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Valuation of Longevity Swaps in a Solvency II Framework

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Valuation of Longevity Swaps in a Solvency II Framework

by

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Abstract

This thesis proposes a model to determine the maximum risk premium paid for a longevity swap by an insurer to hedge longevity risk in specific insurance contracts, considering Solvency II legislation. As longevity risk is an important risk factor next to financial market risks, buying a hedge might be beneficial for insurers. In a longevity swap contract an insurer receives floating leg payments based on realized survival rates and makes predetermined fixed leg payments so that longevity risk in the insurance contract is hedged. As no liquid market for mortality-linked securities exists the risk premium for longevity risk is unknown. Here, Solvency II plays an important role. When Solvency II becomes effective insurers are, among others, obliged to maintain risk based capital buffers and maintain a risk margin for unhedgeable risks such as longevity. The risk margin for longevity, an obligatory margin to raise the best estimate value of a specific insurance contract to a fair value, is assumed to represent the risk premium for longevity. The maximum premium to be paid for the swap is determined such that at inception of the swap contract, the fair value of the insurance contract when a swap is arranged does not exceed the fair value of the contract without the swap arrangement. This leads to a total risk premium which an insurer might pay as a lump sum at inception of the swap contract to its counterparty. But, as the premium in practice is likely included in the fixed leg payments, a method is proposed to spread out the lump sum premium over the fixed leg payments the insurer is obliged to make. The Lee-Carter stochastic mortality model is applied to determine best estimate mortality projections taking into account parameter uncertainty, required for modelling and projecting both legs of the swap. Results indicate that the premium is increasing with the amount of longevity risk in the insurance product. When default risk of the counterparty of the insurer in the swap contract is considered this has to priced by a risk margin as well, since longevity remains the underlying risk factor which cannot be hedged by financial instruments currently available in the market. The risk margin for default risk raises the fair value of the contract under the swap and decreases the value of the swap from the insurer's perspective. Moreover, a decline in the creditworthiness of the counterparty has a negative effect on the premium as the effectiveness of the hedge deteriorates in case default of the counterparty is more likely to occur.

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Contents

1 Introduction

Recent financial crises have shown that assets and liabilities of insurers and pension funds are vulnerable to fluctuations in the financial market, such as declining stock markets, decreasing interest rates and increasing inflation rates. Additional risk factors, influencing the value of specific insurance contracts, are longevity and mortality risk. As of 2014 Solvency II (SII) will become effective and insurance companies face new legislation regarding their financial buffers for the risks they are exposed to. Among others, in contrast to Solvency I the Solvency Capital Requirement (SCR) becomes risk based and all risk factors concerning an insurer's business must be taken into account (see PwC Guide to SII, 2011; QIS5, 2010). Life expectancy of both men and women in the Netherlands has increased over the past century. This brings along a major challenge for insurers and pension funds since they need to take into account uncertainty regarding their estimates of the expected remaining lifetime at certain ages. The major risk factor however might not be the fact that people live longer, or die earlier, than expected, but the fact that the development of future life expectancy is unknown. Hence, next to buffers for financial market risks, SII requires solvency buffers for mortality and longevity risk. This might create an incentive for insurers to set up a hedge in order to mitigate longevity or mortality risk by entering a mortality-linked swap agreement. The goal of this thesis is to determine the maximum premium an insurer might spend on a longevity swap for it to be beneficial to enter such a contract in order to hedge longevity risk. This is the case when the amount an insurer is willing to spend is larger or does not exceed the amount a counterparty demands from the insurer. However, here solely emphasis is put on the amount an insurer is prepared to spend, as the amount a counterparty charges is unknown and depends on the counterparty's beliefs of the exposure to longevity risk of the portfolio for which an insurer requires a hedge or any hedge potential the counterparty has. Note that this analysis is, and can only be, performed due to the lack of a liquid market for mortality-linked securities. When a liquid market would exist, the market price of longevity risk is available and an insurer could determine whether entering a swap is worth the cost charged by the market. Since a liquid market does not exist, the goal is to gain insight in the amount an insurer might spend on the swap by use of the fair value of an insurance contract considering Solvency II legislation.

Insurers have both assets and liabilities on their balance sheet and valueing the latter is difficult as there is no trivial method to determine the market value of current and future liabilities regarding life dependent insurance products. Therefore, Solvency II legislation proposes to determine the value by a mark-to-model approach (Stevens et al., 2010) in which the total value of liabilities must be calculated as the sum of a best estimate of current and future liabilities (BEL) and a risk margin (RM). The total value of liabilities is assumed to be a good representation of the value of a specific insurance contract for which it could be sold to another insurer, and therefore will be referred to as the *fair value* of a specific insurance contract. The risk margin is determined by the required current and future solvency capital and is only obligatory for liabilities subject to unhedgeable risk such as longevity. The risk margin plays an important role in pricing longevity risk, as it reflects the risk premium for longevity risk. As Börger (2009) points out, the required risk margin under SII can serve as a starting point for the pricing of longevity derivatives.

When a mortality-linked swap is entered, SII obliges insurers to maintain a capital buffer for the risk their counterparty cannot meet their obligations regarding swap payments in case of default. This might have a negative effect on the amount an insurer is willing to spend on a swap. Since the fair value of the insurer's liabilities under the swap increases when a risk margin for default risk must be maintained. In this thesis, only the risk that the counterparty of the insurer may default is considered and it is assumed that the insurer does not default before maturity of the swap contract. Note that longevity risk remains the underlying risk factor for a longevity swap exposed to default risk. In case default occurs the insurer is again exposed to longevity risk, which is unhedgeable. Therefore, a risk margin for counterparty default risk is required in order to price counterparty default risk considering a longevity swap.

The mortality-linked market is highly illiquid at this point, and the market is short longevity, i.e. the number of longevity hedge seekers is larger than the number of potential longevity risk buyers (see Section 2.2). In order to enter a mortality-linked contract a counterparty therefore requires a "risk" premium for assuming the risk. On the other hand, at inception of the contract the insurer would only prefer to enter a swap when the fair value of the insurance contract under the swap does not exceed the fair value of the contract when longevity risk is maintained. The premium an insurer prefers to pay for the swap plays an important role in the former, as it influences the fair value of the contract.

To hedge longevity risk with a longevity swap, an insurer receives floating leg payments based on the survival rate in a reference population and makes predetermined fixed leg payments, based on survival rates as well. The price or premium the insurer pays for the swap could be paid at once at inception of the contract, as a lump sum. This lump sum amount is determined such that the fair value of the insurance contract under the longevity swap does not exceed the fair value of the same insurance contract exposed to longevity risk. However, in practice, the premium is likely included in the fixed leg payments. Due to counterparty default risk an insurer might be hesitant to pay the premium as a lump sum. The total amount of premium that follows from the fair value approach is therefore also expressed in terms of mortality rates in line with the payoff structure of a longevity swap. More specifically, the required parallel multiplicative percentage reduction (in case of longevity) in best estimate mortality rates on which fixed leg payments depend is determined, such that the lump sum premium is spread out over the contract term. This gives insight in the fixed leg payments an insurer might accept when a swap is arranged.

According to SII (QIS5, 2010) the total amount of required solvency capital can be calculated by aggregating all $SCRs$ regarding the different risk factors. When aggregating, correlations between different risk factors, for instance longevity and market risk, are taken into account, which introduces diversification effects. In this thesis only the $SCRs$ for longevity and counterparty default are considered and other risk factors such as interest rate risk are excluded. Although it is acknowledged that the interest rate plays an important role in valueing the liabilities. In the base case where a perfect swap is entered and a full hedge is set up, the SCR for longevity risk is completely replaced by the SCR for default risk. Then, diversification effects can be disregarded. When a partial hedge is set up an insurer has to maintain both a SCR for longevity risk as well as for counterparty default risk. This thesis will discuss such a scenario as well.

When more insurance companies and potential counterparties are prepared to enter swap contracts the development of a mortality-linked market might get a boost. Therefore the requirements of such a market to develop and the steps that need to be taken to be able to trade mortality as a commodity are looked into. When mortality is traded as a commodity this is likely to attract new types of investors who seek instruments to diversify their portfolios, since it is typically assumed that mortality and longevity risk are uncorrelated with financial market risks. When different parties enter the mortality-linked security market this might improve liquidity as well as trigger its development. This in turn might lead to lower premium levels, i.e. a lower market price of mortality and longevity risk.

Several attempts have been made to manage longevity and mortality risk by trading these risks on the financial market. Recently Aegon entered a swap with Deutsche Bank AG in order to reduce longevity risk (Gaal, 2012). In 2007 JPMorgan developed the *LifeMetrics* toolbox in an attempt to get mortality and longevity risk traded on the financial market. They introduced the *q-forward* (Coughlan et al., 2007b), a mortality forward contract. Also, an attempt has been made to issue longevity bonds by the EIB and BNP Paribas in 2004. Clearly this market is still in its infancy and a lot has to be done to arrive at a developed, liquid market for mortality securities. New SII legislation may open opportunities since hedging mortality and longevity risks may be beneficial under SII legislation.

The total value of liabilities of insurance companies and pension funds is enormous, the amount of mortality and longevity risk they face is substantial as Loeys et al. (2007) points out. Therefore, the potential market for mortality-linked securities is huge. The fact that there are, at this moment, almost only over-the-counter transactions is likely due to the high illiquidity of the mortality market. According to Loeys et al. (2007), in order to create a form of liquidity the market will have to focus on a single type of product which can be easily used to hedge longevity or mortality risk. More importantly, a large challenge lies in creating the circumstances where needs of both hedgers and investors are satisfied.

Hedgers need an effective hedging instrument for the risk they want to mitigate, whereas investors demand market liquidity to be able to trade assets continuously. One of the advantages of a mortality-linked swap is that a liquid market is not a necessary prerequisite. However, the risk premium for longevity and mortality is unknown which creates difficulties determining the price of such a swap. Therefore, SII legislation is used in this thesis to gain insight in the risk premium for longevity risk.

This thesis is organized as follows. Chapter 2 focuses on the requirements of a market for mortality-linked securities to develop. Chapter 3 states the specific stochastic mortality model used here to predict future mortality, required for estimating both legs of a swap. In Chapter 3 the characteristics of a longevity swap are specified as well. This chapter also elaborates on the solvency requirements of longevity risk as well as the consequences of entering a longevity swap and the associated capital requirements under SII. Chapter 4 contains numerical results for the model specified in the previous chapter. Chapter 5 shows the sensitivity of results found in the previous chapter and Chapter 6 concludes. Finally Chapter 7 contains ideas for future research.

2 Developing a Life Market

Longevity and mortality risk are risk factors that are increasingly important for annuity providers, pension funds and life insurers since these risk factors cannot be pooled away, as De Waegenaere et al. (2010) show. Financial institutions exposed to mortality and longevity risk therefore might prefer to mitigate these risk factors by arranging a hedge. This chapter discusses the difficulties faced in developing a market for securities used to mitigate longevity and mortality risk, which is here referred to as a life market. One can think of difficulties such as the origination of a mortality-linked market, what kind of derivatives should be traded and at what cost these derivatives might be bought and sold. This chapter discusses several of these aspects.

2.1 Mortality and Longevity Risk

This section shortly elaborates on the definition of mortality and longevity risk and the parties or products exposed to these risk factors. In this thesis mortality and longevity risk are defined as the risk that individual mortality rates are higher than expected and the risk that individual mortality rates are lower than expected, respectively. However it is acknowledged that some authors of scientific literature refer to the risk in the distribution of survival rates as "longevity risk", as survival rates are not deterministic but stochastic. This specific, systematic, risk factor is here defined as "life risk", not to be confused with the life (underwriting) risk module for computing the $SCRs$ under SII (see QIS5, 2010). A distinction between mortality and longevity risk is made for two reasons. First, these two risk factors emerge in different financial products and expose their issuers to these two different risk factors. Second, as a consequence, SII makes a distinction between these two risk factors and treats them differently regarding solvency capital requirements. Note that the definition of longevity risk in the scientific literature refers to the systematic risk factor also called parameter or macro-longevity risk (see Hári et al., 2007). The non-systematic risk in survival rates is the risk that an individual's survival rate differs from the expected rates. This risk can be reduced to negligible proportions when the size of the portfolio of insured is large, see De Waegenaere et al. (2010). Life risk as defined here cannot be reduced by increasing the size of the portfolio, as is also the case with our definition of longevity and mortality risk.

Mortality risk is mainly run by life insurers who offer insurance products related to death benefits. Their liabilities increase in value when policyholders demise earlier than expected, in which case contracts are paid out earlier. Since the time value of money forces life insurers to have more assets at an earlier date the value of their liabilities increases. At the same time the total asset value may be smaller than anticipated since the total amount of premiums paid by policyholders decreases due to the early death of policyholders. Hence, the gap between available assets and liabilities becomes larger due to increasing mortality rates.

Pension funds and annuity providers generally face longevity risk, since these institutions guarantee a payment until death. When a pool of participants or policyholders on average lives longer than the expected age of death the provider of annuity income is obliged to make payments for a longer amount of time and therefore the total value of liabilities increases. When the total asset value remains unchanged the increase in total value of liabilities will lead to a deficit or may result in cutting benefits.

In the past, increases in life expectancy were underestimated substantially as Loeys et al. (2007) point out. Additional annuity payments could be made from financial assets due to flourishing financial markets. Interest rates and returns on all kinds of capital market instruments were high enough to compensate for the underestimated mortality rates. Decreased interest rates and returns over the past few years made underestimated longevity risk a larger problem and put longevity risk managment back on the agenda.

2.2 Life Market & Derivatives

In this section emphasis is put on the development and origination of a market for mortality-linked securities; some of the existing literature on this matter is covered. In this thesis longevity swaps are used to hedge longevity risk in specific insurance contracts. Although swaps can be traded over the counter, the development of a market for mortality-linked securities would be preferable for the development of an OTC longevity swap market. Since it will give insight in the market price of longevity and mortality risk causing hedge buyers and sellers to have a greater understanding of the price of these hedging instruments.

As stated in Chapter 1 a financial market on which mortality-linked securities are traded is still in its infancy. This life market has the potential to become a large global market, see for example Blake et al. (2008). Over the past years insurers and pension funds more and more recognize the importance of longevity and mortality risk. Their demand for longevity risk protection increased and they require financial instruments that can provide a hedge to mitigate these risks. Currently, no such instruments are traded and other ways are found to mitigate risk associated with longevity, such as reinsurance. However, as Blake et al. (2008) state, reinsurers do not have the capacity to take over the risk associated with longevity that may arise now and in the future. Hence, Blake et al. (2008) argue that a new financial market is required on which different parties can exchange longevity and mortality risk.

Loeys et al. (2007) state that the existence of a financial market is driven by the presence of "states of the world", i.e. outcomes of economic variables at certain future points in time. Mortality rates are an example of such an economic variable, and a state in which mortality rates are lower than expected is a risk for an insurer who provides annuity payments. Hence, an insurer might want a hedge to mitigate the risk of that certain state occurring. Moreover, Loeys et al. (2007) argue that a new market will only emerge when its benefits exceed the cost of creating it. They state that in general the value of a market depends on the number of agents exposed to a certain state of the world. The value increases when the number of agents with different exposures is large. Loeys et al. (2007) describe the cost as the required capital of bringing together buyers and sellers, the risks to the market makers and taxes incurred by the different parties. In order for a new market to emerge a few conditions must be satisfied according to Loeys et al. (2007). It must provide effective exposure, or hedging to a state of the world that is economically important and that cannot be hedged through existing market instruments, and it must use a homogeneous and transparent contract to permit exchange between agents. They state that longevity meets the basic conditions for a succesful market innovation.

Blake et al. (2008) go deeper into the requirements of a life market and more specifically into the conditions enumerated by Loeys et al. (2007). They state the importance of achieving liquidity. Liquidity could be enhanced by a life market depending on mortality indices which reflect national populations. These indices will serve as the underlying for payoffs of mortality-linked derivatives. However, this brings along a source of basis risk for the hedge buyers, as their insured populations, for which they require a product to mitigate longevity risk, differ from the national populations. This is covered in more detail in Section 2.4.

The establishment of a new market is justified when the number of potential buyers and sellers of the products exchanged on the life market is sufficient. Currently the number of institutions who are short longevity, meaning that the value of their liabilities increases when longevity increases, is larger than the number of institutions who are long longevity. Whose total liability value decreases or asset value increases when longevity increases, as Loeys et al. (2007) point out. Therefore the life market will be net short longevity. For a market to work properly there should be balance between demand and supply of longevity risk. In the life market this is clearly not the case and this influences the price of longevity risk. Buyers of longevity protection will have to pay a premium to attract investors, who otherwise might not want to assume the longevity risk of hedge buyers. This plays an important role in this thesis, as longevity hedge buyers such as insurers will have to pay a premium the question arises how high this premium should be. Here, the value of the swap is determined from the insurer's perspective, such that the insurer is indifferent between hedging and maintaining longevity risk.

According to Blake et al. (2008) the ivolvement of capital markets will lead to reduced costs of longevity risk because of the increase in capacity, together with greater pricing transparency and greater liquidity. This in turn will attract investors seeking assets to diversify their portfolios. Next to economic importance, for a new market to emerge it should offer instruments that provide a hedge for risks that cannot be hegded by already existing products. Loeys et al. (2007) conclude that existing markets do not provide a hedge for longevity and mortality risk. The products traded on the new emerging market should be transparent and homogeneous. The use of indices based on national populations will enhance transparency, but on the other hand causes basis risk. Therefore hedge buyers will have to make a trade-off between hedge-effectiveness and the associated costs.

2.3 Mortality-linked securities

Here some mortality-linked securities are discussed and the longevity bond as well as the mortality-linked swap are covered in more detail. Since the mortality-linked swap plays an important role in this thesis its characteristics are defined in Section 2.3.2. The effectiveness of the hedge depends on several of these characteristics. Some of the problems concerning the development of a life market are related to these, as will be explained in Section 2.3.1.

Loeys et al. (2007) and Coughlan et al. (2007b) argue that, especially in a developing market, in order to create liquidity a limited number of standardized contracts is required. They argue that a mortality forward contract is the simplest longevity and mortality derivative. In a q-forward contract two parties exchange, at maturity, a prespecified fixed mortality rate for a floating rate depending on realized mortality, this product can serve as the first building block of the emerging life market from which other derivatives can be constructed. Such as mortality and longevity (also called survivor) swaps, which will be elaborated on hereafter. Also, mortality options may be traded. Contracts in which the buyer has the right to purchase or sell a product at a predetermined price. The payoff then depends on the underlying (in this case mortality) index and the strike price. Also, mortality caps and floors might be developed. A difficulty for all mortality-linked securities is that they require a reference population or index to serve as underlying. Section 2.4 elaborates further on this matter. An even more important issue is the quantification and pricing of longevity risk and mortality-linked securities. Because there is no liquid market for these instruments the model cannot be calibrated to fit parameters and determine the price, i.e. risk premium for taking on mortality or longevity risk. In Section 2.5 some existing literature on this subject is considered.

2.3.1 Longevity Bond

The longevity bond is one the most basic instruments to hedge longevity risk. The effectiveness of the hedge depends on the characteristics, some of which also hold for swaps as will be argued later. In 2004 a first attempt was made by the EIB to issue longevity bonds. The EIB took over longevity risk from investors by selling the longevity bond and swapped the floating longevity linked obligations with BNP Paribas. BNP Paribas transferred the risk to Partner Re via reinsurance. The index was based on 65-year-old males from the UK and Wales and potential investors were UK pension funds. Coupon payments depended on the number of survivors at each payment date, for details see Blake et al. (2006). It did however not attract enough investors to actually be launched and its issue was withdrawn.

Several problems occurred making the issuance of the bond fail. First, according to Cui (2008) there is no clear view on how longevity risk should be charged. This bond required a risk premium of 20 basis points which was seen as too high by some annuity providers. Second, the reference population included only males which is not representative for a pension fund or insured population as these include women as well. Finally, since its maturity was only 25 years and this was fixed, there is no hedge for the full tail risk, i.e. the risk that many people outlived the last payment from the bond.

Therefore, it might be preferred when the maturity of the bond is dependent on the death of the last individual in the reference population, as Blake and Burrows (2001) state. They introduced their survivor bond earlier, with the following characteristics: beforehand a reference population is specified and a principal is paid by the bond buyer; at each future payment date a proportion of the principal is paid where the proportion is equal to the fraction of survivors in the reference population. Time of maturity depends on the moment the last individual in the reference population deceases. Furthermore, at maturity the principal is not repaid. However, this survivor (longevity) bond structure was pure theoretical. The most important characteristic in favor of the bond proposed by EIB/BNP Paribas is the fact that remaining tail risk is absent. Whether this bond would be preferred over the EIB bond depends highly on the reference population. As this was not specified it remains unclear whether investors would be willing to buy this bond to hedge longevity risk.

Blake and Burrows (2001) argue that the government should issue longevity bonds of some particular form. In certain countries governments issued inflation-linked bonds, they might also be willing to issue longevity bonds. This however might depend on the exposure the government already has to longevity risk. For instance, due to Pay-as-you-go (PAYG) pension systems a government might be reluctant to issue longevity bonds. Then, in case governments are short longevity, for the market to emerge, non-traditional players are required to issue mortality-linked products, as Loeys et al. (2007) state. Hedge funds or investment banks could be potential issuers, they however might require a significant price for offering such products and assuming mortality or longevity risk.

2.3.2 Mortality & Longevity Swaps

Here the longevity swap as used in this thesis is introduced. A mortality swap has similar characteristics but it provides a hedge for a different risk factor. Hence, the payoff structure of a mortality swap is such that mortality risk is hedged and is therefore different from the payoff structure of a longevity sawp. Depending on the source of risk a party determines whether a mortality or longevity swap (MS or LS) is required.

Consider a (vanilla) longevity swap, which provides a hedge against the uncertainty in the distribution of expected survival probabilities. Dowd et al. (2006) develop a longevity swap (called a survivor swap) to hedge longevity risk. Two parties, here an annuity insurer (A) who is exposed to longevity risk in his portfolio of annuitants, and a counterparty (B), agree to exchange payments at certain predetermined points in time for a certain period of time. A pays a preset amount, K_t, at each future payment date t (maturity T), and B pays a floating amount, S_t.

In practice the two parties will only exchange the positive net difference between the two payments. Hence, A pays B the amount $K_t - S_t$ if $K_t > S_t$, and B pays A an amount $S_t - K_t$ if $S_t > K_t$. A requires a payment when the realized number of survivors in his portfolio of insured is higher than anticipated beforehand. A however does not profit when the realized number of survivors turns out to be lower than expected. A (vanilla) MS works in a similar way, but to hedge mortality risk a life insurer requires a payment when realized mortality rates are higher than expected as the value of liabilities increases due to higher mortality.

The preset payments the annuity insurer makes depend on the currently expected future survival rates of a certain population agreed upon at inception of the swap contract. These are derived from expected mortality rates (see Section 3.1). The counterparty pays an amount depending on the realized survival rates of an index or reference population. These payments are stochastic at inception of the contract. Since the market is short longevity, theory (Loeys et al., 2007) suggests that the fixed leg payments will have to incorporate a certain premium to attract counterparties. Therefore the amount K_t the insurer pays will not only reflect expected survival rates but will likely include a premium. In order to model the different legs of the swap a method is required to predict mortality rates. Such a model is introduced in Chapter 3.

Note that the characteristics of a mortality-linked swap are similar to an interest rate swap (IRS). An IRS is a contract in which one party (A) agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a specified number of years. In return, it receives from an other party (B) interest at a floating rate on the same notional principal for the same period of time, see Hull (2008). An IRS could be used to transform a liability or asset. A swap can be arranged with an other party to transform a floating rate loan into a fixed rate loan. The same holds for an asset. In a liquid market, the price (also denoted by the fixed leg rate) of the swap can be determined by the condition that at inception of the contract the value of the swap should be equal for both parties. Once a term structure for the interest rate (yield) is specified, a model to determine the fixed leg rate of the swap is available.

There are some important differces between mortality-linked swaps and IRSs as well. First, regarding the preset payments. Whereas in an IRS the fixed payments are constant, the preset payments the insurer makes in a LS contract decline over time with the survivor index or reference population agreed upon at inception of the swap contract. The preset payments of the MS increase over time since the cumulative number of deaths in a population increases. Second, the floating leg of an IRS is tied to a market interest rate, for instance LIBOR. Whereas the floating leg of a LS (MS) depends on realized survival (mortality) rates of the survivor (mortality) index or reference population. Finally, the market for IRSs is complete and valuation (i.e. determining the fixed rate) can be done on basis of the yield curve, see Schumacher (2010) and Hull (2008) where zero-coupon bonds are used to determine the price of an IRS. Based on an arbitrage-free market, in order to price the zero-coupon bond, a change is made from the physical probability measure to the equivalent risk-neutral measure in order to determine the discount curve. Because the market is complete this measure is unique and by using it to price the zero-coupon bond the expected return and the risk associated with it are taken into account and the fair value of the IRS can be determined. The market for LSs and MSs is currently incomplete and a different valuation methodology is required since no unique equivalent probability measure exists.

An important risk factor that remains when a MS or LS is entered is the difference between the mortality of the insured population and the index or reference population. Assuming a reference population is used, floating leg payments depend on this reference population whereas a perfect hedge requires payments to depend on the insured population. Dowd et al. (2006) note that the amount of basis risk will be smaller when the two population's mortality is positively correlated. Nevertheless, it is a risk factor that remains and may be substantial.

2.4 Choice of Index

The success of any instrument traded on the life market will depend on the underlying index or reference population of the instruments, see for instance Blake et al. (2006). This section elaborates on indices and covers some difficulties in the development of such indices. Note that an index will be determined on the basis of a single, or multiple, reference populations. A method is developed to aggregate the reference populations in a single index. This method should be transparent and consistent, as argued by Sweeting (2010).

As stated earlier, the index must be such that it provides a useful hedge for any situation potential buyers want to hedge, such as variations in age and gender. This however might be difficult to achieve, as the payoff of a hedge for a population of a specific age and gender would preferably depend on a reference population with corresponding age and gender. An index however often consists of the mortality of a specific cohort, as for instance the indices of Deutsche Börse (2012). Sweeting (2010) lists the characteristics a good index should possess. Among others it should be unambiguous, measurable, appropriate, reflective and observable. Next to the criteria of Bailey (1992) which Sweeting (2010) mentions, objectivity and transparency as additional features are important. Especially when smoothing of the reference population is required because of variation in mortality rates that might not be due to deaths or other causes the applied method should be transparent. For variation caused by the finite number of individuals in the population smoothing might be desired. Appropriateness is important as well, a distinction has to be made between a customized and standardized hedge and determine what aspects of both hedges outweighs the other and makes one of the two more appropriate (see further for a more elaborate discussion). Next to some other criteria mentioned before, the index should be specified in advance, i.e. the composition of the index should be known in advance so possible hedges buyers have a good understanding on the underlying.

2.4.1 Customized vs. Standardized Hedge

A hedge without basis risk, where the reference population is equal to the insured population, would be preferred by an insurer setting up a hedge for mortality or longevity risk. Here the differences between such a perfect hedge and a hedge where the insured and reference population differ are considered. The floating leg payments of a longevity swap depend on an index or reference population and the effectiveness of the hedge will as such depend on the index or reference population.

It would be optimal for buyers of longevity and mortality hedges to create a customized hedge. In contrast to a standardized hedge, where the underlying will often depend on a national (sub)population, a customized hedge is set up such that the underlying (reference) population is equal to the insured population (Coughlan et al., 2007a). More specifically, the hedge is generally set up as a cash flow hedge such that the net cash flow is fixed with respect to changes in mortality rates. The maturity of the contract in a customized hedge should optimally be equal to the point the last survivor in the insured population deceases. In practice and in a standardized hedge however, potential counterparties are reluctant to such a contract and time the of maturity is more likely to be a fixed date.

In contrast to a customized hedge, a standardized hedge uses an index which often depends on the mortality experience of a national population, and is structured as a hedge of value. So any change in mortality rates affecting the value of liabilities is offset by a payment from the hedge contract. Stevens et al. (2009) argues that the most natural reference population for the insurer would be the population of insured. But the insurer may then have more information on the mortality of the insured population than the counterparty in the contract. Stevens et al. (2009) argues that when an insurer wants to buy longevity protection, for instance a swap, this might be a signal that the mortality of the insured population is lower than expected which might make the seller of the swap hesitant in entering the agreement, which could make the price of the swap higher.

Coughlan et al. (2007a) note some substantial advantages and disadvantages of custom versus standardized hedges. A customized hedge removes all basis risk but on the other hand is more expensive in the sense that a higher premium is expected to be paid, and set-up costs are higher. Also, since a swap with a custom underlying is set up such that it provides a perfect hedge for a particular insurer it will likely not be of great value for any other party, which deteriorates liquidity. According to Coughlan et al. (2007a) standardized hedges are cheaper. They have lower set-up costs and the risk premium is likely to be lower. Also the maturity of the contract will probably be less far away and is likely to be a fixed future date, which decreases the exposure to counterparty default risk. Next, since a common index is used the product is much more liquid. Nevertheless, the hedge will not be perfect and basis risk remains. Coughlan et al. (2007a) state that the amount of basis risk may be reduced if the correlation between the insured and index population is high. However, one can think of examples where correlation between a national population and an insured population may be small due to socio-economic or demographic differences. They also argue that the amount of basis risk can be minimized through careful construction of the hedge and periodic rebalancing. However, transaction costs may be high for such a strategy. Blake et al. (2008) argue that the presence of basis risk is a barrier that might be the reason insurers and pension funds until now have been reluctant to enter the life market. When this barrier is overcome, the next challenge would be the development of formal spot and derivatives exchanges (especially futures or (q-)forwards) for the development of the futures market. This simple standardized product, Loeys et al. (2007) aim to be the most likely start of a life market. It needs to be a large, liquid and volatile enough spot market in the underlying index to attract hedge sellers (investors) and hedge buyers as Loeys et al. (2007) argue. The higher the number of indices the more customized hedges become, which decreases liquidity. Hence, for a market to develop the number of indices is preferably not too large and the indices available should be such that they provide a hedge for the systematic part of longevity risk.

Note that for large pools of insured where the variety in insureds is large, indices based on national populations might be more appropriate compared to small pools. In the latter, non-systematic longevity risk will be important as well and hedges with indices based on national populations might not be as effective, compared to large pools where the diversity of the insureds is large.

2.4.2 Who determines a Longevity Index?

This section elaborates on which party should determine the index to serve as underlying for mortality-linked securities, and covers several parties currently offering such indices. Who determines the index is a relevant question, but above all, as long as the methodology used for constructing the index is both transparent and consistency is maintained, this might not affect the development of a life market. Moreover, potential investors should have as much information as possible.

JPMorgan launched its LifeMetrics toolkit to facilitate trading of mortality and longevity risk, and provided clear documentation on methodology used; they also present themself as a counterparty for potential buyers. Indices for the UK, US, Germany and the Netherlands are provided since these countries have substantial exposure to longevity risk and have high-quality mortality indices (Loeys et al., 2007). They claim q-forwards are the most basic derivatives and from there on a life market may develop into a more mature market with all sorts of derivatives, also Sweeting (2010) argues that JPMorgan's indices meet a number of the required characteristics. Earlier the attempt of Credit Suisse to create a longevity index for the US population failed due to lack of transparency and since it was inactively marketed, see Blake et al. (2008). Deutsche Börse developed several so called *Xpect* indices for both males and females of different cohorts and countries. Once every two months they provide the number of survivors of each cohort and a best estimate of the development of the number of survivors in the future. For details see Deutsche Börse. To attract investors and speculators, who both enhance and demand liquidity, a liquid market is important. Customized products might emerge when the life market is more developed. For now the market needs to be liquid and provide different agents with simple and transparent products and indices on which payments depend.

This shows that several parties develop and offer indices and it might depend on the party with which the swap is arranged which index will be used to serve as underlying.

The advantage of a swap contract is that liquidity is not a prerequisite for a swap market to develop. Since contracts are sold over-the-counter, only a transparent and up-to-date index or reference population for parties to enter a swap and exchange cash (flows) is required. Hence, next to q-forwards a swap market is likely to develop in early stages of the life market. However, a problem occurring is the pricing of the swap, more precisely, the determination of the premium included in the preset payments of the LS or MS. Wang (2000) proposes a method which transforms an existing probability distribution to an alternative probability distribution where the risk aversion of the market is incorporated. The market price of longevity risk then plays an important role in the transformation of the probability distribution. Whether this can be determined by products offered in the past or existing products is however debatable.

2.5 Quantification and Pricing of Longevity Risk

Next to the difficulties discussed in Section 2.2, the development of a life market is complicated by other factors. Potential counterparties might be hesitant to enter the market because of uncertainty regarding future mortality. Medical improvements and pandemics might cause counterparties to be exposed to possible enormous claims. An even more important factor is that there is no trivial answer to the question how to quantify and price longevity risk. This section puts emphasis on this matter and the approach used in this thesis.

As De Waegenaere et al. (2010) state, several distinctions can be made in quantifying longevity risk. For instance one could quantify longevity risk in annuity portfolios by determining its effect on the probability distribution of the present value of all future payments, for a given, deterministic and constant term structure of interest rates. Also, emphasis can be put on the probability of underfunding of a pension fund. Finally one could quantify longevity risk by determining its effect on the probability of ruin for a portfolio of longevity-linked securities. Where each of these approaches has its own drawbacks. The fact that there is no unambiguous answer to the question how to quantify longevity risk is certainly a reason why potential counterparties for hedge buyers do and did not enter the life market until now. How much should investors be compensated for assuming longevity risk? This also is a highly debated subject and no unambiguous answer is given until now.

2.5.1 Quantification of Longevity Risk

Stevens et al. (2009) quantify longevity risk by determining the minimal required size of the capital buffer, defined as the asset value in excess of the best estimate value of the liabilities, such that the probability that the insurer or pension fund will not be able to cover all future liabilities is sufficiently small. Note that this buffer is not based on SII legislation but solely focuses on meeting future obligations with a certain probability. Stevens et al. (2009) specify different insurance products, i.e. a single life annuity (old age pension), a survivor annuity (partner pension) and a death benefit. Each has its own liability structure and is valued differently depending on survival probabilities of the insured and spouse in case of a survivor annuity. Future liabilities are estimated by discounting projected survival probabilities, to arrive at the best estimate liability (BEL). The buffer is then defined as a percentage of BEL such that the probability of terminal asset value being negative, i.e. the insurer cannot meet all future obligations, is sufficiently small. Stevens et al. (2009) show that offering a combination of different insurance products can be beneficial for the insurer since a smaller buffer may be maintained. The gender mix influences the buffer as well. Since women on average live longer than men the buffer needs to be larger in case the men purchased a partner pension. Also, a riskier investment strategy typically requires a larger buffer since the return on risky assets is much more volatile compared to the volatility of one-year zero-coupon bonds. They also use a longevity swap to hedge longevity risk and calculate the required buffer size. It turns out that in case of a perfect hedge (basis risk is absent) longevity risk can be fully eliminated in case of a single life annuity. In case a product mix is used, the hedge is no longer perfect and a buffer is required. Nevertheless it is much lower than without the hedge. Including basis risk decreases the hedge effectiveness of the swap and a larger capital buffer is required to cover future liabilities.

2.5.2 Utility Pricing Approach

Cui (2008) proposes a method to price longevity risk. An equivalent utility based approach is applied based on the equivalent utility pricing principle which can be used in case of an incomplete market. The idea is that the required risk premium (here the price of longevity risk), also called shadow value in this context, can be interpreted as an additional amount of wealth added to the investor's budget so that he is indifferent between holding the asset to hedge longevity risk compared to the case where the asset is not held. An important aspect here is that the risk premium is investor specific, since it depends on the investor's preferences (CRRA and a modified CARA utility are used). In the context of the equivalent utility pricing principle, the risk premium that can be calculated represents the minimum compensation required by the seller and the maximum price acceptable for the buyer. To maximize expected utility of a shareholder (or manager) of a financial company, who derives utility from dividends and final wealth, asset allocation and dividend decisions have to be optimized. Cui (2008) distinguishes two scenarios, one in which the company issues coupon-based longevity bonds and is thus exposed to longevity risk for which it requires a premium; and another scenario in which no longevity risk is beared by the firm. The results show that risk premia for different maturities, size of initial capital and risk aversion of the insurer (bond issuer), are negative. Meaning that the bond yield is lower than the risk free rate so that the price of a longevity bond is higher than a risk free bond. Risk premia increase with maturity and decrease with initial capital of the bond issuer. Therefore the insurer is compensated for taking on longevity risk.

2.5.3 Securitization

Biffis and Blake (2009) adopt an entirely different method. They argue that until now pricing exercises were mainly based on partial equilibrium arguments which did not emphasize on how supply and demand might equilibrate when longevity exposures are exchanged. They use an equilibrium model, that shows how longevity risk premia are determined by the joint effect of uncertainty in longevity trends and regulatory costs. They allow for assymetric information on longevity exposure between the holder of the risk and potential investors seeking exposure to longevity risk. Transferring longevity exposures to the market via securitization is considered, where the issuer chooses the amount to securitize such that the expected discounted payoff of the securitization is maximized. The greater the exposure to longevity risk indicated by the issuer's private valuation, the greater the fraction of the exposure the issuer will wish to securitize. On the other hand, when investors anticipate that the fraction securitized increases in the private valuation of longevity risk, they will respond by decreasing demand for the security if the fraction securitized increases.

2.5.4 Forward Force of Mortality

There is some literature on modelling the forward force of mortality. Bauer et al. (2009) propose such a model to price longevity-linked securities. They build on earlier research by Bauer (2006) and Cairns et al. (2006) who proposed a framework for the forward force of mortality similar to the forward interest rate. The forward force of mortality is specified as a stochastic differential equation with a drift term similar to the Heath-Jarrow-Morton model (Hull, 2008) which adds uncertainty to the future force of mortality. By specifying an adequate volatility structure the model is fully described, since the drift term of the stochastic differential equation is of the Heath-Jarrow-Morton type. Then according to Bauer and Ruß (2006), given the initial term structure of mortality, one is able to determine expected payoffs of mortality-linked securities under the real world measure. However, in order to price a security we require the expected payoff under a risk-neutral measure. For this one has to specify the market price of risk (Bauer et al., 2007). In the Gaussian case of a deterministic volatility structure, and when a deterministic market price of risk is assumed, this leads to a pricing model. A difficulty arising from this model is that, as stated earlier, no liquid market for mortality-linked securities exists. Therefore it is impossible to determine market based parameters to drive the models and only historical data of mortality rates can be used to calibrate the model parameters. Whether this might cause problems and irregularities when pricing mortality-linked securities is unclear.

2.5.5 SII approach

This thesis follows an approach where SII legislation is applied to determine the maximum "price" of the swap such that the fair value of a specific insurance contract for which a longevity swap is entered does not exceed the fair value of the contract without the longevity hedge. This results in a total premium, an amount an insurer might pay at inception of the swap to the counterparty as a lump sum. The premium has to be paid in order for a potential counterparty to enter the swap and assume longevity risk. On the other hand, for the insurer to enter the swap, the premium paid should logically not exceed the cost of maintaining the capital requirements for longevity risk now and in the future, as Börger (2009). This result is used in this thesis as will become more clear in the next chapter.

An insurer exposed to longevity risk who does not enter a mortality-linked swap values his liabilities by a best estimate of his liabilities plus a risk margin for longevity, to approximate a "market" value. Since the liabilities cannot be valued at market value, as no such market exists, SII states that the market value, hereafter referred to as the fair value of liabilities, consist of a best estimate of liabilities plus a risk margin. The risk margin (also called market value margin) raises the value of liabilities to a transfer value at which the liabilities could be sold to another insurer. It is only required for risks that cannot be hedged in the financial market, as in those cases the market value is known.

When a perfect mortality-linked swap is entered, i.e. basis risk is absent, the liabilities of the insurer implicitly change. Instead of the obligations towards its policyholders, the insurer implicitly has an obligation towards its counterparty in the swap agreement to whom he pays the fixed leg of the swap. He also must maintain a risk margin for counterparty default risk, since default risk where the underlying remains longevity risk cannot be hedged. The sum of these represents the fair value of the insurance contract under the swap, required to determine the total premium that an insurer might pay for a swap.

When this premium is known it is interesting to determine how this amount is paid in terms of the fixed leg payments of the swap. Since in practice, the premium will likely be included in the fixed leg payments instead of being paid as a lump sum at inception of the swap contract. In the model proposed in this thesis, the premium an insurer might pay for a swap is spread out over the expected contract term by use of a parallel reduction of best estimate mortality rates on which fixed leg payments are based. When default risk is not taken into account this parallel reduction can be interpreted as the price of the swap, as it is a direct reflection of the lump sum premium.

When default risk is considered the total premium is affected by the risk margin required for default risk. This is discussed in Section 3.4. Also, the premium paid in the fixed leg is affected when default risk is taken into account, as there is a probability that not all swap payments take place.

The risk profile of the insurer changes when a swap is entered. The insurer substitutes longevity for counterparty default risk for which SII demands other capital requirements. The underlying risk factor remains longevity risk. When default occurs the insurer faces longevity risk again and this cannot be hedged by credit default instruments currently available, as these do not provide a hedge for longevity risk. Arranging a new swap might be very costly when the number of survivors is indeed larger than anticipated beforehand as a new counterparty is likely to charge a high price for such contract.

Due to the change in risk profile solvency capital requirements change. This could result in a capital release at inception of the swap contract which is an additional advantage the insurer might have from entering a mortality-linked swap.

3 Stochastic Mortality Model, Longevity Swap & SII

This chapter treats the mortality model used to fit historic mortality trends as well as predict future mortality. The methods to include uncertainty regarding the parameter estimates of different computation methods are considered, leading to uncertainty in the projected survival probabilities. Also, this chapter elaborates on those parts of SII legislation which are relevant for this thesis. Definitions are given regarding capital requirements insurers have to oblige. Next, emphasis is put on the differences between entering a swap compared to not hedging longevity risk. A model is proposed to determine the value of the insurer's liabilities when a swap contract is arranged. Also a model to determine the required solvency capital for counterparty default risk under SII regarding a longevity swap is proposed. Liabilities, solvency capital and the associated risk margin are defined since these are required for determining the premium an insurer might spend on a longevity swap. The capital requirements under SII play an important role in determining the fair value of a specific insurance contract.

3.1 Stochastic Mortality Model

When two parties agree to enter a longevity swap contract they agree on the future mortality rates on which the payments in the fixed leg depend. As these payments depend on future mortality both parties must have a method to estimate future mortality rates. Definitions and notation regarding mortality models are defined first (De Waegenaere et al., 2010, is followed in this thesis).

The one-year death probability, i.e. mortality rate, is defined as $q_{x,t}^{(g)}$. This represents the probability that an individual belonging to group g, aged x in year t will not survive another year, for $x = x_0, \dots, x_m$ and $t = t_0, \dots, t_n$. Then, the probability that this individual survives another year and reaches age $x+1$ is given by,

$$p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)}, \tag{3.1}$$

which is called the one-year survival rate. The probability that an individual survives for a certain number of years, say τ, is given by the product of one-year survival probabilities

$${}_{\tau}p_{x,t}^{(g)} = \prod_{i=0}^{\tau-1} p_{x+i,t+i}^{(g)}, \tag{3.2}$$

where ${}_1p_{x,t}^{(g)} = p_{x,t}^{(g)}$. The expected remaining lifetime of an individual aged x in year t belonging to group g can be estimated by

$$e_{x,t}^{(g)} = \sum_{\tau \geq 1} {}_{\tau}p_{x,t}^{(g)}. \tag{3.3}$$

When modelling future mortality often the raw central death rate is used. It is defined as

$$m_{x,t}^{(g)} = \frac{D_{x,t}^{(g)}}{E_{x,t}^{(g)}}, \tag{3.4}$$

where $D_{x,t}^{(g)}$ denotes the number of individuals aged x that deceased in group g during year t. $E_{x,t}^{(g)}$ denotes the exposure-to-risk, which is the number of people in g aged x exposed to the risk of dying during year t.

The raw central death rate can be interpreted as the instanteneous rate of death, i.e. the probability that an individual of group g aged x dies in the next ε time units from t, where ε becomes very small. Then one-year death probabilities can be calculated from the central death rate by (De Waegenaere et al., 2010)

$$q_{x,t}^{(g)} = 1 - \exp\left(-m_{x,t}^{(g)}\right), \tag{3.5}$$

with which cumulative survival rates can be computed by using (3.1) and (3.2).

3.1.1 Lee-Carter Model

There are various models to forecast the future mortality of a population. Lee and Carter (1992) (LC) developed a model to predict the central death rate, but other models are possible as well, for instance Cairns et al. (2005) specify a model on the basis of the one-year death probabilities. In this thesis the Lee-Carter model is used, it is defined as

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \tag{3.6}$$

where α_x represents the average level of the log-central death rates over time, β_x quantifies the age-specific sensitivity of the log-central death rates to variations in the time parameter κ_t and κ_t represents the time trend. The error term $\varepsilon_{x,t}$ with mean 0 and variance σ_ε^2 captures age and time-specific variations not captured by the model.

Due to identification problems, the parameters β_x and κ_t must be constrained. The constraints for β_x and κ_t are $\sum_x \beta_x = 1$ and $\sum_t \kappa_t = 0$, respectively. The parameter α_x can easily be estimated by averaging the log-central death rates over time. Lee and Carter (1992) propose to estimate the remaining parameters by Singular Value Decomposition (SVD) and re-estimate the parameter κ_t to fit the observed number of deaths. Since the optimal estimator for κ_t is given by the SVD composition it might not be optimal to adapt this estimate, as the data is already used in equation (3.4).

There are numerous other methods to estimate the parameters. Which method is used depends on the assumptions made. For instance, Brouhns et al. (2002) use the following assumption,

$$D_{x,t} \sim \text{Poi}\left(m_{x,t} E_{x,t}\right), \tag{3.7}$$

where $m_{x,t}$ is as in equation (3.6). Brouhns et al. (2002) use maximum likelihood to derive expressions for the parameters and estimate them by Newton-Rhapson techniques as no analytic solutions exist. Other extensions are binomial (see Cossette et al., 2007), or negative binomial models (see Delwarde et al., 2007) for the number of deaths. The advantage of these models is that they improve the Lee-Carter model on specific characteristics. The original Lee-Carter model for instance implicitly assumes homoskedastic errors. It can be argued that this is unrealistic as the log-central death rate is more variable at older ages than at young ages, as Brouhns et al. (2002) state. A better way to model this would therefore be to assume that the errors are heteroskedastic. This is the case when the number of deaths is assumed to be poisson distributed. The alternative models mentioned before which assume binomial or negative-binomial distributed deaths also resolve this issue.

In order to predict future central death rates the parameters in (3.6) must be estimated. In this thesis the approach of Brouhns et al. (2002) is followed to estimate the parameters of the Lee-Carter model.

Next, an ARIMA proces is defined to fit the estimated time parameter $\hat{\kappa}_t$. Lee and Carter (1992) use a random walk with drift (ARIMA(0,1,0)) but clearly it depends on the estimate of $\hat{\kappa}_t$ which ARIMA proces is required. Box-Jenkins methods can be applied to determine the number of autoregressive and moving average terms. However, often the random walk with drift suffices and will be used in this thesis. It is given by

$$\kappa_t = c + \kappa_{t-1} + \delta_t. \tag{3.8}$$

Here, c is the drift term and δ_t is a white noise process assumed to be normally distributed with mean 0 and variance σ_δ^2 (see De Waegenaere et al., 2010). Future mortality rates can then be obtained by projecting the time parameter $\hat{\kappa}_t$ and plugging the projected values in the expression along with the estimates of the other parameters,

$$\tilde{m}_{x,t_n+k} = m_{x,t_n} \cdot \exp\left(\hat{\alpha}_x + \hat{\beta}_x(\tilde{\kappa}_{t_n+k} - \hat{\kappa}_{t_n})\right). \tag{3.9}$$

Future one-year death and survival probabilities can be estimated by equation (3.5).

3.1.2 Bootstrap

In this section a simulation procedure is considered to take into account uncertainty in the projected mortality rates by the Lee-Carter model proposed in the previous section as well as uncertainty from fitting the Lee-Carter model parameters.

The factor that affects the predicted values the most is the uncertainty regarding the future values of the time parameter κ_t as Lee and Carter (1992) state. Especially the long term predictions of $\tilde{\kappa}_{t_n+k}$ are an important source of uncertainty. This uncertainty can be included by a simulation study in which random numbers are drawn from the normal distribution with mean 0 and variance σ_δ^2. These random numbers can then be included in the predicted out of sample values of $\hat{\kappa}_t$ by using the expression

$$\tilde{\kappa}_{t_n+k} = \hat{\kappa}_{t_n} + \hat{c} \cdot k + \sum_{j=1}^{k} \delta_{t_n+j}, \tag{3.10}$$

where δ_{t_n+j} represent the randomly drawn numbers. Using these values of future κ_t in (3.9) leads to predicted values of future central death rates. According to Lee and Carter (1992) the uncertainty from the parameters $\hat{\alpha}_x$ and $\hat{\beta}_x$ is probably not very large. However, Koissi et al. (2006) argue that the effects from these parameters should not be excluded since especially the short term effects can be substantial. This parameter uncertainty cannot be included easily and therefore a bootstrap procedure is used.

The exact bootstrap method one applies depends on the underlying model. When SVD is used to estimate the Lee-Carter parameters one can apply a residual bootstrap. Then, new data of $m_{x,t}$ is created by drawing random errors from the Lee-Carter model, i.e. $\varepsilon_{x,t} = m_{x,t} - \hat{m}_{x,t}$ where $\hat{m}_{x,t} = \hat{\alpha}_x + \hat{\beta}_x\hat{\kappa}_t$, these are added to the estimated central death rates $\hat{m}_{x,t}$ with which the parameters are estimated again. Future central death rates are predicted as explained above. Then, uncertainty from the model parameters is taken into account.

In case the parameters are estimated on the basis of the Poisson assumption in (3.7) a deviance bootstrap can be used, shown in Koissi et al. (2006). Alternatively the bootstrap procedure of Brouhns et al. (2005) can be applied, which is done in this thesis. The Lee-Carter parameters are estimated by use of the original data and used to estimate the number of deaths by $\hat{m}_{x,t}E_{x,t}$. Next, new data of number of deaths are sampled from the Poisson distribution with parameter $\hat{m}_{x,t}E_{x,t}$ and this new data is used to estimate the Lee-Carter parameters again. This procedure is repeated to include parameter uncertainty.

In addition, in each bootstrap iteration the uncertainty of predicting future values of the time parameter is included when (3.10) is applied. This results in wider confidence bounds around estimates of survival probabilities and expected remaining lifetime compared to the case when only parameter uncertainty is included.

Consider the cumulative survival probabilities of 65-year old Dutch males. Data is obtained from the Human Mortality Database to estimate the Lee-Carter model parameters and predict future central death rates. Calculations are based on sampleperiod $t = 1980, \ldots, 2009$ and age range $x = 0, \ldots, 100$. The residual bootstrap method corresponding with the SVD approach and the Poisson (ML Poisson) related bootstrap method from Brouhns et al. (2005) are applied and 25,000 simulations are performed. One can determine confidence intervals by sorting the simulated one-year survival probabilities. Cumulative survival probabilities can then be determined by use of equation (3.2). Results are shown in Figure 3.1. As shown in the figure, the best estimates (BE), i.e. the mean of the simulated survival rates, of both methods are almost identical and simulated confidence bounds differ for the two bootstrap methods. In this particular case the main reason for this is the variance of the error term δ_t in the ARIMA proces of κ_t in (3.8), as can be seen in Table 3.1. In this thesis the Poisson Maximum likelihood model will be

LC computation method	$\hat{c}$	$\hat{\sigma}_\delta^2$
SVD	-2.0511	6.3292
ML Poisson	-1.9773	4.3648

Table 3.1: Parameters of the ARIMA process in (3.8)

used, as the variance of the error term of the time parameter is smaller compared to the SVD approach.

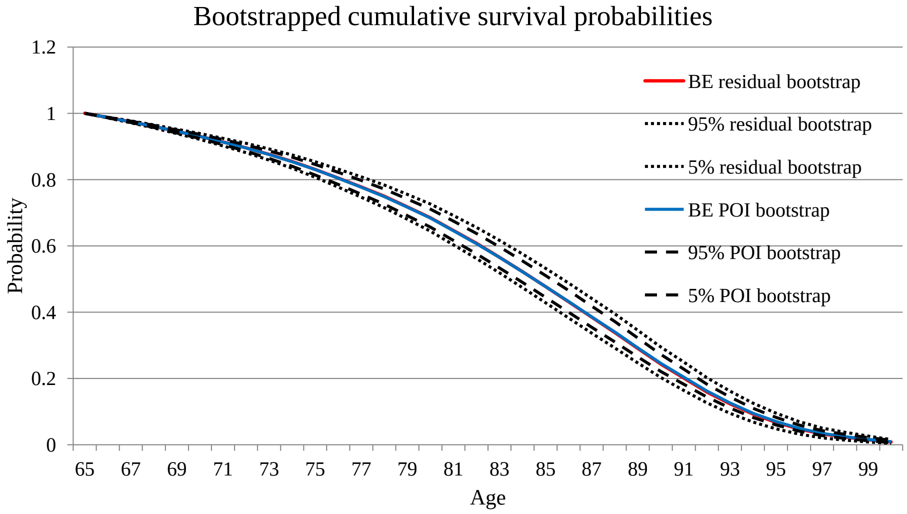


Figure 3.1: Cumulative survival probabilities of 65-year old Dutch males in 2012. Best estimate (BE) and 90% confidence intervals are given for both the residual and Poisson bootstrap method.

3.2 Longevity Swap

In Section 2.3.2 the basics of a longevity and mortality swap were discussed but details regarding the fixed and floating leg payments were omitted. Here a more detailed description of a longevity swap is given. A mortality swap has similar characteristics and the following can easily be adapted to arrive at the payoff structure of a mortality swap. A similar approach as Dowd et al. (2006) is followed but it is extended to incorporate the cumulative nature of both legs of the swap.

The fixed leg payments of a longevity swap decline over time with the number of survivors, more specifically, the payments are based on yearly cumulative survival rates. It is therefore assumed in this thesis that swap payments are made yearly, depending on the type of insurance contract. In practice it will be part of the swap contract to determine the maturity and dates at which payments are made. Here, a general payoff structure is defined with a notional amount N, which is a predetermined amount set at inception of the contract.

In the remainder of this thesis the subscript t in the notation of mortality and survival probabilities in (3.1) - (3.5) will be supressed as the time subscript from now on is required to denote the year in which swap payments take place and later on to specify the value of liabilities in any future year. Since swap payments are not constant and reflect the number of survivors in a specific population at a certain point in time, payments depend on the number of years since inception of the contract. For instance, the payoff after τ years depends on the number of survivors aged x_0, τ years after inception of the contract (in year t_0), ${}_{\tau}p_{x_0,t_0}$. This leads to multiple time subscripts t and τ which might be confusing. For convenience, time t_0 is therefore set to 0. When results are computed it will be specifically mentioned as of which year survival rates are used. Thus, the t year survivor rate from now on will be represented by ${}_tp_{x_0}$.

In Section 2.3.2 the payoff structure of the LS was omitted. Here both legs of the swap are expressed in terms of cumulative survival rates. The fixed leg payments are defined, using (3.1) and (3.2), as

$$K_t \equiv \prod_{i=0}^{t-1} \left(1 - (1-\pi) q^{\mathrm{BE}}_{x_0+i,i}\right) N = {}_tp^{\pi}_{x_0} \cdot N, \quad t \geq 1, \tag{3.11}$$

where the subscript t specifies any future year t. $\pi \in \mathbb{R}$ reflects the parallel reduction of best estimate mortality rates which reflects the premium an insurer pays for a swap. From this point, π will be referred to as the "premium in the fixed leg". As the insurer's fixed leg payments are higher than his best estimate survival rates the insurer makes payments regarding more survivors than expected, and thus pays a premium. q_x^{BE} in (3.11) are the best estimate future mortality rates at inception of the contract, and ${}_tp_{x_0}^{\pi}$ are the premium adjusted best estimate cumulative survival probabilities. These are exactly equal to the best estimate cumulative survival rates when π is equal to zero. Note that $\pi > 0$ corresponds to products with longevity risk and $\pi < 0$ in case of death benefit related insurance products. Since ${}_tp_{x_0}^{\pi}$ is larger than ${}_tp_{x_0}^{\mathrm{BE}}$ for $\pi > 0$, the survival rate for any t is larger than expected, which corresponds to longevity. The opposite holds for $\pi < 0$ and thus corresponds to mortality risk.

Here is chosen for a multiplicative form to incorporate π. This way of modelling the premium is preferred as it is in line with SII. Where the proposed stress to determine the SCR for mortality and longevity risk are modelled similarly as is discussed later, in Section 3.3.1. It is assumed the premium is constant over time and age. This implies a parallel shift in one-year mortality rates as argued before. The shift in cumulative survival and mortality rates is however not parallel as these are the product of one-year mortality rates.

The floating leg payments reflect the realized number of survivors of a certain reference population. At inception of the contract the two parties agree on this population and its size, i.e. the number of individuals in the population. This automatically is the initial exposure which is used to determine the realized mortality and survival rates. In case the two parties agreed that the reference population is equal to the insured population, basis risk is absent. The exposure of the portfolio reduces each year with the number of deaths of the corresponding year. The floating leg payment at t, S_t, is here defined as the estimated cumulative survival rate in the reference population, multiplied by the notional amount. Thus,

$$S_t \equiv {}_tp_{x_0}^{\mathrm{ref}} \cdot N, \quad t \geq 1, \tag{3.12}$$

where the superscript "ref" stands for the reference population. ${}_tp_{x_0}^{\mathrm{ref}}$ is the estimated realized fraction of survivors of the reference population aged x_0 in year 0 up to and including year t. This can for instance be estimated by,

$${}_tp_{x_0}^{\mathrm{ref}} = \prod_{i=0}^{t-1} p_{x_0+i,i}^{\mathrm{ref}} \cong \prod_{i=0}^{t-1} \exp\left(-D_{x_0+i,i}^{\mathrm{ref}}/E_{x_0+i,i}^{\mathrm{ref}}\right), \quad t \geq 1. \tag{3.13}$$

Here $E_{x_0+i,i}^{\mathrm{ref}}$ is the total exposure of individuals aged $x_0 + i$ in year $i > 0$, starting at $t = 0$, of the reference population. $D_{x_0+i,i}^{\mathrm{ref}}$ is the total number of deaths aged $x_0 + i$ in year $i > 0$ of the reference population. Equations (3.1), (3.4) and (3.5) are used to estimate the realized number of deaths. Note that survival rates are not observed, the number of deaths in a population are observed and the exposure decreases with the yearly number of deaths. Then an estimator for the survivor rate is given by the formulas mentioned above.

In order to model the floating leg a reference population has to be determined. When the insured population is taken as the reference population the swap is perfect and basis risk is absent. In this thesis, the insured population is chosen to be the Dutch male population. Therefore, the floating leg can be modelled by best estimate mortality rates of this population. Projected best estimate mortality rates are here determined by use of a stochastic mortality model. The Lee-Carter model described earlier is applied in this thesis.

For a mortality swap a similar method is used. The preset payments of the swap depend on the expected number of deaths of the index population, plus a premium, and the floating payments reflect realized number of deaths.

3.2.1 Payoff Structure Longevity Swap

The payoffs of both legs of a longevity swap were introduced in the previous section. This section illustrates the payoff structure of both legs to gain insight in the payments made when a longevity swap is entered.

Consider a Dutch male, aged 65 in 2012 who receives 1 Euro (in arrears) each year as long as he is alive. Data was obtained from the Human Mortality Database to calculate the Lee-Carter parameters and estimate future survival probabilities. The parameters and projections are based on the sample period $1980, \ldots, 2009$. Since the yearly swap payments are based on cumulative survival probabilities the payments of the fixed leg will look like the bar chart in the left graph of Figure 3.2, where for simplicity the premium π in (3.11) is set to zero. The notional amount is equal to one, which here reflects the payments the insured obtains. The bootstrap procedure (10,000 simulations were performed) of Section 3.1.2 is applied to obtain the best estimate and 90% confidence bounds shown in the figure. The quantiles reflect possible trajectories of realized survival rates, i.e. the floating leg of the swap. Then the payoff of the swap from the insurer's perspective is given in the right graph of Figure 3.2, as it is the difference between the floating leg, here the 95% (5%) quantile, and the fixed leg, here represented by the bar chart. The 95% quantile represents a scenario in which longevity occurs and the insurer obtains positive payments from the swap. Whereas the 5% quantile reflects a scenario in which longevity does not occur and the insurer incurs a loss. At inception of the contract the payoff is zero and as of

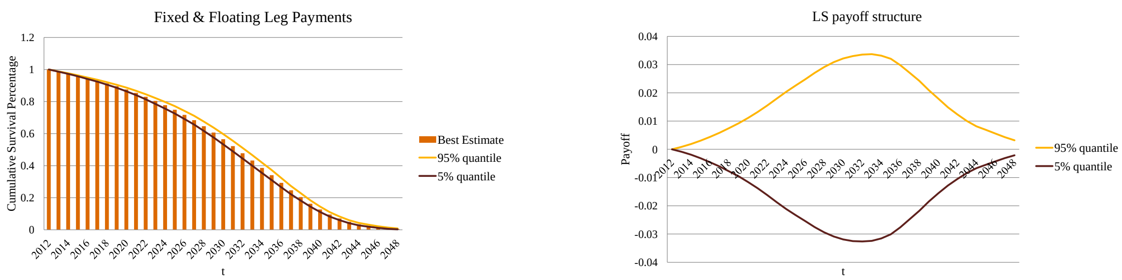


Figure 3.2: LS fixed leg payments, K_t, over time and two possible trajectories of the floating leg, S_t, (left) with the corresponding payoff structure of the two floating leg trajectories of a LS (right).

the first year (net) payments are exchanged. The difference between the floating and fixed leg is first increasing (decreasing) and then decreasing (increasing) for the 95% (5%) quantile since the uncertainty of future survival probabilities first becomes larger as age (time) increases. For high ages, i.e. high t, the probability of survival becomes small and the uncertainty in projections decreases. Therefore the payoff decreases in the 95% scenario. Note that including a positive premium, $\pi > 0$, which is likely in case of a longevity swap, will lower the net payoff in the 95% quantile and makes the payoff in the 5% quantile more negative, since the premium raises the fixed leg payments.

3.3 Solvency II

When SII becomes effective in 2014 European insurers part of the European Economic Area (EEA) face new legislation regarding capital requirements, risk managment standards and disclosure requirements (see PwC Guide to SII, 2011). This section elaborates on the solvency requirements SII obliges insurers to maintain relevant for this thesis.

Implementation of SII is built around three "pillars", see PwC Guide to SII (2011). Pillar I outlines the quantitative and qualitative requirements for the calculation of technical provisions (best estimate liabilities plus required risk margin, see (3.14)). The solvency requirements stated in this pillar are such that they might provide incentives for insurers to mitigate the risks on their balance sheet instead of maintaining solvency buffers. In this thesis merely emphasis is put on Pillar I as it are the quantitative requirements under SII that are relevant. In case a hedge is set up, next to the risk mitigating effect, it might result in a capital release due to the change in risk profile. The capital that comes free can for instance be used to issue new insurance policies which could lead to additional earnings.

Insurers are required to calculate and maintain a Solvency Capital Requirement (SCR) by using the standard formulas (QIS5, 2010) or by developing an internal model. An internal model might be preferred over the standard model as it gives a tool to quantify the risks as well as determine the required solvency capital. The SCR is part of the insurer's own funds and is derived by a value-at-risk approach and should be such that the insurer will meet its obligations over the next year with a probability of 99.5%. Next to the SCR, insurers are also obliged to determine a Minimum Capital Requirement (MCR). The MCR represents a minimum level below which the insurer's resources should not fall over a one-year period. The SCR should be covered by eligible own funds, whereas the MCR should only consist of basic own funds. SII defines own funds as the sum of basic own funds (on balance sheet items) and ancillary own funds (off balance sheet items). Own funds are divided into three so called Tiers. These Tiers represent the nature of the funds and SII states which sort of assets belong to each Tier, see PwC Guide to SII (2011). Eligibility of own funds is based on criteria regarding the percentage of the different tiers of which the own funds may exist. Further details on this matter are omitted in this thesis as they are irrelevant for the performed analysis.

Pillar II sets rules for insurers to develop and demonstrate an adequate system of governance and Pillar III states a new set of EEA-wide reporting and disclosure requirements. Insurers will have to report to their nation dependent supervisor, for instance, for the Netherlands this is De Nederlandsche Bank (DNB), who requires a regular supervisory report (RSR). Also, a public report is required in the form of an annual Solvency and Financial Condition Report (SFCR). The scope of this thesis is on pillar I and further details on the remaining pillars are therefore excluded.

3.3.1 Longevity & Mortality Risk Capital Requirements

As stated earlier, SII states that a capital buffer is required for each of the risks an insurer is exposed to. Next to the evident financial risks, such as interest rate, equity and currency risk, they also face risks regarding the nature of the products they offer. Insurers providing annuity contracts face longevity risk as their liabilities increase due to decreasing mortality rates. Insurers providing death benefit products on the other hand see the value of their liabilities increase when mortality rates increase. Here, solely the capital requirements for longevity risk are covered as these play a role in valueing longevity risk. It is acknowledged that interest rate risk is an important risk factor when the value of the insurer's liabilities is determined. However, the uncertainty regarding interest rate risk is left out.

The idea of the SCR is that insurers have capital available to absorb unexpected losses and are able to meet their obligations with a high degree of certainty. When a funding ratio approach is adopted, as Stevens et al. (2010) propose, the SCR at time t is defined as the difference between the total asset value at time t and the technical provisions, i.e. the fair value of the contract, at time t. Where the value of total assets should be such that the probability of underfunding at time $t+1$ is sufficiently small. As Stevens et al. (2010) show, this definition of the SCR leads to a circularity problem. In order to compute the SCR at t the technical provision at t is required. This however depends on the value of the risk margin which in turn depends on the future values of the SCR. This cirularity can be overcome by a simulation study. However, the number of simulations increases exponentially in the length of the run-off period, which typically can be very long for insurance products (see Stevens et al., 2010).

In this thesis, the premium spend on the swap depends on the fair value of an insurance contract, also referred to as technical provision in SII. The premium is determined by using the fair value of the insurance contract at inception of the swap arrangement. Therefore, only the value of the insurance contract at time $t = 0$ is relevant. The technical provision at $t = 0$ is defined as

$$TP_0 = BEL_0 + RM_0, \tag{3.14}$$

where BEL_0 denotes the best estimate value of liabilities at time $t = 0$ and RM_0 is the risk margin at $t = 0$ which must be added to BEL_0 according to SII to arrive at the fair value of a specific insurance contract, see European Commission Level II (2011). The best estimate of current liabilities highly depends on the nature of the insurance product, but a general expression can be given by

$$BEL_0 = \sum_{t \geq 0} \mathbb{E}_0(L_t) \cdot P_0(t). \tag{3.15}$$

Here L_t denotes the payoff of a certain liability at t, $\mathbb{E}_0(\cdot)$ denotes the expectation with information available up to and including time 0 and $P_0(t)$ is the price of a zero coupon bond at time 0 that matures at time t[1].

[1] Values of BEL_t for $t > 0$ are determined by $BEL_t = \sum_{s>0} \mathbb{E}_t(L_{t+s}) \cdot P_t(s)$. This gives an expression to compute the current best estimate value of liabilities in any future year t. These are required to determine both SCR_t and RM_0.

The risk margin, as calculated under SII, depends on the required solvency capital, SCR. The expression for the risk margin at $t = 0$ is,

$$RM_0 = \sum_{t \geq 0} CoC\% \cdot SCR_t \cdot P_0(t), \tag{3.16}$$

where $CoC\%$ represents the Cost-of-Capital rate, set by the regulator and equal to 6%. Note that SCR_t is unknown at time zero and therefore has to be estimated, methods to do this are described below. Therefore SCR_t are the current estimates of future solvency capital.

The risk margin can be interpreted in different ways. It it calculated as the cost of maintaining the required solvency capital now and in the future and therefore can be interpreted as such. Maintaining the capital buffer (SCR) prevents the insurer to use this capital freely and obtain a higher return than the return usually earned on reserve capital. The holders of the capital, i.e. shareholders, therefore require a premium for not obtaining the return on free capital. This is reflected by the cost-of-capital rate used to calculate the risk margin under SII (see Stevens et al., 2010).

As Brown (2012) and Stevens et al. (2010) argue, it can also be interpreted as a margin that raises the value of liabilities to a transfer or market value. In case an insurer would like to transfer its liabilities to another party the insurer will have to pay a premium to the party taking over the contract due to the uncertain nature of liabilities (Brown, 2012). The risk margin reflects the risk premium the insurer will have to pay. Though, a premium will only have to be paid for risks or liabilities that cannot be hedged or are not at market value yet. Therefore a risk margin is not maintained for interest rate or equity risk. The risk margin does not serve the purpose of a capital buffer in case longevity in the transferred liabilities occurs, since the required solvency capital is meant for this. Its mere purpose is that of a risk premium for a counterparty to take over the insurance contract, possibly exposed to longevity risk. The total amount of capital to be held by an insurer for a specific insurance contract exposed to longevity risk is thus the sum of technical provisions, i.e. the fair value of liabilities, and the required amount of solvency capital (SCR), as can be seen in Figure 3.3. In this thesis the fair value plays the most important role. The SCR is mainly required to determine the risk margin which determines the fair value of the contract.

In this thesis merely longevity risk and counterparty default risk are considered. As there is no market for mortality-linked securities the value of liabilities cannot be determined at market value and thus a risk margin must be maintained for longevity risk. The value of technical provisions is however not a market value, and thus cannot be referred to as such. Therefore, in this thesis, it is referred to as the fair value of the insurer's liabilities, i.e. of a specific insurance contract. The fair value is assumed to be a good approximation of the amount another insurer would charge for taking over the liabilities of a specific insurance contract. It is than implicitly assumed that the risk margin is a good representation of the risk premium for longevity risk. In case a liquid market for mortality-linked securities might develop in the future, liabilities can be valued at market value and a risk margin is no longer required.

Due to the circularity problem (mentioned in Stevens et al. (2010)) and the fact that a simulation study is computationally demanding, SII proposes a different method to calculate the SCR. For instance, the capital requirement for longevity risk may be calculated as the change in net asset value arising from a scenario-based stress. Longevity risk is caused by the unexpected decline of mortality rates. Hence, a longevity stress should be such that mortality rates decline compared to the best estimate mortality rates. The advised stress (QIS5, 2010) is a permanent decrease of 20% in mortality rates, which increases survival probabilities used to determine the BEL. Therefore,

$$SCR_t^{\text{longevity}} = \Delta \text{NAV}_t | \text{longevity stress}. \tag{3.17}$$

The net asset value is given by (Olivieri and Pitacco, 2008 is followed)

$$\text{NAV}_t = A_t - BEL_t, \tag{3.18}$$

where A_t represents the total asset value at t and BEL_t is as before. Hence, the change in net asset value can be written as

$$\Delta \text{NAV}_t = BEL_t^{-20\%} - BEL_t, \tag{3.19}$$

where the superscript -20% refers to the proposed longevity stress[2]. The risk margin computed by the

[2]A similar approach is used for mortality risk, but then a mortality stress has to be applied. The proposed stress is an increase in mortality rates of 15% (QIS5, 2010; European Commission Level II, 2011)

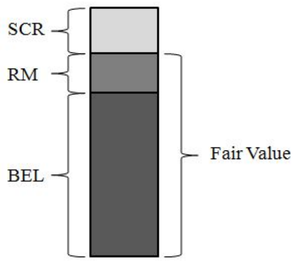


Figure 3.3: Capital requirements for an insurance product exposed to longevity risk. Consisting on the BEL, the SCR for longevity and the associated RM.

SCR as in (3.17) will be later referred to as $RM_0^{(\mathrm{I})}$, as a second method to compute future SCR is considered.

A different method to determine future capital requirements may affect the risk margin, as Börger (2009) points out. An often used method is the “driver approach”. This method assumes that the ratio of SCR and BEL is constant over time and is proposed as a simplification by QIS5 (2010) for (3.17) as it only requires SCR_0 from (3.17) and all future values can be determined by use of the ratio SCR_0/BEL_0. More specifically,

$$SCR_t = \frac{SCR_0}{BEL_0} BEL_t. \tag{3.20}$$

When (3.20) is substituted in (3.16) this leads to an alternative value of the risk margin, which will be later referred to as $RM_0^{(\mathrm{II})}$. Note that BEL_t are the current best estimates of future liabilities and are stochastic.

The differences between the different methods of computing the SCR by (3.17) or (3.20) will become clear later in this thesis. Börger (2009) states that in practice the method considered most exact is the one which uses the best estimate mortality evolution up to time t and stressed mortality as of t to compute SCR_t. This is done if (3.17) is applied for each year t.

The maximum price, i.e. premium paid by the insurer to the counterparty, of the swap is here determined such that the fair value of the insurance contract under the swap does not exceed the fair value of the contract without the swap. This requires a model for the value of the liabilities under the swap contract. Also, since SII obliges insurers to maintain a capital buffer for counterparty default risk and this has to be priced by using a risk margin. Therefore the next section elaborates on a method to determine the solvency capital requirement for default risk in case of a longevity swap where the underlying risk factor remains longevity. A distinction is made between the case when default risk of the counterparty is excluded, in which case solvency capital for default risk can be excluded, and a scenario in which default risk is taken into account. This impacts the fair value of the contract under the swap as a risk margin for default risk must be maintained. The method to determine the premium as well as the parallel reduction in best estimate mortality rates, to include the premium in the fixed leg payments, is described in Section 3.4.

3.3.2 Longevity Swap without Default Risk

This section proposes a method to value the liabilities of the insurer under the swap contract when default risk of the counterparty in the swap contract is excluded.

QIS5 (2010) states that in case of derivative contracts no risk margin for counterparty default risk has to be maintained. Only credit risk regarding reinsurance contracts and special purpose vehicles should be included in the risk margin. It is not trivial where to place a longevity swap. A risk margin is maintained for risk that cannot be hedged. Where interest rate swaps are clearly derivatives and default risk for such contracts can for instance be mitigated by entering credit default swaps this is not the case for longevity swaps. As the underlying longevity risk remains and cannot be hedged by such, or any other, products available in the market today. Therefore it might be seen as a reinsurance contract instead of the obvious derivative. Then, a risk margin for counterparty default risk should be maintained.

In order to determine the maximum premium such that the fair value of the liabilities under the swap does not exceed the fair value without the swap, as well as determining the premium in the fixed leg, i.e. π in (3.11), the value of the insurer's liabilities under the swap contract has to be specified. When default risk is considered, a risk margin for counterparty default risk should be taken into account to price the risk a counterparty does not meet its obligation towards the insurer regarding swap payments.

It is assumed that the hedge from the swap is perfect and the reference population is equal to the insured population. Then there is no basis risk and the floating leg payments from the swap completely hedges the insurer's obligations towards his policyholders. When the number of actual survivors is larger than anticipated the floating leg closes the gap between the required amount to pay policyholders and the payment anticipated beforehand. Then the insurer's liabilities implicitly become the obligations to its counterparty, i.e. the fixed leg payments, in the swap contract. The swap contract can therefore be seen as a change in the insurer's liabilities.

In this section default risk is not taken into account. This means that all swap payments take place and hence the insurer completely hedges the longevity risk embedded in certain insurance contracts. The obligations of the insurer under a perfect swap are then the fixed leg payments as the floating leg exactly matches the required amount to be paid to the insurer's policyholders. Therefore,

$$BEL_t^{\pi} = \sum_{s=t+1}^{T} K_s \cdot P_t(s-t), \tag{3.21}$$

with K_s as in (3.11). The superscript π is added to reflect the fact that BEL_t^{π} computed here depends on the premium π in the fixed leg of the swap. BEL_t^{π} reflects the current best estimate of liabilities in any future year t. Here, as default risk is excluded and the insurer has a deterministic liability stream, it holds that $SCR_t^{\text{cdr}} = 0$, $\forall t$ and therefore $RM_0^{\text{cdr}} = 0$.

In practice the value of π will be set at inception of the swap contract by the two parties. Here the value of π is determined by the fair value of liabilities in case the swap is not entered. The total premium the insurer pays may not exceed the costs of the insurer when no hedge is arranged, the risk margin for longevity represents this amount as Börger (2009) argues. Therefore the total premium spend on the swap may not exceed this quantity. π is determined such that the premium is spread out over the fixed leg payments over the contract term of the swap. As the total premium is the amount such that the fair value of both contracts is equal, π is the maximum premium to be paid in the fixed leg, given the best estimate of future mortality at time zero. Section 3.4 elaborates on this matter in more detail as well as the method to determine π.

3.3.3 Longevity Swap with Default Risk

When a perfect longevity swap agreement is entered the insurer hedges its longevity risk but becomes exposed to counterparty default risk. Here this risk factor is taken into account as it might affect the value of the swap contract from the insurer's perspective.

Reinsurance arrangements, derivative agreements, receivables from intermediaries and other risk mitigating contracts are all subject to counterparty default risk. The risk that the counterparty of the, in this context, insurer defaults before the end of the contract and cannot meets its obligations. SII requires protection for this risk factor in the form of a capital buffer (SCR) and distinguishes between counterparties. The creditworthiness of the counterparty, its rating or solvency ratio as well as the type of contract, i.e. reinsurance, securatizations, derivatives or any other credit exposure affect the size of the SCR associated to counterparty default risk.

Default risk is here reflected by the one-year probability of default of the insurer's counterparty in the swap agreement. Let D be a random variable representing the year of default. The one-year probability of default, $p^{(j)} \in [0,1]$, is assumed constant over time. It depends on the credit or solvency rating of the party. Here $j \in \{\text{AAA}, \text{AA}, \dots, \text{B \& CCC or lower}\}$ is considered. Values are given by SII legislation (QIS5, 2010; PwC Guide to SII, 2011) and are shown in Table 3.2. The probability of default is capped at a maximum of 4.175% for counterparties rated B or below. Default probabilities regarding unrated counterparties with a specific solvency ratio are not taken into account.

The one-year probability of default is constant. This however does not mean that the probability that default occurs two years from now is $p^{(j)}$. Default occurs in year two with probability $p^{(j)}$, but then default did not occur in year one with probability $1 - p^{(j)}$.

Rating j	$p^{(j)}$
AAA	0.002%
AA	0.01%
A	0.05%
BBB	0.24%
BB	1.2%
B & CCC or lower	4.175%

Table 3.2: One-year probabilities of default for counterparties with different credit ratings, according to QIS5 (2010).

For any future year s, two scenarios are taken into account. Default occurs after year s or does not occur at all for the duration of the contract, and default occured during or before year s. Hence,

$$\mathbb{P}(D > s) = (1 - p^{(j)})^s$$
$$\mathbb{P}(D \leq s) = 1 - (1 - p^{(j)})^s.$$

Any year prior to default the insurer has one liability stream, namely the fixed leg of the swap. It must however take into account the possibility of default of its counterparty and this is here taken into account in the valuation of the insurer's current and future liabilities. When the counterparty defaults the insurer has to pay its policyholders according to the realized number of survivors, i.e. he has the payoff structure of the floating leg. Therefore, the best estimate value of liabilities at time t becomes

$$\begin{aligned} BEL_t^{\pi} = &\sum_{s=t+1}^{T} \mathbb{P}(D > s) \cdot K_s \cdot P_t(s-t) \\ &+ \sum_{s=t+1}^{T} \mathbb{P}(D \leq s) \cdot \mathbb{E}_t(S_s) \cdot P_t(s-t). \end{aligned} \tag{3.22}$$

The insurer pays K_s when default occurs after year s or does not occur at all. In case default occured before or at s the insurer expects to pay $\mathbb{E}_t(S_s)$ in year s at time t. The floating leg is reflected by a best estimate of the insured population since it was assumed the swap is perfect and the reference population is equal to the insured population. In practice the floating leg is likely represented by a reference population unequal to the insured population, for instance the national population of a predefined country. Hence, a best estimate of the mortality rates of the reference population should be used to model the floating leg. BEL_t^{π} for $t > 0$ is again a representation of the current best estimate liabilities at time $t > 0$. Therefore BEL_t^{π}, for $t > 0$, takes into account the probability that default did or did not occur in the years before t.

The probability of default is taken into account as the possibility of a counterparty defaulting prior to maturity of the contact might affect the attractiveness to enter the swap contract. Hence, the premium the insurer might be willing to pay.

When an insurer is exposed to default risk the insurer is obliged to maintain solvency capital. The standard model for counterparty default risk in QIS5 (2010) is not applicable for longevity swaps as certain required input to determine the SCR is not available, such as the market value of the swap or risk mitigating effect (see QIS5, 2010). Therefore a method is proposed to determine the SCR for counterparty default risk, SCR_t^{cdr}, for default risk in a longevity swap contract.

The method is based on the fact that the underlying risk factor remains longevity. Even though this is perfectly hedged and default risk takes it place, realized survival rates determine whether the insurer is exposed to default risk. Since default only affects the insurer when the swap has value from the insurer's perspecive. A SCR must be maintained for those scenarios in which the insurer incurs a loss. Here, such a scenario corresponds to the case when default occurs and the number of survivors in the insured population is larger than anticipated. The SCR should be such that in the event of default, shocks regarding longevity in the insured population can be absorbed.

The SCR for counterparty default risk in each year up to maturity is defined here as the difference between expected current and future losses in a stress scenario, and a best estimate. Where the expected loss takes into account the probability that default occurs. At default the insurer loses its obligations towards its counterparty but in return again has an obligation towards its policyholders. Hence, in case of default, the insurer substitutes the fixed leg payments for uncertain floating leg payments. As these are exposed to longevity risk a capital buffer is required for the scenario in which longevity occurs and should be such that in case of default, the insurer can meet its obligations with a high degree of certainty. The loss incurred at a certain time is therefore equal to the present value of the floating leg payments weighted with the probability of default, since the swap is assumed to be perfect. A longevity stress is applied to reflect the losses in a stress scenario, in line with (3.17). Hereafter, the loss in case of default is often referred to as the exposure to counterparty default. Since the insurer is exposed to a loss each year default can occur until the maturity of the swap contract.

In SII the exposure to counterparty default reduces with collateral. However, including collateral in the analysis remains for future research. As before, SCR_t^{cdr} is the current best estimate of future solvency capital and is defined as,

$$SCR_t^{\text{cdr}} = EL_t^{\text{stress}} - EL_t^{\text{BE}}, \tag{3.23}$$

where EL_t represents the expected loss incurred in case the insurer's counterparty defaults during contract year t. A stress is applied to represent the loss in case of a scenario in which losses are high, i.e. the number of survivors in the floating leg is larger than anticipated. This method is in line with the method proposed to determine the SCR for longevity risk in (3.17).

The expected loss can be seen as the total current and future exposure to counterparty default risk. That is, the sum of discounted floating leg payments multiplied by the probability that default occured. The stress enters in year t and therefore it is assumed that for $t > 0$ the best estimate realized up to time t, as before. Therefore,

$$EL_t = \sum_{s=t+1}^{T} \mathbb{P}(D \leq s) \cdot \mathbb{E}_t(S_s) \cdot P_t(s-t), \tag{3.24}$$

where $P_t(s-t)$ is the price of a zero coupon bond at t maturing at time s and T is the year of maturity of the swap contract. The stressed scenario of EL_t can be determined by simulating the floating leg, S_s, and using the 99.5% quantile. A less computationally extensive alternative is applying the proposed longevity shock in SII to the floating leg to represent the stressed scenario. Since the loss becomes larger in case the number of survivors at a certain point in time is larger than anticipated. Depending on the method used to determine future values of SCR_t the associated risk margin can be computed by (3.16).

The required solvency capital for default risk does not depend on the fixed leg since this is deterministic at inception of the swap. π paid in the fixed leg by the insurer is agreed upon at inception and specifies the complete payoff structure of the fixed leg. The only stochastic factor is the floating leg which provides a hedge for longevity risk. Therefore, the SCR for counterparty default risk solely depends on the floating leg.

The stress that is incorporated here only enters through the mortality rates, as the swap payments depend on these. However, at the same time one could for instance also include a stress on the interest rate or probability of default. A drop in the interest rate increases the exposure to counterparty default which makes default even more problematic. A scenario in which it is more likely that a counterparty defaults could also be used, then the probability of default should increase.

This method to determine the SCR for default risk shows that the SCR is increasing when the creditworthiness of the counterparty decreases. Then, when the associated risk margin is determined by (3.16), this is higher for lower rated parties. This means that, in case an insurer has enters a swap with a BB rated counterparty and prefers to transfer his liabilities to another insurer, he will have to pay a higher premium compared to the scenario in which the swap was arranged with a higher rated party.

3.3.4 Method to estimate both Legs of the Swap

In order to estimate both legs of the swap the stochastic mortality model from Section 3.1.1 is applied to determine best estimate future mortality rates. Note that it is than implicitly assumed that both parties in the swap use the same model to predict future mortality. It is acknowledged that this does not necessarily has to be the case. As long as both parties are able to agree on the fixed leg of the swap and an index or reference population on which the floating leg depends a contract can be arranged.

There are two different methods to arrive at the floating leg in a stressed scenario. The first in which the longevity stress from SII is applied, i.e. the 20% shock on one-year mortality rates. The second is a method where the floating leg is simulated to estimate the 99.5% quantile of the expected loss. As the floating leg is stochastic the bootstrap procedure in Section 3.1.2 can be applied to determine the quantile.

First consider the fixed leg of the swap. A best estimate of one-year death probabilities, $q_{x,t}^{\mathrm{BE}}$, can be determined by taking the average of all bootstrap samples of $q_{x,t}$. With these, one-year survival probabilities and cumulative survival probabilities can be calculated which represent fixed leg payments. The premium is included by adjusting the one-year death probabilities as in (3.11).

A method is proposed here to simulate the floating leg. However, in the remainder of this thesis, when results are shown, the proposed longevity stress in QIS5 (2010) is applied to best estimate mortality rates to represent the stressed scenario. It is preferred as it is much less computationally exhaustive. Also, since the risk margin for longevity under the standard model is determined by this stress it is consistent to apply the same stress to determine the required solvency capital for default risk.

Simulating the floating leg is more complex as uncertainty has to be incorporated due to the randomness of realized survival rates at inception of the contract. For clarity the subscript t is added here. In each bootstrap iteration the predicted value $\hat{m}_{x,t}$ from bootstrapped Lee-Carter parameters is used to draw a random number of deaths $D_{x,t}^{\mathrm{ref}}$ for age x in year t from the Poisson distribution with intensity $\hat{\lambda}_{x,t}^{\mathrm{ref}} = E_{x,t}^{\mathrm{ref}} \cdot \hat{m}_{x,t}$. This reflects the number of deaths in the reference population of the floating leg. $E_{x,t}^{\mathrm{ref}}$ represents the number of individuals in the reference population alive at time t. This method is justified by the assumption made earlier in equation (3.7), also, because the Human Mortality Database (2012) uses a similar method to contstruct life tables. The randomly drawn number of deaths are then used to calculate the simulated realized one-year mortality rate for a specific age and time by equation

$$q_{x,t}^{\mathrm{ref}} = 1 - p_{x,t}^{\mathrm{ref}} = 1 - \exp\left(-D_{x,t}^{\mathrm{ref}}/E_{x,t}^{\mathrm{ref}}\right), \tag{3.25}$$

as in (3.13). Cumulative survival rates are then calculated by (3.2).

Repeating the bootstrap procedure provides a large number of simulated survival rates, which represent the floating leg. Together with the best estimate of future survival rates reflecting the fixed leg, the required solvency capital for default risk can be determined.

3.4 Determining the value of the premium

As stated before, the maximum (total) premium to pay for the swap is determined such that the fair value of the insurance contract in which a swap is arranged does not exceed the fair value of the contract where longevity risk is not hedged. The fair value is determined by the capital requirements under SII. Next, the total premium is translated into a parallel one-time percentage reduction in one-year best estimate mortality rates as these determine the payments of the fixed leg of a longevity swap. This is done by modelling the fixed leg as in (3.11) and best estimate liabilities as in (3.21) or (3.22). π denotes this parallel reduction and hereafter will also be referred to as the premium in the fixed leg of the swap. Since this reduction causes the insurer to make payments larger than his best estimate survival rates, with which BEL is determined, from the insurer's perspective a premium is paid. This section elaborates on the method used to determine the total amount of premium an insurer might pay for a swap as well as the method to determine the shift in best estimate mortality rates, π.

3.4.1 Fair Value no Swap vs. Fair Value Swap

In order to determine the amount an insurer might pay for the swap in order to be better of, or at least not to be worse off, compared to the scenario in which longevity risk is not hedged, a distinction must be made between the scenarios in which the insurer enters a swap agreement or not. This is done in this section in order to make clear how the premium is determined.

The insurer would be indifferent when he pays a premium such that the fair value of his liabilities at inception, i.e. the best estimate plus the risk margin, is equal in the scenario in which no hedge is set up and in which is a swap is arranged, respectively. The premium found in that case reflects the maximum premium an insurer might accept. When a higher price is charged the insurer might be worse off at inception of the contract. Then however, only the fair value of the contract is considered. The capital release due to the change in risk profile, results in a possible decrease in the required amount of solvency capital which may also affect the decision to enter a longevity swap.

When a longevity swap contract is not entered the insurer must maintain solvency capital for longevity risk, as well as an associated risk margin. In case a swap is arranged for a certain insurance contract with associated portfolio of insured, and a full hedge is set up, i.e. all the insurer's obligations of this specific portfolio are hedged by the floating leg payments from the swap agreement, solvency capital for counterparty default risk should be maintained and a capital buffer for longevity risk is no longer required. Then a risk margin for default risk prices counterparty default risk and raises the best estimate of liabilities under the swap to a fair value.

A swap agreement obliges the insurer to make fixed leg payments. In practice these will likely be higher than payments based on best estimate survival rates due to the premium the counterparty charges. Here this is modelled as a parallel shift of one-year best estimate mortality rates in line with the stress on mortality rates imposed by SII for calculating the SCR for longevity risk, as was shown in (3.11). The cumulative survival rates do not shift in a parallel manner, as these are not computed by a linear transformation. This results in premium adjusted survival rates on which fixed leg payments are based.

In practice π will be agreed upon at inception of the swap contract and may depend on the beliefs of the counterparty of the exposure to longevity risk of the contract for which the insurer wishes to arrange a swap. Also, the counterparty might have hedge potential from entering the swap in case the counterparty is for instance exposed to mortality risk.

When a perfect hedge is set up, the liabilities of the insurer implicitly change, as argued in Section 3.3.2 and 3.3.3. The liabilities regarding his policyholders remain the same, but when the hedge from the swap is perfect the longevity risk in the portfolio is completely hedged by floating leg payments. Hence, the insurer's liabilities changes to its obligations towards the counterparty in the swap who demands fixed leg payments. The value of the insurer's liabilities is therefore equal to the value of the fixed leg payments. When the swap is not perfect, some longevity risk remains. Then, for the remaining longevity risk a SCR and RM for longevity risk have to be maintained. This is also the case when only a partial hedge is set up. These capital requirements will however be smaller than without the swap, since a fraction of the longevity risk in the contract is hedged.

To determine the total premium an insurer maximally pays for a perfect swap, the fair values of both contracts are equated. That is, the fair value of the contract without the swap (left) and the fair value of the insurance contract for which a swap is entered (right), respectively, in (3.26). Hence,

$$BEL_0 + RM_0^{\text{longevity}} = BEL_0^{\pi} + RM_0^{\text{cdr}} \tag{3.26}$$

The total premium to pay for the swap at inception of the contract are the additional costs in excess of the value of liabilities without the swap plus the risk margin for default risk. Therefore, the right hand side of (3.26) can be written as

$$BEL_0^{\pi} + RM_0^{\text{cdr}} = BEL_0 + RM_0^{\text{cdr}} + \mathbb{E}_0(\Pi). \tag{3.27}$$

Before it was argued that entering a perfect swap implicitly changes the insurer's liabilities to the fixed leg payments, in order to include the premium paid in the fixed leg when the value of the insurer's liabilities is determined. One can also look at it differently. The insurer still has obligations towards its policyholders, reflected by BEL_0, but also pays a premium to its counterparty in the swap contract. This is done in (3.27). Where $\mathbb{E}_0(\Pi)$ is the total premium paid for the swap. Using both (3.26) and (3.27), this is equivalent to writing

$$\begin{aligned}\mathbb{E}_0(\Pi) &= BEL_0^{\pi} - BEL_0 \\ &= RM_0^{\text{longevity}} - RM_0^{\text{cdr}}\end{aligned} \tag{3.28}$$

This shows that the total premium the insurer might pay at inception of the swap contract as a lump sum, should not exceed the cost of maintaining current and future solvency capital, i.e. the difference in risk margins under the two scenarios (swap and no swap respectively), at inception of the contract. In case a counterparty with a specific rating would, in total, charge a higher premium than this amount, maintaining the solvency requirements for longevity risk might be better than entering a swap, with information available up to time zero, when the fair value of the contracts are considered.

As Börger (2009) states, when longevity risk is fully hedged, solvency capital for longevity risk is no longer required and therefore the cost of current and future solvency capital becomes zero. Therefore an insurer might be interested to enter a swap contract when the price does not exceed the cost of current and future solvency capital. This by definition is equal to the risk margin for longevity as can be seen in (3.16). Note that Börger (2009) does not consider counterparty default risk, which is taken into account in this thesis.

When default risk is excluded, as in Section 3.3.2, the premium an insurer maximally pays at inception is equal to the risk margin for longevity, since $SCR_t^{\text{cdr}} = 0$ for all t and therefore $RM_0^{\text{cdr}} = 0$. When default risk is taken into account the amount paid for the swap reduces with the risk margin for counterparty default risk as the fair value of the liabilities under the swap increases.

The value of π for which equation (3.26) is solved when default risk is excluded, i.e. $RM_0^{\text{cdr}} = 0$ and BEL_t^{π} as in (3.21), is the maximum premium in the fixed leg an insurer might accept at inception of the contract in order to be indifferent between entering a swap or maintaining solvency capital for longevity risk. Then, $\mathbb{E}_0(\Pi)$ is perfectly spread out over the contract term taking into account the structure of the fixed leg payments of a longevity swap. This equation can only be solved numerically since no analytic solution exists. π is included in a non-linear way in the variables on the right hand side of the equation, therefore an iterative procedure must be applied to find the value of π that best solves the equation.

When default risk is taken into account the value of π that solves equation (3.26) gives insight in the premium in the fixed leg such that the total premium, $\mathbb{E}_0(\Pi)$, is spread out over the expected number of years before default. Note that in case default risk is considered, $\mathbb{E}_0(\Pi)$ is the most important factor quantifying the premium of the swap at inception. Since the reduction in best estimate mortality rates, π, no longer is such that $\mathbb{E}_0(\Pi)$ is spread out over the entire contract term. Instead it is spread out over the years before default is expected to occur. Therefore, π in (3.11) can no longer be interpreted as the maximum premium an insurer might be willing to pay in the fixed leg of the swap.

The positive probability of default means there is a probability that not all swap payments take place. Intuitively the willingness of the insurer to enter the swap therefore declines as the effectiveness of the hedge deteriorates. This causes the amount an insurer might be prepared to spend on the swap, $\mathbb{E}_0(\Pi)$, to decline.

3.5 Diversification through Partial Hedge

This section elaborates on the consequences of entering a partial hedge for longevity risk. Then, an insurer is obliged to maintain solvency capital for both longevity and counterparty default risk.

As argued in Section 3.3, the required amount of solvency capital is such that insurers meet their obligations in the next year with 99.5% certainty. In practice, insurers are exposed to multiple risk factors. SII proposes to apply an aggregation method for determining the total amount of solvency capital, taking into account correlations, such that the total amount of aggregated solvency capital is sufficient to meet obligations towards the insurer's policyholders in the next year with 99.5% certainty.

The SII standard model (QIS5, 2010) permits insurers to aggregate the $SCRs$ of different risk modules in two stages. First, $SCRs$ are aggregated within a specific risk module, such as longevity and mortality risk in the life risk module. Second, the $SCRs$ of the different risk modules are aggregated, such as the total $SCRs$ of life and market risk. The standard model aggregates on the basis of a variance-covariance method where it is assumed that the underlying loss functions are normally distributed.

Here a method is proposed to determine the required total amount of solvency capital in case a partial hedge is set up by use of a longevity swap. The framework proposed here is in line with the aggregation method of the SII standard model and is applied to aggregate the $SCRs$ for longevity and default risk, which an insurer is obliged to maintain in case of a partial hedge.

When a full hedge is set up by a longevity swap, i.e. all risk regarding longevity in a specific portfolio is hedged by the swap, the solvency capital for longevity risk is no longer required. Instead, a capital buffer for counterparty default risk must be held. Alternatively, an insurer can decide to hedge only part of the porfolio's liabilities. For instance by only including a fraction of the number of policyholders in the swap or by hedging only the policyholders of a specific age in the portfolio. Here, a partial hedge is considered on the basis of the notional of a specific insurance contract for a portfolio of insured. Then the notional amount for which a hedge is set up becomes,

$$N^{\text{hedge}} = (1-\theta) \cdot N, \tag{3.29}$$

where $\theta \in [0,1]$ is the fraction excluded from the hedge. The two boundaries, $\theta = 0$ and $\theta = 1$ are the cases where the hedge is set up for the entire portfolio's notional amount, and no hedge is set up, respectively. For all values in between, the hedge is partial and only a fraction $1-\theta$ of the notional is hedged. Then, both solvency capital for longevity risk and counterparty default risk has to be held. The risk margin is computed for the total amount of required solvency capital. This way of modelling a partial hedge is used as the SCR and RM depend on the notional amount and therefore are sensitive to the value of θ chosen.

Diversification plays an important role due to the aggregation method the standard model (QIS5, 2010) permits. In the set up here, where only longevity and default risk are considered this means that when the variance-covariance method is applied the total amount of required solvency capital can be determined by (QIS5, 2010),

$$SCR_t^{\text{Total}} = \sqrt{(SCR_t^{\text{longevity}})^2 + (SCR_t^{\text{cdr}})^2 + 2 \cdot \rho \cdot SCR_t^{\text{longevity}} \cdot SCR_t^{\text{cdr}}}, \tag{3.30}$$

where $\rho \in (-1,1)$ is the correlation coefficient between longevity and counterparty default risk and both $SCR_t^{\text{longevity}}$ and SCR_t^{cdr} are current estimates of the required amount of future solvency capital, for longevity and default risk respectively.

For $\theta < 1$ the SCR_t^{Total} is likely to be lower than the $SCR_t^{\text{longevity}}$ in case no hedge is arranged, due to the proposed aggregation method and correlation between the two risk factors. Hence, an advantage of entering a partial hedge might be that a smaller amount of solvency capital may be maintained.

As stated above, this aggregation method is based on the underlying assumption that the underlying loss for which the SCR is determined is a normally distributed random variable. The total loss is then determined by summing the normally distributed random variables of all losses. This results in a multivariate normally distributed random variable. The variance of such a distribution, in this case for two risk factors, is as in (3.30), depending on the correlation between the different losses.

For longevity risk one can argue that losses indeed are normally distributed as the mortality experience of a population can both be positive or negative compared to the best estimate mortality with which the value of liabilities is determined. Hence, the loss incurred from realized mortality can both be positive or negative. For default risk, with underlying risk factor longevity, this assumption might be less appropriate as default occurs or not.

Altogether, the downsides and debatable appropriateness of this method in this specific case are acknowledged. Since the standard model in QIS5 (2010) uses a similar approach to aggregate the different solvency buffers of different risk factors this method is applied in order to gain insight and explore the likely advantages in a SII framework when a longevity swap is entered.

The value of ρ is not trivial to determine. According to the standard model (QIS5, 2010), the correlation between default risk and the sub module life risk, which incorporates all risks regarding life such as mortality, longevity, health etc., is 0.25. This however might not be a good approximation of the correlation of longevity and default risk. The proposed correlation coefficients in SII are such that they represent the correlation between the different risk drivers and not so much between the two normally distributed random variables.

It is again assumed that the fraction of liabilities for which a hedge is set up by the swap is hedged perfectly and no basis risk remains. Then the $SCRs$ for both longevity and default risk can be determined by the methods and formulas described before in (3.17) and (3.23) respectively. The adjusted notional amount is used to determine the value of liabilities for both cases. The total risk margin at time zero, RM_0^{Total}, is determined by use of the total amount of required solvency capital in (3.30) by applying (3.16). Then the effect from diversification is also included in the risk margin. The risk premium an insurer might pay for the swap contract then becomes

$$\mathbb{E}_0(\Pi) = RM_0^{\text{longevity}} - RM_0^{\text{Total}}. \tag{3.31}$$

4 Longevity Hedge

In this chapter numerical results are given for the premium an insurer might pay for a swap as a lump sum amount at inception of the swap contract, as well as the corresponding premium in the fixed leg. Results are shown for two different insurance contracts. First an endownment contract is considered where a lump sum payment is made at a future date conditional on the survival of the policyholder to that future point in time. Second, an annuity contract is considered which pays a certain amount on a regular basis as of retirement of the policyholder for as long as the annuitant is alive. Then, characteristics of the contract, such as maturity, notional amount, retirement age, etc. play an important role as these have consequences for the value of the insurer's liabilities and thereby on the value of the swap contract.

In the remainder of this thesis, unless mentioned otherwise, the Lee-Carter parameters are estimated on age range $0, \ldots, 100$ and sample period $1980, \ldots, 2009$. The required data is obtained from the Human Mortality Database (2012), Dutch national mortality data is used through this entire chapter. Best estimate projected cumulative survival rates are determined by a Poisson bootstrap procedure where the Lee-Carter parameters are calculated by Newton Rhapson techniques, due to the assumption that the number of deaths at a specific age and time are Poisson distributed. 50,000 simulations were performed to compute best estimate mortality rates as of 2012.

It is assumed that the hedge by the swap is perfect. Hence, the insured population is taken as the reference population and thus basis risk is absent. Both populations are represented by the Dutch male population. The swap's maturity is equal to the maximum attainable age in the insurance contract, here assumed to be 100. This is done due to the irregularity in mortality rates for ages higher than 100. As the number of observations for ages over 100 becomes small the parameter estimates and projections are not reliable and therefore are excluded.

Results are also computed for mortality rates projected by the Dutch AG (Actuarieel Genootschap, 2012). The mortality rates from the Dutch AG are used as input for best estimate mortality projections. The Dutch AG's mortality projections are determined for ages 0 to 120 and projected for the period 2012 to 2062. The insurance contracts under consideration here all start at age 65 or higher and start in year 2012. Hence, results based on AG projected mortality have maximum attainable age 115 since mortality rates are projected only 50 years into the future.

Furthermore, historic mortality rates based on the data obtained from the HMD are determined and used as well to determine the premium to be paid for a swap. This is done to examine the development of the premium over time. Instead of projecting best estimate mortality rates the documented exposures ($E_{x,t}$) and number of deaths ($D_{x,t}$) by the HMD, for each age and year, are used to estimate realized mortality rates, q_x^{realized}. This is done for the period 1974 to 2009, since the maximum attainable age is again 100 and in order to obtain results for ages 65-100 we require this period where time zero corresponds to 1974. Thus, contracts are considered that start in 1974, i.e. a 65-year old in 1974, who maximally attain age 100 in 2009. So q_x^{BE} in (3.11) is replaced by q_x^{realized} and the floating leg is also modelled by q_x^{realized}. Where q_x^{realized} is computed by use of (3.4) and (3.5) using the observed $E_{x,t}$ and $D_{x,t}$ by the HMD for the period 1974-2009.

When results are shown for mortality rates other than the projections from the Lee-Carter model this is specifically mentioned.

In order to determine the required solvency capital for longevity and counterparty default risk the best estimate mortality rates are stressed. This can be done by two methods as argued in the previous chapter, i.e. the proposed longevity stress from SII or by a simulation study in case the quantile of the simulated distribution is taken. Here, all results are based on the longevity stress proposed in SII as this is less computationally demanding and more in line with the way the premium in the fixed leg, i.e. π in (3.11), is modelled.

4.1 Endowment

Consider an insurer who has a portfolio of M currently x_0-year old Dutch males. The M policyholders get a single payment of, say Z, when they are still alive T years from now at age $x_0 + T$. Hence, the notional N of this contract is $N = M \cdot Z$. Several assumptions have to be made. Premium payments by the policyholders are left out here, payments are made at the beginning of the year T. Further, there is no new inflow in the portfolio. Hence, the only risk factor here is the uncertainty regarding future mortality rates. The insurer wishes to arrange a longevity swap for this portfolio of insureds to hedge the risk that a higher percentage of the policyholders than expected by current best estimate survival rates is alive T years from now. Since there only is one payment from the insurer to the policyholders, if alive, only a single swap payment is required to hedge longevity risk, at maturity T. In case more individuals survived at maturity than expected, the floating leg payment exactly hedges the amount the insurer requires to make payments to its policyholders according to realized survival rates.

First step in calculating the premium, $\mathbb{E}_0(\Pi)$ as well as the premium in the fixed leg, π, is determining the fair value of the contract when the longevity swap is not entered. Then $SCR_t^{\text{longevity}}$ must be maintained for the duration of the contract along with the associated risk margin.

The general expression for the current best estimate of future liabilities, BEL_t, is given by

$$\begin{aligned} BEL_t &= {}_tp_{x_0}^{\text{BE}} \cdot {}_{T-t}p_{x_0+t}^{\text{BE}} \cdot N \cdot P_t(T-t) \\ &= {}_Tp_{x_0}^{\text{BE}} \cdot N \cdot P_t(T-t), \qquad\qquad t = 0, \ldots, T-1, \quad \forall T. \end{aligned} \tag{4.1}$$

Note that the only variable changing with respect to t here is the discount factor, since the insurer may incorporate the fact that a certain percentage does not survive to the next year. Therefore, the probability of survival used to calculate BEL_t is the same for all t (${}_\tau p_{x_0}^{\text{BE}} \cdot {}_{T-\tau}p_{x_0+\tau}^{\text{BE}} = {}_Tp_{x_0}^{\text{BE}}$, for $\tau = 1, \ldots, T-1$).

Next, the general expression for the current best estimate of the value of future liabilities under the longevity stress scenario is required. Note that the stress enters the best estimate of the liabilities once, at time t, and permanently decreases the mortality rates. Thus,

$$BEL_t^{\text{stress}} = {}_tp_{x_0}^{\text{BE}} \cdot {}_{T-t}p_{x_0+t}^{\text{stress}} \cdot N \cdot P_t(T-t), \quad t = 0, \ldots, T-1, \quad \forall T. \tag{4.2}$$

Where ${}_0p_{x_0}^{\text{BE}} = {}_0p_{x_0+T}^{\text{stress}} = 1$ is used for $t = 0$ and $t = T$. The SCR_t for longevity risk can then be computed by (3.17) and the associated risk margin at inception of the contract, $RM_0^{(\text{I})}$, can be calculated by (3.16).

The best estimate value of liabilities increases over time t for fixed T, as the discounting effect becomes smaller when maturity T comes closer. The value of liabilities under the stress scenario proposed by SII is higher than the best estimate of liabilities. The SCR, i.e. the difference between the stressed and best estimate value of liabilities, decreases over time as the impact from the stress is less severe when it is introduced later.

Results regarding the SCR_0 and RM_0 can be found in Table 4.1. Parameters are set as follows: $x_0 = 65$, $T = 20$, $M = 1$, $Z = 1$ and the first year of the contract is 2012. The interest rate with which the value of the zero-coupon bond is calculated is assumed to be flat at 4%. The longevity stress imposed by SII legislation results in an increase of liabilities at $t = 0$ of 16.278% as shown in the table. The RM_0 is a substantial fraction of the best estimate value of liabilities as well, due to the longevity risk in this specific contract. Hence SII lays a burden on the own funds of the insurer.

SCR_0/BEL_0	$RM_0^{(\text{I})}/BEL_0$
16.278%	13.402%

Table 4.1: SCR_0 as in (3.17) and the associated $RM_0^{(\text{I})}$ expressed as percentage of BEL_0 for an endowment contract for a Dutch male aged 65 in 2012, with maturity $T = 20$.

The uncertainty in this contract is caused by the probability that a current 65-year old reaches age 85. As this probability is small, the best estimate value of liabilities is small as well and this is strengted by the effect from discounting. The stress imposed by SII, applied to calculate the liabilities in the stress scenario, impacts the value of liabilities extensively, resulting in the high SCR and associated RM as solvency capital has to be maintained each year until maturity T.

The alternative calculation of the risk margin, by using (3.20), is inappropriate to apply in case of an endownment contract. As the value of liabilities increases over time, a stess which enters at a point in time near to maturity has less impact on the BEL, causing SCR to be lower for t near T. The value of the SCR would therefore be overestimated in case (3.20) would be applied as this approach assumes the ratio SCR_0/BEL_0 is constant over time and is therefore assumed to be representative for the SCR for all t. This is clearly not the case here and thus leads to overestimated results compared to the exact approach of (3.17).

4.1.1 Swap excluding Counterparty Default Risk

Consider the case where an insurer enters a perfect longevity swap to hedge the longevity risk in an endownment contract, and counterparty default risk is excluded. Then, BEL_t^π is as in (3.21). This expression has to be adapted to the special case of the lump sum payment as only one swap payment takes place, at maturity T. Hence

$$BEL_t^\pi = K_T \cdot P_t(T-t), \quad \text{for} \quad t = 0, \ldots, T-1, \quad \forall T. \tag{4.3}$$

Where K_T is as in (3.11). The maximum amount of premium the insurer might spend is here equal to the risk margin for longevity, as was shown in (3.28), since $RM_0^{\text{cdr}} = 0$.

The corresponding value of π for which the BEL_0^π is equal to the fair value of the contract without the swap is a reflection of the premium, $\mathbb{E}_0(\Pi)$. But the interaction between the BEL_0^π and $RM_0^{\text{longevity}}$ plays an important role. The equation to solve for π is (3.26). As default risk is excluded the risk margin for default risk is zero. Thus (3.26) becomes

$$BEL_0 + RM_0 = BEL_0^\pi, \tag{4.4}$$

with BEL_0 as in (4.1) and BEL_0^π as in (4.3). This has to be solved numerically for π.

Consider the endownment contract introduced earlier, where a 65-year old Dutch male in 2012 obtains 1 Euro (in arrears) if he is still alive at maturity $T \geq 1$. The total amount of premium that might be paid at inception of the swap, $\mathbb{E}_0(\Pi)$, is shown in Figure 4.1 for different mortality rates and for all $T \geq 1$. Future mortality projected by the Dutch AG (Actuarieel Genootschap, 2012), observed historic mortality over the period 1974-2009 from the HMD and the best mortality rates projected by using the Lee-Carter model are used to determine $\mathbb{E}_0(\Pi)$. BEL_0 for all T and mortality rates is given in the right graph of Figure 4.1. As best estimate survival rates decline with T and payments further away are more heavily discounted, BEL_0 declines with T.

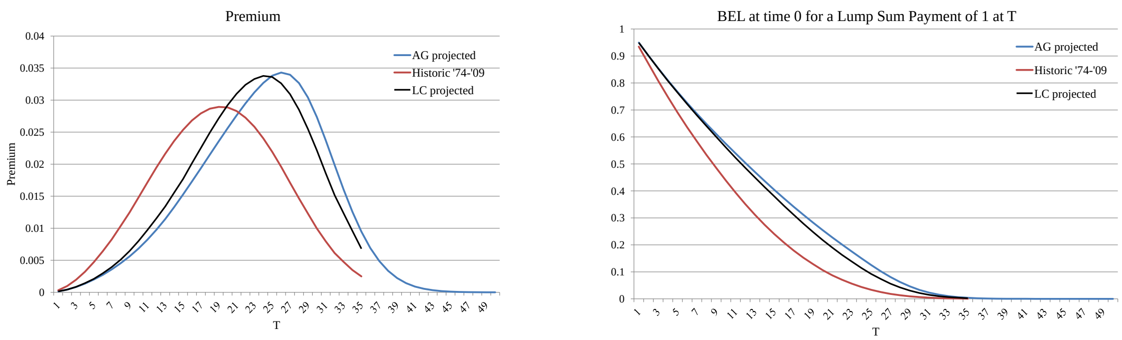


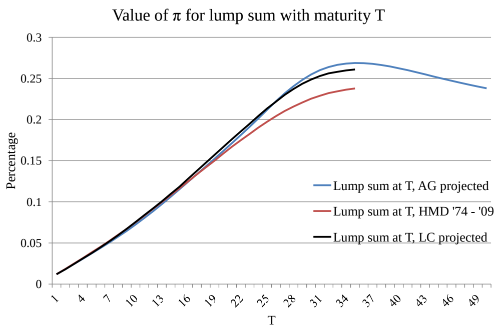
Figure 4.1: The left graph shows $\mathbb{E}_0(\Pi)$ for an endownment contract with a payment of 1 at T if a 65-year old Dutch male in 2012 is alive at T. $\mathbb{E}_0(\Pi)$ is shown for various mortality rates. The right graph shows BEL_0 for all T, where the same mortality rates are applied.

For low values of T the probability of survival is high. The corresponding BEL_0 is therefore close to 1 and the impact of the longevity stress to determine the SCR_t is not very large for contracts with short time to maturity. The associated risk margin is therefore small. Since the risk margin for longevity at time zero reflects the premium, the swap is less valuable for short maturities.

$\mathbb{E}_0(\Pi)$ first increases with T as the risk that the number of survivors is larger than anticipated increases with the time to maturity. The riskier the lump sum payment, i.e. the more longevity risk it entails, the more valuable the swap payment at T is and therefore the premium increases. At a certain point the longevity risk in the lump sum payment reduces as the number of survivors for high values of T becomes low. Therefore the longevity risk in contracts with long time to maturities decreases which lowers the required risk margin and therefore the premium declines as well.

The major part of longevity risk is the systematic risk in mortality projections and the corresponding life expectancy. Hence, the risk that the majority of the individuals attains a higher age than expected by best estimate mortality rates. Therefore, the top of $\mathbb{E}_0(\Pi)$ is highest for T after the remaining years a 65-year old in 2012 is expected to live. For instance, $e_{65,2012} = 18.2$, and as shown in the right graph of Figure 4.1 the value of the swap is largest for $T > 18$.

The left graph in Figure 4.1 also shows that different mortality rates affect the risk margin for longevity. The development of mortality rates and the difference in various projections indicates that the risk in the contract highly depends on the mortality rates used. Since the Dutch AG has projected mortality up to a higher attainable age the survivor rates differ from the rates projected by the Lee-Carter model. The life expectancy of a 65-year old in 2012 is therefore slightly higher. The value of the swap, i.e. the premium, is therefore highest for a higher T when AG mortality rates are applied compared to Lee-Carter projected mortality.



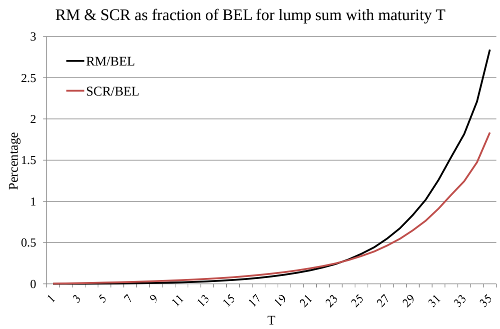


Figure 4.2: The value of π for a lump sum payment if a 65-year old Dutch male in 2012 is alive at maturity T for projected Lee-Carter (LC) mortality rates, projected mortality rates of AG and historic mortality of the HMD for the period 1974-2009 (left). The right graph shows the corresonding SCR_0 as in (3.17) and associated $RM_0^{(\mathrm{I})}$ for longevity risk as a fraction of BEL_0 for a lump sum payment at T based on Lee-Carter projections. $\mathbb{E}_0(\Pi)$ is equal to $RM_0^{\text{longevity}}$ here, since default risk is neglected. Therefore, $RM_0^{(\mathrm{I})}/BEL_0$ also represents $\mathbb{E}_0(\Pi)/BEL_0$.

The value of π for which (4.4) is solved, and the fractions RM_0/BEL_0 and SCR_0/BEL_0 at age 65 in year 2012 for a lump sum that pays at T are given in Figure 4.2, for all T. Also, Table A.1 in Appendix A.2 depicts the values in the left graph of this figure. Table A.2 shows $\mathbb{E}_0(\Pi)/BEL_0 = RM_0/BEL_0$ in the column for $p^{(j)} = 0$, corresponding to the values in Figure 4.2.

The most important finding from the left graph in Figure 4.2 is that there appears to be a term structure for π. More risky contracts demand a higher premium in the fixed leg. However, when the time of maturity becomes high enough the premium in the fixed leg of the swap starts to decrease, as shown by the results based on the mortality projections of the Actuarieel Genootschap (2012). Since the probability to reach an age above 100 for a current 65-year old is very small the value of the swap decreases and the premium to be paid in the fixed leg decreases as well.

The same figure also shows that the term structure of π developed over time, as π computed with historic mortality rates differs from the projected mortality rates of both the Lee-Carter model as well as the projections of the Dutch AG. This is due to the positive development of survivor rates over time.

First consider the premiums in the fixed leg based on Lee-Carter best estimate projected mortality rates. As can be seen in the right graph, lump sum payments further away in the future are more risky as the solvency capital requirement and risk margin as a fraction of best estimate liabilities both increase. This is caused by two effects. First, the BEL_0 is smaller for lump sum payments for higher T. Second, the lower BEL_0 for longer time to maturity makes payments more risky which is reflected by the higher SCR_0 and RM_0 as a fraction of BEL_0. As the BEL_0 is quite small a sudden decrease of mortality has a dramatic effect on the insurer's liabilities. π in BEL_0^π therefore increases with T even though $\mathbb{E}_0(\Pi)$ in Figure 4.1 might decrease for the same values of T.

Interesting is that RM_0/BEL_0 becomes higher than SCR_0/BEL_0 in the right graph of Figure 4.2. The risk margin is the sum of the discounted $SCRs$ multiplied by the $CoC\%$, as T increases so is the number of $SCRs$ that must be maintained to maturity. Then, for large T, the RM_0 becomes larger than the SCR_0 as a fraction of BEL_0. Values of $RM_0^{(\mathrm{I})}/BEL_0$, here equal to $\mathbb{E}_0(\Pi)/BEL_0$ are also given in Table A.2 in the column under $p^{(j)} = 0$, since default risk is neglected here and is considered later in Section 4.1.2.

The results in Figure 4.1 and 4.2 indicate that $\mathbb{E}_0(\Pi)$ and π do not always move in the same direction. Since π is a reflection of both the risk margin as well as the best estimate of liabilities, π keeps increasing with T since the risk margin as a fraction of the corresponding BEL_0 increases with T while $\mathbb{E}_0(\Pi)$ decreases after approximately $T = 24$. When the number of survivors is higher than anticipated this may cause a significant loss as BEL_0 might be insufficient. Especially for long time to maturities. Therefore, the floating leg payment from the swap becomes valuable but a high premium in the fixed leg might have to be paid to hedge this risk.

4.1.2 Swap including Counterparty Default Risk

When a swap contract is entered the insurer is exposed to the risk that the counterparty defaults before maturity of the swap contract. As explained in Section 3.3.3, in case of a perfect swap, liabilities now consist of fixed leg payments weighted with the probability that default does not occur before T, plus the floating leg payment which the insurer is obliged to make to its policyholders in case of default, weighted with the probability that default occured before T.

The general formula to determine the best estimate value of liabilities is given in (3.22). For an endownment contract the current best estimate of future liabilities under the swap becomes

$$BEL_t^\pi = \left(\mathbb{P}(D > T) \cdot K_T + \mathbb{P}(D \le T) \cdot \mathbb{E}_t(S_T)\right) P_t(T-t), \quad t = 0, \ldots, T-1, \quad \forall T. \tag{4.5}$$

Where K_T represents the single fixed leg payment at maturity T and $\mathbb{E}_t(S_T)$ represents the expected floating leg payment S_T at maturity with information available up to time t. The expected floating leg payments are modelled by best estimate mortality rates as the swap is assumed to be perfect.

The required solvency capital for default risk is computed as in (3.23), but is adapted to the endownment contract as there is only one payment, at T, i.e.

$$SCR_t^{\mathrm{cdr}} = \mathbb{P}(D \le T) \cdot \mathbb{E}_t(S_T^{\mathrm{stress}} - S_T) \cdot P_t(T-t), \quad \text{for} \quad t = 0, \ldots, T-1. \tag{4.6}$$

The longevity stress proposed by SII is applied to represent the floating leg payment in a stressed scenario. Hence, the method in (3.17) is applied to determine future values of the SCR. As before, when the stress enters as of year t it is assumed that the best estimate mortality realized up to t. The interest rate is assumed to be flat at 4%.

The risk margin for counterparty default risk increases with the one-year probability of default, dependent on the rating of the counterparty in the swap. The premium an insurer might pay for the swap therefore decreases for less creditworthy counterparties as these require a higher risk margin for default risk. This is shown in Figure 4.3. The structure of $\mathbb{E}_0(\Pi)$ for various T is similar as before in Figure 4.1. First $\mathbb{E}_0(\Pi)$ increases with T as the amount of longevity risk in the lump sum payment increases to a maximum for which longevity risk is largest and therefore the value of a hedge is largest. Then, $\mathbb{E}_0(\Pi)$ decreases as the number of policyholders that survive up to high ages is small, therefore the cost of maintaining solvency capital for this risk is not very high.

In Figure 4.3 the impact of a low rated counterparty becomes clear. As the probability that a low rated counterparty defaults is not negligible, the premium decreases for all T. For short maturities default is not likely to occur and therefore the difference between a good and bad rated counterparty is

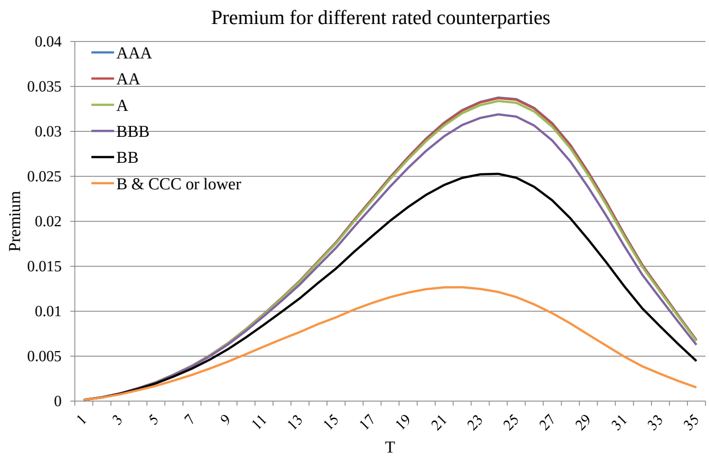


Figure 4.3: $\mathbb{E}_0(\Pi)$ for a swap to hedge a lump sum payment of 1 if a Dutch 65-year old male in 2012 is still alive at T. Several different rated counterparties are considered as counterparty default risk is included. Results are based on mortality projected by the Lee-Carter model.

small. The difference between high and low rated parties increases with T and the difference is largest at the same values of T for which the swap is most valuable. Thereafter, the gap between good and bad rated counterparties decreases again as the general value of the swap decreases for high T.

Figure 4.4 shows $\mathbb{E}_0(\Pi)$ as a fraction of BEL_0 for all T. These values are also given in Table A.2 in Appendix A.2. The results have a similar structure as the right graph in Figure 4.2. Here it also becomes clear that with the deterioration of the counterparty's credit rating the premium decreases. As shown in Figure 4.2, the fraction becomes very high for high maturities, since the BEL_0 for high T becomes very small relative to $\mathbb{E}_0(\Pi)$.

The value of π which solves (3.26) for the endownment contract when default risk is included is equal to the values of π found in the scenario when default risk was excluded. A mathematical proof for this result is given in Appendix A.1. The specific characteristics of the endownment contract are such that the effect of the probability of default does not affect π. That is, the fact that only one payment from the insurer to the policyholders takes place. Therefore only one swap payment is required to hedge the longevity risk in an endownment contract. This characteristic causes π to be independent of the counterparty's credit rating.

An intuitive explanation is the following. The exchange of swap payments takes place at T when default has not occured before or at T. Whether the swap payment takes place is a binary event at inception of the contract. The exchange of swap payments only takes place when default does not occur before or at T. In any other scenario, whether default occured long before T or at T, there is no exchange of swap payments. The expected time to default, which differs among the different rated counterparties, does not affect the premium in this model as the time of default does not matter for the premium that is paid in the fixed leg of the swap at T. As for all differently rated counterparties, when default occurs before T, no swap payment takes place. When default does not occur before or at T, the exchange of swap payments occurs and the insurer makes the fixed leg payment and obtains the floating leg payment hedging his exposure to longevity. The payment obtained from the counterparty is independent of the counterparty's credit rating. Thus the effectiveness of the hedge is equal for all counterparties. Therefore the premium in the fixed leg found in this model is equal for all counterparties.

The value of arranging a swap with a less creditworthy counterparty is lower, which is reflected by the premium $\mathbb{E}_0(\Pi)$. Since the probability that the floating leg payment takes place decreases with the deterioration of the counterparty's credit rating.

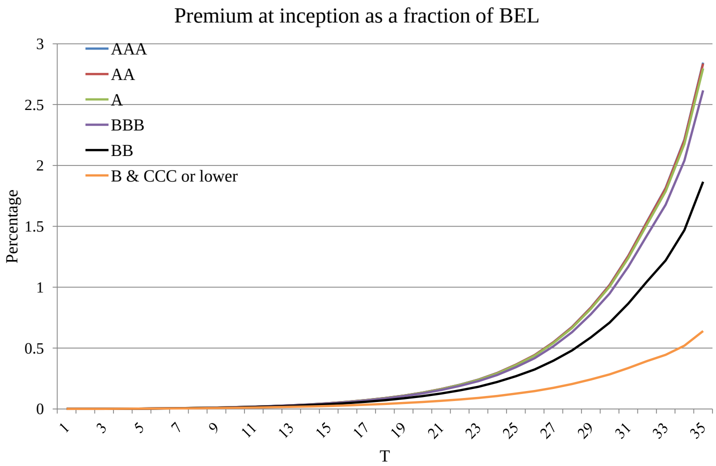


Figure 4.4: $\mathbb{E}_0(\Pi)/BEL_0$ for all T for an endownment contract of 65-year old Dutch male in 2012 who receives a lump sum payment of 1 if alive at T. Results are shown for various rated counterparties.

4.2 Immediate Annuity

A commonly used insurance product is an immediate annuity. A monthly or yearly benefit paid by the insurer to the policyholder as of a certain age (for instance the retirement age) of the policyholder for as long as he is alive. This section elaborates on the risk premium paid by an insurer in case a swap is entered to hedge longevity risk in an annuity contract.

Here, a yearly annuity benefit of Z is considered for a portfolio of M 65-year old Dutch males who can maximally attain age 100. At time 0 (2012) all policyholders are alive. It is assumed that annuity payments are made at the end of each year. Then, at time 0 no cash flow takes place and the first payment is made if the annuitant reaches age 66. The annuity contract can be compared to a portfolio of lump sum payments for age 66 until age 100. The yearly annuity payments are uncertain since not all policyholders survive the upcoming years. Therefore the value of current and future liabilities has to be estimated by (3.15), which is adapted here to represent the liability of an annuity. The following general formula to estimate the current best estimate of future liabilities is applied,

$$\begin{aligned} BEL_t &= {}_tp^{\text{BE}}_{x_0} \cdot N \sum_{s=1}^{100-(x_0+t)} {}_sp^{\text{BE}}_{x_0+t} \cdot P_t(s) \\ &= N \sum_{s=t+1}^{100-x_0} {}_sp^{\text{BE}}_{x_0} \cdot P_t(s-t), \qquad t = 0, \dots, T-1. \end{aligned} \tag{4.7}$$

Where $N = M \cdot Z$, is the notional of the annuity contract. The survival probability in front of the sum in the upper expression on the right hand side of the equation is included to take into account the expected percentage of policyholders alive at time t. At time 0 all policyholders are alive and the best estimate of current and future liabilities is the sum of the present value of expected discounted payments. To calculate BEL_t for $t > 0$ the insurer may take into account that not all policyholders survive to time t and hence does not have to maintain capital for the entire notional N, but only for $N \cdot {}_tp^{\text{BE}}_{x_0}$, i.e. the notional adapted with the expected number of survivors at time t. This holds for all best estimates of future liabilities. For $t = 0$, ${}_0p_{x_0} = 1$ is used. The entire expression can be simplified, as is shown in the latter equality of (4.7).

When longevity risk is considered the SCR is defined as in (3.17) and a stress has to be applied to best estimate mortality rates in order to determine the best estimate of liabilities in the stressed scenario. To determine BEL_t^{stress} one has to distinguish between the time the stress enters. BEL_0^{stress}

is calculated by applying the stress to all mortality rates. BEL_t^{stress} is calculated by using best estimate survival rates to determine the expected number of survivors at time t and stressed survival rates are then used to determine the best estimate of liabilities for the remaining years, as the stress enters in year $t+1$. The general expression is given by

$$BEL_t^{\text{stress}} = {}_tp_{x_0}^{\text{BE}} \cdot N \sum_{s=1}^{100-(x_0+t)} {}_sp_{x_0+t}^{\text{stress}} \cdot P_t(s), \quad t = 0, \ldots, T-1. \tag{4.8}$$

Here, the simplification is not possible as the number of expected survivors at time t, when the stress enters, is computed by best estimate survival probabilities.

The SCR_t for longevity risk for $t > 0$ is determined by the difference between BEL_t^{stress} and BEL_t, or by the method from (3.20). In this entire section $RM_0^{(\text{I})}$ and $RM_0^{(\text{II})}$ correspond to the SCR computed with (3.17) and (3.20) respectively. The corresponding risk margin, RM_0, can be calculated by (3.16). In this section differences between these different methods to determine the required amount of solvency capital in the future will be shown.

Consider first Table 4.2. These results are based on the best estimate mortality projections obtained from the bootstrap procedure and used in Section 4.1. $M = 1$ and $Z = 1$ (Euro, in arrears) are assumed, hence $N = 1$. The interest rate is assumed flat at 4%. As can be seen in the table, the approach where the ratio of SCR_0 and BEL_0 is assumed to be a good representation for future SCR underestimates the SCR computed with the exact method. This leads to a lower risk margin and a lower estimated fair value of liabilities.

BEL_0	SCR_0/BEL_0	$RM_0^{(\text{I})}/BEL_0$	$RM_0^{(\text{II})}/BEL_0$
12.093	6.229%	4.874%	3.542%

Table 4.2: BEL_0 and SCR_0, $RM_0^{(\text{I})}$ and $RM_0^{(\text{II})}$ expressed as percentage of BEL_0 for an annuity payment of 1 each year as long as a 65-year old Dutch male in 2012 is alive. $RM_0^{(\text{I})}$ and $RM_0^{(\text{II})}$ are determined by the SCR in (3.17) and (3.20) respectively.

4.2.1 Swap excluding Counterparty Default Risk

Consider the case that a perfect longevity swap is entered to hedge longevity risk in the immediate annuity contract proposed earlier. Counterparty default risk is omitted for now. The fair value of liabilities under SII is given by (3.14) and the fair value of liabilities when a swap is entered, here (3.21), is set equal to this value in order to determine the maximum premium such that the insurer will not be worse off at inception of the swap contract in terms of fair value. The premium in the fixed leg has to be determined by numerically solving (3.26) for π, where $RM_0^{\text{cdr}} = 0$. The two different versions of calculating future SCR can be applied and hence different values of $\mathbb{E}_0(\Pi)$ and π are found.

An annuity contract starting at a lower age brings along more uncertainty for the annuity provider as the expected number of annuity payments is higher. BEL_0 is higher and solvency capital has to be held for a longer amount of time, which causes the higher value of the risk margin as shown in Table 4.3. Therefore π is higher for lower ages, as shown in Table 4.5.

Table 4.3 shows that the required solvency capital at time 0 increases with age. The applied stress to one-year mortality rates has more effect on the mortality experience of a 67-year old than for a 65-year old in 2012. Stated otherwise, the difference in mortality, $q_x^{\text{stress}} - q_x^{\text{BE}}$, is larger for $x = 67$ compared to $x = 65$, as mortality rates are lower for the former age. When the stressed value of liabilities is determined at time 0, this differs more from the best estimate for an annuity contract starting at age 67 compared to a contract for a 65-year old. This leads to a higher solvency requirement at $t = 0$.

Table 4.4 shows the premium, $\mathbb{E}_0(\Pi)$, as a fraction of the associated BEL_0 for different ages and methods to compute future solvency capital, where default risk is excluded. BEL_0 decreases with the starting age of the contract. The associated risk margin for longevity decreases as well as shown in Table 4.3. Results show that the ratio of the two increases with age. It follows that the difference in risk margins, between for instance a 66 and 65 year old, is smaller than the difference in BEL_0 for a 66 and

Mortality Rates	Age	BEL_0	SCR_0	$RM_0^{(\mathrm{I})}$	$RM_0^{(\mathrm{II})}$
LC projected	65	12.093	0.753	0.589	0.428
	66	11.686	0.764	0.573	0.420
	67	11.263	0.774	0.556	0.412
AG projected	65	12.712	0.741	0.610	0.445
	66	12.302	0.754	0.596	0.438
	67	11.885	0.766	0.581	0.431
Observed 1974-2009	65	9.449	0.910	0.538	0.434
	66	9.065	0.911	0.514	0.420
	67	8.729	0.912	0.494	0.409

Table 4.3: This table shows estimates of BEL_0, SCR_0, $RM_0^{(\mathrm{I})}$ and $RM_0^{(\mathrm{II})}$ for an annuity contract starting at age 65-67 that pays 1 each year if the annuitant is alive. Results are shown for a 65-67 year old Dutch male in 2012 for Lee-Carter and AG projected mortality. Also results are shown for realized mortality from the HMD over the period 1974-2009, where a Dutch male is 65-67 in 1974. $RM_0^{(\mathrm{I})}$ and $RM_0^{(\mathrm{II})}$ are determined by the SCR in (3.17) and (3.20) respectively.

65 year old. However, $\mathbb{E}_0(\Pi)$ decreases with age and thus a hedge for a contract starting at a lower age might be less expensive as the exposure to longevity risk is smaller.

	LC projected		AG projected		1974-2009	
Age	(I)	(II)	(I)	(II)	(I)	(II)
65	4.874%	3.542%	4.795%	3.498%	5.690%	4.592%
66	4.906%	3.596%	4.843%	3.562%	5.673%	4.637%
67	4.935%	3.659%	4.887%	3.624%	5.659%	4.682%

Table 4.4: The table depicts $\mathbb{E}_0(\Pi)/BEL_0$ for the two different risk margins, $RM_0^{(\mathrm{I})}$ and $RM_0^{(\mathrm{II})}$ computed by (3.17) and (3.20) respectively, for various ages and mortality rates.

Table 4.5 depicts results of π when (3.26) is solved numerically. The most important result is that π decreases with age as the longevity risk in an annuity contract declines when such a contract is entered at a higher age in line with the decrease in $\mathbb{E}_0(\Pi)$.

From the results in Table 4.5 can be seen that the decline in the value of π under $RM^{(\mathrm{I})}$ is faster than under $RM^{(\mathrm{II})}$ when age increases. π found with $RM^{(\mathrm{I})}$ are the values of π under the preferred method as future SCR is determined by (3.17). The values of π found with $RM^{(\mathrm{II})}$ are lower compared to $RM^{(\mathrm{I})}$, as the $SCRs$ are underestimated when the simplified driver approach is used. The underestimation has however less effect for higher ages at which the annuity contract starts, as the number of years the solvency capital is required becomes smaller. This is also due to the higher SCR_0 for higher ages at which the annuity contract starts, as shown in Table 4.3. Since SCR_0 increases for higher starting ages of the annuity contract the ratio SCR_0/BEL_0 that is assumed constant over time is larger, causing estimated future SCR to be higher. This results in a higher risk margin. Therefore, the decline in risk margin and thus in the value of π with age is less severe when the driver approach is applied compared to the exact method to determine future SCR.

Results are also given for the projected mortality rates of the Dutch AG. Due to the higher survival probabilities the value of liabilities is higher and therefore the risk margin for longevity is as well, as shown in Table 4.3. The risk margin is the main driver for the higher values of π compared to the Lee-Carter projected mortality. $\mathbb{E}_0(\Pi)$ as fraction of BEL_0 for AG projectd mortality does not differ much from the Lee-Carter projections as can be seen in Table 4.4. In general however, both the premium that might be paid is larger when mortality projections of the Dutch AG are applied compared to the

Age	LC projected $\pi^{(\mathrm{I})}$	$\pi^{(\mathrm{II})}$	AG projected $\pi^{(\mathrm{I})}$	$\pi^{(\mathrm{II})}$	1974-2009 $\pi^{(\mathrm{I})}$	$\pi^{(\mathrm{II})}$
65	15.961%	11.828%	16.711%	12.441%	12.352%	10.093%
66	15.367%	11.476%	16.109%	12.082%	11.837%	9.787%
67	14.741%	11.124%	15.515%	11.721%	11.391%	9.525%

Table 4.5: Numerically found values of π for the two different versions of the risk margin, $RM^{(\mathrm{I})}$ and $RM^{(\mathrm{II})}$. Results are computed for the Lee-Carter (LC) projected mortality rates, the mortality projections of the Dutch AG as well as the observed mortality of Dutch males reported by the Human Mortality Database over the period 1974-2009.

Lee-Carter estimates. All due to the larger exposure to longevity risk due to the higher projected survival rates.

When historic mortality rates are applied, the values of $\mathbb{E}_0(\Pi)$ and π for both methods to determine future SCR ((3.17) and (3.20) respectively) are lower compared to the values computed under the estimated future mortality rates. This was also the case for the endownment contract and again the conclusion is drawn that the longevity risk premium develops over time with the improvement of survival rates.

As π decreases with age, as well as $\mathbb{E}_0(\Pi)$, but the fraction $\mathbb{E}_0(\Pi)/BEL_0$ increases, this shows that only considering the latter might be deceitful when conclusions are drawn on the premium to be paid for a swap. Since the effect of BEL_0 decreasing with age should not be ignored.

4.2.2 Swap including Counterparty Default Risk

When counterparty default risk is included in the analysis the total premium as well as the premium in the fixed leg differ from the earlier found results. This section shows the impact of default risk on the results found in the previous section.

In (3.22) a method was proposed where the probability of default is included in the valuation of the liabilities under the swap. A higher probability of default will result in a decrease in the total number of years the premium in the fixed leg is expected to be paid and it is more likely the insurer will have to make payments equal to the floating leg, i.e. realized survival rates.

The results in Table 4.6 are based on the exact method to determine future SCR and the associated RM, i.e. SCR as in (3.17). When the method from (3.20) is used the risk margin for default risk should be calculated accordingly. But, as this requires BEL_t^{π} for $t > 0$ and this depends on π which is unknown this makes the determination of π more complex. The interest rate is as before assumed to be flat at 4%.

The required solvency capital for counterparty default risk increases in case a swap is entered with a less creditworthy counterparty. The risk margin increases which causes $\mathbb{E}_0(\Pi)$ to decrease, as can be seen from eq. (3.28). When the results in Table 4.6 are compared to the results in Table 4.4 it is clear that the results for high rated parties, i.e. AAA and AA, are very similar to the results when default risk is excluded. Since the probability of default is very small there is a high probability that all swap payments take place and therefore the amount $\mathbb{E}_0(\Pi)$ is spread out over almost the entire contract period which results in a value of π similar to the values under $\pi^{(\mathrm{I})}$ in Table 4.5. Moreover, the required solvency capital and associated risk margin are small for high rated counterparties. Therefore $\mathbb{E}_0(\Pi)$ is very close to the results where default risk was excluded and the probability of default was zero. Entering a swap with a low rated party is less valuable for an insurer which is reflected by the reduction in $\mathbb{E}_0(\Pi)$.

When the annuity contract starts at a higher age, such as 66 or 67, the premium declines as the risk margin for longevity decreases as well as the required risk margin for default risk. However, since the percentage decrease in BEL_0, when age increases, is more severe than the decrease in $\mathbb{E}_0(\Pi)$, the ratio $\mathbb{E}_0(\Pi)/BEL_0$ increases with age. Remarkable is that $\mathbb{E}_0(\Pi)$ is slightly larger when a swap is entered with a party rated B or below for ages 66 and 67, compared to age 65.

In contrast to the results for the endownment contract, π declines with the deterioration of the creditworthiness of the counterparty as depicted in Table 4.6. The expected number of years before default occurs affects the value of π. The single swap payment for an endownment contract only takes

Rating	$\mathbb{E}_0(\Pi)$	65 $\frac{\mathbb{E}_0(\Pi)}{BEL_0}$	π	$\mathbb{E}_0(\Pi)$	66 $\frac{\mathbb{E}_0(\Pi)}{BEL_0}$	π	$\mathbb{E}_0(\Pi)$	67 $\frac{\mathbb{E}_0(\Pi)}{BEL_0}$	π
AAA	0.589	4.872%	15.960%	0.573	4.904%	15.366%	0.556	4.933%	14.740%
AA	0.588	4.864%	15.956%	0.572	4.896%	15.362%	0.555	4.925%	14.736%
A	0.583	4.821%	15.934%	0.567	4.854%	15.342%	0.550	4.884%	14.717%
BBB	0.559	4.622%	15.833%	0.545	4.661%	15.247%	0.529	4.696%	14.627%
BB	0.452	3.740%	15.316%	0.444	3.799%	14.761%	0.434	3.856%	14.170%
B	0.238	1.971%	13.666%	0.239	2.046%	13.220%	0.239	2.122%	12.726%

Table 4.6: This table shows results of π and $\mathbb{E}_0(\Pi)$ when an annuity contract that pays 1 each year as long as a 65, 66 or 67-year old Dutch male in 2012 is alive is hedged by a longevity swap. Results are based on mortality rates projected by the Lee-Carter model. One-year default probabilities corresponding to the different ratings can be found in Table 3.2. Both $\mathbb{E}_0(\Pi)$ and π is determined by the SCR as in (3.17) and the associated $RM_0^{(\mathrm{I})}$.

place when the counterparty did not default before maturity. Whether default occured one or ten years before maturity, in both cases the swap payments do not occur. Making the amount the insurer pays in the fixed leg independent of the expected number of years before default. Net swap payments, hedging the longevity risk in a portfolio of annuities, take place each year. Therefore the expected number of years before default affect the expected number of swap payments that take place and thereby the effectiveness of the hedge. This causes the premium in the fixed leg to decrease when the swap is entered with a less creditworthy counterparty.

When the age at which the annuity contract starts increases, the total number of swap payments that take place decrease. Since the probability of default remains the same, since the contract for a 66 and 67 year old start at $t = 0$ as well, the number of swap payments that is expected to occur is larger for higher ages. This causes the decline in π. A similar explanation holds for the fact that the decline in π with the deterioration of the credit rating of the counterparty is less severe for age 67 compared to age 65.

4.2.3 Capital Release from entering a Longevity Swap

As argued before, entering a swap might be beneficial for an insurer since the amount of solvency capital for default risk the insurer is obliged to maintain is lower than the solvency buffer for longevity risk. Table 4.7 depicts the capital release at inception of the contract as a fraction of the BEL_0 when a perfect swap is entered and a full hedge is set up. A distinction is made between the age at which the annuity contract starts and the credit rating of the counterparty.

For all rated counterparties under consideration, entering a swap results in a capital release. The shock the SCR for default risk should be able to absorb is similar to that of longevity risk. But, the probability of default is taken into account and since this is typically small the probability that the SCR is required is small. This results in a lower capital buffer for default risk compared to longevity risk. The results show that in case a swap is entered with a high rated party the required solvency capital for default risk is a small fraction of BEL_0. This results in a capital release close to the amount of solvency capital that should be maintained for longevity risk.

The capital release at time zero becomes smaller when a swap is arranged with a lower rated counterparty as the required amount of solvency capital for default risk increases. The results show also that the capital release as a fraction of BEL_0 is larger for contracts that start at a higher age. As argued before, the required amount of solvency capital for longevity risk increases with age. The longevity stress applied to determine the SCR for longevity risk has more impact on the mortality rates for higher ages. The absolute capital release, $SCR_0^{\text{longevity}} - SCR_0^{\text{cdr}}$ is therefore larger for a contract that starts at a higher age.

The required amount of solvency capital for default risk at inception of the swap is always smaller than the amount for longevity risk, since the results in the table are all positive fractions. Therefore, even entering a swap with a low rated party is beneficial when the required solvency capital is used to

	Age		
Rating	65	66	67
$p^{(j)} = 0$	6.229%	6.535%	6.876%
AAA	6.227%	6.532%	6.874%
AA	6.218%	6.523%	6.864%
A	6.171%	6.476%	6.816%
BBB	5.956%	6.257%	6.595%
BB	4.987%	5.272%	5.593%
B	2.949%	3.173%	3.428%

Table 4.7: This table depicts results for $\frac{SCR_0^{\text{longevity}} - SCR_0^{\text{cdr}}}{BEL_0}$ for an annuity contract that pays 1 each year a 65-67 year old Dutch male in 2012 is alive. Results are based on Lee-Carter mortality projections. The first row represents the case where default risk is absent and the probability of deafult is zero. The other rows show results for different credit ratings. The associated one-year default probabilities can be found in Table 3.2.

assess the attractiveness of entering a longevity swap.

Due to rounding the results in Table 4.7 for $p^{(j)} = 0$ do not exactly correspond to the value of $SCR_0^{\text{longevity}}/BEL_0$ shown in Table 4.3.

4.3 Comparison of Male and Female Longevity Risk Premiums

Until now only Dutch male mortality data was applied to determine the premium to be paid for the swap. Since the mortality rates of Dutch females differs from men's it is interesting to look at the difference in premium an insurer pays for a swap contract to hedge female longevity risk, as well as the associated premium paid in the fixed leg of the swap. This section shows several results based on female projected mortality and compares the results with the results found earlier for males.

4.3.1 Endownment

The same endownment contract as in Section 4.1 is considered, instead of male policyholders females are considered and projected mortality rates of Dutch females are used to compute results. The differences in mortality rates between males and females are substantial resulting in higher life expectancies for females compared to males. The longevity risk in life related insurance contracts therefore is different compared to males's and the solvency capital requirements and associated risk margin for longevity risk differ accordingly.

First consider the scenario in which default risk is excluded and the premium, $\mathbb{E}_0(\Pi)$ in (3.28) where $RM_0^{\text{cdr}} = 0$, is determined for Dutch females. The left graph of Figure 4.5 shows also the earlier found results for Dutch males to be able to make a comparison.

A distinction is made between projected mortality rates of the Dutch AG and the Lee-Carter model. As for Dutch males, parameter uncertainty as well as uncertainty from forecasting the time parameter of the Lee-Carter model is taken into account by performing 50,000 Poisson bootstrap iterations, as described in Section 3.1.2.

The results in Figure 4.5 indicate that the differences between the projected mortality of both males and females impact the premium that might be paid for a swap. In general, the duration of the contract for females, on average, is higher which results in a shift of the premium to be paid to hedge longevity risk for females.

Because mortality rates for females are lower than male mortality the exposure of the endownment contract with maturity T to longevity risk differs for males and females resulting in different values of $\mathbb{E}_0(\Pi)$. Furthermore, the average life expectancy of females is higher than for males which is reflected by observing that the top of $\mathbb{E}_0(\Pi)$ for females corresponds to higher T compared to males. For the same reason the premium is slightly higher for an endownment contract for females compard to males, i.e. the

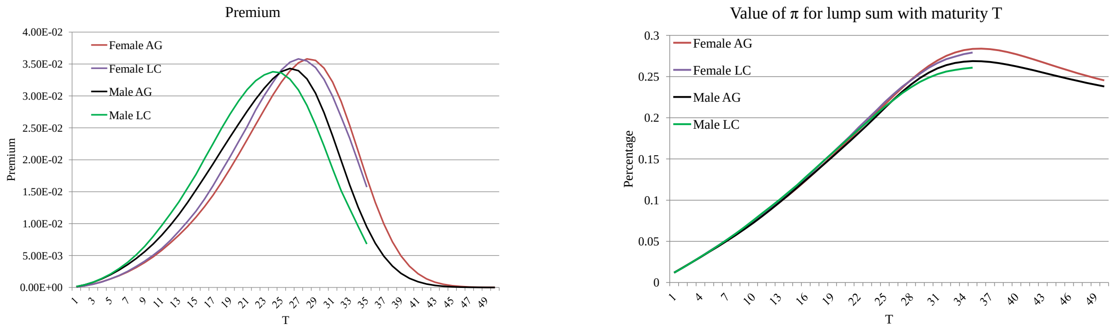


Figure 4.5: The left graph shows $\mathbb{E}_0(\Pi)$ for both Dutch males and females for projected mortality rates of the Dutch AG and the Lee-Carter (LC) model for an endownment contract with maturity T where default risk is excluded. The right graph shows the values of π where the same distinction in projected mortality rates is used as in the left graph.

top of $\mathbb{E}_0(\Pi)$ lies slightly higher. The difference in mortality projections of the AG and Lee-Carter model can be seen here as well. Since the AG projected lower mortality rates the largest part of longevity risk occurs at a later age compared to the Lee-Carter estimates.

The values of π that solve eq. (4.4) are shown in the right graph of Figure 4.5 for both males and females and for the two different mortality projections. The results are very similar for both males and females. Though, since females are expected to live longer the premium an insurer pays in the fixed leg of the swap is slightly higher for high values of T. Since BEL_0 becomes small for long times to maturity, longevity risk has a great impact on the insurer's liabilities. When more policyholders survive than expected the floating leg payment is very valuable for which a higher premium has to be paid. For high T, as before, the probability of survival is so small that a hedge for longevity risk becomes less valuable.

In Figure 4.6 counterparty default risk is taken into account based on female mortality rates projected by the Lee-Carter model.

The results are similar to the results based on male mortality rates. The premium declines when the creditworthiness of the counterparty deteriorates. This is due to the declining effectiveness of the hedge when a swap is arranged with a lower rated counterparty.

4.3.2 Immediate Annuity

This section elaborates on some of the differences in the longevity risk premium to be paid for a longevity swap with respect to an immediate annuity when female mortality projections are applied.

The annuity contract from Section 4.2 is considered. Since females on average live longer compared to males, when the annuity contract starts at age 65, the amount of annuity payments the insurer has to make is expected to be larger for females. This is reflected by a higher value of the insurer's best estimate liabilities, BEL_0, in Table 4.8 compared to the results for males in Table 4.2.

BEL_0	SCR_0/BEL_0	$RM_0^{(\text{I})}/BEL_0$	$RM_0^{(\text{II})}/BEL_0$
13.609	4.811%	4.365%	3.029%

Table 4.8: BEL_0 and SCR_0, $RM_0^{(\text{I})}$ and $RM_0^{(\text{II})}$ expressed as percentage of BEL_0 for an annuity payment of 1 each year as long as a 65-year old Dutch female in 2012 is alive. $RM_0^{(\text{I})}$ and $RM_0^{(\text{II})}$ are determined by the SCR in (3.17) and (3.20) respectively.

The longevity stress applied to determine the required amount of solvency capital for longevity risk therefore has less impact. As such, the solvency buffer at time zero is therefore a lower fraction of BEL_0.

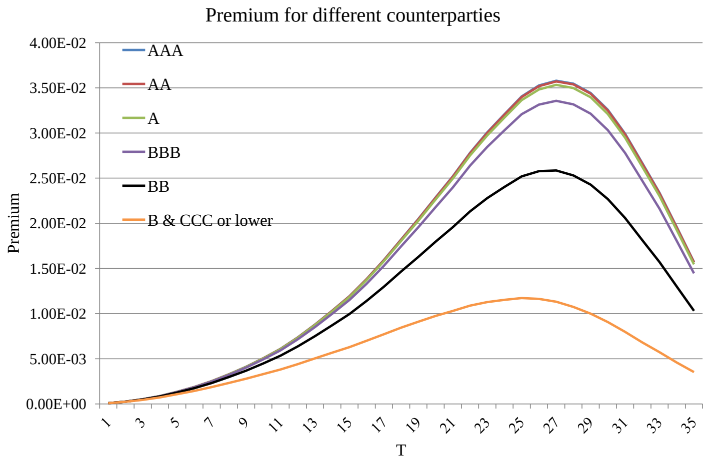


Figure 4.6: $\mathbb{E}_0(\Pi)$ for a lump sum payment of 1 if a Dutch 65-year old female in 2012 is still alive at T. Several different rated counterparties are considered as the probability of default affects the risk margin for counterparty default risk. Results are based on future mortality projected by the Lee-Carter model.

Since the risk margin is a function of the required solvency capital this is lower as well.

Table 4.9 shows results for the total amount of premium such that the fair values of both contracts, i.e. where a swap is entered and no longevity hedge is set up, is equal. The associated premium to be paid in the fixed leg for both the scenario in which default risk is excluded ($p^{(j)} = 0$) and included is shown as well.

	65			66			67		
Rating	$\mathbb{E}_0(\Pi)$	$\frac{\mathbb{E}_0(\Pi)}{BEL_0}$	π	$\mathbb{E}_0(\Pi)$	$\frac{\mathbb{E}_0(\Pi)}{BEL_0}$	π	$\mathbb{E}_0(\Pi)$	$\frac{\mathbb{E}_0(\Pi)}{BEL_0}$	π
$p^{(j)} = 0$	0.594	4.365%	18.280%	0.582	4.401%	17.662%	0.569	4.434%	17.015%
AAA	0.594	4.363%	18.279%	0.582	4.399%	17.661%	0.569	4.432%	17.014%
AA	0.593	4.355%	18.274%	0.581	4.390%	17.657%	0.568	4.424%	17.010%
A	0.587	4.312%	18.251%	0.575	4.349%	17.635%	0.563	4.384%	16.989%
BBB	0.560	4.117%	18.139%	0.550	4.159%	17.530%	0.539	4.198%	16.890%
BB	0.444	3.262%	17.561%	0.439	3.320%	16.990%	0.433	3.377%	16.380%
B	0.219	1.608%	15.655%	0.221	1.674%	15.205%	0.224	1.742%	14.710%

Table 4.9: This table shows results of π and $\mathbb{E}_0(\Pi)$ for an annuity contract that pays 1 each year as long as a 65, 66 or 67-year old Dutch female in 2012 is alive. Results are based on mortality rates projected by the Lee-Carter model. One-year default probabilities corresponding to the different ratings can be found in Table 3.2. Both $\mathbb{E}_0(\Pi)$ and π is determined by $RM_0^{(\mathrm{I})}$ with required solvency capital as in (3.17). $p^{(j)} = 0$ corresponds to the scenario in which default risk is excluded.

Compared to the results for a male in Table 4.6, the price for hedging longevity risk in an annuity contract for females is almost always higher compared to hedging a contract for males. This is due to the higher life expectancy of females and since the duration of the contract is expected to be higher. Resulting in a higher exposure to longevity risk. However there is an exception. When a swap is arranged with a B or lower rated counterparty the premium is lower for females than for males. A possible explanation is that the expected time to default is equal for both males and females, but the life expectancy of a 65-year old female is higher compared to that of a male. Therefore the most important longevity risk, i.e. the risk that a large amount of the insured population attains an age higher than expected, lies less far in the future for males compared to female. Since the probability of default of a B rated party is large, the probability that floating leg payments hedging longevity risk for females are not made is larger compared to males. Therefore the effectiveness of the hedge with a B rated party is likely to be lower for females than for males and therefore a lower premium might be paid.

This section shows the importance of separate reference populations or indices for both males and females to hedge longevity risk in insurance contracts for these respective groups. Floating leg payments based on an index or reference population consisting of male survival rates might be inappropriate to hedge female longevity risk since the floating leg payments are likely to be insufficient when female longevity occurs. Hedge buyers would therefore require separate indices. However, when the insured population is large enough, an unisex index might suffice as well.

4.4 Results for a Partial Hedge

In Section 3.5 a method was proposed to determine the total amount of solvency capital in case a partial hedge for longevity risk was set up by use of a longevity swap. Since the required amount of solvency capital under SII puts a heavy burden on the insurer's own funds the possibly occuring positive effect on the total amount of solvency capital resulting from a partial hedge through diversifation is examined. This section shows several results regarding the required amount of capital at time zero in case a partial hedge is set up as well as the associated premium to be paid for the swap in order not to be worse off at inception of the swap contract.

When a partial hedge is set up the insurer is only exposed to counterparty default risk for the specific part of his liabilities that are hedged by the longevity swap. For the remainder of his liabilities the insurer maintains the required solvency capital for longevity risk. SII permits the aggregation formula in (3.30) as correlation effects between the two risk factors, i.e. longevity and counterparty default risk, may be taken into account. The required solvency capital is determined such that the losses incurred when longevity and default occur can be absorbed.

Since there is no trivial way to determine the correlation between default risk and longevity here all possible correlation coefficients are considered to give insight in the aggregation of the two $SCRs$ when a partial hedge is entered. Note that a negative correlation is highly unlikely, however results are shown for $\rho < 0$ as it gives insight in the effect of negative correlation when a partial hedge is entered. A distinction has to be made between the rating of the specific counterparties, as these have a large impact in the SCR for default risk.

Consider the solvency requirements of a portfolio consisting of an immediate annuity contract as in Section 4.2, with a payment of 1 Euro (in arrears) at the end of each year a Dutch male aged 65 in 2012 is alive, with maximum age 100. Lee-Carter projected future mortality rates are used for the calculations and the interest rate is flat at 4%.

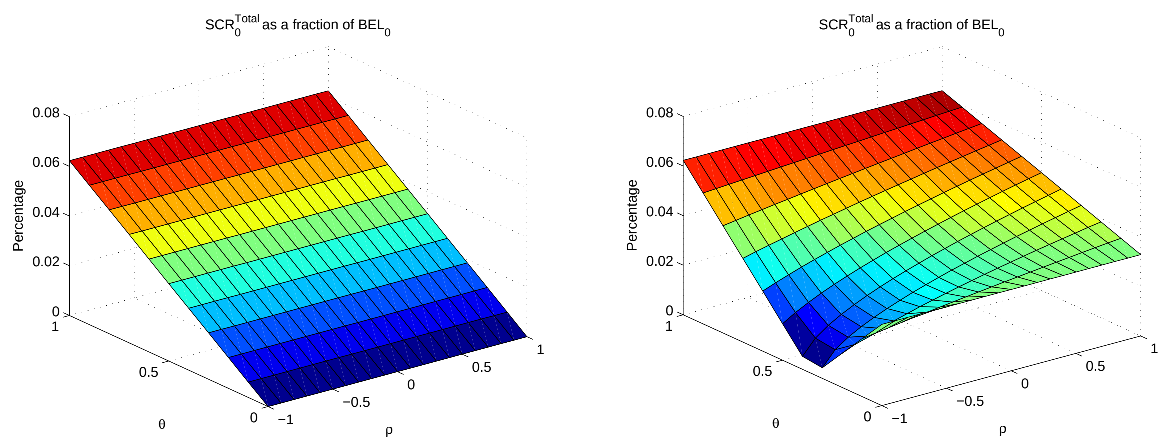


Figure 4.7: SCR_0^{Total} as a fraction of BEL_0 (no swap) for counterparties with rating AAA (left) and B & CCC or lower (right) for different correlation coefficients (ρ) and fraction hedged (θ). $\theta = 1$ and $\theta = 0$ correspond to the scenario of no hedge and a full hedge, respectively.

SCR_0^{Total} as a fraction of BEL_0 when no hedge is arranged (BEL_0 as in eq. (4.7)) is shown in Figure 4.7. $\theta = 1$ corresponds to the scenario in which no swap is entered and therefore for both rated counterparties the fraction $SCR_0^{\text{Total}}/BEL_0$ is equal and corresponds to the results in Table 4.2. In that case the correlation has no effect as SCR_t^{cdr} in (3.30) does not influence SCR_t^{Total}.

First consider a AAA rated counterparty. The one-year probability of default is only 0.002%, since this is the main driver of the required solvency capital for default risk, this is small as well. Namely, 0.002338% of BEL_0. Therefore, a full hedge is always preferred over a partial hedge as indicated by the figure, since the required total amount of solvency capital at time zero is increasing when the fraction hedged decreases, i.e. $SCR_0^{\text{Total}}/BEL_0$ is increasing in θ. Since SCR_0^{cdr} is very small, the effect of diversification is almost zero, which follows from the formula in (3.30). When SCR_0^{cdr} is small, i.e. close to zero, the effect of ρ has almost no impact on the total required solvency capital.

When a swap is entered with a counterparty rated B & CCC or lower, the required solvency capital for default risk at inception of the contract is considerably larger compared to a AAA rated party. Then, as shown in the right graph the effect of a partial hedge becomes more clear, as the correlation coefficient between the two factors has more impact. The effect is largest for $\rho < 0$, but as mentioned before, it is unlikely these two risk factors are negatively correlated. Values of ρ close to zero might be more realistic. Note that for $0 < \rho < 0.3$, a full hedge seems less beneficial in terms of $SCR_0^{\text{Total}}/BEL_0$ than a hedge where 60-70% of the notional is hedged. As the values of the fraction are lower compared to the scenario in which a full hedge is entered and $\theta = 0$.

What becomes clear from both plots is that the diversification effect for a lower rated party is stronger, since SCR_0^{cdr} is larger for a B or lower rated party compared to a AAA counterparty. But more importantly, since SCR_0^{Total} is always smaller for a AAA counterparty for any value of ρ and $\theta < 1$, entering a swap with a AAA party is always preferred over a party with a lower credit rating when the SCR_0^{Total} is used to assess the attractiveness of counterparties. The amount of premium paid will then likely be larger, as shown in Figure 4.8, but the effectiveness of the hedge increases substantially which might be worth paying a higher price for.

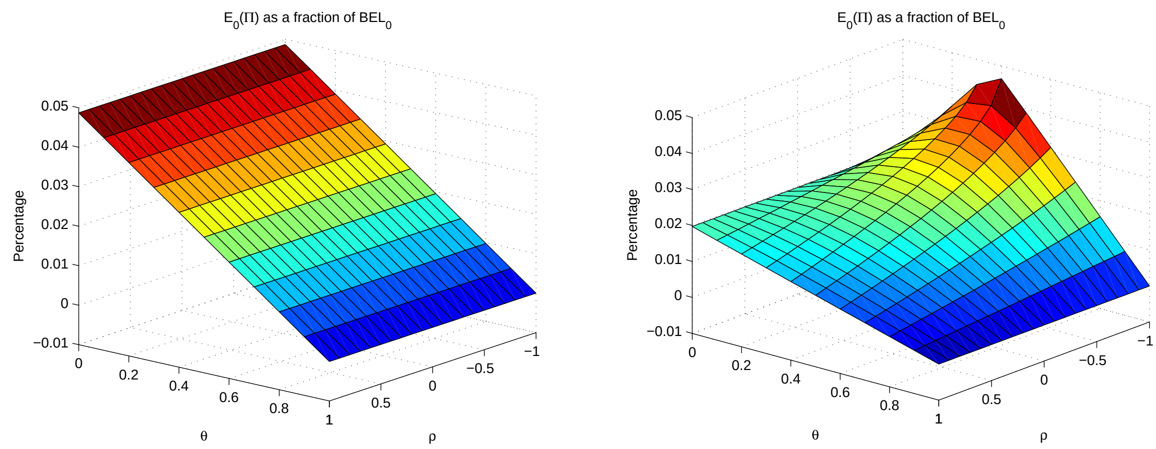


Figure 4.8: $\mathbb{E}_0(\Pi)$ as a fraction of BEL_0 (no hedge) for counterparties with rating AAA (left) and B & CCC or lower (right) for different correlation coefficients (ρ) and fraction hedged (θ). $\theta = 1$ and $\theta = 0$ correspond to the scenario of no hedge and a full hedge, respectively.

The amount an insurer spends on the swap in terms of premium is, by construction, for a AAA rated counterparty decreasing in the fraction not hedged[1]. $\mathbb{E}_0(\Pi)$ in (3.31) decreases as θ increases, as can be seen in Figure 4.8. Since the scenario $\theta = 1$ corresponds to the scenario in which no hedge is arranged, the required total risk margin is equal to the risk margin for longevity risk, which results in $\mathbb{E}_0(\Pi) = 0$. For a AAA rated party entering a full hedge results in the lowest total required risk margin and therefore in the highest amount that might be spend on the swap. The results in this figure for $\theta = 0$ correspond to the row for a AAA (B) rated party in Table 4.6 for a 65-year old.

For a party rated B or below, the effect of diversification becomes clear in the premium that might be paid for the swap as well. The total amount that can be spend on the swap is largest for the same fraction

[1] Note that the axes for both θ and ρ are reversed in Figure 4.8 compared to Figure 4.7.

hedged where the total required solvency capital was lowest in Figure 4.7, namely for θ in between 30 to 40% and a negative correlation coefficient. This of course is by construction as well, since $\mathbb{E}_0(\Pi)$ depends on RM_0^{Total} which is determined by the current and future total amount of required solvency capital. Again, for low positive values of ρ, $\mathbb{E}_0(\Pi)$ for a partial hedge is higher compared to a full hedge. Thus, a higher price may be paid for a partial hedge when $\mathbb{E}_0(\Pi)$ is considered.

Note that $\mathbb{E}_0(\Pi)$ is the maximum premium that an insurer might spend on a swap, such that the insurer is not worse off at inception of the swap contract considering the fair value of both contracts. Arranging a partial hedge leads to diversification effects when the credit rating of the counterparty is low enough, i.e. the probability of default is high enough. The required amount of solvency capital is then lower when a partial hedge is set up compared to a full hedge and this diversification effect is thus included in the associated risk margin which is lower compared to a full hedge. The fact that $\mathbb{E}_0(\Pi)$ is higher for a partial hedge does not imply that a higher price must be paid. In fact, it would seem illogical to pay more for a partial hedge than for a full hedge. But due to diversification the insurer does have more room for negotiating a premium to pay for the swap.

When an insurer determines whether to enter a swap, a trade off has to be made between the capital release and the premium that must be paid. As a hedge is generally set up to mitigate a risk factor, in this case longevity risk, the swap might be set up such that the effectiveness of the hedge is largest. This is the case when a full hedge is arranged with a high rated counterparty. Then, a higher price is likely to be paid, but the capital release is largest in such a scenario, as shown in Table 4.7. Whether the capital release or the price of the swap is more important depends on the preferences of the hedge buyer.

5 Sensitivity Analysis

Certain parameters used to calculate the premium, i.e. both $\mathbb{E}_0(\Pi)$ and π, are assumed to be constant. It therefore is interesting to look at the sensitivity of some results when the chosen values of these parameters are adapted. Also, earlier calculations were based on best estimate projections of mortality rates determined by the Lee-Carter model. One could also simulate survival rates with the bootstrap procedure and determine the premium corresponding to these simulated survival rates. Performing a large number of bootstrap iterations gives an idea of the sensitivity of the premium to mortality rates projected by the Lee-Carter model.

5.1 Sensitivity Analysis for the Endownment Contract

The analysis and results shown in Section 4.1.1 are based on best estimate mortality rates determined by the bootstrap simulation procedure. Each bootstrap iteration, new mortality rates are computed and cumulative survival rates are determined. To obtain best estimate mortality rates the mean is taken over all bootstrapped mortality rates. Liabilities are then computed by use of these best estimate cumulative survival rates.

As stated in the introduction. To gain insight in the sensitivity of $\mathbb{E}_0(\Pi)$ and π, for each bootstrap iteration future survival rates can be estimated with which the premium can be computed. Then, confidence intervals for $\mathbb{E}_0(\Pi)$ and π can be determined when a large number of bootstrap iterations is performed. The parameters are as before for an endownment contract starting at age 65 in 2012 and paying 1 (in arrears) at T if the policyholder is alive. Figure 5.1 shows results of the performed simulation. π and $\mathbb{E}_0(\Pi)$ are both somewhat sensitive to projected mortality rates.

The 99.5% quantiles represent cases in which mortality improved substantially and mortality rates are lower than best estimate mortality rates. Then, the probability that people live longer increases. This causes more uncertainty in the annuity contract, leading to higher $SCRs$ and RMs which is reflected in the higher value of π for this quantile. The opposite holds for the 0.5% quantiles.

The confidence bounds for π are narrow which shows that it is not very sensitive to different mortality rates. The results in Figure 5.1 also show that the uncertainty is small for payments closer to inception of the contract. The uncertainty increases for longer time to maturity, due to the higher uncertainty in mortality projections further away. This results in more variation in the capital requirements for longevity risk. $\mathbb{E}_0(\Pi)$ is more sensitive to the uncertainty from projected mortality than π. Since it is a direct reflection of the longevity risk in an endownment contract it is more sensitive to changing mortality rates, as these serve as main input for the calculation of $\mathbb{E}_0(\Pi)$.

The right graph in Figure 5.1 also shows $\mathbb{E}_0(\Pi)$ in case default risk is taken into account. The differences between the case when default risk is excluded and a swap is arranged with a BB rated party can be found in the level of the premium as shown earlier in Figure 4.3. The sensitivity to different mortality rates results in similar confidence bounds for both cases. The bounds become narrower close to the top of the graph since the slope of $\mathbb{E}_0(\Pi)$ changes.

Consider also the sensitivity to the interest rate. Until now the interest rate was assumed to be flat at 4% to calculate the variables required to determine the premium. A different interest rate might result in different values of the premium in the fixed leg of the swap as well as the total premium, $\mathbb{E}_0(\Pi)$. However, it turns out that changing the interest rate has no effect on the premium in the fixed leg of the swap for an endownment contract. A parallel shift of the interest rate affects the best estimate and stressed liabilities in the same, parallel, manner. Hence, the required solvency capital and the risk

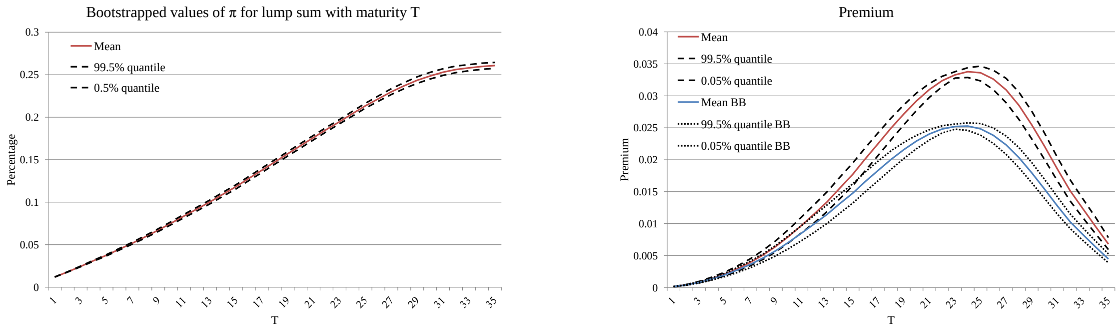


Figure 5.1: Bootstrapped $\mathbb{E}_0(\Pi)$ for an endownment contract with a payment of 1 if a 65-year old Dutch male in 2012 is alive at maturity T when default risk is excluded (red line) and included. A BB rated counterparty (blue line) is considered. Also bootstrapped values of π are shown (left). 99% confidence intervals are based on 1000 bootstrap simulations.

margin are affected in an almost parallel shift as well. Therefore the value of π that solves the equation (4.4) remains almost the same when the interest rate is changed in a parallel manner.

When a real term structure is applied, i.e. the zero curve of September 2012 from DNB is used to construct future zero curves by using forward rates, there appear to be small differences between the premium in the fixed leg computed with this term structure and the flat base case of 4%. The differences are however very small, only at the third decimal the premiums differ and they increase with maturity T and are therefore not shown here. This shows that the premium to be paid in the fixed leg of an endownment contract is quite robust with respect to interest rate changes.

The total premium at inception of the swap contract, $\mathbb{E}_0(\Pi)$, in contrast to π is sensitive to a change in the interest rate. As it is equal to the risk margin for longevity when default risk is excluded, increasing the interest rate decreases the required risk margin and therefore the premium that might be paid for the swap. The opposite holds for a decrease in the interest rate. Figure 5.2 shows this for several interest rate changes.

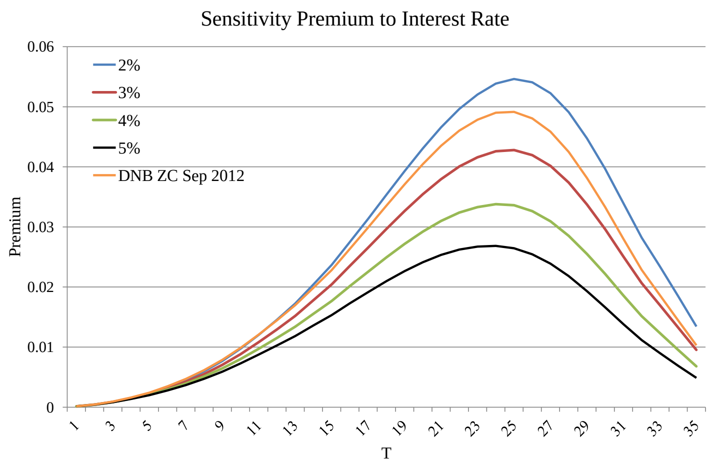


Figure 5.2: $\mathbb{E}_0(\Pi)$ for various interest rates, flat and the DNB Zero Curve of September 2012, for a lump sum payment of 1 at T for 65-year old Dutch males in 2012.

5.2 Sensitivity Analysis for the Annuity Contract

A similar analysis can be performed for the annuity contract for a Dutch male aged 65 in 2012. Parameters, such as the interest rate and maximum age are as before. Then the bootstrapped mortality rates are used to determine the value of the liabilities as well as the SCR and RM for longevity risk. Different mortality rates lead to a different estimate of BEL_0, SCR_t and RM_0. Hence the mortality rates implied by the parallel reduction π, not taking into account counterparty default risk, for an annuity contract differ accordingly. Results are shown in Table 5.1 for different ages as of which the annuity contract starts. The "Exact" approach corresponds to values of the SCR as in (3.17) and the "Driver" approach refers to (3.20). When comparing the results in Table 5.1 with Table 4.5 it becomes clear the results are quite robust, as the mean values of the premium correspond to the values found earlier. The results show that the premium is somewhat sensitive to different mortality projections. However, the confidence bounds are quite narrow, as was the case for the endownment contract in Figure 5.1.

The bootstrapped confidence bounds for the exact approach are wider than the bounds under the driver approach, as the bootstrapped cumulative survival probabilities are used for the calculation of the stressed liabilities for $t > 0$ as well. The bootstrapped survival rates therefore have more impact on the exact calculation of the SCR than on the SCR calculation using the driver approach. Since the driver approach uses the constant fraction SCR_0/BEL_0 to determine future SCR_t, this leads to wider confidence bounds for the exact approach.

Age	Approach	0.5% quantile	Mean	99.5% quantile
65	Exact	15.400%	15.967%	16.543%
	Driver	11.546%	11.828%	12.106%
66	Exact	14.833%	15.367%	15.903%
	Driver	11.205%	11.476%	11.743%
67	Exact	14.248%	14.739%	15.251%
	Driver	10.875%	11.123%	11.361%

Table 5.1: Values of π for an annuity contract for Dutch males aged 65 to 67 in 2012. A bootstrap procedure of 5000 simulation of mortality rates was performed using the Lee-Carter model. The exact approach is based on the calculation of the SCR by (3.17), where the driver approach corresponds to (3.20) to calculate future SCR.

The sensitivity of $\mathbb{E}_0(\Pi)/BEL_0$ to different mortality projections is given in Table 5.2. A distinction is made between the case where default risk is neglected and default risk is taken into account. The sensitivity of π under default risk is considered as well. In this table π for $p^{(j)} = 0$ is excluded as its values are given in Table 5.1 when the exact approach is used.

The results in Table 5.2 show that using different projected mortality rates affects the value of both $\mathbb{E}_0(\Pi)/BEL_0$ and π, as was also the case for the endownment contract shown in Figure 5.1. $\mathbb{E}_0(\Pi)/BEL_0$ is computed by using the bootstrapped mortality rates for both $\mathbb{E}_0(\Pi)$ and BEL_0. Relative to the mean, the confidence bounds for $\mathbb{E}_0(\Pi)/BEL_0$ are larger compared to the quantiles for π. This was also the case for the endownment contract when default risk was excluded. The mean of the simulated values should correspond with the values in Table 4.6. The fact that the results differ is likely caused by the relatively small number of simulations. When more simulations would be performed the mean will converge to the values found in the table where best estimate mortality rates were used based on 50,000 bootstrap simulations.

The sensitivity of the variables of interest, SCR_0, RM_0 and π, to different interest rates is considered below, where default risk is excluded. Default risk is left out to examine the sensitivity to changing interest rates since it will not give more insight in the results. Until now, all results are based on a flat interest rate of 4%, a different interest rate affects the liabilities, solvency capital requirements and risk margin. This might cause the premium in the fixed leg to be affected as well. Earlier the premiums of the endownment contract were not very sensitive to different interest rates. The annuity however is a different contract with different characteristics and here the premium does change. As can be seen in Table 5.3.

	Rating	0.5% quantile	Mean	99.5% quantile
$\mathbb{E}_0(\Pi)/BEL_0$	$p^{(j)}=0$	4.695%	4.873%	5.055%
	AAA	4.693%	4.871%	5.053%
	AA	4.685%	4.862%	5.044%
	A	4.642%	4.819%	5.000%
	BBB	4.447%	4.620%	4.800%
	BB	3.582%	3.739%	3.903%
	B	1.860%	1.970%	2.090%
π	AAA	15.411%	15.963%	16.519%
	AA	15.407%	15.959%	16.515%
	A	15.386%	15.938%	16.494%
	BBB	15.288%	15.837%	16.392%
	BB	14.783%	15.320%	15.860%
	B	13.195%	13.670%	14.136%

Table 5.2: Results are shown for the sensitivity of $\mathbb{E}_0(\Pi)/BEL_0$ when bootstrapped mortality rates are used, for both the case where default risk is disregarded and taken into account. For the latter the sensitivity of π to bootstrapped mortality is depicted as well. Results are based on a 65-year old Dutch male in 2012. RM is computed by the SCR in (3.17). 2500 bootstrap iterations were performed.

	65			67		
Interest rate	$\frac{SCR_0}{BEL_0}$	$\frac{RM_0^{(\mathrm{I})}}{BEL_0}$	π	$\frac{SCR_0}{BEL_0}$	$\frac{RM_0^{(\mathrm{I})}}{BEL_0}$	π
2%	7.477%	6.239%	16.972%	8.134%	6.205%	15.642%
3%	6.819%	5.513%	16.469%	7.472%	5.531%	15.193%
4%	6.229%	4.874%	15.961%	6.876%	4.935%	14.740%
5%	5.702%	4.315%	15.450%	6.339%	4.408%	14.289%
DNB ZC Sep. 2012	6.973%	5.656%	16.520%	7.653%	5.693%	15.264%

Table 5.3: Sensitivity of SCR_0/BEL_0, $RM_0^{(\mathrm{I})}/BEL_0$ and π to change in interest rate. $RM_0^{(\mathrm{I})}$ is determined by the SCR in (3.17). All results are based on mortality projected by the Lee-Carter model.

A lower interest rate increases the level of liabilities as well as the required solvency capital and the associated risk margin for longevity. The increase is however not parallel such that the ratios would remain the same compared to the scenario of the flat interest rate of 4%. Instead, the ratios increase and therefore π that solves (3.26) increases (default risk is disregarded here and thus $RM_0^{\mathrm{cdr}}=0$ and $\mathbb{E}_0(\Pi) = RM_0^{(\mathrm{I})}$). The opposite holds for an increase in the interest rate. The zero curve (ZC) of September of DNB is applied as well to examine the results when a term structure is applied. This term structure was also applied for the endownment contract. The zero curve of September 2012 is used to determine forward rates with which future zero curves are determined. These are then used for determining the value of liabilities and required solvency capital in future years. The results indicate that the interest rates of the current zero curve are quite low, as the risk margin at time zero, as a percentage of BEL_0, is larger compared to the flat interest rate of 3%.

The change in $\mathbb{E}_0(\Pi)$ and π in both the endownment and annuity contract when the interest rate is adapted is not directly a characteristic of the longevity swap but a consequence of the sensitivity of BEL and RM to the interest rates. As these are used to determine the value of the swap for an insurer. When the value of the liabilities changes due to changes in the interest rate the amount of capital available that might be spend on the swap changes. This is reflected by the change in both $\mathbb{E}_0(\Pi)$ and π.

6 Conclusion & Recommendation

Currently there is no liquid market for mortality-linked securities such as longevity or mortality swaps. In case a longevity swap is entered it is likely that a premium has to be paid to a counterparty, in order for a potential counterparty to assume longevity risk. Due to absence of a liquid market the risk premium for longevity cannot be determined by a mark-to-market approach, and is therefore unknown. As such, insurers cannot determine the market value of their liabilities for life insurance contracts. SII therefore obliges insurers to raise the best estimate of the liabilities of such insurance contract to a theoretical market value by adding a risk margin, or market value margin, which serves as a risk premium. The term "market value" however is improper as no market for mortality-linked securities exists. It was therefore referred to as a fair value. A risk margin is required for risks that cannot be hedged in the market and therefore insurance contracts that are exposed to longevity risk require such a margin. The fair value of a contract is assumed to represent the amount an insurer should pay when he transfers the liabilities of a specific contract to another insurer.

This thesis aimed at determining the maximum, or total, premium an insurer might pay to a counterparty such that at inception of the swap the fair value of the insurance contract for which the swap is arranged does not exceed the fair value of the contract without the hedge. A distinction is made between the total premium and the one-time parallel percentage reduction in best estimate mortality rates which spreads this amount over the expected contract term corresponding to the payment structure of the fixed leg payments of a longevity swap.

First a scenario was considered where default risk was excluded from the analysis. Then, the total premium to spend on a swap is reflected by the risk margin for longevity. As this is the premium an insurer likely has to pay for transferring its liabilities to another party. Therefore the risk margin reflects the amount an insurer might be willing to spend on a longevity swap. In case the insurer would spend more on the swap it would have been beneficial to transfer the liabilities to another party by paying the premium equal to the risk margin, assuming such a party exists and agrees that the risk margin is a good compensation for the longevity risk possibly occuring. The parallel percentage reduction in best estimate mortality rates, i.e. the premium in the fixed leg, was then determined by equating the fair value of liabilities to the fair, in this case best estimate, value of liabilities under the swap contract. The shifted mortality rates implied by the fair value of the insurance contract without the swap can then be interpreted as the SII implied fair mortality rates. As these are the mortality rates that, when applied to determine the best estimate value of liabilities, immediately raise the value of liabilities to the fair value of the insurance contract exposed to longevity risk. Therefore, these adjusted mortality rates might even be considered as a starting point for a life market to develop as these incorporate the risk premium for longevity represented by the risk margin, in case the risk margin for longevity would be acknowledged as a good approximation for the risk premium of longevity risk.

Two insurance contracts were considered. An endownment contract, with a single payment in case the policyholder is alive at maturity, and an immediate annuity contract with a yearly payment as long as the annuitant is alive. From the endownment contract it became clear there is a term structure for longevity risk premia. As the premium depends on the time of maturity of the contract. This was also reflected by the parallel reduction in one-year best estimate mortality rates. As these differed for different maturities. The same analysis was performed for an annuity. A distinction was made for the calculation of the amount of required solvency capital which led to different values of the risk margin. One of which is a by QIS5 (2010) permitted simplification. This method turned out to underestimate the required risk margin compared to the, considered to be the more, exact method. The premium that might be paid for the swap therefore differed substantially between the two methods. It was also shown that future best estimate mortality rates have to be adjusted significantly to price the longevity risk in an annuity contract.

When default risk is taken into account the insurer must maintain solvency capital and an associated risk margin for counterparty default risk. Since default risk, in case of a longevity swap where the underlying risk factor remains longevity risk, cannot be hedged. The fair value of the insurer's liabilities under the swap contract was shown to consist of the best estimate liabilities under the swap contract, plus a risk margin for counterparty default risk. A model was proposed based on the payments an insurer expects to make before and after default to arrive at expressions for the best estimate liabilities under the swap as well as the required solvency capital for default risk.

The required solvency capital and risk margin for default risk increase with the deterioration of the credit rating of the counterparty. Because the premium was shown to be the difference between the risk margin for longevity and counterparty default risk, the amount an insurer might pay for a swap decreases when the creditworthiness of the counterparty declines.

The premium in the fixed leg found however does not necessarily change when default risk is considered. Depending on the type of insurance contract, default risk impacts this parallel reduction in different ways. In an endownment contract only one swap payment, at maturity, is required to hedge longevity risk. The time of default does not influence the premium in the fixed leg. In an annuity contract on the other hand, in which the expected time to default influences the number of swap payment that take place, the time of default affects the premium in the fixed leg. When the probability of default increases the expected number of swap payments before default declines. The hedge effectiveness of the swap therefore decreases and thus an insurer likely prefers to pay a lower premium incorporated in the fixed leg payments.

Another important aspect from entering a swap may be a capital release at inception of the swap agreement. Even if the insurer pays a premium equal to the amount such that he is indifferent between entering a full swap and maintaining longevity risk, the capital release due to the lower SCR for default risk compared to longevity risk might be a reason to enter such a swap contract. The capital release is largest for high rated parties as the required solvency capital for default risk is smallest, but a higher premium has likely to be paid. It depends on the preferences of the insurer which of the two weighs more heavily, but a trade off should be made taking into account the goal of entering a longevity swap.

When a market for mortality-linked securities might develop in the future liabilities can directly be valued by for instance market implied mortality rates and a risk margin is no longer required. It will also become clear whether the risk margin and capital requirements for longevity risk required under SII reflected the risk premium investors require for assuming longevity risk properly.

Even though a liquid market for mortality-linked securities is far away, this thesis might give insight in a possible starting point for market consistent mortality rates. Since SII legislation will be introduced in the entire EEA, risk margins for longevity are obligatory for a great number of insurers. In case there would be a method widely accepted to determine the risk margin for longevity, this margin could well serve as a starting point for trading insurance contracts exposed to longevity risk. Then the mortality rates implied by this margin, or the premium incorporated in the fair value of liabilities of specific insurance contracts, may be used as a starting point to trade mortality-linked securities.

An important note regarding the performed analysis has to be made. A model was proposed to determine the required solvency capital for counterparty default risk regarding a longevity swap. This risk was priced by the risk margin for default risk and including default risk in the BEL^{π}. However, the proposed method only prices the risk regarding the payments the insurer has to make given a certain scenario occured, i.e. default or not. This results in a payment of the fixed or floating leg, where longevity risk is priced by a premium in the fixed leg payments. However, the risk of default occuring is not priced and therefore the fair value of the liabilities under the swap contract is somewhat underestimated.

In contrast to longevity risk, the risk of default of a counterparty with a specific rating is priced in the market and therefore can be calibrated by using market data. Hence, so called risk neutral default probabilities reflecting the market price of default risk can be determined and these should have been applied instead of the default probabilities under the physical probability measure in Table 3.2. However, since the risk neutral default probabilities will likely be higher than the default probabilities applied here, the risk margin and fair value increases. Thereby decreasing $\mathbb{E}_0(\Pi)$ as well as π found in this thesis. Hence, using risk neutral default probabilities will have a monotonic impact on the results found and the results still give insight in what the premium an insurer might pay for a swap by use of the fair value of an insurance contract under SII legislation.

7 Future Research

This chapter elaborates on possible extensions for future research. Several assumptions were made to simplify the model as well as make it easier to interpret. A considerable amount of research is performed on longevity risk and topics related to it, such as quantifying longevity risk and possible methods to hedge for this risk factor. Quantifying this risk factor is difficult as no unambiguous method exists. In this thesis the link with SII is made to be able to quantify the amount of longevity risk an insurer faces considering certain insurance products. Here a swap was considered, but other products might have been considered as well, such as longevity bonds.

7.1 Interest Rate Risk

When a longevity swap contract is arranged to hedge longevity risk in certain insurance products this is done to mitigate the amount of longevity risk an insurer is exposed to in specific insurance contracts. The longevity swap considered in this thesis pays a floating amount such that the payments an insurer has to make to its policyholders are precisely hedged, since these payments depend on the number of realized survivors in the reference population. Implicitly, the interest rate risk is then also hedged. Whether this has impact on the required amount of solvency capital for interest rate risk is unclear. It might be interesting to look into this further as the possible advantages of a longevity swap may not be limited to hedging longevity risk.

Then, in case the reference population is unequal to the insured population, basis risk might incorporate both longevity and interest rate risk. It might be difficult to quantify which of the two weighs heaviest and how to mitigate the remaining risk.

7.2 Collateral

Next to maintaining an SCR for counterparty default risk a commonly used tool to mitigate default risk is collateral managment, see International Swaps and Derivatives Association, Inc. (2005). Here a model is suggested similar to the models in Section 3.3.3 to take into account collateral managment. Results are however omitted as it is expected that collateral makes the difference between the premium an insurer might pay for a swap in case default risk is taken into account closer to the premium when default risk is excluded. Hence, the earlier found results may be seen as bounds in between which the results for collateral likely will fall, depending on the amount of collateral posted.

In case of an interest rate swap, the mark-to-market value is determined on a regular basis and the party for which the value of the swap is negative posts collateral. The amount posted as collateral may differ, full collateralization can occur but also other collateral arrangements exist. For instance, collateral is posted only if the mark-to-market value exceeds a certain threshold, then the excess over the threshold is posted as collateral. Alternatively, only a fraction of the value of the swap is posted as collateral. It depends on the type of contract and the agreement between the parties which method is used.

Bilateral collateral posting, meaning that both parties in the swap have the obligation to post collateral, is an important risk mitigator for both parties who entered the longevity swap, as Biffis et al. (2011) state. Posting collateral reduces the risk of lost swap payments in case of default of the counterparty, as the insurer has a claim on the collateral in case the value of the swap is positive from the insurer's perspective. On the other hand, capital posted as collateral cannot be invested freely and can only return the risk free rate. One can also use, as in Johannes and Sundaresan (2003), that the cost of posting collateral is equal to the gain of holding collateral. Hence, if the swap is in the money from the insurer's perspective the insurer is entitled to the yield on collateral posted by its counterparty. However, when the insurer posts collateral the yield on collateral is a cost factor for the insurer and he incurs a loss. In the proposed model here, only the costs of posting collateral are taken into account.

It is unclear how often collateral will be posted when longevity swaps are considered. In the example below collateral is assumed to be posted on a yearly basis. Mortality improvements or deteriorations do not have a volatile structure, hence the assumption of yearly collateral posting. One could however imagine scenarios in which it becomes clear througout the year that a shock in mortality occured. Then the party for which the shock leads to a positive value of the swap might require its counterparty to post collateral, even though the prespecified moment has not yet arrived. Moreover, it is a convenient assumption due to the characteristics of mortality rates. As these are determined on a yearly basis.

An extra default scenario is required compared to valuing the liabilities in (3.22), due to cash flows arising in the year of default when collateral has been posted. The probability of default in year s is given by

$$\mathbb{P}(D = s) = (1 - p^{(j)})^{s-1} \cdot p^{(j)}.$$

The best estimate value of liabilities at time t, taking into account all possible cash flows resulting from posting collateral, then becomes

$$\begin{aligned} BEL_t^{\pi} = & \sum_{s=t+1}^{T} \mathbb{P}(D > s) \left(K_s + \eta \cdot \mathbb{E}_t(C_s^-) \right) \cdot P_t(s-t) \\ & + \sum_{s=t+1}^{T} \mathbb{P}(D = s) \left(-\mathbb{E}_t(C_s^+) - \nu \sum_{\tau=s}^{T} \mathbb{E}_t(S_\tau - K_\tau) \cdot P_s(\tau - s) \right) \cdot P_t(s-t) \\ & + \sum_{s=t+1}^{T} \mathbb{P}(D \leq s) \cdot \mathbb{E}_t(S_s) \cdot P_t(s-t). \end{aligned} \tag{7.1}$$

This formula has a few new components compared to (3.22). It becomes clear that in the year of default, as well as years prior to default several scenarios can occur.

Any time prior to default, the insurer has one certain and possibly second liability stream. Where it is again assumed that the swap is perfect and no basis risk remains. The first liability stream are the fixed leg payments of the swap. The possibly second liability consists of the cost of collateral. When the swap has negative value, i.e. is out of the money, from the insurer's perspective, the insurer posts collateral which leads to additional costs[1]. The cost of collateral is a percentage, η of the collateral posted by the insurer. Hence, $\eta \in (0, 1)$.

When default occurs at s, the insurer has to pay floating leg payments as of that moment. Also, the insurer claims collateral if posted by its counterparty which decreases the value of his liabilities. Next, the insurer has to pay, or receives, a fraction $\nu \in (0, 1)$ of the value of expected future swap payments. This can have both a positive or negative impact on liabilities. In case the swap has positive expected value for the insurer, liabilities decrease. When the expected value of the swap is negative this raises the value of liabilities.

After default, the insurer is back in the case where it did not enter a swap contract and its liabilities are valued by the floating leg payments. Then, an SCR and RM for longevity risk have to be held again in case no new swap can be arranged. The cost of entering a new swap with a new counterparty highly depends on the realized mortality of the insured population. In case the number of insureds that survived to t is larger than anticipated the portfolio is highly exposed to longevity risk. In that case it might be very costly to arrange a new swap. When fewer than expected insureds survived entering a new swap contract might be less costly.

The best estimate value of liabilities when the swap is entered, in (7.1), takes into account all possible scenarios that might occur. Liabilities increase with possibly occuring additional costs and decreases with possible gains. As the probability of default is small the impact of the various components on (7.1) might not be substantial. It is however necessary to take these into account as the value of the insurer's liabilities change, which might affect the attractiveness of the swap contract.

[1] The following notation is used $a^+ \equiv \max(a, 0)$ and $a^- \equiv \max(-a, 0)$

A similar method as in (3.23) can be applied to determine the required solvency capital for default risk. But now the SCR_t^{cdr} is affected by the amount of posted collateral, i.e.

$$SCR_t^{\text{cdr}} = EL_t^{\text{stress}} - EL_t^{\text{BE}} - C_t. \tag{7.2}$$

Where C_t is the fraction of the value of future swap payments posted as collateral and EL_t is as before in Section 3.3.3.

The amount of capital posted as collateral at time t, C_t, is defined as a fraction $\varphi \in (0,1)$ of the value of the swap at time t, as in Biffis et al. (2011). The value of the swap at t from the insurer's perspective is the expected discounted value of the difference of the floating and fixed leg of the swap. Hence,

$$C_t = \varphi \sum_{s=t+1}^{T} \mathbb{E}_t(S_s - K_s) \cdot P_t(s-t). \tag{7.3}$$

Note that C_t is negative when the swap is out of the money from the insurer's perspective, then the insurer posts collateral. C_t is positive in case the counterparty has to post collateral.

This shows that the amount of default risk depends on φ. Default risk is reduced significantly when φ is close to 1 and therefore this scenario is similar to the case when default risk is neglected. Note that the premium an insurer might pay for the swap, $\mathbb{E}_0(\Pi)$, likely increases compared to the case without collateral posting. When the swap is in the money from the insurer's perspective the counterparty posts collateral resulting in a lower SCR_t^{cdr} which causes RM_0^{cdr} to decrease. Then both $\mathbb{E}_0(\Pi)$ and π increase and come closer to the values when default risk was neglected.

Here a simulation study must be performed so that the current estimate of future collateral posting is determined. The floating leg of the swap has to be simulated as the difference between the two legs of the swap determines the amount of collateral that should be posted, as well as the associated costs for the insurer. This however makes the problem much more complex. The amount of collateral posted at $t = 0$ can be determined by simulating the floating leg and determining the present value of the swap. At $t > 0$ however the amount of collateral to be posted should be determined by a new best estimate depending on the realized fraction of survivors at $t > 0$. This means that for each generated floating leg at t one should perform a new simulation. This increases the calculation time exponentially with the number of time steps. A simplification can be made to approximate this method. For $t > 0$ it is assumed that the best estimate fraction of survivors realized up to year t. A consequence is that a new best estimate has to be computed for $t > 0$ by using the previous best estimates as new data to base the new best estimate on. Then the Lee-Carter parameters should be estimated again using the additional data and a new best estimate should be determined. Note that this can only be done by using the Singular Value Decomposition to compute Lee-Carter parameters, as the Poisson model requires data of $E_{x,t}$ and $D_{x,t}$. The SVD approach uses data of $m_{x,t}$ which is used to calculate future mortality and survival rates. This simplification might be a reasonable approximation of the exact method with much lower computation time.

7.3 Other extensions

Considering different mortality models might be an extension for future research. As it mainly served as a tool to estimate future mortality the relatively simple Lee-Carter model was chosen to predict future mortality, but other models might be used as well. The results are sensitive to the mortality rates used, as was shown when projected mortality from the Dutch AG was applied. One might argue that other models are preferred over the Lee-Carter model for predicting mortality and should therefore be used to perform a similar analysis.

It was implicitly assumed that both parties in the swap used the same best estimate mortality rates, as the floating leg was modelled by best estimate mortality from the Lee-Carter model and the same rates were used to determine the parallel reduction in best estimate mortality rates in the fixed leg of the swap. This might be a quite heavy assumption as both parties might have different opinions on how mortality will develop in the future. Hence, both legs of the swap might be reflected by different mortality projections to take this into account.

Also, throughout the entire analysis it was assumed that basis risk was absent. Including this will affect the effectiveness of the hedge and might make insurers more hesitant in entering a longevity hedge contract which may present itself in lower values of the premium an insurer is willing to pay for a swap.

To determine the risk margin for longevity risk the standard model in QIS5 (2010) was applied to compute the required solvency capital for the amount of longevity risk in a specific insurance contract. However, developing an internal model to determine the capital buffer for longevity risk could be an additional extension of this thesis. Insurers will develop an internal model to improve the effectiveness of the required solvency capital compared to the capital requirements according to the standard model. This changes the risk margin, which is the main driver of the value of the swap from the insurer's perspective.

It was assumed that no new longevity swap contract could be arranged in case the counterparty of the insurer defaulted. At the moment this might be a reasonable assumption as only a few of such contracts are arranged. However it does not have to be the case in general. It was mentioned that it will likely depend on the realized mortality in the insured population whether arranging a new swap contract would be expensive. When longevity occured, a new counterparty might require a higher risk premium for assuming longevity risk. Collateral could play an important role here. When longevity occurs, the swap is likely in the money from the insurer's perspective and collateral is posted by the counterparty. In case the counterparty defaults, the collateral obtained by the insurer might be used to finance a new swap contract with another counterparty.

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A Appendix

This appendix contains the proof that π is independent of the probability of default in case of an endowment contract, in case counterparty default risk is taken into account. Also, results for π found for an endowment contract with maturity T for different mortality rates are shown as well as the fraction $\mathbb{E}_0(\Pi)/BEL_0$ in case default risk is excluded and included.

A.1 Proof

Equation (3.26) is used to show that the value of π which solves this equation is independent of the probability of default. (3.26) can be written as

$$BEL_0^{\pi} - BEL_0 = RM_0^{\text{longevity}} - RM_0^{\text{cdr}}. \tag{A.1}$$

First the left hand side of this equation is considered here. This can be rewritten to

$$\begin{aligned} BEL_0^{\pi} - BEL_0 &= (\mathbb{P}(D > T) \cdot K_T + \mathbb{P}(D \leq T) \cdot \mathbb{E}_0(S_T)) \cdot P_0(T) - \mathbb{E}_0(S_T) \cdot P_0(T) \\ &= \mathbb{P}(D > T) \cdot K_T \cdot P_0(T) - \mathbb{E}_0(S_T) \cdot (1 - \mathbb{P}(D \leq T)) \cdot P_0(T) \\ &= \mathbb{P}(D > T) \left(K_T \cdot P_0(T) - \mathbb{E}_0(S_T) \cdot P_0(T) \right). \end{aligned}$$

Where, using (4.1),

$$BEL_0 = {}_T p_{x_0}^{\text{BE}} \cdot N \cdot P_0(T) = \mathbb{E}_0(S_T) \cdot P_0(T).$$

Since the floating leg is reflected by best estimate mortality rates, (3.12) shows that $\mathbb{E}_0(S_T) = {}_T p_{x_0}^{\text{BE}} \cdot N$. The subscript $t = 0$ is supressed in ${}_T p_{x_0}^{\text{BE}}$ as mentioned in Section 3.2. Also, $\mathbb{P}(D > T) = 1 - \mathbb{P}(D \leq T)$ is used above to arrive at the last equality.

The right hand side of (A.1) can also be rewritten. The expression for the required solvency capital for default risk for an endowment contract in (4.6) is used.

$$\begin{aligned} RM_0^{\text{longevity}} - RM_0^{\text{cdr}} &= RM_0^{\text{longevity}} - \sum_{s=0}^{T-1} CoC\% \cdot SCR_s^{\text{cdr}} \cdot P_0(s) \\ &= RM_0^{\text{longevity}} - \sum_{s=0}^{T-1} CoC\% \cdot (\mathbb{P}(D \leq T) \cdot \mathbb{E}_s(S_T^{\text{stress}} - S_T) \cdot P_s(T-s)) \cdot P_0(s) \\ &= RM_0^{\text{longevity}} - \mathbb{P}(D \leq T) \left(\sum_{s=0}^{T-1} CoC\% \cdot (\mathbb{E}_s(S_T^{\text{stress}} - S_T) \cdot P_s(T-s)) \cdot P_0(s) \right) \\ &= RM_0^{\text{longevity}} - \mathbb{P}(D \leq T) \cdot RM_0^{\text{longevity}}. \end{aligned}$$

The last equality holds by construction since the swap is assumed perfect and the floating leg is reflected by best estimate mortality. Also, in this thesis the required solvency capital for default risk highly depends on the underlying remaining risk factor longevity. The difference between the $SCRs$ for longevity and default risk in case of an endowment contract is simply the multiplication with the probability of default for a certain year, since the fixed leg is deterministic at inception of the contract it has no impact on the required amount of solvency capital.

Then (A.1) can be written as

$$
\begin{aligned}
\mathbb{P}(D > T)\left(K_T \cdot P_0(T) - \mathbb{E}_0(S_T) \cdot P_0(T)\right) &= RM_0^{\text{longevity}} - \mathbb{P}(D \leq T) \cdot RM_0^{\text{longevity}} \\
&= RM_0^{\text{longevity}} - (1 - \mathbb{P}(D > T)) \cdot RM_0^{\text{longevity}} \\
&= \mathbb{P}(D > T) \cdot RM_0^{\text{longevity}}
\end{aligned}
$$

Hence

$$K_T \cdot P_0(T) - \mathbb{E}_0(S_T) \cdot P_0(T) = RM_0^{\text{longevity}},$$

which is exactly the equation that is solved for π when default risk is not taken into account. Hence, π that solves (A.1) is equal to π found when default risk is excluded. π is therefore independent of the probability of default in this specific model for SCR_s^{cdr}.

A.2 Results

T	LC	'74-'09	AG	T	LC	'74-'09	AG
1	1.200%	1.200%	1.200%	26	22.172%	20.405%	22.216%
2	1.804%	1.837%	1.821%	27	22.994%	21.041%	23.178%
3	2.451%	2.501%	2.448%	28	23.707%	21.583%	24.052%
4	3.120%	3.162%	3.084%	29	24.344%	22.068%	24.820%
5	3.762%	3.846%	3.728%	30	24.853%	22.529%	25.474%
6	4.475%	4.534%	4.377%	31	25.292%	22.894%	25.996%
7	5.183%	5.229%	5.054%	32	25.611%	23.215%	26.381%
8	5.932%	5.977%	5.751%	33	25.803%	23.430%	26.645%
9	6.716%	6.713%	6.470%	34	25.968%	23.633%	26.802%
10	7.541%	7.510%	7.230%	35	26.077%	23.772%	26.866%
11	8.381%	8.322%	8.030%	36	n.a.	n.a.	26.850%
12	9.212%	9.145%	8.859%	37	n.a.	n.a.	26.767%
13	10.046%	9.971%	9.718%	38	n.a.	n.a.	26.629%
14	10.950%	10.810%	10.614%	39	n.a.	n.a.	26.448%
15	11.832%	11.642%	11.522%	40	n.a.	n.a.	26.234%
16	12.811%	12.491%	12.438%	41	n.a.	n.a.	25.997%
17	13.763%	13.335%	13.356%	42	n.a.	n.a.	25.745%
18	14.736%	14.181%	14.292%	43	n.a.	n.a.	25.485%
19	15.700%	15.004%	15.231%	44	n.a.	n.a.	25.224%
20	16.664%	15.869%	16.176%	45	n.a.	n.a.	24.964%
21	17.618%	16.708%	17.141%	46	n.a.	n.a.	24.711%
22	18.563%	17.489%	18.120%	47	n.a.	n.a.	24.467%
23	19.479%	18.254%	19.120%	48	n.a.	n.a.	24.233%
24	20.426%	19.023%	20.139%	49	n.a.	n.a.	24.012%
25	21.352%	19.736%	21.191%	50	n.a.	n.a.	23.803%

Table A.1: π for an endownment contract that pays 1 at T if 65-year old Dutch male is alive T years from 2012. The table depicts results for various mortality rates, i.e. Lee-Carter projected mortality, historically estimated mortality and projections of the Dutch AG corresponding with Figure 4.2.

				Rating			
T	$p^{(j)}=0$	AAA	AA	A	BBB	BB	B
1	0.016%	0.016%	0.016%	0.016%	0.016%	0.015%	0.015%
2	0.047%	0.047%	0.047%	0.047%	0.047%	0.046%	0.043%
3	0.100%	0.100%	0.100%	0.099%	0.099%	0.096%	0.088%
4	0.177%	0.177%	0.177%	0.177%	0.175%	0.169%	0.149%
5	0.274%	0.274%	0.274%	0.273%	0.271%	0.258%	0.221%
6	0.410%	0.410%	0.410%	0.409%	0.405%	0.382%	0.318%
7	0.579%	0.579%	0.579%	0.577%	0.569%	0.532%	0.430%
8	0.797%	0.797%	0.796%	0.794%	0.782%	0.723%	0.566%
9	1.074%	1.074%	1.073%	1.069%	1.051%	0.963%	0.731%
10	1.427%	1.427%	1.426%	1.420%	1.393%	1.265%	0.932%
11	1.861%	1.861%	1.859%	1.851%	1.813%	1.630%	1.164%
12	2.373%	2.373%	2.370%	2.359%	2.306%	2.053%	1.423%
13	2.979%	2.979%	2.975%	2.960%	2.888%	2.547%	1.711%
14	3.757%	3.756%	3.751%	3.730%	3.632%	3.172%	2.068%
15	4.654%	4.653%	4.647%	4.619%	4.489%	3.883%	2.455%
16	5.842%	5.840%	5.832%	5.795%	5.621%	4.815%	2.953%
17	7.229%	7.227%	7.217%	7.168%	6.940%	5.888%	3.501%
18	8.930%	8.926%	8.914%	8.850%	8.552%	7.185%	4.144%
19	10.958%	10.954%	10.937%	10.854%	10.469%	8.712%	4.873%
20	13.402%	13.397%	13.375%	13.269%	12.773%	10.527%	5.711%
21	16.335%	16.328%	16.301%	16.164%	15.531%	12.677%	6.671%
22	19.877%	19.868%	19.833%	19.659%	18.853%	15.241%	7.778%
23	24.091%	24.080%	24.036%	23.816%	22.796%	18.250%	9.034%
24	29.548%	29.534%	29.477%	29.195%	27.892%	22.115%	10.617%
25	36.410%	36.392%	36.319%	35.957%	34.287%	26.924%	12.537%
26	44.322%	44.299%	44.207%	43.750%	41.638%	32.382%	14.624%
27	54.879%	54.849%	54.731%	54.143%	51.431%	39.613%	17.351%
28	67.475%	67.437%	67.286%	66.536%	63.084%	48.121%	20.443%
29	83.392%	83.343%	83.150%	82.191%	77.779%	58.759%	24.211%
30	101.889%	101.828%	101.584%	100.372%	94.803%	70.931%	28.346%
31	125.937%	125.859%	125.547%	124.000%	116.897%	86.620%	33.574%
32	154.044%	153.946%	153.552%	151.599%	142.643%	104.681%	39.352%
33	181.658%	181.538%	181.059%	178.684%	167.809%	121.964%	44.469%
34	221.340%	221.189%	220.588%	217.608%	203.976%	146.823%	51.921%
35	284.682%	284.483%	283.687%	279.742%	261.719%	186.575%	63.991%

Table A.2: $\mathbb{E}_0(\Pi)/BEL_0$ for an endowment contract that pays 1 at T if 65-year old Dutch male is alive T years from 2012. The table depicts results for the case that counterparty default risk is excluded, i.e. $p^{(j)}=0$, as well as for various counterparties with $p^{(j)}$ as in Table 3.2. The results where default risk is included correspond to the results in Figure 4.4. The column for $p^{(j)}=0$ corresponds to the results for $RM_0^{(\mathrm{I})}/BEL_0$ in Figure 4.2.