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Optimal Investment in Annuities and Life Insurance for Retired Couples

The Role of Side Bequest Motives

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Abstract

Recent empirical evidence shows that a significant amount of wealth is bequeathed at the death of the first spouse. We study both theoretically and quantitatively how couples' retirement portfolio of annuities and life insurance is affected by such side bequests. We show that life-contingent products allow couples to smooth their side bequests as they enable to make the latter independent of which spouse dies first. We also show that side bequests reduce the wealth left to the surviving spouse while raising the demand for life insurance (or reducing the demand for annuities). Quantitatively, we find that side bequests have a sizable impact on the demand for annuities: annuity participation is 28 percentage points lower for married women and 41 percentage points lower for married men. Finally, we find that side bequests modify the distribution of welfare gains from having access to life-contingent products and that incorrectly omitting side bequest motives in the design of optimal portfolio strategies comes at large welfare costs.

Keywords: Life-Cycle Investment, Life Insurance, Annuities, Savings, Retirement Portfolio, Couple Dynamics, Bequest Motives.
JEL codes: C61, D14, D15, G11, G22, G51, G52.

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1 Introduction

Recent empirical evidence shows that couples, at least in the United States, mostly bequeath wealth at two moments in time: at the death of the widowed spouse, but also at the death of the first spouse (Fahle, 2025; De Nardi et al., 2025). Such 'side' bequests pose a tradeoff as the desire to bequeath at the death of the first spouse may come at the expense of the livelihood of the surviving spouse, who may, in addition, face a decrease in income at widowhood. In this paper, we highlight that life-contingent products, such as annuities and life insurance, can potentially help ease this tradeoff, and that the optimal demand for such products can be significantly affected by side bequest motives.

To date, the majority of studies on households' optimal financial decision-making consider only a single time of death and thereby omit the important financial consequences of spousal death. Additionally, the few studies that do account for variations in family size do not account for side bequest motives. In this paper, we try to address this gap. In particular, we try to shed light on the following questions. What are the implications of side bequest motives on the optimal demand for life-contingent products and the resulting welfare gains? To which extent do these products help in addressing the trade-off generated by side bequest motives?

To this end, we develop a life-cycle model of retired couples with investment in annuities and life insurance, in the spirit of Pliska and Ye (2007), in which we introduce side bequest motives as in De Nardi et al. (2025). More specifically, at the death of the first spouse, the couple decides on a side bequest by balancing the direct utility of this early bequest with the indirect utility of a surviving spouse, which accounts for potentially lower income at widowhood. We also account, unlike in Pliska and Ye (2007), for the fact that bequests are luxury goods. We use this framework to study, both theoretically and quantitatively, how side bequests affect the optimal demand for life-contingent products.

We first solve our model analytically in a complete market setup, and identify several mechanisms that underlie the interaction between side bequest motives and the decision to invest into life-contingent products. Firstly, we show that life-contingent products allow couples to smooth their side bequests. Absent life-contingent products, the optimal side bequest level is highly dependent on which spouse dies as, upon widowhood, surviving spouses have different income and longevity. With life-contingent products this is no longer the case: the optimal side bequest at a given point in time is independent of which partner dies first. For instance, if a husband has a larger pension upon widowhood than his wife, absent longevity differences and life-contingent products, the side bequest would be lower in the event of the husband's death. Indeed, in that case, the opportunity cost of leaving a large side bequest would be relatively high because of the lower income of the surviving wife. By purchasing relatively more life insurance or less annuities on the husband's life, the household can equalize resources at widowhood and, therefore, also side bequests.

Secondly, we find that stronger side bequest motives lower the wealth left to the surviving spouse while leading to larger investments in life insurance. While terminal bequests tend to be complementary to the 'provision' motive (i.e the desire to provide for a surviving spouse) for life insurance demand, this result underscores how side bequest motives can decrease the role of the provision motive as a driver of life insurance demand. We confirm this in our quantitative results in which we show that side bequest motives substantially increase the demand for life insurance while lowering the livelihood of a surviving spouse. Interestingly, this contrasts with e.g. Hubener et al. (2014) who find terminal bequest motives to be negligible drivers of life insurance demand relative to the provision motive, highlighting the very different implications of side and terminal bequests for the demand of life-contingent claims.

Thirdly, we challenge Pliska and Ye (2007)'s 'insurance principle', stating that life insurance investment is typically decreasing in wealth. We show that this is not the case in our model: realistic (luxury) bequest motives can cause life insurance investment to increase with wealth. As a result, luxury (side) bequest motives can reduce couples' desire for annuities.

On the quantitative side, we calibrate the model parameters to realistic values, restricting focus to married couples consisting of a husband and wife. We first compare couples who differ only by their side bequest motives, and find that these motives have a sizable impact on consumption and on the demand for life-contingent assets. For instance, in our baseline simulation of a couple in the second permanent income quartile with 300 thousand dollars of wealth at age 65, we find that side bequests lower the consumption of the couple by approximately 3,400 dollars (or about 7%). For these individuals, side bequests have a negligible impact on wealth profiles but raise the demand for life insurance (or reduce the demand for annuities) significantly. Similar qualitative patterns are observed for higher permanent income.

In the aggregate (i.e. considering different permanent income quartiles and a realistic wealth distribution), we find that, with side bequests, annuity participation is 28 percentage points lower for married women and 41 percentage points lower for married men. As a result, the expected present value of income through private annuities is 73% lower for married women and 95% lower for married men. While most of the literature on the so-called annuity puzzle has focused on retired singles, our results highlight that side bequest motives can rationalize a significant share of the low annuity demand of couples. Our results thus complement those of Lockwood (2012) who shows that realistically calibrated terminal bequest motives significantly reduce the annuity demand of single retirees.

We also study how having access to life-contingent products modifies the life-cycle profiles of wealth, consumption and side bequests, and we assess the welfare gains from having access to these products. In our baseline simulation, we find that, absent life-contingent products, the amount of side bequests left differs significantly depending on which spouse dies first. If the husband dies first, the side bequest is significantly lower because of the lower income and longer expected lifetime of his wife. This is especially the case early in retirement where the difference is of around 15% (or about 37 thousand dollars). To satisfy side bequest motives in presence of this asymmetry, the couple limits its consumption and accumulates wealth faster compared to the case with life-contingent assets. Early in retirement, consumption is about 1.8% lower and, by age 80, wealth is about 4.2% higher.

On the welfare side, we find that the average valuation of having access to life contingent products is significant, at about 6% of average wealth. Interestingly, side bequests barely change the average welfare gains but noticeably change who benefits most from having access to these. Absent side bequests, those in the lower permanent income quartiles benefit most from life contingent products as they want to annuitize quite significantly as terminal bequests, in particular, affect them less than richer households (as bequests are luxury goods). With side bequests, lower income households demand less annuities while richer households demand more life insurance. The former group thus benefits less from life contingent claims, as their desired annuity holdings get closer to zero, while for the latter group, the desired holdings of life insurance get further away from zero.

Finally, we quantify the importance of accounting for side bequest motives in the design of financial strategies for retired couples. The welfare loss of incorrectly omitting side bequest motives in this design are large: couples are on average willing to pay 15% of their wealth in order to get access to strategies that align with their side bequest preferences. Additionally, 36% of retired couples would prefer having no access to life-contingent products over subscribing to consumption-investment strategies that do not account for side bequest motives. These findings show that side bequest should be taken into account in the design of retired couples' financial strategies.

The paper is organized as follows. Section 2 discusses related literature. Section 3 present our model setup. Section 4 provides analytical results. Section 5 presents our model calibrations. Section 6 presents numerical results. Section 7 concludes.

2 Related Literature

Our paper builds on the recent literature concerning side bequests. Jones et al. (2020) document substantial declines in retired households' wealth around a time of death, in the U.S. They show that the decline for singles can be explained by increasing medical expenses around the time of death, but that medical expenses fall short in explaining the decline for couples. Instead, the majority of the decline can be attributed to side bequests, which are prominent and sizable. Following these findings, De Nardi et al. (2025) use changes in savings behavior to show that side bequests play an important role in explaining the saving patterns of couples in the U.S. In addition, they find that bequest motives, side and final, are a more important savings motive than medical expenses for retired couples. As medical expenses in the U.S. are generally much higher than in most countries, this finding underlines that the bequest motives of retired couples are a key driver in their financial decision-making. The fact that couples have stronger bequest motives is attributed to (1) bequests being luxury goods and couples being wealthier than singles and (2) the fact that couples leave two bequests.

This paper contributes to three research areas. Firstly, we contribute to the literature on optimal consumption and investment decisions during the life-cycle. This literature is pioneered by Yaari (1965) and Merton (1969, 1971), and extended by Richard (1975) and Pliska and Ye (2007), who introduced optimal investment in life-contingent products. Optimal life insurance and portfolio decisions of households have been studied extensively from many different angles since, including family dynamics (Kwak et al., 2011), inflation risk (Kwak and Lim, 2014), health risk (Koijen et al., 2016; Hambel et al., 2017), and housing decisions (Kraft et al., 2018). Moreover many studies focus on the roles of different preference structures, such as habit formation (Boyle et al., 2022) and time-inconsistency (Chen et al., 2024). In particular, we build on the few studies that introduced couple dynamics into the optimal life insurance framework (Bruhn and Steffensen, 2011; Wei et al., 2020). Different from these studies, we provide a post-retirement focus, model bequests as luxury goods and incorporate side bequest motives.

Secondly, we contribute to the literature on the annuity puzzle. Yaari (1965)'s seminal study shows that households without bequest motives should invest all their wealth in fair annuities. However, very few retired households participate in the private annuity market empirically. Loading factors combined with moderate (luxury) bequest motives go a long way in mitigating the demand for annuities, explaining part of this annuity puzzle (Lockwood, 2012; Arandjelović et al., 2023). Additionally, households having pessimistic survival expectations lead to higher perceived prices, decreasing the desire for private annuities (O'Dea and Sturrock, 2023), though we abstract from survival beliefs in this study. In the context of couples, Hubener et al. (2014)'s portfolio choice model finds that couples' portfolios are heavily weighted in (joint) annuities. However, utility costs from not participating in the private annuity market are low, possibly explaining the low empirical uptake. Hubener et al. (2014) consider only a final bequest, which is not a luxury good. They, therefore, forego the two main reasons for couples' stronger bequest motives, as found by De Nardi et al. (2025). In this paper, we do account for these findings, studying retired couples' demand for annuities, when bequests are luxury goods and side bequests are incorporated. We show that side bequest motives substantially decrease retired couples' desire for annuities, suggesting that side bequest motives play an important role in explaining the annuity puzzle for couples.

Our paper is complementary to the work by Bairoliya et al. (2025) who use life insurance holdings to identify end-of-life motives and the welfare gains from replacing part of the annuity benefits of Social Security with a death contingent payout. Compared to this paper, ours focuses more specifically on side bequests and provides a detailed characterization of the benefits of life-contingent products in the presence of these. In particular, we show that life-contingent products make side bequests independent of which spouse dies first, a mechanism that is absent in Bairoliya et al. (2025) as they consider a setting in which the head of the household always dies first. As a result, they cannot study relative life insurance and annuity holdings on a husband's versus a wife's lives.

3 Model

To study the effects of side bequest motives on the investment in life-contingent products, we extend the model by Pliska and Ye (2007) to couples and adopt the preference structure of De Nardi et al. (2025) to incorporate side bequests. In our model, a couple makes consumption and investment decisions, until either both partners have passed away, which is stochastic, or the final age T is reached. Here, we use a finite horizon because it is mathematically convenient and, provided T is sufficiently large so that living until T is very unlikely, economically insignificant. In addition, at the death of a first partner, the couple decides on a side bequest, trading off the desire to leave a bequest with the needs of a surviving partner. We denote a partner by $i \in \{1, 2\}$ and we use $\not{i}$ to denote i's partner. We first outline our mortality process and the life-contingent product, then we discuss the wealth dynamics and we formalize the household's optimization problem.

3.1 Mortality and Life-Contingent Investment

We denote partner i's time of death by τ_i, which is stochastic. If both partners are alive at time t, partner i's mortality rate is given by $\lambda_t^i = \lambda_C^i(t)$. If partner i is widowed at time t, their mortality rate is given by $\lambda_t^i = \lambda_S^i(t)$. These mortality rates are assumed to be strictly positive and continuous functions of time. We allow for the mortality rates to change after widowhood, as marital status has a strong effect on individuals' life expectancy (De Nardi et al., 2025). Figure 1 visualizes the mortality process, which follows a continuous-time Markov chain. Due to the continuous nature of our model, we exclude the possibility of simultaneous death.[1] Additionally, we do not allow for partners to divorce or for partners to remarry after widowhood.[2]

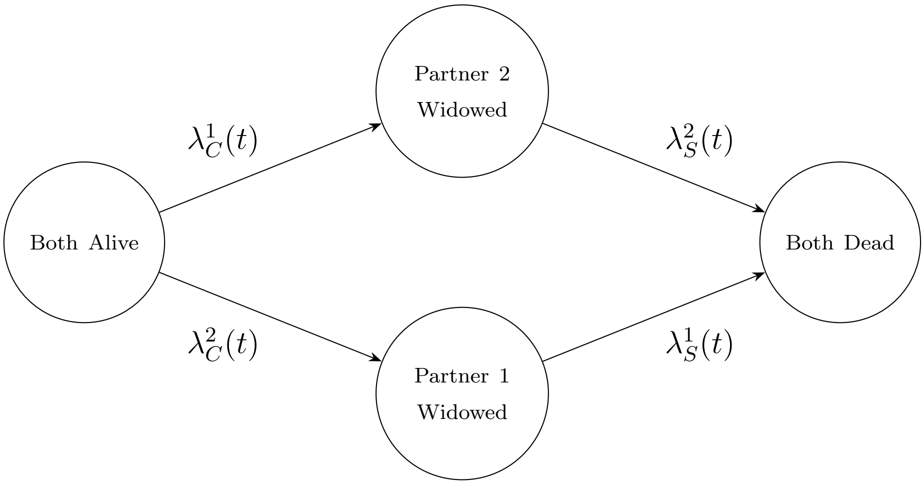

Figure 1: The household structure process

While partner i is alive, the couple can set a life insurance premium p^i, which pays out a lump sum in the event of i's death. These insurance contracts are short-term, as in Richard (1975) and Pliska and Ye (2007). When partner i dies, the life insurance pays out the amount

$$I_t^i = \frac{p_t^i}{\lambda_t^i}.$$

Intuitively, the couple sets the life insurance premium p_t^i at time t and this covers the risk of i dying for the infinitesimal time period $[t, t + \mathrm{d}t)$. Negative values of the premium, $p_t^i < 0$, lower the partner's bequest in return for an income stream now, which we interpret as a desire to invest in annuities. For simplicity, we assume both life insurance and annuities are actuarially fair.[3]

3.2 Wealth Dynamics

While at least one partner is alive, the household needs to decide on consumption and life insurance for this partner. In addition, if both partners are alive, the household needs to decide on side bequest amounts in the event of death. These decisions affect how household wealth evolves over time and in the event of a death. We first outline these dynamics for the couple and then for a widowed partner.

3.2.1 Couple

While both partners are alive, the couple consumes amount c_t, sets life insurance premia (p_t^1, p_t^2) and decides on side bequest amounts in the event of death by partner i, denoted

[1]See Wei et al. (2020) for the implications of simultaneous death on life insurance investment.

[2]HRS data, combined with an event study design, suggest that about 95% of widows/widowers remain singles 6 years after widowhood. Repartnering is thus a rather rare event.

[3]Incorporating loading factors on insurance and annuities can be done similar to Arandjelović et al. (2023), by changing the payoff structure to $I_t^i = \mathbb{1}\{p_t^i > 0\} \frac{p_t^i}{\lambda_{It}^i} + \mathbb{1}\{p_t^i < 0\} \frac{p_t^i}{\lambda_{At}^i}$, with $\lambda_{At}^i \leq \lambda_t^i \leq \lambda_{It}^i$.

by Δ_t^i. Furthermore, the couple receives deterministic income $y_t^C > 0$. The decision to model post-retirement income as deterministic is common, as most retirees' income stems from annuitised pension wealth (see e.g. De Nardi et al., 2025; Mommaerts, 2025).

In the event in which no death occurs, household wealth X earns interest at rate r, decreases by consumption c and the insurance premia (p^1, p^2) and increases with deterministic income y^C:

$$\mathrm{d}X_t = (rX_t + y_t^C - c_t - p_t^1 - p_t^2)\mathrm{d}t. \tag{1}$$

In the event that partner i dies first at time τ_i, wealth jumps according to

$$X_{\tau_i} = X_{\tau_i^-} + I_{\tau_i^-} - \Delta_{\tau_i^-}. \tag{2}$$

That is, wealth after the death equals wealth before death plus the life insurance payout minus the side bequest.

To keep our quantitative model close to the model by De Nardi et al. (2025), we exclude stock investment decisions. Our framework can be extended to incorporate the common Black and Scholes (1973) financial market. However, as our focus is on the interaction between side bequest motives and investment in life-contingent products, we exclude this.

3.2.2 Widowed Partner

After the death of i, when partner $\not i$ is widowed, they consume c_t and set their life insurance $p_t^{\not i}$. In addition, they receive deterministic income $y_t^{\not i} > 0$. We allow for income to differ between widowed partners, as this is empirically common; De Nardi et al. (2025) document substantial heterogeneity by gender, for instance. Wealth evolves according to

$$\mathrm{d}X_t = (rX_t + y_t^{\not i} - c_t - p_t^{\not i})\mathrm{d}t. \tag{3}$$

Finally, at $\not i$'s death, the bequest is given by wealth prior to death plus the life insurance payout:

$$X_{\tau_{\not i}} = X_{\tau_{\not i}^-} + I_{\tau_{\not i}^-} \tag{4}$$

3.3 The Household Problem

Consumption, life insurance and side bequest decisions are made by the household in such a way that the expected future lifetime utility is maximized. The couple weighs both partners' expected lifetime utilities if widowed in its decisions, taking these as given. As such, we first describe the problem of a widowed partner and then describe the problem of the couple.

Widowed Partner At any point in time $t \in [0, T]$, with wealth level $X_t = x$, the problem of widowed partner i is to set a consumption and life insurance strategy $\{(c_s, p_s^i)\}_{s\in[t,T]}$ which maximizes the expected remaining lifetime utility, subject to the wealth dynamics explained above and two additional constraints discussed below. Future utility is discounted exponentially by a factor $\beta \in (0, 1)$. Moreover, utility of consumption accumulates according to the widowed single's utility function $u_S(\cdot)$, until either partner i dies at τ_i or the time horizon T is reached. The final bequest at the end of the planning horizon $(\tau_i \wedge T)$ consists of the final wealth level and the life insurance payout in the event of death ($X_{\tau_i \wedge T}$) and yields a one time utility payoff through the utility function $L_0(\cdot)$. Consequently, the expected remaining lifetime utility derived from a strategy is given by:

$$J^i(t,x;c,p^i) = \mathbb{E}_{t,x}\left[\int_t^{\tau_i \wedge T} \beta^{s-t} u_S(c_s)\mathrm{d}s + \beta^{(\tau_i \wedge T)-t} L_0(X_{\tau_i \wedge T})\right], \tag{5}$$

subject to (3, 4). Moreover, we require the bequest to be non-negative: $X_s + \frac{p_s^i}{\lambda_s^i} \geq 0$ for all $s \in [t,T]$, as heirs are allowed and incentivised to decline a bequest of negative net wealth. Furthermore, we impose a no-borrowing constraint, $X_s \geq 0$ for $s \in [t,T]$. This assumption allows us to isolate the effects of the life insurance product interacting with bequest motives from the effects of alleviating borrowing constraints through insurance. Additionally, debt limits during retirement tend to be low, making this a realistic assumption. The expectation is taken conditional on only partner i being alive at time t with wealth $X_t = x$ and the only source of randomness is τ_i, the random time of death.

We write the set of admissible controls, which satisfy the aforementioned conditions as $\mathcal{A}^i(t,x)$. The value function is then defined as

$$V^i(t,x) = \sup_{(c,p^i)\in\mathcal{A}^i(t,x)} J^i(t,x;c,p^i). \tag{6}$$

We adopt the utility structure of De Nardi et al. (2025). The widowed single's utility of consumption is given by the standard CRRA utility function $u_S(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 1$ denoting the rate of risk aversion. The final bequest utility function is given by $L_0(b) = \phi_0 \frac{(b+\kappa_0)^{1-\gamma}}{1-\gamma}$, where $\phi_0 > 0$ determines the strength of the bequest motive and $\kappa_0 \geq 0$ determines the curvature. If $\kappa_0 > 0$, bequests are luxury goods, making them more valuable at high wealth and less so at low wealth. Bequests are commonly found to be luxury goods post-retirement, justifying setting $\kappa_0 > 0$ (see e.g. De Nardi et al., 2010; Lockwood, 2018).

Couple Then, the problem of the couple is to set consumption, life insurance and side bequest strategies $\{\boldsymbol{\psi}_s\}_{s\in[t,T]} = \{(c_s, p_s^1, p_s^2, \Delta_s^1, \Delta_s^2)\}_{s\in[t,T]}$ to maximize the expected remaining lifetime utility, subject to aforementioned wealth dynamics and several constraints introduced below. We assume the couple consumes as one, but the couple's utility function for consumption differs from the widowed single's, given by $u_C(\cdot)$. At the death of the first spouse i (at $\tau_i < T \wedge \tau_{\not i}$), the couple receives instant utility from the side bequest ($\Delta_{\tau_i}^i$) through the utility function $L_1(\cdot)$ and continues with the widowed problem of partner $\not i$ at time τ_i with the remaining wealth (X_{τ_i}) after the life insurance payout and the side bequest. In case the time horizon is reached ($T < \tau_1 \wedge \tau_2$), final wealth (X_T) gives utility through the final bequest utility function $L_0(\cdot)$. Consequently, the expected future utility of a given strategy $\boldsymbol{\psi}$ is given by:

$$\begin{aligned} J^C(t,x;\boldsymbol{\psi}) = \mathbb{E}_{t,x}\Bigg[&\int_t^{T\wedge\tau_1\wedge\tau_2} \beta^{s-t} u_C(c_s)\mathrm{d}s + \mathbb{1}\{\tau_1 \wedge \tau_2 > T\}\,\beta^{T-t} L_0(X_T) \\ &+ \sum_{i=1}^{2} \mathbb{1}\{\tau_i < T \wedge \tau_{\not i}\}\,\beta^{\tau_i - t}\left(V^{\not i}(\tau_i, X_{\tau_i}) + L_1(\Delta_{\tau_i}^i)\right)\Bigg], \end{aligned}$$

subject to (1-2). We again impose a no-borrowing constraint: $X_s \geq 0$, for $s \in [t,T]$. Additionally, we require both the side bequest and the bequest to a surviving partner to be non-negative: $X_s + \frac{p_s^i}{\lambda_s^i} \geq \Delta_s^i \geq 0$ for $i = 1, 2$ and $s \in [t, T]$. The value function is denoted by

$$V^C(t,x) = \sup_{\boldsymbol{\psi} \in \mathcal{A}^C(t,x)} J^C(t,x;\boldsymbol{\psi}), \tag{7}$$

where $\mathcal{A}^C(t,x)$ is the set of all controls satisfying the above constraints.

The couples' utility function of consumption equals $u_C(c) = 2u_S(\frac{c}{\eta})$, where $\eta \in [1,2]$ determines the economies of scale, so that the utility of the couple spending c equals the utility of two single individuals each spending c/η. As $c < 2 \times c/\eta$ when $\eta < 2$, the couple benefits from economies of scale. These are largest when η is low.

The side bequest utility function is given by $L_1(b) = \phi_1 \frac{(b+\kappa_1)^{1-\gamma}}{1-\gamma}$. The parameter $\phi_1 > 0$ determines the strength of the side bequest motive and $\kappa_1 \geq 0$ determines the extent to which side bequests are luxury goods. De Nardi et al. (2025) argue that differentiating the side bequest utility function from the final bequest utility function is needed to match the bequest behavior observed empirically. In addition, we follow De Nardi et al. (2025), who set the side bequest utility function independent of which spouse dies.

4 Complete Market Solution

The model outlined in the previous section does not admit a closed form solution, due to the no-borrowing constraint and the non-negativity constraints on (side) bequests. In this section, we derive an analytical solution for the problem when we impose only (1-4) and abstract from the remaining constraints. We use this analytical solution to identify several mechanisms underlying life insurance decisions when side bequest motives are present. We begin by defining the solution of a widowed partner. Then, we use the solutions of the widowed partners to determine the solution of the couple.

4.1 Widowed Problem

We consider the widowed problem of partner $i \in \{1, 2\}$. This problem is exactly that of Pliska and Ye (2007), with a scaling term ϕ_0 and a shifting term κ_0 in the bequest utility function L_0. That is, the problem studied by Pliska and Ye (2007) is a special case of the problem we consider here, in which $\phi_0 = 1$ and $\kappa_0 = 0$.

For analytical convenience, we treat income as a state variable, as is done by Hambel (2020), rather than a deterministic stream. Income Y, given current level $Y_t = y$, is given by $Y_s = ye^{\int_t^s \mu_v^i \mathrm{d}v}$, for $s \geq t$ and with μ^i given by $\mu_t^i = \frac{\mathrm{d}}{\mathrm{d}t}\ln(y_t^i)$. This way of writing the income process allows us to more cleanly separate wealth from income and bequest motives. In this section, value functions and policy functions receive the additional argument y, to represent the current income level.

Using the Hamilton-Jacobi-Bellman equation, we arrive at the following proposition:

Proposition 4.1. *The value function in the problem of widowed partner i is given by*

$$V^i(t,x,y) = H^i(t)^\gamma \frac{\left(x + yF^i(t) + A^i(t)\right)^{1-\gamma}}{1-\gamma},$$

where $yF^i(t) = \mathbb{E}_t\left[\int_t^{T\wedge\tau_i} e^{-r(s-t)} Y_s \mathrm{d}s \middle| Y_t = y\right]$, $A^i(t) = \mathbb{E}_t\left[e^{-r((T\wedge\tau_i)-t)}\kappa_0\right]$ *and H^i is defined through an ordinary differential equation, given in Equation (13), in appendix D.1.*

The optimal controls are given by

$$\begin{aligned}\hat{c}(t,x,y) &= \frac{1}{H^i(t)}(x + yF^i(t) + A^i(t)),\\ \hat{p}^i(t,x,y) &= \lambda_S^i(t)\left(-\kappa_0 - x + \phi_0^{\frac{1}{\gamma}}\frac{x + yF^i(t) + A^i(t)}{H^i(t)}\right).\end{aligned} \tag{8}$$

Proof. See appendix D.1. □

Consumption and life insurance policy functions that are affine in wealth are standard in models that use CRRA utility functions with a common risk aversion. In addition, the fact that current wealth (x) is treated the same as the present value of future income ($yF^i(t)$) in the financial decision-making is commonly found in life-cycle models.

The scaling parameter ϕ_0 enters directly in the policy function for insurance ($\hat{p}^i$) and indirectly through the function H^i. A stronger bequest motive decreases consumption at a given state, $\frac{\partial}{\partial \phi_0}\hat{c}(t,x,y) < 0$, and increases the life insurance investment $\frac{\partial}{\partial \phi_0}\hat{p}^i(t,x,y) > 0$.

The shifting parameter, κ_0, enters the solution only through the function $A^i(t)$, which is the time-t expected discounted value of the shifting parameter at the end of the planning horizon. The solution shows that the value $A^i(t)$ is treated the same as current wealth and the present value of future income. The shifting parameter has a positive effect on current consumption: $\frac{\partial}{\partial \kappa_0}\hat{c}(t,x,y) > 0$. As a higher value of κ_0 implies bequests are stronger luxury goods, the fact that κ_0 is treated as additional wealth to spend indicates the partner is willing to trade part of the bequest to finance consumption while alive.

The effect of wealth on consumption is positive ($\frac{\partial}{\partial x}\hat{c}(t,x,y) > 0$), but the effect on life insurance demand is ambiguous. The sign of

$$\frac{\partial}{\partial x}\hat{p}(t,x,y) = \lambda_S^i(t)\left(-1 + \frac{\phi_0^{\frac{1}{\gamma}}}{H^i(t)}\right) \propto \phi_0^{\frac{1}{\gamma}} - H^i(t)$$

is not necessarily positive or negative. Pliska and Ye (2007) show that, if $\phi_0 = 1$ and $\kappa_0 = 0$, the relation between life insurance demand and wealth is negative at all points in time, under realistic assumptions on the model parameters. Hence, they conclude that life insurance is more beneficial to less wealthy households.

However, the parameters of bequest motives for retired households, (ϕ_0, κ_0), estimated in the economic literature tend to be considerably larger than the parametrization used by Pliska and Ye (2007) (see e.g. De Nardi et al., 2010; Lockwood, 2018). The reason for this is that bequests by elderly tend to be luxury goods, leading to a large value of κ_0, and that those retirees that do value bequests, value them strongly, leading to large values of ϕ_0. For the realistic parameters we use, outlined in the next section, we find that the above partial derivative is positive: life insurance is valued more by wealthier households. This aligns with bequests being luxury goods: retirees with lower wealth attach little value to leaving a bequest and, hence, have little incentive to invest in life insurance over annuities. In contrast, wealthier retirees are able to fund their desired level of consumption and have a desire to save for bequests, increasing their demand for life insurance.

4.2 Couple Problem

Next, using the solution of the surviving partner, we continue to solve the couple's problem. We again treat income as a state variable, given by $Y_s = ye^{\int_t^s \mu_v^C \mathrm{d}v}$ for $s > t$ with

$Y_t = y > 0$ and $\mu_t^C = \frac{\mathrm{d}}{\mathrm{d}t}\ln(y_t^C)$. We denote by $\theta_t^i = \frac{y_t^{\not i}}{y_t^C}$ the fraction of income kept after the death of partner i.

We again use the Hamilton-Jacobi-Bellman equation to derive the following proposition:

Proposition 4.2. *The value function in the couple's problem is given by*

$$V^C(t,x,y) = H^C(t)^{\gamma}\frac{\left(x + yF^C(t) + A^C(t)\right)^{1-\gamma}}{1-\gamma}, \tag{9}$$

where

$$\begin{aligned} yF^C(t) &= \mathbb{E}_t\left[\int_t^{T\wedge\max\{\tau_1,\tau_2\}} e^{-r(s-t)}Y_s\mathrm{d}s\big|Y_t = y\right], \\ A^C(t) &= \mathbb{E}_t\left[\mathbb{1}\{\tau_1\wedge\tau_2 < T\}\, e^{-r((\tau_1\wedge\tau_2)-t)}\kappa_1 + e^{-r((T\wedge\max\{\tau_1,\tau_2\}-t)}\kappa_0\right] \end{aligned}$$

and H^C is defined through an ordinary differential equation, given in Equation (16), in appendix D.2.
The optimal controls, for $i = 1, 2$ are given by

$$\begin{aligned} \hat{c}(t,x,y) &= \frac{1}{H^C(t)}(x + yF^C(t) + A^C(t)), \\ x + \frac{\hat{p}^i(t,x,y)}{\lambda_C^i(t)} &= (H^{\not i}(t) + \phi_1^{\frac{1}{\gamma}})\frac{x + yF^C(t) + A^C(t)}{H^C(t)} - y\theta_t^i F^{\not i}(t) - A^{\not i}(t) - \kappa_1 \\ \hat{\Delta}^i(t,x,y) &= -\kappa_1 + \phi_1^{\frac{1}{\gamma}}\frac{x + yF^C(t) + A^C(t)}{H^C(t)}. \end{aligned} \tag{10}$$

Proof. See appendix D.2. □

Again, the policy functions being affine in wealth (x) is standard in the literature. Moreover, we again see that current wealth (x) is treated the same as the present value of future income ($yF^C(t)$).

Our analytical solution provides us with two key insights on the interaction between life insurance decisions and side bequest motives.

Firstly, the side bequest amount $\hat{\Delta}^i$, at given state (t,x,y), does not depend on $i \in \{1,2\}$. Therefore, in the presence of insurance, the optimal side bequest level is irrespective of which partner dies first. Life insurance, absent side bequests, serves as an important tool to smooth consumption after the death of a partner, protecting a surviving partner against a loss in income. Including side bequest motives, life insurance gains a second important purpose, serving as a tool to smoothen out side bequests between partners. Surviving partners differ in longevity risk and income. Therefore, absent life insurance, we would expect side bequests to depend on which partner dies first. The finding that side bequests are equal between partners when life insurance is present, therefore, showcases how life insurance is effective at aiding couples satisfy their side bequest preferences.

Secondly, we infer on the role of ϕ_1, the strength of the side bequest motive, in the insurance and side bequest decisions. In particular, we have the following result:

Corollary 4.3. *For any (t,x,y) and $i = 1, 2$,*

$$\frac{\partial}{\partial\phi_1}\left(x + \frac{\hat{p}^i(t,x,y)}{\lambda_C^i(t)} - \hat{\Delta}^i(t,x,y)\right) < 0.$$

Proof. See appendix D.3. □

The above result implies that household wealth after the death of a partner is decreasing in the strength of the side bequest motive. A stronger side bequest motive, therefore, comes at the cost of the surviving partner. The side bequest motive therefore decreases the role of the provision motive in driving retired couples' demand for life insurance. While the couple could change its financial position in the life-contingent products to mitigate the larger loss in wealth caused by a larger side bequest, this would require too large of a reduction in consumption to be worthwhile.

We finalize this section by discussing the interaction between wealth and life insurance investment by couples. We show that, if $\phi_1 > 0$, there is no set of parameters for which insurance investment is decreasing with current wealth at all points in time:

Corollary 4.4. *If $\phi_1 > 0$, then there is a $\bar{t} < T$ such that for any (t, x, y) with $t > \bar{t}$ and $i = 1, 2$:*

$$\frac{\partial \hat{p}^i}{\partial x}(t, x, y) > 0.$$

Proof. See appendix D.4. □

This result is mainly a consequence of the finite time horizon T and, therefore, not very telling on the interaction between wealth and life insurance over the most relevant part of the life-cycle. However, it does show that, in our model, we cannot find a result on wealth decreasing the position in the life contingent products. Instead, the interaction between insurance and wealth is generally ambiguous and dependent on the bequest motives. For our calibration, explained in the next section, the above partial derivative is positive always.

5 Baseline Calibration

To quantify how side bequest motives affect the investment decisions of retired couples, as well as couples' desire for life insurance, we calibrate our model using realistic parameters that are summarized in Table 1. Doing so, we take parameter estimates from the literature and we estimate mortality rates and income processes using the Health and Retirement Study (HRS)[4] (specifically, the RAND longitudinal and detailed imputations files[5]). Following the literature, we restrict ourselves to married couples consisting of a husband and wife and we change our notation from partners $i \in \{1, 2\}$ to spouses $g \in \{w, h\}$, to denote wife and husband respectively. Furthermore, in our model, we assume wives are three years younger than husbands, as this aligns with the average age difference among retired couples (De Nardi et al., 2025). When we refer to the household's age, we take the husband's age as reference.

Following De Nardi et al. (2025), all monetary values we present are in terms of 2014 USD. The parameter r therefore represents the real interest rate. For the utility

[4] The HRS (Health and Retirement Study) is sponsored by the National Institute on Aging (grant number NIA U01AG009740) and is conducted by the University of Michigan.

[5] The RAND HRS Longitudinal File is an easy-to-use dataset based on the HRS core data. The RAND HRS Detailed Imputations File contains the component and ownership variables that were used to create the income, wealth, and medical expenditures summary measures found in the RAND HRS Longitudinal File. These files were developed at RAND with funding from the National Institute on Aging and the Social Security Administration

Description	Parameter	Value	Source
Utility			
Discount Factor	β	0.97	De Nardi et al. (2025)
Risk Aversion	γ	3.7	De Nardi et al. (2025)
Consumption Economies of Scale	η	1.52	De Nardi et al. (2025)
Bequest Intensity (in 1000s)	ϕ_0	88,711	De Nardi et al. (2025)
Bequest Curvature (in 1000s)	κ_0	3,517	De Nardi et al. (2025)
Side Bequest Intensity	ϕ_1	57,793	De Nardi et al. (2025)
Side Bequest Curvature (in 1000s)	κ_1	211	De Nardi et al. (2025)
Income			
Income	(y^C, y^w, y^h)	See Appendix A	*Authors' Estimations*
Mortality			
Mortality Rates	$\lambda(\cdot)$	See Appendix B	*Authors' Estimations*
Other			
Interest Rate	r	0.04	De Nardi et al. (2025)
Time Horizon (in years)	T	40	*Authors' Choice*
Initial Wealth	X_0	300,000	*Authors' Choice*
Permanent Income Quartile	q	2	*Authors' Choice*
Husband's age at $t = 0$		65	*Authors' Choice*

Table 1: Baseline parameter calibration.

parameters, we use the estimates and calibration by De Nardi et al. (2025). As these estimates are based on a model with a biannual frequency, we transform these parameters to align with a yearly frequency. For details on this transformation, we refer to Appendix E. These parameters make both bequests and side bequests luxury goods and a strong motive to save for wealthy households. In addition, the consumption equivalence scale implies that, in order to have the same marginal utility as a single, the consumption of a couple needs to be 1.64 times as large.[6]

Using data from the HRS we estimate four income and mortality processes, each one representing a quartile of the permanent income (PI) distribution. We consider these different quartiles to identify differences in financial decision-making across the retiree population. Figure 2 shows the income processes of the highest and lowest quartiles, as well as the effect of a death at age 80. The death of a husband leads to a higher loss in income than the death of a wife (in line with De Nardi et al., 2025). Furthermore, we estimate mortality rates for couples, accounting for heterogeneity in permanent income, as this tends to be a strong predictor of mortality. Appendix B provides an explanation of our estimation. Generally, husbands have a shorter longevity than wives and households' longevity rises with PI.

As we want to have a representative household for our baseline problem, we take a household from the second PI quartile ($q = 2$) with an initial wealth of $X_0 = 300,000$, which aligns closely with the median net worth we observe among initial couples younger than 75 in the second PI quartile. This way, we have a good representation of the median retired couple.

[6]Solving $u'_S(c_S) = u'_C(c_C)$ yields $\frac{c_C}{c_S} = \left(\frac{2}{\eta^{1-\gamma}}\right)^{\frac{1}{\gamma}}$, as explained by De Nardi et al. (2025).

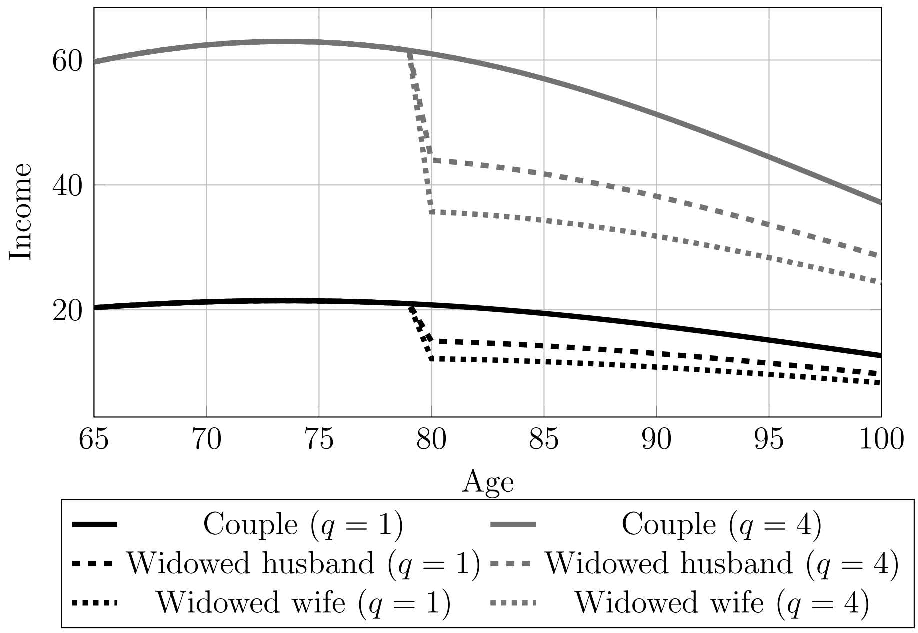

Figure 2: Income paths by marital status and income quartile.

6 Numerical Results

In this section we discuss numerical results of our model. To solve our model, we employ a numerical scheme explained in Appendix C. We begin by studying life-cycle profiles of the household, comparing our baseline problem to a counterfactual, in which the same household has no side bequest motive. Then, we discuss numerical results comparing our baseline problem to a counterfactual in which no insurance is available. This way, we depict how a representative household changes its behavior in the presence of side bequest motives. Furthermore, we discuss the effects of side bequest motives on the decision to invest in annuities and life insurance in aggregate. In particular, we study several key indicators on retired couples' financial strategies, across the four income quartiles. Additionally, we study the welfare gains of getting access to life-contingent products across the retiree population. Finally, we study the welfare implications of incorrectly omitting side bequest motives, to quantify the importance of accounting for side bequest motives in portfolio decisions. Using these analyses, we are able to quantify the effects of side bequest motives on the demand for life-contingent products.

6.1 Life-Cycle Profiles

Comparison of the models with and without side bequests. We start by solving the baseline problem with and without side bequest motives. Figure 3 depicts several key variables over the couple's lifetime, in a simulation in which both spouses remain alive, until the age of 100.[7] We find that the wealth paths evolve similarly between both models. However, a couple with side bequest motives cuts down on consumption considerably: annual consumption decreases by around \$3,000. This is because (1) side bequest

[7]Our actual final age is $T + 65 = 105$ years. In these last years, mortality rates become so large that the couple changes its financial strategies drastically. This makes the presented profiles more difficult to interpret. Given that the vast majority of households experience a death before age 100, we decide to omit the last five years from the graph.

motives induce a savings motive and (2) side bequest motives increase the positions in the life-contingent products, which decreases savings in the event of survival.

A couple with side bequest motives substantially increases its demand for the life insurance to better cope with the loss in wealth caused by side bequests. At retirement, the increase in life insurance on the wife's and on the husband's lives are around \$1,000 and the \$2,000 respectively. Side bequest motives give couples additional reason to leave wealth at the death of a first spouse, as this allows the couple to leave a larger side bequest and provide sufficiently for a surviving spouse, thereby increasing the demand for life insurance. In addition, the couple chooses to take a higher position in the product on the husband's life than the wife's. This aligns with the provision motive, along with the findings by Hubener et al. (2014): a couple's demand for life insurance per spouse's life is driven by the endowment of their surviving spouse when widowed.

Additionally, the increase in the demand for the life insurance product is so large that, while the couple without side bequest motives annuitizes considerably (i.e. has a negative life insurance position), the couple with side bequest motives does not do so on the husband and little on the wife, for most of the couples' life-cycle. Life-cycle frameworks generally predict that life insurance is of greater benefit to working individuals, whereas retirees have a larger demand for annuities (Arandjelović et al., 2023). It is, therefore, expected that in the 'standard' counterfactual without side bequest motives, the retired couple annuitizes considerably. What our simulations show, however, is that realistically calibrated side bequest motives significantly reduce the demand of couples for fair annuities as, in the absence of those, couples would annuitize significantly more. These results complement those of Lockwood (2012) shows that moderate bequest motives go a long way in eliminating retired singles' demand for annuities, even without loading factors.

In appendix F.1, we provide similar life-cycle profiles of couples in the first, third and fourth permanent income quartiles. The aforementioned patterns are present across the retiree population. One exception is the lowest income quartile, which shows considerably different wealth paths for the couples with and without side bequest motives. Without side bequest motives, final bequests, which are luxury goods, provide little reason for less wealthy couples to save. The side bequest motive does, however, still play a role for these couples, increasing their savings rates.

Comparison of the models with and without life-contingent products. Next, we study the effect of getting access to the life-contingent products on key variables over the life-cycle when couples have side bequests. Figure 4 shows model solutions for a couple experiencing no deaths until 100 years with and without access to life-contingent products. As is standard in the absence of life-contingent products, the couple is unable to smooth consumption across its life-cycle: the couple needs to balance consumption today against future consumption and the consequences of uncertain death. As such, the couple's consumption patterns are considerably more responsive to the increasing mortality rate in later years. For most of the couple's life-cycle, wealth evolves similarly. This aligns with the second row of Figure 3, where the life contingent investments drive savings little for this couple ($p_t^w + p_t^h$ is relatively small).

Additionally, solutions align with our analytical results: with access to life-contingent products, side bequests are independent of the spouse at any given time. This stands in contrast to the solution without life-contingent products, where the side bequest in the event of the husband's death is considerably lower, by around \$40,000 (15%) at retirement. Without life-contingent products, the loss in income at death cannot be hedged properly

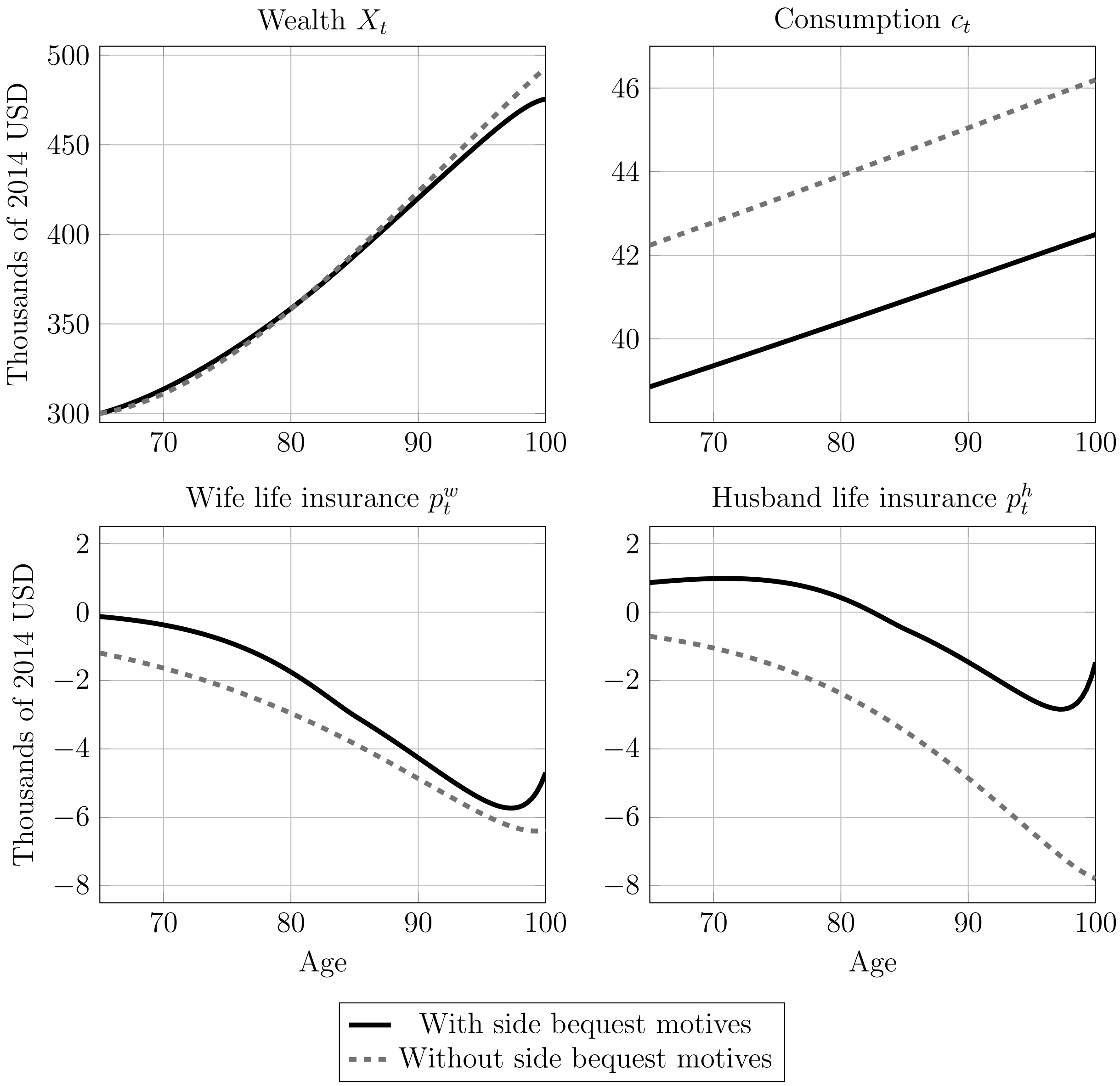

Figure 3: Key variables over the life-cycle with and without side bequests.

and, as a result, the couple needs to adjust its side bequests to partially account for this.

In appendix F.2, we show similar profiles for couples in the first, third and fourth quartiles. For some of the quartiles, the sum of the life-contingent positions $p_t^h + p_t^w$ causes changes in savings' rates. The less wealthy households save more with access to the life-contingent product, due to investment in annuities. In contrast, the couples in the upper two quartiles invest in life insurance, leading to a lower savings rate. Notable is that wealthier couples' side bequests at a given time are much less dependent on which spouse dies: in the highest quartile the difference in side bequests at retirement $(\Delta_0^w - \Delta_0^h)$ is less than \$20,000, which is much less than the \$40,000 in the second quartile. This is especially the case relatively: the couple in the upper quartile has substantially more tangible wealth and higher pension income. The wealthier couples are well enough endowed that the absence of life-contingent products limits their abilities to smooth consumption and side bequests considerably less. Therefore, the welfare gains of the life contingent products can be expected to be smaller for the wealthier couples.

6.2 The Aggregate Effects of Side Bequest Motives on Life-Contingent Investment

In this subsection, we quantify several key indicators on the investment behavior of retired couples in aggregate. First we focus on the demand for annuities, quantifying how side bequest motives reduce the demand for annuities. Then, we will study the effects of side bequest motives on the livelihood of widowed spouses, quantifying how side bequest motives reduce the role of the provision motive in driving life insurance demand.

We consider two key indicators that determine the demand for annuities, both on the extensive and the intensive margins. On the extensive margin, we consider the expected portion of the couple's life, that the couple holds annuities on a spouse's life (EPLA). In terms of our model, this is defined as

$$\mathrm{EPLA}^g = \mathbb{E}\left[\frac{1}{T \wedge \tau_w \wedge \tau_h} \int_0^{T \wedge \tau_w \wedge \tau_h} \mathbb{1}\{p_t^g < 0\}\,\mathrm{d}t\right].$$

The expected value is taken over the stochastic first time of death $(\tau_w \wedge \tau_h)$, as well as the initial value of wealth X_0. First, we compute the above integral for several values of initial wealth $X_0 = x$, giving us the indicator for a couple who enters retirement with wealth x ($\mathrm{EPLA}^g(x)$). Then, for each income quartile, we determine a distribution for X_0 and we compute $\mathrm{EPLA}^g = \mathbb{E}\left[\mathrm{EPLA}^g(X_0)\right]$. This allows us to establish the effects in aggregate across the retiree population, accounting for heterogeneity across and within income quartiles. Further details on the wealth distributions and computation of the indicators are described in Appendix G. We do, however, use a single income process for couples in the same income quartiles, due to the luxury bequest motives.[8]

On the intensive margin, we consider the expected present value of income through private annuities (PVA). In our model, this is defined as

$$\mathrm{PVA}^g = \mathbb{E}\left[\int_0^{T \wedge \tau_w \wedge \tau_h} e^{-rt}\mathbb{1}\{p_t^g < 0\} \cdot (-p_t^g)\mathrm{d}t\right].$$

[8]If $\kappa_0 = \kappa_1 = 0$, the model can be rewritten in terms of the wealth-to-income ratio, which would allow us to account for heterogeneity in wealth and income within income quartiles simultaneously. However, we set bequests as luxury goods, implying that wealth and income remain separate states, so that we only account for wealth heterogeneity within income quartiles.

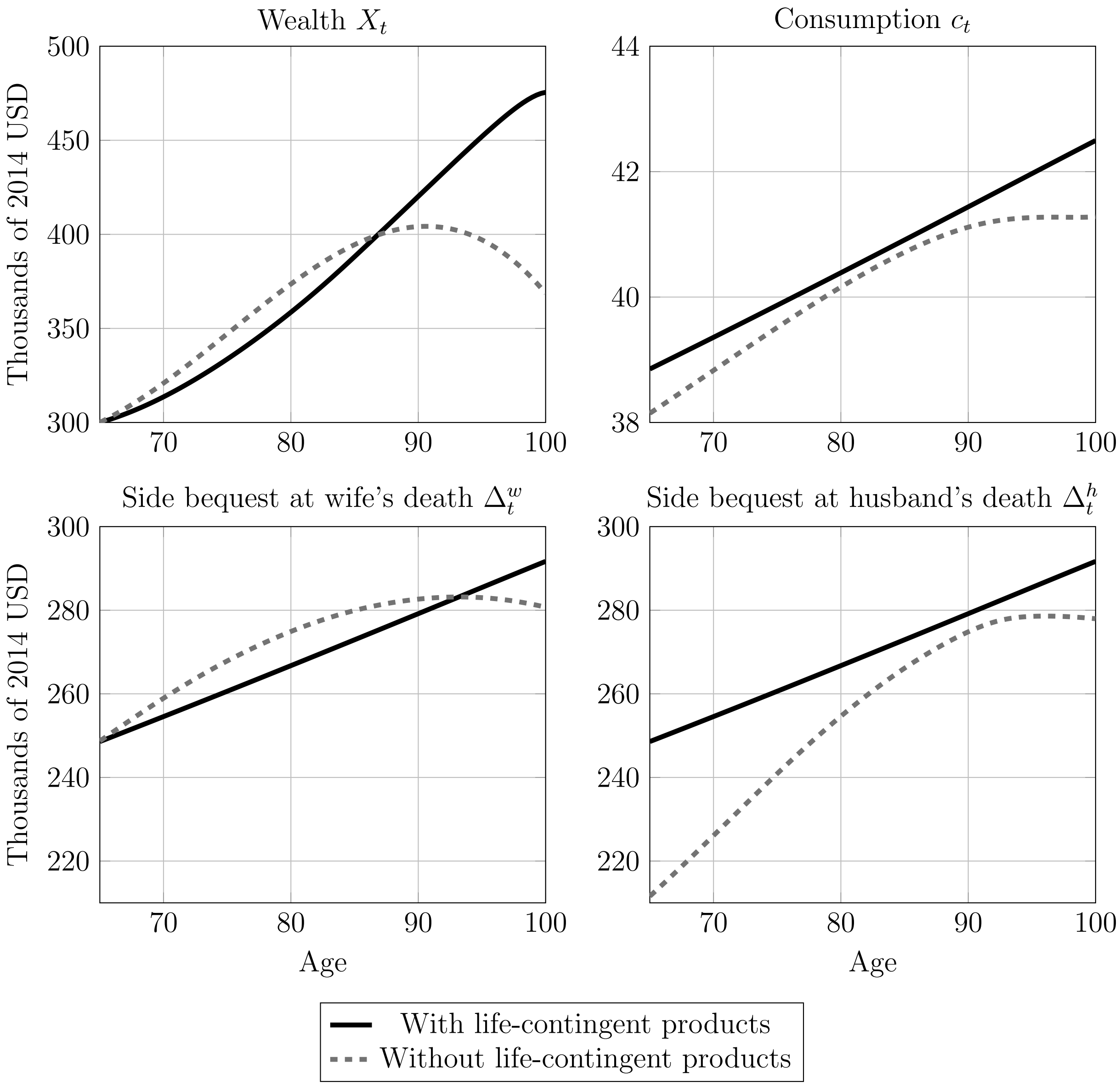

Figure 4: Key variables over the life-cycle with and without life-contingent products

Table 2 presents the indicators, by income quartile, for couples with and without side bequest motives. Without side bequest motives, many households participate in the annuity market: participation in this market is 61% for women and 49% for men. Participation is larger for women because of the provision motive. Moreover, participation is lower or nil in the upper quartiles. This comes from the fact that terminal bequests are strong luxury goods here, therefore, only lowering the annuity demand of richer households. Such bequest motives can therefore remedy Hubener et al. (2014)'s model predictions, in which wealthier couples annuitize substantially.

Side bequest motives further reduce the participation in the annuity market. The effect is sizable for all quartiles in which there was initially some participation in the annuity market. For men, annuity participation almost vanishes, going from 49% to only 8%. For women, participation is about halved, going from 61% to 28%. Overall, these results show that side bequests can rationalize a large share of the low annuity demand of couple households. In particular, they tend not only to affect the richest households but also poorer ones.

On the intensive margin, the present value of annuity income is also significantly reduced by side bequest motives. For instance, for women, annuitized income goes from \$10,804 to \$2,869, a reduction of 73%. For the upper two income quartiles, annuity income is already low without side bequest motives and becomes negligible with those, especially in comparison to the wealth and income of these households. Those in the lowest income quartile have median wealth of \$150,000 and a yearly income of around \$20,000 (Figure 2). For those, the expected discounted annuity income without side bequests of \$25,462+\$11,477=\$36,939 is, in comparison, quite sizable. It is, however, largely reduced with side bequests, by \$27,183 or 74%. For those, in the second quartile, annuity demand decreases by \$22,440 or 90%.

Thus, side bequest motives substantially decrease demand for annuities in couples, both on the extensive margin and intensive margin. Important is that these are fair annuities: even without loading factors, (luxury) side bequest motives eliminate a substantial amount of the participation in annuities. Hence, we conclude that luxury final and side bequest motives go a long way in explaining the annuity puzzle of retirees.

Table 2: Key indicators on annuity demand by income quartile

		Wife ($g = w$)					Husband ($g = h$)				
		Q1	Q2	Q3	Q4	**Avg.**	Q1	Q2	Q3	Q4	**Avg.**
EPLA^g	No SB	1	0.92	0.53	0	**0.61**	0.93	0.95	0.10	0	**0.49**
	SB	0.85	0.43	0.05	0	**0.33**	0.30	0.01	0	0	**0.08**
	Diff.	0.15	0.49	0.48	0	**0.28**	0.63	0.94	0.1	0	**0.41**
PVA^g	No SB	25,462	14,691	3,063	0	**10,804**	11,477	10,313	815	0	**5,651**
	SB	8,672	2,491	312	0	**2,869**	1,080	73	0	0	**289**
	Diff.	16,790	12,200	2,751	0	**7,935**	10,393	10,240	815	0	**5,362**

Notes: The left panel pertains to couples' investments in products contingent on the wife's life, the right on the husband's. 'SB' refers to couples with side bequest motives, 'No SB' refers to couples without side bequest motives. The present value of annuities (PVA) is reported in terms of 2014 U.S. dollars.

Next, we turn to the effects of side bequest motives on life insurance demand. Hubener et al. (2014) find that the provision motive is the key driver of couples' life insurance demand and the pure bequest motive has a negligible effect. In this paper, we have shown that the provision motive is a key driver in couples' investment behavior: couples take a substantially different financial position in the product contingent on the husband's life to the wife's life. However, we have shown that, absent side bequests, couples in the lower quartiles prioritize annuities, while couples in the higher quartiles are the ones holding life insurance. This shows that, when modeling final bequests as luxury goods, the importance of the provision motive driving life insurance demand decreases substantially: only the wealthy couples invest in life insurance and they do not need it for provisional reasons.

Furthermore, we have shown that side bequest motives increase demand for life insurance even further. Our theoretical results suggest that side bequest motives reduce the role of the provision motive: stronger side bequest motives lead to larger losses in wealth at the death of a spouse, decreasing a surviving spouse's resources. Now, we take a quantitative approach to capture the size of this effect. In particular, we compare the expected present value of a surviving spouse's resources in a couple with side bequest motives to the resources of a surviving spouse in a couple without side bequest motives. The indicators we study are the expected present value of savings of a widowed spouse at the first death (EWS):

$$\mathrm{EWS}^g = \mathbb{E}\left[e^{-r(\tau_w \wedge \tau_h)}\left(X_{\tau_w \wedge \tau_h} + \frac{p^g_{\tau_w \wedge \tau_h}}{\lambda^g_C(\tau_w \wedge \tau_h)} - \Delta^g_{\tau_w \wedge \tau_h}\right)\mathbb{1}\{\tau_w \wedge \tau_h < T\}\right],$$

as well as this value in relative terms, using the wealth-to-income ratio:

$$\mathrm{EWSR}^g = \mathbb{E}\left[e^{-r(\tau_w \wedge \tau_h)}\frac{1}{y^{\not g}_{\tau_w \wedge \tau_h}}\left(X_{\tau_w \wedge \tau_h} + \frac{p^g_{\tau_w \wedge \tau_h}}{\lambda^g_C(\tau_w \wedge \tau_h)} - \Delta^g_{\tau_w \wedge \tau_h}\right)\mathbb{1}\{\tau_w \wedge \tau_h < T\}\right].$$

Table 3 reports these indicators. In absolute terms, side bequest motives substantially reduce the resources of a surviving spouse. In aggregate, a couple in the lowest income quartile lowers the savings of a surviving spouse to nearly half of what a couple without side bequest motives would. This constitutes to a difference of \$58,000 in the event of the wife's death and \$60,000 in the event of the husband's. Differences are large in the remaining quartiles as well: a surviving spouse in a couple with side bequest motives holds on average \$120,000 less than a surviving spouse in a couple without side bequest motives.

In relative terms, the results tell the same story. A widower in a couple in the lowest income quartile without side bequest motives inherits a wealth-to-income ratio of 8.7, in contrast to a widower in a couple with side bequest motives, for whom the wealth-to-income ratio at widowhood is 4.7. On average, a widower in a couple with side bequest motives inherits a wealth-to-income ratio that is lower by 4.7 than his counterpart without side bequest motives. For widows, the difference in aggregate is 5.6. These differences imply the wealth-to-income ratio of surviving spouses in couples with side bequest motives are lower by more than 37%.

These quantitative results paint a clear picture: side bequest motives substantially lower the resources of a surviving spouse. In addition, side bequest motives increase the demand for life insurance. We, therefore, find that the pure bequest motive is an important driver of life insurance demand, when side bequests are included and bequests are luxury goods, in contrast to Hubener et al. (2014). The provision motive also plays an important role in driving investment strategies: couples decide to increase the savings of the worse endowed spouse. However, the low demand for life insurance in the lower income quartiles without side bequest motives suggest the role of the provision motive as a driver of life insurance demand is smaller than that in Hubener et al. (2014).

Table 3: Key indicators on surviving spouses' resources by income quartile

		Death by Wife ($g = w$)					Death by Husband ($g = h$)				
		Q1	Q2	Q3	Q4	**Avg.**	Q1	Q2	Q3	Q4	**Avg.**
EWS^g	No SB	127	235	391	586	**335**	146	253	386	571	**339**
	SB	69	132	228	405	**209**	86	150	236	403	**219**
	Diff.	58	103	167	181	**126**	60	103	150	168	**120**
$EWSR^g$	No SB	8.7	11.2	13.4	14	**11.8**	12.4	14.9	16.5	16.8	**15.1**
	SB	4.7	6.3	7.9	9.7	**7.1**	7.4	8.8	10.1	11.8	**9.5**
	Diff.	4	4.9	5.5	4.3	**4.7**	5	5.1	6.4	5	**5.6**

Notes: The left panel pertains to a death by the wife and the right to a death by the husband. 'SB' refers to couples with side bequest motives, 'No SB' refers to couples without side bequest motives. Monetary values are reported in terms of thousands of 2014 U.S. dollars.

6.3 Welfare Effects of Life-Contingent Products

Now, we turn to quantifying the welfare gains of life-contingent products. We do so by determining the willingness-to-pay for access to the life-content products of a couple at the start of retirement. We let V^C denote the value function of the couple in (7) and let $\bar{V}^C$ denote the value function of the same problem without insurance. We then define the welfare gain of life-contingent products, $s(x)$, as the solution $s \geq 0$ to the equation

$$V^C(0, x - s) = \bar{V}^C(0, x).^{9}$$

This means that a couple with wealth $X_0 = x$ at the start of retirement is indifferent between having no access to the life-contingent products and paying $s(x)$. To determine the gains in aggregate, we compute $\mathbb{E}\left[s(X_0)\right]$ for couples in each of the income quartiles with and without side bequest motives. In addition, we present the welfare effects in relative terms, as a fraction of initial wealth, $\mathbb{E}\left[\frac{s(X_0)}{X_0}\right]$.

Table 4 reports the welfare effects of life-contingent products both in absolute and relative terms. Without side bequest motives, actuarially fair life-contingent products, predominantly annuities, yield large welfare gains in the lowest income percentile: a retired couple, on average, is willing to pay 18% of its wealth in order to gain access to life-contingent products. In contrast, life-contingent products provide much smaller welfare gains for couples in the upper two income quartiles. Wealthy households are much less susceptible to mortality/longevity risk, making them able to smooth consumption over the life-cycle well in the absence of life-contingent products. Hence, they have a smaller need for life-contingent products.

Aggregate welfare gains with side bequest motives are very similar to those without. However, the distribution of welfare gains is greatly modified. Couples in the lowest income quartile have substantially lower gains from life-contingent products when side bequest motives are present: on average, a couple with side bequest motives is willing to pay 7% of its wealth for access to life-contingent products, 11 percentage points lower than a couple without side bequest motives. In contrast, welfare gains in the upper two quartiles increase, predominantly driven by a higher demand for life insurance: a couple with side bequest motives in the third (fourth) quartile is willing to pay 3% (4%) more

[9] Since $V^C(t, x) \geq \bar{V}^C(t, x)$ for all $t \in [0, T]$ and $x \geq 0$ and V^C is increasing in x, this equation has a nonnegative solution for s if $V^C(0, 0) \leq \bar{V}^C(0, x)$. If this is not the case, the couple is willing to pay more than it has, so we set $s(x) = x$.

to gain access to life contingent products than a couple without side bequest motives.

Additionally, these welfare effects are measured in terms of tangible wealth. As side bequests substantially decrease widowed spouses' resources, no-borrowing constraints can limit couples' ability to smoothen consumption considerably more in the presence of side bequest motives. So, less wealthy couples with side bequest motives may value tangible wealth more than couples without side bequest motives. This may partially explain the reduction in welfare gains in the lowest income quartile.

Furthermore, the life-cycle profiles provide intuition. Figures F1 and 3 show that representative couples in the lower two quartiles annuitize considerably without side bequest motives, but couples with side bequest motives have little invested in both annuities and life insurance (this effect is stronger for the lowest income quartile). Therefore, the welfare gains of life-contingent products are lower for couples with side bequest motives: even when life-contingent products are available, the couples with side bequest motives use them to a small extent. In contrast, Figures F2 and F3 show that representative couples in the upper two quartiles with side bequest motives have a higher absolute investment in life contingent products. This explains why couples in the upper two income quartiles with side bequest motives yield larger welfare gains from life-contingent products (primarily life insurance).

Thus, the welfare effects suggest that, with side bequest motives, couples in the lowest income quartile, who mainly annuitize, gain considerably less from life-contingent products. In contrast, the welfare gains for wealthier households are considerably larger with side bequest motives, driven predominantly by an increase in demand for life insurance.

Table 4: Welfare gains of life-contingent products by income quartile

		Q1	Q2	Q3	Q4	**Avg.**
Absolute	No SB	15,770	5,424	4,651	12,242	**9,522**
	SB	8,110	4,859	7,635	16,658	**9,316**
	Diff.	7,660	565	-2,984	-4,416	**206**
Relative	No SB	0.18	0.05	0.01	0.03	**0.07**
	SB	0.07	0.04	0.05	0.06	**0.06**
	Diff.	0.11	0.01	-0.04	-0.03	**0.01**

Notes: 'SB' refers to couples with side bequest motives, 'No SB' refers to couples without side bequest motives. Absolute willingness to pay values are reported in terms of 2014 U.S. dollars.

6.4 The Importance of Accounting for Side Bequests

Our analysis on life-cycle profiles and several key indicators show that side bequest motives substantially alter retired couples' financial decision-making. This still leaves a central question: how important is it to account for side bequest motives in the design of financial strategies? In this subsection, we answer this question, by quantifying the welfare cost of incorrectly omitting side bequest motives. In particular, we consider retired couples who do have side bequest motives, but follow the consumption-investment strategies of the same couples without side bequest motives. We denote the expected lifetime utility at time t and wealth x of such a couple by $W(t,x)$. We then define the welfare loss of not correctly accounting for side bequest motives, $l(x)$, as the solution $l \geq 0$ to

$$V^C(0, x - l) = W(0, x).$$

Hence, $l(x)$ represents the willingness to pay of a couple with wealth $X_0 = x$ for the correct strategies. Again, we compute these values in absolute terms, $\mathbb{E}\left[l(X_0)\right]$, and in relative terms, $\mathbb{E}\left[\frac{l(X_0)}{X_0}\right]$. We also compare this counterfactual to couples with side bequest motives who do not have access to life-contingent products. In particular, we compute the fraction of couples who prefer having no access to life-contingent products over subscribing to a consumption-investment strategy that does not account for side bequests, $\mathbb{P}\left(\bar{V}^C(0, X_0) \geq W(0, X_0)\right)$.

Table 5 reports the welfare losses across the income quartiles. The welfare loss of incorrectly omitting side bequest motives in the design of consumption-investment strategies is large: on average, couples are willing to pay 15% of their wealth to get access to consumption-investment strategies that align with side bequest preferences. These values are considerably larger than the welfare gains of life-contingent products reported in Table 4. Aligning with this finding, almost half of retired couples in the lower three income quartiles prefer not having access to life-contingent products over subscribing to a strategy that omits side bequest preferences. These findings show that side bequest motives should be accounted for in the design of retired couples' financial strategies.

Table 5: Welfare loss of using strategies that do not align with side bequest preferences by income quartile

	Q1	Q2	Q3	Q4	**Avg.**
Absolute Welfare Loss	21,197	29,160	14,628	11,677	**19,166**
Relative Welfare Loss	0.20	0.27	0.10	0.04	**0.15**
Fraction Preferring no Investment	0.47	0.50	0.45	0	**0.36**

Notes: Absolute values are reported in terms of 2014 U.S. dollars.

7 Conclusion

In this paper, we examine the effects of side bequest motives on the retired couples' investment in life-contingent financial products. We show that life-contingent products are excellent tools for retired couples to balance side bequests with the livelihood of a surviving spouse. We further show that side bequest motives can substantially alter couples financial decision-making. Side bequests motives are able to explain low demand for fair annuities, aligning with the empirical annuity puzzle. In addition, we find that, in the presence of side bequest motives, the desire to bequeath is a strong driver of couples' life insurance demand, stronger than the desire to protect a surviving spouse's livelihood. Finally, we show that the effect of incorrectly omitting side bequest motives when designing consumption-investment decisions for couples is accompanied by substantial welfare losses. Thus, we show that side bequest motives should be taken into account in the design of financial strategies for retired couples.

Our findings also extend to the growing body of literature documenting the desire to give to one's heirs while alive. While we focus on the death of a spouse in retired couples and the bequest desires that arise around this time, several studies document that retired households have a desire to engage in inter-vivos transfers (see e.g. Kvaerner, 2023; Suari-Andreu et al., 2024). Our findings suggest that optimal financial strategies are influenced strongly by the desire to bequeath before the end of a couples' lifetime. The role of inter-vivos transfers on the financial decision-making of retired households is, therefore, an important point of focus for future research.

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Further

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A Income

We use HRS data, which follow households along waves from 1998 until 2022 at a biannual frequency (waves four through sixteen). We make several sample selection decisions to simplify the estimations we do. Our goal is to have four representative income processes, each one representing a different quartile of the permanent income distribution. We restrict the sample to married households consisting of a husband and a wife, in which both partners are retired. The husband is required to be above 65 years old and the wife between two years older and eight years younger. If households are missing for certain waves but are included in later waves, we remove the waves before such a missing wave occurs. We further require that the couple remains married to the same spouse and that spouses do not remarry after being widowed. We remove the households above the 99th percentile of maximum observed income, to account for outliers and possible measurement errors. As mentioned in the main text, we adjust monetary values by inflation, so that they are reported in 2014 U.S. dollars, aligning with De Nardi et al. (2025). To estimate income profiles from the data we take the same approach as De Nardi et al. (2025).

First, we estimate a general trend regarding age and household structure. Let $\epsilon = \{0 = \text{Both Dead},\ 1 = \text{Widowed Wife},\ 2 = \text{Widowed Husband},\ 3 = \text{Both Alive}\}$ denote the household structure. Then, we denote by inc_{ht} the income of household h when the husband is $t + 65$ years old. Income is computed as the sum of individual income from employer pension or annuities, social security and veterans' benefits or, if this falls below a minimum threshold[10] of \$4100 for singles and \$6150 for couples, the minimum threshold.[11]

A general trend is then estimated by fixed effects according to

$$\ln(\text{inc}_{ht}) = \alpha_h + f_Y(t, \epsilon_{ht}) + \omega_{ht},$$

where α_h is the household fixed effect (capturing permanent income) and ω_{ht} is an error term. The function f_Y contains a quadratic in age t, dummies for whether a spouse is widowed and an interaction between the dummies and age:

$$f_Y(t, \epsilon) = a_1 t + a_2 t^2 + \mathbb{1}\{\epsilon = 1\}\,(a_3 + a_5 t) + \mathbb{1}\{\epsilon = 2\}\,(a_4 + a_6 t).$$

The estimates are given in Table A1.

Then, for each household, we compute a measure for permanent income, which is captured in the α_h term above, using the residuals:

$$R_h = \frac{1}{|\mathcal{T}_h|} \sum_{t \in \mathcal{T}_h} (\ln(\text{inc}_{ht}) - \hat{f}_Y(t, \epsilon_{ht})).$$

The percentile of household h's residual, R_h, in the total residual distribution is then denoted by I_h and represents the household's percentile in the permanent income distribution. Treating I_h as a percentile, we divide the sample into four permanent income quartiles. For each quartile we want an income process and, as the time trend has already been determined and is independent of permanent income, we require a value for y_0^C for each of the quartiles. We do this by regressing R_h on a fifth order polynomial of I_h, denoted by ι. The estimates for ι are given in Table A2.

The couple's income in income quartile $q \in \{1, 2, 3, 4\}$ is then given by $y_t^C = \iota(\frac{2q-1}{8}) e^{f_Y(t,3)}$,

[10] Particularly, the sum of the variables "RwIPENA", "RwISRET" and "RwIVET" for both spouses.

[11] These values correspond to the minimum consumption floor estimated by De Nardi et al. (2025).

the widowed wife's income is $y_t^w = \iota(\frac{2q-1}{8})e^{f_Y(t,1)}$ and the widowed husband's income is $y_t^h = \iota(\frac{2q-1}{8})e^{f_Y(t,2)}$.

Parameter	Estimate
a_1	0.0127
a_2	-0.00075
a_3	-0.6208
a_4	-0.3737
a_5	0.00569
a_6	0.00313

Table A1: Coefficient estimates for the function $f_y(t, \epsilon)$

Variable	Coefficient
Constant	9.18
I	9.82
I^2	-40.83
I^3	89.92
I^4	-92.96
I^5	36.55

Table A2: Coefficient estimates for the function ι

B Mortality

We estimate Gompertz mortality rates. The rates of spouse $g \in \{h, w\}$ in permanent income quartile $q \in \{1, 2, 3, 4\}$ are specified as

$$\lambda_C^g(t; q) = b_{g,q}e^{b_{g,q}t+p_{g,q}+\nu} \quad \text{and} \quad \lambda_S^g(t; q) = b_{g,q}e^{b_{g,q}t+p_{g,q}}.$$

The time trend (b) and baseline mortality (p) parameters account for heterogeneity in income and gender. Furthermore, ν captures the effect of being widowed, in such a way that the mortality rate of a surviving spouse changes by a factor of $e^{-\nu} \approx 1 - \nu$ after widowhood. We fit our mortality rates by matching two-year empirical transition probabilities in a GMM setting. Estimates are given in Table B1 below. Furthermore, Figure B1 shows the estimated survival rates of husbands and wives across the different income quartiles.

	g \ q	1	2	3	4
b	w	0.0390	0.0849	0.0890	0.131
	h	0.1039	0.0932	0.0782	0.1106
p	w	-0.062	-1.937	-2.365	-3.480
	h	-1.849	-1.690	-1.189	-2.294
ν		-0.113			

Table B1: Estimated mortality model parameters.

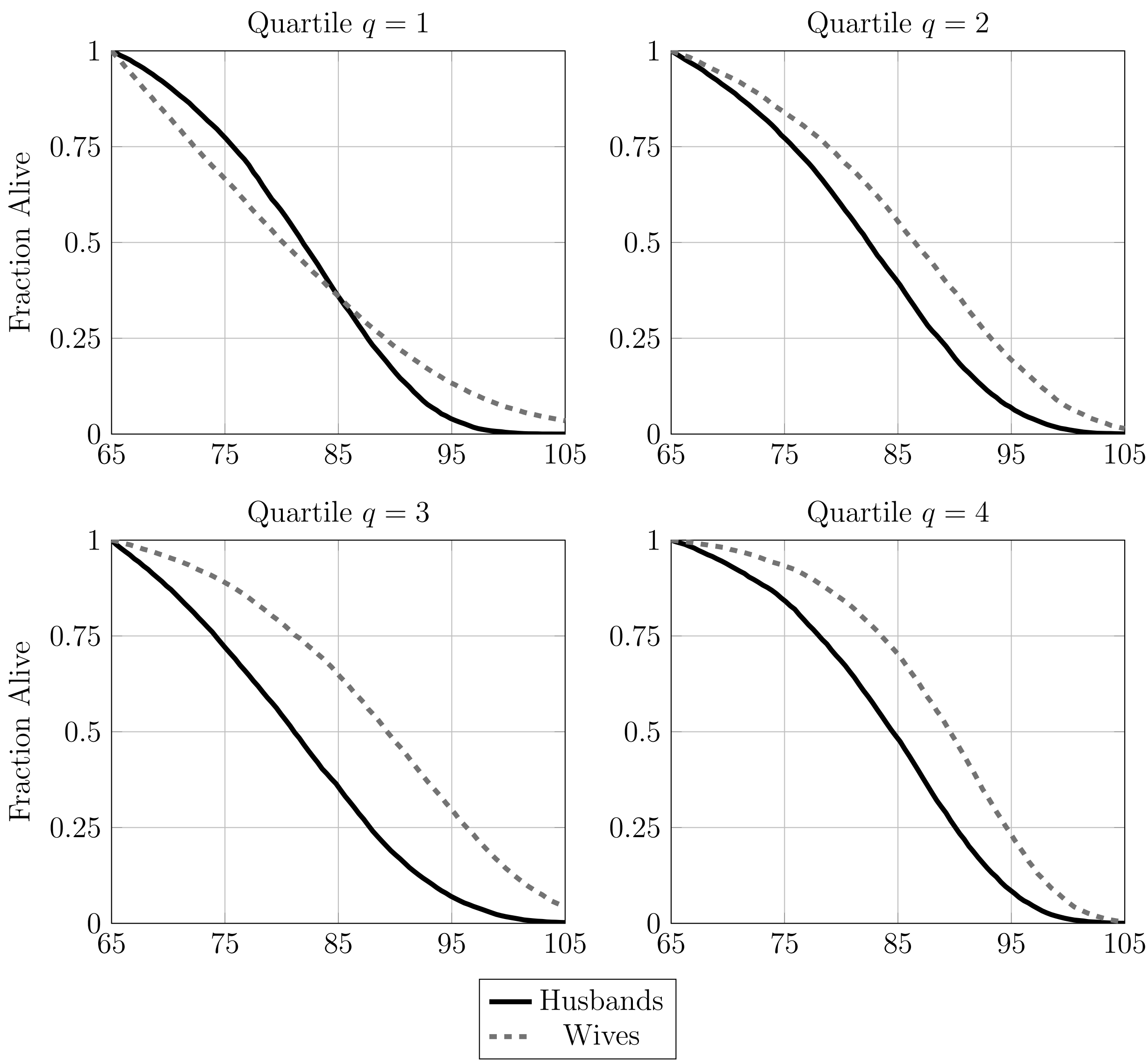

Figure B1: Estimated survival rates. Averages are taken from 10,000 paths of huseholds starting as couples at age 65.

C Numerical Method

In order to solve our model with the non-negativity conditions on (side) bequests and the no-borrowing constraint, numerical methods are required. In addition, to solve our model without insurance we also need numerical methods. This section outlines our numerical method. We first discretize the problem and then we apply an endogenous grid method (Carroll, 2006). We opt for this method, due to its numerical stability and efficiency.

Widowed Partner Following Pliska and Ye (2007), we integrate out the time of death and are left with the following deterministic optimal control problem: At any point in time, $t \in [0, T]$, partner i decides on a consumption and life insurance strategy

$\{(c_s, p^i_s)\}_{s\in[t,T]}$ that solves

$$V^i(t,x) = \sup_{c,p^i} \left\{ \int_t^T \beta^{s-t} e^{-\int_t^s \lambda^i_S(v)\mathrm{d}v} \left[u_S(c_s) + \lambda^i_S(s) L_0\left(X_s + \frac{p^i_s}{\lambda^i_S(s)} \right) \right] \mathrm{d}s + \beta^{T-t} e^{-\int_t^T \lambda^i_S(v)\mathrm{d}v} L_0(X_T) \right\},$$

subject to $X_s \geq 0$, $Z^i_s = X_s + \frac{p^i_s}{\lambda^i_S(s)} \geq 0$ for all $s \in [t,T]$ and

$$\mathrm{d}X_s = (rX_s + y^i_s - c_s - p^i_s)\mathrm{d}s.$$

For any $t \in [0,T)$ and $\delta > 0$ such that $t + \delta < T$, the dynamic programming property reveals that

$$V^i(t,x) = \sup_{c,p^i} \left\{ \int_t^{t+\delta} \beta^{s-t} e^{-\int_t^s \lambda^i_S(v)\mathrm{d}v} \left[u_S(c_s) + \lambda^i_S(s) L_0\left(X_s + \frac{p^i_s}{\lambda^i_S(s)} \right) \right] \mathrm{d}s \right\} + \beta^{\delta} e^{-\int_t^{t+\delta} \lambda^i_S(v)\mathrm{d}v} V(t+\delta, X_{t+\delta}).$$

We then set a discrete time grid with small time steps δ and consider the discrete problem

$$v^i_t(x) = \sup_{c,Z\geq 0,\tilde{x}\geq 0} \left\{ a^i_t u_S(c) + b^i_t L_0(Z) + \beta^\delta s^i_t v_{t+\delta}(\tilde{x}) \right\}$$
$$\text{s.t. } \tilde{x} = x + \delta([r + \lambda^i_S(t)]x + y^i_t - c - \lambda^i_S(t)Z),$$

with the final condition $v^i_T(x) = L_0(x)$. The integration constants read $a^i_t = \int_t^{t+\delta} \beta^{s-t} e^{-\int_t^s \lambda^i_S(v)\mathrm{d}v}\mathrm{d}s$, $b^i_t = \int_t^{t+\delta} \beta^{s-t} e^{-\int_t^s \lambda^i_S(v)\mathrm{d}v} \lambda^i_S(s)\mathrm{d}s$ and $s^i_t = e^{-\int_t^{t+\delta} \lambda^i_S(v)\mathrm{d}v}$.[12] The variable Z represents the legacy: $Z = x + \frac{p}{\lambda^i_S(t)}$. If, in the model, we exclude the investment into life insurance, we add the additional constraint $Z = x$ to the above.
We then derive Euler conditions and iterate backward over these conditions using an exogenous grid for next period's wealth. The policy functions for period t are then found by interpolating[13] over the endogenously determined grid, combined with additional interpolation nodes that incorporate the boundary points. Due to the algorithm's efficiency, we can set a large time and wealth grid, ensuring numerical solutions are very close to the true solutions.

Side Bequests For each discrete time point, we determine the side bequest policy function by solving

$$L^i_t(x) = \sup_{0\leq \Delta^i \leq x} \left\{ L_1(\Delta^i) + v^{\not i}_t(x - \Delta^i) \right\}.$$

We set a grid for wealth after the side bequest and use the first order conditions to determine the corresponding side bequest and wealth before the side bequest. We then enforce the boundary conditions and the policy function is given by interpolating the resulting optimal points.
If we exclude side bequests, $\Delta^i = 0$ is enforced and we simply use $L^i_t(x) = L_1(0) + v^{\not i}_t(x)$.

[12]Numerically $a^i_t \approx \delta$ and $b^i_t \approx \delta\lambda^i_S(t)$. Using these approximations has a negligible effect on the optimal strategies when δ is small, however the value function is approximated poorly even for small δ. We therefore determine these constants using numerical integration.

[13]We use linear interpolation every time we interpolate.

Couple For the couple's problem, we employ the same strategy as in the single's problem. First we discretize to

$$v_t^C(x) = \sup_{c,Z^i\geq 0,\tilde{x}\geq 0} \left\{ a_t^C u_C(c) + \sum_i b_t^{Ci} L_t^i(Z^i) + \beta^\delta s_t^C v_{t+\delta}^C(\tilde{x}) \right\},$$
$$\text{s.t. } \tilde{x} = x + \delta([r + \sum_i \lambda_C^i(t)]x + y_t^C - c - \sum_i \lambda_C^i(t) Z^i),$$

with $v_T^C(x) = L_0(x)$, $a_t^C = \int_t^{t+\delta} \beta^{s-t} e^{-\int_t^s \lambda_C^1(v)+\lambda_C^2(v)\mathrm{d}v} \mathrm{d}s$, $b_t^{Ci} = \int_t^{t+\delta} \beta^{s-t} e^{-\int_t^s \lambda_C^1(v)+\lambda_C^2(v)\mathrm{d}v} \lambda_C^i(s)\mathrm{d}s$ and $s_t^C = e^{-\int_t^{t+\delta} \lambda_C^1(v)+\lambda_C^2(v)\mathrm{d}v}$.
Then, we derive Euler conditions which we use to solve the problem at exogenously chosen grid points for next period's wealth.

D Proofs

D.1 Proposition 4.1

Our solution method is standard, using the Hamilton-Jacobi-Bellman (HJB) equation. First, we derive the HJB equation. Then, we solve the optimality conditions. Next, we make a guess for the value function and substitute this guess into the HJB equation. Then, algebra reveals (several) ordinary differential equations to be solved, from which we can derive the optimal controls.
In line with Pliska and Ye (2007), the value function V^i satisfies the following Hamilton-Jacobi-Bellman equation:

$$\begin{cases} V_t^i - (\lambda_S^i(t) - \ln(\beta))V^i + \sup_{c,p^i} \{\mathcal{H}^i(t,x,y,;c,p^i)\} = 0, \\ V^i(T,x,y) = L_0(x). \end{cases} \tag{11}$$

Here, we use subscripts to denote the partial derivatives of V^i, we suppress the argument (t,x,y) of V^i and the function $\mathcal{H}^i$ is given by

$$\mathcal{H}^i(t,x,y;c,p^i) = u_S(c) + \lambda_S^i(t) L_0\left(x + \frac{p^i}{\lambda_S^i(t)}\right) + y\mu_t^i V_y^i$$
$$+ V_x^i(rx + y - c - p^i).$$

The supremum is attained at the points

$$c = (V_x^i)^{-\frac{1}{\gamma}} \quad \text{and} \quad x + \frac{p^i}{\lambda_S^i(t)} = -\kappa_0 + \left(\frac{V_x^i}{\phi_0}\right)^{-\frac{1}{\gamma}}. \tag{12}$$

If we make the guess $V^i(t,x,y) = \frac{H^i(t)^\gamma}{1-\gamma}(x + yF^i(t) + A^i(t))^{1-\gamma}$ and substitute the above back into the HJB equation we get:

$$\frac{\gamma}{1-\gamma} H_t^i (H^i)^{\gamma-1}\hat{x}^{1-\gamma} + (yF_t^i + A_t^i)(H^i)^\gamma \hat{x}^{-\gamma} - \frac{\lambda_S^i(t) - \ln(\beta)}{1-\gamma}(H^i)^\gamma \hat{x}^{1-\gamma}$$
$$+ \frac{\gamma}{1-\gamma}(1 + \lambda_S^i(t)\phi_0^{\frac{1}{\gamma}})(H^i)^{\gamma-1}\hat{x}^{1-\gamma} + \lambda_S^i(t)\kappa_0 (H^i)^\gamma \hat{x}^{-\gamma} + (r + \lambda_S^i(t))(H^i)^\gamma \hat{x}^{1-\gamma}$$
$$-(r + \lambda_S^i(t))(yF^i + A^i)(H^i)^\gamma \hat{x}^{-\gamma} + y\mu_t^i F^i (H^i)^\gamma \hat{x}^{-\gamma} + y(H^i)^\gamma \hat{x}^{-\gamma} = 0$$

where we write $\hat{x} = x + yF^i + A^i$, we use subscripts to denote time derivatives of (H^i, F^i, A^i) and we suppress the time argument in (H^i, F^i, A^i). This can be further factored into

$$a_H^i(t, H^i, H_t^i)(H^i)^{\gamma-1}\hat{x}^{1-\gamma} + b_F^i(t, F^i, F_t^i)y(H^i)^\gamma \hat{x}^{-\gamma} + c_A^i(t, A^i, A_t^i)(H^i)^\gamma \hat{x}^{-\gamma} = 0.$$

Given that this needs to hold for all (t, x, y) the solution requires that $a_H^i(t, H^i, H_t^i) = b_F^i(t, F^i, F_t^i) = c_A^i(t, A^i, A_t^i) = 0$ for all t. Combined with the boundary condition that $V^i(T, x, y) = L_0(x)$, this yields a differential equation for each of H^i, F^i and A^i.
These differential equations are given by:

$$\begin{cases} H_t^i(t) = H^i(t)\left(-\frac{\ln(\beta)+r(1-\gamma)}{\gamma} + \lambda_S^i(t)\right) - 1 - \lambda_S^i(t)\phi_0^{\frac{1}{\gamma}}, \\ H^i(T) = \phi_0^{\frac{1}{\gamma}}, \end{cases}$$
$$\begin{cases} F_t^i(t) = (r - \mu_t^i + \lambda_S^i(t))F^i(t) - 1, \\ F^i(T) = 0, \end{cases} \tag{13}$$
$$\begin{cases} A_t^i(t) = (r + \lambda_S^i(t))A^i(t) - \lambda_S^i(t)\kappa_0, \\ A^i(T) = \kappa_0. \end{cases}$$

Provided $\lambda_S^i(t)$ and μ_t^i are differentiable in time t, each of these differential equations has a unique solution.
Then, for the identity $yF^i(t) = \mathbb{E}_{t,y}\left[\int_t^{T\wedge\tau_i} e^{-r(s-t)}Y_s \mathrm{d}s\right]$, we refer to Hambel (2020).
Additionally, computing the following expectation, we arrive at the identity for A^i:

$$\mathbb{E}_t\left[e^{-r((T\wedge\tau_i)-t)}\kappa_0\right] = \int_t^T e^{-r(s-t)}\lambda_S^i(s)e^{-\int_t^s \lambda_S^i(v)\mathrm{d}v}\kappa_0 \mathrm{d}s + e^{-\int_t^T \lambda_S^i(v)\mathrm{d}v - r(T-t)}\kappa_0 = A^i(t).$$

The second equality holds, as this is exactly the solution to the ODE of A^i in (13).
The policy functions for consumption and insurance can then be found by combining (12), the guess for V^i and the solution to the ODE's (13). It can be verified that the value function $V^i(t, x, y) = \frac{H^i(t)^\gamma}{1-\gamma}(x + yF^i(t) + A^i(t))^{1-\gamma}$, with (A^i, F^i, H^i) defined in (13), solves the HJB equation (11).

D.2 Proposition 4.2

The HJB equation in the couple problem is given by

$$\begin{cases} V_t^C - (\lambda_C^1(t) + \lambda_C^2(t) - \ln(\beta))V^C + \sup_{\boldsymbol{\psi}}\left\{\mathcal{H}^C(t, x, y, ; \boldsymbol{\psi})\right\} = 0, \\ V^C(T, x, y) = L_0(x), \end{cases} \tag{14}$$

where $\boldsymbol{\psi} = (c, p^1, p^2, \Delta^1, \Delta^2)$ and

$$\mathcal{H}^C(t, x, y; \boldsymbol{\psi}) = u_C(c) + y\mu_t^C V_y^C + V_x^C(rx + y - c - p^1 - p^2) + \sum_{i=1}^2 \lambda_C^i(t)\left(V^{j}(t, x + \frac{p^i}{\lambda_C^i(t)} - \Delta^i, \theta_t^i y) + L_1(\Delta^i)\right).$$

For simplicity, we define $\omega = \frac{2}{\eta^{1-\gamma}}$, so that $u_C(c) = \omega \frac{c^{1-\gamma}}{1-\gamma}$. The FOCs for optimal controls are given by

$$\begin{aligned} c &= \left(\frac{V_x^C}{\omega}\right)^{-\frac{1}{\gamma}}, \\ \Delta^i &= -\kappa_1 + \phi_1^{\frac{1}{\gamma}} (V_x^C)^{-\frac{1}{\gamma}}, \\ x + \frac{p^i}{\lambda_C^i(t)} &= \Delta^i + H^{\not{i}}(t)(V_x^C)^{-\frac{1}{\gamma}} - \theta_t^i y F^{\not{i}}(t) - A^{\not{i}}(t). \end{aligned} \tag{15}$$

Making the guess $V^C(t,x,y) = \frac{H^C(t)^\gamma}{1-\gamma}(x + yF^C(t) + A^C(t))^{1-\gamma}$ and substituting this into the HJB equation we arrive at

$$a_H^C(t, H^C, H_t^C)(H^C)^{\gamma-1}\hat{x}^{1-\gamma} + b_F^C(t, F^C, F_t^C) y (H^C)^\gamma \hat{x}^{-\gamma} + c_A^C(t, A^C, A_t^C)(H^C)^\gamma \hat{x}^{-\gamma} = 0,$$

for specific functions a_H^C, b_F^C and c_A^C and with $\hat{x} = x + yF^C + A^C$. Then, the conditions $a_H^C = 0$, $b_F^C = 0$ and $c_A^C = 0$ are required for the HJB equation to be solved, resulting in a differential equation for H^C, F^C and A^C each:

$$\begin{aligned} &\begin{cases} H_t^C(t) = H^C(t)\left(-\frac{\ln(\beta)+r(1-\gamma)}{\gamma} + \lambda_C^1(t) + \lambda_C^2(t)\right) - \omega^{\frac{1}{\gamma}} - \sum_{i=1}^2 \lambda_C^i(t)[\phi_1^{\frac{1}{\gamma}} + H^{\not{i}}(t)], \\ H^C(T) = \phi_0^{\frac{1}{\gamma}}, \end{cases} \\ &\begin{cases} F_t^C(t) = (r - \mu_t^C + \lambda_C^1(t) + \lambda_C^2(t))F^C(t) - 1 - \sum_{i=1}^2 \lambda_C^i(t)\theta_t^i F^{\not{i}}(t), \\ F^C(T) = 0, \end{cases} \\ &\begin{cases} A_t^C(t) = (r + \lambda_C^1(t) + \lambda_C^2(t))A^C(t) - \sum_{i=1}^2 \lambda_C^i(t)[\kappa_1 + A^{\not{i}}(t)], \\ A^C(T) = \kappa_0. \end{cases} \end{aligned} \tag{16}$$

The proof for the representation of yF^C is again analogous to the one by Hambel (2020), so we omit it here.
The representation of A^C can again be found by computing the expectation directly:

$$\begin{aligned} &\mathbb{E}_t\left[\mathbb{1}\{\min\{\tau_1,\tau_2\} < T\}\, e^{-r(\min\{\tau_1,\tau_2\}-t)}\kappa_1 + e^{-r(T\wedge\max\{\tau_1,\tau_2\}-t)}\kappa_0\right] \\ &= \mathbb{E}_t\left[\left(\sum_{i=1}^2 \mathbb{1}\{\tau_i < T \wedge \tau_{\not{i}}\}\, e^{-r(\tau_i - t)}(\kappa_1 + A^{\not{i}}(\tau_i))\right) + \mathbb{1}\{T < \tau_1 \wedge \tau_2\}\, e^{-r(T-t)}\kappa_0\right] \\ &= \int_t^T \sum_{i=1}^2 \lambda_C^i(s) e^{-r(s-t) - \int_t^s \lambda_C^1(v) + \lambda_C^2(v)\mathrm{d}v}[\kappa_1 + A^{\not{i}}(s)]\mathrm{d}s + e^{-r(T-t) - \int_t^T \lambda_C^1(v)+\lambda_C^2(v)\mathrm{d}v}\kappa_0 \\ &= A^C(t). \end{aligned}$$

It can be verified that the value function $V^C(t,x,y) = \frac{H^C(t)^\gamma}{1-\gamma}(x + yF^C(t) + A^C(t))^{1-\gamma}$, with (A^C, F^C, H^C) defined by (16) solves the HJB equation (14).

D.3 Corollary 4.3

The expression for wealth after the death of partner i is given by

$$x + \frac{\hat{p}^i(t,x,y)}{\lambda_C^i(t)} - \hat{\Delta}^i(t,x,y) = H^{\not{i}}(t)\frac{x + yF^C(t) + A^C(t)}{H^C(t)} - \theta_t^i y F^{\not{i}}(t) - A^{\not{i}}(t).$$

The right-hand side only depends on ϕ_1 through the function H^C, which increases with ϕ_1 following from the solution to (16). As a result, wealth after the death of a partner decreases with ϕ_1.

D.4 Corollary 4.4

We begin by noting that the sign of $\frac{\partial \hat{p}^i}{\partial x}(t,x,y) = \lambda_C^i(t)(-1 + \frac{\phi_1^{\frac{1}{\gamma}} + H^{\not i}(t)}{H^C(t)})$ is equal to the sign of $\chi^i(t) = -H^C(t) + \phi_1^{\frac{1}{\gamma}} + H^{\not i}(t)$. Given that both $H^{\not i}$ and H^C are continuous, so is χ^i. Furthermore, given the boundary condition $H^{\not i}(T) = H^C(T) = \phi_0^{\frac{1}{\gamma}}$ we have $\chi^i(T) = \phi_1^{\frac{1}{\gamma}} > 0$. Continuity of χ^i then implies that there is some $\bar{t}^i \in [0,T)$ such that $\chi^i(t) > 0$ for each $t > \bar{t}^i$. Consequently, with $\bar{t} = \max\{\bar{t}^1, \bar{t}^2\} < T$, we have that $\frac{\partial \hat{p}^i}{\partial x}(t,x,y) > 0$ for all (x,y), $i = 1,2$ and $t > \bar{t}$.

E Adjusting Model Parameters Between Annual and Biannual Frequencies

In our baseline problem we use the parameters estimated by De Nardi et al. (2025) and our model is at an annual frequency. De Nardi et al. (2025) estimate these parameters using a structural life-cycle model with a biannual frequency, however. We, therefore, transform these parameters to annual terms. In this section, we discuss the relation between an annual model and a biannual model. Annual time and parameters will be denoted as normal, but biannual time and parameters will be denoted by a tilde.
Table E1 shows how model parameters relate between an annual and a biannual frequency. The point in time t in years, corresponds to $\tilde{t} = \frac{1}{2}t$ bi-years.
Many parameters and controls in our problem are rates (income y_t, mortality $\lambda(t)$, interest r, discount $\ln(\beta)$, consumption c_t and insurance p_t). The transformation between annual rates and biannual rates is therefore essential. An annual rate z relates to a biannual rate to $\tilde{z}$, so that $\int_0^t z(s)\mathrm{d}s = \int_0^{\tilde{t}} \tilde{z}(\tilde{s})\mathrm{d}\tilde{s}$: the accumulation over $\tilde{t}$ bi-years using the biannual rate equals the accumulation over t years using the annual rate. Integration by substitution reveals that $\tilde{z}(\tilde{t}) = 2z(t)$. This relation reveals the transformations for the mortality rates $(\lambda_S^i, \lambda_C^i)$, the income rate (y_t^i, y_t^C), the interest rate (r) and the discount rate $(\ln(\beta))$. The time horizon T, in years, corresponds to a time horizon of $\tilde{T} = \frac{1}{2}T$ bi-years.
To show the relations for the utility parameters $(\gamma, \eta, \phi_0, \kappa_0, \phi_1, \kappa_1)$, we first show that the single problem at annual frequency can be rewritten into an equivalent problem at biannual frequency using these transformations. Then, we show the same for the couple's problem. This can be found in the next two subsections.
As De Nardi et al. (2025) already report their interest rate (r) and discount factor (β) in annual terms and we estimate mortality rates and income rates ourselves, Table E1 reveals that the only parameters we need to change from De Nardi et al. (2025) are the bequest scaling parameters (ϕ_0, ϕ_1). Their estimates of $(\tilde{\phi}_0, \tilde{\phi}_1) = (6.826 \cdot 10^6, 4447)$ combine with the risk aversion $\gamma = 3.7$ to $(\phi_0, \phi_1) = 2^\gamma(\tilde{\phi}_0, \tilde{\phi}_1) = (88.711 \cdot 10^6, 57.739 \cdot 10^3)$, as we report in Table 1.

Annual Parameter	Bi-Annual Transformation
Time	
t	$\tilde{t} = \frac{1}{2}t$
T	$\tilde{T} = \frac{1}{2}T$
Utility	
β	$\tilde{\beta} = \beta^2$
γ	$\tilde{\gamma} = \gamma$
η	$\tilde{\eta} = \eta$
ϕ_0	$\tilde{\phi}_0 = 2^{-\gamma}\phi_0$
κ_0	$\tilde{\kappa}_0 = \kappa_0$
ϕ_1	$\tilde{\phi}_1 = 2^{-\gamma}\phi_1$
κ_1	$\tilde{\kappa}_1 = \kappa_1$
Income	
y_t^i	$\tilde{y}_{\tilde{t}}^i = 2y_t^i$
y_t^C	$\tilde{y}_{\tilde{t}}^C = 2y_t^C$
Mortality Rates	
$\lambda_S^i(t)$	$\tilde{\lambda}_S^i(\tilde{t}) = 2\lambda_S^i(t)$
$\lambda_C^i(t)$	$\tilde{\lambda}_C^i(\tilde{t}) = 2\lambda_C^i(t)$
Other	
r	$\tilde{r} = 2r$

Table E1: Transformation of Model Parameters, Annual vs. Biannual.

E.1 Single's Problem

The single's problem, when integrating out the time of death, reads for any point in time $t \in [0, T]$ and wealth $X_t = x \geq 0$ as:

$$V^i(t,x) = \sup_{\{c_s, p_s^i\}_{t\leq s\leq T}} \left\{ \int_t^T \beta^{s-t} e^{-\int_t^s \lambda_S^i(v)\mathrm{d}v} \left[u_S(c_s) + \lambda_S^i(s) L_0\left(X_s + \frac{p_s^i}{\lambda_S^i(s)} \right) \right] \mathrm{d}s \right.$$
$$\left. + \beta^{T-t} e^{-\int_t^T \lambda_S^i(v)\mathrm{d}v} L_0(X_T) \right\},$$

subject to[14]

$$\mathrm{d}X_s = (rX_s + y_s^i - c_s - p_s^i)\mathrm{d}s, \text{ for all } s \in [t,T] \text{ and } X_t = x.$$

Substitution over time reveals that

$$V^i(t,x) = \sup_{\{c_s, p_s^i\}_{t\leq s\leq T}} \left\{ \int_{\frac{1}{2}t}^{\frac{1}{2}T} \beta^{2(s-\frac{1}{2}t)} e^{-\int_{\frac{1}{2}t}^s 2\lambda_S^i(2v)\mathrm{d}v} \left[u_S(c_{2s}) + \lambda_S^i(2s) L_0\left(X_{2s} + \frac{p_{2s}^i}{\lambda_S^i(2s)} \right) \right] 2\mathrm{d}s \right.$$
$$\left. + \beta^{2(\frac{1}{2}T-\frac{1}{2}t)} e^{-\int_{\frac{1}{2}t}^{\frac{1}{2}T} 2\lambda_S^i(2v)\mathrm{d}v} L_0(X_T) \right\}.$$

The dynamics of $\tilde{X}_s = X_{2s}$ are

$$\mathrm{d}\tilde{X}_s = 2(rX_{2s} + y_{2s}^i - c_{2s} - p_{2s}^i)\mathrm{d}s = (\tilde{r}\tilde{X}_s + \tilde{y}_s^i - \tilde{c}_s - \tilde{p}_s^i).$$

[14]For the sake of exposition we ignore the non-negativity constraints on wealth and bequests here. The presented relation between the annual and biannual models remains valid if these are included.

Here, $\tilde{c}_s = 2c_{2s}$ and $\tilde{p}^i_s = 2p^i_{2s}$ are the biannual consumption and premium rates and we adopt the notation from Table E1. As a result,

$$V^i(t,x) = \sup_{\{\tilde{c}_s,\tilde{p}^i_s\}_{\frac{1}{2}t\le s\le \tilde{T}}} \Bigg\{ \int_{\frac{1}{2}t}^{\tilde{T}} \tilde{\beta}^{s-\frac{1}{2}t} e^{-\int_{\frac{1}{2}t}^{s} \tilde{\lambda}^i_S(v)\mathrm{d}v} \Bigg[u_S(\frac{1}{2}\tilde{c}_s) + \frac{1}{2}\tilde{\lambda}^i_S(s) L_0\Bigg(\tilde{X}_s + \frac{\tilde{p}^i_s}{\tilde{\lambda}^i_S(s)}\Bigg)\Bigg] 2\mathrm{d}s \\ + \tilde{\beta}^{\tilde{T}-\frac{1}{2}t} e^{-\int_{\frac{1}{2}t}^{\tilde{T}} \tilde{\lambda}^i_S(v)\mathrm{d}v} L_0(\tilde{X}_{\tilde{T}}) \Bigg\}.$$

Given our utility function $u_S(c) = \frac{c^{1-\gamma}}{1-\gamma}$, we can factor the $\frac{1}{2}$ out of the utility function and rewrite to obtain

$$V^i(t,x) = 2^\gamma \sup_{\{\tilde{c}_s,\tilde{p}^i_s\}_{\frac{1}{2}t\le s\le \tilde{T}}} \Bigg\{ \int_{\frac{1}{2}t}^{\tilde{T}} \tilde{\beta}^{s-\frac{1}{2}t} e^{-\int_{\frac{1}{2}t}^{s} \tilde{\lambda}^i_S(v)\mathrm{d}v} \Bigg[u_S(\tilde{c}_s) + \tilde{\lambda}^i_S(s) 2^{-\gamma} L_0\Bigg(\tilde{X}_s + \frac{\tilde{p}^i_s}{\tilde{\lambda}^i_S(s)}\Bigg)\Bigg] \mathrm{d}s \\ + \tilde{\beta}^{\tilde{T}-\frac{1}{2}t} e^{-\int_{\frac{1}{2}t}^{\tilde{T}} \tilde{\lambda}^i_S(v)\mathrm{d}v} 2^{-\gamma} L_0(\tilde{X}_{\tilde{T}}) \Bigg\}.$$

The objective inside the supremum is exactly our problem after the transformation in Table E1, with $\tilde{\phi}_0 = 2^{-\gamma}\phi_0$. The value function relation reads

$$V^i(t,x) = 2^\gamma \tilde{V}^i(\tilde{t},x),$$

where V^i is annual value function in our standard problem and $\tilde{V}^i$ is the biannual value function in our problem under the transformed parameters.

E.2 Couple Problem

The couple's problem reads, using the shorthand notation $\lambda_C = \lambda^1_C + \lambda^2_C$,

$$V^C(t,x) = \sup_{\{\psi_s\}_{t\le s\le T}} \Bigg\{ \int_t^T \beta^{s-t} e^{-\int_t^s \lambda_C(v)\mathrm{d}v} \Bigg[u_C(c_s) \\ + \sum_{i=1}^{2} \lambda^i_C(s) \Bigg(V^{i\!\!/}\Bigg(s, X_s + \frac{p^i_s}{\lambda^i_S(s)} - \Delta^i_s\Bigg) + L_1(\Delta^i_s)\Bigg)\Bigg] \mathrm{d}s \\ + \beta^{T-t} e^{-\int_t^T \lambda_C(v)\mathrm{d}v} L_0(X_T) \Bigg\}.$$

Taking the same steps as in the singles' problem, we arrive at

$$V^C(t,x) = 2^\gamma \sup_{\{\tilde{\psi}_s\}_{\frac{1}{2}t\le s\le \tilde{T}}} \Bigg\{ \int_{\frac{1}{2}t}^{\tilde{T}} \tilde{\beta}^{s-\frac{1}{2}t} e^{-\int_{\frac{1}{2}t}^{s} \tilde{\lambda}_C(v)\mathrm{d}v} \Bigg[u_C(\tilde{c}_s) \\ + \sum_{i=1}^{2} \tilde{\lambda}^i_C(s) \Bigg(\tilde{V}^{i\!\!/}\Bigg(s, \tilde{X}_s + \frac{\tilde{p}^i_s}{\tilde{\lambda}^i_S(s)} - \tilde{\Delta}^i_s\Bigg) + 2^{-\gamma} L_1(\tilde{\Delta}^i_s)\Bigg)\Bigg] \mathrm{d}s \\ + \tilde{\beta}^{T-t} e^{-\int_{\frac{1}{2}t}^{\tilde{T}} \tilde{\lambda}_C(v)\mathrm{d}v} 2^{-\gamma} L_0(\tilde{X}_{\tilde{T}}) \Bigg\}.$$

The controls are given by $\tilde{c}_s = 2c_{2s}$, $\tilde{p}^i_s = 2p^i_{2s}$ and $\tilde{\Delta}^i_s = \Delta^i_{2s}$. The side bequest and bequest utility functions are scaled by $2^{-\gamma}$, showing the relation $\tilde{\phi}_k = 2^{-\gamma}\phi_k$, $k = 0, 1$. We conclude that an annual model with our ordinary parametrization leads to the same solution as a model with parameters transformed according to Table E1.

F Additional Life-cycle profiles by Income Quartiles

In this appendix, we provide life-cycle profiles similar to those in section 6.1, for the first, third and fourth income quartiles. The profiles show couples starting with an initial wealth amount that is close to the median wealth in that quartile, as reported in table G1: the couples in the first, third and fourth quartile start with an initial wealth of 150, 400 and 500 thousand dollars, respectively.

F.1 Comparing Side Bequests with no Side Bequests

In this subsection, we show life-cycle profiles for representative couples in the first, third and fourth income quartiles, which are presented in figures F1, F2 and F3 respectively. As mentioned in the main text, the general patterns stay the same. Firstly, couples with side bequest motives cut down on consumption. Secondly, couples with side bequest motives have a larger incentive to invest in life insurance. Luxury bequests also play a role here: couples in the upper two quartiles have little desire for annuities, even without side bequest motives. This aligns with our theoretical finding that, given strong enough bequest motives, life insurance demand can increase with wealth. Thirdly, couples tend to hold more life insurance on the better endowed spouse ($p_t^h > p_t^w$ for most of the specifications). Finally, side bequest motives have little effect on the savings in the upper two quartiles, but they do affect the savings of the couple in the first quartile. As discussed in the main text, absent side bequest motives, the luxury final bequest motive provides little incentive to save and couples instead favor a higher level of consumption, decreasing their wealth. With side bequest motives, couples' bequest motives are strong enough to provide incentive to save, however.
Furthermore, these profiles provide insight into the mechanisms underlying the welfare effects in section 6.3 in the main text. Without side bequest motives, the couple in the lowest income quartile annuitizes considerably, while the couple with side bequest motives holds few life-contingent products (for most of its life-cycle, that is). In absolute value, holdings in life-contingent products therefore decrease by the inclusion of side bequest. This suggests that the couple with side bequest motives suffers less from having no access to life-contingent products, leading to smaller welfare gains. In contrast, side bequest motives increase the absolute position in life contingent products among the couples in the upper two quartiles. This suggests that in these quartiles, couples have more to gain from life-contingent products. Indeed, Table 4 reports exactly these patterns: side bequest motives decrease the welfare gains of life-contingent products among couples in the lower two income quartiles and increase the welfare gains in the upper two quartiles.

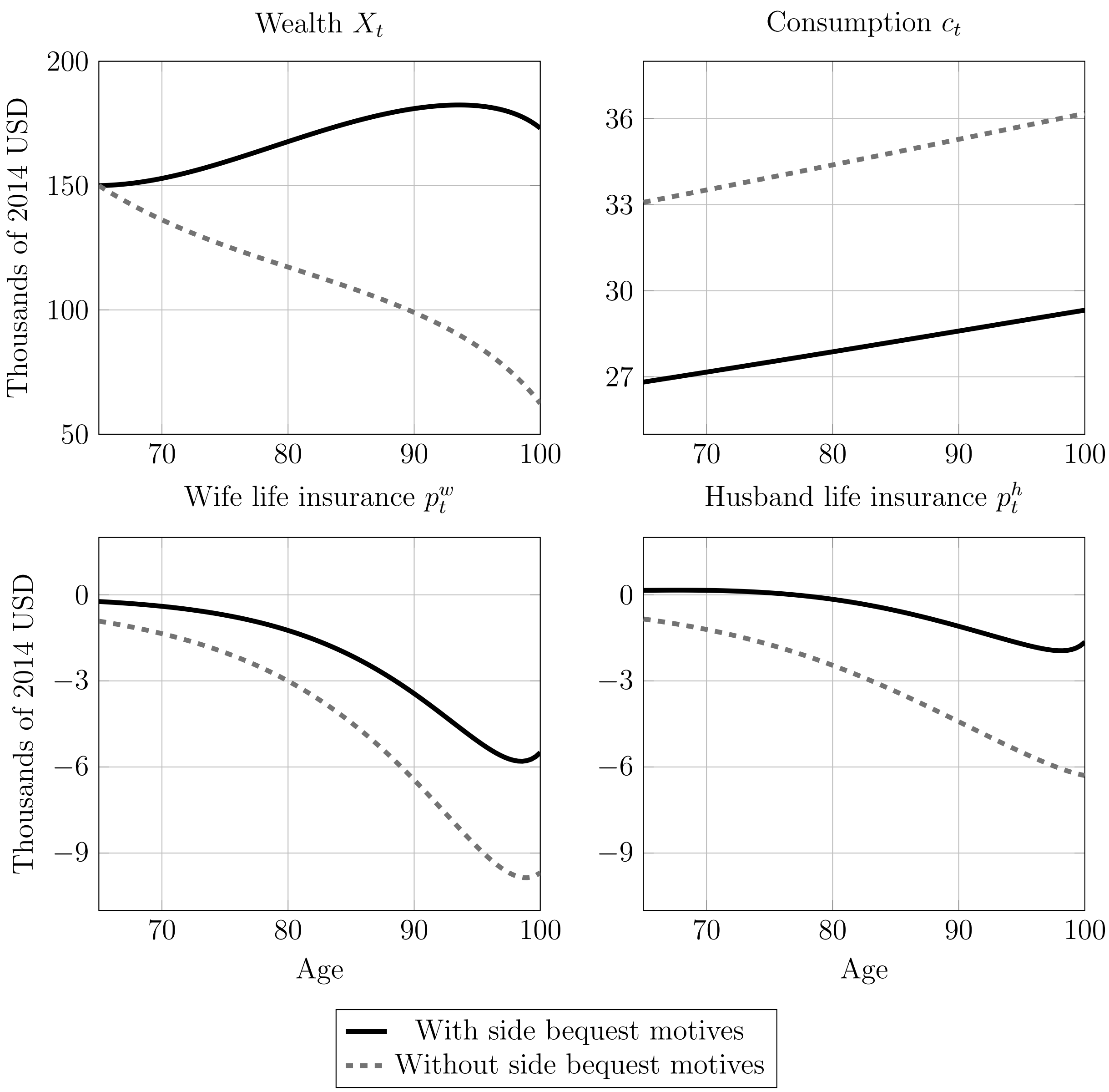

Figure F1: Key variables over the life-cycle, income quartile 1.

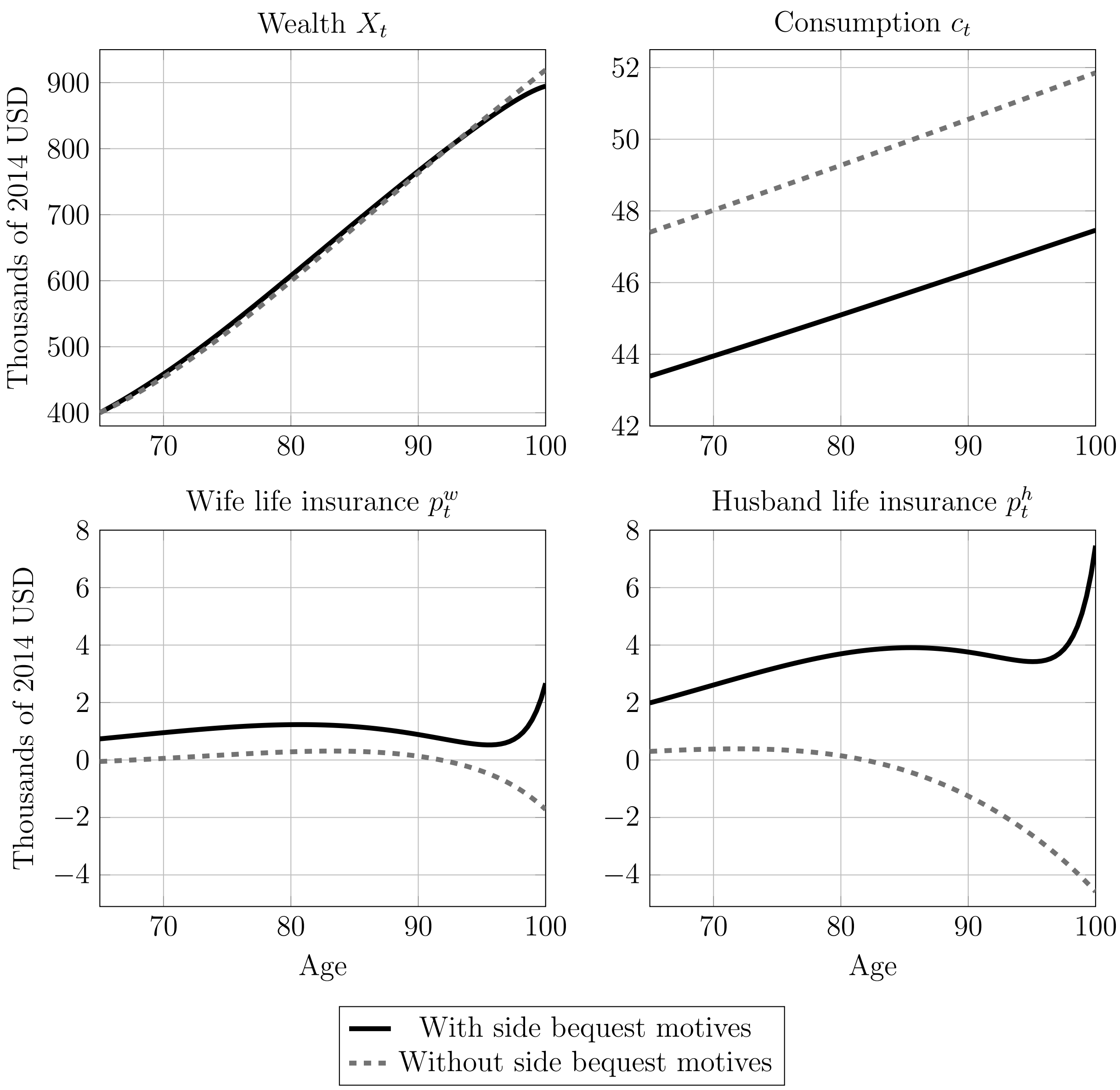

Figure F2: Key variables over the life-cycle, income quartile 3.

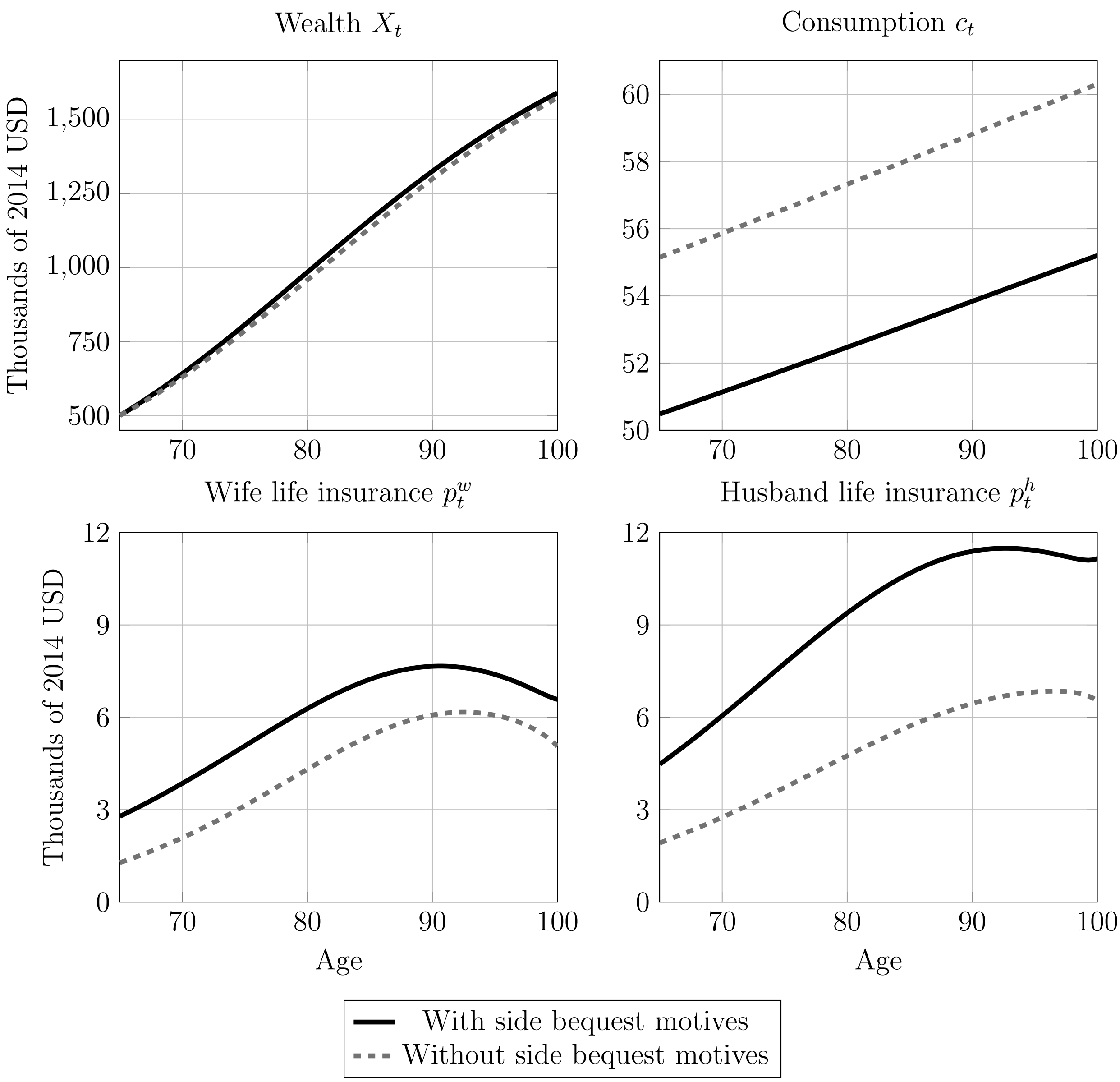

Figure F3: Key variables over the life-cycle, income quartile 4.

F.2 Comparing Life-Contingent Investment with no Investment

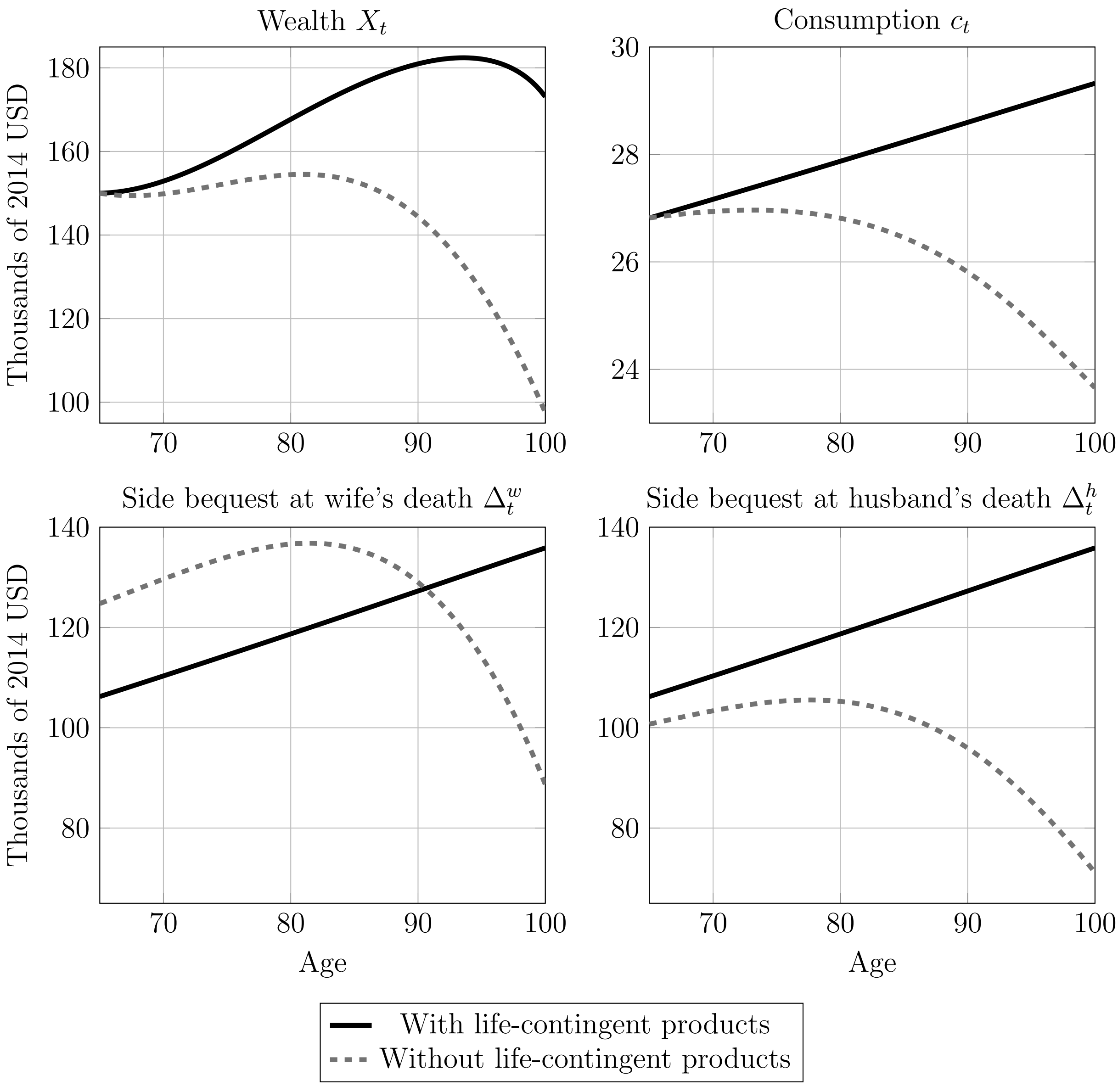

Figure F4: Key variables over the life-cycle, income quartile 1.

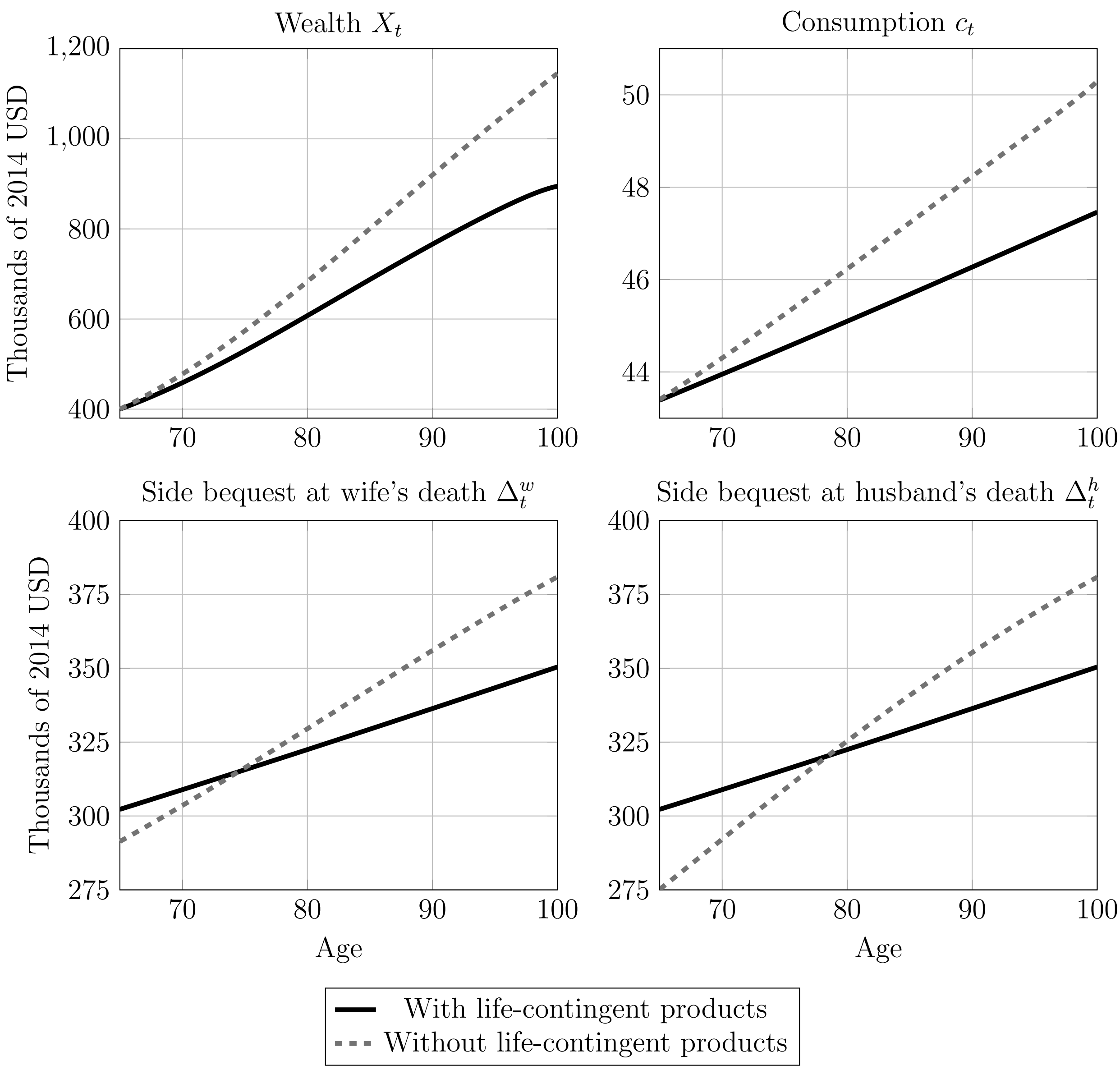

Figure F5: Key variables over the life-cycle, income quartile 3.

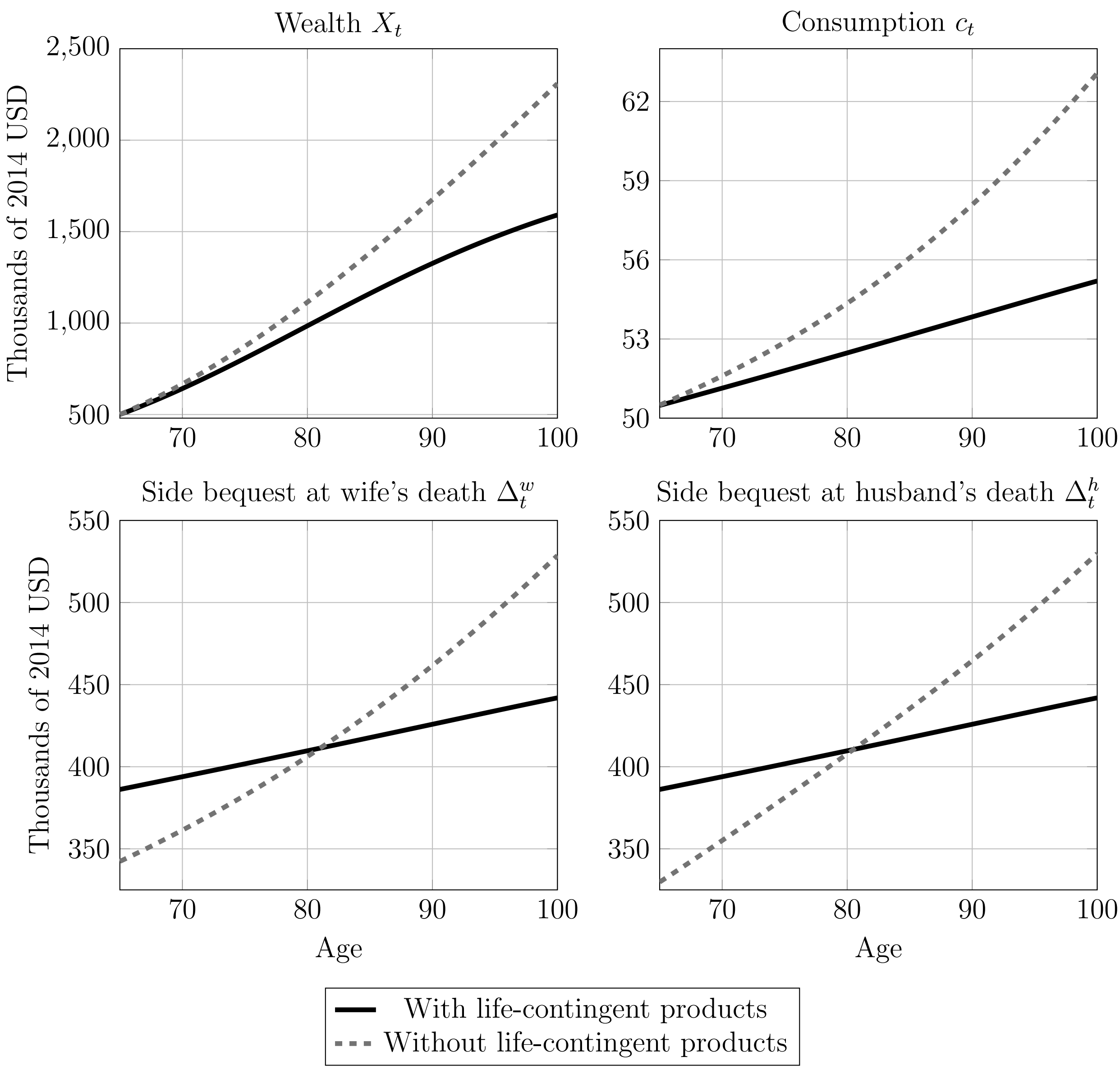

Figure F6: Key variables over the life-cycle, income quartile 4.

G Computation of Initial Wealth and Indicators

To provide quantitative insights in aggregate, we compute several indicators, accounting for heterogeneity across the retiree population by considering four different permanent income quartiles. In addition, we account for heterogeneity within income quartiles by estimating a realistic wealth distribution for each income quartile and considering these indicators aggregated across these wealth distributions.

To do so, we use the HRS data. We use the "HwATOTB" variable in the RAND longitudinal file to define wealth as the sum of the net value of housing and estate wealth, vehicles, businesses, IRAs, stocks, mutual funds, bonds, savings and the money market account less all debt (including mortgages and home loans). For each couple, we take the wealth observed in the first wave they were included and we exclude those households, in which the husband is older than 75 (our initial wealth corresponds to wealth age 65).

We remove the highest 5 percent of observations and couples with negative wealth, to avoid outliers. Then, we compute the 25^{th}, 50^{th} and 75^{th} percentiles of wealth by income quartile. We fit a log-normal distribution to these percentiles: we assume the initial wealth of a couple, X_0, in quartile q is positive and that $\ln(X_0)$ is normally distributed with mean μ_x^q and standard deviation $\sigma_x^q > 0$. Table G1 reports the percentiles and the corresponding parameters. There is a clear gradient, with couples in a higher income quartile being wealthier.

Quartile	Empirical Percentiles			Fitted Parameters	
q	25^{th}	50^{th}	75^{th}	μ_x^q	σ_x^q
1	61,401	160,819	359,922	11.94	1.31
2	137,904	327,264	595,496	12.62	1.10
3	209,999	415,319	794,417	12.93	0.99
4	260,346	532,987	924,766	13.14	0.94

Table G1: The table shows empirical percentiles of wealth by income quartile, as well as the fitted parameters of a log-normal distribution.

Next, we compute the indicators. Taking as example the expected portion of life annuitized (EPLA), we denote

$$\text{EPLA}^g(x) = \mathbb{E}\left[\frac{1}{\tau_h \wedge \tau_w \wedge T}\int_0^{\tau_h \wedge \tau_w \wedge T} \mathbb{1}\{p_t^g < 0\}\,\mathrm{d}t | X_0 = x\right].$$

Solving our model for $X_0 = x$, the above can easily be computed for any $x \geq 0$, via numerical integration. Thus, we find $\text{EPLA}^g(x)$ for a grid of values x. Then, we compute $\text{EPLA}^g = \mathbb{E}\left[\text{EPLA}^g(X_0)\right]$ by numerically integrating over our estimated log-normal distribution for X_0. As we only know the indicator for finitely high x, we set an upperbound at 1.5 million dollars and compute the right-truncated expectation. This method of computing the indicators does assume that, within each income quartile, mortality rates are independent of initial wealth.