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The Inflation Risk Premium

The Impact of the Financial Crisis

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The inflation risk premium: The impact of the financial crisis

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Abstract

This paper examines the inflation risk premium in affine term structure models. By estimating empirical distributions for the inflation risk premium using a new Bayesian methodology, we find a wide range of likely estimates. Credibility intervals for 5 year maturity range from about -95 to 88 basis points in the UK and -4 to 119 basis points in the US during the period of 2004-2012. Our results show that affine term structure models are unable to capture the inflation risk premium accurately. To that end, we use a Bayesian methodology to show how the financial crisis 2008 impacts the uncertainty regarding inflation risk premium. We find a substantial upward shift in the inflation risk premium in the UK while an downward shift in the US. In particular, credibility intervals shift to -105 to 150 in the UK and -50 to 92 basis points in the US.

Keywords: Affine term structure models, real interest rates, inflation risk premia

JEL Classifications: E31, E43, and G12

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1 Introduction

In most debt financing transactions, inflation risk is allocated to the debt holder instead of the debt issuer. In case of such nominal debt financing, the debt holder demands, in addition to the expected inflation rate, a premium associated to the expected inflation risk for that maturity. If uncertainty about future inflation is substantial, debt issuers are forced by markets to issue inflation-linked debt to immunize the holders from inflation risk[1]. For governments and institutional investors, the market inflation risk premium is an important factor in issuing long term debt or hedging liabilities that are exposed to inflation. In particular, the magnitude of the inflation risk premium determines whether governments can be cost efficient by issuing inflation-linked bonds. Due to the increasing market liquidity of inflation-linked bonds in the UK and US, studies have begun to empirically examine the inflation risk premium (e.g. Evans (2003) for the UK, whereas Ang, Bekaert, and Wei (2008), and D'Amico, Kim, and Wei (2010) for the US). Since this premium cannot be directly observed in the market and relies on model specifications, the literature has offered a broad range of likely estimates for the inflation risk premium. However, two additional factors contribute to the difficulty of estimating the premium, namely the small sample including the financial crisis and liquidity issues of the inflation-linked bonds.

In this paper, we take an explicit approach to incorporate uncertainty about model specifications and examine the inflation risk premia in a Bayesian setting to correct for a small sample bias. To explain the large range of inflation risk premia reported in the literature, we determine empirical distributions of the inflation risk premium and present statistical ranges for these estimates. Our analysis starts with extracting the inflation risk premium implied by the nominal and real term structure of interest rates of both the UK and US markets in the period from 2004 to 2012. The typical approach in the literature is to identify real interest rates by inflation-linked bonds, but we use inflation swap rates to determine the real rates instead. Our approach is motivated by liquidity shocks observed in the TIPS market prior to 2003 and during the financial crisis (See e.g D'Amico et al. (2010)). As documented by Haubrich, Pennacchi, and Ritchken (2011), US inflation swap rates responded less sharply to liquidity shocks in the market. Consequently, our estimation of the inflation risk premium is less influenced by liquidity issues. To enhance our model characteristics, we incorporate empirical nominal and real term premia to our measurement equations as well. This approach allows us to verify to what extent affine term structure models are able to capture the dynamics in real term premia.

Theory suggests a link between macroeconomic development and interest rates. Including macroeconomic factors in affine term structure models allowing to capture these dynamics and can improve the explanation of interest rates (Ang and Piazzesi, 2003). Prior literature has focused on the predictability of nominal bonds with the help of information from the macro economy (see e.g. Cochrane and Piazzesi (2005), Joslin, Priebsch, and Singleton (2010), Duffee (2011)). We contribute to the analysis by investigating the effect of macroeconomic factors on the inflation risk premium. The macroeconomic factors in our model have an impact on the unexpected inflation shock that is associated with the inflation risk premium. Consequently, the inflation risk premium depends on the included macroeconomic variables. We estimate

[1] Inflation-linked bonds have been introduced in South American countries to raise long term capital, e.g. Chile in 1951 and Argentina in 1973. Due to high uncertainty about future inflation in these countries, debt holders forced the issuers to bear inflation risk. For a further discussion, see e.g. Garca and Rixtel (2007).

a benchmark model with only inflation data and compare this to two models that add either macroeconomic data or survey data related to macroeconomic developments. In our Macro model we use inflation factors that relate to the current condition of the macro economy and market volatility, whereas in the Survey model we incorporate expectations of the macroeconomic development such as inflation and economic confidence indicators. This approach allows us to link the inflation risk premium in these two models to macroeconomic developments and examine their impact on the inflation risk premium.

To evaluate these models, we first verify whether the Macro and Survey model improve the explanation of real interest rates compared to our benchmark. We confirm the importance of adding macroeconomic expectations to our model in the US during the financial crisis in 2008. Although our Survey model only slightly improves the measurement errors of the real rates across our whole sample, during the financial crisis the Survey model improves the fit by about 5% compared to the benchmark model. As a result of this improvement during the financial crisis, the Survey model leads to a better fit over the whole sample period. The Macro model, on the other hand, does not outperform the benchmark model. This suggests that expectations of the macroeconomy have more explanatory power for the real rate innovations in the US than actual macroeconomic development. Surpringly, we find that our Macro model improves the explanation of real rates in the UK rather than the Survey model. This result is driven by the timing difference of the impact of the financial crisis in the UK. While in the US interest rates started to decrease since 2007, the UK nominal rates rather sharply decreased in 2008. Since the macroeconomic factors improve the explanation of real rates, we investigate the effects on the inflation risk premium among these models.

To assess the small sample bias of the inflation risk premium, we first estimate our models by ignoring parameter uncertainty. Based on this methodology, we observe a declining trend for the inflation risk premium in all models for both markets. Prior to the financial crisis the inflation risk premia was positive in both markets, however during the financial crisis the inflation risk premia became negative. Although we find a decrease in the inflation risk premium during the crisis, it is hard to capture the magnitude of the inflation risk premium due to liquidity issues. After the financial crisis, we observe that rates increase but remain lower than pre-crisis levels. Where the US inflation risk premium returns to a positive level, the inflation risk premium in the UK remains negative. One of the factors leading to lower inflation risk premia in both markets is the drop in the nominal rates after the crisis. Many central banks implemented a zero interest rate policy after the financial crisis to stimulate macroeconomic development. It has been suggested that in order to recover from these low rates, central banks should refrain from deflationary measures. One of the consequences of such policy is an increase of inflation (see e.g. Krugman (1998) and Eggertsson and Woodford (2003)). The inflation risk associated with nominal rates remaining at zero bounds could explain the increasing inflation risk premia after the financial crisis. However, since the post crisis levels of the inflation risk premia remain quite low compared to pre-crisis levels, the markets do not fully reflect the inflation risk associated with low nominal rates.

To investigate the effect of parameter uncertainty on the inflation risk premium, we estimate our models for both markets using a Bayesian approach. Limited data on inflation-linked derivatives is likely to introduce a small sample bias. In particular, the high persistence of interest rates can aggravate this issue in the estimation of affine term structure models (Joslin, Singleton, and Zhu (2011)). Accordingly, we adopt a Bayesian approach that can reduce such biases and assess its impact on the inflation risk premium. Our findings suggest a wide range of likely estimates for the inflation risk premia in both the US and the UK. The 95% credibility intervals of our empirical distributions range from -95 to 88 basis points in the UK, whereas in the US we find a interval of -4 to 119 basis points over the sample period from 2004 to 2012. Although the mean of these distributions for the 5 year risk premium are about -8 basis points for the UK and 74 basis points in the US, the credibility intervals of these distributions include both positive and negative estimates. While these ranges quantify a large dispersion for the estimates of the inflation risk premium, we can conclude that the 5 year inflation risk premium in the US is positive with a probability of 97.2%. In the UK, we find a probability of 42 % that the 5 year inflation risk premium is negative. As a result, credibility intervals show wide ranges for estimates for the inflation risk premium. Hence, there is large uncertainty concerning the point estimates.

Our methodology and findings explain the wide range of estimates found by the affine term structure literature. For example, Ang et al. (2008) find 115 basis points for the 5 year inflation risk premium, whereas D'Amico et al. (2010) find 36 basis points (see e.g. Bekaert and Wang (2010) for an overview in the US.). Since these US estimates fall within the credibility intervals for the inflation risk premia, it is hard to distinguish between the point estimates. Fewer studies have been conducted on the UK market. Evans (2003) estimates a negative inflation risk premium, although Risa (2001) and Joyce, Lildholdt, and Sorensen (2010) find substantial positive inflation risk premium of about 184 and 100 basis points. Our Bayesian methodology confirms their result of wide intervals as well. Since macroeconomic factors can improve the explanatory power of real rates, we examine the impact of the Macro and Survey model on the inflation risk premium. Our models reveal that the addition of macroeconomic factors leads to a wider dispersion of the inflation risk premium. The empirical distributions of the inflation risk premium are especially more platykurtic in the UK than in the US, resulting in larger credibility intervals than in the benchmark. For example, the results of our Macro model suggest a 95% credibility range of -131 to 143 basis points in the UK, whereas in the US our Survey model suggest an interval of -4 to 127 basis points. As a result, the impact of the macroeconomic variables leads to larger uncertainty about the inflation risk premium estimates in our models.

Given the wide dispersion for the estimate of the inflation risk premium, we investigate how the financial crisis impacts the uncertainty of the inflation risk premium. After the nominal interest rates decreased rapidly to relatively low levels for both the UK and US, both markets entered into a low nominal interest rate regime. By attaching more importance to the post crisis observations, we capture a shift in the inflation risk premium for the post crisis regime compared with the credibility intervals of our previous results. In the first part of our analysis we assign equal weights to each observation in our data period. The benefit of our approach is that we can use the entire sample period to identify our model, since discarding observations prior to the crisis and reestimating our model would be infeasible due to limited data. In the US we find a downward effect of about 36 basis points, shifting the mean of the distribution to 38 basis points. The 95% credibility interval ranges from -50 to 92 basis points, increasing the dispersion of the inflation risk premium estimate. As a consequence, negative estimates of the inflation risk premia are more likely. Surprisingly, we find in the UK an upward shift, increasing the mean to about 13 basis points. This leads to a 95% credibility interval from -105 to 150 basispoints. Again, we document a substantial dispersion for the estimate. This new empirical evidence suggests that the low interest rate regime after the financial crisis does not have a similar effect in both markets for the inflation risk premium. While an upward shift, as documented in the UK, would be expected due to the uncertainty of macro inflation risk and low nominal interest rates, our empirical evidence does not support this for the US.

Our contribution to the literature is threefold. First, we are the first to quantify the uncertainty associated with the inflation risk premium and quantify the impact of the financial crisis on the inflation risk premium. This paper demonstrates how to extract the inflation risk premium from an affine term structure model of interest rates (Duffie and Kan, 1996) by using a Bayesian methodology. To this end, we introduce the Chi-squared estimation methodology of Hamilton and Wu (2012), which we apply to a term structure of interest rates with both nominal and real rates. This alternative approach to the typically Maximum Likelihood estimation allows us to employ the Bayesian methodology more easily. Since this methodology employs a two step estimation, we can easily address the issue of small sample bias in our sample. Second, we use a unique dataset of inflation swap rates to identify the real rate for both the UK and US markets. While inflation swap rates have been used by Haubrich et al. (2011), their study only focuses on the US market. Instead, we combine the insights of the macroeconomic literature on nominal interest rates (see e.g. Ang and Piazzesi (2003)) to study the inflation risk premium in both markets. In particular, we expand the literature on the inflation risk premium by showing how similar macroeconomic factors influence cross these markets .

Finally, we contribute to the literature by explicitly including nominal and real term premia. To enhance identification of our structural parameters in our small sample, we use data-implied nominal and real term premia using Campbell-Shiller regressions, and exploit these in our model (Campbell and Shiller, 1991). While others rely on penalizing the maximum likelihood function to generate reasonable term premia (see e.g. Chernov and Mueller (2008)), we explicitly match our model implied and data implied term premia in our Chi-squared estimation methodology. Comparing our term premia to a study that mostly relates to our sample period (Haubrich et al., 2011), we find lower term premia for maturities of 5 and 10 years. As for the coefficients of the Campbell-Shiller regressions for the real returns, we find decreasing coefficients across maturity for both the UK and US market. This evidence is similar to the nominal pattern.

To summarize, we present a novel framework in which we formally quantify the uncertainty associated with estimating the inflation risk premium in affine term structure models. Overall, our results indicate a wide dispersion for the estimate of the inflation risk premium. Therefore, our study raises questions about the economic implications of the inflation risk premia for governments and institutional investors.

The remainder of this paper is organized as follows. Section 2 introduces the term structure model, estimation methodology and the describes the data used in our empirical analysis. Section 3 presents our estimation results without parameter uncertainty and describes the effect of model choice on the real term premia and inflation risk premia. Section 4 analyzes the impact of parameter uncertainty on the inflation risk premia. This section also reveals the shift on the inflation risk premia after the financial crisis. Our conclusions and policy implications follow in Section 5.

2 Methodology

In this section we introduce our Gaussian affine term structure model which is used to identify the inflation risk premium. Subsequently, we describe the minimum Chi-squared methodology to estimate our model. Lastly, we describe our data and the economic factors.

2.1 Discrete Time Gaussian Affine Model

We use monthly frequency in our models. To estimate real interest rates, we incorporate both latent, X_t^L, and economic factors, X_t^{EC}, as state variables. We assume that the nominal bond price with maturity n at time t, $P_t^N(n)$, is exponentially affine in two latent state variables

$$P_t^N(n) = \exp(A_n^N + B_n'^N X_t), \tag{1}$$

where $X_t = [X_t^{EC}, X_t^L]$ denotes a vector with economic variables and latent state variables. We restrict $B_n'^N$ in such a way that only latent state variables can influence the nominal bond prices. Since real bond prices will be dependent on inflation in our framework, we need to incorporate economic factors that explain inflation. We assume that the state variables follow a vector autogressive model of order 1,

$$\begin{bmatrix} X_t^{EC} \\ X_t^L \end{bmatrix} = \begin{bmatrix} \Phi_0^{EC} \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi_1^{EC} & \Phi_1^{EC,L} \\ 0 & \Phi_1^L \end{bmatrix} \begin{bmatrix} X_{t-1}^{EC} \\ X_{t-1}^L \end{bmatrix} + \begin{bmatrix} \Sigma^{EC} & 0 \\ 0 & I_2 \end{bmatrix} \epsilon_t, \tag{2}$$

where we let the economic factors to be correlated with the latent state variables through parameter $\Phi_1^{EC,L}$. We set Φ_0^L to zero and the variance covariance matrix of the latent state variables equal to the identity matrix, I_2, for identification purposes (see e.g. Duffee (2002) and Dai and Singleton (2000). Since we allow for the latent factors to influence the macroeconomic variables, we are able to capture the link between macroeconomic dynamics and the latent factors as for example in Ang and Piazzesi (2003).

In order to derive the no-arbitrage nominal bond prices, we follow the literature of affine term structure models. We postulate the nominal affine pricing kernel as

$$M_{t+1}^N = \exp\left(-r_t^N - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right), \tag{3}$$

where r_t^N denotes the monthly nominal short rate and λ_t denotes the time dependent price of risk. Since we need to price real bonds as well, we need to establish a relation between the nominal and real pricing kernel. The difference between the nominal and real pricing kernel is determined by the impact of realized inflation, PI_t . Thus, the following condition needs to be satisfied between the nominal and real pricing kernel

$$M_t^R = M_t^N PI_t. \tag{4}$$

In order to derive the real pricing kernel, we need to define the inflation process. We specify the inflation process as

$$PI_{t+1} = \exp(\delta_\pi + \delta_{1,\pi}' X_t + \sigma_\pi' \epsilon_{t+1}). \tag{5}$$

The parameter $\delta'_{1,\pi}$ determines the impact of the state variables on inflation. By substituting this process in the definition of the real pricing kernel as given in Equation (4), we can derive the following real pricing kernel

$$M^R_{t+1} = \exp\left(-r^N_t + \delta_{0,\pi} + \delta'_{1,\pi} X_t - \frac{1}{2}\lambda'_t\lambda_t - (\lambda'_t - \sigma'_\pi)\epsilon_{t+1}\right). \tag{6}$$

We can rewrite the real pricing kernel in its typical form as defined in Equation (3) but with equivalent real parameters instead. As a consequence of rewriting, the real short rate consists of four components, namely the nominal rate, expected inflation, an inflation risk premium and a convexity term. The inflation risk premium follows due to the unexpected inflation shock. The convexity term follows due to the lognormality assumption. In mathematical notation, the relation between the nominal and real short rate is described as

$$r^R_t = r^N_t - (\delta_{0,\pi} + \delta'_{1,\pi} X_t) + \sigma'_\pi\lambda_t - \frac{1}{2}\sigma'_\pi\sigma_\pi. \tag{7}$$

The term $\sigma'_\pi\lambda_t$ denotes the inflation risk premium for the short rate, whereas $\frac{1}{2}\sigma'_\pi\sigma_\pi$ is the convexity term. This equation is known as the modern Fisher equation[2].

Furthermore, we assume the monthly nominal short rate to be an affine function of latent state variables similarly as with the nominal bond prices. The nominal short rate can therefore be written as

$$r^N_t = \delta^N_{0,r} + \delta'^N_{1,r} X_t, \tag{8}$$

where δ^N_1 is restricted to incorporate only the impact of the latent factors. Lastly, we assume the price of risk to be affine in the state variable, so that

$$\lambda_t = \Gamma_0 + \Gamma_1 X_t. \tag{9}$$

Substituting the affine assumptions of the short rate and the price of risk into the pricing kernel in Equation (3) leads to the typical form of pricing kernels used in affine term structure models.

Following the no-arbitrage relation between bond prices between two time periods, we can derive the no-arbitrage prices for a nominal bond with maturity n at time t. The no-arbitrage relation can be expressed as

$$P^N_t(n) = E_t\left[M^N_{t+1} P^N_{t+1}(n-1)\right]. \tag{10}$$

By substituting the nominal pricing kernel in Equation (3) and substituting the affine bond prices as defined in (1), a recursion for the coefficients of the bond prices can be derived, which is shown in Appendix A. Using the relation between continuously compounded yields and bond prices, we can express the yield curve as a function of maturity n by the following function

$$Y^N_t(n) = \bar{A}^N_n + \bar{B}'^N_n X_t. \tag{11}$$

Equivalently, we can derive the real yields by using the real pricing kernel. For further details, we refer to Appendix A.

[2] In the derivation of Fisher the difference between the nominal and real yields is only influenced by the inflation component. To distinguish his contribution from the recent affine term structure models, this equation is typically denoted as the modern Fisher equation.

2.2 Term Premia

To enhance identification of the parameters in our model, we incorporate nominal and real term premia. The nominal term premium is defined as the difference between the expected return of the bond minus the nominal short rate. We can express term premium, TP, for a nominal bond in yields as follows

$$TP_t^N(n) \equiv \mathrm{E}_t\left[R_t^N(n)\right] - r_t^N = \left[Y_t^N(n) - \mathrm{E}_t\left[Y_{t+1}^N(n-1)\right]\right](n-1) + \text{Slope}_t(n), \quad (12)$$

where the $\text{Slope}_t(n) = \left[Y_t^N(n) - r_t\right]$. Typically, the Campbell and Shiller (1991) long-horizon regressions are used to analyze bond return predictability and allow to identify the term premia (See e.g. Haubrich et al. (2011)). These regressions explain the changes of the bond yields by the slope of the term structure. Thus, for each nominal bond with maturity n, we can write the regression as

$$Y_{t+1}^N(n-1) - Y_t^N(n) = \beta_0 + \beta_1 \frac{\text{Slope}_t(n)}{n-1} + \eta_{t+1}. \quad (13)$$

This equation allows us to compute the expectation for the bond and determine the nominal term premia as defined in Equation (12) for our sample. By substituting the nominal bonds for real bonds in Equations (12) and (13), we can equivalently determine the real term premia.

2.3 Estimation procedure

We estimate our model using the Chi-squared methodology as proposed by Hamilton and Wu (2012). To identify the latent state variables, we distinguish between yields measured with and without error as proposed by Chen and Scott (1992). In the first step we use an ordinary least squares method (OLS) to estimate the VAR process of the implied state variables and to regress the implied latent state variables on the yields measures with errors. Subsequently, we link these OLS estimates with the coefficients implied by the structural parameters of our model and minimize the distance using a Chi-squared objective function.

Let Y_t^1 denote the vector containing the yields measured without error, and the remaining yields, Y_t^2, will be measured with error. The state and measurement equations can be written as the following system of equations

$$\begin{bmatrix} Y_t^1 \\ Y_t^2 \\ X_{t+1} \\ TP_t \end{bmatrix} = \begin{bmatrix} \bar{A}^1 \\ \bar{A}^2 \\ \Phi_0 \\ \bar{A}^4 \end{bmatrix} + \begin{bmatrix} \bar{B}'^1 \\ \bar{B}'^2 \\ \Phi_1 \\ \bar{B}'^4 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ \Omega_2 \\ \Sigma \\ \Omega_4 \end{bmatrix} \eta_t, \quad (14)$$

where Ω_2 and Ω_4 are diagonal matrices denoting the variance of the error measurement with $\eta_t \sim \mathrm{N}(0, I)$. Coefficients $\bar{A}^4$ and $\bar{B}'^4$ determine the model implied term premia and are reported in Appendix B. We can rewrite this system of equations by substituting the affine relation between yields and factors as given in Equation (11). As a result, we can derive the reduced system of equations

$$
\begin{array}{lcllllll}
Y_t^1 & = & A_1^* & + & \Phi_{11}^*\, Y_{t-1}^1 & + & \Omega_1^*\, \epsilon_{1t}^*, & \\
Y_t^2 & = & A_2^* & + & \Phi_{21}^*\, Y_t^1 & + & \Phi_{2EC}^*\, X_t^{EC} & + \; \Omega_2^*\, \epsilon_{2t}^*, \\
X_t^{EC} & = & \Phi_0^* & + & \Phi_{31}^*\, Y_t^1 & + & \Phi_{3EC}^*\, X_{t-1}^{EC} & + \; \Sigma_{EC}^*\, \epsilon_{ECt}^* \\
TP_t & = & A_4^* & + & \Phi_{41}^*\, Y_t^1 & + & \Phi_{4EC}^*\, X_t^{EC} & + \; \Omega_4^*\, \epsilon_{4t}^*,
\end{array}
\tag{15}
$$

where the reduced parameters A^*, Φ^*, Ω and Σ^* have model coefficients implied equivalents, which are derived in Appendix B. Applying OLS to these equations yields the estimates of the reduced form equations. These reduced form parameters can be used to derive the structural parameters by minimizing the distance between the OLS estimates and the coefficients implied the structural parameters.

To define the Chi-squared objective function, let π denote the vector containing the estimates of the reduced OLS parameters. The estimates of the reduced form parameters are derived from the full information maximum likelihood, so that $\hat{\pi} = \arg\max_{\pi} L(\pi; Y)$ with the function $L(\pi; Y)$ denoting the log likelihood for the entire sample. Following Hamilton and Wu (2012), we define the linearized minimum Chi-squared estimation as

$$
\min_{\theta} T\,[\hat{\pi} - \psi - \Psi\theta]'\, R\,[\hat{\pi} - \psi - \Psi\theta],
\tag{16}
$$

with

$$
\psi = g(\theta_0) - \Psi\theta_0 \text{ and } \Psi = \left.\frac{\partial g(\theta)}{\partial \theta'}\right|_{\theta=\theta_0}.
$$

The function $g(\theta)$ denotes the transformation of the structural parameters θ into a vector of the reduced form parameters as shown in Equation (15). In order to derive the variance of the estimate $\hat{\theta}$, we approximate it by using $T^{-1}\left(\hat{\Psi}'\hat{R}\hat{\Psi}\right)^{-1}$ with $\hat{\Psi} = \partial g(\theta)/\partial\theta'|_{\theta=\hat{\theta}}$. In Appendix C we provide the details on the estimation methodology. Since the minimum Chi-squared estimation and the MLE are asymptotically equivalent, the minimum Chi-squared estimation has the equivalent asymptotic optimality properties of the MLE. Consequently, the variances of the estimates have similar properties as the MLE derived variances.

We motivate this estimation approach by the observation of the difficulties with numerical optimization of the ML function (Aït-Sahalia and Kimmel, 2010). One of the issues is the persistence of the latent factors causing the MLE function to be flat for autoregressive parameters of the latent factors. Consequently, grid searching techniques are quite vulnerable for path dependency and hence finding local minima (see e.g. for discussions Kim (2008) and Joslin et al. (2011)). The Chi-squared method allows to identify the structural parameters by their mapping into reduced form parameters.

Since the minimum Chi-squared approach relies on the initial parameters of the OLS estimation, parameter uncertainty enters at the first stage of our estimation procedure. In order to account for small sample bias, we adopt a Bayesian approach to the first step estimates. To draw from the marginal posterior distribution of the reduced form parameters, we employ a Gibbs sampler. For each set of obtained reduced form parameters, we estimate the associated structural parameters using the Chi-squared objective function. Consequently, we can determine the posterior density of the inflation risk premium. We use an uninformative prior to analyze parameter uncertainty. Additionally, to study the impact of the post crisis period on the inflation risk premium, we adopt a Normal-diffuse prior that adds more weight to the observations in the post crisis period. For details on the Gibbs sampler applied to the first step reduced form parameters, we refer to Appendix D.

2.4 Data

Our data sample ranges from July 2004 up to December 2012 for both markets. While inflation swap rates have been traded since 2001, the market became only mature since the beginning of 2004. In 2004 the aggregated notional amounts in the inflation swap market doubled to about 50 billion Euro, compared to 2003[3]. While Haubrich et al. (2011) use April 2003 as starting date, we exclude data from 2003 and early 2004 for potential liquidity issues.

Inflation swap rates provide a good alternative to the identification of real rates instead of relying on inflation-linked bonds. Although inflation-linked bonds in the US have been available since 1999, studies estimate that early interest rates are substantially affected by liquidity issues. Gürkaynak, Sack, and Wright (2010) and D'Amico et al. (2010) show that a liquidity premium of the TIPS is only negligible after 2004, substantially reducing the sample size. This evidence is in line with the observation of Roush and Ezer (2008) who reports that the TIPS markets increased substantially after 2004-2005. However, the bankruptcy of Lehman Brothers has induced a downward price pressure in the TIPS market as their inventory had to be unwinded (Campbell and Viceira (2009)). As a result of liquidity the larger implied real rates will lead to a smaller inflation risk premium. The bid-ask spreads of inflation-swap rates on the other hand remained quite unaffected by the financial crisis, indicating that inflation swap rates offer a more accurate assessment of the real rate than the TIPS.

To enhance comparability of our results, we employ a similar sample period for the UK market. While not many studies have been conducted on the liquidity of the UK inflation-indexed Gilts, it is argued by Greenwood and Vayanos (2010) that the 2004 UK pension reform had a great effect on the liquidity in the UK inflation-linked bond market. Regulation required pension funds to discount their liabilities at long-term real rates, increasing demand for inflation-indexed Gilts to hedge their exposure. Since liquidity effects in the UK market may be affecting real rates, inflation swap rates can offer an alternative to identify the real rates.

For the US market we use 9 nominal yield series and 5 real yield series, namely the 1, 3 and 6 months Treasury Bill and the zero coupon bonds with maturities of 1, 2, 3, 5, 7 and 10 years. Our data of the US nominal government zero coupon yield curve is taken from Gürkaynak et al. (2010)[4]. To identify the real interest rates we rely on inflation swap rates as suggested by Haubrich et al. (2011). The differences between nominal rates and the inflation swap rates with equivalent maturities that matches those of the real interest rates. The real interest rates are determined using zero coupon inflation swap rates with maturities of 1, 3, 5, 7 and 10 years. For the UK market we employ a similar dataset with an equivalent sample period and maturities. The nominal zero coupon rates are obtained from the UK Central Bank[5]. The

[3]Kerkhof (2005) reports that monthly volumes of inflation swaps traded in the broker market surpassed 3 billion Euro only in 2004.

[4]The US nominal yield data are available on http://www.federalreserve.gov/econresdata/researchdata.htm

[5]The UK nominal yield series can be found on http://www.bankofengland.co.uk/statistics/pages/yieldcurve/default.aspx

inflation zero coupon swap rates of the UK and US are obtained from Datastream/Bloomberg. In Table 1 we present the summary statistics of our sample.

In order to link expected inflation with macroeconomic developments, we employ two additional models with either macroeconomic or survey factors. For the macroeconomic factors we take consumer price inflation, commodity inflation, housing prices, and asset market volatility. We follow Ang and Piazzesi (2003) in selecting the inflation measures and add a measure to capture market volatility. Market volatility in the form of implied volatility can be linked to bond risk premia (see e.g. Rudebusch, Swanson, and Wu (2006)). For the US market, we use the all urban CPI inflation measure published by U.S. Bureau of Labor Statistics, and for the UK the RPI inflation measure. For the commodity prices, we use the average of the global World bank commodity inflation measured for spot prices of the energy and non-energy commodities[6]. For house pricing, we use the average change in monthly house prices. We rely on the US average price of new one-family houses sold during the month and for the UK market on the UK Nationwide Monthly average House price index. For the volatility factor, we MSCI US Minimum Volatility measure and the FTSE 100 Volatility index. These series are obtained from Datastream.

Our survey data consists of four factors in addition to inflation, namely business conditions, consumer confidence, economic optimism, and inflation expectations. Several studies indicate that incorporating forward looking surveys on the state of the macro economy improve the forecast of interest rates (see e.g. Chernov and Mueller (2008) and Moench (2008)). For business conditions we rely on the US Empire State survey on the general business condition and on the UK Retail survey on price expectations. The expectations of the US Consumer Confidence index and UK Consumer Confidence Indicator are used for the consumer confidence factor. For economic optimism, we use data of US TIPP Economic Optimism Index and the UK ZEW indicator of Economic Sentiment. The data of the US University of Michigan Consumer Sentiment on mean expected inflation one year ahead is used for the US inflation expectations and for the UK we incorporate the data of the UK ZEW inflation rate expectation. All the survey data is obtained from Datastream.

While many studies have incorporated inflation surveys (D'Amico et al. (2010)), it is less clear whether those capture the inflation expectations implied by the bond market. As documented by Chernov and Mueller (2008), inflation surveys are prone to overpredict inflation. More specifically, Ang, Bekaert, and Wei (2007) find that when inflation is low the SPF inflation survey tends to under predict inflation[7]. As a consequence of such biases, the inflation risk premium might be estimated less accurately. To measure the impact of survey information on the inflation risk premium, we compare our survey model and the benchmark model in the next section.

3 Empirical results

In this section we estimate both the Macro and Survey model together with a benchmark model. We discuss estimates of the models and their implications for the real rates. To further inves-

[6]The World Bank publishes monthly data on commodity prices, see http://data.worldbank.org/data-catalog/commodity-price-data

[7]The Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia.

tigate the differences, we explore the out-of-sample performance of these three models. Subsequently, we assess the impact of the models on the term premia. Finally, we analyze the inflation risk premia implied by our models.

3.1 Parameter estimates

Tables 2 and 4 report the structural estimates for both the UK and US. We first examine the two market structures using the benchmark models of the UK and US. Based upon our sample period from 2004 to 2012, we establish key differences in the market structures, indicating different dynamics for the inflation risk premia of the two markets. Since both markets are influenced by the financial crisis and experienced low nominal interest rates afterward, the correlation between the nominal short rate is quite substantial (about 85%). However, the level of the annualized nominal short rate in the US is substantially lower (about 1.6 %), whereas in the UK the nominal rate is on average about 2.6%. This difference can be partly explained by the higher level of inflation in the UK. For example, the annualized UK monthly inflation is about 3.3% whereas the US inflation rate is on average 2.3%. As a result of the low nominal interest rates, both markets experience negative real short rates in our sample. While both markets have similar autoregressive coefficients for the first factor ($\Phi_{1,11}$), the persistence of the second factor differs. The half-life in the UK market is about 2.5 years, whereas in the US it is 4 years. Consequently, shocks to the latent pricing factors are more likely to be persistent in the US market than in the UK market. Given the postive estimates for the impact of the pricing factors on the nominal short rate ($\delta_{1,r}$) in both markets, a positive shock to the latent factor will have an positive effect on the nominal short rates.

Tables 2 and 4 also show that the impact of the latent factors on the price of inflation risk differs in the two markets. In the US market the less persistent latent factor drives the price of inflation risk ($\sigma_{\pi,1}$), whereas in the UK there is an interaction between the factors. Although statistically the differences for the impact of the latent factors are hard to capture, inflation levels have a positive effect on the inflation risk premia. Due to the model specification the impact of inflation on the price of risk is estimated negatively ($\sigma_{\pi,CPI}$), resulting in a positive effect. Our model suggests that the inflation level in the US has a larger impact on the inflation risk premium than in the UK. Consequently, we document different dynamics in the prices of inflation risk among the two markets.

Next we turn to the model selection within the two markets. To evaluate the impact of the models on the estimation of real interest rates, we compare the in-sample measurement errors. Note that the specifications of the models are such that they differ in their ability to estimate the real interest rates. Since the financial crisis has substantially influenced the real rates, we evaluate the performance of our models in three periods, namely, prior, during, and post crisis. Tables 3 and 5 report our estimation results for these three periods. From these tables, we can conclude that in the UK the Macro model performs more accurately over the all three periods. In the US the difference between the Survey and Macro models are more difficult to capture. For maturities up to 5 year, the benchmark model outperforms both models in terms of smaller measurement errors. Although the Survey model performs more accurately for the maturities 7 and 10 year, these improvements are only about 1% compared to the Benchmark model. In the UK market the improvements of the UK Macro model are larger (about 5%). As a result, model selection has less impact on the fit of real rates in the US market than in the UK market.

Interestingly, during the financial crisis the Survey model in the US reduces measurement errors by about 5 %. This seems to support that adding surveys factors incorporating market expectations improves the ability of affine models to explain real interest rates. However, in the UK we observe that only for real rates with maturities of 1 and 3 years the Survey factors improve the fit of the real rates compared to the benchmark model. The survey factors in the UK add value prior to the crisis, although the improvements are minor. Our result that the Macro model in the UK improves the fit of the real rates is mainly driven by the improvement of the measurement errors during the financial crisis. Since the macro factors in our model include global indicators, the market timing of the financial crisis is more adequately captured in the UK market. While market rates in the US reacted prior to the bankrupcty of Lehman, the UK rates remained rather stable. While the nominal short rate had a declining trend since August 2007, the UK nominal rate declined rapidly after August 2008. Since these markets responded differently to the crisis, the impact of the economic variable differs substantially among the two markets.

While we find that the addition of survey and macro factors can improve the fit of the real rates, empirical evidence shows that out-of-sample performance of affine term structure models with additional macroeconomic variables is less strong (Ang et al., 2007). To further investigate the effect of the model selection on the real interest rates, we perform an out of sample forecast in the post financial crisis period. Tables 6 and 7 show that the impact of the additional macroeconomic factor is hard to capture. In the US all our indicators show that for short maturities (1 and 3 years) the Macro model outperforms whereas for longer maturities (5, 7, and 10 years) the Survey model improves the out-of-sample forecast. This result suggests that for long maturities real rates can benefit from incorporating market expectations through surveys. However, in the UK the Survey model does not improve the real rates at longer maturities. We do find evidence in the UK that short maturities (1 and 3 years) can benefit from incorporating macro factors, suggesting that short real rates are more influenced by actual macro developments rather than expectations. Similarly, we find in both markets that CPI inflation is more accurately predicted by our Macro model. However, the out-of-sample R-squared measure shows that only in the US the Macro model improves the historical mean.

To summarize, we show that the structures of the governmental bond markets of the US and UK differs, resulting in different underlying mechanics for the inflation risk premium. Consequently, the impact of our model selection may results in different dynamics for the inflation risk premium. While in the US the benchmark model performs reasonably well in-sample, our Macro model substantially outperform the benchmark model in the UK. Therefore, to extract the inflation risk premium we will focus on these models besides our benchmark model.

3.2 Campbell-Shiller regressions and Term premia

To further investigate model selection, we investigate the ability of our models to replicate the Campbell and Shiller (1991) regressions. These regressions as defined in Equation (13) are an application of the Expectation Hypothesis and can therefore be used to test whether the hypothesis holds. The estimates of the impact of the slope on the bond return should be equal to one ($\beta_1 = 1$) according to the theory on the Expectation Hypothesis. However, the literature has documented the stylized fact these coefficients are decreasing with maturity and become negative for longer maturities. Nominal affine term structure models are known to be able to replicate the empirical coefficients of the Campbell-Shiller regression (Dai and Singleton, 2002). While the pattern of the regression coefficients of nominal rates are frequently studied, less is known about characteristics of the coefficients implied by real rates. Therefore, we first estimate the Campbell-Shiller regressions implied by our data and subsequently compare them to our model implied regressions.

The estimates of the nominal coefficients in Campbell-Shiller regressions of both markets reported in Table 8 confirm the typical decreasing pattern of the coefficients across maturity. As expected, our estimates are decreasing and we reject the Expectation Hypothesis only for longer maturities. In the US we reject the hypothesis for 7 and 10 year maturity at a 95% confidence level, while in the UK we only reject for the 10 year maturity. Due to our short sample, the uncertainty of the estimates remains fairly large. To confirm the accuracy of our estimates, we compare our results to Haubrich et al. (2011). Although they use a period of 1982-2010 for the nominal regression, we find similar estimates for shorter maturities although our estimates for 7 and 10 year maturity are twice as small. Since they obtain large standard errors in their estimation as well, it is hard to statistically differentiate between the two sets of estimates. Consequently, our results of the nominal coefficients confirm the range documented in the literature.

Table 8 documents a similar decreasing characteristics for the real coefficients of the Campbell-Shiller regressions as observed in the nominal rates. This empirical observation sheds new light on the behaviour of real coefficients. Haubrich et al. (2011) report coefficients increasing with maturity in their sample of 2003-2010, while our estimates are declining with maturity. Our evidence is supported by the fact that we find the decreasing characteristics in both the UK and US. Table 8 shows that we cannot reject the Expectation hypothesis in the US for all real rates, while in the UK we can only accept the hypothesis for the 1 year maturity. One of the explanations of this results is that the standard errors in the US are substantially larger than in the UK. While standard errors remain large for the real rates, in both markets the uncertainty of the coefficients is larger in the nominal regressions. Due to the large uncertainty, it is hard to capture the link between the different patterns of the nominal and real coefficients within the two markets. For example, we find that the magnitude of nominal coefficients are not similar to the real coefficients. In particular, we observe low nominal coefficients in the US while we observe rather high nominal coefficients in the UK. A reversed pattern holds true for the real coefficients.

Table 9 shows that our estimated benchmark models are able to generate the equivalent decreasing pattern with maturity for both the nominal and real coefficients. Compared to Haubrich et al. (2011), our model performs more adequate on shorter maturities to reproduce the data implied coefficients. Similarly as in their study, it is hard for term structure models to capture both the short and long maturities. Consequently, for longer maturities our model is less able to adequately replicate the data implied estimates, although our estimates fall within the 95% confidence intervals.

Model selection has only a small impact on the ability to replicate the coefficients of the Campbell-Shiller regressions. While in the US the most promising models are the benchmark and the Survey model, both models are less able to capture the data implied coefficients for longer maturities. However, in the UK the benchmark model captures the longer maturities more adequately. Although our previous results on model selection show improvements for the addition of macroeconomic factors, the benchmark model is more able to generate adequate estimates for the coefficients. Therefore, we observe a trade-off between in-sample fitting and the ability to replicate the data implied coeffcients of the Campbell-Shiller regressions.

Next, we examine whether our model is able to replicate the data implied nominal and real term premia. Table 10 reports the mean absolute deviations (MAD) from the data implied term premia and shows that nominal term premia are better captured than the real term premia. For nominal term premia, we observe a smaller difference between short and long maturities than for the real term premia. While in the US we observe that short nominal term premia are more accurately replicated than the long term premia, in the UK our model is able to match both the short and long nominal term premia. Due to the different market structure of the UK, the nominal data implied term premia are less varying over the sample period. Consequently, the deviations of the model implied term premia are reduced.

Table 10 also shows the impact of macroeconomic factors on the estimation of term premia. For example, the benchmark and Survey model are able to replicate the real term premia on short maturities in the US. On the other hand, the Macro model has the ability to improve the long real term premia. This suggests that adding economic variables can influence the ability of the model to replicate real term premia. However, in the UK the Macro model is unable to improve the Benchmark or Survey model. In terms of fitting real rates, the Macro model was outperforming the other two models in the UK, but it is unable to replicate the data implied term premia. Regarding replicating term premia, affine term structure models perform more accurate for nominal term premia than real premia. Part of this can be explained by the uncertainty associated with the estimation of data-implied real term premia. Hence, it is not straightforward to determine whether affine term structure models are able to replicate time varying real term premia.

Although our benchmark models are able to capture the main characteristics of the Campbell-Shiller regressions and term premia, it remains a challenge to accurately replicate all the empirical stylized facts. Model selection substantially affects the performance of the models. While the Benchmark model is able to perform rather well in both the UK and US to replicate the Campbell-Shiller regressions and term premia, our previous results indicated that macroeconomic factor improve the fit and forecasting of the real rates. Therefore, we will investigate the consequences of model selection for the inflation risk premia in the next section.

3.3 Inflation risk premium

An important component of modeling nominal and real interest rate is the inflation risk premium. Inflation risk is associated with unexpected inflation shocks in our model. As a result, we compute the inflation risk premium by taking the difference between the implied break-even inflation with and without unexpected inflation[8]. Therefore, we can write

$$IRP_t(n) = BEI_t^{\sigma_\pi}(n) - BEI_t^{\sigma_\pi=0}(n), \tag{17}$$

where $IRP_t(n)$ is the inflation risk premium for maturity n at time t, $BEI_t^{\sigma_\pi}(n)$ denotes the break-even inflation with unexpected inflation risk and $BEI_t^{\sigma_\pi=0}(n)$ denotes without.

[8]This approach is similar to Chen, Liu, and Cheng (2010) and Haubrich et al. (2011).

Table 11 reports our estimates for the inflation risk premium for both the UK and US markets. For the US benchmark model we find for the 5 year maturity inflation risk premium of 72 basis points, whereas the UK has a negative premium of 45 basis points. Since the estimates for the UK market are negative, this implies that issuance of inflation-indexed Gilts would be costly in the UK. Investigating the impact of the model selection, we can conclude that in the US the models generate a similar inflation risk premium. For the UK market, we show adding macroeconomic factor can have a large impact on the estimate for the inflation risk premium. The Macro model that fits the real rates most accurately identifies a positive risk premium, whereas the Survey model suggests a substantial smaller risk premium than the Benchmark model.

Since the standard error of the average inflation risk premium are quite large, it suggests that there are level shifts in the inflation risk premium across our sample period. To understand the time varying characteristics more adequately, we split the sample into three periods: prior to the Financial crisis in 2008, during and post crisis. We observe in all three models for both markets that the 5 year inflation risk premium was at its highest mark prior to the financial crisis. In our benchmark model, we find estimates in the US of about 115 basis points risk, whereas in the UK we find a positive premium of 30 basis points. During the financial crisis, we estimate negative premia for both markets across all our models. The negative rates do not necessarily correspond to inflation expectations. Although negative rates might imply that the market feared deflation due to the crisis, it could also suggest that the premium is affected by liquidity. Since inflation rates remained quite stable during the financial crisis, our results point at a liquidity shock rather than risk associated with inflation.

After the financial crisis, the inflation risk premium is again increasing but remains substantially lower than pre-crisis levels. For example, our benchmark model generates a 5 year inflation risk premium of 47 basis points in the US and about -104 basis points in the UK. These estimates suggest the UK market responded much stronger to shift in the inflation risk premium due to the financial crisis than the US market. Campbell, Shiller, and Viceira (2009) argue that low inflation risk premia (or even negative risk premia) can be explained by positive correlation between asset returns and inflation in the observed period. Therefore, negative inflation risk premia might indicate low inflation expectations.

We observe similar movement in the inflation risk premia for both markets around the period of the financial crisis. Although there appears to be a different market timing in the drop of nominal interest rates across both markets, from August 2007 to January 2008 the inflation risk premium falls by 50% in both markets. While in the UK market the inflation risk premium again starts to drop around September 2008, the US market remains stable until March 2009. Afterward, the US inflation risk premium continues to drop and remains unstable. By the end of 2011 the US inflation risk premium starts to stabilize around pre-crisis levels (about 97 basis points), while the UK market stabilize around a substantial lower inflation risk premium (about -67 basispoints). Thus, in the UK market the recovery of the inflation risk premium is much less than in the US.

Our estimates for the inflation risk premia fall within the range suggested by earlier research. While the UK and US markets have never been compared with similarly estimated models, our US sample period and inflation-linked derivatives are most related to Haubrich et al. (2011). They find inflation risk premia are about twice as small for the 5 and 10 year maturity, yet their estimates are twice as large for the longer maturities (20 and 30 years). One the novelty of our approach is that we incorporate data-implied term premia. This suggests that they might underestimate unexpected inflation shock at shorter maturities, while they overestimate shocks for longer maturities. Other studies, e.g. D'Amico et al. (2010) report an estimate of 36 basis points for the 5 years inflation risk premium, whereas Ang et al. (2008) estimate 115 basis points. Similar to our results, Buraschi and Jiltsov (2005) estimate a premium of 80 basis points for a 10 year premium. For UK, few comparable studies are available and their estimates of the inflation risk premium differ substantially. All these studies only use inflation-linked Gilts to identify the real interest rates. For example, Evans (2003) estimates a negative inflation risk premium, while on the other hand Risa (2001) reports for the period 1983 and 1999 an inflation risk premium of about 184 basis points. Joyce et al. (2010) on the other hand shows an estimate of about 100 basis points. As a result, the literature suggests a wide range of likely estimates for the inflation risk premium.

To summarize, we show that model selection is an important determinant for the inflation risk premium. Since our models perform differently across various criteria, we are unable to select the most appropriate model to identify the inflation risk premium. Our results might be influenced by a small sample bias. To address this issue and the uncertainty of the estimate of the inflation risk premium, we employ a Bayesian approach in the next section.

4 Parameter uncertainty

To investigate the uncertainty of the range of estimates for the inflation risk premium, we employ a Bayesian methodology. In this way we can address the effect of a short sample period as well. First, we present posterior marginal distributions for the inflation risk premia, which we can use to calculate statistical intervals. These intervals allow us to explain the uncertainty concerning the estimates of the inflation risk premium. Secondly, we analyze on the impact of the financial crisis on the inflation risk premium.

4.1 A range of estimates for inflation risk premium

Figures 1 and 2 report our marginal posterior distributions for the 5 years inflation risk premia in the US and UK. We confirm our previous results on the different characteristics between the inflation risk premia in both markets. The uncertainty concerning the estimate of the inflation risk premium is larger in the UK than in the US. The 95% credibility interval of the UK distribution ranges from -95 to 88 basis points, whereas in the US we observe a range of -4 to 119 basis points. This implies that for both markets we cannot exclude negative inflation risk premia on a 95% credibility interval. However, our results suggests that the US inflation risk premium is substantially more likely to be positive than the UK inflation risk premium.

Next we analyze the impact of parameter uncertainty on the inflation risk premia. The mean of the US distribution is about 74 basis points, which is about about 2 basis points larger than the our previous estimate ignoring parameter uncertainty. Therefore the effect of the parameter uncertainty is not quite large for the US market. In the UK, we observe a larger effect of parameter uncertainty, since the mean of the distribution is about -8 basis points. Since the inflation risk premium in the UK market was substantially affected by the financial crisis, the impact of parameter uncertainty is much larger. As a result, we observe large uncertainty about the point estimates for both markets.

To measure the impact of macroeconomic factors, we also determine the distributions of the inflation risk premium indicated by our Macro and Survey models. Figures 3, 5, 4, and 6 report the marginal distributions for both markets. Our results indicate that the distributions are not substantially altered, although the distributions are more platykurtic. As a result, the uncertainty of the estimates increases. For example, the US Survey model has a larger upper bound for the 95% credibility interval, namely an interval of -4 to 127 basis points. In the UK market the additional macroeconomic factors cause a broadening for interval in both directions. While macroeconomic factors improve our model characteristics as previously shown, it generates larger credibility intervals for inflation risk premium. These results suggest that the identification of inflation risk premium might not be helped by including such variables. Even though from an economic perspective a link between macroeconomic development and the inflation risk is preferred, empirically the addition of macro factors increase the uncertainty of the estimates.

The evidence presented by our Bayesian methodology raises questions about the ability of affine term structure models to adequately pinpoint the value of the inflation risk premium with high precision. Since our empirical distributions include the range of the inflation risk premium suggested by previous literature, it is hard to statistically distinguish between those estimates. To analyze the impact of the financial crisis in 2008 on the inflation risk premia, we extend our Bayesian methodology. In the next section, we investigate the consequences if we add more weight to the observations after the financial crisis in our Bayesian analysis. As a consequence, we can measure the shift in the inflation risk premium caused by the financial crisis.

4.2 The impact of the financial crisis

Since the financial crisis nominal interest rates have dropped to low levels. In the previous sections, we have observed a drop in the inflation risk premium during the financial crisis. In order to analyze the impact of the financial crisis on the range of the inflation risk premium, we extent our Bayesian analysis by adding more weight to post crisis observations. By using a Normal-diffuse prior estimated on a sample period from 2010 up to 2012, we shift the importance of the observations to the financial crisis while maintaining the original sample size. Reestimation of our model on the time period from 2010 up to 2012 is infeasible due to the short sample period. With our approach we can show shifts in the posterior probability distribution caused by the impact of the financial crisis.

Figures 1 and 2 show the impact of the financial crisis for both the UK and US 5 year inflation risk premium. In the US we observe a downward shift in the distribution function for the premium, resulting in a mean of 38 basis points. Although we observe a lower estimate, the uncertainty of the estimate increases as well. The 95% credibility interval in the US ranges from -51 to 92 basis points. As a result, negative inflation risk premium are more likely due to the crisis. As uncertainty about future economic development increases, one would expect that the inflation risk premium would increase as well. Especially since low interest rates in the US would lead to uncertainty about future inflation ((Krugman, 1998) and (Eggertsson and Woodford, 2003)). However, the US governmental bond market does not reflect this uncertainty.

The impact of the financial crisis on the inflation risk premium in the UK is less obvious.

We observe a slightly increase of the mean of the distribution to 13 basis points. Similarly, the 95% credibility interval of the inflation risk premium, ranging from -104 to 150 basis points, shifts upward, although the lower bound of the interval decreases as well. Due to the broadening of the interval, the uncertainty about the estimate of the inflation risk premium increases. As a result, it is hard to determine whether the inflation risk premium is positive of negative in the UK. About 42% of the probability mass in the UK is below 0, indicating that the inflation risk premium could be negative.

The impact of the financial crisis causes a wider range of estimates for the inflation risk premia in the UK and US. We observe that a large downward effect on the US estimates, whereas we show an upward effect for the UK market. In particular, our models seems to indicate that both markets have positive inflation risk premia. However, due to the large uncertainty about these estimates, we cannot statistically reject any of the estimates suggested by earlier studies.

5 Conclusion

Although debt markets with inflation-linked derivatives offer a possibility to capture the inflation risk, the identification remains problematic. Our study quantifies a wide interval of estimates for the inflation risk premium in both the UK and US. This large range suggests that for both markets it remains hard to pinpoint the value of the inflation risk premium with precision. In the US market we find evidence that a 95% credibility interval for the 5 year maturity inflation risk premium of -4 and 119 basis points. This range is even broader in the UK where we observe an interval of about -94 to 88 basis points. The large uncertainty concerning these estimates are robust for different model specification, suggesting that affine term structure models are not able to accurately pinpoint the inflation risk premium.

This study contributes to research about the inflation risk premia by analyzing the inflation risk premium by means of marginal posterior distributions rather than point estimates. While prior research is mainly concerned about estimating the inflation risk premia, it ignores the large uncertainty associated to these estimations. We show using our Bayesian framework that the small sample bias can alter the estimates for the inflation risk premia. Our Bayesian methodology also allows to investigate the effect of the financial crisis on the inflation risk premia. We observe an upward effect in the inflation risk premium due to the financial crisis in the UK market, while we find downward shift in the US markets. Consequently, this shift results in more uncertainty in estimating the inflation risk premium.

Our findings raise a number of questions on the interpretation of inflation risk premium estimates and economic policy based on these estimates. Positive estimates of the inflation risk premia have frequently be used to validate the issuance of governmental inflation-linked bonds. Given the large uncertainty about these estimates, it is questionable whether debt policy should be based on such estimates. Our posterior probability distributions for UK inflation risk premium shows a large probability (42%) that the inflation risk premium is negative. Also, in the US we are unable to reject the hypothesis that the inflation risk premium is negative. Based on these results UK inflation-indexed gilts would be costly to issue. Further work should explore the impact of the financial crisis on the inflation risk premia, as our results point at a positive shift for the UK, while a negative shift in the US.

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A Appendix A: Model derivations

A.1 Coefficients for the Nominal yields

In this section we derive the nominal bond yields for our model in a no-arbitrage framework. In this derivation we follow the typical noarbitrage framework in affine term structure models as derived for example by Duffie and Kan (1996). We substitute the affine bond prices, as defined in Equation (1), in the no-arbitrage relation of the expected bond price. For convenience, we restate this relation

$$P_t^N(n) = E_t\left[M_{t+1}^N P_{t+1}^N(n-1)\right]. \tag{A.1}$$

By substituting the affine bond prices and the dynamics of the nominal pricing kernel in this equation, we derive the following expression for the price of a bond,

$$P_t^N(n) = E_t\left[\exp\left(-r_t^N - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right)\exp\left(A_{n-1}^N + B_{n-1}'^N X_{t+1}\right)\right]. \tag{A.2}$$

Rewriting and determining the expectation, yields the following expression for the bond price

$$P_t^N(n) = \exp\left(-\delta_{0,r} + A_{n-1}^N + B_{n-1}'^N\Phi_0 + (B_{n-1}'^N\Phi_1 - \delta_{1,r}')X_t - B_{n-1}'^N\Sigma\lambda_t + \frac{1}{2}B_{n-1}^N\Sigma\Sigma' B_{n-1}'^N\right). \tag{A.3}$$

Next, we need to substitute the affine function for the price of risk, λ_t. Substituting this, as defined in Equation (9), we arrive at the typical function for the no-arbitrage bond price,

$$P_t^N(n) = \exp\left(-\delta_{0,r} + A_{n-1}^N + B_{n-1}'^N(\Phi_0 - \Sigma\Gamma_0) + \frac{1}{2}B_{n-1}'^N\Sigma\Sigma' B_{n-1}^N + (B_{n-1}'^N(\Phi_1 - \Sigma\Gamma_1) - \delta_1')X_t\right). \tag{A.4}$$

The last step is to determine the recursion for the coefficients by matching the coefficients of the left-hand side with the terms on the right-hand side. In this way we derive the recursion for the no-arbitrage coefficients of the bond prices

$$A_n^N = A_{n-1}^N + B_{n-1}'^N(\Phi_0 - \Sigma\Gamma_0) + \frac{1}{2}B_{n-1}'^N\Sigma\Sigma' B_{n-1}^N - \delta_{0,r}, \tag{A.5a}$$

$$B_n'^N = B_{n-1}'^N(\Phi_1 - \Sigma\Gamma_1) - \delta_{1,r}'. \tag{A.5b}$$

with the initial conditions $A_1^N = -\delta_0$ and $B_1^N = -\delta_1$. In order to derive the coefficients of the yields, using the relation between bond prices and continuously compounded yields, we adjust the coefficients of the bonds as follows

$$\bar{A}_n^N = -\frac{A_n^N}{n}, \tag{A.6a}$$

$$\bar{B}_n^N = -\frac{B_n^N}{n}. \tag{A.6b}$$

This function determines the no-arbitrage coefficients for the affine yields.

A.2 Coefficients for the real yields

In order to derive the coefficients for the real yields, we derive the real equivalent risk and short rate parameters. For convenience of notation, we state the real pricing kernel below again,

$$M_{t+1}^R = \exp\left(-r_t^N + \delta_\pi + \delta_{1,\pi}' X_t - \frac{1}{2}\lambda_t'\lambda_t - (\lambda_t' - \sigma_\pi')\epsilon_{t+1}\right). \tag{A.7}$$

First, we define the real pricing kernel in a similar notation,

$$M_{t+1}^R = \exp\left(-r_t^R - \frac{1}{2}\lambda_t'^R\lambda_t^R - \lambda_t'^R\epsilon_{t+1}\right), \tag{A.8}$$

where r_t^R denotes the instantaneous real rate, and $\lambda_t'^R$ the real price of risk vector. We need to find the equivalent parameters for the real pricing kernel. This can be done by matching the parameters of this kernel with the kernel implied by the relation between the nominal kernel and the inflation process. This relation is given in Equation (4).

We postulate the real price of risk as

$$\lambda_t^R \equiv \lambda_t - \sigma_\pi'. \tag{A.9}$$

Consequently, the real price of risk parameters can be determined from the nominal parameters as follows

$$\Gamma_0^R = \Gamma_0 - \sigma_\pi, \tag{A.10a}$$

$$\Gamma_1^R = \Gamma_1, \tag{A.10b}$$

so that we have that the real price of risk is an affine function of the state variables

$$\lambda_t^R = \Gamma_0^R + \Gamma_1^R X_t.$$

In analogy of the nominal decomposition of the monthly short rate, as in Equation (8), we define the monthly real short rate as

$$r_t^R = \delta_{0,r}^R + \delta_{1,r}'^R X_t. \tag{A.11}$$

To find the relation between the nominal and real parameters of the short rate, we note that the product of the real price of risk can be written as

$$-\frac{1}{2}\lambda_t'^R\lambda_t^R \;=\; (\lambda_t - \sigma_\pi)'(\lambda_t - \sigma_\pi) \;=\; -\frac{1}{2}\lambda_t'\lambda_t + \sigma_\pi\Gamma_0 + \sigma_\pi\Gamma_1 X_t - \frac{1}{2}\sigma_\pi'\sigma_\pi.$$

By matching the real pricing kernel as given in Equation (A.8) with real pricing kernel implied by our relation between the nominal pricing kernel and inflation process, the following restrictions for coefficients of the instantaneous real rate are required

$$\delta_0^R = \delta_{0,r} - \delta_{0,\pi} - \sigma_\pi'\Gamma_0 - \frac{1}{2}\sigma_\pi'\sigma_\pi \tag{A.12a}$$

$$\delta_1'^R = \delta_{1,r}' - \delta_{1,\pi}' - \sigma_\pi'\Gamma_1. \tag{A.12b}$$

Since we have the real equivalent parameters Φ_0^R, Φ_1^R, $\delta_{0,r}^R$, and $\delta_{1,r}'^R$, we use a similar derivation as in the previous section for the recursion of the nominal coefficients. By simply substituting the real equivalent parameters, we obtain the coefficients for the real yields. These coefficients, $\bar{A}_n^R$ and $\bar{B}_n^R$, are the real equivalents of the nominal coefficients as defined in Equation (11).

B Appendix B: Reduced Model derivations

The system of reduced equations, as given in Equation (15), will be derived in this section. We first focus on the equation of the VAR dynamics of the yields measured without error. We start with the state process as defined in Equation (2). Premultiplying this system with $\bar{B}_1'$ and adding $\bar{A}_1$ gives

$$\bar{A}_1 + \bar{B}_1' X_t^L = \bar{A}_1 + \bar{B}_1' \left(\Phi_1^L X_{t-1}^L + I_2 \epsilon_{1t} \right) \tag{B.1}$$

As a result, we can rewrite this equation to a VAR model of the yields measured without errors, Y_t^1,

$$Y_t^1 = \bar{A}_1 + \bar{B}_1' \Phi_1^L \bar{B}_1'^{-1} \left(Y_{t-1}^1 - \bar{A}_1 \right) + \bar{B}_1' \epsilon_{1,t}, \tag{B.2}$$

by use of the definition of Y_t^1 as given in Equation (11). Now we have expressed the time dynamics of the latent factors in yield series measured without error. Rewriting this equation yields the first reduced form regression,

$$Y_t^1 = \underbrace{(\bar{A}_1 - \bar{B}_1' \Phi_1^L \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_1^*} + \underbrace{(\bar{B}_1' \Phi_1^L \bar{B}_1'^{-1})}_{\Phi_{11}^*} Y_{t-1}^1 + \underbrace{\bar{B}_1'}_{\Omega_1^*} \epsilon_{1,t}. \tag{B.3}$$

In this equation the coefficients $\bar{A}_1^*$, Φ_{11}^*, and Ω_1^* will be obtained by OLS estimation.

The second reduced form equation is the impact of the latent factors on the yields measured with errors. For notional convenience, we repeat this equation

$$Y_t^2 = \bar{A}^2 + \bar{B}_2'^{EC} X_t^{EC} + \bar{B}_2'^{L} X_t^L + \Omega \epsilon_{2,t}. \tag{B.4}$$

Since we include real rate series in this equation we need to incorporate the effect of the economic factors as well. Next, we substitute the latent factors with inverse of the yields observed without error,

$$Y_t^2 = \bar{A}^2 + \bar{B}_2'^{L} \left(\bar{B}_1'^{-1} \left(Y_t^1 - \bar{A}_1 \right) \right) + \bar{B}_2'^{EC} X_t^{EC} + \Omega \epsilon_{2,t}. \tag{B.5}$$

Consequently, we derive the following reduced form regression,

$$Y_t^2 = \underbrace{(\bar{A}^2 - \bar{B}_2'^{L} \bar{B}_1'^{-1} \bar{A}_1)}_{A_2^*} + \underbrace{(\bar{B}_2'^{L} \bar{B}_1'^{-1})}_{\Phi_{21}^*} Y_t^1 + \underbrace{\bar{B}_2'^{EC}}_{\Phi_{2EC}^*} X_t^{EC} + \underbrace{\Omega}_{\Omega_2^*} \epsilon_{2,t}. \tag{B.6}$$

We denote the OLS estimates of the coefficients in this equation as $\bar{A}_2^*$, Φ_{21}^*, Φ_{2EC}^* and Ω_2^*.

The economic VAR equation is the third reduced form equation. We use the VAR process implied for the economic state variables as defined in Equation (2) and substitute the inverse of the latent state variables,

$$X_t^{EC} = \Phi_0^{EC} + \Phi_1^{EC} X_{t-1}^{EC} + \Phi_1^{EC,L} \bar{B}_1'^{-1} \left(Y_{t-1}^1 - \bar{A}_1 \right) + \Sigma_{EC} \epsilon_{3,t} \tag{B.7}$$

Rewriting this equation, yields

$$X_t^{EC} = \underbrace{\Phi_0^{EC} - \Phi_1^{EC,L} \bar{B}_1'^{-1} \bar{A}_1}_{A_3^*} + \underbrace{\Phi_{3EC}}_{\Phi_{3EC}^*} X_{t-1}^{EC} + \underbrace{\Phi_1^{EC,L} \bar{B}_1'^{-1} Y_{t-1}^1}_{\Phi_{3L}^*} + \underbrace{\Sigma_{EC}}_{\Sigma_{EC}^*} \epsilon_{3,t} \tag{B.8}$$

The last reduced form equation concerns the model implied term premia. To determine the model implied term premium, we substitute the affine yields in Equation (12) for the term premium and evaluate the expectation. Consequently, we can write for the nominal term premium

$$TP_t^N(n) = \left(n\bar{A}_n^N - (n-1)\bar{A}_{n-1}^N - \delta_{0,r}\right) + \left(n\bar{B}_n^N - (n-1)\bar{B}_{n-1}^N\Phi_1^L - \delta_{1,r}X_t\right) \quad \text{(B.9)}$$

Since the term premium is dependent on the latent factors, we substitute by the yields measured without measurement error. If we allow for measurement error between the model implied term premia and the observed term premia, we obtain the following equation for the nominal term premium

$$\begin{aligned} TP_t^N(n) = & \underbrace{\left(n\bar{A}_n^N - (n-1)\bar{A}_{n-1}^N - \delta_{0,r}^N\right) - \left(n\bar{B}_n^N - (n-1)\bar{B}_{n-1}^N\Phi_1^L - \delta_{1,r}\right)\bar{B}_1^{-1}\bar{A}_1}_{A_4^*} + \\ & \underbrace{\left(n\bar{B}_n^N - (n-1)\bar{B}_{n-1}^N\Phi_1^L - \delta_{1,r}\right)B_1^{-1}}_{\Phi_{4L}^*} Y_t^1 + \underbrace{\Omega_{TP}}_{\Omega_4^*}\epsilon_{4,t}. \end{aligned} \quad \text{(B.10)}$$

For the real term premium, we derive the following equation,

$$\begin{aligned} TP_t^R(n) = & \underbrace{\left(n\bar{A}_n^R - (n-1)\bar{A}_{n-1}^R - \delta_{0,r}^R\right) - \left(n\bar{B}_n^R - (n-1)\bar{B}_{n-1}^R\Phi_1^L - \delta_{1,r}^R\right)\bar{B}_1^{-1}\bar{A}_1}_{A_4^*} + \\ & \underbrace{\left(n\bar{B}_n^{EC,R} - (n-1)\bar{B}_{n-1}^{EC,R}\Phi_1^L - \delta_{1,r}^{EC,R}\right)B_1^{EC,-1}}_{\Phi_{4EC}^*} X_t^{EC} + \\ & \underbrace{\left(n\bar{B}_n^R - (n-1)\bar{B}_{n-1}^R\Phi_1^{EC} - \delta_{1,r}^R\right)B_1^{-1}}_{\Phi_{4L}^*} Y_t^1 + \underbrace{\Omega_{TP}}_{\Sigma_4^*}\Omega_{4,t}, \end{aligned} \quad \text{(B.11)}$$

where $\bar{B}^{EC,R}$ denotes the impact of coefficient $\bar{B}^R$ to the economic variables. The partitioning of the matrices is equivalent as defined in Equation (2).

C Appendix C: MSCE procedure

To minimize the distance between OLS estimates and the coefficients implied by the structural parameters, we employ the Minimum Chi-squared methodology. For notional convenience, we rewrite the reduced system of equations,

$$\begin{array}{lclclclcl} Y_t^1 & = & A_1^* & + & \Phi_{11}^* Y_{t-1}^1 & + & & + & \Omega_1^*\epsilon_{1,t}^* \\ Y_t^2 & = & A_2^* & + & \Phi_{21}^* Y_t^1 & + & \Phi_{2EC}^* X_t^{EC} & + & \Omega_2^*\epsilon_{2,t}^* \\ X_t^{EC} & = & \Phi_0^* & + & \Phi_{3EC}^* X_t^{EC} & + & \Phi_{3L}^* Y_t^1 & + & \Sigma_{EC}^*\epsilon_{3,t}^*. \\ TP_t & = & A_4^* & + & \Phi_{41}^* Y_t^1 & + & \Phi_{4EC}^* X_t^{EC} & + & \Omega_4^*\epsilon_{4,t}^* \end{array} \quad \text{(C.1)}$$

For details of the link between the structural parameters and the reduced form parameters, consult Appendix B.

Applying the minimum Chi-squared estimator, we can directly match reduced form parameters with the coefficients implied by the structural parameters. Some of the parameters can directly be mapped into the OLS estimates, such as Φ_1^{EC} and the measurement error Ω. The other parameters need to be estimated using the Chi-squared estimation. Hence, the parameters used in the MSCE are reduced to

$$\hat{\pi} = \begin{bmatrix} \text{vec}\left(\hat{\Pi}_1\right) \\ \text{vech}\left(\hat{\Omega}_1^*\right) \\ \text{vec}\left(\hat{\Pi}_2\right) \\ \text{vec}\left(\hat{\Pi}_3\right) \\ \text{vec}\left(\hat{\Pi}_4\right) \end{bmatrix}, \tag{C.2}$$

and

$$\hat{R} = \begin{bmatrix} \hat{\Omega}_1^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{1t} Z_{1t}' \\ \frac{1}{2} D_2' \left(\hat{\Omega}_1^{*-1} \otimes \hat{\Omega}_1^{*-1}\right) D_2 \\ \hat{\Omega}_2^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{2t} Z_{2t}' \\ \hat{\Omega}_3^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{3t} Z_{3t}' \\ \hat{\Omega}_4^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{4t} Z_{4t}' \end{bmatrix} \mathbf{I}_n,$$

where

$$\begin{aligned}
Z_{1t} &= \begin{bmatrix} 1 \\ Y_{t-1}^1 \end{bmatrix} \text{ and } Z_{it} = \begin{bmatrix} 1 \\ Y_t^1 \\ X_t^{EC} \\ TP_t \end{bmatrix} \text{ for } i = 2, 3 \\
\hat{\Pi}_i &= \left(\sum_{t=1}^{T} Y_t^i Z_{it}'\right) \left(\sum_{t=1}^{T} Y_t^i Z_{it}'\right)^{-1} \text{ for } i = 1, 2 and 4 \\
\hat{\Pi}_3 &= \left(\sum_{t=1}^{T} X_{t+1}^{EC} Z_{it}'\right) \left(\sum_{t=1}^{T} X_{t+1}^{EC} Z_{it}'\right)^{-1} \\
\hat{\Omega}_1^* &= T^{-1} \sum_{t=1}^{T} \left(Y_t^1 - \hat{\Pi}_1' Z_{1t}\right) \left(Y_t^1 - \hat{\Pi}_1' Z_{1t}\right)' \\
\hat{\Omega}_i^* &= T^{-1} \text{dg} \left(\sum_{t=1}^{T} \left(Y_t^i - \hat{\Pi}_i' Z_{it}\right) \left(Y_t^i - \hat{\Pi}_i' Z_{it}\right)'\right) \\
\hat{\Omega}_3^* &= T^{-1} \sum_{t=1}^{T} \left(X_{t+1}^{EC} - \hat{\Pi}_1' Z_{3t}\right) \left(X_{t+1}^{EC} - \hat{\Pi}_1' Z_{3t}\right)'
\end{aligned}$$

with the matrix function dg (A) is defined such that all elements outside the diagonal of matrix A are all zero and $\mathbf{I}_n$ is the identity matrix with dimension n. For notional convenience we use $Y_t^4 = TP_t$. By minimizing the MSCE, we find the estimates for the structural parameters.

D Appendix D: Bayesian approach

Parameter uncertainty enters in the first stage of the estimation namely the reduced form regressions. Therefore, we adopt a Bayesian methodology for these equations to address parameter uncertainty. In order to obtain the distribution for the reduced form of equations, we follow Bauwens and Lubrano (1996) by rewriting these equations into a system of seemingly unrelated regressions. The reduced form of equations can easily be written in the following form,

$$y_i = X_i\beta_i + \epsilon_i, \tag{D.1}$$

for each $i = 1, .., n$ with n denoting the total number of state variables in the system. If the individual time series included in the model have dimension T, then y_i is a vector with $((T-1) \times 1)$ observations, X_i is a matrix with dimensions $((T-1) \times k_i$ with k_i independent variables, β_i consists of a coefficient vector with k_i elements, and ϵ_i is the vector with the associated errors for each observation $(T-1)$. We rewrite this model in two forms in order to draw parameters from the posterior density. By stacking all the observations for each equation i, we can express Equation (D.1) as

$$y = x\beta + \epsilon, \tag{D.2}$$

where $y = (y_1, .., y_n)$ is a vector with dimensions $((T-1)n \times 1)$, $\beta = (\beta_1, .., \beta_n)$ with a vector of k_n elements, $x = \text{diag}(x_1, .., x_n)$ with dimensions $((T-1)n \times k_n)$, and $\epsilon = (\epsilon_1, .., \epsilon_n)$. In the second approach, we write a VAR specification

$$Y = XB + E, \tag{D.3}$$

with $Y = (y_1...y_n)$ is a matrix with dimensions $((T-1) \times n)$, $X = (X_1...X_n)$ has dimensions $((T-1) \times k_n)$, $B = \text{diag}(\beta_1, .., \beta_n)$ is a matrix with dimensions $(k_n \times n)$ and $E = (E_1...E_n)$ is a matrix with dimensions $((T-1) \times n)$. Next, we use two prior distributions, namely a uninformative and a Normal-Diffuse prior.

D.1 Uninformative Prior

In deriving the posterior density function of the OLS estimates, we assume an uninformative prior. This prior means that we do not impose any prior believe on the parameters of the model. Hence, the prior function is of the form

$$f(\beta, \Sigma) \propto |\Sigma|^{-(n+1)/2}, \tag{D.4}$$

where Σ denotes the variance-covariance matrix of the error in the VAR model. For this uninformative prior, the marginal posterior density of the parameters can be written as

$$\begin{array}{rcl} \beta|\Sigma & \sim & \text{N}(\hat{\beta}, [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}) \\ \Sigma|\beta & \sim & \text{IW}(Q, T-1), \end{array} \tag{D.5}$$

with

$$\begin{array}{rcl} \hat{\beta} & = & [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}x'(\Sigma^{-1} \otimes I_{T-1})y \\ Q & = & (Y - XB)'(Y - XB). \end{array}$$

Since the marginal posterior densities of the two parameters β and Σ are not available, we rely on the Block-Gibbs sampling algorithm (See e.g. Bauwens and Lubrano (1996)). Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity.

Our final sequence consists of 1000 draws from the posterior density. Using these parameters, we determine the structural parameters of our model using the minimum Chi-squared estimation. For each of these set of structural parameters, we determine the implied inflation risk premium and the short rates. We report the distribution of the averages of these time series, which only rely on the observed data.

D.2 Informative Prior

Next, we impose a Normal-diffuse prior on the parameters. Since the weight of recent observations bare more importance, we establish a prior on the impact of the OLS estimates. However, we hold a diffuse prior on Σ in Equation (D.1) or the covariance-variance matrix of the coefficients. Formally, we can write

$$\begin{array}{rcl} f(\beta) & \sim & \mathrm{N}(\beta_{Prior}, \Omega) \\ f(\Sigma) & \propto & |\Sigma|^{-(n+1)/2}, \end{array} \tag{D.6}$$

where β_{Prior} denotes the estimates of the prior. Following Zellner (1971) we can write the marginal posterior distributions as follows,

$$\begin{array}{rcl} \beta|\Sigma & \sim & \mathrm{N}(\hat{\beta}, \hat{\Omega}) \\ \Sigma|\beta & \sim & \mathrm{IW}(Q, T-1), \end{array} \tag{D.7}$$

with

$$\begin{array}{rcc} \hat{\beta}_{OLS} & = & [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1} x'(\Sigma^{-1} \otimes I_{T-1})y \\ \hat{\beta} & = & \hat{\Omega}(\hat{\Omega}^{-1}\beta_{Prior} + [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}\hat{\beta}_{OLS} \\ \hat{\Omega} & = & (\Omega^{-1} + x'(\Sigma^{-1} \otimes I_{T-1})x)^{-1} \\ Q & = & (Y - XB_{OLS})'(Y - XB_{OLS}) + (B - B_{OLS})'X'X(B - B_{OLS}). \end{array}$$

Again we rely on the Block-Gibbs sampling technique to derive the marginal posterior densities of the two parameters β and Σ. Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity. The prior estimates are derived from using the OLS estimates on a sample from January 2010 to December 2012. Our final sequence consists again of 1000 draws from the posterior density. Using these OLS parameters, we determine the structural parameters of our model using the minimum Chi-squared estimation. For each of these set of structural parameters, we determine the implied inflation risk premium and the short rates. We report the distribution of the averages of these time series, which only rely on the observed data.

E Appendix E: Tables

Table 1: Summary Statistics Yields

This table presents the statistics on the main time series used in the paper. The annualized nominal and real yields are presented for both the UK and US. The sample period ranges from July 2004 up to December 2012.

	Mean	St. dev	Min	Max
US				
	Nominal			
1m	1.68 %	1.87 %	0.00 %	5.13 %
3m	1.75 %	1.89 %	0.01 %	5.01 %
6m	1.87 %	1.89 %	0.05 %	5.04 %
1y	2.02 %	1.88 %	0.13 %	5.21 %
2y	2.16 %	1.76 %	0.19 %	5.13 %
3y	2.36 %	1.62 %	0.31 %	5.06 %
5y	2.82 %	1.36 %	0.63 %	5.01 %
7y	3.26 %	1.18 %	1.01 %	5.05 %
10y	3.77 %	1.00 %	1.55 %	5.17 %
	Real			
1y	0.28 %	1.63 %	-2.66 %	4.87 %
3y	0.26 %	1.28 %	-1.80 %	2.42 %
5y	0.50 %	1.14 %	-1.70 %	2.20 %
7y	0.79 %	1.05 %	-1.49 %	2.52 %
10y	1.14 %	0.92 %	-1.06 %	2.91 %
UK				
	Nominal			
1m	2.81 %	2.22 %	0.43 %	5.91 %
3m	2.78 %	2.21 %	0.39 %	5.83 %
6m	2.75 %	2.20 %	0.33 %	5.85 %
1y	2.72 %	2.12 %	0.18 %	5.83 %
2y	2.82 %	1.92 %	0.07 %	5.76 %
3y	3.00 %	1.75 %	0.16 %	5.69 %
5y	3.33 %	1.46 %	0.57 %	5.56 %
7y	3.60 %	1.22 %	1.02 %	5.48 %
10y	3.90 %	0.96 %	1.61 %	5.36 %
	Real			
1y	0.01 %	2.40 %	-3.72 %	4.43 %
3y	0.22 %	1.83 %	-2.61 %	2.82 %
5y	0.46 %	1.50 %	-2.20 %	3.09 %
7y	0.62 %	1.24 %	-1.80 %	2.50 %
10y	0.78 %	0.96 %	-1.31 %	2.16 %

Table 2: Estimation Results US

In this table we present the estimation results of three models using the Minimum Chi Squared approach for the US. All estimates are determined using the full sample ranging from August 2004 up to December 2012. The standard errors are determined by a weighted outer-product as described in Appendix C.

	Benchmark		Macro		Survey	
Parameter	Estimate	Standard error	Estimate	Standard error	Estimate	Standard error
Dynamics latent factors: $X_t = \Phi_1 X_{t-1} + \Sigma \epsilon_t$						
$\Phi_{1,11}$	0.9933	0.0019	0.9939	0.0016	0.9942	0.0002
$\Phi_{1,21}$	-0.0093	0.0015	-0.0083	0.0014	-0.0090	0.0006
$\Phi_{1,22}$	0.9858	0.0013	0.9864	0.0011	0.9857	0.0008
Price of Risk: $\Lambda_t = \Gamma_0 + \Gamma_1 X_t$						
$\Gamma_{0,EC}$	1.3164	0.3070	2.0016	0.3257	1.4412	1.4123
$\Gamma_{0,1}$	0.6435	0.0845	0.6452	0.0943	0.6228	0.0469
$\Gamma_{0,2}$	1.2409	0.3617	1.1414	0.3285	1.2893	0.1102
$\Gamma_{1,11}$	-0.0267	0.0042	-0.0211	0.0032	-0.0229	0.0017
$\Gamma_{1,12}$	0.0263	0.0064	0.0266	0.0055	0.0235	0.0028
$\Gamma_{1,21}$	-0.0457	0.0089	-0.0349	0.0067	-0.0432	0.0057
$\Gamma_{1,22}$	0.0228	0.0038	0.0188	0.0030	0.0199	0.0018
$\Gamma_{1,CPICPI}$	0.0000	0.7097	0.0000	1.1800	0.0000	0.8974
$\Gamma_{1,CPIEC1}$	-	-	0.0000	0.1074	-0.0737	0.0418
$\Gamma_{1,CPIEC2}$	-	-	0.0000	0.0877	-0.0447	0.0288
$\Gamma_{1,CPIEC3}$	-	-	0.0000	0.1064	0.0117	0.0570
$\Gamma_{1,CPIEC4}$	-	-	-	-	-0.0178	0.0299
$\Gamma_{1,CPI1}$	-0.0257	0.0133	-0.0394	0.0069	-0.0249	0.0829
$\Gamma_{1,CPI2}$	0.0353	0.0088	0.0471	0.0061	0.0366	0.0138
Short rate: $r_t^N = \delta_{0,r} + \delta_r' X_t$						
$\delta_r * 12$	0.0168	0.0018	0.0168	0.0018	0.0168	0.0018
$\delta_{r,1} * 12$	0.0010	0.0003	0.0011	0.0003	0.0010	0.0001
$\delta_{r,2} * 12$	0.0017	0.0002	0.0019	0.0002	0.0018	0.0001
Inflation rate: $\pi_t = \delta_{0,\pi} + \delta_\pi' X_t + \sigma_\pi' \epsilon_t$						
$\delta_{0,\pi}$	0.0001	0.0010	0.0000	0.0027	0.0003	0.0052
$\delta_{\pi,CPI}$	0.5064	0.0882	0.1770	0.1075	0.5521	0.1124
$\delta_{\pi,EC1}$	-	-	0.0456	0.0098	0.0108	0.0052
$\delta_{\pi,EC2}$	-	-	-0.0017	0.0080	0.0062	0.0036
$\delta_{\pi,EC3}$	-	-	-0.0098	0.0097	-0.0087	0.0071
$\delta_{\pi,EC4}$	-	-	-	-	-0.0020	0.0037
$\delta_{\pi,1} * 1000$	0.0402	0.0492	0.0427	0.0837	0.0361	0.2957
$\delta_{\pi,2} * 1000$	0.0063	0.0305	0.0160	0.0443	0.0140	0.0489
$\sigma_{\pi,CPI}$	-0.0036	0.0015	-0.0036	0.0003	-0.0033	0.0105
$\sigma_{\pi,1}$	0.0000	0.0027	0.0002	0.0014	0.0000	0.0042
$\sigma_{\pi,2}$	-0.0016	0.0035	-0.0002	0.0031	-0.0017	0.0201

Table 3: Results US Measurement errors

The measurement errors of three estimated models for the US are presented in this table. We report the full sample standard deviation of the measurement error across our entire sample period and the standard deviation of three periods. The first period, denoted as prior the financial crisis, ranges from August 2004 up to August 2008. As starting point for the Financial Crisis in 2008, we take the bankruptcy filling of Lehman Brother in September and we define the crisis period up to August 2009. The period afterward is defined as post crisis and ends in December 2012.

	Full sample	Prior crisis	During crisis	Post crisis
Benchmark				
Nominal				
ω_{1M}	0.0035	0.0038	0.0027	0.0014
ω_{3M}	0.0022	0.0019	0.0026	0.0011
ω_{6M}	0.0012	0.0012	0.0009	0.0007
ω_{2Y}	0.0009	0.0010	0.0010	0.0005
ω_{3Y}	0.0009	0.0009	0.0011	0.0005
ω_{7Y}	0.0009	0.0009	0.0013	0.0007
ω_{10Y}	0.0019	0.0018	0.0025	0.0016
Real				
ω_{1Y}	0.0099	0.0048	0.0168	0.0046
ω_{3Y}	0.0053	0.0026	0.0075	0.0038
ω_{5Y}	0.0043	0.0022	0.0057	0.0033
ω_{7Y}	0.0039	0.0021	0.0055	0.0030
ω_{10Y}	0.0035	0.0021	0.0048	0.0025
Macro				
Real				
ω_{1Y}	0.0106	0.0058	0.0190	0.0062
ω_{3Y}	0.0053	0.0030	0.0075	0.0041
ω_{5Y}	0.0043	0.0024	0.0061	0.0034
ω_{7Y}	0.0039	0.0023	0.0056	0.0030
ω_{10Y}	0.0035	0.0022	0.0048	0.0025
Survey				
Real				
ω_{1Y}	0.0100	0.0052	0.0163	0.0049
ω_{3Y}	0.0053	0.0029	0.0071	0.0039
ω_{5Y}	0.0043	0.0023	0.0056	0.0033
ω_{7Y}	0.0039	0.0022	0.0053	0.0029
ω_{10Y}	0.0034	0.0021	0.0046	0.0025

Table 4: Estimation Results UK

In this table we present the estimation results of three models using the Minimum Chi Squared approach for the UK. All estimates are determined using the full sample ranging from August 2004 up to December 2012. The standard errors are determined by a weighted outer-product as described in Appendix C.

	Benchmark		Macro		Survey	
Parameter	Estimate	Std error	Estimate	Std error	Estimate	Std error
Dynamics latent factors: $X_t = \Phi_1 X_{t-1} + \Sigma \epsilon_t$						
$\Phi_{1,11}$	0.9958	0.0016	0.9955	0.0016	0.9960	0.0009
$\Phi_{1,21}$	-0.0169	0.0023	-0.0171	0.0024	-0.0167	0.0011
$\Phi_{1,22}$	0.9785	0.0016	0.9783	0.0016	0.9784	0.0013
Price of Risk: $\Lambda_t = \Gamma_0 + \Gamma_1 X_t$						
$\Gamma_{0,EC}$	10.3594	4.3597	10.3457	4.0116	10.2048	8.8535
$\Gamma_{0,1}$	1.5049	0.4022	1.4927	0.3715	1.5340	0.2341
$\Gamma_{0,2}$	2.4425	0.8958	2.4586	0.8613	2.4999	0.4876
$\Gamma_{1,11}$	-0.0137	0.0035	-0.0144	0.0039	-0.0134	0.0017
$\Gamma_{1,12}$	0.0391	0.0045	0.0393	0.0047	0.0388	0.0035
$\Gamma_{1,21}$	-0.0427	0.0055	-0.0438	0.0061	-0.0424	0.0037
$\Gamma_{1,22}$	0.0227	0.0036	0.0234	0.0039	0.0224	0.0020
$\Gamma_{1,CPICPI}$	28.3748	13.5459	74.1052	18.7796	12.4202	13.8914
$\Gamma_{1,CPIEC1}$	-	-	0.0000	1.7080	-0.0001	0.4691
$\Gamma_{1,CPIEC2}$	-	-	0.0000	7.1958	0.0003	1.0564
$\Gamma_{1,CPIEC3}$	-	-	0.0001	0.4639	0.0000	1.0128
$\Gamma_{1,CPIEC4}$	-	-	-	-	-0.0002	0.8489
$\Gamma_{1,CPI1}$	-0.1261	0.0428	-0.1227	0.0391	-0.1203	0.0927
$\Gamma_{1,CPI2}$	0.1623	0.0513	0.1777	0.0542	0.1615	0.1084
Short rate: $r_t^N = \delta_{0,r} + \delta_r' X_t$						
$\delta_r * 12$	0.0282	0.0022	0.0282	0.0022	0.0282	0.0022
$\delta_{r,1} * 12$	0.0016	0.0002	0.0015	0.0002	0.0016	0.0000
$\delta_{r,2} * 12$	0.0021	0.0002	0.0020	0.0002	0.0021	0.0000
Inflation rate: $\pi_t = \delta_{0,\pi} + \delta_\pi' X_t + \sigma_\pi' \epsilon_t$						
$\delta_{0,\pi}$	-0.0034	0.0071	-0.0038	0.0056	-0.0031	0.0227
$\delta_{\pi,CPI}$	0.1703	0.1001	-0.0079	0.0933	0.0912	0.0992
$\delta_{\pi,EC1}$	-	-	0.0452	0.0084	0.0044	0.0040
$\delta_{\pi,EC2}$	-	-	0.0085	0.0357	0.0099	0.0091
$\delta_{\pi,EC3}$	-	-	0.0017	0.0023	0.0002	0.0087
$\delta_{\pi,EC4}$	-	-	-	-	0.0231	0.0073
$\delta_{\pi,1} * 1000$	0.0271	0.0918	0.0262	0.0713	0.0295	0.3114
$\delta_{\pi,2} * 1000$	-0.1470	0.1034	-0.1550	0.0877	-0.1458	0.3577
$\sigma_{\pi,CPI}$	-0.0020	0.0010	-0.0020	0.0008	-0.0020	0.0031
$\sigma_{\pi,1}$	-0.0021	0.0019	-0.0004	0.0014	-0.0024	0.0063
$\sigma_{\pi,2}$	0.0031	0.0014	0.0031	0.0006	0.0025	0.0042

Table 5: Results UK Measurement errors

The measurement errors of three estimated models for the UK are presented in this table. We report the full sample standard deviation of the measurement error across our entire sample period and the standard deviation of three periods. The first period, denoted as prior the financial crisis, ranges from August 2004 up to August 2008. As starting point for the Financial Crisis in 2008, we take the bankruptcy filling of Lehman Brother in September and we define the crisis period up to August 2009. The period afterward is defined as post crisis and ends in December 2012.

	Full sample	Prior crisis	During crisis	Post crisis
Benchmark				
Nominal				
ω_{1M}	0.0031	0.0026	0.0036	0.0020
ω_{3M}	0.0022	0.0020	0.0023	0.0014
ω_{6M}	0.0012	0.0013	0.0013	0.0006
ω_{2Y}	0.0009	0.0009	0.0014	0.0006
ω_{3Y}	0.0009	0.0007	0.0014	0.0007
ω_{7Y}	0.0009	0.0007	0.0015	0.0007
ω_{10Y}	0.0017	0.0014	0.0024	0.0017
Real				
ω_{1Y}	0.0116	0.0032	0.0183	0.0061
ω_{3Y}	0.0061	0.0028	0.0101	0.0035
ω_{5Y}	0.0044	0.0024	0.0090	0.0021
ω_{7Y}	0.0032	0.0022	0.0073	0.0016
ω_{10Y}	0.0027	0.0021	0.0054	0.0017
Macro				
Real				
ω_{1Y}	0.0116	0.0051	0.0219	0.0067
ω_{3Y}	0.0058	0.0031	0.0102	0.0035
ω_{5Y}	0.0041	0.0025	0.0084	0.0022
ω_{7Y}	0.0030	0.0023	0.0065	0.0017
ω_{10Y}	0.0026	0.0021	0.0047	0.0017
Survey				
Real				
ω_{1Y}	0.0110	0.0035	0.0172	0.0064
ω_{3Y}	0.0059	0.0028	0.0095	0.0037
ω_{5Y}	0.0042	0.0024	0.0086	0.0023
ω_{7Y}	0.0032	0.0022	0.0070	0.0018
ω_{10Y}	0.0027	0.0020	0.0052	0.0017

Table 6: Out of sample forecasts: US

This table reports the errors of the out-of-sample forecasts of the US using three methods, namely the Root mean squared error (RMSE), Mean absolute deviation (MAD) and the out-of-sample R squared (Out of Sample R^2). We forecast the rates using a fixed window from September 2009 up to December 2012. The bold numbers represent the model that performs the best for each time series.

	RMSE			MAD		
	Benchmark	Macro	Survey	Benchmark	Macro	Survey
1m	0.1942 %			0.1615 %		
3m	0.1597 %			0.1282 %		
6m	0.0844 %			0.0665 %		
1y	0.0839 %			0.0583 %		
2y	0.1495 %			0.1172 %		
3y	0.1861 %			0.1433 %		
5y	0.2450 %			0.1939 %		
7y	0.3052 %			0.2494 %		
10y	0.3617 %			0.3012 %		
1y real	0.8478 %	**0.8175** %	0.8329 %	0.7823 %	**0.6717** %	0.7295 %
3y real	0.4697 %	**0.4655** %	0.4851 %	0.4059 %	**0.3661** %	0.3988 %
5y real	**0.4473** %	0.4522 %	0.4510 %	0.3983 %	0.3743 %	**0.3736** %
7y real	0.4544 %	0.4614 %	**0.4529** %	0.4055 %	0.3912 %	**0.3825** %
10y real	0.4484 %	0.4541 %	**0.4442** %	0.4031 %	0.3913 %	**0.3789** %
CPI	0.2740 %	**0.1942** %	0.2991 %	**0.2113** %	0.2302 %	0.2450 %
	R^2 Out of Sample					
	Benchmark	Macro	Survey			
1m	99.13%					
3m	99.44%					
6m	99.86%					
1y	99.86%					
2y	99.53%					
3y	99.19%					
5y	98.24%					
7y	96.70%					
10y	94.12%					
1y real	79.61%	**82.84** %	82.19 %			
3y real	92.37%	**92.98** %	92.37 %			
5y real	92.23%	92.41 %	**92.45** %			
7y real	91.14%	91.17 %	**91.49** %			
10y real	89.43%	89.45 %	**89.90** %			
CPI	**20.85**%	14.63 %	5.68 %			

Table 7: Results UK Out of sample

This table reports the errors of the out-of-sample forecasts of the UK using three methods, namely the Root mean squared error (RMSE), Mean absolute deviation (MAD) and the out-of-sample R squared (Out of Sample R^2). We forecast the rates using a fixed window from September 2009 up to December 2012. The bold numbers represent the model that performs the best for each time series.

	RMSE			MAD		
	Benchmark	Macro	Survey	Benchmark	Macro	Survey
1m	0.1862 %			0.1531 %		
3m	0.1549 %			0.1283 %		
6m	0.1193 %			0.1034 %		
1y	0.1028 %			0.0799 %		
2y	0.1408 %			0.1194 %		
3y	0.1874 %			0.1552 %		
5y	0.2168 %			0.1638 %		
7y	0.2502 %			0.1898 %		
10y	0.2916 %			0.2280 %		
1y real	1.3176 %	**1.2622** %	1.2631 %	1.1621 %	**1.0901** %	1.1625 %
3y real	0.7072 %	0.7025 %	**0.6881** %	0.6183 %	**0.5813** %	0.6125 %
5y real	**0.4630** %	0.4928 %	0.4793 %	**0.3849** %	0.3963 %	0.4184 %
7y real	**0.3019** %	0.3447 %	0.3299 %	**0.2418** %	0.2780 %	0.2830 %
10y real	**0.2594** %	0.2955 %	0.2849 %	**0.2051** %	0.2408 %	0.2237 %
CPI	0.4161 %	**0.1862** %	0.4486 %	0.3416 %	**0.2854** %	0.3612 %
	R^2 Out of Sample					
	Benchmark	Macro	Survey			
1m	99.60%					
3m	99.72%					
6m	99.83%					
1y	99.87%					
2y	99.72%					
3y	99.43%					
5y	98.96%					
7y	98.06%					
10y	95.85%					
1y real	87.06%	**88.12** %	88.11 %			
3y real	93.54%	93.62 %	**93.88** %			
5y real	**95.81**%	95.25 %	95.51 %			
7y real	**97.34**%	96.53 %	96.82 %			
10y real	**96.67**%	95.67 %	95.98 %			
CPI	-33.79%	**-0.22** %	-55.51 %			

Table 8: Data implied Campbell-Shiller Regressions

This table presents the estimates of the coefficients of the Campbell-Shiller regressions for both the UK and US interest rates. The regressions are defined in Equation (13). The reported standard errors are based on the Newey-West estimator. The p-value is shown for the test with the null hypothesis of a slope coefficient equal to one.

	US		UK	
Maturity	Coef	*p* Value	Coef	*p* Value
Nominal				
1y	0.39 (1.06)	0.57	2.86 (2.60)	0.48
2y	0.09 (1.58)	0.57	−0.07 (1.52)	0.48
3y	−0.89 (1.78)	0.29	−0.87 (1.30)	0.15
5y	−2.42 (1.89)	0.07	−1.44 (1.37)	0.08
7y	−3.35 (2.16)	0.05	−1.81 (1.53)	0.07
10y	−4.34 (2.62)	0.04	−2.31 (1.70)	0.05
Real				
1y	1.38 (0.95)	0.69	0.22 (0.46)	0.09
3y	0.85 (1.14)	0.90	−0.70 (0.79)	0.03
5y	0.61 (1.42)	0.78	−1.86 (0.85)	0.00
7y	0.45 (2.19)	0.80	−1.88 (0.82)	0.00
10y	−1.35 (2.36)	0.32	−2.53 (1.00)	0.00

Table 9: Model implied Campbell-Shiller Regressions

This table presents the model implied estimates of the coefficients of the Campbell-Shiller regressions for both the UK and US. The regressions are defined in Equation (13).

	US	UK
Maturity	Coef	Coef
Nominal		
Benchmark		
1y	0.06	-0.01
	(0.10)	(0.10)
2y	-0.11	-0.25
	(0.07)	(0.06)
3y	-0.28	-0.50
	(0.05)	(0.02)
5y	-0.60	-0.97
	(0.02)	(0.03)
7y	-0.89	-1.33
	(0.01)	(0.06)
10y	-1.26	-1.63
	(0.04)	(0.07)
Real		
Benchmark		
1y	0.94	0.78
	(0.03)	(0.02)
3y	0.98	0.72
	(0.01)	(0.06)
5y	0.99	0.13
	(0.01)	(0.12)
7y	0.98	-0.44
	(0.03)	(0.15)
10y	0.91	-0.92
	(0.07)	(0.16)
Macro		
1y	1.62	1.79
	(0.04)	(0.04)
3y	1.60	1.52
	(0.03)	(0.07)
5y	1.59	1.23
	(0.04)	(0.10)
7y	1.58	0.95
	(0.05)	(0.12)
10y	1.53	0.63
	(0.07)	(0.15)
Survey		
1y	0.83	1.01
	(0.03)	(0.03)
3y	0.87	0.99
	(0.02)	(0.04)
5y	0.88	0.70
	(0.02)	(0.09)
7y	0.87	0.37
	(0.03)	(0.13)
10y	0.83	0.00
	(0.05)	(0.15)

Table 10: Fitting of Term premia

This table presents Mean absolute deviations (MAD) of the the model implied nominal and real term premia for both the UK and US. We show the MAD for all three models. We also report the standard error of the mean deviations.

	US		UK	
Maturity	MAD	Std	MAD	Std
Nominal				
Benchmark				
1y	0.02%	0.01%	0.05%	0.03%
2y	0.03%	0.02%	0.04%	0.03%
3y	0.05%	0.04%	0.05%	0.04%
5y	0.11%	0.09%	0.06%	0.05%
7y	0.19%	0.13%	0.08%	0.07%
10y	0.28%	0.19%	0.11%	0.08%
Real				
Benchmark				
1y	0.12%	0.06%	0.08%	0.07%
3y	0.11%	0.04%	0.14%	0.13%
5y	0.06%	0.04%	0.26%	0.23%
7y	0.11%	0.08%	0.28%	0.25%
10y	0.23%	0.21%	0.36%	0.33%
Macro				
1y	0.16%	0.16%	0.17%	0.17%
3y	0.14%	0.14%	0.18%	0.16%
5y	0.13%	0.13%	0.29%	0.25%
7y	0.15%	0.14%	0.31%	0.27%
10y	0.23%	0.18%	0.39%	0.34%
Survey				
1y	0.14%	0.08%	0.08%	0.06%
3y	0.13%	0.07%	0.15%	0.12%
5y	0.09%	0.06%	0.26%	0.23%
7y	0.14%	0.09%	0.28%	0.25%
10y	0.25%	0.21%	0.36%	0.33%

Table 11: Inflation risk premia

This table present the inflation risk premia of the UK and US. We report the mean and standard error of the series for the 5 and 10 year premium.

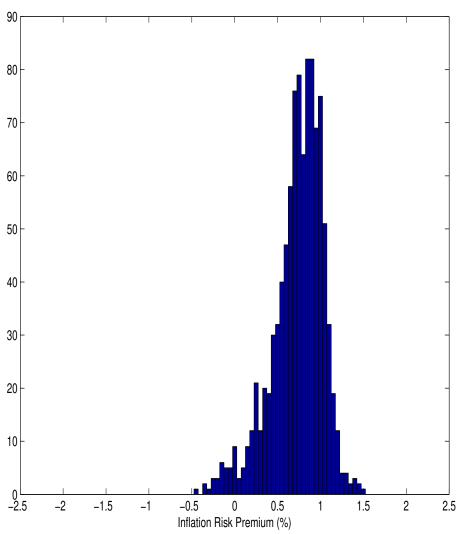
	US		UK	
Maturity	Mean	Std	Mean	Std
Benchmark				
5y	0.72%	0.65%	-0.45%	0.80%
10y	0.69%	0.58%	-0.26%	0.49%
Macro				
5y	0.82%	0.63%	0.13%	0.13%
10y	0.79%	0.56%	0.32%	0.32%
Survey				
5y	0.68%	0.64%	-1.08%	0.87%
10y	0.65%	0.56%	-0.90%	0.53%

F Appendix F: Graphs

Figure 1: Bayesian 5 year inflation risk premium US: Bench model

This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Benchmark model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.74% with a standard deviation of 0.30% and in the second graph it is centered around 0.38% and a standard deviation of 0.37%.

(a) Uninformative Prior



(b) Post Crisis Prior

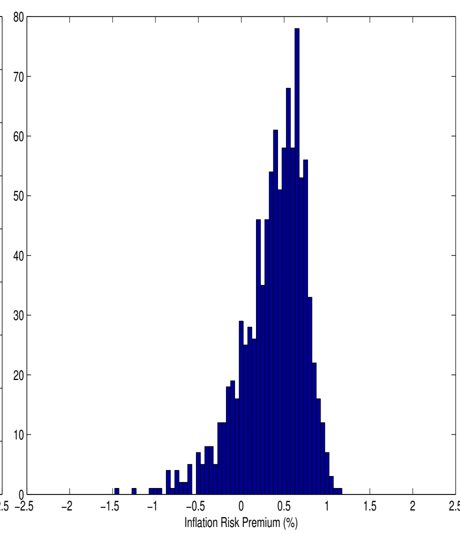


Figure 2: Bayesian 5 year inflation risk premium UK: Bench model

This figure presents the marginal posterior distribution of the mean of the UK 5 year inflation risk premium using the Benchmark model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around -0.08% with a standard deviation of 0.47% and in the second graph it is centered around 0.13% and a standard deviation of 0.61%.

(a) Uninformative Prior (b) Post Crisis Prior

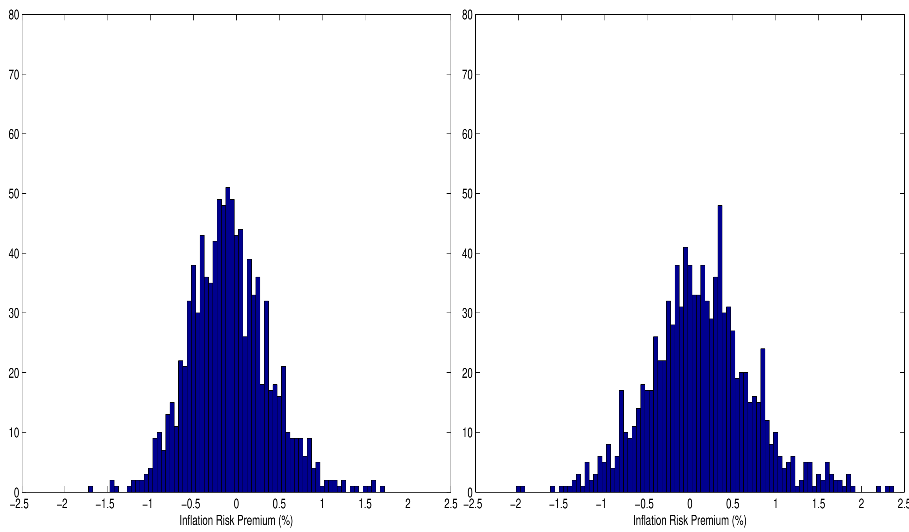


Figure 3: Bayesian 5 year inflation risk premium US: Macro model

This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Macro model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.99% with a standard deviation of 0.43% and in the second graph it is centered around 0.96% and a standard deviation of 0.40%.

(a) Uninformative Prior (b) Post Crisis Prior

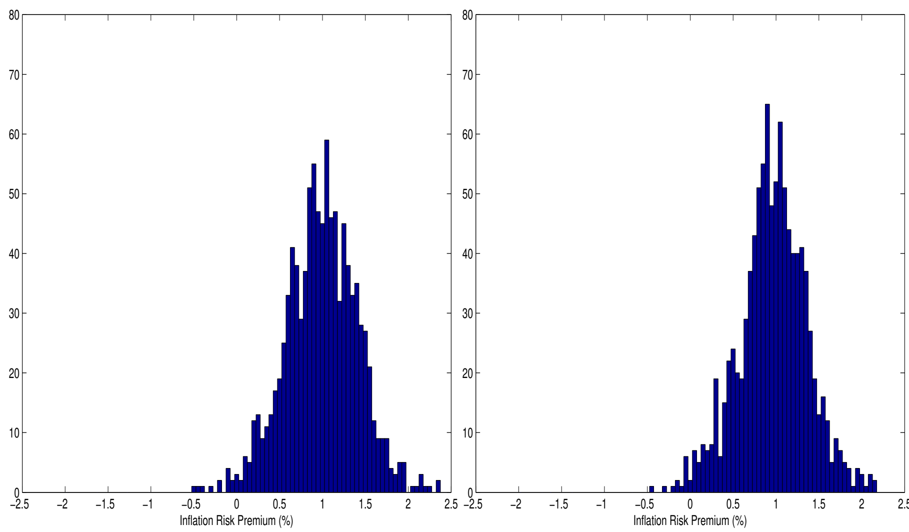


Figure 4: Bayesian 5 year inflation risk premium UK: Macro model

This figure presents the marginal posterior distribution of the mean of the UK 5 year inflation risk premium using the Macro model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.02% with a standard deviation of 0.67% and in the second graph it is centered around 0.49% and a standard deviation of 0.76%.

(a) Uninformative Prior (b) Post Crisis Prior

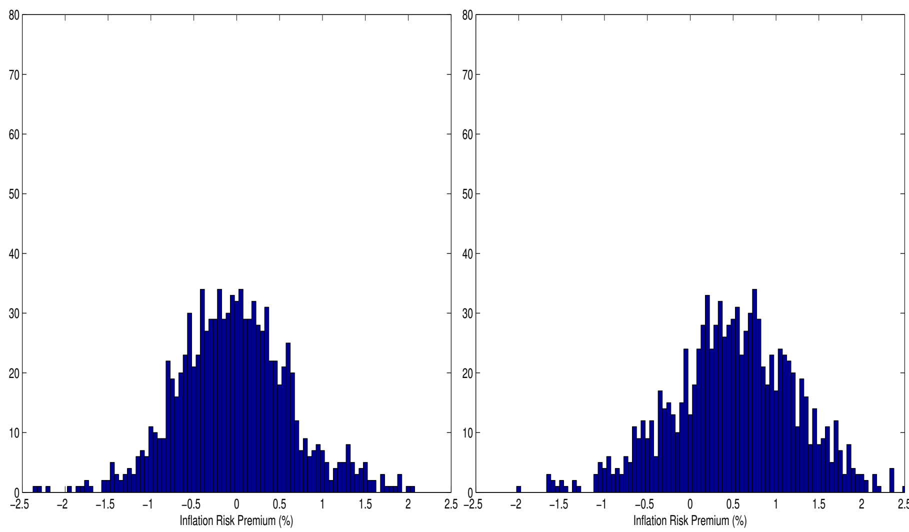


Figure 5: Bayesian 5 year inflation risk premium US: Survey model

This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Survey model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.68% with a standard deviation of 0.31% and in the second graph it is centered around 0.33% and a standard deviation of 0.32%.

(a) Uninformative Prior (b) Post Crisis Prior

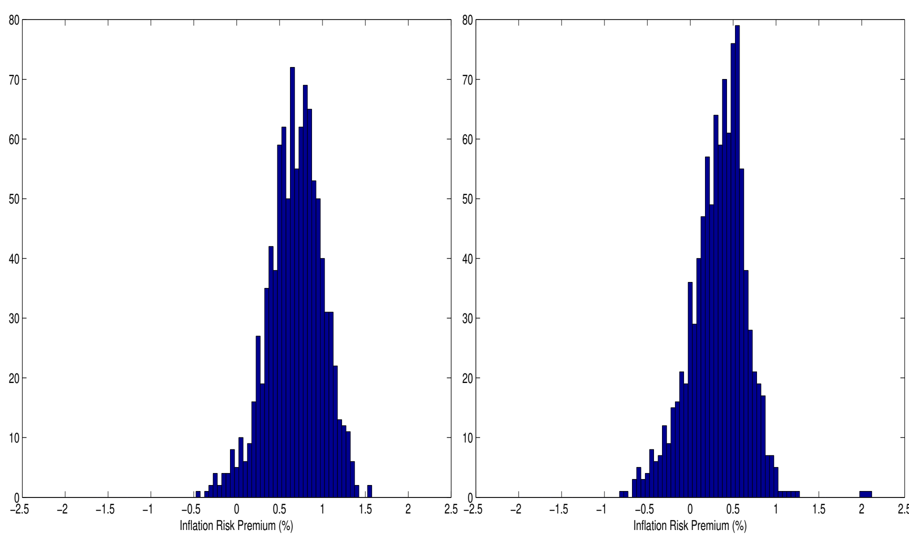


Figure 6: Bayesian 5 year inflation risk premium UK: Survey model

This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Survey model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around -0.03% with a standard deviation of 0.87% and in the second graph it is centered around 0.03% and a standard deviation of 0.97%.

(a) Uninformative Prior   (b) Post Crisis Prior