Investment strategies for the pre-retirement and retirement phase of IDC pensions

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Abstract

A new legislation in the Netherlands makes it possible for participants in DC pension schemes to invest in risky assets after retirement. This thesis considers several aspects of both the investment policy as the pension benefit payment policy in DC pension schemes. First of all, the design of the life-cycle before retirement is investigated. A decreasing life-cycle is preferred above a constant life-cycle. Moreover, the optimal amount of investment risk heavily depends on the risk preferences of the participant: an inadequate average equity exposure can lead to sizeable welfare losses. Furthermore, the influence of the assumed interest rate (AIR) and the effect of financial smoothing on the development of the pension benefit level during retirement is investigated. In case of financial smoothing, a decreasing life-cycle is preferable to prevent an increased amount of investment risk at high ages. Moreover, participants should be aware that financial smoothing around the risk-free rate in combination with the expected return as AIR leads to a high probability of a decrease in the pension benefit level during the first years after retirement. A horizon-dependent AIR can prevent this undesirable effect.

Keywords: individual defined contribution pension scheme, variable annuities, life-cycle investing, assumed interest rate (AIR), smoothing financial shocks

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Beleidssamenvatting

Aanleiding

Om deelnemers in Nederland met een premie- of kapitaalovereenkomst de keuzemogelijkheid te bieden voor een variabel, risicodragend pensioen, is de Wet verbeterde premieregeling[1] tot stand gekomen. Deze wet is op 14 juni 2016 aangenomen door de Eerste Kamer met als beoogde datum van inwerkingtreding 1 september 2016. De wet maakt het voor deelnemers met een premie- of kapitaalovereenkomst mogelijk om door te beleggen na pensioen. Deelnemers krijgen op pensioendatum de keuze tussen een vaste of variabele annuïteit.

Opzet model analyse

Deze scriptie is gericht op het beleggingsbeleid en uitkeringsbeleid voor deelnemers in premieregelingen die kiezen voor een variabele annuïteit op pensioendatum. Hoewel de Wet verbeterde premieregeling betrekking heeft op zowel premie- als kapitaalovereenkomsten, beperk ik me in deze scriptie tot de zuivere premieregeling waarin de beschikbaar gestelde premie wordt belegd tot aan pensioendatum.[2]

Het eerste deel van de model analyse richt zicht op het beleggingsbeleid waarin verschillende life-cycle strategieën met elkaar worden vergeleken. Wat voor life-cycles zijn geschikt voor een deelnemer met bepaalde risicopreferenties en welke life-cycles zijn niet geschikt? Welke aspecten zijn belangrijk voor het toezicht op het gebruik van adequate life-cycles door De Nederlandsche Bank (DNB)? Diverse maten voor de afweging tussen de hoogte en onzekerheid in de pensioenuitkering worden gebruikt om de life-cycle strategieën met elkaar te kunnen vergelijken. Ook worden de pensioenuitkeringen van een variabele annuïteit vergeleken met de pensioenuitkering van een vaste annuïteit.

Naast het beleggingsbeleid is het uitkeringsbeleid tijdens de pensioenperiode relevant voor een variabele annuïteit. Dit wordt onderzocht in het tweede gedeelte van de model analyse. Op pensioendatum dient bepaald te worden hoe het pensioenkapitaal verdeeld wordt over de pensioenperiode. Deze verdeling wordt bepaald door de risicovrije rente in combinatie met een optionele maximale vaste daling (die in verwachting leidt tot een nominaal stabiele uitkering).

Tenslotte wordt het effect van het uitsmeren van financiële schokken onderzocht. Wat voor invloed heeft het uitsmeren van financiële schokken op de volatiliteit in de pensioenuitkering en wat is het effect van uitsmeren op het beleggingsbeleid en de keuze voor een bepaalde vaste daling?

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[1] Wet verbeterde premieregeling (Stb. 2016, 248).

[2] Andere varianten zijn de kapitaalovereenkomst waarbij de premie onmiddellijk wordt omgezet in een aanspraak op kapitaal en de premieovereenkomst waarbij de premie meteen wordt omgezet in een aanspraak op een uitkering. De Wet verbeterde premieregeling heeft geen betrekking op deze laatste variant.

Beleidsrelevante conclusies
In de volgende alinea's worden de belangrijkste bevindingen van deze scriptie gepresenteerd. In deze scriptie worden life-cycles onder andere met elkaar vergeleken door middel van welvaartsanalyses. Het welvaartsverlies van een life-cycle ten opzichte van de optimale life-cycle is in deze scriptie gedefinieerd op basis van het zekerheidsequivalent van de consumptie gedurende de pensioenperiode. Alle welvaartsanalyses zijn gebaseerd op de zeer gangbare veronderstelling over de afweging tussen risico en rendement, namelijk dat de deelnemer een constante risicoaversie heeft.

Beleggingsbeleid
De eerste belangrijke bevinding is dat een life-cycle met een dalend verloop tot betere resultaten leidt ten opzichte van een constante life-cycle. Deze conclusie komt overeen met de standaard life-cycle theorie die stelt dat een goede life-cycle in de opbouwfase gekarakteriseerd wordt door een dalend verloop van de hoeveelheid beleggingsrisico onder andere op basis van de aanname van beperkt risicovol menselijk kapitaal. Het welvaartsverlies van een constante life-cycle ten opzichte van de optimale Merton life-cycle varieert tussen de 4% en 6%, afhankelijk van de risicopreferenties van de deelnemer. Het welvaartsverlies van een constante life-cycle is groter voor een deelnemer die bereid is meer risico te nemen.

Daarnaast kan een premieregeling met beleggingsvrijheid significant toegevoegde waarde hebben voor een deelnemer ten opzichte van een premieregeling zonder beleggingsvrijheid. De optimale hoeveelheid beleggingsrisico is namelijk sterk afhankelijk van de risicopreferenties van de deelnemer. Welvaartsverliezen door het nemen van een inadequate hoeveelheid beleggingsrisico (oftewel toepassing van een inadequate risicoaversie parameter) zijn significant groter dan welvaartsverliezen door toepassing van een suboptimaal dalende life-cycle (bijvoorbeeld een lineair dalende life-cycle in plaats van de optimale Merton life-cycle die wordt gekarakteriseerd door een exponentieel dalend verloop).

Een onjuiste voorlopige keuze voor een vaste of variabele uitkering voor de pensioendatum kan leiden tot een significant welvaartsverlies. Indien een minder risicoaverse deelnemer voorlopig kiest voor een vaste uitkering voor de pensioendatum maar uiteindelijk een variabele uitkering prefereert op pensioendatum, dan bedraagt het welvaartsverlies 4%.

Uitkeringsbeleid
Een interessante conclusie met betrekking tot het uitkeringsbeleid is dat het ongeveer 10 jaar[3] vanaf pensioendatum duurt voordat het gebruik van een vaste daling leidt tot een lagere pensioenuitkering in vergelijking met de pensioenuitkering zonder toepassing van een vaste daling. Pensioenuitvoerders zijn verplicht informatie te verstrekken over de hoogte en risico van de pensioenuitkering op pensioendatum en 10 jaar na de pensioendatum maar niet noodzakelijk over de pensioenuitkering op andere tijdstippen. Dit betekent dat het nadeel van een vaste daling ten opzichte van geen vaste daling (namelijk een lagere pensioenuitkering op hogere leeftijden) mogelijk niet wordt gecommuniceerd naar de deelnemer.

Indien een deelnemer kiest om financiële schokken uit te smeren over meerdere jaren is het wenselijk om een dalende life-cycle gedurende de pensioenperiode te hanteren. Wanneer financiële schokken worden uitgesmeerd over meerdere jaren, leidt een constante life-cycle namelijk tot meer beleggingsrisico op hoge leeftijden vanwege de dalende resterende lev-

[3] De duratie van de pensioenverplichtingen bedraagt ongeveer 10 jaar.

ensduur (beleggingsrisico's worden verschoven in de tijd). Dit betekent dat zowel tijdens de opbouwfase (op basis van de aanname van risicovrij menselijk kapitaal) als tijdens de afbouwfase (op basis van een constante conditionele volatiliteit in de pensioenuitkering) de life-cycle wordt gekenmerkt door een dalend verloop indien de deelnemer kiest om financiële schokken uit te smeren.

Het uitsmeren van financiële schokken rondom de risicovrije rente gecombineerd met een maximale vaste daling leidt tot een significante daling (ongeveer 3% gedurende de eerste 7 jaar voor een uitsmeerperiode van 10 jaar) in de verwachte pensioenuitkering gedurende de eerste jaren van de pensioenperiode. De kans op een daling in de pensioenuitkering gedurende het eerste jaar na pensionering is erg hoog, namelijk 97% voor een uitsmeerperiode van 10 jaar. Aangezien pensioenuitvoerders verplicht zijn informatie te verstrekken over de hoogte en het risico van de pensioenuitkering op pensioendatum en 10 jaar na de pensioendatum maar niet noodzakelijk over de tussenliggende periode, wordt de hoge kans op een daling gedurende de eerste jaren na pensionering mogelijk niet gecommuniceerd naar de deelnemer. Zowel pensioenuitvoerders als deelnemers dienen zich van deze verwachte daling bewust te zijn. De verwachte daling kan onder andere worden opgelost door gebruik te maken van een gepaste horizon afhankelijke vaste daling.

Men dient zich ervan bewust te zijn dat bovenstaande conclusies gebaseerd zijn op een model waarin rente risico niet wordt meegenomen. Hoewel de kwantitatieve resultaten afwijken in een model met rente risico, verwacht ik dat kwalitatieve conclusies met betrekking tot het beleggingsrisico grotendeels ook gelden in een model met rente risico.

Contents

1. Introduction

Over the last few years defined contribution (DC) pension schemes have become increasingly popular in the Netherlands. An important characteristic of a DC pension scheme is that financial risks (i.e. investment risk and interest rate risk) are born by the participant. There exist different types of DC pension schemes. In an individual defined contribution (IDC) pension scheme the participant accrues pension capital by saving and investing on an individual account. The investment returns and premium payments[1] are added to the pension capital on an individual account. In case of a collective defined contribution (CDC) pension scheme the financial assets are collectively owned by all participants (i.e. participants do not have an individual account). This thesis focuses on IDC pension schemes.

Due to poor investment returns during the financial crisis, the prolonged period of low interest rates and a higher life expectancy it is currently very difficult for participants in DC pension schemes to attain an adequate pension. Poor investment returns lead to a lower amount of pension capital at retirement and the last two factors lead to a high price of a fixed lifelong annuity which participants in the Netherlands are currently obliged to buy at retirement. In order to prevent large changes in the expected pension benefit level just before retirement, participants should take less investment risk by applying a decreasing equity exposure over time towards retirement. The development of the equity exposure over the life of the participant is captured in a so called life-cycle. A decreasing life-cycle protects participants against bad stock performances. Moreover, (a part of) the interest rate risk is usually hedged by matching the duration of the bond portfolio with the duration of the pension liabilities. Alternatives for reducing the interest rate risk, for example buying an annuity in different phases before retirement, are also applied in practice.

New legislation[2] allows participants in DC schemes to continue to invest in risky assets after retirement. The new legislation makes it possible to choose either a fixed or a variable annuity at retirement. In case of a fixed annuity, the participant is obliged to convert his complete pension capital into a fixed lifelong annuity at retirement. In case of a variable annuity, the participant continues to invest in risky assets after the retirement age.

Investment after retirement

The main advantage of investing after retirement is that it leads in expectation to a higher pension. This is due to higher expected investment returns during both the accumulation (or pre-retirement) phase as the decumulation (or retirement) phase. The decrease in risky asset investments during the accumulation phase, which starts well before the retirement date, can be delayed in case the participant chooses to invest after retirement since the equity exposure does not have to be zero at retirement anymore.[3] A higher equity exposure leads to higher expected investment returns. Moreover, higher investment returns can be realized during the decumulation phase since the new legislation permits investment in risky assets during the decumulation phase which is currently not yet possible,

[1] After deducting costs, for example administration and asset management costs.

[2] 'Wet verbeterde premieregeling' ('Stb. 2016', 248).

[3] Although the equity exposure is often zero at retirement in practice, it is not obliged to have an equity exposure equal to zero at retirement.

since a participant is obliged to buy a fixed annuity at retirement. This implies that no investment returns after retirement can be realized.

There have been several studies which conclude that investment after retirement is beneficial for participants because of the higher expected pension. Examples are the research reports of LCP (Koopmans and van Ling (2014)) and Ortec Finance (Ortec Finance (2014)). These studies were conducted in preparation of the new legislation. Also academic literature about life-cycle investing proposes investment after retirement. For example, according to the life-cycle theory of Merton (1971), which is further investigated by Bovenberg et al. (2007) and which will be discussed in section 2.1, investment after retirement is optimal as long as the risk aversion of the participant is not infinitely large.

However, investment after retirement also has a downside. The main drawback is that the pension benefit level is not guaranteed anymore but becomes variable due to financial shocks which are the result of investment in risky assets. This implies that the pension benefit level can also decrease. As a consequence, a trade-off between upward potential and downside risk arises. However, when taking both the pension benefit level and the variability in this benefit level into account, investment after retirement is still welfare enhancing compared to no investment after retirement under the assumption of a constant risk aversion as long as the risk aversion of the participant is not infinitely large. In order to reduce the variability in the pension benefit level in the short term, one can smooth financial shocks over several years by applying a certain smoothing period. In case of financial smoothing, shocks in financial wealth are not absorbed immediately in the pension benefit level but are absorbed gradually over the smoothing period. The research report of Ortec Finance (Ortec Finance (2014)) shows that the impact of investment risk on the pension benefit level can be limited by smoothing financial shocks over several years.

In case a participant buys a fixed nominal annuity, he knows at retirement which pension benefits he will receive for sure each year until he dies. If a participant buys a variable annuity, future pension benefits are not known at retirement since these depend on the actual investment returns during retirement. In case of a variable annuity one has to determine at retirement how the initial pension wealth should be divided over the pension payments in the individual years during retirement. The assumed interest rate (AIR) determines how the initial pension wealth is divided. In the new legislation the distribution of the pension wealth is determined by the actual risk-free interest rate[4] and in addition a possible 'fixed' yearly decrease. The AIR thus can be seen as a combination of the risk-free rate and a 'fixed' yearly decrease. Choosing a 'fixed' yearly decrease is equivalent to choosing a certain AIR. [5]

[4] Called 'projectierente' in article 63a, paragraph 7 and 8, 'Pensioenwet', as added by the 'Wet verbeterde premieregeling'.

[5] The equivalence will be shown in section 6.1. The equivalence only holds in case of no financial smoothing, but does not necessarily hold in case of financial smoothing.

Research questions
In this thesis several important aspects of investment after retirement in DC pension schemes will be investigated. This thesis focuses on both the investment policy as the pension benefit payment policy. The AIR and financial smoothing are important factors for the pension benefit payment policy. This can be formulated in three main research questions which are as follows:

1. Which life-cycles are suitable for a participant with certain risk preferences and which life-cycles are inappropriate? Which aspects are important for the supervision of the the Dutch central bank (DNB) on life-cycles implemented by pension providers?

2. What is the influence of the assumed interest rate or 'fixed' yearly decrease on the development of the pension benefit level during retirement and which assumed interest rates are suitable for a participant with certain risk preferences?

3. What is the influence of financial smoothing on the optimal life-cycle design and assumed interest rate?

The first research question focuses on the optimal design of the life-cycle both before and during the retirement period. The performance of several life-cycle strategies will be compared for a participant choosing for a variable annuity at retirement using the risk-free interest rate as discount factor as proposed in the new legislation. Both life-cycles from theory as life-cycles used in practice will be considered. Moreover, I will investigate how sensitive the life-cycles are to the risk preferences of the participant and at which age the life-cycle of a variable annuity starts to deviate from the life-cycle of a fixed annuity. Several indicators of the pension result will be used to compare the performance of the strategies.

The second and third research question focus on the pension benefit payment policy. The AIR and the application of financial smoothing are important factors for the benefit payment policy. The second research question is related to the level of the AIR. The AIR determines the distribution of the available pension wealth over the various pension payments and therefore also the pension benefit level at retirement. A low AIR implies a relatively low benefit level at retirement but a considerable increase in the expected benefit level over time. A high AIR implies a high benefit level at retirement but also a higher probability of a decrease in the pension benefit level. I will also verify whether the maximization of the AIR in the new legislation leads to possible welfare losses for participants.

The third research question focuses on financial smoothing. In case financial shocks are smoothed over several years, the optimal investment policy and optimal benefit payment policy will be different. The alternative life-cycles and AIR's will be derived for participants with different risk preferences and the results will be compared with the life-cycles and AIR's in case of no financial smoothing.

Main findings

Investment policy

According to the standard life-cycle theory of Merton (1971) a suitable life-cycle strategy is characterized by a decreasing equity exposure over time until retirement and a constant equity exposure after retirement (based on the assumption of risk-free human capital and CRRA preferences). Based on the results of the model analysis one can indeed conclude that a decreasing life-cycle leads to better results compared to a constant life-cycle.

An important conclusion from the model analysis regarding the first research question is the difference in size of welfare losses[6]. Welfare losses due to an inadequate amount of investment risk (e.g. in case of inadequate quantification of the risk aversion of the participant) are higher than welfare losses of a suboptimal decreasing life-cycle (e.g. a linearly decreasing life-cycle instead of the optimal Merton life-cycle which exhibits an exponential decrease). This means that it is important to determine the risk profile of the participant.

Finally, temporarily choosing a fixed annuity instead of a variable annuity before retirement also leads to a welfare loss in case the participant prefers a variable annuity at retirement. However, a welfare loss due to no investment after retirement is significantly higher than a welfare loss due to temporarily using an inadequate life-cycle before retirement. This holds for each level of risk aversion.

Pension benefit payment policy

An interesting conclusion related to the second research question is that it takes approximately 10 years[7] after retirement until a low AIR such as the risk-free rate yields a higher replacement rate than a higher AIR. Moreover, the increase in the replacement rate at retirement as a result of a higher AIR is highly dependent on the risk preferences of the participant.

The final research question is related to financial smoothing. In case the participant does not feature CRRA preferences but features internal habit formation, the participant maximizes utility of consumption relative to a reference level which depends on previous consumption. In that case smoothing of financial shocks can be beneficial since financial smoothing reduces the volatility of the change in the pension benefit level. Note that smoothing of financial shocks in combination with a constant equity exposure leads to more investment risk at high ages because of the lower expected remaining lifetime of the participant. This higher investment risk is the result of investment risks being delayed in case of financial smoothing. Therefore, a decreasing equity exposure is preferable in case of financial smoothing. One can conclude that the life-cycle during both the accumulation phase (based on the assumption of risk-free human capital) as the decumulation phase (based on a constant year-on-year volatility of income) is characterized by a decreasing life-cycle in case of financial smoothing.

One has to be aware of the fact that financial smoothing around the risk-free rate in combination with the expected return as AIR implies a significant decrease in the ex-

[6] Welfare losses are calculated in terms of the certainty equivalent consumption during the retirement period based on the common assumption that the participant features CRRA preferences.

[7] The duration of the pension liabilities equals approximately 10 years.

pected pension benefit level during the first years of the retirement period. Moreover, it leads to a very high probability of a decrease in the pension benefit level during the first year after retirement. This can be solved by making use of a suitable horizon-dependent AIR.[8] This AIR is lower than the expected return during the first years after retirement.

Model analysis

I will use the Merton model as benchmark model in the model analysis. This model is based on a simple financial market model with no interest rate risk or inflation risk. In this model an analytic solution for the optimal life-cycle strategy is available and can easily be derived. Subsequently, some strong assumptions in the Merton model will be adapted or relaxed.

The complete life-cycle is divided into three periods: the accumulation period (from age 25 until age 67), the conversion period (from age 57 until the age at death) and the decumulation period (from age 67 until the age at death). The first two periods will be focused on the optimal investment policy while the pension benefit payment policy will be investigated during the retirement period (last period).

This thesis is built up as follows. First of all, a review of the literature on life-cycle investing will be given in chapter 2. In chapter 3, the new legislation will be explained in more detail. The set-up of the model analysis will be explained in chapter 4. In chapter 5 the design of the life-cycle will be investigated and results of the model analysis will be presented while in chapter 6 the benefit payment policy will be considered. The effect of financial smoothing will be investigated in chapter 7. Chapter 8 concludes and recommendations for future research will be given. The appendix includes some mathematical derivations, assumptions of the model analysis and an overview of the acronyms and parameters used in this thesis.

[8] Referred to as BNW AIR in this thesis.

2. Literature life-cycle investing

This chapter presents an overview of the most important literature concerning life-cycle theory. Life-cycle theory is about the optimal saving and investment strategy over an individual's life-cycle based on the risk preferences of the individual. An individual makes two main decisions, namely the saving decision and investment decision. The saving decision is about how to smooth consumption over time by determining the pension contributions and pension benefits, while the investment decision is about how to invest the pension contributions in financial assets. This thesis focuses on the investment decision (see subsection 2.1.3). Another trade-off regarding life-cycle theory is the amount of labor supply. However, it is assumed in this thesis that labor supply is constant (see subsection 2.1.1) and is not included in the decision making of the individual since it is not the focus of this thesis.

The life-cycle strategy is important for a participant in a DC pension scheme since it determines how the accrued pension capital is invested over the life-cycle of the participant. Of course, such a strategy is in line with the participant's characteristics and preferences, such as age and risk aversion. In case the participant chooses for a variable annuity, the life-cycle strategy will be different from the case where the participant chooses for a fixed annuity. The difference between both strategies will be investigated.

First of all, the Merton model will be discussed in section 2.1 and the optimal life-cycle strategy in the Merton model will be derived. It turns out that the optimal equity exposure of total wealth is constant. The Merton life-cycle strategy is the most well-known life-cycle strategy. Subsequently, some of the assumptions in the Merton model are adapted or relaxed in the different subsections of section 2.2.

2.1 Merton model

The basic principles of life-cycle theory have been founded by Merton (1971) and Merton and Samuelson (1974). Amongst others, Teulings and De Vries (2006) and Bovenberg et al. (2007) further explored this theory. Merton (1971) and Merton and Samuelson (1974) derived an optimal saving and investment decision over the life-cycle of an individual under certain assumptions about the financial market, labor market and preferences of the individual.

The financial market in the Merton model is equivalent to the Black-Scholes market in which two investment opportunities exist, namely a risky stock S_t and a risk-free bond B_t. This market can be described via the following stochastic differential equations

$$\frac{dB_t}{B_t} = rdt \tag{2.1a}$$

$$\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t. \tag{2.1b}$$

The stock price S_t follows a geometric Brownian motion and the bond price B_t behaves as an exponential function. Moreover, Z_t is a Brownian motion, r is the constant risk-free interest rate, σ is the standard deviation of stock return and λ is the price of risk of stock returns.

Preferences of the individual feature a positive constant relative risk aversion (CRRA) γ representing both the taste for moderation across time and across economic scenarios. The taste for moderation across economic scenarios is measured by risk aversion parameter γ. If the individual is highly risk averse, he prefers a stable consumption level across the economic scenarios. The taste for moderation across time is measured by the intertemporal elasticity of substitution $1/\gamma$. An individual with a low intertemporal elasticity of substitution prefers a stable consumption level across time.

The time separable utility function of an individual featuring CRRA preferences equals

$$U(C_t) = \begin{cases} \dfrac{C_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \ln(C_t) & \text{if } \gamma = 1. \end{cases} \tag{2.2}$$

The individual maximizes utility over the life-cycle with weights of future expected utilities declining exponentially at the rate of time preference ρ. The time preference represents the impatience of the individual: a marginal unit of consumption tomorrow implies less additional utility than a marginal unit of consumption today for the same consumption level. The maximization function at a certain point in time t is given by

$$\max \mathbb{E}_t \left(\int_t^D \frac{\exp(-\rho(s-t))}{1-\gamma} C_s^{1-\gamma} ds \right). \tag{2.3}$$

An overview of the most important assumptions in the Merton model is presented in the following subsection.

2.1.1 Merton assumptions

An overview of the most important assumptions in the Merton model is presented below.

- Black-Scholes financial market.
- Interest rate and inflation are deterministic.
- Volatility of equity and equity risk premium are constant.
- Continuously-trading perfect market without transactions costs or constraints.
- Human capital is risk-free and individuals earn a constant labor income $Y_t = 1$.
- Labor supply is constant.
- Death is predictable or perfect insurance of micro longevity risk is available and macro longevity risk is absent.
- No bequest motives.
- Preferences of the individual feature a positive constant relative risk aversion (CRRA) γ and the individual maximizes utility over the life-cycle.

Note that the optimal solution in the Merton model only holds in case these assumptions are satisfied.

2.1.2 Default parameters

Some default parameter values are used in the numerical simulations of the Merton model in chapter 2 and in the model analysis in chapter 4. The default coefficient of risk aversion equals $\gamma = 7$. This coefficient represents an individual who is quite risk averse which is reasonable to assume for the accrual of pension benefits (see section 4.3 for further explanation). The rate of time preference equals $\rho = 2\%$[1], risk-free interest rate $r = 1\%$[2], mean stock return $\mu = 7\%$[3] and standard deviation of stock return $\sigma = 20\%$[3]. It is assumed that there is no inflation. Furthermore, the individual starts working at age 25 and retires at age $R = 67$ (i.e. a working life of $T = 42$ years). In subsection 2.1.3 it is assumed that there is no longevity risk: the individual dies at age 85 which implies a retirement period of $P = 18$ years. The total period alive after the starting age has length $D = 65$ years. In the model analysis micro longevity risk is included which implies that the age of death is unknown a priori. This will be further explained in section 4.1.

2.1.3 First-best solution

As already mentioned above, one can derive the optimal saving and investment decision over the life-cycle of an individual in the Merton model.

Investment decision

The investment decision consists of the determination of the optimal amount of total wealth invested in the risky stock. The remaining wealth will be invested in the risk-free bond. Total wealth W_t of an individual consists of financial wealth F_t and human wealth H_t. Financial wealth consists of pension savings and investment returns, which generally increases over time. Since it is assumed that human capital is risk-free (see assumptions subsection 2.1.1), the value of human wealth H_t can be determined by discounting the future wage cash flows at the risk-free interest rate r. This yields the following formula for human wealth[4] at time t

$$H_t = \begin{cases} \frac{Y_t}{r}(1 - \exp(-r(T-t))) & \text{if } t \in [0, T] \\ 0 & \text{if } t \in (T, D]. \end{cases} \tag{2.4}$$

One can see that H_t decreases in t. This makes sense since an older individual has less working years prospectively. The development of total wealth, financial wealth and human wealth over the life-cycle is depicted in figure 2.1.

The optimal exposure of total wealth f to the risky stock depends on the risk aversion of the individual (i.e. preference for consumption smoothing) and the risk premium. It can be derived via two different methods. Merton (1971) and Merton and Samuelson (1974) derived the optimal saving and investment decision using a dynamic programming method for continuous-time portfolio. In this method the optimal investment portfolio is expressed in terms of the solution of the so-called the Hamilton-Jacobi-Bellman (HJB) equation. An alternative solution method is the martingale (Lagrangian) approach which

[1] Average of estimates found by Backus et al. (1995), Cooley and Prescott (1995) and Rouwenhorst (1995).

[2] The nominal interest rate on a zero coupon bond with a maturity of 10 years (i.e. the duration of the pension liabilities) at 31-12-2015 was equal to 1.02%. (`www.dnb.nl/statistiek/statistieken-dnb/financiele-markten/rentes`)

[3] In accordance with the advice of the Committee Parameters: ‘Commissie Parameters 2014’. (`www.rijksoverheid.nl`)

[4] See page 90 in Bovenberg et al. (2007).

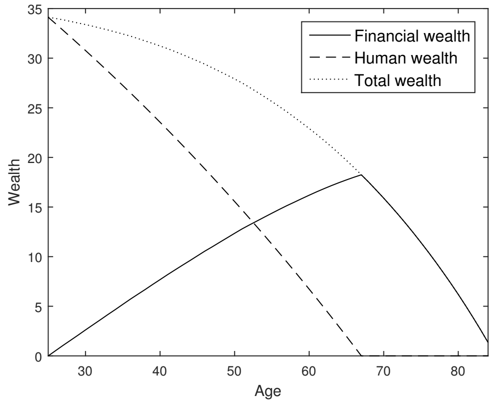

Figure 2.1: Development of expected financial wealth, human wealth and total wealth over the life-cycle.

has been developed by Pliska (1986), Karatzas et al. (1987) and Cox and Huang (1989). The technical derivation of the optimal equity exposure using the martingale approach is stated in appendix A.1. It turns out that the optimal equity exposure of total wealth is constant and does not depend on time and the investment horizon of the individual in the Merton model. The optimal equity exposure of total wealth equals

$$f = \frac{\lambda}{\gamma\sigma} = \frac{\mu - r}{\gamma\sigma^2}. \tag{2.5}$$

Note that a constant optimal equity exposure is related to one of the most famous questions of Fischer Black: *'All stocks half the time or half stocks all the time?'*.[5] From this perspective, the frequently used argument that young individuals should take more investment risk compared to old individuals is not valid. However, when taking the human capital effect (see Bodie et al. (1992)) into account, a constant equity exposure is not optimal anymore. In case human capital H_t is not tradable and risk-free (see assumptions in subsection 2.1.1), human capital acts like a risk-free asset. Therefore, one should adjust the equity exposure of financial wealth f_t^* to the risky stock over time to attain the appropriate constant equity exposure of total wealth. Given the optimal equity exposure of total wealth f to the risky stock as in (2.5), the optimal equity exposure of financial wealth f_t^* to the risky stock is as follows

$$f_t^* = f\frac{W_t}{F_t} = f\left(1 + \frac{H_t}{F_t}\right). \tag{2.6}$$

The optimal exposure of financial wealth to the risky stock decreases over time because of two reasons: human wealth decreases over time since the number of future working years decreases. Young individuals can take more investment risk with their financial wealth since they have an alternative income source, namely labor income. Secondly, financial wealth increases over time since the individual saves part of his labor income.

[5] Kritzman (2000) argues that in terms of utility, a balanced strategy (i.e. half stocks all the time) yields a higher expected utility compared to a switching strategy (i.e. all stocks half the time) which confirms the of the constant equity exposure.

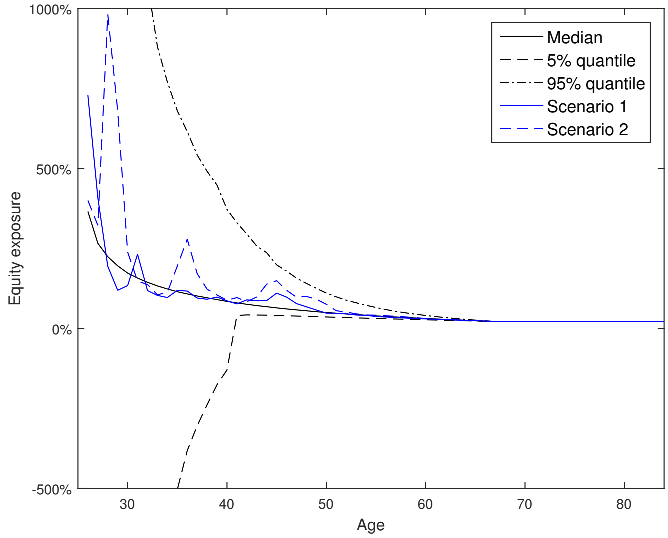

Figure 2.2: Optimal Merton equity exposure of financial wealth f_t^* over the life-cycle.
Besides different quantiles of the optimal equity exposure, the graph also contains the optimal equity exposure in two specific scenarios. For example, the optimal equity exposure at age 36 in scenario 2 equals approximately 250%.

The optimal equity exposure of financial wealth f_t^* over the life-cycle is depicted in figure 2.2. The equity exposure decreases from infinity at the start of the working career to the optimal equity exposure of total wealth f at retirement since the financial wealth as a fraction of total wealth increases from 0 to 1. The median, 5% quantile and 95% quantile of the optimal equity exposure of financial wealth are visualized based on 10,000 simulations of the Merton model with default parameters as listed in subsection 2.1.2. The results are comparable with Bovenberg et al. (2007) (although not identical because of different parameter values). Note that the financial wealth can become negative during the first working years leading to a negative optimal equity exposure (see for example 5% quantile in figure 2.2). Moreover, the optimal equity exposure of financial wealth f_t^* depends on the actual financial wealth F_t and might deviate from the a priori expected equity exposure. Figure 2.2 also contains the development of the optimal equity exposure of financial wealth in two specific scenarios. One can conclude that the optimal exposure is very volatile. This is due to the fact that financial wealth is small during the first working years. In each scenario the optimal exposure of financial wealth f_t^* converges towards the optimal exposure of total wealth f at the end of the working life due to the decline in human wealth H_t. This optimal exposure of total wealth equals $f = 21.4\%$ for an individual with CRRA preferences with risk aversion parameter $\gamma = 7$ and default parameters as described in subsection 2.1.2 (calculated by plugging in the parameter values into (2.5)). Obviously, this exposure will be different for other parameter values.

Note that the human capital effect is reinforced in case the labor income can be partially controlled by the individual. In the Merton model it is assumed that labor supply is constant. However, in case the individual is able to work more in order to compensate for negative financial shocks, the optimal equity exposure is even higher for young individuals.

Saving decision
Besides the optimal equity exposure, Merton (1971) and Merton and Samuelson (1974) also derived the optimal saving (or consumption) decision. A technical derivation is for example given by Bovenberg et al. (2007). Since participants in DC schemes save a fixed percentage (which depends on the age of the participant) of their pension base each period, the saving (or consumption) decision before retirement is exogenous in the model analysis of this thesis (see section 4.1). Moreover, the consumption level during retirement will be determined using the PPR mechanism in case of a variable annuity (see section 4.2). Therefore, I will focus on the investment decision only and not give the derivation of the optimal consumption strategy in the Merton model here.

2.2 Restrictions Merton model

The Merton model is based on certain assumptions (see subsection 2.1.1). To make the model more realistic, it might be better to adapt or relax some of these assumptions. This section gives an overview of the most important restrictions of the Merton model. Some restrictions are discussed in a qualitative way only, while other restrictions are treated quantitatively.

2.2.1 Interest rate risk and inflation risk

Two important risk factors which participants face are interest rate risk and inflation risk. These risk factors are not included in the Merton model, since the interest rate is assumed to be constant and it is assumed that there is no inflation (or inflation is deterministic). In case the interest rate is not constant, the value of bonds and human wealth are effected by interest rate changes. Moreover, the interest rate influences the price of an annuity (i.e. a lower interest rate implies a higher annuity price). The individual can hedge interest rate risk by investing in long-term nominal bonds with a maturity equal to the duration of his pension liabilities. In case of inflation risk nominal bonds are not risk-free anymore. A positive shock to inflation has a negative effect on the real return on a nominal bond. Since a positive shock to inflation is likely to be followed by high inflation in subsequent periods, long-term nominal bonds are riskier than short-term nominal bonds at all horizons. Campbell and Viceira (2005) even concluded that long-term nominal bonds are riskier than stocks for bonds with maturities over 35 years. As a consequence, participants should shorten the maturity of the bond portfolio to hedge large parts of the inflation risk in case inflation indexed bonds are not available.

A model which does take interest rate risk and inflation risk into account is the Brennan Xia model (Brennan and Xia (2002)) in which both the interest rate and inflation follow an Ornstein-Uhlenbeck process which is a mean reverting process. Brennan and Xia (2002) analyzed the asset allocation problem of a finite-horizon investor who can invest in a risky stock and nominal bonds with different maturities. Although inflation indexed bonds are available in some countries (e.g. United Kingdom and Canada), these are only available for a few (long) maturities. Therefore, it is assumed that inflation indexed bonds are not available. Brennan and Xia (2002) determined the optimal portfolio in case only long positions are allowed which can be achieved by investing in a risky stock, a single bond with an optimally chosen maturity and in cash. The maturity of the bond depends on the relative importance of interest rate risk and inflation risk. In case interest rate risk is high, it is optimal to invest in long-term bonds. However, in case inflation risk dominates, it is better to invest in short-term bonds. Moreover, the optimal bond maturity depends on the individual's horizon and the risk aversion of the individual.

Although the introduction of bonds with different maturities leads to interesting solutions about which bonds the individual should choose in case of interest rate risk, this choice has little influence on the equity exposure to the risky stock. The equity exposure is still very much determined by the equity premium.

In the Brennan Xia model risk premia on equity and nominal bonds are constant. As a consequence, the optimal portfolio allocation is time-independent (just as in the Merton model), which implies that the actual interest rate does not influence the optimal portfolio allocation. On the other hand, the optimal portfolio allocation is horizon-dependent in the Brennan Xia model, while the optimal equity exposure is constant in the Merton model. Another difference with the Merton model is the set of investment opportunities. In the Merton model, the investment opportunities are constant since r, μ and σ are deterministic, while these are stochastic in the Brennan Xia model. A consequence of the constant investment opportunities in the Merton model is that one can separate the optimal portfolio allocation during the pre-retirement period from the optimal allocation of pension wealth during the retirement period. Since the investment opportunities are not constant in the Brennan Xia model but vary over time, this separation does no longer hold anymore.

The assumption of a constant risk premium in the Brennan Xia model differs from the assumption that the return on assets is constant. The Committee Parameters[6] has been assigned to quantify the constant expected gross return on assets (7%). If one believes that the risk premium is constant, a decrease in the interest rate implies a similar decrease in the expected return on risky assets. In that case, the benefits of investing in risky assets relative to investing in risk-free bonds do not increase. However, if one beliefs that the expected return on assets is constant, a decrease in the interest rate implies a similar increase in the risk premium which makes assets relatively more attractive. Since the expected return on assets cannot be objectively derived from prices observed on financial markets, it depends on the financial market model used by the investor for his decision making (based on his investment beliefs) whether this benefit of investing in risky assets relative to investing in risk-free bonds is present.

2.2.2 Discrete trading

An important assumption in the Merton model is the assumption that one can trade continuously (see subsection 2.1.1). Obviously, this does not hold in practice. It is more realistic to assume that the life-cycle mix of a participant in a DC scheme is adapted at a monthly or yearly frequency. In this thesis I assume that the life-cycle mix is adapted at a yearly frequency. This leads to the following maximization function of the participant at time t

$$\max \mathbb{E}_t \left(\sum_{s=t}^{D} \exp(-\rho(s-t)) \frac{C_s^{1-\gamma}}{1-\gamma} \right).$$

In the model analysis in chapter 4 different life-cycle strategies are investigated by calculating several indicators of the pension result. One of these indicators is the welfare loss which is calculated in terms of the certainty equivalent consumption. The certainty equivalent consumption can be formulated in several ways. In this thesis the certainty equivalent

[6] 'Commissie Parameters 2014'. (www.rijksoverheid.nl)

consumption is defined as the constant certain consumption level which yields the same ex-ante utility at retirement as the stochastic consumption stream after retirement. The utility before retirement is not calculated since the optimal saving (or consumption) decision is not relevant (the participant saves a fixed percentage). Moreover, the utility is calculated at discrete times only because of the assumption of discrete trading. Finally, since a mortality table is included in the model analysis in chapter 4, the utility each year is multiplied with the survival probability $p_T(t)$, i.e. the probability that the participant is still alive at time t as of time T. The expected utility at retirement (i.e. time T) of the stochastic consumption stream after retirement equals

$$\mathbb{E}_T\left(\sum_{s=T}^{D-1} p_T(s)\exp(-\rho(s-T))\frac{C_s^{1-\gamma}}{1-\gamma}\right) := ELU. \tag{2.7}$$

Setting the expected utility equal to the utility of the certainty equivalent consumption yields the certainty equivalent consumption ce

$$ELU = \sum_{s=T}^{D-1} p_T(s)\exp(-\rho(s-T))\frac{ce^{1-\gamma}}{1-\gamma}$$

$$ce = \left(\frac{ELU}{\sum_{s=T}^{D-1} p_T(s)\exp(-\rho(s-T))/(1-\gamma)}\right)^{1/(1-\gamma)}.$$

The welfare loss of a life-cycle strategy in terms of the certainty equivalent consumption is defined as the relative difference in certainty equivalent consumption compared to the certainty equivalent consumption of the optimal life-cycle strategy.

The restriction of trading at an annual frequency leads to a welfare loss. In case the welfare loss is calculated in terms of the certainty equivalent consumption, discrete trading at an annual frequency leads to a welfare loss of 0.8%[7] of certainty equivalent consumption relative to continuous trading (see Bovenberg et al. (2007)).

Note that while there are no transaction costs in the Merton model, transaction costs have to be paid in case of trading at financial markets in practice. Therefore, one has to make a trade-off between the benefits of adjusting the equity exposure at a higher frequency and the extra costs of the corresponding transactions. This implies that the welfare loss of discrete trading relative to continuous trading will be lower in practice.

2.2.3 Exogenous borrowing constraint

Since young individuals generally have almost no financial wealth but a lot of human capital, the optimal equity exposure of financial wealth can be far above 100% (see figure 2.2). This implies that the individual must go short to acquire the optimal equity exposure. According to Bodie et al. (1992) and Teulings and De Vries (2006), it is optimal for young individuals to borrow up to six times their annual salary (based on the expected equity exposure). However, in reality it is usually not possible to borrow against future labor income due to moral hazard and adverse selection. In the absence of slavery, financial institutions cannot use human capital as collateral to ensure that the individual will

[7] Bovenberg et al. (2007) calculated the certainty equivalent consumption over the complete life-cycle instead of the retirement period only (which is done in this thesis) implying lower welfare losses.

pay back the loan (see Broeders et al. (2009)). Therefore, it makes sense to include an exogenous borrowing constraint which ensures that the exposure to the risky stock does not exceed 100%. Including an exogenous borrowing constraint leads to a welfare loss of approximately 3.0% of certainty equivalent consumption according to Bovenberg et al. (2007).

The median equity exposure over the life-cycle including an exogenous borrowing constraint is depicted in figure 2.3. Up to age 47 the median equity exposure equals 100%. This equity exposure is calculated by minimizing the equity exposure at 0% and maximizing the equity exposure 100%. Note that including an exogenous borrowing constraint is not the same as truncating the equity exposure in the optimal Merton strategy at 100% (see figure 2.3) . A consequence of the borrowing constraint is that the expected financial wealth at a certain age is lower compared to the case without this borrowing constraint. This implies that the size of human wealth relative to financial wealth at a later age is higher in case of a borrowing constraint. As a consequence, the equity exposure of financial wealth in (2.6) is higher. Note that this higher equity exposure does not imply that the participant is willing to take additional risk. The equity exposure of total wealth is still the same.

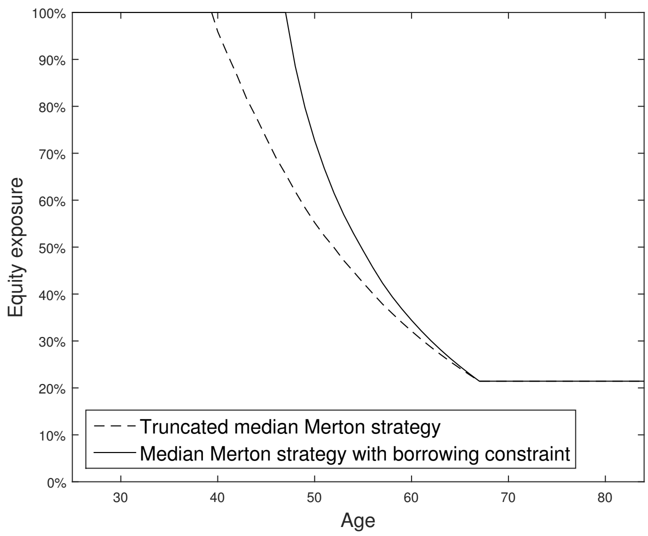

Figure 2.3: The median of the optimal Merton equity exposure of financial wealth over the life-cycle including an exogenous borrowing constraint and the truncated median of the optimal Merton equity exposure.

2.2.4 Human capital

One of the assumptions in the standard Merton model is that labor income is constant, non-tradable and risk-free which implies that human capital acts like a risk-free bond. These assumptions are quite strong and might be relaxed. It is more realistic to include career patterns, i.e. age-dependent labor income, and to use stochastic human capital instead of risk-free human capital. There have been several studies regarding human capital. Some studies assume that human capital is more bond-like, which is in line with the assumption of risk-free human capital in the Merton model, while other studies state that human capital behaves more stock-like. For example Benzoni et al. (2007) investigated the optimal portfolio choice in case labor income and dividends are cointegrated. Due to this cointegration, human capital is more stock-like which implies that young participants should take a substantial short position in stocks which is in contrast with the Merton model which suggests to invest large amounts in stocks at young ages (based on the assumption of risk-free human capital). On the other hand, human capital of old participants is more bond-like in the model of Benzoni because old participants have a shorter period until retirement. As a consequence, the cointegration does not have enough time to act.

In this thesis, I maintain the assumption of risk-free human capital. This assumption is supported by evidence of Cocco et al. (2005) who showed that there is no significant correlation between labor income risk and stock market risk. They investigated the importance of non-tradable labor income with uncertainty by calculating the welfare loss associated with suboptimal portfolio allocations. They found that the loss resulting from a portfolio allocation which ignores labor income risk is an order of magnitude smaller compared to the loss resulting from a portfolio allocation which ignores labor income completely (i.e. constant equity exposure). The only exception is when the possibility of a disastrous labor income realization is allowed. They conclude that, although labor income is risky, the optimal portfolio allocations indicate that labor income which is uncorrelated with equity returns is perceived as a closer substitute for risk-free bonds than for risky assets.

2.2.5 Predetermined pension contribution

In the Merton model the individual is completely free to choose his pension contribution each year. The individual decides to save a certain amount of his labor income (i.e. pension contribution) and consumes the remaining income each period. The optimal pension contribution depends on the actual financial wealth which in turn depends on the actual investment returns from the past. The optimal pension contribution is higher after bad stock returns and lower after good stock returns. However, in practice the pension contribution in IDC schemes is usually based on a certain premium ladder. This premium ladder is determined in an agreement between social partners, which implies that the participant cannot determine the pension contribution himself. The 3% DC fiscal maximum premium ladder is used in this thesis (presented in appendix A.5). The pension contribution depends on the age of the participant (i.e. age-dependent pension contributions) and is predetermined (i.e. exogenous to the model). This restriction implies a different optimal equity exposure. The derivation of the optimal equity exposure in case of predetermined pension contributions is explained below. The derivation is similar to the derivation of the optimal equity exposure in case of fully flexible pension contributions (see subsection 2.1.3).

Since the individual is not able to adjust the pension contribution based on the actual investment returns of the past in case of predetermined pension contributions, the total wealth does not consist of financial wealth and the value of human capital but consists of financial wealth and the value of future pension contributions. $\hat{H}_t$ equals the present value of future pension contributions and is defined in the same way as the present value of human wealth H_t in (2.4)

$$\hat{H}_t = \begin{cases} \frac{Y_t - C_t}{r}(1 - \exp(-r(T-t))) & \text{if } t \in [0,T] \\ 0 & \text{if } t \in (T,D]. \end{cases} \tag{2.9}$$

Note that $Y_t - C_t$ equals the pension contribution at time t. In case it is assumed that human capital is risk-free, the pension contributions are risk-free as well and the pension contributions act like a risk-free asset. Again, one should adjust the exposure of financial wealth f_t^* to the risky asset to attain the appropriate constant equity exposure f of total wealth

$$f_t^* = f\frac{W_t}{F_t} = f\left(1 + \frac{\hat{H}_t}{F_t}\right) = \frac{\lambda}{\gamma\sigma}\left(1 + \frac{\hat{H}_t}{F_t}\right).$$

This optimal equity exposure has been derived by Gollier (2008). Note that the optimal equity exposure of financial wealth in case of predetermined pension contributions is lower than or equal to the optimal equity exposure of financial wealth in case of fully flexible pension contributions due to the fact that the value of future pension contributions $\hat{H}_t$ is lower than the value of human capital H_t. In case of predetermined pension contributions the participant has no flexibility in the pension contribution level implying a lower risk bearing capacity. This results into a lower optimal equity exposure f_t^*.

The median equity exposure is depicted in figure 2.4. The graph corresponding to age-dependent pension contributions contains some remarkable small spikes at certain ages which are not present in the median equity exposure in case of fully flexible pension contributions (see figure 2.3). These spikes are the consequence of the fixed premium percentages during 5 successive years in the premium ladder used in this thesis (presented in appendix A.5). Each time the participant reaches an age with a higher premium percentage in the premium ladder, the discounted value of future pension contributions is higher implying a higher equity exposure. In case the increase of the premium percentage in the premium ladder would be smoother (e.g. premium percentage increases with the same amount each year), the graph of the equity exposure would be smooth as well. The age at which the equity exposure starts to decrease and the equity exposure after retirement do not change when applying a smoother premium ladder.

An alternative to age-dependent pension contributions (as in the above mentioned premium ladder) is an age-independent pension contribution which means that the pension contribution is equal to a constant premium percentage for each age. A transition towards age-independent pension contributions is suggested in the advice of the Social and Economic Council of the Netherlands (see SER (2015)). Using age-independent instead of age-dependent pension contributions has a small impact on the equity exposure (see figure 2.4). Again, the age at which the equity exposure starts to decrease and the equity exposure after retirement are the same. The difference is that the median equity exposure in case of age-independent pension contributions is smooth and is slightly below the median equity exposure in case of age-dependent pension contributions. It makes sense that the equity exposure is slightly lower because the present value of of future pension contributions is lower and financial wealth is higher in case of age-independent pension contributions relative to age-dependent pension contributions. Since the pension contributions in IDC schemes are usually based on a premium ladder, age-dependent pension contributions will be used in the model analysis in chapter 4.

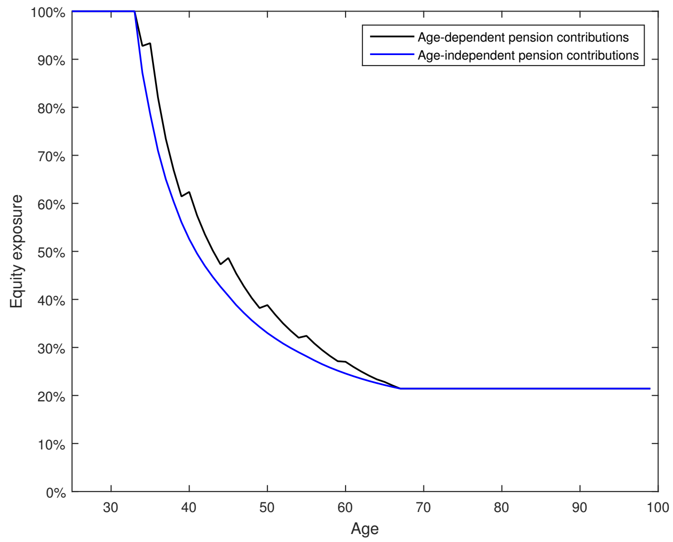

Figure 2.4: The median of the optimal Merton equity exposure of financial wealth over the life-cycle in case of predetermined age-dependent and age-independent pension contributions including an exogenous borrowing constraint.
The age-dependent pension contributions are based on the 3% DC fiscal maximum premium ladder (see appendix A.5) and the age-independent pension contributions equal 15.6%.

Note that it is assumed that the complete pension contribution is paid by the participant. In practice, the employer often contributes a certain amount. When assuming that the labor income after paying the pension contribution is the same, it does not matter for the life-cycle strategy whether the pension contribution is paid by the participant, the employer or by both partially. The value of future pension contributions $\hat{H}_t$ is the same.

The optimal saving decision is an important factor for the welfare of the participant. The restriction of predetermined pension contributions leads to a welfare loss. Bovenberg et al. (2007) for example calculated this welfare loss. They found a welfare loss equal to 3% of certainty equivalent consumption relative to the case with fully flexible pension contributions.[8] Note that the welfare loss also depends on the level of the predetermined pension contribution.[9] Moreover, van den Bleeken et al. (2016) conclude that more freedom of choice regarding pension contributions (e.g. customized pension contributions adjusted to the capital accumulation due to home ownership) implies higher welfare gains compared to more freedom of choice regarding the investment policy (e.g. customized risk profiles relative to the default risk profile).

Since the welfare definition in Bovenberg et al. (2007) and van den Bleeken et al. (2016) differs from the welfare definition in this thesis, the welfare losses cannot be compared directly. The welfare losses in this thesis are calculated in terms of the consumption during

[8] These welfare calculations include an exogenous borrowing constraint, obligatory annuitization and discrete trading at a yearly frequency.

[9] The optimal pension contribution equals 13% in Bovenberg et al. (2007).

the retirement period only (see (2.7)), while the welfare losses in Bovenberg et al. (2007) and van den Bleeken et al. (2016) are calculated in terms of the consumption during the complete life-cycle. Because the retirement period is only a fraction of the complete life-cycle, a welfare loss in terms of the consumption during the complete life-cycle is lower than a welfare loss in terms of the consumption during the retirement period only. Moreover, the discounted value of consumption today is higher than the discounted value of consumption in the future.

2.2.6 State pension

The pension accrual in DC schemes concerns second pillar pension only. Participants who live or work in the Netherlands also receive a basic state pension (AOW) as of the AOW retirement age. This pension is provided by the Dutch government. Just like human capital, basic state pension acts like a risk-free asset (assuming that the participant will receive state pension for sure as of the AOW retirement age). We denote AOW wealth G_t at time t as the present value of future AOW payments the participant will receive during retirement. It is defined in the same way as the present value of human wealth H_t in (2.4)

$$G_t = \begin{cases} \exp(-r(T-t))\frac{A_t}{r}(1-\exp(-r(D-t))) & \text{if } t \leq T \\ \frac{A_t}{r}(1-\exp(-r(D-t))) & \text{if } t > T, \end{cases}$$

with AOW payment A_t at time t.

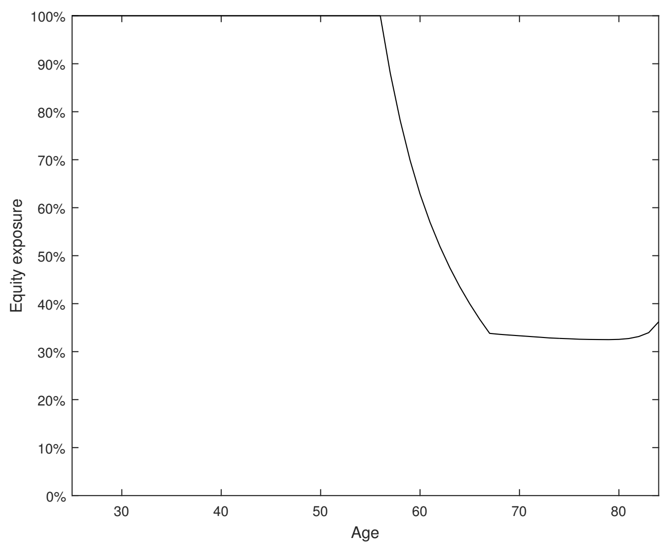

Figure 2.5: The median of the optimal Merton equity exposure of financial wealth over the life-cycle in case state pension is included and including an exogenous borrowing constraint.

The total wealth W_t now consists of financial wealth F_t, human wealth H_t and AOW wealth G_t. The value of AOW wealth increases before retirement because of less discounting. After retirement, the value of AOW wealth decreases because the participant consumes the AOW pension payments. In case state pension is included, the optimal equity exposure changes in different ways: first of all, the equity exposure is higher (see figure 2.5). The equity exposure is higher because state pension acts like a risk-free asset. Note that the upward effect of state pension on the equity exposure depends on the income level of the individual. The upward effect is bigger in case of a lower labor income since then the basic state pension is relatively large compared to the labor income. Moreover, the equity exposure after retirement is not constant anymore but decreases slightly during the first years while it increases during the last years of retirement. This is due to the fact that the total wealth after retirement does not consist of financial wealth only but also contains AOW wealth. During the first years of retirement, AOW wealth decreases at a higher rate than financial wealth while it is the other way around during the last years of retirement.

2.2.7 Longevity risk

There exist two types of longevity risk: micro longevity risk and macro longevity risk. In the Merton model, it is assumed that death is predictable or perfect insurance of micro longevity risk is available and macro longevity risk is absent. The first-best solution in subsection 2.1.3 is based on the assumption that the participant dies at age 85 for sure (see assumptions in subsection 2.1.2). However, both micro longevity risk and macro longevity risk have an impact on the consumption over the life-cycle. Micro longevity risk refers to the uncertainty for an individual about how long he will live, irrespective of changes in mortality. Micro longevity risk can be insured by buying an annuity. The problem of adverse selection arises since the participant generally has more information about his survival probabilities than the insurance company. As a consequence, the annuity market is quite small in countries where insurance of longevity risk is not obligatory (e.g. United States and Australia). However, in the Netherlands participants in DC schemes are obliged to buy an annuity at retirement which reduces the problem of adverse selection. There has been a lot of research regarding the welfare effects of annuities. For example Davidoff et al. (2005) and Poterba (2006) came to the conclusion that annuities are welfare improving for participants in case a wide variety of annuities is available at not too actuarial unfair prices and in case bequest motives and liquidity problems are absent.

One can include micro longevity risk in the Merton model by using a mortality table with survival probabilities. When assuming that the individual lives for sure until the retirement age, including a mortality table has no impact on the optimal equity exposure of financial wealth since the optimal equity exposure is constant after retirement because the total wealth of the individual consists of financial wealth only (i.e. no human wealth after retirement). Including a mortality table does have an impact on the optimal consumption, but since the consumption after retirement is determined using the PPR mechanism (see section 4.2), it is not useful to derive the optimal consumption in the Merton model with micro longevity risk included.

An alternative to an annuity is a draw-down account in which longevity risk is not insured. This is for example the case in the United Kingdom where it is possible since April 2015 to withdraw from your pension capital after retirement as much as you like. In that case the participant bears the longevity risk himself and the pension benefit level can decrease and become low in case the individual becomes old which is of course not desirable. This is shown in the research report of LCP (Koopmans and van Ling (2014)).

Macro longevity risk refers to the uncertainty in the development of future mortality rates due to for example better health care, improved nutrition and habits. Macro longevity risk is especially important for pension funds and insurance companies since macro longevity risk affects the average mortality of a group of participants. In current DC schemes, macro longevity risk is generally insured. The new legislation allows participants to share macro longevity risk within a pool of participants or bear the risk themselves. The advantage is that the participant no longer has to pay a risk premium for this risk to the insurer. The pension benefit is higher in expectation but also becomes more variable. I decided not to focus on macro longevity risk and exclude macro longevity risk from the model analysis since this risk is of second order importance. Steenkamp (2016) concludes that the risks for participants of bearing macro longevity risk themselves are limited. It is also stated in the research report of LCP (van Ling and van Soest (2016)) that insurers do not consider insuring macro longevity risk after retirement as a big issue because of the limited time horizon.

2.2.8 Coefficient of relative risk aversion

Finally, the utility function, which is maximized in the Merton model, uses a constant coefficient of relative risk aversion. However, a lot of empirical evidence exists which rejects this statement. The coefficient of relative risk aversion is not constant in case of loss aversion, a minimum consumption level or habit formation. These characteristics will be discussed below.

Loss aversion

Empirical research (Tversky and Kahneman (1992)) suggests that the preferences of individuals generally feature loss aversion. Loss aversion refers to the phenomenon that individuals are generally more sensitive to losses than to gains. In case of loss aversion human behavior can better be described by a kinked utility function, for example the two-part power utility function considered by Berkelaar et al. (2004) and van Bilsen (2015), rather than the twice differentiable concave CRRA utility function. This is in line with prospect theory. A kinked utility function is characterized by a kink at a certain reference point. Using a kinked utility function leads to a different optimal equity exposure. van Bilsen (2015) for example shows that if the reference level of the individual is constant, the optimal equity exposure of total wealth features a U-shaped pattern. The optimal equity exposure is lower in intermediate economic scenarios relative to good or bad economic scenarios. This is in contrast to the constant optimal equity exposure of total wealth for a CRRA participant (see (2.5) in subsection 2.1.3). The U-shaped pattern of the optimal equity exposure of total wealth does not only hold for an individual featuring risk-averse behavior in the loss domain but also for an individual featuring risk-seeking behavior in the loss domain.

Minimum consumption level

The optimal Merton strategy maximizes utility based on the consumption level at time t. However, the preferences of the individual might be such that utility is only generated in case the consumption level is above a certain minimum consumption level $\bar{C}_t \geq 0$. If this is the case, the maximization function at time t is given by

$$\max \mathbb{E}_t \left(\int_t^D \frac{\exp(-\rho(s-t))}{1-\gamma} (C_s - \bar{C}_s)^{1-\gamma} ds \right). \tag{2.10}$$

According to the corresponding optimal equity exposure it is optimal to first invest in risk-free bonds only such that the minimum consumption level is guaranteed. Subsequently, a fixed fraction of the remaining wealth is invested in risky assets. The optimal equity exposure of financial wealth is given by

$$f_t^* = f\frac{W_t - X_t}{F_t} = f\left(1 + \frac{H_t - X_t}{F_t}\right), \tag{2.11}$$

where X_t represents the discounted value of the minimum consumption over the remaining life-cycle. Including a minimum consumption level leads to a lower optimal equity exposure.

Note that including a minimum consumption level can also be interpreted as if the individual has preferences exhibiting a decreasing coefficient of relative risk aversion instead of a constant risk aversion.

A life-cycle strategy based on the same idea as the minimum consumption level is the constant proportion portfolio insurance (CPPI) strategy explored by Perold and Sharpe (1995). Also in this strategy, the individual wants to receive a minimum consumption level for sure (i.e. the 'floor'). The difference between total wealth and the floor, multiplied with a certain multiplier, is invested in risky stocks.

Important to realize is that the pension accrual in DC pension schemes concerns the second pillar pension only. Participants also receive state pension (AOW) (discussed in subsection 2.2.6) for sure. This can be seen as a minimum consumption guarantee since it provides participants the basic necessities of life. If we assume that the discounted value of the minimum consumption over the remaining life-cycle X_t during the remaining life-cycle is equal to the discounted value of future AOW payments G_t, then the optimal equity exposure of financial wealth is the same as if both state pension and a minimum consumption level are not included in the model. This is shown in the following equation of the optimal equity exposure of financial wealth

$$f_t^* = f\frac{W_t - X_t}{F_t} = f\frac{F_t + H_t + G_t - X_t}{F_t} = f\left(1 + \frac{H_t}{F_t}\right). \tag{2.12}$$

Indeed, the optimal equity exposure of financial wealth is the same as in (2.6). Therefore, I decided to exclude a minimum consumption level and state pension from the model.

Habit formation

Several studies (for example Ferson and Constantinides (1991), Campbell and Cochrane (1999) and Heaton (1993)) provide evidence for habit formation meaning that participants evaluate and adjust consumption relative to a reference (or habit) level. Two types of habit formation exist, namely external habit formation and internal habit formation. In case the individual maximizes (2.10) with $\bar{C}_t$ dependent on previous aggregate consumption, the individual's preferences exhibit external habit formation. The consumption of the individual relative to the consumption level of others is important instead of the absolute consumption level. This characteristic is also known as 'keeping up with the Joneses'. In case of internal habit formation the individual maximizes (2.10) with $\bar{C}_t$ dependent on previous individual consumption. Internal habit formation has been investigated by van Bilsen (2015). Compared to the Merton model, such alternative preferences lead to a shift in consumption from good to bad economic scenarios. As a consequence, the consumption in most economic scenarios lies above the reference level, which is exactly what the individual wants. Another consequence of internal habit formation is the justification of a financial smoothing mechanism in which financial shocks are smoothed over several years. The new legislation allows participants in DC pension schemes to smooth financial shocks over several years. This will be investigated in the model analysis in chapter 4. Note that smoothing of financial shocks has no added value for an individual with CRRA preferences which is assumed in the Merton model.

In case of habit formation, the maximization function (2.10) is not time-separable anymore. The utility at time t does not depend on consumption at time t only, but also on past consumption. As a consequence, the optimal investment and saving problem becomes quite complex and must be solved numerically since an analytic solution does not exist. Because of this complexity, I decided to not take habit formation into account when deriving the optimal investment and saving solution. Habit formation will be considered in chapter 7 in an implicit way by investigating the effects of financial smoothing.

3. New legislation variable pension

This chapter contains an overview of the establishment of and the most important aspects in the new legislation.

Establishment legislation

The foundation of the new legislation was laid in December 2013 when the Parliament adopted a motion of Lodders to make the purchase of a fixed annuity more flexible. Helma Lodders, a member of the Parliament, asked the government to investigate this. Consequently, the consulting companies Lane Clark & Peacock Netherlands B.V. (LCP) and Ortec Finance both published a research report at the request of the government in 2014 with the following conclusion: it is recommended to allow investment after retirement for participants in DC pension schemes. A work group of Netspar came to the same conclusion. In November 2015 the Dutch Ministry of Social Affairs sent a legislation proposal[1] to the Parliament regarding the introduction of variable annuities for participants in DC schemes. After consultation with the industry, several concepts and a consolidation with a legislation proposal of Lodders, the Parliament accepted the consolidated legislation[2] on March 10, 2016 and subsequently the Dutch Senate accepted it on June 14, 2016. The start date of the legislation is September 1, 2016.

Investment after retirement

The new legislation makes it possible for participants in DC schemes to continue to invest after retirement. As already mentioned in the introduction, the most important advantage of investing after retirement is that it yields a higher expected pension. However, the downside is that the pension benefit is not fixed anymore and can also decrease. The participant can choose at retirement whether he prefers a fixed or variable annuity. In case the pension provider does not carry out the preferred type of annuity, the participant has the right to transfer his pension capital to another pension provider with the preferred type of annuity.[3] This option only holds in case the participant chooses the annuity which is not offered by the original pension provider. In case the participant does not make a choice, the default is a fixed annuity.[4]

Variable annuity

The pension benefit level in case of a variable annuity depends on both investment risk and longevity risk: it varies due to investment returns, changes in life expectancy (i.e. macro longevity risk) and biometric returns[5] (i.e. micro longevity risk). Pension providers can offer two different variants of a variable annuity, namely individual allocation and collective allocation[6]. Both variants differ in the way investment risks and longevity risks are incorporated. Note that these variants are broadly in line with two pension contract alter-

[1] 'Wetsvoorstel variabele pensioenuitkering' ('Kamerstukken II 2015-2016', 34344).
[2] 'Wet verbeterde premieregeling' ('Stb. 2016', 248).
[3] Article 80, paragraph 2, 'Pensioenwet', as adjusted by the 'Wet verbeterde premieregeling'.
[4] Article 14d, paragraph 4, 'Besluit uitvoering PW en Wvb', added as a consequence of the 'Wet verbeterde premieregeling'.
[5] The concept of biometric returns will be explained in section 4.2.
[6] Called 'collectief toedelingsmechanisme' in article 1, 'Pensioenwet', as adjusted by the 'Wet verbeterde premieregeling'.

natives[7] which are set up by the Social and Economic Council of the Netherlands (SER) as part of the discussion concerning the future of the pension system in the Netherlands.

In case of individual allocation participants continue to invest after retirement and bear the investment risks themselves completely. In case of collective allocation investment risks are shared within the collective. Participants can already enter the collective as of 10 years before retirement. This entry takes place on a time proportional basis. Note that the participant cannot choose between a fixed or variable annuity at retirement in case he has already entered a collective.

In both variants of a variable annuity, micro longevity risk must be insured or shared within a pool of participants since this is obligatory in the Netherlands. Macro longevity risk can be insured or shared within a pool. In case of individual allocation, the participant can also bear this macro longevity risk himself.

The model analysis in this thesis will be focused on individual allocation. The participant bears the investment risk himself, micro longevity risk is insured and macro longevity risk is excluded from the model analysis (see section 4.1). Modeling collective allocation is much more complex. The report of Ortec Finance (2014) shows that the pension benefits in an individual and collective allocation contract in case of equivalent investment risks are comparable. Moreover, one can perfectly replicate a collective allocation contract (in case of no ex-ante redistributions and a closed financial smoothing mechanism) with an individual allocation contract.

Freedom of investment

For this thesis especially the rules in the new legislation regarding the investment policy are relevant. A distinction is made between DC pension schemes in which participants do not have freedom of investment and pension schemes with freedom of investment.[8] In case the participant has no freedom of investment, the pension provider is fully responsible for an adequate investment policy and must take the age of the participant into account in the implementation of the investment policy.[9] The risk attitude is determined collectively and a uniform life-cycle is used for all participants. In case of freedom of investment the participant can choose between investment policies himself. The risk profile, which specifies how much risk the participant wants and can take, is determined individually. The pension provider must advise the participant to take less investment risk (i.e. use a decreasing life-cycle) as the participant becomes older during the accumulation phase[10]. Investment risks may not increase (unless the risk attitude or risk profile changes) during the decumulation phase. The advice of the pension provider should be in line with the individual risk profile. This is in accordance with the prudent person principle which states that the pension provider should act in the interest of its participants. The prudent person principle is already in force in the current pension legislation.

[7] IV-A and IV-B, see SER (2015).
[8] Social partners decide whether freedom of investment is offered to the participants.
[9] Article 52a, paragraph 1, 'Pensioenwet', as added by the 'Wet verbeterde premieregeling'.
[10] Article 52, paragraph 3, 'Pensioenwet'.

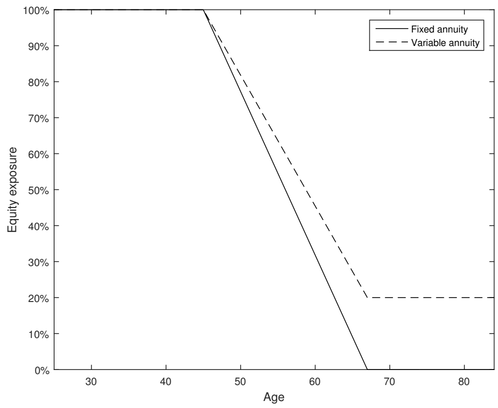

Figure 3.1: Example life-cycle fixed annuity and variable annuity.

Two life-cycles
In case the participant chooses for a variable annuity, the life-cycle strategy is different from the life-cycle strategy for a fixed annuity. The equity exposure can decrease at a slower rate in case of a variable annuity because the equity exposure at retirement should be higher in case of a variable annuity. As a consequence, the life-cycle strategies of both annuities will start to deviate as of a certain age. An example is shown in figure 3.1. The pension provider is obliged to consult participants during the accumulation phase about their preliminary preference for a fixed or variable annuity.[11] Note that the choice of the participant for the type of annuity is not yet binding at this stage. The participant can still make a different choice at retirement. The impact of using an inadequate life-cycle before retirement will be investigated in subsection 5.2.1.

Smoothing financial shocks
Another possibility is the choice to smooth financial shocks during the retirement period. This holds for both individual allocation and collective allocation. It is stated in the legislation that increases and decreases of the pension benefits due to financial shocks can be smoothed over time. In the current version of the legislation, the length of the smoothing period is maximized at 5 years. However, the State Secretary of Social Affairs has expressed the intention to extend this maximum length to 10 years. It is expected that this proposed amendment will be sent to the Parliament later this year via a new legislation proposal. Both lengths of the smoothing period will be investigated in the model analysis. Note that a smoothing period of 10 years generates a level playing field between DC and DB pension schemes on this aspect[12] since it is stated in the financial assessment framework (FTK), which applies to Dutch DB pension funds, that investment shocks can be smoothed over a (maximum) period of 10 years in case of a recovery plan.

Besides the maximization at 10 years, it is also stated in the new legislation that the expected remaining lifetime of the participant should be taken into account when determining the size of the yearly adjustment to the pension benefit level in case of individual allocation. In case of collective allocation the expected remaining lifetime of the collective should be taken into account.

[11] Article 14d, paragraph 5, 'Besluit uitvoering PW en Wvb', added as a consequence of the 'Wet verbeterde premieregeling', 07-07-2016.
[12] Of course, DC and DB pension schemes differ on other aspects.

'Fixed' yearly decrease

As already mentioned in the introduction, it is stated in the legislation that one should use the risk-free rate as discount factor for future pension benefits. In addition, it is allowed to choose for a possible 'fixed' yearly decrease in the pension benefit level which is determined ultimately at retirement.[13] The risk-free rate and the 'fixed' yearly decrease together determine the distribution of the available pension wealth over the various pension payments. A high 'fixed' yearly decrease leads to a high initial benefit level but also to a higher chance of an actual decrease in the benefit level. It is not clearly stated in the legislation whether it is allowed to vary the 'fixed' yearly decrease over the horizon, i.e. horizon-dependent yearly decrease. Both interpretations (horizon-independent and horizon-dependent yearly decrease) will be considered in the model analysis. Note that the choice for a certain 'fixed' yearly decrease only influences the distribution of the pension wealth over the retirement period, but has no impact on the available pension wealth itself.

In the new legislation the 'fixed' yearly decrease is restricted in two ways. First of all, it might not be higher than consistent with the investment policy leading to a nominal stable pension benefit level in expectation. In case the equity exposure after retirement equals 20%, the 'fixed' yearly decrease is maximized at 20% of the difference between the risk-free rate and the parameter of stock returns[14] which equals 7%. Moreover, the 'fixed' yearly decrease is maximized at 35% of the difference between the risk-free rate and the parameter of stock returns. Note that this cap only restricts the 'fixed' yearly decrease and not the actual equity exposure.[15] In the model analysis both restrictions of the 'fixed' yearly decrease will be investigated. Because myopic participants might be temped to take more investment risk in order to increase the 'fixed' yearly decrease (which shifts consumption forward in time), the cap of 35% can be seen as a measure to prevent such potential distortions in the investment behavior of participants.

As mentioned in the introduction, one can use an alternative terminology, namely the assumed interest rate (AIR). The AIR determines how the initial pension wealth will be divided over the various pension payments. It can be seen as a combination of the risk-free rate and a possible 'fixed' yearly decrease. Choosing a 'fixed' yearly decrease is equivalent to choosing a certain AIR.[16]

[13] Called 'uiterlijk op de ingangsdatum van het pensioen vastgestelde, periodieke vaste daling' in article 63a, paragraph 3, 'Pensioenwet', as added by the 'Wet verbeterde premieregeling'.

[14] Maximum expected return, as defined in Article 23a, 'Besluit financieel toetsingskader pensioenfondsen'.

[15] The actual equity exposure is limited by the prudent person rule and risk profile of the participant.

[16] The equivalence will be shown in section 6.1. The equivalence only holds in case of no financial smoothing, but does not necessarily hold in case of financial smoothing.

4. Model analysis

As already mentioned in the introduction the performance of several life-cycle strategies will be measured. Moreover, the impact of the assumed interest rate (AIR) on the pension benefit level during retirement will be investigated in the model analysis. In this chapter the set-up of the model analysis will be described.

This chapter is built up als follows. First of all, a model description is given in section 4.1. This section contains an overview of all assumptions. In section 4.2 it is explained how the pension benefit level of a variable annuity is calculated and the level of risk aversion of the participant is considered in section 4.3. Subsequently, a description of the different time periods into which the complete life-cycle is divided in section 4.4: the accumulation period (1), the conversion period (2) and the decumulation period (3). An overview of these periods is presented in table 4.1. The life-cycle strategies are explained in section 4.5. Finally, the performance indicators of the pension result are listed in section 4.6.

4.1 Model description

The model analysis is based on the assumptions stated in subsection 2.1.1 and the default parameters stated in subsection 2.1.2. In the model analysis we make some additional and/or alternative assumptions which are listed below:

- **Black-Scholes financial market**: in the Black-Scholes model only two investment opportunities exist, namely a risky stock and a risk-free bond. Obviously, more investment opportunities exist in practice like real estate, government bonds and corporate bonds. The risky stock in the Black-Scholes model can be interpreted as a portfolio of different risky investment opportunities. The optimal weights of these investment opportunities in this portfolio will not be derived since it is not the focus of this thesis. Moreover, the interest rate is assumed to be constant and inflation equals zero in the Black-Scholes financial market model which implies that there is no interest rate or inflation risk. As a consequence, the only risk in this model is investment risk. The optimal life-cycle design is completely determined by the equity exposure.[1]
- **Trading frequency**: the life-cycle strategy of the participant can be adapted at a yearly frequency. Euler's discretization method is used to discretize the stochastic differential equations which is elaborated in appendix A.4. There are no transaction costs at the financial market and no additional premia or administrative costs are included in the price of annuities.
- **Offset**: the offset equals €12,953 which is the fiscal minimum AOW offset in the Netherlands in 2016.[2]
- **Career pattern**: a certain career pattern is included. During the first 10 years of the career (age 25-34) the labor income growth rate equals 3%, for age 35-44 the growth rate equals 2%, for age 45-54 the growth rate equals 1% and for the

[1] In practice, there is interest rate risk and inflation risk which implies that hedging interest rate risk and inflation risk is also relevant for the life-cycle. The effect of including a deterministic inflation will be shortly discussed in section 6.2.

[2] www.belastingdienstpensioensite.nl/Overzicht%20AOW-inbouw%20en%20AOW-franchise.htm

remaining working years the growth rate equals 0%. This career pattern is also used to determine the pension premium in a DC pension scheme.[3]

- **Starting salary**: the salary at age 25 equals €23,250. Based on the career pattern described above, this starting salary leads to an average income over the life-cycle of €36,000 which equals the estimated average gross income in the Netherlands in 2016.[4] The participant works the entire period between age 25 and 67 (i.e. no interrupted participation) and there is no employment disability.
- **Pension premium**: the premium is based on the 3% DC fiscal maximum premium ladder[5] for the accrual of old-age pension (partner pension will not be taken into account). This premium ladder is included in appendix A.5. Using a premium ladder implies that the saving decision is exogenous in the model analysis (recall subsection 2.2.5).
- **Micro longevity risk**: micro longevity risk is included using a mortality table with cumulative survival probabilities to make the model more realistic. These survival probabilities are included in appendix A.5. For computational simplicity the additional assumption is made that the participant lives at the retirement age $R = 67$ for sure and cannot become older than $M = 100$ years old. The life expectancy of the participant equals 85 years which is exactly the deterministic age of death in the Merton model in subsection 2.1.2. Macro longevity risk is excluded from the model analysis.
- **Preferences**: in chapter 5 and 6 the individual features CRRA preferences and maximizes utility based on the CRRA utility function (2.2). In chapter 7 the Merton assumptions are extended and habit formation is taken into account.

4.2 PPR mechanism

In the model analysis the so-called ‘Personal Pensions with Risk sharing’ (PPR) mechanism (Bovenberg and Nijman (2015)) will be used to determine the pension benefit level C_t (i.e. the variable annuity payment) during the retirement period each year. In this mechanism certain risks are shared within a pool of participants while participants still have their own personal pension capital (also after retirement).

The PPR mechanism works as follows:

- **Investment function**: investment returns are added to the personal pension capital based on an individual investment policy.

- **Benefit policy function**: withdrawals from the pension capital take place during retirement such that sufficient capital remains for the expected remaining lifetime.

- **Insurance function**: in case a participant dies, the remaining pension capital will be distributed among the other participants in the pool which yields a biometric return for the participants alive.

The biometric return (sometimes called mortality credit) can be interpreted as a bonus for staying alive. It allows participants to enjoy a stable and affordable pension benefit

[3] Article 18a, paragraph 3b, ‘Wet op de loonbelasting 1964’.
[4] www.cpb.nl/cijfer/kortetermijnraming-december-2015
[5] In case a participant with a constant income of €36,000 (i.e. no career pattern) pays pension contributions according to this premium ladder and the return on pension capital equals 3%, the resulting replacement rate at retirement equals 76%.

level during retirement. In return, the capital of a participant who passes away is collected by the insurer. Note that financial returns dominate biometric returns at low ages, while biometric returns become more important than financial returns at high ages. The biometric return can be calculated using the death probabilities.

The mortality table stated in table A.2 in appendix A.5 is used in the model analysis. We assume that the size of the pool of participants is infinitely large since this is easier to model. Note that insurance of micro longevity risk is similar to pooling in an infinite group of participants. Recall that macro longevity risk is not included in the model analysis. In reality, macro longevity risk reduces the biometric return if people live longer than expected. In addition, the biometric return is risky in practice since it depends on previous financial market returns via the pension capital of participants who pass away. In this way, participants are in fact exposed to investment risks of other participants. A possible solution is to pool micro longevity risk within a pool of participants who take comparable investment risks. However, this might lead to difficulties setting up pools with a sufficient number of participants.

The benefit policy function determines the pension benefit level C_t at a certain moment in time and is given by the following formula

$$C_t = \frac{F_t}{pa_t},$$

where the annuity or conversion factor pa_t represents the actuarial fair price of an annuity stream of one euro at time t which will be paid as long as the participant is alive (i.e. lifelong sustainable annuity). F_t equals the available pension wealth at time t. As already mentioned in the assumptions in section 4.1, this price does not contain any additional premia or administrative costs. The annuity factor is defined as follows

$$pa_t = \sum_{j=t+1}^{D} \left(p_t(j) \exp\left(- \sum_{k=t+1}^{j} \mathrm{AIR}_k \right) \right) \quad \text{for } t = T, ..., D-1. \tag{4.1}$$

with assumed interest rate AIR_t, cumulative survival probability $p_t(j)$ and maximum period alive D. The cumulative survival probability $p_t(j)$ is the probability to be still alive at time j as of time t (i.e. the probability that the participant lives $j-t$ more years as of time t).[6]

The AIR represents the rate at which future pension payments are discounted (e.g. the risk-free rate). Future pension payments only have to be paid in case the participant is still alive at that moment in time. One can correct for this by multiplying the discounted value of a future pension payment with the cumulative survival probability $p_t(j)$.

Note that the choice of the AIR is important for the development of the pension benefit level over time. For example, if the risk-free rate is used as AIR and the participant invests a fraction of his wealth in risky assets, the pension benefit level increases over time in expectation. In case the expected return is used as AIR, the expected pension benefit level is constant over time. The AIR will be discussed in more detail in section 6.1.

[6] Bovenberg et al. (2014b) use the biometric return br_t instead of the cumulative survival probability $p_t(j)$ in the definition of the annuity factor which is mathematically equivalent but a bit less intuitive. Moreover, I decided to apply geometric discounting instead of arithmetic discounting (as in Bovenberg et al. (2007)) since geometric discounting in line with the derivation of the optimal AIR in appendix A.2.

4.3 Level of risk aversion

One of the assumptions in the Merton model is the assumption that the preferences of the participant can be represented by a CRRA utility function with risk aversion parameter γ which determines the optimal (dis)saving behavior of the participant and equity allocation. Obviously the preferences of a participant are very personal and γ can differ a lot. Binswanger and Schunk (2008) conducted an empirical research in which they investigated the value of the risk aversion parameter γ assuming that the preferences of the participant can be represented by a CRRA utility function. They conducted a survey among participants in the U.S. and the Netherlands. Their results show that Dutch participants are more risk averse than participants in the U.S. and there is considerable heterogeneity among participants in each country. In the Netherlands, the 25%, 50% and 75% quantile for the risk aversion parameter are equal to 4, 7 and 12, respectively. This can be seen in figure 4.1 (obtained from Binswanger and Schunk (2008).

I decided to consider life-cycle strategies in the model analysis for three levels of risk aversion: risk aversion $\gamma = 4$ (i.e. less risk averse participant), risk aversion $\gamma = 7$ (i.e. default participant) and risk aversion $\gamma = 12$ (i.e. more risk averse participant). In this way a wide range of possible levels of risk aversion among Dutch participants is covered. Moreover, the impact of using a 'wrong' life-cycle strategy, i.e. using a life-cycle strategy applicable to a parameter of risk aversion different from the true risk aversion of the participant, will be investigated in subsection 5.1.1. In practice, participants in DC schemes with freedom of investment have the choice between a number of life-cycle strategies which correspond to different risk profiles. For example, these profiles are called the defensive risk profile, the default or neutral risk profile and the offensive risk profile (see figure 4.2). A life-cycle strategy for the more risk averse participant ($\gamma = 12$) can be interpreted as the defensive profile, the default participant corresponds to the default or neutral profile and the less risk averse participant ($\gamma = 4$) corresponds to the offensive profile. One has to be aware that in practice the life-cycles of risk profiles can differ significantly among different providers (see subsection 4.5.3).

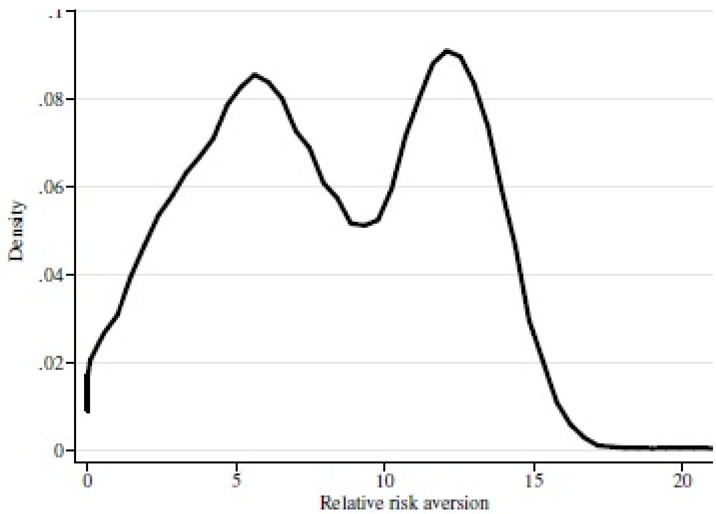

Figure 4.1: The distribution of relative risk aversion in the Netherlands.
Source: Binswanger and Schunk (2008)

4.4 Time periods

In order to investigate the impact of the different life-cycle strategies in a proper and structured way, the complete life-cycle has been split into three time periods. An overview of these periods is given in table 4.1. Period 1 starts at the age at which the participant starts working. The initial financial wealth equals 0. In period 2 and 3 a certain financial wealth, which is reasonable at that moment in time, is assumed at the start of the period. A selection of life-cycle strategies and performance indicators of the pension result is made for each period since not all strategies and indicators are relevant for each period.

	Age	Initial wealth	Focus	Annuity	AIR
Period 1: Accumulation period	25-67	€0	Design life-cycle until retirement	Fixed annuity	Risk-free rate
Period 2: Conversion period	57-100	€175,000	Design life-cycle around and during retirement	Fixed annuity and Variable annuity	Risk-free rate
Period 3: Decumulation period	67-100	€300,000	Distribution pension payments over retirement period Financial smoothing	Variable annuity	- Risk-free rate - Optimal AIR - Expected return - Maximum AIR

Table 4.1: Time periods.

Period 1: accumulation period
The first period is the accumulation period which starts at age 25 (i.e. when the participant starts working) and ends at the retirement age 67. In this period the age at which the decrease in equity exposure should start and the development of this decrease is investigated. It is assumed that the participant buys a fixed nominal annuity at retirement.

Period 2: conversion period
The second period is the period from age 57 (i.e. 10 years before retirement) until the moment of death. This period especially focuses on the decrease in equity exposure just before retirement and the equity exposure during the retirement period. In this period both fixed annuities and variable annuities are considered.

Period 3: decumulation period
The last period is the decumulation period which starts at the retirement age 67 and again ends at the moment of death. In this period the influence of the AIR on the pension benefit level and the impact of financial smoothing will be investigated. The AIR will be explained in more detail in section 6.1 and financial smoothing in section 7.1. The design of the life-cycle will not be investigated in this period since this is already examined in period 2. A constant equity exposure after retirement is assumed in this period.

4.5 Life-cycle strategies

The following life-cycle strategies are considered in the model analysis:
0. Optimal Merton strategy (OM)
1. Median Merton strategy (MM)
2. Linearly decreasing strategy for a variable annuity (LDV)
3. Linearly decreasing strategy for a fixed annuity (LDF)
4. Constant-mix strategy (CM)

These life-cycle strategies are explained in the subsequent subsections. In subsection 4.5.1, some alternative life-cycle strategies are discussed which are not investigated in the model analysis.

4.5.1 Optimal Merton strategy (OM)

This life-cycle strategy is the optimal state-dependent equity exposure in the Merton model including an exogenous borrowing constraint and predetermined pension contribution (as explained in subsections 2.2.3 and 2.2.5). Note that the equity exposure is different in each scenario in this strategy since it depends on the actual financial wealth and might deviate from the a priori expected equity exposure. The optimal equity exposure is higher in case of low financial wealth because of the lower share of financial wealth in total wealth.

State-dependent life-cycle strategies are generally not implemented in practice. However, it is still important to include the optimal state-dependent Merton strategy in the model analysis because it functions as a benchmark because of its optimality. Note that although life-cycle strategies are generally not state-dependent in practice, a participant often has the choice between a number of life-cycle strategies[7] based on different levels of risk aversion. In case of high returns during the first years of the working period, financial wealth is higher than the a priori expected financial wealth. As a consequence, the optimal equity exposure of financial wealth is lower in this scenario. Choosing a more defensive life-cycle strategy would be a wise decision in this scenario. Note that this does not imply that the risk aversion of the participant is different. Since the realized financial wealth at a certain moment in time differs from the a priori expected financial wealth, the optimal equity exposure of financial wealth is also different.

4.5.2 Median Merton strategy (MM)

This life-cycle strategy is the median equity exposure of the optimal state-dependent Merton strategy stated in the previous subsection. This strategy is displayed in figure 2.4 (graph with age-dependent pension contributions). Note that this strategy is state-independent. In practice, state-independent life-cycle strategies are generally implemented. The median Merton strategy is often used as a simplification of the optimal state-dependent Merton strategy.

One has to keep in mind that this median Merton strategy is not optimal. Using the median Merton strategy leads to a welfare loss in terms of certainty equivalent consumption (definition see subsection 2.2.2) relative to the optimal state-dependent Merton strategy. This welfare loss will be calculated in the model analysis and presented in chapter 5.

[7] This only holds in case the participant has freedom of investment.

4.5.3 Linearly decreasing strategy (LD)

The median Merton strategy (stated in the previous subsection) can be approximated by a linearly decreasing strategy. Until a certain age, the participant invests the total or a large fraction of his financial wealth in stocks. At a certain age the participant starts reducing the equity exposure linearly resulting in a low or zero equity exposure at retirement. Such a life-cycle strategy is often applied in practice. Figure 4.2 (taken from van Ling and van Soest (2016)) contains the life-cycle of the default risk profile based on 13 pension providers (both pension insurers and premium pension institutions (PPI)) in the Netherlands. During the first working years, about 75% of financial wealth is invested in risky assets. This is in contrast with the optimal Merton strategy which states it is optimal to invest 100% in risky assets until a certain age. However, the differences between the providers are significant. The equity exposure during the first working years varies between 65% and 90%. The amount invested in risky assets just before retirement is also diverse. It varies between 0% and 27%. Note that these life-cycles correspond to fixed annuities.

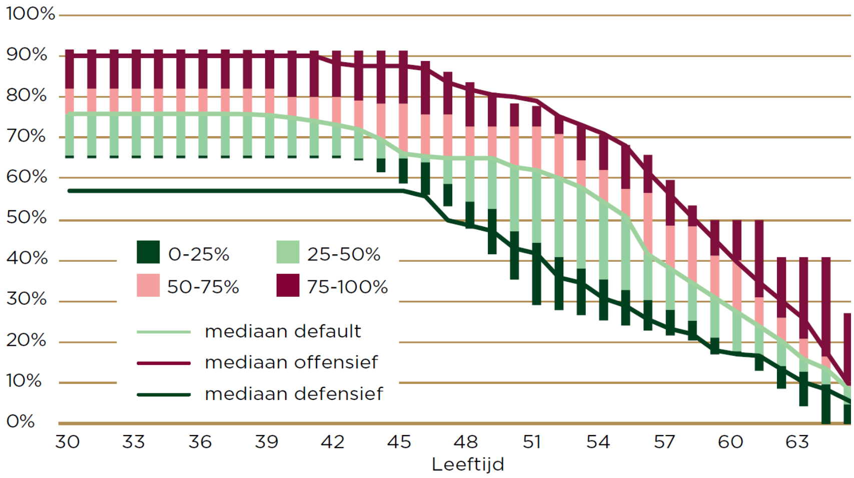

Figure 4.2: Life-cycle of the default risk profile in the Netherlands.
Source: van Ling and van Soest (2016)

Accumulation period
In the model analysis two types of linearly decreasing strategies will be considered, namely linearly decreasing strategies for a fixed annuity (LDF) and for a variable annuity (LDV). In both strategies the participant invests his total financial wealth in stocks until a certain age. After this age, the equity exposure decreases linearly towards 0% at retirement in the linearly decreasing strategy for a fixed annuity (LDF) because the complete pension capital will be converted at retirement in case of a fixed annuity. In case of a variable annuity (LDV) the equity exposure decreases linearly towards the equity exposure at retirement according to the median Merton strategy (MM).

Decumulation period
After retirement two versions (2a and 2b) of the linearly decreasing strategy for a variable annuity (LDV) will be considered. The equity exposure after retirement is constant in strategy 2a (LDVCE) while the equity exposure decreases linearly towards zero at the maximum age in strategy 2b (LDVDE). In the Merton model, it is optimal to maintain a con-

stant equity exposure after retirement. However, it is often suggested to further reduce the equity exposure after retirement. This is for example suggest by van Ling and van Soest (2016) in their report about life-cycles in the Netherlands. In this report, it is proposed to postpone the reduction in equity exposure with 10 years in case of investment risk after retirement. This is depicted in figure 4.3.

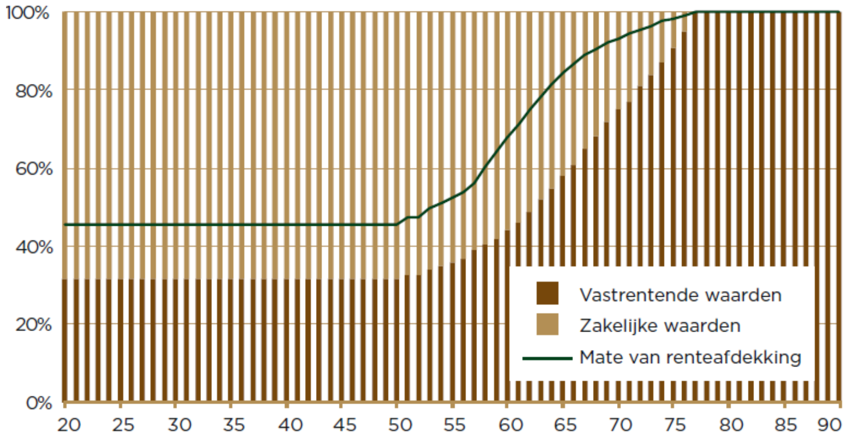

Figure 4.3: Life-cycle in case of investment after retirement.
Source: van Ling and van Soest (2016)

4.5.4 Constant-mix strategy (CM)

A very simple strategy is the constant-mix strategy which is a strategy considered by Perold and Sharpe (1995). A constant-mix strategy maintains a constant equity exposure of financial wealth independent of the age the participant.

As one would expect, such a strategy is not optimal based on the preferences of the participant and the assumptions of the financial market. Although a decreasing life-cycle is commonly applied in practice in the Netherlands, this is not the case in all European countries. The IORPs stress test 2015 for example shows that there are European countries in the DC IORP sector (e.g. Iceland, Spain, Italy and Cyprus) with an (almost) constant asset allocation for all ages.[8] Approximately 30% is invested in risky assets during the accumulation phase. It is interesting to compare the performance of a constant-mix strategy with life-cycle strategies including a decreasing equity exposure over time. This will be investigated in the model analysis in chapter 5.

4.5.5 Alternative life-cycle strategies

Another class of life-cycle strategies are derivative strategies which use assets that directly facilitate risk management. Examples are the option based portfolio insurance approach of Perold and Sharpe (1995) and the collar option approach of Timmermans et al. (2011). In the latter strategy, the participant buys put options and writes call options with a certain strike. In this way the participant sets a floor and cap on his future pension benefit. Derivative strategies will not be considered in the model analysis since this thesis focuses on the trade-off between equity and bonds in life-cycles.

[8] See figure 130 on page 109 in EIOPA (2015).

4.6 Indicators pension result

To measure the performance of the life-cycle strategies, different definitions are used which measure the trade-off between upward potential and downside risk of the pension result. Besides the welfare loss in terms of the certainty equivalent consumption (definition see subsection 2.2.2), which is maximized in Merton's theory, some other indicators are considered. In case of fixed annuities, the mean and variance of the replacement rate at retirement are often used as indicators of the pension result. These indicators are not very appropriate in case of variable annuities since they only consider the pension benefit level at retirement. Therefore, alternative indicators are considered as well. An overview of the indicators is listed below:

A Replacement rate at retirement: pension benefit level (including AOW €12,953) at retirement as percentage of the final salary (€42,074). The median (50% quantile), good case value (95% quantile) and bad case value (5% quantile) are calculated.[9]

B Average replacement rate: weighted average pension benefit level (including AOW) after retirement as percentage of the final salary. The median (50% quantile), good case value (95% quantile) and bad case value (5% quantile) are calculated. The weighted average replacement rate is calculated over the complete retirement period using the relative survival probabilities as weighting factor. The relative survival probability is equal to the cumulative survival probability $p_T(t)$ at time t as of time T divided by the sum of all survival probabilities. More precisely, the relative survival probability is calculated using the following formula

$$wf_t = \frac{p_T(t)}{\sum_{t=T}^{M} p_T(t)}. \tag{4.2}$$

Calculating the weighted average replacement rate instead of the average replacement rate implies that the replacement rates at high ages are less important than the replacement rates at lower ages which is reasonable to assume since less participant will reach a high age.

C Volatility change replacement rate: the volatility of the change in replacement rate during the retirement period. Both the volatility for a five year time horizon from retirement as the weighted average year-to-year volatility (using the relative survival probability (4.2) as weighting factor) are calculated.

D Probability (large) decrease pension wealth: probability that the financial wealth at retirement is less than the financial wealth one or three year before retirement. A large decrease is defined as a decrease which equals more than 5% of the financial wealth one or three years before retirement.

E Welfare loss: loss in utility after retirement measured in terms of the certainty equivalent consumption based on CRRA preferences. The certainty equivalent consumption is the constant certain consumption level which yields the same ex-ante utility after retirement as the stochastic consumption stream after retirement. Note that this is not the same as the consumption level after retirement in case of a fixed annuity. The exact formula of the certainty equivalent consumption is given in subsection 2.2.2. The welfare losses are all calculated relative to the optimal state-dependent Merton strategy (0).

[9] These quantiles are also used in the 'haalbaarheidstoets', article 30b, 'Regeling PW en Wvb'.

F **Growth rate**: average growth rate of the pension benefit level (consumption) over all scenarios. Both the average growth rate from the beginning of the retirement period until the expected age of death (age 85) as the weighted average yearly growth rate (using the relative survival probability (4.2) as weighting factor) are calculated.

G **Probability (large) decrease pension benefit level**: the probability of a decrease in the pension benefit level can be defined in several ways. Two definitions will be used in this thesis:

- The probability of a decrease after 1 or 5 years equals the probability that the benefit level during the 1^{st} or 5^{th} year after retirement is lower than the pension benefit level at retirement.
- The average probability of a decrease equals the weighted average probability that the year to year fluctuation in the benefit level is negative over the complete retirement period using the relative survival probability (4.2) as weighting factor.

Besides the probability of a decrease, one can also calculate the probability of a large decrease which is defined as a decrease which equals more than 5% of the actual pension benefit level.

H **Average relative size decrease**: weighted average relative size of a cut as percentage of the actual pension benefit level of all negative year to year fluctuations using the relative survival probability (4.2) as weighting factor. Using the relative survival probability as weighting factor implies that cuts just after retirement are more important than cuts at high ages.

Note that most indicators are zero in case of a fixed annuity because the pension benefit level is constant after retirement. The following indicators will be calculated in the different time periods:
Period 1: A,D,E
Period 2: A,B,C,E,F,G,H
Period 3: A,B,C,E,F,G,H

The results of period 1 and 2 will be presented and discussed in chapter 5 while the results of period 3 are considered in chapter 6 and 7. Chapter 6 investigates the influence of the AIR on the pension benefit level without financial smoothing. In chapter 7 financial smoothing is included.

5. Results Merton assumptions: investment policy

In this chapter the results of the accumulation period (period 1) and the conversion period (period 2) of the model analysis are presented and discussed (see table 4.1). The design of the life-cycle before retirement is considered in period 1. Different life-cycle strategies are investigated in this period. It is assumed that the participant buys a fixed nominal annuity at retirement. The results are presented in section 5.1. Period 2 focuses on the conversion period. It starts 10 years before retirement and also includes the complete retirement period. In this period several life-cycle strategies are investigated. Both fixed annuities and variable annuities are considered in this period. The results are presented in section 5.2.

5.1 Period 1: accumulation period

The first period focuses on the equity exposure during the accumulation period. The following life-cycle strategies are considered in this period:

0. Optimal Merton strategy (OM)
1. Median Merton strategy (MM)
2. Linearly decreasing strategy for a variable annuity (LDV)
3. Linearly decreasing strategy for a fixed annuity (LDF)
4. Constant-mix strategy (CM)

These strategies are explained in more detail in the subsections of section 4.5.

The strategies will be investigated for three different levels of risk aversion: a default participant ($\gamma = 7$), a less risk averse participant ($\gamma = 4$) and a more risk averse participant ($\gamma = 12$) (see section 4.3). Note that for each level of risk aversion the life-cycles are different. They are displayed in figure 5.1.

Default participant ($\gamma = 7$)
First of all, we focus on the default participant. The equity exposure in the life-cycles for this participant is displayed in the first graph of figure 5.1. Since the optimal Merton strategy (0) is state-dependent, the equity exposure is different in each scenario. As an example, the equity exposure of this strategy in two individual scenarios is displayed in figure 5.1. The same individual scenarios are chosen for the other levels of risk aversion. The decrease in equity exposure in the median Merton strategy (1) starts at age 33. The equity exposure at retirement equals 21.4% (see (2.5)). The constant-mix strategy invests 36% in stocks. This proportion is determined in such a way that the total median equity exposure until retirement (weighted for the available financial wealth) is the same as the total median equity exposure until retirement of the optimal Merton strategy in order to properly compare the constant-mix strategy with the other strategies. Note that this proportion (36%) is significantly lower than the average equity exposure (49%)[1] of all Dutch pension funds.[2] The linearly decreasing strategies (2 and 3) are also determined

[1] A constant-mix strategy of 49% corresponds to a lower risk aversion, namely $\gamma = 5$ instead of $\gamma = 7$.
[2] Report DNB 'Financiele positie pensioenfondsen', 20-05-2016.

such that the median equity exposure until retirement is the same as the equity exposure in the median Merton strategy. This implies that the linearly decreasing fixed strategy (3) starts decreasing the equity exposure at age 35 towards 0% at retirement. Since the reduction of equity exposure in a linear decrease is too flat, the linearly decreasing variable strategy (2) already starts at an equity exposure less than 100%, namely 73% at age 25, in order to have the same median equity exposure until retirement. The equity exposure decreases towards the equity exposure at retirement equal to the equity exposure of the median Merton strategy at retirement, which is 21.4%. Note that such a strategy is not optimal. An exponential decline would yield better results. However, an exponentially declining strategy is almost identical to the median Merton strategy. Also note that the linearly decreasing variable strategy (2) is reasonable close to the frequently applied rule of thumb that one should invest (100 – age)% in risky assets and age% in risk-free bonds.

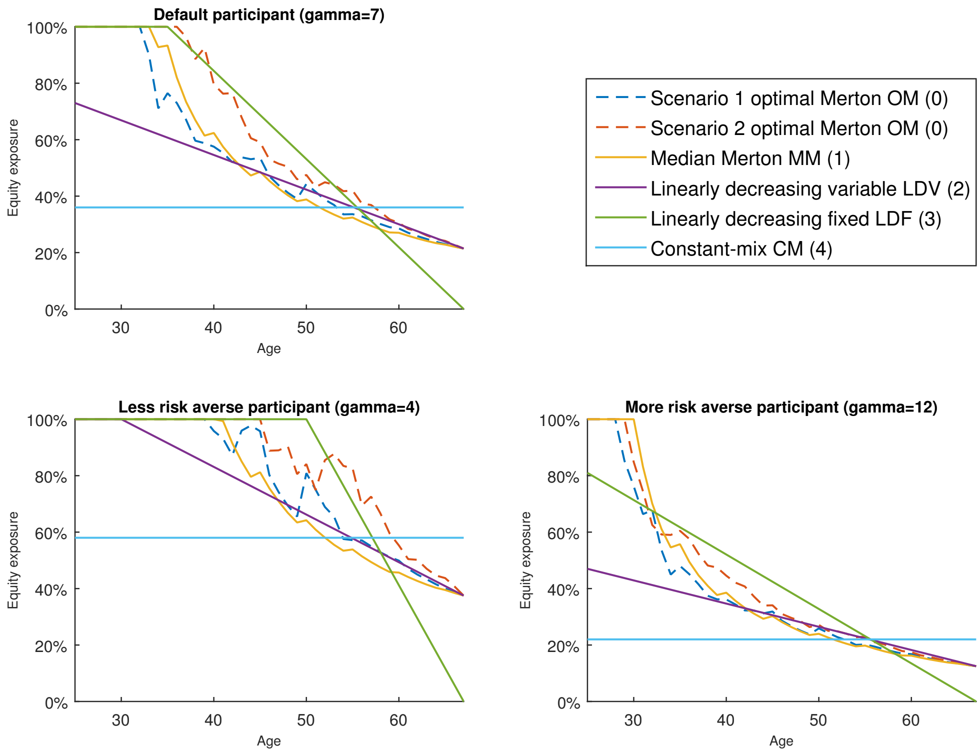

Figure 5.1: Equity exposure of life-cycle strategies in period 1.

Less risk averse participant ($\gamma = 4$)

The equity exposure in the life-cycles for a participant who is less risk averse (i.e. risk aversion parameter $\gamma = 4$) is displayed in the second graph of figure 5.1. The decrease in equity exposure in the optimal Merton strategy (0) of this participant starts a later age. Moreover, the equity exposure at retirement is higher, namely 37.5% (see (2.5)). The proportion invested in stocks in the constant-mix strategy equals 58%. The linearly decreasing fixed strategy (3) starts decreasing the equity exposure at age 50 and the linearly decreasing variable strategy (2) at age 30. Again, these numbers are determined such that the median equity exposure until retirement is the same as the equity exposure in the median Merton strategy for a participant with $\gamma = 4$.

More risk averse participant ($\gamma = 12$)
The equity exposure in the life-cycles for a participant who is more risk averse (i.e. risk aversion parameter $\gamma = 12$) is displayed in the last graph of figure 5.1. The decrease in equity exposure in the optimal Merton strategy (0) of this participant starts an earlier age. Moreover, the equity exposure at retirement is lower, namely 12.5%. The proportion invested in stocks in the constant-mix strategy equals 22%. The linearly decreasing fixed strategy (3) starts at an equity exposure of 81% at age 25 and the linearly decreasing variable strategy (2) starts at an equity exposure of 47%.

Note that since period 1 ends at retirement, only indicators A (replacement rate at retirement) & D (probability (large) decrease pension wealth) as explained in section 4.6 are relevant for this period. These results are presented in table 5.1. Moreover, indicator E (welfare loss) is calculated. The welfare losses are presented in table 5.2.

Life-cycle strategy	OM (0)	MM (1)	LDV (2)	LDF (3)	CM (4)
Less risk averse participant ($\gamma = 4$)					
A Replacement rate at retirement					
5% quantile	65.2%	65.1%	64.9%	63.8%	63.3%
50% quantile (median)	89.1%	87.9%	88.3%	88.6%	87.0%
95% quantile	130.4%	140.5%	140.4%	159.7%	134.7%
D Probability (large) decrease pension wealth					
Decrease last year	22.1%	22.0%	22.3%	0.4%	26.7%
Large decrease last year	13.9%	13.9%	14.2%	0.0%	20.1%
Decrease last 3 years	9.7%	9.7%	10.1%	0.7%	14.7%
Large decrease last 3 years	6.8%	6.8%	7.2%	0.1%	12.1%
Default participant ($\gamma = 7$)					
A Replacement rate at retirement					
5% quantile	66.8%	66.5%	66.0%	66.0%	65.1%
50% quantile (median)	79.7%	79.2%	79.2%	80.0%	78.5%
95% quantile	96.9%	100.3%	100.0%	109.4%	98.6%
D Probability (large) decrease pension wealth					
Decrease last year	11.0%	11.0%	11.3%	0.0%	18.8%
Large decrease (> 5%) last year	3.7%	3.7%	3.9%	0.0%	11.0%
Decrease last 3 years	1.9%	2.0%	2.3%	0.0%	6.1%
Large decrease (> 5%) last 3 years	0.8%	0.8%	1.1%	0.0%	4.3%
More risk averse participant ($\gamma = 12$)					
A Replacement rate at retirement					
5% quantile	67.2%	67.0%	66.7%	66.7%	66.0%
50% quantile (median)	74.2%	74.0%	74.0%	74.5%	73.6%
95% quantile	82.5%	83.4%	83.6%	86.4%	83.2%
D Probability (large) decrease pension wealth					
Decrease last year	2.1%	2.0%	2.3%	0.0%	8.9%
Large decrease (> 5%) last year	0.2%	0.2%	0.2%	0.0%	2.8%
Decrease last 3 years	0.0%	0.1%	0.1%	0.0%	0.8%
Large decrease (> 5%) last 3 years	0.0%	0.0%	0.0%	0.0%	0.4%

Table 5.1: Indicators pension result period 1.

Replacement rate at retirement
When looking at the replacement rate at retirement in table 5.1 one notices that the medians (i.e. 50% quantiles) of all strategies are approximately equal for each level of risk aversion. This makes sense since we determined the life-cycles in such a way that the median equity exposure during the whole accumulation period is the same. The medians are not exactly equal since the distribution of the equity exposure over time is different in each strategy. Moreover, one notices that the 95% quantile of the replacement rate at

retirement of the optimal Merton strategy (0) is the lowest of all strategies which suggests that this strategy has the lowest upward potential. This is the case for each level of risk aversion. Recall that the equity exposure is state-dependent in the optimal Merton strategy (0) which leads to a lower equity exposure after good stock returns. The other life-cycle strategies have a state-independent equity exposure; they do not reduce their equity exposure after good stock returns. On the other hand, the 5% quantile of the replacement rate at retirement of the optimal Merton strategy (0) is the highest, again due to its state-dependency. However, the difference with the 5% quantiles of the other strategies is smaller than the difference between the 95% quantiles. When looking at the box plot of the replacement rate at retirement in figure 5.2, one notices that the differences in replacement rate for the different levels of risk aversion are small for the lower quantiles. This is probably the result of the relatively high equity premium in the financial market which makes it relatively attractive to invest in risky assets. The higher risk due to more investment in risky assets is compensated by the higher expected return on these risky assets. The differences in replacement rate are large for the higher quantiles.

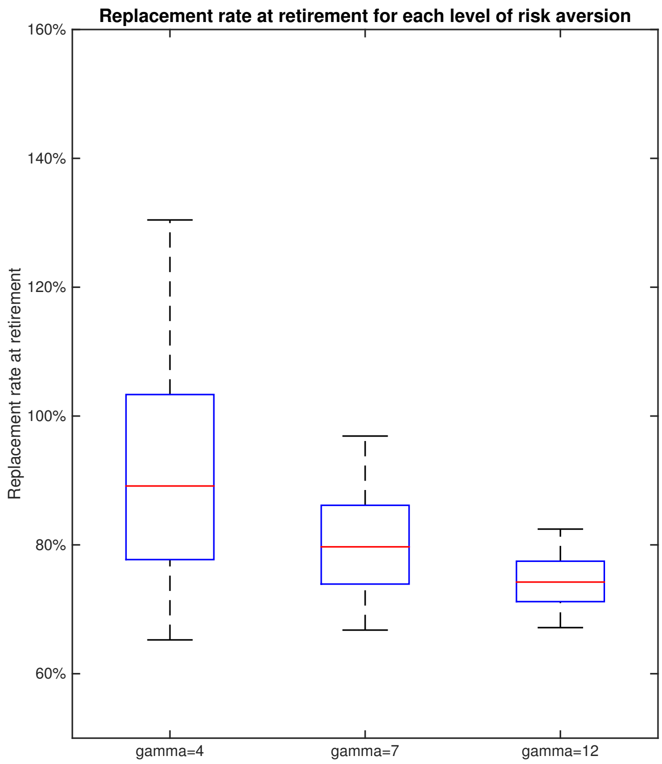

Figure 5.2: Box plot of the replacement rate at retirement in the optimal Merton strategy (0) for each level of risk aversion ($\gamma = 4$, 7 and 12) in period 1. *Red line represents the 50% quantile, edges of the box represent the 25% and 75% quantile and the whiskers extend to the 5% and 95% quantile.*

For the default ($\gamma = 7$) and more risk averse ($\gamma = 12$) participant, the linearly decreasing fixed strategy (3) outperforms the linearly decreasing variable strategy (2) based on the replacement rates at retirement since the replacement rate for all three quantiles is higher in the linearly decreasing fixed strategy (3). This is probably the result of the too low equity exposure during the first working years in the linearly decreasing variable strategy (2) (see figure 5.1). When looking at figure 5.1 one notices that if the participant has a high risk aversion, the median of the optimal Merton strategy exhibits a sharply exponential decrease. This implies that a linearly decreasing strategy is a less accurate approximation of the optimal strategy for a high risk aversion compared to a lower risk aversion.

Note that for the less risk averse participant the 95% quantile of the replacement rate at retirement of each strategy (around 140%) is considerably higher than the fiscal maximum which equals 100% of the pensionable salary.[3] The replacement rate at retirement equals approximately 100% at the 70% quantile. For such a participant, early retirement would be an attractive option in good scenarios. However, one has to keep in mind that the mean stock return in the model $\mu = 7\%$ equals the maximum gross stock return. No additional premia and administrative costs are taken into account which implies that the gross stock return equals the net stock return in our model. Of course, this does not hold in practice. Moreover interest rate risk and inflation risk are not included in this model.

Probability (large) decrease pension wealth

When looking at the probability of a (large) decrease in pension wealth one and three years before retirement one observes that for each strategy the probability of a (large) decrease one year before retirement is higher compared to the probability of a (large) decrease three years before retirement. This is because financial shocks during subsequent years can offset each other.

When comparing the different strategies one notices that the probabilities of a (large) decrease in pension wealth are comparable for the optimal Merton (0), median Merton (1) and linearly decreasing variable (2) strategy for each level of risk aversion. This makes sense since the equity exposure in these strategies just before retirement is similar. The probability of a decrease is (almost) zero for the linearly decreasing fixed strategy (2) since the equity exposure is almost zero just before retirement. All probabilities of a decrease are the highest in the constant-mix strategy. This holds for each level of risk aversion. The difference with the other strategies is especially high for the more risk averse participant. For example, the probability of a decrease during the last year in the constant-mix strategy equals 8.9%, while it is around 2% in the optimal Merton (0), median Merton (1) and linearly decreasing variable (2) strategy.

Welfare losses

Recall that the state-dependent Merton strategy (0) is optimal for a participant with CRRA preferences since this strategy maximizes the lifetime utility of the participant. The other life-cycle strategies are suboptimal and lead to a welfare loss relative to the optimal strategy. A welfare loss is defined as the utility loss after retirement in terms of certainty equivalent consumption (see measurement E in section 4.6). It is interesting to calculate the welfare losses of the life-cycle strategies relative to the optimal strategy. To compare the different life-cycle strategies properly, we assume that the participant buys a fixed nominal annuity at retirement using the risk-free rate as AIR. Note that this is

[3] Article 18a, paragraph 7, 'Wet op de loonbelasting 1964'.

actually not optimal, but recall that period 1 focuses on the life-cycle before retirement: variable annuities will be considered in period 2. Buying a fixed nominal annuity implies that the pension benefit level is constant after retirement.

When looking at the welfare losses in table 5.2, one notices the relative large welfare losses of the constant-mix strategy (4) and the relative small welfare losses of the median Merton strategy (1). Moreover, one notices that the welfare losses for the less risk averse participant are higher compared to the more risk averse participant for each strategy, except for the linear variable (2) strategy. These higher welfare losses make sense since benefits of investing in risky assets are the highest for the less risk averse participant. Recall that a participant with an infinite risk aversion (i.e. $\gamma \to \infty$) does not invest at all. In that case, the welfare loss of each strategy equals zero.

Life-cycle strategy	**MM (1)**	**LDV (2)**	**LDF (3)**	**CM (4)**
$\gamma = 4$	0.5%	1.1%	2.7%	5.8%
$\gamma = 7$	0.5%	1.5%	1.0%	5.0%
$\gamma = 12$	0.3%	1.2%	0.7%	3.6%

Table 5.2: Welfare losses relative to the optimal Merton strategy (0).
The certainty equivalent consumption of the optimal Merton strategy (0) is in fact not optimal since the participant buys a fixed nominal annuity at retirement while it is optimal to buy a variable annuity.

When comparing the linear variable (2) and linear fixed (3) strategy, one notices that the welfare loss of the linear variable (2) strategy is lower for the less risk averse participant, while higher for the default and more risk averse participant. This is line with the difference in performance measured by the replacement rate at retirement (see above).

Welfare losses are often calculated to measure suboptimality of strategies or constraints. Bovenberg et al. (2007) also calculated the welfare loss of a fixed equity exposure relative to the optimal strategy in the Merton model for an individual facing borrowing constraints. They found a welfare loss of 1.7%.[4] The welfare losses of the constant-mix strategy in table 5.2 are significantly higher.

Based on the assumptions about the financial market and the assumption that the participant features CRRA preferences, one can conclude that the decreasing life-cycle strategies (0, 1, 2 and 3) clearly outperform the constant-mix strategy (4). A decreasing equity exposure does not only yield a higher certainty equivalent consumption, but also higher replacement rates for all quantiles compared to a constant equity exposure. Therefore, I decided to not consider the constant-mix strategy anymore in the conversion period (i.e. period 2) and in the retirement period (i.e. period 3). Our results are in line with the analysis of Bonenkamp et al. (2011). They concluded that a decreasing equity exposure leads to an improvement in the trade-off between risk and return compared to a constant equity exposure.

[4] Bovenberg et al. (2007) calculated the certainty equivalent consumption over the complete life-cycle instead of the retirement period only (which is done in this thesis) implying lower welfare losses. Moreover, the fixed equity exposure in Bovenberg et al. (2007) is determined such that the welfare loss is minimized, while the equity exposure of the constant-mix strategy in this thesis is based on another criterium (namely same median equity exposure as median Merton strategy).

5.1.1 Inadequate equity exposure

In table 5.2, the welfare losses of the life-cycle strategies relative to the optimal Merton strategy are presented for three different values for the parameter of risk aversion γ which correspond to different average equity exposures (i.e. a lower γ implies higher average equity exposure). Note that it is assumed that the value for the parameter of risk aversion is the true parameter. Using a life-cycle strategy with an average equity exposure which does not correspond to the true risk aversion parameter obviously leads to welfare losses since this life-cycle strategy is not appropriate for the participant. It is interesting to determine the order of magnitude of such welfare losses because one cannot quantify the risk aversion of a participant very accurately in practice.

The welfare loss for each life-cycle strategy corresponding to the equity exposure of the default participant (i.e. risk version parameter $\gamma = 7$) is calculated for a participant who actually prefers a different equity exposure (i.e. a different risk aversion parameter). $\gamma = 4$ and $\gamma = 12$ are investigated as different levels of risk aversion.

Life-cycle strategy	**OM (0)**	**MM (1)**	**LDV (2)**	**LDF (3)**	**CM (4)**
$\gamma = 4$	6.2%	6.8%	7.4%	6.2%	9.8%
$\gamma = 12$	3.1%	2.0%	3.8%	3.0%	9.0%

Table 5.3: Welfare losses relative to the optimal Merton strategy (0) in case default life-cycles are used.

The welfare losses in table 5.3 are significantly higher than the welfare losses in table 5.2 which is the case for all life-cycle strategies. This implies that a welfare loss due to an inadequate average equity exposure (i.e. inadequate quantification of the risk aversion parameter) is higher than a welfare loss due to inadequate allocation of the equity exposure over the life-cycle (i.e. suboptimal life-cycle design). These results imply that taking into account the risk profile of the participant when determining the investment policy is very important. Moreover, a DC pension scheme in which the participant has freedom of investment has added value for the participant compared to a DC pension scheme in which the participant has no freedom of investment. Just as in table 5.2, one notices that the welfare losses in table 5.3 are significantly higher for the less risk averse participant compared to the more risk averse participant. Of course, the size of the welfare loss also depends on the size of the inaccuracy.

An interesting observation is the worse performance of the optimal Merton strategy (0) (welfare loss of 3.1%) compared to the median Merton strategy (1) (welfare loss of 2.0%) for the more risk averse participant ($\gamma = 12$). Obviously, the optimal Merton strategy (0) outperforms the median Merton strategy (1) in case the correct risk aversion is used, but this does not necessarily hold anymore in case default life-cycle strategies are used. After taking a closer look at the financial wealth at retirement in both strategies, one can conclude that, while the financial wealth in the optimal Merton strategy is higher than the financial wealth in the median Merton strategy in most scenarios (65%), the financial wealth in the optimal Merton strategy is lower than the financial wealth in the median Merton strategy in the extreme scenarios (i.e. bad case and good case scenarios) implying a lower welfare (recall that this participant takes more investment risk than he actually wants). This result suggests that if a pension provider is not able to quantify the risk aversion of the participant accurately, it might be better to implement a static life-cycle strategy such as the median Merton strategy (1) instead of a dynamic life-cycle strategy as the optimal Merton strategy (0). This is the case if the benefits of implementing a dynamic life-cycle strategy for participants of which the true risk aversion is known do not outweigh the losses of implementing a dynamic life-cycle strategy for participants of which the true risk aversion is unknown when performing a cost-benefit analysis. Note that a dynamic life-cycle strategy is expensive to implement in case of freedom of investment and not possible at all in case the participant has no freedom of investment. In case of no freedom of investment the pension provider uses a uniform investment policy for all participants. This investment policy can still be age-dependent.

Time preference parameter

Besides the parameter of risk aversion γ, the time preference ρ also represents the preferences of the participant. One might be interested in the welfare losses of using an inadequate time preference parameter. Recall from subsection 2.1.3 that we focus on the investment decision. The optimal equity exposure in (2.5) does not depend on the time preference of the participant. Therefore, using an inadequate time preference parameter does not lead to welfare losses. The time preference ρ does influence the optimal consumption strategy. However, the optimal consumption strategy is not considered since the saving decision is exogenous in the model analysis.

Welfare losses due to the application of an inadequate time preference parameter have been calculated by Bovenberg et al. (2007). They concluded that the welfare loss due to the application of an inadequate time preference parameter ρ is especially high in case the true time preference of the participant is significantly higher than the time preference parameter which is used to determine the consumption strategy.

Although the optimal consumption strategy is not considered in the model analysis, the pension benefit payment policy (i.e. consumption during the retirement period) is investigated in chapter 6 in the decumulation period (period 3). The time preference parameter influences the AIR which determines how the pension capital is divided over the different pension payments during the retirement period. In that case, using an inadequate time preference parameter can lead to welfare losses. Welfare losses due to the application of inadequate preference parameters in the decumulation period (both the risk aversion parameter as the time preference parameter) will be considered in subsection 6.2.1.

5.2 Period 2: conversion period

The second period focuses on the last years before retirement and the retirement period. It starts 10 years before retirement at age 57 and continues until the moment of death. The following life-cycle strategies are considered in this period:

0. Optimal Merton strategy (OM)
1. Median Merton strategy (MM)

2a. Linearly decreasing strategy for a variable annuity with constant equity exposure after retirement (LDVCE)

2b. Linearly decreasing strategy for a variable annuity with decreasing equity exposure after retirement (LDVDE)

3. Linearly decreasing strategy for a fixed annuity (LDF)

These strategies are explained in more detail in the subsections of section 4.5.

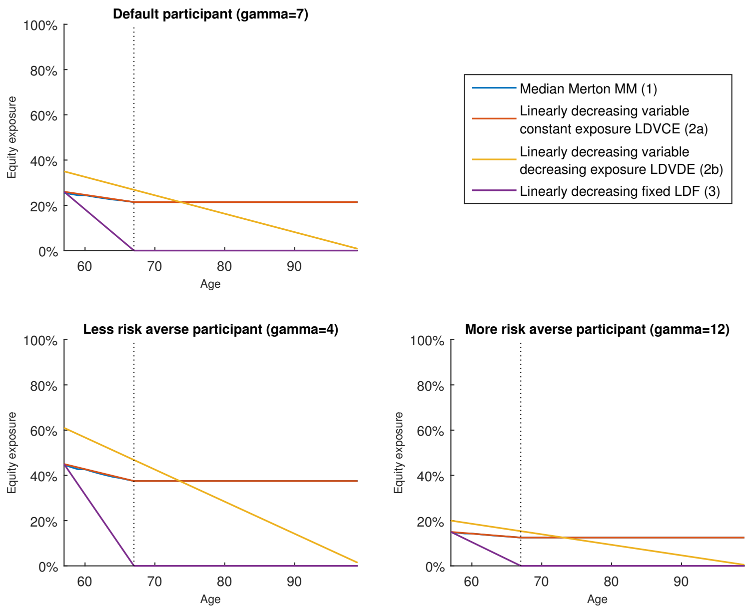

Figure 5.3: Equity exposure of life-cycle strategies in period 2.
Note that the median Merton strategy (1) is not visible since this life-cycle strategy almost completely coincides with the linearly decreasing strategy for a variable annuity with constant equity exposure (2a).

The life-cycles for the different levels of risk aversion are displayed in figure 5.3. As already mentioned in section 5.1, the constant-mix strategy is not investigated in period 2 and 3. Two versions of the linearly decreasing strategy for a variable annuity (2) are considered (see subsection 4.5.3). The equity exposure after retirement is constant in strategy (2a) while the exposure decreases linearly towards zero at the maximum age in strategy (2b). These strategies are determined such that the total median equity exposure as of age 57 (weighted for the available financial wealth) is the same as the total median equity exposure as of age 57 in the optimal Merton strategy. This implies that strategy (2b) starts at a higher equity exposure at age 57 compared to strategy (2a) (see figure 5.3). These linearly decreasing strategies are compared with the optimal Merton strategy (0), the median Merton strategy (1) and the linearly decreasing fixed strategy (3). Note that one cannot determine the linearly decreasing fixed strategy (3) such that the total median equity exposure as of age 57 is the same as in the Merton strategy since this would imply an unrealistic high equity exposure at age 57. Therefore, I decided to start at an equity exposure at age 57 equal to the equity exposure at age 57 in the median Merton strategy (1).

Replacement rate over time

The development of the replacement rate over the retirement period is depicted in figure 5.4. Note that the replacement rate of the median Merton strategy (1) and the linearly decreasing variable strategy with constant equity exposure after retirement (2a) are not visible in this figure, since these replacement rates are (almost) identical to the replacement rate of the optimal Merton strategy (0). While the difference in replacement rate between the fixed annuity and variable annuity strategies is significant in the 50% and 95% quantile, the difference is very small in the 5% quantile.

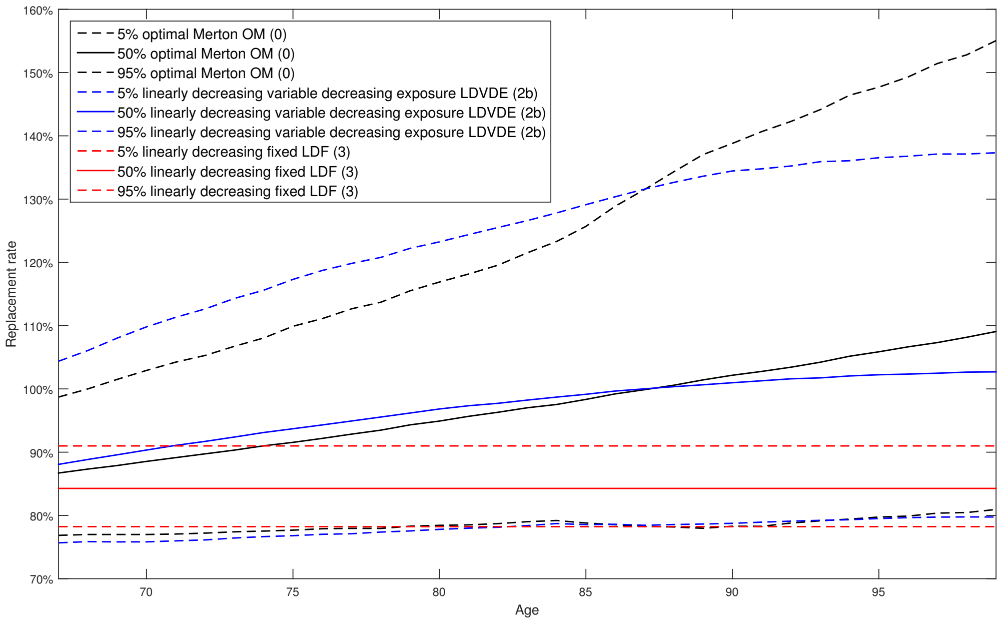

Figure 5.4: Replacement rate over time in period 2 in different life-cycle strategies for the default participant ($\gamma = 7$).

Life-cycle strategy	OM (0)	MM (1)	LDVCE (2a)	LDVDE (2b)	LDF (3)
Less risk averse participant ($\gamma = 4$)					
A Replacement rate at retirement					
5% quantile	73.4%	73.5%	73.8%	71.4%	76.0%
50% quantile (median)	90.7%	90.6%	90.3%	92.9%	86.5%
95% quantile	115.1%	115.5%	114.0%	127.5%	99.2%
B Average replacement rate					
5% quantile	74.7%	74.7%	74.8%	73.4%	76.0%
50% quantile (median)	103.8%	103.7%	103.4%	105.8%	86.5%
95% quantile	154.7%	155.1%	153.3%	165.2%	99.2%
C Volatility change replacement rate					
5-year horizon	10.7%	10.7%	10.7%	13.0%	-
Weighted average year-to-year volatility	4.7%	4.7%	4.7%	4.0%	-
E Welfare loss	-	0.0%	0.0%	0.6%	11.6%
F Growth rate					
Average growth rate until expected age of death	45.3%	45.3%	45.3%	42.4%	-
Weighted average yearly growth rate	2.2%	2.2%	2.2%	1.9%	-
G Probability (large) decrease benefit level					
Probability decrease after 5 years	27.4%	27.4%	27.4%	27.8%	-
Average probability decrease	38.2%	38.2%	38.2%	38.2%	-
Average probability large decrease (> 5%)	16.6%	16.6%	16.6%	13.2%	-
H Average relative size decrease	5.2%	5.2%	5.2%	4.4%	-
Default participant ($\gamma = 7$)					
A Replacement rate at retirement					
5% quantile	76.9%	76.9%	77.0%	75.7%	78.2%
50% quantile (median)	86.7%	86.7%	86.5%	88.1%	84.3%
95% quantile	98.7%	98.9%	98.2%	104.4%	91.0%
B Average replacement rate					
5% quantile	77.9%	77.9%	78.0%	77.2%	78.2%
50% quantile (median)	93.6%	93.6%	93.4%	94.9%	84.3%
95% quantile	115.0%	115.1%	114.4%	119.6%	91.0%
C Volatility change replacement rate					
5-year horizon	5.7%	5.7%	5.7%	6.8%	-
Weighted average year-to-year volatility	2.5%	2.5%	2.5%	2.2%	-
E Welfare loss	-	0.0%	0.0%	0.3%	6.7%
F Growth rate					
Average growth rate until expected age of death	23.9%	23.9%	23.9%	22.6%	-
Weighted average yearly growth rate	1.3%	1.3%	1.3%	1.1%	-
G Probability (large) decrease benefit level					
Probability decrease after 5 years	26.5%	26.5%	26.5%	26.8%	-
Average probability decrease	38.2%	38.2%	38.2%	38.2%	-
Average probability large decrease (> 5%)	7.0%	7.0%	7.0%	5.2%	-
H Average relative size decrease	3.0%	3.0%	3.0%	2.6%	-
More risk averse participant ($\gamma = 12$)					
A Replacement rate at retirement					
5% quantile	78.7%	78.7%	78.8%	78.1%	79.5%
50% quantile (median)	84.4%	84.4%	84.3%	85.2%	83.0%
95% quantile	90.9%	90.9%	90.5%	93.4%	86.6%
B Average replacement rate					
5% quantile	79.4%	79.4%	79.5%	79.1%	79.5%
50% quantile (median)	88.2%	88.2%	88.1%	88.8%	83.0%
95% quantile	98.7%	98.7%	98.3%	100.5%	86.6%
C Volatility change replacement rate					
5-year horizon	3.2%	3.2%	3.2%	3.7%	-
Weighted average year-to-year volatility	1.4%	1.4%	1.4%	1.2%	-
E Welfare loss	-	0.0%	0.0%	0.1%	4.0%
F Growth rate					
Average growth rate until expected age of death	13.4%	13.4%	13.4%	12.4%	-
Weighted average yearly growth rate	0.7%	0.7%	0.7%	0.6%	-
G Probability (large) decrease benefit level					
Probability decrease after 5 years	25.8%	25.8%	25.8%	26.2%	-
Average probability decrease	38.1%	38.1%	38.1%	38.1%	-
Average probability large decrease (> 5%)	1.0%	1.0%	1.0%	0.8%	-
H Average relative size decrease	1.7%	1.7%	1.7%	1.5%	-

Table 5.4: Indicators pension result period 2.
Several indicators are not relevant for the linearly decreasing fixed strategy LDF (3) since a fixed annuity is bought at retirement in this strategy.

The results for period 2 are presented in table 5.4. Compared to period 1 much more indicators of the pension result are relevant since period 2 includes investment after retirement. Note that these results are conditional on the assumed initial financial wealth at age 57 (see table 4.1). When looking at the results in table 5.4 one notices that the results of the optimal Merton (0), median Merton (1) and linearly decreasing variable (2a) strategy are (almost) identical. Only the replacement rates are slightly different, but these differences are negligible. Apparently, the differences in equity exposure during the last years before retirement are too small to have an impact on the pension benefit level.

Replacement rate at retirement & average replacement rate

The greater dispersion in the replacement rate at retirement and average replacement rate in strategy (2b) compared to strategy (2a) is the result of the higher equity exposure before retirement in strategy (2b). This higher equity exposure also leads to a higher volatility of the change in replacement rate during the first five years of the retirement period. On the other hand, a result of the decreasing equity exposure in strategy (2b) is a lower average probability of a large decrease and a lower average relative size of a decrease compared to strategy (2a).

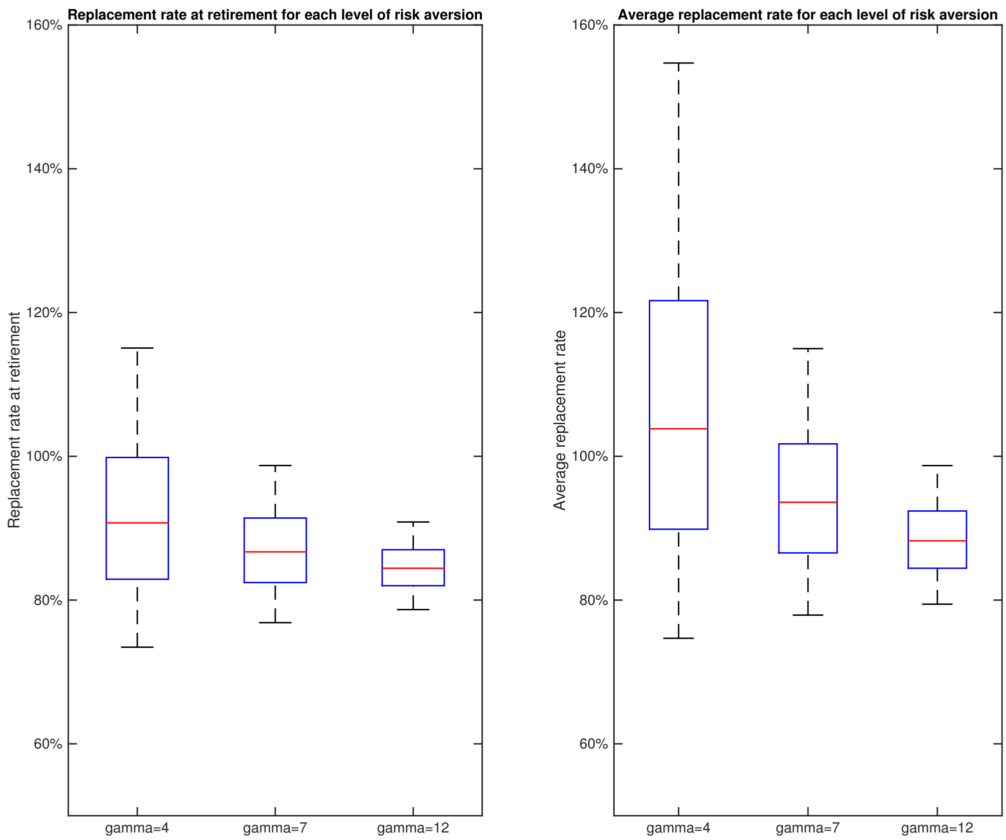

Figure 5.5: Box plot of the replacement rate at retirement and average replacement rate in the optimal Merton strategy (0) for each level of risk aversion in period 2.

Red line represents the 50% quantile, edges of the box represent the 25% and 75% quantile and the whiskers extend to the 5% and 95% quantile.

When comparing the replacement rate at retirement and the average replacement rate for the different levels of risk aversion by looking at the box plot in figure 5.5, one notices that the dispersion in the average replacement rate is significantly higher than in the replacement rate at retirement. The dispersion also depends on the level of risk aversion (i.e. the average equity exposure) to a large extent. The dispersion in the average replacement rate for the less risk averse participant ($\gamma = 4$) is considerable: the 95% quantile is more than two times the 5% quantile.

Other pension results

While the probability of a decrease after 5 years and the average probability of a decrease are comparable for the different risk aversion parameters, the average probability of a large decrease and the average relative size of a decrease differ significantly. For example, the average probability of a large decrease for the less risk-averse participant ($\gamma = 4$) is more than 15 times as high as for the more risk-averse participant ($\gamma = 12$) for each strategy. The difference in the volatility of the change in replacement rate is also significant. When comparing the volatility of the different strategies one notices that the weighted average year-to-year volatility is lower in strategy (2b) compared to strategy (2a) although the volatility on a 5-year horizon from retirement is higher. This holds for all participants. Furthermore, the volatility on a 5-year horizon is approximately twice as high as the weighted average year-to-year volatility.

Welfare loss

The welfare loss of the linearly decreasing fixed strategy (3) compared to the optimal strategy (0) is the highest for the less risk averse participant ($\gamma = 4$) , which makes sense since the benefits of investment after retirement are the highest for this participant. Note that the welfare losses of the linearly decreasing fixed strategy (3) in table 5.4 are higher compared to the welfare losses in period 1 (see table 5.2) for each participant due to the fact that the welfare in the optimal Merton strategy (0) is significantly higher in period 2 because a variable annuity is bought, while a fixed annuity is bought in period 1.

The welfare loss of the linearly decreasing fixed strategy (3) can be split up into two parts: welfare loss because of too little risk taking during the last years before retirement and welfare loss due to no investment risk after retirement (both relative to the optimal Merton strategy (0)). For the default participant ($\gamma = 7$) the welfare loss of the linearly decreasing fixed strategy (3) equals 6.7% (see table 5.4). The welfare loss because of too little risk taking before retirement equals 1.6% while the welfare loss due to no investment risk after retirement equals 5.1%. The welfare loss due to no risk taking after retirement is determined by calculating the welfare loss of the optimal Merton strategy (0) with zero equity exposure after retirement. As a consequence, the remaining welfare loss is a consequence of inadequate (i.e. too little) risk taking before retirement. Note that the welfare loss due to inadequate risk taking before retirement might be underestimated since period 2 investigates the life-cyle design as of age 57 instead of the complete life-cycle. In subsection 5.2.1 the welfare loss of inadequate risk taking before retirement over the complete working period will be investigated. Bovenberg et al. (2007) also calculated the welfare loss of no investment risk after retirement. They found a welfare loss of 0.5%.[5] This welfare loss is much lower than the welfare loss in the model analysis.

[5] Bovenberg et al. (2007) calculated the certainty equivalent consumption over the complete life-cycle instead of the retirement period only (which is done in this thesis) implying lower welfare losses.

5.2.1 Inadequate life-cycle before retirement

As already mentioned in chapter 3 the life-cycle strategy for a variable annuity (2) is different from the life-cycle strategy for a fixed annuity (3). As a consequence, it is desirable to determine the preferences of the participant already before retirement. The pension provider must consult the participants during the accumulation phase about their preliminary preference for a fixed or variable annuity. The fixed annuity is the default in case the participant does not make a choice.[6] In this subsection welfare losses of making an inadequate choice will be calculated.

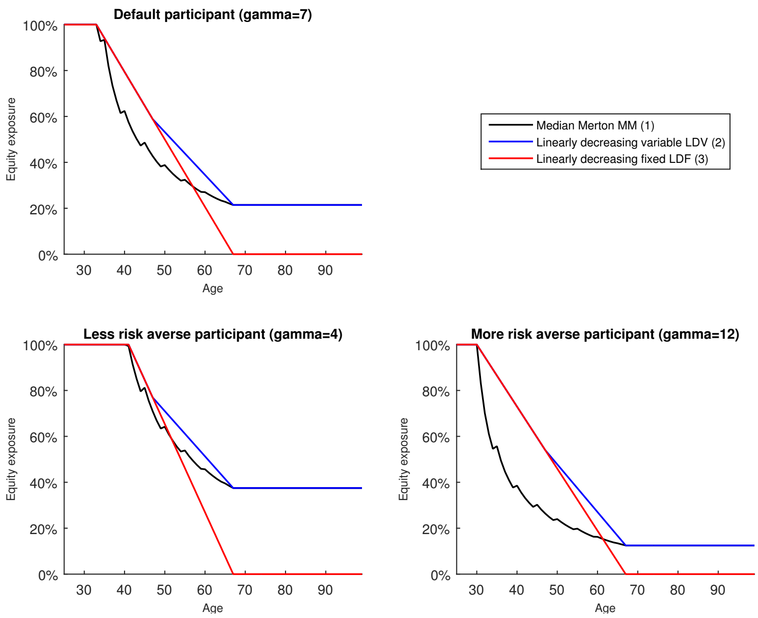

Figure 5.6: Equity exposure in different life-cycle strategies over the complete life-cycle.

Instead of looking at a certain period of the life-cycle, which has been done in sections 5.1 and 5.2, we now consider the complete life-cycle. The decrease in equity exposure in both strategies (linearly decreasing variable annuity (2) and fixed annuity (3)) starts at the same age at which the decrease in equity exposure in the median of the optimal Merton strategy starts. The life-cycles for both annuities start to deviate from each other as of age 47 (i.e. 20 years before retirement). While the linearly decreasing fixed strategy (3) decreases towards zero equity exposure at retirement, the life-cycle for the variable annuity decreases towards the same equity exposure at retirement as in the median of the optimal Merton strategy. This implies that the equity exposure decreases at a higher pace in case of the fixed annuity compared to the variable annuity. The linearly decreasing strategies for both annuities are visualized in figure 5.6 for the three levels of risk aversion. As a benchmark, the median Merton strategy (1) is also included in the graph.

First of all, the welfare losses of the life-cycle strategies for a variable annuity (2) and fixed annuity (3) (see figure 5.6) relative to the optimal Merton strategy are calculated assuming that the participant features CRRA preferences. These welfare losses are presented in table 5.5.

[6] Article 14d, paragraph 4, ‘Besluit uitvoering PW en Wvb’, added as a consequence of the ‘Wet verbeterde premieregeling’.

Life-cycle strategy	LDV (2)	inadequate LDV (2)	LDF (3)
$\gamma = 4$	0.4%	4.2%	13.0%
$\gamma = 7$	0.3%	1.2%	6.7%
$\gamma = 12$	2.2%	1.2%	4.8%

Table 5.5: Welfare losses of the linearly decreasing variable strategy (2), linearly decreasing variable strategy (2) with an inadequate life-cycle before retirement and linearly decreasing fixed strategy (3) relative to the optimal Merton strategy (0).
Inadequate LDV (2) is the life-cycle strategy in which the participant chooses for a fixed annuity (3) at age 47, while choosing for a variable annuity (2) at retirement.

Because the life-cycles for both annuities start to deviate from each other as of age 47, participants should make a preliminary decision for a fixed or variable annuity at age 47. Recall that participants can still deviate from this decision until retirement. It is interesting to calculate the welfare loss of using an inadequate life-cycle before retirement (i.e. preliminary decision at age 47 differs from the ultimate decision at retirement). This has been done by calculating the welfare loss of the life-cycle strategy which uses the linearly decreasing fixed strategy (3) instead of the linearly decreasing variable strategy (2) before retirement and the linearly decreasing variable strategy (2) after retirement relative to the optimal Merton strategy (0). This life-cycle strategy is abbreviated to inadequate LDV (2) in table 5.5.

The welfare losses in table 5.5 are all relative to the optimal Merton strategy (0). This implies that the difference between LDF (3) and inadequate LDV (2) represents the welfare loss due to no investment risk after retirement and the difference between LDV (2) and inadequate LDV (2) represents the welfare loss due to the usage of an inadequate life-cycle strategy before retirement (i.e. linearly decreasing fixed strategy (3) instead of the linearly decreasing variable strategy (2)). One can conclude that a welfare loss due to no investment risk after retirement is significantly higher than a welfare loss due to temporarily using an inadequate life-cycle strategy before retirement. This holds for each level of risk aversion.

A remarkable observation is that the welfare loss of inadequate LDV (2) is lower than the welfare loss of LDV (2) for the more risk averse participant ($\gamma = 12$). This is probably a result of the inaccurate approximation of a linearly decreasing life-cycle strategy. The decrease in equity exposure in the median of the optimal Merton strategy exhibits a sharply exponential decrease which implies that too much investment risk is taken in a linearly decreasing strategy. As a consequence the linear decrease in the fixed annuity strategy (3) is a better approximation of the optimal Merton strategy (0) compared to the linear decrease in the variable annuity strategy (2). This is clearly visible in figure 5.6. The welfare loss of a linearly decreasing equity exposure relative to the optimal equity exposure depends on the risk aversion of the participant. In case of a high risk aversion, the median of the optimal Merton strategy exhibits a sharply exponential decrease which implies a too high equity exposure in the linearly decreasing strategy.

One can also consider the case in which the participant chooses a variable annuity at age 47 but switches to a fixed annuity at retirement. However, this is not relevant since investment after retirement is welfare improving for a participant with CRRA preferences.

6. Results Merton assumptions: pension benefit payment policy

Besides the investment policy, which has been investigated in chapter 5, the pension benefit payment policy is also important for the level of the variable annuity. The benefit payment policy will be investigated in this chapter. The chapter starts with section 6.1 in which the concept of the assumed interest rate is explained in more detail. The influence of the AIR is investigated in the decumulation period (see table 4.1). The results of this period will be presented and discussed in section 6.2.

6.1 Assumed interest rate (AIR)

The assumed interest rate (AIR) determines the distribution of the available pension wealth over time. In this section the impact of the AIR on the pension benefit level is investigated. As already mentioned in chapter 3, the AIR can be seen as a combination of the risk-free rate and a possible 'fixed' yearly decrease. The choice of the 'fixed' yearly decrease is equivalent to the choice of the AIR.[1] The equivalence between the 'fixed' yearly decrease and the AIR does not necessarily hold anymore in case of financial smoothing. However, financial smoothing will not be considered in this chapter (but in chapter 7).

Using a low AIR implies a relatively low pension benefit level at retirement but a considerable expected increase over time. Using a high AIR implies a higher benefit level at retirement but also a higher probability of a decrease in the benefit level. As a rule of thumb, a 1 percentage point higher AIR implies an increase in the benefit level at retirement of approximately 10% compared to the benefit level of a fixed annuity (Bovenberg et al. (2016)). This rule of thumb is based on the duration of the remaining pension benefits which equals approximately 10 years. The exact duration depends on the actual interest rate and life expectancy. The duration of the pension benefits at retirement for the default participant ($\gamma = 7$) in the model analysis equals 10.5 using the financial market parameters stated in subsection 2.1.2 and survival probabilities stated in appendix A.5 (calculation duration see section 7.1).

Optimal AIR

One can derive an optimal AIR if the preferences of the participant can be represented by the CRRA utility (2.2) function. The individual has time preference ρ and maximizes utility with future expected utilities declining exponentially at rate ρ. Since the AIR is relevant for the retirement period, only the utility over the retirement period is maximized. This maximization problem can be solved and yields the following optimal assumed interest rate AIR*

$$\text{AIR}^* = r + \frac{1}{\gamma}(\rho - r) - \frac{1}{2\gamma}\left(\frac{1}{\gamma} - 1\right)\lambda^2. \tag{6.1}$$

The derivation can be found in appendix A.2 and is in line with Grebenchtchikova et al. (2016). When plugging in the parameters of subsection 2.1.2 ($r = 1\%$, $\rho = 2\%$, $\gamma = 7$ and

[1] $\frac{(1+d)^i}{(1+r)^i} \approx \frac{1}{(1+r-d)^i} = \frac{1}{(1+\text{AIR})^i} \;\forall i$
where d denotes the 'fixed' yearly decrease, r the risk-free rate and AIR the assumed interest rate.

$\lambda = (\mu - r)/\sigma = 30\%$), the optimal AIR equals $\text{AIR}^* = 1.7\%$. This corresponds to a constant 'fixed' yearly decrease of approximately 0.7% since the risk-free interest rate equals 1%. In case a participant does not feature CRRA preferences, this AIR is not necessarily optimal anymore and a higher or lower AIR might be preferable. Also other aspects influence the optimal AIR. For example in the presence of inflation, a lower AIR could be more appropriate in order to preserve purchasing power.

When taking a closer look at the formula of AIR^*, one notices that it consists of three terms. The first term is the risk-free rate r. It makes sense that AIR^* is increasing in r since a higher r implies a higher return on wealth so future pension payments can be discounted at a higher rate. The second term contains $\rho - r$ which is the gap between the time preference and the interest rate (i.e. the difference between the cost of waiting (time preference) and the reward of waiting (interest rate)). If $\rho - r$ is higher, then the net cost for waiting is higher which means it is optimal to save less wealth for long-term consumption. Therefore, AIR^* is higher for higher $\rho - r$. The last term consists of λ^2. AIR^* increases in the price of risk λ if $\gamma > 1$. A higher λ implies a higher return on wealth. Discounting at a higher rate is then optimal. Finally, AIR^* also depends on the elasticity of substitution $1/\gamma$. This can be made clear by rewriting (6.1) to the following equation

$$\text{AIR}^* = r + \frac{1}{\gamma}\left(\rho - r + \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)\lambda^2\right).$$

One can take the first order derivative of AIR^* with respect to $1/\gamma$ (derivation see appendix A.2). AIR^* is increasing in the elasticity of substitution $1/\gamma$ (or equivalently decreasing in the risk aversion parameter γ) for most values of γ. Plugging in the parameters of subsection 2.1.2, AIR^* is decreasing in γ for $\gamma > 1.6$. The relation between γ and the optimal AIR is also depicted in figure 6.1. The optimal AIR for the different levels of risk aversion (investigated in the model analysis) is also presented in table 6.1. In case of a high γ, the participant has a high preference for a stable consumption level across time and economic scenarios implying a low optimal equity exposure. As a consequence, one must discount at a lower rate.

In case $\gamma = 1$, AIR^* is equal to ρ. The CRRA utility function is reduced to log utility if $\gamma = 1$. The derivation of AIR^* for this utility function is stated in appendix A.2. This AIR^* does not depend on the interest rate r since the substitution effect of r (i.e. less current consumption in case of a higher r since saving becomes more profitable) and the income effect of r (i.e. more current consumption in case of a higher r since less money has to be saved for future consumption) cancel each other out.

Similar to the optimal equity exposure, AIR^* is constant over time as well. This only holds in the special case of a constant risk-free interest rate r, constant equity exposure (as suggested by the optimal Merton strategy) and no financial smoothing. In a more generalized setting in which one deviates from these assumptions, a constant AIR is no longer optimal. If the interest rate r is not constant over time anymore, AIR^* is obviously also not constant anymore. In case a participant reduces his equity exposure over time during retirement, the AIR should decrease as well because the participant takes less investment risk at high ages (i.e. lower expected investment returns). Finally, a constant AIR is suboptimal in case of financial smoothing (investigated in chapter 7) since then the actual equity exposure to which the pension wealth is exposed also changes over time. In case the AIR is not constant anymore one should refer to the assumed interest curve instead of assumed interest rate. For simplicity the acronym AIR is still used in this thesis.

Assumed interest rates
In section 4.4 the life-cycle has been divided into three periods. In the last period, the effect of the AIR is investigated by applying four different AIR's:

1. Risk-free rate
2. Optimal AIR
3. Expected return
4. Maximum AIR

The relation between γ and the different AIR's is depicted in figure 6.1. Moreover, the AIR's for the different levels of risk aversion (investigated in the model analysis) are presented in table 6.1.

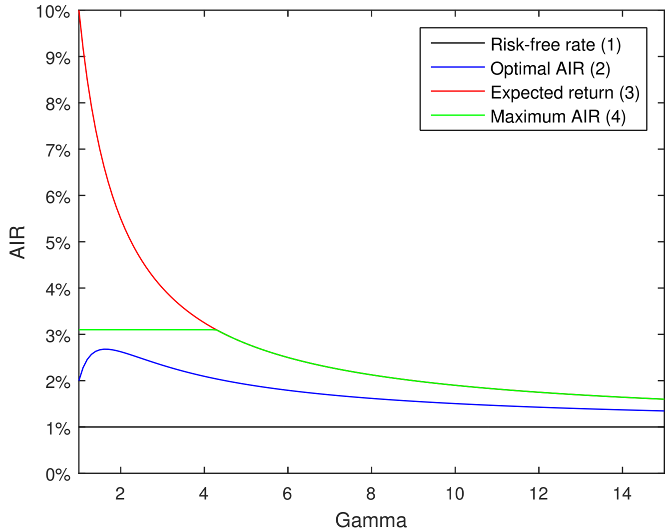

Figure 6.1: Different AIR as function of the risk aversion parameter γ using the default parameter values stated in subsection 2.1.2 for the remaining parameters.

	f	Risk-free rate (1)	Optimal AIR (2)	Expected return (3)	Maximum AIR (4)
$\gamma = 4$	37.5%	1.0%	2.1%	3.3%	3.1%
$\gamma = 7$	21.4%	1.0%	1.7%	2.3%	2.3%
$\gamma = 12$	12.5%	1.0%	1.4%	1.8%	1.8%

Table 6.1: Assumed interest rates (AIR) and optimal equity exposure after retirement f for different levels of risk aversion.

Risk-free rate
First of all, an AIR equal to the risk-free interest rate (1%) will be considered. Recall from chapter 3 that using an AIR equal to the risk-free rate is equivalent to using no 'fixed' yearly decrease. In case the pension wealth would be invested in risk-free bonds only, this AIR would lead to a constant annuity. However, if the participant invests part of his wealth in risky assets the pension benefit level increases over time in expectation. In case the participant does not prefer an increasing consumption pattern during retirement, choosing a higher AIR (i.e. a 'fixed' yearly decrease) than the risk-free rate might be preferable. The optimal AIR in (6.1) indeed lies above the risk-free rate.

Expected return

Another possible AIR is the expected return. Using the expected return as AIR yields a constant expected pension benefit level during retirement. The expected return is given by the following formula

$$\text{Expected return} = r + f(\mu - r) = r + \frac{\lambda^2}{\gamma}, \tag{6.2}$$

where f represents the constant optimal equity exposure[2] (see (2.5)). f depends on the risk preferences of the participant via the risk aversion γ and on the financial market parameters. Obviously, the expected return is decreasing in γ since a high γ implies a low optimal equity exposure. This can be seen in figure 6.1.

Maximum AIR

Recall from chapter 3 that the AIR is maximized in the new legislation. Besides the maximization of the AIR at the expected return based on the actual investment portfolio, there is an additional maximization based on an equity exposure of 35%. This additional maximization implies that, while the participant is still allowed to invest more than 35% of his financial wealth in equity, the maximum AIR is equal to the expected return corresponding to an equity exposure of 35%. The additional maximization leads to an increasing expected pension benefit level in case the equity exposure is above 35%. The maximum AIR is also visualized in figure 6.1. One can conclude that the maximization of the AIR in the new legislation does not lead to a restriction for most participants. It only leads to a restriction for participants with a risk aversion parameter $\gamma < 4.3$ who want to use the expected return as AIR (i.e. participant prefers a constant expected pension benefit level over time). In the distribution of the risk aversion parameter γ among Dutch participants estimated by Binswanger and Schunk (2008), a little more than 25% of all Dutch participants have a risk aversion parameter $\gamma < 4.3$. The maximization in the new legislation does not lead to a restriction for all participants who want to use the optimal AIR. The maximum AIR equals 3.1% while the highest optimal AIR equals 2.7% (see figure 6.1). Note that these statements hold for the financial market parameters as stated in section 2.1.2 and that the maximum AIR heavily depend on the assumed risk premium in the financial market model. If $r = 1\%$ and $\rho > 2.7\%$, then the maximization in the new legislation also leads to a restriction for some participants who want to use the optimal AIR (recall that the optimal AIR (6.1) is increasing in $\rho - r$). In that case, the highest optimal AIR (which corresponds to a low γ, see figure 6.1) lies above the maximum AIR. One can conclude that the new legislation can be restrictive for a participant with a high time preference ρ and low risk aversion γ.

Adverse selection effects

Choosing an appropriate AIR yields additional dimensions of freedom of choice for the participant. Currently freedom of choice is offered to participants via early retirement, part-time pension, high-low pension and the life-cycle (which is related to the risk profile). The first three choices can lead to negative external effects for other participants due to adverse selection. For example, it is beneficial for a low-educated male to choose a high-low pension. Such an adverse selection effect might also occur in case participants can choose the AIR themselves. A high AIR is beneficial for a participant who expects to live a shorter life compared to other participants in his pool. To prevent such adverse selection

[2] After retirement the optimal equity exposure of financial wealth f_t^* is equal to the optimal equity exposure of total wealth f since total wealth consists of financial wealth only.

effects, one should compose homogeneous pools of participants. However, empirical evidence shows that there is currently little evidence for strategic behavior by participants. Numbers of the largest Dutch pension fund ABP (Bovenberg et al. (2014a)) for example show that less men choose for a high-low pension instead of a normal pension, while one would expect the other way around based on the adverse selection effect. The external effects of freedom of choice in current pension plans seem to be limited. Since adverse selection goes beyond the focus of this thesis, I will not take this effect into account.

6.2 Period 3: decumulation period

Period 3 includes the retirement period only. In period 3 the influence of the AIR on the pension benefit level will be investigated instead of the design of the life-cycle itself which has already been investigated in period 2. A constant equity exposure after retirement is assumed which is set equal to the equity exposure after retirement in the optimal Merton strategy. This constant equity exposure equals 21.4% for the default participant ($\gamma = 7$), 37.5% for the less risk averse participant ($\gamma = 4$) and 12.5% for the more risk averse participant ($\gamma = 12$). Moreover, financial smoothing is not taken into account. This will be investigated in chapter 7.

In period 2, I decided to use an AIR equal to the risk-free rate (1%). In period 3, the indicators of the pension result will be calculated for four different AIR (as already explained in section 6.1):

1. Risk-free rate
2. Optimal AIR
3. Expected return
4. Maximum AIR

Although the risk-free rate is the same for each level of risk aversion, the optimal AIR in (6.1) depends on the risk preferences of the participant. The expected return also depends on the risk preferences of the participant via the optimal equity exposure f_t^*. An overview of the AIR's is presented in table 6.1. For each level of risk aversion the optimal AIR is approximately equal to the average of the risk-free rate and the expected return. The table also contains the maximizum AIR which has been explained in section 6.1. Recall that the maximization of the AIR in the new legislation only leads to a restriction for participants with a risk aversion parameter $\gamma < 4.3$ and who prefer a constant expected pension benefit level over time. Therefore, the maximization of the AIR in the new legislation leads to a restriction for the less risk averse participant ($\gamma = 4$). The expected return (3.3%) is higher than the maximum AIR (3.1%). For the other two participants ($\gamma = 7$ and $\gamma = 12$), the maximization of the AIR does not lead to any restrictions which implies that the expected return is equal to the maximum AIR (based on the legislation).

Replacement rate over time
The development of the replacement rate over the retirement period is depicted in figure 6.2. An interesting observation is the replacement rate at age 77. Around this age, the replacement rate is similar for each AIR for each quantile. This implies that it takes approximately 10 years after retirement until the risk-free AIR yields a higher replacement rate than higher AIR's. Note that the duration of the pension liabilities also approximately equals 10 years. While the expected return yields the highest replacement rate compared to the other AIR's before this age, the risk-free AIR yields the highest replacement rate

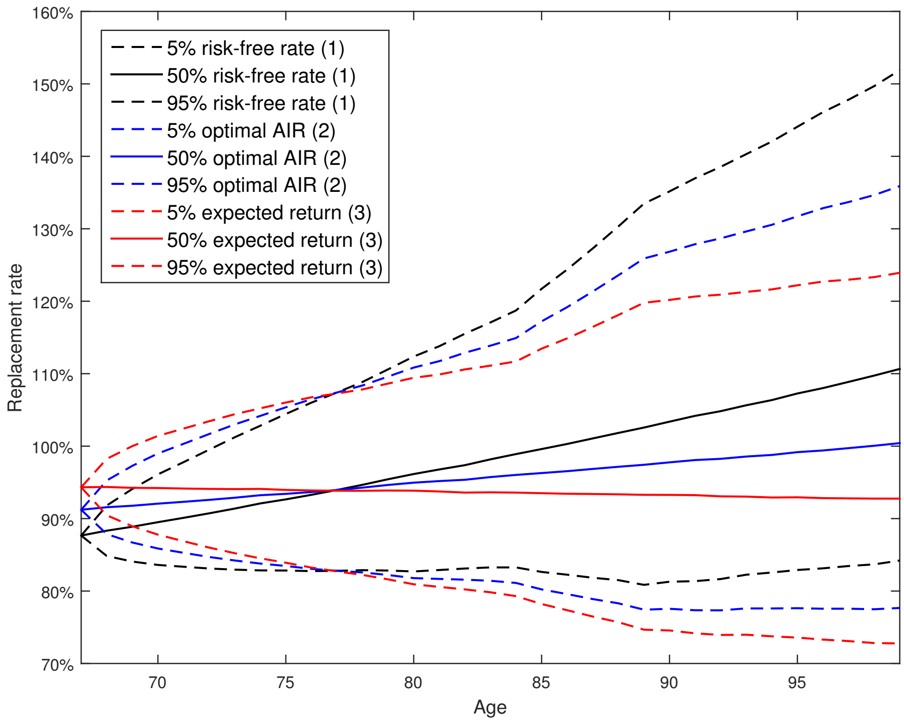

Figure 6.2: Replacement rate over time in period 3 for different AIR for the default participant ($\gamma = 7$).
The replacement over time for the maximum AIR (4) is not displayed in this graph since the maximum AIR (4) is equal to the expected return (3) for the default participant ($\gamma = 7$).

after this age. Important to note is that it is mentioned in secondary legislation[3] that participants should be informed about the level and risk of their pension benefit level at retirement and 10 years after retirement, but not necessarily about the pension benefit level at other moments in time. The results in figure 6.2 reveal that providing information about the pension benefit level after 10 years from retirement is actually relevant since then the drawback of using a higher AIR (namely a lower replacement rate at high ages) becomes visible.

One notices a slight distortion around age 90 in the graphs of the 5% and 95% quantile. This is probably due to the fact that both graphs include the extreme scenarios only. This distortion is not visible in the graph of the 50% quantile.

[3] Article 54, paragraph 2, ‘Besluit uitvoering PW en Wvb’, added as a consequence of the ‘Wet verbeterde premieregeling’.

Assumed interest rate (AIR)	**Risk-free (1)**	**Optimal (2)**	**Expected return (3)**	**Maximum (4)**
Less risk averse participant ($\gamma = 4$)				
A Replacement rate at retirement	87.7%	93.3%	99.5%	98.7%
B Average replacement rate				
5% quantile	79.6%	79.5%	79.4%	79.4%
50% quantile (median)	100.2%	99.0%	98.0%	98.1%
95% quantile	132.2%	129.1%	126.1%	126.4%
C Volatility change replacement rate				
5-year horizon	10.4%	10.3%	10.2%	10.2%
Weighted average year-to-year volatility	4.6%	4.6%	4.5%	4.5%
E Welfare loss	0.8%	-	1.9%	1.9%
F Growth rate				
Average growth rate until expected age of death	45.3%	21.0%	0.0%	2.4%
Weighted average yearly growth rate	2.2%	1.1%	0.0%	0.1%
G Probability (large) decrease benefit level				
Probability decrease after 5 years	27.4%	39.3%	52.9%	51.1%
Average probability decrease	38.2%	43.8%	50.0%	49.1%
Average probability large decrease (> 5%)	16.6%	20.2%	24.6%	24.0%
H Average relative size decrease	5.2%	5.5%	5.8%	5.8%
Default participant ($\gamma = 7$)				
A Replacement rate at retirement	87.7%	91.2%	94.5%	94.5%
B Average replacement rate				
5% quantile	83.2%	83.1%	83.0%	83.0%
50% quantile (median)	94.7%	94.2%	93.8%	93.8%
95% quantile	109.4%	108.4%	107.6%	107.6%
C Volatility change replacement rate				
5-year horizon	5.7%	5.7%	5.7%	5.7%
Weighted average year-to-year volatility	2.6%	2.5%	2.5%	2.5%
E Welfare loss	0.5%	-	1.0%	1.0%
F Growth rate				
Average growth rate until expected age of death	23.9%	10.3%	-0.1%	-0.1%
Weighted average yearly growth rate	1.3%	0.6%	0.0%	0.0%
G Probability (large) decrease benefit level				
Probability decrease after 5 years	26.5%	39.6%	51.8%	51.8%
Average probability decrease	38.2%	44.4%	49.9%	49.9%
Average probability large decrease (> 5%)	7.0%	9.3%	11.7%	11.7%
H Average relative size decrease	3.0%	3.2%	3.3%	3.3%
More risk averse participant ($\gamma = 12$)				
A Replacement rate at retirement	87.7%	89.8%	91.5%	91.5%
B Average replacement rate				
5% quantile	85.1%	85.0%	84.9%	84.9%
50% quantile (median)	91.7%	91.5%	91.4%	91.4%
95% quantile	99.3%	99.0%	98.7%	98.7%
C Volatility change replacement rate				
5-year horizon	3.3%	3.3%	3.2%	3.2%
Weighted average year-to-year volatility	1.5%	1.5%	1.5%	1.5%
E Welfare loss	0.3%	-	0.6%	0.6%
F Growth rate				
Average growth rate until expected age of death	13.4%	5.5%	0.0%	0.0%
Weighted average yearly growth rate	0.7%	0.3%	0.0%	0.0%
G Probability (large) decrease benefit level				
Probability decrease after 5 years	25.8%	39.6%	51.1%	51.1%
Average probability decrease	38.1%	44.7%	49.9%	49.9%
Average probability large decrease (> 5%)	1.0%	1.6%	2.1%	2.1%
H Average relative size decrease	1.7%	1.9%	2.0%	2.0%

Table 6.2: Indicators pension result period 3.

Replacement rate at retirement
Using the optimal AIR or expected return instead of the risk-free AIR implies a higher replacement rate at retirement. The relative increase in replacement rate at retirement depends on the level of risk aversion (i.e. the equity exposure). For example, when using the expected return instead of the risk-free AIR the relative increase in replacement rate at retirement for the less risk averse participant ($\gamma = 4$) of 13.5% (from 87.7% to 99.5%) is approximately three times as large as the relative increase in replacement rate at retirement for the more risk averse participant ($\gamma = 12$) of 4.3% (from 87.7% to 91.5%). Recall from section 6.1 that the pension benefit level at retirement increases with approximately 10% if the AIR increases with 1 percentage point when the duration of the remaining pension benefits equals 10 years. Since the duration in the model analysis equals 10.5, this rule of thumb holds approximately. The increase in replacement rate at retirement is smaller because the replacement rate consists of both the second pillar pension benefit and the first pillar AOW (around 45% of total pension benefit level). The AOW does not increase if a higher AIR is used. As a consequence, the increase in the replacement rate at retirement is also lower.

When looking at the results in table 6.2, one notices that the average replacement rate is comparable for each AIR which makes sense since the AIR does only determine the distribution of the available pension wealth over the various pension payments but has no influence on the amount of pension wealth.

Volatility change replacement rate
Based on the results one can conclude that the AIR has no significant influence on both volatilities of the change in replacement rate (i.e. the volatility on a 5-year horizon and the weighted average year-to-year volatility).

Welfare loss
The welfare loss of using the expected return (i.e. flat expected pension benefit level) for a participant with standard Merton preferences relative to the optimal AIR is about two times as high as the welfare loss of using the risk-free AIR. For example, the welfare loss of using the risk-free AIR equals 0.5% while the welfare loss of using the expected return equals 1.0% for the default participant $\gamma = 7$. It is better to use the risk-free AIR instead of the expected return from a welfare point of view in case of standard Merton preferences. This holds for each level of risk aversion. One has to keep in mind that this result depends on the default parameters stated in subsection 2.1.2. When alternative parameter values are used, the welfare losses change. For example, if one uses a higher ρ (rate of time preference) the welfare loss of using the expected return relative to the optimal AIR decreases while the welfare loss of using the risk free AIR increases. This makes sense since a participant with a high time preference appreciates current consumption more than future consumption. The optimal AIR will be closer to the expected return (see figure 6.1). Therefore, using the expected return as AIR for such a participant is preferable compared to the risk-free AIR. On the other hand, for a participant with a lower ρ, the optimal AIR will be closer to the risk-free AIR (see figure 6.1). In that case, the difference in welfare loss between both AIR (risk-free rate and expected return) becomes even larger. Furthermore, the welfare losses depend on the financial market model parameters μ and r. Finally, one has to keep in mind that these welfare losses are calculated assuming CRRA preferences. In case a participant features habit formation, the welfare loss of the expected return might be even higher since decreases in the pension benefit level are further penalized.

When looking at the welfare losses of the less risk averse participant ($\gamma = 4$), the welfare loss of using the expected return is the same as the welfare loss of using the maximum AIR (1.9%). Apparently, the maximization of the AIR in the new legislation does not have a significant influence on the welfare of the participant.

Average growth rate
Because the maximum AIR is lower than the expected return for the less risk averse participant ($\gamma = 4$), the pension benefit level still grows in expectation from the beginning of the retirement period until the expected age of death with 2.4% in case the maximum AIR is used. Note that for this participant the average growth rate until the expected age of death, in case the risk-free AIR is used, is exceptionally high (45.3%). Such a high growth rate is not necessary for most retirees since the consumption pattern does not increase substantially during retirement in general.

Growth in the pension benefit level can be desirable in case of inflation. Recall that we made the assumption that inflation is zero in the model analysis as stated in section 4.1. In case of inflation the purchasing power decreases over time. A constant pension benefit level in nominal terms is not satisfying anymore since that would imply a decreasing pension benefit level in real terms. A constant pension benefit level in real terms can be achieved by using a lower AIR than the expected return or alternatively including a fixed increase in the pension benefit level. In case one assumes a deterministic inflation level of 1%, an AIR which yields a weighted average yearly growth rate of 1% implies an approximately constant pension benefit level in real terms. One can conclude from table 6.2 that the weighted average yearly growth rate is significantly different for the different levels of risk aversion. In case of a deterministic inflation level of 1% and if we assume that the participant wants to obtain a constant expected pension benefit level in real terms, using the risk-free AIR is (more than) sufficient for the default participant ($\gamma = 7$) and less risk averse participant ($\gamma = 4$) since the weighted average yearly growth rate is higher than 1%. However using the risk-free AIR is not sufficient to obtain a constant expected pension benefit level in real terms for the more risk averse participant ($\gamma = 12$) since the weighted average yearly growth rate equals only 0.7%. Note that in case of inflation the optimal AIR in (6.1) is not optimal anymore for a participant with standard Merton preferences.

Probability (large) decrease benefit level
When looking at the probability of a decrease in the pension benefit level one notices that the influence of the AIR on the average relative size of a decrease is limited. On the other hand, the probability of a decrease after 5 years can be very different. The probability of a decrease after 5 years (almost) doubles when using the expected return instead of the risk-free AIR. This holds for each participant and is a result of the 'fixed' decrease: a higher return is needed to prevent an actual decrease in the pension benefit level.

6.2.1 Inadequate preference parameters

The welfare losses presented in table 6.2 relative to the optimal AIR are based on the assumption that the risk aversion parameter γ and time preference parameter ρ are correctly specified. These parameters represent the CRRA preferences of the participant. Of course, using inappropriate preference parameters leads to welfare losses. It is interesting to determine the order of magnitude of such welfare losses. Note that this subsection is similar to subsection 5.1.1. Welfare losses of suboptimal life-cycle strategies are calculated in subsection 5.1.1 while in this subsection welfare losses as a result of a suboptimal AIR are calculated.

The risk aversion parameter γ influences both the optimal AIR (6.1) as the optimal equity exposure f (2.5) (i.e. it influences both the pension benefit payment policy as the investment policy). Since the expected return depends on f, γ influences the expected return as well. The time preference parameter ρ only influences the benefit payment policy but does not have any impact on the investment policy. The impact of both preference parameters is measured separately. First of all, the welfare losses are calculated for each AIR in case the default risk aversion parameter $\gamma = 7$ is used for a participant who actually has a different risk aversion. Subsequently, the welfare losses are calculated for each AIR in case the default time preference parameter $\rho = 0.02$ is used for a participant who actually has a different time preference. The results are presented in table 6.3.

Assumed interest rate (AIR)	Risk-free (1)	Optimal (2)	Expected return (3)	Maximum (4)
Inadequate risk aversion parameter γ				
$\gamma = 4$	2.0%	1.3%	1.4%	1.4%
$\gamma = 12$	2.7%	4.8%	9.2%	9.2%
Inadequate time preference parameter ρ				
$\rho = 0.01$	0.0%	0.0%	1.5%	1.5%
$\rho = 0.04$	1.2%	0.0%	0.4%	0.4%

Table 6.3: Welfare losses relative to the optimal AIR in case default preference parameters ($\gamma = 7, \rho = 0.02$) are used.

When looking at the welfare losses in table 6.3 one notices that the welfare losses due to application of an inadequate risk aversion parameter are significantly higher compared to the welfare losses due to application of an inadequate time preference parameter. This makes sense since the risk aversion parameter influences both the optimal AIR as the optimal equity exposure, while the time preference parameter influences only the optimal AIR. The welfare losses due to the application of an inadequate time preference are small but make sense: in case the true time preference is higher, a higher AIR is preferred.

An interesting observation is that the welfare losses due to the application of an inadequate risk aversion parameter are much higher for the more risk averse participant ($\gamma = 12$) compared to the less risk averse participant ($\gamma = 4$). In subsection 5.1.1 we came to the opposite conclusion: welfare losses are relatively high for the less risk averse participant ($\gamma = 4$) because the benefits of investment after retirement are the highest for this participant. Recall that in subsection 5.1.1 welfare losses were calculated for the accumulation period (γ influences the investment policy only) while in this subsection the decumulation period is considered (γ influences both the investment policy as the benefit payment policy).

7. Extending Merton assumptions: internal habit formation

Until now we always assumed that the preferences of the participant can be represented by a CRRA utility function. In case of habit formation, this assumption does not hold anymore. The assumption that the participant features CRRA preferences is relaxed in this chapter and internal habit formation will be taken into account. In case of internal habit formation, financial smoothing can be beneficial. The effect of financial smoothing will be investigated in this chapter for the retirement period (period 3) since financial smoothing only takes place during the retirement period. After a short introduction about internal habit formation, the financial smoothing approach applied in the model analysis will be discussed in section 7.1. The results of financial smoothing with a constant equity exposure will be presented and discussed in section 7.2. In section 7.3 the investment policy in case of financial smoothing will be investigated. The results will be presented and discussed in section 7.4. In section 7.5 an alternative financial smoothing approach will be considered.

The CRRA utility function (2.2) (assumed in the Merton model) is a utility function which does not take habit formation into account. Recall from subsection 2.2.8 that several studies provide evidence that preferences of a participant feature internal habit formation which means that the participant maximizes utility of consumption relative to a reference level which depends on previous consumption. In case of internal habit formation, the participant has a preference for smoothing (especially negative) financial shocks over several years.[1] Participants have the possibility to smooth financial shocks over time based on the new legislation. Smoothing financial shocks over time instead of absorbing financial shocks immediately into the pension benefit level (which is considered before) is welfare improving[2] for participants who feature habit formation. Utility functions which take internal habit formation into account are complex (see van Bilsen (2015)). Therefore, I decided not to model such a utility function in an explicit way. However, internal habit formation is investigated in an implicit way in this chapter by investigating the effects of financial smoothing.

Because the assumption that the participant has CRRA preferences is relaxed in this chapter, the terminology of an 'optimal' life-cycle strategy or AIR is not used in this chapter since maximizing the CRRA lifetime utility is not the right optimization problem. Welfare losses corresponding to a CRRA utility are not informative anymore. Instead, the focus lies on indicators such as the volatility of the change in replacement rate during the retirement period and probabilities of a (large) decrease in the pension benefit level. Furthermore, it is not appropriate to compare participants with different levels of risk aversion (represented by the relative risk aversion parameter γ). Instead, one equity exposure will be considered, namely 21.4%. This equity exposure corresponds to the optimal constant equity exposure after retirement for the default participant ($\gamma = 7$) with CRRA preferences.

[1] Smoothing of financial shocks is also used by Dutch DB pension funds as stated in the financial assessment framework (FTK).

[2] Financial smoothing is not beneficial for a participant with CRRA preferences.

7.1 Financial smoothing with constant equity exposure

Investment in risky assets after retirement leads to fluctuations in the pension benefit level. One can reduce the year-to-year volatility of the pension benefit level by smoothing unexpected financial shocks (which are the result of investment in risky assets) over several years. In the new legislation the length of the smoothing period is maximized at $N = 5$ years. However, it is expected that an amendment will be sent to the Parliament to extend this maximum length to 10 years. Both lengths of the smoothing period will be investigated in the model analysis.

While financial smoothing can be desirable for the individual participant, it can also be beneficial from a macroeconomic point of view. It reduces the procyclical effect which is present in case of no financial smoothing. In case of no financial smoothing disappointing stock returns are absorbed immediately in the pension benefit level implying an abrupt decrease in the pension benefit level. This decrease is mitigated in case of financial smoothing.

In section 6.1, different AIR's are considered. The optimal AIR (6.1) is not necessarily optimal anymore in case of financial smoothing since the assumption of CRRA preferences is relaxed in this chapter. Therefore, the effect of financial smoothing will be investigated for the risk-free rate and expected return[3] only. Recall from section 6.1 that the equivalence between the 'fixed' yearly decrease and the AIR does not necessarily hold anymore in case of financial smoothing. In case the 'fixed' yearly decrease[4] does not vary over the horizon (horizon-independent yearly decrease), the equivalence does not hold anymore since the AIR consists of the risk-free interest rate and an additional risk premium[5] which is by definition horizon-dependent in case of financial smoothing. As a consequence, the horizon-independent 'fixed' yearly decrease is not equal to the horizon-dependent risk premium. If the 'fixed' yearly decrease can vary over the horizon (horizon-dependent yearly decrease), the equivalence again holds.

In section 6.2 the equity exposure after retirement is constant as suggested by the optimal Merton strategy. In this section we also assume a constant equity exposure since this section is focused on the benefit payment policy rather than the investment policy. Note that in case of financial smoothing, a constant equity exposure is not optimal anymore. Using a constant equity exposure after retirement in combination with financial smoothing leads to differences in relative size of absorbed financial shocks. During the first years, less risk is absorbed by the participant than would be optimal according to his risk preferences because only a fraction of the financial shock is absorbed. As a consequence, more risk must be absorbed at high ages because larger fractions of the financial shocks have to be absorbed. One has to be aware that financial smoothing shifts investment risks over time but does not reduce the total amount of investment risk. The higher risk absorption at high ages might be undesirable. Therefore a decreasing equity exposure might be preferable in case of financial smoothing. The dependence between financial smoothing and the equity exposure will be investigated in section 7.3 in which a decreasing equity exposure will be considered. This decreasing equity exposure yields a constant

[3] The maximum AIR is equal to the expected return in case of an equity exposure of 21.4%.

[4] It is not clearly stated in the new legislation whether it is possible to use a horizon-dependent yearly decrease (see chapter 3).

[5] Called 'looptijdafhankelijke risico-opslag' in Bovenberg et al. (2016). This additional risk premium is not equal to the constant risk premium of the risky stock.

year-on-year volatility of income during retirement in case of financial smoothing. The results of this decreasing equity exposure will be compared with the results of a constant equity exposure in section 7.4.

To get an idea about the effect of financial smoothing, the development of the replacement rate during the retirement period in one single scenario for different lengths of the smoothing period will be considered. This scenario is depicted in figure 7.1.

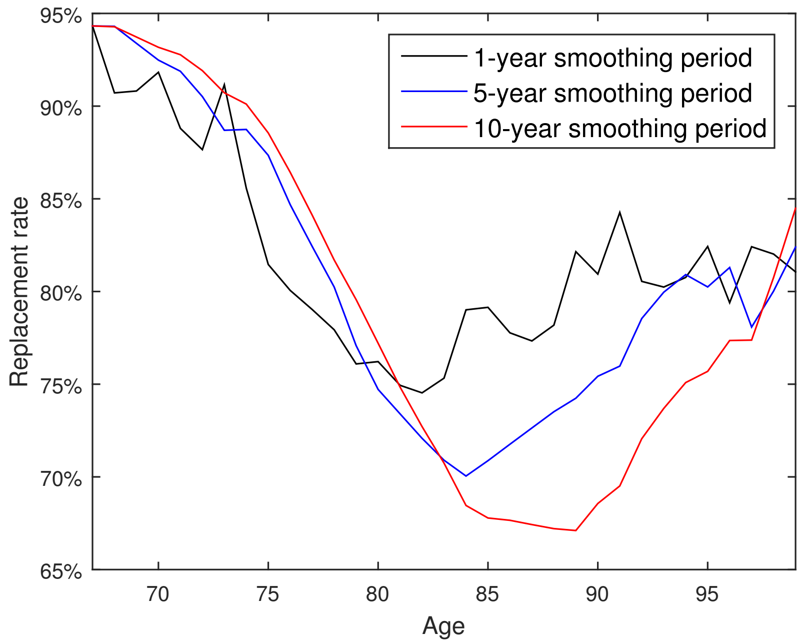

Figure 7.1: Replacement rate during the retirement period in case of financial smoothing around the expected return for different smoothing period lengths, constant equity exposure (21.4%) and expected return as AIR in one individual scenario.
Note that this scenario is characterized by two large negative financial shocks around age 75.

When looking at figure 7.1 one notices that the year-to-year volatility of the replacement rate is lower in case of financial smoothing. The spikes or peaks in the development of the replacement rate in case of no financial smoothing (i.e. 1-year smoothing period) are removed. One notices the limited difference in volatility between the 5-year and 10-year smoothing period. Moreover, as of approximately age 85 the development of the replacement rate over time for the 5-year and 10-year smoothing period is comparable. This is due to the fact that the participant should smooth financial shocks over a smaller period because of lower survival probabilities at high ages.

Growth rate approach

Smoothing of financial shocks can be realized in different ways. I will consider an approach in line with Bovenberg et al. (2014b). In this approach financial smoothing is realized by adjusting the growth rate in the price of the variable annuity after a financial shock. This financial smoothing approach will be denoted by the growth rate approach[6] in the remainder of this thesis. First of all, the concept of this financial smoothing approach will be explained via an example. Subsequently, the technical implementation will be discussed. An alternative financial smoothing approach will be considered in section 7.5.

Example

Suppose an unexpected negative financial shock occurs at age 75. For simplicity, we assume that no unexpected financial shocks occurs during subsequent years. In case of no financial smoothing this financial shock is immediately absorbed in the pension benefit level. In case of financial smoothing the pension benefit level is adjusted with a certain percentage during the subsequent N years. This yearly adjustment is determined in such a way that the budget constraint is still satisfied. As a consequence, the pension benefit level in case of financial smoothing ends at a lower level compared to no financial smoothing. This is illustrated in figure 7.2.

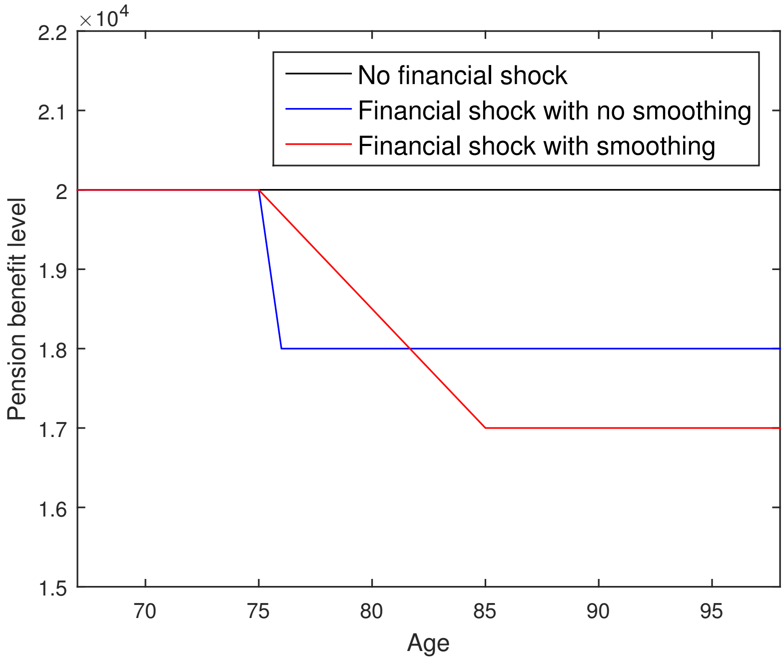

Figure 7.2: Development of the pension benefit level over time in an illustrative example which consists of a single unexpected financial shock at age 75 in case of no financial smoothing and financial smoothing (10-year smoothing period) using the expected return as AIR. After age 75 no more unexpected financial shocks take place.

Note that shocks can cancel each other out, e.g. a negative shock occurs in year t and a positive shock in year $t+1$. This is exactly why the volatility of the change in the pension benefit level is reduced in case of financial smoothing. This effect is not present in the example presented above since this example contains only one unexpected financial shock.

[6] Called ‘groeivoetbenadering’ in Bonenkamp et al. (2011).

Smoothing financial shocks using N-duration

Recall the actuarial fair price of a lifelong sustainable variable annuity in (4.1). In case the expected return is used as AIR, this price implies an annuity with a constant expected pension benefit level during retirement. One can include a certain growth rate in this actuarial fair price which yields an increasing or decreasing expected pension benefit level. In case a growth rate is included, the actuarial fair price of a lifelong sustainable variable annuity equals

$$pa_t = \sum_{j=t+1}^{D} \left(p_t(j) \exp\left(\sum_{k=t+1}^{j} (g_k - \text{AIR}_k) \right) \right) \quad \text{for } t = T, ..., D-1, \tag{7.1}$$

with growth rate g_t at time t. In case of financial smoothing the financial shock is smoothed over several years. This can be realized by adjusting the growth rate g_t in the price of the variable annuity (7.1) after a financial shock during the subsequent N years in such a way that the price still yields a lifelong sustainable variable annuity.[7] The yearly adjustment $x_t\%$ to the growth rate g_t (as a result of previous financial shocks) during the subsequent N years leads to an adjusted growth rate $\hat{g}_t$ which can be calculated as follows

$$(1 + \hat{g}_{t+k}) = \begin{cases} (1 + g_{t+k})(1 + x_t\%) & \text{if } k \leq N \\ (1 + g_{t+k}) & \text{if } k > N. \end{cases}$$

The yearly adjustment $x_t\%$ is in line with the budget constraint and can be approximated with the following formula[8]

$$x_t \approx \frac{\Delta\% F_t}{\text{ND}_t}, \tag{7.2}$$

where $\Delta\% F_t$ equals the percentage change in financial wealth as a result of the unexpected financial shock. A large financial shock requires a large adjustment $x_t\%$. The approximation in (7.2) is accurate for small financial shocks and small time periods but becomes less accurate for large financial shocks or large time periods.

ND_t in (7.2) represents the N-duration of the remaining pension benefit at time t which takes the length of the smoothing period N and the expected remaining lifetime into account. It is given by the following formula[9]

$$\text{ND}_t = \frac{\sum_{j=t+1}^{D} \left(\min(j-T, N) p_t(j) \prod_{k=t+1}^{j} \exp\left(-\sum_{k=t+1}^{j} \text{AIR}_k \right) \right)}{\sum_{j=t+1}^{D} \left(p_t(j) \prod_{k=t+1}^{j} \exp\left(-\sum_{k=t+1}^{j} \text{AIR}_k \right) \right)}. \tag{7.3}$$

The N-duration measures to which extent the participant can recover from a financial shock by taking death probabilities into account. Since there is a probability that the participant dies within N years, the adjustment to the growth rate should be larger than $1/N$ of the financial shock. When the participant becomes older, the recovery capacity is lower because of smaller survival probabilities. This implies a decreasing N-duration. The N-durations for a smoothing period of length $N = 5$ and $N = 10$ are stated in appendix A.5.

[7] Note that a possible 'fixed' yearly decrease is already included in the price of the annuity via the AIR.

[8] See page 20 in Bovenberg et al. (2014b).

[9] This formula is based on the N-duration on page 21 in Bovenberg et al. (2014b). Cumulative survival probabilities are used in this thesis instead of biometric returns and geometric discounting is applied instead of arithmetic discounting (just as in (4.1)).

The decreasing N-duration when the participant becomes older implies an increasing yearly adjustment x_t (see (7.2)) leading to an increasing volatility of the adjustment to the pension benefit level over time in case of a constant equity exposure. This volatility equals the year-on-year volatility of income (i.e. pension benefit level) during retirement: $\text{std}(C_t - \mathbb{E}_{t-1}(C_t))$. This volatility will be denoted by the year-on-year volatility of income from now on.

In case the length of the smoothing period N goes to infinity, shocks are smoothed over the complete remaining lifetime. In that case the N-duration equals the standard duration of the remaining pension benefit since the minimum in the numerator of (7.3) is always equal to $(j - T)$.[10] The duration of the pension benefits at retirement equals 10.5 in the model analysis.

Recall from chapter 3 that the length of the smoothing period N is maximized in the new legislation at 5 years. Moreover, the smoothing period is shorter than 5 years if the life expectancy of the participant is shorter than 5 years in case of individual allocation. The growth rate approach already takes this effect into account since the N-duration measures to which extent the participant can recover from a financial shock.

$\Delta\%F_t$ equals the percentage change in financial wealth as a result of the unexpected financial shock. In Bovenberg et al. (2014b) the unexpected shock is defined as the change in financial wealth relative to the expected financial wealth. This implies that financial shocks are smoothed around the expected return. The risk premium is immediately absorbed in the financial wealth and is not smoothed over several years. In practice, the risk premium on assets is unknown a priori which implies that financial shocks cannot be smoothed around the expected return. Instead, financial shocks can be smoothed around the risk-free rate. In that case, the realized risk premium is included in the financial shock and the compensation for risk taking (i.e. risk premium) is absorbed when the risk is actually taken. Another shortcoming of financial smoothing around the expected return is that it leads to ex-ante redistributions between participants with a different expected remaining lifetime in case of collective allocation.[11] Both smoothing mechanisms (i.e. smoothing around the expected return and smoothing around the risk-free rate) will be investigated in the model analysis. Smoothing around the expected return will be considered in subsection 7.2.1, while the results in case of smoothing around the risk-free rate are presented in subsection 7.2.2.

[10] If $N \to \infty$, then $ND_t = \frac{\sum_{j=t+1}^{D}\left((j-T)p_t(j)\prod_{k=t+1}^{j}\exp\left(-\sum_{k=t+1}^{j}\text{AIR}_k\right)\right)}{\sum_{j=t+1}^{D}\left(p_t(j)\prod_{k=t+1}^{j}\exp\left(-\sum_{k=t+1}^{j}\text{AIR}_k\right)\right)} = \frac{\sum_{j=t+1}^{D}(j-T)V_t^{(j)}}{\sum_{j=t+1}^{D}V_t^{(j)}} = \frac{\sum_{j=t+1}^{D}(j-T)V_t^{(j)}}{V_t}$.

This last expression is indeed equal to the standard duration formula where $V_t^{(j)}$ denotes the discounted value at time t of the pension benefit which will be paid out at time j and V_t equals the total discounted value at time t of all future pension benefits.

[11] This thesis is focused on individual allocation and not on collective allocation.

7.2 Results financial smoothing with constant equity exposure

In this section the results of financial smoothing with a constant equity exposure are presented and discussed. The section starts with the results of financial smoothing around the expected return in subsection 7.2.1. Subsequently, the results of financial smoothing around the risk-free rate are presented in subsection 7.2.2. In subsection 7.2.3 the results of two sensitivity analyses are presented.

7.2.1 Financial smoothing around the expected return

Smoothing period **Assumed interest rate (AIR)** **Equity exposure**	**1-year**	**5-year** **Risk-free rate** **Constant (21.4%)**	**10-year**	**1-year**	**5-year** **Expected return** **Constant (21.4%)**	**10-year**
A Replacement rate at retirement	87.7%	87.7%	87.7%	94.5%	94.5%	94.5%
B Average replacement rate						
5% quantile	83.2%	83.3%	83.5%	83.0%	83.1%	83.3%
50% quantile (median)	94.7%	94.7%	94.7%	93.8%	93.8%	93.8%
95% quantile	109.4%	109.9%	110.0%	107.6%	108.0%	108.1%
C Volatility change replacement rate						
5-year horizon	5.7%	3.4%	2.2%	5.7%	3.5%	2.3%
Weighted average year-to-year volatility	2.6%	1.4%	1.2%	2.5%	1.3%	1.2%
F Growth rate						
Average growth rate until expected age of death	23.9%	24.0%	24.0%	-0.1%	-0.3%	-0.2%
Weighted average yearly growth rate	1.3%	1.2%	1.2%	0.0%	-0.1%	-0.1%
G Probability (large) decrease benefit level						
Probability decrease after 1 year	37.3%	11.9%	4.4%	49.2%	54.9%	56.8%
Probability decrease after 5 years	26.5%	21.7%	10.9%	51.8%	52.8%	54.6%
Probability decrease during last 5 years	26.5%	37.8%	39.5%	51.6%	50.8%	51.3%
Probability large decrease during last 5 years	12.1%	28.2%	31.7%	30.6%	40.4%	41.8%
Average probability decrease	38.2%	27.9%	23.9%	49.9%	52.4%	52.7%
Average probability large decrease (>5%)	7.0%	0.8%	1.1%	11.7%	2.4%	2.5%
H Average relative size decrease	3.0%	1.4%	1.3%	3.3%	1.8%	1.7%

Table 7.1: Indicators pension result in period 3 in case of financial smoothing around the expected return for different smoothing period lengths, different AIR and a constant equity exposure (21.4%).

When comparing the results of no financial smoothing (i.e. 1-year smoothing period) with the results of financial smoothing (i.e. 5-year and 10-year smoothing period) one notices that the differences in average replacement rate and average growth rate are small. However, the volatility of the change in replacement rate is significantly lower in case of financial smoothing. Note that the weighted average year-to-year volatility is similar for the 5-year and 10-year smoothing period. This does also hold for the average relative size of a decrease. Moreover, the expected probability of a large decrease (> 5%) in case of a 10-year smoothing period is even higher compared to a 5-year smoothing period. One can conclude that the benefits of using a 10-year smoothing period instead of a 5-year smoothing period are limited.

Probability (large) decrease benefit level

The average probability of a large decrease (5%) and the average relative size of a decrease are significantly lower in case of financial smoothing compared to no financial smoothing. Although the probabilities of a decrease after 1 or 5 years and the average probability of a decrease are significantly lower in case of financial smoothing with the risk-free rate as AIR, these probabilities are higher in case of financial smoothing with the expected return as AIR compared to no financial smoothing. One can conclude that the benefits of financial smoothing are smaller in case the expected return is used as AIR compared to the risk-free rate as AIR.

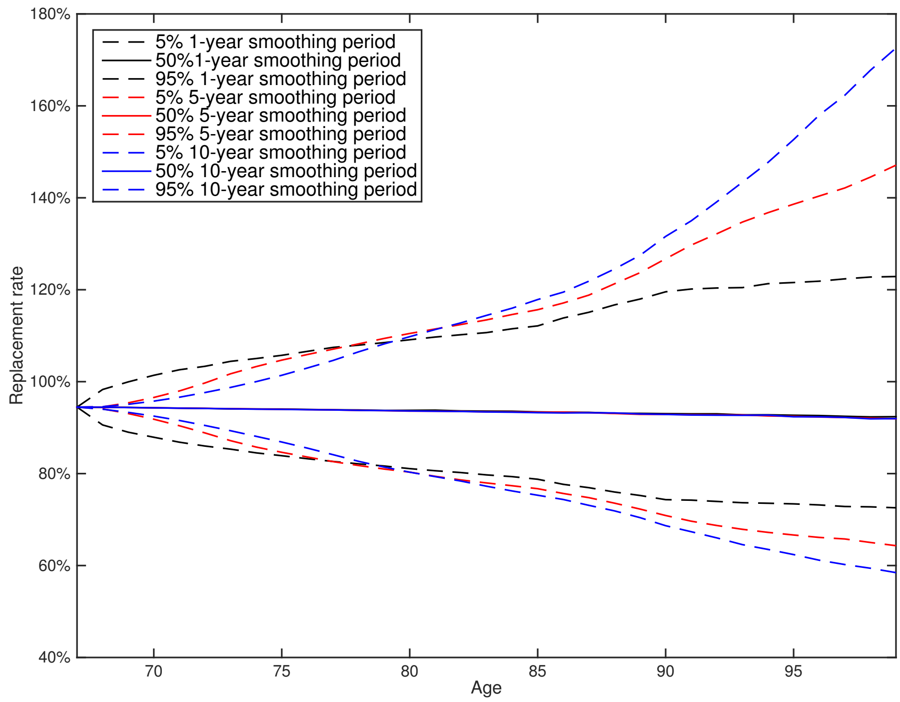

Figure 7.3: Replacement rate over time in case of financial smoothing around the expected return for different smoothing period lengths, expected return as AIR and constant equity exposure (21.4%).

Replacement rate over time

The replacement rate over time in case of financial smoothing for different lengths for the smoothing period is displayed in figure 7.3. One notices that financial smoothing does not have a significant impact on the median replacement rate over time. However, differences are significant for the 5% and 95% quantile, especially at high ages. During the first years after retirement there is less dispersion in the replacement rate in case of financial smoothing (i.e. 5-year and 10-year smoothing period) compared to no financial smoothing (i.e. 1-year smoothing period). This makes sense since the participant only absorbs part of the risk when financial shocks are smoothed. Obviously the difference with no financial smoothing is larger for the 10-year smoothing period compared to the 5-year smoothing period. However, there is no such thing as a free lunch: in return for less dispersion in the replacement rate during the first years after retirement there is more dispersion in the replacement rate at high ages in case of financial smoothing.

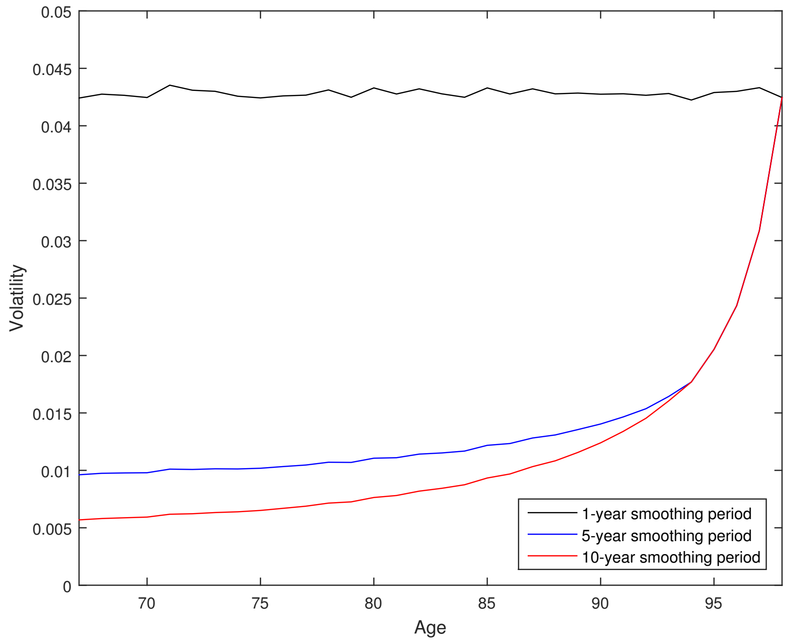

Figure 7.4: Year-on-year volatility of income during the retirement period in case of financial smoothing around the expected return for different smoothing period lengths, expected rate as AIR and constant equity exposure (21.4%).

Financial smoothing around the expected return in combination with a constant equity exposure leads to a lower year-on-year volatility of income due to lower risk absorption during the first years of retirement compared to no financial smoothing. This can be seen in figure 7.4. However, decreasing survival probabilities lead to an increasing yearly adjustment to the pension benefit level over time (see (7.2)), i.e. more risk absorption at higher ages. This implies an increasing year-on-year volatility of income over time. When looking at figure 7.4 we see that during the first years of retirement, the volatility of a 10-year smoothing period is lower compared to the volatility of a 5-year smoothing period. However, when the participant becomes older both volatilities converge towards each other since the capacity to absorb financial shocks for both smoothing periods becomes equivalent. Note that the volatility for both smoothing periods is still below the year-on-year volatility of income in case of no financial smoothing.

7.2.2 Financial smoothing around the risk-free rate

Smoothing period	**1-year**	**5-year**	**10-year**	**1-year**	**5-year**	**10-year**
Assumed interest rate (AIR)	**Risk-free rate**			**Expected return**		
Equity exposure	**Constant (21.4%)**			**Constant (21.4%)**		
A Replacement rate at retirement	87.7%	87.7%	87.7%	94.5%	94.5%	94.5%
B Average replacement rate						
5% quantile	83.2%	83.4%	83.6%	83.0%	83.2%	83.4%
50% quantile (median)	94.7%	95.0%	95.1%	93.8%	94.1%	94.2%
95% quantile	109.4%	110.5%	111.1%	107.6%	108.6%	109.1%
C Volatility change replacement rate						
5-year horizon	5.7%	3.3%	2.1%	5.7%	3.4%	2.1%
Weighted average year-to-year volatility	2.6%	1.4%	1.2%	2.5%	1.4%	1.2%
F Growth rate						
Average growth rate until expected age of death	23.9%	26.1%	26.8%	-0.1%	1.9%	2.8%
Weighted average yearly growth rate	1.3%	1.4%	1.6%	0.0%	0.1%	0.3%
G Probability (large) decrease benefit level						
Probability decrease after 1 year	37.3%	39.6%	38.9%	49.2%	85.1%	96.7%
Probability decrease after 5 years	26.5%	28.4%	28.8%	51.8%	60.6%	79.1%
Probability decrease during last 5 years	26.5%	19.7%	15.3%	51.6%	30.7%	22.9%
Probability large decrease during last 5 years	12.1%	13.2%	10.4%	30.6%	21.7%	16.6%
Average probability decrease	38.2%	27.6%	23.5%	49.9%	51.6%	51.9%
Average probability large decrease (>5%)	7.0%	0.4%	0.4%	11.7%	1.4%	0.9%
H Average relative size decrease	3.0%	1.4%	1.1%	3.3%	1.8%	1.6%

Table 7.2: Indicators pension result in period 3 in case of financial smoothing around the risk-free rate for different smoothing period lengths, different AIR and a constant equity exposure (21.4%).

When comparing the results of financial smoothing around the expected return in table 7.1 with the results of financial smoothing around the risk-free rate in table 7.2, one notices that the replacement rate at retirement is the same. This makes sense since financial smoothing is applied after retirement. Therefore, differences between both smoothing mechanisms will become apparent as of the first year after retirement. Some indicators of the pension result are (significantly) different in case of financial smoothing around the risk-free rate compared with financial smoothing around the expected return. These indicators will be discussed. First of all, one notices a slightly higher average replacement rate for all quantiles in case of smoothing around the risk-free rate. Moreover, the average probability of a large decrease is significantly lower in case of smoothing around the risk-free rate. On the other hand, the probability of a decrease after 1 or 5 years after retirement is significantly higher in case of smoothing around the risk-free rate. In case the expected return is used as AIR, the probability of a decrease during the first year equals 85.1% and 96.7% for a 5-year and 10-year smoothing period respectively. This is due to the fact that only a fraction of the risk premium is absorbed while the complete risk premium is included in the expected return as AIR.

Probability decrease first year

The intuition behind the high probability of a decrease during the first year after retirement in case of financial smoothing with the expected return as AIR can be shown via the following rule of thumb. As an example the 10-year smoothing period is taken. The 'fixed' yearly decrease is equal to the equity exposure times the risk premium in case the expected return is used as AIR (see (6.2)): $f(\mu - r) = 1.29\%$. The fraction of the risk premium absorbed during the first year is equal to the equity exposure times the risk premium divided by the N-duration (N-duration see table A.2): $f(\mu - r)/ND_t = 1.29\%/7.31 = 0.18\%$. This implies that the 'fixed' decrease is actually 1.11% too high in order to generate a constant expected pension benefit level. An actual decrease during the first year takes place if this 1.11% cannot be compensated by the unexpected financial shock. The unexpected financial shock compensates the too high 'fixed' decrease if it increases financial wealth with more than $1.11\% \cdot N$-duration $= 1.11\% \cdot 7.31 = 8.11\%$. The unexpected financial shock is equal to $f\sigma RN$ where RN represents the riskiness of the stock. Because it is assumed that the risky stock follows a geometric brownian motion (see (2.1b)), RN is a standard normal random variable. Therefore, the probability of a decrease during the first year after retirement equals $1 - \Phi(8.11\%/(f\sigma)) = 1 - \Phi(1.89) = 97.1\%$. This probability is approximately equal to the probability found in the model analysis (96.7%).

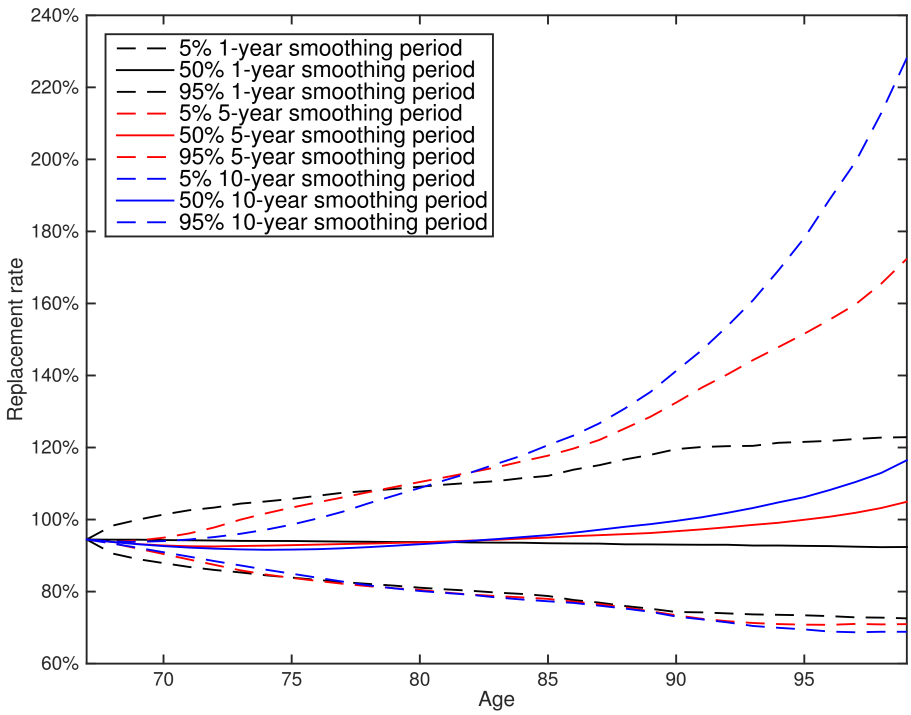

Figure 7.5: Replacement rate over time in case of financial smoothing around the risk-free rate for different smoothing period lengths, expected return as AIR and constant equity exposure (21.4%).

Replacement rate over time

When looking at the replacement rate over time in figure 7.5, one notices a significant increase in the 50% and 95% quantile for both the 5-year and 10-year smoothing period at high ages. This is the result of the higher risk premium absorption at high ages. This increase was not or less present in case of financial smoothing around the expected return since in case of smoothing around the expected return the risk premium is absorbed immediately instead of smoothed over several years.

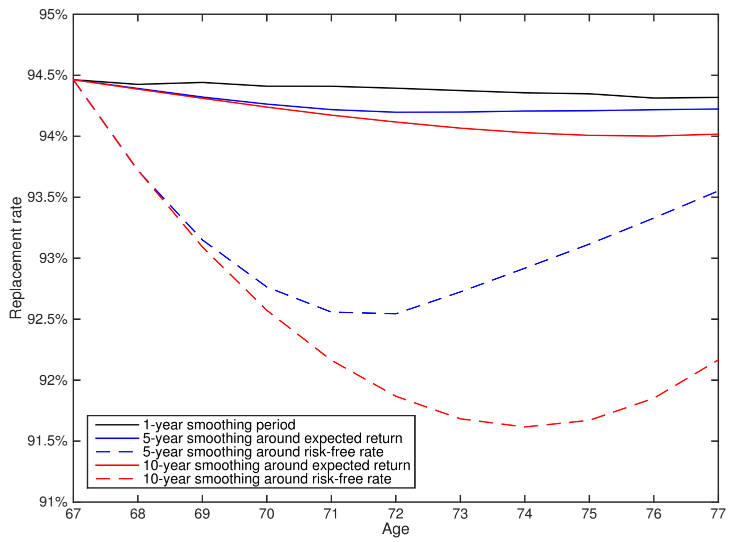

Figure 7.6: Expected replacement rate over time during the first 10 years of retirement in case of financial smoothing around the expected return and risk-free rate for different smoothing period lengths, expected return as AIR and constant equity exposure (21.4%).

Another consequence of smoothing around the risk-free rate is that the expected replacement rate[12] decreases during the first years of retirement. This can be seen more clearly in figure 7.6 which displays the expected replacement rate during the first 10 years of retirement in case of financial smoothing around the expected return and risk-free rate and using the expected return as AIR. In case of financial smoothing around the risk-free rate and using the expected return as AIR, the replacement rate decreases approximately 3% within 7 years in case of a 10-year smoothing period. This is due to the fact that only a fraction of the risk premium is absorbed while the complete risk premium is included in the expected return as AIR. After 7 years, the expected replacement rate starts to increase again. Of course such a significant expected decrease is not desirable. The decrease in the expected replacement rate is larger for the 10-year smoothing period compared to the 5-year smoothing period which makes sense. In case of financial smoothing around the expected return there is also a small decrease in the expected replacement rate, but the size of this decrease is small, namely less than 0.5%. For the 1-year smoothing period (i.e. no financial smoothing) there is no difference between smoothing around the expected return and smoothing around the risk-free rate since financial shocks are immediately absorbed in the pension benefit level.

To prevent the decrease in the expected replacement rate during the first years, one can apply a suitable horizon-dependent yearly decrease.[13] Note that a horizon-dependent yearly decrease is equivalent to a horizon-dependent AIR. This AIR is lower than the expected return during the first years after retirement to correct for the decrease. One can use an alternative financial smoothing approach in which the horizon-dependent AIR,

[12] Note that the expected replacement rate is not exactly equal to the median replacement rate since the pension wealth is log normally distributed (see footnote in appendix A.3).

[13] It is not clearly stated in the new legislation whether it is possible to use a horizon-dependent yearly decrease (see chapter 3).

which leads to a constant expected pension benefit level after retirement, can be derived in an elegant analytic way. This alternative financial smoothing approach and the suitable horizon-dependent AIR[14] will be discussed in section 7.5. The expected decrease during the first years of retirement can also be prevented by using a constant AIR which is lower than the expected return. Using a constant AIR lower than the expected return implies an increase in the expected replacement rate. Note that both solutions imply a (slightly) lower replacement rate at retirement.

In case a participant features habit formation, he especially dislikes decreases in the pension benefit level. Obviously, a decrease in the expected replacement rate is undesirable for such a participant. Both pension providers and participants should be aware of this decrease and might want to prevent it. Recall that participants should be informed about the level and risk of the pension benefit level at retirement and 10 years after retirement[15], but not necessarily about the pension benefit level at other moments in time. This implies that the decrease in the expected pension benefit level is not necessarily visible in the information provided by the pension provider. Of course, pension providers are free to provide more information about the level and risk of the pension benefit level. Although a lower AIR might be a better choice to prevent the undesirable expected decrease, competition incentives might be present among pension providers. Moreover, it seems doubtful whether participants are aware of the expected decrease. In case of myopic behavior of participants, participants might prefer the pension provider who offers the highest AIR since this yields the highest replacement rate at retirement.

Recall from the increasing year-on-year volatility of income over time in figure 7.4 as a result of decreasing survival probabilities in case of a constant equity exposure and smoothing around the expected return. Note that this increasing volatility is also present in case of smoothing around the risk-free rate. The risk premium does not contribute to the year-on-year volatility of income because the risk premium is not random.

7.2.3 Sensitivity analyses

This subsection contains two sensitivity analyses. First of all, the effect of using an alternative mortality table will be investigated. In the second analysis, the results in case of financial smoothing with an increased equity exposure will be compared with the results for the original equity exposure.

Alternative mortality table

A remarkably sharp increase in the year-on-year volatility of income during the last years of the retirement period is present in figure 7.4. This sharp increase is possibly a consequence of the usage of a mortality table (acquired from the 'Centraal Bureau voor de Statistiek', see appendix A.5) which includes death probabilities until age 100. It is assumed that the participant cannot become older than 100 years old. To verify whether this sharp increase is indeed a result of the restricted mortality table, I decided to use an alternative mortality table, namely the projection table of the Dutch 'Koninklijk Actuarieel Genootschap'.[16] This projection table includes death probabilities until age 120. In case this alternative mortality table is used in the model analysis instead of the mortal-

[14] Referred to as BNW AIR in this thesis.

[15] Article 54, paragraph 2, 'Besluit uitvoering PW en Wvb', added as a consequence of the 'Wet verbeterde premieregeling'.

[16] Prognosetafel AG2014. (`www.ag-ai.nl`)

ity table from the 'Centraal Bureau voor de Statistiek', the increase in the year-on-year volatility of income as of age 90 is much more moderate. This can be seen in figure 7.7. Until approximately age 95, the development of the year-on-year volatility of income for both mortality tables is equivalent (compare figure 7.4 and figure 7.7). Only during the last few years, both volatilities differ significantly. A sharp increase is also present in case of the alternative mortality table. However, it occurs at a later age, namely around age 115 which is also a few years before the end of the mortality table.

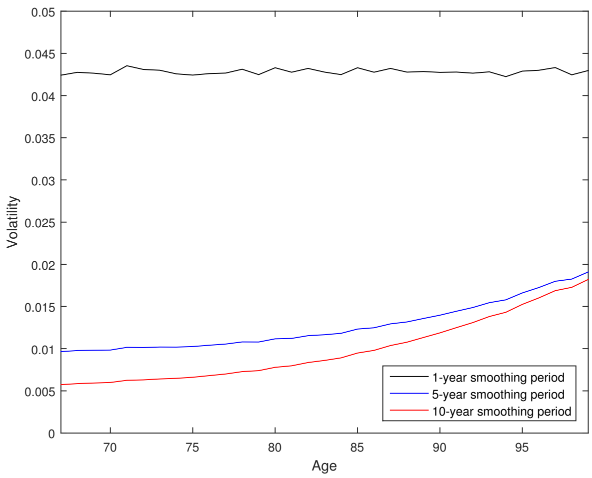

Figure 7.7: Year-on-year volatility of income during the retirement period in case of financial smoothing around the risk-free rate for different smoothing period lengths, expected return as AIR and constant equity exposure (21.4%) using an alternative mortality table.

Note that the year-on-year volatility of income still increases significantly, i.e. the volatility for the 10-year smoothing period doubles between age 67 and 90. The increase in the volatility is not only a consequence of the restricted mortality table but also due to the constant equity exposure. If a decreasing equity exposure is used, a (sharp) increase in volatility will not be present.

I also determined the replacement rate over time using the alternative mortality table. The development of the replacement rate over time is comparable with the replacement rate over time using the original mortality table (see figure 7.3).

Sensitivity analysis: increased equity exposure

As already mentioned before, financial smoothing reduces the volatility of the pension benefit level. When looking at the results in table 7.1 and table 7.2 we see that the weighted average year-to-year volatility decreases from 2.5% in case of no financial smoothing towards 1.2% in case of a 10-year smoothing period. As a consequence, it is sometimes argued that a higher constant equity exposure can be used in case of financial smoothing. However, this argument is not correct. Financial smoothing does not reduce the total amount of investment risk, but only shifts investment risks over time. To investigate the impact of taking more investment risk, I decided to compare the results of no financial smoothing using the original equity exposure (21.4%) with financial smoothing using an increased equity exposure. The increased equity exposure is chosen such that the average year-to-year volatility in case of no financial smoothing using the original equity exposure (2.5%) is the same as the average year-to-year volatility in case of a 10-year smoothing period using an increased equity exposure. This increased equity exposure equals 37.5%. Note that this equity exposure is exactly equal to the optimal equity exposure after retirement for a participant with CRRA preferences and risk aversion parameter $\gamma = 4$. The results are presented in table 7.3. As a benchmark the results for a 10-year smoothing period using the original equity exposure (21.4%) are also included.

Smoothing period **Assumed interest rate (AIR)** **Equity exposure**	**1-year** **Constant (21.4%)**	**10-year** **Expected return** **Constant (21.4%)**	**10-year** **Constant (37.5%)**
A Replacement rate at retirement	94.5%	94.5%	99.8%
B Average replacement rate			
5% quantile	83.0%	83.4%	80.1%
50% quantile (median)	93.8%	94.2%	98.8%
95% quantile	107.3%	109.1%	132.8%
C Volatility change replacement rate			
5-year horizon	5.6%	2.1%	3.7%
Weighted average year-to-year volatility	2.5%	1.2%	2.5%
F Growth rate			
Average growth rate until expected age of death	-0.5%	2.8%	5.9%
Weighted average yearly growth rate	0.0%	0.3%	0.5%
G Probability (large) decrease benefit level			
Probability decrease after 1 year	49.5%	96.7%	97.2%
Probability decrease after 5 years	52.3%	79.1%	81.0%
Probability decrease during last 5 years	51.8%	22.9%	23.4%
Probability large decrease during last 5 years	30.9%	16.6%	19.5%
Average probability decrease	50.2%	51.9%	52.9%
Average probability large decrease (>5%)	11.8%	0.9%	5.6%
H Average relative size decrease	3.4%	1.6%	2.8%

Table 7.3: Indicators pension result in period 3 in case of no financial smoothing using the original constant equity exposure (21.4%) and financial smoothing around the risk-free rate with a 10-year smoothing period, expected return as AIR and using the original constant equity exposure (21.4%) and increased constant equity exposure (37.5%).

When comparing the results of no financial smoothing using the original equity exposure (21.4%) with the results for a 10-year smoothing period using the increased equity exposure (37.5%) one notices that the dispersion in the average replacement rate is much higher in case of a 10-year smoothing period using the increased equity exposure. This makes sense since smoothing of financial shocks does not significantly reduce the dispersion in the average replacement rate. This can also be seen in figure 7.8 which displays the replacement rate over time. One notices that the average probability of a decrease is slightly higher in case of a 10-year smoothing period using the increased equity exposure (37.5%) while the average probability of a large decrease (>5%) and the average relative size of a decrease are lower in case of a 10-year smoothing period using the increased equity exposure (37.5%) compared to no financial smoothing using the original equity exposure (21.4%). Based on the much higher dispersion in the average replacement rate, taking more investment risk in case of financial smoothing is not desirable.

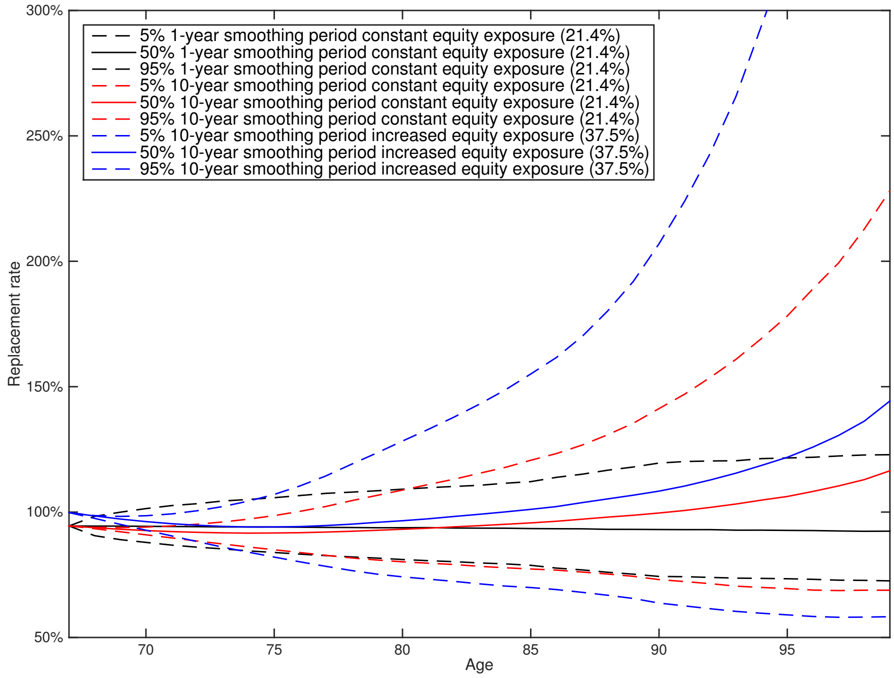

Figure 7.8: Replacement rate over time in case of no financial smoothing using the original constant equity exposure (21.4%) and financial smoothing around the risk-free rate with a 10-year smoothing period using the original constant equity exposure (21.4%) and increased constant equity exposure (37.5%) with the expected return as AIR.

7.3 Financial smoothing with life-cycle investing

In this section the dependence between financial smoothing and the equity exposure will be investigated. As already mentioned, using a constant equity exposure during retirement in combination with financial smoothing is not desirable because this leads to more investment risk at high ages due to decreasing survival probabilities. One can solve this problem by applying a sustainable and efficient investment life-cycle[17] during the retirement period as suggested by Bovenberg et al. (2014b). This implies that the equity exposure is not constant over time anymore. This sustainable and efficient investment life-cycle yields a constant year-on-year volatility over time. To realize this constant volatility of income the equity exposure should be proportional to the N-duration since the yearly adjustment $x_t\%$ is inversely proportional to the N-duration (see (7.2)). A lower N-duration (when the participant is older) implies a lower equity exposure. The equity exposure $\hat{f}_t$ of the sustainable and efficient investment life-cycle equals

$$\hat{f}_t = \xi \cdot \mathrm{ND}_t, \tag{7.4}$$

where ξ is a scale factor which determines the amount of investment risk.[18] $\hat{f}_t$ is decreasing in t since ND_t is decreasing in t. In the model analysis I decided to determine ξ such that the replacement rate at retirement in case of a decreasing equity exposure $\hat{f}_t$ is the same

[17] Denoted by 'duurzaam beleggingsprofiel' on page 21 in Bovenberg et al. (2014b).
[18] See page 21 in Bovenberg et al. (2014b).

as the replacement rate at retirement in case a constant equity exposure is used. When using the constant equity exposure of 21.4%, the scale factor ξ equals 0.033. $\hat{f}_t$ implies a lower expected pension benefit level at high ages and a different expected return which in turn leads to a different N-duration. The new N-duration implies a new $\hat{f}_t$. This is an iterating process. After a few steps the N-duration and $\hat{f}_t$ converge to equilibrium values.

In case a participant prefers a constant year-on-year volatility of income in case of financial smoothing, it is optimal to use a sustainable and efficient investment life-cycle (see (7.4)) during the retirement phase in case of financial smoothing. In section 7.4 the results of this decreasing life-cycle will be compared with the results for a constant life-cycle.

To get an indication of the difference in result between a constant equity exposure and a decreasing equity exposure in case of financial smoothing, the development of the replacement rate during the retirement period in one single scenario for both equity exposures will be considered. This scenario is depicted in figure 7.9.

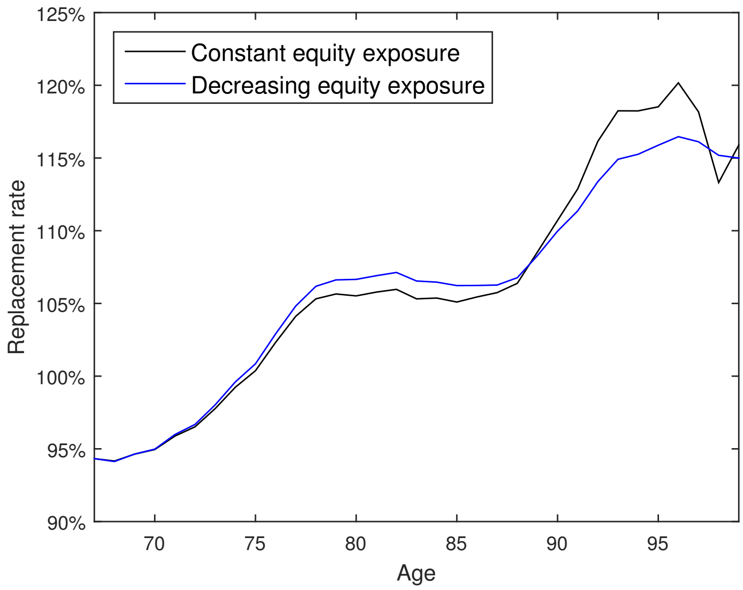

Figure 7.9: Replacement rate during the retirement period in case of financial smoothing around the expected return with a 10-year smoothing period, expected return as AIR and different equity exposures in one individual scenario.

During the first years, the development of the replacement rate over time is similar for both equity exposures. At high ages both replacement rates develop differently. The replacement rate is more volatile in case of a constant equity exposure. The graph of the constant equity exposure contains some peaks which are not present in the graph of the decreasing equity exposure due to the fact that in case of a constant equity exposure more risk must be absorbed at higher ages.

7.4 Results financial smoothing with life-cycle investing

In this section the results of life-cycle investing during the retirement period in case of financial smoothing are presented and discussed. The results for a decreasing life-cycle in case of financial smoothing with a 10-year smoothing period will be compared with the results for a constant life-cycle. Since financial shocks cannot be smoothed around the expected return in practice (because the risk premium on assets is unknown a priori), only financial smoothing around the risk-free rate will be considered in this section.

Figure 7.10 includes the sustainable and efficient equity exposure $\hat{f}_t$ stated in (7.4) which leads to a constant year-on-year volatility of income during the retirement period. For comparison this graph also includes the constant equity exposure which is optimal for a participant with CRRA preferences in case of no financial smoothing.

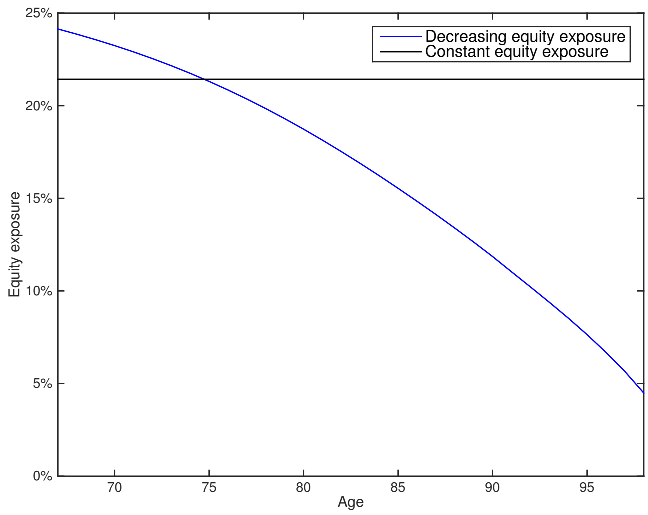

Figure 7.10: Constant equity exposure (21.4%) and decreasing equity exposure (i.e. the sustainable and efficient investment life-cycle $\hat{f}_t$ with $\xi = 0.033$) in case of financial smoothing with a 10-year smoothing period.

Table 7.4 contains the indicators of the pension result in period 3 in case of financial smoothing around the risk-free rate for a 10-year smoothing period using both a constant and decreasing equity exposure (i.e. the sustainable and efficient investment life-cycle) and the risk-free rate and expected return as AIR. While the expected return as AIR is constant in case of a constant equity exposure, this does not hold anymore in case of a decreasing equity exposure. Because the participant reduces his equity exposure over time during retirement, the AIR should decrease as well because the participant takes less investment risk at high ages (i.e. lower expected investment returns). As a consequence, the expected return is horizon-dependent in case of a decreasing equity exposure.[19]

[19] It is not clearly stated in the new legislation whether it is possible to use a horizon-dependent yearly decrease (see chapter 3).

Equity exposure Assumed interest rate	Constant	Decreasing Risk-free	Constant	Decreasing Expected return
A Replacement rate at retirement	87.7%	87.7%	94.5%	94.5%
B Average replacement rate				
5% quantile	83.6%	83.5%	83.4%	83.3%
50% quantile (median)	95.1%	94.9%	94.2%	94.2%
95% quantile	111.1%	110.1%	109.1%	108.6%
C Volatility change replacement rate				
5-year horizon	2.1%	2.3%	2.1%	2.4%
Weighted average year-to-year volatility	1.2%	1.1%	1.2%	1.1%
F Growth rate				
Average growth rate until expected age of death	26.8%	27.7%	2.8%	3.9%
Weighted average yearly growth rate	1.6%	1.4%	0.3%	0.3%
G Probability (large) decrease benefit level				
Probability decrease after 1 year	38.9%	39.5%	96.7%	96.7%
Probability decrease after 5 years	28.8%	29.1%	79.1%	78.6%
Probability decrease during last 5 years	15.3%	14.8%	22.9%	19.8%
Probability large decrease during last 5 years	10.4%	5.9%	16.6%	8.8%
Average probability decrease	23.5%	23.5%	51.9%	50.0%
Average probability large decrease (>5%)	0.4%	0.1%	0.9%	0.3%
H Average relative size decrease	1.1%	1.0%	1.6%	1.4%

Table 7.4: Indicators pension result in period 3 in case of financial smoothing around the risk-free rate with a 10-year smoothing period, different AIR's using a constant equity exposure (21.4%) and decreasing equity exposure (i.e. the sustainable and efficient investment life-cycle).

When comparing the results of a constant equity exposure with the results of a decreasing equity exposure in table 7.4, one notices that the replacement rate at retirement is the same. This makes sense since ξ in (7.4) has been chosen such that the replacement rate at retirement in case of a decreasing equity exposure is the same as the replacement rate at retirement in case of a constant equity exposure. The differences in result between the constant equity exposure and decreasing equity exposure for most indicators are small. This can also be concluded from figure 7.11 which displays the development of the replacement rate over time. One notices that using a decreasing equity exposure instead of a constant equity exposure does not have a significant impact on the replacement rate over time during the first years of retirement. However, differences appear at high ages (above age 87). There is less dispersion in the replacement rate in case of a decreasing equity exposure which was aimed at. This is reflected in the probability of a large decrease (>5%) during the last 5 years which has been halved. As a result, the average probability of a large decrease (>5%) is also much lower.

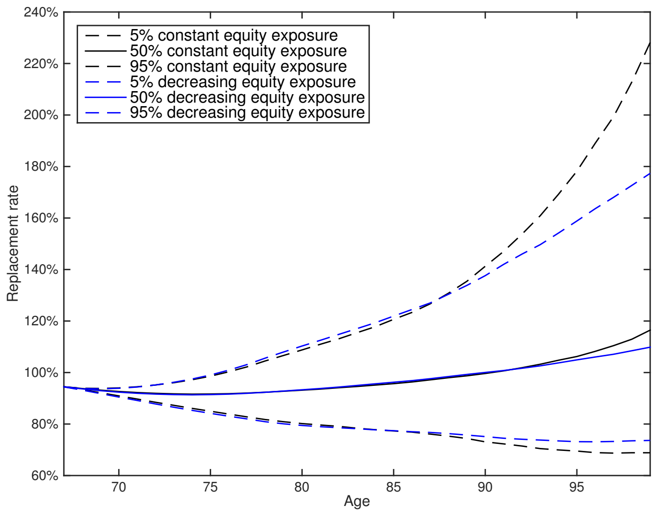

Figure 7.11: Replacement rate over time in case of financial smoothing around the expected return with a 10-year smoothing period, expected return as AIR and different equity exposures.

As already mentioned in section 7.2, the year-on-year volatility of income increases over time due to less recovery capacity at higher ages in case of financial smoothing with a constant equity exposure. Using the sustainable and efficient investment life-cycle (7.4) leads to a constant year-on-year volatility of income. Both volatilities are visualized in figure 7.12. This figure compares the year-on-year volatility of income for both no financial smoothing and smoothing with a 10-year smoothing period and for both a constant equity exposure and decreasing equity exposure (i.e. the sustainable and efficient investment life-cycle).

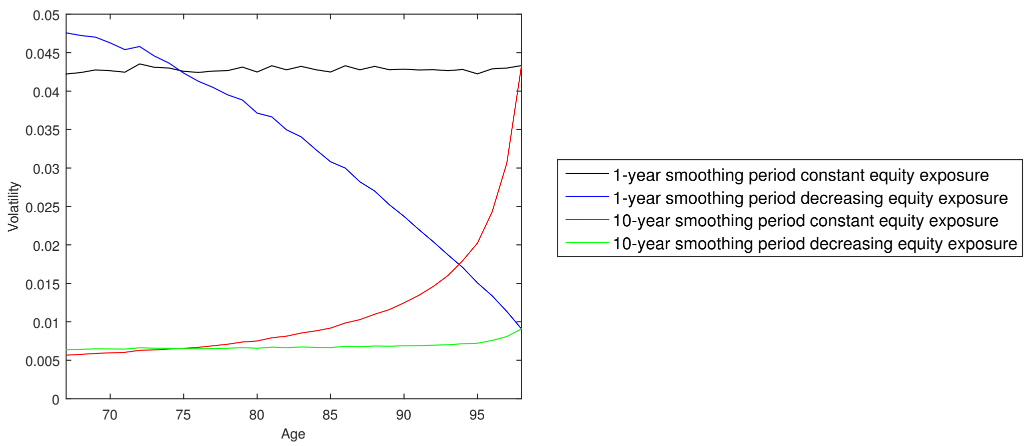

Figure 7.12: Year-on-year volatility of income during the retirement period in case of no financial smoothing and financial smoothing around the risk-free rate with a 10-year smoothing period, expected return as AIR and different equity exposures.

7.5 Alternative financial smoothing approach: money pots approach

A benefit of the financial smoothing approach discussed in section 7.1 (the growth rate approach) is that it reduces the year-to-year volatility of the pension benefit level. This reduction in volatility can also be achieved via an alternative approach which will be considered in this section. Both financial smoothing approaches can be used in practice since it is not clearly specified in the new legislation how the financial smoothing approach should be modeled. In the money pots approach, the initial pension wealth is allocated to money pots. Each money pot corresponds to a different pension benefit payment. The money pots will be invested in a different way depending on the horizon of the pension benefit.[20] The concept of this alternative approach will be discussed below. First of all, an example will be presented to get an idea about the functioning of the money pots approach.

Example
We start in a simplified setting which contains no micro longevity risk. We assume the participant retires at age 67, lives until age 84 and dies for sure at age 85. This means that the participant will receive 18 pension benefit payments. At the retirement age the participant owns a large amount of pension capital. One can allocate this pension capital to 18 money pots which will be spent at different horizons (i.e. $h = 0, ..., 17$). The money in the first pot ($h = 0$) will be consumed immediately. The money in the second pot ($h = 1$) will be partly invested in risky assets for one year and consumed after one year. One can continue in this way for each horizon h.

The participant has to decide how much pension capital should be put into each money pot. Recall from the model analysis that the assumed interest rate (AIR) determines how the available pension wealth is distributed over the various pension payments (via the actuarial fair price of a lifelong sustainable annuity (4.1). The same holds for the money pots approach: the AIR determines how much money is put into each money pot. In case the participant prefers a constant expected pension benefit level over time, less money should be put into money pots which will be consumed at a high age (compared to money pots which will be consumed shortly after retirement) since these money pots can be invested for several years. Moreover, the higher the AIR, the less money is put into money pots which will be consumed at a high age.

Note that in the money pots approach financial shocks are smoothed around the risk-free rate by definition since the compensation for risk taking (i.e. risk premium) is absorbed when the risk is actually taken, namely when the money pot is consumed.

Equity exposure money pots
Since the money in the first pot ($h = 0$) will be consumed immediately, it is not invested in risky stocks. In order to take an appropriate amount of investment risk, the equity exposure of the second money pot ($h = 1$), which contains the pension wealth which will be consumed next year, should be equal to f/N instead of f where f represents the constant equity exposure without financial smoothing. In the same way, the equity exposure of the third money pot ($h = 2$) should be equal to $2f/N$ during the first year and f/N during the second year. This can be done for each horizon h. In case we define $W_T(h)$ as the

[20] In practice, the pension pots will not be invested separately. Instead, the different equity exposures of the money pots are added together and invested as a whole in each period.

pension wealth at retirement (time T) which will be consumed at time $T+h$ (i.e. pension capital allocated to money pot h) and $q_{T+i}(h)$ as the equity exposure at time $T+i$ of the pension wealth $W_T(h)$ then this $q_{T+i}(h)$ is defined as follows

$$q_{T+i}(h) = \begin{cases} f\frac{h-i}{N} & \text{if } h-i < N \\ f & \text{if } h \geq N. \end{cases} \tag{7.5}$$

This holds for $i = 0, ..., h-1$ years after retirement and horizon $h = 1, ..., 17$. The development of $q_{T+i}(h)$ for $h = 17$ (i.e. last pension payment) over time is visualized in figure 7.13. Note that in case of financial smoothing the equity exposure is no longer constant: the equity exposure $q_{T+i}(h)$ implies a decreasing life-cycle strategy.

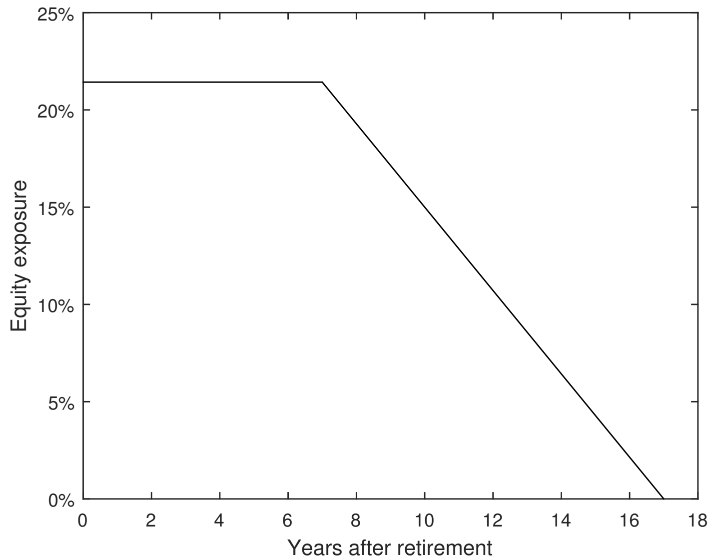

Figure 7.13: Equity exposure of pension wealth which will be consumed at time $T+h$ with $h=17$ in case of financial smoothing for a 10-year smoothing period and equity exposure (21.4%) in the money pots approach.

Assumed interest rate (AIR)

We are interested in the AIR which yields an exact constant expected pension benefit level during the retirement period. Recall from subsection 7.2.2 that the AIR must be horizon-dependent in case of financial smoothing in order to obtain an exact constant expected pension benefit level. It turns out that the appropriate horizon-dependent AIR(h) equals

$$\text{AIR}(h) = r + Q(h)(\mu - r). \tag{7.6}$$

with $Q(h)$ defined as follows

$$Q(h) = \begin{cases} f(h+1)/(2N) & \text{if } h \leq N \\ f(1-(N-1)/(2h)) & \text{if } h \geq N, \end{cases}$$

which holds for $h = 1, ..., 17$. $Q(h)$ represents the average equity exposure of the pension wealth which will be consumed after h years. The derivation of the horizon-dependent AIR(h) in (7.6) can be found in appendix A.3.

The derivation of $Q(h)$ is in line with Bovenberg and Nijman (2011).[21] We call the horizon-dependent $\text{AIR}(h)$ in (7.6) the BNW AIR. The BNW AIR for a 10-year smoothing period and equity exposure (21.4%) is visualized in figure 7.14.

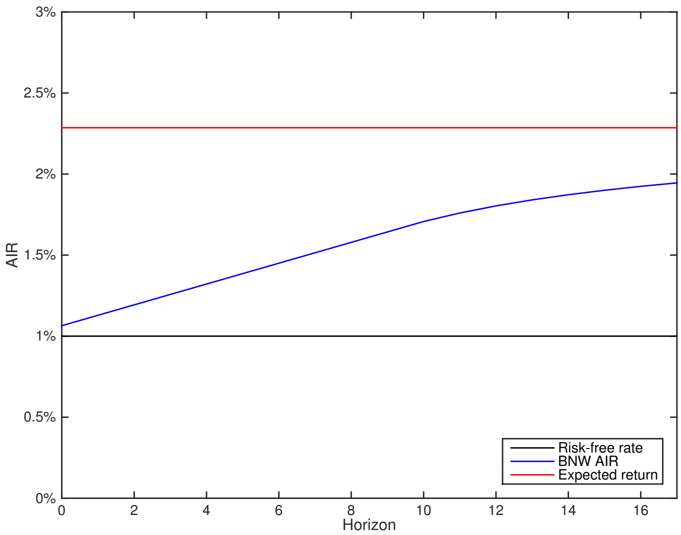

Figure 7.14: Risk-free rate, BNW AIR and expected return in case of financial smoothing with a 10-year smoothing period and equity exposure (21.4%).

It is important to be aware of the difference between $q_{T+i}(h)$ and $Q(h)$. $q_{T+i}(h)$ is equal to the actual market equity exposure at time $T+i$ of the pension wealth $W_T(h)$ which will be consumed at time $T+h$. $Q(h)$ equals the average equity exposure of the pension wealth consumed at time $T+h$ over the horizon h.

Allocation initial pension wealth

The BNW AIR determines how the available pension wealth should be allocated to the pension pots such that it yields an exact constant expected pension benefit level during the retirement period. More concretely, the pension wealth $W_T(h)$ at retirement (time T) which will be consumed at $T+h$ equals

$$W_T(h) = C_{T+h} \exp(-h \cdot \text{AIR}(h)),$$

where $W_T(0)$ equals the constant pension benefit level.

[21] See page 4 in Bovenberg and Nijman (2011).

Micro longevity risk
Recall that we started this section in a simplified setting with no micro longevity risk. In case one includes micro longevity risk, one has to allocate pension capital to money pots at different horizons where the last horizon corresponds to the maximum age of death. Since there is a probability that the participant dies, the pension capital in the money pot which will be consumed at $T + h$ should be corrected with the survival probability $p_T(T + h)$ (i.e. the probability at time T that the participant is still alive at time $T + h$. The pension wealth $W_T(h)$ then equals

$$W_T(h) = p_T(T + h)W_T(0)\exp(-h \cdot \mathrm{AIR}(h)), \tag{7.7}$$

where W_{T+h} equals the (constant) pension benefit level.

Just as in the model analysis, it is assumed that micro longevity risk is shared in an (infinitely large) pool and that participants receive a biometric return for staying alive.

Implicit life-cycle
The equity exposure $q_{T+i}(h)$ in (7.5) equals the equity exposure over time for each money pot with horizon h. In practice, the different money pots will not be invested separately, but a certain fraction of the total pension wealth will be invested. Therefore, we are interested in the equity exposure at time t of the total pension wealth W_t. This equity exposure will be denoted by $\tilde{f}_t$ and will be referred to as the implicit life-cycle of the money pots approach. Note that the implicit life-cycle of the money pots approach decreases over time because the equity exposure of each money pot decreases over time (see figure 7.13). The life-cycle is equivalent to the investment mix of a pension fund in case of financial smoothing as derived by Nijman et al. (2013). The derivation can be found in appendix A.3 with $\tilde{f}_t$ stated in (A.24). $\tilde{f}_t$ is visualized in figure 7.15.

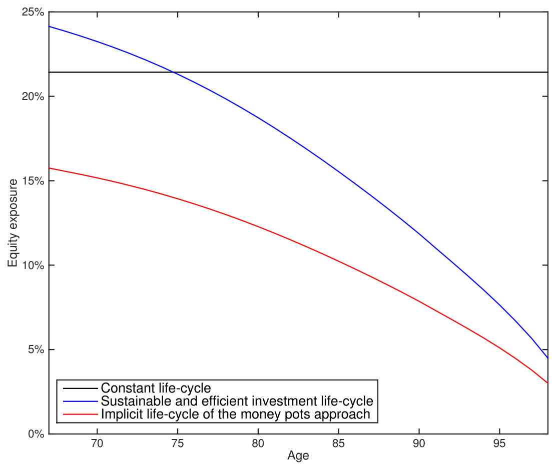

Figure 7.15: Constant life-cycle (21.4%), the sustainable and efficient investment life-cycle $\hat{f}_t$ with $\xi = 0.033$ and the implicit life-cycle of the money pots approach $\tilde{f}_t$ in case of financial smoothing with a 10-year smoothing period.
The sustainable and efficient investment life-cycle is equal to the implicit life-cycle of the money pots approach for $\xi = 0.029$.

In figure 7.15 different equity exposures are compared. The sustainable and efficient investment life-cycle $\hat{f}_t$ of the growth rate approach lies above the implicit life-cycle of the money pots approach $\tilde{f}_t$. However, if ξ in $\hat{f}_t$ in (7.4) is chosen appropriately, both life-cycles are equivalent. This is the case for $\xi = 0.029$. Note that the equivalence of both life-cycles implies that the year-on-year volatility of income is also constant in the money pots approach.

Equivalence financial smoothing approaches

It is shown above that if ξ in $\hat{f}_t$ (7.4) is chosen appropriately, the sustainable and efficient investment life-cycle $\hat{f}_t$ of the growth rate approach and the implicit life-cycle of the money pots approach $\tilde{f}_t$ are equivalent. This gives rise to the presumption that both financial smoothing approaches lead to the same results. If that is indeed the case, using the implicit life-cycle of the money pots approach $\tilde{f}_t$ (or the sustainable and efficient investment life-cycle $\hat{f}_t$ of the growth rate approach with $\xi = 0.029$) in combination with the BNW AIR as AIR also yields a constant expected pension benefit level over time in the growth rate approach.

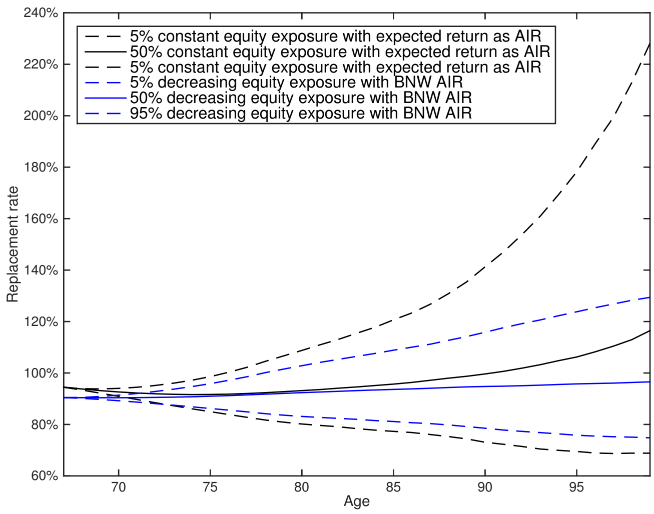

Figure 7.16: Replacement rate over time in case of financial smoothing around the risk-free rate with a 10-year smoothing period, a constant equity exposure (21.4%) with expected return as AIR and a decreasing equity exposure (i.e. the implicit life-cycle of the money pots approach) with the BNW AIR as AIR.

Implementation growth rate approach

I implemented the growth rate approach in case of financial smoothing around the risk-free rate[22] with a 10-year smoothing period applying the implicit life-cycle of the money pots approach $\tilde{f}_t$ in combination with the BNW AIR as AIR. The resulting replacement rate over time is displayed in figure 7.16. For comparison, this figure also contains the replacement rate over time in case of financial smoothing around the risk-free rate with a 10-year smoothing period, a constant equity exposure and the expected return as AIR.

[22] Note that the equivalence between both smoothing approaches does not hold in case of financial smoothing around the expected return in the growth rate approach.

When comparing both replacement rates, one notices that the decrease in the median[23] replacement rate during the first years after retirement is (by approximation) no longer present in case the BNW AIR is used. This is a result of the application of the BNW AIR as AIR instead of the expected return of the actual equity exposure as AIR. One notices that as of age 75 the median replacement rate starts to increase slowly. This result is not in line with the expectations of a constant expected pension benefit level and cannot be clearly explained. Note that the increase is far less significant compared to the increase in the median replacement rate in case of a constant equity exposure with expected return as AIR. The increase is possibly related to the inaccuracy in the implementation of the smoothing approach. The relative importance of biometric returns compared to financial returns at higher ages in combination with the discretization of the model analysis might increase the inaccuracy of the smoothing approach at higher ages. It has to be investigated more extensively to verify whether this is indeed the case. Another recommendation for future research is to implement both financial smoothing approaches (growth rate approach and money pots approach) and verify whether both approaches indeed yield the same results.

[23] Note that the expected replacement rate is not exactly equal to the median replacement rate since the pension wealth is log normally distributed (see footnote in appendix A.3).

8. Conclusion & recommendations

During the last few years the Dutch pension scheme has been put under pressure. The Dutch pension system is currently characterized by a high degree of collectivity and limited scope for freedom of choice and customization for individual participants. Several social developments (such as individualization and labor mobility) have resulted in an increasing demand for freedom of choice and customization. A new legislation in the Netherlands[1] introduces more freedom of choice for participants in DC pension schemes by making it possible to invest after retirement.

In this thesis several aspects of both the investment policy as the pension benefit payment policy in DC pension schemes are investigated. Below the most important conclusions, which give an answer to the research questions formulated in the introduction, are presented.

Investment policy

A decreasing life-cycle is preferred above a constant life-cycle.
It can be concluded from the results of the model analysis that a decreasing life-cycle leads to better results compared to a constant life-cycle. This is line with the standard life-cycle theory of Merton (1971) which states that a suitable life-cycle strategy is characterized by a decreasing equity exposure over time until retirement and a constant equity exposure after retirement (based on the assumption of risk-free human capital and CRRA preferences). The welfare loss[2] of a constant life-cycle relative to the optimal Merton life-cycle lies between 4% and 6%. Welfare losses are higher for less risk averse participants since investment after retirement yields higher benefits for such participants relative to more risk averse participants.

Welfare losses due to an inadequate amount of investment risk are higher than welfare losses due to the implementation of a suboptimal decreasing life-cycle.
An important conclusion from the model analysis is the difference in size of welfare losses. Welfare losses can occur due to the implementation of a suboptimal decreasing life-cycle (e.g. linearly decreasing life-cycle instead of the optimal Merton life-cycle which exhibits an exponential decrease) and due to an inadequate amount of investment risk (e.g. due to inadequate quantification of the risk aversion of the participant). The welfare loss of using a linearly decreasing life-cycle instead of the optimal life-cycle is higher for less risk averse participants because for such participants the optimal life-cycle exhibits a sharp exponential decrease. Welfare losses due to an inadequate amount of investment risk are also higher for less risk averse participants since the benefits of investing in risky assets are higher for such participants. Obviously, the size of the welfare loss depends on the size of the inadequacy. When comparing the different welfare losses, one can conclude that welfare losses due to an inadequate amount of investment risk are higher than welfare losses due to the implementation of a suboptimal decreasing life-cycle. This implies that a DC pension scheme in which the participant has freedom of investment has added value

[1] 'Wet verbeterde premieregeling' ('Stb. 2016', 248).
[2] Welfare losses are calculated in terms of the certainty equivalent consumption during the retirement period based on the common assumption that the participant features CRRA preferences.

for the participant compared to a DC pension scheme in which the participant has no freedom of investment.

A welfare loss due to no investment after retirement is significantly higher than a welfare loss due to temporarily using an inadequate life-cycle before retirement.
Finally, the welfare loss due to no investment risk after retirement lies between 4% and 9%. This confirms the hypothesis that investment after retirement can be welfare improving. Temporarily choosing a fixed annuity instead of a variable annuity before retirement also leads to a welfare loss in case the participant prefers a variable annuity at retirement. This welfare loss is especially significant for the less risk averse participant, namely 4%[2]. However, a welfare loss due to no investment after retirement is significantly higher than a welfare loss due to temporarily using an inadequate life-cycle before retirement. This holds for each level of risk aversion.

Pension benefit payment policy

It takes approximately 10 years after retirement until a low AIR yields a higher replacement rate than a higher AIR.
An interesting conclusion from the pension benefit policy investigation is that it takes approximately 10 years[3] after retirement until a low AIR (such as the risk-free rate) yields a higher replacement rate than a higher AIR. This holds for each equity exposure. If pension providers provide information about the level and risk of the pension benefit level at retirement and 10 years after retirement only[4], the disadvantage of a high AIR (i.e. a lower pension benefit level at high ages) compared to the risk-free rate as AIR will not be communicated to the participant. Moreover, the increase in the replacement rate at retirement because of a higher AIR is highly dependent on the equity exposure after retirement.

A decreasing life-cycle during retirement is desirable for a participant preferring a constant year-on-year volatility of income in case of financial smoothing.
In case the participant does not feature CRRA preferences but features internal habit formation, the participant maximizes utility of consumption relative to a reference level which depends on previous consumption. In that case smoothing of financial shocks can be beneficial since it reduces the year-to-year volatility of the pension benefit level. However, there is no such thing as a free lunch: although the year-to-year volatility of the pension benefit level decreases, more investment risk should be absorbed at high ages. The increased investment risk at high ages is undesirable for a participant preferring a constant year-on-year volatility of income during retirement. For such a participant, a decreasing life-cycle is desirable. In that case the life-cycle during both the accumulation phase (based on the assumption of risk-free human capital) as the decumulation phase (based on a constant year-on-year volatility of income) is characterized by a decreasing life-cycle in case of financial smoothing. An additional benefit of using a decreasing life-cycle in case of financial smoothing (besides the constant year-on-year volatility of income) is the much lower average probability of a large decrease (>5%) in case of a decreasing life-cycle compared to a constant life-cycle after retirement.

[3] The duration of the pension liabilities equals approximately 10 years.
[4] Pension providers are not necessarily required to provide more information, although they are free to do so.

One should use a suitable horizon-dependent AIR to prevent a significant decrease in the expected replacement rate during the first years after retirement in case of financial smoothing around the risk-free rate in combination with the expected return as AIR.
An undesirable effect of financial smoothing is that financial smoothing around the risk-free rate in combination with the expected return as AIR implies a significant decrease in the expected replacement rate during the first years of the retirement period (up to 3% during the first seven years in case of a 10-year smoothing period). Moreover, the probability of a decrease in the pension benefit level during the first year after retirement is very high: it equals 85% in case of a 5-year smoothing period and 97% in case of a 10-year smoothing period. The expected decrease can be prevented by making use of a suitable horizon-dependent AIR.[5] This AIR is lower than the expected return during the first years after retirement to correct for the decrease. The decrease can also be prevented by applying a horizon-independent AIR which is lower than the expected return. Note that both corrections imply a (slightly) lower replacement rate at retirement.

One can use an alternative smoothing approach (the money pots approach) in which the horizon-dependent AIR (BNW AIR), which leads to a constant expected pension benefit level after retirement, can be derived in an elegant analytic way. This horizon-dependent AIR can also be implemented using the growth rate approach. A recommendation for future research is to investigate whether the implementation of both financial smoothing approaches (growth rate approach and money pots approach) yields similar results.

Recommendations for future research
The Merton model, which is used as benchmark model in the model analysis, is based on several assumptions. Although some of these assumptions have been relaxed in the model analysis, there are still restricting assumptions which do not hold in practice. In the subsequent paragraphs recommendations for future research regarding these restricting assumptions are given.

Implement a financial market model including interest rate risk and inflation risk.
The results of the model analysis are based on a model which does not include interest rate risk and inflation risk. In case these risks are included in the model an additional decision needs to be made, namely to which extent interest rate risk and inflation risk should be hedged. Moreover, these risks have an impact on the optimal equity exposure and AIR. The AIR becomes stochastic since it depends on the interest rate. One has to be aware of this possible impact when interpreting the results of this thesis. The impact of interest rate risk and inflation risk on the optimal equity exposure and AIR is an interesting research area for future investigation. Furthermore, the determination of the interest rate and inflation hedge and the choice of bonds (i.e. investment in short-term or long-term bonds) are interesting research topics.

Investigate the impact of more freedom of choice regarding pension contributions.
Since this thesis is related to the a new legislation in the Netherlands for IDC pension schemes[6], most characteristics of IDC pension schemes are taken into account to make the results of the model analysis as realistic as possible. One of these characteristics are predetermined pension contributions. The pension contribution in most IDC schemes is based on a certain premium ladder. This implies that the participant is currently not able to

[5] Referred to as BNW AIR in this thesis.
[6] The new legislation also applies to CDC pension schemes.

adjust the pension contribution based on his preferences and/or actual investment returns.

van den Bleeken et al. (2016) investigated several choice options for participants in pension schemes using a welfare analysis. Based on welfare calculations they conclude that more freedom of choice regarding pension contributions (i.e. customized pension contributions) implies higher welfare gains compared to more freedom of choice regarding the investment policy. It can be interesting to include choice options regarding pension contributions (e.g. customized pension contributions adjusted to the capital accumulation due to home ownership) in the model analysis and to investigate the impact of such a flexibility on the investment and pension benefit payment policy.

One also has to keep in mind that more freedom of choice does not necessarily increase welfare. Since the behavior of participants is characterized by bounded rationality, there also exist welfare losses which result from inadequate choices (e.g. too little pension savings resulting from myopic behavior). As a consequence, a trade-off has to be made between wrong choices due to behavioral imperfections and lack of discretion to tailor individual choices to personal circumstances (see Bovenberg and Nijman (2015)).

Use a utility function which takes habit formation into account.
Another strong assumption in the Merton model is the assumption of a constant risk aversion which is often rejected in empirical research. Instead of assuming that the preferences of the participant can be characterized by a CRRA utility function, alternative utility functions can be explored. Although internal habit formation has been investigated in an implicit way by investigating the effects of financial smoothing in chapter 7, I did not investigate alternative utility functions. Utility functions which take habit formation into account have for example been explored by van Bilsen (2015). The model analysis could be extended by implementing this such alternative utility functions.

Furthermore, empirical research on the minimum consumption level among Dutch participants can provide interesting insights. In the model analysis it is assumed that the minimum consumption level is equal to the state pension. However, I did not investigate to which extent this holds and which impact the minimum consumption level has on the investment policy if this does not hold. In case the minimum consumption level lies above the state pension, the optimal equity exposure is probably lower. Since the participant wants to receive a higher minimum consumption for sure, more investment in risk-free bonds in desirable.

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A. Appendix

The appendix is built up as follows. First of all, some mathematical derivations are presented in appendix A.1, A.2 and A.3. In appendix A.4 the discretization implemented in the model analysis is explained. Appendix A.5 contains the premium ladder and mortality statistics used in the model analysis. The appendix ends with an overview of the acronyms and parameters used in this thesis in appendix A.6.

A.1 Optimal life-cycle in Merton model

The technical derivation of the optimal investment decision via the martingale (Lagrangian) approach is given below. This derivation is in line with Grebenchtchikova et al. (2016).

Recall the utility function of an individual featuring CRRA preferences in (2.2). The individual wants to maximize utility over the life-cycle. This maximization function is presented in (2.3). Solving the maximization function is a dynamic optimization problem. One can reformulate this dynamic optimization problem into a static optimization problem and a representation problem. In the static optimization problem, the optimal total wealth at retirement is determined. The representation problem is solved to determine the investment portfolio which leads to the optimal total wealth at retirement.

First of all, the derivation for an individual with risk aversion parameter $\gamma \neq 1$ will be given. Subsequently, a similar derivation will be given for $\gamma = 1$. In case $\gamma = 1$, the utility function is reduced to log utility.

In the static optimization problem the individual maximizes expected utility from total wealth at retirement W_T given initial wealth W_0

$$\max_{W_T} \mathbb{E}_0 \left(\frac{W_T^{1-\gamma}}{1-\gamma} \right). \tag{A.1}$$

When we assume absence of arbitrage, a complete market and the existence of a pricing kernel M_t with $M_0 = 1$, the martingale method can be applied. The budget constraint which needs to be satisfied equals

$$\mathbb{E}_0 W_T M_T = W_0. \tag{A.2}$$

The Lagrange method is used to determine the optimal W_T. The Lagrange function with Lagrange multiplier η is given by

$$\mathcal{L} = \mathbb{E}_0 \left(\frac{W_T^{1-\gamma}}{1-\gamma} \right) - \eta(\mathbb{E}_0(W_T M_T) - W_0).$$

Taking the first order condition with respect to W_T for each state of the world yields

$$\frac{\partial \mathcal{L}}{\partial W_T} = W_T^{-\gamma} - \eta M_T = 0$$
$$W_T = \eta^{-1/\gamma} M_T^{-1/\gamma}.$$

One can use the budget constraint to find the following expression for the Lagrange multiplier

$$\eta^{-1/\gamma} = \frac{W_T}{M_T^{-1/\gamma}} = \frac{W_0}{\mathbb{E}_0(M_T^{1-1/\gamma})}. \tag{A.4}$$

This Lagrange multiplier can be plugged into the expression of W_T

$$W_T = \frac{W_0 M_T^{-1/\gamma}}{\mathbb{E}_0(M_T^{1-1/\gamma})}. \tag{A.5}$$

We have found the optimal expression for total wealth at retirement. Subsequently, we have to find the investment portfolio which leads to this optimal total wealth at retirement.

In the Merton model, an explicit expression exists for the pricing kernel M_t

$$M_t = \exp\left(-\left(r + 0.5\lambda^2\right)t - \lambda Z_t\right). \tag{A.6}$$

Since M_t represents the pricing kernel, the pricing equation states that the following

$$W_t = M_t^{-1}\mathbb{E}_t(W_T M_T). \tag{A.7}$$

We use this pricing equation and the log normality of the pricing kernel to determine the optimal wealth W_t at time t

$$\begin{aligned} W_t &= M_t^{-1}\mathbb{E}_t\left(\frac{W_0 M_T^{-1/\gamma}}{\mathbb{E}_0(M_T^{1-1/\gamma})}M_T\right) = \frac{W_0}{M_t}\frac{\mathbb{E}_t M_T^{1-1/\gamma}}{\mathbb{E}_0 M_T^{1-1/\gamma}} \\ &= \frac{W_0}{M_t}\frac{M_t^{1-1/\gamma}\exp(-(1-\frac{1}{\gamma})(r+0.5\lambda^2)(T-t)+0.5(1-\frac{1}{\gamma})^2\lambda^2(T-t))}{\exp(-(1-\frac{1}{\gamma})(r+0.5\lambda^2)T+0.5(1-\frac{1}{\gamma})^2\lambda^2 T)} \\ &= W_0 M_t^{-1/\gamma}\exp\left(\left(1-\frac{1}{\gamma}\right)(r+0.5\lambda^2)t - 0.5\left(1-\frac{1}{\gamma}\right)^2\lambda^2 t\right) \\ &= W_0 M_t^{-1/\gamma}\exp\left(\left(1-\frac{1}{\gamma}\right)rt + 0.5\left(1-\frac{1}{\gamma}\right)\frac{1}{\gamma}\lambda^2 t\right). \end{aligned} \tag{A.8}$$

Subsequently, we plug the formula of M_t (A.6) into (A.8)

$$\begin{aligned} W_t &= W_0\exp\left(\frac{1}{\gamma}(r+0.5\lambda^2)t + \frac{\lambda}{\gamma}Z_t + \left(1-\frac{1}{\gamma}\right)rt + 0.5\left(1-\frac{1}{\gamma}\right)\frac{1}{\gamma}\lambda^2 t\right) \\ &= W_0\exp\left(\frac{\lambda}{\gamma}Z_t + \left(r + \frac{\lambda^2}{\gamma} - \frac{\lambda^2}{2\gamma^2}\right)t\right). \end{aligned}$$

This analytic solution of W_t can be written as a stochastic differential equation since it is equal to the solution of a geometric brownian motion

$$dW_t = \left(r + \frac{\lambda^2}{\gamma}\right)W_t dt + W_t\frac{\lambda}{\gamma}dZ_t. \tag{A.9}$$

One can conclude that the optimal exposure of total wealth of the individual to the risk factor Z_t equals $\frac{\lambda}{\gamma}$. Recall from (2.1b) that the exposure of the risky stock S_t to Z_t equals σ. Therefore, the optimal exposure of total wealth W_t to the risky stock S_t equals

$$f = \frac{\lambda}{\gamma\sigma} = \frac{\mu - r}{\gamma\sigma^2}. \tag{A.10}$$

We have derived the optimal investment strategy. The optimal utility can be found by plugging (A.5) into (A.1)

$$\begin{aligned}\mathbb{E}_0\left(\frac{W_T^{1-\gamma}}{1-\gamma}\right) &= \frac{W_0^{1-\gamma}}{1-\gamma}\frac{\mathbb{E}_0 M_T^{1-1/\gamma}}{(\mathbb{E}_0 M_T^{1-1/\gamma})^{1-\gamma}} \\ &= \frac{W_0^{1-\gamma}}{1-\gamma}(\mathbb{E}_0 M_T^{1-1/\gamma})^{\gamma} \\ &= \frac{W_0^{1-\gamma}}{1-\gamma}\exp\left(-\left(1-\frac{1}{\gamma}\right)\left(r+\frac{1}{2}\lambda^2\right)T+\frac{1}{2}\left(1-\frac{1}{\gamma}\right)^2\lambda^2 T\right)^{\gamma} \\ &= \frac{W_0^{1-\gamma}}{1-\gamma}\exp\left((1-\gamma)rT-\frac{1}{2}\left(1-\frac{1}{\gamma}\right)\lambda^2 T\right).\end{aligned} \tag{A.11}$$

Note that the optimal utility has the same structure as a CRRA utility function and that it is increasing in the interest rate r and price of risk λ, which makes sense.

Log utility

A CRRA participant with $\gamma = 1$ wants to maximize the following equation

$$\max_{W_T} \mathbb{E}_0(\ln(W_T)).$$

The Lagrange function is given by

$$\mathcal{L} = \mathbb{E}_0(\ln(W_T)) - \eta(\mathbb{E}_0(W_T M_T) - W_0).$$

Taking the first order condition with respect to W_T for each state of the world yields

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial W_T} &= W_T^{-1} - \eta M_T = 0 \\ W_T &= \eta^{-1}M_T^{-1} \\ \eta^{-1} &= \frac{W_0}{\mathbb{E}_0(M_T^{-1}M_T)} = W_0 \\ W_T &= W_0 M_T^{-1}.\end{aligned}$$

The optimal wealth W_t at time t equals

$$\begin{aligned}W_t &= M_t^{-1}\mathbb{E}_t(W_T M_T) = M_t^{-1}\mathbb{E}_t(W_0 M_T^{-1}M_T) = W_0 M_t^{-1} \\ &= W_0\exp((r+0.5\lambda^2)t+\lambda Z_t) \\ &= W_0\exp((r+\lambda^2-0.5\lambda^2)t+\lambda Z_t),\end{aligned}$$

which can be written as a stochastic differential equation

$$dW_t = (r+\lambda^2)W_t dt + W_t\lambda dZ_t.$$

One can conclude that the optimal exposure of total wealth of the individual to the risk factor Z_t equals λ implying that the optimal exposure of total wealth W_t to the risky stock S_t equals

$$f = \frac{\lambda}{\sigma}.$$

A.2 Optimal AIR in Merton model

The technical derivation of the optimal AIR in the Merton model for an individual with CRRA preferences and no smoothing of financial shocks is given below. This derivation is in line with Grebenchtchikova et al. (2016).

First of all the derivation is given for an individual with risk aversion parameter $\gamma \neq 1$ (just as in appendix A.1). Subsequently, $\gamma = 1$ will be considered.

To find the allocation of available pension wealth at retirement to the different pension payments, the individual maximizes utility over the retirement period with future expected utilities declining exponentially

$$\max \mathbb{E}_T \left(\sum_{h=0}^{P-1} p_T(T+h) \exp(-\rho h) \frac{W_{T+h}(h)^{1-\gamma}}{1-\gamma} \right), \tag{A.14}$$

subject to the budget constraint

$$\sum_{h=0}^{P-1} p_T(T+h) W_T(h) = W_T. \tag{A.15}$$

In these equations, $W_{T+h}(h)$ equals pension wealth (or consumption) at time $T+h$, $W_T(h)$ equals the pension wealth at retirement (time T) needed to obtain $W_{T+h}(h)$ at time $T+h$ and W_T[1] is the total available pension wealth at retirement. This amount W_T needs to be split into proportions used to finance consumption during retirement. Finally, $p_T(T+h)$ equals the survival probability at time $T+h$ as of time T. (A.14) is maximized with respect to the pension wealth at retirement $W_T(h)$.

Consider the static optimization problem at time $T+h$. The martingale method is applied here assuming absence of arbitrage, a complete market and the existence of a pricing kernel M_{T+h} with $M_T = 1$. This problem can be stated as follows

$$\max_{W_T(h)} \mathbb{E}_T \left(\frac{W_{T+h}(h)^{1-\gamma}}{1-\gamma} \right),$$

subject to the budget constraint

$$\mathbb{E}_T(W_{T+h}(h) M_{T+h}) = W_T(h).$$

Note that the discount term $\exp(-\rho h)$ and survival probability $p_T(T+h)$ can be left out of the maximization since these are given for each time $T+h$. This maximization is similar to the maximization (A.1) in appendix A.1 but with different parameters: $W_{T+h}(h)$ instead of W_T and $W_T(h)$ instead of W_0. Just as in the derivation of the optimal life-cycle in the Merton model in appendix A.1, one can apply the Lagrange method here as well. The corresponding optimal utility (similar to (A.11)) is as follows

$$\mathbb{E}_T \left(\frac{W_{T+h}(h)^{1-\gamma}}{1-\gamma} \right) = \frac{W_T(h)^{1-\gamma}}{1-\gamma} \exp\left((1-\gamma) r(T+h) - \frac{1}{2}\left(1 - \frac{1}{\gamma}\right) \lambda^2(T+h) \right). \tag{A.16}$$

[1] At retirement total wealth W_T is equal to financial wealth F_T (i.e. available pension wealth) since human wealth H_T is equal to zero.

This holds for each time $T+h$. Using (A.16) the maximization function (A.14) then becomes

$$\max \sum_{h=0}^{P-1} p_T(T+h)\exp(-\rho h)\frac{W_T(h)^{1-\gamma}}{1-\gamma}\exp\left((1-\gamma)r(T+h)-\frac{1}{2}\left(1-\frac{1}{\gamma}\right)\lambda^2(T+h)\right). \quad \text{(A.17)}$$

To solve the maximization function (A.16) subject to (A.15), again the Lagrange method is used to determine the optimal $W_T(h)$. The Lagrange function with Lagrange multiplier η is given by

$$\mathcal{L} = \sum_{h=0}^{P-1} \frac{W_T(h)^{1-\gamma}}{1-\gamma} p_T(T+h)\exp\left(-\rho h + (1-\gamma)r(T+h) - \frac{1}{2}\left(1-\frac{1}{\gamma}\right)\lambda^2(T+h)\right) - \eta\left(\sum_{h=0}^{P-1} p_T(T+h)W_T(h) - W_T\right).$$

Taking the first order condition with respect to $W_T(h)$ yields

$$\frac{\partial \mathcal{L}}{\partial W_T(h)} = p_T(T+h)\exp(-\rho h + (1-\gamma)r(T+h) - \frac{1}{2}\left(1-\frac{1}{\gamma}\right)\lambda^2(T+h))W_T(h)^{-\gamma} - \eta p_T(T+h) = 0 \quad \text{for } h = 0,\ldots,P-1$$

$$W_T(h) = \eta^{-1/\gamma}\exp\left(\rho h - (1-\gamma)r(T+h) + \frac{1}{2}\left(1-\frac{1}{\gamma}\right)\lambda^2(T+h)\right)^{-1/\gamma}.$$

By leaving out the constant terms in the equation of $W_T(h)$, one can conclude that (A.14) is maximized subject to the budget constraint (A.2) when

$$\begin{aligned} W_T(h) &\propto \exp\left(\rho h - (1-\gamma)rh + \frac{1}{2}\left(1-\frac{1}{\gamma}\right)\lambda^2 h\right)^{-1/\gamma} \\ &= \exp\left(\left(-\frac{\rho}{\gamma} + \left(\frac{1}{\gamma}-1\right)r - \frac{1}{2\gamma}\left(1-\frac{1}{\gamma}\right)\lambda^2\right)h\right) \\ &= \exp\left(-\left(r + \frac{1}{\gamma}(\rho - r) - \frac{1}{2\gamma}\left(\frac{1}{\gamma}-1\right)\lambda^2\right)h\right) \\ &= \exp(-\text{AIR}\cdot h). \end{aligned}$$

The optimal AIR equals

$$\text{AIR}^* = r + \frac{1}{\gamma}(\rho - r) - \frac{1}{2\gamma}\left(\frac{1}{\gamma}-1\right)\lambda^2. \quad \text{(A.18)}$$

It is clear from (A.18) that AIR^* depends on the intertemporal elasticity of substitution $1/\gamma$ (or risk aversion parameter γ). To determine for which values of $1/\gamma$ AIR^* increases in $1/\gamma$ we take the first order derivative of AIR^* with respect to $1/\gamma$

$$\frac{\partial \text{AIR}^*}{\partial 1/\gamma} = \rho - r - \frac{1}{2}\left(\frac{1}{\gamma}-1\right)\lambda^2 - \frac{1}{2\gamma}\lambda^2 = \rho - r + \frac{1}{2}\lambda^2 - \frac{1}{\gamma}\lambda^2.$$

One can conclude that AIR^* increases in $1/\gamma$ if

$$\frac{\rho - r}{\lambda^2} + \frac{1}{2} > \frac{1}{\gamma} \iff \frac{\lambda^2}{\rho - r + \frac{1}{2}\lambda^2} < \gamma.$$

Log utility

A CRRA participant with $\gamma = 1$ maximizes the following function

$$\max \mathbb{E}_T\left(\sum_{h=0}^{P-1} p_T(T+h)\exp(-\rho h)\ln(W_{T+h}(h))\right), \quad \text{(A.19)}$$

subject to the budget constraint. Consider the maximization at time $T+h$

$$\max_{W_T(h)} \mathbb{E}_T(\ln(W_{T+h}(h))).$$

Just as in the derivation of the optimal life-cycle in the Merton model in appendix A.1, one can apply the Lagrange method to determine the optimal $W_{T+h}(h)$

$$W_{T+h}(h) = W_T(h) M_{T+h}^{-1},$$

which yields the following optimal utility

$$\begin{aligned}
\mathbb{E}_T(\ln(W_{T+h}(h))) &= \mathbb{E}_T(\ln(W_T(h) M_{T+h}^{-1})) \\
&= \mathbb{E}_T(\ln(W_T(h)) - \mathbb{E}_T(\ln(M_{T+h})) \\
&= \ln(W_T(h)) - \mathbb{E}_T(\ln(M_{T+h})) \\
&= \ln(W_T(h)) - \mathbb{E}_T(-(r + 0.5\lambda^2)(T + h) - \lambda Z_{T+h}) \\
&= \ln(W_T(h)) + (r + 0.5\lambda^2)(T + h) + \mathbb{E}_T(\lambda Z_{T+h}) \\
&= \ln(W_T(h)) + (r + 0.5\lambda^2)(T + h).
\end{aligned}$$

This holds for each time $T + h$. The maximization function (A.19) then becomes

$$\max \sum_{h=0}^{P-1} p_T(T + h)\exp(-\rho h)(\ln(W_T(h)) + (r + 0.5\lambda^2)(T + h)).$$

The Lagrange function is given by

$$\mathcal{L} = \sum_{h=0}^{P-1} \frac{W_T(h)^{1-\gamma}}{1-\gamma} p_T(T + h)\exp(-\rho h)(\ln(W_T(h)) + (r + 0.5\lambda^2)(T + h)) - \eta \left(\sum_{h=0}^{P-1} p_T(T + h)W_T(h) - W_T \right).$$

Taking the first order condition with respect to $W_T(h)$ yields

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial W_T(h)} &= p_T(T + h)\exp(-\rho h)W_T(h)^{-1} - \eta p_T(T + h) = 0 \quad \text{for } h = 0, ..., P - 1 \\
W_T(h) &= \eta^{-1}\exp(-\rho h).
\end{aligned}$$

By leaving out the constant terms in the equation of $W_T(h)$ one can conclude that (A.19) is maximized if

$$\begin{aligned}
W_T(h) &\propto \exp(-\rho h) \\
&= \exp(-\text{AIR} \cdot h).
\end{aligned}$$

So if $\gamma = 1$ then $\text{AIR}^* = \rho$. Note that this value is also found if $\gamma = 1$ is plugged into (A.18).

A.3 Derivations money pots approach

This appendix contains two derivations. First of all, the derivation of the AIR which yields an exact constant expected pension benefit level during the retirement period (see section 7.5) will be presented. We call this horizon-dependent AIR(h) the BNW AIR. Secondly, the derivation of the implicit life-cycle of the money pots approach is given. This life-cycle is equivalent to the investment mix of a pension fund in case of financial smoothing as derived by Nijman et al. (2013).

Assumed interest rate (AIR)
We are interest in the AIR which yields an exact constant expected pension benefit level during the retirement period. The derivation is in line with Balter and Werker (2016).

We use the same notation as in appendix A.2: $W_{T+h}(h)$ equals the pension wealth (or consumption) at time $T+h$, $W_T(h)$ equals the pension wealth at retirement (time T) needed to obtain $W_{T+h}(h)$ at time $T+h$. W_T is the total available pension wealth at retirement. This amount W_T needs to be allocated to money pots to finance the pension benefit payments during retirement.

We start in a simplified setting (just as in section 7.5) which contains no micro longevity risk, i.e. the participant will receive 18 pension payments after retirement for sure. Moreover, we firstly consider the case without financial smoothing.

Recall the stochastic differential equation of total wealth W_t in (A.9). In the same way, the development of $W_{T+h}(h)$ can be described by

$$dW_{T+h}(h) = (r + f(\mu - r))W_{T+h}(h)d(T+h) + f\sigma W_{T+h}(h)dZ_{T+h},$$

in case the equity exposure f is defined as in (A.10). An explicit expression is available since $W_{T+h}(h)$ follows a geometric brownian motion

$$W_{T+h}(h) = W_T(h)\exp((r + f(\mu - r) - 0.5f^2\sigma^2)h + f\sigma Z_h).$$

Taking the expectation[2] yields

$$\begin{aligned}\mathbb{E}_T[W_{T+h}(h)] &= W_T(h)\exp((r + f(\mu - r) - 0.5f^2\sigma^2)h)\mathbb{E}_T[\exp(f\sigma Z_h)] \\ &= W_T(h)\exp((r + f(\mu - r))h). \end{aligned} \tag{A.21}$$

In case the participant prefers a constant pension benefit level over time, the AIR must be determined such that $\mathbb{E}_T[W_{T+h}(h)] = W_T(0)$.

$$\begin{aligned}\frac{W_T(h)}{W_T(0)} &= \exp(-(r + f(\mu - r))h) \\ &= \exp(-\text{AIR} \cdot h).\end{aligned}$$

So AIR $= r + f(\mu - r)$. Note that this AIR is indeed equal to the expected return (6.2) which yields a constant expected pension benefit level in case of no financial smoothing.

[2] The expectation of $W_{T+h}(h)$ is not exactly equal to the median of $W_{T+h}(h)$ because $W_{T+h}(h)$ is log normally distributed. One should multiply the mean with $\exp(-0.5f^2\sigma^2 h)$ to obtain the median. This second-order difference will be neglected.

Subsequently, the case including financial smoothing is considered. Recall from section 7.5 that the equity exposure in case of financial smoothing is not constant anymore in the money pots approach. The equity exposure $q_{T+i}(h)$ at time $T+i$ of the pension wealth $W_T(h)$ which will be consumed at $T+h$ is defined in (7.5). The equity exposure $q_{T+i}(h)$ is different in each year i and implies a decreasing life-cycle strategy (see figure 7.13). One should replace f in (A.21) by $q_{T+i}(h)$ at time $T+i$. The expectation of the pension wealth $W_{T+h}(h)$ equals

$$\begin{aligned}\mathbb{E}_T[W_{T+h}(h)] &= W_{T+h-1}(h)\exp(r + q_{T+h-1}(h)(\mu - r)) \\ &= W_{T+h-2}(h)\exp(r + q_{T+h-1}(h)(\mu - r) + r + q_{T+h-2}(h)(\mu - r)) \\ &= W_T(h)\exp\left(\sum_{i=0}^{h-1}(r + q_{T+i}(h)(\mu - r))\right),\end{aligned}$$

which implies the following horizon-dependent AIR(h) (similar to the case without financial smoothing)

$$\text{AIR}(h) = \frac{1}{h}\sum_{i=0}^{h-1}(r + q_{T+i}(h)(\mu - r)) = r + \frac{1}{h}\sum_{i=0}^{h-1} q_{T+i}(h)(\mu - r).$$

Subsequently, we plug in $q_{T+i}(h)$ as defined in (7.5). First of all, the case $h \leq N$ is considered

$$\text{AIR}(h) = r + \frac{1}{h}\sum_{i=0}^{h-1} q_{T+i}(h)(\mu - r) = r + \frac{\frac{f}{N} + \frac{2f}{N} + \ldots + \frac{hf}{N}}{h}(\mu - r) = r + \frac{f(h+1)}{2N}(\mu - r).$$

Subsequently, the case $h \geq N$ is consider

$$\text{AIR}(h) = r + \tfrac{1}{h}\sum_{i=0}^{h-1} q_{T+i}(h)(\mu - r) = r + \tfrac{\frac{f}{N} + \frac{2f}{N} + \ldots + \frac{Nf}{N} + f(h-N)}{h}(\mu - r) = r + f\left(1 - \tfrac{N-1}{2h}\right)(\mu - r).$$

One can write AIR(h) (which is called the BNW AIR) as follows

$$\text{AIR}(h) = r + Q(h)(\mu - r).$$

with $Q(h)$ defined as follows

$$Q(h) = \begin{cases} f(h+1)/(2N) & \text{if } h \leq N \\ f(1 - (N-1)/(2h)) & \text{if } h \geq N, \end{cases}$$

which holds for $h = 1, \ldots, 17$.

$Q(h)$ represents the average equity exposure of the pension wealth which will be consumed after h years.

Implicit life-cycle

The derivation of the implicit life-cycle of the money pots approach is presented below. This derivation is in line with the derivation of the investment mix of a pension fund in case of financial smoothing as derived by Nijman et al. (2013).

We want to derive the equity exposure $\tilde{f}_t$ of total pension wealth W_t. Recall the pension wealth $W_T(h)$ at retirement (time T) which will be consumed at $T+h$ in (7.7). This expression does not only hold at time T but can be generalized for the pension wealth $W_t(h)$ at time t

$$W_t(h) = p_t(t+h)W_T(0)\exp(-h \cdot \text{AIR}(h)). \tag{A.23}$$

The collective exposure to risky assets can be found by summing up the equity exposures of the pension wealth $W_t(h)$ allocated to the different money pots and should be equal to the equity exposure of total pension wealth W_t

$$\sum_h q_t(h)W_t(h) = \tilde{f}_t W_t,$$

where $q_t(h)$ equals the equity exposure at time t of $W_t(h)$. The appropriate equity exposure $\tilde{f}_t$ of total pension wealth W_t equals

$$\tilde{f}_t = \frac{\sum_h q_t(h)W_t(h)}{W_t}. \tag{A.24}$$

One can plug the expressions for $q_t(h)$ in (7.5) and $W_t(h)$ in (A.23) into the equation of $\tilde{f}_t$.

A.4 Euler discretization

As already mentioned in subsection 2.2.2, one of the assumptions in the Merton model which does not hold in practice is the assumption regarding continuous trading. In the model analysis, it is assumed that the participant can trade at an annual frequency. All stochastic differential equations are discretized using the Euler scheme. For example, the value of the risky stock (described via stochastic differential equation (2.1b)) at time t is calculated as follows

$$S_t = S_{t-1} + \mu S_{t-1}\Delta t + \sigma S_{t-1}\Delta Z_t = S_{t-1} + \mu S_{t-1}\Delta t + \sigma S_{t-1}\sqrt{\Delta t} RN_t = S_{t-1} + \mu S_{t-1} + \sigma S_{t-1} RN_t,$$

where RN_t is a standard normal random variable.

Another example of discretization is the calculation of the value of future pension contributions $\hat{H}_t$ (the continuous formulation is stated in (2.9))

$$\hat{H}_t = \begin{cases} \sum_{i=0}^{T-t} \exp(-ri)(Y_{t+i} - C_{t+i}) & \text{if } t \leq T \\ 0 & \text{if } t > T. \end{cases}$$

A.5 Assumptions model analysis

Premium ladder

In the model analysis the pension accrual of the participant is determined by the 3% DC fiscal maximum premium ladder for old-age pension.[3] This premium ladder is used in practice in the Netherlands as of 1-1-2015 in many DC pension schemes. I chose the premium ladder for old-age pension only since I do not consider partner pension in this thesis. The premium percentages are displayed in table A.1. A premium percentage of 7.6% at age 25 for example implies a pension contribution at age 25 equal to 7.6% of the pension base. The pension base equals the pensionable salary of the participant minus the offset.

Age	Premium percentage
25-29	7.6%
30-34	8.8%
35-39	10.3%
40-44	11.9%
45-49	13.9%
50-54	16.2%
55-59	19.1%
60-64	22.6%
65-66	25.6%

Table A.1: Premium ladder.

[3] Table 1, appendix IV, 'Besluit loonheffingen, inkomstenbelasting, pensioenen; beschikbare-premieregelingen en premie- en kapitaalovereenkomsten en nettopensioenregelingen'.

Mortality statistics

In the model analysis the cumulative gender neutral survival probabilities of a Dutch individual who is 67 years old at 31-12-2014 are used. These probabilities are acquired from the 'Centraal Bureau voor de Statistiek' published at 31-03-2016[4] and are presented in table A.2. Besides the survival probability the implicit one-year death probability, the expected remaining lifetime and the N-duration are also presented in this table for each age.

For example, the survival probability of 83.53% at age 77 implies that an individual who is currently 67 years old has a 83.53% chance of becoming 10 years older. The one-year death probability of 1.10% at age 67 implies that the individual has a probability of 1.10% that he will die at age 67.

Both the N-duration (definition see section 7.1) for a smoothing period of 5 years and 10 years is presented. The N-durations are calculated using the expected return as AIR. Note that both N-durations converge towards each other when the participant ages.

Age	Survival probability	One-year death probability	Expected remaining lifetime	N-duration ($N = 5$)	N-duration ($N = 10$)
67	100%	1.10%	18.14	4.37	7.31
68	98.90%	1.22%	17.34	4.34	7.22
69	97.69%	1.38%	16.56	4.32	7.13
70	96.34%	1.49%	15.79	4.30	7.04
71	94.90%	1.63%	15.03	4.27	6.94
72	93.35%	1.81%	14.28	4.24	6.83
73	91.66%	1.94%	13.54	4.21	6.71
74	89.88%	2.17%	12.81	4.18	6.59
75	87.93%	2.39%	12.09	4.14	6.46
76	85.83%	2.68%	11.39	4.10	6.32
77	83.53%	3.03%	10.70	4.06	6.17
78	81.00%	3.33%	10.04	4.01	6.02
79	78.30%	3.84%	9.38	3.97	5.86
80	75.29%	4.28%	8.76	3.91	5.69
81	72.07%	4.79%	8.15	3.86	5.51
82	68.62%	5.52%	7.56	3.79	5.32
83	64.83%	6.36%	7.00	3.73	5.13
84	60.71%	7.02%	6.48	3.66	4.93
85	56.45%	7.88%	5.97	3.58	4.73
86	52.00%	9.12%	5.48	3.50	4.52
87	47.26%	10.01%	5.03	3.42	4.31
88	42.53%	11.19%	4.58	3.32	4.09
89	37.77%	12.76%	4.16	3.22	3.87
90	32.95%	14.14%	3.77	3.12	3.63
91	28.29%	16.22%	3.39	3.01	3.39
92	23.70%	17.76%	3.05	2.88	3.14
93	19.49%	19.75%	2.71	2.74	2.90
94	15.64%	21.42%	2.37	2.58	2.64
95	12.29%	23.27%	2.02	2.37	2.37
96	9.43%	26.41%	1.64	2.07	2.07
97	6.94%	27.38%	1.22	1.76	1.76
98	5.04%	31.75%	0.68	1.40	1.40
99	3.44%	100.00%	0.00	1.00	1.00

Table A.2: Mortality statistics.

[4] http://statline.cbs.nl/Statweb/publication/?DM=SLNL&PA=70701ned&D1=a&D2=0&D3=67&D4=l&HDR=G1,\T&STB=G2,G3&VW=T

A.6 Acronyms and formulas

This appendix contains an overview of the acronyms and parameters used in this thesis.

Acronym	Definition
AIR	assumed interest rate
IDC	individual defined contribution
CDC	collective defined contribution
CRRA	constant relative risk aversion
DB	defined benefit
DC	defined contribution
LDF	linearly decreasing strategy for fixed annuity
LDV	linearly decreasing strategy for variable annuity
LDVCE	linearly decreasing strategy for variable annuity with constant equity exposure after retirement
LDVDE	linearly decreasing strategy for variable annuity with decreasing equity exposure after retirement
MM	median Merton strategy
OM	optimal Merton strategy
PPR	Personal Pensions with Risk sharing

Table A.3: Acronyms

Parameter	Definition
B_t	bond price at time t
br_t	biometric return at time t
ce	certainty equivalent consumption at retirement
C_t	consumption at time t
$\bar{C}_t$	minimum consumption level at time t
D	maximum period alive
f	optimal equity exposure of total wealth at time t
f_t^*	optimal equity exposure of financial wealth at time t
$\hat{f}_t$	sustainable and efficient investment life-cycle at time t
$\tilde{f}_t$	implicit life-cycle of money pots approach at time t
F_t	financial wealth at time t
g_t	growth rate in pension benefit level at time t
G_t	present value of future AOW payments at time t
γ	coefficient of relative risk aversion
H_t	present value future labor income at time t
$\hat{H}_t$	present value future pension contributions at time t
λ	price of risk
μ	mean stock return
M	maximum age
N	length smoothing period
ND_t	N-duration at time t
ξ	scaling factor
P	length (maximum) retirement period
$p_T(t)$	cumulative survival probability at time t as of time T
pa_t	actuarial fair price of lifelong sustainable annuity at time t
q_t	one-year death probability at time t
$q_t(h)$	equity exposure at time t of pension wealth which will be consumed at time $t+h$
$Q(h)$	average equity exposure of the pension wealth which will be consumed after h years
r	interest rate
R	retirement age
RN_t	standard normal random variable at time t
ρ	rate of time preference
S_t	stock price at time t
σ	standard deviation of stock return
T	number of working years
$V_t^{(h)}$	market-consistent value at time t of the pension benefit which will be paid h years from now
V_t	market-consistent value at time t of all pension benefits
W_t	total wealth at time t
$W_t(h)$	pension wealth at time t needed for pension benefit at time $t+h$
X_t	present value future minimum consumption levels at time t
Y_t	labor income at time t
Z_t	Brownian motion at time t

Table A.4: Parameters