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Abstract

Dividend derivatives contain information about the expectations that investors have of stock dividends. We employ a state space model to estimate a term structure of discounted risk adjusted dividend growth enclosed in dividend derivative prices. A two state model of the term structure capturing short term mean reversion within a year and a medium term component which reverts at business cycle horizon to a long term constant is superior over a model restricted to a single state variable. The dividend term structure extrapolates to an implied price dividend ratio. This model estimate combined with current dividends captures most of the daily return variation of both the Eurostoxx 50 and the Nikkei 225 indices, despite mean reversion to constant long run growth being reasonably quick. Hence, investors do not seem to change their mind about what happens beyond the next business cycle.

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I. Introduction

Dividends are a key ingredient for valuing stocks. Investors attach a present value to expected future dividends and sum them to arrive at the value of a stock. As Campbell-Shiller (1988) have shown, stock prices thus vary because of changes in expected dividends, changes in interest rates, and changes in risk premiums. However, the expectations of dividends paid in the short run may at least partly be driven by other considerations than those of dividends paid in the distant future. Hence, investors will not only change their expectation of dividends from moment to moment, they may also change them for various maturities relative to each other, similar to a term structure of interest rates. In this paper, we focus on this term structure of expected dividends.

Given that the stock price is simply the sum of the present values of all dividends expected, Michael Brennan called in the late nineties for the development of a market for dividend derivatives: "Since the level of the market index must be consistent with the prices of the future dividend flows, the relation between these will serve to reveal the implicit assumptions that the market is making in arriving at its valuation. These assumptions will then be the focus of analysis and debate.", (Brennan, 1998). Brennan's wish came to life at the beginning of this century, with the introduction of derivatives referring to future dividend payments. These products exchange uncertain future dividends of an underlying stock or stock index in exchange for cash at the time of expiry. As such, they are forward looking in nature as they contain price information about expected dividends corrected for their risk. More precisely, the price of a single dividend future or OTC swap reflects the expected dividend for a given maturity discounted at the risk premium for this maturity. Finding present values of expected dividends only requires discounting these prices at the risk free rate.

In this paper, we use data on these new dividend derivatives to study the dividend term structure for four major stock markets. A key starting point of our analysis is that modelling the dynamics of a single variable, the difference between dividend growth and the sum of risk free rate and the risk premium, is sufficient to describe the entire term structure of discounted dividend derivative prices, and to obtain a total value for the stock index. We call this variable discounted risk adjusted dividend growth. Hence, we do not need to separately assume processes for interest rates, risk premiums and dividend growth rates, the simplicity of which is a major advantage of our approach. The variable can also be interpreted as a growth rate between present values of the dividends expected to be paid in the future.

Inspired by the affine models often used for modelling the term structure of interest rates, we show how to set up a standard affine model for discounted risk adjusted dividend growth. Specifically, our model resembles the interest rate model of Jegadeesh and Penacchi (1996), who use a two factor model, where the first factor reverts to a second factor, which in turn reverts to a long run constant. This model thus distinguishes a short term component, a medium term component and asymptotic growth. We cast this model in state space form and apply the Kalman filter Maximum Likelihood approach to estimate the model using dividend derivative prices of one to ten years. The resulting discounted risk adjusted dividend growth term structure describes the maturity curve of dividend present values in full, including an estimate for long term growth.

Our key contributions are as follows. First, we find evidence that the two factor affine model describes the term structure of dividends well. It captures the dynamics of measured growth rates and it delivers an estimate for long term growth which is economically sensible.

Second, we perform a relative pricing exercise, comparing the calibrated prices of future dividends to the value of the total stock index. Dividend derivatives have maturities up to ten years, but using our term structure model the estimated growth rates beyond that are summed exponentially to arrive at a model based estimate of the price dividend ratio. Together with a market price for current dividends, a comparison is made to the actual stock market. This can also be interpreted as an out of sample test of our model, since the model is estimated using dividend derivatives only, and not the stock index value. We find that most of the variation in the stock market is explained by current dividends and our model implied price dividend ratio, which demonstrates that the stock market can be understood well in terms of the market for dividend derivatives.

Third, we find that the factors driving this term structure have rather strong mean reversion. The first factor has a half-life of 6 months to one year (for reversion to the second factor) and thus captures short term movements in expected dividends. The second factor reverts to a constant at a horizon of business cycle proportion. Then, given the good fit of the stock market, our results show that most of the variation in stock prices is captured by short term and business cycle movements in discounted risk adjusted dividends. The estimate for long term growth is fixed, but this is apparently not a major impediment to capturing observed variation. Investors do not seem to substantially update their day-to-day valuation of dividends beyond the business cycle horizon.

We use data for four markets of dividend derivatives, and contracts that extend out to horizons of up to ten years. Dividend derivative products exist in the shape of futures listed on stock exchanges and as swaps traded over the counter (OTC) between institutions. Minimum criteria for liquidity and transparency restrict the application of daily data in the estimation procedures to listed futures referring to the Eurostoxx 50 and the Nikkei 225 indices. Price data of OTC dividend swaps for the FTSE 100 and the S&P 500 indices are available as well, but they are less liquid and their representativeness of daily variation in dividend expectations is questionable. We therefore perform the same tests for these data using a monthly frequency.

This paper adds to a recent literature that uses dividend derivatives in asset pricing. Our work builds on and complements Binsbergen et al. (2013). They introduce the concept of equity yields, which we use to define our discounted risk adjusted growth measure. However, they do not estimate a pricing model for the term structure of discounted risk adjusted dividend growth nor price the stock market using this model, but instead focus on an empirical decomposition of dividend prices into dividend growth rates and risk premiums. They do this by predicting future dividend growth from current dividend prices, and solving for the term structure of risk premiums implied by this model. They conclude that the term structure for risk premia is procyclical, whereas expected dividend growth is countercyclical. Related to this, various authors (Binsbergen et al., 2012, Cejnek and Randl, 2014, Golez 2014) focus on realized returns of short and long term horizon dividend derivatives or forward dividend prices derived from stock index futures and options, and find evidence for a downward sloping term structure of risk premiums. Wilkens and Wimschulte (2009) compare dividend derivative prices with dividend prices implied by index options. Suzuki (2014) assumes risk premiums are proportional to dividend volatility and then models the dividend growth curve implied by derivative prices using a Nelson-Siegel approach.

In the present value literature, broadly two categories of approaches for the stock market exist, sometimes applied synchronously. Expectations about future dividends are often modelled and tested on historical dividend data combined with diffusion processes. Another strand pursues the decomposition of the present value of future dividends into dividend growth, the risk premium and the risk free rate.

Expectations about the growth in dividends are often estimated by an econometric dividend model given past returns and dividend data. One of the earliest and best known examples is given by Campbell and Shiller (1988), who use vector autoregressive methods to predict returns based on past dividends, and use this to decompose returns into discount rate news and cash flow news. Many other attempts at decomposition of dividend growth and risk premia have since followed (see Cochrane (2011) for an overview). Our approach could be a stepping stone towards a similar decomposition, but making use of forward looking information about expectations instead. These are the implicit assumptions Brennan hinted at revealing in 1998. Furthermore, the emphasis on the shape of the dividend term structure may open an approach to other finance questions. It may help to evaluate various asset pricing models such as long run risk models (Bansal and Yaron, 2004). Potentially, the value premium can be explained by distinguishing loadings on short and long term elements of the term structure. Also, macro variables may be connected to the stock market with the dividend term structure acting as a linking pin.

The remainder of this paper is organized as follows. The next section first deals with the theory of dividend derivatives and their fit into the present value model. It lays out the state space model which parameterizes the dividend term structure. The empirical results are discussed in the subsequent section and the paper summarizes its conclusions in section five. The data of dividend derivatives are not obvious and require a section of their own in the appendix.

II. Theory

This section starts by describing the linkage between dividend derivatives and the present value model. Next, the base model for discounted risk adjusted dividend growth is proposed. The section continues to lay out the state space approach to capturing time- and horizon varying dividend growth.

II.1 Dividend derivatives in relation to the present value model

Dividend derivatives exchange future unknown dividends paid by stock indices[2] for cash. At the transaction date the buyer of a dividend future or OTC swap agrees to pay a cash amount (the derivatives price) for all dividends distributed during a pre-specified period in the future, usually a calendar year. The sum of these dividends determines the payoff of the derivative at the settlement date. The difference between the sum of the realized dividends and the

[2] Dividend derivatives exist referring to single stocks as well. The trading and settlement mechanism is the same for single stock dividend derivatives and stock index dividend derivatives. In this paper we discuss dividend derivatives referring to stock indices only.

transaction price of the derivatives contract is received or paid by the buyer. Typically, dividend derivatives are traded referring to dividends paid in particular calendar years of the same underlying stock index. Often prices of up to ten years into the future are available, which provides an observable term structure of price information about dividends. Figure 1 shows the term structure of derivatives prices on an arbitrary trading day. For example, on January 29, 2014, the price of the Eurostoxx 50 dividend future for the calendar year 2019 was 104.4 euros (figure 1). Buying this contract thus pays out the sum of realized dividends in the year 2019 minus 104.4 euros, to be settled at the end of 2019.

Prices of dividend derivatives reflect both the dividends as investors expect them to be paid at predetermined points in the future and the uncertainty associated with them. Formally, the dividend futures price $F_{t,n}$ equals:

$$F_{t,n} = E_t(D_n) exp\left(-n\theta_{t,n}\right), \tag{1}$$

where $\theta_{t,n}$ is the annualized dividend risk premium and D_n the realized dividend in year $t + n$. Prices $F_{t,n}$ evolve dynamically and differ per horizon at which dividends are paid denoted as n periods into the future, as do both the expectations of dividends $E_t(D_n)$ and the dividend risk premium $\theta_{t,n}$. We define $g_{t,n}$ as the annualized expected growth rate of dividends paid at horizon n starting from current dividends D_t:

$$E_t(D_n) = D_t exp(n g_{t,n}). \tag{2}$$

The present value $P_{t,n}$ is constructed from dividends D_n. In addition to the compensation for risk, the present value of an expected dividend payment compensates for the time value of money by discounting at the risk free rate $y_{t,n}$:

$$P_{t,n} = E_t(D_n)\, exp\left(-n\left(y_{t,n} + \theta_{t,n}\right)\right). \tag{3}$$

Summing the present value of expected dividends and substituting the growth rate for expected dividends as shown in equation (2) delivers the present value accounting identity for the value of the stock index:

$$S_t = \sum_{n=1}^{\infty} P_{t,n} = D_t \sum_{n=1}^{\infty} exp(n(g_{t,n} - y_{t,n} - \theta_{t,n})). \quad (4)$$

This equation shows that a key variable for valuing stocks is the discounted risk adjusted dividend growth[3] rate for each horizon, which we define as:

$$\pi_{t,n} = g_{t,n} - y_{t,n} - \theta_{t,n}. \quad (5)$$

Binsbergen et al. (2013) refer to the negative of this variable as the *equity yield*. Once introduced in the present value equation, the stock index is simplified to:

$$S_t = \sum_{n=1}^{\infty} P_{t,n} = D_t \sum_{n=1}^{\infty} exp(n\pi_{t,n}). \quad (6)$$

The identity shows that the sum of exponential discounted risk adjusted growth rates equals the price dividend ratio. Furthermore, although we speak of growth in dividend present values, equation (6) stresses that risk adjusted growth must be negative at some point in the future to avoid stock valuation from exploding. The data show that the risk adjusted growth rates sometimes rise in the first couple of years ahead, but subsequently they indeed always decline into negative territory.

II.2 The base model

The present value identity (6) links the sum of the present values of all future dividends to stock prices. A test of the compatibility of dividend derivatives and the stock market involves the extrapolation of the dividend term structure into the indefinite future. An estimate of growth rates at horizons extending beyond those observable are therefore required. For this, we propose using a term structure model of the type familiar in the interest rate literature. We start with the standard asset pricing equation for the present value of dividends, where the price equals the expected product of the pricing kernel and the payoff:

[3] We refer to the term discounted risk adjusted dividend growth or the term growth for short.

$$P_{t,n} = E_t\left[D_t exp\left(\sum_{i=1}^{n} m_{t+i}\right) exp\left(\sum_{i=1}^{n} g_{t+i}\right)\right], \tag{7}$$

where m_{t+1} is log pricing kernel and g_{t+1} is the dividend growth rate both for period t to $t+1$. The pricing kernel consists of the one-period risk-free rate y_t and an additional term θ_{t+1} that captures the dividend risk premium:

$$m_{t+1} = -(y_t + \theta_{t+1}), \tag{8}$$

where y_t is observed at time t and reflects the period t to $t+1$. We rewrite the pricing formula (7) by this growth variable:

$$P_{t,n} = D_t\left[E_t exp\left(\sum_{i=1}^{n} g_{t+i} - y_{t+i-1} - \theta_{t+i}\right)\right]. \tag{9}$$

Using a log-linear approximation of equation (9) and substituting definitions (3) & (5) gives an expectations hypothesis like formula:

$$n\pi_{t,n} = lnP_{t,n} - lnD_t = E_t\left(\sum_{i=1}^{n} g_{t+i} - y_{t+i-1} - \theta_{t+i}\right). \tag{10}$$

It then follows that the basic building block of the term structure model is what we denote discounted risk adjusted dividend growth $p_t = E_t(g_{t+1} - y_t - \theta_{t+1})$, as we can rewrite equation (10) to:

$$n\pi_{t,n} = E_t\left(\sum_{i=1}^{n} E_{t+i-1}(g_{t+i} - y_{t+i-1} - \theta_{t+i})\right) = E_t\left(\sum_{i=0}^{n-1} p_{t+i}\right). \tag{11}$$

The crucial question is how to model the evolution of growth rates p_t. The advocated approach that we discuss in the next subsection is decomposition of $\pi_{t,n}$ *by horizon*. Growth rates differ per the number of periods in the future, the pattern of which is the object of this paper.

One may wonder why we model p_t, rather than assuming separate models for its elements dividend growth, risk premium and risk free discount rates. Decomposition of stock prices into dividend growth and risk premia knows many attempts, seminal among which is the VAR based approach by Campbell and Shiller (1988). Information from dividend derivatives is used in the VAR model of Binsbergen et al. (2013). We choose to do the exact opposite of decomposition and instead amalgamate the three variables into one; the proposed model variable is the growth rate of present values of expected dividends p_t. This amalgamation facilitates to focus on the term structure of the growth trajectory alone. Connecting these growth rates via the present value identity to the stock market allows for a judgment call on the relevance of the horizon decomposition without being side tracked by additional assumptions on the constituent variables. In fact, equation (6) shows that such a decomposition is not needed as long as our goal is to price the stock index.

In addition, because the components of p_t are likely to be correlated[4], it should be possible to model p_t with a limited number of state variables. For example, improving projections about the economy may lead investors to believe that dividends will grow faster, and interest rates to rise relative to their earlier assumptions at the same time. There is substantial evidence that in such times risk premia decrease as well. Many such mechanisms will be at work; higher inflation may have similar consequences, shocks to financial stability are also likely to affect dividend expectations, interest rates and risk premia simultaneously.

II.3 The state space model

In order to build a full term structure of discounted risk adjusted dividend growth, we model it in state space form. We first discuss the state equations and then the measurement equations.

State equations

Our modelling approach to execute the decomposition by horizon closely follows Jegadeesh and Pennacchi (1996), who propose a model for estimating Libor futures with an aim to

[4] Bekaert & Engstrom (2010) calculate the correlation between 10 year nominal bond yields and dividend yields in the US over a 40 year period at no less than 0.77. Binsbergen et al. (2013) perform a principal components analysis of equity yields based on dividend derivatives prices. They show that the first two principal components of nominal yields explain about 30% of $g - \theta$ movements.

construct a term structure of interest rates based on three horizons. Their set up is a state space model in which the short term interest rate is a latent variable. The prices of the Libor futures of different horizons are estimated by an equation consisting of the interest rates growth for the three horizons. Instantaneous growth and medium term growth are both state variables, indefinite growth is a constant.

In this paper, we model discounted risk adjusted dividend growth $\pi_{t,n}$ according to the same horizons. Discounted growth p_t is a state variable following a mean reverting process to a medium term growth rate $\tilde{p}_t$ state variable which itself is mean reverting to a long term constant $\overline{p}$:

$$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p, \tag{12}$$

$$d\tilde{p}_t = \psi(\overline{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}. \tag{13}$$

dW_p and $dW_{\tilde{p}}$ are Wiener processes, with σ_p and $\sigma_{\tilde{p}}$ scaling the instantaneous shocks to the state variables. The horizon at which investors adjust their growth expectation from one state to the next is captured by mean reversion parameters φ and ψ.

We now rewrite the state equations (12) and (13) in vector form and then derive the discrete-time implications of the model. Denote $Q_t = \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix}$ the 2×1 vector of the state variables and $\bar{Q} = \begin{pmatrix} \overline{p} \\ \overline{p} \end{pmatrix}$ as the 2×1 vector of the constant indefinite growth rate. In a two equation matrix format, the system becomes:

$$dQ_t = \begin{pmatrix} dp_t \\ d\tilde{p}_t \end{pmatrix} = \left[\begin{pmatrix} -\varphi & \varphi \\ 0 & -\psi \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \psi\overline{p} \end{pmatrix} \right] dt + \begin{bmatrix} \sigma_p & 0 \\ 0 & \sigma_{\tilde{p}} \end{bmatrix} \begin{pmatrix} dW_p \\ dW_{\tilde{p}} \end{pmatrix}. \tag{14}$$

This system of differential equations in matrix notation is:

$$dQ_t = C\left[Q_t - \overline{Q}\right]dt + \Sigma dW, \tag{15}$$

which has the general solution:

$$Q_{t+i} = \overline{Q} + e^{Ai}\left(Q_t - \overline{Q}\right) + \varepsilon_{t+i}, \tag{16}$$

and of which the eigenmatrix solves to:

$$e^{Ai} = \begin{pmatrix} e^{-\varphi i} & (e^{-\varphi i} - e^{-\psi i})\dfrac{\varphi}{\varphi - \psi} \\ 0 & e^{-\psi i} \end{pmatrix}. \tag{17}$$

Substituting the solution for e^{Ai} into (16) results in the state equations:

$$\begin{pmatrix} p_{t+i} \\ \tilde{p}_{t+i} \end{pmatrix} = \begin{pmatrix} 1 + \dfrac{\psi}{\varphi - \psi} e^{-\varphi i} - \dfrac{\varphi}{\varphi - \psi} e^{-\psi i} \\ 1 - e^{-\psi i} \end{pmatrix} \begin{pmatrix} \overline{p} \\ \overline{p} \end{pmatrix} + \begin{pmatrix} e^{-\varphi i} & \dfrac{\varphi}{\varphi - \psi}\left(e^{-\psi i} - e^{-\varphi i}\right) \\ 0 & e^{-\psi i} \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \varepsilon_{t+i}. \tag{18}$$

Measurement equations

The average growth rate of dividend present values from time t to its expiry date at time n corresponds to a function of p_t and $\tilde{p}_t$. The value of a dividend paid n periods ahead equals:

$$P_{t,n} = D_t \, exp\left(n\pi_{t,n}\right), \tag{19}$$

specifying $\pi_{t,n}$ as the average growth rate during *n* periods following current time *t*. The state variables, however, regard only a single period ahead in the estimation procedure. In order to estimate them, $\pi_{t,n}$ needs to be tied to the state variables. p_{t+i} is a single period growth rate starting at $t + i$. This growth rate is multiplied until *n* periods ahead are reached, which is given by the expectations hypothesis in equation (11).

Working out the expectations of p_{t+i} from the dynamic model we find:

$$E_t(p_{t+i}) = e^{-\varphi i} p_t + \frac{\varphi}{\varphi - \psi}\left(e^{-\psi i} - e^{-\varphi i}\right)\tilde{p}_t + \left(1 + \frac{\psi}{\varphi - \psi} e^{-\varphi i} - \frac{\varphi}{\varphi - \psi} e^{-\psi i}\right)\overline{p}. \tag{20}$$

Summing growth rates for n periods into the future, and adding a measurement error, the measurement equations are:

$$lnP_{t,n} - lnD_t \ = n\pi_{t,n} = \sum_{i=0}^{n-1} e^{-\varphi i}\, p_t + \frac{\varphi}{\varphi - \psi}\left(\sum_{i=0}^{n-1} e^{-\psi i} - \sum_{i=0}^{n-1} e^{-\varphi i}\right)\tilde{p}_t \tag{21}$$

$$+\left(\frac{\psi}{\varphi-\psi}\sum_{i=0}^{n-1}e^{-\varphi i}-\frac{\varphi}{\varphi-\psi}\sum_{i=0}^{n-1}e^{-\psi i}+n\right)\overline{p}+\eta_{t,n}.$$

II.4 The single state model

We benchmark the ability of the two state model to fit the dividend term structure by a state space model with a single state variable. In essence, the medium term state variable is set to the long term constant estimate[5], rendering the same estimation equations as a Vasicek model:

$$dp_t=\varphi(\overline{p}-p_t)dt+\sigma_p dW_p. \tag{22}$$

Its state equations are:

$$p_{t+i}=\overline{p}+(p_t-\overline{p})e^{-\varphi i}+\varepsilon_{t+i} \tag{23}$$

and measurements equations are:

$$lnP_{t,n}-lnD_t=n\pi_{t,n}=\ n\overline{p}+(p_t-\overline{p})\sum_{i=0}^{n-1}e^{-\varphi i}+\eta_{t,n}. \tag{24}$$

III. Empirical Results

The estimation methodology uses prices of dividend derivatives referring to four major stock markets: Eurostoxx 50 and Nikkei 225 dividend futures and S&P 500 and FTSE 100 OTC dividend swaps. Dividend futures were introduced in 2008 to the European market and in 2010 in Japan. Maturities extend out to ten years with annual intervals. Price data are available on a continuous basis from the relevant stock exchanges, with full transparency. Liquidity is good for shorter maturities and reasonable for the longest dated futures. Trading normally occurs on a daily basis and we apply the estimation procedure to daily prices. Liquidity and transparency of OTC swaps is less developed and we limit the analysis to

[5] Alternatively, this model can be described as a nested two-state model with medium-term mean reversion parameter ψ constraint to infinity.

monthly data as a consequence. The data and their application are described in the appendix in detail. We first discuss the results using the two dividend futures datasets.

We estimate a cross section of measurement variables for up to 8 periods[6] on the discounted risk adjusted dividend growth term structure. The base for the growth rates would be D_t, however, current dividends are not observable. In view of the daily data that we deploy, current dividends are not well described by the dividend index to which the derivatives refer. These indices capture the dividends paid in the 12 months preceding observation date t, whereas we are in fact looking for a value that reflects dividends as if they had to be paid on t itself. The data section in the appendix contains further analysis on this topic.

Consequently, we remove the unobserved current dividend D_t from equation (19) and instead subtract the first discounted derivatives price from the longer maturity prices to arrive at growth rate measurements. The term structure then consists of a present value that is observed for the first period and growth rates that are modelled for all subsequent periods. Subtracting the first period present value gives the following measurement equation for growth rates:

$$
\begin{aligned}
lnP_{t,n} - lnP_{t,1} = n\pi_{t,n} - \pi_{t,1} = \sum_{i=1}^{n-1} e^{-\varphi i} p_t + \frac{\varphi}{\varphi-\psi}\left(\sum_{i=1}^{n-1} e^{-\psi i} - \sum_{i=1}^{n-1} e^{-\varphi i}\right)\tilde{p}_t \\
+\left(\frac{\psi}{\varphi-\psi}\sum_{i=1}^{n-1} e^{-\varphi i} - \frac{\varphi}{\varphi-\psi}\sum_{i=1}^{n-1} e^{-\psi i} + n - 1\right)\overline{p} + \eta_{t,n}.
\end{aligned}
\tag{25}
$$

The full system of measurement equations is now given as follows:

$$
\begin{pmatrix} lnP_{t,2} - lnP_{t,1} \\ lnP_{t,3} - lnP_{t,1} \\ \vdots \\ \vdots \\ lnP_{t,n} - lnP_{t,1} \end{pmatrix} = \begin{pmatrix} \left(\frac{\psi}{\varphi-\psi} e^{-\varphi} - \frac{\varphi}{\varphi-\psi} e^{-\psi} + 1\right)\overline{p} \\ \left(\frac{\psi}{\varphi-\psi}\sum_{i=1}^{2} e^{-\varphi i} - \frac{\varphi}{\varphi-\psi}\sum_{i=1}^{2} e^{-\psi i} + 2\right)\overline{p} \\ \vdots \\ \vdots \\ \left(\frac{\psi}{\varphi-\psi}\sum_{i=1}^{n-1} e^{-\varphi i} - \frac{\varphi}{\varphi-\psi}\sum_{i=1}^{n-1} e^{-\psi i} + n - 1\right)\overline{p} \end{pmatrix} + \tag{26}
$$

[6] Only 8 annual growth periods are retrieved from 10 expiry dates. See the data section in the appendix for details.

$$\begin{pmatrix} e^{-\varphi} & \frac{\varphi}{\varphi-\psi}\left(e^{-\psi}-e^{-\varphi}\right) \\ \sum_{i=1}^{2} e^{-\varphi i} & \frac{\varphi}{\varphi-\psi}\sum_{i=1}^{2}\left(e^{-\psi i}-e^{-\varphi i}\right) \\ \vdots & \vdots \\ \vdots & \vdots \\ \sum_{i=1}^{n-1} e^{-\varphi i} & \frac{\varphi}{\varphi-\psi}\sum_{i=1}^{n-1}\left(e^{-\psi i}-e^{-\varphi i}\right) \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \begin{pmatrix} \eta_{t,2} \\ \eta_{t,3} \\ \vdots \\ \vdots \\ \eta_{t,n} \end{pmatrix}.$$

State equations (18) and measurement equations (26) together form the system of which the variables are estimated by maximum likelihood. The procedure is recursive by means of a Kalman filter (Jegadeesh and Pennacchi, 1996).

The error variance terms are assumed to be the same for all measurement equations (σ_η), except for the first one (which we denote σ_ε). The definition of the first derivative to expire (set to a constant maturity of one year following the observation date) differs slightly from subsequent derivative prices due to an alternative weighting scheme as explained in the appendix.

III.1 Estimation results

Table 1 provides the results of the two state model and a benchmark single state model for both the Eurostoxx 50 and Nikkei 225 dividend markets – the two markets for which listed data with sufficiently long horizons of 10 years exist. Estimations are performed on daily data. For robustness, we perform the same tests with monthly data, shown in table 4. None of the parameter estimates and test coefficients change meaningfully relative to the daily dataset.

Pricing errors

Before we discuss the parameters of the growth rate model, we first establish that the two-state model fits the data well. To this end, we calculate mean absolute errors for the measurement equations (26). Given that they are specified for log prices of dividend futures, these mean absolute errors can be interpreted as relative pricing errors.[7] The first measurement equation produces a mean absolute pricing error of 0.015 (1.5%) and

[7] These errors are thus not annualized. Transformed to annual growth rates, the errors are even smaller.

measurement errors of subsequent expiries are between 0.0025 and 0.005 (figures 7 and 8). The error levels are clearly small, confirming a good fit of the model to the data.

Mean reversion estimates

The adjustment from one state to the next occurs at a pace that is broadly expected. The mean reversion towards medium term growth φ attains levels which translate to a half-life expressed in less than a year. Eurostoxx 50 adjustment at 1.43 is nearly twice as fast as Nikkei 225 adjustment at 0.76, which is due to the global credit crisis in 2008/09 being included in the Eurostoxx 50 data period and not in the Nikkei 225 data period.[8] Mean reversion towards the long run constant ψ is broadly measured in half-lives of 2 to 4 years[9], a space of time that comes close to that of a business cycle. All mean reversion parameters are significant at the 1% level. The estimates for φ and ψ are positive, which implies that the growth rate is stationary and thus tends to a long-term constant.

These results for medium term mean reversion suggest that investors do not look further ahead than the next business cycle when establishing the value of future dividends. The model imposes the long-run growth rate to be constant, while the speed at which medium-term growth adjusts to it is estimated from the data. The interpretation is that investors change their opinion about growth only as far ahead as the anticipated business cycle. We do not formally link an economic interpretation to the three growth stages, but given the estimates of the mean reversion parameters some intuition can be provided. Instantaneous growth can be thought of as the expectation of the immediate future. Shocks to risk aversion and to the volatility of the current business climate are likely to influence investors' valuations of dividends several months ahead, but perhaps not much further. Developments in the business cycle, on the other hand, such as credit conditions, investment growth and monetary policy set the stage for the business cycle influencing dividend expectations over a longer period ahead, measured in several years. Structural factors such as population growth and technological progress determine how investors perceive the long run, extending from the business cycle horizon into the indefinite future.

Structural developments should be slow moving, if at all, and are approximated by imposing asymptotic constancy. Thus, at horizons extending well beyond business cycles,

[8] Estimating the model for the Eurostoxx 50 data over a partial data period that coincides with the Nikkei 225 data period yields mean reversion parameters that are close to those found for the Nikkei 225: $\varphi = 0.7942$ and $\psi = 0.1737$.

[9] Applying the two state model to interest rates finds the opposite pattern; short mean reversion is slower than medium term mean reversion (Jegadeesh and Penacchi, 1996).

investors have opinions of economic and financial variables, but they do not change them once taken together. This means that any rise in long maturity interest rates is exactly offset by a rise in long-term dividend growth or a fall in long term risk premia. Mean reversion towards such a constant implies therefore that a horizon exists at which investors effectively never change their opinion about present value growth.

Discounted risk adjusted dividend growth rates

Given the mean reversion estimates, the instantaneous state variable reflects short term movements in risk adjusted growth, the medium term state variable reflects an assessment of the business cycle, while $\bar{p}$ depicts a structural level which can be linked closely to the dividend yield. Figures 3 to 6 provide estimates of expected growth rates by recalculating the state variables by means of the measurement equations (26) into 1 year growth and 1 year forward 4 year growth of discounted risk adjusted dividends. Forward growth rates imply the level of growth expected after the 1 year growth rate has materialized.

1 year growth is mostly determined by the instantaneous state variable. Figure 3 shows that it is highly volatile for the Eurostoxx 50, with the global credit crisis in 2008/09 showing a decline by nearly half and during the Eurozone sovereign debt crisis in 2011 by a quarter. Outside these periods, it moves between broadly – 10 and + 5 percent. Nikkei 225 dividends move in the same range (Figure 5). The period following the announcement of "Abenomics" in 2012[10] portrays high optimism with growth rates attaining 10 percent and more.

Given the values found for the mean reversion parameters, the medium term state variable largely determines 1 year forward 4 year growth depicted in figures 4 and 6. In Europe it circles around the long run constant between – 2 and – 6 percent. The sovereign debt crisis in 2011 shows a somewhat more negative rate than the global credit crisis. Investors apparently expected that the serious short term blow to dividends in 2008/09 would not be corrected or reversed (by positive growth) afterwards. However, the less negative blow in 2011 would be followed by a period more negative than the long run constant, implying that investors expected that the European sovereign debt crisis would bear consequences for the business cycle.

The volatility of forward growth rates provides further insight into the relation between the risk and the maturity of the dividends. Both the Eurostoxx 50 and the Nikkei 225

[10] Late 2012 the government of Shinzo Abe proclaimed a policy of monetary and fiscal expansion combined with economic reform. The two main consequences for financial markets were a substantial weakening of the Japanese Yen and a rise in the stock market.

dividend markets portray declining volatility in growth rates as maturities increase (figure 2). This coincides with findings in other studies that the risk premium term structure is downward sloping and is line with the mean reverting behavior of growth rates (see for example Binsbergen et al., 2013).

Long term growth and dividend yields

The economic interpretation of the long term discounted risk adjusted dividend growth constant is briefly recapitulated. The present value identity is recalled as:

$$S_t = \sum_{n=1}^{\infty} P_{t,n} = D_t \sum_{n=1}^{\infty} exp(n\pi_{t,n}). \tag{27}$$

The dividend price ratio is obtained by rearranging the identity to:

$$\frac{D_t}{S_t} = \frac{1}{\sum_{n=1}^{\infty} exp(n\pi_{t,n})}. \tag{28}$$

In the Gordon growth representation of the identity, $\pi_{t,n}$ is a horizon invariant constant:

$$\frac{D_t}{S_t} = -\bar{p}, \tag{29}$$

and therefore the long term estimate for p is the negative of the steady state dividend yield of the stock market.

Seen in this light, there is an economic rationale in the estimates from the state space model for the long term growth constant. The levels found equal near – 3 percent in Japan and Europe, which appears reasonable relative to dividend yields. Table 2 contains some metrics for comparison. The average dividend yield in Europe was 3.8 percent and in Japan it was 2.3 percent during the data period. The average 1 year forward 4 year growth rate also deviates less than 1 percent from the average dividend yield, but the average short term growth rate deviates substantially more. A superficial conclusion is that the business cycle stood close to the long term average during the data period, but sentiment was more negative in Europe and more positive in Japan[11]. Seen in this light, the estimates for long term growth seem a fair assessment of the long term cash run rate of the stock market. It is noteworthy that the estimates are produced without input from the stock market itself.

It is also important to observe that the state space model estimates long term growth to be negative as present value theory requires stock valuations to be finite. The flexibility of the model would allow for positive values, but the curvature estimated correctly signals that dividend present values must decline at a horizon that is sufficiently long.

The interpretation of the assumption that long run discounted risk adjusted dividend growth is constant is not that investors do not change their opinion about what value to attach to dividend present values far into the future. The value ascribed to dividends expected ten years and twenty years from today is influenced by the estimate of present values in the near term and medium term. But the value of the twenty year dividend does not change relative to that of the ten year dividend regardless of changes in near and medium term expectations – the relationship between them is (approximately) fixed. Therefore, long-run constancy excludes mean reversion to levels. Dividend levels attained in the past are not a target for investors to project their long term expectations onto. Only long term growth is.

The Dividend Term Structure

Equipped with model estimates for the growth parameters, a Dividend Term Structure (DTS) can be calibrated. The DTS depicts the present values that investors attach to expected dividends per horizon n expressed as a proportion of total present values:

$$DTS_n = \frac{\hat{P}_{t,n}}{\sum_1^\infty \hat{P}_{t,n}}. \tag{30}$$

The value for $\hat{P}_{t,1}$ is the discounted price of the derivative expiring one year from t. The values for subsequent expiries $n \geq 2$ are calibrated from the estimated growth parameters. Figure 9 shows that the average term structure of the Nikkei 225 starts sloping upwards, and then becomes downward sloping as the horizon increases. The transition is slow given the low mean reversion and the moderate levels of the estimated averages for instantaneous and medium term growth. The Eurostoxx 50 DTS, by contrast, is strongly negatively sloping at the outset, but adjusts to the long term growth path rather quickly. The first dividend point on

[11] In fact, in particular in Japan it turned more positive during the data period.

the Eurostoxx 50 DTS is therefore high, which translates into an equally high current dividend yield. The Nikkei 225 first dividend points are lower on average, which fits with the positive slope at the start of their DTS. It is also in line with the fact that the estimate for long term growth is somewhat higher than the first dividend point.

Its DTS indicates that the fundamental value of the European stock market is more front loaded, or more heavily weighted towards the near future, than the Japanese stock market. The surface below the calibrated DTS equals one by definition. The present values of dividends cross over after about thirty years into the future. Relative to the European stock market, the present value of Japanese dividends beyond the cross over makes up for their lower contribution before it.

Other long term growth estimates

Giglio et al. (2014) compare prices of houses of different contractual ownership to arrive at a very long term discount rate. Leased housing reverts to the owner of the land after the lease expires, while freehold housing remains with the owner of the house indefinitely. The difference in price between the two for comparable properties equals today's present value put to ownership once the lease has expired. At lease expiries of over one hundred years, this provides an interesting comparison to the estimates for long term discounted risk adjusted dividend growth.

The discounts Giglio et al. find in the data equate to a value for asymptotic growth of around – 2% for periods of 100 years and more. This level makes sense economically and is also reasonably close to the long term discounted risk adjusted dividend growth estimates[12].

III.2 Reconciliation to the stock market

The second part of our research agenda is to analyze the implications of the model for the value of the stock market. Given that we estimate the model using dividend derivative data only, this can be seen as an out-of-sample test of the model. Alternatively, if one takes the model assumptions for granted, it can be seen as a relative pricing exercise of the dividend derivative prices versus stock market levels.

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[12] It is clear that not the *level* of the rents D, but only the *growth* of rents (being part of p) matters for establishing the lease discount. We can therefore consider growth in rents with or without maintenance cost, depreciation and taxes assuming they stay constant in proportion to rents over the very long term considered.
Another aspect is the convenience provided to the occupier of a house. Growth comparisons should be made only for sufficiently remote horizons. Since the notion of convenience yield is that there is a benefit to the current user that a future user cannot currently enjoy, nearer horizon comparisons are distorted.

The present value model incorporates expected index dividends which can be extrapolated from the estimated dividend term structure. This provides the following estimate for the stock market:

$$\hat{S}_t = D_t \sum_{n=1}^{\infty} exp(n\hat{\pi}_{t,n}) = D_t \widehat{PD}_t, \tag{31}$$

with the summation of growth rates equal to the estimated dynamic price dividend ratio $\widehat{PD}_t$. It is a well known and critical problem of the present value model that it depends on a reasonable estimate for the expected growth and the risk premium of dividends. Historical analysis of dividend growth followed by risk premium decomposition provides such estimates. (Campbell and Shiller, 1988). Binsbergen et al. (2013) execute the decomposition by making use of the price data of dividend derivatives.

A key contention in this paper is that for the purpose of the present value model reconciliation, without decomposition of dividend expectations into growth and risk premia, decomposition of risk adjusted discounted dividend growth by horizon alone is very informative. Successful reconciliation of dividend derivative price information to the stock market has not been performed often. Suzuki (2014) builds a Nelson Siegel model of the Eurostoxx 50 dividend growth term structure and makes assumptions about the level for longer dated values. These include a fixed level imposed at 4% for discounted growth after 25 years. Under these conditions, Eurostoxx 50 dividends reconcile well with the stock market. The state space model renders an estimate for the long term growth path of the present value of dividends independent from stock market information, while it also captures the shape and the dynamics of the term structure up to the medium term. The entirety of the present value term structure is thus described by a handful of variables from two markets in a single estimation procedure.

The fit of the reconciliation to the stock market acts as a joint check on the validity and the robustness of the two state model and the present value identity. To that end, the growth parameter of present values is estimated by the state space model by means of the conditional measurement equation:

$$
\begin{aligned}
n\hat{\pi}_{t,n} = \sum_{i=0}^{n-1} e^{-\varphi i} p_t + \frac{\varphi}{\varphi-\psi}\left(\sum_{i=0}^{n-1} e^{-\psi i} - \sum_{i=0}^{n-1} e^{-\varphi i}\right)\tilde{p}_t \\
+\left(\frac{\psi}{\varphi-\psi}\sum_{i=0}^{n-1} e^{-\varphi i} - \frac{\varphi}{\varphi-\psi}\sum_{i=0}^{n-1} e^{-\psi i} + n\right)\overline{p}.
\end{aligned} \tag{32}
$$
[13]

All variables are taken as estimated by the state space model applied to dividend derivative data. Current dividends in (33) are approximated by the value of the first constant maturity derivative $F_{t,1}$, which is discounted at the risk free rate.[14] This is a better approximation for investors' estimate of current dividends than twelve month historical dividends. We thus get for the model implied stock market level:

$$
\hat{S}_t = F_{t,1}\, exp\left(-y_{t,1}\right)\left(1 + \sum_{n=2}^{\infty} exp\left(n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}\right)\right) = F_{t,1}\, exp\left(-y_{t,1}\right)\left(1 + \widehat{PD}_t^1\right), \tag{33}
$$

in which $n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}$ are the fitted values, estimated as a single variable, of the measurement variables in equations (26) and $\widehat{PD}_t^1$ represents the estimate for the price dividend ratio as implied by the sum of exponential growth rates, where growth starts from the present value of the dividend derivative expiring one year following the observation date[15].

Stock level reconciliation

We now discuss the empirical results of the reconciliation with the stock market levels. In the European market, the two state model overestimates the stock index at a reasonably constant level distance to the actual stock index for most of the data period (Figure 10). There is no

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[13] Subtracting the first growth rate $\pi_{t,1}$ from equation (32) provides an alternative representation which can be directly applied to the present value identity in equation (33);

$$
\begin{aligned}
n\hat{\pi}_{t,n} - \hat{\pi}_{t,1} = \sum_{i=1}^{n-1} e^{-\varphi i} p_t + \frac{\varphi}{\varphi-\psi}\left(\sum_{i=1}^{n-1} e^{-\psi i} - \sum_{i=1}^{n-1} e^{-\varphi i}\right)\tilde{p}_t \\
+\left(\frac{\psi}{\varphi-\psi}\sum_{i=1}^{n-1} e^{-\varphi i} - \frac{\varphi}{\varphi-\psi}\sum_{i=1}^{n-1} e^{-\psi i} + n - 1\right)\overline{p}.
\end{aligned}
$$

[14] We interpolate between 10 annual derivatives expiry dates to arrive at prices of derivatives with constant maturities. These 9 prices of derivatives with a constant maturity of 1 to 9 years provide 8 growth rates. See the data section in the appendix for further details.

[15] The stock index estimate is approached by numeric summation, which is approximated by:

$$
\hat{S}_t \approx F_{t,1}^{CM}\, exp\left(-y_{t,1}\right)\left[1 + \sum_{n=2}^{\bar{n}} exp\left(n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}\right) + \frac{exp\left(\bar{n}\hat{\pi}_{t,\bar{n}}\right)}{-\bar{\tilde{p}}}\right].
$$

In the estimations $\overline{n}$ is set at 50 years. Unless reduced to single digits, the number of years which $\overline{n}$ is set to is not material to the stock index estimates.

clear trend among the state variables driving the estimated valuation away or towards the stock index. The historical dividend yield (3.7%) is somewhat higher than the negative of the long term estimate (– 2.9%) and the index is overestimated at some 20 to 30 percent except during the outbreak of the global credit crisis[16]. The level estimate of the stock index is highly sensitive to the long term growth parameter. For the mean squared errors of this level comparison to be minimized, the estimate for long term discounted growth would have to be closer to the historical dividend yield, or about 0.7% higher[17]. This falls at around two standard deviations of the coefficient error of the long term growth estimate as is shown in figure 10.

Dividend present values underestimate the Nikkei 225 index level at the beginning of the data period, but the gap closes from 2012 onwards. Short term growth ranges between – 0.20 and + 0.05 percent initially, but at the onset of Abenomics in late 2012, it turns strongly positive (figure 11). At – 2.7 percent, long term growth is more pronounced than the historical Japanese dividend yield (2.1%), which contributes to the underestimation. Here again, the model implied estimate varied for two standard deviations of the long run constant broadly spans the observed index prices.

The half-value time of the two mean reversion mechanisms taken together may be labelled as the *attachment point* between the state variables and the long term constant. At distances in the future shorter than the attachment point, the state variables dominate, beyond it their impact on growth is materially transferred to the long term constant. The state variables therefore dominate the position of the attachment point on the x-axis of the DTS. For example, positive values of the state variables will push the attachment point upwards and produce higher levels of present values of dividends for all of the remaining dividend term structure. Consequently, the state variables not only shape the DTS, they influence present values beyond their own horizon and therefore heavily affect the stylized value of the stock market.

Dynamic reconciliation

Following the present value model, *stock returns* are the consequence of investors changing their expectation of future dividends. The dynamics of stock indices can be retrieved from the present value model estimate as provided in equation (33). The present value of the first

---

[16] Market participants consider Eurostoxx dividend derivatives prices around the turn of 2008/09 as unrepresentative of dividend expectations due to one sided interests.

[17] The long term growth duration of the Eurostoxx is 28 and of the Nikkei it is 31. A 0.01% estimation error of long term growth produces an stock value error of 11 and 49 index points, respectively (Nikkei at 15,000).

dividend amount to be paid over the year to come is the starting point of the growth term structure. The first dividend is observable and the growth path of discounted risk adjusted dividends starting after it is a model implied estimate. The dynamic fit as well as the relative importance to stock returns of the first derivative on the one hand and the growth path on the other requires testing. For this reason the estimated returns of the stock market is split into its drivers. Equation (33) is repeated with logs denoted in lower case as a regression equation:

$$\Delta ln(S_t) = \alpha + \beta_F \Delta ln(F_t) + \beta_y \Delta y_t + \beta_{\widehat{PD}} \Delta ln\left(\widehat{PD}_t^1\right) + \varepsilon_t. \qquad (34)$$

Stock index log returns are regressed by OLS on the log return of the first constant maturity derivative F_t, changes in the 1 year risk free rate Δy_t and the log returns of the estimated price dividend ratio $\widehat{PD}_t^1$, which is the sum of the normalized dividend present values of the state space model. The beta's[18] of the returns of the first dividend and the price dividend ratio are predicted to be close to + 1, while the beta of the risk free rate is expected at – 1. Data are daily.

Eurostoxx 50 and the Nikkei 225 index returns respond well to the prediction of the present value model, shown in table 3. The model is quite capable of explaining variation in stock returns, reaching an R-squared of above 50 percent. Although we cannot benchmark this explanatory power, it appears substantial given that the model does not incorporate any direct information of the stock market. Each of the regressors add considerably to the explanatory power, while the constant is close to zero. Both stock markets appear highly sensitive to changes in the first constant maturity derivative. The daily beta's are in the order of 0.7 for the Nikkei 225 to 0.9 for the Eurostoxx 50. The beta of the price dividend ratio is close to 0.9 and 0.7 respectively. In the case of Japan, most of the explanatory power comes from the price dividend ratio, in Europe it is evenly divided between short term dividends and the price dividend ratio.

The 1 year zero interest rate brings the price of the first derivative to its present value. Its relevance seems limited and the expected beta is – 1. It is highly significant in the estimates for the Eurostoxx 50, but reaches values of 0.15 to 0.20. In Japan, the risk free beta is closer to zero and not significantly different from it.[19]

[18] Coefficients are expected below 1 due to the errors in the regressor estimates increasing their variance.
[19] The impact of short term dividends and the price dividend ratio is mitigated by negative coefficients found once lags are added to the set of regressors (not shown here). This suggests that the stock market overreacts to shocks, which is corrected in the following day.

III. 3   Robustness

The single state model

The two state model distinguishes instantaneous from medium term growth. Its ability to fit the dividend present value term structure is benchmarked by a state space model with a single state variable[20], in which the medium term state variable is set to the long term constant estimate[21]:

$$dp_t = \varphi(\overline{p} - p_t)dt + \sigma_p dW_p, \tag{35}$$

Figures 7 and 8 show estimation errors of the single state model in comparison to the two state model. While still not substantial, single state estimation errors are larger. Table 1 contains the estimated parameters and figures 3 to 6 depict the forward discounted risk adjusted dividend growth rates as delivered by the single state model. Overall, the parameter estimates are significant and attain reasonable levels, but the two state model is superior.

The Eurostoxx 50 estimate for mean reversion at 1.90 is even higher than the short term mean reversion in the two state model, its standard error is twice as large. This adjustment speed implies a half-value time of instantaneous growth of only 4 months. Long term growth is slightly lower and its standard error is smaller than in the two state model. The 1 year growth rate is less volatile, which mirrors the quick fading of the instantaneous growth state variable. For the Nikkei 225, the picture is rather different. Mean reversion attains a value in the middle ground of the two parameters in the two state model. Long term growth is somewhat lower, but again economically sensible. Standard errors are smaller for both parameters. The 1 year growth rate largely overlaps with that of the two state model.

The fit of the models measured by estimation errors is reduced in the single state variation relative to the two state model. The absolute measurement errors are on average always bigger in the single state model than in the two state model, in most cases by a factor of 2 to 3 (figures 7 and 8). The better fit of the model is also indicated by the log likelihood statistics. Per observation the log likelihood contribution is at least a third higher in the two state model than it is in the single state variation. Benchmarking against the single state model

[20] A more basic model would be a zero state model. However, such a Gordon Growth model with a horizon independent but time-varying growth parameter as a benchmark is omitted here. Its fit is poor and its theoretical basis is absent. Moreover, it does not pick up the curvature required for negative long term growth rates. It thus often estimates positive growth, which is not stationary and akin to negative discounting.

[21] Alternatively one can depict this model as a nested two state model with medium term mean reversion parameter ψ constraint to infinity.

indicates that it appears plausible to distinguish between investors gauging the immediate future on the one hand and their considerations about the business cycle level on the other, as catered for in the two state model.

The alternative model

Our modelling approach focuses directly on discounted risk adjusted growth $p_t = E_t(g_{t+1} - y_t - \theta_{t+1})$. We thus incorporate discounting at the risk free rate when valuing future dividends. An obvious alternative to this approach would be to model $p_t = E_t(g_{t+1} - \theta_{t+1})$ using a term structure model to value dividend derivatives, and subsequently discount it at observed interest rates to calculate the present values. This latter step requires the assumption that interest rates and $g_t - \theta_t$ are independent. The alternative growth parameter thus equals dividend growth without discounting[22]:

$$z_{t,n} = g_{t,n} - \theta_{t,n} = \pi_{t,n} + y_{t,n}, \tag{36}$$

And it then follows that this alternative risk adjusted growth variable can be expressed as the growth rate of the dividend derivatives term structure:

$$(n-1)z_{t,n} = lnF_{t,n} - lnF_{t,1}. \tag{37}$$

Using this pricing equation, one can again specify a two state model, in this case for one period growth z_t, and estimate this using the Kalman filter in the same way as described above. To calculate model implied stock market levels, the observed yield $y_{t,n}$ for maturity n, is used to obtain present values of dividends become:

$$P_{t,n} = D_t \, exp\big(n(z_{t,n} - y_{t,n})\big). \tag{38}$$

As mentioned, this model assumes independence of interest rates and growth rates. In the real world, however, correlation between the risk free rate, dividend growth and the dividend risk premium is expected since often the same drivers apply: economic growth, the investment cycle, slack in the labor market and other economic variables will affect all of them. For estimating the term structure model, such correlation is not a problem if $z_{t,n}$ is the

[22] Referred to by Binsbergen et al. (2013) as *forward equity yield*.

subject of state space estimation instead of $\pi_{t,n}$, but it will cause misestimation of the implied stock market levels. Specifically, it is easy to show that this separation of the two correlated variables would produce overestimation of the stock index in equation (6).

Turning to the results, the long term estimate for risk adjusted growth $\bar{z}$ is estimated rather high, at 1.7 percent for the Eurostoxx 50 and 0.1 percent for the Nikkei 225 (see table 5 in the appendix). The standard errors of the long term estimate for undiscounted growth are higher relative to discounted growth. They read 0.0081 against 0.0051 for the Nikkei 225 and 0.0116 against 0.0039 for the Eurostoxx 50. Reconciling the dividend market to the stock market based on these estimates overstates the stock market by a large margin and reduces the fit of the dynamic return reconciliation (table 7). The coefficient of the estimated price dividend ratio often attracts the wrong sign. The data confirm the advantage of estimating risk adjusted dividend growth *after* discounting at the risk free rate.

OTC data

We retain price data of dividend swaps from several investment banks[23] for the markets under investigation, and also for the S&P 500 and the FTSE 100. These data extend back to December 2005. OTC prices for dividend derivatives are not readily observable as are, for example, interest rate swaps, money market derivatives or foreign exchange derivatives. Investment banks daily update their pricing sheets, but days and even months may go by without a single trade taking place. The data set of OTC prices for dividend swaps, therefore, is impacted by the model investment banks use for pricing them. If the OTC market does not regularly trade, it seems likely that fitting the state space model to its price data is akin to mimicking the models used by the investment banks. We nonetheless perform the same set of estimations and reconciliations as above on the OTC price data of dividend swaps referring to the S&P 500 and the FTSE 100 indices. The results shown are restricted to monthly frequencies, as the daily data are stale. The results are shown in the appendix (tables 6 and 8).

The two state model produces a high estimate for the long run growth constants of S&P 500 dividends. Indeed, at – 1.3 percent for this constant, the S&P 500 present value as estimated by the model (equation (33)) overestimates its observed values by a factor of 2. Both mean reversion parameters attain reasonable levels, but they attract fairly large standard errors. In the case of the FTSE 100, the two state model estimate for long run growth equals – 5.3 percent, with a standard error even exceeding that level in absolute terms. At the same

[23] Deutsche Bank, Goldman Sachs and Credit Suisse.

time, the second mean reversion parameter comes out low at 0.04, which translates into a half-value time running into several decades. At such slow moving mean reversion, the role of the long run constant is essentially taken over by the medium term state variable. The single state estimate for long run growth is more reasonable at – 3.3 percent.

The variation in the modelled price dividend ratio produced by these estimates does not depend on the long run constant and the dynamic reconciliation to the stock indices demonstrates that it has meaningful explanatory power. Table 8 shows that the model produces a coefficient between 0.2 and 0.3, with good significance for both the S&P 500 and the FTSE 100. Overall explanatory power is reasonable with adjusted R^2 reaching 0.35 – 0.40. However, the coefficient estimate for the modelled price dividend ratio as well as its explanatory power appear weak relative to the first dividend price F_t.[24]

IV. Conclusion

This paper proposes a method to extract information about the expectations that investors entertain of stock dividends from dividend derivatives. We import the information as dividend growth discounted for risk as well as the time value of money as a single parameter into a state space model with a short term and medium term state variable and a long run constant characterizing its trajectory. The two state variables shape a term structure of dividend growth which fits the data well and they determine the dynamics of the price dividend ratio thus estimated. Applied to the Eurostoxx 50 and the Nikkei 225, most of the variation of the stock market can be traced back to the model and short term dividends together. We conclude that dividend derivatives and stock prices line up well enough to consider the information contained in one market for use of understanding the other. Several inferences from these findings can be drawn.

The importance of risk free discounting of risk adjusted dividend growth

The estimation of the dividend term structure improves as a result of discounting risk adjusted dividend growth for the time value of money. Without doing so, the estimates of dividend growth are not economically sensible and reconcile poorly to the stock market. An explanation could be that investors determine the constituent components of the growth in dividend present values based on overlapping information. In that case, the correlation

[24] Similar regressions based on daily estimates and stock index data produce R^2 of less than 5 percent.

between these constituents troubles the reconciliation to the stock market once they are put together to do so.

Horizon matters

The distance into the future considered by investors affects the precision of the estimates. At the extreme, the Gordon Growth model assumes a constant discount rate, with poor fit and explanatory power. But even when short term variation in growth expectations is introduced as a single state model, it still is significantly outperformed by a two state model. The short term state variable reflects a horizon of under one year and the medium term state variable a horizon of several years. Deploying two states next to each other allows some distinction between sudden occurrences and those at business cycle proportions. Pursuing different explanations for the two states, or in other words, finding different determinants of how investors think of the short and the medium term, seems an appropriate research avenue.

Long term growth is constant

The state space model imposes the return to a constant mean level of growth in the long run. This assumption is loosely interpreted as that investors do not change their opinion about the sequence of present values of dividends in the long run, which seems very restrictive intuitively. Nevertheless, small estimation errors, the explanatory power of the reconciliation of the model to stock returns and the near unity of the coefficients of the short term dividend and the price dividend ratio add credence to the imposition that long run growth is no source of stock market variation. Interest rates are a part of discounted risk adjusted dividend growth, and they are observable to investors. Under the assumption that they do not change their opinion about discounted risk adjusted dividend growth $\bar{p}$ in the long run, then any interest rate variation is balanced by risk adjusted dividend growth expectations $g - \theta$ at these long horizons.

Listed data are preferred

We perform the estimations using prices of OTC dividend swaps as well as of listed dividend futures. The prices produced by the OTC market are relevant, but are of little significance and explanatory power, while dividend futures provide intuitive and highly significant results. Not only do the long run estimates come out poorly, also the added value of the two state model is not confirmed by OTC prices. It seems to us that these should be interpreted with caution when applied in present value analysis. Fortunately, the set of listed data will only expand as time passes.

Follow up

Being able to understand most of the stock market dynamics from a single model with only three variables, of which one year ahead dividend prices are actually observable, seems attractive enough to pursue finding the drivers of the other two. This is a direction of further research worth pursuing: to which fundamental variables can the variation of short term and medium term growth in discounted risk adjusted dividends be ascribed?[25] Armed with such linkages, the ability to describe stock market dynamics will improve. At the same time, Cochrane (2011) is clear in his assertion that: "We do not have to *explain* discount rates – relate expected returns to betas and understand their deep economics – in order to *use* them.". Opportunities are plentiful.

[25] Binsbergen et al. (2013) advocate the same.

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VI. Appendix: dividend derivatives data

Deriving the observed growth path of discounted risk adjusted dividend derivatives prices calls for several processes to make them suited for use in the state space approach. This section describes dividend derivatives, the data and these processes.

Dividend derivatives

Dividend derivatives exchange the value of a dividend index for cash at set expiry dates. The price of the derivative is set at the transaction date t and settled at the expiry date n. The difference between the transaction price and the amount of dividends actually paid is the amount settled between buyer and seller.

The transaction price reflects the growth path expected from the current level of dividends and the premium required for the risk of the actual payment differing from what is expected (equation (1)). It is a risk adjusted price and equals the present value of a dividend once the time value of money is accounted for.

The dividend index measures the amount of dividends paid by the companies constituent to a stock index during a calendar year[26]. At the end of the year, the index is the fixing at which the dividend derivative is settled. Manley and Mueller-Glissmann (2008) provide an overview of the market for dividend derivatives and its mechanisms.

Dividend derivatives data

Listed futures on dividends paid by the companies in the Eurostoxx 50 and the Nikkei 225 indices are the subject of this paper. These indices are widely accepted as representative for large cap firms based in Europe and Japan. They are chosen as dividend futures have been traded on them for a meaningful period with reasonable liquidity and good transparency.

Dividend futures are available for other markets as well. They are referenced to dividends of the FTSE 100, Hang Seng and Hang Seng China Enterprises, although not the S&P 500[27]. They started trading in 2008 on the Eurostoxx 50 and have expanded in region and horizon since then. Nonetheless, only for the Eurostoxx 50 and the Nikkei 225 dividend indices futures are traded with a maturity range of up to ten years. The other markets extend out to no longer than four years. The purpose of this paper is to estimate a term structure of

[26] Derivatives relating to dividends paid by individual companies exist as well.
[27] It is not clear why no exchange has opened up to listing futures on dividends of a US stock index.

dividend risk adjusted growth, for which longer dated maturities are required. We therefore exclude the shorter maturity dividend futures markets from the dataset.

Before 2008, dividend derivatives existed as dividend swaps traded over-the-counter (OTC) only. They date back to 2002, well before the onset of listed futures. Maturities extend out to ten years and more for Eurostoxx 50, Nikkei 225, FTSE 100 and the S&P 500. We obtained dividend swap price data from several investment banks for all four stock indices mentioned[28], but there are problems. Before 2005, prices are stale and, throughout the data period prices, not always consistent which each other among suppliers. Moreover, turnover is low and not transparent[29].

We nonetheless perform the estimations with datasets both of listed dividend futures at daily frequency and OTC dividend swaps at monthly frequency. The main conclusion from these results is that it shows that OTC price data originate from pricing models and may or may not reflect market prices. Table A1 contains a detailed description of the data.

Constant maturities

Dividend derivatives usually expire at a fixed date near the end of the calendar year[30] and therefore their time to maturity shortens by one day for each day that passes. For application in the state space model, growth rates of a constant horizon are required. The horizons of the measurement equations regard annual increments, the state equations regard one day increments. To obtain growth rates from prices with constant maturities, we interpolate derivatives with adjacent expiry dates. The interpolation is weighted by a scheme which reflects the uneven distribution of dividends through the year. For example, in the spring season 60% of the Eurostoxx 50 dividends of full index year are paid in a matter of a few weeks (Figure A1).

Derivatives prices which have a constant horizon from any observation date are constructed from observed derivatives prices. Such Constant Maturity (CM) derivative prices $F_{t,n}^{CM}$ take the following shape, attaching the seasonal pattern of the dividend index as weights to the observed derivatives prices w_i, with i standing for the day in the dividend index year, i = 1 being the first day of the count of the dividend index[31]:

[28] Deutsche Bank, Goldman Sachs and Credit Suisse.
[29] Dividend swaps are said not to trade daily, "sometimes not even for months". Turnover figures are not public.
[30] The Nikkei 225 dividend index runs until the last trading day in March.
[31] which is the first trading day following the expiry date of a dividend derivatives contract.

$$F_{t,n}^{CM} = (1 - w_i)F_{t,n} + w_i F_{t,n+1}. \tag{39}$$

The weight w_i of the dividend index reflects the cash dividend amount paid as a proportion of the total amount during a dividend index year. The average of the years 2005 to 2013 is taken. $F_{t,n}$ is the observed price of the derivative which expires n^{th} in line into the future from the observation date onwards, $F_{t,n+1}$ expiring the following year. This weighting scheme reduces the impact of the n^{th} derivative to expire on the constant maturity derivative as time passes by the proportion w_i of dividends that have actually been declared. Its complement $(1 - w_i)$ is the proportion that remains to be declared until the expiry date and is therefore an expectation of undeclared dividends for year n at the observation date. In order to produce a derivative price with constant maturities, this undeclared amount is balanced by the proportion of the price of the derivative expiring the year after. In so doing, the constant maturity price reflects no seasonal pattern, while still accounting for the seasonal shift in impact from the n^{th} derivative to the next. For example, during the dividend season in Spring, the weight is shifted more quickly from the first to the second derivative[32] than in other parts of the year[33].

The first to expire constant maturity derivative

The weighting scheme in equation (39) will be applied to obtain all CM derivatives prices, except for the first CM derivative, because the proposed approach carries measurement problems for it. The expected dividend to be delivered at the expiration of the first derivative $E_t(D_1)$ is the sum of the dividend index DI_t as it accretes throughout the year and its unknown complement $E_t(UD_1)$.

$$E_t(D_1) = DI_t + E_t(UD_1) \tag{40}$$

For CM derivatives with horizons longer than the first, the weight w_i is the average seasonal pattern in the preceding decade, which may not necessarily resemble that of a particular dividend index year $^{DI_t}/_{E_t(D_1)}$. The difference between the two is shown in Figure A2; in April 2013 the payments of Eurostoxx 50 dividends had already reached 33% of the annual total, while on average in the years 2005 to 2013 it stood at 20%. This advance dropped below

[32] First and second derivatives is shorthand for the derivatives that are first and second to expire.

[33] A linear weighting scheme would reflect the adjacent derivative prices unevenly. For example, half way through the dividend index year already 80% of annual dividends is declared and paid. Linear weighting would then overemphasize the information contained in the price of the derivative that is the soonest to expire.

ten percent not until a month later. In general, dividend payments in 2012 and 2013 seem to have taken place earlier in the calendar year than usual in the preceding years.

Weighting the first derivative by the average of the preceding decade when dividends realize sooner in the year than the average, as was the case in April 2013, overemphasizes the importance of that first derivative to the one year CM derivative. This first CM derivative will then contain backward looking information as well underemphasize the unrealized proportion of the contemporaneous dividend index both to the tune of the difference between the historical average and the realized dividend index. To avoid this issue, the first CM derivative is construed by defining the weight as the proportion of the dividend index that has been realized of the total expected dividend for that year only:

$$F_{t,1}^{CM} = F_{t,1} - DI_t + \frac{DI_t}{D_1} F_{t,2}. \tag{41}$$

For building a first CM derivative with a constant one year horizon as a stochastic variable, we include unknown $E_t(UD_1)$ and exclude known DI_t. The expectation of full year dividends is proxied by the equivalent observation. Later CM derivatives do not weight variables which have already been partly realized, hence the weighting issue of the first CM derivative does not reoccur. For $n \geq 2$, the prices of CM derivatives remain constructed as in the weighting equation (39).

The growth rate of derivative prices is projected from a stable starting point, from a base consisting of a CM derivative which does not come closer to expiry as time passes:

$$n\pi_{t,n} = ln\left(F_{t,n}^{CM}\right) - ny_{t,n} - ln(D_t). \tag{42}$$

Calculating seasonal weights for different dividend index years

Expiry years do not have the same number of trading days every year or across markets. Not only do trading holidays differ, also the expiry date is set to the third Friday in December in every expiry year. This day falls anywhere between the 15th and the 21st of December[34] and the number of trading days fluctuates accordingly.

In order to establish a seasonal pattern for w_i that is correct for the actual number of trading days in each expiry year, realized dividends are normalized and averaged. First, the amount of dividends paid on a given day is expressed as a percentage of the total dividends

[34] With exception of the Nikkei 225.

paid in the matching dividend index year. Next, for each expiry year these percentages are normalized to a set number of trading days. Finally, they are averaged. For calculating the values in the weighting equation, they are rescaled to the actual number of trading days in the dividend index year in question. This approach guarantees that in every expiry year, weight w_i starts at zero and ends the year at 100%, regardless of the number of trading days.

Current dividends

At the heart of the present value model are the discounted values of risk adjusted dividends. These present values $P_{t,n}$ take current dividends D_t as the starting point from which growth is projected forward at growth rate $\pi_{t,n}$ (equations (3) and (5)). It is sometimes assumed[35] that current dividends can be reasonably approximated by realized dividends. For daily data as applied in this paper, however, this assumption causes issues.

The asset underlying dividend derivatives is the amount of cash dividend thrown off by a stock or a stock index during the year in which the derivative expires. The index companies pay dividends throughout the calendar year[36] which implies that taking realized dividends as current dividends at a certain day of the year would require looking back for twelve months. The dividend paying capacity of index companies are unlikely to stay constant for a year, however.

To take a strong example, around the days of the Lehman bankruptcy on the 15th of September 2008, the one year dividend history of Eurostoxx 50 companies amounted to 154. Due to the bankruptcy, investors would have changed their opinion strongly downwards about the dividend that companies would pay if they would have had to pay on these days. Even if dividends reflect the past year of earnings, company management is likely to reduce dividends if their near term outlook changes for the worse by precautionary motive. After Lehman, taking a dividend history of twelve months would then overestimate the approximation of current dividends as they stood in the fall of 2008. In the weeks following the default, the Eurostoxx 50 dividend future expiring in 2009 dropped from 140 to 100. Therefore, if twelve month realized dividends are used as current dividends, the shortest horizon observation for growth from 2008 to 2009 on the dividend curve would attain a strongly negative figure even though the actual growth expectation, starting from a level that would have been revised downwards, could be flat or even positive.

[35] E.g. Binsbergen et al. (2013), Cejnek and Randl (2014).

[36] In fact, the dividend index year usually runs from the first working day following the third Friday in December until and including the third Friday in December of the following year. Dividend derivatives also apply the third Friday of December as the expiry date.

This problem rules out considering the dividend index itself, or a rolling twelve month estimate of it, as a starting point from which to calculate the growth rate until the first derivative to expire. The first derivative to expire, of course, does contain investor expectations about dividends to be paid in the remaining period until the first expiry date. But this information regards the period from the observation date until the expiry date; it is not a reflection of dividend expectations on the observation date itself.

To avoid these data difficulties, we propose an alternative base. In lieu of an estimate for current dividends, we use dividend derivatives with one year remaining life to expiry $F_{t,1}$ discounted at the one year risk free rate $y_{t,1}$ as the base from which to calculate growth rates:

$$P_{t,1}^{CM} = F_{t,1}^{CM} exp\left(-y_{t,1}\right), \tag{43}$$

and the first year of growth is deducted accordingly. Discounted risk adjusted dividend growth rates are then given by:

$$n\pi_{t,n} - \pi_{t,1} = ln\left(F_{t,n}^{CM}\right) - ny_{t,n} - \left(ln\left(F_{t,1}^{CM}\right) - y_{t,1}\right). \tag{44}$$

As a consequence of estimating $n\pi_{t,n} - \pi_{t,1}$ as a single variable, we do not account for the first year of discounted dividend growth as part of the dividend term structure. At the same time, the one year dividend present value $P_{t,1}^{CM}$ includes short term derivatives prices which encompass investor expectations extending from the observation date until a year later. Although growth for the first year is not observed, the one year discounted derivative price is included in the present value identity ensuring that no information is lost when reconciling the model estimates to the stock market.

VII. Tables and Figures

Table 1

Benchmark Model of Discounted Risk Adjusted Dividend Growth:
$\pi_{t,n} = g_{t,n} - \theta_{t,n} - y_{t,n}$.
Estimates using **listed Dividend Futures**

	Two state		Single state	
	$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi(\overline{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi(\overline{p} - p_t)dt + \sigma_p dW_p$	
	Eurostoxx 50	**Nikkei 225**	**Eurostoxx 50**	**Nikkei 225**
Sample period	4 August 2008 – 10 July 2014	17 June 2010 – 10 July 2014	4 August 2008 – 10 July 2014	17 June 2010 – 10 July 2014
$\overline{p}$	-0.0294	-0.0272	-0.0329	-0.0328
	(0.0039)	(0.0051)	(0.0020)	(0.0034)
φ	1.4336	0.7555	1.9006	0.2905
	(0.3196)	(0.2413)	(0.6328)	(0.0344)
ψ	0.2985	0.1762		
	(0.1119)	(0.0530)		
σ_p	1.09×10^{-03}	9.16×10^{-05}	2.95×10^{-03}	1.16×10^{-05}
	(1.52×10^{-03})	(1.36×10^{-04})	(5.73×10^{-03})	(2.23×10^{-05})
$\sigma_{\tilde{p}}$	1.18×10^{-05}	2.31×10^{-06}		
	(2.58×10^{-05})	(6.68×10^{-06})		
σ_ε	5.04×10^{-04}	2.50×10^{-04}	2.07×10^{-03}	3.37×10^{-04}
	(6.94×10^{-04})	(3.61×10^{-04})	(3.94×10^{-03})	(6.19×10^{-04})
σ_η	3.89×10^{-05}	1.69×10^{-05}	3.13×10^{-04}	2.09×10^{-04}
	(1.47×10^{-05})	(6.57×10^{-06})	(1.23×10^{-04})	(6.54×10^{-05})
Log Likelihood per contribution	24.48	29.13	18.24	21.65

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted risk adjusted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years. The sample of the Eurostoxx 50 data contains five measurement equations from its begin until 13th May 2009 due to a lack of data. σ_η measures the standard deviations of the second until the eight measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section. Standard errors in parentheses.

Table 2

Key results from two state space model and historical data

		Eurostoxx 50	Nikkei 225
Sample period		4 August 2008 – 10 July 2014	17 June 2010 – 10 July 2014
Estimated LT growth	$\bar{p}$	– 2.9 %	– 2.7 %
Average historical dividend yield	$\frac{D_t}{S_t}$	3.8 %	2.3 %
Average estimated 1 year growth	p_t	– 9.8 %	1.1 %
Average estimated 1 year forward 4 year growth	$\tilde{p}_t$	– 3.7 %	– 1.6 %
Average calibrated first dividend point	$\frac{\widehat{P}_{t,1}}{\sum_1^\infty \widehat{P}_{t,n}}$	3.1 %	2.1 %

Table 3

Reconciliation of two state present value model constituent returns to stock market returns: **listed Dividend Futures**

	Eurostoxx 50				Nikkei 225			
Constant	0.0005	0.0001	0.0006	-0.0001	-0.0002	0.0009	0.0004	0.0001
	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0003)	(0.0004)	(0.0004)	(0.0003)
$\Delta\ln(F_t)$	0.8976	0.9923			0.7202	0.6783		
	(0.0350)	(0.0437)			(0.0548)	(0.0785)		
Δy_t	0.1453		0.2067		0.0190		-0.0662	
	(0.0131)		(0.0181)		(0.0659)		(0.0979)	
$\Delta\ln(\widehat{PD}_t^1)$	0.6784			0.7094	0.9045			0.8960
	(0.0245)			(0.0307)	(0.0278)			(0.0300)
$Adj.R^2$	0.531	0.253	0.078	0.259	0.550	0.069	0.000	0.471

The modelled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (34) $\Delta ln(S_t) = \alpha + \beta_F \Delta ln(F_t) + \beta_y \Delta y_t + \beta_{\widehat{PD}} \Delta ln(\widehat{PD}_t^1) + \varepsilon_t$, in which $\Delta ln(S_t)$ is stock index log returns, $\Delta ln(F_t)$ is the log return of the first constant maturity dividend derivative, Δy_t is the change in the one year zero swap rate and $\Delta ln(\widehat{PD}_t^1)$ is the first differenced log of the sum of the normalized present value of dividends as estimated in the two (i = 2) or single (i = 1) state space model. Standard errors in parentheses.

Table 4

Benchmark Model of Discounted Risk Adjusted Dividend Growth:
$\pi_{t,n} = g_{t,n} - \theta_{t,n} - y_{t,n}$.
Estimates using **listed Dividend Futures** (**monthly**)

	Two state		Single state	
	$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi(\overline{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi(\overline{p} - p_t)dt + \sigma_p dW_p$	
	Eurostoxx 50	**Nikkei 225**	**Eurostoxx 50**	**Nikkei 225**
Sample period	August 2008 – July 2014	June 2010 – July 2014	August 2008 – July 2014	June 2010 – July 2014
$\overline{p}$	-0.0291	-0.0277	-0.0326	-0.0323
	(0.0039)	(0.0043)	(0.0021)	(0.0036)
φ	1.5155	0.7860	1.8647	0.2899
	(0.4636)	(0.2527)	(0.6628)	(0.0377)
ψ	0.2923	0.1784		
	(0.1102)	(0.0495)		
σ_p	7.30×10^{-02}	1.46×10^{-03}	1.24×10^{-01}	3.10×10^{-04}
	(8.31×10^{-02})	(1.83×10^{-03})	(2.24×10^{-01})	(4.12×10^{-04})
$\sigma_{\tilde{p}}$	2.98×10^{-04}	8.05×10^{-05}		
	(6.45×10^{-04})	(1.18×10^{-04})		
σ_ε	4.51×10^{-04}	2.30×10^{-04}	2.02×10^{-03}	3.10×10^{-04}
	(7.42×10^{-04})	(3.66×10^{-04})	(4.43×10^{-03})	(6.38×10^{-04})
σ_η	3.22×10^{-05}	1.28×10^{-05}	3.04×10^{-04}	2.09×10^{-04}
	(1.66×10^{-05})	(6.31×10^{-06})	(1.41×10^{-04})	(7.48×10^{-05})
Log Likelihood per contribution	22.21	27.41	16.81	20.33

Maximum Likelihood estimates are based on monthly averages of daily prices of dividend futures and interest rates. Measurement equations capture discounted risk adjusted dividend growth starting in one year following the observation date. The estimates include eight measurement equations: from one to eight years. The sample of the Eurostoxx 50 data contains five measurement equations from its begin until May 2009 due to a lack of data. σ_η measures the standard deviations of the second until the eight measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section. Standard errors in parentheses.

Table 5

Alternative Model of Undiscounted Risk Adjusted Dividend Growth:
$z_{t,n} = g_{t,n} - \theta_{t,n} = \pi_{t,n} + y_{t,n}$
Estimates using **listed Dividend Futures**.

	Two state		Single state	
	$dz_t = \varphi(\tilde{z}_t - z_t)dt + \sigma_z dW_z$ $d\tilde{z}_t = \psi(\overline{z} - z_t)dt + \sigma_{\tilde{z}} dW_{\tilde{z}}$		$dz_t = \varphi(\overline{z} - z_t)dt + \sigma_z dW_z$	
	Eurostoxx 50	**Nikkei 225**	**Eurostoxx 50**	**Nikkei 225**
Sample period	4 August 2008 – 10 July 2014	17 June 2010 – 10 July 2014	4 August 2008 – 10 July 2014	17 June 2010 – 10 July 2014
$\overline{p}$	0.0174	0.0008	-0.0014	-0.0168
	(0.0116)	(0.0081)	(0.0030)	(0.0034)
φ	1.4356	0.6940	1.3605	0.2942
	(0.2710)	(0.1898)	(0.4330)	(0.0380)
ψ	0.1816	0.1886		
	(0.0800)	(0.0522)		
σ_p	1.10×10^{-03}	7.64×10^{-05}	8.16×10^{-04}	1.23×10^{-05}
	(1.40×10^{-03})	(1.07×10^{-04})	(1.67×10^{-03})	(2.43×10^{-05})
$\sigma_{\tilde{p}}$	5.57×10^{-06}	2.77×10^{-06}		
	(1.12×10^{-05})	(7.40×10^{-06})		
σ_ε	5.93×10^{-04}	2.34×10^{-04}	3.43×10^{-03}	3.89×10^{-04}
	(8.34×10^{-04})	(3.22×10^{-04})	(7.93×10^{-03})	(7.40×10^{-04})
σ_η	3.88×10^{-05}	1.46×10^{-05}	6.88×10^{-04}	2.41×10^{-04}
	(1.55×10^{-05})	(5.27×10^{-06})	(3.55×10^{-04})	(7.53×10^{-05})
Log Likelihood per contribution	24.41	29.55	15.58	21.11

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted risk adjusted dividend growth starting in one year following the observation date. The estimates include eight measurement equations: from one to eight years. The sample of the Eurostoxx 50 data contains five measurement equations from its begin until 13th May 2009 due to a lack of data. σ_η measures the standard deviations of the second until the eight measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section. Standard errors in parentheses.

Table 6

Benchmark Model of Discounted Risk Adjusted Dividend Growth:
$\pi_{t,n} = g_{t,n} - \theta_{t,n} - y_{t,n}$.
Estimates using **OTC Dividend Swaps (monthly)**

	Two state		Single state	
	$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi(\overline{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi(\overline{p} - p_t)dt + \sigma_p dW_p$	
	S&P 500	**FTSE 100**	**S&P 500**	**FTSE 100**
Sample period	Dec 2005 – June 2014	Dec 2005 – June 2014	Dec 2005 – June 2014	Dec 2005 – June 2014
$\overline{p}$	-0.0130	-0.0534	-0.0099	-0.0334
	(0.0084)	(0.0639)	(0.0027)	(0.0015)
φ	1.0341	1.6440	0.3555	1.7675
	(0.6843)	(0.5815)	(0.0587)	(0.5603)
ψ	0.1812	0.0439		
	(0.1455)	(0.1434)		
σ_p	2.40×10^{-03}	2.94×10^{-02}	3.80×10^{-02}	3.80×10^{-02}
	(3.89×10^{-03})	(4.41×10^{-02})	(6.22×10^{-02})	(6.22×10^{-02})
$\sigma_{\tilde{p}}$	6.88×10^{-05}	2.57×10^{-05}		
	(1.50×10^{-04})	(5.17×10^{-05})		
σ_ε	2.86×10^{-04}	3.95×10^{-04}	8.89×10^{-04}	8.89×10^{-04}
	(1.20×10^{-04})	(6.26×10^{-04})	(1.97×10^{-03})	(1.97×10^{-03})
σ_η	6.16×10^{-05}	3.03×10^{-05}	2.00×10^{-04}	2.00×10^{-04}
	(1.62×10^{-05})	(1.10×10^{-05})	(7.64×10^{-05})	(7.64×10^{-05})
Log Likelihood per contribution	22.61	23.72	19.79	19.29

Maximum Likelihood estimates are based on monthly averages of daily prices of OTC dividend swaps and interest rates. Measurement equations capture discounted risk adjusted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years. σ_η measures the standard deviations of the second until the eight measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section. Standard errors in parentheses.

Table 7

Reconciliation of the **Undiscounted** two state present value model constituent returns to stock market returns:

	Eurostoxx 50				Nikkei 225			
Constant	0.0006	0.0001	0.0006	0.0004	-0.0002	0.0001	0.0004	0.0003
	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)
$\Delta\ln(F_t)$	0.9186	0.9923			0.7435	0.6783		
	(0.043)	(0.0437)			(0.0768)	(0.0785)		
Δy_t	0.1425		0.2067		0.0817		-0.0662	
	(0.0165)		(0.0181)		(0.0928)		(0.0979)	
$\Delta\ln(\widehat{PD}_t^1)$	-0.0203			-0.053	0.2259			0.1952
	(0.007)			(0.0079)	(0.0286)			(0.0295)
$Adj.R^2$	0.298	0.253	0.078	0.028	0.120	0.069	0.000	0.042

The modelled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (34) $\Delta ln(S_t) = \alpha + \beta_F \Delta ln(F_t) + \beta_y \Delta y_t + \beta_{\widehat{PD}} \Delta ln(\widehat{PD}_t^1) + \varepsilon_t$, in which $\Delta ln(S_t)$ is stock index log returns, $\Delta ln(F_t)$ is the log return of the first constant maturity dividend derivative, Δy_t is the change in the one year zero swap rate and $\Delta ln(\widehat{PD}_t^1)$ is the first differenced log of the sum of the normalized present value of dividends as estimated in the two (i = 2) or single (i = 1) state space model. Standard errors in parentheses.

Table 8

Reconciliation of two state present value model constituent returns to stock market returns:

OTC Dividend Swaps (monthly data)

	S&P 500				FTSE 100			
Constant	0.0000	0.0000	0.0003	0.0002	0.0002	0.0005	0.0032	0.0014
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0033)	(0.0034)	(0.0040)	(0.0037)
$\Delta\ln(F_t)$	0.6933	0.6634			0.5685	0.6273		
	(0.1004)	(0.1012)			(0.1069)	(0.1033)		
Δy_t	0.0013		0.0323		0.0026		0.0320	
	(0.0174)		(0.0207)		(0.0155)		(0.0173)	
$\Delta\ln(\widehat{PD}_t^1)$	0.2680			0.2244	0.2364			0.2804
	(0.0688)			(0.0838)	(0.0588)			(0.0667)
$Adj.R^2$	0.392	0.298	0.023	0.066	0.354	0.262	0.023	0.142

The modelled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (34) $\Delta ln(S_t) = \alpha + \beta_F \Delta ln(F_t) + \beta_y \Delta y_t + \beta_{\widehat{PD}} \Delta ln(\widehat{PD}_t^1) + \varepsilon_t$, in which $\Delta ln(S_t)$ is stock index log returns, $\Delta ln(F_t)$ is the log return of the first constant maturity dividend derivative, Δy_t is the change in the one year zero swap rate and $\Delta ln(\widehat{PD}_t^1)$ is the first differenced log of the sum of the normalized present value of dividends as estimated in the two (i = 2) or single (i = 1) state space model. Standard errors in parentheses.

Table A1

		Eurostoxx 50	S&P 500	FTSE 100	Nikkei 225
Number of companies in the index		50	500	100	225
Currency		Euro	US$	GBP	JPY
Market capitalization in US$ per 7th May 2014		US$ 3.3 trillion	US$ 17.2 trillion	US$ 3.1 trillion	US$ 2.7 trillion
Data period	Dividend swaps	N/A	19 December 2005 – 13 June 2014	19 December 2005 – 13 June 2014	N/A
	Dividend futures	4 August 2008 – 10 July 2014	N/A	N/A	17 June 2010 – 10 July 2014
Source of the data	Dividend swaps	N/A	OTC	OTC	N/A
	Dividend futures	Eurex	N/A	N/A	Singapore exchange
Average number of trading days		256	252	253	245
Liquidity		Good	Low	Low	Reasonable
Expiry horizon	Dividend swaps	N/A	10 years	10 years	N/A
	Dividend futures	10 years	N/A	4 years	10 years
Expiry date		3rd Friday of December	3rd Friday of December	3rd Friday of December	Last trading day in March
Data frequency		Daily	Daily	Daily	Daily
Stock index ticker		SX5E	SPX	UKX	NKY
Dividend index ticker		DKESDPE	SPXDIV	F1DIVD	JPN225D

Figure 1. Eurostoxx 50 Dividend Futures Prices
Price curve of dividend futures on an arbitrary day, for purpose of illustration. Expiries occur on the third Friday in December of each expiry year.

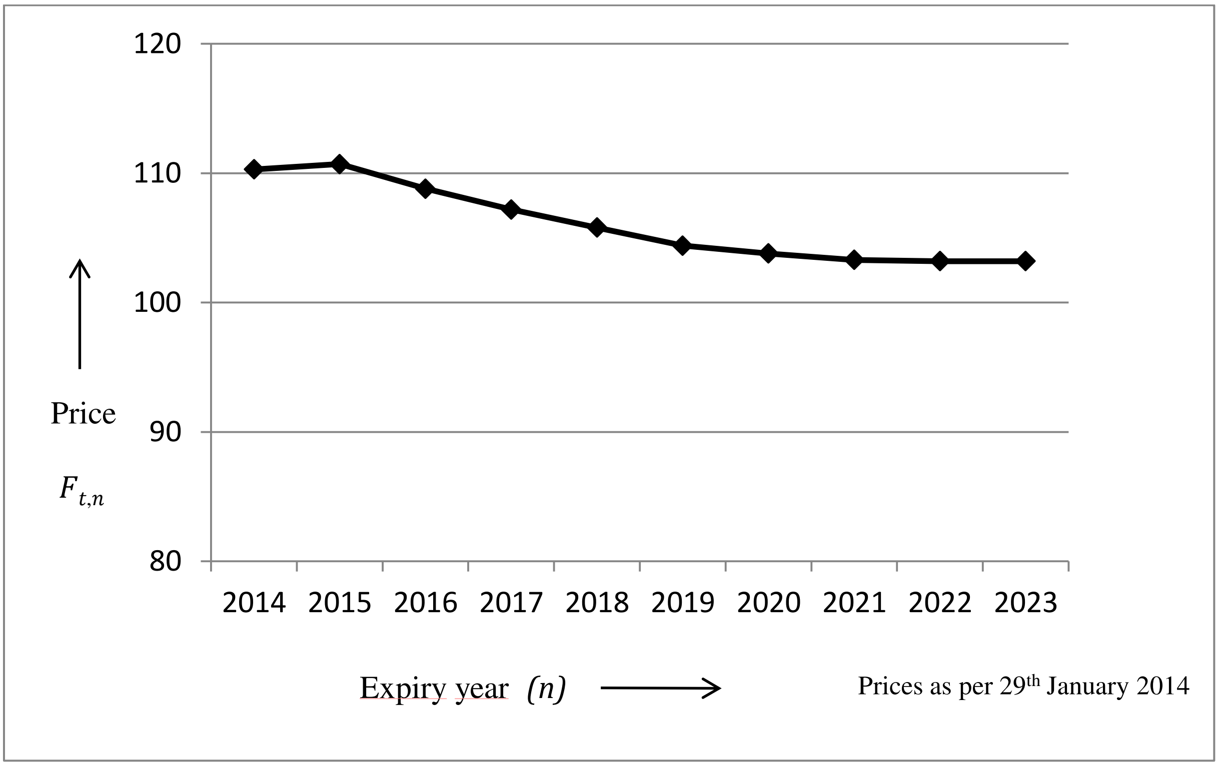
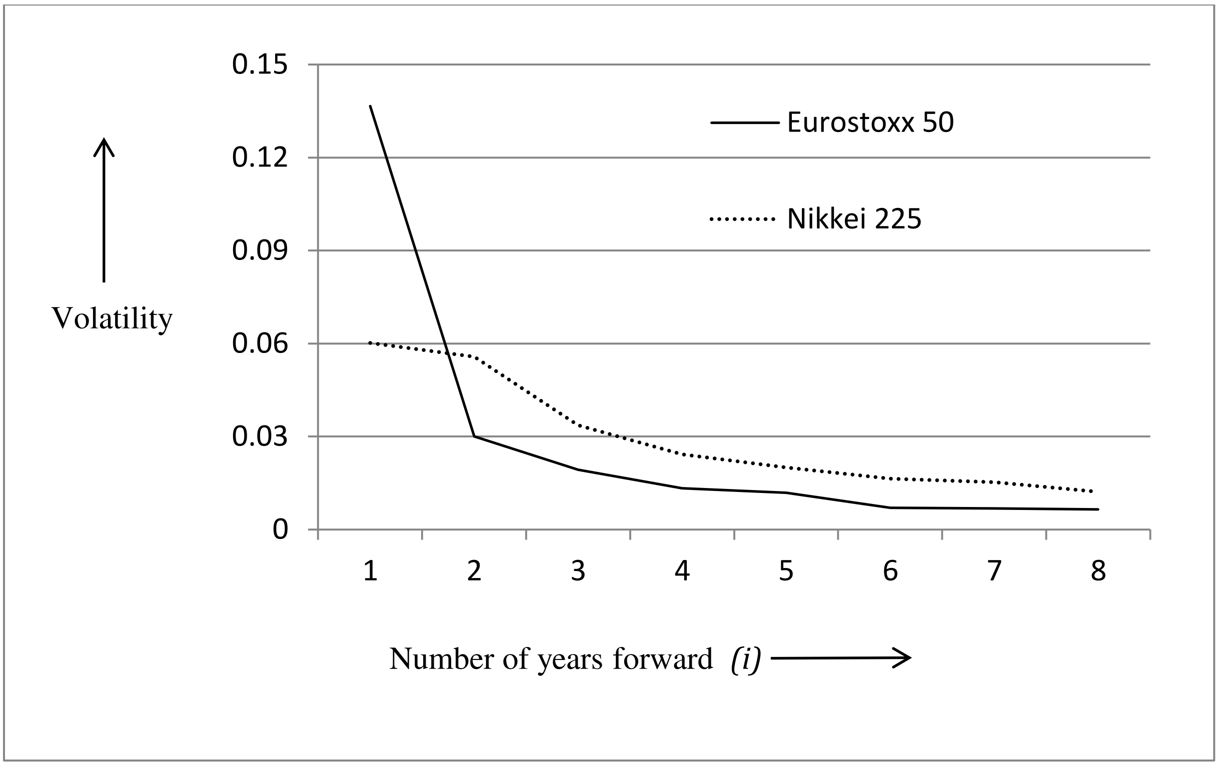


Figure 2. Discounted Risk Adjusted Dividend Growth Volatility Term Structure
$p_{i \to i+1}$ = volatility from $n = i$ to $n = i + 1$ (= single period forward growth rates).



Calibrated growth rates

Figures 3 to 6 show calibrated growth rates of discounted risk adjusted dividends. Figures 3 and 5 contain the 1 year growth rate. Figures 4 and 6 contain average annual growth rates of the 4 years following the first year of growth: $p_{t,t+1 \rightarrow t+5}$.

Figure 3. Eurostoxx 50: 1 year growth

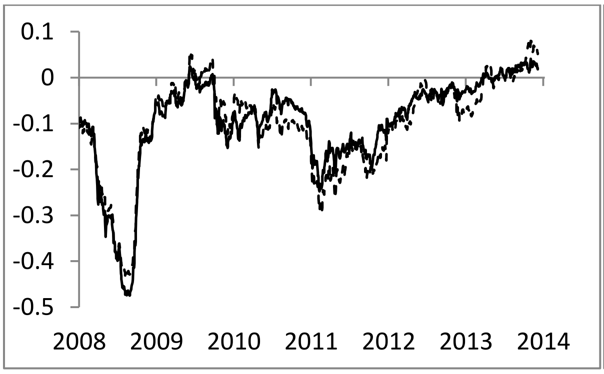


Figure 4. Eurostoxx 50: 1 year forward 4 growth

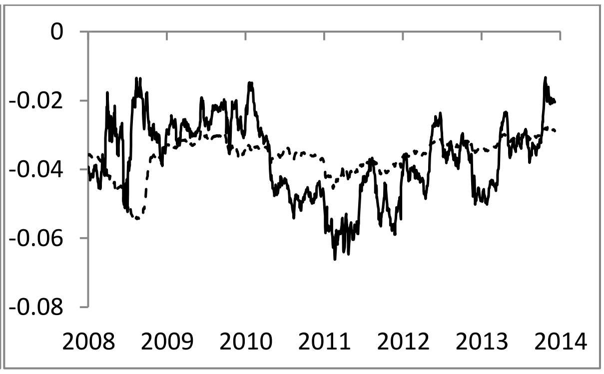


Figure 5. Nikkei 225: 1 year growth

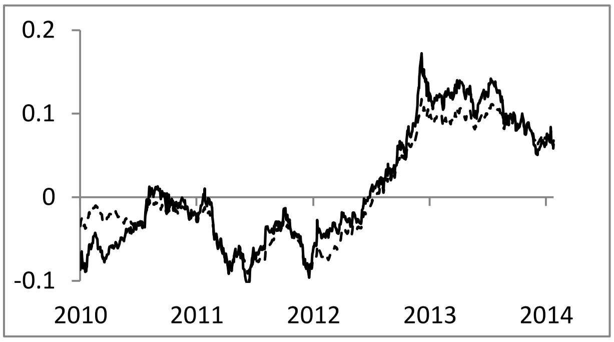
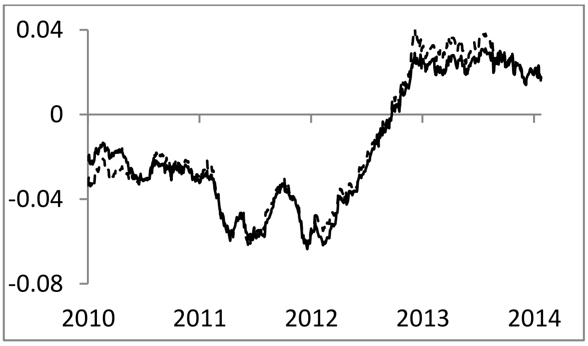


Figure 6. Nikkei 225: 1 year forward 4 year growth



Two state model ———— Single state model - - - -

Mean absolute estimation errors

Figures 7 and 8 depict the average of the absolute estimation error of the two state and the single state benchmark model. The measurement variables are discounted dividend risk adjusted growth rates of 1 to 8 years.

Figure 7. Eurostoxx 50

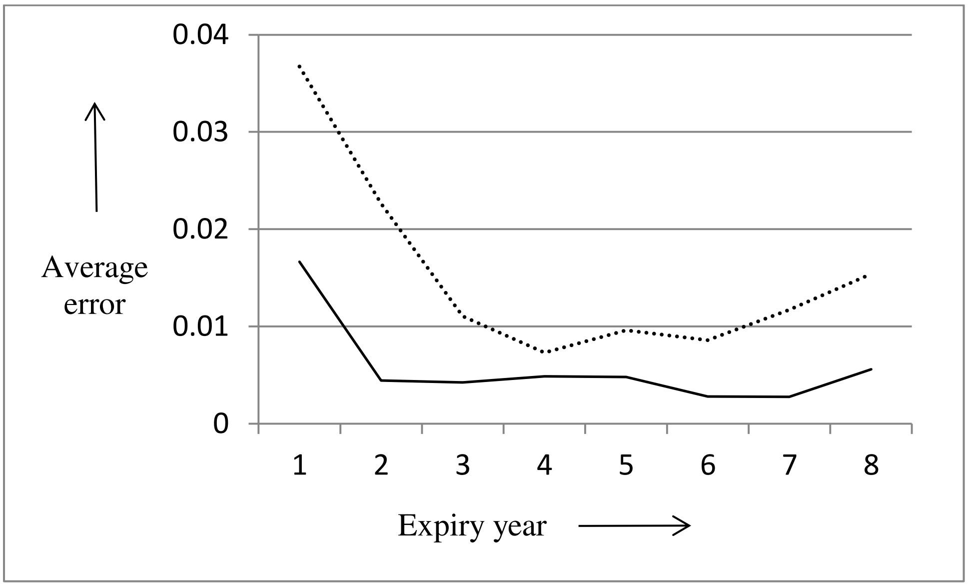
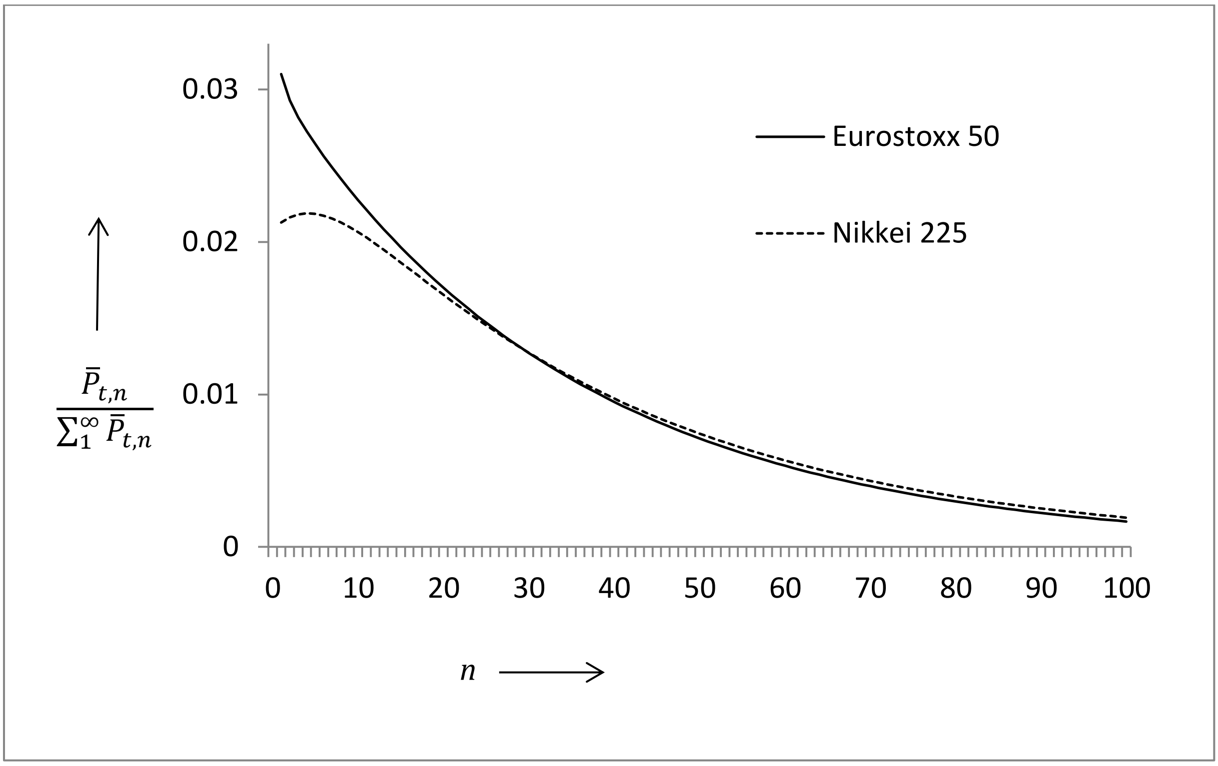


Figure 8. Nikkei 225

Two state model ———— Single state model ....

Figure 9. Calibrated average Dividend Term Structure
The average of calibrated present values of dividends per expiry year $\bar{P}_{t,n}$ is divided by the sum of the averages. This represents the average dividend yield per expiry year in present value terms.



Stock level reconciliation

Figures 10 & 11 portray the present value model estimates for the level of stock indices as described in $\hat{S}_t = F_{t,1}\, exp\left(-y_{t,1}\right)\left(1 + \sum_{n=2}^{\infty} exp\left(n\hat{\pi}_{t,n} - \hat{\pi}_{t,1}\right)\right)$ (equation (33)) in relation to stock market observations S_t. The model estimates are shown with a 2σ bound surrounding it, set at levels of the estimate for long term growth plus and minus 2σ, all other things equal.

Figure 10. Eurostoxx 50 level reconciliation

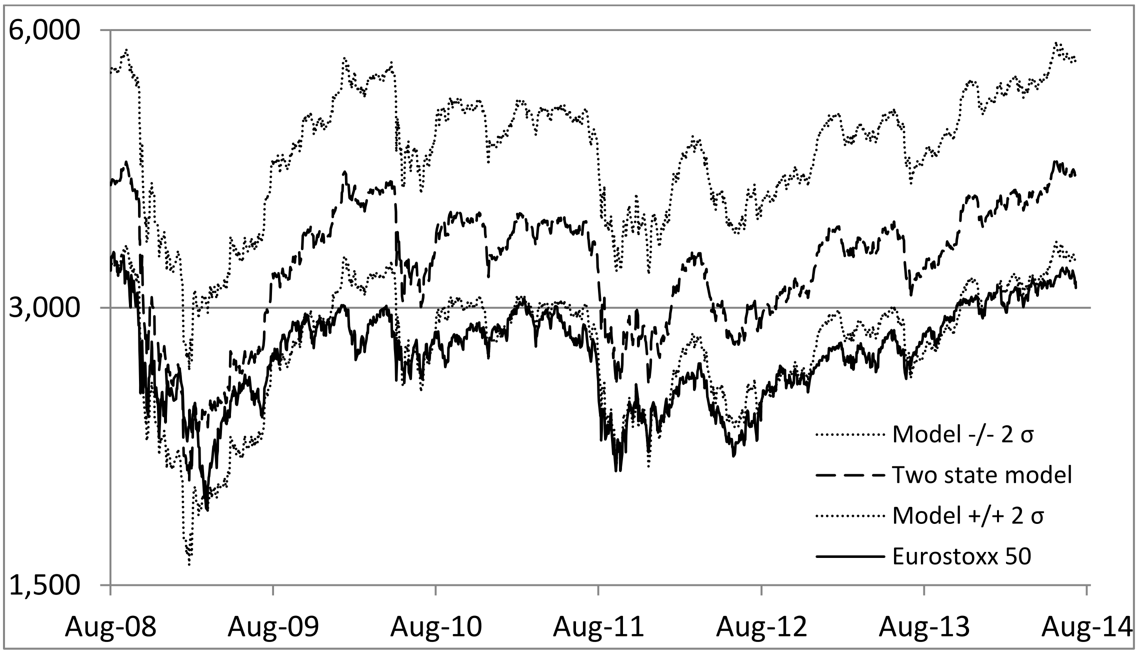


Figure 11. Nikkei 225 level reconciliation

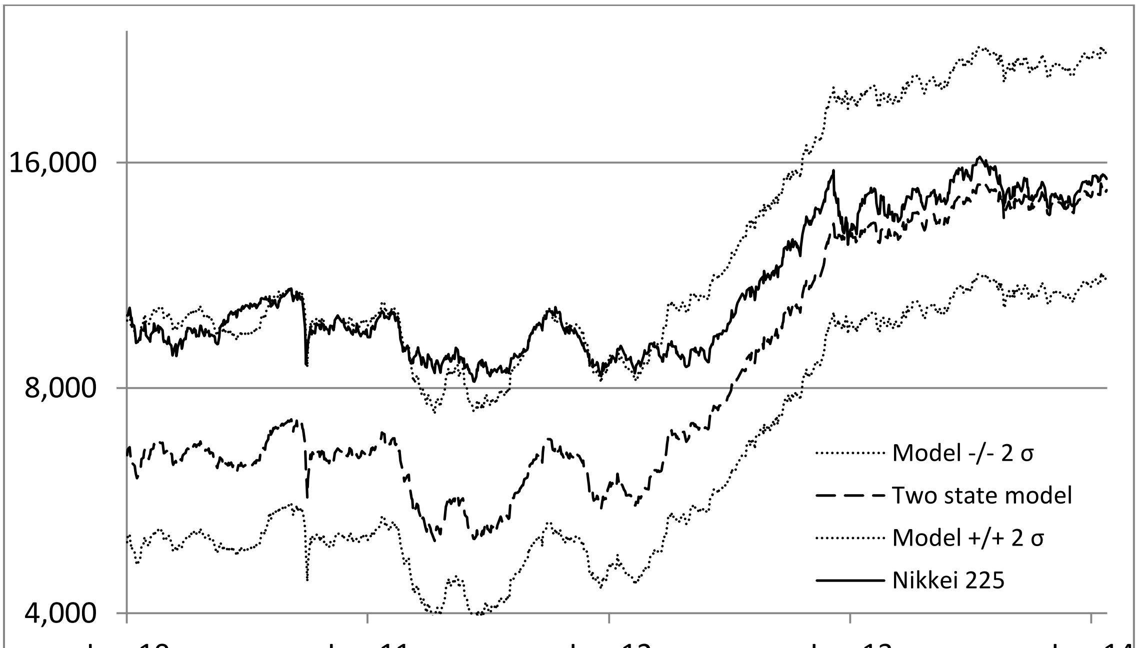


Figure A1. Proportion of dividend payments throughout the Eurostoxx 50 dividend index year. The first trading day of a dividend index year is the Monday following the third Friday of December. The chart depicts the average of the years 2005 to 2013.

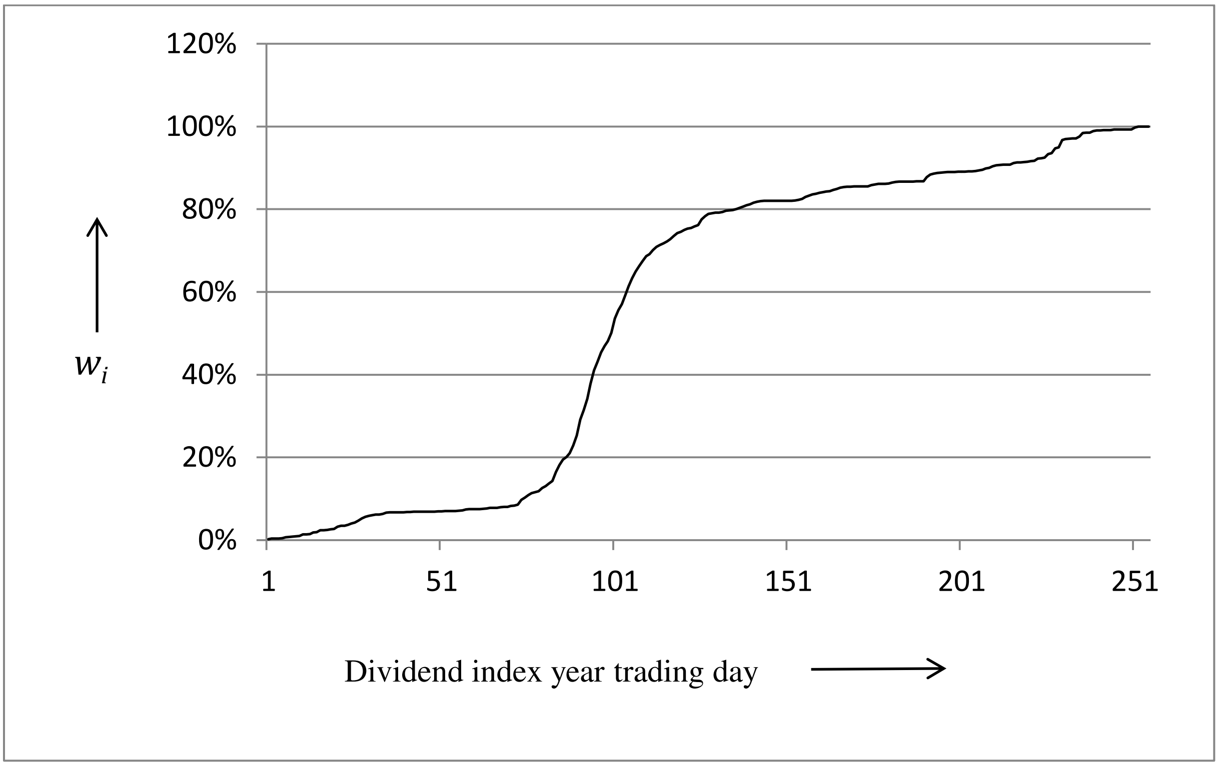


Figure A2. The difference between the proportion of annual dividends paid out at a given date and their average over the period 2005 to 2013 (Eurostoxx 50).