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Risk Decision Support System for Asset Liability Management

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RISK DECISION SUPPORT SYSTEM

FOR

ASSET LIABILITY MANAGEMENT

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Abstract

This paper outlines a risk decision support system designed to arrive at well-substantiated policy decisions using asset liability management (ALM) models. The risk decision support system explicitly takes into account that 1) there are multiple risk and return measures that are all important to some extent, 2) there are multiple stakeholders with different (and typically to some extent conflicting) objectives and 3) there is model uncertainty and therefore (economic) stress tests should also play a role in the decision making process.

The method is based on techniques from the operations research literature on multi-criteria decision making and group decision support to map outcomes generated by any ALM model into policy preferences of (the stakeholders of) the organization in question. Literature on robust control theory is used to guide us how to make good decisions when the ALM model is regarded as just an approximation to an unknown and unspecified model that actually generates the data.

The risk decision support system can be used theoretically to obtain insight in for example the influence of a change in future regulation on policy choices or to help design future (pension) contracts. In practice experience learned us that the method facilitates discussions and decision making substantially since the approach helps parties to focus on interests (objectives), rather than positions (alternatives).

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Keywords: ALM, MCDM, AHP, GDSS, risk attitude, stakeholders, solidarity, robustness

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Outline

1. Introduction

Asset liability management, typically used by for example pension funds and insurance companies, is challenging. There are stakeholders with different objectives, there are many policy instruments and there is a lot of uncertainty on for example future economic scenarios, mortality and regulation. The situation is complicated and the consequences of (the lack of) policy decisions are uncertain but can be significant.

1.1 ALM models

Asset liability management (ALM) models are there to support the decision maker. These models are available at any chosen degree of complexity, but the main idea is simple: model the future assets and the future liabilities, where both may depend on the same underlying variables e.g. future interest rates. Once all future assets and liabilities are modeled just vary the policy parameters and assess the impact of the change in the policy parameters on some sub-set of pre-defined risk and return measures. Then choose the policy alternative that gives the best results.

1.2 Practical concerns on the use of ALM models

Three – to some extent interrelated - concerns on the use of ALM models are addressed in this paper.

1) **An ALM-model will generate many numbers.** To illustrate: consider the simple case when there are 10 policy alternatives from which one is to be chosen, and one would like to take into account 5 risk/return measures on 2 time horizons. Then you already have 100 (10x5x2) numbers to look at.

   One of the key aspects in making choices is however the limited human ability to process information. As stated by Hogarth (1987): "We simply cannot handle all information inherent in complex decision problems and, in particular to make the many kinds of tradeoffs implied by choices involving several conflicting dimensions." Therefore, just the ALM model and intuitive judgment of the results is not enough. **One needs 'decision aids'**.

   We must admit that it sounds logical that the decision maker has decision aids, **such as well-defined objectives and constraints** that result in an easy search for the optimal policy. But in ALM context the formulation of objectives and constraints (or a so called 'risk attitude') is a problem that is still hardly addressed, see also the discussions in Slater e.a. (2002), Chapman (1999) and Boender (2009a,b) on this issue.[2]

[2] One solution could be the use of the utility function of the decision maker. See for example Blake (1998), Poterba e.a. (2005) who -in a simplified context- calculate the optimum pension scheme or investment portfolio, given different values of the risk-aversion parameters. There are a few problems to implement this kind of approaches: the main problems are that 1) the context is too simplified for practical applications and 2) how can you determine the actual risk aversion parameters? In practice we observe nowadays the use of questionnaires and business games. These tools can make the decision maker aware of the risks and their impact, but how can you capture the results in a well-defined risk attitude?

2) In ALM context there are **multiple stakeholder groups with non-common objectives**[3] **which makes it difficult** (if not impossible) **to establish a consensus** on all aspects of the decision, although consensus might be possible on many aspects of the decision.

We state that effective decision making in ALM context **requires 'decision aids' that take into account that there are multiple decision makers** (groups of stakeholders) so that all stakeholders are represented in making the final decision and implementation is facilitated.

To illustrate this statement consider the following[4]. What happens if the results of the ALM study are shown to the group of decision makers in an unstructured way? It is not unlikely that some aspects of the decision will be addressed several times, while other aspects will not be discussed at all. Some of the decision makers will already have made up their minds and just use the discussion to strengthen their position rather than share ideas and gain a better insight into the tradeoffs that have to be made. The discussion will continue until the scheduled decision moment approaches, at which point the leader of the discussion group attempts to obtain a consensus on the best alternative.

It is a pity to say that in practice this is a common way to make ALM decisions.

3) The third practical concern on the use of ALM models is that there is a lot of **uncertainty on the quality of the ALM models** and their input parameters. The credit crunch took everyone by surprise; the real outcomes were much worse than the ALM models had promised given the chosen policies. This could be due to model misspecification: do we still trust our ALM model?

In practice we observe two 'solutions' that cope with this issue:

a) Build an even more sophisticated or so-called 'hyper' ALM model.
But if you did not trust the old model, can you really trust this new model?[5]

b) Stress testing.
Perform all necessary calculations under different (stress) scenarios. But keep in mind that a simple problem as mentioned in concern 1 already resulted in 100 outcomes of risk and return measures. By adding 2 extra stress conditions this number will grow from 100 to 300 outcomes. These extra numbers will no doubt give extra insight, but the question remains how to deal with all this extra information?

We state that next to the ALM model we **need 'decision aids' that acknowledge model misspecification and can take into account several (economic) (stress) scenarios.**

[3] Which can also be seen as one explanation of the practical difficulty to formulate a risk attitude.
[4] The discussion here is based on Dyer and Forman (1992)
[5] This third concern on the use of ALM models could also be somehow related to the first two concerns (lack of a well defined risk attitude and multiple groups of stakeholders). Even if all groups of stakeholders have consensus on the objectives and constraints and use the same ALM model, they might prefer different policy variants because of different concern on misspecification of the model. This latter aspect is interesting but left for further research. In this paper we assume a common concern on misspecification of the model among the stakeholder groups.

1.3 Risk decision support system as a 'decision aid'

This paper outlines a risk decision support system designed to arrive at well-substantiated policy decisions using ALM models. Note that we are not concerned *which* ALM model is used, but *how* an ALM model is used.

To build a risk decision support system for ALM we tackle the three concerns on the use of ALM models as mentioned above:

1) We recognize that an ALM problem is actually a so-called **multi-criteria decision making** problem: there are multiple risk- and return measures (criteria) that are all important to some extent and need to be taken into account when choosing a policy alternative (the decision).

   Therefore we propose to use techniques from the operations research literature on multi-criteria decision making, see for an overview also Carlos e.a. (2007). In this paper we have chosen a method called the Analytical Hierarchy Process (AHP) as developed by Saaty (1980).

   AHP **allows us to structure the complex decision and to develop a risk attitude of the decision maker, such that we can map outcomes generated by any ALM model into policy preferences**.

2) In ALM context it can be difficult to achieve consensus among all stakeholder groups (decision makers). To take all stakeholder groups explicitly into account when making ALM decisions we can use techniques from literature on **group decision support**.[6]

   Group decision support in the context of multi-criteria decision making can be found in e.g. Saaty (1987), Dyer and Forman (1992) and Lai e.a. (2001).

   The resulting risk decision support system for ALM helps to **structure the group decision so that the discussion concentrates on objectives rather than on the alternative policy choices**. A structured analysis assures that discussion will continue until all available information has been considered and a consensus choice is reached on the policy that most likely is to achieve the stated objectives.

3) Anno 2010 an ALM model is no longer fully 'trusted' by the decision makers. Literature on **robust control theory**, see for example Hansen and Sargent (2006), is used to guide us how to make good decisions when the ALM model is regarded as an approximation to an unknown and unspecified model that actually generates the data.

   In ALM context the main concerns are typically on the economic parameters. The risk decision support system is designed to **choose a *robust* policy that works well under a set of (economic) (stress) scenarios** or alternative ALM models.

[6] We also called our 'decision aid' a 'Risk Decision Support System', which is a combination of 'Group Decision Support System' and the nowadays popular term in ALM context 'Risk management'.

1.4 Outline of this paper

The remainder of this paper is organized as follows:
Section 2 describes the risk decision support system.
Section 3 demonstrates the risk decision support system with a numerical example for a pension fund.
Section 4 summarizes.

2. Risk decision support system

This section outlines how to build a risk decision support system for asset liability management. This system should help the decision maker to develop a risk attitude that enables him to choose a well-substantiated policy variant given the outcomes of an ALM model. We take into account that there are several stakeholder groups, who judge on multiple risk and return measures calculated under several possible realistic (economic) (stress) scenarios.

Notice that we are not concerned which ALM model is chosen. We are also not concerned which risk and return measures the decision maker chooses to take into account. These are just input variables for the proposed method. What the method does is rank the risk and return measures, such that the policy alternatives can be ranked in preference.

If one does not want (or does not have time) to read this paper in detail, then one can skip this section (or only read the summary in §2.5) and continue with section 3: the numerical example.

2.1 This section

The first step in the risk decision support system is to obtain the risk attitude (or risk/return profile as we call it) of the decision makers: we try to capture the objectives and constraints of the decision maker in terms of preferences for risk- and return measures in the ALM model.

To do this we look at literature on decision making given multiple criteria. According to Carlos e.a. (2007) there are two main lines of thought regarding multi-criteria decision making:

- The French school is based on an outranking relation built up using pair wise comparisons of the criteria and alternatives under study.
- "The American multi attribute utility theory school is based on the formulation of an overall utility function and its underlying assumption is that such a utility function is available or can be obtained through an interactive process." Carlos e.a. (2007)

In practice we have seen several unfruitful attempts that are in line with the American school. Here we will use elements from a French school method, namely the Analytic Hierarchy Process as developed by Saaty (1980). We first compare the risk and return measures pairwise in preference, then compare all alternative policies pairwise with respect to each of the risk and return measures. To add a level of stakeholders we use ideas from literature on group decision support in the context of multi-criteria problems. Paragraph 2.2 and 2.3 are on this subject: in paragraph 2.2 we structure the ALM problem into a hierarchy, in paragraph 2.3 we set up the risk/return profiles of the stakeholders.

Given paragraph 2.2 and 2.3 we can already rank alternative policies in preference if we just use the ALM model with given parameters. But can we also take into account model (or parameter) uncertainty? In ALM context typically several economic stress tests are performed. Paragraph 2.4 shows how also these stress tests can be taken into account in the decision making process. This paragraph is based on ideas from robust control theory: how to make good decisions when the model is regarded as merely an approximation.

Paragraph 2.5 summarizes how to obtain the 'best' policy using the risk decision support system. Paragraph 2.6 poses some concluding remarks on the use of risk decision support systems.

2.2 Structure the ALM problem

Why perform an ALM study? The goal is typically to choose a well-substantiated ('optimal') policy for the (near) future. This is clearly a decision problem with multiple criteria and often multiple decision makers or stakeholder groups. We use aspects from the Analytic Hierarchy Process as a first step is structuring the problem. First, we decompose the decision problem into several elements:

- Risk and return measures
  The goal 'to choose a policy' is achieved by satisfying the objectives to the maximum extent possible. Risk and return measures will be used to evaluate how well each policy alternative satisfies the objectives. Examples of risk and return measures are: chance of insolvency and expected profit.
- Policy alternatives
  The policy alternatives available to reach the goal. For example different investment mixes and overlays, or different contribution schemes.
- (Stress) Scenarios
  Model or parameter uncertainty can be represented by including (stress) scenarios. For example several economic scenarios like a 'goldilocks' economy and a 'risk neutral' economy can be taken into account.
- Stakeholders
  Decisions are often made through group consensus, yet sometimes it is difficult for all members of the group to meet or for each member's opinion to be heard during a meeting. A level of stakeholders can be added to the model.

In using the analytic hierarchy process we need a hierarchic structure to represent the problem. A hierarchy of the ALM problem is schematically given in figure 1[7].

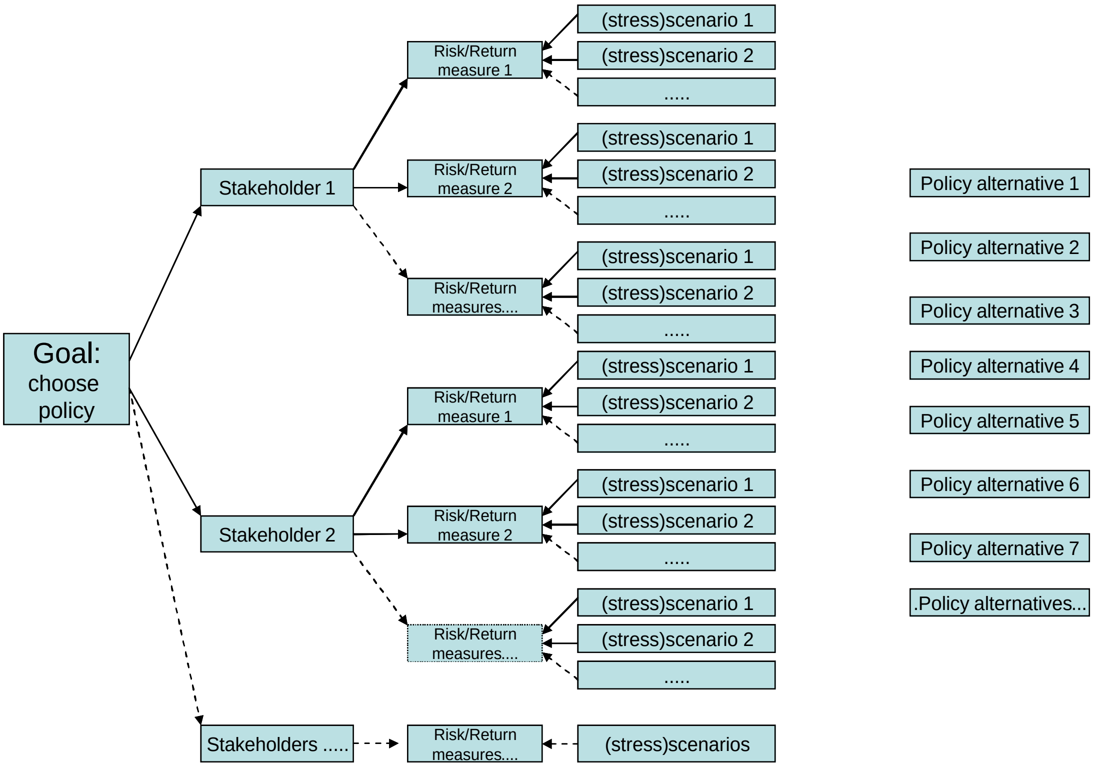

Figure 1: ALM decision making schematically

[7] Alternative orders of the decomposition are also possible, this is just the one we chose, see also Saaty (1987) on different types of hierarchies for practical problems.

2.3 Set up risk/return profiles

The next step is to evaluate the relative importance of the risk and return measures with respect to the overall goal of obtaining the 'best' policy. In the hierarchic structure in figure 1 we do this exercise for each of the stakeholders although it could also be done for the organization as a whole, and then there is just one level (the 'stakeholder' level) less in the hierarchy.

2.3.1 Risk/return profiles

Each of the stakeholders establishes priorities for the risk and return measures by judging them in pairs for their relative importance. We chose to use 'verbal' pairwise relative comparison[8]:we describe the relative preference of one risk measure over the other by using a nine point scale consisting of five words (1: equal importance, 3: moderate importance of one over another, 5: strong importance, 7:very strong importance and 9: extreme importance) and four intermediate levels (e.g. 8: between strong and extreme).

The pairwise comparisons result in a comparison matrix. Element a_{ij} gives the relative importance of risk/return measure *i* over *j*. To complete the matrix $a_{ij}=1/a_{ji}$ and the diagonal elements equal unity. The comparison matrix is positive and reciprocal. The risk/returnprofile: a vector with relative priority weights for each of the risk/returnmeasures, is obtained by calculating the normalized principal eigenvector of the matrix which is (in most problems) the only derived scaling that makes use of all dominance information given in the comparison matrix.

An alternative to this approach of obtaining a risk/retum profile is just to assign weights to the different risk and return measures immediately. But making pairwise comparisons has some advantages over this 'simpler' method. Because more comparisons are made than required to calculate the relative priorities the comparison matrix contains some redundancy. As Dyer and Forman (1992) formulate it: "this redundancy is in a sense used to 'average' errors of judgment in a manner analogous to averaging errors when estimating a population mean. The errors of judgment include errors in translating from imprecise words to the numbers that are used to represent these words in the algorithm."

Also related to this 'redundancy' of the judgments is the possibility of obtaining a measure of inconsistency. The comparison matrix is said to be consistent if for all *i, j, k* $a_{ij}a_{jk}=a_{ik}$. To calculate the extent of inconsistency we can calculate an inconsistency ratio: the ratio of the difference between the number of risk and return measures and the principal eigenvalue of the comparison matrix and the average of the difference between the number of risk and return measures and the principle eigenvalues of a large number of randomly chosen comparison matrices, see Saaty (1987) for details.

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[8] The analytical hierarchy process as used here accommodates pairwise comparisons either verbally, numerically or graphically. We choose for 'verbal' pairwise comparisons for the ALM context. 'Advantages are that relative verbal judgments are easier to discuss, justify and agree upon. …. Using inexact words alleviate the discomfort that many people feel when forced to put hard numbers to subjective feelings. Further makes it easier for group members to reach a consensus.' Dyer and Forman (1992)

Another advantage of this analytic hierarchy process setup is that other criteria than ALM risk and return measures, for which no quantifiable scale exists, can be taken into account.

One of the drawbacks of the pairwise comparison method is that it can take considerable time since the number of judgements to be made if there are *n* risk and return measures is $n(n-1)/2$. To save time one could use results from Harker (1987) on calculating priorities when some judgments are missing. The tradeoff is obviously one between accuracy and time.

2.3.2 Policy preferences per risk/return measure

The next step is to evaluate the relative preferences for the policy alternatives with respect to each risk and return measure criteria. This could again be done by setting up pairwise comparison matrices, but this would be a lot of work. Since our ALM risk and return measures are actual measurements we just use the actual values for comparison between policy alternatives.

One can assume for example a linear or diminishing marginal utility curve to assign priorities for the different risk and return measure with respect to the policy alternatives. The lower the probability of insolvency for a policy variant, the more the policy variant is preferred when looking with respect to this risk measure.

2.3.3 Stakeholders

Both the risk and return profiles and relative preferences for the policy alternatives with respect to each return measure are ratio scale numbers and thus can be meaningfully synthesized (trough multiplication) to derive an overall prioritization and ranking of the policy alternatives for each of the stakeholders.

But how can we reach consensus among all stakeholders on one policy alternative? Literature on group decision support gives us several possibilities see a.o. Saaty (1987), Dyer and Foreman (1992) and Lai et al (2002).

One could try to reach consensus in an earlier stage of the process by letting all decision makers meet as a group and strive for consensus in both constructing the hierarchy and the risk/return profiles. But in ALM context we typically have non-common objectives and it could be interesting to see the differences in policy preferences and the degree of solidarity between stakeholders. Therefore we choose to separate the model by players: we added an extra ‘stakeholder’ level to the hierarchy in figure 1.

The problem in both of the mentioned setups is how to get consensus on either the risk/return profile or the preferred policy alternative respectively. Literature suggests the following methods: pairwise comparison of the decision makers, voting, compromising or taking a geometric mean of the individual judgments/policy preferences.

We think that in practice compromising through discussion or taking a geometric mean over the stakeholders are the most promising methods. When taking a geometric mean of course consideration must be given to the weights of individual players. Literature on group decision support suggests several possibilities: take each stakeholder as equally important, or do pairwise comparisons about relative importance of the players. The latter will be quite difficult in practice. In the context of ALM we suggest that the weights for the individual players could be given by for example the stakeholders share in the provision.

2.4 Stress scenarios: likelihood or robustness?

ALM models are not perfect. In practice most concerns are on the quality of the economic scenario generators. Traditional economic scenario generators are based on Gaussian distributions for market returns and (constant) historical volatilities and correlations. These models may show the consequences of worst case scenarios under 'normal economic circumstances', but fail to capture (plausible) extreme market conditions such as the credit crunch where for example the volatilities and correlations between asset categories suddenly change over time.

Often stress tests are performed to asses the impact of more extreme but still plausible shocks. How can we take the results of the stress tests into account when making policy decisions?

One solution would be to put a prior over the different economic stress scenarios and thereby creating a 'hypermodel'. Using this hypermodel we can just use the results from paragraph 2.2 and 2.3 to obtain the 'optimal' policy. But the question is whether the decision makers are able to put a prior over the stress scenarios. And even if they can, the decision makers might also want decisions to be 'robust' to whatever prior they can imagine over the set of stress scenarios. How can we set up 'robust' decision rules?

We look at robust control theory to find answers, see a.o. Gilboa and Schmeidler (1989) and Hansen and Sargent (2008). Robust control theory tells us how to make good decisions when the model in use is regarded as an approximation to an unknown and unspecified model that actually generates the data. The robust decision rule we propose for the risk decision support system is a *max min decision rule* such that the decision maker *maximizes* his objective by choosing a policy alternatives when a hypothetical 'malevolent nature' *minimizes* the same objective by choosing one of the economic (stress) scenarios. In this way we construct a lower bound on the performance of the return measures under different policy alternatives[9].

Where does the robust 'max min decision rule' enter our risk decision support system?

- Without the max min decision rule we need to compare each policy alternative in preference according to each of the risk and return measures which can be done by

[9] We refer to Hansen and Sargent (2008) pp.14-16 for an excellent discussion why to use a max-min decision rule and whether this rule is too cautious.

The max min decision rule results in a policy variant that keeps extreme (but plausible) losses 'as small as possible', given the potential economic (stress) scenarios. In pension fund context this decision rule is supported by the conclusion reached in several papers that stakeholders of pension funds show extreme risk aversion or (myopic) loss aversion (see for example Rooij et al (2007) and Benartzi and Thaler (1999)).

The prospect theory on which this latter position is based states that people are not good at analyzing complex situations with uncertain future consequences (such as pensions). 'Loss aversion' means that people attach greater importance to losses than to gains of equal size. People take current capital as their frame of reference and prefer to avoid losses even if, in absolute terms, there are prospects of equally large gains that would compensate for and/or outweigh those losses.

assuming for example a linear or diminishing marginal utility (paragraph 2.3.2). After that multiply these preferences with the risk/return profiles to obtain the 'optimal' policy.

- With the max min decision rule however we no longer use the utility of the actual measurements of the risk and return measures but we use max min results taken over the stress scenarios instead. So for each policy alternative we now have the max min results for each of the risk and return measures, and we multiply them with the risk/return profiles to obtain a 'robust and reliable' policy[10].

Formulas and a summary of the risk decision support system including stress scenarios are given in Box 1.

[10] The policy obtained by using robust control theory is not necessarily optimal under all circumstances, simply because you do not know what future will bring.

2.5 The 'best' policy: summary of the risk decision support system

Here we summarize the risk decision support system, in formulas the summary is given in Box 1. The risk decision support system starts with a pairwise verbal 'preference' comparison of all risk and return measures with respect to the overall objective of each of the stakeholders. This result in a **(1) risk/return profile** for each of the stakeholders which is a vector with the relative priority weights attached to each of the risk and return measures.

The next step is to obtain the **(2) preference for each of the policy variants with respect to each of the risk and return measure**. If no stress tests are taken into account, or the decision maker puts a prior over the stress tests, this preference is obtained by the actual measurements and assuming for example linear or diminishing marginal utility. If the decision maker wants a *robust decision given several stress scenarios*, then the preference for policy alternatives with respect to a risk or return measure is obtained by a max min decision rule.

Then **multiply (1)** the risk/return profiles with **(2)** the preference for each for policy alternatives with respect to each of the risk and return measures, **to obtain a ranking of the policy alternatives per stakeholder**.

To obtain a **consensus policy** for the organization as a whole 1) set up a **discussion** with as input the policy preferences of each of the stakeholder groups, or 2) take a **geometric mean** over the stakeholders where weights are for example given by their shares in the present value of the pension liabilities.

Box 1: Risk decision support system in formulas
Determine risk/return profile using a comparison matrix for each of the stakeholders $g = 1,...,g_{\max}$. Calculate and normalize the principle eigenvectors of the matrices. This gives the risk/return profiles, a vector with elements $P_g(R_r)$, for each of the stakeholders

Then preferred policy obtained by taking a geometric mean is then given by:

$$\max_b \sum_g w_g \left[\sum_r P_g(R_r) \{ \min_e (R_r(B_b, E_e) - \max_b \min_e R_r(B_b, E_e)) \} \right]$$

Where E_e, are the identified economic (stress) scenarios (either deterministic or stochastic), B_b the policy alternatives, $R_r(B_b, E_e)$ the return measures, w_g the weights over the stakeholders, and the 'max' and 'min's within the brackets need to be reversed in case of a risk measure.

2.5 Concluding remarks on the risk decision support system

The risk decision support system comes up with the preferred policy variant given the ALM model. But of course one should always carefully examine all results, such as the consistency of the comparison matrix and the worst case results from the stress scenarios.

Decision making is a process in itself as opposed to an event. Iteration might be required when for example during the decision making one discovers that there are possibly attractive policy alternatives that were not taken into account at first.

It is also wise to perform a sensitivity analysis to obtain more insight in the results, and to obtain information on the relative importance of the different assumptions or how possible changes in assumptions will affect results.

3. Numerical example

In this section we demonstrate the risk decision support system for ALM. We first structure the ALM problem, then show 1) how to define a risk attitude (or risk/return profile) for each of the stakeholder groups, and 2) how to incorporate different (stress) scenarios, as to obtain a well-substantiated policy decision for the organization.

The example is based on a stylized pension fund and a simple ALM model, but the method can easily be applied to any practical ALM model. Paragraph 3.5 gives some remarks on the use of risk decision support systems for ALM in practice.

Since the focus in this paper is on the *way we use ALM models* and *not on the ALM model itself*, the latter is given at the end of this section in box 2.

3.1 Structure the problem

The pension fund board wants to choose next year's policy as to obtain the 'best possible' pension benefits for all of its members. We first structure the problem: Who are the stakeholders? Which risk and return measures and which economic (stress) scenarios have to be taken into account? And finally, what are the policy alternatives?

- Stakeholders
  In the pension fund we identify three stakeholder groups: the active members, the pensioners and the sponsor[11]. Typically these stakeholder groups have different objectives. The active members are still young and willing to take risks on the short horizon to reach their long horizon goal of obtaining high benefits from their date of retirement on. The pensioners are already retired and want stability (low risks) on the short horizon and are keen on their purchasing power on the short horizon. The long horizon is less interesting from the pensioners' perspective since they are probably dead already by that time. The sponsor pays an upfront contribution which should be low according to his objectives.
- Risk and return measures
  A policy choice is achieved by satisfying (to the maximum extent possible) the objectives. The risk and return measures that are used to evaluate how well each alternative policy variant satisfies the objectives are calculated using the ALM model. We take into account 1) the expected value of the liabilities, 2) the 5^{th} percentile of the liabilities, 3) the expected value of the 'buffer' (assets minus liabilities), 4) the 5^{th} percentile of the buffer, 5) upfront contribution. All of these risk and return measures are considered in today's currency, and calculated on two time horizons: 5 and 40 years[12].

[11] Other stakeholders are for example former members (here taken together with active members), who left the pension fund before retirement date, the regulator and the public (the public perception of pension funds and their role in society).
[12] In a practical ALM study the number of risk and return measures will typically be larger; our choice here is just for illustrative purposes. The number and variety in risk and return measures should be large enough to capture the situation, but small enough to be sensitive to changes.

- <u>(Stress) scenarios</u>
  There are three economic scenarios that should be taken into consideration: a pessimistic, an optimistic, and an expected scenario (details are given in box 2).
- <u>Policy alternatives</u>
  There are ten policy alternatives under consideration which differ in terms of investment mix, the indexation policy and the upfront contribution made by the sponsor (details are given in box 2).

Given the policy alternatives and economic (stress) scenarios we calculate all outcomes for the risk and return measures using the ALM model.

In total we have ((4 measures x 2 time horizons+1) x 10 policies x 3 scenarios =) 270 outcomes of risk and return measures as shown in figure 2.

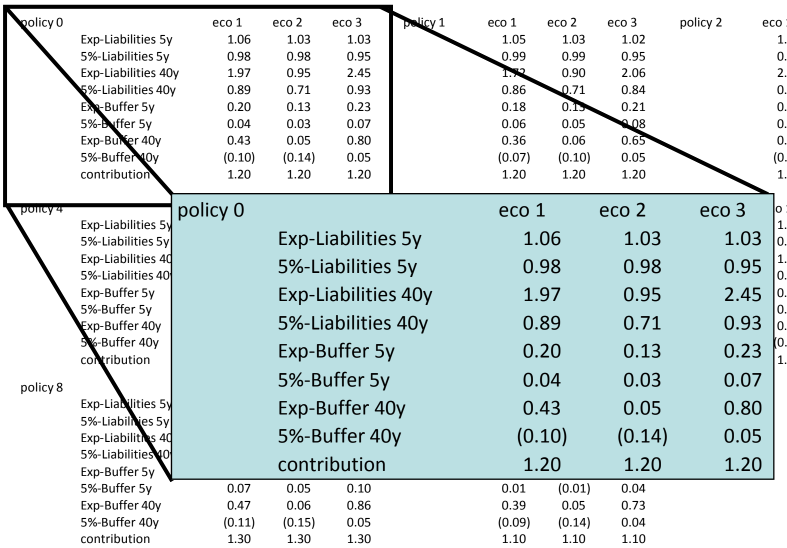

Figure 2: ALM results

Given this range of risk and return measures, the problem is choosing an optimum policy: what is really important for the pension fund? The risk and return measures often move in different directions. When one risk or return measure gives a good outcome, another will have a poor outcome in the same circumstances. Moreover, a policy that is optimum for one stakeholder group may not be optimum for the others.

3.2 Risk/return profiles

To determine which policy best meets the objectives and constraints, it is necessary to define the pension fund's risk attitude or 'risk/return profile' – the importance which the pension fund attaches to the outcomes of the different risk and return measures. We do this by pairwise comparisons of the risk and return measures using a nine point scale consisting of five words (1: equal importance, 3: moderate importance of one over another, 5: strong importance, 7: very strong importance and 9: extreme importance) and four intermediate levels (e.g. 8: between strong and extreme).

For all stakeholder groups pairwise comparison of the risk and return measures results in a 'comparison matrix'. Element a_{ij} in the matrix gives the importance of the left measure (i) relative to the top measure (j). The comparison matrix of the active members is given in figure 3.

Active	Exp liabilities 5y	5% liabilities 5y	Exp buffer 5y	5% buffer 5y	Exp liabilities 40y	5% liabilities 40y	Exp buffer 40y	5% buffer 40y	Extra single
Exp liabilities 5y	1	3	3	2	1/5	1	1/3	1	3
5% liabilities 5y	1/3	1	1	3	1/7	1/3	1/3	1/2	2
Exp buffer 5y	1/3	1.00	1	3	1/3	1/6	1/5	1/2	2
5% buffer 5y	1/2	1/3	1/3	1	1/9	1/5	1/7	1/3	1
Exp liabilities 40y	5	7	3	9	1	5	2	7	6
5% liabilities 40y	1	3	6	5	1/5	1	2	5	4
Exp buffer 40y	3	3	5	7	1/2	1/2	1	3	5
5% buffer 40y	1	2	2	3	1/7	1/5	1/3	1	3
Extra single premium	1/3	1/2	1/2	1	1/6	1/4	1/5	1/3	1

Figure 3: Comparison matrix of the active member of the pension fund from which the risk/return profile can be calculated

The comparison matrix gives us information on 'consistency' of the preference relations filled in by the stakeholder group. A comparison matrix is 'consistent' if for all i, j, k $a_{ij}a_{jk}=a_{ik}$. For example (Expected buffer 40y, expected liability 5y)=3 while (Expected liability 5y, expected liability 40y)=1/5, therefore to be consistent (Expected buffer 40y, expected liability 40y) = (Expected buffer 40y, expected liability 5y) x (Expected liability 5y, expected liability 40y) = 3 x 1/5 = 3/5. But it has value 1/2, which is smaller than it should be to be consistent, but notice that (expected liability 40y, Expected buffer 40y) is 2 is more than 5/3 and thus there is a tendency to compensate.

Inconsistency will always occur in practical problems, errors include for example errors in translating from imprecise words to the numbers that represent these words in the algorithm. The decision process set up here allows for some inconsistency. We can calculate a measure of consistency, the 'consistency ratio', which equals 0.08 for the active members' comparison matrix. According to the rule of thumb this ratio is acceptable since it should be smaller than 0.1 (see Saaty, 1987).

Now that we checked the matrix for consistency we can calculate the risk attitude or **risk/return profile**: a vector with weights attached to each of the risk and return measures which represent the preference for each of the measures relative to the other measures. The risk/return profile is given by the normalized principle eigenvector of the comparison matrix.

For each of the stakeholders the risk/return profiles are given by:

	Active	Pensioner	Sponsor
Exp liabilities 5y	9%	26%	5%
5% liabilities 5y	6%	16%	3%
Exp buffer 5y	6%	17%	9%
5% buffer 5y	3%	12%	6%
Exp liabilities 40y	28%	9%	6%
5% liabilities 40y	18%	7%	4%
Exp buffer 40y	19%	7%	11%
5% buffer 40y	9%	5%	9%
Extra single premium	3%	2%	48%

Figure 4: risk return profiles of the stakeholders

In using the risk/return profiles identified in this way, we can rank different policy alternatives by preference for each of the stakeholder groups, this is done in paragraph 3.4. But first we look at the economic (stress) scenarios in paragraph 3.3.

Some concluding remarks on the risk and return profiles:

- As mentioned in paragraph 2.3.1 a drawback of the pairwise comparison method to set up the risk/return profile is that it takes considerable time to fill the matrix especially when the number of risk and return measures is large. There are methods to reduce the number of judgments to be made, but fewer judgments means less accuracy of the results.
- An advantage of the setup used here is that other criteria than ALM risk and return measures, for which no quantifiable scale exists, can be taken into account.
- In this example we choose to set up a risk/return profile for each of the stakeholders, but the group of decision makers can also decide to just fill in one consensus risk/return profile.
- Note that in deriving the priorities as we do above the utility curve of the decision makers is implicitly taken into account.

3.3 Stress scenarios

In deciding on the policy alternative, the pension fund board would like to take into account three economic (stress) scenarios: a pessimistic, an optimistic, and an expected scenario (details are given in box 2).

This can be done in several ways. The first is to assign a probability to each of the three economic scenarios so that we can take the weighted average over the risk and return measures. Another way to take into account the stress scenarios is to use a *robust* decision rule. The idea behind this rule is that the decision maker wants the ultimate decision to be robust to whatever prior he can imagine over the set of economic stress scenarios.

The robust decision rule in our risk decision support system is a max min decision rule that we use to construct a lower bound on the performance of the return measures under different policy alternatives. To explain the decision rule we first look at just one return measure: the expected liabilities at a 5 year horizon. The results[13] for just this particular return measure under the three different economic scenarios for all policy alternatives are given in the matrix in figure 5.

	economy E1	economy E2	economy E3	minimum	punishment
Policy 0	1.06	**1.03**	1.03	1.03	0.07
Policy 1	1.05	1.03	**1.02**	1.02	0.08
Policy 2	1.07	**1.04**	1.05	1.04	0.06
Policy 3	1.09	1.07	**1.05**	1.05	0.05
Policy 4	1.09	**1.05**	1.06	1.05	0.05
Policy 5	1.03	1.01	**0.99**	0.99	0.10
Policy 6	1.03	**1.00**	1.00	1.00	0.10
Policy 7	1.10	**1.07**	1.07	1.07	0.03
Policy 8	1.14	**1.10**	1.11	**1.10**	0.00
Policy 9	1.00	0.97	**0.97**	0.97	0.13
		The maximum of the minima:		1.10	

Figure 5: robustness, a max min decision rule

To perform the min max decision rule we first look at each of the policy alternatives and determine for each of them the worst-case performance on the expected liabilities (grey areas). The results are given in the column labeled 'minimum'. If we want to maximize the minimum performance of this return measure we should choose policy alternative 8: for policy 8 the minimum expected liabilities are 1.10.

Policy 8 has the least downward exposure on the expected liabilities on a 5 year horizon, i.e. it is the most robust policy alternative in this example for this return measure.

Other policy alternatives perform less than policy 8: the column 'punishment' gives 'punishment scores' to the other policy alternatives for this return measure. This column gives the difference

[13] Here the minimum results are attained in only two of the economic scenarios, but for other return measures economic scenario 1 also attains minimum results.

between the minimum for the policy under consideration and 1.10 (the maximum minimum performance attained under policy 8).

The end result is a preference relation for policy alternatives with respect to one return measure. This exercise is repeated for all of the risk and return measures. Note that for risk measures we need a min max rule instead of a max min rule. Thus we obtain a preference relation for each of the policy alternatives with respect to each of the risk and return measures.

3.4 The 'best' policy alternative

T**o obtain a ranking of the policy alternatives per stakeholder** we **multiply (1)** the risk/return profiles with **(2)** the preference for each for policy alternatives with respect to each of the risk and return measures. See figure 6, the lower the 'weighted average' the more preferred is the policy.

	Risk/return profile active members	Punishment policy 1
Exp liabilities 5y	9%	0.07
5% liabilities 5y	6%	0.06
Exp buffer 5y	6%	0.03
5% buffer 5y	3%	0.03
Exp liabilities 40y	28%	0.56
5% liabilities 40y	18%	0.33
Exp buffer 40y	19%	0.04
5% buffer 40y	9%	0.04
Extra single premium	3%	0.10
	weighted average:	0.24

Figure 6 Score of policy alternative 8 where all risk and return measures are taken into account and using the risk/return profile of the active members.

The scores of all policy alternatives for each of the stakeholders result in the ranking of the policy alternatives in figure 7 (1= most preferred policy, 10=least preferred policy).

	Active	Pensioner	Sponsor	Total
policy 0	6	4	5	3
policy 1	9	6	4	8
policy 2	3	5	6	1
policy 3	1	7	10	10
policy 4	5	3	7	7
policy 5	7	9	3	9
policy 6	8	8	2	6
policy 7	4	2	8	5
policy 8	2	1	9	2
policy 9	10	10	1	4

Figure 7 Ranking of the policy alternatives (1= most preferred policy, 10=least preferred policy)

To obtain an overall policy for the pension fund as a whole[14] one could set up a discussion with as input the policy preferences of each of the stakeholder groups, or one could take a geometric mean over the stakeholders where weights for the active members and pensioners are for

[14] See paragraph 2.3.3 for other options

example given by their shares in the provision and the weight of the sponsor is given by the upfront contribution.

The last column in figure 7 gives the results after taking the geometric mean over the stakeholders.

The ‘best’ consensus policy according to the risk decision support system is given by policy 2: 10% more invested in risky assets as opposed to the current policy (which is policy 0).

3.5 Risk decision support system in practice

Over the last year we used risk decision support systems similar to the one in this paper in practice to obtain insight in our ALM results as to ultimately better advise pension fund boards on next years investment policy. Some remarks before putting a risk decision support in practice for your own company are relevant:

- Experience learns that it is useful to determine the risk/return profile of the decision makers in an interactive context using a business game that also shows the tradeoffs that have to be made in an ALM context. This requires some extra time investment compared to the situation where we determine the risk/return profile a priori but it adds value to the knowledge of the ultimate decision makers and we noticed that objectives and risk attitude changes as a result of learning and experience.
- We also advise to evaluate the risk/return profiles over time, since the environment in which the situation is embedded influences perceptions and also because preferences of the customer may change over time.
- With respect to the stress scenarios and the use of a robust decision rule it is important to look at all the results to see whether not too much weight is put on a very unlikely scenario.
- The robust decision rule can also be applied when other then ‘economic’ stress tests are preformed. Even the results under different ALM models can be taken into account.
- A different ‘risk decision support system’ where we do not need to determine the policy alternatives upfront is given in another paper of the authors titled ‘Inverse ALM’ Joseph e.a.(2010).

Box 2: The ALM model

In section 3 we work through an example of a stylized pension fund. In this box we introduce the simple ALM model we used. To model the liabilities we were inspired by Grosen and Jørgensen (2000):

$$L_{t+1} = L_t(1+r)\left[1+\max\left\{\alpha\left(\frac{A_t}{L_t}-\gamma\right), i_{uncond}\right\}\right]$$

where i_{uncond} is the unconditional indexation, • the fraction of buffer granted for indexation, A_t /L_t the funding ratio at t and • is the minimum funding ratio above which indexation is allowed. The value of the liabilities at time $t+1$ is determined by the liabilities at time t where the liabilities will only grow by indexation and interest.

We now turn to the dynamics on the asset side of the balance sheet. The pension fund has a yearly rebalancing portfolio with some percentage, denoted by w_s, invested in risky assets and (1- w_s) is invested in riskless bonds. The dynamics of the interest rate and the equity market are given by a (discrete) Black-Scholes model:

$$\Delta\beta_t = r\beta_t\Delta t$$
$$\Delta S_t = \mu S_t\Delta t + \sigma S_t \Delta W_t$$

where r denotes the instantaneous short interest rate, S the risky assets, μ the expected return and σ the standard deviation of the risky assets. Inflation is assumed to be constant.

Input parameters

Policy: The policy parameters in the example are: i_{uncond} **,** •, •, and A_0 and the percentage of stocks in the asset mix. In this example the pension fund's current policy is: i_{uncond} =0**,** • =25%, •=110%, A_0=120% and w_s =50%.

Economic (stress) scenarios: For each policy variant and each time horizon, we calculated the outcomes of the risk measures under the various stochastically generated economic scenarios. The economic scenarios may reflect normal expectations, but may also point to economic crisis or extreme welfare.

In the example there are three possible economic scenarios. The parameters of the (discrete version of the) Black-Scholes model for the three economic scenario's are given in table B1.

	Economy description	µ	•	% interest	% inflation
E_1	pension's fund vision of the future	8%	10%	4%	1%
E_2	pessimistic	3%	7.5%	2%	1%
E_3	optimistic	10%	10%	4%	2%

The policy variants

We consider 10 policy variants which are specified in table B2.

	Policy description	% stocks	i_{uncond}	•	•	A_0
B_0	the fund's current policy	50	0	25	110	120
B_1	10% less percentage stocks	40	0	25	110	120
B_2	10% more percentage stocks	60	0	25	110	120
B_3	2% more unconditional indexation	50	2	25	110	120
B_4	10% more alpha	50	0	35	110	120
B_5	10% less alpha	50	0	15	110	120
B_6	5% more gamma	50	0	25	115	120
B_7	5% less gamma	50	0	25	105	120
B_8	10% more single-premium at t=0	50	0	25	110	130
B_9	10% less single-premium at t=0	50	0	25	110	110

Table B2 Current policy and nine other variants.

4. Summary

Theoretical and practical ALM models are becoming more and more sophisticated (and thus complicated). This can be seen as a good development on its own, but we think that it is also important to **focus on the way we use ALM models** since this may in our opinion even be more important than the level of sophistication achieved in the ALM model at hand.

The objective of this paper is to facilitate ALM decision making. We propose an easy method to 1) define a risk attitude taking into account multiple stakeholder groups, and 2) incorporate ALM model uncertainty (for example different stress scenarios) as to obtain policy preferences for the organization under consideration. The method recognizes that an ALM problem is complex, there are multiple criteria, multiple stakeholders and the asset/liability management model is an approximation.

The first contribution of this paper is linking the 'actuarial' asset liability management problem with literature on Multi-Criteria Decision Making and Group Decision Support from Operations Research, thereby facilitating the users of ALM models to formulate a well-defined risk attitude and to arrive at a well-substantiated policy decision.

Our second contribution is to recognize that there are techniques in Robust Control Theory that help decision makers to make good decisions even if he is uncertain on the quality of the model, and to apply these techniques in the ALM context.

In short the risk decision support system works as follows. The method starts from the perceptions of the various groups of stakeholders and the importance each attaches to different risk and return measures which we obtain from pairwise comparisons. Next, it ranks alternative policies according to stakeholder preferences taking also into account the results under (economic) (stress) scenarios. Finally, the stakeholder groups will have to agree on one overall policy choice, where all stakeholders feel comfortable to the degree of solidarity between stakeholder groups. This can be reached by consensus, voting, geometric mean or pairwise comparisons of the stakeholders. We illustrated the method with a numerical example.

The risk decision support system can be used theoretically to obtain insight in for example the influence of a change in future regulation on policy choices or to help design future (pension) contracts. In practice experience learned us that the method facilitates discussions and decision making substantially since the approach helps parties to focus on interests (objectives), rather than positions (alternatives).

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