15.8;26.0)

The table presents the expected remaining lifetime with the 95% confidence interval for parameter risk for historical time periods from 1900 to 2000. For the expected remaining lifetime in 2025, we present two intervals. The narrower represents the 95% prediction interval of the expected life expectancy due to the fact that the future conditional expectation is a random variable, while the wider includes parameter risk.

market risk are independent under the risk-neutral measure,[10] and that the price of longevity risk is zero.[11] We denote $T_x = T_{x,0}$ for the current remaining lifetime of an individual with age x. Then, the current market value of an annuity for an x-year-old equals:

$$a_x = \sum_{\tau=\max\{65-x,0\}}^{110-x} \mathbb{E}[1_{(T_x \geqslant \tau)}] P_0^{(\tau+1)}, \tag{13}$$

where $\mathbb{E}[1_{(T_x \geqslant \tau)}]$ denotes the expected value at $t = 0$ of one unit to be paid at time $\tau + 1$ if the annuitant is still alive, and $P_0^{(\tau+1)}$ denotes the current market value of one unit to be paid at time $\tau+1$, i.e. the market value of a zero-coupon bond maturing at time $\tau + 1$.

It follows from (11) that:

$$\mathbb{E}[1_{(T_x \geqslant \tau)}] \cong \mathbb{E}\left[\exp\left(-\sum_{s=0}^{\tau-1} \frac{D_{x+s,s}}{E_{x+s,s}}\right)\right]. \tag{14}$$

Now, (14) can be simulated by means of (4) and (5). To determine the market value of the annuity, it now only remains to specify the term structure of interest rates at $t = 0$. We will use the term structure of interest rates implied by the model presented in Section 5.2.

Table 3 shows the market value of the annuity, as a function of age, for ages varying from 25 to 85 based on period tables (first column), and based on forecasted mortality rates (second column), for men and for women. When forecasts are made, the uncertainty in future mortality rates is taken into account (i.e., we allow for randomness in (4) and (5)). Note that the market value of the annuity increases with age until the age of 65, and starts to decrease after that. This is due to the fact that for individuals that are not yet retired, the probability that they will reach retirement increases when they get older. Moreover, discounting plays a more important role for the young. Once a person has reached retirement age, the market value of the remaining pension payments obviously decreases with age. In comparing the first and the second columns, we see that the market value of the annuity based on period life tables underestimates,[12] the annuity value based on forecasted death probabilities by 7.7% for a 25-year-old man and 8.8% for a 25-year-old woman. For the 65-year-old, the corresponding numbers are 0.4% and 1.7%, respectively.

Table 3
Market value of annuities

Age	Men		Women	
	Period table	Projected table	Period table	Projected table
25	0.872	0.944	1.038	1.139
30	1.193	1.279	1.418	1.541
35	1.633	1.733	1.939	2.086
40	2.238	2.350	2.654	2.827
45	3.079	3.198	3.643	3.840
50	4.255	4.373	5.023	5.240
55	5.918	6.022	6.950	7.177
60	8.279	8.356	9.606	9.831
65	10.403	10.441	11.969	12.179
70	8.669	8.677	10.333	10.508
75	6.897	6.881	8.490	8.617
80	5.191	5.151	6.535	6.593
85	3.723	3.675	4.643	4.680

The table shows the market value of the annuity, as a function of age, based on period tables (first column), and based on forecasted mortality rates (second column), for men and for women.

[10] There might be some correlation between mortality and financial market factors, however we think it is negligible.

[11] This assumption is quite common in the literature. See e.g. Schrager (2006).

[12] There are some exceptions for elderly men. Due to the MA structure in (4) and (5) the predicted level of log mortality for elderly men in 2004 is higher than the level estimated for 2003, at the end of the sample period. The level correction, the fact that discounting amortizes the effect of longer term mortality improvement, and the relatively short time horizon for mortality improvement for the elderly yield a lower annuity value with projected life tables, than with period life tables.

5. Effect of longevity risk on funding ratio uncertainty

In this section we investigate the effect of micro- and macro-longevity risk on the probability distribution of the funding ratio in the future. The funding ratio at time $T (FR_T)$ is defined as the market value of the assets at time $T (A_T)$ divided by the market value of the liabilities at time T (L_T), and can be seen as a measure of solvency. Formally, FR_T is defined as follows:

$$FR_T = \frac{A_T}{L_T}. \tag{15}$$

In the sequel we quantify the effect of micro-longevity, macro-longevity, and parameter risk on the probability distribution of the funding ratio at a given time horizon T. In Section 5.1, we first describe the characteristics of the fund. Then, in Section 5.2, we present the model for the pricing kernel, which is needed to determine and forecast the market value of assets, and, specifically, the term structure of interest rates. In Section 5.3 we use the term structure model to simulate the market value of assets and liabilities, and to determine characteristics of the corresponding funding ratio distribution.

5.1. Fund characteristics

We consider two types of annuity funds: (i) an annuity fund consisting of 65-year-old who are about to annuitize their wealth at retirement, and (ii) a representative fund, whose age and gender composition is the portrayal of the Dutch population at the beginning of 2004.[13] In both cases, we choose the retrospective approach, i.e. there are no new entrants into the fund, and no rights are built up or premiums are paid after time $t = 0$. Furthermore, we assume that the maximum attainable age is $x_T = 110$, that all participants enter at $x_0 = 25$ and retire when they become 65. Consequently they contribute to the fund for maximum 40 years. We consider a nominal defined benefit fund where the right built up by a policyholder increases linearly with the amount of time he/she spent contributing to the fund, i.e. an annuitant with age x at time $t = 0$ has built up the right to receive a yearly payment of

$$C_x = \min\left\{\frac{x - x_0}{40}, 1\right\} * Q \tag{16}$$

after retirement, where Q denotes the yearly nominal pension payment to a person who participated for 40 years in the fund.

5.2. Pricing kernel and term structure of interest rates

Assuming that mortality risk is not priced, we postulate, following Campbell et al. (1997), that the one-period nominal pricing kernel $(M^{\$}_{t+1})$ satisfies \$

$$-\log M^{\$}_{t+1} = \alpha + \delta r^{(1)}_t + \beta^{r^{(1)}} \varepsilon^{r^{(1)}}_{t+1}, \tag{17}$$

where $\alpha \in \mathbb{R}$, $\delta \in \mathbb{R}$, and $\beta^{r^{(1)}} \in \mathbb{R}$ are constants, where the 1-year rate $r^{(1)}_{t+1}$ follows a mean reverting process

$$r^{(1)}_{t+1} = \mu_r + \gamma(r^{(1)}_t - \mu_r) + \varepsilon^{r^{(1)}}_{t+1}, \tag{18}$$

where the mortality part is modeled by Eqs. (4) and (5), and where the error terms of the mortality model and of the 1-year-rate process satisfy

$$\begin{pmatrix} \varepsilon^{r^{(1)}}_{t+1} \\ \psi_{t+1} \\ \xi_{t+1} \end{pmatrix} |\mathcal{F}_t \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{r^{(1)}_t} & 0 & 0 \\ 0 & \Sigma_\psi & 0 \\ 0 & 0 & \Sigma_\xi \end{pmatrix}\right). \tag{19}$$

Using now that the time t price of any nominal time $t+1$ payoff X_{t+1} can be obtained via

$$P_t = \mathbb{E}_t[M^{\$}_{t+1} X_{t+1}], \tag{20}$$

this implies that the time t zero-coupon bond price with time-to-maturity τ, $P^{(\tau)}_t$, is exponentially affine in the short rate $r^{(1)}_t$, i.e.

$$P^{(\tau)}_t = \mathbb{E}_t[M^{\$}_{t+1} \times \cdots \times M^{\$}_{t+\tau}] = \exp(-A_\tau - B_\tau r^{(1)}_t), \tag{21}$$

where A_τ and B_τ are constants that can easily be determined recursively from the underlying model parameters (see Campbell et al. (1997)).

We observe[14] the Dutch 1-year euro (previously guilder) interest rate swap middle rate between 1975 and 2004 on a yearly frequency, which is used to proxy the 1-year zero-coupon yield.[15] By using the 1-year rate as the factor which drives the term structure and observing the 10-year yield with error[16] (proxied by the 10-year benchmark yield observed between 1979 and 2004), the first order autoregressive parameter for the short rate is estimated to be 0.75 with a mean of 5.4% p.a. and a standard deviation of 1.8%. The model implies a term premium of 1.2% on the 50-year bond. Since the model-implied long rates at the beginning of 2004 are below the observed long rates, and correct representation of the long end of the term structure is important in case of pension fund valuation, we calibrate the model on the 10-year rate instead of the 1-year rate. This proceeds as follows:

1. We use an equivalent representation of the term structure model using a rotation of the underlying factor, where the nominal 10-year yield $(r^{(10)}_t)$ replaces the role of the 1-year interest rate $(r^{(1)}_t)$. The 10-year rate then follows a mean reverting process[17]

$$r^{(10)}_{t+1} = \mu_{r^{(10)}} + \gamma^{r^{(10)}}(r^{(10)}_t - \mu_{r^{(10)}}) + \varepsilon^{r^{(10)}}_{t+1}, \tag{22}$$

[13] CBS Netherlands, see Appendix B for more details.

[14] The source of data for all interest rate related time-series is Datastream.

[15] The zero-coupon yield data is available for the period starting only from year 1997, which is very short to estimate its time-series properties. The euro/guilder interest rate swap market might contain some counterparty risk, however, the depth and the quality of the market in London is likely to make the counterparty risk limited. The comparison of the zero-yield with the swap rate in the period between 1997 and 2004 yielded a deviation of at most 0.1% point, also suggesting that the swap rate is likely to be a good proxy.

[16] For more details, see Ang and Piazzesi (2003).

[17] Due to the unavailability of sufficient 10-year zero-coupon bond data, we were not able to estimate the dynamics of the 10-year yield directly. The

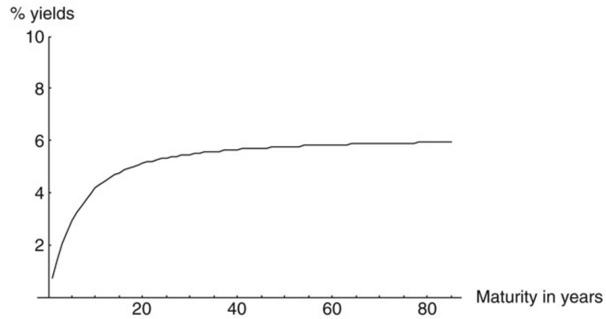

Fig. 1. The term structure of interest rates, January 2004. The figure shows the term structure of interest rates in January 2004, if the 10-year yield is 4.2% p.a.

with parameters $\mu_{r^{(10)}}$, $\gamma^{r^{(10)}}$, $\varepsilon_{t+1}^{r^{(10)}} \sim N(0, \sigma^2_{r_t^{(10)}})$, which can be uniquely determined from the parameters of the term structure model driven by $r_t^{(1)}$. Consequently, the financial market (and the term structure of interest rates) is now driven by the 10-year nominal yield ($r_t^{(10)}$):

$$P_t^{(\tau)} = \exp(-A_\tau^{(10)} - B_\tau^{(10)} r_t^{(10)}). \tag{23}$$

2. In order to fit the observed 10-year nominal yield perfectly, we recalculate α and δ in (17) such that the model driven by the 10-year rate yields an identity for the 10-year yield in (23), e.g. $A_{10}^{(10)} = 0$ and $B_{10}^{(10)} = 10$. The re-estimation yields $A_\tau^{(10)}$ and $B_\tau^{(10)}$ for all τ.

The term structure of interest rates for January 2004 is illustrated in Fig. 1.

5.3. *The funding ratio distribution*

In this section, we quantify the effect of micro- and macro-longevity risk on the probability distribution of the funding ratio at a given time horizon.

First, in order to focus exclusively on longevity risk, we eliminate financial market risk by assuming that the expected liabilities are hedged with cash flow matching; i.e. the initial asset portfolio consists of zero-coupon bonds paying out the initial expected value of future liabilities. Due to non-systematic and systematic deviations in mortality, the realized pension benefits typically deviate from the expectation (i.e. the payoff of the zero-coupon bonds). We assume that the surplus is reinvested in, and the deficit is financed with, 1-year zero-coupon bonds. The value of the assets at time $t+1$ therefore equals the value of the bond portfolio at time $t+1$, minus realized pension payments at the end of period t, i.e.

$$A_{t+1} = A_t(1 + R_{t+1}) - \sum_{i: x_i + t \geqslant 65} 1_{(T_{x_i} \geqslant t)} C_{x_i}, \tag{24}$$

where R_{t+1} denotes the return on the bond portfolio between time t and $t+1$. Since the asset portfolio consists of zero-coupon bonds with different maturities, the market value of the asset portfolio at time T can be simulated by means of the mortality forecast model in (4) and (5), and the term structure model in (21). To isolate the effect of longevity risk, we eliminate interest rate risk by letting the term structure of interest rates move deterministically to its long term average,[18] i.e. we set $\varepsilon_t^{r^{(10)}} = 0$, for all t.

The market value of the liabilities at time T is the sum of the present value of the future cash flow stream over all individuals who are still alive at time T. Let us denote I for the initial number of participants in the fund, x_i for the age at time $t = 0$ of participant i, and T_{x_i} for the current remaining lifetime of participant i, $i = 1, \dots, I$. We assume that mortality risk and financial market risk are independent, and that the price of longevity risk is zero. Then, the market value of the pension fund's liabilities at time T is currently a random variable given by:

$$L_T = \sum_{i=1}^{I} 1_{(T_{x_i} \geqslant T)} \sum_{\tau=\max\{65-(x_i+T),0\}}^{110-(x_i+T)} \mathbb{E}_T[{}_\tau p_{x_i+T,T}] P_T^{(\tau+1)} C_{x_i}, \tag{25}$$

where $1_{(T_{x_i} \geqslant T)}$ denotes the indicator random variable that is equal to one if participant i is still alive at time T, and zero otherwise, and $P_T^{(\tau+1)}$ denotes the market value, at time T, of a zero-coupon bond maturing at time $T + \tau + 1$. Simulation of the value of the liabilities at time T involves:

1. Simulation of death rates for all ages and for $t = 1, \dots, T$, using (4) and (5).
2. Simulation of $1_{(T_{x_i} \geqslant T)}$ for all participants, given the simulated death rates.
3. Determination of

$$\mathbb{E}_T[{}_\tau p_{x_i+T,T}] = \mathbb{E}_T \left[\exp\left(-\sum_{s=0}^{\tau-1} \mu_{x_i+T+s,T+s} \right) \right] \tag{26}$$

$$\cong \mathbb{E}_T \left[\exp\left(-\sum_{s=0}^{\tau-1} \frac{D_{x_i+T+s,T+s}}{E_{x_i+T+s,T+s}} \right) \right], \tag{27}$$

for every participant for which $1_{(T_{x_i} \geqslant T)} = 1$, and given the simulated death rates at time T. A closed form expression for (26) is not available. Because determination of (26) through simulation of the future value of the liabilities for every scenario generated in steps 1. and 2. is computationally intensive, we use a projection method introduced in the American option pricing literature; see e.g. Longstaff and Schwartz (2001). This method speeds up the calculations to a large extent.[19]

In the remainder of this section, we use the simulation procedure to determine characteristics of the probability distribution of the funding ratio in the future. First, in order

dynamics of the 10-year yield is derived from the model driven by the one-period yield. However, the characteristics of the longer term yields observed on the market are partly incorporated into the model (through A_τ and B_τ) driven by the one-period yield, because we used a proxy for the 10-year yield (10-year benchmark yield observed with error) in order to estimate the term premium. Consequently, the model-implied dynamics of the 10-year yield also reflect the characteristics of the observed (proxied) 10-year yield to a certain extent.

[18] We assume that the future term structures of interest rates are in line with the implied forward rates of the today observed term structure.

[19] Details are available upon request from the corresponding author.

Table 4
Funding ratio distribution characteristics, 65-year-old men

		Men								
		Micro			Micro + Macro			Micro + Macro + Parameter		
		500	5000	10 000	500	5000	10 000	500	5000	10 000
$T = 1$	$\text{StDev}[FR_T]/E[FR_T]$	0.007	0.002	0.001	0.013	0.011	0.011	0.033	0.032	0.032
	$Q(0.025)$	0.988	0.996	0.997	0.976	0.978	0.979	0.940	0.940	0.941
	$Q(0.975)$	1.015	1.004	1.003	1.027	1.023	1.022	1.070	1.068	1.067
	$E[FR_T \mid FR_T < Q(0.025)]$	0.987	0.996	0.997	0.971	0.974	0.974	0.927	0.928	0.928
	$E[FR_T \mid FR_T > Q(0.975)]$	1.017	1.005	1.004	1.032	1.028	1.027	1.088	1.087	1.088
$T = 5$	$\text{StDev}[FR_T]/E[FR_T]$	0.021	0.007	0.005	0.039	0.033	0.033	0.094	0.090	0.090
	$Q(0.025)$	0.962	0.988	0.991	0.929	0.940	0.940	0.841	0.848	0.851
	$Q(0.975)$	1.042	1.014	1.009	1.083	1.068	1.068	1.225	1.223	1.217
	$E[FR_T \mid FR_T < Q(0.025)]$	0.956	0.985	0.989	0.916	0.928	0.929	0.818	0.821	0.820
	$E[FR_T \mid FR_T > Q(0.975)]$	1.051	1.016	1.011	1.102	1.086	1.084	1.293	1.287	1.284

The table shows the standard deviation of the funding ratio relative to its expectation, the 2.5% quantile, the 97.5% quantile, and the expected shortfall with respect to these quantiles for an annuity portfolio which consists of men with the age of 65 for maturities $T = 1$ and $T = 5$, for three different fund sizes (500, 5000, and 10 000), and for several (combined) risk sources (micro-, macro-longevity and parameter risk).

to illustrate the effect of portfolio size, we consider portfolios of different sizes with identical annuities. Conditional on any given survival rates, the annuitants' survival distributions are independent. In order to gain some insight into the differences between the mortality risk profiles of men and women, we consider annuity funds for 65-year-old men and annuity funds for 65-year-old women. We consider fund sizes ranging from 500 to 10 000 participants, and maturities of 1 and 5 years. In each case, the initial funding ratio is assumed to be equal to 1.

Table 4 yields the standard deviation of the funding ratio relative to its expectation, the 2.5% quantile $Q(0.025)$, the 97.5% quantile $Q(0.975)$, and the expected shortfall with respect to these quantiles,[20] for an annuity portfolio which consists of men with the age of 65 for maturities $T = 1$ and $T = 5$, and for three different fund sizes. In order to assess the relative importance of micro- and macro-longevity risk, we determine these characteristics without (columns 1–3) and with (columns 4–6) macro-longevity risk. The last three columns present the results when also parameter risk is included.[21] In the absence of macro-longevity risk, we assume that the evolution of death rates is deterministic and given by (4) and (5) with $\xi_t = 0$ and $\psi_t = 0$, for all t.

The riskiness in the future funding ratio increases with maturity, which is a natural consequence of the fact that the uncertainty in survival up to time T becomes larger. As the fund size increases, micro-longevity risk in relative terms decreases to zero, due to the pooling effect. In contrast, macro-longevity risk does not become negligible; it is almost independent of portfolio size. In absence of parameter risk, longevity risk (micro+macro) implies that with probability 2.5%, a fund of size 10 000 will face underfunding of at least 6% at a 5-year horizon (i.e. $Q(0.025) = 0.94$). Including parameter risk, the corresponding level of underfunding becomes as high as 15%.

Table 5 presents the results for an annuity fund of 65-year-old women. The combined micro- and macro-longevity risk is smaller for the annuity fund of 65-year-old women compared to men with the same age. If only micro- and macro-longevity risk are considered, women contribute less to the overall risk of the annuity portfolio than men. The additional risk in the parameter estimates is also larger for 65-year-old cohort of men, which is best reflected in the distribution of the future funding ratio in a 5-year horizon.

Now we turn to analyze the case of a representative fund, where the age and gender composition reflects the one observed in the Dutch population at the beginning of 2004. The age and gender distribution of the Dutch population is given in Appendix B. We allow for correlation between the latent processes of the mortality models which are separately estimated for men and for women. The analysis on the Dutch data implies a correlation of 0.846 between the first factors, and 0.858 between the second factors. The relatively high correlations imply that the shocks which drive the latent processes for men and women are similar. Note that the fact that the latent processes are highly correlated does not imply that the future survival probabilities of men and women move together, because (i) the age-specific sensitivities are different for men and women, (ii) particular age-specific error terms influence the mortality rates.

Table 6 presents distributional characteristics of the funding ratio for a Dutch representative fund with maturity of 1 or 5 years, and several fund sizes. The contribution of micro- and macro-longevity risk to the overall riskiness in the funding ratio is substantial. For a maturity of 5 years, micro- and macro-longevity risk imply that the standard deviation of the funding ratio is about 3.7% of its expected value for a fund with 500 participants. It decreases to 2.9% of the expected value if the

[20] Whereas pension funds are mostly interested in longevity risk, shorter than expected lifetime of the policyholders plays an important role in the risk management of life insurance companies. Our mortality model allows for improvement as well as deterioration of future survival rates. Therefore, we consider risk measures that quantify the effect of shorter than expected and longer than expected lifetime on the riskiness of the future funding ratio distribution.

[21] We draw a large number of realizations of the estimated parameters, using the robust covariance matrix of the maximum likelihood estimator. For each parameter realization, we calculate the characteristics of the funding ratio distribution, such as quantiles, variances or expected shortfall. This yields the simulated distribution of these risk measures. Depending on the risk measure, we determine either the 95% quantile or the 5% quantile of this simulated distribution.

Table 5
Funding ratio distribution characteristics, 65-year-old women

		Women								
		Micro			Micro + Macro			Micro + Macro + Parameter		
		500	5000	10 000	500	5000	10 000	500	5000	10 000
$T = 1$	$\text{StDev}[FR_T]/E[FR_T]$	0.005	0.001	0.001	0.010	0.008	0.008	0.017	0.017	0.017
	$Q(0.025)$	0.992	0.997	0.998	0.982	0.984	0.984	0.968	0.969	0.969
	$Q(0.975)$	1.010	1.003	1.002	1.020	1.017	1.017	1.035	1.033	1.034
	$E[FR_T \mid FR_T < Q(0.025)]$	0.992	0.997	0.998	0.979	0.981	0.981	0.963	0.964	0.963
	$E[FR_T \mid FR_T > Q(0.975)]$	1.011	1.004	1.003	1.024	1.020	1.020	1.040	1.039	1.038
$T = 5$	$\text{StDev}[FR_T]/E[FR_T]$	0.014	0.004	0.003	0.029	0.025	0.025	0.039	0.036	0.036
	$Q(0.025)$	0.973	0.991	0.994	0.947	0.953	0.953	0.931	0.934	0.935
	$Q(0.975)$	1.030	1.009	1.006	1.059	1.051	1.052	1.083	1.076	1.076
	$E[FR_T \mid FR_T < Q(0.025)]$	0.969	0.990	0.993	0.938	0.945	0.946	0.919	0.923	0.924
	$E[FR_T \mid FR_T > Q(0.975)]$	1.036	1.010	1.007	1.073	1.062	1.062	1.101	1.091	1.091

The table shows the standard deviation of the funding ratio relative to its expectation, the 2.5% quantile, the 97.5% quantile, and the expected shortfall with respect to these quantiles for an annuity portfolio which consists of women with the age of 65 for maturities $T = 1$ and $T = 5$, for three different fund sizes (500, 5000, and 10 000), and for several (combined) risk sources (micro-, macro-longevity and parameter risk).

Table 6
Funding ratio distribution characteristics, Dutch population

		NL population								
		Micro			Micro + Macro			Micro + Macro + Parameter		
		500	5000	10 000	500	5000	10 000	500	5000	10 000
$T = 1$	$\text{StDev}[FR_T]/E[FR_T]$	0.009	0.003	0.002	0.014	0.012	0.012	0.022	0.020	0.020
	$Q(0.025)$	0.985	0.995	0.996	0.973	0.977	0.977	0.962	0.964	0.965
	$Q(0.975)$	1.019	1.006	1.004	1.029	1.024	1.024	1.044	1.040	1.040
	$E[FR_T \mid FR_T < Q(0.025)]$	0.983	0.994	0.996	0.967	0.971	0.971	0.956	0.960	0.960
	$E[FR_T \mid FR_T > Q(0.975)]$	1.024	1.007	1.005	1.036	1.031	1.031	1.051	1.045	1.044
$T = 5$	$\text{StDev}[FR_T]/E[FR_T]$	0.023	0.007	0.005	0.037	0.029	0.029	0.058	0.053	0.053
	$Q(0.025)$	0.959	0.986	0.991	0.934	0.946	0.947	0.901	0.910	0.911
	$Q(0.975)$	1.048	1.015	1.010	1.077	1.060	1.058	1.120	1.113	1.113
	$E[FR_T \mid FR_T < Q(0.025)]$	0.953	0.984	0.989	0.923	0.938	0.939	0.888	0.898	0.899
	$E[FR_T \mid FR_T > Q(0.975)]$	1.058	1.017	1.012	1.093	1.071	1.070	1.144	1.130	1.130

The table shows the standard deviation of the funding ratio relative to its expectation, the 2.5% quantile, the 97.5% quantile, and the expected shortfall with respect to these quantiles for an annuity portfolio, which consists of an annuity population portraying the composition of the Dutch population with people older than 24. We report the risk measures for maturities $T = 1$ and $T = 5$, for three different fund sizes (500, 5000, and 10 000), and for several (combined) risk sources (micro-, macro-longevity and parameter risk).

fund is large (10 000 participants). Due to pooling effects, micro-longevity risk then becomes negligible. The uncertainty increases even further if parameter uncertainty of the estimates is also incorporated in the analysis. For a large fund (10 000 participants), the standard deviation of the funding ratio in a 5-year horizon is then 5.3% of the expected value. The results show that if financial market risk is perfectly hedged so that uncertainty in future lifetime is the only source of risk, pension funds are exposed to a substantial amount of uncertainty. The problem raises a hedging demand.

6. Management of longevity risk

Longevity bonds could potentially be used to hedge the future liabilities of a pension fund. However a longevity bond is a tool to hedge only against the macro-longevity risk, micro-longevity risk is not covered. In the previous section we saw that, as the size of the fund gets large, micro-longevity risk does not play an important role in the future uncertainty. However, for very small funds it is an important risk source. This fact already creates a mismatch between the realized liabilities of the fund and the payoff of the longevity bonds. In addition, there are other sources of mismatch related to the standardized features of longevity bonds. The longevity bonds which were issued in the UK are linked to an age group with a fixed maturity. The payoff of the product is linked to the actual evolution of a group of people (birth-cohort), which does not necessarily reflect the actual age composition of the fund. Moreover, the market for longevity bonds is very illiquid. Consequently, macro-mortality risk cannot be hedged perfectly with longevity bonds. Therefore, we analyze alternative strategies insurance companies and pension funds can use to reduce both macro- and micro-longevity risk. We determine the size of the buffer required to reduce the probability of underfunding to an acceptable level. To concentrate on mortality risk we filter out all other uncertainties. Specifically, we assume that the expected liabilities are fully matched with cash flow matching at date zero, and that the term structure of interest rates moves deterministically to its long term average. We consider pension funds with different sizes for which the age and

Table 7
Calibrated solvency buffer, VaR

T	N	Micro (%)	Micro + Macro (%)	Micro + Macro + Parameter (%)
$T = 1$	500	1.455	2.624	3.760
	1000	1.086	2.412	3.671
	2500	0.723	2.256	3.582
	5000	0.497	2.210	3.515
	10 000	0.358	2.179	3.485
$T = 5$	500	3.163	5.178	8.016
	1000	2.331	4.826	7.618
	2500	1.509	4.486	7.282
	5000	1.056	4.269	7.281
	10 000	0.774	4.238	7.172

The table presents the percentage of the initial liability value that has to be invested in a 1- or 5-year bond (depending on the maturity) in order to meet the Value at Risk solvency requirement in (28) with $\varepsilon = 0.025$, with several (combined) risk sources (micro-, macro-longevity and parameter risk).

Table 8
Calibrated solvency buffer, expected shortfall

T	N	Micro (%)	Micro + Macro (%)	Micro + Macro + Parameter (%)
$T = 1$	500	1.637	3.190	4.397
	1000	1.240	2.986	4.241
	2500	0.832	2.825	4.044
	5000	0.596	2.791	3.986
	10 000	0.423	2.784	3.961
$T = 5$	500	3.792	6.282	9.211
	1000	2.788	5.774	8.789
	2500	1.772	5.406	8.492
	5000	1.264	5.223	8.383
	10 000	0.909	5.141	8.393

The table presents the percentage of the initial liability value that has to be invested in a 1- or 5-year bond (depending on the maturity) in order to meet the expected shortfall solvency requirement in (29) with $\varepsilon = 0.025$, with several (combined) risk sources (micro-, macro-longevity and parameter risk).

gender composition is that of the Dutch adult population at the beginning of 2004. In each case, the initial funding ratio is assumed to be equal to one.

First, we calibrate the size of the buffer such that the $VaR_{1-\varepsilon}$ (the Value at Risk at the $(1-\varepsilon) * 100\%$ level) of the funding ratio at time T is equal to one, i.e.

$$\Pr\left(\frac{A_T + B_T}{L_T} < 1\right) = \varepsilon, \tag{28}$$

where B_T denotes the size of the buffer at time T. We assume that the pension fund invests its buffer in a T-period risk-free zero-coupon bond, and express the buffer size at time $t = 0$ as a percentage c of the initial market value of the liabilities, i.e. $B_0 = cL_0$ and $B_T = cL_0/P_0^{(T)}$.

Table 7 presents the percentage c of the initial liability value that has to be invested in a 1- or 5-year bond (depending on the maturity) in order to meet the solvency requirement in (28) with $\varepsilon = 0.025$.

Next, we calibrate the size of the initial buffer such that the expected shortfall of the funding ratio with respect to the $VaR_{1-\varepsilon}$ is 1 at maturity T, i.e.

$$\mathbb{E}\left[\frac{A_T + B_T}{L_T}\,\middle|\,\frac{A_T + B_T}{L_T} < VaR_{1-\varepsilon}\right] = 1. \tag{29}$$

with $\varepsilon = 0.025$.

Tables 7 and 8 illustrate the importance of micro-longevity, macro-longevity and parameter risk. Depending on the risk measure (VaR or Expected Shortfall), the combination of micro- and macro-longevity risk implies that a large pension fund which is currently funded has to reserve between 4.2% and 5.1% of the initial value of the liabilities to meet the solvency requirement in a 5-year horizon. Smaller funds have to reserve even more due to the extra randomness related to micro-longevity risk. If parameter risk is included in the analysis, the initial funding ratio for large funds then has to be 107.2% in order to meet the solvency requirement in (28) and 108.4% in order to meet the solvency requirement in (29).

7. Effect of combined longevity and market risk

In Section 5 we have assumed that the liabilities are matched with cash flow matching, so that we can isolate the effect of longevity risk on funding ratio uncertainty. In this section we include financial market risk, and determine the relative importance of micro- and macro-longevity risk in the presence of market risk.

We consider several alternative asset compositions consisting of stocks and bonds with different maturities. When dealing with stocks, we postulate that the excess stock return in excess of the short rate follows a random walk with drift,

Table 9
Distribution of future funding ratio with market risk and longevity risk combined, $T = 1$

		NL population						
		Micro				Micro + Macro + Parameter		
		500	5000	10 000	infinity	500	5000	10 000
Perfect hedge of market risk	$\text{StDev}[FR_T]/E[FR_T]$	0.009	0.003	0.002	0.000	0.022	0.020	0.020
	$Q(0.025)$	0.985	0.995	0.996	1.000	0.962	0.964	0.965
	$Q(0.975)$	1.019	1.006	1.004	1.000	1.044	1.040	1.040
	$E[FR_T \mid FR_T < Q(0.025)]$	0.983	0.994	0.996	1.000	0.956	0.960	0.960
	$E[FR_T \mid FR_T > Q(0.975)]$	1.024	1.007	1.005	1.000	1.051	1.045	1.044
Static duration hedge	$\text{StDev}[FR_T]/E[FR_T]$	0.014	0.012	0.011	0.011	0.024	0.023	0.023
	$Q(0.025)$	0.971	0.976	0.976	0.976	0.954	0.957	0.956
	$Q(0.975)$	1.027	1.022	1.021	1.020	1.048	1.044	1.044
	$E[FR_T \mid FR_T < Q(0.025)]$	0.965	0.971	0.971	0.972	0.949	0.952	0.953
	$E[FR_T \mid FR_T > Q(0.975)]$	1.033	1.025	1.025	1.024	1.056	1.050	1.050
100%–0%–0%	$\text{StDev}[FR_T]/E[FR_T]$	0.013	0.010	0.010	0.010	0.023	0.022	0.021
	$Q(0.025)$	0.978	0.982	0.983	0.983	0.959	0.962	0.963
	$Q(0.975)$	1.029	1.022	1.022	1.021	1.051	1.046	1.046
	$E[FR_T \mid FR_T < Q(0.025)]$	0.974	0.979	0.979	0.979	0.953	0.956	0.955
	$E[FR_T \mid FR_T > Q(0.975)]$	1.034	1.026	1.025	1.025	1.059	1.054	1.053
50%–50%–0%	$\text{StDev}[FR_T]/E[FR_T]$	0.009	0.003	0.002	0.000	0.022	0.020	0.020
	$Q(0.025)$	0.986	0.996	0.997	1.000	0.963	0.965	0.965
	$Q(0.975)$	1.020	1.007	1.005	1.001	1.045	1.042	1.041
	$E[FR_T \mid FR_T < Q(0.025)]$	0.984	0.995	0.997	1.000	0.957	0.961	0.961
	$E[FR_T \mid FR_T > Q(0.975)]$	1.025	1.008	1.006	1.001	1.052	1.046	1.045
37.5%–37.5%–25%	$\text{StDev}[FR_T]/E[FR_T]$	0.072	0.071	0.071	0.072	0.073	0.072	0.072
	$Q(0.025)$	0.909	0.910	0.910	0.907	0.902	0.904	0.904
	$Q(0.975)$	1.195	1.195	1.193	1.197	1.195	1.190	1.190
	$E[FR_T \mid FR_T < Q(0.025)]$	0.892	0.894	0.894	0.894	0.884	0.886	0.886
	$E[FR_T \mid FR_T > Q(0.975)]$	1.243	1.242	1.242	1.242	1.241	1.238	1.239
25%–25%–50%	$\text{StDev}[FR_T]/E[FR_T]$	0.139	0.138	0.138	0.141	0.137	0.136	0.136
	$Q(0.025)$	0.819	0.818	0.817	0.813	0.819	0.823	0.823
	$Q(0.975)$	1.384	1.386	1.384	1.393	1.370	1.365	1.365
	$E[FR_T \mid FR_T < Q(0.025)]$	0.786	0.787	0.787	0.787	0.789	0.789	0.789
	$E[FR_T \mid FR_T > Q(0.975)]$	1.483	1.483	1.483	1.483	1.461	1.459	1.459

The table shows the standard deviation of the funding ratio relative to its expectation, the 2.5% quantile, the 97.5% quantile, and the expected shortfall with respect to these quantiles for an annuity portfolio, which consists of an annuity population portraying the composition of the Dutch population with people older than 24. We report the risk measures for maturity $T = 1$, for several fund sizes (500, 5000, 10 000, and infinitely large fund), and for several (combined) risk sources (micro-, macro-longevity and parameter risk) under alternative investment strategies. The investment strategies are as follows: (i) expected liabilities are cash flow hedged; (ii) liabilities are duration hedged; (iii) assets are invested exclusively in 5-year bonds; (iv) 50% of the assets is invested into 5-year, and 50% in 10-year bonds; (v) 37.5% is invested into 5-year, 37.5% in 10-year bonds, and the rest is invested into stocks; (vi) 25% is invested in 5-year, 25% in 10-year bonds, while the rest is invested in stocks.

independently[22] of the short rate process and the mortality driving factors. This can easily be included in our market valuation model, presented in Section 5.2. We again consider pension funds of different sizes for which the age and gender composition reflects that of the Dutch population at the beginning of 2004, and assume that the initial funding ratio is one.

7.1. Data

The term structure models for the interest rates and the mortality rates we use in this section are identical to the ones estimated and introduced in the previous sections. The stock market index is measured by the total return index of the Dutch market calculated by Datastream for the period between 1983 and 2004. The excess return over the short rate is estimated to be 6.2% with a volatility of 23.9% p.a.[23]

7.2. Uncertainty in the future funding ratio

We investigate the effect of an imperfect hedge of investment risk combined with micro-, macro-longevity and parameter on the future distribution of the funding ratio for five different investment strategies: (i) liabilities are 'perfectly' hedged: expected liabilities are hedged with cash flow matching initially; (ii) liabilities are duration hedged, based on the McCauley duration; (iii) assets are invested exclusively in

[22] We calculated a 0.2 sample correlation between the 1-year excess stock return and the one-period interest rate in the period of 1985 and 2004 in the Netherlands, which is not significantly different from zero. Ang et al. (2005) documented a −0.05 correlation between the excess stock return and the one-period short rate by using quarterly US data from 1926 and 1998, which also supports the independence assumption.

[23] Fama and French (2002) suggest that the equity premium estimated from fundamentals (for instance, the dividend or earnings growth rates) can be much lower than the equity premium produced by the average stock return. For simplicity, to calculate the excess return we used the average stock return in the sample from 1983 and 2004 and no fundamentals.

Table 10
Distribution of future funding ratio with market risk and longevity risk combined, $T = 5$

		NL population						
		Micro				Micro + Macro + Parameter		
		500	5000	10 000	infinity	500	5000	10 000
Perfect hedge of market risk	$\text{StDev}[FR_T]/E[FR_T]$	0.023	0.007	0.005	0.000	0.058	0.053	0.053
	$Q(0.025)$	0.959	0.986	0.991	1.000	0.901	0.910	0.911
	$Q(0.975)$	1.048	1.015	1.010	1.000	1.120	1.113	1.113
	$E[FR_T \mid FR_T < Q(0.025)]$	0.953	0.984	0.989	1.000	0.888	0.898	0.899
	$E[FR_T \mid FR_T > Q(0.975)]$	1.058	1.017	1.012	1.000	1.144	1.130	1.130
Static duration hedge	$\text{StDev}[FR_T]/E[FR_T]$	0.038	0.032	0.031	0.031	0.069	0.065	0.064
	$Q(0.025)$	0.919	0.930	0.931	0.931	0.872	0.878	0.877
	$Q(0.975)$	1.065	1.053	1.051	1.051	1.137	1.124	1.122
	$E[FR_T \mid FR_T < Q(0.025)]$	0.903	0.916	0.916	0.917	0.854	0.863	0.864
	$E[FR_T \mid FR_T > Q(0.975)]$	1.081	1.064	1.062	1.061	1.159	1.147	1.148
100%–0%–0%	$\text{StDev}[FR_T]/E[FR_T]$	0.033	0.024	0.024	0.023	0.062	0.057	0.057
	$Q(0.025)$	0.954	0.967	0.968	0.968	0.906	0.916	0.915
	$Q(0.975)$	1.082	1.063	1.062	1.061	1.148	1.137	1.137
	$E[FR_T \mid FR_T < Q(0.025)]$	0.943	0.957	0.957	0.959	0.891	0.899	0.900
	$E[FR_T \mid FR_T > Q(0.975)]$	1.096	1.072	1.071	1.069	1.175	1.159	1.157
50%–50%–0%	$\text{StDev}[FR_T]/E[FR_T]$	0.023	0.007	0.005	0.002	0.058	0.053	0.053
	$Q(0.025)$	0.964	0.991	0.995	1.000	0.907	0.915	0.916
	$Q(0.975)$	1.054	1.021	1.016	1.009	1.129	1.119	1.120
	$E[FR_T \mid FR_T < Q(0.025)]$	0.958	0.989	0.993	0.998	0.892	0.903	0.904
	$E[FR_T \mid FR_T > Q(0.975)]$	1.064	1.024	1.019	1.010	1.152	1.138	1.137
37.5%–37.5%–25%	$\text{StDev}[FR_T]/E[FR_T]$	0.179	0.177	0.177	0.172	0.176	0.175	0.175
	$Q(0.025)$	0.825	0.825	0.825	0.832	0.819	0.826	0.826
	$Q(0.975)$	1.622	1.621	1.619	1.605	1.615	1.602	1.601
	$E[FR_T \mid FR_T < Q(0.025)]$	0.779	0.782	0.782	0.791	0.782	0.787	0.786
	$E[FR_T \mid FR_T > Q(0.975)]$	1.759	1.755	1.755	1.717	1.725	1.717	1.716
25%–25%–50%	$\text{StDev}[FR_T]/E[FR_T]$	0.346	0.346	0.345	0.335	0.333	0.331	0.331
	$Q(0.025)$	0.660	0.660	0.658	0.669	0.667	0.668	0.668
	$Q(0.975)$	2.398	2.404	2.406	2.381	2.362	2.356	2.356
	$E[FR_T \mid FR_T < Q(0.025)]$	0.586	0.587	0.587	0.601	0.604	0.608	0.608
	$E[FR_T \mid FR_T > Q(0.975)]$	2.785	2.782	2.782	2.689	2.637	2.626	2.624

The table shows the standard deviation of the funding ratio relative to its expectation, the 2.5% quantile, the 97.5% quantile, and the expected shortfall with respect to these quantiles for an annuity portfolio, which consists of an annuity population portraying the composition of the Dutch population with people older than 24. We report the risk measures for maturity $T = 5$, for several fund sizes (500, 5000, 10 000, and infinitely large fund), and for several (combined) risk sources (micro-, macro-longevity and parameter risk) under alternative investment strategies. The investment strategies are as follows: (i) expected liabilities are cash flow hedged; (ii) liabilities are duration hedged; (iii) assets are invested exclusively in 5-year bonds; (iv) 50% of the assets is invested into 5-year, and 50% in 10-year bonds; (v) 37.5% is invested into 5-year, 37.5% in 10-year bonds, and the rest is invested into stocks; (vi) 25% is invested in 5-year, 25% in 10-year bonds, while the rest is invested in stocks.

5-year bonds; (iv) 50% of the assets is invested into 5-year and 50% in 10-year bonds: the interest rate elasticity of the liabilities matches the elasticity of the assets, based on the term structure model; (v) 37.5% is invested into 5-year, 37.5% in 10-year bonds, and the rest is invested into stocks; (vi) 25% is invested in 5-year, 25% in 10-year bonds, while the rest is invested in stocks. We investigate the 1- and 5-year horizons. The (classical) duration of the annuity portfolio is about 13 years initially, therefore the assets used to hedge the liabilities with duration matching consist of 10% 5-year and 90% 15-year bonds.

Tables 9 and 10 show the simulated distributional characteristics of the funding ratio at $T = 1$ and $T = 5$, for the above mentioned investment strategies. Because micro-longevity risk becomes negligible when the portfolio size is infinitely large, the fourth column yields the effect of different investment strategies on funding ratio uncertainty in absence of longevity risk.

If we compare the duration hedge to the asset composition of 50% 5-year and 50% 10-year bonds, we see that the relative standard deviation of the funding ratio is higher for the duration hedge. Apart from the fact that the duration of the liabilities is matched initially and the asset portfolio is not rebalanced in order to match the duration of the liabilities in the subsequent years, another reason why duration matching does not perform so well is related to the limitations of duration hedging. The interest rate sensitivity of the liabilities matches the interest rate sensitivity of the 6-year zero-coupon bond based on the term structure model we use, which explains the underperformance of the duration hedge when it is compared to the alternative bond portfolios. The 50% 5-year and 50% 10-year bond portfolio matches the interest rate sensitivity of the liabilities fairly well based on the term structure model, which explains the good hedging performance for both horizons.

As expected, investment risk gets relatively important when the fraction of stocks increases. Intuitively, one would expect that adding a source of uncertainty by including macro-longevity and parameter risk into the analysis would increase funding ratio uncertainty, as measured by the relative standard deviation for instance. However, this is not always the case. Although the increase in terms of funding ratio uncertainty when macro-longevity and parameter risk is introduced is indeed quite pronounced for the case where financial risk is perfectly hedged, it becomes relatively less important if investment risk increases. If the assets of the fund consist of 50% stocks, then macro-longevity risk and parameter risk even decrease the relative standard deviation of the funding ratio. This occurs because the introduction of macro-longevity risk and parameter risk implies that the distribution of the liabilities becomes more skewed to the right. When a substantial amount of stocks are included in the assets, the uncertainty in the asset value is high, so that the effect of increased uncertainty in the liabilities on the distribution of the numerator of the funding ratio (asset value minus realized pension payments) becomes negligible. As a consequence, the increased skewness of the denominator (value of future liabilities) can indeed result in a decrease of the spread of the funding ratio distribution.

8. Conclusions

Uncertainty in the future survival probabilities contributes significantly to the riskiness of the future funding ratio. In the absence of financial market risk, a large pension fund that is currently exactly funded, and wants to reduce the probability of underfunding in a maturity of 5 years to 2.5% has to hold a buffer of about 7%–8% of the initial value of the liabilities.

If market risk is also considered, the contribution of mortality risk to the overall risk of the future funding ratio becomes relatively less important. The relative importance of longevity risk decreases if the fraction of stock investments in the asset portfolio increases. However, it is not negligible.

The mortality model we considered has the potential of both future mortality improvement and mortality deterioration. We believe that we cannot exclude the risk in mortality deterioration in the future, which would significantly affect the risk of the portfolio of life insurance companies. However, given the downward sloping trend in the future, improvement is more likely than deterioration. The construction of a model which implies improvement with a large probability, and deterioration with a smaller probability is a topic for further research.

Appendix A. Parameter estimates of the mortality model

Parameter estimates of the two-factor moving average mortality model introduced in Section 2 (see Tables A.1 and A.2). As outlined in Hári et al. (in press), in finite samples the estimator for the mean of the latent process ($\widehat{\mu}_u$) might deviate from zero, which reflects the model's difficulty in estimating the long run trend a. We therefore estimate the long run trend

Table A.1
Parameter estimates for men

	Coefficients					
Ξ	−0.602 *0* (0.140)					
	−0.127 −0.261 (0.118) (0.358)					
Σ_Ψ	*1* *0*					
	0 *1*					
Age group (x)	A_x	a_x	$B_{1,x}$	$B_{2,x}$	MA(1): Θ_x	ME: $\sigma_{\xi,x}$
1–4	−0.032 (0.009)	−0.032 (0.014)	0.040 (0.064)	−0.118 (0.031)	−0.576 (0.118)	0.114 (0.009)
5–9	−0.024 (0.008)	−0.024 (0.014)	0.042 (0.066)	−0.114 (0.023)	−0.407 (0.137)	0.106 (0.008)
10–14	−0.021 (0.010)	−0.021 (0.016)	0.055 (0.083)	−0.137 (0.026)	−0.633 (0.067)	0.082 (0.006)
15–19	−0.020 (0.012)	−0.020 (0.020)	0.071 (0.103)	−0.167 (0.034)	−0.513 (0.070)	0.076 (0.005)
20–24	−0.019 (0.013)	−0.020 (0.022)	0.082 (0.114)	−0.182 (0.041)	−0.554 (0.251)	0.107 (0.017)
25–29	−0.018 (0.013)	−0.019 (0.022)	0.087 (0.112)	−0.179 (0.043)	−0.677 (0.079)	0.061 (0.006)
30–34	−0.017 (0.012)	−0.018 (0.020)	0.087 (0.101)	−0.161 (0.042)	−0.665 (0.061)	0.053 (0.004)
35–39	−0.016 (0.010)	−0.016 (0.017)	0.083 (0.085)	−0.135 (0.039)	−0.728 (0.072)	0.038 (0.003)
40–44	−0.014 (0.008)	−0.014 (0.014)	0.078 (0.067)	−0.106 (0.036)	−0.739 (0.045)	0.037 (0.002)
45–49	−0.012 (0.007)	−0.013 (0.011)	0.073 (0.051)	−0.081 (0.034)	−0.538 (0.074)	0.035 (0.002)
50–54	−0.011 (0.005)	−0.011 (0.009)	0.069 (0.038)	−0.060 (0.033)	−0.472 (0.093)	0.033 (0.002)
55–59	−0.009 (0.004)	−0.009 (0.007)	0.067 (0.029)	−0.045 (0.032)	−0.458 (0.074)	0.031 (0.002)
60–64	−0.008 (0.004)	−0.008 (0.006)	0.065 (0.022)	−0.033 (0.032)	−0.383 (0.091)	0.025 (0.002)
65–69	−0.006 (0.003)	−0.006 (0.005)	0.064 (0.017)	−0.024 (0.032)	−0.559 (0.107)	0.025 (0.002)
70–74	−0.005 (0.003)	−0.005 (0.005)	0.064 (0.014)	−0.017 (0.032)	−0.538 (0.131)	0.021 (0.002)
75–79	−0.004 (0.003)	−0.004 (0.005)	0.065 (0.011)	−0.012 (0.032)	−0.538 (0.079)	0.025 (0.003)
80–84	−0.003 (0.003)	−0.003 (0.005)	0.065 (0.009)	−0.007 (0.032)	−0.481 (0.060)	0.034 (0.004)
85+	−0.002 (0.003)	−0.002 (0.005)	0.066 (0.008)	−0.002 (0.033)	−0.379 (0.117)	0.047 (0.004)
Log-Likelihood	4024.39					

The table shows the parameter estimates of the two-factor mortality model for men in (4) and (5).
Note: This table reports QML estimates and standard errors of the two-factor affine mortality model. Standard errors are in parenthesis. Normalized coefficients are written with italics.

Table A.2
Parameter estimates for women

	Coefficients					
Ξ	−0.587 *0* (0.118) −0.209 −0.113 (0.113) (0.264)					
Σ_Ψ	*1* *0* *0* *1*					
Age group (x)	A_x	a_x	$B_{1,x}$	$B_{2,x}$	MA(1): Θ_x	ME: $\sigma_{\xi,x}$
1–4	−0.033 (0.009)	−0.033 (0.018)	0.044 (0.02)	−0.116 (0.022)	−0.578 (0.095)	0.113 (0.009)
5–9	−0.024 (0.01)	−0.024 (0.021)	0.040 (0.021)	−0.140 (0.021)	−0.412 (0.098)	0.105 (0.006)
10–14	−0.022 (0.011)	−0.022 (0.022)	0.043 (0.025)	−0.148 (0.024)	−0.609 (0.09)	0.095 (0.007)
15–19	−0.023 (0.011)	−0.023 (0.022)	0.049 (0.028)	−0.146 (0.027)	−0.642 (0.088)	0.075 (0.007)
20–24	−0.024 (0.01)	−0.024 (0.021)	0.053 (0.029)	−0.136 (0.029)	−0.736 (0.07)	0.074 (0.006)
25–29	−0.023 (0.009)	−0.023 (0.019)	0.053 (0.027)	−0.122 (0.028)	−0.702 (0.091)	0.057 (0.005)
30–34	−0.020 (0.008)	−0.020 (0.017)	0.051 (0.022)	−0.105 (0.025)	−0.698 (0.11)	0.058 (0.007)
35–39	−0.018 (0.007)	−0.018 (0.014)	0.048 (0.017)	−0.086 (0.021)	−0.702 (0.064)	0.050 (0.005)
40–44	−0.015 (0.005)	−0.015 (0.011)	0.045 (0.013)	−0.068 (0.016)	−0.636 (0.075)	0.041 (0.003)
45–49	−0.013 (0.004)	−0.013 (0.009)	0.045 (0.01)	−0.052 (0.013)	−0.627 (0.111)	0.043 (0.004)
50–54	−0.011 (0.004)	−0.011 (0.007)	0.046 (0.008)	−0.039 (0.011)	−0.591 (0.106)	0.043 (0.004)
55–59	−0.011 (0.003)	−0.011 (0.006)	0.050 (0.007)	−0.029 (0.011)	−0.694 (0.068)	0.043 (0.004)
60–64	−0.010 (0.003)	−0.010 (0.005)	0.055 (0.006)	−0.021 (0.011)	−0.635 (0.089)	0.034 (0.004)
65–69	−0.010 (0.003)	−0.010 (0.005)	0.060 (0.007)	−0.015 (0.011)	−0.691 (0.099)	0.027 (0.002)
70–74	−0.009 (0.003)	−0.009 (0.005)	0.066 (0.007)	−0.011 (0.012)	−0.775 (0.107)	0.019 (0.002)
75–79	−0.008 (0.003)	−0.008 (0.005)	0.071 (0.008)	−0.009 (0.013)	−0.713 (0.103)	0.019 (0.003)
80–84	−0.006 (0.003)	−0.006 (0.006)	0.076 (0.009)	−0.008 (0.013)	−0.667 (0.127)	0.025 (0.004)
85+	−0.004 (0.003)	−0.004 (0.006)	0.079 (0.01)	−0.008 (0.015)	−0.519 (0.145)	0.045 (0.006)
Log-likelihood	3961.59					

The table shows the parameter estimates of the two-factor mortality model for women in (4) and (5).
Note: This table reports QML estimates and standard errors of the two-factor affine mortality model. Standard errors are in parenthesis. Normalized coefficients are written with italics.

by

$$\widehat{a} = \widehat{A} - \widehat{B}\widehat{\mu}_u, \tag{A.1}$$

where $\widehat{A} \in \mathbb{R}^{na}$ is the estimated trend by the model.

Appendix B. Age and gender distribution of the Dutch population

See Fig. B.1.

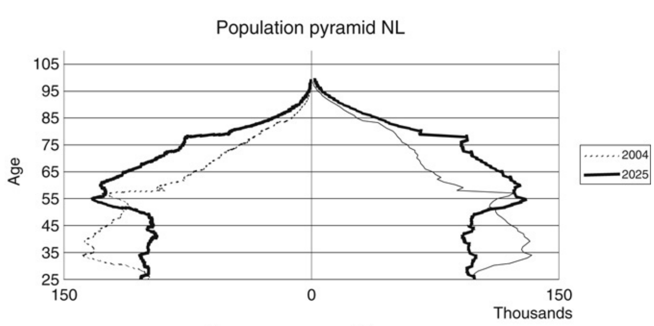

Fig. B.1. Population by age and gender in January 2004. The figure shows the population pyramid for the Netherlands at the beginning of 2004 and the expected population pyramid for 2025 for people older than 24. Men constitute 48.8% of the population with the age older than 24 in 2004. The expected number of people in the cohorts that are alive in 2025 is calculated by applying the two-factor mortality model, which was used through the paper. Since all the cohorts of the pyramid that are older than 24 in year 2025 are alive in year 2004 already, we do not make additional assumptions on the number of newly born between 2004 and 2025. We assume there is no migration.

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