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How Often Should Investors Rebalance Their Portfolio When Trading Costs Are Present?

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Master Thesis Finance

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Abstract

It will be examined what the optimal trading strategy is for an investor when trading costs are present. This will be done for different portfolios. The trading costs are assumed to be proportional to the amount of each asset traded. Amongst other findings, it was found that for periodic rebalancing the yearly interval and for no-trade region rebalancing the 1% interval provide the investor, with risk aversion A = 10, with the highest utility as well as the highest absolute return. When the different scenarios were compared to the benchmark scenario, which is a scenario without rebalancing, it was seen that the returns of the benchmark cases are most of the time higher. However, due to the fact that the standard deviation of these scenarios is also higher, the higher return is offset and there are scenarios with rebalancing which still provide the investor with a higher utility.

Chapter 1: Introduction

When investing in assets, investors start with a specific weight in each asset. However, when time passes these weights change due to fluctuations in price. To bring assets back in line, and thus to maintain the initial risk and return characteristics, investors rebalance their portfolio. Some investors rebalance their portfolio each time an asset deviates from the initial weight. Under the assumptions that the mean and the variance are constant and no trading costs are present, the previous strategy is optimal. However, when trading costs are present this changes. Also when momentum is assumed, which means that it is assumed that the price of an asset is more likely to keep moving in the same direction rather than to change its direction, it is not optimal to rebalancing constantly. However, momentum is not assumed and thus excluded from this research. As mentioned before trading costs can change the decision when to rebalance because trading costs diminish the net return. Therefore, the effect of trading costs on the rebalancing decision is examined in this research.

In practice, determining how often to rebalance your portfolio is difficult and complex. Current literature determines that there are 2 main strategies for rebalancing a portfolio: periodic rebalancing and no-trade region rebalancing. Periodic rebalancing means that a portfolio is rebalanced once a period, which can be for example, daily or yearly. In no-trade region rebalancing investors rebalance an asset when the weight deviates more than a pre-determined percentage from the initial weight. These two rebalancing strategies will be investigated in this research and compared to each other and the scenario in which no rebalancing takes place.

The main research question that will be investigated is ‘how often should investors rebalance their portfolio when trading costs are present?’. This is analyzed by using the two different rebalancing strategies mentioned before. Due to the fact that the decision also depends on other factors, there are different scenarios for the following parameters: trading cost and rebalancing policy. Rebalancing policy can be either that there will be rebalanced back to the initial weight or to either a specific threshold or the closest boundary, depending on which rebalancing strategy is used.

Researching this main question is interesting due to the fact that in the current literature how often to rebalance is only investigated for portfolios consisting of one or two assets, whereas a lot of investors have bigger portfolios with many different assets.

There are 3 different portfolios investigated in this research. The first portfolio only includes one risky and one risk-free asset whereas the other two portfolios include 4 risky assets as well as one risk-free asset. The assets in the second two portfolios are chosen to be diversified because this represents a portfolio of an investor with more assets. For these assets the return will be retrieved from DataStream. Trading cost data is gathered from previous papers on this topic. In this study only data from the United States is used.

The approach that is followed in this research is based on Monte Carlo simulations by means of Excel. However, first of all the portfolio weights are calculated by calculating the tangency portfolio which is the portfolio maximizing the Sharp Ratio. This approach was used for portfolio 1 and portfolio 2. For portfolio 3 a different approach is taken: it is assumed that the return and volatility of all risky assets are the same and the weights are evenly distributed. After determining the weights of the assets in the portfolio Monte Carlo simulations for all different scenarios can be done. Finally, the results of the Monte Carlo simulation are inserted in the utility function, calculating the utility for an investor with a specific risk aversion. In this way it can be determined which rebalancing strategy is preferred by the investor.

Chapter 2 starts with discussing the current state of the literature on rebalancing portfolios. Afterwards, chapter 3 gives a detailed explanation of the methodology of this research, of which a step-by-step plan can be found in Appendix A.1. Chapter 4 discusses and analyzes the results of this research. Finally, the research will be ended with a brief conclusion in chapter 5, which also includes some ideas for future research.

Chapter 2: Literature Review

First of all section 1.1 will give an introduction about rebalancing; what it is and why investors do this. Section 2.2 will elaborate on the downside of rebalancing: trading costs. Section 2.3 will discuss the difference for institutional traders as opposed to individual traders. Section 2.4 introduces different rebalancing strategies, where section 2.5 and 2.6 discuss the periodic rebalancing strategy and the no-trade region rebalancing strategy in more detail, respectively. Section 2.7 will give another perspective on rebalancing as opposed to the most common strategies: a behavioural perspective.

2.1 Introduction

When investing in assets investors start with a specific weight in each asset, however when time passes this weight changes. This is due to changes in the financial markets and the values and prices of the assets traded in these markets. These movements in the market lead to shifts in the allocation of the funds in the investors' portfolio. This shift in allocation results in the fact that some assets in the portfolio will be overweighted and others will be underweighted when compared to the initial allocation of the investor. A well-diversified investor is not willing to hold a disproportionate share of wealth in any of the assets in his portfolio (unknown, 2007). This is because this would mean that the investment incurs larger risk than what the investor intended as well as the fact that the portfolio might be less diversified. Therefore, the investor wants to rebalance his portfolio at regular intervals to bring the assets back in line to the initial allocation and thus to not become over-/under invested in one type of asset. Leibowitz and Bova (2011) argue that rebalancing is actually nothing more than a desired return to a relatively fixed long-term portfolio strategy. However, holding constant weights is of course not always optimal due to the fact that the investor's optimal choice might not be really optimal in the environment. This is due to the fact that the optimal choice for each investor differs according to their personal circumstances such as need for liquidity and age. Next to that, the environment also changes which might change the optimal portfolio in the market. However, for the purpose of this research it is assumed that it is always aimed to hold the initial portfolio weights approximately constant over time.

Generally, rebalancing is just buying and/or selling enough assets to restore the original weights the investor intended to have in his portfolio. When one asset begins to dominate the portfolio, this asset should partly be sold and another asset should be purchased instead. According to Buetow et al (2002) rebalancing a fund can be understood as a contrarian strategy. In their example, being a two-asset class case, the rebalancing strategy sells equities after they have outperformed and purchases equities after they have underperformed. Vice versa holds for bonds. This is confirmed by Leibowitz and Bova (2011) who rebalance the portfolio in their example by buying risky assets when they are 'cheap' after a fall and by selling 'expensive' assets after their prices rose. Summarizing, rebalancing demands to partly sell better performing assets and purchase less performing assets from its proceeds (Dichtl et al, n.d.). However, a risk which investors face is human nature because the investor has to take money from assets who have performed well and reinvest these proceeds in an asset which had lagged behind, which sounds contrary and is sometimes difficult for humans due to heuristics in human nature (Masters, 2003).

There are several benefits for investors when rebalancing their portfolio. First of all, rebalancing reduces risk significantly compared to a buy-and-hold strategy, however it does not (or only slightly) increase mean returns (Dichtl et al, n.d.). The same return as well as a reduction of the risk is an improvement for the portfolio and thus one of the advantages of rebalancing. Next to that, it also reduces concentration risk and downside risk (Bouchey et al, 2012). Second, it is argued by Willenbrock (2011) that "if diversification is the only 'free lunch' of investing, then the diversification return from rebalancing is the only 'free dessert'". By this he means that even though a buy-and-hold portfolio is diversified in the beginning, eventually it will become concentrated due to shifts in the market, as opposed to a rebalanced portfolio in which you make sure your portfolio is diversified at all times. Thus, rebalancing creates extra return from diversification. Third, some researchers, amongst others Bouchey et al (2012), have argued that there is a 'rebalancing premium', which yields extra return by creating positive portfolio growth even though the overall asset growth is flat by constantly buying low and selling high. However, this 'rebalancing premium' is not taken into account in this analysis.

2.2 Trading costs

Trading costs are non-zero in practice. Trading continuously in a risky asset would be subject to infinite trading costs (Liu, 2004). This thus affects the decision whether or not and how often to rebalance. Trading costs are the costs of buying and selling assets in the market. These costs diminish the net return which is earned by investing. This makes the choice of rebalancing more difficult due to the fact that the investor has to consider the costs relative to the additional return of rebalancing. As argued by Lovell and Arnott (1989) portfolio rebalancing can be seen as a simple trade-off: the cost of trading versus the cost of not trading. The cost of not trading includes the fact that the portfolio traded is not completely optimal anymore, for example assets being over-/underpriced, not fitting the needs of the client and/or being poorly diversified. Considering this trade-off, Maretno et al (2005) found that rebalanced portfolios outperform portfolios without rebalancing being present. This is the main reason why investors consider rebalancing their portfolios even though it involves trading costs.

Nevertheless, it is found by Magill and Constantinides (1976), Barclay et al (1998) and Liu (2004) that trading costs lead to trading less frequently, and thus longer holding periods, as well as lower trading volume. This is also confirmed by de Jong and Driessen (2013) who argue that the best response to trading costs is to rebalance your portfolio infrequently instead of continuously. They even found that an investor only needs a 0.2% extra annual return on a risky asset to be compensated for a 1% transaction cost. However, this highly depends on the composition of the portfolio. Masters (2003) found that the benefit of rebalancing rises geometrically whereas the costs of rebalancing are linear which means that when you sell twice as much this will approximately cost twice as much. However, in the current literature there is still disagreement about the magnitude of the effect of trading costs. [1]

Next to reduced trading volume and longer holding periods of assets it is also found that the higher the transaction costs are, the lower the amount invested in the risky asset (de Jong and Driessen,

[1] *It should be noted the results of the researches stated above and below might not be compared completely because the results are depended on the assumptions made in the different researches. The results may deviate when different assumptions are made about the assets in the portfolio, markets, trading costs and rebalancing policy.*

2013). Amihud et al (2005) argue that this is the case because the trading costs attached to the risky asset make the risk-free asset more attractive to invest in. This leads to the fact that the risk-free rate will decrease and the equity premium will increase. However, this effect is only significant when the trading costs are large enough or when the quantity of the assets traded is small. Another effect of trading costs is that the higher the trading costs are the less likely it is that investors will respond to information signals in the market (Amihud et al, 2005).

Due to the fact that trading costs are highly economically significant it is important to know what influence they have, which is mentioned before, and what they exist of. Trading costs are made up of different components, the most obvious being brokerage commission and price impact. Sun et al (2006) state that the tracking error, the expected future costs of your current actions, costs of personnel and technological resources also need to be taken into account. Bikker et al (2007) found that also delay costs and opportunity costs should be included. In addition, Keim and Madhayan (1997) argue that trading costs can be divided into two components: explicit costs and implicit costs. The explicit costs are the most transparent and easy to quantify costs, such as brokerage commission and taxes. The implicit costs consist essentially of the price impact of the trade, also called 'market impact'. This can be defined as "the deviation of the transaction price from the unperturbed price that would prevail had the trade not occurred" (Keim and Madhayan, 1997). The height of this cost is dependent on two components: one size related component and one component which is related to the bid-ask spread. Furthermore, Bikker et al (2007) found that also price volatility, momentum, trade type, trading venue and industry sector have an effect on the market impact cost. They also argue that when it takes longer to execute a trade, the market impact costs will be lower, although the volatility will be higher. Lastly, price impact on the market is most likely to be smaller when the market is more liquid and has a larger market capitalization (Keim and Madhayan, 1997). Also, the bid-ask spread is lower when the market is more liquid.

Apart from that, Keim and Madhayan (1997) also argue that trading costs are influenced by investment style as well as the market on which the assets trade. It was found that it makes especially a difference whether assets are traded at an exchange using an auction mechanism or an exchange which operates as a dealer market. Moreover, the fact that trading costs differ across investment styles is due to the fact that different investments styles thus have a different trading purpose. A technical trader looks for short-term price movements and an indexer trades at the market closing price. Both have the need to trade quickly which will lead to higher trading costs. This is opposed to a value trader who looks for long-term value and thus trades less incurring less trading costs. In their quantative research they found that technical and index traders have 0.61% and 0.45% higher trading costs compared to value traders, respectively. Due to the fact that this is a substantial amount they argue that trading costs should never be studied in isolation of investment style. However, even within the different investment styles there is heterogeneity in trading costs. They attribute this to differences in trading skills.

In research papers, assumptions about the trading costs have to be made. According to de Jong and Driessen (2013) there can be distinguished between three different types of assumptions on trading costs: fixed trading costs, proportional trading costs and quadratic trading costs. They argue that a different assumption on trading costs creates a different optimal rebalancing strategy. Liu (2004) found in his research that when fixed transaction costs are assumed, and thus the cost is independent of the size of the trade, the optimal trading strategy for each asset is to trade to the same target as soon as the amount in the asset is beyond a pre-specified constant range. For proportional trading costs the optimal trading strategy is to trade as soon as the asset falls below (buy) or rises above (sell) back to certain pre-specified boundaries. Most researchers assume linear proportional trading costs in their studies. Additionally, Leland (1999) states that the trading costs may also differ between buying and selling. Besides, Keim and Madhayan (1997) found in their sample that buys are more costly sells. This is unlike Bikker et al (2007) who found that trading costs are, on average, 27.4 bp for buys and 37.5 for sells. However, in almost all studies these are assumed to be equal.

2.3 Institutional Traders

However, there are some differences for individual traders and institutional traders. Institutional traders account for a big part of the international stock holdings (e.g. in 2001 they owned more than 50% of total

US equities) meaning that they have an impact on the market. On the one hand, this can be an advantage for them due to the fact that their size allows them to negotiate more attractive commission rates than individuals (Bhide, 1992). This is confirmed by Keim and Madhayan (1997) who found that costs are inversely related to market capitalization. On the other hand, this can also disadvantage them because when institutions trade this may have a price effect and this may cause additional trading costs, called 'market impact costs'. This happens when execution prices are less favorable than benchmark prices due to price effects (Bikker et al, 2007). Next to that, institutional investors face restraints on active stockholding compared to individuals, however, they face lower taxation of trading gains and low deductibility of losses (Bhide, 1992). Summarizing, there is quite a difference in trading costs of institutional investors than individuals.

2.4 Different Rebalancing Strategies

As can be seen, trading costs have a large impact on rebalancing and determining how often to rebalance is difficult and complex. The market changes continuously and as already established trading continuously when trading costs are present is not optimal. However, the investor still has to adapt his portfolio to the changing market environment, otherwise he will be exposed to a lot of unintended risk. The loss of not rebalancing can be reduced by trading more often, but trading more often will lead to higher transaction costs and thus the optimal investment strategy is to minimize the sum of the tracking error and the trading costs (Leland, 1999). However, as there is agreement on the benefits of rebalancing there is currently little agreement on the right rebalancing strategy. This is due to the fact that in these researches a lot of assumptions are made and for a model with a lot of assets the problem is not yet solved as opposed to a model with only 1 or 2 assets. Nowadays, investors mostly rely on ad hoc heuristics to determine when and how often to rebalance (Donohue and Yip, 2003). The most widely used rebalancing heuristics, in academic literature as well as used by portfolio managers, are either periodic or no-trade region rebalancing (Dichtl et al, n.d.).

Periodic rebalancing, also called strict frequency rebalancing by Buetow et al (2002), implies that the investor rebalances his portfolio at regular time intervals. This can be for example weekly, monthly or yearly. It is advised that investors should rebalance their portfolios at least once a year (Unknown, 2007). When applying the no-trade region rebalancing, which is also called interval rebalancing by Buetow et al (2002), the investor rebalances his portfolio whenever the asset ratio deviates more than a specific percentage from the initial weight. This percentage is pre-specified and triggers action of the investor. However, for both strategies the investor also needs a rebalancing policy to indicate how far he has to rebalance: back to the initial weight or, for example, only half way. Also, a combination of periodic and no-trade region rebalancing can be used. Sometimes there is not even a strategy and investors only rebalance when action is dictated by the funds cash in- or outflows (Buetow et al, 2002). Often the validation of a specific rebalancing strategy cannot be found in a theoretical framework but is rather due to the desire for simplicity and pragmatism (Sun et al, 2006). Still, the results for each of the rebalancing methods make intuitive sense because it can be seen that infrequent trading incurs lower trading costs, but higher tracking error costs (Sun et al, 2006).

2.5 Periodic Rebalancing

More specifically, the periodic rebalancing strategy is the most popular rebalancing strategy where the investor adjusts the weights in his portfolio at regular time intervals, which can be daily, weekly, monthly, quarterly or yearly. The popularity of this strategy is due its simplicity. Within this strategy rebalancing yearly is most common. However, different studies found different results for the best period after which to rebalance. Buetow et al (2002) found that daily monitoring was the best interval leading to the highest improvement of performance. In contrast, Dichtl et al (n.d.) argue that the most optimal rebalancing interval varies between quarterly and yearly, due to the fact that these intervals have the lowest annual volatility, which is 9.73%. They tested this among different horizons and countries. This is in accordance with another research who found that the average annual return will increase from 7.575%

to 9.519% with a stable variance when the investor rebalances his portfolio approximately annually (Unknown, 2007).

Nevertheless, when compared to other rebalancing strategies periodic rebalancing is evaluated as not being optimal. Leland (1999) argues that even though the periodic rebalancing strategy is popular, it is not optimal, especially when proportional trading costs are assumed. Other strategies with the same value of tracking error can reduce the expected turnover by about 50%. Next to that, also accuracy is less when periodic rebalancing is used as opposed to other rebalancing strategies. The most important drawback however is that periodic rebalancing is independent of market behavior (Sun et al, 2006). The investor rebalances at specific time intervals even though market behavior has not changed and thus it might not be necessary to rebalance. Masters (2003) argues that the strategy is too mechanistic, which he explains by giving the following example: when an asset rises in July and declines in November it does not make sense to rebalance every six months due to the fact that the investor fails to protect himself against the increase/decrease in the asset value.

2.6 No-trade Region Rebalancing

Nowadays a strategy that is studied often and which is said to be more optimal than the periodic rebalancing strategy is the strategy with a ‘no-trade’ region, called ‘no-trade region rebalancing’. In this method a ‘no-trade’ region will be determined, which is a band around the initial asset weights (Leland, 1996). When the asset is inside the no-trade region no rebalancing will take place, but when the asset is outside of the no-trade region there should be rebalanced back to a level at the boundary or within the no-trade region. Buetow et al (2002) suggest to monitor the portfolio daily and rebalance when the boundaries are exceeded. Some investors choose to rebalance all the way back to the initial weight, other investors only to the nearest boundary of the no-trade region and others something in between these two extremes. Amihud et al (2005) argue that this depends on which kind of trading costs are assumed. When fixed trading costs are assumed it is more optimal to trade all the way back to the initial weights, whereas for proportional trading costs it is more optimal to trade back to the nearest boundary of the no-trade region.

The most important determinant of the no-trade region is obviously trading cost. The higher the trading costs, the wider the no-trade region (Amihud et al, 2005; de Jong and Driessen, 2013; Leland, 1996). Liu (2004) found that even if trading costs are only small this can lead to no-trade regions of substantial size due to the fact that holding a suboptimal portfolio is not very costly. More specifically, Leland (1996) found that the variation in the size of the no-trade region can be approximated by the cube root of trading costs or equivalently proportional to the standard deviation to the 2/3 power. Also, risk aversion influences the no-trade region. When risk aversion increase the no-trade region will shift to the risk-free asset as well as the fact that the no-trade region will shrink in size (Constantinides, 1986; Liu, 2004; Donohue and Yip, 2003). Liu (2004) found that the more risk averse the investor is the less frequently he should trade whereas Masters (2003) argues that the frequency increases when the investor is more risk averse. Next to that, there is disagreement about whether variance affects the no-trade region. According to Constantinides (1986) it does not, whereas Leland (1996) argues that doubling the variance does increase the no-trade interval. Other determinants influencing the no-trade region identified by Leland (1996) are the risk-free interest rate, the difference in expected return between different assets in the portfolio and the cost per unit of the tracking error. As with the trading costs these parameters also change the no-trade region with the cube root of parametric changes, meaning that the change in the no-trade region is proportional to the cuberoot of the parametric change in the specific variable. The sensitivity of the width of the no-trade region also depends on the type of trading costs assumed. Boundaries, when fixed transaction costs are present as opposed to proportional costs, change less when the costs are increased (Liu, 2004). In addition, the no-trade region differs when the assets in the portfolio are correlated as opposed to being uncorrelated due to the fact that these assets are substitutes of each other as well as the fact that the diversification effect is smaller (Liu, 2004). The size of the no-trade region shrinks when assets are correlated (Donohue and Yip, 2003).

According to Donohue and Yip (2003) it can be concluded that the no-trade region rebalancing strategy systematically seems to find the right balance between the costs of tracking error and trading costs, leading to a maximization of utility and risk/return ratio. This is confirmed by findings in literature on the performance of this strategy. Some examples are given as follows. The strategy with no-trade region is found to decrease trading costs as well as turnover by about 50% (Leland, 1996; Sun et al, 2006). Buetow et al (2002) found in their research that rebalancing beyond the 5% interval does not add significant value however there still might be a slight benefit when raising the interval to 10%, but above that it does not make sense. Next to that, the frequency of trading is not only determined by the width of the no-trade region, it is also affected by the location of the no-trade region.

As can be noticed the no-trade region rebalancing strategy depends on a lot of factors and is quite complex due to its different dimensions. Therefore, currently most research on the no-trade region is done by using a portfolio of only two assets. However, Liu (2004) suggests that when there are more than two assets which are uncorrelated and proportional trading costs are assumed it is possible to establish the no-trade boundaries for each asset separately.

Buetow et al (2002) argue that one of the most important criteria for evaluating which rebalancing strategy to choose should be risk control. Their argument for this is that when portfolios are not rebalanced they lose their diversification benefit by floating towards a higher percentage in highly risky assets than initially intended. Thus, by choosing a rebalancing strategy the investor should make sure that this does not lead to higher risk or when it does that this is compensated with higher return as well. The other two criteria they mention are trading costs and implementation simplicity.

The benefits which can be gained from rebalancing also differ dependent on the portfolio the investor holds. When there are more assets in the portfolio and the weights among the assets are more evenly distributed the benefit from rebalancing should be higher (Bouchey et al, 2012). Also, the correlation between the assets in the portfolio influences the effect of the rebalancing strategy. Rebalancing add most value when the assets are perfectly negatively correlated (Buetow et al, 2002).

2.7 Behavioral Perspective

Lovell and Arnott (1989) distinguish themselves in their theory on how often and when to rebalance a portfolio. They look at rebalancing from a more behavioral, customer centered perspective. It was argued that they consider rebalancing when this is demanded by important changes in the objectives of a client because each client is unique and so should be the managers' understanding of his clients' needs. Examples of such client circumstances are change in wealth, change in time horizon, change in liquidity requirements, change in law, regulation or tax and unique preferences. When there are significant changes in these client needs trading costs should be taken for granted.

Chapter 3: Methodology

First, in section 3.1 a short summary of the overall process is described. Section 3.2 discusses how the portfolios used in the research are established and which assets they include. Afterwards in section 3.3, it is described how the weights are allocated to the assets in the portfolios by using mean-variance optimization. Section 3.4 discusses the trading costs and their assumptions. Next to that, the other inputs which are needed for the simulation are discussed in section 3.5. Furthermore, it is described how the simulation for the benchmark scenario, periodic rebalancing and no-trade region rebalancing are set up in section 3.6, 3.7 and 3.8 respectively. In section 3.9 the robustness tests and sensitivity analysis are discussed. Finally, section 3.10 gives an overview of all the assumptions made in this research.

3.1 Introduction

The different strategies, periodic and no-trade region rebalancing, will be investigated when trading costs are present. Using the different strategies with the same transaction costs and the same portfolios the return will be simulated for a horizon of 10 years. This will be done by using Monte Carlo simulations in excel. There will be run 1,000 simulations of which an average return (mean), median, volatility and different percentiles will be extracted. Afterwards the returns as well as the volatilities of the different strategies can be compared and it can be determined which strategy is the most optimal for the investor in which scenario.

3.2 Asset allocation

First a portfolio of only one risky asset is examined after which the amount of assets will be built up. In total 3 portfolios will be examined: (1) 1 risky asset and 1 risk-free asset, (2) 4 diversified risky assets and 1 risk-free asset and (3) 4 diversified risky assets with the same return and volatility , and 1 risk-free asset.

Due to the fact that in the scenario with a portfolio of only one risky asset it is hard to diversify an index will be used. A stock based on an index tracks the specific index. The index which will be used here is the S&P 500 (SPY) which consists of the 500 largest firms in the US based on market capital. This is a solid equity core which is broadly diversified across different sectors and individual stocks. The S&P 500 is the most known and reliable stock index in the US. Therefore, it also is one of the world's most heavily traded securities, making the stock very liquid and cheap to trade. These are the reasons for choosing this stock for portfolio 1 to use as a diversified stock.

The risk-free asset in all portfolios is a US Treasury bill, called Vanguard Long-Term Treasury Inv (VUSTX). It consists for 98.98% of US Treasuries which are all AAA rated. The rest, 1.02%, consists of cash. Next to that, 80.38% of the US Treasuries in the portfolio are long-term, between 20 and 30 years. Also, the US Treasuries of which the asset consists are backed up by the US government. Summing all of this up, leads to the fact that this asset is considered to be almost risk-free. Therefore, in this case it is assumed to be risk free and used as the risk-free asset in all 3 portfolios.

Portfolio 2 and 3 consist of a combination of individual assets of companies being active in different market sectors. This is done due to the fact that the assets should be diversified because this would represent a portfolio with different assets. In table 3.1 below it can be found for each asset in which industry sector the company operates. The decision for choosing individual assets for portfolio 2 and 3 have been made due to the following. When using mean-variance optimization where volatility is minimized it is difficult to make a reasonable portfolio which consists of a combination of indexes, ETFs and individual assets because their volatilities differ too much and the risk-return trade-off is not in line. This leads to the fact that the weights for the individual assets will be 0% and the weights for the indexes very high. This is mainly due to the fact that the return and volatilities are calculated by using the average of previous years. It is especially difficult to estimate returns of individual assets which are less stable than indexes due to the fact that indexes are already more spread because of the fact that it includes more assets. A decision had to be made between only indexes or only individual assets due to the fact that when combining individual assets with indexes a reasonable portfolio cannot be made. Also, when analyzing different indexes which could be in the portfolio the differences in risk and return were still too large and would lead to a portfolio out of proportion. Therefore, the decision was made to only include individual assets next to the risk-free asset which leads to a reasonable portfolio.

As can be seen in the table below, portfolio 2 and 3 consist of the same assets. For portfolio 2 the average return of the assets is calculated. This return data can be found in DataStream and the average will be taken over the past 20 years. For some assets this is not possible due to the fact that the asset exists for less than 20 years. In that case, as most years of data is used as possible. However, it is also interesting to investigate a scenario which is not influenced by the historical data of the assets in the portfolio, because historical data causes differences between the assets. Therefore, for portfolio 3 it will be assumed that the return and the volatility of all the assets are the same. The numbers for return and volatility will be determined by taking the average of the averages of each asset for return and volatility which were calculated for portfolio 2. This leads to a daily return of 0.0460% and a daily volatility of 1.6300%. Next to that, it is also assumed that the weights among the risky and risk-free part are evenly distributed, meaning that both parts are 50% of the complete portfolio. Assuming the same return and volatility for all assets automatically leads to the fact that the weights of all assets within the risky part of the portfolio are the same. All assumptions above lead to the differences between portfolio 2 and 3 even though the assets in the portfolios are the same.

The portfolios include the following stocks/funds:

Portfolio	*Name*	*Ticker*	*Type/Industry Sector*
Portfolio 1			
1 Risky Asset +	SPDR S&P 500	SPY	S&P 500 index
1 Risk-free Asset	Vanguard Long-Term Treasury Inv	VUSTX	Treasury Bill index
Portfolio 2 and 3			
4 Risky Assets +	Vanguard Long-Term Treasury Inv	VUSTX	Treasury Bill index
1 Risk-free Asset	Wal-Mart Stores	WMT	FMCG
	AON Class A	AON	Insurance
	Procter & Gamble	PG	Consumer Goods
	Coca Cola Company	KO	Food and Beverage

Table 3.1: Portfolio construction

3.3 Tangency Portfolio

First of all, the initial weights of the assets in the different portfolios have to be determined. This will be done by using mean-variance optimization which maximizes the Sharpe ratio ($= \frac{E(Rp)}{\sigma p}$) and leads to the tangency portfolio. This portfolio combines the optimal combination of risky assets that maximize return at each level of risk with a risk-free asset. The tangency portfolio can be calculated by using excel. The inputs which are needed are the returns of the assets. First, the average return for each asset in the portfolio will be calculated as well as the volatility. Also, the correlation matrix and variance-covariance matrix will be calculated. This can be done by using the excel formulas which can be found in the step-by-step plan for the methodology in Appendix A.1. The formulas in this chapter as well as the appendix contain Greek characters of which the explanations can be found in Appendix A.2.

Afterwards, a vector will be made for the weights and next to that the portfolio expected return and the portfolio standard deviation will be calculated using the following formulas:

$$E(Rp) = w^{T\mu} \qquad \sigma p = (w^T \sum w)^{1/2}$$

Using this information, solver in excel is used to extract the weights of the assets in the portfolio when maximizing the Sharpe Ratio of the portfolio. However, some constraints have to be added to the solver: (1) the sum of the weights must be equal to 100% and (2) short selling is not allowed so the individual weights should be higher than 0%. No short-sales are allowed due to the fact that it leads to extreme positions in some assets which are not realistic. The weights which the solver calculates are the weights for the risky part only. However, the portfolio also contains the risk-free asset and thus this needs to be taken into account. The division between the risky and risk-free part of the portfolio can be calculated by using the following formula, where the solution for w^* will give the weight in the risky part of the portfolio:

$$w^* = \frac{E(Rp) - Rf}{A\sigma p^2}$$

The return and the volatility of the complete portfolio are calculated using the following formulas:

$$E(Rc) = Rf + \frac{[E(Rp) - Rf]^2}{A\sigma p^2} \qquad \sigma c = \frac{[E(Rp) - Rf]}{A\sigma p^2}$$

'A' in these formulas is the risk aversion of the investor, which is assumed to be 10. When choosing 10 as risk aversion a portfolio is created which could be a real-life portfolio investors would hold. A high risk aversion, such as 10, is needed because it explains the return of assets in the market as well as it is the reason why only a small amount of people invests in stocks.

The weights for the risky individual assets in the complete portfolio can be calculated by multiplying their weight which resulted from the solver by w*. The weight of the risk-free asset is 100% - w^*. This will result in the weights for each of the assets in the complete portfolio. To check whether everything went correct, the weights should add up to 100%.

3.4 Trading Costs

In this research proportional trading costs are assumed. This means that the cost of each individual asset is a percentage of the amount of the asset that is rebalanced. Thus, the higher the price of the asset, the higher the cost. Proportional trading costs are the most convenient to use due to the fact that only one percentage needs to be determined and it is easy to incorporate in the research. The other options were fixed trading costs or quadratic trading costs. Fixed trading costs are difficult due to the fact that for each individual asset a different trading cost has to be determined and it has to rely on solid arguments. Next to that, it is fixed and thus does not automatically adjust when other variables change. Quadratic costs are the most complex of the three and it is thus difficult to incorporate it in the research. Next to that, it is in this case not better than using proportional costs. Therefore, it resulted in the fact that it is assumed that the trading costs are proportional.

Next to that, it is assumed that the costs of buying and the costs of selling are equal. This is done for simplicity reasons because when this would not be the case that would make the research highly complex whereas that is not necessary. However, due to the fact that it is difficult to determine one number for trading costs for different assets, the simulations will be done with 2 different numbers for trading costs, which are: 0.005 and 0.20 percent point of the rebalancing value.

These are based on the papers of Constantinides (1986), Keim and Madhayan (1997) and Morton and Pliska (1995). The amounts for the cost that are chosen are quite extreme and far apart. However, this is due to the fact that in the above mentioned papers they do use very different trading costs in each paper. All of the following mentioned trading costs are stated in percent point. Morton and Pliska (1995) assume trading costs ranging from 0.001 to 0.005, whereas Keim and Madhayan (1997) use a range for trading costs from seller and buyer initiated trades ranging from 0.08 to 0.87. Constantinides (1986) is somewhat in the middle of these two by using trading costs ranging from 0.005 to 0.2. None of the papers mentions their reasons for choosing these specific levels of trading costs. This makes it hard to determine one trading cost which can be used in this research. Therefore, an educated guess should be made. On the one hand, the 0.87 is assumed to be too extreme for trading costs, because this would lead to a huge amount of trading costs. On the other hand, 0.001 is considered too low. Therefore, one trading costs which is somewhat in the middle is chosen, which is 0.2 and the other one will be 0.005. This is due to the fact that 2 out of 3 papers use this as trading costs and next to that it is not as extremely low as 0.001. Both these trading costs are used because it shows what impact the level of trading costs has on the rebalancing decision and its return. These trading costs are only for the risky assets in the portfolio. The trading costs for risk-free assets are assumed to be zero due to the fact that the asset does not experience any risk compared to the risky asset, which is the basis of the trading cost.

3.5 Inputs

The overall benchmark scenario with which all other scenarios will be compared is a 10 year horizon in which no rebalancing will take place. For each of the three different portfolios a benchmark scenario will be made. The other scenarios differ from the baseline case according to some parameters. These are: rebalancing strategy and rebalancing policy. Rebalancing policy can be explained as how far there will be rebalanced, meaning in this research either back to the initial weight or to a specific interval. When comparing the other scenarios to this benchmark scenario it can first of all be determined whether rebalancing the portfolio in the specific case is efficient. Next to that, for each rebalancing strategy the benchmark scenario is the first case which is analyzed. The other scenarios will be compared to this case in terms of return, standard deviation, input parameters and assumptions. Also, the different rebalancing scenarios can be compared with each other. After establishing the other scenarios their robustness needs to be checked before determining the best strategy.

One of the inputs that is needed for the Monte Carlo simulation is the probability distribution of asset returns. It is found that the distribution at each point in time is identical and independent from what happened in the past (Bouchey et al, 2012). Therefore, it can be assumed that the probability distribution of the returns is normal. The other inputs for the Monte Carlo simulation are expected return, volatility and the initial weights. The volatility can be calculated in excel using the past returns. All inputs which are used for the Monte Carlo simulation can be found in Appendix A.4.

3.6 Benchmark Scenario

To simulate a portfolio using a Monte Carlo simulation an initial investment of $1,000,000 for the whole portfolio is assumed. To determine the initial investment for the different assets the total initial investment is multiplied by the weight of the asset in the specific portfolio. Together with the average return and the volatility these are the inputs needed to start the simulation. A time horizon of 10 years is set up in the excel sheet. However, the period should be in trading days instead of years due to the fact that the simulation has to be done for each day seperately. This means that, when assuming an average of 260 trading days per year, there are 2,600 trading days in 10 years. This also means that the daily return and daily volatility should be used. The simulated return for each day is calculated, which is repeated for each of the 2,600 trading days. Each result yields a possible one day return value. Then, the ending value needs to be calculated. This is done by multiplying the simulated return by last periods' value. This process is repeated for each asset in the portfolio separately by using its own initial investment, return and volatility.

When this is done for all assets the final ending values are added up for all the different assets, leading to the ending value of the complete portfolio.

Afterwards, a simulation iteration can be set up which executes the simulation 1,000 times. This means that every time the whole process is repeated for all the individual assets and the ending value which was added up is adjusted accordingly, giving 1,000 different ending values for the complete portfolio. The simulation iteration is done by using the ‘data table’ function in excel and yields a result which generates 1,000 different ending values. Finally, the mean, median, volatility and two percentiles are calculated from these 1,000 values, giving the expected result of the portfolio over a 10 year horizon. The 5% and the 25% percentile will be calculated. For example, the 5% percentile generates the minimum value which the investor will have with a 95% chance after 10 years. These are the steps that need to be taken to create the benchmark scenario in which no rebalancing takes place.

3.7 Periodic Rebalancing

For the periodic rebalancing strategy 4 different intervals of rebalancing will be assumed. First of all, the different portfolios will be simulated using a daily rebalancing interval. Afterwards, also monthly, quarterly and yearly rebalancing will be simulated. For all scenarios there are two rebalancing policies. There is the possibility to rebalance back to the initial weights or to a 0.5% threshold, meaning that the weight to which the asset will be rebalanced back is either 0.5% lower or higher than the initial weight depending on whether the current weight is below or above the initial weight. This threshold is chosen taking into account the deviation between the initial weights of the assets in the portfolio and the current weights. When looking at this, it was seen that most of the time there was a maximum deviation of around 0.5% for the small assets whereas this was 1% for the larger assets. Therefore, a threshold of 0.5% was chosen.

Using the periodic rebalancing strategy means that you have to rebalance at the regular pre-specified time intervals. Normally, yearly rebalancing would mean that once every 365 days rebalancing would take place. However, due to the fact that only trading days are concerned this is not the case. The same holds for monthly and quarterly rebalancing. It is assumed that there are 260 trading days in a year and thus for yearly rebalancing once every 260 days there will be rebalanced. For monthly rebalancing the following calculation, 2600 trading days / (10*12), shows that there are 21.6667 days per month. Therefore, it is assumed that there will be 22 days in one month meaning that there will be 118 months in 10 years instead of 120. For quarterly rebalancing calculation shows that there should be 54.1667 days per quarter (2600 trading days / (12*4)). In this case it is assumed that there are 54 days in one quarter, meaning that there will be 48 full quarters in 10 years.

The process for the simulation is the same as the basic simulation stated before. Whether there should be rebalanced depends on the period because there has to be rebalanced every pre-specified numbers of days which is mentioned before. For example, for monthly rebalancing every 22 days there has to be rebalanced. When there will be rebalanced, the weight of the asset will be set equal to the initial weight again or the initial weight plus or minus the threshold, depending on the rebalancing policy which is used. In all other cases the weight will stay as was calculated by the simulation in that period. Afterwards, the costs of rebalancing will be allocated to each period when rebalancing took place and afterwards this amount will be added up over the whole investment horizon. This cost will be subtracted from the return leading to a net return. Finally, the total equity, total cost of rebalancing and total net return of all the individual assets should be added up to determine the total return and cost of the complete portfolio. Afterwards the simulation iteration needs to be done and the solutions which were also calculated for the benchmark scenario. This is done for all the different time intervals. A set-up of the excel columns of the periodic rebalancing simulation including excel formulas can be found in Appendix A.3.

3.8 No-trade Region Rebalancing

For the no-trade region rebalancing strategy also 3 different rebalancing intervals will be assumed. These are 1%, 2% and 5% intervals, meaning that the portfolio will be rebalanced when the weight deviates more than the specific interval. Also for this strategy there are two possible rebalancing policies:

rebalance back to the initial weights or back to the closest boundary. The closest boundary is then the interval which is assumed.

As with the benchmark scenario the returns and resulting values for each asset in each period are calculated. The new weight is calculated in the same way as for periodic rebalancing: by dividing the ending value by the total initial investment. Next to that, initially for each asset a range in which the weight is allowed is established. The boundaries are calculated by subtracting and adding the intervals from and to the initial weight. The weight after each period should fall into this interval otherwise rebalancing should take place. This means that if the weight within the interval the weight stays that weight, however if it is outside the interval the weight should be rebalanced back to the initial weight (or the closest boundary, depending on the rebalancing policy).

The new weight should be included in the simulation by using the ending value. The new weight should be multiplied by the total investment to create a new ending value of the whole portfolio. This new ending value is used as the starting point for the next period. Finally, to determine the net return the costs should be subtracted from the ending value of equity previously calculated. Also in this case the total equity, total cost of rebalancing and total net return of all the individual assets should be added up. From these results the simulation iteration should be set up. Next to that, again the solutions will be calculated. The same process is repeated for the different intervals. A set-up of the excel columns of the no-trade region rebalancing simulation including excel formulas can be found in Appendix A.3.

After the results are calculated for all the simulations from all different portfolios and scenarios (also for periodic rebalancing), the utility for the specific investor with risk aversion A = 10 will be calculated. This will be done by means of the following formula:

$$U = E(Rp) - \frac{1}{2}\, A\, \sigma^2$$

As can be seen from the formula above the utility takes into account the return and the standard deviation of the portfolio and thus the utility can be used to compare the different scenarios and portfolios to each other. The utility refers to the total satisfaction which is received from consuming a good or service, in this case meaning that total satisfaction equals the wealth obtained from the specific portfolio.

3.9 Robustness Test and Sensitivity Analysis

After all the results of the Monte Carlo simulation for all the different assets are computed a robustness test has to be done. The purpose of this robustness test is to check whether the results of the research are stable in different scenarios. This is especially important when the results are compared due to the fact that not all results might have the same stability. However, some robustness checks are already done due to the fact that the research includes portfolios which consist of different (types of) assets. One other robustness checks will also be done: different holding period. For this an investment horizon of 5 years will be taken instead of 10 years. However, due to the fact that doing this for all 3 portfolios for all 4 scenarios is not achievable due to time constraints, this will only be done for one portfolio but for different scenarios including all the different parameters. Therefore, it is chosen to do the robustness checks for portfolio 2 for scenario 1, 0.00005 trading costs and rebalancing back to the initial weight, and scenario 4, 0.02 trading costs and rebalancing back to a threshold, both for periodic – and no-trade region rebalancing.

To determine the best strategy correctly it is also important to do a sensitivity analysis due to the fact that some results might be highly sensitive to changes in the assumptions on parameters made. The sensitivity analysis is already included in the research by using 3 different portfolios, 2 different trading cost rates, 2 different rebalancing policies and for both the periodic and the no-trade region based rebalancing strategy 4 and 3 different intervals are included, respectively.

3.10 Summary of assumptions

To sum up, the assumptions that are made in this research are:

- Short sales are not allowed
- Trading costs are proportional
- Trading costs for buys and sells are equal
- Trading costs are equal for the different rebalancing policies
- The securities pay no dividends
- Taxes on capital gains are zero
- The probability distribution of returns is normal
- A year consists of 260 trading days, a month of 22 trading days and a quarter of 54 trading days
- The correlations between different assets are neglected
- The return and volatility of all the risky assets in portfolio 3 are the same

Chapter 4: Results

First, it is introduced how the results will be analyzed in paragraph 4.1. Afterwards, it will be analyzed in section 4.2 which rebalancing range within periodic as well as no-trade region rebalancing has the highest utility. In section 4.3 the different scenarios for each portfolio are compared to the benchmark scenario in which no rebalancing takes place. Then section 4.4 will discuss whether periodic or no-trade region rebalancing is more optimal for all portfolios overall. Section 4.5 will go into this in more detail discussing for each portfolio specifically which strategy with which rebalancing range provides the investor with the highest utility. To check the stability of the results, section 4.6 will analyze the results of the robustness test and sensitivity analysis. In section 4.7 it is discussed whether the results of this research are in line with the current literature on this topic. Finally, section 4.8 will mention the limitations of the research.

4.1 Introduction

The results of all the simulations of the different portfolios and scenarios can be found in Appendix A.5. These include the return, median and standard deviation of the wealth, the costs of rebalancing and the return on the portfolio. A general result that can be observed when looking at these outcomes, specifically the return and the standard deviation, is that when going from daily to yearly rebalancing the return as well as the standard deviation increase. A higher return is more profitable for the investor, however a higher standard deviation is more risky and thus more costly. Therefore, it cannot be determined which rebalancing strategy would be most optimal for the investor by looking at the return and the standard deviation directly. Therefore, the utility for each case should be calculated. The utilities of the different scenarios and portfolios can be found in table 4.1. The table shows the utilities for each scenario with each portfolio and rebalancing strategy. The columns determine the portfolio whereas the rows determine the scenarios in the first place as well as the rebalancing strategy. The same is also done for the scenario without rebalancing and the robustness test later on in this chapter.

	Portfolio 1	Portfolio 2	Portfolio 3
Scenario 1 (Rebalancing back to the initial weight and costs = 0.00005)			
Daily rebalancing	0.050362	0.080034	0.063198
Monthly rebalancing	0.051249	0.083440	0.084563
Quarterly rebalancing	0.050584	0.084574	0.084384
Yearly rebalancing	0.054215*	0.087746*	0.087663*
1 % no-trade region rebalancing	0.050939*	0.045114*	0.043202*
2% no-trade region rebalancing	0.050510	0.032700	0.034266
5% no-trade region rebalancing	0.033547	0.029497	0.029382
Scenario 2 (Rebalancing back to the initial weight and costs = 0.02)			
Daily rebalancing	0.051933	0.075108	0.077963
Monthly rebalancing	0.054529*	0.084299	0082900
Quarterly rebalancing	0.049892	0.085307*	0.081443
Yearly rebalancing	0.048782	0.085299	0.084611*
1 % no-trade region rebalancing	0.052490*	0.045266*	0.042440*
2% no-trade region rebalancing	0.048361	0.035096	0.033547
5% no-trade region rebalancing	0.051642	0.027327	0.029398
Scenario 3 (Rebalancing back to the closest boundary and costs = 0.00005)			
Daily rebalancing	0.050942	n.a.	n.a.
Monthly rebalancing	0.052104	0.077743	0.078349
Quarterly rebalancing	0.050819	0.080394	0.088169
Yearly rebalancing	0.052794*	0.084968*	0.090199*
1 % no-trade region rebalancing	0.050483	0.063975*	0.062162*
2% no-trade region rebalancing	0.052516	0.057959	0.058248
5% no-trade region rebalancing	0.055884*	0.055009	0.057334
Scenario 4 (Rebalancing back to the closest boundary and costs = 0.02)			
Daily rebalancing	0.052241	n.a.	n.a.
Monthly rebalancing	0.048878	0.078238	0.076407
Quarterly rebalancing	0.051192	0.080074	0.085416
Yearly rebalancing	0.053459*	0.086947*	0.086963*
1 % no-trade region rebalancing	0.051307*	0.061388*	0.061765*
2% no-trade region rebalancing	0.050726	0.056053	0.057122
5% no-trade region rebalancing	0.050764	0.056689	0.056567

Table 4.1 Utilities for investor with risk aversion A = 10, for all portfolios and scenarios. *The wealth that is used to calculate the return used for the utility is the wealth after subtracting the costs of rebalancing and the initial investment. * indicates the highest utility value for the combination of the portfolio, scenario and the rebalancing strategy. Underlined is the rebalancing strategy with the highest utility for the combination of the portfolio and the scenario.*

4.2 Rebalancing strategy with the highest utility

The table above can be used to compare the utilities of the different intervals of the rebalancing strategies within the portfolios for each scenario. This can be done by looking at the column portfolio 1 and then comparing the utilities of the periodic rebalancing of, for example, scenario 1 with each other. The same should also be done for no-trade region rebalancing. This is done for all the scenarios and portfolios. The stars in the table indicate the result of this: a star is added to the rebalancing strategy which has the highest utility within each portfolio and scenario. When looking at portfolio 1, it can be seen that for 3 out of 4 scenarios the star concerning the periodic rebalancing strategy is located at yearly rebalancing, thus meaning that yearly rebalancing provides the investor with the highest utility. Only for scenario 2 the star can be found at monthly rebalancing, which means that in this case the utility for monthly rebalancing is higher than the utility for yearly rebalancing. For no-trade region rebalancing it can also be seen that 3 out of 4 scenarios give the same result, saying that the 1% threshold is the most optimal rebalancing strategy. The same can be analyzed for portfolio 2 and portfolio 3. For portfolio 2 it can also be seen that yearly rebalancing and 1% no-trade region rebalancing provide the investor with the highest utility. In this case scenario 2 is the only exception on the previous statement because it shows that for periodic rebalancing quarterly rebalancing has the highest utility. For portfolio 3 every scenario gives the same result, being that yearly and 1% no-trade region rebalancing are the most optimal periodic and no-trade region rebalancing strategy, respectively.

To sum up, amongst all portfolios and scenarios there are only two deviations concerning periodic rebalancing and only one deviation concerning no-trade region rebalancing. By deviations it is meant that another strategy than yearly or 1% no-trade region rebalancing is optimal. Therefore, it means that it is agreed upon that whatever portfolio or scenario is concerned the most optimal strategies are yearly rebalancing and 1% no-trade region rebalancing. Especially for periodic rebalancing it can be concluded that rebalancing too often, like daily rebalancing, is thus not optimal. The same conclusion can be drawn when looking at the absolute values of the net returns, which can be found in Appendix A.5.

However, these results are more unstable due to the fact that there are more deviations where yearly and 1% no-trade region rebalancing are not the most optimal strategies.

4.3 Without rebalancing versus with rebalancing

Also the results of the rebalancing strategies have to be compared to the benchmark case which is without rebalancing. When looking at the absolute net returns of the rebalancing strategies and the benchmark scenario without rebalancing, which can be found in Appendix A.5, it can be seen that for portfolio 1 the results are extremely close, however for portfolio 2 and 3 the results are highly different from each other. Especially for no-trade region rebalancing there is a difference of around $ 500,000 between with and without rebalancing strategies. For periodic rebalancing this is only $ 200,000 all the time.

In all cases, when looking at the absolute net returns, the return without rebalancing is higher than when there will be rebalanced. However, when comparing other results for rebalancing strategies as opposed to not rebalancing, it is found that for without rebalancing the standard deviation is a lot higher than for the scenarios with rebalancing. This thus means that the risk is higher when no rebalancing takes place. This is logical because rebalancing makes sure that the weights in assets do not become too extreme. For no-trade region rebalancing the weights always stay within a boundary which is determined by the interval and for periodic rebalancing once every period the weights are brought back to the initial weights, which also holds them closer to the initial weight as opposed to no rebalancing at all. This will not be the case when no rebalancing takes place.

However, due to the fact that the standard deviation is different, the results for the returns of the rebalancing strategies as opposed to the benchmark scenario without rebalancing cannot be compared to each other. To be able to compare them, the utilities have to be calculated which was already done for the rebalancing strategies in table 4.1. The utilities for the benchmark scenario without rebalancing can be found in table 4.2 below.

	Portfolio 1	Portfolio 2	Portfolio 3
Without Rebalancing	0.048544	0.086198	0.085626

Table 4.2 Utilities for the 3 different portfolios without rebalancing taking place. *The wealth that is used to calculate the return used for the utility is the wealth after subtracting the costs of rebalancing and the initial investment*

When comparing these utilities to the utilities of the rebalancing strategies, meaning comparing the utilities in table 4.2 to the utilities in table 4.1 with their respective portfolio, it can be seen that for portfolio 1 for all strategies and scenarios the utilities for strategies with rebalancing are higher except for scenario 1 5% no-trade region rebalancing. So, even though the absolute values of equity are higher for the benchmark scenario without rebalancing, the strategies with rebalancing provide the investor with a higher utility due to the fact that the higher equity is offset by the higher standard deviation for the case without rebalancing.

However, the results of this comparison are different for both portfolio 2 and portfolio 3. For these portfolios the benchmark scenario without rebalancing provides the investor with the highest utility in most of the cases. For portfolio 2 there are only 3 cases for which without rebalancing does not have the highest utility. These cases are: scenario 1 yearly and 1% no-trade region rebalancing and for scenario 4 yearly rebalancing as well. For portfolio 3 there are 4 cases for which without rebalancing does not provide the investor with the highest utility, which are yearly rebalancing in scenario 1, 3 and 4 as well as quarterly rebalancing in scenario 3.

To conclude, even though the absolute net returns of the strategies without rebalancing are higher than for the strategies with rebalancing, the utility of almost all strategies with rebalancing for portfolio 1 as well as some cases of the other portfolios are higher due to the fact that their standard deviation is lower.

4.4 Overall no-trade region versus periodic rebalancing

For portfolio 2 and 3 it is found that periodic rebalancing always outperforms no-trade region rebalancing a lot in terms of net return. However, for portfolio 1 it is more ambiguous because, as can be found in the column of portfolio 1 in table 4.1, the utilities are really close as well as that sometimes periodic rebalancing and sometimes no-trade region rebalancing provides a higher utility for the investor. For the other two portfolios a huge difference in terms of utility between periodic and no-trade region rebalancing can be seen in table 4.1. This might be caused by the difference between the absolute net return values of periodic and no-trade rebalancing which was around $500,000 as mentioned in the previous paragraph.

4.5 Best strategy per portfolio

Not all scenarios can be compared to each other due to the fact that they are based on different assumptions, such as trading costs, which cannot be determined by the investor himself but depend on the situation. Other variables also differ, however they are choices the investor can make, such as whether to rebalance back to the initial weight or to a specific interval. This means that scenario 1 and 3 as well as 2 and 4 can be compared because these are only based on decisions the investor can make instead of variables that are determined by the environment. Also, the portfolios cannot be compared to each other because they are based on different assets and/or returns and volatilities.

For portfolio 1 the above analysis results in the fact that when the trading costs are 0.00005 that it is most optimal for the investor to rebalance the portfolio with 5% no-trade region rebalancing by rebalancing back to the initial weight. This result is found when comparing the utilities of scenario 1 and scenario 3 in table 4.1. When the trading costs are 0.02 the investor should rebalance back to the initial weight on a monthly basis. For portfolio 2 and 3 the same analysis concerning the utilities in table 4.1 can be done. The result of this is that for portfolio 2 and 0.00005 costs the investor should rebalance back to the initial weight with a yearly rebalancing strategy. When the costs are 0.02 the most optimal strategy is yearly rebalancing back to the specific interval. The same result is found when portfolio 3 is analyzed: for both cost scenarios rebalancing back to the specific interval with yearly rebalancing is most optimal when looking at the utilities.

4.6 Robustness test and sensitivity analysis

As can been seen from the results of the robustness checks, which can be found in Appendix A.7 as well as table 4.3, decreasing the holding period of portfolio 2 to 5 years will not affect the choice for the optimal rebalancing strategy concerning the highest utility. For no-trade region rebalancing every time the 1% interval is the best choice, which was also the case with the assumptions chosen in the research. For periodic rebalancing, previously, there were 3 out of 4 cases which showed that the utility level of yearly rebalancing is the highest. This however is for the robustness test only the case for scenario 4, because scenario 1 shows that monthly rebalancing would provide the investor with a higher utility, which was not the case for either of the portfolios for scenario 1 before. Still, this is the only different result that is shown in the robustness test and therefore it can still be concluded that the research is mostly stable. However, the robustness checks are not done for all scenarios and therefore this cannot be concluded with 100% certainty.

Portfolio 2	**Holding Period = 5**
Scenario 1 (Rebalancing back to the initial weight and costs = 0.00005)	
Daily rebalancing	0.036619
Monthly rebalancing	0.038168*
Quarterly rebalancing	0.037049
Yearly rebalancing	0.038041
1 % no-trade region rebalancing	0.012366*
2% no-trade region rebalancing	0.009844
5% no-trade region rebalancing	0.010149
Scenario 4 (Rebalancing back to the closest boundary and costs = 0.02)	
Daily rebalancing	n.a.
Monthly rebalancing	0.034557
Quarterly rebalancing	0.036038
Yearly rebalancing	0.036607*
1 % no-trade region rebalancing	0.016943*
2% no-trade region rebalancing	0.015444
5% no-trade region rebalancing	0.016678

Table 4.3 Utilities for investor with risk aversion A = 10, for the robustness checks. *The wealth that is used to calculate the return used for the utility is the wealth after subtracting the costs of rebalancing and the initial investment. * indicates the highest utility value for the combination of the portfolio, scenario and the rebalancing strategy.*

As already mentioned in the methodology a sensitivity analysis is already included in the research itself due to the fact that 3 different portfolios, 2 different trading cost rates, 2 different rebalancing policies and for both the periodic and the no-trade region rebalancing 4 and 3 different intervals are included, respectively. As was already discussed in the previous paragraphs the results over the different scenarios and parameters are stable.

4.7 Similarities and differences with the current literature

The current literature is quite diverse about the periodic rebalancing interval which gives the investor the highest profit. Some papers argue that yearly rebalancing is the best whereas others argue that daily or quarterly rebalancing is better. However, yearly rebalancing is the most commonly used method even though it is not the most optimal, especially when proportional trading costs are assumed (Leland, 1999). This research actually finds that yearly rebalancing is the most optimal strategy in most cases and therefore it is not in congruence with the current literature.

About the most optimal interval for the no-trade region rebalancing almost no statements are made in the current literature and thus the results of this research cannot really be compared to the current literature on this aspect. The only statement in the current literature that was made, by Donohue and Yip (2003), was that the no-trade region rebalancing strategy systematically seems to find the right balance between the costs of tracking error and trading costs, leading to a maximization of utility and risk/return ratio. Especially, for portfolio 2 and portfolio 3 this is not confirmed with this research because for these portfolios it is found that periodic rebalancing is always more optimal. However, as mentioned before, this might be due to the large difference in the absolute return between the two strategies. As already mentioned in paragraph 4.4, the optimal strategy for portfolio 1 is more ambiguous and this thus does not oppose the statement of Donohue and Yip, however it also does not confirm it.

For the rebalancing policy Amihud et al (2005) argues that for proportional trading costs it is more optimal to trade all the way back to the nearest boundary. The same conclusion cannot be drawn from this research because when the utilities of scenarios 1 and 3 as well as scenarios 2 and 4 are compared with each other it can be seen that both rebalancing policies are optimal in 50% of the cases. Therefore, no conclusion about the most optimal rebalancing policy can be drawn based on this research.

Also, it is mentioned that the higher the trading cost is, the wider the no-trade region. However, the only result in this research that confirms this statement is when scenario 1 (low costs) and scenario 3 (high costs) for portfolio 1 are compared. Here it can be found that under the low cost scenario the highest utility is present at the 1% no-trade region interval, whereas for the high costs scenario the highest utility is present at the 5% no-trade region interval. From this analysis it can be concluded that due to the fact that the trading costs are higher it is less optimal to rebalance more often and thus the 5% interval provides the investor with a higher utility for the scenario with higher trading costs, which implies a wider no-trade region. However, this result is not confirmed for any of the other scenarios and/or portfolios.

4.8 Limitations

At the start of this research a portfolio with 10 assets, of which 1 risk-free asset and 9 risky assets, was also investigated the same way as is done with the other portfolios. Only for periodic rebalancing back to the initial weight feasible results could be found, whereas for the other scenarios the results deviated extremely and were thus unstable and infeasible. Therefore, this portfolio has been excluded from the research. Next to that, portfolios with different weights have been analyzed, however these also lead to infeasible results. Also, as you might have noticed, the daily rebalancing of portfolio 2 and 3 to a specific threshold are missing in the results in the Appendix A.5. This is also due to the fact that unrealistic results were found. Therefore, these results have been excluded from the analysis as well.

Chapter 5: Conclusion

By means of Monte Carlo simulations it has been investigated how often investors have to rebalance their portfolios when trading costs are present. Three different portfolios were established and the simulations were done for several scenarios with different trading costs and rebalancing policies. The different scenarios and rebalancing strategies are compared to each other and the benchmark scenario in terms of mainly utility but absolute values as well.

It can be concluded that for periodic rebalancing yearly rebalancing is the most optimal because it has the highest utility as well as absolute return in all most all of the scenarios and portfolios. For no-trade region rebalancing the most optimal range is the 1% range. The robustness tests confirms these results because also for these scenarios yearly periodic rebalancing and 1% no-trade region rebalancing provide the investor with the highest utility. Next to that, it was found that for portfolio 2 and portfolio 3 periodic rebalancing is always more optimal than no-trade region rebalancing. Also, even though the absolute returns of without rebalancing are higher compared to the scenarios with rebalancing for portfolio 1, the utilities show that rebalancing is more optimal for the investor. This is due to the fact that the scenario without rebalancing does not only have a higher return, but also has a higher standard deviation, which increases risk and can thus be more costly.

Due to the fact that in the current literature there is quite some ambiguity about what the most profitable rebalancing strategy is, it cannot really be compared to the current literature. Some papers argue that indeed yearly rebalancing is the most profitable, whereas others argue that daily or quarterly rebalancing is better. However, currently yearly rebalancing is most often used by investors. Next to that, the current literature states that no-trade region rebalancing is more optimal than periodic rebalancing, which is not supported by this research. Also, the argument that when proportional trading costs are assumed it is more profitable to rebalance back to the closest boundary or a specific threshold is not confirmed by the results of this research.

A lot more research could be done on this topic to extend the general knowledge on rebalancing strategies. However, this was out of scope for this research. Especially, a lot of different parameters can be analyzed which might have an influence on the decision of the rebalancing strategy. It might also be good to establish which variables have the biggest influence on the decision of the rebalancing strategy. Next to that, research on rebalancing portfolios with a lot of different assets is currently lacking because most current papers only analyze portfolios with 1 or 2 assets. Therefore, this would be something which could be investigated in the future.

Appendix

A.1 Step-by-step plan for methodology

Computing a tangency portfolio

- Retrieve return data of all assets in the portfolios from DataStream
- Determine optimal weights using mean-variance optimization
  - Calculate returns from stock prices by: $= \frac{(P_t - P_{t-1})}{P_{t-1}}$
  - Calculate average (daily) return for each asset
  - Compute the yearly average return from the daily return by using:

$$= (1 + \mu\, daily)^{260} - 1$$

  - Compute the standard deviation (volatility) by using the excel function: $stdev.s$
  - Annualize the standard deviation[2]: $\sigma \text{ anual} = \sigma \text{ daily} \sqrt{260}$
  - Compute variance covariance matrix using excel:
    - Compute the correlation matrix by using the excel function: $= correlation$
    - Put the annualized volatilities in a row
    - Compute VCV matrix by using: $= mmult(transpose(annualized\ \sigma), annualized\ \sigma) * correlation\ matrix$
  - Set up an equally weighted portfolio and calculate:
    - Portfolio mean return: $\mu_p = w'\mu$
    - Portfolio standard deviation: $\sigma_p = \sqrt{(w^T \Sigma\, w\,)}$
    - Sharp ratio: $\frac{\mu}{\sigma}$
  - Use solver to compute the tangency portfolio, which combines the optimal combination of risky assets that maximize return at each level of risk with a risk-free asset.
  - In solver: maximize the Sharpe Ratio of the portfolio, while having the following constraints:

[2] *260 is chosen due to the fact that this is the average amount of trading days in a year.*

    - Sum of the weights should be equal to 100%
    - No short-sales are allowed (meaning: weights of the individual assets should be equal or higher than zero)
- Solver comes up with the weight for each asset in the Global Minimum Variance Portfolio. However, this is only the risky part of the portfolio and it should be calculated using the investors risk aversion how large the risky part of the portfolio should be.
- Take a risk aversion of 10 and use the following formula's to determine the division between the risky part and risk-free asset:

$$w^* = \frac{E\,(Rp)-Rf}{A\sigma p^2} \qquad E(Rc) = Rf + \frac{[\,E(Rp)-Rf]^2}{A\sigma p^2} \qquad \sigma c = \frac{[\,E(Rp)-Rf]}{A\sigma p}$$

- Afterwards multiply the weights of the individual asset which came out from the tangency portfolio by w* to obtain the weights of the complete portfolio.
- This process should be repeated for portfolio 1 and portfolio 2. For portfolio 3 this is not done because due to the assumptions made the weights are already identified.

The (basic) simulation is set up in the following way:

- The inputs of each asset which are needed: current investment, average return, standard deviation (volatility) and time horizon.
- The total current investment for the whole portfolio is $1,000,000, so for each individual investment the weight should be multiplied by this amount to determine the initial investment in the specific asset.
- The average return and volatility were already calculated when the tangency portfolio was calculated.
- The time horizon for each portfolio and rebalancing strategy is 10 years. The average trading days in a year is assumed to be 260, and thus for 10 years 2600 trading days are

assumed. (This means that the daily return and volatility should be used, as opposed to when calculating the tangency portfolio where the yearly return and volatility are used).

These inputs should be separately stated for all the assets in the portfolio.

- First of all, the basic situation without rebalancing should be simulated and the ending value and return of this should be determined. This is done by making a column with the time horizon in days. Next to it, make a column in which the returns can be randomly simulated. This can be set up using the following excel command: $= norm.inv(rand(\,), \mu, \sigma)$. This will generate a possible return for each of the periods. This return will be randomly chosen by using the mean and the standard deviation of the asset.
- In the next column put the ending value which results from the given random return. This can be computed the following way: $= NEV_{t-1} * \ (1 + \ \mu)$
- To carry out the simulation the data table function of excel should be used. This can be found under the tab 'data' and then go to what-if analysis. ($data > what - if\ analysis > data\ table$). First of all, the result you want to make the simulation about should be put above the column where you want to have the simulated results. On the left side of that column there should be the numbers of how many simulations you would like to make, which is 1,000 in this case. Then, select the column with the numbers 1-1,000 as well as the column next to it in which you would like to have the results as well as the result you want to simulate which you have put above. Afterwards click on data table and excel will simulate the results for you.
- The final step is to calculate the solutions of the simulation. The solutions (and the adequate excel commands) which are needed are the following:

  - Mean return: $= average\ (simulation\ iteration)$

  - Median return: $= median\ (simulation\ iteration)$

- Volatility (standard deviation): $= stdev.s\ (simulation\ iteration)$

- Percentiles (5% and 25%): $= percentile.inc\ (simulation\ iteration, percentile)$

Periodic Rebalancing

- Even though periodic rebalancing is done, return data will still be simulated daily using 2600 trading days even when rebalancing should be done yearly. This can be done using the excel command mentioned in the basic simulation previously.
- Also, just like in the basic simulation the ending values have to be calculated by multiplying last periods ending value by 1 plus the simulated return.
- A separate column at the end should be made which calculates the total amount in the complete portfolio after each period. This is done by adding all the ending values in the specific period of all the individual assets.
- The ending values of each period of each individual asset in the portfolio should be added up to determine the total ending value of the whole portfolio.
- To calculate the new weight of the asset in the portfolio the amount in the specific asset should be divided by the total ending value of the complete portfolio in the specific period, which determines the weight.
- Then it has to be determined for each period whether or not there has to be rebalanced. For daily rebalancing this is easy because in every period there has to be rebalanced. For the other periodic intervals it is more difficult due to the fact that you have trading days instead of normal days. Therefore some assumptions are made:

  * *Monthly rebalancing*: 2600 trading days / (10*12) = 21.6667 days per month. However, for simplicity it is assumed that there will be 22 days in one month meaning that there will be 118 months instead of 120.

* *Quarterly rebalancing*: 2600 trading days / (12*4) = 54.1667 days per quarter. Here it is assumed for simplicity that there are 54 days in one quarter meaning that there will be 48 full quarters.

* *Yearly rebalancing*: it is assumed that there are 260 trading days in one year.

For this a column is made in which it is indicated with either yes or no whether there will be rebalanced on that day.

- After that there will be a column with the new weight. The weight stays the same as the weight calculated from the simulated return when there is not rebalanced and moves back to the initial weight (or a threshold of 0.5%) when there is rebalanced in the specific period. This can be summarized using the following excel command:

(a) There will be rebalanced back to the initial weight:

$= if(\theta = yes, \omega_i, \omega^*)$

(b) There will be rebalanced back to a threshold of 0.5%

$$= if(\theta = yes, if(\omega \leq \omega_i, \omega_i - threshold, \omega_i + threshold), \omega)$$

- Afterwards the new ending value will be calculated by multiplying the new weight by the ending value of the complete portfolio.
- Also, the costs of the rebalancing should be calculated. The costs rebalancing are determined by a percentage of the rebalancing value. The excel command used for this is the following:

$$= ABS((NEV - EV) * c)$$

- Finally, the ending value and costs of rebalancing of the complete time horizon should be calculated. The ending value is the value of the portfolio at trading day 2600. For the rebalancing costs the costs made in the whole investment horizon should be added up.
- In the end, when this is done for all the individual assets in the portfolio this should be added up for all of them. From this the simulation iteration can be computed using the following function in excel: $data > what - if\ analysis > data\ table$
- To conclude, make the same table with solutions as was explained in the basic simulation.

- This should be done for all different portfolios and all scenarios.[3]

No-trade region rebalancing:

- Simulate the daily return for 10 years (meaning 2600 trading days) with the following excel command: $= norm.inv(rand(\,), \bar{\text{u}}, \sigma)$.
- Calculate the ending value at the end of each period by multiplying last periods ending value by 1 plus the simulated return.
- From this new ending value calculate the new weight of the asset. The process for this is the same as was used for periodic rebalancing: dividing the ending value of the specific asset by the total ending value of the complete portfolio.
- Due to the fact that this is no-trade region rebalancing the lower and upper bound should be determined. This is the weight plus and minus the percentage range. The new weight should be within these two boundaries, otherwise there should be rebalanced.
- Whether there should be rebalanced and getting the new (rebalanced or not) weight immediately the following excel command should be used:

  a) There will be rebalanced back to the initial weights

  $= if(and(\omega \geq \lambda, \omega \leq \varphi, \omega, \omega_i)$

  b) There will be rebalanced back to the closest boundary

  $= if(and(\omega \geq \lambda, \omega \leq \varphi, \omega, if(\omega \leq \lambda, \lambda, \varphi)))$

  This means that when the simulated weight is smaller than the lower bound rebalancing will take place to the lower bound, and if the simulated weight is larger than the upper bound rebalancing will take place to the upper bound.

[3] *The different periods of periodic rebalancing can be implemented by only changing that part of the process. The same holds for the different trading cost. The exact same process is repeated for all the different portfolios, where the only difference is the extension of the assets and their different weights.*

** Both scenarios will be composed and analyzed separately. Furthermore, the process for computing the end result of the portfolios with either of the rebalancing policies used is the same, however there might be some slight adjustments for some excel commands.*

- Now, the new starting value for the next period can be calculated. This should be done by multiplying the new weight by the ending value of the complete portfolio in that period.
- Finally, it should be calculated how much it costs to rebalance. The costs rebalancing are determined by a percentage of the rebalancing value. This can be calculated by using the following excel command:

$$= ABS((NEV - EV) * c)$$

- This process should be repeated for all the different assets in the portfolio separately. After this is done the complete portfolio should be simulated by adding all the ending values and the sum of rebalancing costs for all the different assets in the portfolio. This number should then be used to make the following simulation iteration.
- The simulation iteration should be made for the ending values, rebalancing costs and the return 1,000 times. In each when case subtracting the costs from the return the net return can be calculated. From the simulation iteration as well as the values for net return the average, median, standard deviation and percentiles should be calculated (as is stated above in the basic simulation).
- This should be done for all different portfolios and all scenarios.[4]

[4] *The different ranges of no-trade region rebalancing can be implemented by only changing that part of the process. The same holds for the different trading cost. The exact same process is repeated for all the different portfolios, where the only difference is the extension of the assets and their different weights.*

A.2 Summary of Greek characters

Greek Character	***Definition***
i	*Current Investment*
I	*Total initial investment*
t	*Time horizon*
c	*Rebalancing costs*
$\bar{u}$	*Average Return (daily) (%)*
μ	*Return (%)*
r	*Return ($)*
R	*Net Return ($)*
σ	*Standard deviation of return (daily)*
ω	*Weight*
ω*	*New weight*
NEV	*New Ending Value*
CEV	*Ending Value Complete Portfolio*
EV	*Ending Value*
θ	*Rebalancing (yes/no)*
λ	*Lower bound*
φ	*Upper bound*
p	*Percentiles*

Table A.1: the Greek symbols for all input and output data which is generated

A.3 Template of excel sheet for the different rebalancing strategies (including the excel commands)

Input table

	Risk-free	***Risky***	
	Ticker Asset	*Ticker Asset*	*Ticker Asset*
Weight (ωi)	*=weight calculated by mean-variance optimization*		
Current Investment (i)	*= ωi * I*		
Average Return (daily) (ū)	*= average daily return past 20 years*		
St. Dev of Return (daily) (σ)	*= average daily volatility past 20 years*		
Time horizon (in days) (t)	*2600*		
Rebalancing Costs (c)			
0.00005			
0.02			
Total Initial Investment (I)	*$1,000,000*		

Table A.1: The input table including all inputs needed to simulate all rebalancing strategies

The input table is the same for all the different rebalancing strategies. When the portfolio contains more assets, the table is extended with more columns on the right side which includes the other assets.

Periodic Rebalancing

Ticker Asset	Period	Return % (μ)	Ending Value (EV)	Weight (ω)	Rebalancing (yes/no) (θ)
	1 2 …	$= norm.inv(rand(\,), \bar{\mu}, \sigma)$	$= NEV_{t-1}$* (1+ μ)	$= EV/CEV$	*depends on the interval (daily, monthly, quarterly or yearly)*

New weight (ω*)	New ending value (NEV)	Costs of Rebalancing (c)	Ending value complete portfolio (CEV)
$= \boldsymbol{if}$(θ=yes,ωi,$\boldsymbol{\omega_t}$) or $= \boldsymbol{if}(\boldsymbol{\theta} = \boldsymbol{yes}, \boldsymbol{if}(\boldsymbol{\omega} \leq \boldsymbol{\omega_i}, \boldsymbol{\omega_i} - \boldsymbol{threshold}, \boldsymbol{\omega_i} + \boldsymbol{threshold}), \boldsymbol{\omega})$	$= \omega^*$ * CEV	$= if$(θ=yes, c,0)	$= sum\ of\ NEV\ of\ individual\ assets$

Table A.2: The input table including all inputs needed to simulate all rebalancing strategies

The last two columns only apply for the risky assets, not for the risk-free asset. This table should be made for all individual assets.

Next to the columns for the individual assets which are stated above an column for the Total Ending Value (TEV) should be made at the end which includes the sum of the ending values of all individual assets.

No-trade Region Rebalancing

Ticker Asset	Period	Return % (μ)	Ending Value (EV)	Weight (ω)	Lower bound (λ)
	1 2 …	$= norm.inv(rand(\,), \bar{\mu}, \sigma)$	$= NEV_{t-1}$* (1+ μ)	$= EV/CEV$	$= \omega i - volatility\ interval$

Upper bound (φ)	New weight (ω*)	New ending value (NEV)	Costs of Rebalancing (c)	Ending value complete portfolio (CEV)
$= \boldsymbol{\omega i} + \boldsymbol{volatility\ interva}$	$= if(and($ ω >= λ, ω<= φ), ω, ωi) or $= if(and(\omega \geq \lambda, \omega \leq \varphi), \omega, \text{if}(\omega \leq \lambda, \lambda, \varphi)$	$= \omega^{*}$ * CEV	$= if($ θ=yes, c,0)	$= sum\ of\ NEV\ of\ individual\ assets$

Table A.3: The input table including all inputs needed to simulate all rebalancing strategies

The last two columns only apply for the risky assets, not for the risk-free asset. This table should be made for all individual assets.

Next to the columns for the individual assets which are stated above an column for the Total Ending Value (TEV) should be made at the end which includes the sum of the ending values of all individual assets.

Solutions

Solutions	Equity	Costs of Rebalancing	Net Return
Mean		$= average(simulation\ iteration)$	
Median		$= median(simulation\ iteration)$	
St. Dev		$= stdev.s(simulation\ iteration)$	
Percentiles (p)			
5%		$= percentile.inc(simulation\ iteration, p)$	
25%		$= percentile.inc(simulation\ iteration, p)$	

Table A.4: The input table including all inputs needed to simulate all rebalancing strategies

The solution table is the same for all rebalancing strategies.

A.4 Inputs for the individual assets of all portfolios

Asset	Daily return	Daily volatility
VUSTX	0.0063%	0.65%
SPY	0.0349%	1.22%

Table A.5: daily returns and volatilities of the individual assets for Portfolio 1

Asset	Daily return	Daily volatility
AON	0.0528%	1.92%
PG	0.0446%	1.46%
WMT	0.0489%	1.67%
KO	0.0377%	1.45%
VUSTX	0.0063%	0.65%

Table A.6: daily returns and volatilities of the individual assets for Portfolio 2

Asset	Daily return	Daily volatility
AON	0.0460%	1.63%
PG	0.0460%	1.63%
WMT	0.0460%	1.63%
KO	0.0460%	1.63%
VUSTX	0.0063%	0.65%

Table A.7: daily returns and volatilities of the individual assets for Portfolio 3

Output tangency portfolio

Portfolio 1

Complete Portfolio	
w*	20.36%
Risk-free	79.64%
E(Rc)	7.30%
σ c	4.00%

Table A.8: Weights, return and volatility for the complete portfolio 1

Weights individual assets	
SPY	20.36%
VUSTX	79.64%
Total	**100.00%**

Table A.9: Individual asset weights within portfolio 1

Portfolio 2

Risky Portfolio	
AON	19.61%
PG	32.19%
WMT	25.14%
KO	23.06%
Σw	100%
μ p	12.62%
σ p	14.69%
μ/σ	85.92%

Table A.10: Tangency portfolio 2

Complete Portfolio	
w*	50.82%
Risk-free	49.18%
E(Rc)	7.23%
σ c	7.47%

Table A.11: Weights, return and volatility for the complete portfolio 2

Weights individual assets	
AON	9.97%
PG	16.36%
WMT	12.78%
KO	11.72%
VUSTX	49.18%
Total	**100.00%**

Table A.12: Individual asset weights within portfolio 2

Portfolio 3

Weights individual assets	
AON	12.50%
PG	12.50%
WMT	12.50%
KO	12.50%
VUSTX	50.00%
Total	**100.00%**

Table A.13: Individual asset weights within portfolio 3

Complete Portfolio	
w*	50.00%
Risk-free	50.00%
E(Rc)	8.41%
σ c	8.64%

Table A.14: Weights, return and volatility for the complete portfolio 2

A.5 Output simulations

Benchmark scenario: Without Rebalancing

	Equity	*Net Return*
Mean	$1,802,792.74	$802,792.74
Median	$1,595,650.08	$595,650.08
St. Dev	$796,687.85	$796,687.85
Percentiles		
5%	$919,427.90	-$80,572.10
25%	$1,237,315.97	$237,315.97

Table A.15: Output portfolio 1 without rebalancing

	Equity	*Net Return*
Mean	$2,254,526.41	$1,254,687.46
Median	$2,090,523.02	$1,090,795.80
St. Dev	$886,431.22	$886,238.74
Percentiles		
5%	$1,235,204.40	$235,345.81
25%	$1,650,663.84	$651,077.02

Table A.16: Output portfolio 2 without rebalancing

	Equity	*Return*
Mean	$2,241,280.52	$1,241,280.52
Median	$2,082,315.81	$1,090,229.44
St. Dev	$864,869.99	$877,520.56
Percentiles		
5%	$1,224,740.54	$310,363.08
25%	$1,648,111.12	$688,662.53

Table A.17: Output portfolio 3 without rebalancing

Scenario 1: Assumptions: Rebalancing back to initial weight, Costs: 0.00005

Periodic Rebalancing Portfolio 1

Daily Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,697,667.62	$474.12	$697,193.50
Median	$1,608,051.32	$463.40	$607,494.19
St. Dev	$622,213.64	$104.76	$622,213.02
Percentiles			
5%	$879,601.07	$322.55	-$120,949.54
25%	$1,251,050.92	$402.14	$250,597.76

Table A.18: Output daily rebalancing portfolio 1 scenario 1

Monthly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,694,905.22	$102.35	$694,802.88
Median	$1,592,185.26	$98.20	$592,070.71
St. Dev	$603,842.08	$25.06	$603,842.67
Percentiles			
5%	$903,166.38	$69.40	-$96,977.64
25%	$1,262,134.04	$83.93	$262,051.53

Table A.19: Output monthly rebalancing portfolio 1 scenario 1

Quarterly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,678,585.10	$64.83	$678,520.26
Median	$1,598,205.99	$62.52	$598,136.59
St. Dev	$587,675.51	$17.17	$587,675.93
Percentiles			
5%	$920,549.93	$41.53	-$79,521.72
25%	$1,257,979.66	$52.78	$257,905.56

Table A.20: Output quarterly rebalancing portfolio 1 scenario 1

Yearly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,761,118.71	$31.57	$761,087.14
Median	$1,659,345.51	$29.63	$659,325.94
St. Dev	$661,715.83	$12.28	$661,715.63
Percentiles			
5%	$900,714.66	$16.30	-$99,319.08
25%	$1,306,270.59	$22.93	$306,241.87

Table A.21: Output yearly rebalancing portfolio 1 scenario 1

Periodic Rebalancing Portfolio 2
Daily Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,931,486.33	$1,090.92	$930,413.45
Median	$1,893,340.59	$1,077.55	$892,267.71
St. Dev	$510,038.33	$180.95	$510,038.33
Percentiles			
5%	$1,192,691.07	$817.80	$191,618.19
25%	$1,572,074.18	$962.22	$571,001.29

Table A.22: Output daily rebalancing portfolio 2 scenario 1

Monthly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,987,790.85	$240.14	$993,459.31
Median	$1,921,159.22	$235.74	$912,682.88
St. Dev	$533,895.47	$42.23	$564,027.48
Percentiles			
5%	$1,220,710.80	$179.32	$220,468.47
25%	$1,592,806.39	$209.78	$592,564.06

Table A.23: Output monthly rebalancing portfolio 2 scenario 1

Quarterly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,994,798.84	$155.09	$994,646.05
Median	$1,908,807.92	$150.66	$908,655.13
St. Dev	$545,712.80	$29.59	$545,712.80
Percentiles			
5%	$1,264,910.36	$113.75	$264,757.57
25%	$1,605,181.73	$134.20	$605,028.94

Table A.24: Output quarterly rebalancing portfolio 2 scenario 1

Yearly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$2,027,027.71	$89.93	$1,026,933.80
Median	$1,968,212.56	$98.77	$968,118.64
St. Dev	$546,761.12	$20.32	$546,761.12
Percentiles			
5%	$1,261,950.45	$53.68	$261,856.53
25%	$1,597,171.33	$71.90	$597,077.41

Table A.25: Output yearly rebalancing portfolio 2 scenario 1

Periodic Rebalancing Portfolio 3
Daily Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,798,888.88	$1,109.69	$797,779.19
Median	$1,744,514.71	$1,094.04	$743,469.56
St. Dev	$575,862.11	$190.90	$575,851.90
Percentiles			
5%	$1,137,739.66	$832.61	$136,467.98
25%	$1,291,754.81	$968.20	$290,565.40

Table A.26: Output daily rebalancing portfolio 3 scenario 1

Monthly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,972,965.21	$237.41	$972,727.80
Median	$1,911,847.17	$234.68	$911,597.00
St. Dev	$504,186.49	$40.50	$504,186.03
Percentiles			
5%	$1,247,755.40	$176.36	$247,505.53
25%	$1,612,706.88	$207.99	$612,470.58

Table A.27: Output monthly rebalancing portfolio 3 scenario 1

Quarterly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,993,172.07	$152.81	$993,043.86
Median	$1,909,660.02	$150.23	$909,531.81
St. Dev	$546,270.74	$28.46	$546,270.74
Percentiles			
5%	$1,261,846.22	$111.36	$261,718.01
25%	$1,598,917.75	$132.39	$598,789.54

Table A.28: Output quarterly rebalancing portfolio 3 scenario 1

Yearly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$2,039,105.88	$74.97	$1,040,586.15
Median	$1,962,870.54	$73.01	$964,293.39
St. Dev	$574,178.47	$17.49	$572,628.98
Percentiles			
5%	$1,256,158.61	$50.18	$267,271.91
25%	$1,624,744.34	$62.92	$624,686.42

Table A.29: Output yearly rebalancing portfolio 3 scenario 1

No-trade Region Rebalancing Portfolio 1
1% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,694,870.86	$165.69	$694,705.17
Median	$1,586,332.77	$159.61	$586,162.15
St. Dev	$608,800.39	$36.79	$608,798.44
Percentiles			
5%	$904,327.48	$114.46	-$95,818.20
25%	$1,257,150.00	$140.50	$256,970.08

Table A.30: Output 1% no-trade region rebalancing portfolio 1 scenario 1

2% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,707,244.82	$92.31	$707,152.51
Median	$1,591,295.82	$88.96	$591,233.30
St. Dev	$635,692.40	$23.44	$635,692.69
Percentiles			
5%	$878,122.13	$58.81	-$121,975.04
25%	$1,240,981.00	$76.28	$240,900.16

Table A.31: Output 2% no-trade region rebalancing portfolio 1 scenario 1

5% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,533,351.86	$3,544.77	$529,807.10
Median	$1,403,744.29	$3,449.91	$400,787.86
St. Dev	$623,411.63	$908.15	$623,430.34
Percentiles			
5%	$751,607.30	$2,280.28	-$251,448.31
25%	$1,085,429.78	$2,901.22	$81,333.34

Table A.32: Output 5% no-trade region rebalancing portfolio 1 scenario 1

No-trade Region Rebalancing Portfolio 2
1% threshold:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,672,064.08	$181.60	$671,882.47
Median	$1,543,518.83	$176.67	$543,312.03
St. Dev	$664,448.26	$41.37	$664,450.35
Percentiles			
5%	$876,094.43	$122.58	-$124,059.11
25%	$1,200,853.55	$152.19	$200,662.21

Table A.33: Output 1% no-trade region rebalancing portfolio 2 scenario 1

2% threshold:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,525,392.78	$89.97	$525,302.81
Median	$1,404,445.09	$86.82	$404,337.82
St. Dev	$629,764.13	$24.14	$629,763.10
Percentiles			
5%	$749,443.18	$57.68	-$250,658.27
25%	$1,093,072.20	$73.52	$092,960.22

Table A.34: Output 2% no-trade region rebalancing portfolio 2

5% threshold:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,495,609.82	$39.51	$495,570.31
Median	$1,377,976.93	$37.51	$377,935.44
St. Dev	$633,396.37	$13.67	$633,396.50
Percentiles			
5%	$752,367.68	$21.63	-$247,665.49
25%	$1,053,816.51	$30.30	$53,753.20

Table A.35: Output 5% no-trade region rebalancing portfolio 2 scenario 1

No-trade Region Rebalancing Portfolio 3
1% threshold:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,606,076.15	$178.73	$605,897.42
Median	$1,519,552.86	$171.87	$519,374.24
St. Dev	$589,710.33	$42.86	$589,710.42
Percentiles			
5%	$829,736.67	$120.54	-$170,435.74
25%	$1,181,478.56	$147.67	$181,351.01

Table A.36: Output 1% no-trade region rebalancing portfolio 3 scenario 1

2% threshold:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,544,390.50	$90.80	$544,299.69
Median	$1,442,211.15	$87.95	$442,146.29
St. Dev	$635,041.51	$24.65	$635,041.21
Percentiles			
5%	$727,171.28	$57.14	-$272,936.18
25%	$1,087,819.78	$72.56	$87,677.46

Table A.37: Output 2% no-trade region rebalancing portfolio 3 scenario 1

5% threshold:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,484,555.20	$39.04	$484,516.16
Median	$1,371,854.19	$37.54	$371,827.83
St. Dev	$617,564.29	$13.02	$617,564.06
Percentiles			
5%	$698,171.14	$20.67	-$301,877.39
25%	$1,053,369.39	$30.44	$53,323.02

Table A.38: Output 2% no-trade region rebalancing portfolio 3 scenario 1

Scenario 2: Assumptions: Rebalancing back to initial weight, Costs: 0.002

Periodic Rebalancing Portfolio 1

Daily Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,743,260.07	$18,719.36	$724,540.71
Median	$1,636,672.08	$17,968.58	$619,853.27
St. Dev	$640,673.40	$4,071.30	$640,634.26
Percentiles			
5%	$888,052.23	$13,183.64	-$131,103.95
25%	$1,311,721.42	$15,768.55	$291,932.30

Table A.39: Output daily rebalancing portfolio 1 scenario 2

Monthly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,734,160.74	$4,058.87	$730,101.86
Median	$1,650,568.42	$3,951.54	$646,999.60
St. Dev	$607,961.14	$942.15	$607,960.02
Percentiles			
5%	$899,155.51	$2,743.85	-$105,614.25
25%	$1,321,658.63	$3,393.01	$317,681.62

Table A.40: Output monthly rebalancing portfolio 1 scenario 2

Quarterly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,682,273.95	$2,592.20	$679,681.75
Median	$1,575,672.04	$2,493.56	$572,679.46
St. Dev	$601,288.45	$679.18	$601,273.25
Percentiles			
5%	$890,836.19	$1,681.25	-$112,908.13
25%	$1,262,625.58	$2,093.17	$259,627.83

Table A.41: Output quarterly rebalancing portfolio 1 scenario 2

Yearly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,679,057.95	$1,293.91	$677,764.04
Median	$1,560,923.61	$1,202.69	$560,085.85
St. Dev	$616,352.68	$506.50	$616,344.27
Percentiles			
5%	$860,319.30	$681.60	-$140,671.93
25%	$1,257,254.46	$936.23	$254,586.60

Table A.42: Output yearly rebalancing portfolio 1 scenario 2

Periodic Rebalancing Portfolio 2

Daily Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,942,547.94	$44,093.74	$901,064.38
Median	$1,876,810.03	$43,730.18	$835,326.47
St. Dev	$547,693.55	$7,473.39	$547,693.55
Percentiles			
5%	$1,180,418.40	$33,157.16	$138,934.85
25%	$1,566,846.66	$38,671.29	$525,363.10

Table A.43: Output daily rebalancing portfolio 2 scenario 2

Monthly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,989,522.00	$9,502.08	$993,532.98
Median	$1,911,095.90	$9,294.15	$907,564.80
St. Dev	$545,931.13	$1,668.01	$548,719.16
Percentiles			
5%	$1,236,542.29	$7,061.64	$228,728.34
25%	$1,608,820.00	$8,363.68	$601,006.05

Table A.44: Output monthly rebalancing portfolio 2 scenario 2

Quarterly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$2,019,951.05	$6,171.65	$1,013,133.97
Median	$1,930,454.50	$6,080.22	$925,095.12
St. Dev	$568,911.18	$1,131.84	$565,807.20
Percentiles			
5%	$1,254,625.63	$4,481.03	$249,266.25
25%	$1,613,210.73	$5,382.23	$608,861.77

Table A.45: Output quarterly rebalancing portfolio 2 scenario 2

Yearly Rebalancing:

	Equity	*Cost of Rebalancing*	*Net Return*
Mean	$1,997,605.85	$2,396.70	$995,209.15
Median	$1,925,380.54	$2,396.70	$922,983.84
St. Dev	$533,327.07	$679.33	$533,327.07
Percentiles			
5%	$1,253,436.42	$2,396.70	$251,039.72
25%	$1,618,461.00	$2,396.70	$616,064.30

Table A.46: Output yearly rebalancing portfolio 2 scenario 2

Periodic Rebalancing Portfolio 3

Daily Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,964,874.36	$44,724.33	$920,150.03
Median	$1,891,189.06	$44,133.67	$847,305.37
St. Dev	$529,910.90	$7,643.21	$530,130.43
Percentiles			
5%	$1,220,630.28	$33,332.81	$174,826.39
25%	$1,591,428.41	$39,270.97	$547,342.88

Table A.47: Output daily rebalancing portfolio 3 scenario 2

Monthly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,968,498.83	$9,532.83	$958,966.00
Median	$1,896,017.94	$9,393.33	$886,029.59
St. Dev	$509,810.86	$1,659.90	$509,837.07
Percentiles			
5%	$1,234,560.26	$7,144.62	$225,292.87
25%	$1,608,348.86	$8,347.50	$599,179.73

Table A.48: Output monthly rebalancing portfolio 3 scenario 2

Quarterly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,961,743.38	$6,200.83	$954,668.39
Median	$1,866,640.24	$6,067.36	$859,565.25
St. Dev	$529,596.75	$1,187.47	$529,596.75
Percentiles			
5%	$1,248,136.34	$4,460.37	$241,061.34
25%	$1,590,853.99	$5,363.31	$583,778.99

Table A.49: Output quarterly rebalancing portfolio 3 scenario 2

Yearly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,982,987.98	$3,015.64	$979,821.73
Median	$1,905,684.26	$2,920.09	$902,518.02
St. Dev	$517,127.57	$745.80	$517,127.57
Percentiles			
5%	$1,288,388.07	$1,990.22	$285,221.82
25%	$1,607,085.37	$2,476.54	$603,919.12

Table A.50: Output yearly rebalancing portfolio 3 scenario 2

No-trade Region Rebalancing Portfolio 1

1% threshold:

	Equity	***Cost of Rebalancing***	***Return***
Mean	$1,735,649.07	$6,605.60	$729,043.48
Median	$1,617,276.42	$6,366.08	$609,096.62
St. Dev	$638,946.97	$1,530.32	$638,980.95
Percentiles			
5%	$905,776.74	$4,512.31	-$100,463.21
25%	$1,278,790	$5,532.35	$273,110.27

Table A.51: Output 1% no-trade region rebalancing portfolio 1 scenario 2

2% threshold:

	Equity	***Cost of Rebalancing***	***Return***
Mean	$1,696,510.02	$3,738.19	$692,771.83
Median	$1,566,816.63	$3,628.68	$563,129.46
St. Dev	$646,777.60	$951.04	$646,786.75
Percentiles			
5%	$894,610.90	$2,404.28	-$109,254.36
25%	$1,245,512.00	$3,043.92	$241,349.78

Table A.52: Output 2% no-trade region rebalancing portfolio 1 scenario 2

5% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,707,389.81	$1,595.89	$705,793.93
Median	$1,587,959.19	$1,497.71	$586,087.96
St. Dev	$615,383.21	$604.91	$615,417.42
Percentiles			
5%	$888,663.07	$814.70	-$112,719.82
25%	$1,299,360	$1,160.67	$298,012.55

Table A.53: Output 5% no-trade region rebalancing portfolio 1 scenario 2

No-trade Region Rebalancing Portfolio 2

1% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,665,843.43	$7,286.97	$658,556.45
Median	$1,552,796.67	$7,030.75	$546,511.12
St. Dev	$641,778.35	$1,797.53	$641,718.14
Percentiles			
5%	$838,825.73	$4,809.36	-$167,576.04
25%	$1,208,006.47	$6,035.43	$200,075.82

Table A.54: Output 1% no-trade region rebalancing portfolio 2 scenario 2

2% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,532,286.55	$3,546.70	$528,739.86
Median	$1,427,821.88	$3,408.16	$424,579.59
St. Dev	$596,288.12	$928.19	$596,292.62
Percentiles			
5%	$735,247.68	$2,292.47	-$269,128.56
25%	$1,094,224.90	$2,896.93	$090,725.72

Table A.55: Output 2% no-trade region rebalancing portfolio 2 scenario 2

5% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,499,686.95	$1,546.10	$498,140.85
Median	$1,376,686.12	$1,466.94	$375,230.94
St. Dev	$670,600.71	$530.38	$670,631.22
Percentiles			
5%	$687,220.37	$818.08	-$314,383.92
25%	$1,017,006.30	$1,176.83	$016,048.36

Table A.56: Output 5% no-trade region rebalancing portfolio 2 scenario 2

No-trade Region Rebalancing Portfolio 3
1% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,607,654.55	$7,019.02	$600,635.53
Median	$1,513,477.76	$6,822.01	$505,910.28
St. Dev	$593,648.20	$1,691.98	$593,688.78
Percentiles			
5%	$829,720.14	$4,719.65	-$177,859.44
25%	$1,180,032.57	$5,778.77	$173,055.03

Table A.57: Output 1% no-trade region rebalancing portfolio 3 scenario 2

2% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,533,351.86	$3,544.77	$529,807.10
Median	$1,403,744.29	$3,449.91	$400,787.86
St. Dev	$623,411.63	$908.15	$623,430.34
Percentiles			
5%	$751,607.30	$2,280.28	-$251,448.31
25%	$1,085,429.78	$2,901.22	$081,333.34

Table A.58: Output 2% no-trade region rebalancing portfolio 3 scenario 2

5% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,478,631.62	$1,557.51	$477,074.10
Median	$1,358,810.38	$1,469.57	$357,347.11
St. Dev	$605,133.28	$527.46	$605,138.65
Percentiles			
5%	$741,303.14	$847.70	-$260,507.07
25%	$1,038,241.70	$1,169.75	$036,876.12

Table A.59: Output 5% no-trade region rebalancing portfolio 3 scenario 2

Scenario 3: Assumptions: Rebalancing back to closest boundary/threshold, Costs: 0.0005
Periodic Rebalancing Portfolio 1
Daily Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,728,223.71	$436.29	$727,787.42
Median	$1,614,104.05	$422.06	$613,697.60
St. Dev	$660,855.77	$95.50	$660,854.05
Percentiles			
5%	$917,256.74	$306.64	-$83.327,60
25%	$1,265,192.05	$364.38	$264,758.00

Table A.60: Output daily rebalancing portfolio 1 scenario 3

Monthly Rebalancing:

		Return	Cost of Rebalancing	Net Return
Mean		$1,713,070.42	$76.45	$712,993.97
Median		$1,599,240.43	$73.64	$599,163.27
St. Dev		$619,595.86	$18.92	$619,596.46
Percentiles				
	5%	$879,568.13	$50.48	-$120.498,79
	25%	$1,275,575.74	$62.60	$275,516.39

Table A.61: Output monthly rebalancing portfolio 1 scenario 3

Quarterly Rebalancing:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,690,728.13	$53.22	$690,674.91
Median		$1,587,318.92	$51.06	$587,264.97
St. Dev		$604,135.62	$15.62	$604,135.46
Percentiles				
	5%	$888,915.34	$32.65	-$111.133,01
	25%	$1,253,770.84	$42.78	$253,719.33

Table A.62: Output quarterly rebalancing portfolio 1 scenario 3

Yearly Rebalancing:

		Return	Cost of Rebalancing	Net Return
Mean		$1,716,589.89	$28.86	$716,561.02
Median		$1,628,791.10	$26.56	$628,768.69
St. Dev		$614,200.02	$12.53	$614,200.16
Percentiles				
	5%	$892,084.28	$13.23	-$107.944,34
	25%	$1,290,905.11	$20.24	$290,883.24

Table A.63: Output yearly rebalancing portfolio 1 scenario 3

Periodic Rebalancing Portfolio 2
Monthly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,931,420.56	$166.10	$931,265.61
Median	$1,856,908.75	$163.05	$856,753.79
St. Dev	$554,689.13	$32.25	$554,689.13
Percentiles			
5%	$1,167,822.59	$118.59	$167,667.63
25%	$1,538,270.14	$144.57	$538,115.18

Table A.64: Output monthly rebalancing portfolio 2 scenario 3

Quarterly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,964,359.29	$120.42	$964,198.53
Median	$1,887,641.75	$119.49	$887,480.99
St. Dev	$566,137.01	$16.78	$566,137.01
Percentiles			
5%	$1,161,726.11	$94.73	$161,565.35
25%	$1,549,172.44	$115.66	$549,011.68

Table A.65: Output quarterly rebalancing portfolio 2 scenario 3

Yearly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$2,015,379.26	$63.07	$1,015,337.57
Median	$1,945,982.75	$60.81	$945,941.06
St. Dev	$575,599.94	$16.29	$575,599.94
Percentiles			
5%	$1,219,232.41	$40.50	$219,190.72
25%	$1,601,979.19	$51.12	$601,937,50

Table A.66: Output yearly rebalancing portfolio 2 scenario 3

Periodic Rebalancing Portfolio 3

Monthly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,962,742.81	$167.20	$962,575.61
Median	$1,875,041.14	$163.60	$874,888.58
St. Dev	$598,472.29	$32.12	$598,470.65
Percentiles			
5%	$1,189,970.10	$119.30	$189,781.90
25%	$1,554,140.11	$144.90	$553,990.69

Table A.67: Output monthly rebalancing portfolio 3 scenario 3

Quarterly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$2,077,752.18	$115.21	$1,077,656.33
Median	$2,001,440.51	$112.46	$1,001,344.66
St. Dev	$626,052.94	$23.36	$626,052.94
Percentiles			
5%	$1,247,405.30	$81.09	$247,309.45
25%	$1,643,401.77	$98.59	$643,305.92

Table A.68: Output quarterly rebalancing portfolio 3 scenario 3

Yearly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$2,063,883.22	$75.14	$1,065,542.49
Median	$1,972,398.83	$72.24	$972,340.15
St. Dev	$571,110.44	$19.12	$571,939.24
Percentiles			
5%	$1,285,718.20	$48.29	$285,659.51
25%	$1,677,643.69	$61.68	$677,585.00

Table A.69: Output yearly rebalancing portfolio 3 scenario 3

No-trade Region Rebalancing Portfolio 1

1% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,710,284.10	$84.57	$710,199.53
Median	$1,579,323.14	$82.47	$579,241.87
St. Dev	$640.892,32	$20.72	$640.893,36
Percentiles			
5%	$913,120.68	$55.18	-$86.968,51
25%	$1,270,193	$69.67	$270,151.15

Table A.70: Output 1% no-trade region rebalancing portfolio 1 scenario 3

2% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,715,841.90	$47.90	$715,794.00
Median	$1,613,942.36	$46.05	$613,886.71
St. Dev	$617,465.48	$13.96	$617,465.46
Percentiles			
5%	$939,670.70	$28.93	-$60,393.62
25%	$1,285,206.00	$37.65	$285,144.28

Table A.71: Output 2% no-trade region rebalancing portfolio 1 scenario 3

5% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,743,247.21	$21.94	$743,225.27
Median	$1,656,928.20	$19.66	$656,910.46
St. Dev	$607,258.24	$10.90	$607.257,77
Percentiles			
5%	$911,967.31	$8.57	-$88.050,83
25%	$1,293,588.00	$14.00	$293,560.89

Table A.72: Output 5% no-trade region rebalancing portfolio 1 scenario 3

No-trade Region Rebalancing Portfolio 2

1% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,829,485.51	$109.11	$829,376.40
Median	$1,729,148.61	$106.15	$729,057.35
St. Dev	$615,828.61	$24.23	$615,828.76
Percentiles			
5%	$1,046,426.03	$76.57	$46,321.94
25%	$1,393,937.59	$92.18	$393,804.28

Table A.73: Output 1% no-trade region rebalancing portfolio 2 scenario 3

2% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,779,084.11	$57.27	$779,026.84
Median	$1,689,583.87	$55.64	$689,522.69
St. Dev	$631,562.26	$14.46	$631,561.65
Percentiles			
5%	$969,888.52	$36.47	-$30.146,36
25%	$1,327,470.09	$47.42	$327,406.34

Table A.74: Output 2% no-trade region rebalancing portfolio 2 scenario 3

5% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,763,493.43	$27.29	$763,466.14
Median	$1,651,935.14	$25.59	$651,909.72
St. Dev	$653,262.93	$11.47	$653,263.19
Percentiles			
5%	$889,869.35	$11.61	-$110,172.44
25%	$1,311,047.98	$19.23	$311,024.11

Table A.75: Output 5% no-trade region rebalancing portfolio 2 scenario 3

No-trade Region Rebalancing Portfolio 3

1% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,825,659.35	$106.99	$825,552.36
Median	$1,722,907.37	$103.07	$722,801.45
St. Dev	$638,644.55	$23.61	$638,643.62
Percentiles			
5%	$1,000,122.12	$75.36	$7.22
25%	$1,368,364.00	$90.39	$368,272.42

Table A.76: Output 1% no-trade region rebalancing portfolio 3 scenario 3

2% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,807,223.60	$56.85	$807,166.75
Median	$1,678,397.11	$54.75	$678,351.64
St. Dev	$670,355.82	$15.07	$670,355.66
Percentiles			
5%	$987,331.00	$35.89	-$12,741.66
25%	$1,349,405.89	$46.03	$349,351.24

Table A.77: Output 2% no-trade region rebalancing portfolio 3 scenario 3

5% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,791,911.35	$27.60	$791,883.75
Median	$1,659,292.46	$25.72	$659,254.35
St. Dev	$661,124.42	$11.31	$661,124.68
Percentiles			
5%	$969,588.30	$11.64	-$30.454,31
25%	$1,307,944.73	$19.55	$307,912.27

Table A.78: Output 5% no-trade region rebalancing portfolio 3 scenario 3

Scenario 4: Assumptions: Rebalancing back to closest boundary/threshold, Costs: 0.002
Periodic Rebalancing Portfolio 1
Daily Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,705,984.93	$17,563.13	$688,421.80
Median	$1,623,417.00	$17,015.74	$606,116.74
St. Dev	$576,226.91	$3,870.12	$576,220.32
Percentiles			
5%	$939,063.36	$11,977.30	-$80.380,04
25%	$1,290,621.46	$14,914.71	$274,069.24

Table A.79: Output daily rebalancing portfolio 1 scenario 4

Monthly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,677,534.39	$3,017.05	$674,517.33
Median	$1,576,994.27	$2,910.84	$574,468.19
St. Dev	$609,508.87	$764.03	$609,485.24
Percentiles			
5%	$880,678.39	$1,971.55	-$123,007.87
25%	$1,225,160.78	$2,481.84	$222,846.99

Table A.80: Output monthly rebalancing portfolio 1 scenario 4

Quarterly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,719,295.40	$2,157.80	$717,137.60
Median	$1,597,240.90	$2,055.46	$595,365.24
St. Dev	$640,677.55	$616.07	$640,658.37
Percentiles			
5%	$920,714.24	$1,335.66	-$81,963.92
25%	$1,279,918.53	$1,717.71	$277,957.88

Table A.81: Output quarterly rebalancing portfolio 1 scenario 4

Yearly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,740,794.14	$1,183.24	$739,610.90
Median	$1,629,238.82	$1,084.66	$628,628.88
St. Dev	$640,333.34	$547.06	$640,345.41
Percentiles			
5%	$919,069.81	$551.73	-$81,591.94
25%	$1,278,647.70	$816.70	$277,542.23

Table A.82: Output yearly rebalancing portfolio 1 scenario 4

<u>Periodic Rebalancing Portfolio 2</u>

Monthly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,946,683.36	$6,701.01	$940,403.90
Median	$1,875,236.33	$6,533.10	$868,956.87
St. Dev	$562,176.05	$1,301.71	$562,176.05
Percentiles			
5%	$1,175,813.71	$4,841.00	$169,534.25
25%	$1,519,068.50	$5,814.25	$512,789.04

Table A.83: Output daily rebalancing portfolio 2 scenario 4

Quarterly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,947,899.42	$5,663.22	$942,236.20
Median	$1,873,148.40	$4,402.39	$867,485.18
St. Dev	$531,966.99	$867.63	$531,966.99
Percentiles			
5%	$1,237,263.49	$5,663.22	$231,600.27
25%	$1,567,571.58	$5,663.22	$561,908.36

Table A.84: Output quarterly rebalancing portfolio 2 scenario 4

Yearly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$2,033,379.95	$2,529.78	$1,030,734.59
Median	$1,963,389.98	$2,435.67	$960,744.62
St. Dev	$567,908.15	$683.90	$567,908.15
Percentiles			
5%	$1,236,827.73	$1,568.36	$234,182.37
25%	$1,631,896.75	$2,041.57	$629,251.39

Table A.85: Output yearly rebalancing portfolio 2 scenario 4

Periodic Rebalancing Portfolio 3

Monthly Rebalancing:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,915,623.44	$6,640.42	$908,983.02
Median	$1,843,067.97	$6,518.18	$836,443.10
St. Dev	$538,386.82	$1,250.43	$538,347.16
Percentiles			
5%	$1,179,950.72	$4,799.53	$171,679.92
25%	$1,526,743.84	$5,748.53	$519,987.59

Table A.86: Output monthly rebalancing portfolio 3 scenario 4

Quarterly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$2,024,960.93	$4,532.93	$1,021,563.10
Median	$1,942,614.74	$4,414.99	$939,216.90
St. Dev	$578,629.81	$876.20	$578,629.81
Percentiles			
5%	$1,234,078.66	$3,325.73	$230,680.82
25%	$1,631,429.76	$3,902.12	$628,031.92

Table A.87: Output quarterly rebalancing portfolio 3 scenario 4

Yearly Rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$2,017,196.19	$3,004.62	$1,014,706.63
Median	$1,946,603.02	$2,905.74	$944,113.46
St. Dev	$538,664.11	$699.41	$538,664.11
Percentiles			
5%	$1,273,927.90	$2,028.27	$271,438.33
25%	$1,645,032.97	$2,491.42	$642,543.41

Table A.88: Output yearly rebalancing portfolio 3 scenario 4

<u>No-trade Region Rebalancing Portfolio 1</u>
1% threshold:

	Equity	***Cost of Rebalancing***	***Return***
Mean	$1,691,378.37	$3,442.37	$687,936.00
Median	$1,594,091.48	$3,332.66	$591,140.80
St. Dev	$591,334.78	$811.66	$591,374.71
Percentiles			
5%	$890,954.32	$2,319.97	-$112.635,95
25%	$1,264,912	$2,874.88	$262,143.34

Table A.89: Output 1% no-trade region rebalancing portfolio 1 scenario 4

2% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,685,357.73	$1,884.52	$683,473.21
Median	$1,576,828.20	$1,804.55	$574,908.90
St. Dev	$593,637.85	$549.74	$593,653.78
Percentiles			
5%	$907,951.21	$1,148.74	-$93,455.59
25%	$1,262,969	$1,512.00	$261,570.15

Table A.90: Output 2% no-trade region rebalancing portfolio 1 scenario 4

5% threshold:

	Equity	Cost of Rebalancing	Return
Mean	$1,711,290.86	$856.11	$710,434.76
Median	$1,605,520.87	$771.55	$604,711.23
St. Dev	$636,870.26	$436.67	$636,858.93
Percentiles			
5%	$884,980.99	$331.46	-$116,219.59
25%	$1,282,096	$562.57	$281,383.90

Table A.91: Output 5% no-trade region rebalancing portfolio 1 scenario 4

No-trade Region Rebalancing Portfolio 2

1% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,818,150.87	$4,297.53	$813,853.34
Median	$1,722,534.80	$4,161.88	$717,881.71
St. Dev	$632,425.89	$948.04	$632,416.05
Percentiles			
5%	$1,002,680.59	$2,953.55	-$1,008.17
25%	$1,365,395.06	$3,662.58	$360,627.38

Table A.92: Output 1% no-trade region rebalancing portfolio 2 scenario 4

2% threshold:

	Equity	Cost of Rebalancing	Net Return
Mean	$1,767,310.91	$2,289.63	$765,021.28
Median	$1,662,123.94	$2,191.63	$659,597.89
St. Dev	$639,524.15	$603.13	$639,522.49
Percentiles			
5%	$909,498.31	$1,442.87	-$92,842.21
25%	$1,326,094.19	$1,886.22	$324,029.79

Table A.93: Output 2% no-trade region rebalancing portfolio 2 scenario 4

5% threshold:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,780,928.08	$1,072.59	$779,855.49
Median		$1,668,450.04	$993.46	$666,631.47
St. Dev		$652,616.86	$447.23	$652,632.75
Percentiles				
	5%	$956,639.25	$461.75	-$44,765.99
	25%	$1,305,251.38	$756.06	$304,252.58

Table A.94: Output 5% no-trade region rebalancing portfolio 2 scenario 4

No-trade Region Rebalancing Portfolio 3

1% threshold:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,826,947.59	$4,320.08	$822,627.51
Median		$1,722,082.09	$4,217.25	$718,375.97
St. Dev		$640,306.64	$949.47	$640,271.00
Percentiles				
	5%	$1,000,122.12	$2,950.71	-$4.201,75
	25%	$1,368,364.00	$3,669.67	$364,342.56

Table A.95: Output 1% no-trade region rebalancing portfolio 3 scenario 4

2% threshold:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,792,406.77	$2,267.16	$790,139.61
Median		$1,673,245.84	$2,183.74	$670,684.87
St. Dev		$661,661.03	$589.63	$661,692.85
Percentiles				
	5%	$964,738.45	$1,433.09	-$36,783.54
	25%	$1,333,406.25	$1,854.72	$330,964.38

Table A.96: Output 2% no-trade region rebalancing portfolio 3 scenario 4

5% threshold:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,819,234.63	$1,075.62	$818,159.01
Median		$1,676,805.21	$1,011.61	$674,764.43
St. Dev		$710,631.68	$461.29	$710,615.46
Percentiles				
	5%	$955,187.39	$475.47	-$45,625.08
	25%	$1,336,217.01	$757.36	$333,773.84

Table A.97: Output 5% no-trade region rebalancing portfolio 3 scenario 4

A.6 Utility tables

	Portfolio 1	**Portfolio 2**	**Portfolio 3**
Scenario 1 (Rebalancing back to the initial weight and costs = 0.00005)			
Daily rebalancing	0.150409	0.153997	0.163308
Monthly rebalancing	0.151259	0.149178	0.184586
Quarterly rebalancing	0.150590	0.151406	0.184397
Yearly rebalancing	<u>0.154218*</u>	<u>0.153578*</u>	<u>0.187427*</u>
1 % no-trade region rebalancing	0.150955*	0.151654*	0.14322*
2% no-trade region rebalancing	0.150519	0.150915	0.134275
5% no-trade region rebalancing	0.133903	0.150849	0.129386
Scenario 2 (Rebalancing back to the initial weight and costs = 0.02)			
Daily rebalancing	0.153803	0.179256	0.182447
Monthly rebalancing	<u>0.154935*</u>	0.18405	0.183855
Quarterly rebalancing	0.15015	<u>0.185812*</u>	0.182151
Yearly rebalancing	0.148911	0.185539	<u>0.184928*</u>
1 % no-trade region rebalancing	0.153152*	0.14599*	0.143145*
2% no-trade region rebalancing	0.148735	0.135451	0.133903
5% no-trade region rebalancing	0.151804	0.127483	0.129554
Scenario 3 (Rebalancing back to the closest boundary and costs = 0.00005)			
Daily rebalancing	0.150986	n.a.	n.a.
Monthly rebalancing	0.152112	0.177758	0.178366
Quarterly rebalancing	0.150824	0.180410	0.188178
Yearly rebalancing	<u>0.152797*</u>	<u>0.184972*</u>	<u>0.190080*</u>
1 % no-trade region rebalancing	0.150491	0.163986*	0.162173*
2% no-trade region rebalancing	0.152521	0.157965	0.158254
5% no-trade region rebalancing	0.155887*	0.155012	0.157337
Scenario 4 (Rebalancing back to the closest boundary and costs = 0.02)			
Daily rebalancing	0.153997	n.a.	n.a.
Monthly rebalancing	0.149178	0.178866	0.177069
Quarterly rebalancing	0.151406	0.180640	0.185755
Yearly rebalancing	<u>0.153578*</u>	<u>0.187212*</u>	<u>0.187212*</u>
1 % no-trade region rebalancing	0.151654*	0.161817*	0.162195*
2% no-trade region rebalancing	0.150915	0.156282	0.157351
5% no-trade region rebalancing	0.150849	0.156797	0.156674

Table A.98 Utilities for investor with risk aversion A = 10, for all portfolios and scenarios. *The wealth that is used to calculate the return used for the utility is the wealth before subtracting the costs and the initial investment. * indicates the highest utility value for the combination of the portfolio, scenario and the rebalancing strategy. Underlined is the rebalancing strategy with the highest utility for the combination of the portfolio and the scenario.*

Holding Period = 5	
Scenario 1 (Rebalancing back to the initial weight and costs = 0.00005)	
Daily rebalancing	0.136640
Monthly rebalancing	0.138178*
Quarterly rebalancing	0.137055
Yearly rebalancing	0.138044
1 % no-trade region rebalancing	0.121937*
2% no-trade region rebalancing	0.118257
5% no-trade region rebalancing	0.116667
Scenario 4 (Rebalancing back to the closest boundary and costs = 0.02)	
Daily rebalancing	
Monthly rebalancing	0.134837
Quarterly rebalancing	0.136225
Yearly rebalancing	0.136712*
1 % no-trade region rebalancing	0.130363*
2% no-trade region rebalancing	0.128197
5% no-trade region rebalancing	0.129459

Table A.99 Utilities for investor with risk aversion A = 10, for the robustness checks. *The wealth that is used to calculate the return used for the utility is the wealth after subtracting the costs of rebalancing and the initial investment. * indicates the highest utility value for the combination of the portfolio, scenario and the rebalancing strategy.*

A.7 Output Robustness Checks

Robustness check: holding period to 5 years

Portfolio 3, Scenario 1

Daily rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,401,935.30	$206.75	$401,728.55
Median	$1,391,204.51	$38.55	$391,165.96
St. Dev	$266,602.87	$215.24	$266,598.81
Percentiles			
5%	$1,018,799.41	$21.88	$18,777.53
25%	$1,208,519.71	$28.99	$208,490.72

Table A.100: Output robustness test holding period daily rebalancing portfolio 3 scenario 1

Monthly rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,421,054.30	$100.09	$420,954.21
Median	$1,398,806.48	$99.24	$398,709.64
St. Dev	$280,257.78	$12.87	$280,257.97
Percentiles			
5%	$1,025,363.60	$80.33	$25,254.72
25%	$1,226,262.72	$91.41	$226,154.67

Table A.101: Output robustness test holding period monthly rebalancing portfolio 3 scenario 1

Quarterly rebalancing:

	Equity	***Cost of Rebalancing***	***Net Return***
Mean	$1,406,443.15	$63.92	$406,379.23
Median	$1,377,442.96	$62.85	$377,377.38
St. Dev	$267,924.33	$9.20	$267,924.30
Percentiles			
5%	$1,004,912.45	$50.29	$4,854.63
25%	$1,226,119.96	$57.30	$226,047.76

Table A.102: Output robustness test holding period quarterly rebalancing portfolio 3 scenario 1

Yearly rebalancing:

		Equity	*Cost of Rebalancing*	*Net Return*
Mean		$1,418,218.76	$30.96	$418,187.80
Median		$1,376,892.09	$30.08	$376,854.70
St. Dev		$274,868.95	$7.71	$274,868.98
Percentiles				
	5%	$1,025,663.02	$20.34	$25,629.20
	25%	$1,221,954.74	$25.41	$221,926.66

Table A.103: Output robustness test holding period yearly rebalancing portfolio 3 scenario 1

1% threshold:

		Equity	*Cost of Rebalancing*	*Net Return*
Mean		$1,276,848.15	$78.62	$276,769.52
Median		$1,233,362.77	$77.34	$233,286.49
St. Dev		$339,043.03	$14.49	$339,043.30
Percentiles				
	5%	$793,276.65	$57.81	-$206,795.93
	25%	$1,027,941.08	$68.27	$27,868.81

Table A.104: Output robustness test holding period 1% no-trade region rebalancing portfolio 3 scenario 1

2% threshold:

		Equity	*Cost of Rebalancing*	*Net Return*
Mean		$1,250,352.47	$39.98	$250,312.48
Median		$1,202,000.49	$39.40	$201,966.61
St. Dev		$368,183.85	$9.08	$368,184.19
Percentiles				
	5%	$749,867.56	$27.12	-$250,172.69
	25%	$986,414.45	$33.35	-$13,625.10

Table A.105: Output robustness test holding period 2% no-trade region rebalancing portfolio 3 scenario 1

5% threshold:

		Equity	*Cost of Rebalancing*	*Net Return*
Mean		$1,229,218.69	$16.75	$229,201.94
Median		$1,187,041.22	$16.49	$187,025.71
St. Dev		$353,698.54	$6.16	$353,698.45
Percentiles				
	5%	$748,245.76	$7.26	-$251,760.74
	25%	$981,093.48	$12.18	-$18,924.08

Table A.106: Output robustness test holding period 5% no-trade region rebalancing portfolio 3 scenario 1

Portfolio 3, Scenario 4

Monthly rebalancing:

		Equity	*Cost of Rebalancing*	*Net Return*
Mean		$1,386,688.17	$2,792.00	$383,896.17
Median		$1,364,807.87	$2,770.37	$362,132.09
St. Dev		$276,850.22	$404.72	$276,863.11
Percentiles				
	5%	$970,703.27	$2,196.25	-$31,442.31
	25%	$1,184,325.11	$2,489.15	$181,144.71

Table A.107: Output robustness test holding period monthly rebalancing portfolio 3 scenario 4

Quarterly rebalancing:

		Equity	*Cost of Rebalancing*	*Net Return*
Mean		$1,398,964.46	$1,870.75	$397,093.71
Median		$1,376,392.10	$1,836.66	$374,592.04
St. Dev		$270,985.97	$316.04	$270,988.11
Percentiles				
	5%	$983,904.28	$1,423.03	-$18,307.38
	25%	$1,215,709.25	$1,652.31	$213,721.28

Table A.108: Output robustness test holding period quarterly rebalancing portfolio 3 scenario 4

Yearly rebalancing:

		Equity	*Cost of Rebalancing*	*Net Return*
Mean		$1,406,598.41	$1,050.97	$405,547.44
Median		$1,390,505.91	$1,019.35	$389,057.30
St. Dev		$280,978.86	$289.90	$280,975.17
Percentiles				
	5%	$999,280.50	$626.25	-$1,587.00
	25%	$1,192,490.73	$846.02	$191,496.00

Table A.109: Output robustness test holding period yearly rebalancing portfolio 3 scenario 4

1% threshold:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,361,569.37	$1,829.90	$359,739.47
Median		$1,302,764.39	$1,803.71	$301,110.67
St. Dev		$340,411.54	$322.15	$340,414.03
Percentiles				
	5%	$902,018.58	$1,361.91	-$99,484.23
	25%	$1,117,290.66	$1,598.70	$115,221.88

Table A.110: Output robustness test holding period 1% no-trade region rebalancing portfolio 3 scenario 4

2% threshold:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,341,454.40	$973.73	$340,480.67
Median		$1,302,465.02	$952.80	$301,453.44
St. Dev		$344,930.34	$243.89	$344,923.90
Percentiles				
	5%	$854,714.11	$615.81	-$146,131.57
	25%	$1,093,790.56	$795.48	$92,846.86

Table A.111: Output robustness test holding period 2% no-trade region rebalancing portfolio 3 scenario 4

5% threshold:

		Equity	Cost of Rebalancing	Net Return
Mean		$1,353,217.16	$415.92	$352,801.24
Median		$1,303,474.74	$386.52	$303,027.55
St. Dev		$342,423.21	$209.63	$342,425.08
Percentiles				
	5%	$883,569.10	$110.91	-$117,090.10
	25%	$1,099,167.93	$269.12	$98,527.83

Table A.112: Output robustness test holding period 5% no-trade region rebalancing portfolio 3 scenario 4

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