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Variable Annuity and Interest Rate Risk

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Financial Decision of a Retiree

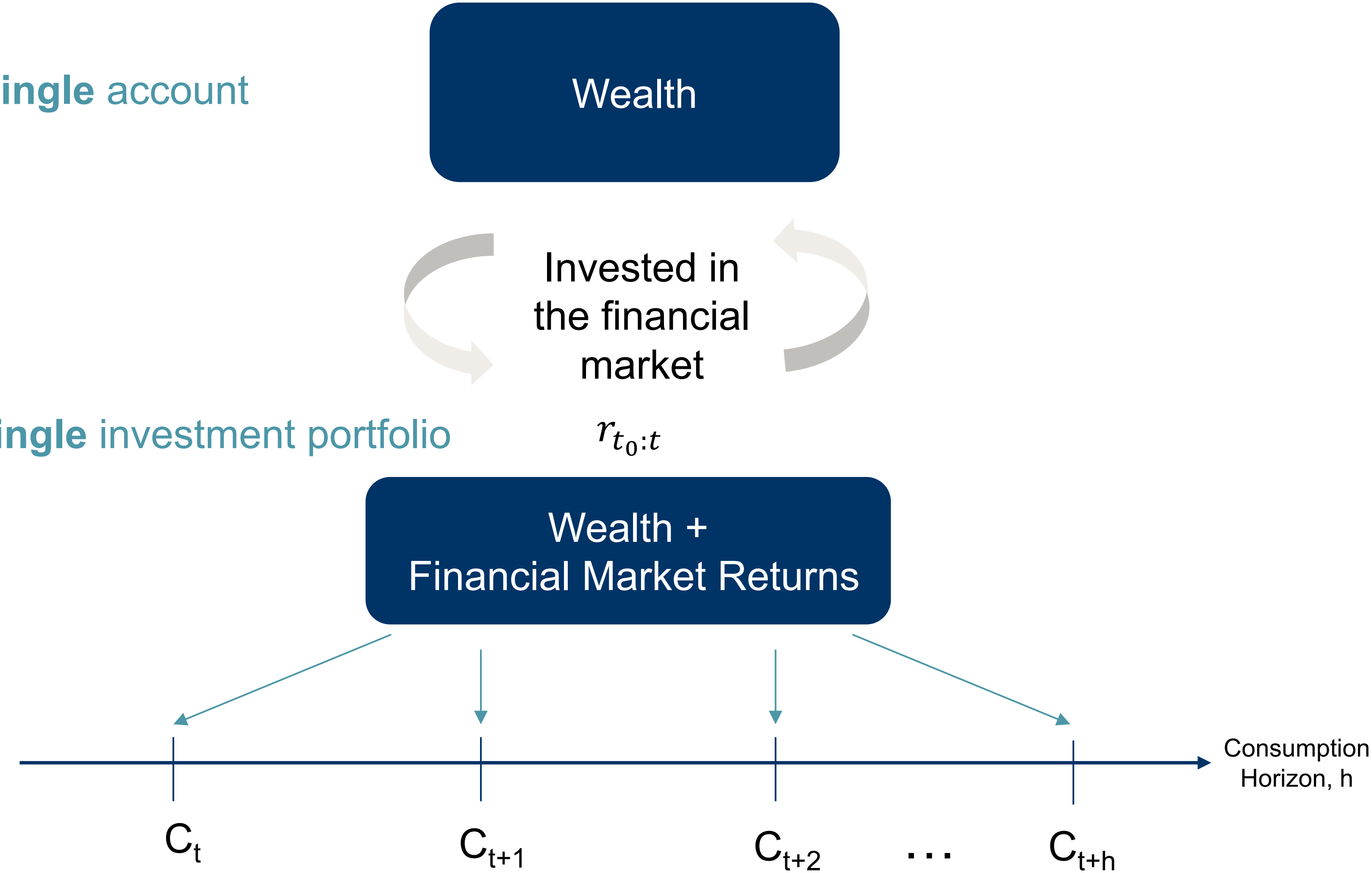
How much to consume?

How to invest?

Intertemporal Consumption and Investment Problem

Single account

Single investment portfolio



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Stock Market and Interest Rate Risk

Merton's (1973) solution:

1. Investment Rule θ_{t+h}

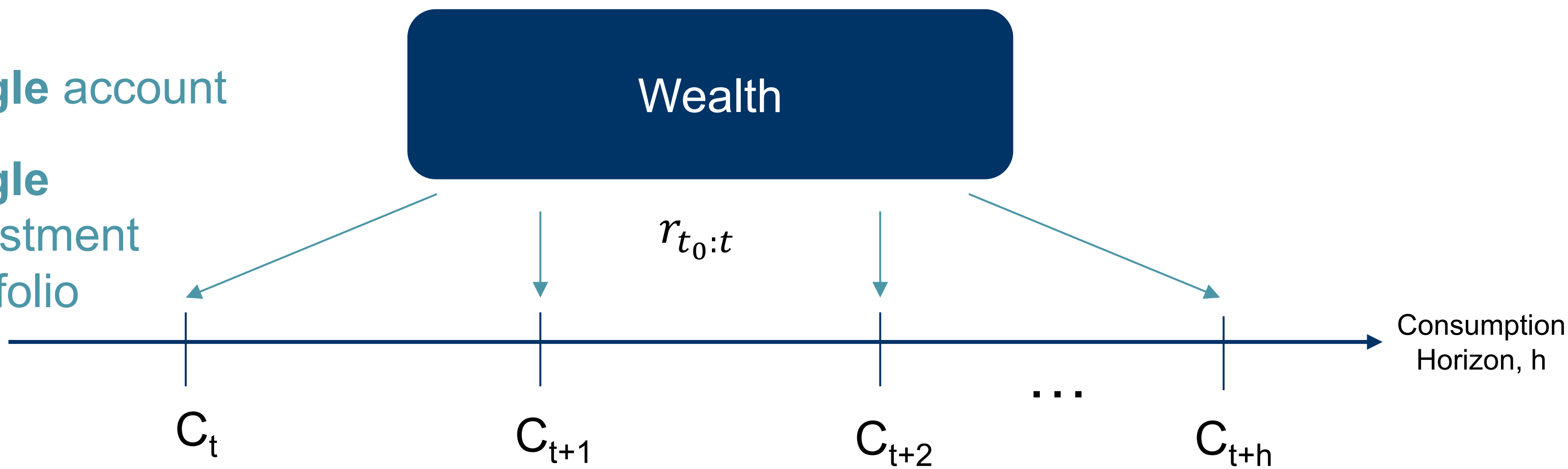
Function of the financial assets' expected returns, standard deviation of returns, and the individual's level of risk aversion.

2. Consumption Rule C_{t+h}

Function of realized financial market outcome.

Single account

Single investment portfolio



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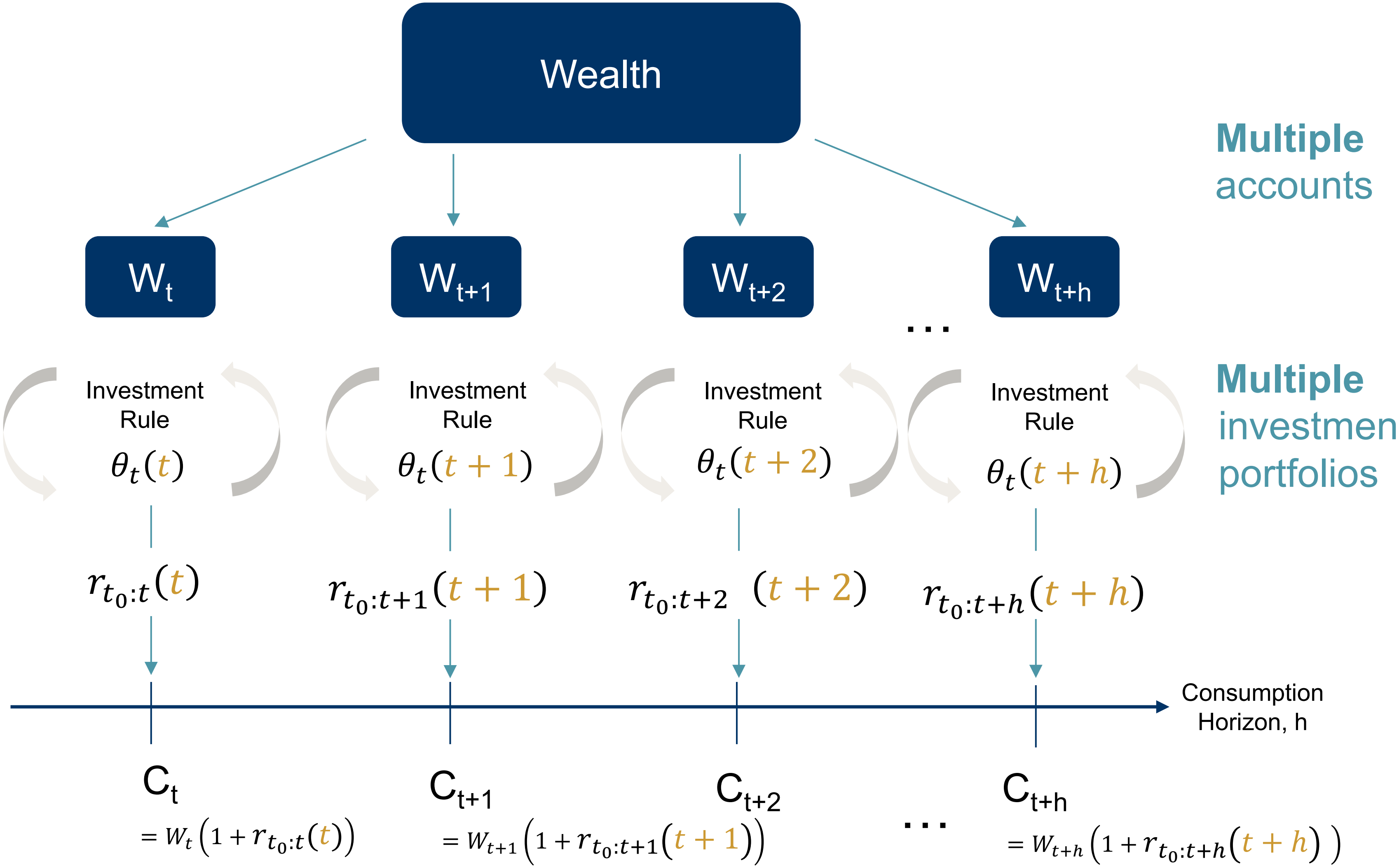
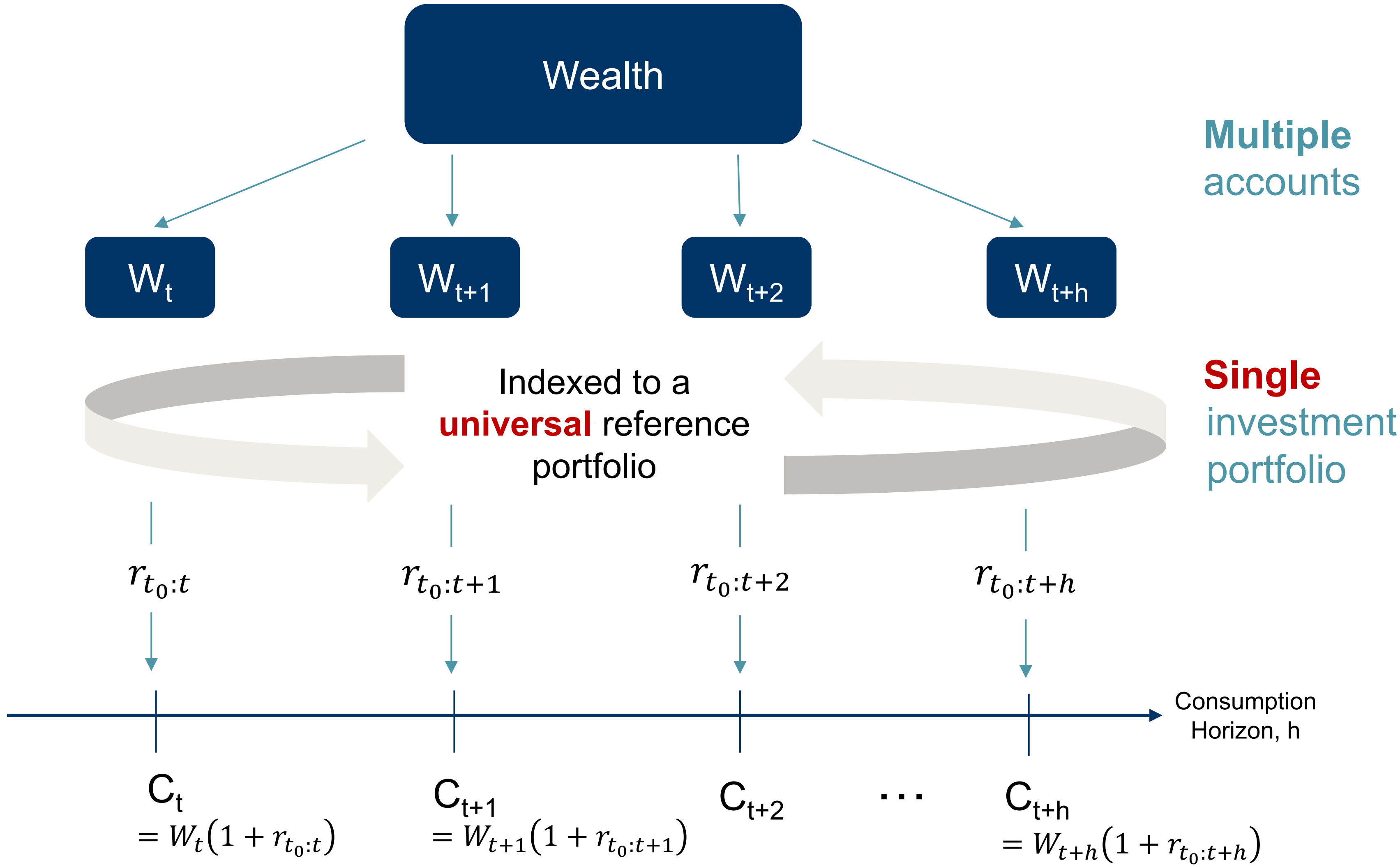
Two-Stage Formulation

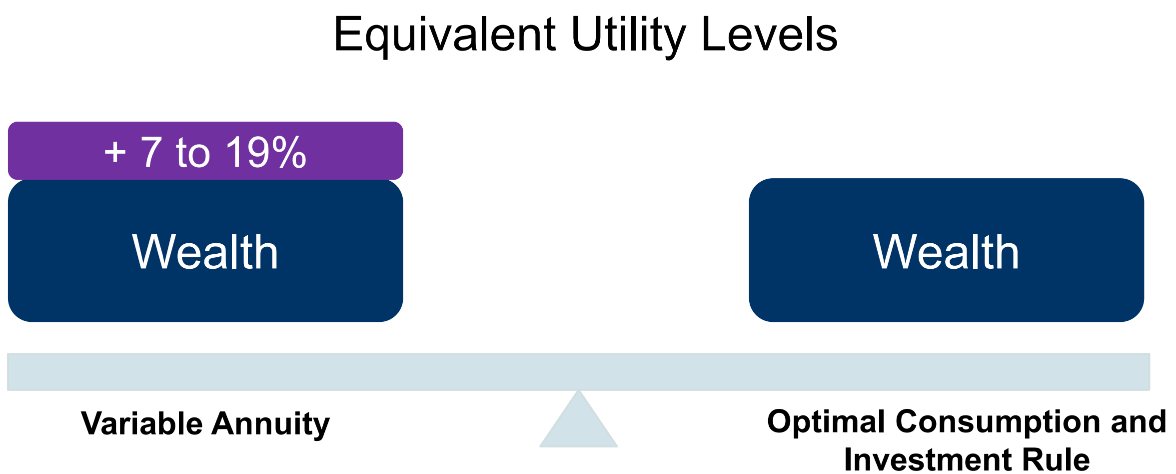


Variable Annuity



Variable Annuity Welfare Losses

The variable annuity fails to optimally hedge interest rate risk at every consumption period.



A straightforward **refinement of unit-linked contracts** (e.g., variable annuity) to allow dependence of the unit on the consumption period **improves individual welfare**.

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Setup

Financial Market

- Interest rate (Vasicek, 1977):

$$dr_t = \kappa(\mu_r - r_t)dt + \sigma_r dZ_{r,t}$$

- Stock market index:

$$dS_t = S_t(r_t + \lambda_S \sigma_S)dt + S_t \sigma_S dZ_{S,t}$$

- Constant τ maturity bond fund; and,
- Stochastic discount factor:

$$dM_t = -M_t r_t dt + M_t \phi_r dZ_{r,t} + M_t \phi_S dZ_{S,t}$$

Individual Preference

- Certain lifetime between ages 65 in year t_0 to 110 in year T.
- Initial endowment, $W_{t_0} = 1$
- Annual retirement benefits, C_{t_0+h}, $h = 0, \dots, T - t_0$.
- CRRA Utility: $\int_0^{T-t_0} e^{-\beta h} \frac{{C_{t_0+h}}^{1-\gamma}}{1-\gamma} dh$, $\gamma = 2, \dots, 10$.

Optimal Consumption

$$C^*_{t_0+h} = W_{t_0} X^*_{t_0}(h, T - t_0)\, Y^*_{t_0+h}(h)$$

Initial endowment

Division of wealth over the consumption horizon, h

Indexation Portfolio

Stage 1: Choose $X^*_{t_0}$

Stage 2: Choose $Y^*_{t_0+h}$

The two decision variables $X^*_{t_0}$ and $Y^*_{t_0+h}$ allude to the two-stage formulation.

Optimal Investment

$$\theta^*_{S,t_0+u}(h) = -\frac{\phi_S}{\gamma\sigma_S}$$

$$\theta^*_{B,t_0+u}(h) = \frac{\phi_r}{\gamma\sigma_r B(\tau)} + \left(1 - \frac{1}{\gamma}\right)\frac{\hat{B}(h-u)}{B(\tau)}$$

Time-invariant speculative demands

Time-varying (h & u) hedge demand

The bond hedge demand is a weighted average over the horizon:

$$\hat{B}(h-u) = \int_0^{h-u} X^*_{t_0}(l, h-u)\, B(l)\, \mathrm{d}l$$

Two-Stage (2S) Formulation

$$C_{t_0+h}^{2S} = W_{t_0} X_{t_0}^{2S}(h, T-t_0)\, Y_{t_0+h}^{2S}(h)$$

Stage 1: Choose $X_{t_0}^{2S}$ given any investment policy that defines $Y_{t_0+h}^{2S}$.

Stage 2: Regard every date $t_0 + h$ as a **standalone terminal wealth utility maximization problem** to determine the investment policy that defines $Y_{t_0+h}^{2S}$.

$$\theta_{S,t_0+u}^{2S}(h) = -\frac{\phi_S}{\gamma\sigma_S}$$

$$\theta_{B,t_0+u}^{2S}(h) = \frac{\phi_r}{\gamma\sigma_r B(\tau)} + \left(1-\frac{1}{\gamma}\right)\frac{B(h-u)}{B(\tau)}$$

Both formulations are equivalent.

Variable Annuity

$$C_{t_0+h}^{VA} = W_{t_0} \frac{\exp\left(-a_{t_0}(h) \times h\right)}{\int_0^{T-t_0} \exp\left(-a_{t_0}(t) \times t\right) \mathrm{d}t} Y_{t_0+h}^{VA}$$

Initial endowment

Division of wealth over the consumption horizon, h

Indexation Portfolio

Similar three-term representation.

$a_{t_0}(h)$ is the Assumed Interest Rate.

Variable Annuity Indexation Portfolio

Variable Annuity	Two-Stage
$Y^{VA}_{t_0+u}$	$Y^{2S}_{t_0+u}(h)$
depends only on the planning horizon.	depends on the planning and consumption horizons.
$\left\{\theta^{VA}_{t_0+u}\right\}^{T-t_0}_{u=0}$	$\left\{\left\{\theta^{2S}_{t_0+u}(h)\right\}^{h}_{u=0}\right\}^{T-t_0}_{h=0}$

Dependence on both horizons arise from the bond hedge demand:

$$\theta^{2S}_{B,t_0+u}(h) = \frac{\phi_r}{\gamma\sigma_r B(\tau)} + \left(1-\frac{1}{\gamma}\right)\frac{B(h-u)}{B(\tau)}$$

<u>Consequence</u>

VA fails to optimally hedge for interest rate risk for all consumption periods.

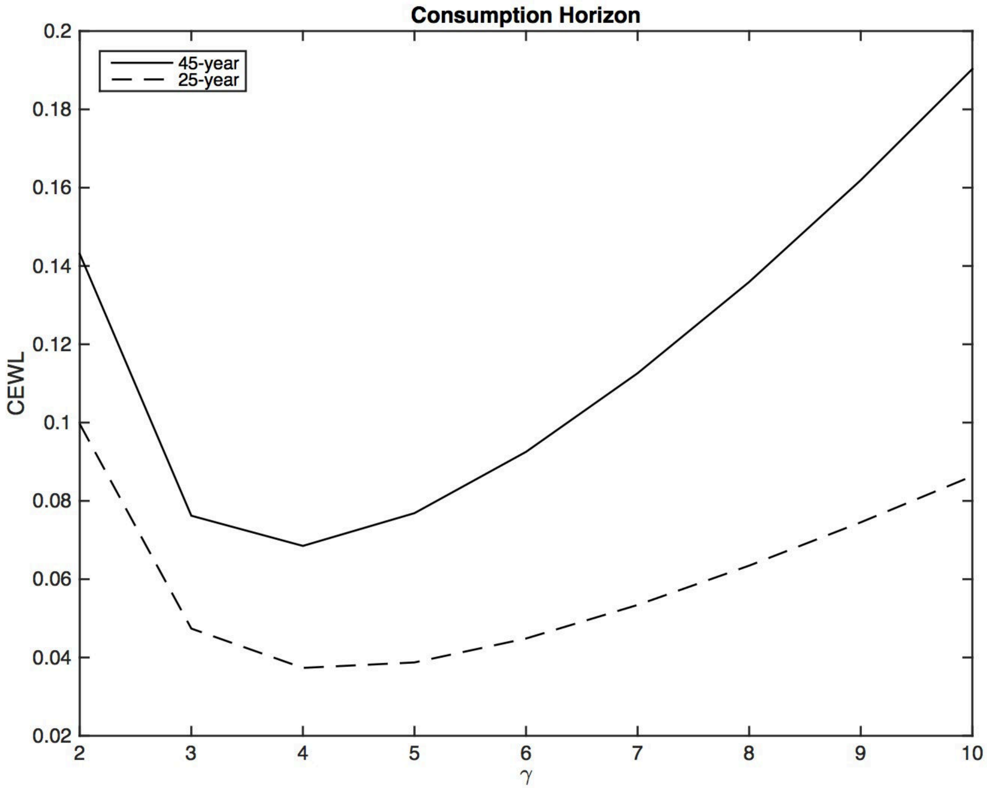
Welfare Losses (1/2)

Financial market parameters calibrated to U.S. data.

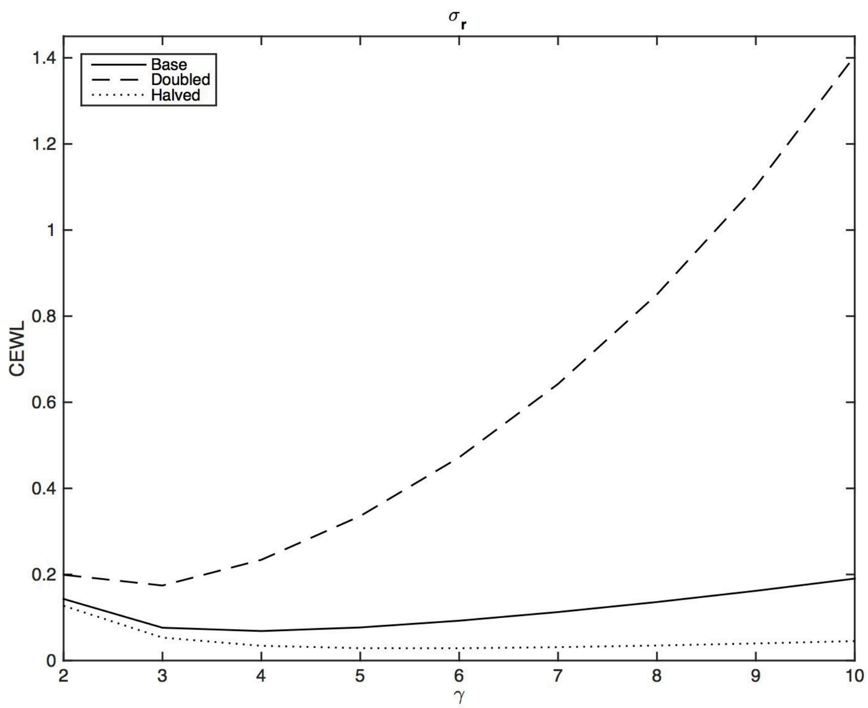
Quantify welfare losses with the **Certainty Equivalent Wealth Loading (CEWL)**.

- Proportional loading on the initial wealth that the individual requires to purchase a VA* that yields the same expected utility as with the 2S.

Non-monotonicity with respect to risk aversion level, γ.



Welfare Losses (2/2)



Welfare losses are more severe when there is:

- Lower subjective discount factor;
- Higher interest rate risk;
- Lower interest rate mean reversion rate, or mean reversion rate level, and;
- Higher correlation between interest rate and stock market risks.

Conclusion

The optimal consumption and investment problem is **equivalent** to a **two-stage problem** that:

1. Selects the **optimal division of initial wealth** over the consumption horizon.
2. Sets the **optimal investment policy** for each portion of divided wealth.

Constraining the investment policy to vary only along the planning horizon, the variable annuity fails to optimally hedge interest rate risk at every consumption period.

The **utility loss** arising from the VA's deficient interest rate risk hedge is **substantial**: 7-19%.

Applicable to **all unit-linked contracts** without guarantees:

- Pension contract design discussions in Australia (CIPR) and Europe (PEPP), especially the Netherlands (PPR) and Denmark.

CIPRs: Comprehensive Income Products for Retirement.
PEPP: Pan-European Personal Pension Product.
PPR: Personal Pension with Risk-Sharing (Bovenberg and Nijman, 2016; Bovenberg and Nijman, 2017).

Appendices

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Financial Market Parameters

Parameter	Estimate	Standard Error
Stock Return Process: $\mathrm{d}S_{t_0+u} = (r_{t_0+u} + \lambda_S \sigma_S) S_{t_0+u} \mathrm{d}u + \sigma_S S_{t_0+u} \mathrm{d}Z_{S,t_0+u}$		
σ_S	0.158	0.004
λ_S	0.467	0.164
Short Rate Process: $\mathrm{d}r_{t_0+u} = \kappa(\mu_r - r_{t_0+u}) \mathrm{d}u + \sigma_r \mathrm{d}Z_{r,t_0+u}$		
μ_r	0.036	0.043
κ	0.067	0.003
σ_r	0.017	0.001
λ_r	-0.350	0.177
Stochastic Discount Factor Process: $\mathrm{d}M_{t_0+u} = -r_{t_0+u} M_{t_0+u} \mathrm{d}u + \phi_S M_{t_0+u} \mathrm{d}Z_{S,t_0+u} + \phi_r M_{t_0+u} \mathrm{d}Z_{r,t_0+u}$		
ρ_{Sr}	0.120	0.049
ϕ_S	-0.516	
ϕ_r	0.412	

Maximum likelihood parameter estimates of the interest rate and the stock return, obtained by implementing the Kalman filter on monthly US government bond yields of 3-month, 1, 5, and 10-year maturities, and the return on the CRSP value- weighted stock index, from August 1971 till December 2014. Standard errors are by the outer product of gradients.

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