

# Explaining the Level of Credit Spreads: Option-Implied Jump Risk Premia in a Firm Value Model

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We study whether option-implied jump risk premia can explain the high observed level of credit spreads. We use a structural jump-diffusion firm value model to assess the level of credit spreads generated by option-implied jump risk premia. Prices and returns of equity index and individual options are used to estimate the jump parameters. We further calibrate the model to historical information on default risk and the equity premium. The results show that incorporating option-implied jump risk premia brings predicted credit spread levels much closer to observed levels. The introduction of jumps also helps to improve the fit of the volatility of credit spreads and equity returns. (*JEL* G12, G13)

## 1. Introduction

Corporate bonds are defaultable, and thus, trade at higher yields than default-free government bonds. However, it has been difficult to reconcile this observed difference in yields (the credit spread) with the historically observed default losses of corporate bonds, especially for investment-grade firms (Elton et al. 2001). In particular, Huang and Huang (2003, henceforth HH) analyze a wide range of structural firm value models that build on the seminal contingent-claims analysis of Merton (1974). HH show that these models typically explain only 20%–30% of the observed credit spreads for these firms. In response to this credit spread “puzzle” (Amato and Remolona 2003), a number of authors have

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recently incorporated jump risk premia into the analysis. As discussed below, the existing evidence on the relevance of jump risk premia is inconclusive.

The contribution of this article is to use information on the market price of downward jump risk embedded in index put options to estimate a structural jump-diffusion firm value model. We investigate the out-of-sample predictions of the estimated model for credit spreads. As a result, we study whether the price of jump risk embedded in corporate bonds is consistent with the price of jump risk in index options. This is a natural question since index put options constitute the prime liquid market for insurance against systematic jumps, precisely the type of jumps that corporate bond investors are exposed to. A short index put option tends to pay off particularly badly when the stochastic discount factor is very high, and therefore, commands a large risk premium. Furthermore, interpreting a corporate bond as a default-free bond plus a short position in a put option on the firm value (Merton 1974), we effectively test empirically whether the jump risk premium embedded in this firm value put is in line with the jump risk premium embedded in equity index put options.

Our analysis leads to three novel findings. First, the results indicate that structural models are useful for the pricing of credit risk, in contrast to the conclusions of previous work. Second, we provide evidence that option-implied jump risk premia generate credit spread levels that are quite close to observed spreads, and thus, relate the credit spread puzzle to the level of average index option returns. Third, we show that incorporating jumps in a firm value model is important to improve the fit of observed option prices and returns, credit spread volatilities, and equity volatilities.

There are several reasons why the relationship between the prices of the firm value put and the equity index put is not obvious. First, equity index put options have much shorter maturities and are less “out-of-the-money” than the firm value put. Second, equity index put options have the equity index as the underlying asset, while the embedded corporate bond put option has the firm value as the underlying asset. Third, there is a debate on whether corporate bond and equity (option) markets are integrated (Collin-Dufresne, Goldstein, and Martin 2001; Cremers et al. 2008; and Ericsson, Jacobs, and Oviedo 2005).

Recent empirical work has revealed a number of intriguing stylized facts about the prices of equity index options (see Bates 2003 for a survey). It is now well accepted that the underlying index is subject to jumps to returns and volatility,<sup>1</sup> generating market incompleteness. Moreover, this incompleteness seems to be priced (Buraschi and Jackwerth 2001).<sup>2</sup> The goal of our paper is not to explain the source and nature of the jump risk premium. In fact, some

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<sup>1</sup> See Ait-Sahalia (2002), Andersen, Benzoni, and Lund (2002), and Eraker, Johannes, and Polson (2003) for recent contributions to this literature.

<sup>2</sup> Evidence of priced jump risk in index options is presented in Ait-Sahalia, Wang, and Yared (2001), Bakshi, Cao, and Chen (1997), Bates (2002), Pan (2002), and Rosenberg and Engle (2002), among others.

studies on equity index options (Pan 2002, Bondarenko 2003, and Driessen and Maenhout 2007) argue that it is hard to reconcile the size of option-implied risk premia with rational equilibrium models. Instead, our goal is to examine the implications of the observed size of this option-implied jump risk premium for the level of credit spreads.

Our model has the following structure. The asset value of each firm in the S&P 100 follows a jump-diffusion process. Allowing for common and firm-specific jumps, the jump sizes are drawn from a double-exponential distribution. We incorporate a risk premium for the common but not for the firm-specific jumps since only common jumps should be priced in equilibrium. Given assumptions about the debt structure, default boundary, and recovery rule, the corporate bond and equity can be priced under the risk-neutral measure. Pricing the equity of all firms in the index in this way and aggregating constructs the S&P 100 index, so that in the final step, index put options can be priced. S&P 100 index options are thus viewed as options on a portfolio of 100 firm value call options.

The parameters of the model (e.g., governing the recovery rule, the default boundary, the asset risk premium and diffusion volatility, the cross-firm diffusion correlation, the jump processes, etc.) are calibrated to the following moment conditions: the default probability (per rating category), the average par recovery rate, the equity risk premium, the leverage ratio, equity correlations, equity option prices, and expected option returns. This ensures that the model matches estimates for the expected loss and the equity premium, which is important since otherwise a Merton (1974) diffusion model could generate high credit spread levels (either by increasing default risk or by increasing the diffusion risk premium). By calibrating our jump-diffusion model to both the equity premium and expected option returns, we can disentangle the diffusion and the jump risk premia, without changing the total equity risk premium. Data on S&P 100 options and stock options for 69 firms are used to identify the parameters related to the common and firm-specific jump processes. Importantly, the model is not calibrated to credit spread data. Once the model is estimated, the model-implied credit spread is calculated and compared to the observed average credit spread level, for either corporate bonds or credit default swaps.

Our estimates show that both common and firm-specific jumps are relevant. Firm-specific jumps are relatively large and frequent, while common jumps are less frequent and somewhat smaller: the common jump intensity equals 5.6% per year with a mean jump size of  $-7.4\%$ . Consistent with the option pricing literature (e.g., Pan 2002), we find a large risk premium on common jump risk, since the jump risk premium corresponds to a relative risk aversion parameter of 9.23. Under the risk-neutral measure, the annualized jump intensity is 20.3% and the mean jump size is  $-23.3\%$ . This jump risk premium reflects the strongly negative average return on equity index put options. Relative to a model without jumps, the addition of priced jumps enables the model to price options much better and to obtain a closer fit of the distribution of index returns. The model

also predicts that individual stock-option prices are higher for lower-rated firms, as observed in our sample.

Most importantly, the jump-diffusion model generates an out-of-sample prediction of 71 basis points for the credit spread on a 10-year bond of a typical A-rated firm. Across ratings, we obtain credit spreads ranging from 48 basis points for AAA to 512 basis points for B. The jump-diffusion model explains a reasonably large fraction of the observed corporate bond credit spreads. Furthermore, the credit spreads predicted by the model come close to observed spreads on credit default swaps.

Besides the out-of-sample prediction of credit spread levels, we also test the model along other dimensions. First, we show that the common jump process does not generate too much negative skewness in the actual equity index return distribution. Second, the jump-diffusion model somewhat underestimates the volatility of both equity returns and credit spread changes, although the fit is much better than for a model without jumps. This underestimation is caused by our matching of the historical default rates, which constrains the level of volatility in the model. Finally, the amount of default correlation generated by the model is quite similar to empirical estimates of default correlations.

The remainder of the paper is structured as follows. Section 2 discusses the contribution of our article to the existing literature. Section 3 presents the theoretical model. The calibration methodology is explained in Section 4. Section 5 describes the options data and discusses the parameter estimates. In Section 6, we compare the model-implied credit spreads with observed levels of corporate bond and CDS spreads. Section 7 studies default correlations, the volatility of credit spread changes and equity returns, and other out-of-sample predictions. The conclusion follows in Section 8.

## **2. Related Literature**

This article is closely related to the work of HH: our specification for the firm value process is the same as in HH, except that we distinguish between common and firm-specific jumps. However, our paper differs from HH in several major ways. First and most importantly, we estimate both the jump process and the jump risk premium from data on equity returns and option prices. To this end, we model the joint behavior of all firms in the equity index. In contrast, HH do not model this joint process and mostly focus on pure diffusion models. They do consider a jump-diffusion model, but do not estimate the parameters associated with the jump process. In Section 5.3, we discuss this in more detail. Second, HH only focus on credit spread implications and do not assess the model implications for equity and option prices. Our results show that allowing for jumps improves the fit not only for credit spread levels, but also for equity returns and option prices. Finally, we study the implications for the volatility of credit spread changes and perform a comparison of empirical versus model-implied default correlations.

Collin-Dufresne, Goldstein, and Helwege (2003) argue that jump risk premia are unlikely to explain the level of credit spreads. However, in their paper, the jump risk premium is not estimated, so that the empirical relevance of jump risk remains unclear. Berndt et al. (2004) and Driessen (2005) estimate a jump risk premium from CDS spreads and corporate bond spreads, respectively. Their results suggest that large jump risk premia are needed to explain the level of credit spreads. However, their jump risk premia are essentially fitted to the spread level. Instead, we provide an out-of-sample test of the importance of the jump risk premium. This has the advantage that our estimates are not affected by tax effects that impact corporate credit spreads or by liquidity premia in credit markets. Also, in contrast to Driessen and Berndt et al., our model does not necessarily have jumps directly to default: both the jump intensity and the jump size are estimated from equity and option data.

Eom, Helwege, and Huang (2004) and Ericsson, Reneby, and Wang (2005) analyze structural firm value models using firm-level data. However, they do not incorporate jump risk and jump risk premia, and their estimation methodology does not impose that the expected loss is matched to the data (as is done in HH and in this article). Carr and Wu (2005) use data on credit default swaps and equity options to estimate a jump-diffusion model where stock prices can jump to zero in case of default. They find evidence for a risk premium associated with the time variation in the jump intensity.

Finally, several authors have studied the determinants of credit spread variation, including, among other explanatory variables, implied volatilities of equity options (e.g., Collin-Dufresne, Goldstein, and Martin 2001, Hull, Nelken, and White 2004, and Cremers et al. 2008). In general, these articles document a significant relationship between equity option prices and credit spreads. However, these studies focus on explaining the empirically observed variation in credit spreads and do not analyze the pricing of default risk through structural models nor the impact of jump risk premia on the level of credit spreads.

### 3. The Model

We model the dynamics of all  $N$  firms in the stock market index. The firm value is exposed to correlated diffusion shocks, common jumps, and firm-specific jumps. The dynamics of the asset value  $V_{j,t}$  of firm  $j$ , where  $j \in \{1, \dots, N\}$ , under the physical measure are given by the following jump-diffusion process:

$$\frac{dV_{j,t}}{V_{j,t-}} = (\pi + r - \delta) dt + \sigma dW_{j,t} + dJ_t - \lambda \xi dt + dJ_{j,t}^f - \lambda_f \xi_f dt, \quad (1)$$

where  $\pi$  is the total firm value risk premium,  $r$  is the risk-free rate, and  $\delta$  is the payout rate (resulting from both coupon payments and dividends). This specification is also used by HH, except that we allow for firm-specific jumps. The Brownian motions affecting firms  $j$  and  $k$  are correlated according to a

correlation parameter  $\rho$ :

$$E [dW_{j,t}dW_{k,t}] = \rho dt, \quad \rho \in [-1, 1] \text{ for } j \neq k. \quad (2)$$

The common jump process  $J_t$  has the following structure:

$$J_t = \sum_{i=1}^{N_t} (Z_i - 1), \quad (3)$$

where  $N_t$  is a standard Poisson process with a jump intensity  $\lambda$  and  $\ln(Z_i)$  has a double-exponential distribution with density given by:

$$p_u \eta_u e^{-\eta_u \ln(z_i)} 1_{[\ln(z_i) \geq 0]} + (1 - p_u) \eta_d e^{\eta_d \ln(z_i)} 1_{[\ln(z_i) < 0]}, \quad (4)$$

where  $\eta_u, \eta_d > 0$  and  $0 \leq p_u \leq 1$ . Hence, any jump occurring is a downward jump with probability  $1 - p_u$ , and the associated jump size distribution is exponential with parameter  $\eta_d$ . The mean jump size is  $\xi \equiv E [Z_i - 1] = \frac{p_u \eta_u}{\eta_u - 1} + \frac{(1 - p_u) \eta_d}{\eta_d + 1} - 1$ . The processes  $W_{j,t}$  and  $N_t$ , as well as the random variables  $\{Z_i\}$ , are independent. Zhou (2001) proposes a similar jump-diffusion model for the value of the firm using a lognormal distribution for the jump size. The firm-specific jump processes, denoted  $J_{j,t}^f$  for firm  $j$ , are modeled in exactly the same way as the common jump process. Both the jump sizes and the counting processes underlying the common and firm-specific jumps (across firms) are assumed to be independent.

As discussed below in more detail, we ensure that we do not overstate the jump risk in the model by imposing that the historical default probability for each rating category is matched by the model. All parameters are assumed to be identical across firms in the model, in order to maintain parsimony. The only ex-ante heterogeneity concerns the initial firm value  $V_{j,0}$ , which depends on the rating of the firm.

We allow for risk premia on both the correlated diffusion shocks and the common jumps. We assume that firm-specific jumps do not carry a risk premium, since these jumps are diversifiable. Then, under a risk-neutral measure  $Q$ , the asset value is assumed to follow:

$$\frac{dV_{j,t}}{V_{j,t}} = (r - \delta) dt + \sigma dW_{j,t}^Q + dJ_t^Q - \lambda^Q \xi^Q dt + dJ_{j,t}^f - \lambda_f \xi_f dt, \quad (5)$$

where  $J_t^Q = \sum_{i=1}^{N_t^Q} (Z_i^Q - 1)$  with  $N_t^Q$  having jump intensity  $\lambda^Q$  and  $\ln(Z_i^Q)$  has a double-exponential distribution with parameters  $p_u^Q$ ,  $\eta_u^Q$ , and  $\eta_d^Q$ . The mean jump size is now:

$$\xi^Q \equiv E^Q [Z_i^Q - 1] = \frac{p_u^Q \eta_u^Q}{\eta_u^Q - 1} + \frac{(1 - p_u^Q) \eta_d^Q}{\eta_d^Q + 1} - 1. \quad (6)$$

The jump risk premium, an important economic parameter in our model, is thus given by  $\lambda\xi - \lambda^Q\xi^Q$ . The total firm value risk premium  $\pi$  is the sum of this jump risk premium and the diffusion risk premium  $\pi_d$ , so that we have  $\pi = \pi_d + \lambda\xi - \lambda^Q\xi^Q$ . We follow HH, who invoke the equilibrium analysis of Kou (2002) to motivate the following transformation from the physical to the risk-neutral measure:  $\lambda^Q = \lambda E[Z_i^{-\gamma}]$ , where the risk premium parameter  $\gamma$  can be interpreted as the coefficient of relative risk aversion of the representative agent in an equilibrium model. The same risk premium parameter dictates the mapping of the jump size parameters  $p_u$ ,  $\eta_u$ , and  $\eta_d$  from the double-exponential distribution under the physical measure to the double-exponential distribution under the risk-neutral measure:

$$p_u^Q = \frac{p_u\eta_u/\eta_u^Q}{p_u\eta_u/\eta_u^Q + (1 - p_u)\eta_d/\eta_d^Q}, \eta_u^Q = \eta_u + \gamma, \text{ and } \eta_d^Q = \eta_d - \gamma.$$

Each firm  $j$  has a single long-maturity coupon bond outstanding, maturing at  $T$  with face value  $F$ . This assumption is made for simplicity. The coupon rate is chosen so that the associated default-free bond trades at par. We assume that default occurs when the asset value  $V_t$  drops to, or below, an exogenous default boundary  $V_t^*$ . At maturity, for the default event to be well defined, we impose that  $V_T^* = F$ . Given that we only model a single bond, the default boundary would be 0 before maturity (since the coupon is automatically paid through the payout rate  $\delta$ ). However, to mimic a richer setting with multiple bond issues maturing at different points in time (where default can occur at these different points in time), we allow for a nonzero default boundary before maturity. Setting the default boundary for times  $t < T$  at  $V_t^* = F$  enables us to study the term structure of credit spreads up to the final maturity date  $T$ . As discussed by Black and Cox (1976) and Leland (1994), a formal justification for the constant default boundary is to assume that the bond covenant has a positive net worth provision. In practice, many bonds do not have such covenants. Although an endogenous default boundary (Black and Cox 1976 and Leland 1994) would be more appealing, Leland (2004) and Huang and Huang (2003) show that exogenous and endogenous default models generate quantitatively similar implications for default probabilities and credit spreads, respectively.<sup>3</sup>

Another important ingredient of the model is the recovery rule. We follow Leland (1994) and assume that if default occurs (at time  $\tau$ ), the bankruptcy costs are a fraction  $\alpha$  of the firm value at default  $V_\tau$ . That is, bondholders recover  $(1 - \alpha)V_\tau$  upon default.<sup>4</sup> In case of a pure diffusion model, it is clear that  $V_\tau = F$ . However, in a model with jumps,  $V_\tau$  can jump to a value below the default boundary  $F$ . Our model thus implies that recovery is particularly low when default was caused by a downward jump. Also, since common downward

<sup>3</sup> An exogenous default boundary generates higher short-term default probabilities. However, unreported results show that our model still underestimates short-term default probabilities.

<sup>4</sup> We thank an anonymous referee for suggesting this model of recovery.

jumps carry a jump risk premium, our model generates risk-neutral par recovery rates that are below actual par recovery rates. In Section 6.2, we discuss the effect of this recovery assumption on credit spreads.

Given the asset value process, default boundary process, and recovery rule, the model can be used in the standard fashion to price the corporate bond under the risk-neutral measure and to obtain its credit spread. Relative to HH, a novelty of our paper is that we also price each firm's equity. The equity values for all firms are added up to obtain the stock market index, which is used to price index options. Hence, the index option is priced under the risk-neutral measure as a compound option, namely an option on a portfolio of  $N$  call options. It is precisely from the prices of different index options that the crucial parameters concerning the common jump process and the jump risk premium are estimated, as is explained in detail in the next section.

#### **4. Calibration Methodology**

This section describes the calibration strategy for the two models that we analyze. The first model is the jump-diffusion model, which is the main focus of the paper. Second, we also calibrate a diffusion-only model to highlight the relative importance of jumps. Table 1 summarizes the calibration setup and contains all target values. Generally speaking, the calibration methodology is designed to fit historical information in equity and option prices, as well as historical default and recovery rates. In all cases, we strive to obtain long-term averages for the relevant calibration inputs, as our goal is to analyze the unconditional implications of a jump risk premium for credit spread levels. Importantly, the historically observed level of credit spreads is not included as one of the calibration targets, allowing us to compare our model's out-of-sample forecast to observed credit spread levels.

We model the joint firm value process of 100 firms in order to construct an equity index that closely resembles the S&P 100 index. We allow the initial firm value to depend on the rating of the firm. We use the rating distribution of all firms in the S&P 100 index as of February 2006, which is as follows: 5% (AAA), 16% (AA), 42% (A), 25% (BBB), 5% (BB), and 7% (B). The median and mode of this rating distribution are the A rating. Unreported results show that we obtain very similar results along all dimensions if we assume that all firms in the index are rated A.<sup>5</sup>

The calibration methodology consists of three steps. The first two steps apply to both the diffusion-only and jump-diffusion model, while the third step is only relevant for the jump-diffusion model. We now describe these three steps.

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<sup>5</sup> Over time, the composition of firms in the S&P 100 index changes. For example, poorly performing firms drop out of the index. We neglect such changes in the composition of the S&P 100 index. To price options and assess the equity return distribution, we only need to simulate the behavior of the stock index one month ahead in our model, so that such composition changes are unlikely to seriously affect our results.

**Table 1**  
**Target values in calibration**

	Target	Model	
		Jump-diffusion model	Diffusion-only model
Equity risk premium (A)	5.99%	Perfect fit	Perfect fit
Leverage ratio (A)	31.98%	Perfect fit	Perfect fit
Average recovery rate	51.31%	Perfect fit	Perfect fit
Stock return correlation	25.39%	Perfect fit	Perfect fit
10-year default prob (AAA)	0.77%	Perfect fit	Perfect fit
10-year default prob (AA)	0.99%	Perfect fit	Perfect fit
10-year default prob (A)	1.55%	Perfect fit	Perfect fit
10-year default prob (BBB)	4.39%	Perfect fit	Perfect fit
10-year default prob (BB)	20.63%	Perfect fit	Perfect fit
10-year default prob (B)	43.91%	Perfect fit	Perfect fit
OTM index put price ( $\times 100$ )	0.847	Imperfect fit	Out of sample
ATM index put price ( $\times 100$ )	2.073	Imperfect fit	Out of sample
ITM index put price ( $\times 100$ )	4.392	Imperfect fit	Out of sample
OTM expected put return	-46.52%	Imperfect fit	Out of sample
ATM expected put return	-24.78%	Imperfect fit	Out of sample
ITM expected put return	-14.81%	Imperfect fit	Out of sample
Individual option prices	Table 3B	Imperfect fit	Out of sample

The table contains target values used for the calibration of the diffusion-only model and jump-diffusion model (see Section 4). The table also summarizes the calibration strategy for each model. “Perfect fit” means that the calibration procedure is designed to perfectly fit this calibration target. “Imperfect fit” indicates that the calibration target is fitted by a nonlinear least-squares procedure. “Out of sample” means that the calibration target is not included in the calibration procedure.

#### 4.1 First two steps of the calibration methodology

In the first step, some parameters that are common to the two models considered, the diffusion-only model and the jump-diffusion model, are fixed at reasonable levels. Here we follow HH and fix the risk-free rate  $r$  and the payout rate  $\delta$  at 8% and 6%, respectively. HH choose the risk-free rate using historical data for Treasury yields and the payout rate as a weighted average of bond coupons and equity dividend rates. We focus on coupon-paying corporate bonds with 10 years to maturity. The face value  $F$  of the debt is normalized to 1.

In a second step we exactly match 10 moment restrictions. Of these 10 restrictions, 9 are the same as in HH. First of all, we calibrate to historical estimates of the cumulative 10-year default probability for all ratings from AAA to B, based on Moody’s data for 1970–1998.<sup>6</sup> For example, for an A-rated firm, the historical estimate for the 10-year default probability is 1.55%. Second, for A-rated firms, we calibrate to a historical estimate for the par recovery rate of 51.31% (Keenan, Shtogrin, and Sobehart 1999). The third calibration restriction involves an equity premium for A-rated firms of 5.99% per year (derived from the results of Bhandari 1988). Fourth, we calibrate to the firm leverage ratio, defined as the market price of the corporate bond divided by the firm value at time zero. Standard and Poor’s (1999) report a leverage ratio of 31.98% for A-rated firms. The intuition of these calibration restrictions is as follows. First, the leverage ratio is informative about the initial firm value,

<sup>6</sup> Throughout the paper, we assume that Moody’s and S&P ratings are the same.

the jump and diffusion risk premia are related to the equity premium, and the observed par recovery rate can be used to calibrate the recovery parameter  $\alpha$ . Further, the default probabilities depend strongly on the initial firm values and the firm value volatility. Importantly, the calibration approach ensures that both the expected loss on a corporate bond and the equity premium are matched to the data.

Since HH do not model the interaction between firms, they do not need to calibrate to cross-firm return dependence. We calibrate to the equity return correlation of S&P 100 firms, which is estimated by calculating the full correlation matrix for S&P 100 stocks over the 1996–2002 period (using data on daily equity returns) and subsequently taking the average across all correlations. This average correlation equals 25.4%.

As discussed above, to place the results for the jump-diffusion model in perspective, we also consider a pure diffusion model. For this model, the number of parameters<sup>7</sup> is equal to the number of moment restrictions discussed above, so that a perfect fit can be obtained.<sup>8</sup> The jump-diffusion model contains additional parameters, and consequently, its calibration involves a third step, which is described in the next subsection.

#### 4.2 Third step of the calibration methodology

The jump-diffusion model has nine additional parameters: the common and firm-specific jump intensities  $\lambda$  and  $\lambda_f$ , the probabilities  $p_u$  and  $p_u^f$  that the (common and firm-specific, respectively) jump is upward, the common and firm-specific jump size parameters  $\eta_u$ ,  $\eta_d$ ,  $\eta_u^f$ , and  $\eta_d^f$ , and the common jump risk premium parameter  $\gamma$ . In addition to the 10 calibration restrictions that are used for the diffusion-only model, we include 15 additional restrictions. First, we calibrate the model to the prices of S&P 100 index options and individual equity options. For both the index and individual options, we use three strike levels and calibrate to out-of-the-money, at-the-money, and in-the-money (OTM/ATM/ITM) option prices. We allow for rating heterogeneity at the firm level by calibrating to individual option prices across firms with different ratings. We have sufficient individual stock-option data for three rating categories: AA, A, and BBB. We standardize for the level of the stock price by dividing the option prices by the price of the underlying stock. For each rating category, we average the standardized individual option prices across all firms in this rating category and over all weeks in our sample. In total, this gives 12 additional moment restrictions (three index options, and  $3 \times 3$  individual options). The index option prices capture the distribution of stock index returns, and are therefore informative about the common jump intensity

<sup>7</sup> The parameters are the initial firm value for each rating category (six parameters), the recovery parameter  $\alpha$ , the firm value volatility  $\sigma$ , the expected excess return on the firm value  $\pi$ , and the diffusion correlation  $\rho$ .

<sup>8</sup> In the diffusion-only model, the firm value correlation is equal to the equity return correlation in continuous time. To speed up the calculations, we thus directly use a value for  $\rho$  of 25.4%. This procedure neglects a discretization error, which is expected to be small.

and the common jump size. Similarly, the individual stock-option prices are informative about the firm-specific jump intensity and the firm-specific jump size.

Finally, we calibrate to the expected returns on S&P 100 equity index options for the three moneyness levels. In the next section, we describe how these expected returns are estimated from historical data on S&P 100 index options and the index level. By including both option returns and option prices, we calibrate to information related to the actual as well as the risk-neutral behavior of equity prices. Moreover, by calibrating to both the equity premium and the expected option returns, we can disentangle the diffusion risk premium from the jump risk premium, since options generally have different loadings on diffusion and jump risk than equity. We will impose that the diffusion risk premium and the jump risk premium are both nonnegative, consistent with risk-averse investors, by requiring  $0 \leq \lambda\xi - \lambda^Q\xi^Q \leq \pi$ .

For this jump-diffusion model, the number of parameters is lower than the number of restrictions. We impose that the jump-diffusion model perfectly fits the 10 moments that are used for the diffusion model: the default probability (for each rating category), the par recovery rate, the leverage ratio, the equity premium, and the equity correlation. The remaining restrictions (the option prices and option returns) are fitted by minimizing the sum of squared percentage differences between the observed and model-implied restrictions (see Table 1 for an overview).

## **5. Estimation Results and Model Fit**

This section presents the parameter estimates and discusses the fit of the moment restrictions in the calibration. First, we discuss the option data.

### **5.1 Option data**

We use option data from OptionMetrics, consisting of options on the S&P 100 index and individual stocks, which are traded on CBOE (Chicago Board Options Exchange). The dataset contains daily end-of-day bid and ask quotes for options with various strike prices and maturities. The sample runs from January 1996 until September 2002.<sup>9</sup> We use data on individual stock options for different firms. These firms are chosen such that we have a matching sample of individual corporate bonds with available price data.

We focus on short-maturity put options that have on average one month to maturity as these typically have the largest trading volume (Bondarenko 2003). For each day and for each stock (and for the S&P 100 index), we collect the price of a put option whose remaining maturity is closest to one month and whose strike price is closest to the ATM level. Similarly, we collect the prices

<sup>9</sup> The equity premium estimate over the 1996–2002 period is 5.96% per year (using S&P 100 returns and the 3-month T-bill rate), which is extremely close to the equity premium of 5.99% (obtained from a longer sample) that we use in the calibration.

of put options whose strike price is closest to the 8% OTM and ITM levels for individual options, and closest to the 4% OTM and ITM levels for index options. We divide each option price by the price of the underlying stock (or index) and take the average over the full sample period. This gives average ITM, ATM, and OTM option prices (as a percentage of the underlying value), which we average across firms in the same rating category using the average rating of a firm over the 1996–2002 period. This way, we have individual option prices for 11 AA-rated, 36 A-rated firms, and 22 BBB-rated firms.<sup>10</sup>

Next, we construct estimates for the expected option returns. On each day, we construct the return to holding the option to maturity. Using the S&P 100 index option prices and daily data for the value of the S&P 100 index, we construct a time series of overlapping monthly option returns from which we obtain the average option return over the full sample.

The resulting estimates can be found in the “target” column in Table 1. The OTM, ATM, and ITM index option prices are equal to 0.85%, 2.07%, and 4.39% of the underlying index value, respectively. The average monthly put return equals about –15% for the ITM option, –25% for the ATM option, and –47% for the OTM option. While using a different sample period, these numbers are in line with average option returns reported in Bondarenko (2003) and Driessen and Maenhout (2007). Bondarenko constructs monthly S&P 100 option returns, holding the one-month options until expiration. Using a longer sample period, he reports an average monthly return of –58% for OTM options and –39% for ATM options. Driessen and Maenhout analyze returns on equity index options that are not held until expiration, and report an average monthly return on OTM put options of about –41%.

## 5.2 Parameter estimates

This subsection discusses the parameter estimates for the diffusion-only model and the jump-diffusion model. We also compare the estimation results to findings in the existing work.

**5.2.1 Diffusion-only model.** We start with the model without jumps, which may be viewed as a Longstaff-Schwartz (1995) model with constant interest rates. Because the number of parameters to be estimated equals the number of restrictions, this model achieves a perfect fit for the following target values: the 10-year default probabilities (for all rating categories), the equity risk premium, the leverage ratio, the average par recovery rate, and the stock return correlation.

The intuition for the estimates in Table 2 is straightforward. Given the 5.99% matched target value for the equity risk premium, combined with a leverage ratio of roughly 1/3, the firm or asset value risk premium  $\pi$  of 4.11% can be understood as follows. As a first-order approximation, the drift of the equity value process in a pure diffusion model can be written as the firm value drift

<sup>10</sup> We could not obtain sufficient data for option prices on stocks with AAA ratings or below-BBB ratings.

**Table 2**  
**Parameter estimates**

Model	Jump-diffusion model	Diffusion-only model
Firm value risk premium $\pi$	4.32%	4.11%
Firm value volatility $\sigma$	19.88%	19.95%
Recovery of firm value at default $(1 - \alpha)$	0.533	0.513
Diffusion correlation $\rho$	28.15%	25.39%
Initial firm value (AAA) $V_{AAA,0}$	3.705	3.523
Initial firm value (AA) $V_{AA,0}$	3.317	3.203
Initial firm value (A) $V_{A,0}$	3.056	3.072
Initial firm value (BBB) $V_{BBB,0}$	2.387	2.290
Initial firm value (BB) $V_{BB,0}$	1.582	1.561
Initial firm value (B) $V_{B,0}$	1.264	1.221
Jump intensity $\lambda$	0.056	–
Probability of upward jump $p_u$	0.000%	–
Upward jump size parameter $\eta_u$	–	–
Downward jump size parameter $\eta_d$	12.518	–
Risk premium par. $\gamma$	9.234	–
RN jump intensity $\lambda^Q$	0.203	–
Mean jump size $\xi$	–7.40%	–
RN mean jump size $\xi^Q$	–23.34%	–
Jump risk premium $\lambda\xi - \lambda^Q\xi^Q$	4.32%	–
Firm-specific jump intensity $\lambda_f$	0.219	–
Prob. of firm-specific upward jump $p_{u,f}$	0.408	–
Upward firm-specific jump size parameter $\eta_{u,f}$	11.976	–
Downward firm-specific jump size parameter $\eta_{d,f}$	10.088	–

The table reports estimates of the model parameters (obtained using the calibration procedure described in Section 4) for the jump-diffusion model and the diffusion-only model.

$(\pi + r - \delta)$  multiplied by the firm-value-to-equity ratio (roughly 3/2) and the delta of equity (close to 1). This results in an equity value drift of roughly 9%. Given a risk-free rate  $r$  of 8%, this is consistent with an equity risk premium of 6% if the dividend rate is around 5%. A dividend rate of 5% is reasonable for our model since the coupon rate is 8% and the total payout rate of the firm (weighted sum of coupons and dividends) equals 6%. In fact, using weights of 1/3 and 2/3 for debt and equity, respectively, produces exactly a 5% dividend rate. The firm value volatility  $\sigma$  of 19.95% is consistent with an individual equity return volatility of around 30%, due to the leverage effect. The initial firm value  $V_{A,0}$  of 3.072 is clearly driven by the leverage ratio of roughly 1/3, since the face value of the debt is normalized to 1 and the debt is not very risky. The recovery fraction  $(1 - \alpha)$  equals 51.31% since at default the firm value is always equal to the default boundary  $F$ . Finally, as discussed before, the firm value diffusion correlation  $\rho$  equals the stock return correlation in this model, since jumps are absent.

**5.2.2 Jump-diffusion model.** Next, we add priced systematic jumps and firm-specific jumps to the model. As before, we constrain the estimation to perfectly fit the historical default probability and the recovery rate, so that the credit risk in the model is not inflated. Table 2 shows that the recovery fraction  $(1 - \alpha)$  is equal to 0.533 for the jump-diffusion model. The average actual par recovery rate then matches the calibration target of 51.31%, because the

average firm value at default is 96.3% of the face value under the actual measure  $P$  for an A-rated firm.

The parameter estimates reveal an important role for jumps and jump risk premia. We first discuss the parameters of the common jump process. The jump intensity of 0.056 translates to a common jump hitting the economy every 18 years. The estimate of the probability that a jump is upward equals 0, so that the model only generates downward jumps. Intuitively, this is because the most important mismatch in the diffusion model is in the left tail of the equity index return distribution, both under the risk-neutral and actual probability measure (as discussed below in more detail). It thus turns out to be optimal to only incorporate downward common jumps in the model, in order to maximize the amount of negative skewness in the equity index return distribution. Note that the amount of negative skewness that can be incorporated is limited by fitting the actual default probability. The estimate for the downward jump size parameter  $\eta_d$  of 12.52 corresponds to a mean jump size  $\xi$  of  $-7.4\%$ .

The jump risk premium parameter  $\gamma$  is estimated at 9.23. Judging from the jump intensity and the mean jump size under the risk-neutral measure implied by  $\gamma$ , it is clear that  $\gamma = 9.23$  is nontrivial. Under the risk-neutral measure, common jumps with mean size  $-23.3\%$  hit the economy every 5 years ( $\lambda^Q = 0.203$ ). The effect of  $\gamma$  is summarized by the jump risk premium  $\lambda\xi - \lambda^Q\xi^Q$  of 4.32%. Given the total firm value risk premium  $\pi$  of 4.32%, this implies that the estimated diffusion risk premium equals 0%. That is, the restriction imposed in the calibration that the diffusion risk premium be nonnegative is binding. Without this restriction, the diffusion risk premium estimate would be slightly negative at  $-0.55\%$ .

It seems hard to explain these estimates for the jump and diffusion risk premia from representative-agent equilibrium models with standard expected utility preferences. This follows from the analysis of Kou (2002), who shows that, under certain conditions, the risk premium parameter  $\gamma$  can be interpreted as the relative risk aversion parameter of a representative agent. A coefficient of relative risk aversion of 9.23 clearly reflects the high jump risk premium embedded in index option prices. However, in an expected-utility equilibrium model (as in Naik and Lee 1990), such high risk aversion is unlikely to generate a zero value for the diffusion risk premium. Several articles in the options literature come to a similar conclusion (Pan 2002, Bondarenko 2003, and Driessen and Maenhout 2007): it is hard to explain the observed size of risk premia embedded in options prices using standard expected-utility equilibrium models (where both jump and diffusion risk would be priced). For example, Pan (2002) estimates diffusion and jump risk premia from equity index options and finds that only the jump risk premium is statistically significant. In this article, we analyze whether option prices and corporate bonds exhibit similar risk premia, taking the empirically observed risk premium as given.

Next, we turn to the estimates for the firm-specific jump parameters (Table 2). Both upward and downward jumps are relevant for explaining the

individual stock-option prices across strikes: conditional on a jump occurring, the probability that the jump is upward is about 41%. Relative to common jumps, firm-specific jumps occur more often (once every 5 years), and are slightly larger. The expected downward jump size equals about  $-9\%$ , while the expected upward jump size is  $7.7\%$ . As discussed in Section 5.3, these firm-specific jumps help to capture the skewness and the kurtosis that is present in individual stock returns and reflected in individual option prices. Also, given that the model is calibrated to exactly fit historical default rates, incorporating firm-specific jumps is important since one would otherwise overstate the size and frequency of common jumps.<sup>11</sup>

**5.2.3 Comparison to previous work.** In this subsection, we compare the common jump parameter estimates with results from previous work. This comparison is not straightforward, since we model jumps in the firm value, whereas existing work has focused on jumps in equity prices. However, as argued above, given the leverage ratio and delta of equity, we can multiply firm value shocks with a ratio of roughly 1.5 to obtain shocks to the equity price of A-rated firms. This would generate a mean equity jump size of about  $-11\%$  under the actual measure, and of about  $-35\%$  under the risk-neutral measure. This can be compared with the recent work of Pan (2002) and Eraker (2004), who both estimate a jump-diffusion model with stochastic volatility from equity index returns and prices of equity index options. They restrict the jump intensity to be the same under the actual and the risk-neutral measures, and report an average jump intensity of about 0.36 (Pan) and 0.50 (Eraker) per year,<sup>12</sup> which is much larger than our estimates. Pan reports mean jump sizes of  $-0.3\%$  and  $-18\%$  under the actual ( $P$ ) and the risk-neutral ( $Q$ ) measures, while Eraker reports mean jump sizes of  $-0.38\%$  ( $P$ ) and  $-2.00\%$  ( $Q$ ), respectively. These values are somewhat smaller than our estimates. One reason for this difference may be that Pan and Eraker also include stochastic volatility, which is negatively correlated with equity returns. This already captures some of the negative skewness in the return distribution. However, most important for our purpose is the total jump risk premium. The estimates of Pan imply a jump risk premium of 6.39% per year, while our firm value jump risk premium of 4.32% generates an equity jump risk premium of 5.99%. Without the nonnegativity restriction on the diffusion risk premium, the model would generate an equity jump risk premium of 6.75%. Thus, our estimate for the jump risk premium is very much in line with Pan's estimate. Comparing with Eraker is more difficult, since he also includes a volatility risk premium. His estimates imply a jump risk premium of 0.85%.

Next, we compare our jump parameter estimates with Ait-Sahalia, Wang, and Yared (2001). They propose a peso-problem interpretation for the difference

<sup>11</sup> Without firm-specific jumps, the estimate for the common jump intensity under  $P$  is 0.306, with a mean jump size of  $-9.34\%$ .

<sup>12</sup> These values are obtained by multiplying the parameter  $\lambda$  with  $\bar{v}$  in Pan (2002), and by multiplying the parameter  $\lambda_0$  in Eraker (2004) with 252 (the number of trading days).

between the option-implied stock price distribution and the probability distribution generated by a one-factor diffusion model with time-varying volatility. Fixing the jump size at  $-10\%$ , they show that a risk-neutral intensity of  $1/3$  per year generates the smallest difference between the option-implied and model-implied risk-neutral probability distributions. These numbers can directly be interpreted as estimates of risk-neutral jump parameters. Our jump estimates under the risk-neutral measure are somewhat different, since they generate jumps that are more negative, but occur less often. This could be due to a difference in the modeling approach (our model does not have a fixed jump size) or sample period, but also to a difference in calibration methodology: Ait-Sahalia, Wang, and Yared (2001) focus on fitting the skewness and kurtosis of the stock price distribution, while we calibrate to option prices and returns.

### **5.3 Fit of moment restrictions**

In this subsection, we analyze the calibration fit of option prices and returns. For the diffusion model, this is a direct out-of-sample test, since the model parameters are not calibrated to option prices and returns. In Table 3, Panel A shows that this model generates expected option returns that are much less negative than the observed average option returns. For example, the OTM expected option return predicted by the model equals  $-19.9\%$ , while the empirical average is  $-46.5\%$ . Therefore, the model does not capture the risk premia embedded in observed option returns, in line with evidence in the option pricing literature that additional risk factors are priced.

The diffusion-only model also generates option prices that are well below the observed option prices. For example, over our 1996–2002 sample, the average ATM index option price equals  $2.07\%$ , as a fraction of the underlying value. The diffusion-only model generates a price of  $1.48\%$ . Similarly, the model underprices individual options: for A-rated firms, the average ATM option price is  $3.50\%$ , while the model predicts  $3.01\%$  (Table 3, Panel B).

Table 3 also shows that, in relative terms, the underpricing is most severe for OTM options. Because we consider put options, this means that the pure diffusion model generates an index return distribution that lacks the considerable degree of negative skewness that is embedded in the observed option prices (i.e., the volatility skew). This apparent mispricing of index options occurs even though the pure diffusion model endogenously produces stochastic volatility for the S&P index return, because the well-known leverage effect makes equity volatilities stochastic since equity is a call option on the firm value. As an intuitive illustration, we calculate the Black-Scholes implied volatilities, both from observed index put prices and from the index put prices generated by the model. For the latter, the implied volatilities are  $13.7\%$  (ATM) and  $14.5\%$  (OTM), exhibiting a very slight implied-volatility skew. This finding of a very slight skew is consistent with Toft and Prucyk (1997), who only find a significant implied skew when firms have high leverage. However, the observed implied volatilities for the ATM and OTM index options are  $19.1\%$  and  $21.6\%$ ,

**Table 3**  
**Option pricing implications**

*Panel A: Index options*

	Observed	Jump-diffusion model	Diffusion-only model	HH (2003) example 1	HH (2003) example 2
OTM put price ( $\times 100$ )	0.847	0.553	0.308	0.430	0.387
ATM put price ( $\times 100$ )	2.073	1.782	1.478	1.471	1.293
ITM put price ( $\times 100$ )	4.392	4.363	4.084	4.170	4.136
OTM Exp. put return	-46.52%	-50.07%	-19.92%	-33.05%	-64.38%
ATM Exp. put return	-24.78%	-23.94%	-14.11%	-21.09%	-33.41%
ITM Exp. put return	-14.81%	-13.10%	-9.24%	-10.71%	-15.66%

*Panel B: Individual options*

	Observed	Jump-diffusion model	Diffusion-only model
AA-rated firms: OTM put price ( $\times 100$ )	0.915	0.716	0.501
AA-rated firms: ATM put price ( $\times 100$ )	3.164	3.130	2.911
AA-rated firms: ITM put price ( $\times 100$ )	8.436	8.549	8.322
A-rated firms: OTM put price ( $\times 100$ )	1.086	0.750	0.567
A-rated firms: ATM put price ( $\times 100$ )	3.496	3.214	3.010
A-rated firms: ITM put price ( $\times 100$ )	8.694	8.659	8.395
BBB-rated firms: OTM put price ( $\times 100$ )	1.285	0.898	0.708
BBB-rated firms: ATM put price ( $\times 100$ )	3.742	3.451	3.277
BBB-rated firms: ITM put price ( $\times 100$ )	8.948	8.762	8.578

This table reports empirical estimates and model implications for option prices and expected option returns. Panel A reports average option prices and average monthly option returns for S&P 100 index options, as observed over the 1996–2002 sample period and as implied by the jump-diffusion and the diffusion-only models, and for two examples of jump-diffusion models presented in Huang and Huang (2003). The OTM (ITM) index options are 4% out-of-the-money (in-the-money). Panel B reports the average prices of individual stock options over the 1996–2002 period and the model-implied prices. The observed option prices are averaged according to the rating category of the underlying stock. The OTM (ITM) stock options are 8% out-of-the-money (in-the-money). Option prices are expressed as a fraction of the price of the underlying asset.

respectively.<sup>13</sup> In sum, the diffusion-only model misses both the overall level of option prices and the volatility skew.

The model with jumps fits expected index option returns and option prices much better (Table 3). The most important improvement of the jump-diffusion model over the diffusion-only model is in fitting expected option returns. Due to the jump risk premium, the model predicts an expected return on OTM puts of -50.1%, which is close to the empirical counterpart of -46.5%. For ATM and ITM options, there is also a large improvement in the fit of option returns. Thus, allowing for a jump risk premium helps considerably in fitting expected option returns, without increasing the total equity premium.

Due to the jump risk premium, the jump-diffusion model also generates index option prices that are much closer to observed prices than the diffusion-only model. Still, observed option prices are even higher. Translating the prices in Table 3, Panel A into implied volatilities, the model generates 16.5% (ATM) and

<sup>13</sup> These implied volatilities are calculated by inverting the average observed option prices, assuming an annual interest rate of 3.5% and dividend rate of 2.5%.

18.0% (OTM), versus 13.7% and 14.5%, respectively, for the diffusion-only model, compared to observed implied volatilities of 19.1% and 21.6%, respectively. The jump-diffusion model generates an implied skew that is much larger than the diffusion model, and closer to the observed implied skew. Turning to individual options, Table 3, Panel B shows that adding both systematic and firm-specific jumps also considerably improves the fit of option price levels. For ITM options, the fit is particularly good as a result of including firm-specific upward jumps. Allowing for common and firm-specific downward jumps also leads to a better fit of OTM options. Still, the jump-diffusion model underestimates the level of OTM equity option prices, because the size and frequency of the downward jumps is restricted by calibrating the model to default probabilities.

In sum, the jump-diffusion model fits option prices and returns much better than the diffusion-only model. In particular, the fit of average index option returns is very good, while the jump-diffusion model somewhat underprices options. In other words, the model provides a very good description of the difference between the risk-neutral and the actual equity index return distributions, while underestimating the dispersion of the risk-neutral distribution. In Section 7.3, we will see that the model also underestimates the dispersion of the actual equity index return distribution, and we will discuss this result in more detail. For our purpose, the very good fit of option returns is most important, since the associated jump risk premium is a key determinant of credit spread levels.

Another interesting aspect of the model fit in terms of option prices concerns the variation of individual option prices across ratings. Empirically, Table 3, Panel B shows that lower-rated firms have higher option prices. Even though we restrict all parameters to be the same across ratings, our firm-value model also generates variation of option prices across ratings. This is because we allow the current firm value (and thus the leverage ratio) to vary across ratings. The results in Panel B of Table 3 show that the model indeed generates higher option prices for lower-rated firms. Still, the empirically observed variation across ratings is slightly larger, suggesting that the parameters of the firm value processes differ across ratings.<sup>14</sup>

Finally, we compare our estimated jump-diffusion model with the two examples of jump-diffusion models presented by HH. As discussed above, HH do not estimate the common jump parameters, but consider two examples for the common jump parameters. In both examples, HH impose that the total risk premium equals the jump risk premium, which is thus in line with the outcome of our estimation. HH's first example has a common jump intensity of  $\lambda = 3$ , and symmetric and small jumps:  $p_u = 0.5$  and  $\eta_u = \eta_d = 30$ . The second example has less frequent but very large jumps, which are still symmetric:  $\lambda = 0.1$ ,  $p_u = 0.5$ , and  $\eta_u = \eta_d = 5$ . These two parameter sets used

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<sup>14</sup> A model with, for example, firm-specific regime switches in some of the parameters may help to explain this variation better. We leave this for future research.

by HH are very different from our estimates, which imply more downward than upward jumps (generating negative skewness) and jump sizes that are in between the two cases presented by HH. We examine the implications of their parameters for options as follows. Since HH do not incorporate firm-specific jumps, we calibrate all parameters of the jump-diffusion model using our calibration approach, but impose that the common jump parameters are given by one of the two examples of HH.

In Table 3, Panel A presents the equity index option prices and returns generated by the two examples of HH. The results for option prices show that our calibrated jump-diffusion model generates a much better fit. For ATM index options, the diffusion-only model actually outperforms the HH examples with jumps. Unreported results show that the calibrated jump-diffusion model also outperforms the two HH examples in terms of fitting individual option prices. Turning to the average option returns, the results show that example 1 of HH (frequent small jumps) generates option returns that are less negative than in the data, while example 2 (infrequent large jumps) generates option returns that are too negative. Overall, our calibrated jump-diffusion model fits the observed data best. Given that HH impose that the total firm value risk premium equals the jump risk premium, the difference with our results is fully due to differences in the jump size distribution and jump frequency. This illustrates the importance of calibrating the jump-diffusion model to equity and option data.

## **6. Implications for Credit Spread Levels**

We now turn to the main results of the paper, namely the out-of-sample implications of the models for the credit spread level. We first discuss the credit spread data that we use for this analysis, and then present the results for credit spread levels in Section 6.2 and for credit spread term structures in Section 6.3.

### **6.1 Credit spread data**

Table 4 presents the average credit spread levels for 10-year bonds as used by HH, based on Lehman data for 1973–1993. We also consider a different sample of Datastream data on Lehman corporate bond indices, which runs from 1983 until 2002.<sup>15</sup> The second column of Table 4 contains the time-series averages for these credit spreads, which are very similar to the levels reported by HH.

We further validate our results using a different sample of corporate bond prices from Bloomberg for 1996–2002 for those 69 firms (524 corporate bond issues) for which we have individual stock option data available. This allows a detailed study of the implications for the term structure of credit spreads.<sup>16</sup>

<sup>15</sup> Datastream reports (average) yields for both intermediate and long-maturity government and corporate bond indices, as well as the average maturity for each index. We first subtract the appropriate Treasury yield from the yield of each corporate index to obtain credit spreads. Subsequently, we interpolate between the intermediate and long-maturity credit spreads to obtain 10-year maturity credit spreads.

<sup>16</sup> We only use bonds with constant, semiannual coupon payments. Bonds with embedded put or call options or sinking fund provisions are excluded. As in Duffee (1999), observations on bond prices with remaining

**Table 4**  
**Observed corporate bond and credit default swap spreads**

Source period maturity	Corporate bond spreads			Credit default swap spreads	
	HH 1973–1993 10-year	Lehman index 1983–2002 10-year	LMN 2001–2002 5-year	LMN 2001–2002 5-year	LMN extrapolated 5-year
AAA	63	66	–	–	–
AA	91	92	104	51	45
A	123	115	151	80	65
BBB	194	171	229	156	132
BB	320	332	428	358	268
B	470	548	–	–	–

The table contains average credit spread levels (in basis points) across rating categories, (i) as reported by Huang and Huang (2003), (ii) estimated using a 1983–2002 sample of credit spread data for Lehman indices, and (iii) as reported by Longstaff, Mithal, and Neis (LMN 2005). In addition, the table reports average credit default swap spreads (in basis points), as reported by LMN, and an extrapolation of these spreads to the 1973–1993 sample period of HH (see Section 6).

Besides corporate bond price data, we also use Bloomberg data on the 6-month US T-Bill, and the most recently issued US Treasury Bonds with maturities closest to 2, 3, 5, 7, and 10 years.

We extract a term structure of zero-coupon credit spreads using these bond prices at the rating level, as in Elton et al. (2001). Each week, we assign the corporate bonds to rating-maturity buckets, with maturity intervals of 1–3 years, 3–5 years, 5–7 years, 7–9 years, and 9–11 years. We then assume that the term structure of zero-coupon corporate rates is flat within each maturity interval, allowing the estimation of the term structure for each rating category. The term structure of default-free zero-coupon interest rates is estimated in the same way, so that par-coupon credit spreads per rating category can readily be calculated. We only have sufficient data for AA, A, and BBB ratings. Figures 1–3 depict the observed par-coupon credit spread term structures. In line with several other papers (e.g., Duffee 1999 and Elton et al. 2001), we find upward sloping credit spread term structures for these investment-grade firms.

Finally, we also corroborate our results using data on credit default swaps (CDS). Recently, several articles have analyzed CDS spreads, e.g., Longstaff, Mithal, and Neis (LMN 2005) and Blanco, Brennan, and Marsh (2005). These CDS spreads may provide a better measure of default risk than spreads on corporate bonds, since CDS spreads are not influenced by tax effects and have become more liquid than corporate bonds. Unfortunately, only recent data for CDS spreads are available. LMN report average corporate bond and CDS spread levels for the period March 2001 until October 2002, presented in the third and fourth columns of Table 4.<sup>17</sup> These CDS spreads are clearly much lower than corporate bond credit spreads, and LMN attribute the difference to tax and

maturity less than one year are dropped. Most bonds are senior unsecured. We only include other bonds, such as subordinated bonds, if they have the same rating as the senior unsecured bonds of the particular firm.

<sup>17</sup> LMN report results for different default-free term structures. We use the results that are based on the Treasury yield curve, for contracts with 5 years to maturity (see their Table 2).

Figure 1: Term structure of AA credit spreads

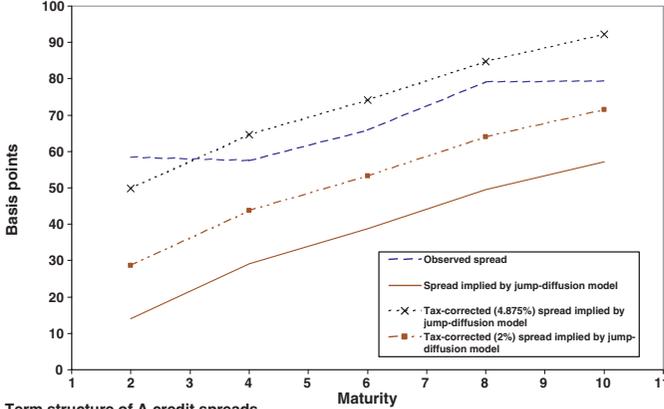


Figure 2: Term structure of A credit spreads

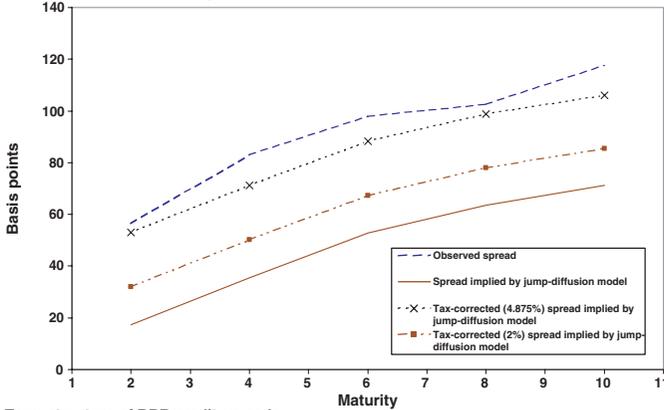
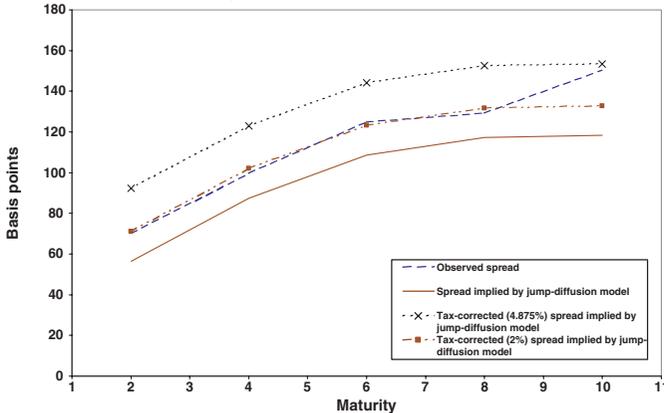


Figure 3: Term structure of BBB credit spreads



**Figures 1–3**

**Term structure of credit spreads**

These figures depict observed term structures of par-coupon credit spreads, constructed using Bloomberg corporate bond price data from 1996 to 2002, for three rating categories, AA (Figure 1), A (Figure 2), and BBB (Figure 3). Section 6 describes how the par-coupon spreads are constructed. The figures also contain the par-coupon credit spreads implied by the jump-diffusion model, with and without a correction for a tax on corporate bond coupons using tax rates of 4.875% and 2%, respectively (Elton et al. 2001).

liquidity effects. Because credit spreads on corporate bonds were historically high during the 2001–2002 sample period, we also provide extrapolated CDS spreads for the full 1973–1993 period of HH in Table 4.<sup>18</sup> In the next subsection, we compare these spread levels with the model-implied credit spreads.

## 6.2 Credit spread levels

We first briefly discuss the results for the diffusion-only model, which generates very low credit spreads. For example, the 10-year credit spread equals 27 basis points for A-rated debt (Table 5, Panel A). This number is similar to what HH report, across different structural firm value models and across a variety of different parameterizations. Empirically, the credit spread for 10-year debt of this rating is at least four times as large (depending on the sample period). For other ratings, the model-implied credit spreads are also well below the observed spreads (Table 5, Panel A). The relative difference between model-implied and observed credit spreads is large especially for investment-grade firms. Even when looking at CDS data, Table 4 shows that the observed 5-year spreads range from 45 basis points (AA) to 132 basis points (BBB), while the model (unreported) generates 5-year par-coupon spreads between 4 basis points (AA) and 37 basis points (BBB).

Next, we turn to the results for the jump-diffusion model. The option-implied jump risk premium brings the 10-year credit spread level from 27.1 basis points to 70.7 basis points for A-rated firms (Table 5, Panel A). Given that the 10-year default rate equals 1.55% for A-rated firms, this is rather encouraging. Similarly, for other rating categories, a large increase in the credit spread is obtained. Part of the increase in credit spreads relative to the diffusion-only model is due to the risk premium on recovery rates (as discussed in Section 3): the jump-diffusion model generates an average risk-neutral par recovery rate of 47.9%, which is below the average actual par recovery rate of 51.3%. Unreported results show that this recovery risk premium accounts for about five basis points of the total credit spread of A-rated firms.

In order to directly test whether our model predictions are compatible statistically with the average credit spread in our 1983–2002 sample, we compute the 95% confidence interval (using Newey-West) for the estimated average credit spread (Table 5). For investment-grade ratings, the model-implied spread is outside this 95% confidence interval, showing that in our model credit risk does not fully explain observed credit spreads on corporate bonds.<sup>19</sup> However, many authors have demonstrated the presence of nondefault-related factors in credit spreads, like taxes and liquidity. To illustrate this, we also calculate the

<sup>18</sup> We attempt to correct for this by assuming that the ratio of CDS to corporate spreads is constant. We scale the CDS spreads by the ratio of the corporate bond spreads reported by HH to the corporate bond spreads reported by LMN.

<sup>19</sup> Note that the model-implied credit spreads are based on parameter estimates, which contain estimation error. Our procedure of comparing the model-implied spread with the empirical confidence interval is thus conservative in the sense that we reject the jump-diffusion model too often.

**Table 5**  
**Model implications for credit spread levels and the volatility of credit spread changes**

*Panel A: 10-year corporate bond credit spread level*

Rating	Lehman index data Avg. (2.5% LB)	Jump-diffusion model	Diffusion model	Jump-diffusion model <i>Tax corrected</i>	Diffusion model
AAA	66.3 bp (55.0 bp)	47.6 bp	11.7 bp	82.7 bp	46.9 bp
AA	91.9 bp (75.2 bp)	56.5 bp	19.4 bp	91.6 bp	54.6 bp
A	115.4 bp (97.4 bp)	70.7 bp	27.1 bp	105.7 bp	62.2 bp
BBB	171.0 bp (147.7 bp)	118.3 bp	68.4 bp	152.3 bp	102.7 bp
BB	332.6 bp (281.4 bp)	309.0 bp	247.9 bp	339.9 bp	280.4 bp
B	547.8 bp (475.6 bp)	512.3 bp	478.7 bp	535.9 bp	502.9 bp

*Panel B: Standard deviation of monthly credit spread changes*

Rating	Lehman index data Avg. (2.5% LB)	Jump-diffusion model	Diffusion model
AAA	10.3 bp (7.2 bp)	4.1 bp	1.5 bp
AA	10.9 bp (6.6 bp)	4.4 bp	2.3 bp
A	11.5 bp (6.9 bp)	6.3 bp	2.8 bp
BBB	16.0 bp (10.6 bp)	11.2 bp	6.7 bp
BB	41.2 bp (30.4 bp)	24.4 bp	20.9 bp
B	58.8 bp (41.2 bp)	52.3 bp	37.6 bp

Panel A reports model implications for the level of the credit spread on a 10-year par-coupon corporate bond, as well as the average credit spread of Lehman corporate bond indices over a 1983–2002 sample period. In brackets, we report the value for the 2.5% confidence level of the average credit spread, calculated using the standard error (based on Newey-West with 12 monthly lags). We also report tax-corrected credit spread levels, using the tax correction on corporate bond coupons of Elton et al. (2001), with a tax rate of 4.875%. Panel B reports the standard deviation of monthly changes in the credit spreads on Lehman corporate bond indices (1983–2002), and as implied by the models. The model-implied standard deviation is calculated by simulating credit spreads for 100 firms per rating category, and subsequently calculating the monthly variation of the average spread across firms. For the empirical standard deviation, we also calculate the standard error and the corresponding 2.5% confidence level value (using the delta-method).

model-implied credit spreads including the tax effect of Elton et al. (2001), which involves a state tax on corporate bond coupons that does not apply to Treasury bond coupons. Using a tax rate of 4.875% (reported by Elton et al. 2001) as the midpoint of the range of maximum state taxes] and following their procedure, Table 5 reports the tax-corrected model-implied credit spreads. In this case, the spreads implied by the jump-diffusion model are inside the 95% confidence interval for five out of six rating categories. The spreads implied by the diffusion-only model are still outside the confidence interval for 5 out of 6 ratings. These results illustrate that the tax effect is a candidate for explaining the residual error, but we do not want to claim here that the tax rate of 4.875% is the appropriate marginal tax rate faced by the marginal investor. The size of the tax effect is debated: some investors (e.g., pension funds) are tax exempt (Grinblatt 2001) and tax arbitrage could potentially neutralize the tax effect. Empirically, there is mixed evidence for the presence of a tax effect: Elton et al. (2004) find a positive relation between credit spreads and coupon rates, which is consistent with a tax effect, while Longstaff, Mithal, and Neis (2005) find only weak evidence for a coupon-effect on spreads. In Section 6.3, we analyze

the effect of a smaller tax rate of 2%. As an alternative or complement to the tax effect, liquidity effects may well explain (part of) the residual difference between observed and model-implied credit spreads.<sup>20</sup>

Next, we compare the model-implied 5-year spreads with the observed 5-year credit default swap spreads. The model generates 5-year spreads between 31 basis points (AA) and 95 basis points (BBB). Comparing this with the last column of Table 4, we see that our model still underestimates CDS spreads, but much less so than the pure diffusion model. Of course, CDS spreads may contain a liquidity premium, so that it is not clear that the firm value model should fully match the observed CDS levels. In addition, the CDS spread averages are estimated using a short sample period.

These results provide evidence that the severe mispricing of corporate bonds by existing structural models can be remedied to a large extent by introducing priced jump risk. Intuitively, having jumps allows the model to generate more negative skewness in the firm value return distribution. Furthermore, having jump risk premia enables the model to generate higher credit spread levels.

Finally, we again compare our results with the two jump-diffusion examples in HH, now focusing on credit spread levels. Allowing for firm-specific jumps, we use the same calibration approach for the HH examples, as discussed in Section 5.3: we fix the common jump parameters and calibrate all remaining parameters in the usual way. For HH example 1, this generates a 10-year A spread of 23.2 bp, while example 2 renders a spread of 93.6 bp for an A-rated firm,<sup>21</sup> while the model calibrated in this article generates a spread of 70.7 bp. Given that all these credit spread levels reflect the same jump risk premium, this again illustrates that the choice of jump parameters is nontrivial and has a considerable impact on the credit spread levels.

### **6.3 Term structure of credit spreads**

So far, we have focused on explaining 5- and 10-year credit spread levels. However, previous work has shown that short-maturity credit spreads are even harder to explain using firm value models. Given that we have incorporated a default boundary that equals the face value of the debt during the 10-year maturity period, we can directly value (hypothetical) bonds with shorter maturities. For each maturity, we choose a coupon rate such that the associated default-free bond trades at par and calculate the credit spread relative to a default-free bond with the same maturity and coupon.

The results in Figures 1–3 show that incorporating jumps and a jump risk premium helps to generate larger short-maturity credit spreads. While the model without jumps generates short-maturity credit spreads that are in most cases essentially zero (not reported), the jump-diffusion model generates nonzero credit

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<sup>20</sup> See, for instance, Chen, Lesmond, and Wei (2007), and De Jong and Driessen (2005).

<sup>21</sup> In their model without firm-specific jumps, HH report values of 26.1 bp and 101.8 bp in their Tables 8 and 9, respectively.

spreads. We compare these with observed term structures of credit spreads, constructed using Bloomberg data on individual corporate bonds of different maturities (see Section 6.1). The figures show that the empirically observed shapes of the credit spread term structure are very similar to the model-implied shapes. We also graph the impact of the tax correction of Elton et al. (2001), applying the tax rate of 4.875% used in Section 6.2, and, for comparison, a tax rate of 2%. Interestingly, with a tax rate of 4.875%, the model overestimates credit spread levels for AA- and BBB-rated firms, while the model with a 2% tax rate underestimates credit spread levels for AA and A firms. This actually suggests that it is unlikely that the tax effect can fully explain the residual pricing errors of the jump-diffusion model, as was also discussed in Section 6.2.

## **7. Tests of Other Out-of-sample Implications**

In this section, we perform several additional out-of-sample analyses to examine the implications of the jump-diffusion model. We assess its implications for (i) the term structure of default probabilities, (ii) the volatility of credit spread changes, (iii) the distribution of equity index returns, (iv) default correlations, and (v) the behavior of credit spreads before and after the 1987 crash.

### **7.1 Term structure of default probabilities**

In this subsection we study the term structure of default probabilities generated by the model. In particular, we analyze the relative importance of jumps versus diffusion shocks for generating default events.<sup>22</sup> We want to verify that default events are not only the result of jumps, since in that case the model would generate a very irregular time series of default rates; therefore, we generate two term structures of default probabilities for each rating category: the “total” default probability generated by the jump-diffusion model and the term structure of default probabilities that is obtained when we set all jump intensities (both common and firm-specific) equal to zero in the jump-diffusion model (without changing the other parameter values in the model).

Figures 4 and 5 depict the results for investment-grade firms (with similar results for speculative-grade firms). The results indicate that a considerable part of the default probability is generated by diffusion shocks. Even for AAA-rated firms, about half of the default probability is due to diffusion shocks. The main reason for the moderate impact of jumps is that the jump intensities under the actual measure are modest. For lower ratings, diffusion shocks account for an even larger fraction of the total default probability. These results show that even though our model contains large jumps, diffusion shocks and jumps both play an important role in the model.

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<sup>22</sup> Leland (2004) argues that including jump risk is important to generate a realistic term structure of default probabilities.

Figure 4: Cumulative default probabilities

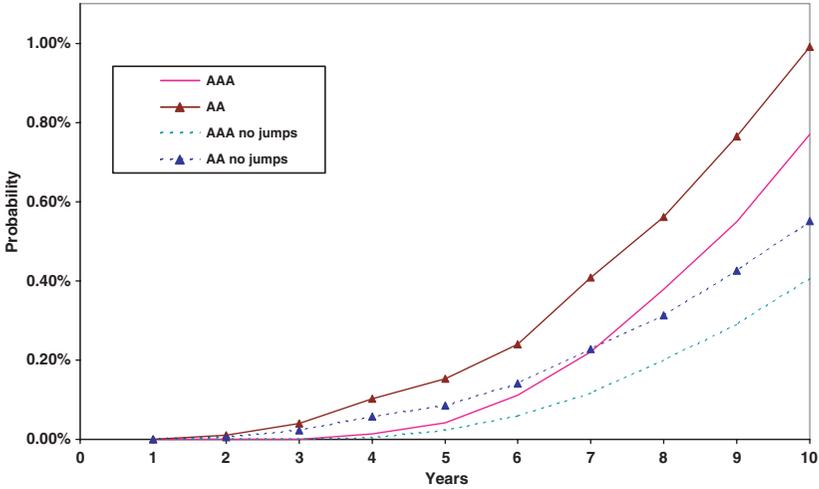
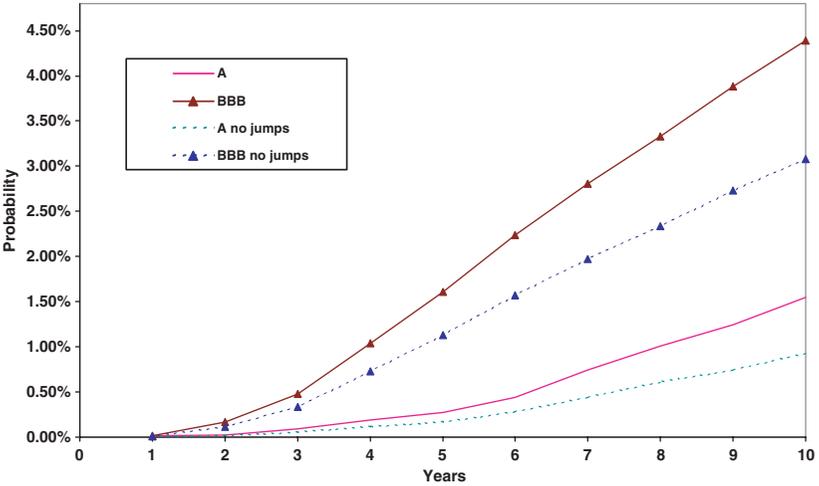


Figure 5: Cumulative default probabilities



Figures 4 and 5  
Cumulative default probabilities

Figures 4 and 5 depict cumulative default probabilities for four rating categories, AAA to BBB, as generated by the jump-diffusion model. The figures also contain the cumulative default probability that is generated by diffusion shocks (denoted “no jumps”). This is calculated by setting the jump intensities equal to zero in the jump-diffusion model, without changing the other parameter values.

## 7.2 Volatility of credit spread changes

Another way to assess the performance of the jump-diffusion model is to analyze to what extent the model fits the observed variation of credit spreads. This is particularly interesting since we also study how the model fits the variation of equity returns.

We focus on the systematic variation in credit spreads. Empirically, we measure this using the Lehman corporate bond index data for 1983–2002, as discussed in Section 6.1. For each rating category, we calculate the standard deviation of the monthly change in the average credit spread of all bonds in the appropriate corporate bond index. Table 5, Panel B contains these numbers. For example, for the A-rated index, we find a monthly standard deviation of the credit spread change of 11.5 bp. We also calculate a confidence interval for the estimated standard deviations, using the central limit theorem for the variance estimator and subsequently applying the delta method. In Table 5, Panel B reports the 2.5% lower bound of the 95% confidence interval. For the A-rated index, this lower bound is 6.9 basis points.

To compare these results with the model-implied standard deviations, we simulate monthly changes in the firm values for the jump-diffusion and diffusion-only model (given the initial levels of the firm values), from which we obtain monthly changes in the credit spreads. Given that we focus on the systematic variation of corporate bond indexes, we simulate for each rating category credit spread changes for 100 firms, and calculate the monthly standard deviation of the average credit spread change across these 100 firms.

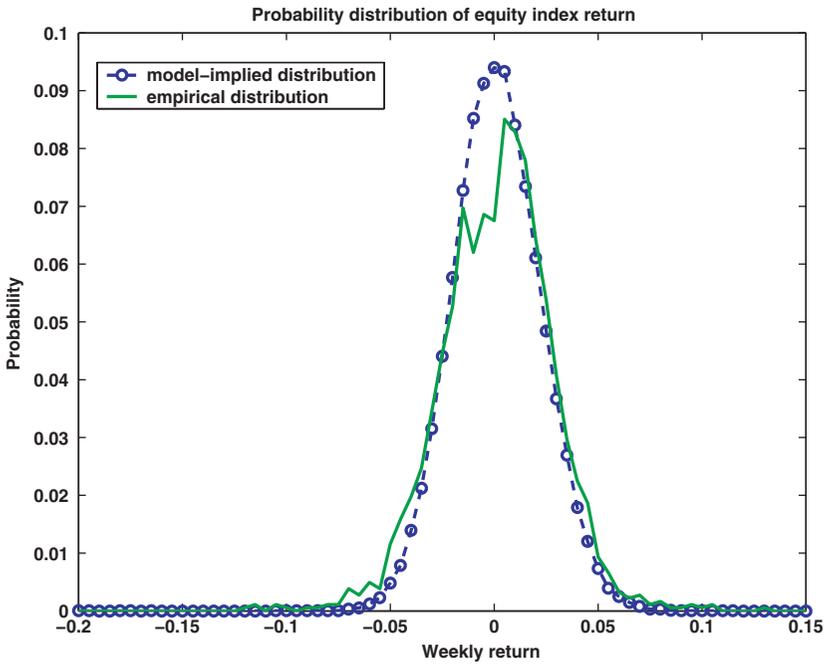
In Table 5, Panel B shows that the diffusion-only model significantly underestimates the variability of credit spreads for all ratings. The jump-diffusion model still underestimates volatility for four out of six rating categories, but does much better in economic terms. For example, for the A rating, the diffusion-only model generates a monthly standard deviation of 2.8 bp, while the jump-diffusion model predicts 6.3 bp. In the next subsection, we will see that the jump-diffusion model also underestimates the equity volatility and discuss this in more detail.

### **7.3 Equity return distribution**

In this subsection, we study the model implications for the equity index return distribution, focusing on the index since, for our purpose, the systematic variation is most important.

We first calculate the index return distribution implied by the jump-diffusion model by simulating weekly changes in the firm value and the corresponding equity value. We focus on a weekly frequency using 250,000 simulations, since for daily returns even more simulations would be needed to reliably estimate the impact of jumps on the distribution. We compare this with the empirical distribution of the S&P 100 index return estimated from daily data for the 1996–2002 period, using overlapping weekly returns.

Figure 6 presents both the empirical and model-implied distribution of equity index returns. The graph shows that the empirical distribution clearly has more dispersion than the model-implied counterpart. Both the left and right tail are fatter for the empirical distribution, as evidenced by the annualized standard deviation, which equals 19.3% empirically as compared to 15.5% for the model.



**Figure 6**  
**Probability distribution of the equity index return**

The figure shows the probability distribution of the weekly return on the S&P 100 index, as generated by the jump-diffusion model (using 250,000 simulations). The figure also contains the empirical distribution of this return, calculated from daily S&P 100 return data for 1996–2002, using overlapping weekly returns.

Next, we focus on the left tail of the distribution, since this is the most relevant part of the distribution for modeling default risk. Specifically, we want to make sure that even deep in the left tail the jump-diffusion model does not generate “too much” risk. Table 6 reports empirical and model-implied cumulative probabilities, as well as confidence intervals for the empirical cumulative probabilities. These are calculated by bootstrapping the daily observed returns and reconstructing a series of overlapping weekly returns in each bootstrap simulation. Table 6 shows that even for a weekly return of  $-10\%$ , the jump-diffusion model does not overstate the observed risk. In fact, the model predicts a left tail that is thinner than the observed left tail, although for the  $-10\%$  to  $-8\%$  return levels, the difference is not significant.

In Section 5.3, we already saw that the jump-diffusion model somewhat underestimates equity option prices, or equivalently, that the model underestimates the dispersion of the equity return distribution under the risk-neutral measure  $Q$ . Here, we find the same result for the distribution under the actual measure  $P$ . This is consistent with our finding that the jump-diffusion model gives an accurate description of expected equity index option returns, which describe the transformation from  $P$  to  $Q$ . A similar result holds for corporate

**Table 6**  
**Implications for the equity index return distribution**

Return	Empirical	Empirical	Jump-diffusion model
	Cumulative probability	95% Confidence interval	Cumulative probability
-10%	0.33%	(0.00%; 0.44%)	0.02%
-9%	0.38%	(0.00%; 0.55%)	0.02%
-8%	0.55%	(0.03%; 0.93%)	0.03%
-7%	1.04%	(0.27%; 1.65%)	0.07%
-6%	1.81%	(0.93%; 2.97%)	0.23%
-5%	3.35%	(2.42%; 5.38%)	0.94%

This table reports cumulative probabilities of the weekly equity index return distribution, as implied by the jump-diffusion model. We also report empirical cumulative probabilities, using weekly overlapping returns constructed from daily data on S&P 100 returns for 1996–2002. We bootstrap the daily data to construct bootstrap series of overlapping weekly returns. Using 1000 bootstrap simulations, we then calculate the confidence interval for the empirical cumulative probabilities.

bonds: the jump-diffusion model generates credit spread levels that are close to observed levels, but underestimates the volatility of credit spread changes.

The main reason for the underestimation of volatilities is the restriction to match the default probability. This highlights a tension between the observed level of default rates and the observed volatility of equity and corporate bond prices. As pointed out by HH, one interpretation of this result is that asset prices exhibit excessive volatility (Shiller 1981). Such excessive volatility could, for example, be caused by liquidity-related shocks. Collin-Dufresne, Goldstein, and Martin (2001) argue that corporate bond prices are partially driven by local demand/supply shocks, while Chordia, Roll, and Subrahmanyam (2000) show that equity prices are exposed to systematic liquidity shocks.

#### 7.4 Default correlations

In this subsection, we perform an explorative analysis of default correlations. This is interesting since the inclusion of common jumps could have a significant effect on default correlations. To analyze this, we compare empirical estimates of default correlations, obtained by Lucas (1995), with the default correlations that are implied by the diffusion and jump-diffusion models analyzed in this paper. Lucas uses Moody's default data on more than 4000 issues for 1970–1993. To obtain the model-implied correlations, we use simulations and calculate default correlations under the actual probability measure.

Before we present the results, it is important to emphasize that this is merely an explorative analysis. Our model setup explicitly focuses on S&P 100 firms, while the empirical estimates of Lucas (1995) are based on a much larger sample of firms. It is likely that the cross-firm correlations of S&P 100 firms are higher than the correlations of other typically smaller firms. In addition, even though Lucas uses more than 4000 issuers in his analysis, there is obviously estimation error present in his estimates of default correlations. As noted by Lucas, default rates are close to zero especially for short horizons, so that

**Table 7**  
**Default correlations**

Empirical estimates Lucas (1995)						
	AAA	AA	A	BBB	BB	B
AAA	1%					
AA	2%	1%				
A	2%	2%	2%			
BBB	2%	1%	1%	0%		
BB	4%	3%	4%	2%	8%	
B	9%	8%	9%	6%	17%	38%
Jump-diffusion model						
	AAA	AA	A	BBB	BB	B
AAA	3.4%					
AA	3.4%	3.0%				
A	4.3%	4.3%	4.5%			
BBB	4.6%	4.8%	5.7%	5.9%		
BB	4.9%	5.6%	6.9%	8.5%	12.3%	
B	4.3%	4.7%	6.2%	8.0%	13.2%	15.2%
Diffusion-only model						
	AAA	AA	A	BBB	BB	B
AAA	3.2%					
AA	3.3%	2.5%				
A	3.4%	3.2%	3.4%			
BBB	3.6%	4.1%	4.4%	5.0%		
BB	4.2%	4.9%	5.5%	7.5%	11.9%	
B	3.5%	4.1%	5.2%	7.3%	12.3%	14.5%

This table reports 10-year default correlations of firm pairs in different rating categories, as estimated empirically by Lucas (1995) and as implied by the jump-diffusion model and the diffusion-only model.

few default observations are available. For this reason, we focus on a 10-year horizon for our analysis of default correlations.

Table 7 reports the 10-year default correlations implied by the diffusion and jump-diffusion models, as well as the estimates reported by Lucas (1995). Default correlations are presented for pairs of firms in different rating categories. Focusing on the diffusion-only model first, the model overestimates default correlations for high-rated firms. For example, the AAA–AAA default correlation is estimated at 1% by Lucas (1995), while the diffusion-only model generates a default correlation of 3.2%. For low-rated firms, the diffusion-only model actually generates default correlations that are too low. The most extreme case is the B–B default correlation, for which the model generates a value of 14.5%, while the empirical estimate equals 38%.

Turning to the jump-diffusion model, incorporating common downward jumps clearly increases default correlations relative to the diffusion-only model. Quantitatively, however, the effect is relatively small. For example, the AAA–AAA correlation increases from 3.2% to 3.4% when jumps are added, while the B–B correlation increases from 14.5% to 15.2%. Comparing the results with the empirical estimates, the model overestimates default correlations for

high ratings (and more so than the diffusion model), and underestimates default correlations for low ratings (but less so than the diffusion model). In sum, we conclude that, although the jump-diffusion model does not generate a perfect fit, the default correlations it implies do not seem excessively high, especially given the caveats discussed above.

### **7.5 Credit spreads and the crash of 1987**

As a final additional test of the jump-diffusion model, we examine a subtle implication stemming from a structural break in option prices around the stock market crash of 1987 (Rubinstein 1994). Remarkably, before the crash, the implied volatilities of stock index options reflected a volatility smile, which changed into a volatility skew after the crash. This skew has not disappeared to date. If this change in option prices reflects a structural increase in risk premia, credit spreads should have been systematically lower before the 1987 crash. For example, if options and corporate bonds were priced by the diffusion-only model before the crash, and by the jump-diffusion model afterward (which would be consistent with the observed change in option prices), we should observe that credit spreads of A-rated firms were about 44 bp lower before the crash (27.1 versus 70.7 bp in Table 5).

To test for a systematic change in credit spreads around the crash, we again use the Lehman corporate bond index data, in this case an all-maturity index incorporating all investment-grade bonds, starting in 1973. We find that the average spread from 1973 to October 1987 equals 94.0 bp, while the average spread from November 1987 to December 2005 equals 106.1 bp. Even though the average spread is higher after the crash by about 12 bp, this difference is not significantly different from zero given a standard error of 14.0 bp on the average spread increase.<sup>23</sup> Also, given this standard error, the increase of 44 bp predicted by the model is outside the 95% confidence interval of the observed increase of 12 bp.<sup>24</sup> Changing the sample periods by deleting the first and last few years of data does not change this result.

Given that the results presented in this paper show that option prices and corporate bonds reflect similar jump risk premia after the crash of 1987, the findings in this subsection suggest that option prices and corporate bonds did not reflect similar risk premia before the 1987 crash. This would imply that index options were mispriced relative to corporate bonds before the crash. An alternative explanation for this result is that option markets were not mature and liquid enough to properly reflect the distribution of the underlying asset prices. After the crash, options became more popular and liquidity of option markets improved.

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<sup>23</sup> This *t*-statistic is calculated using Newey-West with 12 monthly lags for both subsamples.

<sup>24</sup> Here, we compare the investment-grade data with the implications of the model for A-rated firms.

## 8. Conclusion

We calibrate a structural firm value model with priced jump risk to information in equity and option prices. The estimated model with priced jump risk explains a significant part of observed credit spread levels. In line with existing work, we find that a model without jumps generates much lower credit spreads. We show that such a model also has a worse fit of equity and option prices. Incorporating jumps also helps to better fit observed volatilities of equity and credit spreads. Furthermore, we illustrate that carefully calibrating the jump parameters is important to obtain a good fit of option prices and returns, and realistic predictions of credit spread levels. Most importantly, our results suggest that prices of jump risk embedded in corporate bonds and credit default swaps on the one hand, and in equity and equity options on the other hand, are close to each other.

Given that our calibration to option data resulted in a large value for the jump risk premium, understanding the size and source of the jump risk premium is an interesting topic for future research. This is challenging since recent work has argued that rational pricing models may have difficulties explaining the option-implied jump risk premia (Bondarenko 2003 and Driessen and Maenhout 2007).

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