Intergenerational Risk Sharing and Prospect Theory

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Abstract

There is overwhelming evidence that prospect theory offers a more realistic description of people's preferences than traditional expected utility theory. Still, the paradigm of expected utility dominates many areas of economics; one of them is pension policies. This paper explores pension policies through the lens of prospect theory and draws three conclusions. First, although for small risks the gains from risk sharing are larger under prospect theory than under expected utility theory, the opposite holds true for shocks taken from a more realistic income distribution. This result holds true for different specifications of prospect theory. Second, pension funds should only share positive risks between generations. In general, negative risks should not be shared and, if possible, allocated fully to future generations. Thirdly, the finding that optimal policies share only positive risks between generations further reduces the gains from risk sharing under prospect theory relative to those under expected utility theory.

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Prospect Theory, Behavioral Economics, Pension Policies, Intergenerational Risk Sharing, Public Finances

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1 Introduction

It is no exaggeration to state that prospect theory is a cornerstone of behavioral economics. Kahneman and Tversky′s original paper (Kahneman and Tversky, 1979) and its later extension (Tversky and Kahneman, 1992) have laid the basis for a large and broad literature. Two elements explain the success of prospect theory. First, as it is a theory of decision-making under risk, it is relevant in a wide range of areas. Second, dozens of experiments have delivered empirical evidence that accords with the theory.

Quite surprisingly however, prospect theory has not become part of the standard toolkit of economists yet. Important applications have been made in various fields, like that of finance, insurance and health economics. But, to cite Barberis (2013), there are relatively few well-known and broadly accepted applications of prospect theory in economics. The case of pension economics may be illustrative. Most papers that explore pension policies are based on traditional expected utility theory (Ball and Mankiw, 2007; Bovenberg et al., 2007; Gollier, 2007; Bohn, 2009; Bonenkamp and Westerhout, 2014; Bonenkamp et al., 2017); there are only a few papers that are based on prospect theory (Fabbro, 2010; Popescu, 2014).[1]

This paper puts prospect theory central. It explores the implications of prospect theory for pension policies and, particularly, how these differ from the implications of expected utility theory. It addresses three sets of questions. First, what is the role of risk and risk sharing under prospect theory and expected utility theory? Is the result robust to different representations of prospect theory? Second, should pension funds react to positive and negative shocks in a symmetric way? Does this depend on the type of pension scheme in place, PAYG or funded? Thirdly, what do optimal pension policies imply for the gains from intergenerational risk sharing under prospect theory relative to expected utility theory?

There is some earlier literature on the welfare gain of intergenerational risk sharing by pension funds. Boeijen *et al.* (2016) provides a recent overview. This study finds that estimates of this welfare gain are driven by assumptions on preferences, the economic environment and the type of pension contract. The model in the present paper differs significantly from many others in the literature. Therefore, we will not try to relate our find-

[1] It would be an exaggeration to say that expected utility theory dominates the field of pension polices. There are many papers that explore hyperbolic discounting and quasi-hyperbolic discounting in relation to pension policies (Frederick et al., 2002; Thaler and Benartzi, 2004). Furthermore, Binswanger (2007) studies pension policies adopting lexicographic loss aversion preferences.

ings to the earlier literature but, instead, focus on the relation between prospect theory and expected utility theory.

By applying prospect theory to government policies that pertain to different age groups in society, our paper moves far beyond existing applications of prospect theory that focus on individuals or groups of individuals. One may question whether prospect theory is meant to be applied in the way we apply it here. For our analysis assumes homogeneity of preferences: all agents have prospect-theory preferences characterized by the same parameter values. Not only introspection, but also empirical evidence tells us that this assumption is not realistic (Abdellaoui et al., 2010; Bruhin et al., 2010). Furthermore, policymakers are assumed to know the preferences of the persons they represent. One argument to defend our approach is that many analyses of public policies that are based on expected utility theory adopt a similar approach. The approach also seems to be obvious when one wants to expose what the two theories have in common and in which aspects they differ. Furthermore, once the relation between prospect theory and expected utility theory has become more transparent, it is easier to study similar questions without making the homogeneity assumption.

The structure of this paper is as follows. Section 2 explores the relation between prospect theory and expected utility theory from the angle of risk and risk sharing. Section 3 quantifies the effects of risk. Sections 4 to 6 discuss optimal pension policies under prospect theory and, in particular, their reaction to positive and negative shocks. Section 4 does so for the contemporaneous and section 5 for the intertemporal allocation over generations; section 6 integrates the two. Section 7 makes some concluding remarks.

2 The impact of risk

To explore the implications of risk under prospect theory, we start to define the theory and, particularly, how it differs from expected utility theory. The description will be brief: for a more detailed (and excellent) discussion of prospect theory, the reader is referred to Barberis (2013).

Basically, if we define welfare or expected utility as the weighted average of the utilities attached to different outcomes, prospect theory introduces two modifications. The first regards the shape of the utility function that connects utilities to outcomes. The second concerns the weights attached to different utilities: whereas expected utility theory uses probabilities as weights, prospect theory adopts decision weights.

Let us look at figure 1. This figure shows the shape of the utility function that is familiar from expected utility theory: utility as an increasing and concave function of consumption and defined for positive values of consumption only. Important for our purpose are the factors that are missing from figure 1: consumption in the past, consumption of others and earlier expectations of consumption. Most applications of expected utility theory assume such factors to be irrelevant.[2]

[Figure 1 about here.]

[Figure 2 about here.]

Now move to the case of prospect theory. Figure 2 illustrates utility under prospect theory. Three differences with figure 1 are visible. The first is found on the x-axis. Under prospect theory, utility relates not to the level of consumption, but to the change in consumption relative to some reference position (reference dependence).[3] To be sure, prospect theory is not very explicit about the reference position. It may relate to history (one's past consumption, for example) or to earlier expectations (Loewenstein and Prelec, 1992). Whatever the case, under prospect theory it is consumption in deviation from this reference position that generates utility. Second, the utility function adopted by prospect theory consists of two parts. For gains (consumption higher than the reference level), the function is concave, as in the case of expected utility theory; for losses (consumption lower than the reference level) however, the utility function is convex.[4] Thirdly, the utility function exhibits loss aversion: the losses part of the utility function is steeper than the gains part for equally large changes of consumption in absolute terms.

For all three aspects, *i.e.* reference dependence, the convex-concave shape of the utility function and loss aversion, there is abundant empirical evidence. The same holds true for probability weighting, the replacement of probabilities with decision weights. The evidence is in two forms. Laboratory experiments yield direct evidence about decision-making processes (for

[2]Exceptions to the rule include habit formation (Constantinides, 1990) and 'Keeping up with the Joneses' preferences (Liu and Turnovsky, 2005).

[3]A large part of the literature on prospect theory bases utility on income rather than consumption. Here, we use consumption as we want to compare prospect theory with expected utility theory. The choice for consumption is without implications: wording both theories in terms of income rather than consumption would give the same results.

[4]In figure 2 a zero change in consumption corresponds to zero utility. This is not more than a normalization.

a review, see Kahneman and Tversky, 1979, Kahneman et al., 1991, or Rabin, 1998). Indirect evidence comes from field data. In particular, prospect theory has been found to be helpful explaining phenonema that are more difficult to understand from an expected-utility perspective (for a review, see Starmer, 2000 or Barberis, 2013).

In order to study the impact of risk, we now move to the marginal utility functions. Here, we will make two simplifications for transparency. One is that marginal utility is linear, the other is that - in case of prospect theory - decision weights equal the corresponding probabilities.[5] In later sections, we will explore in detail the role of the curvature of the utility function and also that of probability weighting.

Figure 3 exhibits marginal utility in the case of expected utility theory. It assumes there are two states of nature, a good state and a bad state, both with 50 percent probability. The figure shows the effect of consumption risk. The riskiness of consumption implies a utility loss equal to area ABGF in the bad state and a utility gain equal to area BCHG in the good state. Given that the marginal utility function is linear, the loss in expected utility due to consumption risk equals the triangle ABD (or BEC). Risk sharing policies that enable an increase in consumption in the bad state at the cost of an equally large reduction of consumption in the good state will make this triangle shrink: the graphical manifestation of the welfare gain from risk reduction.

[Figure 3 about here.]

[Figure 4 about here.]

Now move to the case of prospect theory. In order to proceed, we have to define the reference level of consumption. Here, we follow Kőszegi and Rabin (2006) and assume that the household takes as reference position the expected level of consumption. As expected consumption averages consumption in the good state and bad state, the good state thus represents a consumption gain and the bad state a consumption loss.

As figure 4 shows, the marginal utility function has now two distinct parts. In addition, two more aspects are worth mentioning. First, marginal utility is increasing in the losses part and decreasing in the gains part. This reflects the convex-concave nature of the prospect-theory utility function. Second, marginal utility corresponding to a given consumption loss is higher

[5]Decision weights and probabilities do not play a direct role in figures 1 to 4, but they do play a role in calculating expected utilities.

than the marginal utility corresponding to an equally large consumption gain. Obviously, this is the counterpart of the losses part being steeper than the gains part in figure 2 and reflects the loss aversion that characterizes a prospect-theory consumer.

What is the impact of risk? Risk produces a utility loss in the bad state equal to area AB0F and a utility gain in the good state equal to area CDE0. Hence, the net loss in expected utility that is due to consumption risk equals half the parallelogram ABCG. Risk sharing policies that enable an increase in consumption in the bad state at the cost of an equally large reduction of consumption in the good state will makes this parallelogram shrink. This is the graphical manifestation of the welfare gain from risk reduction under prospect theory.

We draw two conclusions from our graphical analysis. First, risk sharing is welfare-increasing under both theories of economic decision-making, but the nature of the welfare gain is different. Under expected utility theory, the welfare gain from risk sharing relates to the concavity of the utility function. Under prospect theory it is loss aversion that renders risk sharing beneficial. Second, the welfare loss of consumption risk is second-order in case of expected utility theory, but first-order in case of prospect theory.[6] This can be seen by checking what happens to the welfare loss due to risk when one lets the difference between the good and bad state become arbitrarily small. Under expected utility theory, the welfare loss from risk vanishes; under prospect theory, it remains positive. Hence, for a sufficiently small amount of risk, prospect theory will ascribe a larger welfare loss to this risk than expected utility theory does. For larger amounts of risk, no conclusion can be made without parameterizing the utility functions.

3 The potential gains from intergenerational risk sharing

In order to gain insight into the quantitative implications of risk under the two theories, we have to parameterize our utility functions. We will approach this in two ways. First, we assume two states, relating to the graphical analysis in figures 3 and 4 of the previous section. Second, we repeat the analysis, now assuming there are multiple states by making 10,000 draws from an income distribution.

[6] Aizenman (1999) makes the same point regarding generalized expected utility.

3.1 A two-state distribution

Our two-state approach is very similar to the graphical analysis of the previous section. Indeed, we distinguish again between a good state and a bad state that have equal probability (1/2), we again take the expected level of consumption as the reference level in case of prospect theory and we again abstract from probability weighting. Different from the graphical analysis is that we now assume that utility is of the isoelastic type, which is closer to the preferences that are commonly assumed in the literature.

Hence, for the case of expected utility theory, the utility function looks as follows,

$$u_{EU} = \frac{c^{\gamma_{EU}}}{\gamma_{EU}} \tag{1}$$

where c denotes consumption, u denotes utility, EU refers to expected utility theory and $\gamma_{EU} < 1$ measures the risk aversion of the household, defined as $1 - \gamma_{EU}$.

The extension to expected utility is straightforward,

$$E(u_{EU}) = \frac{1}{2}\frac{c_B^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{2}\frac{c_G^{\gamma_{EU}}}{\gamma_{EU}} \tag{2}$$

where we use B and G to refer to the bad state and good state respectively.

In order to arrive at a meaningful expression for the loss due to consumption risk, we calculate the certainty-equivalent of consumption. This is defined as the reduction of a certain amount of consumption in relative terms that would produce the same welfare loss as the welfare loss that occurs because the state of the economy and the associated level of consumption are unknown a priori. Using Υ to denote this certainty-equivalent of consumption and $E(c)$ to denote average consumption, the following equation formalizes this definition,

$$\frac{(E(c)(1-\Upsilon_{EU}))^{\gamma_{EU}}}{\gamma_{EU}} = E(u_{EU}) \qquad \longrightarrow$$

$$\Upsilon_{EU} = 1 - \left\{ \frac{E(u_{EU})}{\left(\frac{E(c)^{\gamma_{EU}}}{\gamma_{EU}}\right)} \right\}^{\frac{1}{\gamma_{EU}}} \tag{3}$$

where $E(u_{EU})$ follows from equation (2).

To represent prospect theory, we also assume isoelastic preferences,

$$\begin{aligned} u_{PT} &= (c - E(c))^{\gamma_{PT}^{+}} \qquad && c - E(c) \geq 0 \\ &= -\lambda(E(c) - c)^{\gamma_{PT}^{-}} \qquad && c - E(c) \leq 0 \end{aligned} \tag{4}$$

where $0 < \gamma_{PT}^{+}, \gamma_{PT}^{-} < 1$, the index PT refers to prospect theory and the parameter $\lambda > 1$ denotes the loss aversion index. The argument of the utility function is the difference between the actual and some reference level of consumption in absolute terms. As mentioned above, we take the expected level of consumption as the reference level.

The specification in equation (4) generalizes the standard Tversky and Kahneman (1992) convex-concave S-shaped utility function. Concretely, equation (4) does not impose that γ_{PT}^{+} and γ_{PT}^{-} are equal. Indeed, the equation allows for a different curvature of the losses and gains part of the utility function, which accords with recent empirical estimates of the utility function (Abdellaoui et al., 2010).

Recall that we abstract initially from probability weighting. Then, it is straightforward to derive the following expression for expected utility under prospect theory:

$$E(u_{PT}) = -\frac{1}{2}\lambda(E(c) - c_B)^{\gamma_{PT}^{+}} + \frac{1}{2}(c_G - E(c))^{\gamma_{PT}^{-}} \tag{5}$$

Finally, we need to define the certainty-equivalent of consumption. This definition is similar to the one we used in case of expected utility theory. Note that risk implies a welfare loss, so we need to invoke the losses part of the utility function to calculate the certainty-equivalent,

$$\begin{aligned} -\lambda(E(c)\Upsilon_{PT})^{\gamma_{PT}^{-}} &= E(u_{PT}) \qquad \longrightarrow \\ \Upsilon_{PT} &= \left(\frac{-E(u_{PT})}{\lambda E(c)^{\gamma_{PT}^{-}}}\right)^{1/\gamma_{PT}^{-}} \end{aligned} \tag{6}$$

where $E(u_{PT})$ follows from equation (5).

Let us assume the following parameter values for our benchmark calculation. Average consumption equals 1 and the good state and bad state imply 10 percent deviations, so that $c_B = 0.9$ and $c_G = 1.1$. We fix γ_{EU} to the value of -1, which comes down to a coefficient of risk aversion equal to 2. Estimates of this parameter are very heterogeneous; a value of 2 is a sort of mid value (Campbell, 1996). Next, following Tversky and Kahneman (1992), we choose $\gamma_{PT}^{+} = \gamma_{PT}^{-} = 0.88$ and $\lambda = 2.25$.

[Table 1 about here.]

The upper line in table 1 provides our results for the benchmark case. In this case, the welfare loss from risk is 1 percent under expected utility theory and 2.3 percent under prospect theory. Hence, in order to avoid the lottery that consumption will be 10 percent higher or lower than expected, the household is willing to accept a 1 percent lower level of consumption under expected utility theory. Under prospect theory, he is more risk-averse and is willing to accept a 2.3 percent reduction of consumption.

In order to explore how this result relates to numerical assumptions, we run a number of alternative simulations. Simulations (2) and (3) in table 1 assume a much lower and higher value for the coefficient of relative risk aversion in the case of expected utility theory: 1.2 and 5 respectively. These simulations reduce and increase the certainty-equivalent of the welfare loss under expected utility theory; in the latter case, the welfare loss is about the same size as the welfare loss under prospect theory. Simulations (4) and (5) deviate from the benchmark by assuming a much lower and higher value for the loss aversion index: 1.5 and 3.0 respectively. This changes the certainty-equivalent under prospect theory, but does not change the result that the welfare loss from consumption risk is higher under prospect theory. Only if we abandoned loss aversion completely ($\lambda = 1$), the ranking would be reversed (in table 1, the certainty-equivalent welfare loss under prospect theory would be reduced to zero).

Two more simulations explore the role of γ_{PT}^{-} and γ_{PT}^{+}. Simulation (6) assumes different values for the losses and gains parts. In particular, it retains the 0.88 value for γ_{PT}^{+} but assumes $\gamma_{PT}^{-} = 1$. This accords with empirical evidence that the losses part of the utility function may be closer to linear than the gains part (Abdellaoui *et al.*, 2010). As simulation (6) illustrates, this accentuates the loss aversion in the prospect theory specification and blows up the certainty-equivalent welfare loss from 0.023 to 0.037. Simulation (7) again assumes γ_{PT}^{-} and γ_{PT}^{+} are equal, but, unlike the benchmark simulation, assumes they adopt a unitary value. This allows us to see whether the conclusion from Benartzi and Thaler (1995) that the nonlinearity of the utility function may not be crucial, also applies in our case. As shown in table 1, the result is mixed. The certainty-equivalent of consumption does increase from 0.023 to 0.028. Still, this effect is dwarfed by that of loss aversion.

The last four simulations assume different changes in consumption. In particular, the change in consumption of 10 percent as assumed in the benchmark case is replaced with changes of 1 percent, 5 percent, 20 percent and 30 percent respectively. These simulations have a dramatic impact upon the certainty-equivalents. The certainty-equivalent under prospect theory increases up to 20 times the certainty-equivalent under expected utility theory in the 1 percent case. At the other end of the spectrum, when consumption deviations from the reference position are as large as 30 percent, the certainty-equivalent under prospect theory becomes smaller than that under expected utility (0.070 versus 0.090). This pattern of results reflects the observation made above that the welfare loss from risk is second-order in the case of expected utility theory and first-order in the case of prospect theory.

3.2 A multi-state distribution

The dependence of results on the assumed deviations of consumption is a compelling argument to adopt a more detailed and more realistic income distribution. Here, we replace the assumption of a hypothetical two-state distribution with that of a more realistic multi-state distribution. Expected utility is the natural generalization of that in equation (2) in case of expected utility theory and of that in equation (5) in case of prospect theory,

$$E(u_{EU}) = \frac{1}{n}\sum_{i=1}^{n}\frac{c(i)^{\gamma_{EU}}}{\gamma_{EU}} \tag{7}$$

$$\begin{aligned} E(u_{PT}) &= -\lambda\frac{1}{n^-}\sum_{i=-n^-}^{-1}(E(c)-c(i))^{\gamma_{PT}^-} \\ &+ \frac{1}{n^+}\sum_{1}^{i=n^+}(c(i)-E(c))^{\gamma_{PT}^+} \end{aligned} \tag{8}$$

where n, n^- and n^+ are related by definition, $n \equiv n^- + n^+$.[7] The certainty-equivalents Υ_{EU} and Υ_{PT} are calculated as before, *i.e.* by plugging the results for $E(u_{EU})$ and $E(u_{PT})$ from equations (7) and (8) into equations (3) and (6) respectively.

We construct our income distribution in the following way. We assume one euro is invested in an asset of which the return follows an independent lognormal distribution. The holding period is 30 years. 30 years is a rough measure of the average distance between pension contributions and pension benefits. Moreover, in section 4 of the paper we will construct a 2-period overlapping-generations model, the unit period of which can be taken to be 30 years. The asset is defined as a portfolio that consists of bonds and equity in equal amounts. Adopting the estimates in Campbell and Viceira (2002) for the mean and standard deviation of the real interest rate on bonds and the mean and standard deviation of the return on equity, we

[7]The simulations for expected utility theory and prospect theory use the same set of 10,000 draws from the income distribution.

assume that our asset has an expected annual rate of return of 5.3% and an annual standard deviation of 8.1%. This procedure results in a lognormal consumption distribution with expected value 4.7 and standard deviation 2.1. In our simulations, we report the expected value and standard deviation of the log of consumption. This expected value, denoted μ, equals 1.46; the standard deviation, denoted σ, equals 0.42.

Having specified the distribution of income, we take 10,000 draws from this distribution. The number of 10,000 is sufficiently large in the sense that alternative seeds produce very similar outcomes. For γ_{EU}, γ_{PT}^{-}, γ_{PT}^{+} and λ we take the same values as in the previous subsection. As before, we perform simulations for the benchmark parameterization and some alternative parameterizations. In particular, the parameter configurations in simulations (1) to (7) are the same as in table 1. Furthermore, table 2 also includes two simulations that change the spread of the income distribution.[8]

[Table 2 about here.]

The upper line in table 2 displays the results for the benchmark simulation. These results are interesting for two reasons. First, the welfare loss from consumption risk is much larger if we replace the artificial two-state distribution with a more realistic multi-state distribution. Under expected utility theory, the welfare loss increases from 1.0 percent to 16.2 percent. Under prospect theory, the change is more nuanced, but still large: from 2.3 to 8.2 percent. Second, under the multi-state distribution, the welfare loss under prospect theory is smaller than under expected utility theory. This relates to the fact that risk sharing implies a first-order welfare loss under prospect theory and second-order under expected utility theory.

The alternative simulations show that both results are robust. In all simulations (2) to (7) the welfare effects become larger under both theories. The result that the welfare effect under prospect theory is smaller than under expected utility theory is also robust, except if the income distribution is very concentrated. In simulation (9), featuring a 0.15 value for the standard deviation of the log of income rather than 0.42 in the benchmark simulation is the welfare loss under prospect theory much larger than under expected utility theory; the two effects differ a factor two. Again, this finding can easily be explained by referring to the observation that risk sharing implies

[8] We do not include simulations that change the value of μ. Changing the value of μ will not have an effect upon the results as it changes the income distribution equiproportionally without any change upon the coefficient of variation of the consumption distribution.

a first-order welfare loss under prospect theory and second-order under expected utility theory.

All the results obtained thus far can be easily interpreted in terms of their assumptions. One question remains, however, which is how our results can be reconciled with those obtained by Benartzi and Thaler (1985)? That paper found that prospect theory can explain the equity premium puzzle, something that expected utility theory cannot. This suggests that risk aversion is stronger under prospect theory, even in case of a multi-state income distribution and contrary to our results. The explanation is probably that the explanation offered in Benartzi and Thaler (1995) relies on the combination of prospect theory and the frequent evaluation of investors of their portfolios. Without the latter element, prospect theory cannot explain the equity premium and the contradiction between the two papers disappears.

3.3 Probability weighting

One aspect that we did not elaborate thus far is probability weighting. In their analysis of the equity premium puzzle, Benartzi and Thaler (1995) concluded that the crucial element of prospect theory is loss aversion; probability weighting (like nonlinear utility) plays only a moderate role. In other applications however, some results are ascribed to probability weighting (*e.g.*, Barberis and Huang, 2008; Sydnor, 2010). Therefore, let us bring in probability weighting and explore how it affects our results.

We follow Benartzi and Thaler (1995) by adopting the specification of Tversky and Kahneman (1992). This specification replaces the probabilities in equation (8) with decision weights π: π^- for the losses part and π^+ for the gains part of the utility function. Let us use a hat above the utility variable to denote that it refers to probability weighting:

$$\begin{aligned} E(\hat{u}_{PT}) &= -\lambda \sum_{i=-n^-}^{-1} (E(c) - c(i))^{\gamma_{PT}^-} \pi^-(c(i)) \\ &+ \sum_{1}^{n^+} (c(i) - E(c))^{\gamma_{PT}^+} \pi^+(c(i)) \end{aligned} \tag{9}$$

The decision weights of the various outcomes are connected to the corresponding probabilities by linking cumulative decision weights to cumulative probabilities. This is done separately for the losses part and the gains part. For the losses part, $\pi^-(c(i))$ is calculated for all values of $i = -n^-, -1$ by the following formulas,

$$\pi^-(c(i)) = \Pi^-(c(i)) - \Pi^-(c(i-1))$$
$$\Pi^-(c(i)) = \frac{P^-(c(i))^{\epsilon^-}}{\left(P^-(c(i))^{\epsilon^-} + (1 - P^-(c(i)))^{\epsilon^-}\right)^{1/\epsilon^-}} \tag{10}$$
$$P^-(c(i)) = \frac{i + n^- + 1}{n^-}$$

where $\Pi^-(c(-n^- - 1)) = 0$.

Similarly, for the gains part, $\pi^+(c(i))$ is calculated for all values of $i = 1, n^+$ by the following formulas,

$$\pi^+(c(i)) = \Pi^+(c(i)) - \Pi^+(c(i-1))$$
$$\Pi^+(c(i)) = \frac{P^+(c(i))^{\epsilon^+}}{\left(P^+(c(i))^{\epsilon^+} + (1 - P^+(c(i)))^{\epsilon^+}\right)^{1/\epsilon^+}} \tag{11}$$
$$P^+(c(i)) = \frac{i}{n^+}$$

where $\Pi^+(c(0)) = 0$.

The corresponding certainty-equivalent of consumption, $\hat{\Upsilon}_{PT}$, can be derived straightforwardly by combining $E(\hat{u}_{PT})$ in equation (9) with equation (6).

The parameters ϵ^- and ϵ^+ are equal to 0.69 and 0.61 respectively; we will perform simulations to study the role of different values.

We make one correction to our original dataset. Instead of taking 10,000 draws that imply a positive or negative deviation of consumption from its mean, we now calculate the corresponding absolute values of all draws and apply them both to the positive and negative domain (antithetic variates). Hence, we double the number of draws. The motivation is to avoid that our results will be driven by one or more outliers that figure only in the positive or negative domain since probability weighting attached a large weight to outliers. In our sample, the numerical correction is of little importance however: applying this correction to the calculation of Υ_{PT} for the benchmark configuration in table 2 would result in a certainty-equivalent loss of 0.075 rather than the 0.082 that is reported in table 2.

Table 3 reports the results for the relevant parameter configurations in table 2. It adds two simulations that take different values for ϵ^- and ϵ^+ (simulation (7) halves the difference between ϵ^- (ϵ^+) and one; simulation (8) increases this difference to 150 percent of its value in the benchmark configuration). For comparison, we repeat the results for Υ_{PT} from table 2.

[Table 3 about here.]

Comparing the results for $\hat{\Upsilon}_{PT}$ with those for Υ_{PT} indicates that the impact of probability weighting is negligible - in line with the earlier finding by Benartzi and Thaler (1995). This holds true for the benchmark parameter configuration as well as for the other configurations. The reason might be that probability weighting mainly affects the weights of the tails of the distribution of which the role in the whole distribution is relatively minor.

4 Optimal pension fund policies; the case of a PAYG scheme

The results thus far are clear and intuitive as they match things we know from theory as well as results from earlier analyses. Still, the results obtained thus far do not provide a good picture of risk sharing policies. This is because underlying the calculations in the previous section is the implicit idea that risk can be eliminated. However, this is not what pension policies can achieve. To be sure, pension fund policies can share risks such as to reduce their aggregate welfare loss. Pension fund policies can in no way completely eliminate the risks that are exogenous to the economy, however.

There is a second element missing in the analysis in the previous section. That is that participants in pension schemes experience risk sharing policies at two points in their life cycles. In retirement, they gain from risk sharing when risk sharing policies are targeted at reallocating risks from the old to the young. When they are in their working years, the same participants suffer from risk sharing policies however. An appropriate analysis of risk sharing policies therefore includes the effects of risk sharing policies upon people in both stages of their lives (or the effects upon both working and retired generations, which amounts to the same thing). Accounting for the two sides of risk sharing may result in much smaller net welfare gains.

A further issue that needs to be addressed is that we have made the implicit assumption thus far that pension funds will be willing to organize risk sharing. But it is unclear whether this is the right assumption. In particular, in some cases it may be optimal for pension funds not to apply intergenerational risk sharing, as we will show below. Again, this suggests that the calculations thus far overstate the gains from risk sharing.

Having explored the potential gains from risk sharing, this section asks the question how large are the gains from risk sharing policies that pension funds can be expected to adopt? To answer it, we take a two-step approach.

In the first step, we ask what optimal risk sharing policies look like. In the second step, we explore what this implies in terms of welfare gains from risk sharing policies. As before, we do the same steps for prospect theory and expected utility theory. In both cases, we resort to the utility functions of the previous section.

Moreover, this section assumes that the pension scheme is a PAYG scheme which resembles many real-world pension schemes. We will explore the case of a funded pension scheme in the next section. The present analysis of a PAYG scheme will form the basis for that, so we start with the PAYG case and, more precisely, with the case of expected utility theory.

4.1 Expected utility theory

In order to describe real-world pension policies, we have to distinguish between different generations. Indeed, pension policies distribute between generations, so the balance between generations must play in role in the decision making of the pension fund. For the case of a PAYG scheme, it suffices to have two generations: an old and a young generation. Expected utility theory then suggests that the pension fund maximizes the following welfare function V:

$$V_{EU} = \frac{c_o^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{c_y^{\gamma_{EU}}}{\gamma_{EU}} \tag{12}$$

Here, Δ weighs the utility of the young generation versus that of the old generation. We impose $\Delta \geq 0$. The indices o and y refer to the old and young generation respectively.

Consumption of the young generation is determined by his wage income, w, and the transfer made to the contemporaneous old generation, z. Consumption by the old is determined by his income, y, and the transfer made by the young generation. The income of the old is stochastic, whereas that of the young is taken to be non-stochastic. We will return to that assumption in our concluding comments. Hence, we have the following,

$$c_y(i) = w - z(i) \tag{13}$$

$$c_o(i) = y(i) + z(i) \tag{14}$$

where we use index i to refer to a specific simulation.

The pension scheme chooses the transfers between the two generations, z, after it has observed a shock in the income of the old generation. Transfers thus follow from the standard first-order condition of optimal policies,

$$c_o(i) = \Delta^{1/(1-\gamma_{EU})} c_y(i) \tag{15}$$

if the corresponding second-order condition is met.

The second derivative of V_{EU} with respect to $z(i)$ reads as

$$(\gamma_{EU} - 1)\left[c_o(i)^{\gamma_{EU}-2} + \frac{1}{\Delta}c_y(i)^{\gamma_{EU}-2}\right] \tag{16}$$

As this expression is always negative, the second-order condition for optimal policies is met and the solution given by the first-order condition is the optimal one. What does this first-order condition tell us? That optimal policies imply that the marginal utilities of consumption of the young and of the old should be equal to one another. In the special case $\Delta = 1$, this amounts to consumption smoothing: $c_o(i) = c_y(i)$ (Bohn, 2009).

Exact expressions for the intergenerational transfer, consumption of the old and consumption of the young follow when we combine the first-order condition (15) with the budget constraints of the old and the young, (14) and (13) respectively:

$$z^*_{EU}(i) = \Phi_{EU}w - (1 - \Phi_{EU})y(i) \tag{17}$$

$$c^*_{0,EU}(i) = \Phi_{EU}(w + y(i)) \tag{18}$$

$$c^*_{y,EU}(i) = (1 - \Phi_{EU})(w + y(i)) \tag{19}$$

Here, we use asterisks to indicate that the variables correspond to optimal pension fund policies. Furthermore, $\Phi_{EU} \equiv \Delta^{1/(1-\gamma_{EU})}/(1+\Delta^{1/(1-\gamma_{EU})})$, so that $0 < \Phi_{EU} < 1$.

Equations (18) and (19) show that shocks in y are allocated to the young and the old according to the social discount factor Δ. In the extreme case where $\Delta = 0$, $\Phi_{EU} = 0$ and the young absorb all the risks; in the other extreme case where $1/\Delta = 0$, $\Phi_{EU} = 1$ and risks are absorbed entirely by the old. In the special case where Δ equals $1, \Phi_{EU} = 1/2$ and risks are shared equally between the young and the old.

To evaluate the welfare gains from risk sharing policies, we now calculate a certainty-equivalent differential. In particular, we first calculate the certainty-equivalents corresponding to optimal policies and a policy of zero risk sharing and then calculate the difference between the two. The calculation of certainty-equivalents is the same as in the previous section, except that now the certainty-equivalents adjust consumption of *two* generations in all states of the world. We refer the reader to the appendix for the equations.

4.2 Prospect theory

In case of prospect theory the natural extension of the utility function to a two-generation economy reads as follows:

$$\begin{aligned} V_{PT} &= (c_o - E(c_o))^{\gamma_{PT}^{+}} + \frac{1}{\Delta}(c_y - E(c_y))^{\gamma_{PT}^{+}} \\ & \qquad\qquad c_o - E(c_o), c_y - E(c_y) > 0 \qquad (20) \\ &= -\lambda\Big[(E(c_o) - c_o)^{\gamma_{PT}^{-}} + \frac{1}{\Delta}(E(c_y) - c_y)^{\gamma_{PT}^{-}}\Big] \\ & \qquad\qquad c_o - E(c_o), c_y - E(c_y) \leq 0 \end{aligned}$$

Note that the two generations will in general have different reference positions. To elaborate these reference positions, we use the equations for consumption of the two generations specified above (equations (13) and (14)). This yields $E(c_y) = w$ and $E(c_o) = E(y)$.

Compared with expected utility theory, the case of prospect theory adds a dimension since an analysis of this case requires us to distinguish between an adverse shock and a positive shock. Let us start with the adverse shock.

An adverse shock

The utility function determines how the loss will be allocated over the two generations (the case where one generation absorbs more than the shock and the other actually benefits from the shock can be dismissed, as we will explain below). Hence, the function that the pension funds seeks to maximize is made up of the losses parts of the utility functions of the young and the old,

$$V_{PT}(i) = -\lambda(E(y) - c_o(i))^{\gamma_{PT}^{-}} - \frac{1}{\Delta}\lambda(w - c_y(i))^{\gamma_{PT}^{-}} \qquad (21)$$

where we have made use of the expressions for $E(c_y)$ and $E(c_o)$.

Consumption of the young and the old is again described by equations (13) and (14). The first-order condition for optimal policies now reads as follows:

$$c_o(i) - E(y) = \Delta^{1/(1-\gamma_{PT}^{-})}(c_y(i) - w) \qquad (22)$$

However, this condition is irrelevant as the second-order condition for optimal policies is not met. Indeed, due to the convex shape of the losses part of the utility function, the second-order derivative $\partial^2 V_{PT}(i)/(\partial z(i))^2$,

$$-\lambda\gamma_{PT}^{-}(\gamma_{PT}^{-} - 1)\Big[(E(y) - c_o(i))^{\gamma_{PT}^{-}-2} + \frac{1}{\Delta}(w - c_y(i))^{\gamma_{PT}^{-}-2}\Big] \qquad (23)$$

is always positive.

Hence, it will be one of the two corner solutions that solves the optimization problem: $(c_o(i) - E(y), c_y(i) - w) = (dy(i), 0)$ or $(c_o(i) - E(y), c_y(i) - w) = (0, dy(i))$ where $dy(i) \equiv y(i) - E(y)$ denotes the shock. It can easily be shown that the former (latter) solution is optimal if $\Delta < (>)1$, whereas the two solutions are equal in utility terms if $\Delta = 1$.

Let us for now focus on the case $\Delta = 1$, as this seems to be a natural benchmark case. In this case we have an indeterminacy. What we can do to get at a unique solution is to resort to factors outside the model. For example, we may argue that the old will generally be less able to absorb negative income changes and may be more adverse to such shocks (Bohn, 2009). Alternatively, we may consider that transfers between generations will be costly, if only because of administration costs. We may also refer to the change in numbers of retired and working persons to argue that the old should bear the risks (Bonenkamp *et al.*, 2017).

We prefer not to make a choice however, for two reasons. First, to rely on any of these outside arguments is a little *ad hoc*. Second, if we impose $\Delta = 1$, the choice we make has no relevance whatsoever for the welfare gains from risk sharing. To be sure, the fact that the pension fund chooses not to apply any risk sharing is important for welfare; which generation bears all the risk does not play a role however.

We choose to adopt the latter argument and let $(c_o(i) - E(y), c_y(i) - w) = (dy(i), 0)$ describe optimal pension fund policies in case of an adverse shock. We will redo the calculation on the basis of the alternative assumption, however.

Hence, for z, c_o and c_y we have one of the following two solutions,

$$z^{*,-}_{PT}(i) = 0; \qquad E(y) - y(i) \tag{24}$$

$$c^{*,-}_{o,PT}(i) = y(i); \quad E(y) \tag{25}$$

$$c^{*,-}_{y,PT}(i) = w; \qquad w + y(i) - E(y) \tag{26}$$

where we have added a superscript - to indicate that the solutions apply to the $dy < 0$ case.

The message of equations (24) to (26) is clear: in case of an adverse shock, a pension fund that acts on behalf of participants that have prospect theory preferences will not let both the old and the young generation bear part of the shock. Rather, it will give full protection to one of the two generations, which boils down to zero risk sharing. The contrast with expected utility theory could not be larger.

Before continuing to study the case of a positive shock, we have to solve one more problem. Particularly, how can we be sure that a corner solution where one of the two generations suffers a zero loss dominates a hypothetical solution where this generation would enjoy a (small) consumption gain (at the expense of a bigger loss for the other generation)? The answer is that we cannot on theoretical grounds alone. It depends on the interaction between diminished sensitivity (the convex-concave shape of the utility function) and loss aversion. But we can exclude the hypothetical solution on the basis of empirical evidence: the empirical case for loss aversion is much more convincing than for diminished sensitivity.

A positive shock

Let us now focus on the case of a positive shock. In this case, the social welfare function that is based on the gains parts of the utility function applies:

$$V_{PT}(i) = (c_o(i) - E(y))^{\gamma_{PT}^{+}} + \frac{1}{\Delta}(c_y(i) - w)^{\gamma_{PT}^{+}} \tag{27}$$

The first-order condition is the same as in equation (22). The second-order derivative is different, however, and has opposite sign:

$$\frac{\partial^2 V_{PT}(i)}{(\partial z(i))^2} = \gamma_{PT}^{+}(\gamma_{PT}^{+}-1)\left[(c_o(i)-E(y))^{\gamma_{PT}^{+}-2}+\frac{1}{\Delta}(c_y(i)-w)^{\gamma_{PT}^{+}-2}\right] < 0 \tag{28}$$

In the case of a positive shock, the interior solution thus describes the solution to the optimization problem. Combining this first-order condtion (22) with the two consumption equations (13) and (14), we derive the following solution for z, c_o and c_y,

$$z_{PT}^{*,+}(i) = -(1 - \Phi_{PT})dy(i) \tag{29}$$

$$c_{o,PT}^{*,+}(i) = E(y) + \Phi_{PT}dy(i) \tag{30}$$

$$c_{y,PT}^{*,+}(i) = w + (1 - \Phi_{PT})dy(i) \tag{31}$$

where $\Phi_{PT} \equiv \Delta^{1/(1-\gamma_{PT}^{+})}/(1 + \Delta^{1/(1-\gamma_{PT}^{+})})$, so that $0 < \Phi_{PT} < 1$, and the superscript $+$ is used to indicate that the solutions apply to the $dy(i) > 0$ case.

In case of a positive shock, the pension fund does share risks between the young and the old. This bears resemblance to the case of expected utility theory. Still, equations (29) to (31) look very different from equations (17) to (19). The reason is that the utility functions of expected utility theory and prospect theory have different arguments. The responses of z, c_o and c_y to shocks in y are very similar, however. Apart from a difference in the curvature of the two utility functions, *i.e.* γ_{EU} and γ_{PT}^{+}, these responses are identical.

Welfare

In order to derive the certainty-equivalent of optimal risk sharing policies, we adopt the same procedure as in the case of expected utility theory. Again, we relegate the equations to the appendix.

4.3 Results

The upper line of table 4 reports the results for the benchmark parameter configuration. We make two observations. First, the welfare gains from risk sharing policies are much smaller than the welfare losses from consumption risk that we calculated above. This relates to the different nature of the calculations. Above, we calculated the effects of a complete elimination of risk that people face in one stage of their lifecycle. Now, we calculate the effects of policies that only reallocate risk and include their effects on people in both stages of their lifecycle.

The second observation is that the ranking of welfare effects is as in table 2. That is, the welfare gains from risk sharing pension policies are larger under expected utility theory than under prospect theory. Moreover, the gap between the two has increased. In table 2 the certainty-equivalent under expected utility theory is about two times as large as the certainty-equivalent under prospect theory. In table 4, the factor has doubled to 4. As explained above, prospect theory implies less risk sharing than expected utility theory. Indeed, under expected utility theory, optimal policies imply risk sharing, irrespective the deviation of consumption from its earlier expectation. Under prospect theory, pension schemes apply risk sharing only if consumption is higher than expected.

The other lines of table 4 display the results of our alternative simulations. Most results are intuitive, but the effect of changing the loss aversion index λ is opposite from what one might expect: a higher (lower) loss aversion lowers (increases) the certainty-equivalent of risk sharing under prospect theory. Probably, this is due to the asymmetric risk sharing policies of the pension fund. The effect is not particularly large however. Adding or subtracting a third to or from the loss aversion index does not change the result from the benchmark case that risk sharing policies generate a smaller welfare gain under prospect theory than under expected utility theory. Finally, the simulation on the lower line of table 4 is interesting. Even if we take σ equal to 0.15 is the welfare gain from risk sharing under prospect theory smaller than under expected utility theory - contrary to the result in table 2.

We add a simulation that adds the Tversky and Kahneman (1992) specification of probability weighting.

[Table 4 about here.]

5 Optimal pension fund policies; the case of a funded scheme

The last section focussed on PAYG pension schemes. The next question is whether the results that we obtained apply to the case of funded pension schemes as well.

In the case of a funded scheme, risk allocaton takes the form of intertemporal transfers that imply the accumulation of wealth or debt. Let us for expositional clarity focus entirely upon this dynamic element. Hence, we include only two generations: the young generation, defined as the generation that is young at the time of the shock and the future generation, defined as the generation that will be born into the following period.

Like before, we start with expected utility theory.

5.1 Expected utility theory

To obtain a utility function, we take the utility function of the previous section, equation (12), but use as arguments the consumption of the young and the future generation,

$$V_{EU,t} = \frac{c_{y,t}^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{c_{y,t+1}^{\gamma_{EU}}}{\gamma_{EU}} \tag{32}$$

where we have added a time index as the model in this section is a dynamic one.

We write consumption of the young generation as wage income minus pension savings, denoted a. Consumption of the future generation equals next period's wage income, pension savings and the return on these savings. Hence, we have the following:

$$c_{y,t}(i) = w_t(i) - a_t(i) \tag{33}$$

$$c_{y,t+1}(i,j) = w_{t+1}(i)+a_t(i)(1+r(j)_{t+1}) = (E(w_t)+a_t(i))(1+r(j)_{t+1}) \quad (34)$$

We use r to denote the rate of return on savings and will assume limited liability: $r > -1$. Further, note the index j which refers to the stochastic element in period $t+1$. In addition, we assume that future wage income is perfectly correlated with the rate of return on savings. This is reflected in the second line of equation (34). This assumption simplifies the exposition of our argument, but is not crucial for our purposes.

The problem for the pension scheme is to maximize $V(i)_{EU}$ in equation (32), using pension savings a as policy instrument. As before, we can derive the first-order condition and the second-order condition of the optimization problem, confirm that the latter condition is met and elaborate the former condition. We do not write down the solution of this maximization problem, but immediately present the implied expressions for pension savings, consumption of the young and the future generation,

$$a^*_{EU,t}(i) = (1-\Psi_{EU})w_t(i) - \Psi_{EU}E(w_t) \quad (35)$$

$$c^*_{y,EU,t}(i) = \Psi_{EU}\Big(w_t(i) + E(w_t)\Big) \quad (36)$$

$$c^*_{y,EU,t+1}(i,j) = (1-\Psi_{EU})\Big(w_t(i) + E(w_t)\Big)(1+r(j)_{t+1}) \quad (37)$$

where $\Psi_{EU} \equiv \psi_{EU}/(1+\psi_{EU})$ and $\psi_{EU} \equiv (\Delta/E((1+r_{+1})^{\gamma_{EU}}))^{1/(1-\gamma_{EU})}$. Note that $\psi_{EU} \geq 0$ and $0 \leq \Psi_{EU} \leq 1$. The similarity of equations (35) and (37) with the corresponding equations for the PAYG case, (18) and (19), is obvious.

To evaluate the welfare gains from risk sharing under expected utility theory, we follow the procedure as detailed in the previous section (equations (44) to (48)). See the appendix.

5.2 Prospect theory

The social welfare function to apply in the case of prospect theory is obvious. It is the natural counterpart of the one specified in the previous section, equation (20):

$$\begin{aligned} V_{PT,t} &= (c_{y,t} - E(c_{y,t}))^{\gamma^+_{PT}} + \frac{1}{\Delta}(c_{y,t+1} - E(c_{y,t+1}))^{\gamma^+_{PT}} \\ &\qquad c_{y,t} - E(c_{y,t}), c_{y,t+1} - E(c_{y,t+1}) > 0 \qquad (38) \\ &= -\lambda\Big[(E(c_{y,t}) - c_{y,t})^{\gamma^-_{PT}} + \frac{1}{\Delta}(E(c_{y,t+1}) - c_{y,t+1})^{\gamma^-_{PT}}\Big] \\ &\qquad c_{y,t} - E(c_{y,t}), c_{y,t+1} - E(c_{y,t+1}) \leq 0 \end{aligned}$$

Unlike the case of expected utility theory, we will investigate the case of prospect theory in two steps. The first step assumes that period $t+1$ is riskless. Subsequently, we will explore the consequence of adding period $t=1$ uncertainty in step 2.

As before, we will start to explore the case of an adverse shock.

An adverse shock

The losses part of the welfare function now reads as follows:

$$V_{PT,t}(i) = -\lambda\Big[(E(c_{y,t}) - c_{y,t})^{\gamma_{PT}^{-}} + \frac{1}{\Delta}(E(c_{y,t+1}) - c_{y,t+1})^{\gamma_{PT}^{-}}\Big] \qquad (39)$$

Here, $E(c_{y,t})$ and $E(c_{y,t+1})$ represent the reference positions of the young and future generation. Note that the latter is not uniquely defined. We can take the expectation to be unconditional, *i.e.* formed before the realization of the shock in period t, or conditional,*i.e.* formed after the period-t shock has realized. We will adopt the first definition and elaborate the case of an adverse shock. After that, we will return to this issue.

In this case, the solution to the optimization problem is similar to the one in case of a PAYG scheme. Again, the second-order condition of optimal policies indicates that the first-order condition features a local minimum. Hence, optimal policies are defined by a corner solution. Which of the two corner solutions yileds highest utility is again ambiguous. The case $\Delta = (1+r_{t+1})^{\gamma_{PT}^{-}}$ seems a natural benchmark. Then, the two candidate solutions yield equally high utility. As before, we will not bring in outside factors and leave it undetermined which of the two solutions will be chosen. If $\Delta = (1+r_{t+1})^{\gamma_{PT}^{-}}$, the choice of corner solution is not relevant for welfare analysis.

We have to discuss the choice of reference position however, as it is not obvious that an unconditional expectation is better than a conditional one. In particular, as the expectation held by the future generation refers to the next period, one could reason that a conditional definition would be more obvious. Then, the expectation of the consumption of the future generation would take into account what shock occurred in the previous period and which part was absorbed by the then young generation. It is a straightforward exercise for the future generation to figure out what the budget constraint of the pension fund implies for their consumption.

The immediate implication of this definition is that, absent any shock in the future period $(t+1)$, the future generation will exactly consume the amount it expected. As a consequence, for the future generation the welfare loss will be zero, no matter how much the young generation has reduced his consumption. Then, it will be obvious that optimal policies imply that the young generation does not adjust his consumption. The future generation then has to absorb the full shock but, given that this is known and reflected in its earlier expectation, does not imply a welfare loss.

This outcome is peculiar, but follows naturally from the definition of a conditional expectation of future consumption[9] and the chosen utility function[10].

Hence, we have the following solutions for a_t, $c_{y,t}$ and $c_{y,t+1}$:

$$a^{*,-}_{PT,t}(i) = w_t(i) - E(w_t); \quad 0 \tag{40}$$

$$c^{*,-}_{y,PT,t}(i) = E(w_t); \quad w_t(i) \tag{41}$$

$$c^{*,-}_{y,PT,t+1}(i) = w_t(i)(1+r_{t+1}); \quad E(w_t)(1+r_{t+1}) \tag{42}$$

As outlined above, we now add uncertainty regarding period $t+1$. The effect of this is that the expected utility corresponding to the solution that lets the future generation absorb the shock changes from $-\lambda(E(c_{y,t})-c_{y,t})^{\gamma^-_{PT}}$ to $-\lambda E[(E(c_{y,t}) - c_{y,t})^{\gamma^-_{PT}}]$. Given the convexity of the utility function under prospect theory, uncertainty decreases the expected disutility (or increases expected utility) that derives from consumption in period $t+1$. Hence, if absent period $t+1$ uncertainty the two corner solutions generate equal levels of utility, adding period $t+1$ uncertainty implies it is optimal to let the future generation bear all of the shock.

In the end, we thus have the result that the funded pension scheme allocates an adverse shock fully to the future generation. Why? Because the future is uncertain. Hence, consumption may be even worse if the future features another adverse shock. But consumption of the future generation may also drop less or not at all. Being in the convex losses part of the utility function, the pension fund is risk seeking and is willing to take the risk of very low consumption in the hope of avoiding the expected consumption loss (Shefrin and Statman, 1985).

[9] Actually, things are even more strange. Absent any shock, it is optimal under the chosen definition of reference position to increase consumption of the young to the maximum extent. That is the point where the consumption of the future generation would be pushed to nil (negative consumption is technically infeasible). Such a policy would increase the utility enjoyed by the young without any disutility for the future generation.

[10] Kőszegi and Rabin (2006) and Barberis (2013) argue that utility in the case of prospect theory should have at least two arguments, namely both the deviation of consumption from its reference position and the level of consumption. The problem with the conditional expectation of consumption would probably disappear when one adopts such a utility function, but to investigate it is beyond the scope of the current paper.

A positive shock

To find out the optimal way to absorb a positive shock, the pension fund adopts the gains part of the prospect-theory utility function:

$$V_{PT,t}(i) = (c_{y,t} - E(c_{y,t}))^{\gamma_{PT}^{+}} + \frac{1}{\Delta}(c_{y,t+1} - E(c_{y,t+1}))^{\gamma_{PT}^{+}} \tag{43}$$

As reference position, we adopt the unconditional expectation of consumption. As the derivation is similar to the one presented in the previous section, we turn immediately to the derived expressions for the intertemporal transfer, consumption of the young and consumption of the future generation:

Welfare

See the appendix.

6 Optimal pension fund policies; the general case

This section integrates the last two sections. It focuses on a pension scheme that can allocate risks to an old generation, a young generation and a future generation. It will be no surprise that the analysis of this general case offers no new insights. We perform this analysis only to see how the welfare effects for this general case compare with those for the two partial cases analyzed before.

Like before, we start with expected utility theory.

6.1 Expected utility theory

6.2 Prospect theory

As before, we will start to explore the case of an adverse shock.

An adverse shock

A positive shock

Welfare

7 Concluding remarks

The issue of intergenerational risk sharing by pension funds and its associated welfare gains is important. It is a major argument in the policy discussion in the Netherlands about pension reform. Surprisingly, little attention has been paid thus far to prospect theory, whereas there is an abundant literature that shows that prospect theory offers a more realistic model of decision-making under risk than traditional expected utility theory.

What we find is that the welfare gain from risk sharing is smaller under prospect theory than under expected utility theory. This has to do with the curvature of the utility functions under the two theories and with the implications of convex-concave utility for risk sharing policies of pension funds. In our assessment, we have necessarily made a host of assumptions that cannot easily be verified. In order to reduce the uncertainty that results from this, we have i) stayed as close as possible to existing parameter estimates as can be derived from the earlier literature and ii) provided a detailed sensitivity analysis.

One may of course object that still the results are uncertain. We could have introduced more sources of uncertainty or adopted a model with a shorter time period. We agree. It should be stressed however that our results relate to fundamental aspects of prospect theory. Therefore, we do not think that our results will be easily rejected.

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Appendix

To calculate the certainty-equivalent of risk sharing policies, we need to calculate two certainty-equivalents. One relates to optimal pension policies, the other one relates to the case where risk sharing policies are absent. By subtracting the former from the latter, we derive the certainty-equivalent of optimal risk sharing policies. We apply this approach to a number of cases. The first is that of PAYG schemes and expected utility theory.

We start by writing down the expression for welfare under optimal policies,

$$E(V_{EU}^*) = \frac{1}{n}\sum_{i=1}^{n}\Big[\frac{(c_{o,EU}^*(i))^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{(c_{y,EU}^*(i))^{\gamma_{EU}}}{\gamma_{EU}}\Big] \tag{44}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\Big[\frac{(\Phi_{EU}(w+y(i)))^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{((1-\Phi_{EU})(w+y(i)))^{\gamma_{EU}}}{\gamma_{EU}}\Big]$$

where the second line follows from using equations (18) and (19).

The corresponding certainty-equivalent of consumption can be calculated as in equation (3), although now the certainty-equivalent applies to consumption in all states of nature for two generations:

$$\Upsilon_{EU}^* = 1 - \left\{\frac{E(V_{EU}^*)}{\Big(\frac{E(y)^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{w^{\gamma_{EU}}}{\gamma_{EU}}\Big)}\right\}^{\frac{1}{\gamma_{EU}}} \tag{45}$$

Next, we elaborate the same expression for welfare in case the pension fund does not apply any risk sharing, *i.e.* $z = 0$:

$$E(V_{EU}|z=0) = \frac{1}{n}\sum_{i=1}^{n}\Big[\frac{y(i)^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{w^{\gamma_{EU}}}{\gamma_{EU}}\Big] \tag{46}$$

The expression for the certainty-equivalent of consumption is again similar:

$$(\Upsilon_{EU}|z=0) = 1 - \left\{\frac{E(V_{EU}|z=0)}{\Big(\frac{E(y)^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{w^{\gamma_{EU}}}{\gamma_{EU}}\Big)}\right\}^{\frac{1}{\gamma_{EU}}} \tag{47}$$

Finally, the certainty-equivalent of consumption that corresponds with optimal risk sharing policies follows from subtracting Υ_{EU}^* from $(\Upsilon_{EU}|z=0)$:

$$\tilde{\Upsilon}_{EU} = (\Upsilon_{EU}|z=0) - \Upsilon_{EU}^* \tag{48}$$

The second application is that of a PAYG scheme in the case of prospect theory. First, we calculate welfare and the corresponding certainty-equivalent of optimal policies,

$$\begin{aligned}
& E(V^*_{PT}) \\
= & -\lambda \frac{1}{n^-} \sum_{i=-n^-}^{-1} \Big[(E(y) - c^-_{o,PT}(i))^{\gamma^-_{PT}} + \frac{1}{\Delta}(w - c^-_{y,PT}(i))^{\gamma^-_{PT}} \Big] \pi^-(y(i)) \\
+ & \frac{1}{n^+} \sum_{i=1}^{n^+} \Big[(c^+_{o,PT}(i) - E(y))^{\gamma^+_{PT}} + \frac{1}{\Delta}(c^+_{y,PT}(i) - w)^{\gamma^+_{PT}} \Big] \pi^+(y(i)) \\
= & -\lambda \frac{1}{n^-} \sum_{i=-n^-}^{-1} (E(y) - y(i))^{\gamma^-_{PT}} \pi^-(y(i)) \qquad (49) \\
+ & \frac{1}{n^+} \sum_{i=1}^{n^+} \Big[(\Phi_{PT})^{\gamma^+_{PT}} + \frac{1}{\Delta}(1 - \Phi_{PT})^{\gamma^+_{PT}} \Big] (y(i) - E(y))^{\gamma^+_{PT}} \pi^+(y(i))
\end{aligned}$$

$$\Upsilon^*_{PT} = \left(\frac{-E(V^*_{PT})}{\lambda \big(E(y)^{\gamma^-_{PT}} + \frac{1}{\Delta} w^{\gamma^-_{PT}} \big)} \right)^{1/\gamma^-_{PT}} \qquad (50)$$

where the second line in equation (49) follows from using equations (25), (26), (30) and (31).

Second, we calculate welfare and the corresponding certainty-equivalent for the case in which the pension fund does not engage in any risk sharing policies:

$$\begin{aligned}
E(V_{PT}|z=0) \;=\; & -\lambda \frac{1}{n^-} \sum_{i=-n^-}^{-1} (E(y) - y(i))^{\gamma^-_{PT}} \pi^-(y(i)) \\
& + \frac{1}{n^+} \sum_{i=1}^{n^+} (y(i) - E(y))^{\gamma^+_{PT}} \pi^+(y(i)) \qquad (51)
\end{aligned}$$

$$(\Upsilon_{PT}|z=0) = \left(\frac{-E(V_{PT}|z=0)}{\lambda \big(E(y)^{\gamma^-_{PT}} + \frac{1}{\Delta} w^{\gamma^-_{PT}} \big)} \right)^{1/\gamma^-_{PT}} \qquad (52)$$

Thirdly, we combine the two certainty-equivalents to derive at the certainty-equivalent of risk sharing policies:

$$\tilde{\Upsilon}_{PT} = (\Upsilon_{PT}|z=0) - \Upsilon^*_{PT} \qquad (53)$$

The third and fourth application refer to the funded pension scheme. The expressions are similar to the equations for the PAYG scheme (equations (44) to (53)); only the input variables are different. Hence, we have in the case of expected utility theory,

$$E(V_{EU}^*) = \frac{1}{n}\sum_{i=1}^{n}\Big[\frac{(c_{y,t,EU}^*(i))^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{(c_{y,t+1,EU}^*(i))^{\gamma_{EU}}}{\gamma_{EU}}\Big] \tag{54}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\Big[\frac{(2\Phi_{EU}w_t(i))^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{(2\big(1-\Psi_{EU}\big)w_t(i)(1+r_{t+1}))^{\gamma_{EU}}}{\gamma_{EU}}\Big]$$

$$\Upsilon_{EU}^* = 1 - \left\{\frac{E(V_{EU}^*)}{\Big(\frac{E(y)^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{w^{\gamma_{EU}}}{\gamma_{EU}}\Big)}\right\}^{\frac{1}{\gamma_{EU}}} \tag{55}$$

$$E(V_{EU}|z=0) = \frac{1}{n}\sum_{i=1}^{n}\Big[\frac{y(i)^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{w^{\gamma_{EU}}}{\gamma_{EU}}\Big] \tag{56}$$

$$(\Upsilon_{EU}|z=0) = 1 - \left\{\frac{E(V_{EU}|z=0)}{\Big(\frac{E(y)^{\gamma_{EU}}}{\gamma_{EU}} + \frac{1}{\Delta}\frac{w^{\gamma_{EU}}}{\gamma_{EU}}\Big)}\right\}^{\frac{1}{\gamma_{EU}}} \tag{57}$$

$$\tilde{\Upsilon}_{EU} = (\Upsilon_{EU}|z=0) - \Upsilon_{EU}^* \tag{58}$$

where the second line of equation (54) follows from using equations (35) and (37).

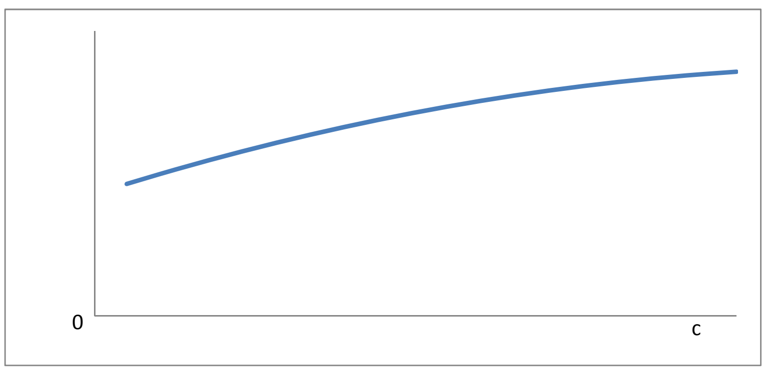

Figure 1: Expected utility theory: utility as a function of the level of consumption

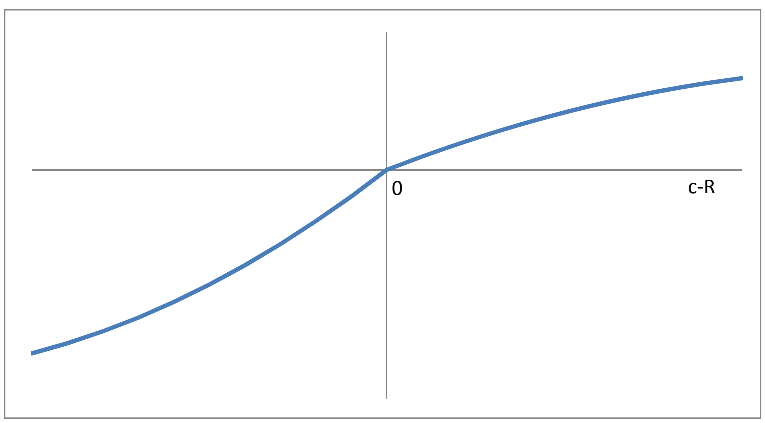

Figure 2: Prospect theory: utility as a function of the change in consumption

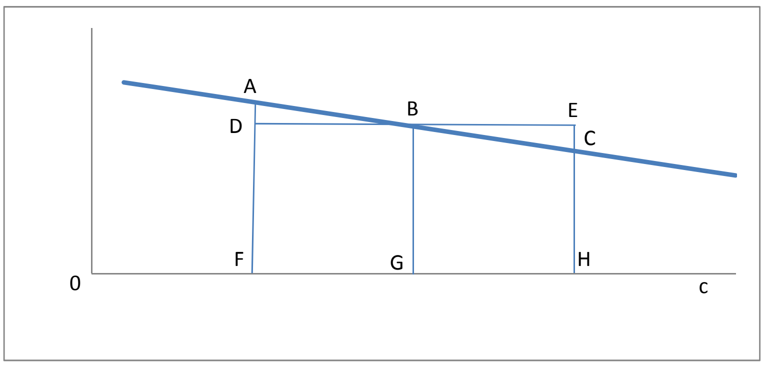

Figure 3: Expected utility theory: marginal utility as a function of the level of consumption

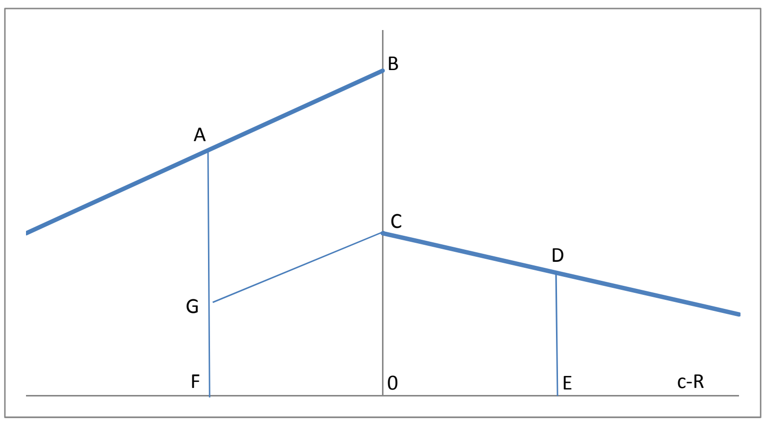

Figure 4: Prospect theory: marginal utility as a function of the change in consumption

Table 1: Welfare loss due to consumption risk under prospect and expected utility theory - the case of a two-state consumption distribution

Simulation		Υ_{EU}	Υ_{PT}
(1)	Benchmark values	0.010	0.023
(2)	As in (1), but $\gamma_{EU} = -0.2$	0.006	0.023
(3)	As in (1), but $\gamma_{EU} = -4.0$	0.024	0.023
(4)	As in (1), but $\lambda = 1.5$	0.010	0.013
(5)	As in (1), but $\lambda = 3.0$	0.010	0.029
(6)	As in (1), but $\gamma_{PT}^{-} = 1.0$	0.010	0.037
(7)	As in (1), but $\gamma_{PT}^{-} = \gamma_{PT}^{+} = 1.0$	0.010	0.028
(8)	As in (1), but $c_B = 0.99$, $c_G = 1.01$	0.000	0.002
(9)	As in (1), but $c_B = 0.95$, $c_G = 1.05$	0.003	0.012
(10)	As in (1), but $c_B = 0.8$, $c_G = 1.2$	0.040	0.047
(11)	As in (1), but $c_B = 0.7$, $c_G = 1.3$	0.090	0.070

Table 2: Welfare loss due to consumption risk under prospect and expected utility theory - the case of a multi-state consumption distribution

Simulation		Υ_{EU}	Υ_{PT}
(1)	Benchmark values	0.162	0.082
(2)	As in (1), but $\gamma_{EU} = -0.2$	0.101	0.082
(3)	As in (1), but $\gamma_{EU} = -4.0$	0.357	0.082
(4)	As in (1), but $\lambda = 1.5$	0.162	0.050
(5)	As in (1), but $\lambda = 3.0$	0.162	0.099
(6)	As in (1), but $\gamma_{PT}^{-} = 1.0$	0.162	0.101
(7)	As in (1), but $\gamma_{PT}^{-} = \gamma_{PT}^{+} = 1.0$	0.162	0.093
(8)	As in (1), but $\sigma = 0.3$	0.086	0.057
(9)	As in (1), but $\sigma = 0.15$	0.022	0.028

Table 3: Welfare loss due to consumption risk under prospect theory - the role of probability weighting

Simulation		Υ_{PT}	$\hat{\Upsilon}_{PT}$
(1)	Benchmark values	0.082	0.086
(2)	As in (1), but $\lambda = 1.5$	0.050	0.037
(3)	As in (1), but $\lambda = 3.0$	0.099	0.113
(4)	As in (1), but $\gamma_{PT}^{-} = 1.0$	0.101	0.128
(5)	As in (1), but $\gamma_{PT}^{-} = \gamma_{PT}^{+} = 1.0$	0.093	0.109
(6)	As in (1), but $\sigma = 0.3$	0.057	0.061
(7)	As in (1), but $\sigma = 0.15$	0.028	0.030
(8)	As in (1), but $\epsilon^{-} = 0.85$, $\epsilon^{+} = 0.80$	0.082	0.079
(9)	As in (1), but $\epsilon^{-} = 0.55$, $\epsilon^{+} = 0.40$	0.082	0.067

Table 4: Welfare loss under expected utility and prospect theory - optimal policies for a PAYG pension scheme

Simulation		$\tilde{\Upsilon}_{EU}$	$\tilde{\Upsilon}_{PT}$
(1)	Benchmark values	0.024	0.006
(2)	As in (1), but $\gamma_{EU} = -0.2$	0.001	0.006
(3)	As in (1), but $\gamma_{EU} = -4.0$	0.148	0.006
(4)	As in (1), but $\lambda = 1.5$	0.024	0.008
(5)	As in (1), but $\lambda = 3.0$	0.024	0.004
(6)	As in (1), but $\sigma = 0.3$	0.012	0.004
(7)	As in (1), but $\sigma = 0.15$	0.003	0.002
(8)	As in (1), but $\epsilon^- = 0.69$ and $\epsilon^+ = 0.61$		