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The Equity Risk Premium and Pension Ambition

The Effect of Parameter Uncertainty

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The equity risk premium and pension ambition: the effect of parameter uncertainty

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Abstract

We model uncertainty of financial parameters and examine its impact on the replacement rate in a DC pension contract. To this end, we develop a novel Bayesian framework that reveals substantial reduction in the lower percentiles for the replacement rate at retirement. We identify that the key factor driving our results is the uncertainty of the equity risk premium. Our model shows that a time-varying contribution scheme based on observed interest rates and previous equity return can partially compensate for the effect of parameter uncertainty.

Keywords: Pension contracts, model uncertainty, equity risk premium
JEL Classifications: D91, H55

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1 Introduction

A central theme of pension contracts is to accumulate sufficient wealth for participants to realize their pension ambition. In a defined contribution (DC) plan, the risk of attaining a lower replacement rate at retirement depends ex ante on the risk profile of the participant. A high risk profile will lead to higher expected returns, yet will increase the uncertainty about future replacement rates at retirement. In both public and private US pension plans, more than three quarters of pension wealth is invested in equity markets (Munnell and Soto, 2007). Such investment strategies will substantially increase uncertainty about future replacement rates. However, all existing models to quantify uncertainty in replacement rates are based on the assumption that financial parameters such as the equity risk premium and the volatility in equity returns are known a priori.

In this paper, we extend the analysis of uncertainty by introducing an additional source of risk, namely uncertainty of financial parameters. Since financial parameters are unknown and are typically estimated with large errors, participants have to form beliefs about ex ante projections for the financial market. Recent literature has shown that different beliefs about future projections can substantially affect the optimal investment strategy of the portfolio, because equity returns do not necessarily have a decreasing volatility over the investment horizon (Pastor and Stambaugh, 2012; Hoevenaars, Molenaar, Schotman, and Steenkamp, 2013). Regardless of the investment strategy, ignoring parameter uncertainty will lead to an incorrect risk analysis of future replacement rates. As a result, participants may invest in portfolios that do not correspond with their risk perception.

This paper highlights the complex interaction between real pension ambitions and minimum required contribution levels in a financial market with parameter uncertainty. To set a real ambition in our pension contract, the participant aims to acquire a real variable annuity at retirement with a desired replacement rate. Based on the risk preference of the participant, we establish an investment strategy to achieve this pension ambition. This approach is similar to the ambition of most DC plans in which participants can purchase an annuity at retirement.[1] Since DC contracts offer no guarantee for the desired replacement rate, sufficient capital accumulation to achieve the participant's ambition relies only on his pension contributions and the performance of his investment portfolio. Ignoring parameter uncertainty may introduce overly optimistic beliefs of future portfolio returns. As a result, participants will underestimate the uncertainty in replacement rates and will set lower contribution rates.

To assess parameter uncertainty, we focus on two questions. First, we analyze the impact of parameter uncertainty on the replacement rates when the participant uses a fixed contribution scheme based on historical portfolio performance. We find that the major factor of parameter uncertainty driving the risk of the replacement rates in our pension contract is the equity risk premium. Although parameter uncertainty is an important additional source of risk for the participant, economic uncertainty causes the largest fluctuations in the replacement rates at retirement. Second, we verify whether a time-varying contribution scheme based on a combination of the term structure of interest rates and the historical equity risk premium estimated over a large sample period, enables the participant to respond to parameter uncertainty. We

[1] An example of a DC contract with a pension ambition in the US is a target benefit plan. In these plans, contributions are set such that projected pension entitlements can be achieved, however the actual level of the participant's benefit depends on the accrued pension wealth at retirement.

find that if parameter uncertainty observed in our sample period corresponds to future parameter uncertainty, the time-varying contribution scheme can partially compensate the participant for parameter uncertainty. However, when structural shifts occur in the financial parameters, the risk of replacement rates at retirement substantially increases in this contribution scheme.

Predictions of economic conditions to determine pension contributions rely on parameter and model assumptions. Ignoring parameter uncertainty in these projections will underestimate the risk involved with future pension entitlements. Estimates for the equity risk premium can be substantially different between various sample periods, suggesting large parameter uncertainty about the ex ante equity risk premium (Jagannathan, McGrattan, and Scherbina, 2001). We estimate the historical equity risk premium to be about 5.0% using an extensive time period of 1952 and 2012, yet uncertainty about this estimate is rather large with a standard error of 2.0%. When this historical equity risk premium is used to set contribution levels and uncertainty of the estimate is ignored, the participant will underestimate his risk of achieving a lower replacement rate at retirement. We show that the equity risk premium is the major factor for parameter uncertainty in our pension contract.

To evaluate the effect of ignoring parameter uncertainty, we compare our pension contract in a setting with parameter uncertainty and without. We set the pension ambition of a participant to acquire a real variable annuity at retirement with a replacement rate of 70% relative to his end wage. We motivate this ambition by the findings of Munnell and Soto (2007) who report median replacement rates in the US for DB pension plans ranging from 76% for workers covered by social security and 81% for workers without social security. The participant uses a fixed pension contribution scheme based on historical portfolio performance, which requires him to yearly contribute 6.9% of his wage. While this contribution scheme is naive as he ignores both parameter uncertainty and economic shocks, it allows us to examine the effect of parameter uncertainty on the replacement rates. On average the participant achieves his ambition of 70%, however this strategy introduces large risk even without parameter uncertainty. With 2.5% probability, the participant will obtain a replacement rate of 35.3%. However, if the participant takes into account parameter uncertainty of the equity risk premium, his 2.5% percentile of replacement rate's distribution is far lower, namely a replacement rate of 26.6%. As a result, his risk perception of the replacement rate at retirement is substantially overestimated when parameter uncertainty about the equity risk premium is ignored.

Introducing parameter uncertainty to other financial parameters does not substantially affect the risk of the participant. The 2.5% percentile of replacement rate at retirement drops to 26.5% when parameter uncertainty is assumed for all financial parameters. This result shows that uncertainty of the equity risk premium is the key factor for parameter uncertainty in DC pension contracts. Therefore, ignoring uncertainty regarding the equity risk premium will lead to underestimation of the replacement rate's risk at retirement. As a consequence, the participant may set too low pension contributions due to his distorted risk perception.

Next, we examine whether a time-varying contribution scheme can compensate for the effects of parameter uncertainty. The intuition behind this contribution rate is to spread out differences between the present value of the price of the real variable annuity and current accumulated pension wealth over the remaining future expected wage of the individual. To set the required contribution level, we assume the participant sets the equity risk premium to our estimated historical equity risk premium. Although the estimate might be too optimistic or pessimistic about future equity return, this contribution scheme allows the participant to respond to economic shocks or parameter uncertainty. We find that the time-varying contribution scheme can partially compensate for the effects of uncertainty. Ignoring parameter uncertainty in this strategy results in a lower bound for the replacement rate of 45.2% at a 2.5% percentile. Allowing for parameter uncertainty of all financial parameters, the risk of the replacement rate at a 2.5% percentile drops to 42.1%. Incorporating parameter uncertainty causes a large shift in the risk perception of the participant. If he would like to set his risk level at a 2.5% percentile of the replacement rate distribution, incorporating parameter uncertainty requires him to shift to a 1.3% percentile when the distribution ignores parameter uncertainty. To compensate for parameter uncertainty, the pension contribution rate needs to increase relatively to the case without parameter uncertainty at the start of the accumulation phase. For example, the contribution rate at age 25 increases from 5.8% to 6.5%, whereas at age 60 the contribution decreases from 11.6% to 10.7%. Due to the front loaded contribution, the participant requires less contribution at older ages compared to the contribution rate that ignores parameter uncertainty. While the time-varying contribution reacts to parameter uncertainty, it can only partially compensate for the replacement rate risk as it still affects the risk perception of the participant.

One of the factors explaining the large impact of parameter uncertainty is the risk profile of the participant. The participant in our benchmark contract has a high risk profile with a fixed investment strategy of 60% equity and 40% bonds, which resembles the average US pension funds investment strategy.[2] To investigate the effects of a lower risk profile, we set the portfolio to 30% equity and the remaining invested in bonds.[3] While the absolute size of the effect of parameter uncertainty is smaller due to a lower exposure to the equity premium, parameter uncertainty causes a similar relative shift in risk perception as in the benchmark case. The 2.5% percentile of the replacement rate's distribution at retirement drops from 51.0% to 48.4%, if parameter uncertainty is introduced. As a result, the risk perception of the participant shifts from a 2.5% percentile to a 1.4% percentile of the replacement rate's distribution that ignores parameter uncertainty. Hence, lowering the equity exposure in the portfolio leads to a similar shift in the participant's risk perception about the replacement rate at retirement.

A limiting factor of the time-varying contribution scheme's ability to compensate for parameter uncertainty is the occurrence of a permanent shock in our financial parameters. In that case, the historical equity risk premium might not be a good estimate for ex ante equity performance. Bansal and Lundblad (2002) report that the ex ante equity risk premium has globally decreased over time, so that the historical equity risk premium might therefore be an optimistic ex ante estimate. To account for the fact that economic regimes may shift our financial parameters, we investigate this effect by estimating a prior that is benchmarked on the period between 2000 and 2012. This period is not only distinct in that it features the financial crisis in 2008, it also includes the aftermath of the dot-com bubble in which partial recovery for the equity market and interest rates was observed. To incorporate the effect of this period in our predictions, we use a Normal-Diffuse prior calibrated to this period, so that our model adds more

[2]An alternative to a fixed risk profile is to decrease the equity exposure over time. Using a linear decreasing exposure to equity during the accumulation phase, starting at age 25 with 90% equity and 10% 5-year maturity bonds and ending with 20% equity exposure at age 65 and the remainder in bonds, results in a similar impact of parameter uncertainty on the replacement rates for participants.

[3]This conservative approach is, for example, taken in the Netherlands, where the average invesment strategy of pension funds is a mix of 32% equity and 68% bonds portfolio. We obtain the Dutch and US percentages from the report of Towers Watson (2013) by excluding other investments.

value to observations after the year 2000. As a consequence, both our estimates and parameter uncertainty are substantially affected by that subsample period. Our Bayesian model reveals that the equity risk premium is negatively influenced by about 57 basis points, lowering the estimate from 5.0% to 4.5%. The inflation rate increases by 10 basis points to about 3.6%. The uncertainty regarding our financial parameters remains large, and only slightly increases if we incorporate our prior view. Consequently, the economic projections of the financial market are substantially affected by our prior view. However, the participant cannot observe this shift and continues to use the historical equity risk premium in his time-varying contribution scheme.

Allowing for an unobserved shift in our financial parameters limits the ability of the time-varying contribution scheme to mitigate the effects of parameter uncertainty. Our previous results indicate that the 2.5% percentile of the replacement rate distribution at retirement is 42.1% when parameter uncertainty is incorporated with an uninformed prior. However, if we apply our prior view to our model, the 2.5% percentile of the replacement rate's distrubtion drops substantially to 38.0%. This replacement rate corresponds to a 1.2% percentile in the distribution that ignores the shift in financial parameters. Therefore, the shift in financial parameters has a large impact on the risk perception of the participant. Since the participant is uninformed about the regime shift in financial parameters, the use of the historical equity risk premium to set his pension contributions is far too optimistic. He anticipates high future equity returns, so that for younger ages the contribution level is similar as in our model without a shift in financial parameters. After age 45, however, the time-varying contribution scheme responds to changes in the economic regime by increasing the contribution level since the accumulated pension wealth is not sufficient to reach the participant's ambition. For example, at age 45 and 55 the contributions increase by 7% and 17% respectively, due to the lower portfolio returns. This result shows that prudent estimation of the equity risk premium is required to absorb shocks using a time-varying contribution scheme, especially for younger age cohorts. Ignoring shifts in the financial parameters may, therefore, negatively affect the steering capacity of a time-varying contribution scheme.

Our paper contributes to the literature in three ways. First, we address the impact of parameter uncertainty on replacement rates in a DC pension setting with real variable annuities. It is widely observed that pension funds are switching from a DB structure to a DC contract without guarantees of a sponsor. Since insufficient capital at retirement will directly lower replacement rates for participants, risk analysis of replacement rate using future projections will receive considerably more attention. We show that parameter uncertainty can substantially affect these future projections and can lead to underestimation of replacement rate risk at retirement. Second, to implement parameter uncertainty, we extend the Minimum Chi-squared approach of Hamilton and Wu (2012) to the estimation of our financial market. This approach allows us to straightforwardly implement our no-arbitrage financial market which is a discrete time adaptation of the models used in Brennan and Xia (2002a), and Campbell and Viceira (2001a) and simplifies the estimation of the marginal posterior densities.

In terms of the broader pension literature, our paper builds on the insights of the life cycle literature (see e.g., Cocco, Gomes, and Maenhout (2005), and Viceira (2001)) by considering parameter uncertainty. Our results indicate that the key source of parameter uncertainty for the replacement rate risk at retirement is the estimate of the equity risk premium. Since the literature has shown that high equity exposure at young age is optimal from a portfolio optimization perspective, the participant will be affected by parameter uncertainty of the equity risk premium. Life cycle models ignoring parameter uncertainty may therefore underestimate the risk of replacement rates at retirement and set inadequate contribution levels to achieve the participant's pension ambition.

The remainder of this paper is organized as follows. Section 2 relates our approach to regulatory issues for US pension plans. Section 3 introduces our financial market, pension contract, estimation methodology and parameter uncertainty. Section 4 discusses the results of the pension contract if parameter uncertainty is ignored. Section 5 shows the impact of parameter uncertainty on the replacement rate and discusses whether a time-varying contribution scheme can compensate for this effect. Section 6 shows that shifts in the economic regime limits the ability of the time-varying contribution scheme to compensate for parameter uncertainty in terms of replacement rate risk. Our conclusions follow in Section 7.

2 Pension ambition and contribution

In this section, we discuss the intuition of our methodology to determine the required contribution rate for our DC pension contract. In particular, we explain how uncertainty affects the contribution rate to achieve the pension ambition and relate this to choices in US pension regulation.

To define the pension ambition in pension plans, the rights of the participant at retirement need to be analyzed. In our DC pension contract, we set the pension ambition of a participant to purchasing a real variable annuity at retirement with his desired replacement. This setting is similar to US target benefit plans which sets an ambition for the participant to accrue pension wealth, although the actual benefit of the participant depends on the accumulated pension wealth in their account at retirement.

An important factor in setting contribution rates is the equity risk premium. The life cycle literature has shown that the equity risk premium is an important driver of wealth accumulation (see e.g., Cocco et al. (2005) and Viceira (2001)). Since most pension plans invest in equity to benefit from the higher expected return, the discount rate needs to account for equity return (Cochrane, 2011). When the discount rate ignores the higher expected equity returns, pension contributions will be larger than required based on the expected portfolio returns. Hence, the amount of pension wealth will overshoot the pension ambition in expectation. In order to account for equity returns, we raise the discount rate with the percentage invested in equity times the equity risk premium (see e.g., Nijman and Werker (2012)). When we allocate the required pension contribution based on this discount rate to a self-replicating portfolio, our portfolio will in expectation achieve the pension ambition at retirement based on the no-arbitrage argument.

Economic uncertainty of equity returns, however, can still substantially affect the replacement rate at retirement. When the equity risk premium is known, the previous approach allows us in expectation to accrue sufficient pension wealth to achieve the desired replacement rate. Only in case the participant invests in real bonds with a maturity corresponding to the remaining years before retirement, uncertainty about replacement rates can be eliminated. Since such an investment strategy requires high contribution levels, the participant may want to include equity in his portfolio to lower the contribution rate according to his risk averseness. An additional source of risk occurs if the future equity risk premium deviates from the estimated historical equity risk premium. For example, if the participant is optimistic about future equity returns and discounts using a high estimate of the equity risk premium, his portfolio return will be lower than expected. If the contribution rate is not adjusted during the accumulation phase, the participant will fall short of the desired amount of pension wealth at retirement. Consequently, a wrong risk perception of the participant may lead to the underestimation of the replacement rate at retirement and may lead to low contribution levels.

In DB funds, there is a similar risk in attaining the pension ambition for participants. DB funds typically offer nominal pension entitlements with additional Cost of Living Adjustments (COLAs) which are entitled to the participant in case the assets of the funds permit such benefits. Consequently, sufficient pension wealth accumulation is necessary to protect the participants from inflation risk and possible default risk of the fund. Newly accrued rights need to be valued taking into account the fund's portfolio allocation in order to determine the appropriate contribution rate for participants. Similar as in DC contracts, a wrong risk perception of the funds may lead to large exposure to the equity risk premium. Such a strategy will increase uncertainty about future replacement rates.

US pension regulation on contribution rates differs among types of pension plans. Since DC pension plans do not guarantee pension entitlements to participants, regulation concerning the level of contribution is typically not applicable to DC type of funds. However, DC funds are obliged under the Employee Retirement Income Security Act (ERISA) to provide benefit statements for participants.[4] These benefit statements are based on a number of actuarial assumptions. To determine benefit statements, the US department of Labor has set forward a few key assumptions which are based on long term projections on inflation and portfolio return.[5] These projections ignore uncertainty, so that the participants may form inadequate risk perceptions about the replacement rate at retirement. As a result, participants may contribute insufficiently to realize their pension ambition.

For DB funds, regulation is different for public and private funds. Regarding public US pension funds, the Governmental Accounting Standards Board[6] (GASB) holds the view that the pension contribution should be set using the long-term expected rate of return (GASB, 2012). Similarly, Canadian public state and local funds use a similar setting to set their contribution rate by investment return (CIA, 2010). While regulation on private US funds for the regulatory discount rate is split in accounting and contributions purposes, the regulator has chosen not to disentangle these two functions for public funds. Conceptually, regulation on accounting aims at transparency and comparison across funds, whereas guidelines on contribution influence the levels of contributions for the participant. These assumptions on long-term projections are set by the Board of Actuaries rather than observed market rates. As a result of using such projections in their contribution rates, these funds are also at risk for both economic uncertainty and parameter uncertainty of the equity risk premium.

Recent literature on DB pension funds have shown that public state pension plans are severely underfunded due to their reliance on discounting using high portfolio returns (Novy-Marx and Rauh, 2009, 2011). Due to perverse regulation, these funds have substantially

[4]Under ERISA section 105 all private funds that are governed by ERISA need to report the benefit levels to their participant periodically. Most DC funds fall into the scope of ERISA, except for some specific pension plans.

[5]A number of key assumptions are an inflation rate of 3%, nominal portfolio return of 7% and a nominal contribution increase of 3%. For more information, see: http://www.dol.gov/ebsa/newsroom/fsanprm.html

[6]Public pension funds for the state and local government fall within the scope of the guidelines of the Government Accounting Standards Board (GASB).

increased their equity exposure, allowing participants to benefit from low contribution rates (Novy-Marx, 2013). The risk of not achieving sufficient pension wealth for the participant is transfered to the state, because public pension entitlements are guaranteed under federal and case law (Brown and Wilcox, 2009). As a consequence of the underfunding in these pension plans, the state may be less able to bear this risk. While the literature has pointed out that using actuarial parameters will introduce large economic uncertainty, these funds are also suspect to parameter uncertainty. In case of such underfunding, our framework can serve as a lower bound for the contribution rates if the pension ambition corresponds to a real variable annuity and the state is not offering guarantees.

Private US DB funds rely on the Internal Revenue Service (IRS) to determine their contribution levels, which publishes discount rates based on investment grade corporate bonds. To understand the regulatory choice for US private pension funds, the specification of the pension ambition in the contract is of importance. DB private pension funds entitlement are guaranteed by the Pension Benefit Guaranty Corporation (PBGC) which requires a mandatory contribution of the funds based on corporate bond yields to ensure the protection of the participants' rights. Since the pension ambition is guaranteed at retirement, the contribution levels follow the nominal term structure of interest rates. Such contribution scheme allows to capture economic shocks through the term structure of interest rates.

Regulation on projections in DC plans is less developed as in DB plans, which raises an important question about the adequate contribution rate in DC contracts. Since actuarial standards are typically based on long-term sample periods, we set a fixed contribution based on our historical portfolio performance to analyze the risk of economic uncertainty and parameter uncertainty. To contrast this approach, we use a time-varying contribution scheme based on the term structure of interest rates and the historical equity risk premium. The latter approach resembles the contribution scheme in private funds, except for that fact that equity risk is explicitly incorporated in the determining the contribution level.

3 Financial market and pension contract

3.1 Financial market

Before we introduce our Bayesian methodology, we first explain our financial market without parameter uncertainty. In this case the investor knows the parameters of the financial market, although he will experience economic shocks to his investment portfolio. We further develop our Bayesian methodology in section 3.3.

Our financial market consists of nominal bonds and equity. We use a discrete time model that relates to continuous time equivalents such as Brennan and Xia (2002b), and Campbell and Viceira (2001b). In line with this literature, we assume that two latent state variables capture the nominal interest rate movements. In addition, we assume the equity risk premium is constant. As a result, the expected equity return is dependent on our state variables.

To value the pension contracts of the participants, we focus on an asset choice of bonds and equity. In order to determine the no-arbitrage prices of these assets, we establish a nominal pricing kernel. First, we define the monthly nominal short rate, r_t^N, as a function of two latent state variables X_t

$$r_t^N = \delta_{0,N} + \delta_{1,N}' X_t, \tag{1}$$

with the restriction $\delta_{0,N} > 0$. Second, we postulate the inflation process in terms of price levels that is driven by monthly expected inflation, π_t as

$$\frac{I_{t+1}}{I_t} = \exp(\pi_t + \sigma_\pi' \epsilon_t). \tag{2}$$

As a result, the realized inflation process consists of a shock $\sigma_\pi' \epsilon_t$ and the expected inflation rate. The inflation process allows to link the nominal and real pricing kernels in the financial market. We define monthly expected inflation as an affine transformation of the state variables,

$$\pi_t = \delta_\pi + \delta_{1,\pi}' X_t, \tag{3}$$

restricting the parameter $\delta_\pi > 0$. The state variables, X_t, that determine the time dynamics in our financial market are assumed to be following a first order vector autoregressive model,

$$\begin{bmatrix} X_{1,t+1} \\ X_{2,t+1} \end{bmatrix} = \begin{bmatrix} \phi_1 & 0 \\ \phi_{2,1} & \phi_2 \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \epsilon_t, \tag{4}$$

For the equity returns, we decompose the returns in the monthly short rate plus an equity risk premium. We assume the following process

$$R_t = r_t^N + \eta + \sigma_R' \epsilon_t, \tag{5}$$

where r_t^N denotes the monthly nominal short rate and η is the constant equity risk premium.

To derive the no-arbitrage bonds prices, we specify an affine nominal pricing kernel, M_t^N,

$$M_{t+1}^N = \exp\left(-r_t^N - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right), \tag{6}$$

with λ_t denoting the price of risk. We assume that the price of risk is affine in the state variables,

$$\lambda_t = \Gamma_0 + \Gamma_1 X_t. \tag{7}$$

Since λ_t is dependent on the state variables, we establish time-varying bond risk premiums. For the equity risk premium, we have the following restriction on the parametrization

$$\eta = \sigma_s' \lambda_t. \tag{8}$$

To satisfy this restriction, the constraints $\sigma_s'\Gamma_0 = \eta$ and $\sigma_s'\Gamma_1 = 0$ need to hold. Following Koijen, Nijman, and Werker (2010), we choose the elements on the last row of parameters ($\Gamma_{0,4}$, $\Gamma_{1,41}$ and $\Gamma_{1,42}$) in such way that these constraints hold. The equity return depends on the state variables through the nominal short rate, whereas the equity risk premium is constant through time.

Lastly, we derive the no-arbitrage yield curve. We assume exponential affine bond prices in the state variables, i.e.

$$P_t^N = exp(A^N + B'^N X_t) \tag{9}$$

Solving the no-arbitrage relation of the bond prices, we can establish a recursion for the coefficients of the yields. The no-arbitrage relation is as follows

$$P_t^N(n) = E_t\left[M_{t+1}^N P_{t+1}^N(n-1)\right]. \tag{10}$$

Substituting the nominal pricing kernel and the assumption of exponential affine bond prices, results to a recursion for the no-arbitrage coefficients for the yields.

$$Y_t^N(n) = \bar{A}_n^N + \bar{B}_n'^N X_t, \tag{11}$$

where the coefficients $\bar{A}_n^N$ and $\bar{B}_n'^N$ are functions of maturity n. In Appendix A, we derive the recursions for the nominal yields.

3.2 Pension contract

The objective of our pension contract is to ensure a sufficient standard of living after retirement for the participant. At retirement the participant will purchase an actuarial fair variable real annuity with his accumulated pension wealth. The real ambition of this pension contract is therefore to achieve an amount of pension wealth that is equivalent to the desired real replacement rate of the participant. At the start of the pension contract, the participant sets his retirement ambition at 70% of his real expected end wage at age 64. We set our ambition based on the findings of Munnell and Soto (2007), who report in the US median replacement rates of DB pension schemes ranging from 76% for workers covered by social security and 81 % for workers without social security.

The actuarial fair price of the real variable annuity at retirement age 65, PA_{65}, which entitles the participant to a replacement rate of 70% of his end wage at age 64, Z_{64}, can be denoted as

$$PA_{65} = \sum_{k=65}^{99} 0.70 Z_{64} \exp\left(-k(Y_t^R(k) + 0.2\eta)\right) p_k. \tag{12}$$

The variable annuity is based on a 20% equity exposure, which is denoted by 0.2η. If the participant wants a real fixed anuity, then this exposure drops to 0%. To determine the surival probability p_k, we use the mortality data of the US for cohort 2010[7].

To achieve this real ambition, the participant accumulates pension wealth in his saving account, comparable to an Individual Retirement Account (IRA). Since the participant is not in a DC fund, he ignores mortality risk during the accumulation phase. The individual's risk preference will influence his investment strategy. A higher risk aversion leads to lower equity holdings, so that the risk of not attaining the desired replacement rate decreases. We assume his risk aversion to be low by setting the investment strategy to a 60% equity and 40% bonds portfolio. We rely on data of the average equity and bonds holdings in the US to set his portfolio (Towers Watson, 2013). They find that US pension funds' portfolios consist of about 66% equity with the remaining wealth of 34% invested in bonds. A lower risk profile is for example taken in the Netherlands, where the average portfolio consists of 32% equity and 68% bonds. This investment strategy will be used to analyze the consequences of a low risk profile. Each year the accrued pension wealth of a participant, W_t, is affected by the portfolio return,

[7] We obtained the dataset from www.mortality.org and used the 2010 cohort to calibrate our model.

R_t^P. In addition, the participant contributes a fraction of his wage Z_t, resulting in the following dynamics for the pension wealth

$$W_{t+1} = W_t R_t^p + \beta_t Z_t, \text{ for } t = 25, .., 64. \tag{13}$$

The pension contribution β_t is a fraction of the real wage of the individual. The participants starts without any pension wealth, rendering the initial condition $W_{25} = 0$.

We employ two contribution schemes to accrue sufficient pension wealth to purchase the variable annuity. First, we set a fixed contribution level based on the historical performance of the portfolio. To determine the fixed minimal contribution at age 25, the price of the annuity at retirement age 65 is divided by the sum of yearly salaries multiplied by the real portfolio return up to retirement, which yields the following contribution rate

$$\beta^{\text{fixed}} = \frac{PA_{65}}{\sum_{t=25}^{64} E[Z_t] \exp\left((64-k)E[R_t^p]\right)}. \tag{14}$$

The average wage $E[Z_t]$ is assumed to be known, since the participant uses the average wage pattern known for his level of education. The underlying idea of this contribution scheme is that past historical performance, $E[R_t^p]$, is similar to future portfolio returns, so that in expectation the pension ambition can be achieved. Even when the parameters of our financial market are known to the participant, economic shocks occurring in the financial market can lead to large uncertainty for the replacement rate at retirement. This strategy will also ignore possible parameter uncertainty.

Second, we determine a time-varying contribution scheme that allows the participant to react to economic shocks or parameter uncertainty. Each year the participant verifies whether his pension wealth grows sufficiently compared to the present value of his annuity. To value the annuity for his desired replacement rate annuity, we use the real term structure of interest rates to discount its price. Since the participant invests in equity, we need to account for the equity risk premium in our contribution. Otherwise the contribution level will be too high, resulting in overshooting his pension ambition. Therefore, we add 60% of the equity risk premium to the real interest rates, 0.6η, since he will allocate 60% of his pension wealth to stocks. To determine the present value of the annuity during the accumulation phase, we use a similar methodology as in Equation (12). We first discount the pension payments during retirement, but use the real forward curve at time t instead of the yield curve, so that we are able to determine the amount of pension wealth required to purchase the real variable annuity at retirement. Next, we discount this required pension capital with the yield curve to determine its present value of the annuity at time t. Therefore, the present value of the real ambition before retirement is

$$\begin{aligned} PV_t^{\text{Annuity}} &= \exp\left(-(65-t)(Y_t^R(65-t) + 0.6\eta)\right) \\ &\quad \sum_{k=65}^{99} 0.7E[Z_{64}] \exp\left(-k(FR_t^R(65-t, k-t) + 0.2\eta)\right) p_k, \end{aligned} \tag{15}$$

where $FR_t^R(65-t, k-t)$ denotes the real forward rate at time t for the pension payments from time period $65-t$ to $k-t$. Economics shocks will enter the term structure of interest rates, affecting the present value of the pension ambition.

Next, we determine the present value of the future expected wage pattern at time t. The present value is the sum of the expected wage, $E[Z_k]$, so that we can write

$$PV_t^{\text{Wage}} = \sum_{k=t}^{64} E[Z_k] \exp\Big(-k(Y_t^k + 0.6\eta)\Big). \tag{16}$$

Combining the present value of the wage and the ambition, we can determine to the time-varying pension contribution as follows,

$$\beta_t = \frac{PV_t^{\text{Annuity}} - W_t}{PV_t^{\text{Wage}}},. \tag{17}$$

The underlying idea of the pension contribution is that the difference between the present value of the real ambition and current wealth will be spread out over the remaining future income of the participant. Thus, if pension wealth fall short compared to the present value of the annuity, the contribution will increase. Economics shocks to the term structure will affect the valuation as well. In case the yields decrease, the price of the annuity will go up.

Our specification of the pension contract allows for a number of individual choices. The underlying motivation is based on the global trend of DB shifting to DC schemes. For example, in 2012 the DC funds have grown 4% world wide relatively to DB funds in terms of assets since 2010. (see e.g., Towers Watson (2011) and Towers Watson (2013)). As a result, the participant needs to bear the risk about the replacement rate himself. Individual choices can help to overcome economic shocks and lower potential risk. For example, retirement age could be postponed in order to acquire a higher replacement rate at a later age. Also, lowering the risk profile may influence the risk of the replacement rate at retirement.

3.2.1 Real wage growth

We benchmark the real wage growth of the individual in our model to Cocco et al. (2005). Using their characteristics for a High school individual, who did not obtain a college degree yet did obtain a high school degree, we calibrate the real income process and capture the income pattern over the lifetime for the participant in our framework. We assume the following polynomial function

$$Z_t = \alpha_0 + \alpha_1 t + \alpha_2 \frac{t^2}{10} + \alpha_3 \frac{t^3}{100}, \tag{18}$$

where we set $\alpha_1 = 0.1682$, $\alpha_2 = -0.0323$, and $\alpha_3 = 0.0020$ which follow from Cocco et al. (2005). They decompose the variance in transitory and permanent shocks with respectively the variances $\sigma_\epsilon = 0.0738$ and $\sigma_u = 0.0106$. We choose the parameter α_0 in such a way that the wage at age 25 for the individual is \$20,000.

An important observation is that the real wage growth patterns differ among different levels of education. While all groups have an upward shaped salary pattern, the highest salaries are obtained at different ages. For example, the lowest educated level (No high school degree) reaches its highest real salary at age 38, whereas for the highest (College degree) and average educated groups reach this highest salary at a later age, at age 45 and 44 respectively. As a result, the lowest educational group has a rapid growing salary at young age. Moreover, the salary pattern of the average educational level remains relatively fairly high after reaching its peak compared to the other groups. Consequently, a pension ambition based on the end wage will be more costly compared to the other two educational groups.

3.3 Estimation

3.3.1 Data

Our data sample ranges from January 1952 up to December 2012. We use 6 nominal yield series, namely the 3 and 6 months Treasury Bills and the US Treasury bonds with maturities 1, 2, 3, 5 and 10 year. The data on the 3-month and 6-month T-Bills are taken from the Federal Reserve Bank of St. Louis.[8] The other yields up to February 1991 are from the McCulloch and Kwon and extended using the data in Bliss (1997) to December 1998. From January 1999, we use the data from the Federal Reserve bank of New York.[9] Inflation data is obtained from the US CPI All urban price level from the Bureau of Labor Statistics. The monthly equity return is determined by the logarithmic return on the CRSP value-weighted NYSE/Amex/Nasdaq index data.

3.3.2 System of equations

We estimate our model using a system of reduced equations based on the methodology presented by Hamilton and Wu (2012). We invert the yields measured without error to identify the latent factors and subsequently we regress these factors on the remaining yields, inflation and equity return. Details on the derivation are provided in Appendix A.

The reduced form of equations is used to estimate the reduced form parameter, which can be summarized as

$$\begin{aligned}
Y_t^1 &= \underbrace{(\bar{A}_1 - \bar{B}_1' \Phi_1^L \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_1^*} + \underbrace{(\bar{B}_1' \Phi_1^L \bar{B}_1'^{-1})}_{\Phi_{11}^*} Y_{t-1}^1 + \underbrace{\bar{B}_1'}_{\Omega_1^*} \epsilon_{1,t} \\
Y_t^2 &= \underbrace{(\bar{A}^2 - \bar{B}_2'^L \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_2^*} + \underbrace{(\bar{B}_2'^L B_1'^{-1})}_{\Phi_{21}^*} Y_t^1 + \underbrace{\bar{B}_2'^{EC}}_{\Phi_{2EC}^*} X_t^{EC} + \underbrace{\Omega}_{\Omega_2^*} \epsilon_{2,t} \\
\frac{I_{t+1}}{I_t} &= \underbrace{\delta_{0,\pi} - \delta_{0,\pi} \bar{B}_1'^{-1} \bar{A}_1}_{A_3^*} + \underbrace{\delta_{0,\pi} \bar{B}_1'^{-1} Y_t^1}_{\Phi_{31}^*} + \underbrace{\Sigma_\pi}_{\Sigma_\pi^*} \epsilon_{3,t} \\
R_t &= \underbrace{\eta + \delta_{0,r} - \delta_1' \bar{B}_1'^{-1} \bar{A}_1}_{A_4^*} + \underbrace{\delta_1' \bar{B}_1'^{-1} Y_t^1}_{\Phi_{41}^*} + \underbrace{\sigma_R}_{\Sigma_R^*} \epsilon_{4,t}.
\end{aligned} \tag{19}$$

This system of equation can be estimated with OLS resulting in the reduced form estimates of our model. Subsequently, these estimates can be matched with the structural parameters of our model using a Minimum Chi-squared approach. Minimization of the Chi-squared value function gives the estimates for the structural parameters.

To estimate the effect of parameter uncertainty, we apply our Bayesian methodology to the reduced form of the system of equations in Equation (19). Our estimation strategy is to

[8] http://research.stlouisfed.org/fred2/

[9] The nominal yield data are available on http://www.federalreserve.gov/econresdata/researchdata.htm

employ a Block-Gibbs sampling algorithm to draw from the marginal posterior probability distribution of the reduced form coefficients and subsequently link these coefficients with the associated structural parameters using the Minimum Chi-squared approach. Depending on the prior distribution, we can evaluate the pension contract for each set of parameters in our financial market incorporating the effect of parameter uncertainty. We refer to Appendix A.4 for the details on the estimation strategy. Since we incorporate parameter uncertainty in the reduced form equations, our approach allows for a straightforward implication without having to determine the marginal posterior distributions of the structural coefficients. Therefore, our approach can be easily adopted to determine parameter uncertainty for pension contracts.

We estimate our model without an inflation risk premium due to limited data of inflation-linked derivatives. Recent empirical studies suggest a wide range for the magnitude of the inflation risk premium (See e.g., D'Amico, Kim, and Wei (2010) and de Roode (2013)). In addition, liquidity effects in the inflation-linked bond market, especially at the start of the issuance and during the financial crisis, substantially reduce the available data range to estimate the inflation risk premium from such derivatives. To refrain from the uncertainty about the magnitude of the inflation risk premium, we set the inflation risk premium to zero.

3.3.3 Financial market

Table 1 presents the estimates of our financial market based on our extensive sample from 1950-2012. Both our latent factors in our model exhibit a high degree of persistence, since the estimates of $\Phi_{1,11}$ and $\Phi_{1,22}$ are fairly high. As a consequence, the half-life of the first factor is 7 years whereas the second factor has a shorter half-life of 1 year. Consequently, the second factor is less persistent than the first. Although our estimation suggests that the factors are negatively correlated, uncertainty remains an issue since the standard error of $\Phi_{1,21}$ is quite large. Moreover, we find that the monthly 5 year bond return is positively correlated with inflation, whereas the equity return is negatively correlated. However, since both assets are negatively correlated with inflation innovations, bonds do not offer a better hedge against inflation risk than equity.

To further analyze our financial market, we turn to the nominal risk premia. The average nominal yield curve is upward sloping with nominal term premia for a bond with maturities 1, and 5 year of about 45 basis points and 143 basis points respectively. As a result, the average nominal yield curve in our financial market is relatively high compared to recent periods observed after 2000. In Figure 1, we present the nominal yield for the 3 months and 5 year maturity. In addition, we show the relatively high average yield curve over our sample period in Figure 1. We observe a high average equity return during our sample as well, leading to an annual equity risk premium of about 5%. As a consequence of relatively high average equity rates during this sample period, our pension premium will be downward affected. Furthermore, we observe an average annual inflation of 3.5%, suggesting that inflation is an important factor in diminishing nominal returns.

In terms of model fit, our model is able to replicate the average yield curve within 95% confidence levels. The measurement errors in Table 1 show that our model is able to capture the long term nominal yields better than the short maturities. For example, we report a standard error of about 38 basis points for the 3 months yield, whereas for the 2 year maturity the error is about 12 basis points. Also, we find positive correlation between nominal bond and equity returns, similarly as in Sangvinatsos and Wachter (2005). While our model is able to capture the characteristics of the financial data rather well, the uncertainty about equity risk premium is large. Therefore, we use our Bayesian approach to verify this effect on the replacement rate of our pension contract.

4 Pension contract with known financial market parameters

In this section, we analyze the uncertainty in the replacement rate at retirement, assuming there is no uncertainty about the parameters of our financial market. The participant will only be affected by economic uncertainty of the financial market, so that we can analyze the risk of achieving low replacement rates at retirement for our fixed and time-varying contribution schemes. To investigate the impact of risk profile, we also verify the replacement rate risk if the participant has a low equity exposure.

4.1 Benchmark pension contract

Table 2 shows that economic uncertainty in the equity returns affects the ability of the fixed contribution scheme to achieve the desired replacement rate of the real variable annuity. The fixed contribution based on the historical performance return of the participant's portfolio requires a yearly contribution of 6.85% and leads to an average replacement rate of 70.0%. However, the fixed contribution scheme introduces substantial uncertainty about the replacement rate. The lower bound of the 95% confidence interval of the replacement rate is about 35.2%. This result indicates that a fixed contribution scheme can harm the real ambition of the participant, because it prohibits the participant to respond to economic shocks. In bad economic scenarios in which equity returns are low, the participant remains optimistic about his future returns and refrains from increasing his pension contribution.

The time-varying contribution scheme can partially compensate for economic shocks. Table 2 shows that the average replacement rate is 75.8%, which is larger than the desired replacement rate. One of the reasons causing the higher replacement rate is the time-varying dynamics of the interest rates. In case the term structure of interest rates decreases early in the accumulation phase, the contribution will increase due to the high present value of the real ambition. If later on, during the accumulation phase, the pension wealth exceeds the present value of the real ambition, the time-varying contribution scheme would imply to withdraw pension wealth from the account. However, since we do not allow the participant to withdraw from his pension account, the participant slightly overshoots his desired pension ambition.

To achieve the pension ambition, the average time-varying contribution is increasing with the age of the participant. Since his investment strategy is not a self-replicating portfolio, the pension contribution needs to compensate this difference near the end of the accumulation phase. A self-replicating strategy would imply that the participant invests his pension contribution at age 25 in 40% bonds with a maturity of 40 years instead of a fixed maturity bond portfolio of 5 years. While the time-varying contribution scheme responds to shortages of pension wealth compared to the present value of the annuity due to equity shocks, the pension contribution is for young ages most sensitive to changes in the term structure. If pension wealth falls short due to low performance of equity returns at older ages, the contribution scheme will react by substantially increasing the contribution level. As a result, the 95% confidence intervals of the pension contributions are quite wide near retirement.

One important constraint for the time-varying contribution scheme is that the contributions are restricted to the interval of 0% and 30%. This implies that the participant is prohibited from withdrawing pension wealth during the accumulation phase and can only contribute at most 30% of his yearly wage to his pension account. These arbitrary fixed bounds are necessary, because contributions higher than 30% are economically unjustifiable. Otherwise these high contributions would have a large impact on the consumption of the participant. As a result, these fixed bounds lower the volatility in the pension contribution, but also reduce the steering capacity of the time-varying contribution. If the participant is not restricted by these fixed bounds, the 2.5% percentile of the replacement rate at retirement would be 51.9%. This is 15% higher than when the contribution is restricted with fixed bounds, which results in a replacement rate of 45.2%. However, Figure 2 shows that in order to achieve this lower replacement rate risk, the contribution rates during the accumulation phase may become extremely high if the participant is not restricted by fixed bounds. For example, the 95% confidence interval of the contribution at age 55 is wider than the fixed interval of 0% and 30%, as it ranges from -34.1% to 54.6%. Such contribution rates are economically unjustifiable, so that fixed bounds to restrict contributions are required.

To investigate whether the constraints of fixed bounds for the contribution are restrictive, Table 2 presents the percentage of scenarios in which the fixed interval is binding. At age 55, 43.4% of the scenarios hits the lower bound of the contraint at 0%, whereas the upper bound of 30% is binding in 14.3% of the scenarios. Near retirement, at age 64, this increase to a probability of 55% hitting the 0% lower bound and 38.8% the upper bound of 30%. These results suggest that the lower bound which restricts the participant from withdrawing pension wealth is more important than the upper bound. While the restriction of the lower bound is frequently binding, it can economically be justified since the additional accumulated pension wealth can be used either for a higher replacement rate or for smoothing economic shocks. However, the arbitary upper bound of the fixed interval would be dependent on the time preference of the individual for future consumption. Also, the probability of hitting the upper bound is a measure that indicates whether a pension amibition with a desired replacement rate is costly. For example, at age 64 with a probability of 14.3% the pension ambition with a real replacement rate of 70% is set too high. Therefore, in such scenarios the participant may want to reduce the pension ambition in favour of current consumption.

4.2 Alternative individual specifics

Our results on the risk of replacement rates are sensitive to three factors, namely level of education, risk profile and retirement age. Since our main findings are not materially affected, we only analyze the impact of a low risk profile. For the impact of education level and retirement age, we refer to Appendix D.

4.2.1 Risk profile

An important factor driving the pension contribution level is the risk preference of the participant. To investigate its effect on the contribution level and the replacement rates, we switch the investment strategy to a low risk profile by altering the portfolio to 30% equity and 70% bonds.

Table 3 shows that a low risk profile decrease the risk of not achieving the desired replacement rate. To realize the pension ambition with lower equity holdings, the fixed contribution increases by 41.9% from a yearly premium of 6.85% to 9.72%. As a result of lower the equity exposure, the lower bound of the 95% confidence interval of the replacement rate increases to 43.2%. Not surprisingly, the lower risk profile leads to an improvement in the risk of low replacement ratee, since the 2.5% percentile of replacement rate is 35.2%. While a low risk profile improves the uncertainty about the replacement rate, it requires a substantial increase in contribution.

Regarding the effect of lower equity holdings, Table 3 shows that time-varying contribution scheme is less capable of improving the uncertainty about the replacement rate at retirement. The lower bound of the 95% confidence interval changes from a replacement rate of 44.5% to 48.3%. This result indicates that reducing the equity holdings from 70% to 30%, only slightly improves the worst-case scenario with a 2.5% probability, since the participant will only slightly reduce the uncertainty of the replacement rate at retirement. Investors with a low risk profile are therefore still substantially exposed to economic uncertainty.

5 Impact of parameter uncertainty on the pension contract

To assess parameter uncertainty in the pension contract, we first introduce the effects of parameter uncertainty on our estimates of the financial market. We show that the equity risk premium is the most important factor attributing to parameter uncertainty in our model. Therefore, we analyze this effect separately by first evaluating our pension contract with parameter uncertainty restricted to only the equity risk premium. Subsequently, we generalize our model to account for parameter uncertainty for all financial parameters.

5.1 Parameter uncertainty and the financial market

To incorporate parameter uncertainty in our financial market, we adopt a Bayesian approach with an uninformed prior. For details on the estimation approach, we refer to Appendix A.4. By using an uninformative prior, we do not hold ex ante views on the projections of the financial market other than the observed data.

Figure 3 shows the dispersion of the equity risk premium by means of the estimated marginal posterior distributions. Since the estimate of the equity risk premium has a large standard error, these distributions have a wide support. For example, we find a 95% credibility interval ranging from 1.34% to 8.75 %. This result suggests that in projections of the financial market a wide range of estimates may be used. As a result of this parameter uncertainty, the mean of the stock return process can be substantial higher or lower in our scenarios. In particular, we are interested in the case that mean stock returns diminish and whether our time-varying contribution can partially compensate this effect on the portfolio returns.

The parameter uncertainty of inflation and the term structure of interest rates is far less substantial than the uncertainty regarding the equity risk premium. For example, Figure 4 shows the impact of parameter uncertainty on the average inflation rate is not widely dispersed. Since the inflation estimate has a lower uncertainty with a standard error of about 0.14%, the interval for the mean of the inflation process is much smaller. However, scenarios with a combination of a low equity risk premium and a high inflation level diminish real returns of the investor's portfolio more rapidly.

5.2 Parameter uncertainty restricted to the equity risk premium

Parameter uncertainty mostly affects the equity risk premium in our financial market. Therefore, we estimate our pension contract in this section with parameter uncertainty restricted to only the equity risk premium. As a consequence of parameter uncertainty, the average stock return in our model is uncertain for the participant. We assume that the participant is not aware of parameter uncertainty and only observes historical equity performance. Therefore, he relies on the estimated historical equity risk premium to set his minimal contribution level, which is for the fixed contribution scheme a yearly contribution of 6.85%. In case of a low equity risk premium, the participant is too optimistic about his future projections of the average stock return. However, at retirement the participant purchases a fair priced annuity based on the equity risk premium in the financial market. As a result, parameter uncertainty leads in this scenario to pension contributions that are too low.

Table 4 shows that employing parameter uncertainty about the equity risk premium puts the participant at risk, causing lower replacement rates at retirement, if a fixed contribution scheme is used. The 2.5% percentile of the replacement rate at retirement drops 24.4% compared to our previous results ignoring parameter uncertainty, from a replacement rate of 35.2% to 26.6%. This result shows that ignoring parameter uncertainty leads to the underestimation of risk in the replacement rate at retirement.

Next, we verify whether the time-varying contribution is able to partially subdue the effects of parameter uncertainty of the equity risk premium. Table 4 shows that the risk is far lower by employing a time-varying contribution scheme. For example, the lower bound of the 95% credibility interval decreases from 45.2% to only 41.3%, compared to model without parameter uncertainty. This results shows that only partial effects of the parameter uncertainty concerning the equity risk premium can be compensated by increasing the pension contribution.

One of the underlying factors are the restrictions on the pension contributions, requiring pension contributions to be between the fixed interval of 0% and 30%. The time-varying contribution scheme allows shocks in the portfolio return to be spread across the remaining years. As a consequence of the high present value of future wage at young years, the pension contribution is mostly affected by interest rate movements. At older ages, the present value of future wage dimishes more rapidly, so that the contribution mechanism steers more strongly. As a result, near retirement the restrictions become more important. For example, Table 4 shows that after age 55 the probability of hitting the fixed upper bound of 30% increases due to parameter uncertainty. As a result, near retirement, at age 64, he is less likely to hit the upper bound. This shows that parameter uncertainty forces the time-varying contribution scheme to react earlier because of the lower portfolio returns.

5.3 Parameter uncertainty for all financial parameters

Next, we verify whether parameter uncertainty about inflation and the term structure adds to the uncertainty of our previous reported results. We focus on the risk of the replacement rate and whether the time-varying premium can still partly compensate for these effects.

Table 5 shows that the uncertainty about the replacement rate at retirement increases only slightly compared to our previous result. Using a fixed contribution scheme lowers the 2.5% percentile from a replacement rate of 26.6% to a rate of 26.4%. This result shows that the uncertainty about the inflation and interest rates has a lower impact on the replacement rate. Thus, uncertainty about the equity risk premium is the most important factor for parameter uncertainty in our pension contract.

Adopting a time-varying contribution scheme actually decreases the risk about the replacement rate at retirement. Table 5 shows that the lower bound of 95% credibility interval increases from a replacement rate of 41.3% in case of restricting parameter uncertainty to only the equity risk premium to a lower bound of 42.1%. This increase can be explained by the higher pension contributions at the start of the accumulation phase. At younger ages, Table 5 shows that the participant increases his pension contribution due to the parameter uncertainty entering in the term structure of interest rates. For example at age 25, the average pension premium increases from 5.83 % to 6.51%, which is an increase of 11.7%. Simiarly, at age 35 the participant increases his contribution by 8.2% to 7.52% instead of 6.95%. As a result, near retirement the participant can lower his contribution rate.

The front loading of the pension contribution in the time-varying contribution scheme also leads to a lower probability of hitting the fixed upper bound of 30%. Table 5 shows the participant is less likely to hit the premium bound near retirement. With a probability of 35.6% the participant will hit the upper bound at age 64 when parameter uncertainty affects all financial parameters. In our previous result, this was 39.1%. This result suggests that incorporating only uncertainty about the equity risk premium may overestimate the effect of parameter uncertainty on the replacement rate for the time-varying contribution scheme.

Altering the risk profile of the investment portfolio leads to a similar conclusion. For the time-varying contribution scheme, the 2.5% percentile of the replacement rate at retirement is about 48.4% when parameter uncertainty affects all financial parameters. Ignoring parameter uncertainty results in a 2.5% percentile of 51.0%, which causes a similar shift in risk perception as in the benchmark case. In case of a low risk profile, the 2.5% percentile of the replacement rate's distribution that incorporates parameter uncertainty corresponds to a 1.4% percentile in case parameter uncertainty is ignored. A similar shift in risk perception can be observed for the benchmark case, where parameter uncertainty causes a shift from the 2.5% percentile to the 1.3% percentile. Therefore, the effect of parameter uncertainty has a similar effect on the risk perception regardless of the risk profile of the individual.

6 Economic regimes and parameter uncertainty

Our previous results are based on the underlying idea that uncertainty about financial parameters is captured by our full sample period. However, economic regimes may switch throughout our sample, substantially altering projections. For example, Figure 1 shows the declining trend of interest rates of US Treasury bonds after the year 2000. As a result, using average interest rates over the full sample period might lead to too optimistic projections. To analyze the impact of more recent economic regimes, we incorporate a prior view in this section. We first analyze the impact to our financial market and subsequently, verify its effect on the contribution schemes of our pension contract.

6.1 Impact of the 21st century on market projections

During the first decade of the 21st century, financial markets experienced rapid developments due to globalization. This period is characterized by two important events for the financial markets, namely the crash of the dotcom bubble in 2001 and the financial crisis in 2008. Due to these distinct features, we investigate the effect of this period on the projections of our financial market. By attaching more importance to observations from 2000 up to 2012 in our Bayesian framework, we are able to incorporate a shift in the economic regime for our projection without harming the estimation of our financial parameters. To this end, we calibrate a Normal-Diffuse prior in our Bayesian estimation to economic conditions observed in this period. The tightness of our Normal-Diffuse prior is calibrated to this period, so that the uncertainty of our prior view corresponds to the observed uncertainty in this period. This allows us to incorporate a market shift observed in this period without excluding observations.

Since the equity risk premium has the largest effect on our pension contract, we first analyze Figure 3 that presents the marginal posterior distribution of the equity risk premium. We show that incorporating our prior view leads to a decrease of 58 basis points in the average equity risk premium. This results suggests that projections of the average equity returns will be quite lower than the previous estimate. The dispersion of the estimate is slightly larger when incorporating a prior view, suggesting that the uncertainty about the estimate is similar compared to our previous result. For the other economic variables, the impact of a market shift is quite smaller. For example, Figure 4 shows that the mean inflation rate is shifted upward by 12 basis points. A higher inflation rate will cause that real returns on the investment portfolio decrease. In combination with a lower equity risk premium, this could lead to an increase in pension contributions. All together, these results imply more severe market conditions for the participant. Ignoring such developments in projections may cause that contribution scheme are too optimistic about future economic projections.

To measure the risk of using an ex post estimate of the equity risk premium for ex ante future stock returns, we assume that the participant's contribution is based on the historical estimated equity risk premium as previously used. In case of a fixed contribution rate, the risk of the replacement rate at retirement will substantially increase. Table 6 shows that the lower bound of the 95% credibility interval of the replacement rate at retirement drops to 21.7%, which is lower than our previous result (26.5%), using the parameter uncertainty with an uninformative prior. Since the fixed premium is based on historical performance, it substantially overestimates the future returns of the investment portfolio.

Next, we verify whether the time-varying contribution can partially compensate for the shift of the economic regime. In the time-varying premium, the participant uses the historical equity risk premium to value future stock returns. As a result of this overvaluation, the risk about the replacement rate at retirement increases. Table 6 shows that the time-varying contribution cannot compensate for the shift in the parameter uncertainty, since the lower bound of the 95% credibility interval drops from 42.1% to 37.6%. Consequently, the time-varying contribution scheme is less able to react to a shift in an economic regime.

To compensate for the lower portfolio return, the pension contribution is in general higher throughout the accumulation phase. However, Table 6 shows that at the start of the pension contract the economic projections are slightly better than in our previous case of parameter uncertainty. To summarize the contribution effects, Figure 5 shows the contributions for parameter uncertainty with an uninformative prior and with the informative prior. Incorporating the regime of the 21st century has strong upward impact on the average contribution after age 45. In particular, the upward effect can be seen for the median contribution rates. The median contribution based on the uninformative prior decreases to zero before age 60, whereas the median contribution using the prior view increases up to age 62 and starts declining after. Consequently, the ability of the time-varying contribution scheme to achieve the desired replacement rate decreases when economic regime alters.

7 Conclusion

Uncertainty about financial parameters complicates the participants' planning to achieve his pension ambition. When parameter uncertainty is ignored, the risk of lower replacement rates at retirement is underestimated. To investigate the main factor of uncertainty in financial parameters, we adopt a Bayesian methodology that captures uncertainty of the equity risk premium, inflation and term structure of interest rates. Based on our estimated financial parameters, we set a fixed yearly pension contribution that on average achieves the real pension ambition in our DC pension contract. The effects of parameter uncertainty affects our pension contract which allows us to analyze its effect on the replacement rate at retirement. Based on this framework, we demonstrate that uncertainty of the equity risk premium is the key factor driving the parameter uncertainty for the risk in the replacement rate at retirement.

Next, we verify whether a time-varying contribution scheme based on a combination of the term structure of interest rates and a historical estimate of the equity risk premium can partially compensate for parameter uncertainty. We show that although the participant either underestimates or overestimate future stock returns by using a historical equity risk premium, the time-varying contribution scheme partially compensates the risk of the replacement rate at retirement. However, our model reveals that the time-varying contribution scheme is less able to compensate for uncertainty in replacement rates if the equity risk premium is affected by a permanent unobserved shock. As a result of this shock, the participant's contributions are based on overly optimistic beliefs about future equity returns. Unobserved changes in financial parameters may, therefore, introduce risk in the time-varying contribution rate if the equity risk premium is set inadequately.

Our results imply that DC pension funds that rely on fixed yearly contributions to achieve a fixed real pension ambition, may introduce substantial replacement rate risk for participants.

Even when time-varying contribution schemes are employed, risk in replacement rates can be substantial if the equity risk premium is set overly optimistically. To reduce replacement rate risk, regulators implementing contribution schemes similar to our time-varying contribution, need to address whether the ex post estimate of the equity risk premium is adequate to be used for future projections. Since pension wealth accumulation of young cohorts are particularly affected by inadequate estimates of the equity risk premium, regulators may want to impose upper limits of the equity risk premium to set pension contributions.

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A Appendix A: Model derivations

A.1 Nominal yields

In this section we derive the nominal bond yields in a no-arbitrage framework. We substitute the affine bond prices, as defined in equation (9), in the no-arbitrage relation of the expected bond price. For convenience, we write this relation here

$$P_t^N(n) = E_t\left[M_{t+1}^N P_{t+1}^N(n-1)\right]. \tag{A.1}$$

By substituting the affine bond prices and the dynamics of the nominal pricing kernel in this equation, we derive the following expression for the price of a bond,

$$P_t^N(n) = E_t\left[\exp\left(-r_t^N - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right)\exp\left(A_{n-1}^N + B_{n-1}'^N X_{t+1}\right)\right]. \tag{A.2}$$

Rewriting and determining the expectation, yields the following expression for the bond price

$$P_t^N(n) = \exp\left(-\delta_0 + A_{n-1}^N + B_{n-1}'^N\Phi_0 + (B_{n-1}'^N\Phi_1 - \delta_1')X_t - B_{n-1}'^N\Sigma\lambda_t + \frac{1}{2}B_{n-1}^N\Sigma\Sigma' B_{n-1}'^N\right). \tag{A.3}$$

Next, we need to substitute the affine function for the price of risk, λ_t. Substituting this, as defined in Equation (7), we arrive at the typical function for the no-arbitrage bond price,

$$P_t^N(n) = \exp\left(-\delta_0 + A_{n-1}^N + B_{n-1}'^N(\Phi_0 - \Sigma\Gamma_0) + \frac{1}{2}B_{n-1}'^N\Sigma\Sigma' B_{n-1}^N + (B_{n-1}'^N(\Phi_1 - \Sigma\Gamma_1) - \delta_1')X_t\right). \tag{A.4}$$

For estimation reasons as suggested in Hamilton and Wu (2011), we introduce parameters for time dynamics process of the state variables under the Q-measure. These parameters are defined as

$$\Phi_0^Q \equiv \Phi_0 - \Sigma\Gamma_0, \Phi_1^Q \equiv \Phi_1 - \Sigma\Gamma_1. \tag{A.5}$$

The last step is to determine the recursion for the coefficients by matching the coefficients of the left-hand side with the terms on the right-hand side. In this way we derive the recursion for the no-arbitrage coefficients of the bond prices

$$A_n^N = A_{n-1}^N + B_{n-1}'^N\Phi_0^Q + \frac{1}{2}B_{n-1}'^N\Sigma\Sigma' B_{n-1}^N - \delta_0, \tag{A.6a}$$

$$B_n'^N = B_{n-1}'^N\Phi_1^Q - \delta_1'. \tag{A.6b}$$

with the initial conditions $A_1^N = -\delta_0$ and $B_1^N = -\delta_1$. In order to derive the coefficients of the yields, using the relation between bond prices and continuously compounded yields, we adjust the coefficients of the bonds as follows

$$\bar{A}_n^N = -\frac{A_n^N}{n}, \tag{A.7a}$$

$$\bar{B}_n^N = -\frac{B_n^N}{n}. \tag{A.7b}$$

This function determines the no-arbitrage coefficients for the affine yields.

A.2 System of reduced equations

The system of reduced equations will be derived in this section. We first focus on the equation of the VAR dynamics of the yields measured without error. We start with the state process as defined in Equation (4). Premultiplying this system with $\bar{B}_1'$ and adding $\bar{A}_1$ gives

$$\bar{A}_1 + \bar{B}_1' X_t^L = \bar{A}_1 + \bar{B}_1' \left(\Phi_1^L X_{t-1}^L + I_2 \epsilon_{1t} \right) \tag{A.8}$$

As a result, we can rewrite this equation to a VAR model of the yields measured without errors, Y_t^1,

$$Y_t^1 = \bar{A}_1 + \bar{B}_1' \Phi_1^L \bar{B}_1'^{-1} \left(Y_{t-1}^1 - \bar{A}_1 \right) + \bar{B}_1' \epsilon_{1,t}, \tag{A.9}$$

by use of the definition of Y_t^1 as given in Equation (11). Now we have expressed the time dynamics of the latent factors in yield series measured without error. Rewriting this equation yields the first reduced form regression,

$$Y_t^1 = \underbrace{(\bar{A}_1 - \bar{B}_1' \Phi_1^L \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_1^*} + \underbrace{(\bar{B}_1' \Phi_1^L \bar{B}_1'^{-1})}_{\Phi_{11}^*} Y_{t-1}^1 + \underbrace{\bar{B}_1'}_{\Omega_1^*} \epsilon_{1,t}. \tag{A.10}$$

In this equation the coefficients $\bar{A}_1^*$, Φ_{11}^*, and Ω_1^* will be obtained by OLS estimation.

The second reduced form equation is the impact of the latent factors on the yields measured with errors. For notional convenience, we repeat this equation

$$Y_t^2 = \bar{A}^2 + \bar{B}_2'^{EC} X_t^{EC} + \bar{B}_2'^L X_t^L + \Omega \epsilon_{2,t}. \tag{A.11}$$

Since we include real rate series in this equation we need to incorporate the effect of the economic factors as well. Next, we substitute the latent factors with inverse of the yields observed without error,

$$Y_t^2 = \bar{A}^2 + \bar{B}_2'^L \left(\bar{B}_1'^{-1} \left(Y_t^1 - \bar{A}_1 \right) \right) + \bar{B}_2'^{EC} X_t^{EC} + \Omega \epsilon_{2,t}. \tag{A.12}$$

Consequently, we derive the following reduced form regression,

$$Y_t^2 = \underbrace{(\bar{A}^2 - \bar{B}_2'^L \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_2^*} + \underbrace{(\bar{B}_2'^L B_1'^{-1})}_{\Phi_{21}^*} Y_t^1 + \underbrace{\bar{B}_2'^{EC}}_{\Phi_{2EC}^*} X_t^{EC} + \underbrace{\Omega}_{\Omega_2^*} \epsilon_{2,t}. \tag{A.13}$$

We denote the OLS estimates of the coefficients in this equation as $\bar{A}_2^*$, Φ_{21}^*, Φ_{2EC}^* and Ω_2^*.

The measurement equation for inflation follows a similar derivation. We start with the definition of inflation and substitute the model implied equivalents,

$$\frac{I_{t+1}}{I_t} = \delta_{0,\pi} + \delta_{0,\pi} \left(\bar{B}_1'^{-1} \left(Y_t^1 - \bar{A}_1 \right) \right) + \sigma_\pi \epsilon_{3,t} \tag{A.14}$$

Rewriting this equation, yields

$$\frac{I_{t+1}}{I_t} = \underbrace{\delta_{0,\pi} - \delta_{0,\pi} \bar{B}_1'^{-1} \bar{A}_1}_{A_3^*} + \underbrace{\delta_{0,\pi} \bar{B}_1'^{-1} Y_t^1}_{\Phi_{31}^*} X_{t-1} + \underbrace{\Sigma_\pi}_{\Sigma_\pi^*} \epsilon_{3,t} \tag{A.15}$$

We retrieve the OLS estimates of the coefficients in this equation as $\bar{A}_3^*$, Φ_{31}^*, and Σ_π^*.

Lastly, we need to retrieve the equity risk premium from the equity return process. For notational convenience, we repeat the definition of this process.

$$R_t = r_t^N + \eta + \sigma_R \epsilon_{4,t}, \tag{A.16}$$

where r_t^N denotes the monthly nominal short rate and η the equity risk premium. By substituting the definition of the nominal short rate and we derive the reduced form equation,

$$R_t = \eta + \delta_{0,r} - \delta_1' \bar{B}_1'^{-1} \bar{A}_1 + \delta_1' \bar{B}_1'^{-1} Y_t^1 + \sigma_R \epsilon_{4,t}. \tag{A.17}$$

Consequently, the OLS estimates can be mapped as follows,

$$R_t = \underbrace{\eta + \delta_{0,r} - \delta_1' \bar{B}_1'^{-1} \bar{A}_1}_{A_4^*} + \underbrace{\delta_1' \bar{B}_1'^{-1} Y_t^1}_{\Phi_{41}^*} X_t + \underbrace{\sigma_R}_{\Sigma_R^*} \epsilon_{4,t} \tag{A.18}$$

A.3 Bayesian Approach

We apply a Bayesian methodology to our OLS estimates. In this step the parameter uncertainty enters since these reduced OLS estimates are used to estimate the structural parameters. To obtain the distribution for the reduced form of equations, we follow Bauwens and Lubrano (1996) by rewriting these equations into a system of seemingly unrelated regressions. The reduced form of equations can easily be written in the following form,

$$y_i = X_i \beta_i + \epsilon_i, \tag{A.19}$$

for each $i = 1, .., n$ with n denoting the total number of state variables in the system. If the individual time series included in the model have dimension T, then y_i is a vector with $((T-1) \times 1)$ observations, X_i is a matrix with dimensions $((T-1) \times k_i$ with k_i independent variables, β_i consists of a coefficient vector with k_i elements, and ϵ_i is the vector with the associated errors for each observation $(T-1)$. We rewrite this model in two forms in order to draw parameters from the posterior density. By stacking all the observations for each equation i, we can express Equation (A.19) as

$$y = x\beta + \epsilon, \tag{A.20}$$

where $y = (y_1, .., y_n)$ is a vector with dimensions $((T-1)n \times 1)$, $\beta = (\beta_1, .., \beta_n)$ with a vector of k_n elements, $x = \text{diag}(x_1, .., x_n)$ with dimensions $((T-1)n \times k_n)$, and $\epsilon = (\epsilon_1, .., \epsilon_n)$. In the second approach, we write a VAR specification

$$Y = XB + E, \tag{A.21}$$

with $Y = (y_1 ... y_n)$ is a matrix with dimensions $((T-1) \times n)$, $X = (X_1 ... X_n)$ has dimensions $((T-1) \times k_n)$, $B = \text{diag}(\beta_1, .., \beta_n)$ is a matrix with dimensions $(k_n \times n)$ and $E = (E_1 ... E_n)$ is a matrix with dimensions $((T-1) \times n)$.

A.3.1 Uninformative Prior

In deriving the posterior density function of the OLS estimates, we assume an uninformative prior. This prior means that we do not impose any prior believe on these parameters of the model. Hence, the prior function is of the form

$$f(\beta, \Sigma) \propto |\Sigma|^{-(n+1)/2}, \tag{A.22}$$

where Σ denotes the variance-covariance matrix of the error in the VAR model. For this uninformative prior, the marginal posterior density of the parameters can be written as

$$\begin{aligned} \beta|\Sigma &\sim \mathrm{N}(\hat{\beta}, [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}) \\ \Sigma|\beta &\sim \mathrm{IW}(Q, T-1), \end{aligned} \tag{A.23}$$

with

$$\begin{aligned} \hat{\beta} &= [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1} x'(\Sigma^{-1} \otimes I_{T-1})y \\ Q &= (Y - XB)'(Y - XB). \end{aligned}$$

Since the marginal posterior densities of the two parameters β and Σ are not available, we rely on the Block-Gibbs sampling algorithm (See e.g., Bauwens and Lubrano (1996)). Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity as in Bansal and Kiku (2011).

Our final sequence consists of 2000 draws from the posterior density. Using these parameters, we calculate the associated means and variance-covariance matrices of the various horizons. For each of these moments, we determine the optimal allocation strategy. We report the average of portfolio holdings for various horizons and a 95% confidence bounds of these allocations. This procedure results in optimal portfolio allocations that only rely on the observed data.

A.3.2 Informative Prior

Next, we impose a Normal-diffuse prior on the parameters. Since the weight of recent observations bare more importance, we establish a prior on the impact of the OLS estimates. However, we hold a diffuse prior on Σ in Equation (A.19) or the covariance-variance matrix of the coefficients. Formally, we can write

$$\begin{aligned} f(\beta) &\sim \mathrm{N}(\beta_{Prior}, \Omega) \\ f(\Sigma) &\propto |\Sigma|^{-(n+1)/2}, \end{aligned} \tag{A.24}$$

where β_{Prior} denotes the estimates of the prior. Following Zellner (1971) we can write the marginal posterior distributions as follows,

$$\begin{aligned} \beta|\Sigma &\sim \mathrm{N}(\hat{\beta}, \hat{\Omega}) \\ \Sigma|\beta &\sim \mathrm{IW}(Q, T-1), \end{aligned} \tag{A.25}$$

with

$$
\begin{array}{rcl}
\hat{\beta}_{OLS} & = & [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}x'(\Sigma^{-1} \otimes I_{T-1})y \\
\hat{\beta} & = & \hat{\Omega}(\hat{\Omega}^{-1}\beta_{Prior} + [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}\hat{\beta}_{OLS} \\
\hat{\Omega} & = & (\Omega^{-1} + x'(\Sigma^{-1} \otimes I_{T-1})x)^{-1} \\
Q & = & (Y - XB_{OLS})'(Y - XB_{OLS}) + (B - B_{OLS})'X'X(B - B_{OLS}).
\end{array}
$$

Again we rely on the Block-Gibbs sampling technique to derive the marginal posterior densities of the two parameters β and Σ. Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity. The prior estimates are derived from using the OLS estimates on a sample from January 2000 to December 2012.

B Appendix B: Alternative individual specifics

In this section, we analyze the impact of educational level and retirement age in case there is no parameter uncertainty.

B.1 Education level

We explore the differences of education level of the participant as it influences the wage pattern during the accumulation phase. In previous results, the educational level was set to high school level. In this section, we compare the results in case the participant has either a low educational level (no high school) and a high educational level (College level). We use the definitions of Cocco et al. (2005) and calibrate our model using their coefficients $\alpha_1 = 0.1684$, $\alpha_2 = -0.0353$, and $\alpha_3 = 0.0023$ for No high school and $\alpha_1 = 0.3194$, $\alpha_2 = -0.0577$, and $\alpha_3 = 0.0033$ for College.

Table 7 shows that the low and high educational levels require a lower pension contribution than the average educated group in our benchmark model. The fixed contribution level is 5.76% for the low educated group, whereas it is 6.66% for the high educated group. While all groups share the characteristics that the end wage is lower than the average wage, the end wage of the average educated group is only 0.13% lower than the average wage. For the other two groups this difference is much larger, respectively 11.6% for the low educated group and 8.3% for the high educated group. As a result of the wage pattern, both groups require less pension contribution as their ambition level is relatively lower compared to the average wage.

Table 7 confirms our previous observation that a fixed pension contribution can lead to large uncertainty about the replacement rate at retirement. For both educational levels, the lower bound of the 95% confidence interval of the replacement rate is similar to the benchmark level. Likewise, the time-varying contribution scheme is only partial able to compensate for the effect of economic shocks. However, the 95% confidence interval of the contribution is less wide, suggesting that the low and high educational groups experience less volality in their contributions.

Regarding the importance of restrictions on the time-varying contribution, Table 7 shows that the high and low educated groups are less restricted by the fixed contribution bounds. However, the restriction of the fixed lower bound (at 0%) for the low educational level remains more important than in the benchmark case. For the low educated group, the fixed lower bound of 0% is binding in 58.2% of the scenarios at age 64, whereas in the benchmark this is 55.0%. This result indicates that restriction on withdrawal for pension wealth is more important for this group to achieve the pension ambition. Since lower educated groups might have less ability to address financial decisions, these resctriction are even more important to convey to this group.

B.2 Retirement age

Retirement age is an important factor since it gives the individual the opportunity to partially overcome economic shocks. By increasing the retirement age in case of bad scenarios, the participant will prolong the accumulation phase and purchase the real variable annuity with his desired replacement rate. To analyze the effect of retirement age, we extend the accumulation phase by two years in case the 70% replacement rate is not met. In case the desired replacement rate is achieved at age 66, we will assume the participant retires.

Our model shows that in 57.4% of the scenarios the participant can retire with a replacement rate of 70% or more. This result suggests that the decision to delay retirement is likely to occur. Within one year, the participant can retire in 20.7% of the scenarios at age 66 due to a lower annuity price and additional savings. Thus, the probability that the participant can retire at age 65 or 66 with the desired replacement rate is 78.1%. In 8.3% of the scenarios, the participant can retire at age 67 with a replacement rate larger than 70%. Thus, there is a probability of 13.6% the participant has to retire with a replacement rate lower than his desired ambition at age 67. In these bad scenarios, the average replacement rate is 59.5% when the participant retires at age 67. These results show that the participant can increase his probability to retire with a real replacement rate from 57.4% to 86.4% in case he is willing to work one or two additional years. In particularly, delaying retirement to age 66 increases the probability substantially with 20.7%, so that the marginal contribution of working one additional year after age 66 is far lower. This shows that delaying retirement can only partly compensate for the bad scenarios and that delaying retirement beyond age 66 does not have a substantial impact on the replacement rates.

C Appendix C: Tables

Table 1: Estimation of financial market

This table presents the estimation results of the financial market based on a sample period of January 1952 to December 2012. For the short rate, inflation rate and the equity return process, we show the annualized terms.

Parameter	Estimate	Standard error
Dynamics latent factors: $X_t = \Phi_1 X_{t-1} + \Sigma\epsilon_t$		
$\Phi_{1,11}$	0.9917	0.0053
$\Phi_{1,21}$	-0.0279	0.0135
$\Phi_{1,22}$	0.9473	0.0127
Price of Risk: $\Lambda_t = \Gamma_0 + \Gamma_1 X_t$		
$\Gamma_{0,1}$	-0.0878	0.0230
$\Gamma_{0,2}$	-0.0119	0.0281
$\Gamma_{1,11}$	0.0109	0.0104
$\Gamma_{1,12}$	0.0332	0.0016
$\Gamma_{1,21}$	-0.0170	0.0127
$\Gamma_{1,22}$	-0.0294	0.0121
Short rate: $r_t^N = \delta_{0,r} + \delta_r' X_t$		
δ_r	0.0488	0.0026
$\delta_{r,1} \times 100$	0.5106	0.0520
$\delta_{r,2} \times 100$	0.1980	0.1263
Inflation rate: $\pi_t = \delta_{0,\pi} + \delta_\pi' X_t + \sigma_\pi' \epsilon_t$		
$\delta_{0,\pi}$	0.0354	0.0107
$\delta_{\pi,1} \times 100$	0.3471	0.0595
$\delta_{\pi,2} \times 100$	0.2092	0.0948
σ_π	0.0109	0.0009
Equity return: $R_t = r_t^N + \eta + \sigma_R' \epsilon_t$		
η	0.0504	0.0193
σ_R	0.1508	0.0012
Measurement error Yields: $Y_t^N = \bar{A}_n + \bar{B}_n' X_t + \Omega\eta_t$		
ω_{3M}	0.0040	
ω_{6M}	0.0021	
ω_{2Y}	0.0012	
ω_{10Y}	0.0028	

Table 2: Pension contract without parameter uncertainty

This table presents the replacement rates, and contribution for two contribution schemes. The fixed contribution scheme is set to a yearly rate of 6.85% and is based on the historical performance of the portfolio without taking into account economic shocks. The time-varying contribution scheme is determined by the term structure of interest rates and the estimated historical equity risk premium. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% confidence intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	70.0 %	76.8 %	
	median	65.3 %	73.4 %	
	L.B.	35.3 %	45.2 %	
	U.B.	124.2 %	127.5 %	
Average Contribution	25		5.83 %	
	35		6.96 %	
	45		7.69 %	
	50		8.57 %	
	55		9.88 %	
	60		11.61 %	
	64		12.67 %	
			L.B.	U.B.
C.I. interval Contribution	25		4.07 %	8.01 %
	35		0.00 %	23.55 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		10.40 %	0.90 %
	45		28.80 %	2.90 %
	50		35.80 %	6.40 %
	55		43.40 %	14.30 %
	60		48.60 %	26.80 %
	64		55.00 %	38.80 %

Table 3: Impact of risk profile on the pension contract

This table presents the replacement rates, and contribution for two contribution schemes. The fixed contribution scheme is set to a yearly rate of 9.72% and is based on the historical performance of the portfolio without taking into account economic shocks. The low risk portfolio consists of 30% equity and 70% bonds. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% confidence intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	71.4 %	73.1 %	
	median	69.4 %	71.6 %	
	L.B.	42.4 %	51.0 %	
	U.B.	111.7 %	103.0 %	
Average Contribution	25		8.65 %	
	35		9.87 %	
	45		10.11 %	
	50		10.86 %	
	55		11.37 %	
	60		13.04 %	
	64		13.16 %	
			L.B.	U.B.
C.I. interval Contribution	25		6.14 %	11.71 %
	35		0.00 %	30.00 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		6.50 %	2.90 %
	45		22.00 %	5.50 %
	50		28.80 %	10.80 %
	55		35.60 %	18.30 %
	60		42.60 %	30.70 %
	64		53.40 %	40.50 %

Table 4: Pension contract with parameter uncertainty of the equity risk premium

This table presents the replacement rates, and contributions for two contribution schemes when the equity risk premium is uncertain. The parameter uncertainty is based on an uninformed prior using the sample period from 1952-2012. The time-varying contributions are based on the term structure of interest rates and the historical equity risk premium. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% credibility intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	73.1 %	80.2 %	
	median	64.0 %	74.1 %	
	L.B.	26.6 %	41.3 %	
	U.B.	170.2 %	154.8 %	
Average Contribution	25		5.83 %	
	35		6.95 %	
	45		7.64 %	
	50		8.53 %	
	55		9.82 %	
	60		11.54 %	
	64		12.48 %	
			L.B.	U.B.
C.I. interval Contribution	25		4.08 %	8.01 %
	35		0.00 %	23.60 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		10.41 %	0.89 %
	45		30.00 %	2.80 %
	50		37.99 %	6.61 %
	55		45.01 %	14.51 %
	60		49.84 %	26.89 %
	64		56.32 %	39.09 %

Table 5: The pension contract with parameter uncertainty of all financial parameters

This table presents the replacement rates, and contribution for two contribution schemes when all financial parameters are affected by parameter uncertainty using an uninformative prior. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% credibility intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	75.0 %	83.8 %	
	median	65.2 %	76.4 %	
	L.B.	26.5 %	42.1 %	
	U.B.	177.3 %	166.0 %	
Average Contribution	25		6.51 %	
	35		7.52 %	
	45		7.97 %	
	50		8.64 %	
	55		9.42 %	
	60		10.74 %	
	64		11.31 %	
			L.B.	U.B.
C.I. interval Contribution	25		4.49 %	9.07 %
	35		0.00 %	25.52 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		10.82 %	1.35 %
	45		30.68 %	3.76 %
	50		39.45 %	7.73 %
	55		47.57 %	14.52 %
	60		53.44 %	25.63 %
	64		60.18 %	35.61 %

Table 6: Prior view and parameter uncertainty

This table presents the replacement rates, and contribution for two contribution schemes. In this setting, parameter uncertainty affects all financial parameters and is incorporated using a Normal-Diffuse prior calibrated on the period from Jan 2000 to Dec 2012. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% credibility intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	65.1 %	76.6 %	
	median	56.0 %	70.5 %	
	L.B.	21.7 %	38.0 %	
	U.B.	160.4 %	150.4 %	
Average Contribution	25		6.38 %	
	35		7.65 %	
	45		8.55 %	
	50		9.65 %	
	55		11.08 %	
	60		13.17 %	
	64		14.09 %	
			L.B.	U.B.
C.I. interval Contribution	25		4.59 %	8.60 %
	35		0.00 %	24.14 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		8.36 %	1.04 %
	45		25.60 %	3.53 %
	50		32.99 %	8.01 %
	55		40.02 %	17.49 %
	60		44.84 %	32.80 %
	64		50.86 %	44.82 %

Table 7: Impact of educational level on the pension contract

This table presents the replacement rates (Repl. rate), and contribution for two contribution schemes when parameter uncertainty is ignored. The low educational level correspond to no high school and high educational level to College level. The fixed yearly contribution for the low educational level is 5.76%, whereas the contribution for the high educational level is 6.66%. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% confidence intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

		Educational level					
		Low			High		
Contribution Scheme		Fixed	Varying		Fixed	Varying	
Repl. rate at 65	mean	69.9 %	78.3 %		70.1 %	77.0 %	
	median	65.0 %	74.7 %		65.4 %	73.6 %	
	L.B.	35.2 %	45.8 %		36.0 %	44.9 %	
	U.B.	125.6 %	131.3 %		123.1 %	126.8 %	
Average Contribution	25		4.88 %			5.69 %	
	35		5.97 %			6.69 %	
	45		6.79 %			7.26 %	
	50		7.71 %			8.19 %	
	55		9.09 %			9.58 %	
	60		10.83 %			11.36 %	
	64		11.69 %			12.55 %	
			L.B.	U.B.		L.B.	U.B.
C.I. interval Contribution	25		3.38 %	6.76 %		4.02 %	7.76 %
	35		0.00 %	21.14 %		0.00 %	21.39 %
	45		0.00 %	28.29 %		0.00 %	27.49 %
	50		0.00 %	30.00 %		0.00 %	30.00 %
	55		0.00 %	30.00 %		0.00 %	30.00 %
	60		0.00 %	30.00 %		0.00 %	30.00 %
	64		0.00 %	30.00 %		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)		L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %		0.00 %	0.00 %
	35		12.40 %	0.50 %		7.40 %	0.50 %
	45		32.40 %	2.30 %		26.70 %	1.40 %
	50		40.50 %	5.70 %		34.00 %	5.50 %
	55		46.60 %	12.70 %		42.90 %	12.40 %
	60		52.00 %	24.10 %		49.10 %	25.50 %
	64		58.20 %	36.20 %		55.50 %	39.30 %

Figure 1: Yield curve

This figure present two time series of yields and the average yield curve for the sample period of Jan 1952 to Dec 2012. For the yields, we plot both the 3-months and 5 years bond yield.

(a) Yield rates (b) Average yield curve

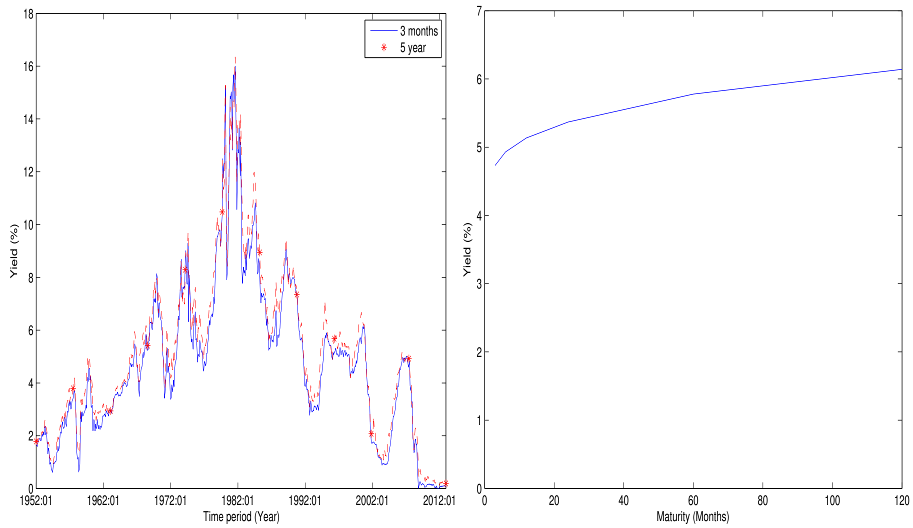


Figure 2: Pension contribution without restrictions to the contribution level

This figure presents the contribution during the accumulation phase if parameter uncertainty is ignored. Our second graph denotes the corresponding pension wealth over the life time. The time-varying contribution is not restricted by the fixed bounds of 0% and 30% as in the benchmark contract. Real wealth is denoted in dollars and the contribution in percentage of the annual wage.

(a) Contribution (b) Real pension wealth

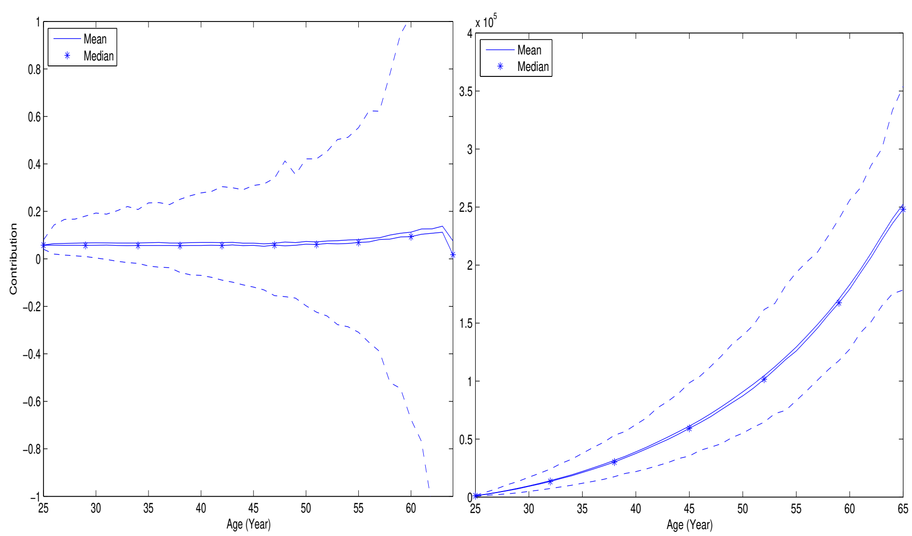
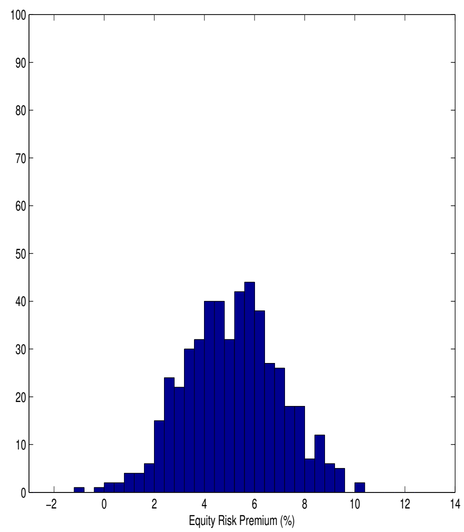


Figure 3: Equity risk premium

This figure presents the marginal posterior distribution of the equity risk premium. In the first graph, an uninformative prior is assumed, which assigns equal weight to all observations. In the second graph, the Normal-diffuse prior adds more weight to the period from Jan 2000 up to Dec 2012. The distribution in the first graph is centered around 5.09% with a standard deviation of 1.95% and in the second graph it is centered around 4.61% and has a standard deviation of 1.98%.

(a) Equal weight



(b) Post 2000 regime

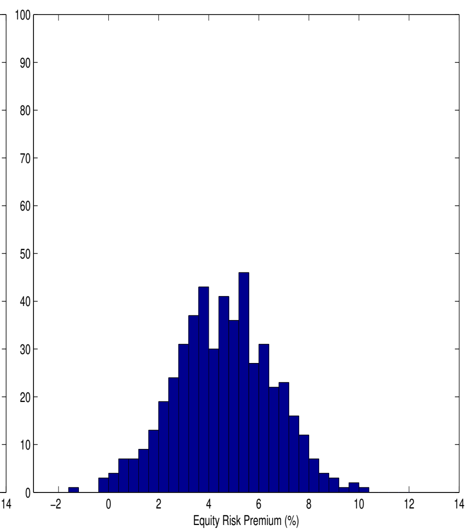


Figure 4: Inflation rate

This figure presents the marginal posterior distribution of the mean inflation rate. In the first graph, an uninformative prior is assumed, which assigns equal weight to all observations. In the second graph, the Normal-diffuse prior adds more weight to the period from Jan 2000 up to Dec 2012. The distribution in the first graph is centered around 3.54% with a standard deviation of 0.14% and in the second graph it is centered around 3.61% and has a standard deviation of 0.14%.

(a) Equal weight (b) Post 2000 regime

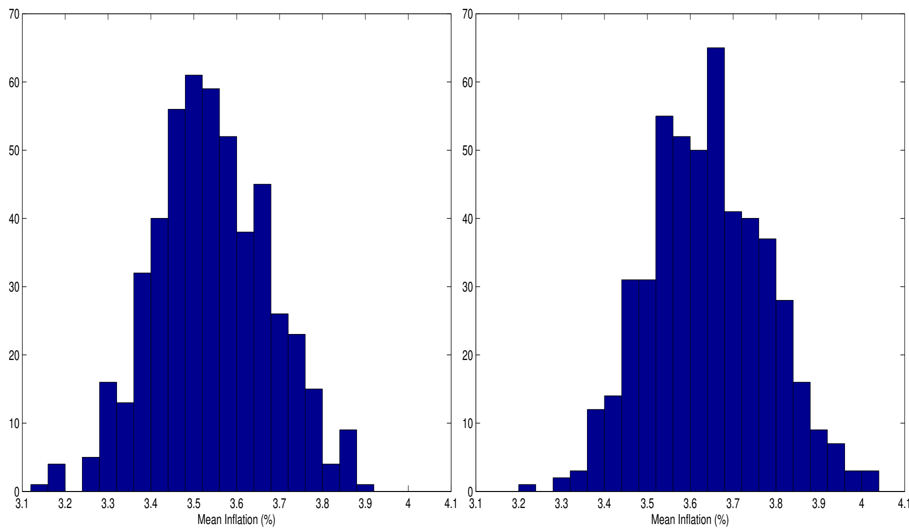


Figure 5: The effect of an informative prior on the contribution levels

This figure presents the mean, median, and 95% credibility interval of the contributions levels for two different regimes on parameter uncertainty. In the first regime (blue line), parameter uncertainty is estimated with an uninformative prior, which holds equal weights to all observations. In the second regime (red line), an informative prior is used that assigns more weight to the period from Jan 2000 to Dec 2012.