Discussion of paper Kristy Jansen

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Outline

Summary of paper

- Paper studies optimal asset allocation with illiquid assets
- Illiquidity defined as periods of “non-trading”
- Re-write optimisation problem in terms of total wealth $(W_t + X_t)$ and fraction in illiquid assets ξ_t
- Assume CRRA utility
- Study impact of illiquid asset on optimal investment strategy

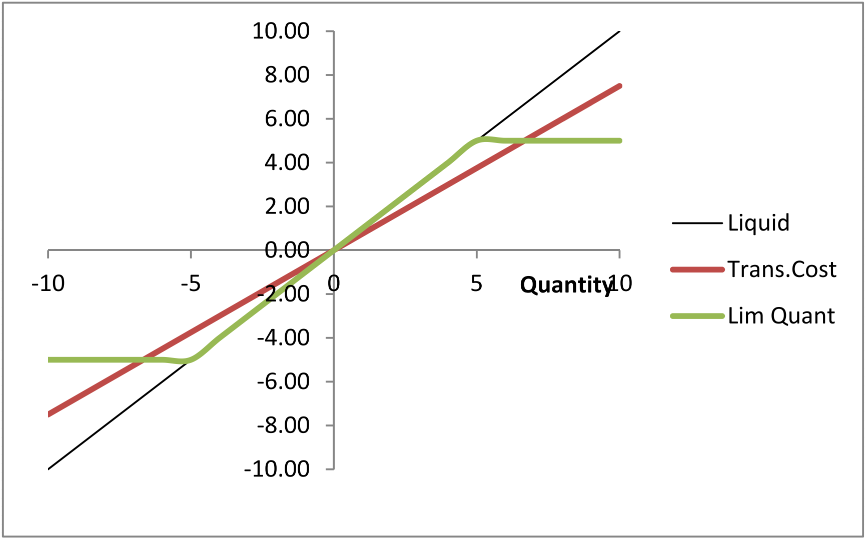
Definition Illiquid

We have different definitions of “illiquid” in the literature:

- Buy/sell illiquid asset at worse price. Like transaction costs.
- Restriction on traded quantity of illiquid asset.
- Restriction on trading time of illiquid asset.

All three definitions could be unified into one concept: price-function depends on traded quantity and on time. With numerical computation, all three cases can be computed.

Price-function



Choice Illiquid

- Why do you choose the “non-traded” definition of illiquidity?
- Provide more motivation for your choice!

Change of Variables (1)

- You perform a change of variables from (W_t, X_t) to $(W_t + X_t, \xi_t)$ where $\xi_t = X_t/(W_t + X_t)$
- You then factor the value-function as

$$V(t, W_t + X_t, \xi_t) = (W_t + X_t)^{1-\gamma} \cdot H(t, \xi_t)$$

  which you motivate by citing [Merton, 1971].
- However, I cannot find this factorisation in Merton...

Change of Variables (2)

- You do a proof by induction that this factorisation holds for all time-points
- Core idea of proof:

$$\begin{aligned} V(t-h) &= \mathbb{E}_{t-h}[V(t)] = \mathbb{E}_{t-h}\left[(W_t + X_t)^{1-\gamma}\ H(t,\xi_t)\right] \\ &= (W_{t-h} + X_{t-h})^{1-\gamma}\ \mathbb{E}_{t-h}\left[\left(\tfrac{W_t+X_t}{W_{t-h}+X_{t-h}}\right)^{1-\gamma} H(t,\xi_t)\right] \\ &= (W_{t-h} + X_{t-h})^{1-\gamma}\ \mathbb{E}^*_{t-h}\left[H(t,\xi_t)\right] \\ &= (W_{t-h} + X_{t-h})^{1-\gamma}\ H(t-h,\xi_{t-h}) \end{aligned}$$

Change of Variables (3)

- However...
- You take conditional expectations given $\mathcal{F}_{t-h}$
- $\mathbb{E}^*_{t-h}[H(t,\xi_t)] = \mathbb{E}^*[H(t,\xi_t) \mid W_{t-h} + X_{t-h}, \xi_{t-h}]$
- You must rule out the case: $H(t-h, W_{t-h} + X_{t-h}, \xi_{t-h})$
- In your current proof you achieve this by imposing that the control policies are functions of ξ_t only, e.g. $\theta(t,\xi)$
- But you are now using a *smaller class* of control policies compared to the original problem: $\theta(t, W, X)$. This may lead to sub-optimal control policies.
- How do you compute these conditional expectations numerically? Your description in Section 2.2.2 is a bit vague...

Interpretation of p

- At the horizon T you have a probability p that the asset X is liquid, and $(1-p)$ that the asset sells at the lower value $d\,X_T$
- At all earlier time-steps you also have these probabilities
- What is the economic interpretation of this?
- What happens with p when you make smaller time-steps? What happens in the limit for $h \to 0$?

Case $p = 0$

- We do have an interesting case for $p = 0$: asset X cannot be traded at all in $[0, T]$. Economically reasonable case.
- Obtain initial position X_0 at $t = 0$ and then hold position until time T. Liquidate position for $d\,X_T$
- Economically viable when excess return on X compensates for non-trading and discount d
- I believe we can solve this case in "semi explicit" form using [Cox and Huang, 1989] approach
- Also benchmark for your numerical computations

Solving case $p = 0$ with Cox-Huang

- For example, consider non-consumption case for $p = 0$, then we have

$$\max_{X_0, W_T} \mathbb{E}\Big[U\big(W_T + X_0(de^{R_T^X})\big)\Big]$$
$$\text{s.t. } e^{-rT}\mathbb{E}^{\mathbb{Q}}[W_T] + X_0 = w_0$$

- Notice the “static” formulation, where we choose initial illiquid position X_0 and final liquid wealth W_T.
- In complete market we can replicate every possible W_T.
- We can “integrate out” the non-tradeable return R_T^X to obtain the modified utility: $\hat{U}(W_T, X_0) := \mathbb{E}[U(W_T + X_0(de^{R_T^X}))|\mathcal{F}_T^S]$
- $\hat{U}()$ will have higher risk-aversion due to “background risk” X_T
- Solve optimal liquid W_T for $\hat{U}()$ with reduced budget $w_0 - X_0$ using [Cox and Huang, 1989]

Explicit solution for $p = 0$ and quadratic util

- Example: quadratic util, $U(x) = -(x-K)^2$ with saturation point K
- Log-normal $e^{R_T^X}$ with drift ν and vol τ and assume uncorrelated
- $\hat{U}(W_T, X_0) = \mathbb{E}[-(W_T + X_0(de^{R_T^X}) - K)^2 | \mathcal{F}_T^S] = -(W_T - K)^2 - 2(W_T - K)X_0 de^{\nu T} - X_0^2 d^2 e^{(2\nu+\tau^2)T}$
- FOC: $\partial \hat{U}()/\partial W_T = -2(W_T - K) - 2X_0 de^{\nu T} = 2\lambda S_T^{-(\mu-r)/\sigma}$
- Optimal wealth: $W_T^* = K - X_0 de^{\nu T} - \lambda(X_0)\, S_T^{-(\mu-r)/\sigma}$ where $\lambda(X_0)$ is solved to satisfy budget constraint for each X_0
- Final step: find the optimal X_0 that maximises utility
- Note: optimal wealth W_t^* is *not* a function of ξ_t in this case

References I

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