# Macroeconomic and welfare implications of different pension benefit arrangements

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**Abstract**

We analyze the trade-off among insurance, labor supply and savings incentives arising in the design of pay-as-you-go (PAYG) pension benefits. We consider two extreme types of pension benefits: i) a flat benefit (FL) system that pays the same pension regardless of the amount of previous contributions and ii) a notional defined contribution (NDC) system in which benefits are perfectly linked to previous contributions. The FL system promotes lower labor supply, lower consumption inequality and generally crowds out capital less. If the level of pension contributions is low, the FL system brings a higher welfare due to higher consumption insurance. At higher levels of contributions, the NDC system leads to higher welfare due to lower labor supply distortions. General equilibrium effects tilt the welfare result in favor of FL pensions in dynamically efficient economies. The analysis suggests that pension benefit design should depend on the size of the pension system and the size of idiosyncratic risk. The welfare results can explain the contrasting reforms of pension benefits implemented in different countries, as well as the empirical correlations between pension benefit progressivity, on the one hand, and the size of pension systems and idiosyncratic risk, on the other hand.

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# 1 Introduction

Pay-as-you-go pension (PAYG) systems around the world differ significantly regarding the link between an individual's life-time earnings and the pension benefits he receives. Countries like Germany, Italy, France and Poland switched to a Notional Defined Contribution (NDC) pension system or a points system in which pension benefits are perfectly linked to contributions paid. One of the main aims of these reforms was to improve labor supply incentives of agents. In sharp contrast, the UK recently eliminated the earnings related part of the pension system, keeping only the flat benefit (FL) pension that pays retired agents the same pension benefit regardless of life-time contributions.

The relationship between life-time earnings and pension benefits determines the *progressivity* of the pension system. If the replacement rate does not vary with life-time earnings, then the pension system entails *zero progressivity*. This is the case of pure NDC and points based pension systems[1]. However, if the replacement rate decreases with life-time contributions - as is the case under the FL pension system for example - then the pension system is *progressive*. Cross-country data show that there exists: i) a negative relationship between the size of the PAYG pension system and its progressivity and ii) a positive relationship between income inequality prevailing in an economy and the progressivity of pension systems. Starting with Conde-Ruiz and Profeta (2007), the literature explained these stylized facts as the outcome of voting on both the size of the pension system and its progressivity.

In this paper we analyze the trade-off among consumption insurance, labor supply and savings incentives that determines the welfare of agents in economies with different PAYG pension benefit arrangements. A high degree of pension progressivity amplifies the labor supply distortions brought by the pension system, lowering the labor supply of old agents. However, progressive pension benefits offer insurance against the idiosyncratic earnings shocks that agents face during their working life. The implications for savings are not straightforward. On the one hand, because old agents work less, they need to make more savings when young in order to achieve the same level of old age consumption. On the other hand, a pension system with high progressivity offers insurance against idiosyncratic earnings shocks so agents need to make less precautionary savings.

We make three contributions to the literature. First, we show that, if we abstract from general equilibrium effects working through factor prices, the negative relationship between the size of the pension system and pension progressivity can be substantiated from a welfare point of view. More precisely, if the level of contributions is small, a high progressivity of pension benefits is preferable from a welfare point of view because of the insurance provided against idiosyncratic earnings shocks. As the size of the pension contribution increases, the

[1] NDC and points pension systems can also entail some degree of progressivity by including provisions such as minimum pensions or contributions caps. However, cross-country data shows that the degree of progressivity remains low even with such provisions.

welfare gain from insurance is outweighed by the increasing welfare losses from labor distortions, so less progressive pension systems become desirable. This also implies that countries in which the level of idiosyncratic earnings risk is high should implement more progressive pension systems. Hence the positive correlation between the level of income inequality and the progressivity of pension systems is also sustained from a welfare point of view.

Second, we highlight two important general equilibrium effects working through factor prices that tilt the balance in favor of pension systems with high progressivity. The first general equilibrium effect works through the amount of individual savings. Because old agents work less under pension systems with high progressivity of benefits, they also save more when young in order to achieve the same level of old age consumption. In general equilibrium, the higher level of savings under a pension system with high progressivity implies a lower difference between the return of capital and the return of the pension system in economies that are dynamically efficient in the sense of Cass (1972)[2]. Consequently, pension systems with high progressivity have a less negative impact on welfare. The second general equilibrium effect appears due to the fact that a higher distortion on labor supply implies a lower distortion on savings. Moreover, labor distortions increase (quadratically) with the level of pension contribution. Consequently, because pension systems with very progressive benefits distort labor supply more, they also distort savings less and the distortions decrease (quadratically) with the level of pension contribution. Through factor prices, the lower savings distortions have a positive effect on welfare and this effect increases with the level of pension contribution. The general equilibrium effects highlighted in the paper are extremely important because they imply that pension systems with high progressivity may be desirable even in economies with low or no idiosyncratic earnings risk.

Third, our analysis has an important policy implication. It suggests that the move towards pension systems with no progressivity like the NDC pension system is not necessarily welfare enhancing for two reasons: i) it reduces the insurance against idiosyncratic earnings risks and ii) can worsen the capital crowding out exactly because it restores the work incentives of the elderly. The results of the paper indicate that the optimal degree of progressivity embedded in a pension system: i) can never be zero and ii) depends on the size of the uninsurable idiosyncratic earnings risk and the level of the pension system.

We obtain the above results in a tractable general equilibrium model with two overlapping generations, incomplete markets, endogeneous labor supply of old agents and a PAYG pension system. In the first period of their life agents supply labor inelastically, pay the contribution to the PAYG pension system and save in risk free capital. In the second period of their life, agents are hit by idiosyncratic earnings shocks, chose how much to work and receive the pension benefit. We consider that pension contributions are proportional with earnings, but pension benefits can be either FL or NDC.

[2]An economy is dynamically efficient in the sens of Cass (1972) if the return on capital is higher than the return of the pension system.

In order to determine the macroeconomic and welfare implications of pension progressivity, we compare the steady states of two economies with extreme pension system arrangements: i) a flat benefit (FL) system and ii) a NDC system. Because the contribution to the NDC system is nondistortionary for labour supply, old agents work more than under the FL system. Hence, aggregate labour is higher in the NDC system. However, consumption inequality is also higher in the NDC system, because pension benefits are perfectly linked with the agents' life-time labor income. The impact of the pension benefit arrangement on consumption inequality and labor supply determines the relative welfare of the two pension systems, if we abstract from general equilibrium effects working through factor prices. The FL system brings a higher welfare than the NDC system at low levels of pension contributions due to the insurance it provides. As the size of the pension contribution increases, the welfare losses from labor supply distortions increase outweighing the welfare gains from insurance and the NDC system eventually provides a higher welfare. The threshold contribution up to which the FL system brings a higher welfare increases with the size of the idiosyncratic productivity risk.

The impact of the pension benefit arrangement on savings depends on two opposing factors. On the one hand, because old agents work more under the NDC pension system, they need to save less for their consumption. On the other hand, due to the existence of idiosyncratic earnings shocks, agents make precautionary savings. The level of precautionary savings is higher in the NDC pension system because this type of pension system does not provide insurance against shocks. We show that the former effect dominates the latter: the NDC pension system crowds out capital more than the FL system if labor supply is relatively elastic. If labor supply is relatively inelastic, the NDC pension system still crowds out capital accumulation more than the FL pension system as long as the level of idiosyncratic risk is not very high.

Hence, an important finding of the paper is that the strengthening of work incentives in the NDC pension system can come at the expense of lower savings. This outcome impacts steady state welfare through general equilibrium effects. In economies that are dynamically efficient in the sense of Cass (1972), since the FL pension system crowds out capital formation less it also ensures that the difference between the return on the investment in capital and the return on the investment in the pension system is lower than under the NDC pension system. Hence, it has a less negative impact on welfare. The general equilibrium effect reinforces the welfare gains from consumption insurance of the FL pension system at low levels of contributions. As the level of the contribution increases, compared to the NDC pension system, the FL pension system brings higher welfare losses due to labor supply distortions but lower welfare losses through general equilibrium effects. Which of the two effects dominates is a quantitative question. A calibrated version of the model shows that general equilibrium effects dominate and the FL pension system brings a higher welfare at any level of pension contributions.

The present paper considers both the benefits of PAYG pension systems stemming from better insurance in economies with incomplete markets and the costs of labor supply and savings distortions. Storesletten et al. (2004) show that the US pension system reduces consumption variance with 20% due to the particular, convex shaped link between pension contributions and pension benefits. Imrohoroglu et al. (1995) obtain that, due to its consumption insurance property, a PAYG with a replacement rate of 30% is welfare improving in the US. However, these papers do not consider the labor supply distortions of PAYG pension contributions. Gruber and Wise (1998), Erosa et al. (2012), Wallenius (2013), Alonso-Ortiz (2014), Bagchi (2015) show how different pension system arrangements impact on the labor supply of agents, but do not consider the insurance properties of the pension systems.

Our paper is also closely related to the literature analyzing pension reforms in economies with incomplete markets. Nishiyama and Smetters (2007) analyze the consequences of a 50% privatization of the US pension system. They conclude that this is welfare decreasing because of the lower insurance of idiosyncratic productivity shocks. However, if the privatization is accompanied by an increase in the progressivity of pension benefits and is financed by a consumption tax, then it can produce efficiency gains. Also focusing on the US pension system, Huggett and Parra (2010) determine the pension benefit function that is optimal from an ex-ante point of view. They consider benefit functions that are constant, linear or quadratic with respect to life-time income. The results show that the optimal benefit function entails more progressivity than the one currently in place. Fehr and Habermann (2008) and Fehr et al. (2013) show that reforming the current German pension benefit system towards more progressivity is welfare improving because of better insurance of idiosyncratic earnings risk.

We depart from this literature on pension reforms in a number of ways. First, we explicitly analyse the impact of the pension benefit arrangement on labor supply, savings and consumption inequality. Second, we develop a very tractable model that enables us to compare the ex-ante utility of pension systems in economies that differ along three dimensions: benefit progressivity, level of pension contributions and the size of the idiosyncratic earnings risk. In this way, we aim to explain the cross country heterogeneity in pension benefit arrangements. Finally, we point to the fact that not only the insurance property of pension systems with progressive benefits, but also their impact on welfare through general equilibrium effects can render them more efficient.

The setup of our model is related to Harenberg and Ludwig (2015). In contrast to them, we consider the labor supply decision of old agents, but abstract from aggregate shocks. However, the most important difference is that we also consider a NDC pension system and study its macroeconomic and welfare implications in comparison with the FL pension system.

The rest of the paper is structured in the following way. Section 2 presents cross-country data on the relationship between pension progressivity, on the one hand, and the size of the pension system and of the after tax income inequality, on the other hand. In Section

3 we present the model. Section 4 compares the steady states of the economies with a FL and a NDC pension system, while section 5 illustrates the welfare implications of pension progressivity in a calibrated version of the model. Section 6 concludes the paper. All the proofs are presented in the Appendix.

## 2 A cross-country analysis of pension arrangements

In this section we analyze the empirical relationship between the progressivity of pension systems, on the one hand, and the size of the pension system and of the idiosyncratic earnings risk, on the other hand. To this end, we use cross-country data for the following countries: Sweden, France, Germany, Netherlands, Austria, Belgium, Finland, Italy, Spain, UK, US, Canada, Australia, New Zealand, Japan.

As a proxy for the size of the idiosyncratic earnings risk, we consider the Gini coefficient of disposable income (income after taxes and transfers) for the population aged 18-64 (working age). We also consider the earnings inequality among retired agents by analysing the Gini coefficient of disposable income for the population aged over 65. Both data sets are obtained from the OECD database and we compute the average of the data recorded for 1974-2012. The pension progressivity index is taken from OECD Pensions at a Glance (2005)[3]. The size of the pension system is proxied by the gross replacement rate of the agent with an average income provided by OECD Pensions at a Glance (2005).

Figure 1 presents the relationship between the progressivity and the size of the pension system. The figure documents a considerable heterogeneity in the progressivity of pension systems and it indicates a strong negative correlation between pension progressivity and the size of the pension system.

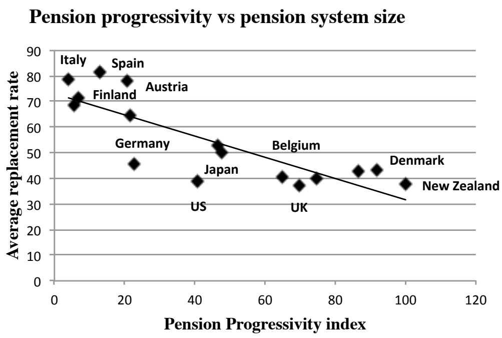



Figure 1

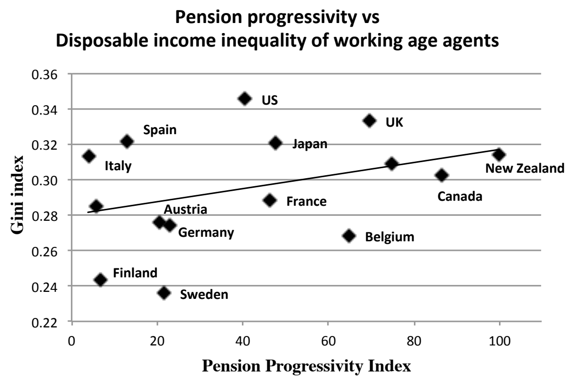



Figure 2

[3]In the paper we analyze only the implications of pension benefit progressivity. However, the OECD pension progressivity index also considers the progressivity of pension contributions.

Figure 2 shows the relation between disposable income inequality of working age agents and pension progressivity. The relationship between the two variables is positive, indicating that countries with a higher income inequality have more progressive pension arrangements.

Finally, figure 3 relates the progressivity of pension systems with the ratio between the Gini coefficient of retirement age population and the Gini coefficient of working age population. We notice a slight negative correlation between the two variables. We take this as an indication that higher pension progressivity reduces disposable income inequality of old agents compared to the disposable income inequality of young agents.

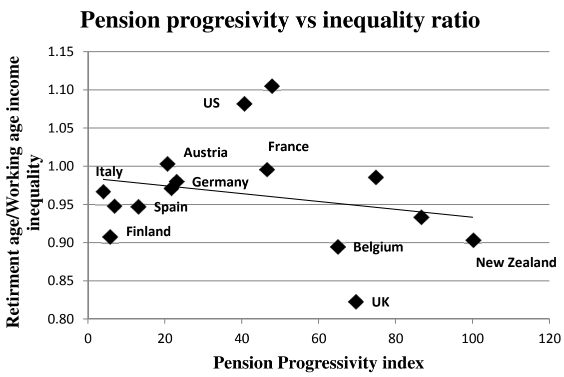



Figure 3

In this paper, we show that the relation between pension benefit progressivity, on the one hand, and the size of the pension system and of the idiosyncratic earning risk, on the other hand can be substantiated from a welfare point of view.

## 3 The model

We consider an economy with two overlapping generations: young ($y$) and old ($o$). There are $N^y$ young agents and $N^o$ old agents. Population grows at rate $n = \frac{N^y}{N^o} - 1$. Young agents can be thought of as the group of prime-aged workers (25-55 years) who decide how much to consume ($c^y$) and save ($a$). They supply labor inelastically. Old agents can be considered as the group of old workers (55-85 years old). They decide how much to consume ($c^o$) and work ($l^o$), but they do not save. We assume that agents face no uncertainty regarding the length of their life and that they die with no assets.

Agents have the same earnings $w\bar{z}$ when young, where $w$ is the average wage per unit of efficiency. When old, they are hit by an idiosyncratic earnings shock $z$ with mean $E(z) = \bar{z}$ and variance $var(z) = \sigma^2$. The earnings shock stands for the different labor market experiences and health status of old agents. At this point, we make no assumptions regarding the distribution of the idiosyncratic shock.

### 3.1 Households

Agents decide how much to consume, save and work maximizing their expected utility.

$$\max_{c_t^y, c_{t+1}^o(z), l_{t+1}^o(z), a_t} u(c_t^y) + \beta E u(c_{t+1}^o(z), l_{t+1}^o(z))$$
$$c_t^y + a_t = w_t z(1-\tau_t)$$
$$c_{t+1}^o(z) = a_t(1+r_{t+1}) + w_{t+1} z l_{t+1}^o(z)(1-\tau_{t+1}) + b_{t+1}(z)$$
$$l_{t+1}^o \in [0,1)$$

We denote by $w_t$ the wage per unit of efficiency and by $r_t$ the interest rate at time $t$. The contribution to the pension system is $\tau_t$, while the pension benefit is represented by $b_t$. We assume that the pension benefit can depend on the contributions paid by agents and, hence, on the labour supplied by agents[4].

We consider a form for life-time utility that is consistent with a balanced growth path:

$$u(c^y) + \beta E u(c^o, l^o) = \begin{cases} \frac{(c^y)^{\theta(1-\sigma)}}{1-\sigma} + \beta E \frac{((c^o)^\theta (1-l^o)^{1-\theta})^{1-\sigma}}{1-\sigma} & \text{, if } \sigma \neq 1 \\ \theta ln(c^y) + \beta E(\theta ln(c^o) + (1-\theta) ln(1-l^o)) & \text{, if } \sigma = 1 \end{cases} \tag{1}$$

where $\sigma$ is the inverse of the intertemporal elasticity of substitution and $1-\theta$ is the leisure share. The parameter $\theta$ is linked to the Frisch elasticity of labor supply: $\epsilon = \frac{1-l(z)}{l(z)} \frac{1-\theta(\sigma-1)}{\sigma}$, where $l(z)$ is the labor supply of an agent with idiosyncratic earnings shock $z$.

The first order conditions of the household's problem are:

$$\frac{-u_l(c_{t+1}^o(z), l_{t+1}^o(z))}{u_c(c_{t+1}^o(z), l_{t+1}^o(z))} = \left( w_{t+1} z(1-\tau_{t+1}) + \frac{\partial b_{t+1}(z)}{\partial l_{t+1}^o(z)} \right) \tag{2}$$
$$u_c(c_t^y) = \beta(1+r_{t+1}) E[u_c(c_{t+1}^o(z), l_{t+1}^o(z))] \tag{3}$$

If pension benefits are linked to pension contributions, a higher labor supply when old increases pension benefits, i.e. $\frac{\partial b_{t+1}(z)}{\partial l_{t+1}^o(z)} > 0$. Relation (2) shows that such a pension system lowers the distortion of the intratemporal consumption-leisure choice.

### 3.2 The pension system

The government runs a PAYG pension system. The contributions to the pension system come from a proportional tax on labor income $\tau_t$. Pension benefits can be arranged either as a flat benefit (FL) or they can be perfectly related to life-time pension contributions (a Notional Defined Contribution (NDC) pension).

[4] In means tested pension systems, the pension benefit can also depend on the size of an agent's savings, hence distorting the intertemporal consumption-savings decision. In the present paper we abstract from this possibility.

1. Under the flat benefit (FL) or Beveridge pension system, the benefit received from the pension system does not depend on the previous contributions of the agent. Hence, this is a pension system with high progressivity. The intratemporal consumption-leisure trade-off of equation (2) becomes:

$$\frac{-u_l(c^o_{t+1}(z), l^o_{t+1}(z))}{u_c(c^o_{t+1}(z), l^o_{t+1}(z))} = w_{t+1}z(1 - \tau_{t+1}) \tag{4}$$

2. Under the Notional Defined Contribution (NDC) system, benefits are proportional to the contributions made throughout the lifetime. Hence, there is no pension benefit progressivity. The consumption-leisure trade-off from equation (2) becomes:

$$\frac{-u_l(c^o_{t+1}(z), l^o_{t+1}(z))}{u_c(c^o_{t+1}(z), l^o_{t+1}(z))} = w_{t+1}z \tag{5}$$

   Hence, the NDC pension system does not impose any distortions on the labor supply of old agents.

   In line with the way that the NDC system is arranged in practice, the return of the NDC part $r^P_t$ is given by the growth rate of the aggregate wage bill:

$$1 + r^P_t = \frac{w_t L_t}{w_{t-1} L_{t-1}} \tag{6}$$

   Aggregate labor in the economy is represented by $L_t$. The return of the NDC pension system, $r^P_t$, is set in such a way that the budget of the pension system is balanced in the stationary steady state.

### 3.3 Firms

Firms operate a technology $F(A_t, K_t, L_t)$. Profit maximization yields the following conditions:

$$r_t + \delta = \frac{\partial F(A_t, K_t, L_t)}{\partial K_t} \tag{7}$$

$$w_t = \frac{\partial F(A_t, K_t, L_t)}{\partial L_t} \tag{8}$$

where $A_t$ is the total factor productivity, $K_t$ is the capital stock, $\delta$ is the depreciation rate of capital.

### 3.4 The equilibrium

In the following, we define the stationary competitive equilibrium of the economy described in sections 3.1-3.3.

**Definition 1.** *Given a certain arrangement of the PAYG pension system represented by* $\eta$*, a stationary competitive equilibrium of the model is represented by a set of time-invariant allocations* $\{c^y, c^o(z), l^o(z), a\}$*, prices* $\{w, r\}$ *and policies* $\{\tau, b(z)\}$ *such that:*

1. *Agents take optimal decisions given prices and policies (equations* (2) *and* (3)*);*

2. *Firms take the optimal decision given prices (equations* (7) *and* (8)*);*

3. *The budget of the pension system balances:*

$$w\tau L = N^o \int b(z)dF(z)$$

4. *Capital market clears:*

$$K = N^o a$$

5. *Labour market clears:*

$$L = N^y \bar{z} + N^o \int z l^o(z)dF(z)$$

In the following sections, all variables will be expressed in terms of labor per capita $l = \frac{L}{N^o}$ and capital to labor ratio $k = \frac{K}{L}$.

### 3.5 A closed form solution

We impose a number of assumptions on the parameters of the model in order to obtain an analytical solution.

**Assumption 1.** *Capital depreciates fully (*$\delta = 1$*), the technology is of Cobb-Douglas type* $F(A_t, K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}$ *and there is no technological progress (*$A_t = A_{t+1} = A$*). Preferences of agents are represented by the life-time utility described in relation* (1) *with* $\sigma = 1$*.*

With the above simplifying assumptions we obtain a closed form solution for the model described in sections 3.1-3.4. The resulting aggregate labor and capital per labor ratio are presented in Proposition 1.

**Proposition 1.** *Under Assumption 1, the level of labor (*$l = \frac{L}{N^o}$*) and capital to labor ratio (*$k = \frac{K}{L}$*) in the stationary competitive equilibrium of the economies with a FL pension system* *and a NDC pension system, respectively, are equal to:*

$$l^{FL} = \frac{(1+n+\theta)(1-\alpha)(1-\tau)\bar{z}}{1-\alpha\theta-(1-\alpha)\theta\tau} \tag{9}$$

$$k^{FL} = \left(\frac{A\bar{z}(1-\alpha)(1-\tau)s^{FL}}{l^{FL}}\right)^{\frac{1}{1-\alpha}} \tag{10}$$

$$l^{NDC} = \frac{(1+n+\theta-\tau(1-\theta)(1+n))(1-\alpha)\bar{z}}{1-\alpha\theta} \tag{11}$$

$$k^{NDC} = \left(\frac{A\bar{z}(1-\alpha)(1-\tau)s^{NDC}}{l^{NDC}}\right)^{\frac{1}{1-\alpha}} \tag{12}$$

*where*

$$s^{FL} = \frac{\beta\Gamma^{FL}}{\theta+\beta\Gamma^{FL}} \tag{13}$$

$$\Gamma^{FL} = \frac{\alpha}{1-\alpha}l^{FL}E\left[\frac{1}{\frac{\alpha}{1-\alpha}l^{FL}+\tau l^{FL}+z(1-\tau)}\right] \tag{14}$$

$$s^{FL} = \frac{\beta\Gamma^{NDC}}{\theta+\beta\Gamma^{NDC}} \tag{15}$$

$$\Gamma^{NDC} = \frac{\alpha}{1-\alpha}l^{NDC}E\left[\frac{1}{\frac{\alpha}{1-\alpha}l^{NDC}+z+\bar{z}\tau(1+n)}\right] \tag{16}$$

Labor supply is decreasing in the pension contribution rate $\tau$ in both types of pension systems. Equations (4) and (5) show that the pension contribution distorts the intratemporal choice between leisure and consumption in the FL pension system, but not in the NDC pension system[5]. However, it lowers the accumulation of savings under both types of pension systems. The smaller amount of capital leads, in equilibrium, to a lower demand for labor also in the case of the NDC pension system.

The pension contribution impacts the capital to labour ratio through three channels: i) the distortion of the intertemporal savings-consumption decision - reflected by the term $1-\tau$ -, ii) the savings rate $s$ and iii) the labor supply $l$. An increase in the contribution to the pension system lowers the capital to labour ratio through the distortion of intertemporal choices and through the lower savings rate. However, an increase in the contribution to the pension system also lowers labour supply and this triggers an increase in savings and, hence, in capital.

[5]The contribution to the NDC pension system would distort the intratemporal decision of young people, if they supplied labor elastically. However, it would still be the case that the NDC pension system imposes lower labor supply distortions than the FL pension system.

# 4 A comparison between the FL and NDC pension economies

In this section, we compare the steady states of two economies with extreme types of pension benefit arrangements: i) a FL system and ii) a NDC system. We focus on the differences between the two economies in terms of aggregate labor, capital to labor ratio, consumption inequality and welfare. To make the economies with the two types of pension systems comparable, we consider that the same level of pension contributions $\tau$ prevails in both economies.

## 4.1 Macroeconomic variables

Proposition 2 presents the impact of the pension benefit arrangement on aggregate variables.

**Proposition 2.** *Under Assumption 1, comparing aggregate labor and the capital to labor ratio under the FL and NDC pension systems, we obtain the following results:*

1. *Aggregate labour is higher in the economy with a NDC pension system, i.e.* $l^{NDC} > l^{FL}$*;*
2. *In an economy without idiosyncratic productivity risk, the NDC pension system crowds out capital formation more than the FL pension system, i.e.* $k^{NDC}_{\sigma=0} < k^{FL}_{\sigma=0}$*;*
3. *For each* $(\alpha, \tau)$*, there exists* $\theta^*(\alpha, \tau) \in (0, 1]$ *such that:*
   - *if* $\theta < \theta^*(\alpha, \tau)$ *(labor supply is relatively elastic), the capital to labor ratio increases with respect to the size of the idiosyncratic risk more under the FL pension system than under the NDC pension system, i.e.* $\frac{\partial k^{FL}}{\partial \sigma^2} > \frac{\partial k^{NDC}}{\partial \sigma^2}$*. Consequently, the NDC pension system crowds out capital formation more than the FL pension system at any level of idiosyncratic earnings risk, i.e.* $k^{NDC} < k^{FL}, \forall\ \sigma$*;*
   - *if* $\theta > \theta^*(\alpha, \tau)$ *(labor supply is relatively inelastic), the capital to labor ratio increases with respect to the size of the idiosyncratic risk more under the NDC pension system than under the FL pension system, i.e.* $\frac{\partial k^{FL}}{\partial \sigma^2} < \frac{\partial k^{NDC}}{\partial \sigma^2}$*. In this case the FL pension system can crowd out capital formation more than the NDC pension system, i.e.* $k^{NDC} > k^{FL}$*, when the size of idiosyncratic earnings risk is high enough.*

The fact that aggregate labor is higher under the NDC pension system is intuitive, since this type of pension system does not distort the consumption-leisure decision.

The capital prevailing under the two pension systems is determined by two opposing effects. On the one hand, because people work more under the NDC pension system, they can insure the same old-age consumption level by making lower savings. Consequently, in an economy with no idiosyncratic shocks the capital to labor ratio in the NDC pension system is lower than in the FL pension system, i.e. $k^{NDC}_{\sigma=0} < k^{FL}_{\sigma=0}$.

On the other hand, as we show in the Appendix, capital is increasing in the size of the idiosyncratic risk under both types of pension systems. This is because agents make higher precautionary savings. If labor supply is relatively inelastic ($\theta > \theta^*$), capital increases faster with respect to $\sigma^2$ under the NDC system. This effect is intuitive. An increase in the size of the idiosyncratic risk triggers higher precautionary savings. The effect is more pronounced in an economy with an NDC pension system because this offers lower consumption insurance (Proposition 3). Hence, capital can be higher in the NDC pension economy than in the FL pension economy if the size of idiosyncratic risk is large enough so that the effect through higher precautionary savings dominates.

If labor supply is relatively elastic ($\theta < \theta^*$), agents adjust their labor supply significantly when the size of the idiosyncratic risk is higher: agents with high earnings work even more and agents with low earnings work even less. The flexible adjustment of labor supply makes agents increase their savings for precautionary reasons to a lesser degree. Consequently, when $\theta < \theta^*$ the NDC pension system economy crowds out capital formation more than the FL pension system at any level of idiosyncratic productivity risk.

Next, we investigate the insurance properties of the two pension systems.

**Proposition 3.** *Under Assumption 1, consumption inequality is lower in the FL pension system economy than in the NDC pension system economy. Both types of pension systems lower consumption inequality compared to an economy with no pension system.*

The FL pension system lowers consumption inequality because it provides a return that decreases with the earnings of agents. This comes from the fact that benefits are flat, but contributions are proportional to earnings. The NDC pension system, however, does not provide direct consumption insurance, because the pension benefits are perfectly linked to the contributions. Compared to an economy without a pension system, the reduction in consumption inequality in the NDC pension system is brought about by the fact that the capital crowding out also reduces labor demand. Hence, all types of agents retire earlier in the economy with a NDC pension system, lowering their exposure to idiosyncratic earnings shocks.

### 4.2 Welfare

We turn to assessing the impact that the different macroeconomic and consumption inequality outcomes have on the ex-ante utility of agents living under the two types of pension systems. We consider the implication of increasing the contribution to the pension system for the utility that agents have at birth:

$$SWF = U(c^y) + \beta EU(c^o, l^o)$$

The following proposition presents the trade-off among consumption insurance, labor supply and capital accumulation that defines the welfare in the two economies.

**Proposition 4.** *Under Assumption 1, the change in welfare caused by a marginal increase in the contribution to the pension system is composed of the following three terms:*

1. *the welfare loss from lower consumption due to the higher contribution to the pension system ($\omega_1'$);*
2. *the welfare gain from higher consumption due to the higher pension benefit ($\omega_1''$);*
3. *the impact of general equilibrium effects ($\omega_2$).*

| | *FL* | *NDC* |
|---|---|---|
| $\omega_1'$ | $-\frac{\theta}{1-\tau} - \frac{\beta\theta}{1-\tau}$ | $-\frac{\theta}{1-\tau} - \frac{\beta\Gamma^{NDC}}{1-\tau}$ |
| $\omega_1''$ | $\beta\frac{1-\alpha}{\alpha}\Gamma^{FL}$ | $\beta\frac{1-\alpha}{\alpha}\bar{z}(1+n)\frac{\Gamma^{NDC}}{l^{NDC}}$ |
| $\omega_2$ | $\frac{\partial ln(k^{FL})}{\partial\tau}\Phi^{FL} + \frac{\partial ln(s^{FL})}{\partial\tau}\frac{\beta\tau\Gamma^{FL}(1-\alpha)}{\alpha}$ | $\frac{\partial ln(k^{NDC})}{\partial\tau}\Phi^{NDC}$ |
| $\Phi$ | $\theta\alpha(\beta+1) - \beta(\alpha+\tau(1-\alpha))\frac{1-\alpha}{\alpha}\Gamma^{FL}$ | $\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{NDC}$ |

$$\omega_1 = \omega_1' + \omega_1'' \tag{17}$$

$$\frac{\partial SWF}{\partial\tau} = \omega_1 + \omega_2 \tag{18}$$

An increase in the contribution to the pension system produces a reallocation in the income sources for consumption: net wages of young and old agents decrease while pension benefits increase. The welfare loss due to lower consumption from net wages is measured by the term $\omega_1'$, while the welfare gain of consumption from higher pension benefits is given by $\omega_1''$. The term $\omega_1''$ increases with $\Gamma$ and, hence, with the size of the idiosyncratic risk and of consumption inequality in the economy (see proof of Proposition 2). Intuitively, the higher is consumption inequality in the economy, the more valuable is the insurance provided by the pension system and the higher is the present value of the additional pension benefit agents will receive. We call $\omega_1 = \omega_1' + \omega_1''$ *the consumption reallocation effect*.

The last component of the marginal welfare change, $\omega_2$, captures the impact of the higher contribution to the pension system working through *general equilibrium effects*. A higher contribution reduces savings and, hence, crowds out capital formation. In general equilibrium, the lower level of capital impacts on welfare through factor prices: wages decrease and the return on capital increases. If the economy is dynamically efficient, capital crowding out has a negative impact on welfare.

As Proposition 2 shows, the NDC system crowds out capital formation more than the FL system as long as the size of idiosyncratic risk is not very high:

$$\frac{\partial ln(k^{NDC})}{\partial\tau} < \frac{\partial ln(k^{FL})}{\partial\tau} < 0 \tag{19}$$

From this point of view, the NDC system brings higher welfare losses through general equilibrium effects than the FL pension system in dynamically efficient economies.

The sign and size of the impact that capital crowding out has on welfare depends on the term $\Phi$.

Let us first analyze the impact of a higher pension contribution on welfare through general equilibrium effects at the marginal introduction of the pension system, i.e. $\tau = 0$. In this case $\Phi^{FL} = \Phi^{NDC}$ and the term $\Phi$ has the following interpretation. A marginal introduction of the pension system decreases the capital to labor ratio. On the one hand, this leads to a higher return on savings and increases consumption when old. The welfare gain of this effect is given by $\beta(1-\alpha)\Gamma$. On the other hand, a decrease in the capital to labor ratio leads to a lower wage, decreasing consumption in both periods of life. The welfare loss from lower wages is given by $\theta\alpha(\beta+1)$. Hence, the sign of $\Phi$ indicates the net impact of the capital crowding out effect on welfare.

In an economy with no idiosyncratic earnings shocks, capital crowding out has a negative effect on welfare if $\Phi_{\sigma=0,\tau=0} > 0$. We prove in Appendix 2 that $\Phi_{\sigma=0,\tau=0} > 0$ also implies that the economy is dynamically efficient in the absence of idiosyncratic shocks. Hence, if the economy is dynamically efficient then the introduction of the PAYG pension system has a negative impact on welfare through general equilibrium effects. This is a standard result in the literature.

The opposite is true if $\Phi_{\sigma=0,\tau=0} < 0$: the economy is dynamically inefficient and capital crowding out has a positive effect on welfare.

Let us analyze the case when the pension contribution is increased past the point of the marginal introduction, i.e. $\tau > 0$. In this case, it is straightforward to show that the impact of capital crowding out on welfare is higher under the NDC pension system than under the FL pension system, i.e. $\Phi^{NDC} > \Phi^{FL}$. This is mainly due to the the following term that appears only in the case of the general equilibrium effect of the FL pension system on welfare:

$$\Omega = -\beta\tau\frac{(1-\alpha)^2}{\alpha}\Gamma^{FL} \tag{20}$$

This term ensures that, as $\tau$ increases, the FL system brings lower (higher) welfare losses (gains) through general equilibrium effects than the NDC pension system.

The intuition for the term $\Omega$ is the following. The contribution to the FL pension system distorts both labor supply and capital accumulation. In contrast, the NDC pension system does not distort labor supply and all the distortions fall on capital accumulation. As the contribution to the FL pension system $\tau$ increases, the distortions on labor supply increase (quadratically) with the level of pension contribution. However, this implies that distortions of capital accumulation decrease with the level of pension contribution. Hence, the negative impact on welfare through general equilibrium effects decreases in the FL pension system.

### 4.3 Welfare comparison

In order to determine the relative impact on welfare of the two pension systems, we compare the sizes of $\omega_1$ and $\omega_2$. For ease of exposition, we focus in this section on the relevant case when the economy is dynamically efficient in the absence of idiosyncratic shocks. This is the case if the parameters of the model fulfil the restriction $\Phi_{\sigma=0,\tau=0} > 0$. We consider the case of a dynamically inefficient economy in Appendix 3. We start by analyzing the two terms of the marginal welfare change in an economy with no uncertainty ($\sigma = 0$).

**Proposition 5.** *Under Assumption 1, in a dynamically efficient economy with no idiosyncratic earnings risk ($\Phi_{\sigma=0,\tau=0} > 0, \sigma = 0$) a marginal increase in the contribution to the pension system has:*

1. *a higher (lower) positive (negative) impact on welfare through the consumption reallocation effect in the case of NDC system than in the case of FL system, i.e. $\omega_1^{NDC}|_{\sigma=0} \geq \omega_1^{FL}|_{\sigma=0}$. At the marginal introduction of the pension system, the impact on welfare of the two pension systems is the same, i.e. $\omega_1^{NDC}|_{\sigma=0,\tau=0} = \omega_1^{FL}|_{\sigma=0,\tau=0}$;*

2. *a lower negative impact on welfare through general equilibrium effects in the case of the FL pension system ($\omega_2^{FL}|_{\sigma=0} > \omega_2^{NDC}|_{\sigma=0}$).*

Proposition 5 shows that in the absence of idiosyncratic shocks, at the marginal introduction of the pension system ($\tau = 0$), the impact on welfare through the consumption reallocation effect is the same in the case of the two pension systems ($\omega_1^{FL}|_{\sigma=0,\tau=0} = \omega_1^{NDC}|_{\sigma=0,\tau=0}$). However, for $\tau > 0$, the NDC pension system brings a higher (lower) welfare gain (loss) through consumption reallocation because it distorts labor supply less.

General equilibrium effects favor the FL pension system at all levels of pension contributions ($\omega_2^{FL}|_{\sigma=0} > \omega_2^{NDC}|_{\sigma=0}$). Two effects determine this result. First, the FL pension system crowds out capital formation less than the NDC pension system and this has a less negative impact on welfare through prices. Second, the impact of the FL system on welfare through general equilibrium effects contains the additional term $\Omega$ identified in equation (20). This term is positive and increases with the level of pension contributions.

In conclusion, in the absence of idiosyncratic shocks, at the marginal introduction of the pension system, the FL system dominates the NDC system from a welfare perspective. As $\tau$ increases, the consumption reallocation effect and the general equilibrium effects act in opposite direction favoring the NDC system and the FL system, respectively.

Proposition 6 shows that the consumption reallocation effect increases with the size of idiosyncratic risk. Hence, welfare losses brought by pension systems decrease. Also, the consumption reallocation effect increases more in the case of the FL pension system. While in the absence of idiosyncratic shocks, at the marginal introduction of the pension system ($\tau = 0$), the consumption reallocation effect is the same under the two pension systems, in the presence of idiosyncratic earnings risk, the FL pension system brings lower welfare losses than the NDC system. This is due to the better insurance provided by the FL pension system. However, as $\tau$ increases, we expect the welfare gains from insurance to increase and the losses from labor distortions to increase. The size of the idiosyncratic earnings risk matters: the higher the earnings risk is, the higher will be the welfare gains from consumption reallocation in the case of the FL pension system.

In the absence of idiosyncratic earnings risk, the welfare losses brought by the pension systems through general equilibrium effects are lower under the FL system. In the presence of idiosyncratic risk, the difference between the two pension systems is expected to decrease. This is because the difference in capital crowding out between the NDC pension system and the FL pension system lowers as the size of idiosyncratic risk increases (Proposition 2) if labor supply is not very elastic. However, it is unclear whether a high level of idiosyncratic earnings risk could lead to lower welfare losses through capital crowding out in the NDC pension system. Also, whether this will dominate the higher welfare brought by the FL pension system through insurance is a quantitative question. We explore this in Section 5.

**Proposition 6.** *A higher level of idiosyncratic earnings risk increases (decreases) the welfare gains (losses) from the consumption reallocation effect under the FL system more than under the NDC system, i.e.* $\frac{\partial\left(\omega_1^{FL}-\omega_1^{NDC}\right)}{\partial\sigma^2} > 0$.

In conclusion, if we abstract from general equilibrium effects, we obtain that the FL pension system brings a higher welfare at low levels of pension contributions due to consumption insurance, while the NDC pension system brings a higher welfare at high levels of pension contributions due to lower labor supply distortions.

## 5 A numerical example

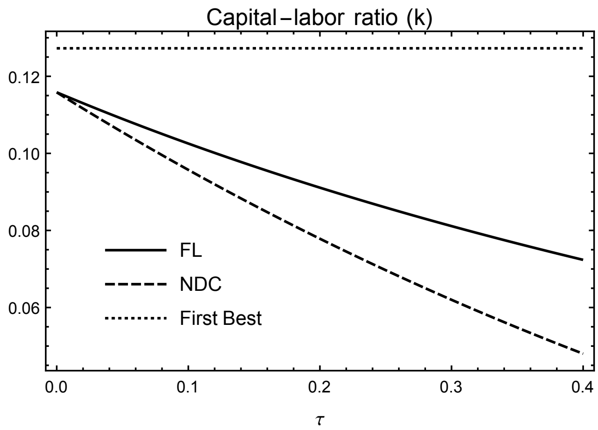
In the previous section we established that the FL pension system brings a higher welfare at low levels of pension contributions in dynamically efficient economies. It is unclear what happens when the level of pension contribution is higher. We analyze this using a calibrated version of the model in this section.

We consider that the idiosyncratic productivity shock has a lognormal distribution $z = e^y, y \sim N(\mu, \sigma^2)$. We assume that a period of the model represents 30 years. Following the relevant literature (Storesletten et al. (2004), French (2005), Kaplan (2012)), we set the locational parameters of $z$ by assuming the following values at yearly frequency: $\mu_y = 0$ and $\sigma_y^2 = 0.014$. The value for the variance of the earnings shock is consistent with estimates for the persistent part of the earnings process. Since we considered young agents to be homogeneous, we abstract from the permanent earnings shocks that affect agents when entering the labor market. We calibrate $\beta = 0.99^{30}$ and $\alpha = 0.3$ in line with Song (2011), Gonzalez-Eiras and Niepelt (2008), Harenberg and Ludwig (2015). The yearly population growth rate is set to

0.8%, a value consistent with the UN population projections for the US. The leisure share is set to $1 - \theta = 0.58$, implying a labor disutility of 1.38, similar to the value considered in Alonso-Ortiz and Rogerson (2010).

For the set of parameters we chose, the capital to labor ratio is below the first best level in the absence of the pension system (Figure 4). Hence, the economy is dynamically efficient. The NDC pension system crowds out capital more than the FL pension system at every level of contribution to the pension system ($\tau$). In Proposition 2 we pointed out that if labor supply is not very elastic, the NDC pension system could crowd out capital less than the FL pension system if the size of the idiosyncratic risk is very high. We find that this does not happen for a realistically calibrated value of $\sigma$.

Figure 4: Capital to labor ratio




## 5.1 Welfare analysis

Figure 5 presents the difference in ex-ante utility between the FL and the NDC pension economies at different levels of pension contribution. The welfare change caused by a marginal increase in the contribution to the pension system is shown in Figure 6.

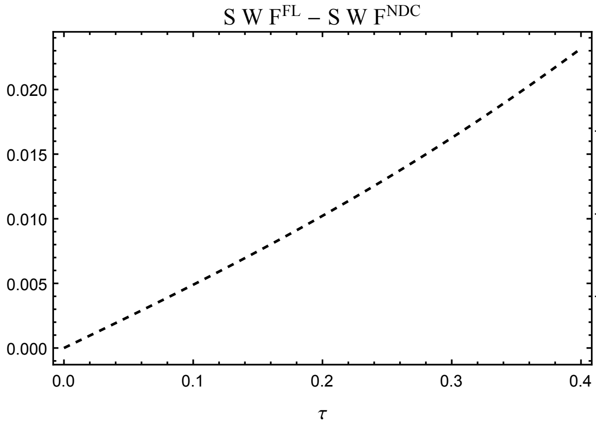




Figure 5: Difference in welfare

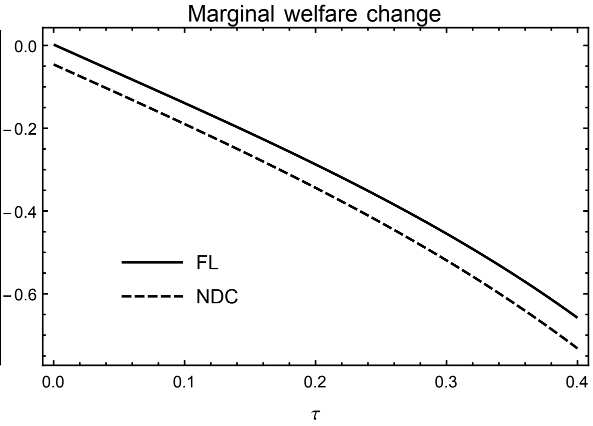




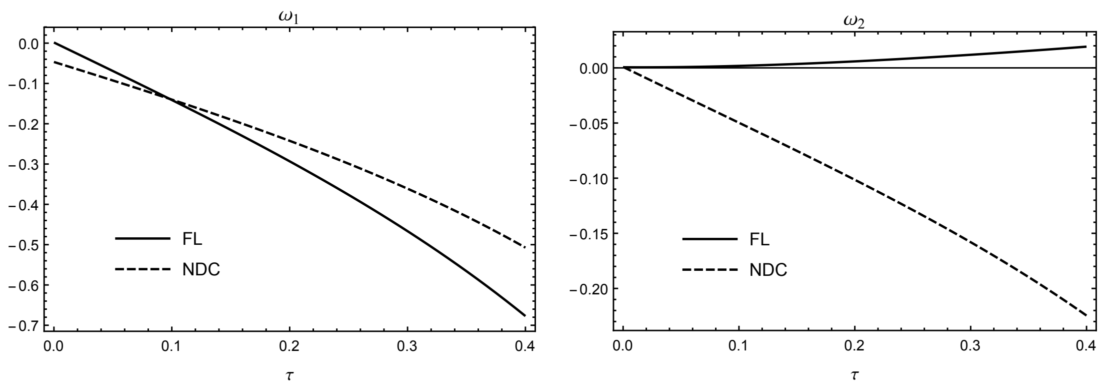
Figure 6: Marginal welfare change

The FL pension system brings a higher welfare than the NDC pension system at all levels of pension contribution ($SWF^{FL} - SWF^{NDC} > 0$). To see why this happens, we analyse the two components of the marginal welfare change identified in Proposition 4. These are illustrated in Figure 7, panels (a) and (b).

First, as anticipated by Proposition 4, the FL pension system has a lower negative impact on welfare through consumption reallocation ($\omega_1^{FL} > \omega_1^{NDC}$) when the level of contribution is small (Figure 7, panel (a)). In our calibration, a marginal increase of the FL pension system leads to lower welfare losses if the contribution is less than 10%. This is due to the insurance that the FL pension provides. As the level of pension contribution increases, the welfare loss from the higher labor supply distortions of the FL pension system increases and the welfare gain from insurance decreases. Consequently, for high levels of pension contributions, the marginal welfare loss from consumption reallocation becomes higher in the FL system ($\omega_1^{FL} < \omega_1^{NDC}$).

Figure 7: The components of marginal welfare change

(a) Consumption reallocation effect ($\omega_1$) (b) General equilibrium effect ($\omega_2$)



Second, as Figure 7 panel (b) shows, the FL pension system has a less negative impact on welfare through general equilibrium effects at any level of pension contribution ($\omega_2^{FL} > \omega_2^{NDC}$). This result was anticipated by Proposition 5 and is due to the fact that NDC pension system crowds out capital formation more than the FL pension. Moreover, as the level of pension contribution $\tau$ increases, the negative impact on welfare through general equilibrium effects amplifies under the NDC pension system, but decreases under the FL pension system. Two effects contribute to this result: i) the difference in capital to labor ratio between the two pension systems increases with the level of pension contribution (Figure 4 and Proposition 2) and ii) the impact of the FL pension system on welfare through general equilibrium effects contains the additional term $\Omega$ (relation (20)) that is always positive and increases at all levels of pension contributions.

Overall, general equilibrium effects ensure that the FL pension system brings a higher welfare also at high levels of pension contributions.

## 5.2 Comparison with a partial equilibrium model

In this section we analyze the role played by general equilibrium effects in determining the relative welfare of FL and NDC pension systems. We consider a version of the model in which the interest rate and the wage do not change when we vary the contribution to the pension system. We label this a partial equilibrium analysis and we consider it relevant for small open economies.

Figure 8: General equilibrium

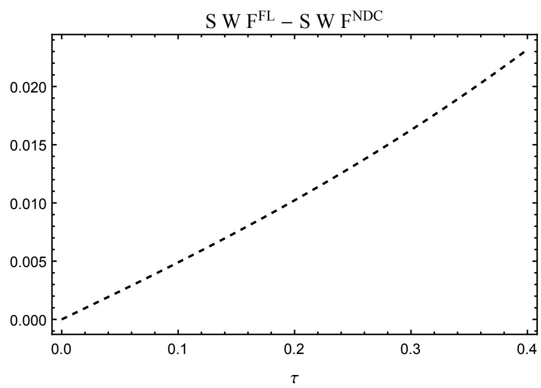
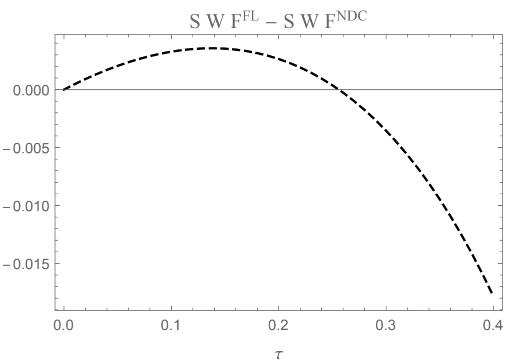



Figure 9: Partial equilibrium




Figures 8 and 9 present the difference between the welfare in the FL pension economy and the NDC pension economy in a general equilibrium and in a partial equilibrium setting, respectively. In the general equilibrium setting, the FL pension system brings a higher welfare than the NDC pension system at any level of pension contribution. In partial equilibrium, the welfare difference is given by the difference in $\omega_1$ between the FL and the NDC pension economies. The welfare is higher in the FL pension system for low levels of pension contributions. As the level of pension contribution increases, the welfare of the FL pension system becomes lower due to the labor supply distortions.

The conclusion is that general equilibrium effects play a decisive role in the welfare comparison between the two types of pension systems in dynamically efficient economies and they may tilt the balance in favour of the FL pension system even at high levels of pension contributions.

### 5.3 Optimal combination of FL and NDC pension

In this section we analyze whether a combination of FL and NDC pension system is more efficient. Hence, we consider that pension benefits are arranged as a linear combination of a FL and an NDC pension benefit:

$$b_t(z) = \eta b_t^{FL} + (1-\eta) b_t^{NDC}(z) \tag{21}$$

$$b_t^{NDC}(z) = w_{t-1}\bar{z}\tau_{t-1}(1+r_t^P) + w_t z l_t^o(z)\tau_t \tag{22}$$

(23)

The term $b_t^{FL}$ defines the part of the pension benefit that does not depend on the previous pension contributions, i.e. the flat benefit part. The term $b_t^{NDC}(z)$ defines the part of the pension benefit that is perfectly linked to previous contributions.

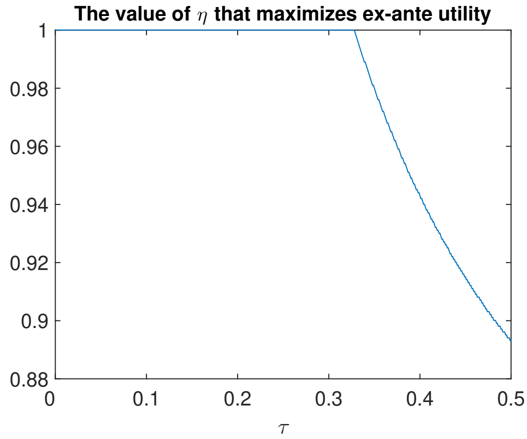
To determine the optimal combination of FL and NDC pensions at each level of pension contributions, we compute the value of $\eta$ that maximizes the ex-ante utility of agents:

$$\max_{\eta \in [0,1]} SWF(\sigma, \tau, \eta) = U(c^y) + \beta EU(c^o, l^o)$$

The consumption of young agents and the consumption and labor supply of old agents ($c^y$, $c^o$, $l^o$) are the the ones prevailing in the stationary steady state of the economy presented in Proposition 2. The coefficient $\eta$ is a parameter of the pension system that determines the degree of pension benefit progressivity: a higher $\eta$ implies a higher progressivity of pension benefits.

We vary $\tau$ between $[0, 0.5]$. Figure 10 presents the optimal value of $\eta$ at each level of pension contribution $\tau$.

Figure 10: Optimal value of $\eta$




## 6 Conclusion

In this paper we analyzed the macroeconomic and welfare implications of two extreme types of pension benefits arrangements: the FL and NDC pension systems. We found that the FL system leads to lower aggregate labor, but also to lower consumption inequality. As long as the level of idiosyncratic risk is not very high, the FL system also crowds out capital formation less than the NDC system.

Comparing the welfare under the two pension systems, if we abstract from general equilibrium effects, the FL pension system brings a higher welfare at low levels of pension contributions. As the level of pension contribution becomes higher, the NDC pension system brings a higher welfare due to the lower labor supply distortions. This result can explain why countries with a high size of the PAYG pension system like Germany, Italy, France and Poland switched to a perfect link between pension contributions and pension benefits. Countries with a small size of the pension system like the UK prefer a FL pension system because of the insurance it provides. The impact of the pension systems on welfare through general equilibrium effects tilt the balance in favor of the FL pension system in dynamically efficient economies.

For future research, it may prove useful to verify whether the results hold also in a more detailed model. For example, it would be interesting to analyze how other tax and transfer programs ran by the government interact with the PAYG pension system. Progressive income taxes also offer some degree of insurance, thus reducing the importance of the FL pension system.

The tractability of our model makes it an extremely useful tool to analyse other issues related to pension benefit arrangements. One straightforward extension of the present paper is to determine what (non)linear combination of the FL or NDC pension system would be better from a welfare perspective at different levels of idiosyncratic risk and of pension contributions.

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# Appendix 1

## Proof of Proposition 1

With the preferences considered in Assumption 1, using (2) and the budget constraints of the agents and government, we find closed form solutions for consumption and labor as a function of savings and prices:

| FL | NDC |
|---|---|
| $c^y = w\bar{z}(1-\tau) - a$ | $c^y = w\bar{z}(1-\tau) - a$ |
| $c^o(z) = \theta(a(1+r) + wz(1-\tau) + w\tau l)$ | $c^o(z) = \theta(a(1+r) + wz + w\bar{z}\tau(1+n))$ |
| $l^o(z) = \theta - (1-\theta)\frac{a(1+r)+w\tau l}{wz(1-\tau)}$ | $l^o(z) = \theta - (1-\theta)\frac{a(1+r)+w\bar{z}\tau(1+n)}{wz}$ |

In the above relations we replace the formula for the gross rate of return from the firm's problem:

$$1 + r = \frac{\alpha}{1-\alpha}\frac{l}{a}w$$

and obtain:

| FL | NDC |
|---|---|
| $c^y = w\bar{z}(1-\tau) - a$ | $c^y = w\bar{z}(1-\tau) - a$ |
| $c^o(z) = \theta w(\frac{\alpha}{1-\alpha}l + z(1-\tau) + \tau l)$ | $c^o(z) = \theta w(\frac{\alpha}{1-\alpha}l + z + \bar{z}\tau(1+n))$ |
| $l^o(z) = \theta - (1-\theta)\frac{\frac{\alpha}{1-\alpha}l+\tau l}{z(1-\tau)}$ | $l^o(z) = \theta - (1-\theta)\frac{\frac{\alpha}{1-\alpha}l+\bar{z}\tau(1+n)}{z}$ |

The expressions for consumption are substituted in (3).

We solve for the level of assets:

$$a = \frac{\beta\Gamma}{\beta\Gamma+\theta}\bar{z}(1-\tau)(1-\alpha)\left(\frac{a}{l}\right)^{\alpha} \Rightarrow l = s\bar{z}(1-\tau)(1-\alpha)k^{\alpha-1} \tag{24}$$

where $\Gamma^{FL} = \frac{\alpha}{1-\alpha}lE\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z(1-\tau)+\tau l}\right]$, $\Gamma^{NDC} = \frac{\alpha}{1-\alpha}lE\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z+\bar{z}\tau(1+n)}\right]$ and $s = \frac{\beta\Gamma}{\beta\Gamma+\theta}$.

We aggregate individual labor supply in order to obtain a closed form expression for it:

$$L = N^y z + N^o \int_i z_i l_i dF(z_i) \Rightarrow$$

$$l^{FL} = \frac{(1+n+\theta)(1-\alpha)(1-\tau)\bar{z}}{1-\alpha\theta-\theta\tau(1-\alpha)} \tag{25}$$

$$l^{NDC} = \frac{(1+n+\theta-(1-\theta)\tau(1+n))(1-\alpha)\bar{z}}{1-\alpha\theta} \tag{26}$$

We obtain the capital to labor ratio by substituting (26) in (24).

## Proof of Proposition 2

For simplicity, we will prove the Proposition for $n = 0$ and $E(z) = \bar{z} = 1$.

Using (9) and (11), we get:

$$l^{NDC} - l^{FL} = \frac{(1-\alpha)(1-\alpha(1-\tau)-\tau)\tau\theta(1-\theta)}{(1-\alpha\theta)(1-\alpha\theta(1-\tau)-\tau\theta)} > 0$$

Using (10) and (12), we show that capital accumulation is lower in the NDC pension system in absence of idiosyncratic productivity shocks ($\sigma = 0$) at all levels of pension contributions:

$$sgn\left(k^{NDC}_{\sigma=0} - k^{FL}_{\sigma=0}\right) = sgn\left(\left(\frac{k^{NDC}_{\sigma=0}}{k^{FL}_{\sigma=0}}\right)^{1-\alpha} - 1\right) =$$

$$= sgn\left(-\frac{\tau\theta(1+\alpha\beta)(1-\theta)(1+\alpha+\tau(1-\alpha)))}{(1-\alpha\theta-\tau\theta(1-\alpha))(\theta(1+\alpha)+\alpha\beta(1+\theta)+\tau(\theta(1-\alpha)-\alpha\beta(1-\theta)))}\right) = -$$

Using the logarithm version of (10) and (12) , we show that, in absence of idiosyncratic productivity shocks ($\sigma = 0$), an increase in the contribution to the pension system reduces capital under the NDC pension system more than under the FL pension system:

$$\frac{\partial(ln(k^{NDC}_{\sigma=0}) - ln(k^{FL}_{\sigma=0}))}{\partial\tau} =$$

$$-\frac{\theta(1+\alpha\beta)(1-\theta)[(1-\alpha)(\theta+1)(1-\tau)\alpha\beta\tau + (\theta(1+\alpha)+\alpha\beta(\theta+1)+\tau\theta(1-\alpha))(1+\alpha(1-\tau)+\tau))]}{(1-\alpha)(1-\tau)(\theta(1+\alpha)+\alpha\beta(\theta+1)+\tau\theta(1-\alpha))(\theta(1+\alpha)+\alpha\beta(\theta+1)+\tau(\theta(1-\alpha)-\alpha\beta(1-\theta)))(1-\alpha\theta-\tau\theta(1-\alpha))} < 0$$

We compute the change of the capital stock with respect to the variance of the idiosyncratic risk:

$$\frac{\partial k^{1-\alpha}}{\partial\sigma^2} = \frac{A(1-\alpha)(1-\tau)}{l}\frac{\partial s}{\partial\sigma^2} = \frac{A(1-\alpha)(1-\tau)\theta\beta}{l(\theta+\beta\Gamma)^2}\frac{\partial\Gamma}{\partial\sigma^2}$$

The relationship above holds for both the FL and the NDC economy.

We determine the sensitivity of $\Gamma$ with respect to the size of the idiosyncratic shock by taking a second order Taylor expansion around $(E(z) = 1, \tau)$ and differentiating with respect to $\sigma^2$:

$$\frac{\partial\Gamma^{FL}}{\partial\sigma^2} \approx \left.\frac{\partial^2\Gamma^{FL}}{\partial z^2}\right|_{\sigma=0} = \frac{\alpha}{1-\alpha}l^{FL}\frac{(1-\tau)^2}{\left(\frac{\alpha}{1-\alpha}l^{FL}+\tau l^{FL}+1-\tau\right)^3} > 0 \tag{27}$$

$$\frac{\partial\Gamma^{NDC}}{\partial\sigma^2} \approx \left.\frac{\partial^2\Gamma^{NDC}}{\partial z^2}\right|_{\sigma=0} = \frac{\alpha}{1-\alpha}l^{NDC}\frac{1}{\left(\frac{\alpha}{1-\alpha}l^{NDC}+1+(1+n)\tau\right)^3} > 0 \tag{28}$$

$$\frac{1}{l^{FL}}\frac{\partial\Gamma^{FL}}{\partial\sigma^2} - \frac{1}{l^{NDC}}\frac{\partial\Gamma^{NDC}}{\partial\sigma^2} \approx \frac{\alpha\tau P(\theta)}{(1-\alpha)(1-\tau)(1+\tau+\alpha(1-\tau))^3} \tag{29}$$

where

$$P(\theta) = 1 - 3\theta + 3\theta^2(1 - (1-\tau)(1-\alpha)^2) + \theta^3(-3\alpha(1-\tau)(\alpha(1-\tau)+\tau) - \tau^2 + \alpha^3(2 - 3\tau + \tau^2))$$

From relations (27) and (28), we obtain that capital accumulation increases with the level of idiosyncratic risk in both economies.

It is straightforward to show that:

$$P(0) = 1 > 0, P(1) = -(1-\alpha)^3(2-\tau)(1-\tau) < 0$$
$$P(\theta) = 0 \text{ has discriminant } \Delta = -27(1-\alpha)^6(1-\tau)^2\tau^2 < 0$$

Consequently $P(\theta) = 0$ has one real solution $\theta^*(\alpha, \tau)$ and $\theta^*(\alpha, \tau) \in (0, 1]$.

We distinguish 2 cases:

- $\theta < \theta^*(\alpha, \tau)$: $P(\theta) > 0$ and capital increases faster in the FL economy than in the NDC economy when the size of idiosyncratic risk increases. Hence for these values of labor supply elasticity, the NDC pension system crowds out capital formation more than the FL pension system at any level of idiosyncratic productivity risk.
- $\theta > \theta^*(\alpha, \tau)$: $P(\theta) < 0$ and capital in an NDC pension system economy increases faster than in an FL pension system economy. Hence, although $k^{NDC}(0, \tau) < k^{FL}(0, \tau)$, a sufficiently high increase in the level of idiosyncratic risk may lead to a lower capital to labor ratio in the case of the FL pension system.

## Proof of Proposition 3

We consider as a measure of consumption inequality the ratio of two quantiles of the consumption distribution:

$$c_{inequality} = \frac{c_H}{c_L}$$

where $z_H - z_L = 2\sigma$.

Using the closed for solutions for $c^H$ and $c^L$ obtained in the proof of Proposition 2, we obtain the following relations:

$$c^{FL}_{inequality} - c^{NDC}_{inequality} = -\frac{2(1-\alpha)\sigma\tau\theta(\tau + 1 + \alpha(1-\tau))}{(1 + \tau + \alpha(1-\tau) - \sigma(1-\alpha\theta))(1 + \tau + \alpha(1-\tau)(\sigma\theta + 1) + \theta - \sigma(1-\tau))} < 0$$
$$c^{FL}_{inequality} - c^{FL}_{inequality}|_{\sigma=0} = -\frac{2(1-\alpha)\sigma\tau(1+\theta)}{(1 + \alpha - \sigma + \alpha\sigma\theta)(1 - \sigma + \tau\sigma\theta + \alpha(1-\tau)(1+\sigma\theta))} < 0$$
$$c^{NDC}_{inequality} - c^{NDC}_{inequality}|_{\sigma=0} = -\frac{2(1-\alpha)\sigma\tau(1-\alpha\theta)}{(1 + \alpha - \sigma + \alpha\sigma\theta)(1 + \alpha - \sigma + \tau - \alpha\tau + \alpha\sigma\theta)} < 0$$

## Proof of Proposition 4

We write all the macroeconomic variables in terms of the $\tau, k$ and $s$:

$$a = sw\bar{z}(1-\tau) = s\bar{z}(1-\tau)(1-\alpha)k^{\alpha} \Rightarrow 1 = \frac{s\bar{z}(1-\tau)(1-\alpha)k^{\alpha-1}}{l}$$
$$c^{y} = \bar{z}(1-s)(1-\tau)(1-\alpha)k^{\alpha}$$

| FL | NDC |
|---|---|
| $c^{o}(z) = \theta(1-\tau)(1-\alpha)k^{\alpha}(s\bar{z}\alpha k^{\alpha-1} + z + \tau s\bar{z}(1-\alpha)k^{\alpha-1})$ | $c^{o}(z) = \theta(1-\alpha)k^{\alpha}(s\bar{z}\alpha k^{\alpha-1}(1-\tau) + z + \bar{z}\tau(1+n))$ |
| $l^{o}(z) = \theta - (1-\theta)s\bar{z}k^{\alpha-1}\frac{\alpha+\tau(1-\alpha)}{z}$ | $l^{o}(z) = \theta - (1-\theta)\frac{s\bar{z}(1-\tau)\alpha k^{\alpha-1}+\bar{z}\tau(1+n)}{z}$ |

The ex ante utility of an agent is:

$$SWF(\tau, k, s) = U(c^{y}) + \beta E(U(c^{o}(z), l^{o}(z)))$$

The change in ex ante utility following a marginal introduction of a pension system is:

$$\begin{aligned}\frac{\partial SWF}{\partial \tau} &= \frac{\partial U(c^{y})}{\partial c^{y}}\left(\frac{\partial c^{y}}{\partial \tau} + \frac{\partial c^{y}}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial c^{y}}{\partial s}\frac{\partial s}{\partial \tau}\right) + \\ &+ \beta E\left[\frac{\partial U(c^{o}(z), l^{o}(z))}{\partial c^{o}(z)}\left(\frac{\partial c^{o}(z)}{\partial \tau} + \frac{\partial c^{o}(z)}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial c^{o}(z)}{\partial s}\frac{\partial s}{\partial \tau}\right)\right] + \\ &+ \beta E\left[\frac{\partial U(c^{o}(z), l^{o}(z))}{\partial l^{o}(z)}\left(\frac{\partial l^{o}(z)}{\partial \tau} + \frac{\partial l^{o}(z)}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial l^{o}(z)}{\partial s}\frac{\partial s}{\partial \tau}\right)\right]\end{aligned}$$

We plug in the formulas for $c^{y}, c^{o}(z), l^{o}(z)$. We group the terms of the derivative in order to obtain the term $\omega_1$ of Proposition 4:

$$\begin{aligned}\omega_1 &= \frac{\partial U(c^{y})}{\partial c^{y}}\frac{\partial c^{y}}{\partial \tau} + \beta E\left[\frac{\partial U(c^{o}(z), l^{o}(z))}{\partial c^{o}(z)}\frac{\partial c^{o}(z)}{\partial \tau} + \frac{\partial U(c^{o}(z), l^{o}(z))}{\partial l^{o}(z)}\frac{\partial l^{o}(z)}{\partial \tau}\right] \\ \omega_1^{FL} &= -\frac{\theta}{1-\tau} - \frac{\beta\theta}{1-\tau} + \beta\frac{1-\alpha}{\alpha}\Gamma^{NDC} \\ \omega_1^{NDC} &= -\frac{\theta}{1-\tau} - \frac{\beta\Gamma^{NDC}}{1-\tau} + \beta\frac{1-\alpha}{\alpha}\frac{\Gamma^{NDC}}{l^{NDC}}\end{aligned}$$

The formulas for the term $\omega_2$ are:

$$\omega_2 = \frac{\partial k}{\partial \tau}\left(\frac{\partial U(c^y)}{\partial c^y}\frac{\partial c^y}{\partial k} + \beta E\left[\frac{\partial U(c^o(z), l^o(z))}{\partial c^o(z)}\frac{\partial c^o(z)}{\partial k} + \frac{\partial U(c^o(z), l^o(z))}{\partial l^o(z)}\frac{\partial l^o(z)}{\partial k}\right]\right) +$$
$$+ \frac{\partial s}{\partial \tau}\left(\frac{\partial U(c^y, l^y)}{\partial c^y}\frac{\partial c^y}{\partial s} + \beta E\left[\frac{\partial U(c^o(z), l^o(z))}{\partial c^o(z)}\frac{\partial c^o(z)}{\partial s} + \frac{\partial U(c^o(z), l^o(z))}{\partial l^o(z)}\frac{\partial l^o(z)}{\partial s}\right]\right)$$
$$\omega_2^{FL} = \frac{\partial ln(k^{FL})}{\partial \tau}\left(\theta\alpha(1+\beta) - \beta(\alpha + \tau(1-\alpha))\frac{1-\alpha}{\alpha}\Gamma^{FL}\right) + \frac{\partial ln(s^{FL})}{\partial \tau}\frac{\beta\Gamma^{FL}\tau(1-\alpha)}{\alpha}$$
$$\frac{\partial ln(s^{FL})}{\partial \tau} = (1-\alpha)\frac{\partial ln(k^{FL})}{\partial \tau} + \frac{1}{1-\tau} + \frac{\partial ln(l^{FL})}{\partial \tau}$$
$$\omega_2^{NDC} = \frac{\partial ln(k^{NDC})}{\partial \tau}\left(\theta\alpha(1+\beta) - \beta(1-\alpha)\Gamma^{NDC}\right)$$

## Proof of Proposition 5

1. We compute for $\sigma = 0$ the difference between the impact on welfare through consumption reallocation triggered by a marginal increase in the FL and NDC pension system, respectively.

$$(\omega_1^{FL} - \omega_1^{NDC})|_{\sigma=0} = \frac{2\beta\tau\theta P(\tau)}{(1-\tau)^2(1+\alpha(1-\tau)+\tau)(1+\tau+\alpha)} \leq 0 \tag{30}$$

   where $P(\tau) = \tau^2(1-\alpha)^2 + 2\tau\alpha(1-\alpha) - (1-\alpha^2)$. The result follows from the fact that $P(\tau) < 0$ for $\tau \in [-\frac{1+\alpha}{1-\alpha}, 1]$.

2. We compute the difference between the impact on welfare through general equilibrium effects triggered by a marginal increase in the FL and NDC pension system, respectively.

$$\omega_2^{FL} - \omega_2^{NDC} = \frac{\partial ln(k^{FL})}{\partial \tau}\left(\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{FL}\right) -$$
$$\frac{\partial ln(k^{NDC})}{\partial \tau}\left(\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{NDC}\right) + \beta\tau\Gamma^{FL}\frac{\theta(1-\alpha)^2}{\alpha(1-\alpha\theta-\tau\theta(1-\alpha))} \tag{31}$$

   From Proposition 2, we know that:

$$\Gamma_{\sigma=0}^{NDC} \leq \Gamma_{\sigma=0}^{FL} \leq \Gamma_{\sigma=0,\tau=0} \Rightarrow$$
$$\Phi_{\sigma=0}^{NDC} \geq (\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{FL})_{\sigma=0} \geq \Phi_{\sigma=0,\tau=0} > 0 \tag{32}$$

   Also from Proposition 2, we know that the NDC pension system crowds out capital formation strictly more than the FL pension system when $\sigma = 0$:

$$0 > \frac{\partial ln(k^{FL})}{\partial \tau} > \frac{\partial ln(k^{NDC})}{\partial \tau} \tag{33}$$

From relations (31), (32), (33) it follows that $0 > \omega_2^{FL}|_{\sigma=0} > \omega_2^{NDC}|_{\sigma=0}$.

## Proof of Proposition 6

For simplicity, we will prove the Proposition for $n = 0$.

To prove this Proposition, we use the same Taylor expansion as in the proof of proposition 2. Specifically, we employ relations (27) and (28) to compute the sensitivity of $\omega_1^{FL}$ and $\omega_1^{NDC}$ with respect to the variance of the idiosyncratic earnings risk $\sigma^2$. We obtain the following:

$$\frac{\partial \omega_1^{FL}}{\partial \sigma^2} = \beta \frac{1-\alpha}{\alpha} \frac{\partial \Gamma^{FL}}{\partial \sigma^2} \approx \left. \frac{\partial^2 \Gamma^{FL}}{\partial z^2} \right|_{\sigma=0} = \beta l^{FL} \frac{(1-\tau)^2}{\left(\frac{\alpha}{1-\alpha} l^{FL} + \tau l^{FL} + 1 - \tau\right)^3}$$

$$\frac{\partial \omega_1^{NDC}}{\partial \sigma^2} = \beta \frac{\partial \Gamma^{NDC}}{\partial \sigma^2} \left( -\frac{1}{1-\tau} + \frac{1-\alpha}{\alpha} \frac{\bar{z}}{l^{NDC}} \right) \approx$$

$$= \beta \left( -\frac{1}{1-\tau} + \frac{1-\alpha}{\alpha} \frac{1}{l^{NDC}} \right) \frac{\alpha}{1-\alpha} l^{NDC} \frac{1}{\left(\frac{\alpha}{1-\alpha} l^{NDC} + 1 + (1+n)\tau\right)^3}$$

We expand the expressions above:

$$\frac{\partial \omega_1^{FL}}{\partial \sigma^2} = \beta \frac{(1-\alpha)(1+\theta)(1-\alpha\theta-(1-\alpha)\theta\tau))^2}{(1+\alpha+\tau-\alpha\tau)^3} > 0$$

$$\frac{\partial \omega_1^{NDC}}{\partial \sigma^2} = \beta \frac{((1-\alpha)(1-\tau)-2\alpha\theta)(1-\alpha\theta)^2}{(1-\tau)(1+\alpha+\tau-\alpha\tau)^3} > 0$$

We compute the difference between the two sensitivities:

$$\frac{\partial \omega_1^{FL}}{\partial \sigma^2} - \frac{\partial \omega_1^{NDC}}{\partial \sigma^2} = \frac{\beta\left[(1-\tau)(1-\alpha)(1+\theta)(1-\alpha\theta-(1-\alpha)\theta\tau))^2 - ((1-\alpha)(1-\tau)-2\alpha\theta)(1-\alpha\theta)^2\right]}{(1-\tau)(1+\tau+\alpha(1-\tau))^3} =$$

$$= \frac{\beta\left[(1-\tau)(1-\alpha)\theta\left[(1-\alpha\theta)^2 + (1-\alpha)^2\tau^2\theta(1+\theta) - 2(1-\alpha\theta)\tau(1+\theta)(1-\alpha)\right] + 2\alpha\theta(1-\alpha\theta)^2\right]}{(1-\tau)(1+\tau+\alpha(1-\tau))^3} =$$

$$= \frac{\beta\left[(1-\tau)(1-\alpha)\theta P(\tau) + 2\alpha\theta(1-\alpha\theta)^2\right]}{(1-\tau)(1+\tau+\alpha(1-\tau))^3}$$

The minimum of the numerator is given by the minimum of $P(\theta)$ that is achieved for $\tau_{min} = \frac{1-\alpha\theta}{(1-\alpha)\theta}$. The value of the numerator in $\tau_{min}$ is $(1-\alpha\theta)^2\left(2\alpha\theta + \frac{1-\theta}{\theta}\right) > 0$. Hence the difference between the two sensitivities is positive.

# Appendix 2

# Appendix 3

In this Appendix we consider the welfare of the FL and NDC pension systems in an economy that is dynamically inefficient in the absence of idiosyncratic shocks ($\Phi_{\sigma=0,\tau=0} < 0$). Under Assumption 1, in an economy with no idiosyncratic risk ($\sigma = 0$) a marginal increase in the contribution to the pension system has a higher positive impact on welfare through general equilibrium effects in the case of the NDC pension system, ($\omega_2^{NDC} > \omega_2^{FL}$) if the level of pension contribution $\tau$ is low.

For this proof we consider first the case $\tau = 0$. Equation (31) becomes:

$$(\omega_2^{FL} - \omega_2^{NDC})_{\sigma=0,\tau=0} = \left( \frac{\partial ln(k^{FL})}{\partial \tau} - \frac{\partial ln(k^{NDC})}{\partial \tau} \right) \Phi_{\sigma=0,\tau=0} < 0 \tag{34}$$

Increasing $\tau$ further has the following effects:

- the crowding out effect under the NDC pension system worsens faster than under the FL pension system (see Proof of Proposition 2):

$$0 > \frac{\partial ln(k^{FL})}{\partial \tau} > \frac{\partial ln(k^{NDC})}{\partial \tau} \tag{35}$$

- however, the impact of the general equilibrium effect on welfare also shrinks faster in the NDC pension system:

$$\Gamma_{\sigma=0}^{NDC} \leq \Gamma_{\sigma=0}^{FL} \leq \Gamma_{\sigma=0,\tau=0} \Rightarrow 0 > \Phi_{\sigma=0}^{NDC} \geq \Phi_{\sigma=0}^{FL} \geq \Gamma_{\sigma=0,\tau=0} \tag{36}$$

- $\omega_2^{FL}$ has an additional component $\beta\tau\Gamma^{FL}\frac{\theta(1-\alpha)^2}{\alpha(1-\alpha\theta-\tau\theta(1-\alpha))}$ that is positive and increasing with the level of pension contributions $\tau$.

From the above we conclude that, when $\Phi_{\sigma=0} < 0$, we can have a higher welfare gain from general equilibrium effects in the NDC system, but only for low levels of $\tau$.