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Analysis of Longevity Risk

A Contribution to the Financial Assessment Framework (FTK)

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A Contribution to the Financial Assessment Framework (FTK)

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1 Introduction

The Financial Assessment Framework (FTK) from DNB demands that pension funds cover for risks by insuring a risk margin. The risks are briefly divided over six pillars where the sixth pillar, the S6, contains longevity risk (TSO). We will establish that that the current model that determines the TSO does not meet the requirements. We will also provide a new table with TSO values by using the Lee Carter model.

When people live longer than expected, an unfavorable situation for a pension fund arises since it will have to pay more to its policy holders than anticipated. The value of the obligations increases when the life expectancy turns out to be higher. This is why especially the positive deviations for the members (they live longer) are of importance for pension funds. The risk of underestimating the future payments due to this situation is what we call longevity risk.

Longevity risk for pension funds is the risk due to the unforeseen increase in life expectancy of its policy holders which leads to higher obligations.

The current FTK originates from January 1, 2007 and soon a new FTK must be set in place. Thus far the S6 is one of the least influential pillars out of the six. However, recent changes in the mortality rates show that longevity risk might be underestimated. The underlying calculations for the S6 date from 2004 and should therefore be updated with the most recent data as well as new estimation techniques.

The relevance of an investigation to longevity risk, next to the pressure from the new FTK, is emphasized by the media. In the Dutch financial newspaper (*Financieele Dagblad*) from June 2, 2014[1] Frank van Berkum and Mischa Borst outline the importance of a projection for future mortality rates that includes the uncertainty around them. The AG projection table is the only reference to future mortality rates and it provides us with estimated mortality rates. At the recent releases of a new AG projection table, the previous rates proved to be underestimated. Pension funds need to raise their provisions as a direct consequence: An estimated €10bn last year and three years ago an extra provision of €50bn had to be installed, increases of roughly 1% and 6% respectively. Total technical provisions in the first quarter of 2014 was 1,075tn (CBS).

Pension funds experience several unintended effects due to the change of future mortality rates. Both the underlying model as the unpredictability of the death rates determine the uncertainty. Van Berkum and Borst state that this uncertainty should be included in the new projection table such that pension funds can cover for longevity risk gradually and stay in control.

A new AG projection table by the Dutch Royal Actuarial Association will most likely provide future mortality rates as well as uncertainty intervals. It is expected to be issued in September 2014, which is unfortunately too late for

[1] http://fd.nl/incoming/823481-1406/krant-20140602–0-008-012.pdf/BINARY/Krant-20140602+-0-008-012.pdf

this research to challenge the results.

In this master's thesis longevity risk is investigated. The report provides an overview of the current modeling techniques and how uncertainties are included. The main research question goes a step further and links the risk of longevity to what the impact is for a pension fund in the form of a risk margin.

2 Mortality

2.1 Background

Rising life expectancy is a prolonged trend. Due to the improvements of welfare, e.g. outcomes of medical, societal and political structures, it is that people live longer. This is good news but also raises social and economic concerns as public health care for the elderly and the restructuring of pensions.

Life expectancies for both sexes have increased significantly, although progresses have occurred at an uneven rate. In figure 1 is shown that the increasing longevity in the Netherlands for men is currently around 79 and for women 83 years. Five decades years ago these numbers were about 71 and 77 respectively.

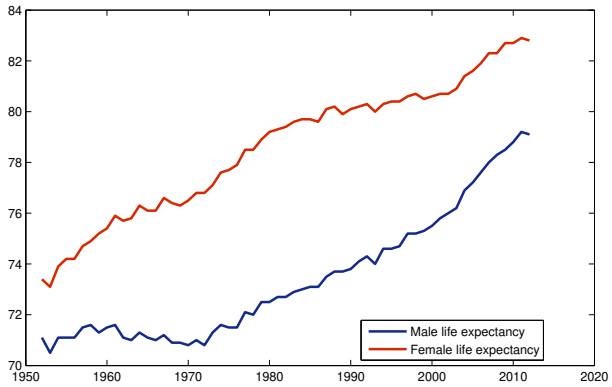


Figure 1: Life expectancy for a 0-year old. From CBS Statline.

The figure shows a clear difference between men and women mortality. This result traces back to behavioral reasons. Given this difference and that the development of mortality is therefore different as well, we make a distinction between male and female mortality in the rest of this report.

Longer life expectancy reflects that a good quality of life is being maintained over a longer time span. Van Bodegom et al. [21] explain that differences in life expectancy between countries exists because of differences in the prevalence of deceases as well as behavioral and societal aspects. Mortality for young and middle aged persons is minimized in all developed countries. However, Dutch life expectancy for adult and old ages has not improved as much as in some other countries over the past 50 years. This stagnation is mainly due the decline of the mortality rate of the age-group 65 years and above, see figure 2. For Dutch men of 65 years and older this can be largely explained by the high historic prevalence of smoking in the Netherlands, whereas this effect is less for women [14].

Note that the expected life age is higher for a 65-year old than for a 0-year old. Once a person reached the age of 65 he/she has more chance of reaching

a higher age compared to the life expectancy for the entire population. The expectation conditional on reaching the age of 65 is higher than someone who has not reached the age of 65 yet.

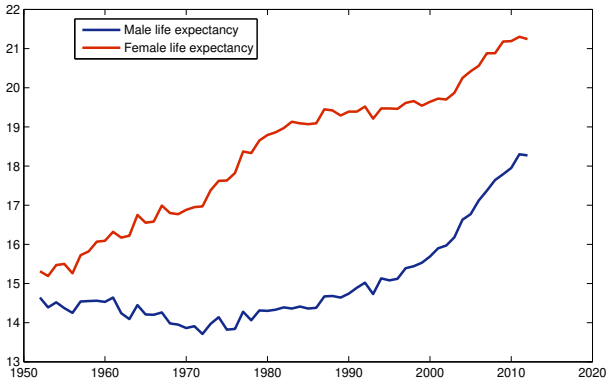


Figure 2: Life expectancy for a 65-year old. From CBS Statline.

Following the trend in mortality improvement we expect life expectancy to rise even further. Progress made in health care is one of the main reasons approving this. From the point of view of a pension fund the rise in life expectancy requires more provisions. Pension benefits are paid until death. Hence, if the retirement age stays the same, the more a pension fund has to pay for a longer life of their policy holders.

Problems might arise when people live even longer than estimated. The last years show a upward trend in mortality rates that is steeper than the average improvement over the preceding decades. This has not been foreseen and expectations had to be adjusted. It is desirable to quantify longevity risk such that pension funds can incorporate this risk within their policies to avoid underestimation of future costs.

2.2 Mortality Data

Life expectancy increases as a population dies at a later age. This is the case when the probabilities of dying during a lifetime decrease. So has the probability of dying at the age of 50 halved from 0,54% in 1950 to 0,28% in 2012. The survival rate is higher and therefore the life expectancy.

Mortality data are defined by the central death rate

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}}, \tag{1}$$

where $D_{x,t}$ are the empirical number of deaths of lives aged x in calendar year t and $E_{x,t}$ is the total population of lives aged x in year t called exposure-to-risk. Usually $E_{x,t}$ is taken as the average size of the population during the year.

There are two Dutch mortality data sets to our disposal: The Human Mortality Database[2] (HMDB) and Statistics Netherlands Statline database[3] with extension (CBS). Both databases distinguish between male and female. The data from HMDB are exposure-to-risk and deaths of years ranging from 1850 to 2009 and ages from 0 to 110. Log mortality rates are calculated by equation 1, given that $\ln(m_{x,t}) = 0$ if $D_{x,t} = 0$. Raw mortality rates are taken from CBS Statline and it ranges from year 1950 to 2012. The AG performed an extension to this data such that ages from 0 to 125 are covered. This is further explained in the next section.

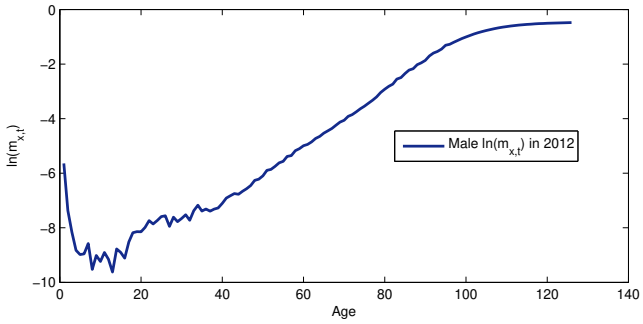


Figure 3: Logarithmic mortality rates, men, 2012 (CBS)

Figure 3 shows the natural logarithm of raw mortality rates $\ln(m_{x,t})$ in 2012. Notice the effect of infant mortality at the younger ages and the increase of the mortality probabilities after a minimum at the teenage years. At mature ages the graph increases in a linear way, implying that the probability of death grows exponentially. From now on mortality rates refer to the natural logarithm of mortality.

In figure 5 and 4 the log mortality rates for 25 and 65 year old's are depicted. HMDB has 100 years more data, while the data from CBS is more updated. The overlapping data is very similar but not exactly the same.

Since people get older, (log) mortality rates gradually decrease. Next to that we observe two spikes in the data from HMDB: In 1918 and around the Second World War. In 1918 the Spanish flu occured which resulted in a one year exceptional high mortality rate. During the Second World War mortality rates rose again. For men mortality rates attained a level similar to the Spanish flu. Woman clearly participated less in the war such that female increase in mortality remained relatively limited. This effect is less present for 65 year old's than for 25 year old's.

With the available data sets want to create future values for the probability of survival that are used to determine life expectancies. The relation between the one year survival probabilities $p_{x,t}$ and mortality rates is given by the following

[2]http://www.mortality.org/
[3]http://statline.cbs.nl/statweb/

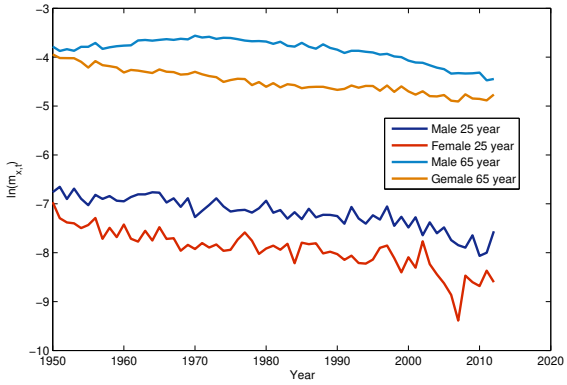


Figure 4: CBS mortality rates for 25 and 65 year old's (1950-2012)

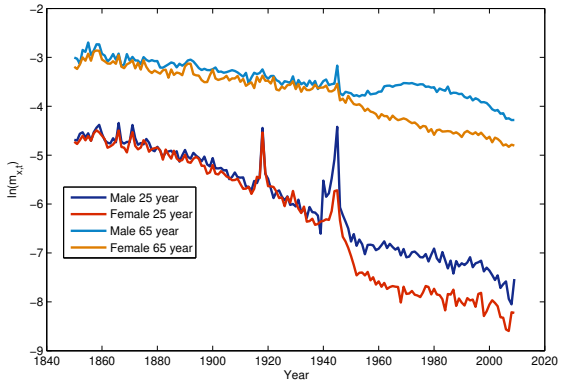


Figure 5: HMDB mortality rates for 25 and 65 year old's (1850-2009)

structure.

$$\mu_{x,t} = \frac{d}{ds}\ln({}_sp_{x,t}) \qquad \Leftrightarrow \qquad p_{x,t} = \exp(-\int_{s=0}^{1} \mu_{x+s,t}ds) \tag{2}$$

The forces of mortality $\mu_{x,t}$ are assumed to be constant over a one year time period

$$\mu_{x+s,t} = \mu_{x,t} \qquad \text{for} \qquad 0 \leq s < 1, \tag{3}$$

such that

$$p_{x,t} = \exp(-\mu_{x,t}). \tag{4}$$

The estimated force of mortality is equal to the instantaneous rate of mortality $m_{x,t}$:

$$\widehat{\mu}_{x,t} = m_{x,t}. \tag{5}$$

Combining these results with equation 1 we get estimated survival probabilities based on the mortality rates from the data.

$$\widehat{p}_{x,t} = \exp(-m_{x,t}). \tag{6}$$

3 Mortality Rate Modeling

In this section we discuss three models that are used to estimate and forecast mortality rates. First the AG table is introduced, which is the standard reference for Dutch survival probabilities for the next fifty years. However, the AG model does not quantify the uncertainty around future mortality rates which is needed to compute longevity risk. Second is the model that was applied for the S6 in 2007 to determine future mortality rates and the uncertainties around the predicted trend. And the third method is the Lee Carter model which is frequently used for estimating mortality data. Lee and Carter propose a method to forecast and include variation as well. Finally, the progress within the field of mortality modeling is discussed by means of several other models.

3.1 The AG Projection Table

Given is the most recent AG projection table 2012-2062 (*AG prognosetafel 2012-2062*) [1] provided by the Dutch actuarial association (*Actuarieel Genootschap*). The table contains estimated mortality probabilities until 2062. The method that the AG used in 2012 to derive the AG projection table has been carefully documented by the Dutch actuarial association [1] and will be briefly outlined.

3.1.1 Using a Projection Table

The AG projection table contains two specific tables: q_x and q_y, the male respectively female chances of dying within one year. The tables look as follows: The female table q_y is similar. The values in the AG projection table are de-

x,t	**2011**	**2012**	...	**2062**
0	0,0035533	0,0034244	...	0,0007617
1	0,0003727	0,0003540	...	0,0000381
⋮	⋮	⋮	⋱	⋮
120	0,6051180	0,6051180	...	0,6051180

Table 1: AG projection table (q_x)

noted $q_{x,t}$ for male and female, where $q_{x,t}$ is the probability that a life dies within year t for a person with age x. It contains values from ages 0 to 120 and years ranging from 2011 until 2062. Assumed is that for years after 2062 there will not be any mortality improvement any longer, such that the rates of 2062 can be used.

In some cases it is more intuitive to use the one-year probability that a person survives, denoted by $p_{x,t}$. Obviously,

$$p_{x,t} = 1 - q_{x,t}. \tag{7}$$

Given that the table is correct it is possible to determine ${}_sp_{x,t}$ that denotes how likely it is that a person lives another s years. Pension funds are interested

in knowing, for example, the probability that a 30-year old working person makes it to the retirement age of 67 such that he will receive his first pension payment. That is ${}_{37}p_{30,0}$ and $t = 0$ represents (the beginning of) the current year 2014. For a person to make it until the age of x, he needs to survive every single year until he reaches x. We can write the probability that a life aged x will survive at least s years as the product of one-year survival probabilities:

$$ {}_sp_{x,0} = p_{x,0} \cdot p_{x+1,1} \cdot \cdots \cdot p_{x+s-1,s-1} \tag{8} $$

Note that in year $x + s$ the person of interest is also s years older. Hence calculating ${}_sp_{x,t}$ requires a diagonal multiplication through a $p_{x,t}$ table that can be derived from the AG table by using equation 7.

We assume that $t = 0$ at the beginning of 2014 such that we can make use of the most accurate data that is appropriate to use the AG projection table. 2062 is the end of the prognosis table, hence we can estimate whether a life survives the year 2062. Here we assume that the s-year survival probabilities until the end of 2062 and beginning of 2063 are equal.

3.1.2 Data Extension for Old Ages

Mortality data extracted from CBS Statline includes death rates for ages 0 to 94. The AG applies a method to expand this to the age of 125 by using the logistic-type model introduced by Kannistö [12]. This technique to extrapolate mortality is called the closing of mortality tables.

The mortality rates have been adjusted to the two year average and are smoothed by the Van Broekhoven algorithm, see *Prognosetafel AG2012-2062* [1]. Here we will skip these steps.

The force of mortality for ages above 94 is modeled as

$$ m_{x,t} = \frac{\phi_{1,t} \exp(\phi_{2,t} \cdot x)}{1 + \phi_{1,t} \exp(\phi_{2,t} \cdot x)}. \tag{9} $$

First the death rates from CBS are transformed by equation 6 to mortality rates. Then the time dependent parameters $\phi_{1,t}$ and $\phi_{1,t}$ are estimated by OLS on the interval $x = 80, \ldots, 94$ where

$$ logit(m_{x,t}) = \log\left(\frac{m_{x,t}}{1 - m_{x,t}}\right) = \log(\phi_{1,t}) + \phi_{2,t} \cdot x. \tag{10} $$

Again by (6) and formula 9, $q_{x,t}$'s can now be created for the old ages as well. The extended table to the age of 125 is the CBS data that is referred to in this research. Table 3 shows the effect of the technique by Kannistö for the high ages in 2012.

3.1.3 Forecasting with Short and Long Term Trend

The AG method incorporates that the recent mortality improvement is stronger than the preceding trend. This short term trend is assumed to converge to the

long term trend. The short term trend dominates mortality rates for the first twenty years. After that the rates move towards the flatter long term trend line that reflects a mortality rate with less improvement.

In both 1988 and 2002 the trend of mortality rates has significantly changed. The change from 1988 onwards determines the long term trend and the short term trend is determined solely over the last ten years of available (CBS 1988-2011) data.

The short and long term trends are calculated as the average mortality improvement over the years 1988 respectively 2002 to 2011:

$$stt_x = \sqrt[9]{\frac{q_{x,2011}}{q_{x,2002}}} \tag{11}$$

$$ltt_x = \sqrt[23]{\frac{q_{x,2011}}{q_{x,1988}}} \tag{12}$$

For the ages until 100, almost every x the $stt_x < ltt_x$, see appendix B.

The short term trend (*stt*) converges to the long term trend (*ltt*). The end point of the projection table, named the *goaltable*, can therefore be constructed on the basis of the long term trend alone:

$$\widehat{q}_{x,2062} = q_{x,2011} \cdot ltt^{51}. \tag{13}$$

Next the goaltable will be smoothed by the Van Broekhoven algorithm for the ages 3 to 120. The value in the goaltable is restricted to the value of *2011* as maximum such that mortality improvement will always occur.

Intermediate values for the AG projection table can be derived as follows:

$$\widehat{q}_{x,t} = q_{x,2011} \cdot \prod_{j=1}^{t} stt \cdot \exp(j \cdot \delta_x) \tag{14}$$

$$= q_{x,2011} \cdot stt^t \cdot \exp\left(\delta_x \cdot \frac{t(t+1)}{2}\right) \tag{15}$$

for $x = 0, \ldots, 120$, $t = 1, \ldots, 50$ and δ_x is calibrated such that the goaltable is the same as the projection to $t = 51$. Some algebra will prove that

$$\delta_x = \frac{\ln \widehat{q}_{x,2062} - \ln q_{x,2011} - 51 \ln stt}{\frac{51(51+1)}{2}} \tag{16}$$

and

$$\widehat{q}_{x,t} = q_{x,2011} \cdot stt^t \cdot \left(\frac{\widehat{q}_{x,2062}}{q_{x,2011} \cdot stt^{51}}\right)^{\frac{t(t+1)}{51(51+1)}}. \tag{17}$$

This formula insures that the values slowly approach the goaltable at $t = 2062$.

Again some constrictions need to be taken into account. First, intermediate values can not get below the value of the goaltable, such that mortality improvement is guaranteed for all years. Second, to prevent at old ages that mortality rates will increase first and then decrease towards the goaltable, the projections will be set equal to the goaltable for $x = 100, \ldots, 125$.

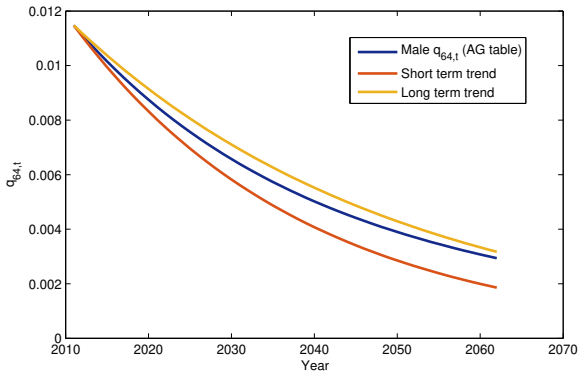


Figure 6: AG projected mortality rates for a 64-year old and the short and long term trend estimated by 11 and 12.

3.1.4 Including Uncertainty

The AG has highlighted the strong need for uncertainty estimates by means of a stochastic model, yet was not able to incorporate this within the method in 2012. The next edition of the AG projection table will most likely include a form of refinement.

Looking at equation 17 uncertainty can be added in the *stt* and *ltt* variables, assumed we take the model as given. The short term trend is determined over the last 10 years and its variance can be derived as well. However, the amount of data points is relatively small and will give a large variance. The values for $\widehat{q}_{x,t}$ remain the best estimates.

Another remark can be made to the use of smoothing by the Van Broekhoven algorithm. Since this takes away some irregularity within the data, it is inappropriate to apply it before determining an accurate level of uncertainty of mortality rates.

3.2 Old TSO Model

In the current FTK from 2007 the calculation of the S6 could not make use of the AG projection table, since the AG did not publish this yet. The model is documented by Van Dalen [22]. It predicts future mortality rates based only on last year's rates. That means that an assumption needs to be made about the progress of future mortality rates such that we apply an extrapolation from last year onwards. A general assumption is that

$$q_{x,t+1} = q_{x,t} \cdot [1 - \rho_{x,t}] \tag{18}$$

where $\rho_{x,t}$ is the mortality improvement for a x-year old compared to the year before.

They assumed that $\rho_{x,t} = \rho$ which implies that mortality improvement is independent of age and fixed over time. It seems unrealistic that the old ages run by the same extent of mortality improvement as other age cohorts, therefore a linear decline of ρ towards 0 is applied for the last 20 ages. Hence, for the ages $x_{min} = 80$ to $x_{max} = 100$, a gradual transition from mortality improvement of ρ to zero is applied:

$$\begin{aligned} \rho_{x,t} &= \rho && \text{if } x < x_{min} \\ \rho_{x,t} &= \rho \cdot \frac{x_{max} - x}{x_{max} - x_{min}} && \text{if } x_{min} \leq x < x_{max} \\ \rho_{x,t} &= 0 && \text{if } x \geq x_{max} \end{aligned}$$

It also excludes the possibility that survival rates for high ages become unrealistic.

To incorporate uncertainty into future development of mortality rates is assumed that ρ is normally distributed, i.e. $\rho \sim N(\mu_\rho, \sigma_\rho^2)$. According to the documentation, the expected value of this distribution is tweaked in such a way that the mean life expectancy in 2050 corresponds to the given life expectancy in 2050 given by the CBS prognosis and the standard deviations match the given 5% and 95% percentiles. For men is the mean $\mu_\rho = 0,945\%$ and standard deviation $\sigma_\rho = 0,835\%$, for women $\mu_\rho = 0,575\%$ and $\sigma_\rho = 0,960\%$.

The expected value for the mortality rate in s years is

$$\mathbb{E}[q_{x,T+s}] = q_{x,T} \cdot (1 - \mu_\rho)^s \tag{19}$$

and simulations are performed as

$$q^*_{x,T+s} = q_{x,T} \cdot (1 - (\mu_\rho + \sigma_\rho \cdot \epsilon_\rho))^s \,, \tag{20}$$

where the asterisk indicates a simulated variable. ϵ_ρ is a random standard normal distributed variable. For convenience it is assumed that the variation is perfectly correlated for both male and female ρ.

Extrapolation is performed from the AG survival table 1995-2000 (GBM/V) data for ages 0 to 99, hence this table is the $q_{x,T}$ and $T = \textit{2002}$.

With these simulated mortality rates it is possible to determine the s-year survival probabilities as well as the life expectancy. The average number of years that an x-year old at t will survive, the expected curtate future lifetime, is determined as [9]

$$e_{x,t} = \sum_{s \geq 1} {}_sp_{x,t} \tag{21}$$

in which the sum of the forecasted s-year survival probabilities can either be estimations or simulations. In $t = \textit{2002}$ the life expectancy at birth for a man is 80,35 years and there is a 50% change of dying between the age of 77,01 and 83,46 (25 and 75 percentiles). For 65-year old's in 2002 life expectancy remains 15,15 (14,75-15,55) years.

If we want to apply this model to current data, new assumptions need to be made about the distribution of ρ and the starting point of the extrapolation.

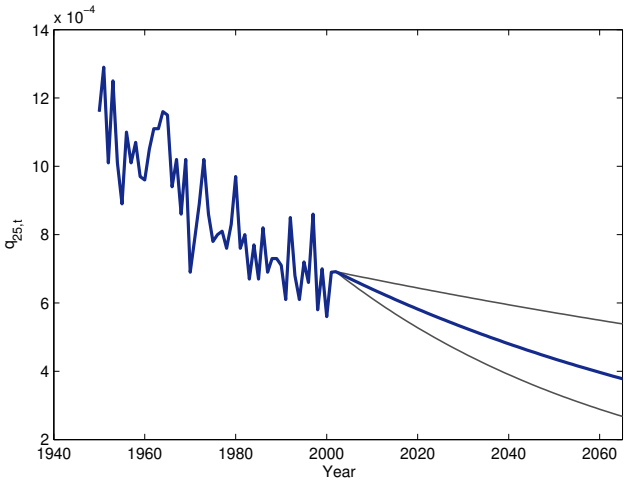


Figure 7: Forecasted mortality rate for a 25 year old man: The best estimate and the 25% and 75% percentiles. Based on 10.000 estimations.

3.3 The Lee Carter Model

The current international benchmark model for estimating mortality rates is the Lee Carter model. In 1992 R.D. Lee and L. Carter [6] proposed the following model, a log-bilinear form of the central death rate:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t} \qquad \text{for } x = 0, \ldots, X \text{ and } t = 1, \ldots, T. \tag{22}$$

It captures the age-dependent factor by α_x as well as the decreasing dynamics over time that is different for all ages. The consistent rise of life expectancy is incorporated in the model by κ_t. We can call κ_t the dependent trend as β_x is the sensitivity to the trend for age x. $\epsilon_{x,t}$ is assumed to be white noise.

3.3.1 Identification Problem

First we need to cope with an identification problem. The parameter set $(\alpha_x, \beta_x, \kappa_t)$ forms the same model as $(\alpha_x, \beta_x/c, \kappa_t \cdot c)$ for any constant $c \neq 0$ and does therefore not find unique solutions. To overcome this problem two constraints are imposed. It can be shown that $\sum_{x=0}^{X} \beta_x = 1$ and $\sum_{t=0}^{T} \kappa_t = 0$ ensure identifiability.

Proof. The Lee Carter model is identifiable if for two different parameter sets, say $(\alpha_x, \beta_x, \kappa_t)$ and $(\overline{\alpha}_x, \overline{\beta}_x, \overline{\kappa}_t)$, can not lead to an identical model. Given are equation 22 and the constraints $\sum_{x=0}^{X} \beta_x = 1$ and $\sum_{t=1}^{T} \kappa_t = 0$.

$$\sum_{t=1}^{T} (\alpha_x + \beta_x \kappa_t) = \sum_{t=1}^{T} (\overline{\alpha}_x + \overline{\beta}_x \overline{\kappa}_t)$$

$$\Leftrightarrow T \cdot \alpha_x + \beta_x \sum_{t=1}^{T} \kappa_t = T \cdot \overline{\alpha}_x + \overline{\beta}_x \sum_{t=1}^{T} \overline{\kappa}_t.$$

Substituting $\sum_{t=1}^{T} \kappa_t = 0$ implies that $\alpha_x = \overline{\alpha}_x$.

$$\sum_{x=0}^{X} (\alpha_x + \beta_x \kappa_t) = \sum_{x=0}^{X} (\overline{\alpha}_x + \overline{\beta}_x \overline{\kappa}_t) = \sum_{x=0}^{X} (\alpha_x + \overline{\beta}_x \overline{\kappa}_t)$$

$$\Leftrightarrow \sum_{x=0}^{X} \alpha_x + \kappa_t \sum_{x=0}^{X} \beta_x = \sum_{x=0}^{X} \alpha_x + \overline{\kappa}_t \sum_{x=0}^{X} \overline{\beta}_x.$$

Substituting $\sum_{x=0}^{X} \beta_x = \sum_{x=0}^{X} \overline{\beta}_x = 1$ implies $\kappa_t = \overline{\kappa}_t$. These two results combined make that on a term level also $\beta_x = \overline{\beta}_x$.

Here is proven that two parameter sets can only lead to the same model when the parameter sets are equal. Hence with the constraints

$$\sum_{x=0}^{X} \beta_x = 1 \qquad \text{and} \qquad \sum_{t=1}^{T} \kappa_t = 0$$

the Lee Carter model is identifiable. □

3.3.2 Lee Carter Estimations

The estimation for α_x is defined as the average mortality for every age:

$$\hat{\alpha}_x = \frac{\sum_{t=1}^{T} \ln(m_{x,t})}{T}. \tag{23}$$

Proof. Given is equation 22. Then

$$\begin{aligned} \sum_{t=1}^{T} \ln(m_{x,t}) &= \sum_{t=1}^{T} (\alpha_x + \beta_x \kappa_t + \epsilon_{x,t}) \\ &= T \cdot \alpha_x + \beta_x \sum_{t=1}^{T} \kappa_t + \sum_{t=1}^{T} \epsilon_{x,t}. \end{aligned}$$

Substituting $\sum_{t=1}^{T} \kappa_t = 0$ and taking the expectation yields

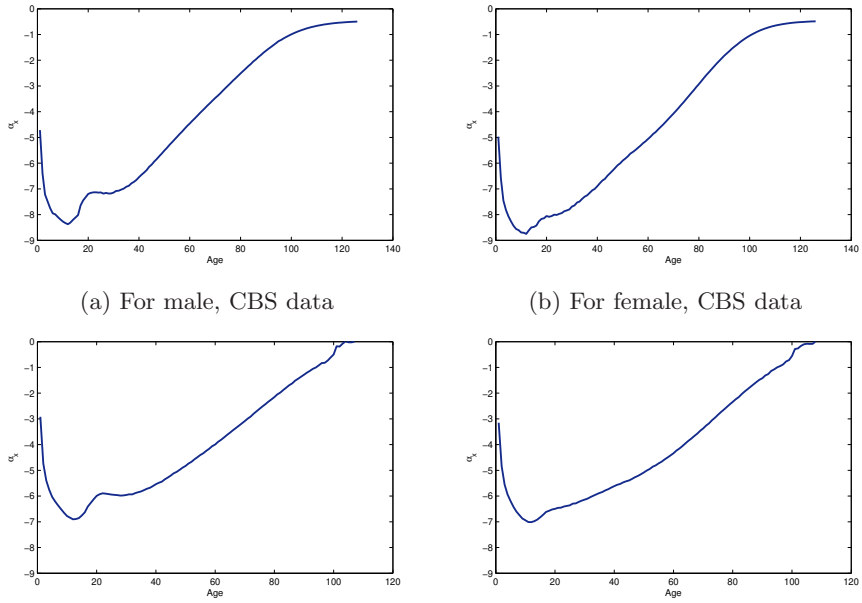
$$\hat{\alpha}_x = \frac{\sum_{t=1}^{T} \ln(m_{x,t})}{T}$$

as the mean of the error terms is zero. □

The next step is to perform a least squares estimation on the β and κ vectors, such that $\sum_{x=0}^{x} \sum_{t=0}^{T} \left(\ln(m_{x,t}) - \hat{\alpha}_x - \beta_x \kappa_t\right)^2$ is minimized. Ordinary least squares (OLS) can be obtained from Singular Value Decomposition (SVD or First Principal Component). The mortality data is summarized by making use

of only the first single principal component with other variation ignored. This is especially useful for the purpose of forecasting. The SVD code is available in the MATLAB software; there are plenty of good references [13][19].

SVD ensures that the constraint $\sum_{t=1}^{T} \kappa_t = 0$ suffices by construction. This means that after estimation by SVD the vector β_x remains to be divided by its sum and hence the κ_t vector must be multiplied by this sum.



(a) For male, CBS data

(b) For female, CBS data

(c) For male, HMDB data

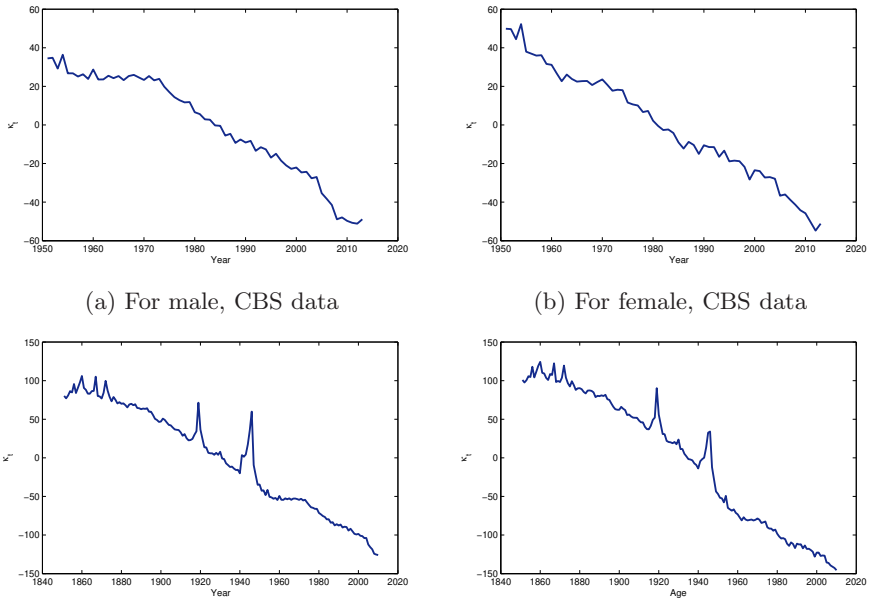
(d) For female, HMDB data

Figure 8: Lee Carter estimations for α_x

The estimations are distinguished in four types: The combinations of male/ female and CBS/ HMDB data. See figures 8, 9 and 10 for the estimated α_x, κ_t and β_x.

The α_x estimates in figure 8 represent the means for every age with use of 60 years of data. If we compare these estimates with the data from 2012 in figure 3 a smoothing effect is visible. The estimates for the HMDB data are structurally lower since the older data with higher expected mortality is included resulting in a lower mean over all years. An other interesting result is the peak at 18 to 24 for men. This age group is more vulnerable for largely preventable causes of death as (motor vehicle) accidents and suicide resulting in an higher mortality rate.

The estimates for κ_t represent the progress of mortality over time. The actual forces of mortality change according to an overall mortality index $\widehat{\alpha}_x$. Spot the similarities to the raw mortality rates (figure 4 and 5). The outliers from the Spanish flu and during the Second World War are equally visible.



(a) For male, CBS data

(b) For female, CBS data

(c) For male, HMDB data

(d) For female, HMDB data

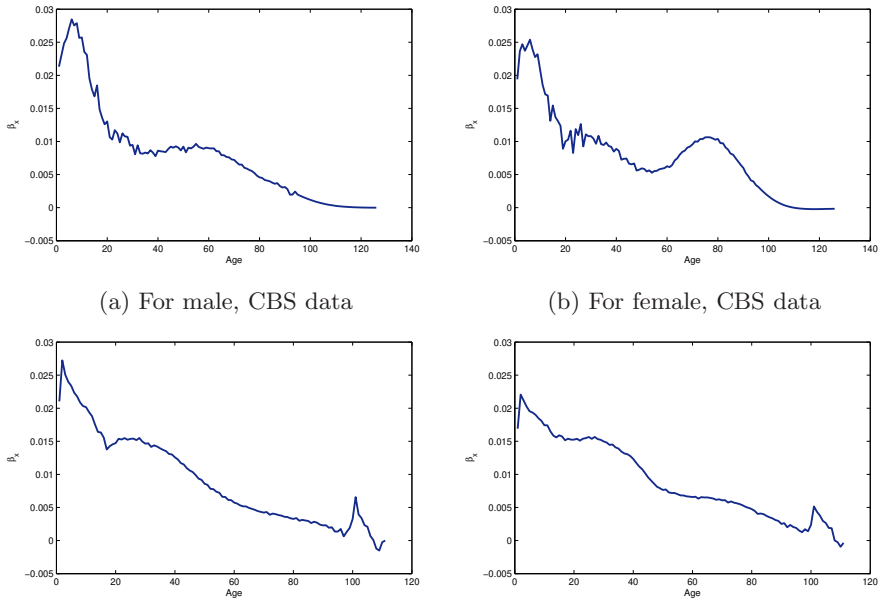
Figure 9: Lee Carter estimations for κ_t

The sensitivity to the mortality improvement κ_t is different for every age, represented by β_x. The shape of the β_x profile tells which rates decline rapidly and which slowly over time in response of change in κ_t. It is especially high for young ages and decreases from 0,25 to 0 for old ages. Around the age of 20 the sensitivity to the mortality is less since mortality around this age is remarkably high and therefore less coherent with the general improvement over the years. A significant peak around the age of 100 is featured for the HMDB data. This is due to the fact people reaching the age of 100 has only been documented recently for this database. Hence the sensitivity towards the time trend is relatively strong.

Lee and Carter also provide a re-estimation technique such that the observed number of deaths matches the predicted:

$$D_t = \sum_{x=0}^{X} E_{t,x} e^{\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t}. \tag{24}$$

This re-estimation is not applied since there does not exists a valid argument that it would lead to a more reliable estimation.



(a) For male, CBS data

(b) For female, CBS data

(c) For male, HMDB data

(d) For female, HMDB data

Figure 10: Lee Carter estimations for β_x

3.3.3 Forecasting Lee Carter

The progress of future mortality rates are based on the course of past data. The estimates for α_x and β_x within the Lee Carter model are assumed to be independent of time such that they will not change in the future. Lee and Carter propose to forecast the time-dependent trend of $\hat{\kappa}_t$ by a random walk with drift:

$$\widehat{\kappa}_t - \widehat{\kappa}_{t-1} = \theta + \omega_t, \tag{25}$$

where $\omega_t \sim N(0, \sigma_\omega^2)$. They tested several ARIMA specifications and found that an ARIMA(0,1,0) was the most appropriate to fit their US mortality data (1933-1987). Brouhns et al. (2002) [2] found that for Dutch death rates from 1950 (1970 for men) to 2000 and for ages 60 to 98, the ARIMA(0,1,0) is a good model for both men and women. Moreover, Tuljapurkar et al. [20] found that the decline in mortality has been relatively constant in developed countries. All in all, it suggests that a random walk with drift is an appropriate assumption for our investigation.

θ is known as the drift parameter and it's maximum likelihood estimate is

$$\widehat{\theta} = \frac{\widehat{\kappa}_T - \widehat{\kappa}_1}{T-1}. \tag{26}$$

Hence $\widehat{\theta}$ only depends on the first and last estimations of κ_t which implies that extrapolation in the Lee Carter model is simply a line through $\widehat{\kappa}_1$ and $\widehat{\kappa}_t$ and

all other points are ignored. This makes the drift parameter very sensitive to the calibration period.

To forecast κ_t k years after the last observation, at time $T+k$, the procedure continues iteratively. We obtain that

$$\widehat{\kappa}_{T+k} = \widehat{\kappa}_T + k\widehat{\theta} + \sum_{l=1}^{k} \omega_{T+l-1} \tag{27}$$

The variable ω_T is assumed to be independent of ω_{T+i} for $i \neq 0$ with identical variance. Because of this property the variation at $t = T + k$ can be derived directly. Since ω_T is normally distributed with variance σ_ω^2, the sum $\sum_{l=1}^{k} \omega_{T+l-1}$ has variance $k\sigma_\omega^2$. Consequently, the standard errors for the forecast increase with the square root of the forecast horizon.

$$\widehat{\kappa}_{T+k} = \widehat{\kappa}_T + k\widehat{\theta} + \sqrt{k}\omega_T. \tag{28}$$

Furthermore, the MLE of the variance for ω_T and $\widehat{\theta}$ are given [10]

$$\widehat{\sigma}_\omega^2 = \frac{1}{T-1} \sum_{t=1}^{T} (\widehat{\kappa}_{t+1} - \widehat{\kappa}_t - \widehat{\theta})^2 \tag{29}$$

and

$$\mathrm{var}(\widehat{\theta}) = \frac{\sigma_\omega^2}{T-1}. \tag{30}$$

By substitution we obtain the *estimated* forecast of log mortality:

$$\begin{aligned} \ln(\widehat{m}_{x,T+k}) &= \widehat{\alpha}_x + \widehat{\beta}_x \widehat{\kappa}_{T+k} && (31)\\ &= \widehat{\alpha}_x + \widehat{\beta}_x(\widehat{\kappa}_T + k\widehat{\theta}) && (32)\\ &= \ln(\hat{m}_{x,T}) + k\widehat{\beta}_x\widehat{\theta} && (33)\\ &= \ln\left(\hat{m}_{x,T} \cdot \exp(k\widehat{\beta}_x\widehat{\theta})\right), && (34) \end{aligned}$$

where $\widehat{\theta}$ is defined as in equation 26.

3.3.4 Forecast Uncertainty

Variation in the future trend can be attained by including parameter uncertainty in the mortality improvement θ by drawing $\widehat{\theta}$ from a distribution:

$$\theta^* = \widehat{\theta} + \frac{\sigma_\omega}{\sqrt{T-1}} \cdot \epsilon_\theta, \tag{35}$$

where the standard deviation of $\widehat{\theta}$ is equal to $\frac{\sigma_\omega}{\sqrt{(T-1)}}$ by equation 29.

We can also apply a second randomization of ω_T such that it includes the estimation error. Since we know that its distribution is normal with mean zero

$$\omega_k^* = \widehat{\sigma}_\omega \cdot \epsilon_\sigma. \tag{36}$$

ϵ_θ and ϵ_σ are independent randomized standard normal distributed numbers. With this information we can create possible paths for mortality rates by using θ^* and ω_T^* instead of $\widehat{\theta}$ and ω_T in (28).

Equivalently, randomizing is performed as

$$\widehat{\kappa}_{T+k} = \widehat{\kappa}_T + k\left(\widehat{\theta} + \frac{\widehat{\sigma}_\omega}{\sqrt{T-1}} \cdot \epsilon_\theta\right) + \sqrt{k}\left(\widehat{\sigma}_\omega \cdot \epsilon_\sigma\right) \tag{37}$$

$$= \widehat{\kappa}_T + k\widehat{\theta} + \widehat{\sigma}_\omega \left[\frac{k}{\sqrt{T-1}} \cdot \epsilon_\theta + \sqrt{k} \cdot \epsilon_\sigma\right]. \tag{38}$$

When the number of observations minus one is higher than the forecast horizon, the variation induced by the parameter uncertainty of θ is smaller than the variation from the yearly volatility. The term in brackets is the sum of two independent normal distributions and is therefore normally distributed as well:

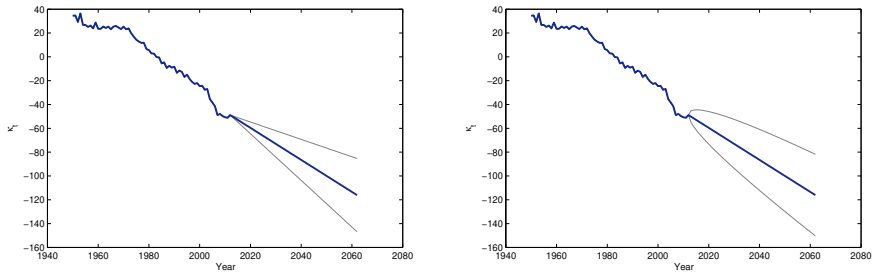
$$\left[\frac{k}{\sqrt{T-1}} \cdot \epsilon_\theta + \sqrt{k} \cdot \epsilon_\sigma\right] \sim N\left(0, \frac{k^2}{T-1} + k\right). \tag{39}$$

Both drivers of uncertainty increase with the length of forecasting k. Uncertainty induced by an inaccurate estimation of the trend reduces when more data is available in order to make a better approximation. Their separate effects are shown in figure 11.

It is possible to forecast the prediction interval analytically by CLT or by means of simulations. For the analytical 90% confidence interval of $\widehat{\kappa}_{T+k}$ we can apply

$$\widehat{\kappa}_T + k\widehat{\theta} - z \cdot \widehat{\sigma}_\omega \sqrt{k + \frac{k^2}{T-1}} < \widehat{\kappa}_{T+k} < \widehat{\kappa}_T + k\widehat{\theta} + z \cdot \widehat{\sigma}_\omega \sqrt{k + \frac{k^2}{T-1}}, \tag{40}$$

where z is the $1 - \frac{1}{2}(1 - 90\%) = 0,95$ percentile of a standard normal distribution for a 90% confidence interval, i.e. $z = 1,645$. For a 50% confidence interval $z = 0,6745$.

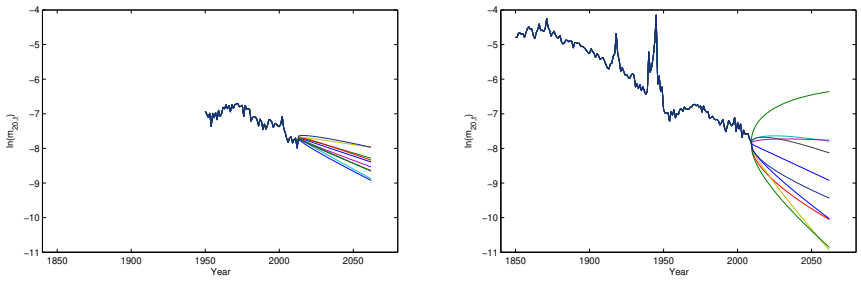


(a) Variation from θ (b) Variation from yearly volatility

Figure 11: Estimated κ_t and its 90% confidence interval using male CBS data. Variation is induced by one of the two risk drivers.

In line with AG projection table, we project the mortality rates until 2062. To achieve possible paths for the mortality rates we can randomize ϵ_θ and ϵ_σ. Depicted in figure 12 are the estimated mortality and 10 possible paths for a man aged 20.

Again it matters which data set is used. Estimated mortality for a 20 year old man in 2062 equals $\ln(\widehat{m}_{20,2063}) = -8.3929$ for CBS data and -8.9249 for HMDB data. The paths for the HMDB data seem less accurate. This comes from the fact that the data contains more volatile years what results in a higher variance of ω_T. The results for female mortality rates are similar, only lower.



(a) CBS data

(b) HMDB data

Figure 12: Projected mortality rates by Lee Carter for a 20 year old man.

3.3.5 Prediction in Number of People

Pension funds need to know at what age their members will still be alive, so that they can incorporate future payments in their current financial plans. We can compare the Lee Carter forecasts with the AG table to make an estimation of how many people will survive to 2063 based on a member base of 100.000 people. The variability from the estimations is the variation induced by longevity risk.

We have obtained estimates for the Lee Carter model in order to apply the formulas given in this section for the forecast. With equations 6 and 8 the predicted mortality rates are transformed to s-year survival rates ${}_sp_{x,0}$ from now (= 2014) onwards. The expected number of survivors of age x in year s, given the current population of size $n_{x,0} = 100.000$ is

$$\widehat{n}_{x,s} = n_{x,0} \cdot {}_sp_{x,0}. \tag{41}$$

Table 2 and 3 give the expectations and the 90% confidence interval of the predicted number of survivors based on the analytically derived confidence interval of κ_{T+k}.

Over time the number of survivors becomes less and this effect is becomes more noticeable for the high ages. Furthermore, the different types of estimation give a variety of results. In section 5.2 we will put more emphasis on the dependence of the model and the underlying data set.

	2014	2040	2063
20-year old's	100.000	98.969	94.070
		[98.066 98.407 98.683]	[83.992 88.164 91.277]
		[97.127 98.797 99.481]	[73.035 84.791 91.601]
40-year old's	100.000	93.373	46.448
		[87.827 90.009 91.802]	[22.178 28.429 34.945]
		[81.723 87.939 29.126]	[10.009 20.183 32.802]
60-year old's	100.000	55172	7
		[32.985 37.148 41.497]	[28 44 67]
		[21.008 29.682 38.821]	[0 1 2]

Table 2: Projection of a current male member base of 1.000.000 to 2063 with CBS data. First line: Estimate using the AG table. Second line: Best estimate and analytical approximation for a 90% confidence interval of the Lee Carter model with CBS data. Third line: with HMDB data. All values are rounded to the nearest integer.

	2014	2040	2063
20-year old's	100.000	99.234	92.412
		[98.778 99.023 99.218]	[90.446 92.975 94.806]
		[98.324 99.211 99.623]	[85.151 92.104 95.868]
40-year old's	100.000	93.425	54.439
		[92.209 93.451 94.480]	[46.543 59.347 69.843]
		[89.460 93.105 95.506]	[23.008 37.696 52.181]
60-year old's	100.000	64.945	22
		[56.913 64.805 71.520]	[103 183 297]
		[37.532 48.427 58.437]	[2 12 50]

Table 3: Equivalent of table 2 for woman.

In the Lee Carter model, the number of men in 2063 is approximately half the number of the predicted number of women in 2063. Men's forecasts also seem to be underestimated compared to AG projection table estimates.

Aside from the model, the data used for this forecast is different as well. The AG uses CBS data from 1988 onwards where we choose to use the whole CBS and HMDB data sets. The last part of the data has a stronger downwards sloping trend resulting in a higher mortality improvement for men.

The 90% confidence interval does not get relatively smaller for a larger member base, since the mortality rates and the trend risk are not dependent on the population number. The uncertainty increases only by the projection period. The size of the trend risk is independent of the number of participants and dependent on the forecast horizon. Neither within an age group, nor between different age groups the trend risk is diversifiable. For this reason it is possible to add the confidence intervals of the age groups.

3.4 Alternative Models

In the literature we find alternative approaches for modeling mortality rates. Cairns et al. [4] discuss a wide range of extrapolative stochastic mortality models that have been proposed over the last 20 years by evaluating their modeling and forecasting quality.

3.4.1 Additional Components

One option is to include a second principal component at the original Lee Carter model (M1). Renshaw and Haberman [16] proposed this in 2003. Lee Carter with two principal components would go by model 2 given in table 4, basically an extra β times κ vector is added to describe the data. The first two principal components of SVD should be taken in this case. Though it has been confirmed by Tuljapulkar [20] that the first principal component accounts for high percentages of the variance for developed countries. Girosi and King [10] show that 89% of the variance of Dutch male mortality data across all causes is explained by the First Principal Component.

A significant improvement to the Lee Carter model according to Cairns et al. are cohort effects. A new model complements the Lee Carter model with a (approximate) birth year variable and takes the form

$$ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + F(\gamma_{t-x}) + \epsilon_{x,t}, \tag{42}$$

where $F(\gamma_{t-x})$ is a function of the cohort effects and $(t-x)$ represents the cohort year.

Renshaw and Haberman (2006) [17] propose a model that includes the three main effects: Age, time (year) and birth year (M3). Forecasting in this model is done linearly on κ_t and γ_{t-x}, equivalently to the proposed forecast in original Lee Carter. However, parameter estimation is more difficult since the three factors are constrained by the relationship

$$cohort = period - age. \tag{43}$$

Alpha is typically estimated as in the original Lee Carter SVD approach. Other parameters are estimated by iterative processes.

Cairns et al. state that model from Renshaw and Haberman lacks robustness. They propose a better model [4] that estimates the logistic transformation rather than the log of the death rate: M4. Model 5 is a generalized version of their model that adds a quadratic age-period effects as well as cohort effects and is supposed to give good results. $\overline{x}$ is the mean over the ages, $\widehat{\sigma}_x^2$ the corresponding variance and $(\kappa_t^{(1)}, \kappa_t^{(2)})$ is assumed to be a bivariate random walk with drift.

3.4.2 Poisson Assumption

A main drawback of the OLS estimation via SVD is that the errors are assumed to be homoscedastic. However, the logarithm of the observed force of mortality is much more variable at older ages because of the much smaller absolute number

	Reference	Model
M1	Lee and Carter (1992)	$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$
M2	Renshaw and Haberman (2003)	$\ln(m_{x,t}) = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)} + \beta_x^{(2)} \kappa_t^{(2)} + \epsilon_{x,t}$
M3	Renshaw and Haberman (2006)	$\ln(m_{x,t}) = \alpha_x + \beta_x^{(1)} \kappa_t + \beta_x^{(2)} \gamma_{(t-x)} + \epsilon_{x,t}$
M4	Cairns, Blake, Dowd (2006)	$logit(q_{x,t}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \overline{x}) + \epsilon_{x,t}$
M5	Cairns, Blake, Dowd (2006)	$logit(q_{x,t}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \overline{x}) + \kappa_t^{(3)}((x - \overline{x})^2 - \widehat{\sigma}_x^2) + \gamma_{(t-x)} + \epsilon_{x,t}$

Table 4: Alternative mortality rate models.

of deaths. The Poisson assumption discussed by Brouhns et al. [3] appears to be plausible in order to circumvent the problems associated with the OLS method.

The Poisson error setting is a widespread assumption within mortality modeling in which

$$D_{x,t} \sim Poisson(E_{x,t} \cdot \mu_{x,t}) \tag{44}$$

and $\mu_{x,t}$ is estimated by a mortality rate model. Hence the error term applies directly on the number of deaths. The parameters from the Lee Carter model in this setting can no longer be estimated via SVD, but require a form of MLE.

Brouhns et al. applied the Lee Carter model within the Poisson setting to Dutch death rates (1950-2000) for ages 60 to 98 and κ_t is assumed to be an ARIMA(0,1,0). The Poisson error setting accounts for slightly more variability than its SVD counterpart and results in lower forecasts [3].

Brouhns et al. also provide an application to calculating longevity risk within the Poisson setting using Lee Carter [2]. The confidence interval is constructed by combining two different sources of uncertainty: Sampling errors in the parameters α_x, β_x and κ_t given their multivariate normal distribution and forecast errors in the projected ARIMA parameters of κ_t.

Wilmoth (1993) [24] uses weighted least squares with weights $w_{x,t} = D_{x,t}$ instead of OLS within the original Lee Carter setting. Empirical studies reveal that this has the effect of bringing the parameter estimates into close agreement with the Poisson response based estimates.

3.4.3 Bayesian Estimation

Cairns et al. (2008) state that the issue of parameter uncertainty along with model uncertainty are important issues that require substantial further research. Improvements for parameter estimation find place within the Bayesian framework. In Bayesian models the parameters are considered random variables and therefore these models can assist in determining the variability of the parameters.

Also G.L. Cairns [5] underlines the important role of Bayesian stochastic mortality models with respect to considering longevity risk. He describes in his PhD thesis that Bayesian methods give better fits for both the Lee Carter model as well as age-period-cohort models.

Czado et al. (2004) [7] provide Bayesian analysis for the original Lee Carter model with the Poisson setting by Brouns et al. Also Koissi and Shapiro [13] incorporate variation in the age-specific parameters by constructing likelihood-based models and applying bootstrap and Bayesian technique to construct confidence intervals for the parameters.

4 Solvability Requirement Derivations

The TSO represents the magnitude of reserves that pension funds would have to hold in order to meet their future contractual liabilities. The TSO is given in percentages of the provisions and can be seen as an solvability requirement. The provisions are directly related to the annuities of pensions and life expectancies of the policy holders. Pension funds can not only rely on the expected future payments, but they should also install a risk margin to cover for uncertainties. The S6 in the FTK by DNB prescribes the volume of this risk margin for insurance technical risks. The TSO, that is part of the S6, accounts for longevity risk.

A pension scheme may include three elements [23]: old-age pension, partner's pension and disability pension. The contents of a pension scheme is decided by trade unions and employer organizations. The disability pension is excluded from this research.

4.1 Required Provisions Old-Age Pension

The present value of annuities for the old-age pension are comparable to a pension fund that pays 1 every year to their retired participants. The old-age pension benefit is paid from the retirement age, set at 67, until death. We will continue to use 67 but in the old TSO calculations the retirement age was 65.

Current income influences the magnitude of the pension and so the required capital. However, it is not necessary incorporating a salary scheme in the calculation since the TSO is in percentages and we assume at this point that every retiree receives the same pension.

Using the projected life tables generated in the preceding section, we deduce present values based on the price of an immediate life annuity sold to an individual aged x in year t [2]:

$$a_x(t) = \sum_{k \geq 0} {}_kp_{x,t} \cdot d_k. \tag{45}$$

where

$$\begin{aligned} {}_kp_{x,t} &= \prod_{j=0}^{k} p_{x+j,t+j} && (46)\\ &= p_{x,t} \cdot p_{x+1,t+1} \cdots p_{x+k+1,t+k-1} && (47) \end{aligned}$$

and

$${}_0p_{x,t} = 1. \tag{48}$$

Payments start directly (prenumerando) and are made yearly at the beginning of each year. For an old-age pension the payments start after the retirement age of 67 (deferred payments), hence $x + k \geq 67$. The underlying data determines k: $x+k \leq 125$ for CBS data, $x+k \leq 110$ for HMDB data and $x+k \leq 99$ within

the old model. The projection lies beyond the scope of k years, i.e. mortality rates are projected until every current participant has died.

d_k is the discount factor at year k defined by the recent (31-7-2014) nominal interest term structure (zero coupon) for pension funds (z_k) provided by DNB[4]. In the old calculations the term structure from 30-12-2002 was used.

$$d_k = \frac{1}{(1+z_k)^k}. \tag{49}$$

The term structure is given for 60 years ahead and we assume that the one-year forward rates remain the same thereafter,

$$f_t = \frac{d_{t-1}}{d_t} - 1 \tag{50}$$

and

$$f_t = f_{60} \qquad \text{for} \qquad t \geq 60. \tag{51}$$

Furthermore, t is *2014* (old model: 2002) for calculating the present value. The present value of all pension agreements is simply multiplying $a_x(t)$ by the number of participants of age x, using the expected values for ${}_sp_{x,t}$. Analytically derived or simulated ${}_sp_{x,t}$ (${}_sp^*_{x,t}$ where the asterisk indicates derivation by simulation) can be applied to above formula such that confidence intervals can be derived.

For a pension fund we get the present value of old-age pension agreements

$$PV(OP) = \sum_{x\geq 0} \left[\sum_{k\geq 0} {}_kp_{x,0} \cdot d_k \cdot \mathbb{1}_{x+k\geq 67} \right] \cdot n_{x,0}, \tag{52}$$

where also simulated $p^*_{x,t}$ can be used to derive the simulated present value PV^*. It is the sum of the expected discounted payments over all ages. Values are not rounded; we only consider probabilities and euros, not persons.

The present value of the life annuities indicate a clear maximum value at the retirement age of 67. In the picture $t = 0$ at 2014. Once a life gets closer to the retirement age the present value increases and afterwards decreases due to two reasons:

1. For the young ages, it is every year more likely that a life reaches the retirement age and hence more likely to receive payments,
   i.e.
   $$ {}_{67-x}p_{x,t} > {}_{67-(x-1)}p_{x-1,t}; $$

2. After the retirement age, expected future expenses become less every year. Ignoring the term structure,
   $$ a_x = 1 + p_{x,1} \cdot a_{x+1} \qquad \text{for} \qquad x \geq 65 $$

[4] www.statistics.dnb.nl/usr/statistics/excel/t1.3nm.xls

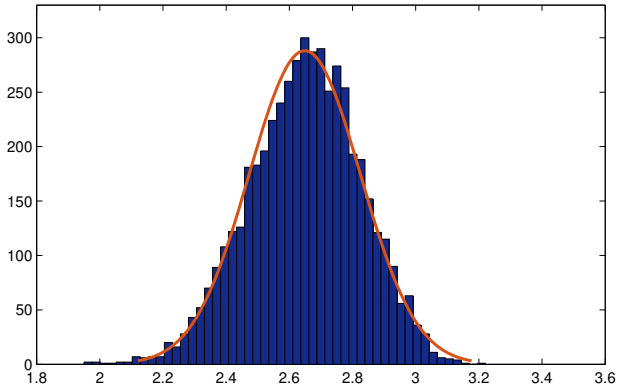


Figure 13: Present values for a 20-year old men on the basis of 5.000 simulations (by Lee Carter estimates). Normal approximation has mean 2,649 and standard deviation 0,176.

such that

$$a_x > a_{x+1}.$$

This effect is much stronger than the increase of the conditional life expectancy, which would imply again more payments.

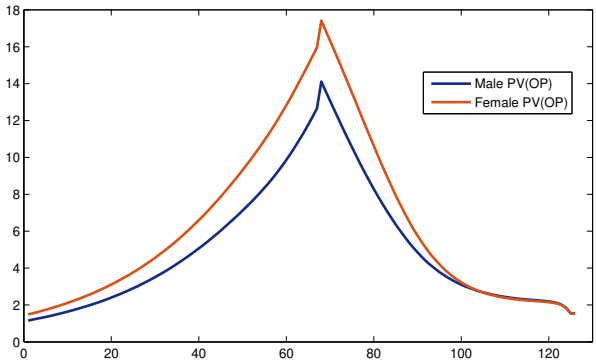


Figure 14: Expected present values of the old-age pension for a person aged x (by Lee Carter estimates).

By including uncertainty in the old model and the Lee Carter model and using simulated values for ${}_sp_{x,t}$ in formula 52 we get results from table 5.

Apparently the AG table expects that more payments should be made to men than predicted by the Lee Carter model, the opposite applies to women.

	Men	Women
20-year old	3,155	3,224
	[1,118 1,259 1,393]	[1,307 1,448 1,576]
	[2,531 2,653 2,770]	[3,362 3,517 3,658]
40-year old	6,421	6,807
	[2,516 2,712 2,908]	[3,041 3,238 3,428]
	[5,218 5,406 5,590]	[6,797 7,044 7,274]
60-year old	12,191	13,372
	[7,218 7,404 7,594]	[8,705 8,901 9,098]
	[10,196 10,404 10,611]	[13,167 13,512 13,845]

Table 5: Present values in units of the old age pension. First line: Estimate by AG projection table. Second line: Simulated [25 50 75] percentiles by the old method. Third line: Analytical [25 50 75] percentiles by the Lee Carter model (CBS data).

This is in line with the expected survivors from table 2 and 3. The old model is significantly underestimates the payments.

The addition of the confidence intervals is possible since the trend risk is not diversifiable. Estimates and uncertainties can be added. Also, linear multiplication is possible for any number of members. For example: In the Lee Carter model, the present value of the annuity payments of a male member's base of 50 20-year old's, 20 40-year old's and 30 60-year old's is $50 \cdot [2,531; 2,653; 2,770] + 20 \cdot [2,531; 2,653; 2,770] + 30 \cdot [10,196; 10,404; 10,611] = [483,05; 498,83; 512,23]$.

4.2 Required Provisions Partner's Pension

A partner's pension is paid out to the living partner after death. This kind of benefit depends on the salary and maximum attainable number of years of service. The partner's pension is on *funded basis* or on a *risk basis*. For a partner's pension on funded basis the partner can only count on the part that has been funded and that is dependent on the numbers of years of employment. In the second case, the partner will receive a yearly allowance comparable to the pension scheme, but if the employment relationship is terminated the partner's pension is canceled as well.

The partners pension is assumed to be 70% of the old-age pension. In the calculations we subdivide the old-age pension (OP) with partner's pension (NP) in four types with differences in financing method (risk (r) or funded (f) basis) and magnitude of partner's pension (accrued (a) or projected (p) OP). The partner's pensions is therefore dependent on the survival probabilities of both partner's. The person that will receive the OP payments is referred to as *first life* and his or her partner is referred to as *second life*.

We start here with NP on an funded basis and where OP is accrued, i.e. the age of 67 is reached. With this type of pension plan a yearly NP will be

paid to the second life after the first life has died. The second life receives NP for the rest of his/her life as if the OP was fully funded. In comparison with OP the PV of the NP is dependent on the year of death on the condition that the partner is still alive instead of the retirement age. For a pension fund the present value of this kind of pension plan is

$$PV(OP + 70\% \cdot NP_{f,a}) = PV(OP) + 70\% \cdot PV(NP_{f,a}) \tag{53}$$

$$= PV(OP) + 70\% \cdot \sum_{x \geq 0} \left[\sum_{k \geq 0} {}_kq_{x,0} \cdot {}_kp_{y,0} \cdot v_k^k \right] \cdot n_{x,0} \tag{54}$$

and $PV(OP)$ is defined as in equation 52. The subscript x refers to the first life and y to the survival probability of the second life. We assume that both lifes are of the same age, so $x = y$ and that the second life is of the other sex. Here again it is necessary to distinguish between NP that is related to men and women, since their mortality rates significantly differ.

Partner's pension on a risk basis is paid if and only if the first life is in active service of employment. The right to partner's pension is therefore only valid during the next year. The first life's probability of dying within the coming year and the probabilities of survival for the partner are therefore needed to compute the present value.

$$PV(OP + 70\% \cdot NP_{r,a}) = PV(OP) + 70\% \cdot \sum_{x \geq 0} \left[q_{x,0} \sum_{k \geq 0} {}_{\cdot k}p_{y,0} \cdot v_k^k \right] \cdot n_{x,0} \tag{55}$$

Projected OP means that the retirement age has not been reached yet, meaning that the maximum attainable years of employment is yet to be reached. A factor is needed to account for the amount of NP that will be paid in case of decease as the OP that has not attained his (maximum) value yet and so will be the amount of NP. The ratio between the two options:

$$R_{p/a} = \frac{\text{projected}}{\text{accrued}} = \max\left(1, \frac{67 - 21}{x - 21}\right). \tag{56}$$

The maximum is attained at the retirement age of 67; the projected partner's pension will reach the amount of the accrued partner's pension. Assumed is that from the age of 21 to 67 onwards the pension is being funded. This has changed compared to the old calculation. In those days pension was funded between the minimum age of 25 and retirement age 65.

We obtain

$$PV(OP + 70\% \cdot NP_{r,p}) = PV(OP) + 70\% \cdot R_{p/a} \cdot NP_{r,r} \tag{57}$$

$$= PV(OP) + 70\% \cdot R_{p/a} \cdot \sum_{x \geq 0} \left[q_{x,0} \sum_{k \geq 0} {}_{\cdot k}p_{y,0} \cdot v_k^k \right] \cdot n_{x,0} \tag{58}$$

and

$$PV(OP + 70\% \cdot NP_{f,p}) = PV(OP) + 70\% \cdot \left[NP_{f,a} + (R_{p/a} - 1) \cdot NP_{r,a}\right] \tag{59}$$

$$= PV(OP) + 70\% \cdot \sum_{x \geq 0} \left[\sum_{k \geq 0} \left({}_k q_{x,0} + (R_{p/a} - 1) \cdot q_{x,0} \right) \cdot {}_k p_{y,0} \cdot v_k^k \right] \cdot n_{x,0}. \tag{60}$$

$PV(NP_{f,p})$ exists of a part that assumes the funded pension at this point in time. The second part compensates for the years to go until the retirement age in which the PV will increase.

The partner's pension adds some diversification advantage to the pension plan with old-age pension. A decrease in the mortality rate of the first life results in a higher OP since he/she is expected to live longer. It also results in a lower expected NP due to the same reason. A negative deviation in the mortality rate is correlated with a positive deviation in NP. This hedge reduced the uncertainty and therefore the confidence interval of the pension plan including partner's pension.

	Man	**Women**
OP	7,726	8,157
	[6,212 6,417 6,617]	[8,101 8,379 8,641]
$OP + 70\% \cdot NP_{f,p}$	9,378	9,486
	[8,812 9,007 9,192]	[9,228 9,458 9,672]
$OP + 70\% \cdot NP_{f,a}$	9,369	9,475
	[8,782 8,978 9,162]	[9,206 9,436 9,650]
$OP + 70\% \cdot NP_{r,p}$	7,741	8,178
	[6,264 6,470 6,670]	[8,138 8,417 8,679]
$OP + 70\% \cdot NP_{r,a}$	7,735	8,169
	[6,242 6,447 6,647]	[8,122 8,401 8,663]

Table 6: Present values for 5 types of pension plans for one 45-year old. First line: Estimation by AG projection table. Second line: Analytical [5 50 95] percentiles using Lee Carter.

The NP adds value to the pension plans. The present value of NP on a risk basis is not very large since it only pays in case of dead within the next year. This probability is, at least for a 45-year old, small. The payments are expected to be significant for the funded basis. However, by the diversification advantage, the total $OP + 70\% \cdot NP$ confidence interval becomes smaller. The difference with respect to the the AG projection table remain also for the pension plan with NP included.

Estimates can be added. Addition of intervals for men and women is only possible when the variation in Lee Carter is perfectly correlated for men and

women. It is thinkable that the deviation from the trend for the sexes is different, hence ϵ_θ and ϵ_σ are not the same for male and female mortality rates. We will assume they are both random and therefore simulations are required to determine the correct confidence interval for a couple, which will be smaller than the sum of the two.

4.3 TSO and Old Results

The difference between the 75% percentile and the best estimate for the present values of a pension plan determines the longevity risk (TSO). This proportion is interpreted as the percentage of the value of the liabilities that is required to cover for unforeseen events that lead to a shift in the mortality rate trend. Hence it is not to compensate for the expected mortality improvement which is incorporated in the present value of the insurance liabilities.

$$\text{TSO} = \frac{75\% \text{ percentile}}{\text{Best estimate}} - 1 \tag{61}$$

The TSO percentages in the table provided by DNB are rounded up.

Tables 7 and 8 show the TSO percentages within the old model for 45-year old men and women. The five types of pension schemes are considered. The TSO for the Lee Carter estimates are saved for the results section.

Men	Best estimate	75% percentile	TSO
OP	3,368	3,570	6,02%
$OP + 70\%NP_{f,p}$	4,710	4,893	3,89%
$OP + 70\%NP_{f,a}$	4,687	4,870	3,90%
$OP + 70\%NP_{r,p}$	3,420	3,622	5,95%
$OP + 70\%NP_{r,a}$	3,396	3,599	5,98%

Table 7: Present values annuities for 5 types of pension plans for one 45-year old men. The best estimate and 75% percentile, based on 10.000 simulations and the corresponding TSO for the old model.

Women	Best estimate	75% percentile	TSO
OP	4,053	4,251	4,88%
$OP + 70\%NP_{f,p}$	4,786	4,984	3,86%
$OP + 70\%NP_{f,a}$	4,782	4,968	3,87%
$OP + 70\%NP_{r,p}$	4,089	4,288	4,85%
$OP + 70\%NP_{r,a}$	4,073	4,271	4,87%

Table 8: Equivalent of table 7 for women.

Not just the present values but also in the solvency margin occur differences. Apparently whether OP is accrued or projected does not matter for the TSO, as is shown in the given table from [8] as well. The present value of a NP on risk basis is very small and therefore contributes nothing to the TSO. The addition of the partner's pension on accrual basis to the pension contract causes a lower

TSO than the OP on his own. Of course the present value of OP with NP is higher, but the TSO becomes smaller due to a diversification effect.

We should consider the differences in the mortality between men and women as we observe differences in the TSO. A larger share of male participants would lead to higher TSO percentages than for solely female participants. This is specifically noticeable for the young ages. For simplicity the participants in the model pension plan(s) are assumed to consist of as many men as women.

In the picture below are shown the TSO percentages for the 5 types of pension plan for an insured couple (a man and a woman) of the same age. From the age of 65 the remaining effect of the variation in mortality rates is as low that the type of pension plan does not matter any longer for the size of TSO.

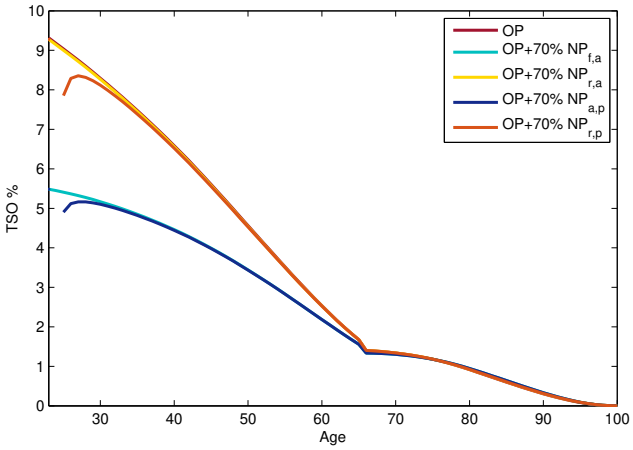


Figure 15: TSO percentages for an insured couple, to age and pension plan.

The percentages for the TSO were derived using the model by Van Dalen [22] from section 3.2. Unfortunately, the available documentation at DNB provides us with inconclusive information, but we were able to reproduce the original TSO table with small differences. A copy of the table by [8] is given here, see table 9, in which all the red numbers indicate a 1% higher value than our recalculations. In the original calculations there appears to be used a different discount rate than the term structure of 2002, which accounts at least for part of the differences compared to the former calculations.

Age	OP	$OP + 70\%NP_{f,p}$	$OP + 70\%NP_{f,a}$	$OP + 70\%NP_{r,p}$	$OP + 70\%NP_{r,a}$
30	10%	6%	6%	9%	10%
35	9%	6%	6%	9%	9%
40	8%	5%	5%	8%	8%
45	7%	5%	5%	6%	7%
50	5%	4%	4%	5%	5%
55	4%	3%	3%	4%	4%
60	3%	3%	3%	3%	3%
65	2%	2%	2%	2%	2%
70	2%	2%	2%	2%	2%
75	2%	2%	2%	2%	2%
80	2%	2%	2%	2%	2%
85	1%	1%	1%	1%	1%
90	1%	1%	1%	1%	1%

Table 9: TSO (percentages) to age and pension plan from [8]. Red marked values indicate an one percent difference to the results of the described method.

5 Discussions

5.1 Risk Identification

The uncertainty concerning mortality rate forecasting distinguishes different types. We focus on the identification of longevity risk. Here we discuss all risks concerning mortality modeling.

Risk can be divided into two types which both must be included in a proper forecast [4]: Systematic and unsystematic mortality risk. Systematic mortality risk is the type of risk that effects all individuals in the same way and is therefore undiversifiable. Trend risk is typically systematic risk. Unsystematic mortality risk occurs even if the true progress of mortality is known, since the number of deaths will be random due to individual volatility. The larger the population the smaller this type of risk becomes due to diversification.

1. Model risk

The choice of the correct model is of importance of the modeling of the (projected) mortality rates. It is impossible to know whether the selected model reflects the real world dynamics correctly. The most important dynamics must be accounted for while maintaining an understandable and appealing form. Model risk also applies to the choice of risk factors included in the model.

2. Parameter risk

Parameter risk is induced by the uncertainty around a parameter. The underlying data set affects the results of the parameters, as we have seen. The data set needs to be representative and large enough to determine the parameters properly. Preferably the estimated parameters are robust to the scale and richness of the data set. This type of risk is therefore diversifiable in such a way that the mis-estimation of the parameters can be minimized by using a proper data set [18]. Implementation of parameter risk can also be achieved by Bayesian estimation.

3. Trend risk

Trend risk is our main concern, it represents the longevity risk we strive to quantify. Trend risk does not account for individual deviations from the trend, merely the yearly shocks that have their effect on the total population. As the trend is estimated by the parameter θ it is a form of parameter risk. By the inclusion of the two randomizing terms in the forecasting structure of the Lee Carter model we account for trend (θ) risk and for the risk concerning (the forecasting model of) the parameter κ_t.

4. Basis risk

Basis risk occurs if the given model does not reflect to the actual properties of the portfolio. For example, it is necessary to distinguish between men and women considering their distinct mortality improvements. The S6 covers for risks for

a model pension fund, which is applicable to any other pension fund under the general assumptions. Fund specific information can be used to determine the longevity risk more accurately and hence to minimize the basis risk.

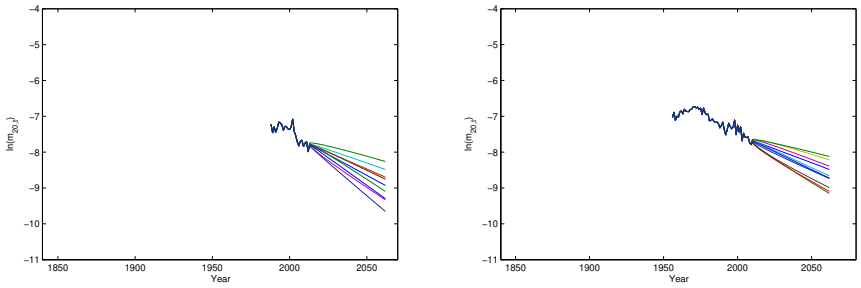
5. Volatility risk

Even if everything is know about the past and we are able to predict the future with the correct model and parameters, still then future individual deviations happen by unforeseen events since the future is uncertain by definition. Volatility risk covers for the individual stochastic deviations and is therefore diversifiable. The binomial approximation over the AG projection table in appendix F provides a good example of this type of risk.

5.2 Robustness to Data

Until this point we have seen that not only the model but the data set (CBS or HMDB) effects the forecast as well. The next two examples provide again different forecasts while maintaining the Lee Carter model. The underlying data sets have been rightfully changed.

The first example makes use of only the last years from the data set to forecast mortality rates. Considering that the last years are the most reliable years to determine the near future. We again will forecast until 2063, that is for the HMDB data a projection of 53 years and for the CBS data it is a projection of 50 years. Therefore we choose to use a calibration of the last 53 years from HMDB. For CBS we estimate from 1988 onwards; a convenient choice since the AG projection table makes use of it as well.



(a) CBS data from 1988

(b) HMDB data from 1956

Figure 16: Projected mortality rates by Lee Carter for a 20 year old man, with a shorter calibration period.

For the CBS case it seems that the new estimated trend over the later years is steeper: The mortality improvement is higher than in the 1950's an 1960's. This is the effect from a $\widehat{\theta}$ that is very sensitive to the calibration period since it is only estimated by κ_1 and κ_T. The expected mortality rate in 2063 in figure 12a is -8.393, in figure 16a it is -8.878 and the AG table provides a value

of $ln(m_{20,2063}) = -9.034$ (6). Our new estimation comes a lot closer to the prediction value from the AG.

The variance over the data from 1988 until 2012 is relatively smaller due to the fact that this trend seems to be more accurate in describing the data in the later years. However, the number of data points is much less such that the future of the mortality improvement has more uncertainty. In the short run this overrules the effect of the smaller variance and results in a more insecure forecast.

Figure 4 and tables 2 and 3 explain this observation. As the mortality rates in the early years seem to rise, the recent years show a downward trend. If only the data from 1988 onwards is used we end up with different mortality rates than for data over a longer time. The mortality improvement in the later years is stronger resulting in a higher forecast for the survival probabilities. Hence we expect more people to survive per year. We also see that the mortality rates for the older generation have this effect in a greater extent than the younger generation. Actually the expected number of men aged 40 (2014) surviving until 2063 - using Lee Carter for all CBS data - is almost half of what we expect as we use the AG projection table that makes use from 1988 onwards.

For woman there is this difference between the mortality improvement of the early and later observations is not significant for the young ages.

In the example from HMDB the early years include some extremes, such as the Second World War, those are now excluded. This causes the variation to be smaller. By absence of such outliers a trend is better fitted for the data.

Another point of view is the removal of those outliers and maintaining the rest. In this example we delete the years 1918 and 1940-1945 from the data such that the trend would fit the data better. Now obviously the variation around the predicted trend is less, but not as tight as in figure 16 that makes use of years from 1956 onwards. The results for the other age categories and females match with observation.

One might argue that a world war will not occur anymore or that an event similar to the Spanish flu is handled by the variation anyway. On the other hand, we can never completely exclude certain events. Catastrophes, think about a deadly virus for example, are not foreseeable. Deleting years results in a more precise forecast, while the original data set (most likely) leads to a more realistic one.

The model choice by the AG is based on the fact that the recent years have a significant different trend than the preceding observations and that this recent trend will set through the next years. However, it seems to be inconsistent with regard to convergence towards the long term trend. Suppose there are some realizations that exactly match the predictions and the technique would be applied again. The long term trend, estimated over the 1988 and the data point from 2016, results in a stronger trend than the *ltt* from 12. As the *stt* again dominates the early predictions, the *ltt* will eventually approach the *stt*. In contrast to the purpose of this modeling technique, the long term trend actually converge to the short term trend that is estimated from 2002 onwards. This technique might therefore be too shortsighted as it ignores a big part of

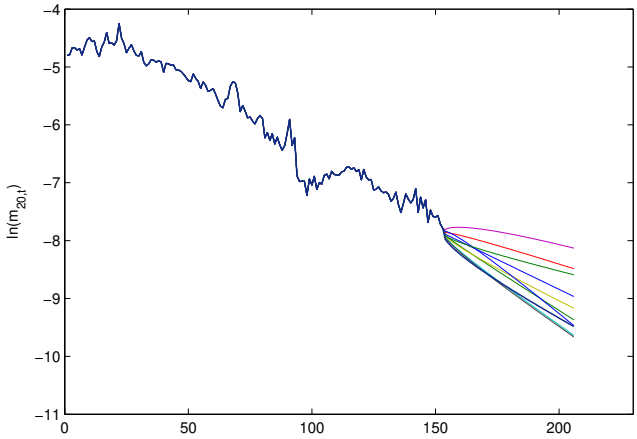


Figure 17: Projected mortality rates by Lee Carter for a 20 year old man, Spanish flu and Second World War deleted.

the data.

Concluding, selecting data comes down to personal opinion. Recent years show a different trend compared to the last 50 or 100 years. More data usually results in better estimations and hence forecasts. However, recent development might be more reliable for estimating the nearby future.

5.3 Discussion on Old TSO Model

First we look at the performance of the old TSO model over the 10 years. Secondly, we have applied the old TSO model according to the documentation, though we question some assumptions on which the model is based and find that the old model overestimates the longevity risk.

5.3.1 Performance over 10 Years

How did the model actually perform when we compare it to 10 years of new information? Now that CBS data is available until 2012 we are able to look at the performance of the model with respect to the forecasting of the death rates $q_{x,t}$. It is also possible to forecast from 2012 onwards and observe the consequences of new information. We use the model described earlier with updated data: An extra 10 years of mortality rate data, ages 0 until 125 such that people are allowed to live after the age of 100 and the extrapolation is performed from the CBS 2012 values. Mortality improvement after the age of 100 is zero. We also assume that the new data does not affect the rate of mortality improvement ρ.

Interesting to see is that the model underestimated the mortality improvement quite severe in the years 2002 to 2012. If this trend would set trough, pension funds will suffer from a lot of unforeseen payments. Next, we observe that the forecast is very dependent on the last data point. Suppose the extrapolation started at the 2011 point, the death rate prediction would also be much lower. This indicates a pitfall to the robustness of this method.

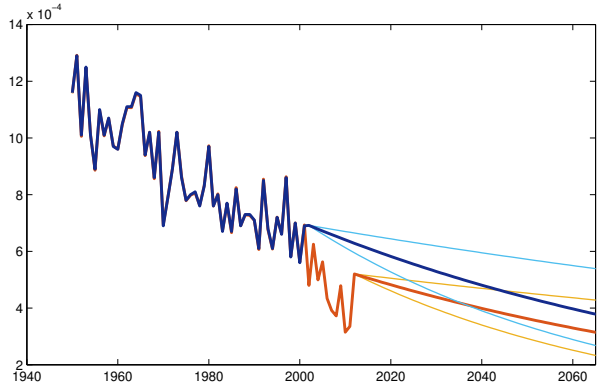


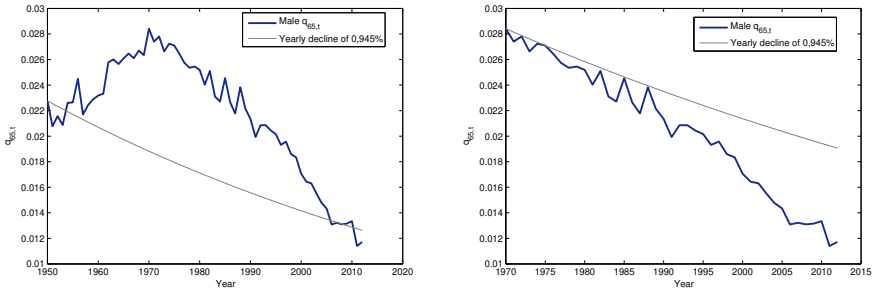
Figure 18: Forecasted mortality rate (old model) for a 25 year old man: The best estimate and the 25% and 75% percentiles. The red forecast is computed with 10 years extra data.

The question arises whether a fixed mortality improvement of 0,945% for all ages is realistic. In figure 19a and 19b is shown what this assumption would have done with past data for a 65-year old. If we take 1950 as basis and reduce the value of $q_{65,1950}$ every year by 0,945% then in 2012 the predicted value is almost the same as the data provided by the CBS. For intermediate values however it appears not to be a good estimate since it underestimates the mortality rates a lot. The factor to fit the first and last observations would be

$$\widetilde{\rho}_{x=65} = 1 - \sqrt[62]{\frac{q_{65,2012}}{q_{65,1950}}} \approx 1.07\%. \tag{62}$$

Another possibility is to take 1970 - as it is the maximum value - as a basis. In that case a yearly 0,945% decline overestimates the rate of mortality (underestimates the probability of survival) in every year. The mortality improvement in this time interval is approximately 2,09%. Hence we might underestimate the size of ρ for a 65-year old, but based on all provided data it seems a good estimate.

Following this approach we compare the results with the estimated mortality probabilities given by the AG projection table. The short and long term trend represent respectively 3,89% and 3,15% per year for a 65-year old. Female mortality improvement has according to the AG a *stt* of 2,06% and *ltt* of 1,20% which is also higher than the assumed $\mu_\rho = 0,575\%$.



(a) (b)

Figure 19: Process of $q_{65,t}$ (CBS) with begin point 1950(a) and 1970(b) against a yearly decline of 1%

According to the table provided by the AG the old model underestimates the probability of survival. The assumption made about the mortality improvement ρ with fixed mean μ_ρ and standard deviation σ_ρ is restrictive.

The average mortality improvement in figure 20 over the years 1950 to 2012 suspects that ρ is not fixed for all age. A better specified mortality improvement can be attained by introducing ρ_x that is dependent on the age x, with age-specific mean and standard deviation. It is estimated by

$$\widehat{\rho}_x = 1 - \sqrt[62]{\frac{q_{x,2012}}{q_{x,1950}}}. \tag{63}$$

The resulting graph is not smooth due to the estimation based on solely the

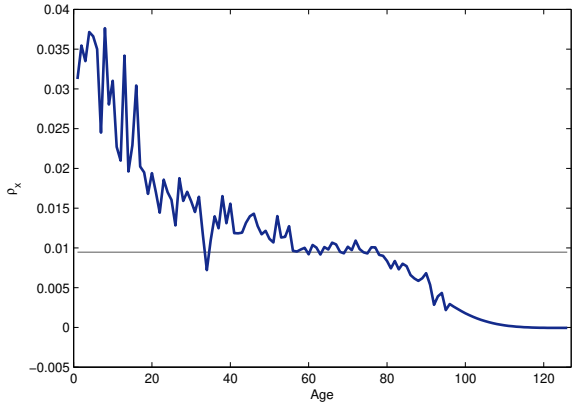


Figure 20: Estimated ρ_x for man against the given estimator of 0,945%.

first and last observation. For the ages 60 to 80 the assumption of a mortality

improvement of $0,945\%$ looks most reasonable. But especially for the young ages the mortality improvement is higher, which has great effect on the expected life expectancy and the present values of the pensions, see table 5.

5.3.2 Variation Assumptions

The incorporation of uncertainty in the old TSO model has been done by the assumption that $\rho \sim N(\mu_\rho, \sigma_\rho^2)$, such that the s-year forward expectation and variance equal

$$\mathbb{E}[q_{x,T+s}] = q_{x,T} \cdot \mathbb{E}[(1-\rho)^s] = q_{x,T} \cdot (1-\mu_\rho)^s \tag{64}$$

and

$$\begin{aligned}
\text{var}[q_{x,T+s}] &= \mathbb{E}[q_{x,T+s}^2] - \mathbb{E}[q_{x,T+s}]^2 && (65)\\
&= \mathbb{E}[q_{x,T}^2 \cdot (1-\rho)^{2s}] - q_{x,T}^2 \cdot \mathbb{E}[(1-\rho)^s]^2 && (66)\\
&= q_{x,T}^2 \cdot \left(\mathbb{E}[(1-\rho)^{2s}] - \mathbb{E}[(1-\rho)^s]^2\right) && (67)\\
&= q_{x,T}^2 \cdot \text{var}[(1-\rho)^s]. && (68)
\end{aligned}$$

which requires the knowledge of the noncentral moments of the normal distribution[5]. Given is that

$$\mathbb{E}[(\rho)^s] = \sum_{j=0}^{[s/2]} \binom{z}{2j} (2j-1)!! \cdot \sigma^{2j} \mu^{s-2j} \tag{69}$$

and

$$x!! = \prod_{i=o}^{[x/2]-1} (x-2i) \tag{70}$$

with

$$0!! = 1 \qquad \text{and} \qquad (-1)!! = 1. \tag{71}$$

Formula 69 can be substituted in equation 65 if it is written out in terms of $\mathbb{E}[(\rho)^s]$. It makes the variance hardly analytically tractable and therefore simulations are required to determine the confidence intervals of the distribution of $(1-\rho)^s$.

Moreover, the way of using the variation ϵ_ρ is restrictive. It is drawn once for one forecasting path. We think however that a variation of the mortality improvement μ_ρ is not fixed over time, but rather $\epsilon_{\rho,t}$ is more appropriate and hence must be *yearly* randomized for every year as

$$q_{x,T+s}^* = q_{x,T} \cdot (1 - (\mu_\rho + \sigma_\rho \cdot \epsilon_{\rho,T+1})) \cdots (1 - (\mu_\rho + \sigma_\rho \cdot \epsilon_{\rho,T+s}))\,. \tag{72}$$

The old model also assumes that both male and female mortality variations are perfectly correlated. This assumption is not necessarily valid. We will

[5] http://www.johndcook.com/blog/2012/11/06/general-formula-for-normal-moments/

assume the same as for the Lee Carter model: No correlation between male and female ϵ_ρ.

The two new properties to lead to a lower variation, hence a lower TSO. Compare figure 21 to 7. All updated TSO solvency margins from the table correspond a value of 1% (rounded to above).

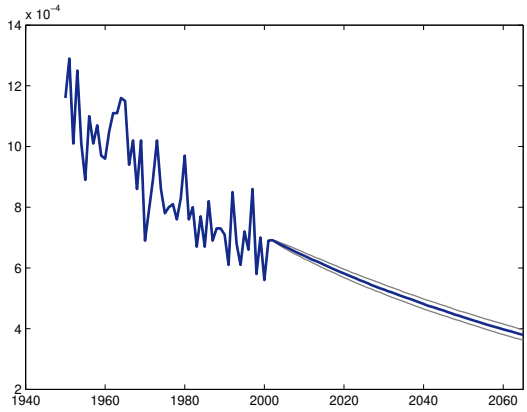


Figure 21: Forecasted mortality rate for a 25 year old man: The best estimate and the 25% and 74% percentiles. With renewed variation assumptions on the old TSO model. Based on 10.000 simulations.

5.4 Model Selection

All of the critique on the old TSO model asks for a more understandable model to determine mortality (improvement) rates: The Lee Carter model.

The challenge is to select the right data and modeling techniques such that we obtain the best quantification of longevity risk. Choices should be made that are acceptable within the context of mortality rates. The (forecasting) model should be consistent with historical trends and variability. The model we are looking for must be in correspondence with the most appealing results from the data, namely

1. For all ages mortality rates have declined;
2. At specific ages, improvement rates seem to have varied over time;
3. Improvement rates have been significantly different at different ages;
4. Mortality rates reveal significant volatility from one year to the next.

The Lee Carter model captures all those influential features and, compared to the old TSO model, it does not lead to problems regarding the normality

assumption in the mortality improvement (trend in κ_t) since it works on a log level.

The (updated) old model struggles with risk identification: It does not separate pure trend risk from yearly volatility and can therefore not identify the magnitude of the TSO correctly. In the Lee Carter model longevity risk is captured by trend risk and the yearly volatility induced by the estimation of the forecasting model for κ_t.

The biggest difference with the projection from the AG is that we (with the Lee Carter model) use all available CBS data, e.g. 1950-2012, and that we account for solely one persistent trend. We are also able to identify longevity risk, something that is not possible with the AG method (yet).

6 New TSO Results

A pension fund can cover for longevity risk if it holds the recommended TSO solvency margin. In this section the results of the Lee Carter approach are given.

In comparison with the old TSO method the Lee Carter method results in lower TSO values since the Lee Carter method identifies the trend and its variation better.

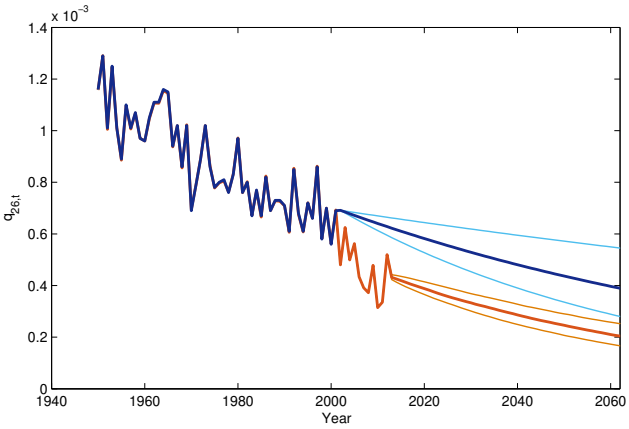


Figure 22: Forecasted mortality rate for a 25 year old man: The best estimate and the 25% and 75% percentiles. In blue the old method. The red forecast is computed with 10 years extra data and the Lee Carter method.

5.000 simulations were used to derive figure 23. It is assumed that the member's base consists of as many men as women. The variation around the trend for men and women is not correlated which means that intervals from table 5 can not simply be added.

The percentages in the new TSO table (10) can be used to determine appropriate solvency margins covering longevity risk. The TSO depends on the mean age of the participants in the pension fund and the type of pension plan. A complete table of unrounded TSO percentages can be found in appendix G.

We evaluated the TSO percentage for a model pension fund to show that the mean age can be used as an estimation for the total TSO. The model pension fund consists of 20 25-, 40 45-, 20 65- and 5 85-year old men and the same amount of women. The mean age within the fund is 47,4. We derive the TSO percentages with the Lee Carter for this fund as in table 11. These values are compared with the corresponding TSO values form the table at 47 years and to the weighted mean of the TSO values from the table.

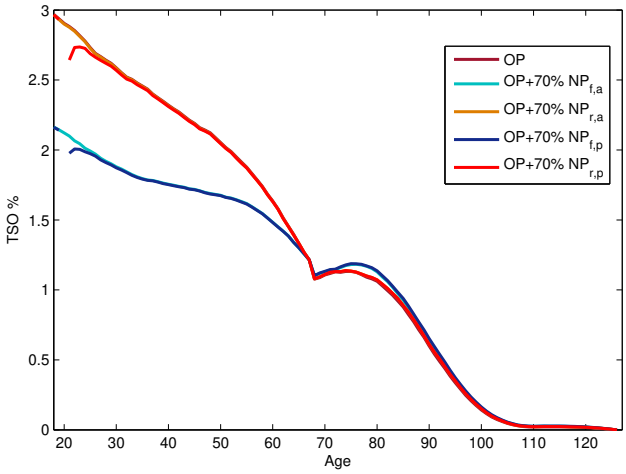


Figure 23: TSO percentages for an insured couple, to age and pension plan. With the new method.

Age	OP	$OP + 70\%NP_{f,p}$	$OP + 70\%NP_{f,a}$	$OP + 70\%NP_{r,p}$	$OP + 70\%NP_{r,a}$
30	3%	2%	2%	3%	3%
35	3%	2%	2%	3%	3%
40	3%	2%	2%	3%	3%
45	3%	2%	2%	3%	3%
50	3%	2%	2%	3%	3%
55	2%	2%	2%	2%	2%
60	2%	2%	2%	2%	2%
65	2%	2%	2%	2%	2%
70	2%	2%	2%	2%	2%
75	2%	2%	2%	2%	2%
80	2%	2%	2%	2%	2%
85	1%	1%	1%	1%	1%
90	1%	1%	1%	1%	1%

Table 10: TSO (percentages) to age and pension plan.

	OP	$OP + 70\%NP_{f,p}$	$OP + 70\%NP_{f,a}$	$OP + 70\%NP_{r,p}$	$OP + 70\%NP_{r,a}$
Lee Carter TSO	1,82 %	1,59 %	1,59 %	1,81 %	1,82 %
TSO (mean age)	2,12 %	1,69 %	1,69 %	2,11 %	2,12 %
Weighted TSO	2,06 %	1,61 %	1,61 %	2,06 %	2,06 %

Table 11: TSO percentages for a model pension fund.

7 Conclusion

The solvency margins for longevity risk that are the TSO percentages for the S6 within the new FTK are updated by using the Lee Carter model. The old method can not hold and the understandable and generally accepted Lee Carter model is selected for its qualities.

Powerful estimations suppress the uncertainties. Assumptions and predictions about 100 years in the future are by definition uncertain, even with rich knowledge of the past. Estimating the future mortality rates is one thing, but acknowledging the magnitude of the uncertainties is a second. It will be interesting to see the future developments of the AG projection table as it will predict the mortality rates and provide uncertainty intervals.

8 Possible Further Research

1. The model could be improved by including additional variables such as a second principal component (2SVD) or the cohort effect. The relevance of both effects should first be established for Dutch data.

2. Bayesian analysis seems a remarkable progress in identifying the parameter risk of mortality rates. This technique will result in an higher TSO (S6) as it will include more uncertainty within the model.

3. The pension fund specifications affect the magnitude of longevity risk. We have talked about age (of first and second life), sex, type of insurance, number of participants in a fund, salary (equivalently: amount of pension) and the interest rate structure. Each factor has its specific effect on the TSO. When assumptions are invalid that might have significant effect on the on the required solvability for a pension fund. It is possible to include more variables in the model.

4. The TSO defines together with the NSA the insurance technical risk (S6) within the new FTK. The uncertainty about the mortality improvement trend is covered by an solvability buffer TSO. The NSA is there to cover for volatility risk; negative variations that happen on a day-to-day basis. How this part of the S6 will be defined is yet uncertain. In appendix F is given based on the binomial approximation of the AG projection table values. It indicates the magnitude of volatility risk.

References

[1] Actuarieel Genootschap, Utrecht. *Prognosetafel AG2012-2062*, September 2012.

[2] Natacha Brouhns, Michel Denuit, and Jeroen K. Vermunt. Measuring the longevity risk in mortality projections. June 2002.

[3] Natacha Brouhns, Michel Denuit, and Jeroen K. Vermunt. A poisson log-bilinear regression approach to the construction of projected lifetables. *Insurance: Mathematics and Economics*, (31):373–393, 2002.

[4] Andrew J.G. Cairns. Modelling and management of mortality risk: A review. May 2008.

[5] George Lindsay Cairns. *A Bayesian approach to modeling mortality, with applications to insurance*. PhD thesis, University of Glasgow, 2013.

[6] R.D. Carter and L.R. Lee. Modeling and forecasting u.s. mortality. *Journal of the American Statistical Association*, 87:659–671, 1992.

[7] Claudia Czado, Antoine Delwarde, and Michel Denuit. Bayesian poisson log-bilinear mortality projections. September 2004.

[8] De Nederlandsche Bank (DNB), Amsterdam/Apeldoorn. *Advies inzake Onderbouwing Parameters FTK*, October 2006.

[9] Hans U. Gerber. *Life Insurance Mathematics*. Springer, third edition, 1997.

[10] Federico Girosi and Gary King. Understanding the lee-carter mortality forecasting method. September 2007.

[11] Yili Hong. On computing the distributuion function for the sum of independent and non-identical random indicators. Technical report, Virginia Tech, Blacksburg, VA 24061, USA, April 2011.

[12] V. Kannisto. *Development of Oldest-Old Mortality, 1950-1990: Evidence from 28 Developed Countries*. Odense Monografhs on Population Aging. 1. Odense University Press, Odense, Denmark, 1994.

[13] Marie-Claire Koissi and Arnold F. Shapiro. The lee-carter model under the condition of variables age-specific parameters. August 2008.

[14] WJ Nusselder and JP Mackenbach. Lack of improvement of life expectancy at advanced ages in the netherlands. *International Journal of Epidemiology*, 29:140–148, 2000.

[15] Pensioen- en Verzekeringskamer. *Consultatiedocument Financieel Toetsingskader*, October 2004.

[16] A.E. Renshaw and S. Haberman. Lee-carter mortality forecasting with age-specific enhancement. *Insurance: Mathematics and Economics*, (33):255–272, 2003.

[17] A.E. Renshaw and S. Haberman. A cohort-based extension to the lee-carter model for mortality reduction factors. *Insurance: Mathematics and Economics*, (38):556–570, 2006.

[18] S. J. Richards. Mis-estimation risk: Measurement and impact. *Longevitas Ltd.*, 2014.

[19] Tim Schulteis. Master thesis - actuarial sciences. Master's thesis, Maastricht University, August 2009.

[20] S. Tuljapurkar, N. Li, and C. Boe. A universal pattern of mortality decline in the g 7 countries. *Nature*, (405):789–792, 2000.

[21] D. van Bodegom, L. Bonneux, F.M. Engelaer, J. Lindenberg, J.J. Meij, and R.G.J. Westendorp. *Dutch life expectancy from an international perspective*. Leyden Academy, October 2010.

[22] Arjan van Dalen. Opslag voor het trendrisico sterfte in het ftk tabellenboek verklaard. *De Actuaris*, January 2005.

[23] Ministerie van Sociale Zaken en Werkgelegenheid. *The old age pension system in the Netherlands*. Rijksoverheid, June 2008.

[24] J.R. Wilmoth. Computational methods for fitting and extrapolating the lee-carter model of mortality change. Technical report, Department of Demography, University of California, Berkeley, 1993.

A The S6

The Financial Assessment Framework (FTK) dating from 2007 demands that pension funds take longevity risk into account by insuring a financial buffer. The TSO, element of the S6, that has been established by DNB prescribes the volume of this buffer.

Two documents survey basic information about the goals, the underlying reasoning and the proposed S6 percentages. These are *Consultatiedocument Financieel Toetsingskader* [15] and *Advies Inzake Onderbouwing Parameters FTK* [8] that proposes modifications to [15].

A.1 Position of Insurance Technical Risk within the FTK

According to DNB, the required solvability for a pension fund must prevent that the fund can not longer fulfil its obligations when unforeseen events occur. It is based on the idea that a pension fund should have a capital requirement such that it holds enough capital to provide protection for the company to cover all liabilities in the following year at a 97,5% confidence level. This level corresponds to a 75% confidence level on the provisions regarding insurance technical risk.

The total financial risk for a pension fund is differentiated into six risk drivers. Those six pillars define the required solvability.

S1 Interest risk and inflation risk

S2 Equity and real estate risk

S3 Currency risk

S4 Commodities risk

S5 Credit risk

S6 Insurance technical risk

For each of these a calculation is made to establish the required buffer capital. The cumulative value of these amounts in addition to the fair value of the current liabilities is the capital that is required to maintain the solvency margin.

The six pillars of risk are combined using the formula

$$\text{Total required solvability} = \sqrt{S1^2 + S2^2 + 2 \cdot \rho \cdot S1 \cdot S2 + S3^2 + S4^2 + S5^2 + S6^2}. \tag{73}$$

The factors are independent of each other (are fully diversified), therefore the total required solvability is lower than the sum over the individual risk pillars. Assumed is some correlation between S1 and S2; ρ is 0,65 for a pension fund.

A.2 Standard Calculations for Fund Specific S6

Ideally, the calculations to determine the S6 exactly reflect the risk profile of the pension fund and this asks for an investment in a intern model. DNB constructed a model in 2004 based on a model pension fund. It was revised in 2006 and has not been updated afterwards. With the help of this model DNB provided tables for a standard S6 value.

The model from DNB has as its dependent variables the retirement age, the average age within the pension fund and it distinguishes between types of pension plan. Next to this it sets a minimum to the number of participants within the fund to use the S6 table as representable values.

The formula that determines the required solvability for insurance technical risk is dependent on the number of participants in the pension fund n by

$$\text{S6} = \frac{c_1}{\sqrt{n}} + \frac{c_2}{n} + \sqrt{TSO^2 + \left(\frac{NSA}{\sqrt{n}}\right)^2}, \tag{74}$$

where TSO is the longevity risk and NSA covers for negative stochastic deviations.

The first part of the formula is defined as process (or avoidable) risk

$$\text{Process risk} = \frac{c_1}{\sqrt{n}} + \frac{c_2}{n}, \tag{75}$$

where c_1 is the base percentage and c_2 a term that corrects for the skewness of that distribution.

The S6 is a percentage of the technical provisions. A lower average age requires a higher S6, since the uncertainty of the assumptions regarding the future have more effect.

Due to unsystematic risk, included in the NSA, the expected pension payments from a pension fund to its members vary more in a smaller fund than in a fund with a lot of people. Because of this a smaller fund requires a higher S6 to cover this risk.

In the calculations for S6 there is distinguished between old-age pension (OP) without partner's pension (NP) and OP with NP in four types with differences in financing method (risk or funded basis) and magnitude of partner's pension (NP accrued or projected). In the types with partner's pension a diversification advantage occurs since a negative deviation in the mortality rate is correlated with a positive deviation in NP. This hedge reduces the risk and therefore the S6.

The values for the parameters c_1, c_2, TSO and NSA as well as the minimum requirement of n can be found in five tables, we refer to pages 35-37 in [8]. The tables consist of values arranged by age cohort of 5 years and type of pension plan. The average age of the members from a fund should be used to read the table and intermediate values require linear interpolation.

When multiple types of pension plans are present, the values for separate

pension plans (numbered i=1,...,5) can be added as follows[6]:

$$\text{Process risk} = \sqrt{\sum_i \text{Process risk}_i^2} \tag{76}$$

$$TSO = \sum_i TSO_i \tag{77}$$

$$NSA = \sqrt{\sum_i NSA_i^2} \tag{78}$$

and should be implemented to equation 74 to derive the S6.

[6] http://www.toezicht.dnb.nl/2/50-202316.jsp

B stt_x and ltt_x

Age x	stt_x	ltt_x
0	0,966	0,972
1	0,963	0,965
2	0,943	0,948
3	0,957	0,969
4	0,927	0,953
5	0,957	0,960
6	0,987	0,940
7	0,946	0,963
8	0,938	0,950
9	1,037	0,979
10	0,968	0,985
11	0,925	0,964
12	0,967	0,977
13	0,953	0,978
14	0,979	0,975
15	0,937	0,976
16	1,004	0,992
17	0,948	0,968
18	0,945	0,978
19	0,920	0,967
20	0,933	0,979
21	0,977	0,984
22	0,967	0,973
23	0,959	0,967
24	0,955	0,983
25	0,961	0,967
26	0,949	0,973
27	0,974	0,971
28	0,992	0,976
29	0,984	0,987
30	0,960	0,985
31	0,951	0,973
32	0,980	0,982
33	0,935	0,973
34	0,991	0,983
35	0,950	0,974
36	0,954	0,974
37	0,974	0,987
38	0,988	0,988
39	0,963	0,986
40	0,966	0,986
41	0,969	0,976
42	0,985	0,987
43	0,940	0,967
44	0,965	0,977
45	0,957	0,972
46	0,963	0,977
47	0,961	0,984
48	0,970	0,983
49	0,968	0,985
50	0,967	0,976
51	0,974	0,980
52	0,968	0,976
53	0,970	0,973
54	0,976	0,978
55	0,957	0,978
56	0,982	0,982
57	0,974	0,979
58	0,973	0,974
59	0,983	0,979
60	0,961	0,970
61	0,967	0,971
62	0,966	0,972
63	0,963	0,972
64	0,965	0,975
65	0,961	0,968
66	0,977	0,976
67	0,959	0,971
68	0,956	0,970
69	0,961	0,973
70	0,956	0,973
71	0,956	0,971
72	0,958	0,972
73	0,955	0,972
74	0,955	0,973
75	0,960	0,976
76	0,963	0,978
77	0,963	0,977
78	0,959	0,977
79	0,966	0,980
80	0,965	0,982
81	0,970	0,983
82	0,976	0,985
83	0,972	0,985
84	0,966	0,985
85	0,969	0,988
86	0,972	0,988
87	0,972	0,988
88	0,973	0,991
89	0,984	0,998
90	0,981	0,995
91	0,980	0,995
92	0,991	0,998
93	0,990	0,998
94	0,985	0,998
95	0,989	1,000
96	0,990	1,001
97	0,991	1,001
98	0,993	1,002
99	0,994	1,003
100	0,995	1,003
101	0,996	1,003
102	0,996	1,004
103	0,997	1,004
104	0,998	1,004
105	0,998	1,004
106	0,999	1,004
107	0,999	1,004
108	0,999	1,004
109	1,000	1,004
110	1,000	1,004
111	1,000	1,004
112	1,000	1,004
113	1,000	1,004
114	1,001	1,004
115	1,001	1,004
116	1,001	1,003
117	1,001	1,003
118	1,001	1,003
119	1,001	1,003
120	1,001	1,003

Table 12: The short term trend and long term trend by calculations 11 and 12.

C Values for α_x

Age x	Male	Female
0	-4,716	-4,971
1	-6,405	-6,587
2	-7,219	-7,452
3	-7,486	-7,819
4	-7,745	-8,066
5	-7,949	-8,263
6	-7,989	-8,435
7	-8,092	-8,559
8	-8,183	-8,604
9	-8,268	-8,695
10	-8,326	-8,704
11	-8,372	-8,748
12	-8,309	-8,615
13	-8,202	-8,496
14	-8,112	-8,480
15	-8,019	-8,424
16	-7,645	-8,260
17	-7,447	-8,168
18	-7,326	-8,141
19	-7,195	-8,054
20	-7,155	-8,085
21	-7,134	-8,056
22	-7,131	-8,002
23	-7,143	-8,011
24	-7,137	-7,973
25	-7,182	-7,939
26	-7,155	-7,869
27	-7,181	-7,835
28	-7,178	-7,800
29	-7,144	-7,688
30	-7,080	-7,644
31	-7,072	-7,533
32	-7,028	-7,494
33	-6,990	-7,391
34	-6,916	-7,303
35	-6,881	-7,250
36	-6,785	-7,152
37	-6,729	-7,060
38	-6,648	-6,997
39	-6,546	-6,895
40	-6,447	-6,781
41	-6,367	-6,678
42	-6,273	-6,608
43	-6,141	-6,487
44	-6,055	-6,391
45	-5,939	-6,294
46	-5,838	-6,187
47	-5,729	-6,082
48	-5,624	-6,005
49	-5,511	-5,902
50	-5,408	-5,823
51	-5,292	-5,740
52	-5,190	-5,629
53	-5,084	-5,565
54	-4,975	-5,490
55	-4,871	-5,405
56	-4,769	-5,320
57	-4,660	-5,226
58	-4,551	-5,150
59	-4,461	-5,058
60	-4,354	-4,968
61	-4,260	-4,866
62	-4,156	-4,788
63	-4,053	-4,685
64	-3,958	-4,589
65	-3,859	-4,493
66	-3,758	-4,386
67	-3,665	-4,282
68	-3,565	-4,176
69	-3,469	-4,073
70	-3,383	-3,966
71	-3,288	-3,852
72	-3,184	-3,739
73	-3,085	-3,628
74	-2,991	-3,511
75	-2,894	-3,393
76	-2,794	-3,278
77	-2,703	-3,160
78	-2,607	-3,041
79	-2,510	-2,926
80	-2,421	-2,810
81	-2,329	-2,685
82	-2,237	-2,569
83	-2,151	-2,461
84	-2,050	-2,351
85	-1,971	-2,240
86	-1,881	-2,130
87	-1,798	-2,026
88	-1,711	-1,925
89	-1,630	-1,824
90	-1,556	-1,735
91	-1,484	-1,648
92	-1,409	-1,553
93	-1,333	-1,472
94	-1,257	-1,398
95	-1,209	-1,313
96	-1,150	-1,240
97	-1,095	-1,171
98	-1,042	-1,106
99	-0,994	-1,046
100	-0,948	-0,990
101	-0,905	-0,938
102	-0,866	-0,890
103	-0,829	-0,846
104	-0,796	-0,806
105	-0,765	-0,770
106	-0,736	-0,736
107	-0,710	-0,706
108	-0,686	-0,679
109	-0,664	-0,655
110	-0,644	-0,633
111	-0,626	-0,613
112	-0,610	-0,596
113	-0,595	-0,580
114	-0,582	-0,566
115	-0,569	-0,554
116	-0,558	-0,543
117	-0,548	-0,533
118	-0,539	-0,525
119	-0,531	-0,517
120	-0,524	-0,510
121	-0,517	-0,504
122	-0,512	-0,499
123	-0,506	-0,494
124	-0,501	-0,490
125	-0,497	-0,486

Table 13: Estimated values for α_x (Lee Carter model) for CBS data.

Age x	Male	Female
0	-2,919	-3,148
1	-4,740	-4,835
2	-5,400	-5,555
3	-5,760	-5,941
4	-6,034	-6,193
5	-6,211	-6,422
6	-6,367	-6,608
7	-6,512	-6,742
8	-6,653	-6,873
9	-6,776	-6,940
10	-6,839	-7,010
11	-6,904	-7,012
12	-6,892	-6,976
13	-6,855	-6,915
14	-6,754	-6,826
15	-6,633	-6,729
16	-6,411	-6,616
17	-6,267	-6,573
18	-6,123	-6,522
19	-5,995	-6,498
20	-5,930	-6,457
21	-5,893	-6,452
22	-5,906	-6,407
23	-5,922	-6,383
24	-5,941	-6,360
25	-5,953	-6,294
26	-5,962	-6,284
27	-5,982	-6,227
28	-5,976	-6,178
29	-5,958	-6,138
30	-5,934	-6,101
31	-5,935	-6,038
32	-5,888	-5,983
33	-5,851	-5,931
34	-5,820	-5,882
35	-5,768	-5,841
36	-5,714	-5,779
37	-5,677	-5,734
38	-5,612	-5,668
39	-5,539	-5,614
40	-5,483	-5,562
41	-5,442	-5,530
42	-5,350	-5,468
43	-5,281	-5,427
44	-5,205	-5,391
45	-5,116	-5,335
46	-5,063	-5,269
47	-4,981	-5,217
48	-4,898	-5,153
49	-4,831	-5,083
50	-4,737	-5,008
51	-4,671	-4,946
52	-4,574	-4,859
53	-4,490	-4,807
54	-4,404	-4,727
55	-4,338	-4,663
56	-4,243	-4,575
57	-4,160	-4,500
58	-4,061	-4,419
59	-3,996	-4,336
60	-3,892	-4,234
61	-3,815	-4,153
62	-3,724	-4,059
63	-3,624	-3,958
64	-3,537	-3,857
65	-3,444	-3,768
66	-3,355	-3,669
67	-3,268	-3,577
68	-3,176	-3,464
69	-3,087	-3,373
70	-2,988	-3,262
71	-2,909	-3,175
72	-2,796	-3,053
73	-2,704	-2,954
74	-2,609	-2,846
75	-2,515	-2,741
76	-2,427	-2,644
77	-2,330	-2,540
78	-2,245	-2,437
79	-2,153	-2,339
80	-2,047	-2,239
81	-1,976	-2,155
82	-1,877	-2,043
83	-1,785	-1,937
84	-1,692	-1,846
85	-1,623	-1,754
86	-1,513	-1,660
87	-1,440	-1,568
88	-1,362	-1,480
89	-1,283	-1,418
90	-1,204	-1,313
91	-1,135	-1,259
92	-1,055	-1,138
93	-1,004	-1,077
94	-0,918	-0,996
95	-0,831	-0,947
96	-0,820	-0,885
97	-0,733	-0,770
98	-0,612	-0,716
99	-0,495	-0,557
100	-0,178	-0,284
101	-0,190	-0,264
102	-0,077	-0,165
103	0,011	-0,103
104	-0,022	-0,082
105	-0,025	-0,086
106	0,000	-0,084
107	0,059	0,018
108	0,072	0,022
109	0,011	0,061
110	0,013	0,022

Table 14: Estimated values for α_x (Lee Carter model) for HMDB data.

D Values for β_x

Age x	Male	Female
0	0,0213	0,0194
1	0,0231	0,0237
2	0,0248	0,0247
3	0,0256	0,0237
4	0,0271	0,0245
5	0,0285	0,0254
6	0,0275	0,0239
7	0,0279	0,0228
8	0,0257	0,0232
9	0,0257	0,0208
10	0,0235	0,0185
11	0,0231	0,0171
12	0,0195	0,0169
13	0,0178	0,0131
14	0,0168	0,0154
15	0,0185	0,0137
16	0,0149	0,0131
17	0,0136	0,0124
18	0,0126	0,0089
19	0,0130	0,0100
20	0,0107	0,0103
21	0,0103	0,0116
22	0,0117	0,0083
23	0,0112	0,0119
24	0,0099	0,0110
25	0,0112	0,0126
26	0,0108	0,0093
27	0,0107	0,0111
28	0,0094	0,0108
29	0,0095	0,0108
30	0,0081	0,0104
31	0,0094	0,0097
32	0,0082	0,0109
33	0,0082	0,0096
34	0,0083	0,0095
35	0,0082	0,0098
36	0,0087	0,0093
37	0,0083	0,0092
38	0,0078	0,0085
39	0,0086	0,0089
40	0,0085	0,0086
41	0,0085	0,0072
42	0,0084	0,0074
43	0,0088	0,0074
44	0,0092	0,0066
45	0,0091	0,0065
46	0,0093	0,0067
47	0,0091	0,0056
48	0,0086	0,0057
49	0,0092	0,0059
50	0,0084	0,0058
51	0,0091	0,0055
52	0,0089	0,0056
53	0,0092	0,0053
54	0,0096	0,0055
55	0,0092	0,0056
56	0,0090	0,0058
57	0,0089	0,0059
58	0,0091	0,0060
59	0,0090	0,0063
60	0,0089	0,0061
61	0,0090	0,0064
62	0,0085	0,0071
63	0,0085	0,0074
64	0,0079	0,0080
65	0,0079	0,0084
66	0,0077	0,0086
67	0,0076	0,0091
68	0,0073	0,0092
69	0,0072	0,0096
70	0,0068	0,0100
71	0,0065	0,0100
72	0,0065	0,0104
73	0,0060	0,0104
74	0,0059	0,0106
75	0,0057	0,0107
76	0,0055	0,0106
77	0,0052	0,0105
78	0,0048	0,0103
79	0,0046	0,0104
80	0,0045	0,0097
81	0,0042	0,0097
82	0,0041	0,0091
83	0,0040	0,0088
84	0,0038	0,0082
85	0,0036	0,0080
86	0,0037	0,0075
87	0,0033	0,0070
88	0,0030	0,0065
89	0,0031	0,0060
90	0,0028	0,0052
91	0,0019	0,0047
92	0,0020	0,0041
93	0,0024	0,0040
94	0,0020	0,0034
95	0,0018	0,0031
96	0,0016	0,0027
97	0,0015	0,0024
98	0,0013	0,0020
99	0,0012	0,0017
100	0,0010	0,0014
101	0,0009	0,0012
102	0,0008	0,0009
103	0,0007	0,0007
104	0,0006	0,0005
105	0,0005	0,0004
106	0,0005	0,0003
107	0,0004	0,0002
108	0,0003	0,0001
109	0,0003	0,0000
110	0,0002	-0,0001
111	0,0002	-0,0001
112	0,0002	-0,0002
113	0,0001	-0,0002
114	0,0001	-0,0002
115	0,0001	-0,0002
116	0,0001	-0,0002
117	0,0000	-0,0002
118	0,0000	-0,0002
119	0,0000	-0,0002
120	0,0000	-0,0002
121	0,0000	-0,0002
122	0,0000	-0,0002
123	0,0000	-0,0002
124	0,0000	-0,0002
125	0,0000	-0,0002

Table 15: Estimated values for β_x (Lee Carter model) for CBS data.

Age x	Male	Female	Age x	Male	Female	Age x	Male	Female
0	0,0210	0,0169	37	0,0131	0,0131	74	0,0039	0,0056
1	0,0272	0,0221	38	0,0130	0,0128	75	0,0038	0,0055
2	0,0251	0,0212	39	0,0126	0,0123	76	0,0036	0,0053
3	0,0240	0,0203	40	0,0123	0,0117	77	0,0036	0,0051
4	0,0234	0,0195	41	0,0117	0,0112	78	0,0034	0,0049
5	0,0224	0,0193	42	0,0115	0,0108	79	0,0033	0,0048
6	0,0218	0,0190	43	0,0109	0,0102	80	0,0034	0,0045
7	0,0209	0,0185	44	0,0106	0,0096	81	0,0030	0,0040
8	0,0203	0,0181	45	0,0104	0,0091	82	0,0031	0,0041
9	0,0202	0,0174	46	0,0099	0,0086	83	0,0030	0,0040
10	0,0194	0,0174	47	0,0094	0,0081	84	0,0030	0,0037
11	0,0188	0,0165	48	0,0092	0,0079	85	0,0026	0,0035
12	0,0176	0,0159	49	0,0086	0,0077	86	0,0028	0,0033
13	0,0164	0,0156	50	0,0084	0,0077	87	0,0027	0,0031
14	0,0163	0,0159	51	0,0078	0,0073	88	0,0024	0,0030
15	0,0155	0,0157	52	0,0078	0,0072	89	0,0023	0,0025
16	0,0138	0,0152	53	0,0074	0,0072	90	0,0023	0,0026
17	0,0143	0,0154	54	0,0072	0,0070	91	0,0019	0,0020
18	0,0146	0,0152	55	0,0066	0,0068	92	0,0020	0,0024
19	0,0147	0,0153	56	0,0066	0,0068	93	0,0014	0,0021
20	0,0154	0,0154	57	0,0061	0,0067	94	0,0014	0,0019
21	0,0153	0,0151	58	0,0061	0,0066	95	0,0017	0,0015
22	0,0155	0,0154	59	0,0057	0,0066	96	0,0006	0,0013
23	0,0152	0,0155	60	0,0056	0,0066	97	0,0013	0,0017
24	0,0154	0,0157	61	0,0053	0,0063	98	0,0019	0,0014
25	0,0154	0,0154	62	0,0052	0,0066	99	0,0033	0,0024
26	0,0152	0,0157	63	0,0052	0,0065	100	0,0066	0,0052
27	0,0155	0,0153	64	0,0049	0,0065	101	0,0039	0,0044
28	0,0150	0,0152	65	0,0048	0,0064	102	0,0034	0,0038
29	0,0147	0,0150	66	0,0046	0,0064	103	0,0023	0,0030
30	0,0147	0,0148	67	0,0044	0,0062	104	0,0021	0,0026
31	0,0141	0,0145	68	0,0043	0,0062	105	0,0007	0,0019
32	0,0144	0,0145	69	0,0042	0,0061	106	0,0001	0,0019
33	0,0142	0,0141	70	0,0043	0,0061	107	-0,0012	0,0000
34	0,0139	0,0139	71	0,0039	0,0057	108	-0,0015	-0,0003
35	0,0137	0,0135	72	0,0041	0,0059	109	-0,0002	-0,0009
36	0,0135	0,0132	73	0,0040	0,0057	110	0,0000	-0,0004

Table 16: Estimated values for β_x (Lee Carter model) for HMDB data.

E Values for κ_t

Year t	Male	Female
1950	34,47	49,87
1951	34,79	49,67
1952	29,21	44,44
1953	36,40	52,23
1954	26,77	37,96
1955	26,73	36,94
1956	25,09	35,92
1957	26,26	36,14
1958	23,85	31,62
1959	28,75	31,17
1960	23,53	26,78
1961	23,59	22,67
1962	25,50	26,10
1963	24,25	23,83
1964	25,35	22,46
1965	23,21	22,71
1966	25,34	22,79
1967	25,93	20,69
1968	24,60	22,20
1969	23,31	23,58
1970	25,34	20,78
1971	23,09	17,74
1972	23,89	18,26
1973	19,84	18,02
1974	16,98	11,61
1975	14,39	10,64
1976	12,83	10,07
1977	11,71	6,66
1978	11,87	7,26
1979	6,59	2,19
1980	5,61	-0,51
1981	2,90	-2,61
1982	2,70	-2,37
1983	-0,25	-4,16
1984	-0,43	-9,05
1985	-5,51	-12,24
1986	-4,61	-8,79
1987	-9,33	-10,41
1988	-7,54	-15,00
1989	-9,11	-10,52
1990	-8,27	-11,44
1991	-13,40	-11,50
1992	-11,52	-16,51
1993	-12,69	-13,30
1994	-16,90	-18,85
1995	-14,93	-18,49
1996	-18,49	-18,76

Year t	Male	Female
1997	-20,97	-21,71
1998	-22,73	-28,28
1999	-22,06	-23,47
2000	-24,60	-23,93
2001	-24,27	-27,22
2002	-27,65	-27,04
2003	-26,98	-27,88
2004	-35,39	-36,67
2005	-38,33	-36,02
2006	-41,48	-38,78
2007	-48,89	-41,33
2008	-47,92	-44,19
2009	-49,71	-45,76
2010	-50,72	-50,28
2011	-51,15	-54,75
2012	-48,88	-51,15
2013	-50,2	-52,8
2014	-51,57	-54,41
2015	-52,91	-56,04
2016	-54,25	-57,67
2017	-55,60	-59,30
2018	-56,94	-60,92
2019	-58,29	-62,55
2020	-59,63	-64,18
2021	-60,98	-65,81
2022	-62,32	-67,44
2023	-63,66	-69,07
2024	-65,01	-70,70
2025	-66,35	-72,33
2026	-67,70	-73,96
2027	-69,04	-75,59
2028	-70,39	-77,22
2029	-71,73	-78,85
2030	-73,07	-80,48
2031	-74,42	-82,11
2032	-75,76	-83,74
2033	-77,11	-85,36
2034	-78,45	-86,99
2035	-79,80	-88,62
2036	-81,14	-90,25
2037	-82,48	-91,88
2038	-83,83	-93,51
2039	-85,17	-95,14
2040	-86,52	-96,77
2041	-87,86	-98,40
2042	-89,21	-100,03
2043	-90,55	-101,66

Table 17: Estimated (1950-2012) and forecasted (2013-2139) values for κ_t (Lee Carter model) for CBS data.

Year t	Male	Female
2044	-91,90	-103,29
2045	-93,24	-104,92
2046	-94,58	-106,55
2047	-95,93	-108,18
2048	-97,27	-109,80
2049	-98,62	-111,43
2050	-99,96	-113,06
2051	-101,31	-114,69
2052	-102,65	-116,32
2053	-103,99	-117,95
2054	-105,34	-119,58
2055	-106,68	-121,21
2056	-108,03	-122,84
2057	-109,37	-124,47
2058	-110,72	-126,10
2059	-112,06	-127,73
2060	-113,40	-129,36
2061	-114,75	-130,99
2062	-116,09	-132,62
2063	-117,44	-134,24
2064	-118,78	-135,87
2065	-120,13	-137,50
2066	-121,47	-139,13
2067	-122,81	-140,76
2068	-124,16	-142,39
2069	-125,50	-144,02
2070	-126,85	-145,65
2071	-128,19	-147,28
2072	-129,54	-148,91
2073	-130,88	-150,54
2074	-132,22	-152,17
2075	-133,57	-153,80
2076	-134,91	-155,43
2077	-136,26	-157,06
2078	-137,60	-158,68
2079	-138,95	-160,31
2080	-140,29	-161,94
2081	-141,63	-163,57
2082	-142,98	-165,20
2083	-144,32	-166,83
2084	-145,67	-168,46
2085	-147,01	-170,09
2086	-148,36	-171,72
2087	-149,70	-173,35
2088	-151,05	-174,98
2089	-152,39	-176,61
2090	-153,73	-178,24
2091	-155,08	-179,87
2092	-156,42	-181,50
2093	-157,77	-183,12
2094	-159,11	-184,75
2095	-160,46	-186,38
2096	-161,80	-188,01
2097	-163,14	-189,64
2098	-164,49	-191,27
2099	-165,83	-192,90
2100	-167,18	-194,53
2101	-168,52	-196,16
2102	-169,87	-197,79
2103	-171,21	-199,42
2104	-172,55	-201,05
2105	-173,90	-202,68
2106	-175,24	-204,31
2107	-176,59	-205,94
2108	-177,93	-207,56
2109	-179,28	-209,19
2110	-180,62	-210,82
2111	-181,96	-212,45
2112	-183,31	-214,08
2113	-184,65	-215,71
2114	-186,00	-217,34
2115	-187,34	-218,97
2116	-188,69	-220,60
2117	-190,03	-222,23
2118	-191,37	-223,86
2119	-192,72	-225,49
2120	-194,06	-227,12
2121	-195,41	-228,75
2122	-196,75	-230,38
2123	-198,10	-232,00
2124	-199,44	-233,63
2125	-200,79	-235,26
2126	-202,13	-236,89
2127	-203,47	-238,52
2128	-204,82	-240,15
2129	-206,16	-241,78
2130	-207,51	-243,41
2131	-208,85	-245,04
2132	-210,20	-246,67
2133	-211,54	-248,30
2134	-212,88	-249,93
2135	-214,23	-251,56
2136	-215,57	-253,19
2137	-216,92	-254,82
2138	-218,26	-256,44
2139	-219,61	-258,07

Table 18: Continued.

Year t	Male	Female
1850	80,20	100,85
1851	77,15	97,63
1852	80,70	100,48
1853	86,43	105,69
1854	84,79	104,71
1855	95,59	118,02
1856	83,96	104,18
1857	90,95	111,68
1858	97,86	119,16
1859	106,05	124,31
1860	90,61	110,09
1861	88,09	109,23
1862	83,16	103,62
1863	83,03	100,86
1864	86,38	108,55
1865	86,68	107,36
1866	105,11	122,42
1867	80,14	98,30
1868	79,56	99,61
1869	76,89	98,24
1870	84,24	103,83
1871	99,80	119,51
1872	87,37	103,92
1873	79,22	96,71
1874	73,23	92,50
1875	78,77	99,22
1876	75,13	94,32
1877	70,50	88,05
1878	72,10	90,05
1879	70,04	90,28
1880	70,65	88,95
1881	68,34	85,41
1882	65,39	83,52
1883	69,15	87,14
1884	70,06	87,34
1885	68,19	86,70
1886	69,38	84,82
1887	64,67	78,70
1888	64,22	80,36
1889	63,07	79,93
1890	63,90	80,95
1891	63,43	80,09
1892	64,21	81,60
1893	59,68	76,03
1894	59,76	75,07
1895	55,49	70,34
1896	50,61	66,19
1897	49,05	63,02
1898	46,52	62,29
1899	47,01	62,20
1900	50,75	66,12
1901	49,03	63,65
1902	45,68	61,78
1903	42,70	55,49
1904	42,08	56,20
1905	39,30	53,61
1906	36,81	52,13
1907	36,35	51,90
1908	36,02	51,90
1909	32,97	48,35
1910	28,62	46,04
1911	30,75	46,02
1912	25,35	40,15
1913	22,69	37,24
1914	23,10	36,98
1915	24,56	41,48
1916	30,03	49,22
1917	34,38	52,10
1918	71,46	90,24
1919	36,66	56,06
1920	24,96	43,39
1921	13,72	30,86
1922	13,45	30,64
1923	6,50	21,93
1924	5,87	20,58
1925	5,82	20,14
1926	3,93	18,90
1927	6,49	20,57
1928	3,91	17,60
1929	7,92	23,66
1930	-0,91	11,34
1931	-1,57	11,40
1932	-7,08	5,27
1933	-9,39	1,89
1934	-11,82	-1,62
1935	-11,29	-2,12
1936	-13,94	-3,27
1937	-15,78	-7,15
1938	-15,43	-9,25
1939	-20,08	-13,73
1940	3,58	-4,42
1941	1,41	-1,69
1942	4,33	0,16
1943	17,35	13,54
1944	35,72	32,54
1945	59,95	34,12
1946	-9,35	-12,16
1947	-22,63	-28,36
1948	-34,91	-43,56
1949	-34,60	-46,77
1950	-42,56	-51,67
1951	-42,19	-52,48
1952	-48,24	-57,69
1953	-41,54	-49,30
1954	-50,60	-64,81
1955	-51,17	-66,70
1956	-52,97	-68,52
1957	-51,90	-66,78
1958	-54,60	-71,24
1959	-49,34	-73,25
1960	-54,35	-77,17
1961	-54,43	-81,02
1962	-52,43	-76,97
1963	-53,71	-79,94
1964	-52,44	-81,07
1965	-54,48	-80,42
1966	-52,85	-79,91
1967	-52,70	-81,18
1968	-53,39	-80,19
1969	-54,44	-78,53
1970	-53,06	-80,23
1971	-55,25	-84,36
1972	-54,31	-83,48
1973	-57,73	-82,50
1974	-61,08	-90,07
1975	-64,02	-91,63
1976	-64,77	-91,67
1977	-66,07	-94,37
1978	-66,10	-92,94
1979	-71,31	-98,31
1980	-73,15	-101,93
1981	-75,40	-104,56
1982	-76,62	-104,06
1983	-79,73	-105,35
1984	-79,49	-111,19
1985	-84,04	-114,04
1986	-83,34	-109,62
1987	-87,50	-111,49

Table 19: Estimated (1850-2009) and forecasted (2010-2124) values for κ_t (Lee Carter model) for HMDB data.

Year t	Male	Female
1988	-85,91	-116,85
1989	-87,72	-111,16
1990	-86,45	-112,20
1991	-90,24	-111,99
1992	-89,26	-117,28
1993	-89,66	-113,52
1994	-94,20	-118,05
1995	-91,88	-117,75
1996	-95,07	-119,51
1997	-98,48	-121,67
1998	-99,31	-128,04
1999	-98,51	-122,75
2000	-100,94	-122,73
2001	-101,34	-127,07
2002	-104,05	-126,45
2003	-103,83	-126,82
2004	-112,21	-135,58
2005	-115,63	-136,20
2006	-118,47	-139,45
2007	-124,25	-141,02
2008	-125,26	-142,70
2009	-125,96	-145,63
2010	-127,26	-147,18
2011	-128,55	-148,73
2012	-129,85	-150,28
2013	-131,15	-151,83
2014	-132,44	-153,38
2015	-133,74	-154,93
2016	-135,04	-156,48
2017	-136,33	-158,03
2018	-137,63	-159,58
2019	-138,93	-161,13
2020	-140,22	-162,68
2021	-141,52	-164,23
2022	-142,82	-165,78
2023	-144,11	-167,33
2024	-145,41	-168,88
2025	-146,71	-170,43
2026	-148,00	-171,98
2027	-149,30	-173,54
2028	-150,60	-175,09
2029	-151,89	-176,64
2030	-153,19	-178,19
2031	-154,49	-179,74
2032	-155,78	-181,29
2033	-157,08	-182,84
2034	-158,38	-184,39
2035	-159,67	-185,94
2036	-160,97	-187,49
2037	-162,27	-189,04
2038	-163,56	-190,59
2039	-164,86	-192,14
2040	-166,16	-193,69
2041	-167,45	-195,24
2042	-168,75	-196,79
2043	-170,05	-198,34
2044	-171,34	-199,89
2045	-172,64	-201,44
2046	-173,94	-202,99
2047	-175,23	-204,54
2048	-176,53	-206,09
2049	-177,82	-207,64
2050	-179,12	-209,19
2051	-180,42	-210,74
2052	-181,71	-212,29
2053	-183,01	-213,84
2054	-184,31	-215,39
2055	-185,60	-216,94
2056	-186,90	-218,49
2057	-188,20	-220,04
2058	-189,49	-221,59
2059	-190,79	-223,14
2060	-192,09	-224,69
2061	-193,38	-226,24
2062	-194,68	-227,79
2063	-195,98	-229,34
2064	-197,27	-230,89
2065	-198,57	-232,44
2066	-199,87	-233,99
2067	-201,16	-235,54
2068	-202,46	-237,09
2069	-203,76	-238,64
2070	-205,05	-240,19
2071	-206,35	-241,74
2072	-207,65	-243,30
2073	-208,94	-244,85
2074	-210,24	-246,40
2075	-211,54	-247,95
2076	-212,83	-249,50
2077	-214,13	-251,05
2078	-215,43	-252,60
2079	-216,72	-254,15
2080	-218,02	-255,70
2081	-219,32	-257,25
2082	-220,61	-258,80
2083	-221,91	-260,35
2084	-223,21	-261,90
2085	-224,50	-263,45
2086	-225,80	-265,00
2087	-227,10	-266,55
2088	-228,39	-268,10
2089	-229,69	-269,65
2090	-230,99	-271,20
2091	-232,28	-272,75
2092	-233,58	-274,30
2093	-234,88	-275,85
2094	-236,17	-277,40
2095	-237,47	-278,95
2096	-238,77	-280,50
2097	-240,06	-282,05
2098	-241,36	-283,60
2099	-242,66	-285,15
2100	-243,95	-286,70
2101	-245,25	-288,25
2102	-246,55	-289,80
2103	-247,84	-291,35
2104	-249,14	-292,90
2105	-250,43	-294,45
2106	-251,73	-296,00
2107	-253,03	-297,55
2108	-254,32	-299,10
2109	-255,62	-300,65
2110	-256,92	-302,20
2111	-258,21	-303,75
2112	-259,51	-305,30
2113	-260,81	-306,85
2114	-262,10	-308,40
2115	-263,40	-309,95
2116	-264,70	-311,51
2117	-265,99	-313,06
2118	-267,29	-314,61
2119	-268,59	-316,16
2120	-269,88	-317,71
2121	-271,18	-319,26
2122	-272,48	-320,81
2123	-273,77	-322,36
2124	-275,07	-323,91

Table 20: Continued.

F Binomial Approximation Using the AG table

The numbers in the AG table can be treated as independent success probabilities drawn from a Bernouilli distribution. As there is a $q_{x,t}$ chance of not making it to the next year (failure probability=$1-p$) and a $p_{x,t}$ chance of surviving (success probability=p). An population of Bernouilli distributed variables with success probability p forms a multivariate distribution also known as the binomial distribution:

$$n_{x,t+1} \sim Bin(n_{x,t}, p_{x,t}).$$

Here $n_{x,t+1}$ is the number of survivors after 1 year for a population aged x. In this setup we assume that every person in the population has exactly the same probabilities of making it through the years, independent of each other (iid random variables). The one-year probability of exactly ν survivors within year t is given by

$$\mathbb{P}(\nu) = \binom{n_{x,t}}{\nu} p_{x,t}^{\nu} \cdot (1 - p_{x,t})^{n_{x,t}-\nu}. \tag{79}$$

Equivalently for a population, it is possible to determine how many are expected to survive until 2063. The mean of a binomial distribution is given as $\mathbb{E}[n_{x,t+1}] = n_{x,t} \cdot p_{x,t}$ with variance $\sigma^2_{x,t+1} = n_{x,t} \cdot p_{x,t}(1 - p_{x,t})$. Hence the expected number of survivors of age x in year s, given the current population size $n_{x,o}$, is

$$\widehat{n}_{x,s} = \mathbb{E}[n_{x,s}] = n_{x,0} \cdot {}_sp_{x,0} \tag{80}$$

with

$$\sigma^2_{x,s} = n_{x,0} \cdot {}_sp_{x,0}(1 - {}_sp_{x,0}). \tag{81}$$

A population that contains multiple age groups is a combination of binomial distributed variables with non-identical success probabilities. The sum of binomial variables that are independent but not identical is called a Poisson binomial distribution [11]. The expectation and variance are

$$\widehat{n}_s = \mathbb{E}[n_s] = \sum_{x=0}^{110} \mathbb{E}[n_{x,s}] = \sum_{x=0}^{110} n_{x,0} \cdot {}_sp_{x,0} \tag{82}$$

and

$$\sigma^2_s = \sum_{x=0}^{110} \sigma^2_{x,s} = \sum_{x=0}^{110} n_{x,0} \cdot {}_sp_{x,0}(1 - {}_sp_{x,0}). \tag{83}$$

The age x can attain a maximum of 110 to be able to use the AG projection table.

Suppose the population exists of 1000 20-year old men (at the beginning of 2014). According to the AG projection table and equation 8 and 80, it is expected that 940 of them will still be alive at (the start of) 2063. In other words, it is the most likely that 940 people survive. Of course it might also be possible that only 939 survived, or 945. That does not directly mean that the

table is incorrect. It is an outcome that is just less likely than the expected value of 940.

One approach is an analytical approximation interval justified by the Central Limit Theorem (CLT). We assume that the error around ${}_sp_{x,0}$, the deviation from the estimation, is normally distributed and ${}_sp_{x,0}$ is binomial. The 90% confidence interval for the number of survivors in year s is given:

$$\widehat{n}_s - \frac{1}{2} - z \cdot \sigma_s < n_s < \widehat{n}_s + \frac{1}{2} + z \cdot \sigma_s. \tag{84}$$

A 90% confidence level requires z to be the $1 - \frac{1}{2}(1 - 90\%) = 0.95$ percentile of a standard normal distribution, i.e. $z = 1.645$. Hence with 90% certainty the number of survivors lies within this interval. Note the application of a continuity correction.

Table 21 gives 16 examples of populations and their expected interval for number of survivors in 2063. The populations vary in sex, age group 20- or 40-year old's and number of participants.

Obviously the number of survivors of a group 10 20-year old's can not be 11. CLT is known to perform poorly with a sample size less than 30 or where ${}_sp_{x,0}$ is close to 0 or 1, explaining this irregularity.

The Lee Carter estimations and intervals for women are similar to binomial forecasting using the AG projection table. The CBS estimates for the older ages are slightly higher than before and the HMDB estimates are lower in the Lee Carter model. However, the intervals are much larger: Trend risk is higher than risk coming from individual volatility.

A second option is to apply simulations to find values that can tell us how likely certain outcomes are. Established is that the survivors are binomial distributed. We make use of the binomial inverse to reconstruct paths. Every year the computer randomly picks a number between 0 and 1 that can be translated to the probability of observing at least an Y amount of successes (survivors) during year s in $n^*_{x,s}$ independent trials where $p_{x,s}$ is the probability of success in each trial, this results in $n^*_{x,s+1}$. Each Y is a positive integer less than or equal to $n_{x,s}$. We start with $n_{x,0}$ and it goes on for $s = 1, \ldots, 49$ until 2063 is reached and the value for $n^*_{x,2063}$ is determined.

See figure 24 how this evolves for a group of 1000 20-year old men. Some series are obviously more likely to occur than others. Moreover, two opposite extreme observations within one path (for example 10 deaths in 1 year and the upcoming years none) might lead to an outcome after 49 years that is very close to the expected value. A large number of those paths give a clear view about the course over time.

Note that the number of survivors shows outcomes rounded to the nearest integer in figure 24, however there has been no rounding in between for the calculations made for table 21.

The results in 2063 are summarized by percentiles again in table 21. Where a 5% percentile covers for a lowerbound on the number of survivors and the 95% percentile an upperbound. In other words, outcomes lower than the 5%

$\mathbf{n_{x,0}}$	**1**	**10**	**1.000**	**100.000**
20-year old's	1 (0,2) [0 1 1]	9 (7,*11*) [8 10 10]	940 (928,954) [929 941 953]	94.070 (93.947,94.193) [93.968 94.087 94.196]
40-year old's	0 (*-1*,1) [0 0 1]	5 (2,8) [2 5 7]	464 (438,490) [438 464 489]	46.448 (46.188,46.708) [46.200 46.466 46.742]
60-year old's	0 (*-1*,1) [0 0 0]	0 (*-1*,1) [0 0 0]	0 (*-1*,1) [0 0 0]	7 (2,12) [3 5 11]

Table 21: Projection of a current male member's base to 2063. First line: Estimation by AG table. Second line: Analytical approximation for a 90% confidence interval. Third line: [5 50 95] percentiles using binomial forecasting based on 1000 simulations. All numbers are rounded to the nearest integer.

20-year old's	1 (0,2) [0 1 1]	9 (7,*11*) [8 9 10]	924 (910,938) [910 924 937]	92.412 (92.274,92.550) [92.275 92.416 92.560]
40-year old's	1 (0,2) [0 1 1]	5 (2,8) [3 5 8]	544 (518,570) [519 544 569]	54.439 (54.179,54.699) [54.204 54.455 54.695]
60-year old's	0 (*-1*,1) [0 0 0]	0 (*-1*,1) [0 0 0]	0 (*-1*,1) [0 0 1]	22 (14,30) [15 22 30]

Table 22: Equivalent of table 21 for a female member's base.

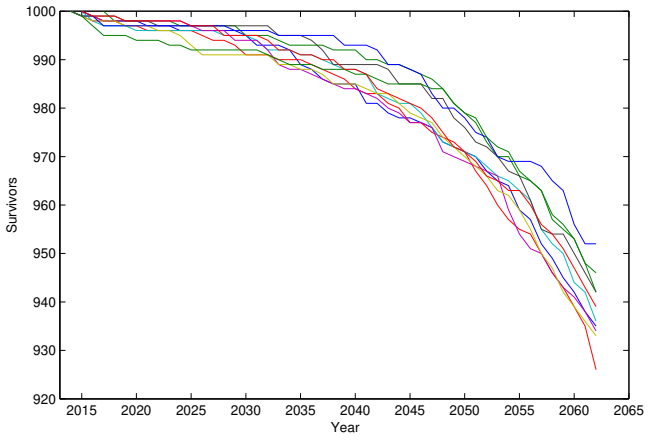


Figure 24: Number of survivors. Male, 20-year old's, n_0=1000 (10 simulations)

or higher than the 95% percentile are unlikely, i.e. they will not occur with a confidence level op 90%, so it will not occur under regular circumstances.

The percentiles from the simulations and the analytical approximations are the same. Deviations come from populations that are not large enough and/or the number of simulations has been insufficient to achieve asymptotic results.

With identical properties, a female member base expects more survivors than a male member base, since the survival probabilities for women are persistently slightly higher in the AG projection tables.

We observe the 90% confidence interval getting relatively smaller as the population size increases. For a member base of 100.000 20-year old men the number of survivors in 2063 lies $0,13\%$ under or above the estimate, for a member base of 1000 it is $1,3\%$ under or above the estimate. Hence for a member base that is 100 times bigger, a confidence interval approximately 10 times smaller. This ratio comes from the dependence on the square root of $\sum_{x=0}^{110} n_{x,0}$ within the standard deviation $\sqrt{\sigma_s^2}$. In other words, the larger the population the smaller the estimation error.

Note that the expected number of survivors for a combination of age groups is equal to the sum of the survivors for individual age groups. However, the 90% confidence intervals and the simulations can not be added. It can be analytically calculated by making use of equation 83. Here size effects play a role for the confidence interval since $\sqrt{n_1} + \sqrt{n_2} > \sqrt{n_1 + n_2}$ for $n_1, n_2 \neq 0$.

For the simulations we made use of the probability mass distribution (pmf) of binomial distributed variables. The pmf of a Poisson binomial random variable is not straightforward [11].

G Complete TSO Table

Age x	OP	$OP + 70\%NP_{f,p}$	$OP + 70\%NP_{f,a}$	$OP + 70\%NP_{r,p}$	$OP + 70\%NP_{r,a}$
0	3,300 %	2,366 %	2,417 %	3,208 %	3,208 %
1	3,288 %	2,446 %	2,455 %	3,273 %	3,273 %
2	3,278 %	2,441 %	2,445 %	3,272 %	3,272 %
3	3,264 %	2,427 %	2,430 %	3,260 %	3,260 %
4	3,252 %	2,412 %	2,414 %	3,249 %	3,249 %
5	3,245 %	2,393 %	2,394 %	3,243 %	3,243 %
6	3,233 %	2,377 %	2,378 %	3,230 %	3,230 %
7	3,215 %	2,361 %	2,362 %	3,213 %	3,213 %
8	3,208 %	2,344 %	2,345 %	3,206 %	3,206 %
9	3,189 %	2,324 %	2,325 %	3,188 %	3,188 %
10	3,173 %	2,311 %	2,312 %	3,171 %	3,171 %
11	3,142 %	2,291 %	2,292 %	3,140 %	3,140 %
12	3,116 %	2,272 %	2,273 %	3,114 %	3,114 %
13	3,085 %	2,254 %	2,256 %	3,083 %	3,083 %
14	3,053 %	2,232 %	2,233 %	3,051 %	3,051 %
15	3,021 %	2,208 %	2,209 %	3,019 %	3,019 %
16	2,996 %	2,186 %	2,188 %	2,993 %	2,993 %
17	2,969 %	2,162 %	2,164 %	2,965 %	2,965 %
18	2,937 %	2,142 %	2,145 %	2,932 %	2,932 %
19	2,904 %	-	2,122 %	-	2,899 %
20	2,882 %	1,977 %	2,100 %	2,643 %	2,876 %
21	2,853 %	2,007 %	2,066 %	2,732 %	2,848 %
22	2,817 %	2,004 %	2,045 %	2,736 %	2,812 %
23	2,778 %	1,987 %	2,012 %	2,726 %	2,774 %
24	2,732 %	1,974 %	1,993 %	2,688 %	2,727 %
25	2,690 %	1,956 %	1,970 %	2,662 %	2,686 %
26	2,667 %	1,929 %	1,941 %	2,640 %	2,663 %
27	2,642 %	1,911 %	1,919 %	2,620 %	2,637 %
28	2,619 %	1,893 %	1,901 %	2,598 %	2,615 %
29	2,587 %	1,872 %	1,880 %	2,569 %	2,583 %
30	2,552 %	1,856 %	1,863 %	2,536 %	2,548 %
31	2,521 %	1,838 %	1,844 %	2,507 %	2,517 %
32	2,504 %	1,818 %	1,824 %	2,492 %	2,501 %
33	2,479 %	1,803 %	1,808 %	2,466 %	2,475 %
34	2,457 %	1,791 %	1,796 %	2,446 %	2,453 %
35	2,435 %	1,783 %	1,786 %	2,424 %	2,431 %
36	2,400 %	1,779 %	1,782 %	2,389 %	2,396 %
37	2,373 %	1,771 %	1,773 %	2,364 %	2,370 %
38	2,348 %	1,760 %	1,763 %	2,338 %	2,344 %
39	2,320 %	1,752 %	1,755 %	2,310 %	2,316 %
40	2,294 %	1,745 %	1,748 %	2,285 %	2,290 %
41	2,272 %	1,737 %	1,740 %	2,262 %	2,267 %
42	2,242 %	1,731 %	1,733 %	2,232 %	2,237 %

Table 23: TSO (percentages) for all ages, unrounded.

Age x	OP	$OP + 70\%NP_{f,p}$	$OP + 70\%NP_{f,a}$	$OP + 70\%NP_{r,p}$	$OP + 70\%NP_{r,a}$
43	2,213 %	1,720 %	1,723 %	2,205 %	2,209 %
44	2,190 %	1,715 %	1,719 %	2,181 %	2,185 %
45	2,159 %	1,706 %	1,709 %	2,151 %	2,154 %
46	2,143 %	1,694 %	1,697 %	2,135 %	2,138 %
47	2,123 %	1,685 %	1,689 %	2,114 %	2,118 %
48	2,089 %	1,680 %	1,682 %	2,080 %	2,083 %
49	2,053 %	1,673 %	1,676 %	2,045 %	2,048 %
50	2,018 %	1,661 %	1,663 %	2,011 %	2,013 %
51	1,989 %	1,654 %	1,657 %	1,982 %	1,984 %
52	1,951 %	1,642 %	1,644 %	1,943 %	1,945 %
53	1,913 %	1,628 %	1,631 %	1,905 %	1,907 %
54	1,877 %	1,614 %	1,617 %	1,871 %	1,872 %
55	1,832 %	1,592 %	1,595 %	1,826 %	1,827 %
56	1,787 %	1,568 %	1,570 %	1,781 %	1,782 %
57	1,741 %	1,544 %	1,546 %	1,736 %	1,737 %
58	1,684 %	1,513 %	1,514 %	1,679 %	1,680 %
59	1,636 %	1,481 %	1,484 %	1,631 %	1,632 %
60	1,580 %	1,449 %	1,452 %	1,577 %	1,577 %
61	1,514 %	1,418 %	1,420 %	1,509 %	1,509 %
62	1,456 %	1,385 %	1,386 %	1,452 %	1,453 %
63	1,398 %	1,339 %	1,340 %	1,392 %	1,392 %
64	1,333 %	1,301 %	1,301 %	1,333 %	1,333 %
65	1,268 %	1,258 %	1,257 %	1,269 %	1,269 %
66	1,216 %	1,215 %	1,213 %	1,214 %	1,214 %
67	1,078 %	1,103 %	1,100 %	1,079 %	1,079 %
68	1,088 %	1,121 %	1,118 %	1,093 %	1,093 %
69	1,107 %	1,132 %	1,129 %	1,111 %	1,111 %
70	1,118 %	1,146 %	1,142 %	1,126 %	1,126 %
71	1,129 %	1,148 %	1,146 %	1,131 %	1,131 %
72	1,125 %	1,165 %	1,160 %	1,131 %	1,131 %
73	1,132 %	1,179 %	1,174 %	1,138 %	1,138 %
74	1,132 %	1,187 %	1,181 %	1,135 %	1,135 %
75	1,126 %	1,187 %	1,183 %	1,128 %	1,128 %
76	1,109 %	1,183 %	1,179 %	1,114 %	1,114 %
77	1,090 %	1,172 %	1,169 %	1,100 %	1,100 %
78	1,079 %	1,159 %	1,153 %	1,089 %	1,089 %
79	1,062 %	1,137 %	1,129 %	1,072 %	1,072 %
80	1,030 %	1,102 %	1,094 %	1,043 %	1,043 %
81	0,996 %	1,067 %	1,060 %	1,011 %	1,011 %
82	0,961 %	1,026 %	1,019 %	0,975 %	0,975 %
83	0,920 %	0,982 %	0,972 %	0,939 %	0,939 %
84	0,878 %	0,939 %	0,929 %	0,890 %	0,890 %

Table 24: Continued.

Age x	OP	$OP + 70\%NP_{f,p}$	$OP + 70\%NP_{f,a}$	$OP + 70\%NP_{r,p}$	$OP + 70\%NP_{r,a}$
85	0,825 %	0,886 %	0,874 %	0,843 %	0,843 %
86	0,773 %	0,828 %	0,817 %	0,788 %	0,788 %
87	0,715 %	0,770 %	0,762 %	0,733 %	0,733 %
88	0,661 %	0,714 %	0,701 %	0,673 %	0,673 %
89	0,599 %	0,652 %	0,644 %	0,612 %	0,612 %
90	0,541 %	0,595 %	0,585 %	0,555 %	0,555 %
91	0,488 %	0,538 %	0,528 %	0,505 %	0,505 %
92	0,439 %	0,485 %	0,474 %	0,453 %	0,453 %
93	0,384 %	0,426 %	0,417 %	0,398 %	0,398 %
94	0,337 %	0,373 %	0,365 %	0,351 %	0,351 %
95	0,291 %	0,324 %	0,316 %	0,302 %	0,302 %
96	0,247 %	0,278 %	0,271 %	0,259 %	0,259 %
97	0,208 %	0,235 %	0,229 %	0,218 %	0,218 %
98	0,173 %	0,198 %	0,192 %	0,183 %	0,183 %
99	0,142 %	0,163 %	0,158 %	0,151 %	0,151 %
100	0,115 %	0,133 %	0,129 %	0,122 %	0,122 %
101	0,092 %	0,108 %	0,104 %	0,098 %	0,098 %
102	0,074 %	0,087 %	0,084 %	0,079 %	0,079 %
103	0,060 %	0,071 %	0,068 %	0,064 %	0,064 %
104	0,046 %	0,055 %	0,053 %	0,050 %	0,050 %
105	0,036 %	0,044 %	0,042 %	0,039 %	0,039 %
106	0,029 %	0,035 %	0,033 %	0,031 %	0,031 %
107	0,024 %	0,030 %	0,028 %	0,026 %	0,026 %
108	0,021 %	0,027 %	0,026 %	0,024 %	0,024 %
109	0,020 %	0,026 %	0,025 %	0,022 %	0,022 %
110	0,021 %	0,027 %	0,026 %	0,023 %	0,023 %
111	0,021 %	0,027 %	0,026 %	0,024 %	0,024 %
112	0,021 %	0,027 %	0,026 %	0,023 %	0,023 %
113	0,021 %	0,027 %	0,026 %	0,024 %	0,024 %
114	0,021 %	0,027 %	0,026 %	0,024 %	0,024 %
115	0,021 %	0,027 %	0,025 %	0,023 %	0,023 %
116	0,020 %	0,025 %	0,024 %	0,022 %	0,022 %
117	0,019 %	0,025 %	0,023 %	0,021 %	0,021 %
118	0,018 %	0,023 %	0,022 %	0,020 %	0,020 %
119	0,016 %	0,021 %	0,020 %	0,018 %	0,018 %
120	0,015 %	0,019 %	0,018 %	0,016 %	0,016 %
121	0,012 %	0,016 %	0,015 %	0,014 %	0,014 %
122	0,010 %	0,012 %	0,012 %	0,011 %	0,011 %
123	0,007 %	0,008 %	0,008 %	0,008 %	0,008 %
124	0,003 %	0,004 %	0,004 %	0,004 %	0,004 %

Table 25: Continued.

H Definitions

AG Dutch Royal Actuarial Association (Koninklijk Actuarieel Genootschap)

AG projection table AG Prognosetafel 2012-2062

ARIMA Autoregressive integrated moving average model

CBS Statistics Netherlands (Centraal Bureau voor de Statistiek)

CBS Statline Electronic databank of Statistics Netherlands

CLT Central limit theorem

DNB Dutch central bank (De Nederlandsche Bank)

FTK Financial assessment framework (Financieel toetsingskader)

GBM/V AG survival tables male/female (AG overlevingstafels gehele bevolking man/vrouw 1995-2000)

HMDB Human mortality database

Longevity risk Langlevenrisico

ltt Long term trend (langetermijntrend)

MLE Maximum likelihood estimator

NP Partner's pension (nabestaandenpensioen)

NSA Negative stochastic deviations (Negatief stochastische afwijkingen)

OLS Ordinary least squares

OP Old-age pension (ouderdomspensioen)

Retirement age Pensioenleeftijd (67)

stt Short term trend (kortetermijntrend)

SVD Singular value decomposition, First principal component

S6 Risk margin for insurance technical risk

TSO Longevity risk (Toekomstige sterftetrendonzekerheid)

α_x Mortality index for age x

$\widehat{\alpha}_x$ Estimated α_x

β_x Sensitivity of the mortality rate for age x to κ_t in the Lee Carter model

$\widehat{\beta}_x$ Estimated β_x

$D_{x,t}$ Number of deaths of lives aged x in year t

d_k Discount factor

$E_{x,t}$ Total population of lives aged x in year t (exposure to risk)

$e_{x,t}$ Life expectancy for a life aged x at year t

ϵ_ρ Randomized standard normal distributed variable

$\epsilon_{\rho,t}$ Randomized standard normal distributed variable

ϵ_σ Randomized standard normal distributed variable

ϵ_θ Randomized standard normal distributed variable

$\epsilon_{x,t}$ White noise term

k Forecast horizon

κ_t Mortality improvement parameter in year t in the Lee Carter model

$\widehat{\kappa}_t$ Estimated or forecasted κ_t

ltt_x Long term trend of mortality improvement for age x

$m_{x,t}$ Instantaneous rate of mortality for lives aged x in year t

$\widehat{m}_{x,t}$ Forecasted $m_{x,t}$

μ_ρ Mean ρ

$\mu_{x,t}$ Force of mortality for lives aged x in year t

$\widehat{\mu}_{x,t}$ Estimated force of mortality

n Number of participants in a pension fund

$n_{x,t}$ Number of (living) people in a population of age x in year t

$\widehat{n}_{x,t}$ Estimated $n_{x,t}$

$n^*_{x,t}$ Simulated $n_{x,t}$

$p_{x,t}$ Probability of surviving year t for a life aged x

$\widehat{p}_{x,t}$ Forecasted $p_{x,t}$

${}_sp_{x,t}$ Probability of surviving the next s years for a life aged x at year t

${}_sp^*_{x,t}$ Simulated ${}_sp_{x,t}$

PV Present value of life annuities

PV^* Simulated PV

$q_{x,t}$ Probability of dying in year t for a life aged x

$\widehat{q}_{x,t}$ Forecasted $q_{x,t}$

$q^*_{x,t}$ Simulated $q_{x,t}$

${}_sq^*_{x,t}$ Simulated ${}_sq_{x,t}$

ρ Mortality improvement

$\rho_{x,t}$ Mortality improvement for lives aged x in year t compared to the year before

$\widehat{\rho}_x$ Estimated mortality improvement for lives aged x

s Forecast horizon

stt_x Short term trend for mortality improvement for age x

σ_ρ Standard deviation of ρ

$\sigma_{x,t}$ Standard deviation of $n_{x,t}$

σ_ω Standard deviation of ω_t

$\widehat{\sigma}_\rho$ Estimated σ_ρ

$\widehat{\sigma}_\omega$ Estimated σ_ω

T Number (*last year*) of data points

θ Drift term of κ_t

$\widehat{\theta}$ Estimated θ

θ^* Simulated θ

X Given maximum age

z Value of standard normal distribution for desired percentile

ω_t Yearly difference between observed and real θ

ω^*_t Simulated ω_t