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A Value-Based Approach to Pension Redesign in the US State Plans

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A value-based approach to pension redesign in the US state plans

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Abstract

This paper explores the financial sustainibility of a typical U.S. state defined-benefit pension fund under the continuation of current policies and under alternative policies, such as alternative contribution, indexation and investment allocation policies. We explore the "classic" asset-liability management (ALM) results, which indicate that a policy of conditional indexation may substantially improve the financial position of the fund. We also investigate the value-based ALM results, which provide a market-based evaluation of the net benefits of the contract to the various stakeholders. All participant cohorts under our simulation horizon derive a substantial net benefit from the pension contract, implying that tax payers make substantial contributions to this pension arrangement. The aforementioned measures can be instrumental in alleviating the burden on the tax payer, though this will happen at the cost of a reduction in the value of the contract to the participants.

1 Introduction

Over the recent past the U.S. has witnessed a trend away from defined-benefit (DB) towards defined-contribution (DC) pension plans. However, an exception to this trend are the state pension plans.[1] The state pension plans manage the pensions of the state civil servants. Despite the ageing of the population those plans still largely operate on a DB basis, although it is clear that in most of the cases the promises made to the fund participants cannot be honored in the absence of drastic measures to guarantee the financial sustainability of their funds. However, the pension promises made to the state employees cannot be so easily reneged upon (see Brown and Wilcox, 2009). In fact, these promises may even get priority to the states' debt holders when

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[1] See (Brown et al., 2011) for an economics-based perspective on the financial aspects of state and local public pensions in the U.S in the June 2011 special issue of the Journal of Pension Economics and Finance on US state and local pension plans.

a state goes bankrupt. Hence, in many instances, the state's pension fund threatens the state's public finances, possibly resulting in large claims to its tax taxpayers and, to the extent that taxes cannot be raised, resulting in a crowding out of other public services. A recent overview of funding ratios (ratios of assets over liabilities) of U.S. state plans (Munnell et al., 2013) illustrates the urgency of the funding problem. The paper comes to an estimated aggregate funding ratio of 73% for a sample of 109 state plans and 17 locally administered plans. Almost a quarter of them have a funding ratio below 60%. A small fraction of 6% have a funding ratio above 100%. This reporting of funding ratios is based on the GASB standards which prescribe that assets are reported on an actuarially smoothed basis, while the discount rate for the liabilities typically is set at around 8 percent, reflecting the expected long term investment return on assets. These standards have been criticized by financial economists (Novy-Marx and Rauh (2009), Bader and Gold (2003)) who claim future streams of benefit payments should be discounted at a discount rate reflecting their degree of risk. As the state pension benefits are protected under most state laws, these payments can be seen as guaranteed and so this would plead for discounting future benefits against the risk-free interest rate. Doing so would lead to a severe fall in the already low funding ratio of state plans. In the last years, some plans have relaxed the guaranteed character of benefits by suspending the cost-of-living adjustment but accrued pension promises will be paid out as defined by the plan. However, the recent default of Detroit is an instructive case of whether pensions should be seen as a contractual obligation that cannot be diminished or impaired, or whether those with pension rights should be treated like other creditors so that pension cuts cannot be ruled out. A U.S. Bankruptcy Judge declared in December 2013 that pensions legally can be cut (Bomey and Priddle, 2013).

In this paper we explore the financial sustainability of a typical U.S. state defined-benefit (DB) pension plan under unchanged fund policies and under alternative policies aimed at alleviating the burden of the fund's current policies on the tax payer. We analyse both the qualitative and quantitative effects of those alternative policies. We explore the classic ALM approach, which shows the future distribution of the funding ratio, i.e. the ratio of assets over liabilities, and we study the so-called "value-based ALM" approach, in which we provide a market-based valuation of all cash flows resulting from the pension contract. This way we can value the conseqences of the various interventions for the different stakeholders, i.e. the different cohorts of fund participants and tax payers. The policy changes are always a zero-sum game, implying that the total value of the contract to all stakeholders together is unchanged and interventions may only shift value from some stakeholders to others.

We consider a variety of intervention measures. The first set of measures concerns adjustments in the contribution rate to the fund, but leaves the benefit levels unchanged. Obviously, an increase in the part of the contribution paid by the fund participants alleviates the burden on the tax payers. The second set of measures addresses the benefit levels and specifically concentrates on reducing the indexation of pension entitlements to inflation. We also entertain the possibility that indexation to inflation is conditional on the funding ratio, i.e. the value of the fund's assets over its liabilities. The third set explores changes in the pension fund's asset portfolio.

We simulate a representative pension fund over a period of three-quarters of a century, thereby exploring the consequences of the existing contract and changes to the contract for both currently-living generations and future-born generations. The classic ALM results reveal that increasing the share of the amortization cost that is contributed, speeding up the amortization payment or halving indexation to CPI inflation leads to substantial long-term improvement of the funding ratio. A policy that is of particular interest is a policy of indexation to inflation that is conditional on the funding ratio. A funding ratio below one leads to less-than-full or even zero indexation, while a funding ratio above one leads to more than full indexation. Such type of indexation policy has become popular among Dutch pension funds. This way of using indexation has a tendency of improving the financial position of the fund, but at the same time of compressing the spread in the distribution of possible funding ratios and thereby also of compressing the spread in the distribution of contribution rates, as its amortization component is linked to the funding ratio. Hence, a well-designed policy of conditional indexation may be of particular interest for policymakers concerned with a redesign of the U.S. system of state pension plans.

Our value-based ALM results show that the current pension contract provides a substantial net benefit, thus accounting for both contributions and the payment of pension benefits, to all cohorts of fund participants, implying a substantial burden on the tax payers of which all cohorts are net contributors. Increasing the amortisation contribution rate and speeding up amortisation without addressing the benefit level improves the residual value of the fund at the end of the projection horizon, but does not affect the contract value to the participants over our simulation horizon. A doubling of the contribution rate by the participants does affect their benefit from the contract. As a group participants that have not yet entered the labour force lose about a quarter of their net benefit, while older participants lose less then 10%. Reducing benefits by reducing indexation can be quite effective too. Halving the indexation to CPI inflation reduces the contract value by around 20% for both young and old participants, while making indexation conditional on the funding ratio is even more effective. Changes in the pension fund's investment portfolio do not affect the participants values, though they do lead to minor shifts in value between tax payers and the fund's residual value.

To the best of our knowledge this paper is the first attempt at applying value-based ALM to U.S. state pension funds. It is closely related to Novy-Marx and Rauh's (2011) careful analysis of the value of the pension promises made by U.S. state pension funds under different assumptions about the riskiness of these promises, for example whether these promises have identical or higher seniority of state debt. In view of the fact that current promises are unlikely to be honoured in full, it is highly relevant to explore the value of these promises under different default scenarios. Other related contributions conduct a classic ALM analysis of pension funds modelled in line with Dutch pension arrangements. These include Ponds and van Riel (2009) and Beetsma and Bucciol (2010, 2011a and 2011b).

The remainder of this paper is structured as follows. Section 2 presents the model, including a description of the demography, the pension fund and the calculation of the fund's benefits and liabilities. Sections 3 - 5, respectively, describe the data, our economic scenario generator and the valuation of the cash flows following from the pension contract. Section 6 summarises the baseline settings, while Section 7 discusses the simulation results for the classic ALM and the value-based ALM. Finally, Section 8 concludes the main text of this paper.

2 The model

This section describes the demography, wage developments and the pension fund. The survival probabilities are deterministic. Hence, there is no longevity risk.

2.1 The population

The pension fund population consists of people from 25 to 99 years old. Hence, individuals enter the fund at age 25. Further, we assume that they retire at age a_R. The number of male and female participants of age a at time t is denoted as M_t^a and F_t^a respectively, where $a \in [25, 99]$. The population size for the rest of the simulation for each age group is determined using the projections of survival probabilities. That is, $M_{t+n}^{a+n} = q_{a,t}^{m,n} M_t^a$ and $F_{t+n}^{a+n} = q_{a,t}^{f,n} F_t^a$, where $q_{a,t}^{m,n}$

and $q^{f,n}_{a,t}$ is the probability that a person aged a in period t will survive another n years, for males and females respectively.

2.2 Wages

Crucial for the calculation of the pension liabilities are the wage developments. The wage level of the cohort of age a at time t is W^a_t and it is updated each time period. We do not take into account idiosyncratic wage risk, hence we assume a uniform wage level within each cohort. The wage levels across cohorts will be set to follow a certain career profile. For now we set wage levels of males and females equal, but one can allow for different wage levels or career profiles depending on gender. The nominal wage level is determined as follows:

$$W^a_t = W^{a-1}_{t-1} w^{a-1}_{t-1},$$

where w^{a-1}_{t-1} is the gross wage growth rate from period $t-1$ to t for generation becoming age a in period t. It can be disaggregated into the economy-wide gross wage growth rate w_{t-1} and a component $\tilde{w}^{a-1}_{t-1}$ attributable to the progression of the individual career:

$$w^{a-1}_{t-1} = w_{t-1} \tilde{w}^{a-1}_{t-1}. \tag{1}$$

We refer to $\tilde{w}^{a-1}_{t-1}$ as the promotion rate from time $t-1$ to t for somebody becoming age a in period t. Note that $\tilde{w}^{a-1}_{t-1}$ is always positive if the career profile has an upward sloping shape. Promotion rates differ across age but the career profile remains constant over time, hence the ratio of the wage levels of two workers of different age is kept constant over time. Economy-wide wage growth rate w_{t-1} is stochastic and modeled in the VAR model as explained in section 4.

Given the current wage rate, based on (1), we calculate the wage expected i periods into the future of an individual who is currently of age a as:

$$\mathrm{E}_t\left[W^{a+i}_{t+i}\right] = W^a_t \prod_{j=1}^{i} \mathrm{E}_t\left[w^{a+j}_{t+j}\right], \forall i \in \{1, a_R - 1 - a\}, \tag{2}$$

where, using (1),

$$\mathrm{E}_t\left[w^{a+j}_{t+j}\right] = \mathrm{E}_t\left[w_{t+j}\right] \mathrm{E}_t\left[\tilde{w}^{a+j}_{t+j}\right], \quad j \geq 1.$$

2.3 The pension fund

State pension funds can differ in many respects. Some of the differences are parametric, such as the value of discount rate to be applied when calculating the liabilities, while other differences are more fundamental. We model a pension fund based on the most common features across the entire population of pension funds.[2]

2.3.1 Assets

Market value of assets The market value of the fund's assets at the beginning of the next year, A_{t+1}, is equal to the asset value A_t at the beginning of this year multiplied by its gross

[2]The most common features are provided in the Public Plans Database by Center for Retirement Research at Boston College (CRR, 2012)

rate of return R_t plus the net money inflow times the gross return of on average half a year over which it is invested (hence square root of the gross return):

$$A_{t+1} = A_t R_t + (C_t - B_t) R_t^{1/2}. \tag{3}$$

where C_t is the total amount of contributions received (calculated below) and B_t the total amount of benefits paid out. Since the benefits and contributions are (usually) paid on a monthly basis but our model runs on a yearly basis, we assume that the payment of benefits and contributions takes place in the middle of the calendar year and hence the net money inflow is invested on average for half a year.

Actuarial assets Pension fund assets in the U.S. are not measured as their market value when they are used as an input for pension policy. Rather, pension funds in the U.S. apply a certain smoothing procedure to come up with an actuarial value A_t^{act} of their assets in period t. [3] Its detailed calculation is laid out in the appendix A. In short, the actuarial value of the assets at the end of the year is equal to the actuarial value at the end of previous year, plus the net cashflows into the fund, plus the projected return on the assets, and a recognition of the smoothed excess of actual above expected investment income. In the special case that initial actuarial assets equal the initial market value of the assets, $A_0^{act} = A_0$ and the smoothing period is shrunk to a single period, one has $A_t^{act} = A_t$ for all $t \geq 0$. Hence, the process for actuarial assets coincides with that of the market value of the assets.

2.3.2 Calculation of the liabilities

This section we will formalize the calculation of the liabilities, closely following Munnell et al. (2008b) and Novy-Marx and Rauh (2011). Total liabilities L_t are calculated as the present value of the projected future benefit payments to all current pension fund participants based on accumulated accrual up to now, adjusted for the survival probabilities and discounted back to period t. The group of participants includes the employees and the retired. The total liabilities are the sum of the liabilities L_t^m and L_t^f to the male and female participants. The liabilities to each gender, in turn, are calculated by multiplying the individual liabilities $L_t^{a,\zeta}$ by the number of people of that gender in the cohort and then summing over the cohorts. Here, $L_t^{a,\zeta}$ denotes the liability at time t to a participant of gender ζ in a cohort of age a, where $\zeta \in \{f; m\}$. Hence, the total liabilities are calculated as:

$$\begin{aligned} L_t = L_t^m + L_t^f &= \sum_{a=25}^{99} \left(M_t^a L_t^{a,m} + F_t^a L_t^{a,f} \right) \\ L_t^{a,\zeta} &= \sum_{i=\max(a_R - a, 0)}^{99-a} \mathrm{E}_t \left[K_{t+i}^{a+i} \right] \left(\tilde{R}_t^{(i)} \right)^{-i} q_{a,t}^{\zeta,i} \end{aligned} \tag{4}$$

where $\mathrm{E}_t \left[K_{t+i}^{a+i} \right]$ is the expectation at time t of the pension payout i years ahead for a participant of age a at time t, $q_{a,t}^{\zeta,i}$ stands for the probability for an individual of gender ζ and age a to survive i more years, and $\tilde{R}_t^{(i)}$ is the interest rate used to discount cash flows materializing i periods into the future to the current period t. We notice that $\tilde{R}_t^{(0)} = 1$, $p_{a,t}^{\zeta,0} = 1$ and $\mathrm{E}_t [K_t^a] = K_t^a$. U.S. public pension plans usually use a discount rate that is flat over the entire projection horizon and equal to the expected return $\bar{R}_t$. Summarizing, the liabilities to a particular cohort depend

[3] Based on email conversations with Jean-Pierre Aubry, Assistant Director of State and Local Research at the Center for Retirement Research at Boston College.

on the years it will (still) receive benefits, the level of the benefits, the discount rate and the survival probabilities.

Under plausible parameter settings the value of the liabilities peaks at around retirement age. The reason is that the workers close to retirement have reached the maximum time span over which pension rights can be accumulated, while most of their benefits occur relatively near into the future so that the effect of discounting is relatively limited. Younger individuals have fewer years of accruals, while any given benefits occur further into the future. Older individuals expect to receive fewer remaining benefit payments. Moreover, the average of the latest wages during their career tends to be lower for older individuals and this effect is usually not compensated by the COLA they have received while in retirement.

Pensioners To protect the living standards of the retired as much as possible, pensioners receive a cost-of-living adjustment (COLA), which we denote as π_t. The COLA usually covers a certain share of inflation and it is often capped at a predetermined level (CRR, 2012):

$$\pi_t = 1 + \min\{\alpha \; cpi, cap\},$$

where *cpi* stands for the inflation rate in the consumer price index, α is the fraction of inflation compensated and *cap* is the maximum indexation rate (both *cpi* and *cap* expressed as a fraction of unity).

The current payout and the expected payout i periods from now to current retirees equal the current pension rights B_t^a and the current pension rights adjusted for the expected future indexation, respectively:[4]

$$\begin{aligned} \mathrm{E}_t\left[K_t^a\right] &= B_t^a, \forall a \in \left[a_R, 99\right], \\ \mathrm{E}_t\left[K_{t+i}^{a+i}\right] &= B_t^a \prod_{j=1}^{i} \mathrm{E}_t\left[\pi_{t+j}\right], \forall i \in \left[1, 99-a\right], \forall a \in \left[a_R, 98\right], \end{aligned}$$

where B_t^a equals the pension rights at the moment of retirement increased by the past indexation since then:

$$B_t^a = B_{t-(a-a_R)}^{a_R} \prod_{j=0}^{a-(a_R+1)} \pi_{t-j}, \forall a \in \left[a_R+1, 99\right].$$

For example, the pension rights of a 70 year old person (who retired at age 65) are equal to the benefit calculated when that person reached the age of 65, plus 5 years of indexation that have been awarded since then.

A common feature of the state pension plans in the US is that they are based on the average wage preceding the moment of exit from the workforce. Concretely, the benefit for somebody retiring in year $t-(a-a_R)$ can be calculated as the product of the accrual rate ε, the number of years (40) in the workforce and the average wage level over the past z years:

$$B_{t-(a-a_R)}^{a_R} = \frac{40\varepsilon}{z} \sum_{l=1}^{z} W_{t-(a-a_R)-l}^{a_R-l}, \forall a \in \left[a_R, 99\right].$$

[4]COLAs are only included in the liabilities for pensioners if there exists an explicit policy rule on how the indexation is awarded. It is not taken into account if indexation is determined on an ad-hoc basis.

The averaging period z varies from one to five years, with majority of public plans applying a three–year average (Munnell et al., 2012).

Because the pension rights for the retired are already determined and, hence, cannot be affected by future service or salary increases, they are independent of the method we use to recognize the liabilities.

Workers The calculation of the expected payouts to current employees is similar to that of pensioners, i.e. as the pension rights B_t^a at time t for someone of age a adjusted for future indexation during retirement:

$$\mathrm{E}_t\left[K_{t+i}^{a+i}\right] = B_t^a, i = a_R - a, \forall a \in [25, a_R - 1],$$
$$\mathrm{E}_t\left[K_{t+i}^{a+i}\right] = B_t^a \prod_{j=a_R+1-a}^{i} \mathrm{E}_t\left[\pi_{t+j}\right], \forall i \in [a_R + 1 - a, 99 - a], \forall a \in [25, a_R - 1].$$

Determining the pension rights B_t^a is not as obvious for the working population as it is for the pensioners. Unlike pensioners, the active participants continue working and, hence, they are expected to accrue more years of service and (in most cases) to have a salary at retirement that is higher than the current salary. Hence, depending on the method used, their pension rights B_t^a can be recognized in different ways. We will now discuss the various ways in which B_t^a can be determined.

Methods for recognising liabilities There are different methods for recognizing liabilities. In this section we will explain the so-called *Accrued Benefit Obligation* (ABO) method, the *Projected Benefit Obligation* (PBO) method and the *Projected Value of Benefits* (PVB) method.[5] The latter is also known as the Present Value of Future Benefits method.

Under the *ABO method*, only the pension rights accrued up until time t are taken into account. Hence, B_t^a is based on the number of years since entry into the workforce, of the average of the past z wage levels:

$$B_t^a = \frac{(a-25)\varepsilon}{\min(a-25, z)} \sum_{l=1}^{\min(a-25,z)} W_{t-l}^{a-l}, \forall a \in [26, a_R],$$
$$B_t^{25} = 0.$$

The youngest cohort of age 25 has just entered the fund and has no rights accrued yet. The benefits of the other young cohorts who do not yet have z years in service are based on the average of the available wage history. For the cohorts that have at least z years of service, the benefit is the product of years in the workforce, the accrual rate and the average pay over the past z years. Hence, for an individual worker the ABO pension rights increase with each additional year of service.

Under the *PBO method*, we also take into account the effect of expected future salary increases on the rights earned up to now. Hence, under this method B_t^a is the projected benefit level at retirement when expected future salary advances are taken into account:

[5] There is also the Vested Benefit Obligation (VBO), which considers only employees who have served long enough (given that many plans feature a vesting period). However, this variant is not meaningful in our context, because we assume that everybody works for 40 years.

$$B_t^a = \frac{(a-25)\varepsilon}{z} \sum_{l=1}^{z} \mathrm{E}\left[W_{t+(a_R-a)-l}^{a_R-l}\right], \forall a \in [26, a_R - 1],$$
$$B_t^{25} = 0.$$

Again, the youngest cohort, the 25 years old, has no accrual yet and, hence, it has no pension rights in terms of the PBO. However, for given age and assuming the overall wage level is expected to increase, the pension rights for the other working cohorts are higher in PBO than in ABO terms. When the worker nears the retirement age, the difference of rights in PBO and ABO terms shrinks and the two measures merge for a given individual at the moment of his retirement.

Finally, the state jobs are relatively secure so the pension fund might in addition consider the rights that the employees will acquire in the future if they continue working in this job until retirement. The *PVB method* takes this into account. Therefore, it defines the pension rights B_t^a including future accrual due to new service:

$$B_t^a = \frac{40\varepsilon}{z} \sum_{l=1}^{z} \mathrm{E}\left[W_{t+(a_R-a)-l}^{a_R-l}\right], \forall a \in [25, a_R - 1]. \tag{5}$$

Note that in this case the generation that has just entered the labor force already has a steak in the pension liabilities, as opposed to ABO and PBO. At any given age, rights computed according to the PVB method exceed rights computed according to ABO or PBO, because PVB takes into account both expected future increases in the wage level and expected future accrual. All three measures merge for a given individual at the moment of his retirement.

Rights to be accrued by future employees that replace current employees are not taken into account in any of the methods. Notice that in any given period t under all three methods pension rights increase proportionally with years of service if the career profile of the wage is flat, while pension rights increase more than proportionally with years of service when the career profile of the wage is positive, i.e. the salary rises with age.

2.4 Contributions and benefits

2.4.1 The funding method

With new contributions paid to the pension fund, additional accrual is generated and liabilities increase. The value of the additional pension rights earned in a given year is called the normal cost (NC). The fund's actuaries determine the NC based on the liability recognition method used by the pension fund. The two most common methods[6] are the so-called *projected unit credit* method, which is mostly used in the private sector, and the so-called *entry age normal costing* (EAN) method, which is dominant in public plans and which is related to the PVB. Here, we explain only the EAN method.

Under the EAN method the employer's annual NC associated with an individual participant is calculated as a level payment throughout the projected years of service needed to pay the PVB obligation. Since accrued benefits due to salary growth increase more than linearly, the level normal cost rate implies a component of front-loading, because the employer is pre-paying

[6]Based on email conversations with Matthew Smith, Washington State Actuary

some of the benefits to be earned in the future (Munnell et al., 2008b). The EAN method is the method we will use in our calculations.

The level payment of NC can be calculated in dollar terms or as a level percentage of projected salary levels. Here, we will explain the more common level percentage EAN method. The normal cost *rate* (NCR) as a level percentage of the salary of an active participant is calculated at the entry age as the ratio of the present value of future benefits and the present value of career salary levels:

$$NCR_t^{25,\zeta} = \frac{L_t^{25,\zeta}}{PVW_t^{25,\zeta}}, \quad \zeta \in \{f; m\}$$

where $L_t^{25,\zeta}$ is the liability to someone of gender ζ who enters the labor force in period t as calculated in (4) on the basis of the PVB method (as in (5), i.e. taking into account future accrual associated with future service), and $PVW_t^{25,\zeta}$ is the present value of all future wages throughout the participant's career as projected at entry. Hence, the normal cost rate is the percentage payment of a worker's projected career salary needed to cover the cost of the projected benefits for that worker.

The $PVW_t^{a,\zeta}$ associated with a worker of age a and gender ζ is determined as follows:

$$PVW_t^{a,\zeta} = \sum_{i=0}^{a_R-1-a} \mathrm{E}_t\left[W_{t+i}^{a+i}\right]\left(\tilde{R}_t^{(i)}\right)^{-i} q_{a,t}^{\zeta,i},$$

where the expected wage level $\mathrm{E}_t\left[W_{t+i}^{a+i}\right]$ is calculated according to (2). Hence, the PVW_t of the entire population of working participants is:

$$PVW_t = PVW_t^m + PVW_t^f = \sum_{a=25}^{a_R-1}\left(M_t^a PVW_t^{a,m} + F_t^a PVW_t^{a,f}\right),$$

The NC attributed to an active participant in a given year is his current salary times the NCR determined at the time $t-(a-25)$ when he entered the fund:

$$NC_t^{a,\zeta} = W_t^a \times NCR_{t-(a-25)}^{25,\zeta}.$$

The present value of future normal cost ($PVFNC$) of a worker are the normal costs that will be recognized throughout the remaining years of service:

$$PVFNC_t^{a,\zeta} = PVW_t^{a,\zeta} \times NCR_{t-(a-25)}^{25,\zeta}.$$

Hence, this is the product of the present value of future wages and NCR determined at the time when he entered the fund. The so-called *accrued* liability (L_{acr}) for this participant will then be the difference between the present value of benefits and the present value of the future normal cost:

$$L_{acr,t}^{a,\zeta} = L_t^{a,\zeta} - PVFNC_t^{a,\zeta}.$$

If we follow the individual worker over time, we will see that the present value of future normal cost will decrease, as there will be fewer years remaining to pay the normal cost. The accrued liability associated with an individual therefore increases over time. The accrued liability for pensioners is simply equal to the present value of future benefits, since they should have paid the whole normal cost before reaching the retirement age. Therefore,

$$\begin{aligned} PVFNC_t^{a,\zeta} &= 0, \forall a \in [a_R, 99], \\ L_{acr,t}^{a,\zeta} &= L_t^{a,\zeta}, \forall a \in [a_R, 99]. \end{aligned}$$

The total accrued liability is then the sum of the individual accrued liability over genders and cohorts:

$$L_{acr,t} = L^m_{acr,t} + L^f_{acr,t} = \sum_{a=25}^{99} \left(M^a_t L^{a,m}_{acr,t} + F^a_t L^{a,f}_{acr,t} \right).$$

To evaluate their funding position, pension funds calculate the Unfunded Actuarial Accrued Liability ($UAAL$) as the difference between the total accrued liability and the actuarial value of assets:

$$UAAL_t = L_{acr,t} - A^{act}_t.$$

2.4.2 Contributions

The annual contribution to the pension fund is paid by both the employer and the employees. The two payments combined form the total contribution. We determine the annual total contribution as a sum of two components: the normal cost and the amortization payment for the $UAAL$.[7] The amortization payment, like the normal cost, can be determined as a dollar amount or a percentage of projected salaries. We will use a level dollar amount amortization method. The amortization payment cannot be negative. Hence, if there is a funding surplus, the normal cost covering contribution cannot be reduced. We assume a smoothing period of u years for the amortization of the $UAAL$, implying that the required amortization payment in period t is:

$$AMORT_t = \begin{cases} \frac{1}{u} UAAL_t & \text{if } UAAL_t \geq 0, \\ 0 & \text{if } UAAL_t < 0. \end{cases}$$

Munnell et al. (2008a) show that only about half of all plans pay the annually required contribution. Hence, in our simulations we include a parameter λ of the fraction of the required amortization payment that is actually paid. Then the total amount of contributions that should be paid to the pension fund by all participants and the employer in year t is:

$$C_t = \sum_{a=25}^{a_R-1} \left(NCR^{25,m}_{t-(a-25)} M^a_t W^a_t + NCR^{25,f}_{t-(a-25)} F^a_t W^a_t \right) + AMORT_t.$$

The total contribution rate c_t is expressed as a percentage of the total wage sum in year t:

$$c_t = \frac{C_t}{\sum_{a=25}^{a_R-1} (M^a_t + F^a_t) W^a_t}. \tag{6}$$

In other words, first the aggregate required amount of contributions is determined. This is then spread over cohorts in such a way that everybody pays the same contribution *rate*. Usually, the contribution is paid by both the employer (denoted by superscript E), and the employee (henceforth denoted by the superscript P of participant) together. Typically, the employee pays a fixed contribution rate, while the employer pays the remaining part:

$$c_t = c^P_t + c^E_t.$$

It is also possible that the employee contribution is a fixed proportion of actuarially required contribution, in which case c^P_t may change over time.

[7] Based on email conversations with Matthew Smith, Washington State Actuary

2.4.3 Total benefits

The total amount of benefits to be paid out in year t is:

$$B_t = \sum_{a=a_R}^{99} B_t^a (M_t^a + F_t^a).$$

2.5 Sponsor support

Forecasting the development of an underfunded pension fund over a long horizon implies that there exist possible scenarios in which the fund's assets get depleted. Therefore, we assume that whenever the pension fund's assets become unsufficient to pay the net outflow of money, the employer makes up for the deficit by replenishing the assets. This additional support, ES, comes on top of the employer's regular contribution:

$$ES_t = \begin{cases} 0 & \text{if } A_t \geq B_t - C_t, \\ (B_t - C_t) - A_t & \text{if } A_t < B_t - C_t, \end{cases}$$

so that the sponsor support covers the difference between the assets and the net outflow. From then on the assets are depleted and the fund is run on a pay-as-you-go basis. If the total benefits to be paid are higher than the total contribution amount received, the tax payers continue paying sponsor support. The additional sponsor support is included in the total contribution account in the calculation of actuarial assets (equations (13), (14) and (15)).

2.6 The funding ratio

The evaluation of the pension fund's financial position is usually based on the funding ratio (FR), which we calculate here as the ratio of actuarial assets over liabilities:

$$FR_t = \frac{A_t^{act}}{L_t}.$$

The funding ratio measures the financial health of the pension fund, and is an important quantity to guide the policies of the fund.

3 Data

The economic scenario generator described in the following section is based on historical data spanning the third quarter of 1971 up to and including the last quarter of 2012. We use historical time series of the short interest rate, stock returns, price inflation and wage inflation, which are the state variables of the VAR model presented below, and zero coupon yields of maturities from 1 to 10 years for the term structure we use to value the cash flows.

For the term structure we use the U.S. Treasury yield curve zero-coupon rates from the Finance and Economics Discussion Series of the Federal Reserve Board. The 3-month Treasury rate series are composed of two parts.The time series until 1982 are provided in the Handbook of Monetary Economics (McCulloch, 1990). The remaining part from 1982 onwards is available through the Federal Reserve Economic Data. For stock returns we use the value-weighted NYSE/Amex/Nasdaq return including dividends, which is available from the Center for Research in Security Prices. For price inflation we use the CPI index also provided by the Center for Research in Security Prices. Finally, wage inflation is the compensation of employees, wages and salary accruals, retrieved from the Federal Reserve Economic Data. All data are quarterly.

4 The economic scenario generator

An economic scenario is one possible realization of economic variables over a certain period of time in the future. It is generally not known which scenario will occur in reality, hence the ALM model makes use of a number of possible scenarios to evaluate the performance of an object of interest under all these realizations. Here we use a Vector Auto-Regressive (VAR) model to generate scenarios. We estimate the VAR on historical data for the U.S. Then, using the estimated VAR, we generate one set of 5000 scenarios and use this same set for all our analyses. Hence, different policies are examined in the same set of scenarios.

Specifically, we use a first-order VAR model:

$$X_{t+1} = \begin{bmatrix} y_{t+1} \\ xs_{t+1} \\ cpi_{t+1} \\ w_{t+1} \end{bmatrix} = \alpha + \Gamma X_t + \varepsilon_{t+1}, \tag{7}$$

where

$$\alpha = (\mathrm{I} - \Gamma)\mu, \tag{8}$$

and where μ is the unconditional mean of the vector X_t for all t and the noise term ε_{t+1} follows a multivariate normal distribution:

$$\varepsilon_{t+1} \sim \mathbb{N}(0, \Sigma). \tag{9}$$

Here, y_{t+1} is the short interest rate, xs_{t+1} is the excess return on stocks, cpi_{t+1} stands for US consumer price inflation, and w_{t+1} is wage growth.The variables are in natural logarithms. Equation (9) implies that:

$$\mathrm{E}_t \left[X_{t+1}\right] = (\mathrm{I} - \Gamma)\mu + \Gamma X_t. \tag{10}$$

The empirical values $\hat{\alpha}$ and $\hat{\Gamma}$ for α, respectively Γ, in equation (7) are determined by regressing the historical data of state variables on their lagged values using multivariate OLS. $\hat{\Sigma}$ is then the variance of the residuals, i.e. the difference between the observed and the estimated values of the variables.

Given the values $\hat{\Gamma}$ and $\hat{\alpha}$, the estimate $\hat{\mu}$ of the average μ of X can be obtained from (8) as:

$$\hat{\mu} = (\mathrm{I} - \hat{\Gamma})^{-1}\hat{\alpha}. \tag{11}$$

The values of $\hat{\alpha}$, $\hat{\Gamma}$, $\hat{\Sigma}$ and $\hat{\mu}$ are used for the pricing of the cash flows – see below.

5 Valuation

A pension plan is a financial contract whose payoff depends on market contingencies. For example, the contract might specify that benefits are higher and/or contributions are lower when market outcomes are good, and vice versa. In that case the implicit return on the pension deal depends on the plan specifics as well as the market conditions. If we perceive the pension contract as a combination of contingent claims, we can value the pension deal using the derivative pricing techniques of risk-neutral valuation introduced by Black and Scholes (1973). Specific valuation issues of contingent claims in pension contracts are discussed in Hoevenaars and Ponds (2008), Lekniute (2011) and Ponds and Lekniute (2011).

To value a contract we use the scenarios for the underlying variables produced by our scenario generator. One can calculate the market value of a contingent claim by using deflators and real-world scenarios, or risk-neutral scenarios. Both alternatives should result in the same outcome. Here, we will use risk-neutral valuation. We want to compare the claims of different stakeholders of the pension fund. Therefore, we will calculate the value of the net benefits for each cohort of pension fund participants, the value of the contributions paid by the employer, i.e. the tax payers and the residual assets in the fund. We will value the cash flows over a horizon of T years.

The value V_0^{Pa} of the pension contract to participants of cohort a at the starting moment $t = 0$ is:

$$V_0^{Pa} = \mathrm{E}_0^Q \left[NB_1^a (R_1^f)^{-0.5} + \sum_{t=2}^{T} \left(NB_t^a \prod_{j=1}^{t-1} (R_j^f)^{-1} (R_t^f)^{-0.5} \right) \right],$$

where $R_j^f = 1 + r_j^f$ is the gross return on a short-term risk-free bond and r_j^f the corresponding (net) rate of return in time period j. E_0^Q is the risk-neutral expectation under the $\mathbb{Q}$ measure of the cashflows discounted against the risk-free rate. Further, NB_t^a is the net benefit, which is contribution for workers and benefit for pensioners, hence NB_t^a is negative for workers and positive for retirees. Therefore for a cohort that is already retired at $t = 0$, $V_0^{Pa} > 0$. NB_t^a occurs in the middle of time period t, thus it is discounted only for a half of the last period. Similarly, the value of the employer's (tax payer's) contributions is calculated as

$$V_0^E = \mathrm{E}_0^Q \left[-C_1^E (R_1^f)^{-0.5} \sum_{t=2}^{T} \left(-C_t^E \prod_{j=1}^{t-1} (R_j^f)^{-1} (R_t^f)^{-0.5} \right) \right].$$

Finally, at the end of evaluation horizon the pension fund generally has some assets left, of which the value is analogously calculated as:

$$V_0^A = \mathrm{E}_0^Q \left[A_T \prod_{j=1}^{T} (R_j^f)^{-1} \right].$$

In line with the literature (e.g., see Cochrane, 2005), we assume that the stochastic discount rate $-m_{t+1}$ for the real-world scenario's is given by the following function of the state vector generated by our VAR model and the shocks to this state vector:

$$-m_{t+1} = e'X_t + \frac{1}{2}(\beta_0 + \beta_1 X_t)'\Sigma(\beta_0 + \beta_1 X_t) + (\beta_0 + \beta_1 X_t)'\varepsilon_{t+1}, \tag{12}$$

where β_0 and β_1 are a vector and a matrix of parameters respectively, and e indicates the position of the short rate in the state vector:

$$e = (1, 0, 0, 0, 0)'.$$

The Appendix B shows the details for the pricing of the cash flows and the transformation to the risk-neutral scenario needed to calculate the value of the pension contract.

6 Baseline settings for our U.S. state plan

Our baseline simulation is based on the most common characteristics of the U.S. state pension plans. The relevant data are available in the Public Plans Database (PPD) of the Center for Retirement Research at Boston College. Later we will perform a sensitivity analysis for the scheme parameters. This section will discuss the baseline assumptions.

As far as the demography is concerned, we will use Dutch survival rates to project the future population structure. We choose Dutch data, because we do not have the corresponding U.S. figures available. Hence, the Dutch demographic structure will represent the demographic structure of some pension fund. This approach does not seem unreasonable in view of the fact that the various U.S. state pension funds feature quite a wide range of demographic structures. Ageing in the Dutch population is stronger than for the U.S. population at large. However, this is also the case for the state pension funds.

Individuals enter the labour market at the age of 25 and retire at the age of 65. Hence, a full career means that individuals work for 40 years. We distinguish between men and women in terms of survival rates but we assume no gender differences in wage levels. The process for wages is based on Dutch data. The economy features annual averages for the stock return of 11.7%, 10-year Treasury bonds of 6.3%, nominal wage growth of 5.7% and price inflation of 3.7%. These figures are obtained as averages of all the numbers generated for all the scenarios.

The initial funding ratio of the fund, FR_0, will be a design parameter in the model. Based on the value of the initial funding ratio, and given the initial liabilities, we obtain the initial actuarial assets of the fund:

$$A_0^{act} = FR_0 L_0.$$

We assume that the fund has an initial funding ratio of 75% based on actuarial assets calculated by averaging the excess returns of the last 5 years and using the entry age normal actuarial method for liability recognition. Although more asset classes exist, we simplify the asset mix somewhat and assume that the fund invests 50% of its assets in fixed income and the other 50% in risky assets. Based on this allocation the average annual return on the fund's portfolio is 9%.

The calculation of the fund's liabilities is based on the following actuarial assumptions. Career profiles are calibrated to Dutch career paths, assuming that the shape of age-wage profile does not change over time, but that each year all wages increase with a common growth rate. The benefit factor, or accrual rate, is 2% for each additional year of service, while the earnings base for the retirement benefit is the average of the final three years of pay during the career. Projected benefits are heightened up with an assumed salary increase of 5.4% for each working year and with an assumed rate of annual price inflation of 3.5% for years in retirement. Note that the salary increase used in the projections does not necessarily have to be equal to the realization or even the average of the wage growth in the economy. It is based on the average that the funds in the sample use. Future retirement benefits are discounted at a fixed rate of 8% a year, which is the current median for U.S. state pension funds. Hence, the implicit assumption is that the funds assets earn an average annual return of 8%. We assume the initial accrued pension rights, B_0^a, to have been fully indexed up to $t = 0$.

Contributions to the fund are calculated as follows. The annual required contribution (ARC) is set to the normal cost (NC) plus the required amortization payment, of which we assume that it cannot be used to lower the contribution in case of a fund surplus (one-sided policy). The normal cost is calculated as a level percentage of the projected career salary based on the EAN actuarial method. The amortization payment is determined by spreading the unfunded liability amount in equal annual payments over the next 30 years (with a moving 30 years window, hence, we use the so-called open amortization period). The employee pays a fixed 6% contribution of her salary, while the employer pays the remainder of the required contribution. The actual contribution is set to 100% of the normal cost, plus 50% of the required amortization payment. Our data show that, assuming that the normal cost is paid in full, the actual amortization payment is on average 10% of the required amortization payment. However, these data are for the year 2009, the year with the worst economic performance since the Great Depression and a year with unusually large declines in the value of stocks. This suggests that the share of the required amortization payment paid in that year may have been unusually low. Because of the lack of potentially more representative data we thus assume that 50% of the required amortization payment is paid in a year. However, below we will examine variations in which less or more of the required amortization payment is paid in a year.

To evaluate the magnitude of the problem in the U.S. we set the population of our pension fund equal to the number of participants in the state plans reported in the PPD of the Center for Retirement Research at Boston College. To do that we scale the Dutch population of ages 25-99 in 2011, by factor 1.11 which then yields a fund of approximately 13 million participants, as reported in the PPD. In fact, the number of participants in the fund that we simulate is essentially irrelevant, although it helps to give an impression of the amounts of money at stake in the reforms. Notice that the PPD does not include all the public plans, therefore one should view our estimates as a lower bound of the problem.

7 The reforms

We consider a number of variations on the baseline plan, which we refer to as Plan 0.0, to explore what different policy changes imply for the contract values of the various stakeholders. This would give us leads about the effectiveness of different measures in increasing the financial sustainability of the pension fund.

We consider three groups of measures, which we summarise in Table 1. The first set of measures, Plans 1.1−1.4, addresses the contribution rate. Plans 1.1 and 1.2 vary the fraction of the required amortization payment actually paid, Plan 1.3 shortens the period over which the amortization payment is spread, while Plan 1.4 doubles the contribution payment by the participants from 6% to 12%, but leaving the total contribution rate unchanged. In other words, in Plan 1.4 the financial health of the pension fund is unaffected, while the burden on the employers, i.e. the tax payers, is alleviated.

The second set of alternatives addresses the degree of indexation. Under all plans, as under the baseline, when CPI inflation is negative, then indexation is always full (thus negative). Plan 2.1 halves indexation when CPI inflation is positive. Under Plan 2.2, if CPI inflation is positive, then indexation to CPI inflation is conditional on the level of the funding ratio FR. Specifically, if $cpi \geq 0$, we index by 0 if $FR < 0.5$ and by $(2 * FR - 1) * cpi$ if $FR \geq 0.5$. In other words, if CPI inflation is positive, the degree of indexation is set at zero for funding ratios below one half and it increases linearly with the funding ratio for funding ratios equal to or larger than one half. A funding ratio above unity implies more than full indexation. This way of providing conditional indexation is closely related to the way most Dutch pension funds index their pension entitlements, (see Ponds and van Riel (2009), Beetsma and Bucciol (2011b)). We set contributions such that the normal cost is calculated under the assumption of full indexation. We do this in order to see the effect purely of the reduction in indexation. Otherwise, the normal cost included in the calculation, which gives the cost-covering contribution, would fall in response to the reduction in indexation, and, hence, the comparison would be contaminated by

Case	Description
0.0	Baseline
Contribution	
1.1	0% amortization paid
1.2	100% amortization paid
1.3	amortization spread over 10 years
1.4	participants' contribution rate doubled to 12%
Indexation	
2.1	indexation is $0.5 * CPI$
2.2	conditional indexation
Portfolio composition	
3.1	100% stocks
3.2	0% stocks

Table 1: Summary of alternative policies

the response of the normal cost; the effect of the reduction in indexation on the financial health of the fund would essentially be undone by reduced contributions.

The third group of measures concerns the composition of the fund's asset portfolio, which we vary from zero to 100% stocks.

8 Results

8.1 The "classic" ALM results

This subsection reports a number of measures generated by the simulations of our 5000 future economic scenarios. Because in this subsection we do not apply our market-based valuation, we refer to this case as "classic" ALM. In the sequel we always confine ourselves to a horizon of 75 years, as this allows us to take into account a number of cohorts that have not entered the fund at the start of the evaluation horizon. Moreover, it is a common horizon for pension policy evaluation in the US.

Table 2 reports for the different plans the 5%, 50% and 95% percentiles after 25 years for the funding ratio FR_t, the "pension result" PR_t, the average total contribution rate (c), the average normal cost payment (cNC) and the average amortization payment ($cAmort$). The pension result is defined as the ratio of cumulative granted indexation to cumulative price inflation, i.e. $PR_t = \prod_{\tau=0}^{t}(1+\pi_\tau)/\prod_{\tau=0}^{t}(1+cpi_\tau)$. Hence, it measures the deterioration in the purchasing power of the benefits. The three contribution measures are expressed in percentages of wage sum. Table 3 reports all corresponding numbers after 75 years.

First we discuss the results for the baseline. We observe a wide spread in the development of the funding ratio over the 75 years horizon. In particular, the median funding ratio after 75 years is 0.46, while the fifth percentile for the distribution of the funding ratio is 0, which esentially means the fund is run on a pay-as-you-go basis, where benefits are paid from participants' contributions and sponsor's support. This substantial expected deterioration of the funding ratio is not surprising in view of the fact that only 50% of the required amortization costs is paid when the fund's financial health is poor. The pension result is by construction equal to unity because full indexation is always given, including a negative adjustment in case of deflation.

	0.0	**1.1**	**1.2**	**1.3**	**1.4**	**2.1**	**2.2**	**3.1**	**3.2**
FR 5%, t=25	23%	3%	38%	48%	23%	32%	39%	16%	15%
FR 50%, t=25	57%	38%	71%	80%	57%	69%	73%	91%	23%
FR 95%, t=25	131%	115%	144%	155%	131%	151%	135%	436%	38%
PRp 5%, t=25	100%	100%	100%	100%	100%	54%	37%	100%	100%
PRp 50%, t=25	100%	100%	100%	100%	100%	67%	60%	100%	100%
PRp 95%, t=25	100%	100%	100%	100%	100%	80%	101%	100%	100%
c 5%, t=25	17%	17%	17%	17%	17%	17%	17%	17%	26%
c 50%, t=25	23%	17%	25%	25%	23%	21%	20%	18%	28%
c 95%, t=25	28%	17%	35%	40%	28%	26%	25%	29%	29%
cAmort 5%, t=25	0%	0%	0%	0%	0%	0%	0%	0%	9%
cAmort 50%, t=25	6%	0%	8%	8%	6%	4%	3%	1%	11%
cAmort 95%, t=25	11%	0%	18%	23%	11%	9%	8%	12%	13%
cNC 5%, t=25	17%	17%	17%	17%	17%	17%	17%	17%	17%
cNC 50%, t=25	17%	17%	17%	17%	17%	17%	17%	17%	17%
cNC 95%, t=25	17%	17%	17%	17%	17%	17%	17%	17%	17%

Table 2: Classic ALM measures over 25 year horizon

	0.0	**1.1**	**1.2**	**1.3**	**1.4**	**2.1**	**2.2**	**3.1**	**3.2**
FR 5%, t=75	0%	0%	36%	56%	0%	21%	44%	0%	0%
FR 50%, t=75	46%	0%	108%	143%	46%	132%	104%	255%	0%
FR 95%, t=75	500%	325%	644%	744%	500%	788%	220%	7141%	0%
PRp 5%, t=75	100%	100%	100%	100%	100%	17%	8%	100%	100%
PRp 50%, t=75	100%	100%	100%	100%	100%	25%	44%	100%	100%
PRp 95%, t=75	100%	100%	100%	100%	100%	37%	1030%	100%	100%
c 5%, t=75	17%	17%	17%	17%	17%	17%	17%	17%	29%
c 50%, t=75	24%	17%	17%	17%	24%	17%	17%	17%	30%
c 95%, t=75	31%	17%	34%	34%	31%	27%	23%	30%	31%
cAmort 5%, t=75	0%	0%	0%	0%	0%	0%	0%	0%	13%
cAmort 50%, t=75	7%	0%	0%	0%	7%	0%	0%	0%	13%
cAmort 95%, t=75	14%	0%	17%	18%	14%	10%	7%	14%	14%
cNC 5%, t=75	17%	17%	17%	17%	17%	17%	17%	17%	17%
cNC 50%, t=75	17%	17%	17%	17%	17%	17%	17%	17%	17%
cNC 95%, t=75	17%	17%	17%	17%	17%	17%	17%	17%	17%

Table 3: Classic ALM measures over 75 year horizon

We now discuss the variations on the baseline setting. Under all alternatives the component of the contribution rate based on the normal cost remains unchanged at approximately 17% under the full indexation assumption. When no amortization payments are made (Plan 1.1), the total contribution rate is equal to normal cost. However, this leads to a dramatic expected deterioration of the funding ratio. Even the median funding ratio after 75 years is 0. By contrast, a shift to 100% amortization under Plan 1.2 tends to produce a slow improvement of the funding ratio, since the median funding ratio after 75 years is 1.08. The spread in the distribution of the funding ratio is still quite high, but extremely low funding ratios are highly unlikely at the end of the horizon. The flip side of the improvement in the financial position of the fund is, of course, that the total contribution rate rises. The median contribution rate after 25 years is 25%, hence 2 percentage points above the median in the baseline and 8 percentage points above the median in Plan 1.1. In the long run, however, the improved financial position of the fund allows contribution rates again to fall. After 75 years, no amortization payment is needed anymore in plan 1.2, whereas the base plan with a half of required amortization paid still requires 7% amortization. Reducing the amortization period to 10 years has a similar and, in fact, even stronger positive effect on the fund's financial position. Compared to Plan 1.2 the distribution of funding ratios after 75 years shifts even further upwards. Of course, the contribution rates in the meantime will be even higher, with a ninety-fifth percentile of 40% after 25 years, as compared to 35% in plan 1.2 and 28% in the base plan. However, at the end of the full horizon, the median contribution rate for the amortization has fallen to zero and, hence, the total contribution rate is 7 percentage points below the level under the baseline Plan 0.0. The reported numbers under Plan 1.4 are identical to those under the baseline. Funding ratios, total contributions and amortization contributions are identical, though of the total contributions now twice as much as before is paid by the participants.

The second set of alternative measures deals with changes in the indexation rate. Plan 2.1 always yields lower indexation, implying an expected improvement in the financial position of the funding over the long horizon. Although a long-term halving of the indexation rate has a substantial effect on the purchasing power of the pension benefits, the improvement in the funding ratio is not as spectacular as one might a priori expect, because the total contribution in the meantime falls below that under the baseline. While the normal cost rate remains the same, the amortization contribution rate is lower than under baseline. After 25 years, the median contribution rate is 2 percentage points lower than under the baseline. At the end of the full horizon, the median amortization contribution rate has fallen to zero and the difference with the baseline has risen to 7 percentage points.

A shift from the baseline to conditional indexation (Plan 2.2) also tends to improve the financial position of the fund. As compared to Plan 2.1 where indexation is halved, the spread in the distribution of the funding ratios becomes smaller, with the fifth percentile under Plan 2.2 at the end of the horizon located at 0.44 instead of 0.21 under Plan 2.1. The compression in the distribution of funding ratios is not surprising, because the policy is specifically aimed at stabilising the funding ratio with smaller indexation given when it is low and higher, possibly more than full, indexation given when it is high. As a result, the distribution of the total contribution rates is also more compressed. At the end of the full horizon, the median contribution rate only contains the normal contribution rate.

Our third set of measures looks at change in the pension fund's asset portfolio. Not surprisingly, a policy of investing 100% of the portfolio in stocks, Plan 3.1, raises the median funding ratio compared to the baseline, but it also raises the spread in funding ratios. After 75 years there is a non-negligible probability that the assets are down to zero. A complete shift out of stocks, Plan 3.2, has the opposite effect. Interestingly, we see that even the ninety-fifth percentile of the funding ratio after 75 years is zero under this plan. This is not surprising, as the normal cost is based on liabilities calculated with a discount rate higher than the expected portfolio return, implying a systematic shortage of resources on the side of the fund.

Overall, it can be concluded that under many of the settings studied here the pension plan is likely to be unsustainable. The factors contributing to this conclusion are the fund's initial underfunding and the long amortization period in combination with the amortization contribution rate being lower than the required rate.

8.2 The value-based ALM results

The classic ALM results discussed above provided us with a number of interesting results regarding the effectiveness of alternative policies to strengthen the financial sustainability of our pension fund. Value-based ALM is useful to compare different plans by assigning a market value of the plan to its various stakeholders. Shifting from the baseline plan to another plan we can see what is the magnitude of the shifts in plan values across the various stakeholders.

A_0 V_0^E	V_0^P
	V_0^R

Table 4: The balance sheet.

To analyze the value shifts for different stakeholders we can employ the balance sheet shown in Table 4.The left side of it reports the initial assets of the pension fund (A_0) and the present value of all tax payers contributions (V_0^E) paid over the 75 years of evaluation. On the right side we find the present value of the net benefit to participants (V_0^P), which consists of net benefits received over the evaluation horizon plus the pension rights at the end of the evaluation horizon that were accrued in exchange for the contributions paid. To evaluate the pension rights we use the PBO method, which takes into account the current service and projected wage growth. What remains is the present value of the residue of the fund (V_0^R), which is the difference between the present value of the final assets and the present value of final pension rights of the participants. Specifically, the values of the participants are computed by taking into account the projected benefits at the end of the evaluation horizon based on the contributions up to then. For example, the value that we calculate for someone who enters the fund at $t = 70$ includes 5 years of contributions and the value of the benefits to be paid beyond the evaluation horizon that emanate from these contributions. We calculate the present value of these benefits at $t = 75$ and then treat this number as a cash flow at $t = 75$ that is valued in the same way as the other cashflows over the evaluation horizon.

We evaluate the baseline contract (denoted by a tilde) and changes relative to the baseline at moment $t = 0$. Specifically, $\tilde{V}_0^P$ denotes the aggregate baseline value of all participating cohorts alive during the evaluation, $\tilde{V}_0^{Pa}$ the baseline value of the individual cohort of age a, $\tilde{V}_0^E$ the baseline value for the employer, i.e. tax payers, by $\tilde{V}_0^R$ the baseline residual value of the fund at the end of the evaluation period, all discounted back to the period 0.[8]

Reforms are zero-sum games in value terms across the funds' stakeholders:

$$\Delta V_0^P + \Delta V_0^E + \Delta V_0^R = 0.$$

[8]Alternatively, the residual value could be attributed to the tax payers. However, in the sequel we keep it separate from the tax payers' value based on the employer's contributions and employer's support ES.

where V_0 is the value of the new plan and $\Delta V_0 \equiv V_0 - \tilde{V}_0$ the change in value from the baseline. For example, if the contributions of the tax payers to the pension fund increase, this implies that the value of the residue, the value of the net benefits to the participants or both must increase. The initial assets from the balance sheet do not appear in this expression, as we start with the same initial asset value under all pension plan alternatives, hence the change in the value of the initial assets is always 0. We are also interested in the relative change ΔRV_0 in the values of the various stakeholders, computed as:

$$\Delta RV_0 = \frac{V_0 - \tilde{V}_0}{|\tilde{V}_0|} * 100\%,$$

We report the results from the value-based ALM in a number of ways. In particular, we report values for different cohorts of participants, different cohorts of tax payers and the residual value of the fund. The values for the different cohorts of participants follow directly from the cash flows associated with the pension contract. The aggregate cashflows in a year from all the tax payers are immediately available. However, calculating the values for the various cohorts of tax payers requires an assumption about the allocation of the aggregate cash flows across these cohorts. We assume that the demographic structure of the tax payer population is proportional to that of the participant population and that the cash flows assigned to the different tax payer cohorts are in proportion to shares of the various cohorts in the total population of tax payers.

While values in dollars are based on aggregates per cohort, relative changes in value are the same for the total population in a cohort and for an individual within a cohort, since the cohort population is held constant across the alternatives.

8.2.1 Baseline

Figure 1 shows for the baseline Plan 0.0 the value of the contract $\tilde{V}_0^{Pa}$ in billions of dollars for the various cohorts of fund participants and tax payers. The youngest generation is the one to be born in 50 years (age -50 at $t = 0$), whereas the oldest is 99 years old at the start of the simulation period. The figure gives an impression of size of the order of magnitude of the sizable amounts at stake for the various cohorts of stakeholders of a pension fund of about 800,000 participants. More importantly, though, the figure shows the relative distribution of the net benefits across the various participant cohorts and the burden on the cohorts of tax payers. It is useful to notice that the positive and negative areas in the figure do not sum to zero, because the residual value of the fund after 75 years is non-zero. We see that the value of the contract is positive for all participant cohorts. This is not surprising for cohorts that have already been working for a while, i.e. the cohorts older than 25, because their contributions are "sunk", while the benefits are still ahead of them. Indeed, we see that the benefits increase with the age of the participant up to the late forties, reflecting an increasing amount of sunk contributions. As of the age of the late forties, the contract value starts falling with age. For those that are already retired, i.e. of age 65 and over at $t = 0$, the amount of future benefits to be received is shrinking with age. In addition, older cohorts have been accumulating lower entitlements, ceteris paribus, because the real wage levels were lower during their work career than for younger cohorts.

For the cohorts of age 25 and younger at $t = 0$, the positive contract value is more remarkable, because in a completely actuarially-neutral system, those who have not been contributing yet would experience zero value from their pension contract: the value of their future benefits must be offset by the value of their future contributions. This value is only zero for the cohort born 50 years from now, because over the evaluation horizon this cohort will pay zero contributions and thus build up zero entitlements. For future generations that are born earlier, the value of the

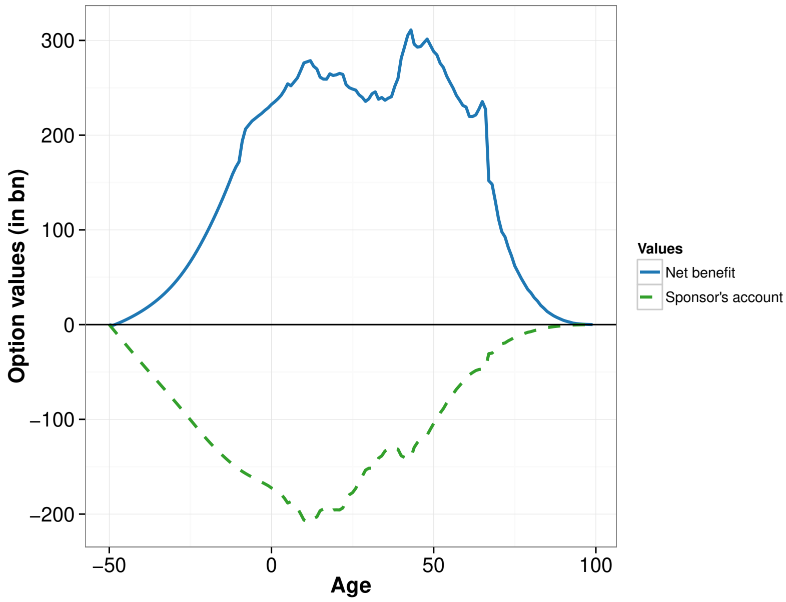

Figure 1: Net benefit and Sponsor payment option values in Plan 0.0

benefits they receive over the evaluation horizon plus the value of the accrued entitlements at the end of the evaluation period exceeds the value of the contributions made during the evaluation period. This net benefit can only be financed out of two sources: a reduction in the residual value $\tilde{V}_0^R$ of the fund or contributions from the taxpayers. From the figure we observe that the various cohorts of tax payers make substantial contributions to the fund – the value of their involvement with the fund $\tilde{V}_0^{T^a}$ is negative for all cohorts of tax payers.

8.2.2 Outcomes of the reforms

We report changes in dollar value and relative value for groups of cohorts of participants and tax payers. Negative relative values denote a deterioration for the particular stakeholders under consideration. The results are reported in Table 5 (changes in dollar values) and Table 6 (relative changes in values) for two groups of participants and two groups of tax payers. The groups of the young (superscript Y) comprise all cohorts that are younger than 25 or yet unborn at $t = 0$, while the groups of the old (superscript O) comprise all cohorts of age 25 or older at $t = 0$. The table also reports dollar and relative changes in the value of final residue at the end of the evaluation horizon. As explained above, for each plan, the sum of value changes, $\Delta V_0^{P,Y} + \Delta V_0^{P,O} + \Delta V_0^{E,Y} + \Delta V_0^{E,O} + \Delta V_0^R$ must be zero. Due to numerical inaccuracies small deviations from zero are possible.

Under Plans 1.1 – 1.3 neither contributions made by the plan participants nor the indexation rules are changed, so $\Delta V_0^{P,Y} = \Delta V_0^{P,O} = 0$. A reduction in the amortization payment (Plan 1.1) benefits current tax payers, because they see the amortization contribution rate fall. The value of the final residue falls, in line with the expected deterioration of the funding ratio over time. This also has an effect on the future cohorts of tax payers, as they have to cover the deficit with employer support payments if assets get depleted completely. An increase in the amortization payment, Plan 1.2, means higher contribution payments by the tax payers, so both young and old tax payers experience a loss of value. A similar conclusion holds for a shortening of the period over which the amortization takes place, Plan 1.3. The residue value improves, but at the cost of higher contributions by tax payers, who see the value of their stake in the

Case	Description	$\Delta V_0^{P,Y}$	$\Delta V_0^{P,O}$	$\Delta V_0^{E,Y}$	$\Delta V_0^{E,O}$	ΔV_0^{R}
Contribution						
1.1	0% amortization paid	0	0	-285	370	-66
1.2	100% amortization paid	0	0	-963	-1114	2045
1.3	amortization 10 years	0	0	-1878	-2024	3830
1.4	part. contr. rate doubled	-2579	-826	2077	1328	0
Indexation						
2.1	indexation is 0.5*CPI	-1859	-2755	2225	661	1732
2.2	conditional indexation	-2655	-4285	3090	957	2905
Portfolio composition						
3.1	100% stocks	0	0	-99	-140	221
3.2	0% stocks	0	0	-6	69	-62

Table 5: Level effects of plan changes. Changes are in bilions.

Case	Description	$\Delta RV_0^{P,Y}$	$\Delta RV_0^{P,O}$	$\Delta RV_0^{E,Y}$	$\Delta RV_0^{E,O}$	ΔRV_0^{R}
Contribution						
1.1	0% amortization paid	0%	0%	-3%	7%	-1%
1.2	100% amortization paid	0%	0%	-10%	-23%	32%
1.3	amortization 10 years	0%	0%	-20%	-41%	59%
1.4	part. contr. rate doubled	-24%	-7%	22%	27%	0%
Indexation						
2.1	indexation is 0.5*CPI	-17%	-23%	23%	13%	27%
2.2	conditional indexation	-24%	-35%	32%	19%	45%
Portfolio composition						
3.1	100% stocks	0%	0%	-1%	-3%	3%
3.2	0% stocks	0%	0%	0%	1%	-1%

Table 6: Relative effects of plan changes. Negative numbers imply a deterioration of the value for that stakeholder.

pension arrangement fall. A doubling of the contribution rate by the participants reduces the total contribution paid by the tax payers and, thus, shifts value from both groups of participants to the tax payers. The financial position of the fund is unchanged throughout and, hence, the change in the final asset value is zero.

Changes in indexation policy shift value across groups of participants. Halving indexation, Plan 2.1, lowers the benefits for the participants and, thus shifts value from the participants to the tax payers, who have to pay smaller amortization and employer support contributions. In addition, the long-run expected improvement in the funding ratio and lower accrued pension rights relative to its baseline raises residue value of the fund. Conditional indexation, Plan 2.2, has qualitatively similar, but quantitatively larger, value-shift effects across the stakeholders. This is because under Plan 2.2 no indexation is given a times when it is most valuable, and more is given when it is less valuable.

Our third set of changes concerns changes in the composition of the pension assets portfolio. In all of the previous pension plan variations the asset mix was kept constant at 50% fixed income and 50% risky assets. When the portfolio allocation is the same across plans, the total amount of risk remains unchanged, but it is shifted among stakeholders due to changes in policy, like the contribution or indexation rules. Changing the asset mix, on the other hand, changes the risk in the pension fund. Under a symmetrical contract this should not lead to value transfers, as in such contract higher volatility is rewarded with higher expected returns. However, the pension plan policy in question is not symmetrical from the tax payers' perspective. When bad returns materialize, tax payers have to cover the deficit by increased amortization and employer support payments. However, good returns do not necessarily lead to lower contribution payments as the contribution can never fall below the normal cost level. To sum up, the contribution payments are limited on the downside but unlimited on the upside. When riskier investments are made (Plan 3.1) the upwards potential of the returns goes to the residue of the pension fund, but the downwards risk is allocated to the tax payers in form of higher amortization and employer support payments. Therefore we see negative value changes for tax payers and an increase in residue value. In the case of a derisking the portfolio (Plan 3.2), the opposite effects can be seen. A much lower downside risk of portfolio returns results in a lower probability of high amortization support payments, hence the current generations of the tax payers are better off. On the other hand, a less risky asset mix means lower expected returns, and hence a negative effect on the residue value. The future cohorts of tax payers are also hurt by lower expected returns, as they have to cover the deficit when assets are depleted by means of employer support. Since neither the indexation policy nor the participant contribution rules are changed, the fund participants are unaffected by changes in portfolio composition.

Figures 8 - 10 depict the value consequences of contract changes for individual cohorts of participants and tax payers. In line with the figures reports in the above tables, we see indeed that moving from the baseline plan to Plan 1.1 with zero amortization older tax payers are better off as part of the contribution burden is shifted to the future, when younger cohorts of tax payers have to make up for the losses of the fund. Moving to 100% amortization, Plan 1.2 reduces the value for all tax payers, but ensures that the residual value of the fund is raised. Speeding up amortization, Plan 1.3 also affects all tax payer cohorts over the simulation horizon, while doubling the contribution from participants affects all participants' values negatively, but all tax payers' values positively. Not surprisingly, a shift from full inflation to compensation of only half of the CPI, i.e. a shift from Plan 0.0 to 2.1, lowers the net benefit value for all participants. Given that inflation is mostly positive, getting only half of the inflation indexed leads to lower pension benefit payments and consequently a lower value of the contract. Similarly, since indexation tends on average to be lower under conditional indexation, Plan 2.2 also affects the value for the participants in a negative way. The flip side is that all tax payer cohorts benefit from the two plan changes. Shifts in the portfolio of the pension fund are completely neutral in value terms for all participants, while we see small changes in value for the tax payers. Some of the future-born tax payers benefit from a move to a 100% equity portfolio, while the other cohorts all suffer from this shift. The opposite is the case for a shift to a 100% bonds portfolio.

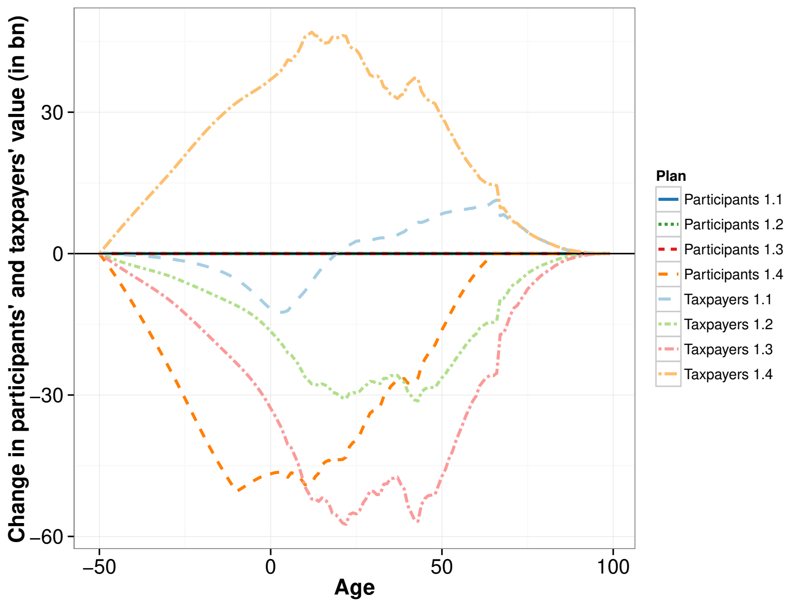

Figure 2: Change in values in the contracts in the Contribution group

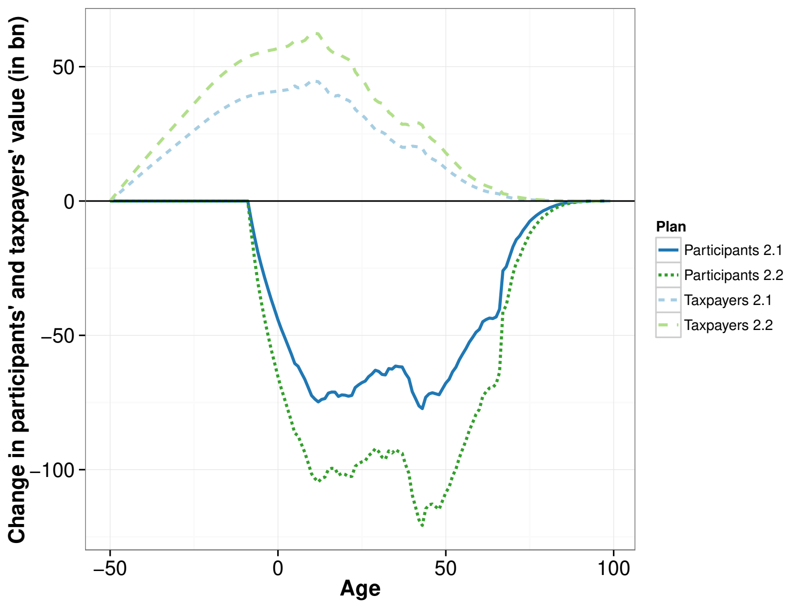

Figure 3: Change in values in the contracts in the Indexation group

9 Concluding remarks

This paper has explored the financial sustainability of a typical U.S. state DB pension fund under unchanged policies. The results confirm what is generally feared, namely that current pension policies are unlikely to be financially sustainable. Therefore, we explored a number

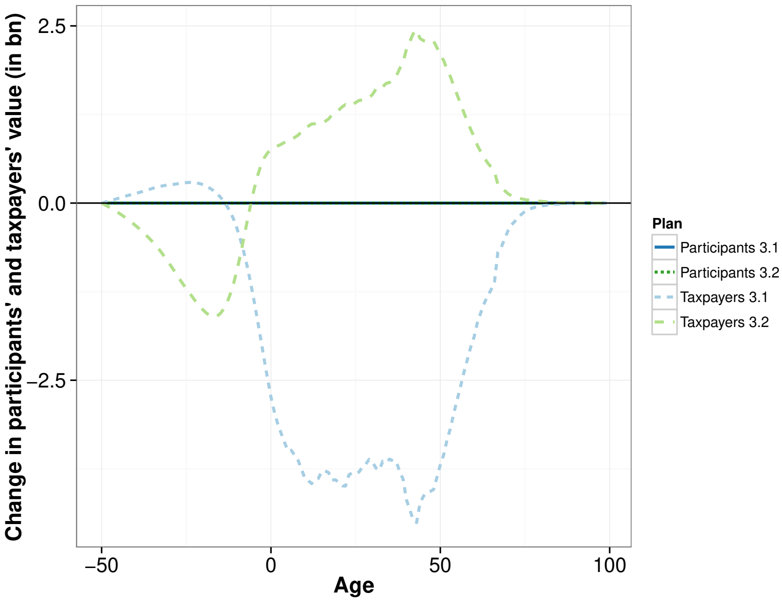

Figure 4: Change in values in the contracts in the Portfolio composition group

of alternative policies, involving changes in the investment portfolio composition of the pension fund, the structure of contributions and policies aimed at reducing benefits, in particular policies that reduce indexation. Some of these measures can indeed be quite effective at improving the fund's financial position. Indexation conditonal on the funding ratio is quite promising, because it can improve the fund's financial position, while still keeping the potential spread in the funding ratio limited. The reason is that indexation rises with the size of the funding ratio and may even become more than full for large funding ratios. We also applied a market-based valuation approach to the cash flow generated by the fund and we found that all cohorts of participants experience substantial benefits at the expense of all cohorts of tax payers. Policies that raises the contribution from participants or that reduce indexation of benefits are quite effective at shifting part of the participants' benefit back to the tax payers. We estimate the effect of halving the indexation to CPI inflation at a contract value reduction of around 20% for the tax payers, while conditional indexation lowers the participants' value by around 30%.

A somewhat general conclusion from our analysis could be that the substantial net benefit of the promises to fund participants at the cost of tax payers is an argument for fund boards and participants to initiate steps to improve their funding by raising contributions or by cutting back on benefit levels. The looming financial burden on tax payers and the prospective deterioration of public services may chase away individuals of working age from specific states. An outcome with falling spending on public goods and rising taxes merely paid to finance civil servants' retirement benefits will become politically unsustainable and might result in a state default that whipes out all claims on tax payers.

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Appendices

A Calculation actuarial assets

Define the Investment Income Amount Of Immediate Recognition ($IIAOIR$) as a target investment income based on the *expected* return $\bar{R}_t$:

$$IIAOIR_t = A_t(\bar{R}_t - 1) + (C_t - B_t)(\bar{R}_t^{1/2} - 1). \tag{13}$$

The so-called Investment Income Market Total ($IIMT$) denotes the actual realization of investment income, i.e. the difference between the market value of assets at the end of the year and the market value of assets at the beginning of the year, less the net cash inflow associated with contributions and benefits. From equation (3) we get:

$$IIMT_t = A_{t+1} - A_t - (C_t - B_t) = A_t(R_t - 1) + (C_t - B_t)(R_t^{1/2} - 1). \tag{14}$$

Finally, the Investment Income Amount For Phased In Recognition ($IIAFPIR$) is realized investment income in excess of expected investment income:

$$IIAFPIR_t = IIMT_t - IIAOIR_t.$$

Hence, $IIAFPIR_t$ is positive when the actual investment return exceeds its expected value, and vice versa. The usual smoothing procedure to calculate actuarial assets involves taking the average of the excess investment incomes over the past. We define the Total Recognized Investment Gain ($TRIG$) as the average of $IIAFPIR$ over a smoothing horizon of v years:

$$TRIG_t = \frac{1}{v}\sum_{i=0}^{v-1} IIAFPIR_{t-i}.$$

Then, the actuarial value of the assets at the beginning of year $t+1$ will be determined by raising the actuarial value of assets at the beginning of year t with the net money inflows (contribution payments minus the benefit payouts), investment income of immediate recognition and the smoothed value of the excess investment income over the past v years:

$$A_{t+1}^{act} = A_t^{act} + (C_t - B_t) + IIAOIR_t + TRIG_t. \tag{15}$$

B Pricing according to risk-neutral scenarios

This appendix provides the details of our valuation of the cash flows associated with the pension contract.

B.1 General pricing framework

Consider some derivative (e.g., the pension contract) with a payoff of Z_τ at time τ, which is a function of the path $X_1, X_2, ...X_\tau$ of the state vector. The price of the derivative at time t is then given by

$$P_t = \sum_{\tau=t+1}^{\infty} \mathrm{E}_t \left[Z_\tau \exp \sum_{s=t+1}^{\tau} m_s \right].$$

In a complete market it is possible to sell the derivative at time $t+1$ for its price P_{t+1}. Hence the following must hold:

$$P_t = \mathrm{E}_t\left[P_{t+1}\exp m_{t+1}\right], \tag{16}$$

where P_{t+1} is the total price based on the total return index where any payoff is reinvested in the same index. If we use lower-case letters to denote log-values so that

$$p_t = \log P_t,$$

knowing that X_{t+1} has a Gaussian distribution, and using the properties of the log-normal distribution, we can derive from equation (16):

$$\begin{aligned}\exp p_t &= \mathrm{E}_t\left[\exp p_{t+1}\exp m_{t+1}\right] = \mathrm{E}_t\left[\exp(p_{t+1}+m_{t+1})\right]\\ &= \exp\left(\mathrm{E}_t\left[p_{t+1}+m_{t+1}\right] + 1/2\,\mathrm{Var}_t\left[p_{t+1}+m_{t+1}\right]\right),\end{aligned}$$

Hence,

$$p_t = \mathrm{E}_t\left[p_{t+1}+m_{t+1}\right] + 1/2\,\mathrm{Var}_t\left[p_{t+1}+m_{t+1}\right]. \tag{17}$$

Note that

$$\mathrm{E}_t\left[m_{t+1}\right] = -e'X_t - \frac{1}{2}(\beta_0+\beta_1X_t)'\Sigma(\beta_0+\beta_1X_t). \tag{18}$$

$$\mathrm{Var}_t\left[m_{t+1}\right] = (\beta_0+\beta_1X_t)'\Sigma(\beta_0+\beta_1X_t). \tag{19}$$

We will now apply our pricing framework to the various assets that are relevant for us.

B.1.1 Nominal bonds

Denote by $p_t^{(n)}$ the period-t price of a zero coupon bond that matures at time $t+n$ and pays one unit of currency at maturity date. We assume that it is an affine function of the state variables:

$$p_t^{(n)} = -D_n - H_n'X_t. \tag{20}$$

The nominal yield $Y_t^{(n)}$ satisfies the following relationship:

$$\left(1+Y_t^{(n)}\right)^{-n} = P_t^n. \tag{21}$$

If we denote $\ln(1+Y_t^{(n)})$ as $y_t^{(n)}$, it follows from equations (20) and (21) that

$$\ln\left(1+Y_t^{(n)}\right) = -\frac{1}{n}\ln P_t^n,$$

$$y_t^{(n)} = -\frac{1}{n}p_t^{(n)} = \frac{1}{n}D_n + \frac{1}{n}H_n'X_t. \tag{22}$$

Applying the pricing equation (17) to zero-coupon bonds and using equation (20) we get:

$$\begin{aligned}p_t^{(n)} &= \mathrm{E}_t\left[p_{t+1}^{(n-1)} + m_{t+1}\right] + 1/2\,\mathrm{Var}_t\left[p_{t+1}^{(n-1)} + m_{t+1}\right]\\ &= -D_{n-1} - H_{n-1}'\,\mathrm{E}_t\left[X_{t+1}\right]\\ &+ \mathrm{E}_t\left[m_{t+1}\right]\\ &+ \frac{1}{2}Var_t\left[-H_{n-1}'X_{t+1}\right]\\ &+ \frac{1}{2}\,\mathrm{Var}_t\left[m_{t+1}\right]\\ &+ \mathrm{Cov}_t\left[-H_{n-1}'X_{t+1}, m_{t+1}\right].\end{aligned} \tag{23}$$

Using (18), (19) and (10) we can rewrite (23) as

$$\begin{aligned}
p_t^{(n)} &= -D_{n-1} - H'_{n-1}\left((I-\Gamma)\mu + \Gamma X_t\right) \\
&- e'X_t - \frac{1}{2}(\beta_0+\beta_1 X_t)'\Sigma(\beta_0+\beta_1 X_t) \\
&+ \frac{1}{2}\left(H'_{n-1}\Sigma H_{n-1}\right) \\
&+ \frac{1}{2}(\beta_0+\beta_1 X_t)'\Sigma(\beta_0+\beta_1 X_t) \\
&+ H'_{n-1}\Sigma(\beta_0+\beta_1 X_t),
\end{aligned}$$

where the last part follows from

$$\begin{aligned}
\text{Cov}_t\left[-H'_{n-1}X_{t+1}, m_{t+1}\right] &= \text{Cov}_t[-H'_{n-1}\left((I-\Gamma)\mu + \Gamma X_t + \varepsilon_{t+1}\right), \\
&- e'X_t - \frac{1}{2}(\beta_0+\beta_1 X_t)'\Sigma(\beta_0+\beta_1 X_t) - (\beta_0+\beta_1 X_t)'\varepsilon_{t+1}] \\
&= \text{E}_t\left[(-H'_{n-1}\varepsilon_{t+1})(-(\beta_0+\beta_1 X_t)'\varepsilon_{t+1})\right] \\
&= \text{E}_t\left[(H'_{n-1}\varepsilon_{t+1})(\varepsilon'_{t+1}(\beta_0+\beta_1 X_t))\right] \\
&= H'_{n-1}\,\text{E}_t\left[\varepsilon_{t+1}\varepsilon'_{t+1}\right](\beta_0+\beta_1 X_t) \\
&= H'_{n-1}\Sigma(\beta_0+\beta_1 X_t).
\end{aligned}$$

We can further rewrite $p_t^{(n)}$ as

$$\begin{aligned}
p_t^{(n)} &= -D_{n-1} - H'_{n-1}(I-\Gamma)\mu - H'_{n-1}\Gamma X_t \\
&- e'X_t \\
&+ \frac{1}{2}H'_{n-1}\Sigma H_{n-1} \\
&+ H'_{n-1}\Sigma\beta_0 + H'_{n-1}\Sigma\beta_1 X_t \\
&= -D_{n-1} - H'_{n-1}(I-\Gamma)\mu + \frac{1}{2}H'_{n-1}\Sigma H_{n-1} + H'_{n-1}\Sigma\beta_0 \\
&- (e' + H'_{n-1}\Gamma - H'_{n-1}\Sigma\beta_1)X_t.
\end{aligned}$$

The last equation is already of the affine structure as in equation (20) with parameters

$$\begin{aligned}
D_n = D_{n-1} + H'_{n-1}(I-\Gamma)\mu - \frac{1}{2}H'_{n-1}\Sigma H_{n-1} - H'_{n-1}\Sigma\beta_0, \\
H_n = e + (\Gamma - \Sigma\beta_1)'H_{n-1}.
\end{aligned} \tag{24}$$

For $n = 0$ we have that:

$$p_t^{(0)} = \ln P_t^{(0)} = \ln 1 = 0,$$

which is given by equation (20) with parameters

$$\begin{aligned}
D_0 &= 0, \\
H_0 &= 0.
\end{aligned} \tag{25}$$

Using (25) in (24) we get

$$\begin{aligned}
D_1 &= 0, \\
H_1 &= e.
\end{aligned} \tag{26}$$

This implies

$$p_t^{(1)} = -e'X_t = -y_t^{(1)}, \tag{27}$$

because e indicates the position of the short rate in the state vector. The deflator is calibrated on the short rate so that this constraint is satisfied.

B.1.2 Investable assets

The excess return on investable assets other than the nominal bond is defined as follows and can be rearranged using (27)

$$r_{t+1} - y_t^{(1)} = \ln\left(\frac{P_{t+1}}{P_t}\right) - y_t^{(1)} = p_{t+1} - p_t + p_t^{(1)}, \tag{28}$$

$$p_{t+1} = r_{t+1} - y_t^{(1)} + p_t - p_t^{(1)}. \tag{29}$$

Using (25) in (23) we get

$$p_t^{(1)} = \mathrm{E}_t\left[m_{t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}\right]. \tag{30}$$

From (17), (28) and (30) it follows that

$$\begin{aligned}
\mathrm{E}_t\left[r_{t+1} - y_t^{(1)}\right] &= \mathrm{E}_t[p_{t+1}] - \left(\mathrm{E}_t\left[p_{t+1} + m_{t+1}\right] + 1/2\,\mathrm{Var}_t\left[p_{t+1} + m_{t+1}\right]\right)\\
&\quad + \mathrm{E}_t\left[m_{t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}\right]\\
&= \mathrm{E}_t[p_{t+1}] - \left(\mathrm{E}_t\left[p_{t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[p_{t+1}\right] + \mathrm{Cov}_t\left[p_{t+1}, m_{t+1}\right]\right)\\
&= -\frac{1}{2}\mathrm{Var}_t\left[p_{t+1}\right] - \mathrm{Cov}_t\left[p_{t+1}, m_{t+1}\right].
\end{aligned}$$

Using (29) and the fact that p_t and $p_t^{(1)}$ are known at time t, so that

$$\mathrm{Var}_t[p_{t+1}] = \mathrm{Var}_t[r_{t+1} - y_t^{(1)} + p_t - p_t^{(1)}] = \mathrm{Var}_t[r_{t+1} - y_t^{(1)}], \tag{31}$$

we get

$$\mathrm{E}_t\left[r_{t+1} - y_t^{(1)}\right] = -\frac{1}{2}\mathrm{Var}_t\left[r_{t+1} - y_t^{(1)}\right] - \mathrm{Cov}_t\left[r_{t+1} - y_t^{(1)}, m_{t+1}\right]. \tag{32}$$

The excess return on assets other than the nominal bond can also be written as

$$r_{t+1} - y_t^{(1)} = e_i' X_{t+1} = e_i'\left((\mathrm{I} - \Gamma)\mu + \Gamma X_t + \varepsilon_{t+1}\right),$$

where e_i is a unit vector representing the location i of investable assets in the state vector. It follows from equation (32) and (12) that

$$\begin{aligned}
e_i'\left((\mathrm{I} - \Gamma)\mu + \Gamma x_t\right) &= -\frac{1}{2}\mathrm{Var}_t\left[e_i'\varepsilon_{t+1}\right] - \mathrm{Cov}_t\left[e_i'\varepsilon_{t+1}, -(\beta_0 + \beta_1 x_t)'\varepsilon_{t+1}\right]\\
&= -(\frac{1}{2}e_i'\Sigma e_i) - \mathrm{E}_t\left[(e_i'\varepsilon_{t+1} - 0)(-(\beta_0 + \beta_1 x_t)'\varepsilon_{t+1} - 0)\right]\\
&= -(\frac{1}{2}e_i'\Sigma e_i) - \mathrm{E}_t\left[-e_i'\varepsilon_{t+1}\varepsilon_{t+1}'(\beta_0 + \beta_1 x_t)\right]\\
&= -(\frac{1}{2}e_i'\Sigma e_i) + e_i'\Sigma(\beta_0 + \beta_1 x_t),
\end{aligned}$$

$$e_i'\left((\mathrm{I} - \Gamma)\mu + \Gamma X_t + \frac{1}{2}\Sigma e_i - \Sigma\beta_0 - \Sigma\beta_1 X_t\right) = 0.$$

$$e_i'\left(\left((\mathrm{I} - \Gamma)\mu - \Sigma\beta_0 + \frac{1}{2}\Sigma e_i\right) + \left((\Gamma - \Sigma\beta_1)X_t\right)\right) = 0.$$

This is satisfied for all values of X_t if:

$$\begin{aligned}
\left((\mathrm{I} - \Gamma)\mu - \Sigma\beta_0\right)_i + \frac{1}{2}\Sigma_{ii} &= 0,\\
\left(\Gamma - \Sigma\beta_1\right)_{i*} &= 0,
\end{aligned} \tag{33}$$

where i represents the i-th element of a vector, ii represents the element in i-th row and i-th column, while $i*$ stands for i-th row of a matrix. Hence, the first equation yield one condition, whereas the second equation yields as many conditions as there are state variables. It follows that the conditions in (33) must be satisfied for investable assets. These conditions determine the parameters of the parameters β_0 and β_1 of the discount factor.

B.2 Parameter optimization

The empirical values $\hat{\alpha}$ and $\hat{\Gamma}$ for α and Γ in equation (7) are determined by regressing the state variables on their lagged values, using the multivariate OLS. Then, $\hat{\Sigma}$ is the variance of the residuals, i.e. the difference between the observed and the estimated values. We use these values of $\hat{\alpha}$, $\hat{\Gamma}$ and $\hat{\Sigma}$ in the sequel.

Given the values $\hat{\Gamma}$ and $\hat{\alpha}$, the estimate $\hat{\mu}$ of the average μ can be obtained from (8):

$$\hat{\mu} = (\mathrm{I} - \hat{\Gamma})' \hat{\alpha}. \tag{34}$$

The empirical values $\hat{D}_n$ and $\hat{H}_n$ for D_n and H_n in equation (22) are obtained by regressing the historical time series of zero-coupon yields on the time series of the state variables using multivariate OLS.

The optimal $\tilde{\beta}_0$ and $\tilde{\beta}_1$ are obtained by an optimization procedure with several restrictions imposed. The model parameters $\tilde{D}_n$ and $\tilde{H}_n$ are implied by these values of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ through the recursion in equation (24).

B.2.1 Optimization for β_1

We obtain $\tilde{\beta}_1$ by minimizing the criterion function given by the euclidean norm derived from the following restriction:

Constraint 1: we impose that the exposures of the nominal yields towards the state variables for maturities $n = [15, 30]$ as predicted by the model coincide to those obtained through the historical data ($\tilde{H}_n = \hat{H}_n$).

We want to satisfy the constraint by finding an optimal $\Sigma\beta_1$ value that minimizes the criterion function of the following form:

$$\sum_{n \in \tau} \sum_{sv} \left(e + (\hat{\Gamma} - \Sigma\beta_1)' H^{sv}_{n-1} - \hat{H}^{sv}_{n} \right)^2, \tag{35}$$

where $\tau = \{15, 30\}$ H^{sv}_n indicates the interest rates exposure to the state variable sv for maturity n.

The optimization is done by varying the rows of matrix $\Sigma\beta_1$ that do not correspond to investable assets. The rows of matrix $\Sigma\beta_1$ corresponding to investables are set to be equal to the respective rows of $\hat{\Gamma}$ (from (33)).

The matrix $\Sigma\beta_1$ with some rows variable is then used in (24) to calculate the exposures of nominal yields to state variables (H_n), and these are compared to values of regression parameters ($\hat{H}_n$), the differences being minimized.

When the optimal values $(\tilde{\Sigma\beta_1})$ of matrix $\Sigma\beta_1$ are found, $\hat{\Sigma}$ is used to extract $\tilde{\beta}_1$:

$$\tilde{\beta}_1 = \hat{\Sigma}^{-1}(\tilde{\Sigma\beta_1}) \tag{36}$$

The optimal values $\tilde{\beta}_1$ and $\tilde{H}_n$ implied by it via (24) are used in the remaining optimization procedure and further in the model.

B.2.2 Optimization for β_0

We obtain $\tilde{\beta}_0$ by minimizing the criterion function given by the euclidean norm derived from the following restrictions:

Constraint 1. $\tilde{D}_n$ must be equal to $\check{D}_n$ implied by the current term structure and $\tilde{H}_n$ for maturities $\{15, 30\}$. Using $\tilde{H}_n$ and the current observation of the state variables, we calculate the current yield curve, as predicted by the model without the intercept. Then the intercept $\check{D}_n$ is calculated to match the current yield curve by setting it equal to the difference between the observed current yield curve and the one predicted by the model without a constant. This is done for maturities $n = \{15, 30\}$ quarters, following recursion in (24).

Constraint 2. For each investable asset, $\hat{\alpha}$ obtained from the regression must satisfy the first restriction of (33). From (33) and (8) we get that $\left(\alpha - \hat{\Sigma}\beta_0\right)_i + \frac{1}{2}\hat{\Sigma}_{ii} = 0$.

We want to satisfy these constraints by finding an optimal β_0 value that minimizes the criterion function of the following form:

$$\sum_{n\in\tau}\left(\left(D_{n-1} + \tilde{H}'_{n-1}(I-\hat{\Gamma})\hat{\mu} - \frac{1}{2}\tilde{H}'_{n-1}\hat{\Sigma}\tilde{H}_{n-1} - \tilde{H}'_{n-1}\hat{\Sigma}\beta_0\right) - \check{D}_n\right)^2 + \sum_i\left(\left(\hat{\Sigma}\beta_0\right)_i - \frac{1}{2}\hat{\Sigma}_{ii} - \hat{\alpha}_i\right)^2, \tag{37}$$

where $N = \{15, 30\}$.

The optimal value $\tilde{\beta}_0$, and $\tilde{D}_n$ implied by it, are used further in the model.

B.3 Fixed-income returns

The model generates scenarios for the term structure of interest rates. The fixed-income portfolio returns have to be extracted from this information. A unit coupon bearing bond priced at par satisfies the equation

$$1 = cp\sum_{t=1}^{\tau}\left(R_0^{(4t)}\right)^{-4s} + 1\left(R_0^{(4\tau)}\right)^{-4\tau},$$

where cp is the annual coupon payment and τ is the bond maturity in years. The coupon of a par bond with τ years maturity can then be determined in the following way:

$$cp = \frac{1 - \left(R_0^{(4\tau)}\right)^{-4\tau}}{\sum_{t=1}^{\tau}\left(R_0^{(4t)}\right)^{-4t}}.$$

When the coupon is determined, the cashflows of the bond are known. Hence, by discounting them with the interest rate term structure at the beginning of the period and the term structure at the end of the period the return can be calculated by subtracting the former value from the latter value. The interest rate term structure follows from the affine structure model. The bond return is thus obtained as follows:

$$\left(cp\sum_{t=1}^{\tau}\left(R_1^{(4t-1)}\right)^{-(4t-1)}+\left(R_1^{(4\tau-1)}\right)^{-(4\tau-1)}\right)-\left(cp\sum_{t=1}^{\tau}\left(R_0^{(4t)}\right)^{-4t}+\left(R_0^{(4\tau)}\right)^{-4\tau}\right)$$

For the fixed-income portfolio we assume a portfolio consisting of constant 10 years maturity bonds that are priced at par. Constant maturity implies that the portfolio is rebalanced at the beginning of every time period.

C Risk-neutral sampling

The price of a derivative paying a cashflow Z (which is a function of the path $X_1, X_2, ...X_\tau$ of the state vector) at time τ is

$$P_0 = \mathrm{E}_0\left[Z_\tau \exp\sum_{t=1}^{\tau} m_t\right].$$

Using (12) this equation can be rewriten as

$$\begin{aligned} P_0 = \int & Z(\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau)\exp\left[-\sum_{t=1}^{\tau} e' X_{t-1}\right] \\ & \exp\left[-\sum_{t=1}^{\tau}\left(\frac{1}{2}(\beta_0+\beta_1 X_{t-1})'\Sigma(\beta_0+\beta_1 X_{t-1})+(\beta_0+\beta_1 X_{t-1})'\varepsilon_t\right)\right] \\ & \frac{1}{(2\pi)^{K\tau/2}|\Sigma|^{\tau/2}}\exp\left[-\sum_{t=1}^{\tau}\frac{1}{2}\varepsilon_t'\Sigma^{-1}\varepsilon_t\right] d\varepsilon_1 d\varepsilon_2 ... d\varepsilon_\tau. \end{aligned}$$

Hence,

$$\begin{aligned} P_0 = \int & Z(\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau)\exp\left[-\sum_{t=1}^{\tau} y_{t-1}^{(1)}\right]\frac{1}{(2\pi)^{K\tau/2}|\Sigma|^{\tau/2}} \\ & \exp\left[-\sum_{t=1}^{\tau}\left(\frac{1}{2}\left(\varepsilon_t+\Sigma(\beta_0+\beta_1 X_{t-1})\right)'\Sigma^{-1}\left(\varepsilon_t+\Sigma(\beta_0+\beta_1 X_{t-1})\right)\right)\right] d\varepsilon_1 d\varepsilon_2 ... d\varepsilon_\tau. \end{aligned}$$

This integral can be evaluated numerically in a Monte Carlo simulation by drawing a number of time series $\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau\}$ from the multivariate normal probability distribution

$$f = \frac{1}{(2\pi)^{K\tau/2}|\Sigma|^{\tau/2}}\exp\left[-\sum_{t=1}^{\tau}\left(\frac{1}{2}\left(\varepsilon_t+\Sigma(\beta_0+\beta_1 X_{t-1})\right)'^{-1}\left(\varepsilon_t+\Sigma(\beta_0+\beta_1 X_{t-1})\right)\right)\right], \quad (38)$$

calculating for each time series the derivative payoff discounted with a risk-free rate

$$Z(\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau)\exp\left[-\sum_{t=1}^{\tau} y_{t-1}^{(1)}\right]$$

and taking a simple average over the drawings. Hence, we have transformed the original problem in which we draw from the multivariate normal density function of ε_i and discount cash flows at the stochastic discount factor into a problem in which we draw from from the multivariate normal density function with mean $-\Sigma(\beta_0 + \beta_1 X_{i-1})$ and the same original variance-covariance matrix Σ, but discount cash flows at the risk-free rate.

The marginal distribution of $\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau$ for $t \leq \tau$ can be obtained by integrating over the remaining variables. Now, consider $t = \tau - 1$. For fixed values of $\varepsilon_1, \varepsilon_2, ..., \varepsilon_{\tau-1}$, variable ε_τ has a normal distribution, so it can be integrated over analytically:

$$f(\varepsilon_1, \varepsilon_2, ..., \varepsilon_{\tau-1}) = \frac{1}{(2\pi)^{(\tau-1)K/2}|\Sigma|^{(\tau-1)/2}} \exp\left[-\sum_{t=1}^{\tau-1}\left(\frac{1}{2}\left(\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})\right)' \Sigma^{-1} \left(\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})\right)\right)\right]. \tag{39}$$

By repeating this procedure for $\varepsilon_{\tau-1}, \varepsilon_{\tau-2}, ..., \varepsilon_1$ we find that

$$f(\varepsilon_1, \varepsilon_2, ..., \varepsilon_t) = \frac{1}{(2\pi)^{Kt/2}|\Sigma|^{t/2}} \exp\left[-\sum_{s=1}^{t}\left(\frac{1}{2}\left(\varepsilon_s + \Sigma(\beta_0 + \beta_1 X_{s-1})\right)' \Sigma^{-1} \left(\varepsilon_s + \Sigma(\beta_0 + \beta_1 X_{s-1})\right)\right)\right]. \tag{40}$$

The conditional density $f(\varepsilon_t|\varepsilon_{t-1}, ..., \varepsilon_1)$ is obtained using the Bayes' formula

$$f(\varepsilon_t|\varepsilon_{t-1}, ..., \varepsilon_1) = \frac{f(\varepsilon_t, ..., \varepsilon_1)}{f(\varepsilon_{t-1}, ..., \varepsilon_1)}.$$

Using (39) and (40) we get

$$f(\varepsilon_t|\varepsilon_{t-1}, ..., \varepsilon_1) = \frac{1}{(2\pi)^{K/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\left(\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})\right)' \Sigma^{-1} \left(\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})\right)\right],$$

which is normally distributed as:

$$\varepsilon_t|\varepsilon_{t-1}, ..., \varepsilon_1 \sim \mathbb{N}(-\Sigma(\beta_0 + \beta_1 x_{t-1}), \Sigma).$$

We can draw $\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau$ from the distribution $f(\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau)$ by first drawing ε_1 from its marginal distribution, then drawing ε_2 from the conditional distribution $f(\varepsilon_2|\varepsilon_1)$, and so on. Scenarios are produced in pairs using antithetic variables so as to increase the efficiency of the simulation.

D Generating scenarios

The economic scenarios are generated using the parameters $\hat{\alpha}$, $\hat{\Gamma}$, $\hat{\Sigma}$, $\tilde{\beta}_0$,$\tilde{\beta}_1$, $\tilde{D}_n$ and $\tilde{H}_n$.

First, the development of the state variables is modeled for a chosen number of years in the future, by using (7). The initial value of the state vector is set at the average μ. After the state vectors have been simulated, the development of the term structure of interest rate yields can be simulated by using (22).

The same procedure is followed for both the real-world and risk-neutral scenarios. The only difference lies in the dynamics of shock ε. For the real-world simulation the error terms are drawn from a normal distribution $\varepsilon_{t+1} \sim \mathbb{N}(0, \Sigma)$ (from (9)). Under the risk-neutral scenarios, the noise term is normally distributed with $\varepsilon_{t+1} \sim \mathbb{N}(-\Sigma(\beta_0 + \beta_1 X_t), \Sigma)$. Hence, the transformation from a real-world scenario to risk-neutral one is done by merely changing the mean of the distribution.