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What Explains the Diverse Funding Ratios of Public Sector Pension Funds

MSc Thesis 2010-043

What explains the diverse funding ratios of public sector pension funds

by

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A thesis submitted in partial fulfillment

of the requirements for the degree of

Master of Science in Economics and Finance of Aging

Faculty of Economics and Business Administration

Tilburg University

October 2010

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Chapter 1

Introduction

1.1 Background

Fiscal sustainability problems have been brought into the focus of public attentions since the credit crisis and the following economic recession. The most recent is Southern Europe's fiscal crisis that draws a public concern to the fiscal viability of Greece, Portugal, Spain and Italy. No better than Southern Europe, most states and local governments in the U.S. are striving against huge fiscal deficits. A recent study has found that, for the fiscal year 2010, at least 48 states are facing huge budget shortfalls that on average amount to 24 percent of state budgets (Lav and McNichol, 2009[14]). While the current fiscal crisis is triggered by the recent economic recession, severe long-term fiscal challenges are still to be addressed even after economic rebounds.

One often overlooked contributor to the surging concerns of fiscal sustainability is the severe underfunding of public sector pension plans. As for the local public sector plans in the U.S., it has been reported that though the average level of funding ratios were over 80% before the financial crisis, many public sector pension funds in the U.S. are now seriously underfunded (Munnell at al, 2008[17], 2010[18]). The relevance of public sector pensions to the fiscal sustainability challenges hinges on the differences from their counterparts in the private sector. As for the profiles of pension membership, public sector pension participants are older, more risk averse, less mobile and more unionized than those in private sector pensions.

General experience from researches has also shown that public sector pensions offer more generous retirement benefits to their members and tend to be less adequately funded than private sector pensions (Palacios and Whitehouse 2006[21]).

The generosity of public sector pension plans over private sector pension plans is reflected in many respects and due to many reasons. Researchers both in the U.S. and the U.K. have identified the fact that the coverage ratio of public sector pension pensions are much higher than private sector pensions (Munnell, Aubry and Muldoon, 2008[17]; Disney et al, 2009[8]). In the UK, the generosity of public sector pensions can also be explained by much lower effective retirement age of public sector workers (Disney et al, 2009[8]). Moreover, generous defined benefit plans remain dominant in the public sector while last few decades witnessed the revolution from defined benefit plans to defined contribution plans in the private sector. Munnell et al (2007)[16] attribute the staying power of the defined benefit plans in American public sector to the mentioned profile of public sector pension participants and different regulation environment facing public sector pension sponsors. Since pensions, as deferred wage, are an important component of remuneration package, such a divergence between the pension arrangements for public sector and private sector may reflect the human resource consideration of public sector to attract and retain talents. To be competitive in labor market, public sector employers have to offer a total remuneration package that is comparable to what is offered in the private sector. However, public sector employees are regarded less paid than private sector employees in terms of gross wage. Therefore, generous public sector pensions are arranged in such a way that they guarantee the security, integrity, independence and attractiveness of a career in civil service (Disney et al, 2009[8]).

Regarding the funding of public sector pensions compared to that of private sector, historical experience tells that public sector pensions were more often inadequately funded with funding ratio less than 100% while private sector plans subject to stricter solvency constraints. Additionally, public sector pension sponsors are too often less than transparent in reporting their funding status and economic cost of their projected future benefits. Another outstanding difference is that public

sector employees are less vulnerable to contribution increase, benefits cut and default on their pension benefits.

Most of the above issues relating to public sector pensions are explored and strengthened, broadly and specifically, by an international survey on public sector pensions in the coming section of this thesis.

1.2 Motivation and Research Questions

This thesis firstly targets at international evidence for the main characteristics and funding status of public sector pension plans. Then, the heart of this thesis is located at the funding issues of public sector pension plans.

1.2.1 Why funding of public sector pensions matters?

Government, as it is the typical case, plays a role of the largest employer in the economy. Pension liabilities in the public sector can make up a substantial part of the total debt position of the economy. The funding status of public sector pensions should be concerned by investors holding government bonds and rating organizations.

Mainly driven by demographic pressure and the strong power of public sector labor unions, pension benefits have been increasing steadily while economic cycle is fluctuating over time. Taxpayers' pocket tends to be the last resort for public sector pension deficits when the tax base is shrinking. In face of huge deficits in public sector pension funding and fiscal stress, the government would either raise tax or cut other public expenditures when taxpayers' marginal utility from consumption is at highest level. However, the government is more likely to expand benefits or cut contributions during economic booming and fiscal prosperity where public servants' marginal utility from consumption is extremely low. American evidence has been found that replacement rate, during the last three decades, for

typical state employees and teachers have increased by about 10 percent(Clark and Craig, 2009[5]). Therefore, any insight into the funding of public sector pensions provides critical implications for intergenerational transfers and associated intertemporal social welfare.

1.2.2 Why funding ratios matter?

Funding ratio indicates the funding status of a specific pension plan. The ratio of the asset set aside for a public sector pension fund over the fair value of its accrued future benefits defines the funding ratio in this thesis. This research starts with an international overview of public sector pensions which concludes a diversity of funding ratios by public sector pension funds across countries. However, explanations for such a diversity of funding ratios by public sector plans are rare in the literature. To fill in the blank, this thesis addresses the following research questions:

1. What drives the diverse funding ratios of public sector plans from the perspective of politics?

2. Is there optimal funding ratio for public sector plans?

To answer the first question, this thesis first looks back into the literature on pension politics for theoretical inputs. A general analysis based on the standard theory of pension politics is given in combination with the findings of next chapter. As for the second question, a simplified model is constructed to track the optimal funding ratios for public sector pensions analytically.

1.3 Thesis Organization

This thesis will be structured as follows. After the introduction, Chapter two conducts an international examination of public sector pension plans and provides evidence for a diversity of funding ratios. Chapter three gives some explanations, positively and normally, for the diversity of funding ratios by public sector pensions. Chapter four introduces a simplified two-period model based on mean-

variance utility to investigate the optimal funding ratios for public sector pensions plans. Finally, Chapter five presents some concluding thoughts.

Chapter 2

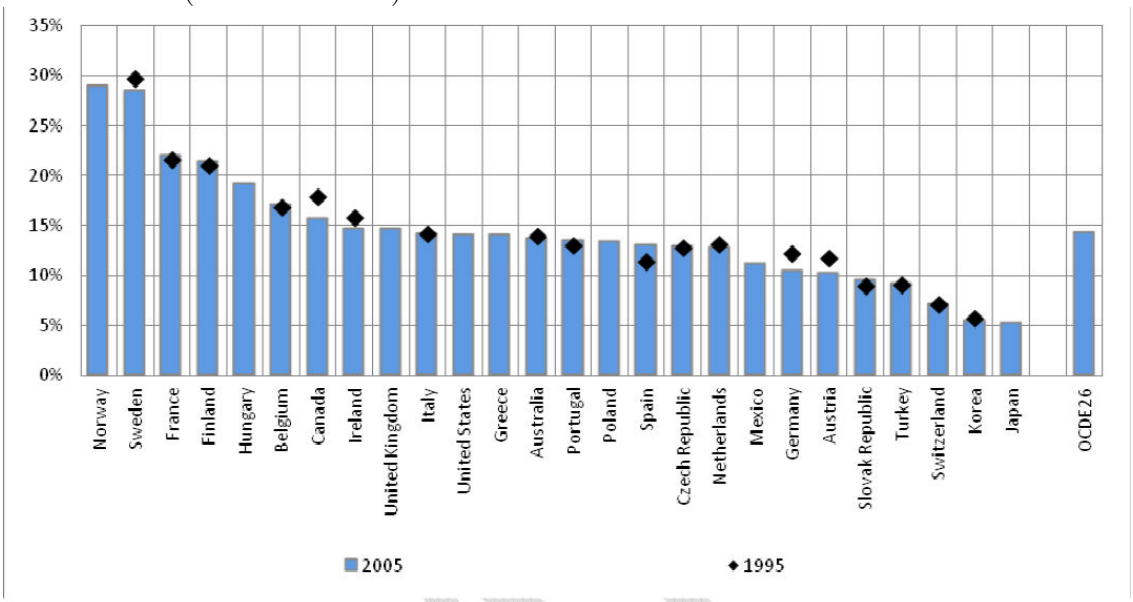
International Evidence

The first section of this chapter provides an extensive examination of public sector pension plans across various countries. The second section lies in the core of this section where funding status of a number of funded defined-benefit public plans is investigated intensively.

2.1 An international overview of public sector pensions

Public sector pensions post a major fiscal and policy challenge for the economy for many reasons. Four main reasons are found and confirmed by a general description of public sector pensions. Firstly, the state is often the largest employer in the country and thus commits to pay a huge amount of retirement benefits. Secondly, Most of the generous defined benefit schemes survived in the public sector while an unidirectional transition from defined benefit to defined contribution was dominating in private sector for the last three decades. Thirdly, a majority of OECD countries are financing pensions for public sector employees directly out of their tax revenues just as the pay-as-you-go financing mechanism. Fourthly, there is an absence of international agreement on standardizing the reporting and disclosure of public sector pension liabilities. Therefore, public sector pension sponsors may hide huge pension deficits via manipulation of reporting and disclosure.

Figure 2.1: **Employment of general government as a percentage of the labor force** (1995 and 2005)



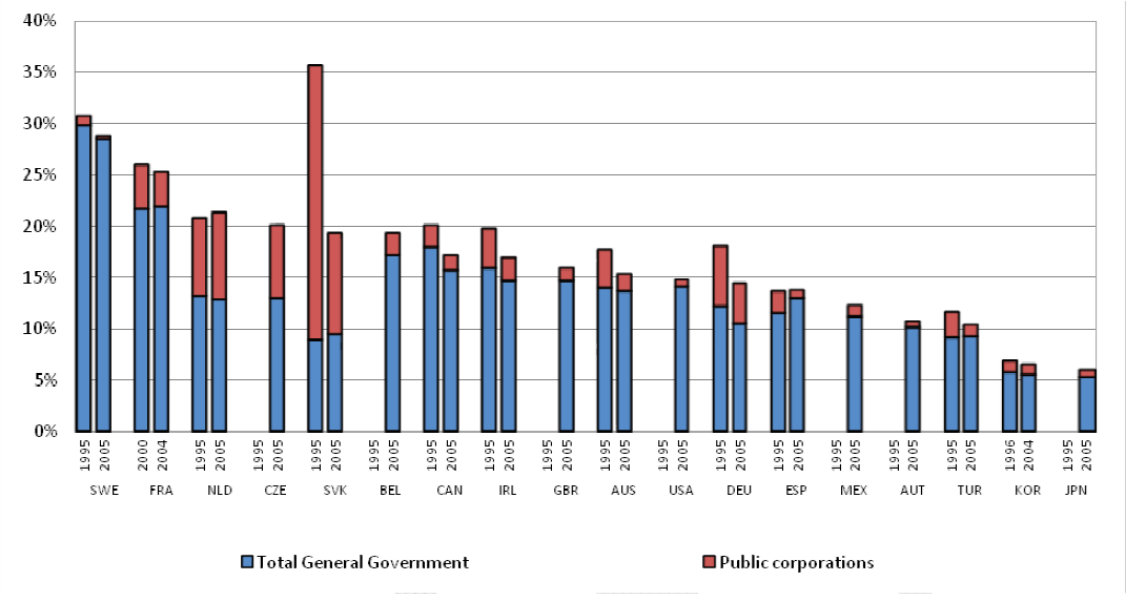
Source: Government at a Glance 2009

2.1.1 Size of public sector as a percentage of total labor force

As shown in figure 2.1, a substantial part of total labor force in investigated OECD countries is made up by government servants with an average level roughly at 15%. The highest level is found in Norway and Sweden where government workers amount to nearly 30% of each total labor force. The opposite is found in Japan and South Korea where government employees only account for about 5% of the total employment in each country.

Figure 2.2 takes employees of state-owned enterprises into account and the combined employment as a share of total labor force only has increased by few percent in most countries compared to the level in Figure 2. Moreover, the combined public sector employment level was relatively stable during that decade except for Slovakia where the number of public sector employees dropped dramatically.

Figure 2.2: **Changes in employment in general government and public corporations as a percentage of the labor force** (1995 and 2005)



Source: Government at a Glance 2009

2.1.2 Classification of pension plans for OECD government workers

There are three critical criteria classifying public sector pension schemes clearly. By financing mechanism, public sector pension schemes, as general pension schemes, can be divided into funded schemes and PAYG schemes. Those funded schemes are featured by pension assets that are invested before the maturity of the benefits it will pay for. In contrast, pension benefits of PAYG schemes are directly paid out by government general revenues without any pension assets as prefunding. By administration, the second classification concerns whether a public sector pension scheme substitutes the national scheme as a standalone pension scheme or complements the general pension system. The last criterion is benefit formula categorizing public sector pension schemes as defined benefit schemes, defined contribution schemes and hybrid schemes. Defined benefit schemes are characterized

by the sponsor guaranteed pension benefits. In defined benefit schemes, the government, as the sponsor, does not bear any obligation beyond the fulfillment of specified contribution payment and thus no future liabilities are built. Hybrid schemes can be recognized as a combination of defined benefit schemes and defined contribution schemes that share the characteristics of each type and may vary significantly across schemes.

A brief summary on main features of pension arrangement for public sector workers in OECD countries is presented in Table 2.1. It gives a clear overview of public sector pension plans and their three classifications based on their financing mechanisms, administrations and pension formulas.

As seen in Table 2.1, a majority of OECD countries operating unfunded PAYG pensions for their public sector employees. In most circumstance, public sector pension plans in OECD countries are complementary to the national pension system. Most notably, OECD countries, with very few exceptions, offer generous defined benefit pension arrangements for their public sector workers.

2.1.3 Pension disclosure

There is available data on public sector pension expenditure in many OECD countries. However, international comparability is impeded by problematic way of reporting. The OECD's SOCX data base reports pension expenditure of autonomous public sector funds in Australia (partially), Canada, Denmark, the Netherlands, Sweden and the United Kingdom a private spending item. In this reporting standard, all social benefits not directly paid by government budget are categorized as private spending item.

In contrast to private sector pensions, an internationally standardized reporting of public sector pensions, especially on pension liabilities is still lacking even though a reporting system SNA urged by OECD and IMF. Governments in some countries, like Canada and Australia, are required to disclose their underfunded pension benefits for public servants on their balance sheets. This implies that gov-

Table 2.1: **Main features of pension arrangement for public sector workers in OECD countries**

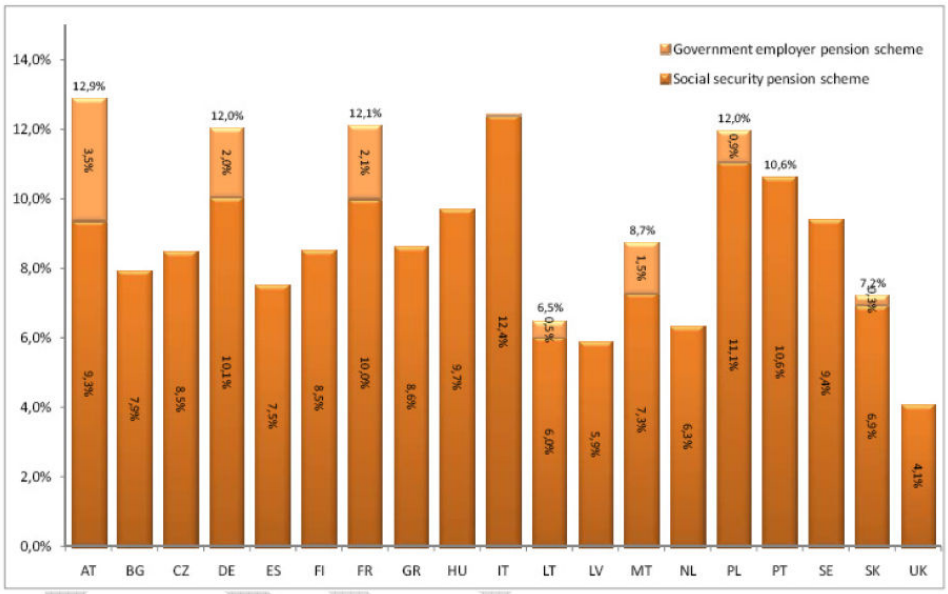
Country	Administration	DB or DC or hybrid	Financing Method
Australia	Standalone	DB and Hybrid	Funded
Austria	Complementary	DB	Unfunded
Belgium	Standalone	DB	Unfunded
Canada	Complementary	DB	Funded
Chile	Complementary	DC	Funded
Denmark	Complementary	DC	Funded
Finland	Standalone	DB	Partially Funded
France	Standalone	DB	Unfunded
Germany	Standalone	DB	Unfunded
Greece	Complementary	DB	Unfunded
Iceland	Complementary	DC	Funded
Italy	Complementary	DB	Unfunded
Japan	Complementary	DB	Unfunded
Mexico	Complementary	DB	Unfunded
Netherlands	Complementary	DB	Funded
Norway	Complementary	DB	Funded
Spain	Complementary	DB and DC	Funded and unfunded
Sweden	Complementary	DB and DC	Funded
Switzerland	Complementary	DB	Funded
United Kingdom	Complementary	DB	Funded and unfunded
United States	Complementary	DB and DC	Funded and unfunded

ernment has made a commitment to pay the pension benefit.

2.2 A comparison of funding ratios of funded plans

Although there is an international majority running unfunded public sector pensions, many countries are either undergoing pressures reforming their public sector pensions from pay-as-you-go to full funding or on the agenda of such reforms. A comparison of the funding position of funded DB public sector plans

Figure 2.3: **Pension expenditure for government workers and social security compared**(as % of GDP, 2006)



Source: Müller et al. (2009)

is still of great research significance. However, the international comparability of public sector pensions is jeopardized by their nontransparent reporting and divergent valuation methods. With adjustments by unifying valuation methods, the results demonstrate that public sector pension plans generally tend to be underfunded with a variety of funding ratios, except for unjustifiable high discount rate.

Depending on availability, representative public sector plans from six countries are investigated. Data are mostly obtained from official annual reports, financial statements and actuarial reports in the year 2008 or 2009. Table 2.2 gives the list of the examined public sector pension plans.

The Emergency Service Superannuation Scheme (ESS) is the representative of Australian public sector pension funds. Two typical Canadian public sector pension plans are investigated-the Ontario Teachers' Pension Plan (OTPP) and the Public Service Pension Plan. There are two separate Dutch public sector funds, ABP for government and public education sector and PfZW for public health care sector. As for Sweden, Swedish federal government plan (SPV) is chosen for com-

Table 2.2: **Public sector pension plans in comparison**

Country	Public sector plans	
Australia	Emergency Service Superannuation Scheme	
Canada	Ontario Teachers' Pension Plan	Public Service Pension Plan
Netherlands	ABP	PfZW
Sweden	Swedish federal government plan	
UK	West Yorkshire Local Government Plan	
US	All public sector plans	

parison. In the U.K., only local government plans are capitalized. However, there are 99 akin local government funds which are locally administrated defined benefit plans. The West Yorkshire Local Government plan is selected as the representative of the 99 British local government plans. Information regarding the U.S. public sector pension funds is aggregated together with funding indicators from Novy-Marx& Ruah (2009)[20].

2.2.1 Valuation methods in practice

Funding ratio, also known as funding level, is calculated by the ratio of pension asset value over pension liability value. The valuation of pension asset is marked to market by convention in both sectors while the valuation methods of public pension liabilities for public sector pensions differ substantially across countries.

Pension benefits are accrued according to pension formulas which are based on the years of employment, pre-specified annual accrual rate and wage income

throughout the career. Most public sector pension plans index their pension benefits to either wage growth or price inflation. Pension payments are to be fulfilled when the accrued benefits matures at the retirement age. The liability of a pension fund is accounted as the sum of a stream of discounted future pension payments to the present.

There is an academic debate on the appropriate methods for valuing pension liabilities (Exley et al. 1997[9], Bader and Gold 2003[1], Kortleve et al. 2006[11], Waring 2009[30]). The actuarial approach was widely adopted until the last two decades when the economic approach became increasingly favorable and substituted the actuarial approach in the private sector. Nowadays, private sector plans are obliged to report their pension liabilities in fair value terms by economic approach for the sake of transparency and comparability. However, most public sector plans still apply the actuarial approach to discount their future pension payments.

The actuarial approach discounts pension liabilities by an arbitrary fixed rate which is mostly determined by rules of thumb. The discount rate could be the expected return on pension asset as that is the case in the U.S..Historically, a fixed rate of 4% was used by Dutch pension fund for contribution setting and liabilities valuation. This fixed rate has been assumed to be the long term difference of asset return (7%) over the wage growth (3%).

On the other side, the economic approach recognizes the pension liabilities by mark-to-market principle. The goal of the economic approach is to promote a transparent view on the funding status and associated risk of a pension fund. The superiority of the economic method over the actuarial method is that it recognizes the risk of pension funding by market value and thus deters any unjustified benefit increase or contribution cut which are often induced by underestimated cost and risk.

However, the debate on which approach is appropriate for public sector pension plans is beyond the interest of this thesis. For later comparison of funding ratios, both approaches have been applied since they have their own cons and pros. Ad-

mittedly, the government's infinite horizon and supreme power to tax justify the actuarial approach which ignores fluctuations and solvency considerations in the short term. However, the discretion of the actuarial approach impedes the comparability across plans and an insight into the solvency status of a public sector pension which may indicate to what extent taxpayers are indebted. Therefore, funding ratios by the economic approach are at a better position to visualize the comparability of public sector pension financial positions.

The choices of discount factor and valuation method are informed in Table 2.3. Australian public sector plans employ discount rates around 8% which are based on the long-term expectation of pension asset return. The two Canadian public sector pension funds differ in discounting their liabilities. The OTPP recognizes its pension liabilities by a discount rate derived from the long-term Canadian government real-return bonds and a surcharge of 50 basis points which reflects the assumed credit risk of Province of Ontario. In contrast, the Canadian public service plan appoints an actuarial discount rate ranging from 6% to 6.7% to value future pension payout. Dutch public sector pension funds are required to report the fair value of their nominal liabilities which exclude any future indexation. Dutch pension supervisor appoints corresponding nominal swap rates as the discount rates for pension payments of different terms. The nominal swap rate, at the end of 2008, matching the duration of the liabilities of the two Dutch plans is 3.56%. The Swedish federal government plan chooses a prudent discount at 1.9% with the consideration of future indexation. The indexation rate for the year 2008-2009 is 2.8% which implies a nominal discount rate around 4.7%. An rough estimation, derived from the results of West Yorkshire plan, for all the 99 plans is also included in the table. The complexity of UK regularity on the choice of discount rate for local government plans is that they apply separate discount rates for financial reporting and contribution setting. As for funding disclosure, the nominal discount rate, which is often at the level of 6%, is the sum of nominal gilt yield and an assumed surcharge of outperformance on pension asset. However, the contribution is calculated according to two discount rates that treat liabilities before and after members' retirement differently. Most state and local government plans in the United States set the discount rates around 8% following the prescription by

Table 2.3: **Reported discount rates and associated valuation method**

Country	Plan	Asset bln euros end 2008	Liabilities bln euros end 2008	Discount rate	Discount method
Australia	ESS	8.5	13.8	%8.00	Actuarial method
Canada	OTPP	62.1	68.6	%4.00	Quasi fair value
	Public service	70.0	66.8	%6.00	Actuarial method
Netherlands	ABP	175.6	195.7	%3.57	Fair value
	PfZW	71.3	78.7	%3.55	Fair value
Sweden	SPV	20.0	19.0	%4.75	Quasi fair value
UK	West Yorkshire	4.4	5.5	%6.00	Actuarial method
	(Approx)all	317.8	542.1	%6.00	Actuarial method
US	All plans	1374.3	2110.8	%8.00	Actuarial method

Government Accounting Standards Boards (GASB) that discount rate should be based on the long term expectation of pension asset performance.

2.2.2 Revaluation methodology and results

Besides the divergent discount rates applied by each plan, diversity in demographic and economic assumptions such as mortality table and assumed wage growth, regulation and disclosure rules, and specific actuarial method all lead to incomparability of the examined public sector pension plans. Although diversity in these aspects are not detectable and adjustable, I believe that an international comparison by unifying the methods of valuation still carries embody fruitful information and insight on financial status of public sector pension plans.

The key formula for revaluations of pension liabilities is adopted from Biggs (2009)[2] and Novy-Marx& Ruah(2008)[19]:

$$L_{Variant} = L_{reproted}(\frac{1 + r_{reported}}{1 + r_{variant}})^{Duration}$$

Where $L_{Variant}$ stands for the officially reported liabilities value, $r_{reported}$ represents the discount rate reported by each plan, and $r_{variant}$ is the discount rate implied by alternative approach.

The term duration in the formula is the money-weighted average maturity of future cash flows. It also approximates the elasticity of pension liabilities with respect to interest rate. For instance, the duration of a pension is normally around 15 years in practice. A immediate parallel drop of interest rate by 1% implies a dramatic increase in the pension liabilities by roughly 15%. The actual duration of each plan cannot be observed since it depends on the maturity and structure of future pension payments. Here, every plan is assumed to have an identical duration of 15 years. The rationale is that durations of defined benefit plans rarely deviate from 15 years significantly. However, it should be admitted that a duration of 15 years is neither a reasonable approximation for a very mature nor a very young plan.
Funding positions by three methods of each plan are displayed in Table 2.4. The descriptions of the employed three methods are:
1. Reported ratio: Funding ratios officially reported by each scheme.
2. Actuarial approach: This method, following the U.S. practice, is based on an assumed discount rate of 8% which reflects the American's expectation of annualized long term pension asset return.
3. Fair value: This method, inspired by Dutch plans, uses a market discount rate to account for pension liabilities. Dutch pension industry refers discount rates to nominal swap rates since the market of government bonds is not deep enough for the industry. 30-year nominal swap rate, which roughly has the same duration of 15 years as a typical pension fund, is used as the market discount rate for nominal liabilities.

Table 2.5 describes the 30-year rates at the end of 2008 for relevant countries. For the methods applying expected long term return on pension asset as discount rate, public sector pension funds in Canada, Sweden, and the Netherlands enjoy funding ratios far above the full funding level of 100%. Even favored by this accounting method, public sector funds in the U.S., the U.K. and Australia are still

Table 2.4: **Funding positions for different method of valuation**

Country	Plan	Liabilities as % of GDP			Funding ratios		
		Reported	Fair value	ER	Reported	Fair value	ER
Australia	ESS	%1.9	%3.9	%1.9	%60.5	%30.4	%60.9
Canada	OTPP	%16.4	%	%18.1	%90.5	%82.4	%159.5
	Public service	%6.7	%9.8	%5.1	%104.9	%71.7	%138.8
Netherlands	ABP	%32.1	%32.1	%17.1	%89.8	%89.8	%166.2
	PfZW	%12.9	%12.8	%6.9	%90.7	%91.0	%170.5
Sweden	SPV	%5.6	%7.3	%3.5	%104.8	%80.7	%166.4
UK	West Yorkshire	%0.3	%0.4	%0.2	%79.6	%54.2	%105.4
	(Approx)all	%28.9	%42.4	%21.8	%58.6	%39.9	%77.6
US	All plans	%20.9	%44.1	%20.9	%65.1	%30.9	%65.1

Table 2.5: **Swap rates 2008**

Australia	Canada	Netherlands	Sweden	UK	US
%3.05	%3.35	%3.57	%2.91	%3.32	%2.77

struggling with extremely low funding ratios. Strikingly, the fair value method lowers the funding ratios substantially. All the public sector funds turn out to be severely underfunded by a mark-to-market view. The most pessimistic cases are told by the funding ratios in the U.S., the U.K. and Australia. Less pessimistic funding ratios in Canada, Sweden and the Netherlands range from 70% to 90%.

The results make clear that the public sector pension plans are generally underfunded by a comparable and transparent mark-to-market valuation and that all valuation methods generate a diversity of funding ratios by public sector pensions. However, one may suspect the validity of the first argument since these sad ratios are derived from values during or just after the financial crisis. It might be true that public sector pension funds, in the Netherlands, Sweden and Canada, are financially sound and healthy during peaceful periods. Nevertheless, public sector funds in the U.K., the U.S. and Australia are most likely to be underfunded given

extremely low funding ratios. To dispel this doubt, it could be necessary to look at the historical funding ratios by public sector funds. However, data for most funds is neither complete nor accessible.

Alternatively, it is helpful to compare the funding position of public sector funds with their counterparts in the private sector where pension funds are always required to be fully funded by supervision. Data from Certificated General Accountants Association of Canada shows that aggregated nominal funding ratio for Canadian private sector pensions was 77% on 31st December of 2008. The aggregated nominal funding ratio of Dutch pension funds on that date was about 98% on the website PensioenThermometer by Hewitt Inc. At the end of 2008, the largest Swedish occupational pension fund for its private sector, Alecta, reported a nominal funding ratio of 112%. Comparing to the ratios in Table 2.4, all these figures confirm that underfunding tends to be a rule in public sector pension funds with the exception of OTTP whose funding (82.4%) is slightly better than its private peers (77%).

Chapter 3

What drives the diversity?

3.1 Introduction

As last section arrives at spectrum of funding ratios by public sector funds, there could be a galaxy of factors determining such diverse funding ratios. Naturally, one may first attribute the diversity of funding ratios by public sector funds to the stochasticity of financial markets. However, this assumed link is weakened in the context that all these public sector pension funds are taking efficient positions in the international financial market. An experienced pension fund would take advantage of an internationally efficient portfolio to diversify its idiosyncratic risk away as much as possible. Therefore, the remaining diversifying power of funding ratio from financial markets could be negligible given that public sector pension funds are mainly exposed to the world wide systematic risk.

Since the observed diversity is not by randomness, there must be some fundamental drivers behind it. A latest study by Munnell, Aubry and Quinby (2010)[18] explores the influence on the funding diversity of American public sector plans from four factors: funding discipline, plan governance, plan characteristics, and the fiscal situation of the state. Differently, this section delivers more fundamental explanations, positively and normatively, for the observed diversity of funding ratios in public sector pensions. It firstly looks at the history of literatures on pension politics and applies the standard theory to the general political environment of public sector pensions. Then, the interest and arguments shift to the second research

question "Is there an optimal funding ratio".

3.2 Pension politics perspective

3.2.1 Literature review on pension politics

The development of pension politics origins from the scholarship about the impact of political institutions on public policies. Three typologies particularly have captured the essence of institutional framework for pension politics. In a number of pioneering scholarships, Aren Lijphart (1984[12], 1999[13]) categorized democracies as majoritarian and consensus democracies. The key disparity between the two named democratic systems is to the extent which executive power is concentrated or dispersed. A majoritarian democracy is marked by concentrated political power in the executive which is often correlated with a politically dominating party as the case in the United Kingdom. The consensus systems are at the other extreme where political power is shared by executive, legislative and judicial power. Countries with bicameralism, federalism, and multi-party governments, like Switzerland and Belgium often fall into this category. The core argument on this typology is that two types of democracies tend to issue policies with divergent characteristics. Consensus systems have a better performance in controlling inflation, protecting the environment and promoting more generous social security. Therefore, consensus democracy appears to be "'kinder"' and "'gentler"' than majoritarian democracies.

The basic division between concentrated and dispersed executive power is also intensively analyzed by G. Bingham Powell's (2000)[22] work on competing "'visions of democracy"'. The majoritarian vision leads to electoral rules and legislative procedures that resulting at clear majority and its overwhelming power to enact legislation. The goal of this vision is to produce decisive political action and clear accountability. However, the political decisiveness ness and accountability is not at the heart of the combating proportional vision. The proportional vision favors political inclusiveness which is reflected by proportional representation for the

electoral system and power-sharing arrangements. From this view, majoritarian governments are supposed to be more efficient in pushing reforms while proportional governments better stand for the interest of voters.

The third typology was introduced by Persson and Tabellini (2002[24]; see also Persson et al. 2000[23]) who described the institutional framework by two variables: the electoral system and the executive regime. The electoral system is divided into single-member district system and proportional representation system. The difference hinges on that rules whether politicians are required to maximize the number of voters or districts. The distinction of executive regime is between presidential and parliamentary regimes. This distinction is relevant to public spending and associated taxation policy in that the directly elected president is more accountable for tax increase than parliaments in the eyes of voters.
Though these typologies are starting points of analysis of the impact of political institutions on public policies with fruitful merits, they still suffer from several shortcomings when applied in field of pension politics. Many countries with political systems that predict social security expansions ended up with pension retrenchments. Moreover, the agreement that most Western European political systems are "'consensus democracies"' with a "'proportional vision"' cannot explain the dynamics of pension policy changes.

Veto points and veto players theory comes into being and generally fits well to the reality of pension politics. Both perspectives provide insightful understandings of pension reforms by focusing on how policy-making is obstructed by political institutions and political preferences. In viewpoint of veto points theory introduced by Immergut (1990)[10], legislative process is a chain of political decisions taken by a series of political conflicts and competitions. The political institution and electoral results indicate the likelihood that a pension policy will be vetoed during the policy-making. The likelihood of veto depends on the interest groups' political preferences and their accessibility to the decision-makers at different decision points. Therefore, the probability of veto and the potential influence of certain interest group are at the heart of veto point theory.

The paralleling veto players theory emphasizes how legislative process is influenced by the structure of political institutions. There are two types of "'Veto players"' defined by Tsebelis (1995[26]; 1999[27]; 2002[28]).Institutional veto players, similar to "'veto points"' in veto points theory, are constitutional participants, such as presidents and second chambers, whose agreement is required for decision-taking during the legislation process. "'partisan veto players"' are the partisan majorities deciding whether the institutional veto players are relevant in during policy-making. The theory generally puts forward that the possibility of legislation's passage is inversely related to the number of veto players and the maximal distance of their policy preferences.

However, the veto points and veto players theory does not tell the whole story of pension politics without the incorporation of the new elements which shaped pension politics exclusively. One important new ingredient is the "'blame avoidance"' thesis proposed by Weaver (Weaver 1986[29], Pierson and Weaver 1993[21]). The most radical argument of this thesis is that politicians tend to avoid blame by making cuts less transparent in response to the public exaggerated aversion to pension retrenchment. Another one is called the "'Varieties of Capitalism"' approach which links the development of financial markets to pension politics, especially in the context of Europe (Clark and Whiteside 2003[5]). Many other factors also explain the dynamics of pension policy exclusively: the role of labor union, demographic and budget pressure, and policy legacies, to name but a few.

3.2.2 The politics of public sector pensions behind the diversity

Although the academic history of pension politics put most emphasis on the policy reform of general pension system, the standard theories together with the new ingredients still can be applied to the field of public sector pensions which subject to similar political context.

One vital explanation relates the underfunding and the diversity of funding

ratios to the interaction between asymmetric risk sharing and related political configurations of veto points and veto players. The conception of asymmetric risk sharing between the public sector and the private sector explains the observed underfunding. The risk of a pension fund can be divided symmetrically into downside risk and upside risk. The downside risk refers to the possibility of pension deficit while the upside risk refers to the likelihood of pension surplus. A fair risk sharing contract claims that the risk of a pension fund should be taken symmetrically by stakeholders. More specifically, taking a downside risk should be rewarded by a corresponding upside risk. Therefore, asymmetric risk sharing means a stakeholder who takes a downside or upside risk is not rewarded or punished by an equal upside/downside risk. For public sector pension funds, the downside risk and upside risk are distributed between taxpayers and public sector employees as they are the ultimate stakeholders. A public pension shortfall, as the realization of downside risk, can be either bailed out by taxpayers' pocket or compensated by pension participants via benefits cut or contribution increase. Similarly, a public sector pension surplus, as the realization of upside risk, can be either spent on taxpayers' well-being or on the public sector employees by cutting contribution or increasing benefits. In most cases, the risk of public sector pension fund is asymmetrically shared by the two sectors in that most of the upside risk is granted to public sector employees while the corresponding downside risk is at the cost of taxpayers' money. As mentioned in the previous chapter, the government often plays the role of the largest employer of the economy. Moreover, workers of the public sector are much more unionized than their peers in the private sector. These two natures of the public sector contribute to the strong political influence of public sector employees on the decision of public sector pension risk sharing. For example, in the U.S., the payments of state and local pension benefits are protected at the constitution level. Whenever there is a pension deficit, the government would, at the cost of taxpayers, increase tax rate or cut other public expenditures to fulfill the payments of pension benefits to its employees. Under the optimistic circumstances with a pension surplus, the public sector workers would claim on the surplus for benefits increasing or contribution cutting. However, taxpayers are not rewarded for the downside risk bearing when they cannot get access to the pension surplus. Therefore, funding surplus gives pressure to increase benefits and associated funding cost

while funding deficit has little effect on the payments of pension benefits. It also explains the general experience that public sector pensions are expanding steadily during economic boom and relatively stable during recessions. In order to maintain the affordability and sustainability of public sector pension funds, a sensible government has the incentive to take a deliberate underfunding to reduce the risk of overfunding. Moreover, as argued by the "blame avoidance" theory, politicians tend to avoid blame by making underfunding less transparent in response to the aversion of public sector workers to underfunding. This point may well explain the divergent accounting methods and reporting of public sector pension fund.

However, the analyzed asymmetric risk sharing alone cannot explain the observed diversity without the incorporation of the veto points and veto players theory. The underfunding of public sector pension plans is partially attributed to the asymmetric risk sharing of public sector pension funds. Accordingly, the diversity of underfunding may reflect the divergent patterns of asymmetric risk sharing in different countries. The extent to which the funding risk of public sector pension funds is asymmetrically shared between taxpayers and public sector employees may be mainly determined by the political setup of veto points and veto players. Thus, the diversity of veto points and veto players setup may contribute to the diverse underfunding of public sector pension plans. However, the crux of this argument is that how the funding risk of public sector pension fund is shared between the two sectors should be decided in the political arena of legislation procedures. If it's not the case, the public sector workers would always shift the financial burden of pension deficit to taxpayers and skim the pension surplus without any blockage from taxpayers who cannot get the crucial access to the decision making. It is common for most countries that the changes of pension benefits both for the public sector and private sector are very politically sensitive and required to take legislative process. Thus, taxpayers often enjoy the political accessibility to the decision on how pension deficit and surplus are distributed between the two sectors. Basically, the veto points and veto players theory predicts that the possibility of legislation's passage is inversely related to the number of veto points and veto players and the maximal distance of their policy preferences. Given the conflicting interests of the two sectors regarding the risk sharing of public sector pension funds, the number

of veto points and veto players becomes the most important variable predicting the likelihood of legislative passage of benefits increase and cut. In a political system featured by many veto points and veto players, there are more channels for taxpayers to block any legislative claim from the public sector for pension benefits increase and pension bailout. Thus, there is less asymmetric risk sharing in that the public sector labor unions are confronted with more barriers when shifting pension deficit and skimming surplus. As for the government, it has less incentive to underfund the pension plans for the public sector because that any unjustified expansion and bailout of public sector pensions are subject to the scrutiny and the veto power of the private sector. In a political system with less veto points and veto players, public sector employees enjoy less interference from taxpayers in expanding their pension benefits and shifting the financial burden of underfunding to taxpayers due to limited accessibility and veto power of taxpayers. The risk sharing might be extremely asymmetric when the public sector labor unions can easily shift the burden of underfunding and skim the bonus of overfunding without significant blockage from taxpayers. In this case, the government has a strong incentive to underfund the public sector pension plans to minimize the likelihood of overfunding and thus to control the funding cost in the long run. Hence, the observed underfunding of public sector pension plans may vary substantially across diverse political configurations of veto points and veto players.

This argument is further strengthened by our findings on the financial positions of a number of OECD public sector pension funds in preceding chapter. High funding ratios of the Dutch and the Swedish public sector funds do not seem to be a coincidence with their political environment in terms of veto points and veto players. The privatization of the Dutch and Swedish public sector pensions reflects their multiple veto players politics. In these two countries, public sector pension funds are operated in a very similar way to private sector pension funds. Take the Dutch funds for illustration. As discussed at the end of Section 2, funding ratios of Dutch public sector funds (ABP 89.8% and PfZW 91.0%) are very close to the nationwide average (around 98%). This is because that Dutch public sector pension funds are subject to the same legislation and supervision as private sector funds. The Dutch electoral system stands out as the most proportional in

Western Europe. Even though there is no practical veto point in the Netherlands, such a competitive electoral system nourishes an extremely broad parliamentary representation with many partisan veto players. Not surprisingly, public sector pension funds are under strict scrutiny from all interest groups and treated almost the same as private sector funds. That is also why the financial reporting of the Dutch public sector pension funds is the most transparent among the investigated funds. In such a political system, the Dutch public sector labor unions suffer from substantial veto power from taxpayers when claiming for pension bailout and benefits increase. With much less asymmetric risk sharing, the Dutch government is less willing to underfund the public sector pension plans to keep them sustainable.

On the other side, the British pension politics exemplifies the fewer veto points and veto players cases. The British single-party governments are usually supported by stable parliamentary majorities. The British governments contain a single veto player, and are not impeded by institutional veto points in that the House of Lords cannot veto but delay legislation. Hence, the British political system can be categorized as the single veto player and no veto points type. In such political system, public sector labor unions can get access to the funding surplus and pass the burden of underfunding to taxpayers easily without significant blockage from taxpayers. Moreover, the accounting standard of British public sector pension funds is extremely complex and opaque as informed in the former chapter. With a politically strong public sector, the British government has to underfund the public sector pension plans substantially, by an opaque and complex accounting method, to deter the increasing unaffordability. Hence, the British single veto player and no veto points politics goes well with its observed severe underfunding of public sector pension plans.

Another promising explanation for the diverse underfunding comes from the pressure on the government to reform public sector pension plans. The generosity of public sector pensions has been climbing dramatically over private sector pensions since the economic boom in 1990s. During the last three decades, there was a structural transition from DC to DB plans in the private sector while most generous DB plans remain in the public sector. Much public debate is focused

on the generosity of public sector pension plans over private sector pension plans. The government has also gradually realized the unaffordability of public sector DB pensions and feels much pressure from the private sector to reform the public sector pension funds. Meanwhile, the government has also met with strong political resistance from the public sector labor unions against structural or even parametric reforms on the generous public sector pensions. However, public sector labor unions have to concede to reforms that rebuild the sustainability of public sector pensions in face of irreversible underfunding. Thus, the government may manipulate an irreversible underfunding deliberately to shift the reforming pressure to public sector labor unions. The level of underfunding may be determined by the difference of pension benefits between the public sector and the private sector. As for an extremely generous public sector pension plan, the government may undergo tremendous pressure to bring the public sector pension plan in line with private sector pension plans and manipulate a severe underfunding to transfer the pressure to the public sector workers. For those public sector pension plans with similar benefit level to private plans, the private sector puts less pressure on the government to reform which brings less incentive to underfund public sector pension plans.

This hypothesis can be tested by previous evidence for a diversity of funding ratios. In the Netherlands and Sweden, public sector pension funds are operated in the same legislative and supervisory framework of private sector funds. More importantly, the retirement benefits for the Dutch and the Swedish government workers are slightly more generous than that for their compatriots in the private sector. In Sweden, the scheme examined provides slightly less generous benefits than the scheme for most private sector white-collar workers. However, it is more generous than the schemes provided for blue-collar workers which account for the majority of private sector pension plans. The small difference of pension arrangement between the public sector and the private sector gives little pressure on the Dutch and the Swedish government to reform and associated underfunding. Hence, the convergence of public funds and private funds in these two countries suits well with their high funding ratios of public sector plans in the former report. On the other side, the U.S. , Australia, and the U.K. are typical countries that offer much

more generous DB pension arrangement for their public sector servants but stingy DC pension for their private sector workers. The divergence of pension benefits for public sector servants and private sector workers brings much pressure on the governments in these countries to reform. Therefore, the governments in the U.S., Australia, and the U.K. are most likely to pass the pressure to their public sector servants by severe underfunding as shown in Table 2.4. The two Canadian public sector pension funds offer moderately more generous benefits than private sector pension funds. The Canadian government experienced moderate pressure from the private sector to reform public sector funds and thus take moderate underfunding to transfer the pressure to Canadian public sector workers.

The difference in the horizons of politicians may also account for the diversity of underfunding by public sector pension funds. The horizons of politicians are usually much shorter than the maturity of pension benefits due to politicians shorter term of office. Typically, the pension benefits mature roughly in 30 years while an administration can hold its position only for several years. This horizon gap favors those politicians who exploit risk taking and shift the corresponding cost to their following administrations. By risk taking, speculative politicians can prefund public sector pension plans with a cost that is less than the financially fair amount. However, the corresponding cost of this risk taking will be passed to future administrations. Speculative politicians may hide the cost by very intransparent and complex accounting standard for "blame avoidance". In this case, pension deficits will not be recognized until the maturity of pension payments which may be far beyond the term of speculative politicians. Taking speculative politicians who seek votes from public sector during elections for instance. Speculative politicians, during elections, can ask public sector labor unions for votes by "promising" them a very generous pension expansion. Even though they have very limited financial resources for such a generous expansion, they can capitalize the public sector pension fund in a very risky portfolio and discount the pension liabilities without much recognition of funding risk. The public sector pension funds are financially sound by such accounting standard while they are severely underfunded in financially fair value terms. However, the pension shortfall will only be realized during future administrations. Moreover, the diversity of under-

funding may depend on the diversity of the horizons of politicians which determine the politicians' incentive for political speculations. Politicians with longer horizons are more concerned with the sustainability of public sector pension plans and less willing to speculate the public sector pension plans by underfunding.

All in all, there could be numerous explanations from the perspective of pension politics for the observed diverse level of underfunding. Some of these factors may be complementary, some may be combating, and some may be interwoven with each other.

3.3 Is there an optimal funding ratio?

The optimality of public sector pension funding is another focus of this thesis. Hereby, three optimal drivers are presented and the last two are modeled in the following chapter.

Considerations of optimal intergenerational risk sharing and related intergenerational transfers imply an optimal level of underfunding. Since shortfalls of public sector pension plans at maturity are more often financed by contemporary tax revenue (just as the PAYG mechanism), the deliberate underfunding resulting in shortfalls can be treated as an implicit PAYG component. A implicit PAYG component in the public sector pensions can be regarded as an another instrument, in addition to the public debt and first pillar PAYG, to implement non-overlapping generational risk sharing and transfers.

The remaining important factors that drive the funding optimally are the risk profiles and risk preferences involved in the public sector pensions. Generally speaking, these risks mainly include financial market risk, wage growth risk and demographic risk. Each country tends to have its specific risk-return tradeoff profiles of financial market risk and wage growth risk. In the modern portfolio theory, it might be optimal to manipulate a fraction of underfunding in a pension fund for potential diversification benefits (Dutta et al. 2000[7]). Level of optimal funding of

public sector pension plans also reflects the specific risk preference of each country.

Chapter 4

A Model in a risky world

4.1 Introduction

Very limited attentions have been put on the optimal funding of public sector pensions in the pension literature. D'Arcy et al (1999)[6] are the only few contributors to the modeling of optimal funding ratios by public sector pensions plans. However, their model is based on an unrealistic deterministic setup ignoring the risks involved and agents' risk preference.

In this chapter, taking into account of risk and taxpayers' risk preference ,a simplified two-period overlapping model is developed to analytically track the optimal funding ratio by funded public sector DB plans. The government can either collect an ex-ante funding tax or impose an ex post PAYG tax to meet the promised retirement benefits at the pension maturity. Basically, this model explores the optimal balancing between funding and PAYG financing which reflects the optimality of public sector pension funding and taxation policy.

To keep the model tractable, the model is situated in virtual two-sector economy where stochastic variables are all assumed to be normally distributed and pension maturity takes one period. Inter-temporal budget constraints for the model are decomposed into two components-the PAYG component and the funding component. The objective functions are optimizing the expected next period utility of taxpayers whose utility function is assumed to be constant absolute risk averse.

4.2 The model

4.2.1 Notations and definitions

G_t National total output at the beginning of period t

T_t Total tax paid for public sector pension benefits at the beginning of period t

F_t Funding contribution of the public sector pension set aside at the beginning of period t

B_t Public sector pension benefit matured at the beginning of period t

$PV[B_{t+1}]$ Present fair value of public sector pension benefits matured at the beginning of period t+1

I_t^{pri} Private sector net income at the beginning of period t

I_t^{pub} Public sector total income at the beginning of period t

W_t^{pub} Public sector net wage income at the beginning of period t

γ Arrow-Pratt absolute risk aversion coefficient of the private sector taxpayers

g_t National wage growth rate as well as the indexation rate during period t with a volatility of σ_g and mean of μ_g

r_t Financial market return during period t with a volatility of σ_r and mean of μ_r

D_t Deflator for the valuation of pension liabilities at period t and its expectation is the risk free rate $E[D_t] = r_f$

R_t Funding residue of the public sector pension at the beginning of period t

$\rho_{r,g}$ Correlation between the financial market return r_t and the wage growth rate g_tt

FR_t Funding ratio at the beginning of period t as the ratio of funding contribution C_t over present value of pension liabilities $PV[B_{t+1}]$

4.2.2 The simple two-sector economy and inter-temporal budget constraints

There are two sectors in an economy where a private sector is paying for the total income of a public sector and no tax other than the public sector pension tax is imposed. The income profiles of the two sector workers are shown by following equations:

Private sector total net income at the beginning of period t,

$$I_t^{pri} = G_t - T_t - W_t^{pub}$$

Public sector total net income as at the beginning of period t,,

$$I_t^{pub} = PV[B_{t+1}] + W_t^{pub}$$

There is a strong assumption behind the simple economy that public sector net wage income ,W_t^{pub}, is invulnerable to the financial health of its pension. In other words, the funding position of public sector pension fund does not affect the net wage income of public sector workers which follows the dynamics:

$$W_{t+1}^{pub} = W_t^{pub}(1 + g_t)$$

The risk sharing pattern of this economy turns out to be that taxpayers are taking all the funding risk symmetrically while public sector employees are at no risk of their deferred wage. This risk sharing principle generally holds in reality. Public sector pensions in countries like Germany, Australian and Sweden are exemplary noncontributory plans whose funding status does not have any impact on the net wages of their participants. Even for other contributory plans, employee contribution rates are all at very low levels (less than 10%) and insensitive to their financial situations. Therefore, this analysis is kept away from risk sharing between the two sectors since public sector workers are always well protected by the their overwhelming political power.

The government can manipulate a PAYG component implicitly in the public sector pension fund by a high discount rate underestimating the funding cost. In the real world, the funding contribution paid by the employer, which is also actually invested in the financial market , is often less than the financially fair cost. $F_t < PV[B_{t+1}]$. Thus, a funding deficit, in terms of financial expectation, prevails in the balance sheets of public sector pensions. The following equation shows a funding position with a pension shortfall.

$$R_{t+1} = F_t(1 + r_t) - B_{t+1} < 0$$

Part of the funding cost, as the funding deficit denoted by $B_{t+1} - F_t(1 + r_t)$, is to be immediately met by a PAYG tax in the next period. The merits of this deliberate underfunding may be that PAYG financing can provide a secure asset and associated diversification benefits for the stakeholders in the private sector. The pension budgets are inter-temporally constraint by the following equations:

- $T_1 = B_1 + F_1$
- $T_2 = B_2 - F_1(1 + r_1) + F_2$
- ...
- $T_t = B_t - F_{t-1}(1 + r_{t-1}) + F_t$
- ...
- $T_T = B_T - F_{T-1}(1 + r_{T-1})$

For the first period, B_1 can be regarded as the windfall gain of the first generation of public sector retirees when the pension is introduced to the economy. Total public sector pension tax T_1 is paying for both the windfall gain and funding contribution, F_1, of the next period benefit , B_2.

For the last period, the pension fund has to be closed such that total pension tax, T_T, is only paying for the funding deficit that occurred at the last period, $B_T - F_{T-1}(1 + r_{T-1})$. No other funding contribution, say F_{T+1}, is set since the plan is closed virtually.

Given the focus on developed public sector pensions, the analytical power of this model is located in the budget constraints for every period other than the first and the last. Moreover, this budgeting system is stable and satisfies the no-ponzi-game condition which is demonstrated by the equation below:

$$\sum_{t=1}^{T} PV[T_t] = \sum_{t=1}^{T} PV[B_t]$$

The related inter-temporal budget constraints is generalized by the equation below:

$$T_t = B_t - F_{t-1}(1 + r_{t-1}) + F_t \tag{4.1}$$

Total pension tax T_t can be divided into two parts-funding component and PAYG component. In priority, funding deficit at the beginning of period t , $B_t - F_{t-1}(1 + r_{t-1})$, has to be firstly compensated by an instant taxation which can be interpreted as the PAYG component. The working generations at period t are paying for part of the retirement benefits of public sector retirees living at the same period. If the plan is purely financed on PAYG basis, the budget constraint is driven by PAYG and changed to $T_t = B_t$. The left pat of the total tax revenue, F_t, is invested in the financial market for the next period benefit, B_{t+1}. F_t is the funding contribution paid by the working generations at period t and prefunding part of the retirement benefits for the public sector retirees at next period. If the plan is fully funded, the budget constraint is driven by financial accumulation and changed to $T_t = F_t = PV[B_{t+1}]$.

A further assumption is that the pension benefit is indexed to the expected wage growth rate and predetermined by the first period level B_1:

$$B_{t+1} = B_t(1 + \mu_g) = B_1(1 + \mu_g)^t \tag{4.2}$$

The design of benefit dynamics is mainly driven by the unhedgeable nature of wage growth risk in the financial market. Moreover, the diversification benefit of indexation risk, hedging capability of PAYG financing against indexation, and pension liability pricing are beyond the focus and analytical power of this model. Hence, a prudent funding contribution can only be kept abreast with the expected wage growth rate μ_g. The dynamics of the funding contribution takes the following form:

$$F_{t+1} = F_t[1 + E(g_{t+1})] = F_t(1 + \mu_g) \tag{4.3}$$

Then, the discounting of pension benefits is implied by the equation below:

$$PV[B_{t+1}] = E[B_{t+1}D_t] = B_{t+1}E[D_t] = \frac{B_{t+1}}{1 + r_f}$$

Combing equation 1 and 3, the inter-temporal budget constraint for the public sector pension fund is rearranged as:

$$T_{t+1} = B_{t+1} - F_t(1 + r_t) + F_{t+1} = B_{t+1} + (\mu_g - r_t)F_t \tag{4.4}$$

4.2.3 Optimization problem with mean variance utility

The objective function is straightforwardly optimizing the expected utility from national disposable income of the next period while bearing the inter-temporal budget constraints such that the public sector pension benefits are well kept at a predetermined level by the government. For mathematical tractability, the applied specific utility function is the standard exponential utility function with constant absolute risk aversion (ARRA) and shown as below:

$$U(C) = -e^{-\gamma C} \tag{4.5}$$

In this function, γ is the Arrow-Pratt absolute risk aversion coefficient and C is instant consumption level at period t. By some derivations and with the assumption that stochastic variables are normally distributed, the expected exponential utility function takes the mean-variance pattern as below: (Appendix A gives a detail on the derivation)

$$E[U(C_{t+1})] = \mu - \frac{\gamma\sigma^2}{2} \tag{4.6}$$

By this function, μ is the mean consumption level and σ is the standard deviation of consumption. Essentially, this mean-variance utility depends on the preference over the trade-off between expected consumption level and associated risk. Specifically, the expected utility of the agent is increasing with the expectation of his consumption and decreasing with the variance. The rate of the decrease with respect to the variance is larger by a more risk averse agent.

In the analysis, the optimized mean-variance utility is based on the taxpayers' disposable income at the next period. The utility level of public sector workers is fixed by their predetermined pension benefits and their invulnerability to the funding status of public sector pension fund.

Taxpayers' disposable income at the next period

$$I_{t+1}^{pri} = G_{t+1} - T_{t+1} - W_{t+1}^{pub} = G_{t+1} - W_{t+1}^{pub} - B_{t+1} + (r_t - \mu_g)F_t$$

Predetermined benefit level of public sector workers as a function of time

$$B_{t+1} = B_t(1 + \mu_g) = B_1(1 + \mu_g)^t$$

where B_1 is the predetermined level of public sector pension benefits at the first period. According to the indexation rule, the predetermined level of pension benefits at period t+1 is $B_1(1 + \mu_g)^t$.

The government, as the agent standing on taxpayers' feet, has to choose an optimal taxation policy to maximize taxpayers' expected utility of the next period at every period t (except the last two periods) repeatedly. Meanwhile, the government has also to take care of its employees by its commitment of the predetermined level of pension benefits. Therefore, The most desirable taxation policy achieves the optimal allocation of the predetermined public sector pension benefits between the funding component and the PAYG component. The optimization is generalized as the followings:

$$\begin{aligned}
\max_{F_t} E[U(I_{t+1}^{pri})] \\
s.t\ I_{t+1}^{pri} = G_{t+1} - W_{t+1}^{pub} - T_{t+1} \\
G_{t+1} = G_t(1+g_t) \\
W_{t+1}^{pub} = W_t^{pub}(1+g_t) \\
T_{t+1} = B_{t+1} - F_t(1+r_t) + F_{t+1} \\
B_{t+1} = B_t(1+\mu_g) = B_1(1+\mu_g)^t \\
F_{t+1} = F_t(1+\mu_g)
\end{aligned}$$

Combining the budget constraints, The mean-variance version is summarized as :

$$\max_{F_t} \mu_{I_{t+1}^{pri}} - \frac{\gamma \sigma^2_{I_{t+1}^{pri}}}{2} \tag{4.7}$$

$$s.t\ I_{t+1}^{pri} = G_{t+1} - W_{t+1}^{pub} - B_{t+1} + (r_t - \mu_g)F_t \tag{4.8}$$

$$B_{t+1} = B_1(1+\mu_g)^t \tag{4.9}$$

$$\tag{4.10}$$

where $\mu_{I_{t+1}^{pri}}$ is the expected national disposable income and $\sigma_{I_{t+1}^{pri}}$ is its standard deviation.

4.3 Optimization result and implications

Although the choice between funding and PAYG financing is to be optimized, the optimal funding ratio, as the target of this chapter, has not been explicitly involved in the optimization problem. Hereby, proposition 1 is introduced for the incorporation of the funding ratio:

Proposition 1
The solution of the following optimization in measurement of 1 unit

$$\max_{C} E[U(C)]$$

is the same as the solution of the optimization measured by $1/k$ unit

$$\max_{k*C} E[U(k*C)]$$

where k is a deterministic scaler of the measurement and the absolute risk aversion coefficient is scaled by $\frac{1}{k}$ such that

$$E[U(k*C)] = \mu_{k*C} - \frac{\frac{\gamma}{k}\sigma_{k*C}^2}{2}$$

Proof: see Appendix B
Implied by Proposition 1, the initial optimization shares the monotone property of the following transformed optimization problem:

$$\max_{FR_t} \mu - \frac{PV[B_{t+1}]\gamma\sigma^2}{2}$$

subject to

$$\frac{I_{t+1}^{pri}}{PV[B_{t+1}]} = \frac{G_{t+1} - W_{t+1}^{pub}}{PV[B_{t+1}]} + FR_t(r_t - \mu_g) - (1 + r_f)$$
$$B_{t+1} = B_1(1 + \mu_g)^t$$

where μ is the mean of $\frac{I_{t+1}^{pri}}{PV[B_{t+1}]}$ and σ^2 is the variance of $\frac{I_{t+1}^{pri}}{PV[B_{t+1}]}$ and $PV[B_{t+1}]\gamma$ is the new scaled absolute risk aversion coefficient.

The intuition behind this monotone transformation is that the solution of this optimization should not be changed by different measurements. Take an illustration, the optimization of national disposable income in trillion euro term should not be different from that in billion euro term with certain appropriate adjustment of the absolute risk aversion coefficient. The adjustment of risk preference parameter is due to the increasing relative risk aversion of the underlying exponential utility function. Without the adjustment, the agent would be ridiculously more risk averse in a gamble of 1 trillion euros than the same gamble of 1000 billion euros.

In the transformed optimization, every variable is measured by its relative value with respect to the present value of pension benefits. Hence, the targeted funding ratio at the beginning of period t, $\frac{F_t}{PV[B_{t+1}]}$, becomes new the choice variable. Instead of I_{t+1}^{pri}, the relative magnitude of national disposable income with respect to the present value of pension liabilities, $\frac{I_{t+1}^{pri}}{PV[B_{t+1}]}$, is the new measurement of the national disposable income. The "transformed" intuition is: the government, in every period except $T-1$ and T, has to repeatedly make the same decision on the funding ratio FR_t to give the most favorable next period aggregate disposable income in terms of level and risk and guarantee the predetermined level of pension benefits for public sector workers. Finally, the optimization result is calculated as follows: (see Appendix C for derivation)

$$FR_t = \frac{\mu_r - \mu_g - \gamma(G_t - W_t^{pub})\sigma_g\sigma_r\rho_{g,r}}{PV[B_{t+1}]\gamma\sigma_r^2} \tag{4.11}$$

4.3.1 Comparative analysis and implications

A further rearrangement of the expression may present the result in a much more understandable way:

$$FR_t = \frac{\mu_r - \mu_g}{\gamma'\sigma_r^2} - \frac{\frac{G_t - W_t^{pub}}{PV[B_{t+1}]}COV(g,r)}{\sigma_r^2}$$

where $COV(g,r) = \sigma_g\sigma_r\rho_{g,r}$ and $\gamma' = PV[B_{t+1}]\gamma$

Thus, the scaler effect on the absolute risk aversion coefficient is subtracted from the analysis of the result. As we can see from the rearranged equation, the optimal funding ratio is driven by a number of factors some of whose effects are interactive. Basically, the optimal funding ratio is determined by two components-a "speculative component" and a "hedging component".

The "speculative component", as denoted by $\frac{\mu_r - \mu_g}{\gamma'\sigma_r^2}$, reflects the attractiveness of capital investment over PAYG financing to the taxpayers without taking into account of their hedging possibilities. The expected return of PAYG financing is a crucial parameter in the "speculative component". A higher expected return of PAYG financing will challenge the attractiveness of investment in financial market since the PAYG financing provides a favorable substitution for financial investment to generate a higher expected disposable income of taxpayers at the next period. If the expected return of PAYG financing is higher than that of financial investment, pure PAYG financing may be the most favorable for taxpayers. A higher expected return of financial asset over PAYG financing favors capital investment since it contributes to a higher expected disposable income of taxpayers at the next period. However, a higher volatility of financial asset investment may jeopardize its speculative attractiveness substantially. The reason is very simple that a more volatile financial market can result at a more fluctuating disposable

income of taxpayers who are both interested in the expectation and the variance of their disposable income. The risk preference in the "speculative component" reflects taxpayers' willingness to take risk in the financial market to get a higher level of expected disposable income while bearing a higher variance as a penalty of risk taking.

The "hedging component", as denoted by $-\frac{\frac{G_t-W_t^{pub}}{PV[B_{t+1}]}COV(g,r)}{\sigma_r^2}$, reflects the demand of taxpayers to stabilize their aggregate disposable income by hedging the two sources of risk interactively. The critical parameter in this "hedging component" is the correlation between the financial market risk and the wage growth risk which decides the direction of their co-movement. If the correlation is positive, taxpayers should take less position in the financial market in that the interactive effect of wage growth volatility and financial market volatility increases the variance of their disposable income. On the other side, the interactive effect of wage growth volatility and financial market volatility will reduce the variance of taxpayers' disposable income if the correlation is negative. Hence, a negative correlation between the wage growth risk and financial risk provides extra opportunities for investment in financial market while positive correlation gives penalty on extra risk taking in the financial market. The ratio $\frac{COV(g,r)}{\sigma_r^2}$, which takes the same form as the "β" in the Capital Asset Pricing Model, measures the co-movement of the PAYG financing risk and the financial market risk. A higher co-movement ratio indicates a greater hedging opportunity to stabilize the disposable income of taxpayers. Another important ratio $\frac{G_t-W_t^{pub}}{PV[B_{t+1}]}$ reflects the leverage effect of the tax base on hedging taxpayers' disposable income at the next period. The tax base, as denoted by $G_t - W_t^{pub}$, is often much larger than the size of public sector pension plan which is denoted by $PV[B_{t+1}]$. A larger tax base, also as more wage income of taxpayers, can provide greater hedging opportunity to stabilize taxpayer's disposable income at the next period. Because taxpayers have already taken a tremendous position in the wage growth risk which is the background risk inherently in their wage income. Given such an inherent exposure to wage growth risk, taxpayers enjoy a significant opportunity to hedge their background risk by taking an appropriate position in the financial market. Thus, the ratio $\frac{G_t-W_t^{pub}}{PV[B_{t+1}]}$ play as the role of leverage in hedging taxpayers' disposable income.

The net effect of each parameter on the optimal funding ratio are also interesting. It is not surprising that a higher expected return of financial market favors a higher funding ratio and that a higher wage growth rate favors a lower funding ratio. The net effects of the two parameters are reflected in the "speculative component". The financial market will be more attractive for pension investment with higher expected return and thus higher funding ratio is preferable. Similarly, PAYG financing, with a higher expected return, is more attractive and favor a lower funding ratio. The net effect of taxpayers' risk preference is also driven by the "speculative component". However, it is contingent on the relation between expected financial market return and wage growth rate. As indicated by the ratio $\frac{\mu_r - \mu_g}{\gamma' \sigma_r^2}$, the net effect of risk preference is negative when the financial return is larger than the wage growth in expectation and vice versa. Given the fact that the expected rate of return in financial market is higher than the expected wage growth rate in most countries, more risk averse taxpayers would prefer a lower funding ratio. This is because more risk averse taxpayers are more sensitive to the variance of their disposable income and thus they are less willing to take risk in the financial market. The net effect of the correlation between wage growth and financial market return is driven by the "hedging component" and depends on its sign. If the correlation is positive, a higher correlation will lower the optimal funding ratio and vice versa. Because taking more risk in the financial market will be penalized by higher variance of taxpayers' disposable income when the wage and financial market are often moving in the same direction. Since the correlation for 30 years is positive in most countries, it can be generally concluded that a higher correlation between wage growth and financial market makes a lower funding ratio. The net effect of wage growth volatility is also driven by the the "hedging component". A higher volatility of taxpayers' wage growth gives taxpayers strong demand to hedge their background risk for a stable disposable income. Thus, when the correlation is positive, taxpayers prefer lower funding ratios because that taking more positively correlated risk in the financial market will further amplify the variance of their disposable income. However, the net effect of financial volatility, which hinges on the confliction between the "speculative component" and the "hedging component" when the correlation is positive, is ambiguous. On one hand, a more

volatile financial market will be less attractive for pension investment and thus favor a lower funding ratio. On the other hand, the magnitude of the co-movement with wage growth will be decreased by a more volatile financial market. Then, the hedging capability of the two sources of risk will be smaller which in turn favors a higher funding ratio. The net effects of taxpayers' wage income denoted by $G_t - W_t^{pub}$ and the size of public sector pension plans indicated by $PV[B_{t+1}]$ are reflected in the hedging leverage ratio. As discussed before, a higher wage income of taxpayers implies a higher background risk taken and stronger hedging demand of taxpayers. Taxpayers prefer less funding when the correlation is positive because that more funding in interaction with their background risk will amplify the variance of their disposable income. Similarly, a larger public sector pension fund will shrink the effects of background risk leverage. Because public sector pension liabilities of taxpayers can be seen as their negative position in wage growth risk. A larger public sector pension liabilities contributes to less background risk taking and less incentive to hedge taxpayers' disposable wage income. There, taxpayers will favor higher funding ratio with a larger public sector pension size when the correlation is positive.

4.3.2 A numerical illustration

This part provides a numerical illustration for a more complete analysis of the model and the funding of public sector pension funds. Since the pension maturity is accounted for one period in the model, the data is recalculated on 30 years basis. Based on the model, the optimal funding ratios of public sector pension funds for the U.S. and the U.K. are calculated. Besides the availability of the data, the two countries both have well-developed financial markets that are deep enough for their public sector pension funds. Therefore, the data on their financial markets more truly reflects the financial risk their public sector pension funds are taking. However, as public sector pension funds in other economies mostly have global portfolios, the valuation for these countries may be extremely mismatched unless the data on their specific portfolio returns is available. The long term data on equity markets and GDP growth is from Matsen and Thogersen (2001)[15]

It is unrealistically assumed that public sector pension funds are fully investing in the equity markets whose returns represent the financial market return in the model. The previous estimated liabilities in Chapter 2 seems to be a good approximation for the present value of pension benefits in the model due to that a 15-year duration corresponds to a maturity roughly in 30 years. The data on the wage bill of public sector workers is either rare or incomplete. However, it is assumed that the replacement rate as the ratio of pension benefits over previous pensionable wage income is 75%. The public sector wage income can be approximated by the formula:

$$W_t^{pub} = \frac{B_{t+1}}{75\%} = \frac{L_{reproted}(1 + r_{reported})^{duration}}{75\%}$$

where $L_{reproted}$ is the reported public sector pension liabilities and $r_{reported}$ is the corresponding reported discount rate.

In combination of previous data on pension liabilities, the hedging demand leverage can be calculated. Moreover, the real risk free rate is assumed to be 2% for both countries which is in line with the real returns on short-term money market instruments reported by Campell (2001)[3]. Finally, the absolute risk aversion coefficients for the calculation of both countries are selected over the spectrum from 3 to 5.5. Table 4.1 reports the historical statistics of key variables , the assumptions for r_f and γ', and the numerical results.

In the results, optimal funding ratios are more often between 0% and 100% which implies that the "speculative component" is more often overwhelming. In addition, the optimal funding ratio shows very sensitive to the risk aversion parameters. This may implies that the optimal funding ratios may also vary significantly across countries.

Table 4.1: **Values of the key parameters and numerical results**

Historical statistics of key variables				Optimal funding ratios	
Country and period	US 1891-1998	UK 1919-1998	γ'	US	UK
μ_r	%6.93	%7.41	3	%78.82	%126.83
μ_g	%2.96	%1.88	3.5	%55.75	%103.92
σ_r	%18.67	%21.69	4	%38.45	%86.74
σ_g	%5.77	%3.50	4.5	%25.00	%73.38
$\rho_{r,g}$	0.112	0.087	5	%14.23	%62.69
r_f	%2.00	%2.00	5.5	%5.43	%53.94

Chapter 5

Conclusion

Public sector pension plans brings a major political and fiscal challenge to the economy. Firstly, the government, as the largest employer in the economy, faces large commitments to provide retirement benefits for its employees. Secondly, pension arrangements provided for public sector workers are usually much more generous than that for private sector workers. Thirdly, public sector pension plans are mostly financed on PAYG basis. Even among those funded plans, full funding position is rare or absent. Therefore, PAYG and underfunded pension plans for public sector workers may take a huge amount of implicit pension debt that accounts for a substantial part of the debt position of the economy. Fourthly, the financial disclosure of public sector pension is far from transparent. The nontransparency of public sector pension reporting may hide huge pension deficits and manipulations from political incentives.

This thesis delivers an extensive examination of public sector pension schemes over a number of OECD countries which provides solid evidence for these highlighted reasons. Most OECD countries have special pension arrangements for public sector employees. On average, 15% of the total labor force in OECD countries is employed by the public sector. In general, public sector pension plans in are more generous than private plans due to the fact that generous DB plans are still staying in the public sector of most countries. Moreover, the reporting of public sector pension schemes in OECD countries is rather divergent and nontransparent. It is mainly because that public sector pension plans in most countries are subject

to special supervisory regulations that are different from private sector schemes. It is also found that a majority of public sector pension schemes in OECD countries are running on PAYG basis.

This thesis also explores the recent funding positions of public sector pension plans by a comparison of funding ratios of selected funds in several countries. However, a fair comparison of funding status of public sector pension funds over countries is hindered by their distinct valuation principles. In order to achieve such a fair comparison, adjustment and recalculation based on economic approach have been made to each investigated scheme. We find that all the examined public sector pension funds are underfunded by the fair value principle. The funding ratios of the public sector pension funds in the U.S., Australia, and the U.K. are the lowest while Dutch funding ratios are the highest. Funding ratios of Swedish and Canadian public sector pension funds show up in in-between positions. These findings demonstrate that public sector pension funds, in general, are underfunded with a diversity of funding ratios.

This finding delivers meaningful messages and implications for the funding issues of public sector pension funds from the perspective of pension politics. The diversity of underfunding ratios can be partially explained by the diversity of asymmetric risk sharing patterns and associated veto points and veto players setup. The asymmetric risk sharing reflects that fact that public sector employees can always claim on their pension funding residue while the pension deficits are always bailed out by taxpayers. The political configuration of veto points and veto players mainly decides public sector employees' accessibility to the pension residues and taxpayers' vulnerability to the funding deficits. Our analysis concludes that a public sector pension fund in a political environment featured by multiple veto points and veto players is likely to enjoy high funding ratio. This conclusion is further confirmed by the funding ratios and current political situations in the Netherlands, Sweden and the U.K.. This diversity of underfunding can also be explained by the different pressures on governments to reform the public sector pension funds. Given the unfair pension arrangements between the public sector and private sector, the government faces huge pressures to keep the public sector pension in line with

the private sector pension. However, the government may also be confronted with strong resistances from the public sector union which usually play a significant role in politics. Under this circumstance, the government may take deliberate underfunding to shift the reforming pressures financially to the public sector employees. Therefore, the diversity of underfunding may also be driven by the unfairness of pension arrangements between the public sector and private sector. In general, this hypothesis is strengthened by the funding ratios of all investigated countries. Another important explanation refers the diversity of underfunding to the difference of the horizon gaps between pension maturity and politicians. A political term is much shorter than the maturity of pension benefits. Also, the horizon of a politician differentiates by different personalities and different phase of its political life. Thus, politicians with different horizons have different incentives for risk taking of public sector pension funds which, in turn, determine the level of underfunding.

Besides a positive perspective from pension politics, a model based analysis of the optimality of funding ratios also yields meaningful insights into the funding issues of public sector pension funds. We have constructed a two-period overlapping generational model in a risky world where random variables are normally distributed. We put our analysis on a virtual DB public sector pension fund whose benefits are indexed to expected wage growth. The public sector employees are assumed to be immune to the financial health of their pension fund while private taxpayers are at the risk symmetrically. The government has the discretion to choose the financing ratios of PAYG and funding to achieve the most favorable taxation policy that optimizes the expected aggregate utility of the taxpayers from their total disposable income. The result from our model shows that the optimal funding ratio of public sector pension fund consists of two components-a "speculative component" and a "hedging component". For a typical dynamically inefficient economy where correlation between capital return and wage growth is positive, expected capital return always has a positive effect on the optimal funding ratio. However, risk preference, expected wage growth rate and its volatility, and the correlation also show negative effects on the optimal funding ratio. Importantly, the "hedging component" is leveraged by a ratio that represents the relative size of tax base to the public sector pension funds. The net effect of financial volatility

depends on the relation between the "speculative component" and the "hedging component". Financial volatility generates negative effects when the "speculative component" dominates. We have also made numerically illustrations, based on the U.S. and the U.K. data, to explore the model more completely. For plausible values and parameters, results of both countries often lie between 100% and 0%. The numerical results also infer that the "speculative component" weights over the "hedging component" more often and optimal funding ratios are very sensitive to the risk aversion coefficient. Thus, even the optimal funding ratios of public sector pension funds may vary significantly across countries.

To sum up, this investigation of the financial positions of public sector pension fund provides fruitful information for the investors holding government bonds and rating organizations. As we estimated, the implicit pension debt for public sector workers can make up a substantial part of the national total debt position. Further insights into the observed diverse underfunding of public sector pension funds give profound implications for policy making, both politically and economically. Due to politically strong labor unions and demographic pressures, public sector pension benefits have been climbing steadily. However, the economic cycle is rather volatile compared to the steadily increasing benefits of public sector pension. Therefore, public sector pension funding has significant influence on the intergenerational transfers and inter-temporal social welfare. Politically, the government is trapped into a dilemma between reforming pressures from the private sector and resistances and expansion pressures from the public sector. Economically, the government has to decide the most desirable funding of public sector pension for taxpayers while guaranteeing the security, integrity, independence and attractiveness of its employment. However, political justice is not always the economic justice. There are discrepancies between the optimalities of economics and politics in the field of public sector pension funding. Driven by uncountable interactive factors, the reality of public sector pension funding is far from the optimalities of both economics and politics.

Admittedly, the analysis of thesis suffers from many limitations that call for further researches. The lack of data on historical funding ratios of public sec-

tor pension funds may still call the conclusions of the comparison into question since the funding ratios compared are based on values during the financial crisis. If historical data on public sector pension funds provided, it may be necessary to conduct empirical researches to identify the analyzed effects of pension politics and other possible factors on the funding ratios of public sector pension funds. Also, the simple model introduced in the later chapter is constraint by many technical limitations. For mathematical tractability, the model has to be based on many strong assumptions and is kept away from many interesting issues. The applied utility function is based on constant absolute risk aversion. Random variables are all assumed to be normally distributed with time invariant parameters and unpredictable. The model only concerns two sources of macroeconomic risk-wage growth risk and capital market risk-while demographic risk is left out of the model. The optimal risk sharing pattern between the public sector and the private sector is also untouched in the analysis. Intergenerational effects of public sector pension risk sharing and inter-temporal social welfare are absent in the analysis. Therefore, further researches may be interested in extending the model with more parsimonious utility function and dynamics of random variable to incorporate demographic risk and cover the issues of inter-sector and intergenerational welfare. Simulations for the extended models are also at the interest.

Appendix A

Derivation of mean-variance utility function

The original utility function is exponential as follows:

$$U(C) = -e^{-\gamma C}$$

where C is the agent's consumption level and γ is the Arrow-Pratt absolute risk aversion coefficient of the agent. The consumption level, C, is assumed to be normally distributed with mean ,μ , and standard deviation, σ. The density of C is given by the function,

$$f(C) = \frac{e^{-\frac{(C-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

Then, the expected exponential utility is given by,

$$\begin{aligned} E[U(C)] &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} -e^{-\gamma C} e^{-\frac{(C-\mu)^2}{2\sigma^2}} dC \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} -e^{-(\gamma C + \frac{(C-\mu)^2}{2\sigma^2})} dC \end{aligned}$$

Because that,

$$\gamma C + \frac{(C-\mu)^2}{2\sigma^2} = \frac{(C-\mu+\gamma\sigma^2)^2}{2\sigma^2} + \gamma(\mu - \frac{\gamma\sigma^2}{2})$$

Substituting in E[U(C)] gives,

$$E[U(C)] = \frac{e^{-\gamma(\mu-\frac{\gamma\sigma^2}{2})}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} -e^{-\frac{(C-\mu+\gamma\sigma^2)^2}{2\sigma^2}} dC$$

Moreover, it holds for all μ' that

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(C-\mu')^2}{2\sigma^2}} dC = 1$$

Therefore, this relationship also holds for $\mu' = \mu - \gamma\sigma^2$ such that,

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(C-\mu+\gamma\sigma^2)^2}{2\sigma^2}} dC = 1$$

Substituting in E[U(C)], it follows that

$$E[U(C)] = e^{-\gamma(\mu-\frac{\gamma\sigma^2}{2})}$$

Hence, the objective of the agent is to maximize the expression

$$\mu - \frac{\gamma\sigma^2}{2}$$

Therefore, the agent is interested in maximizing the so-called mean-variance utility which is his mean consumption level minus the variance multiplied by half of his risk preference.

Appendix B

Proof of Proposition 1

The original optimization is

$$\max_{C} E[U(C)]$$

Suppose the unit in this optimization is measured by k euro. Then, let's introduce an identical optimization but the unit is measured by 1 euro. Therefore, every value in the new measurement is scaled by k compared to the original optimization and the transformed optimization is shown as the following:

$$\max_{k*C} E[U(k*C)]$$

where k is a non-zero deterministic scaler
Identical to the derivation in Appendix A, the expected utility of the consumption in terms of 1 euro takes the mean-variance form:

$$E[U(k*C)] = k*\mu - \frac{\gamma' k^2 * \sigma^2}{2}$$

The first order condition of this identical optimization with the same choice variable, $k*C$, but measured by 1 euro is

$$\frac{\partial E[U(k*C)]}{\partial k*C} = \frac{\partial k*\mu}{\partial k*C} - \frac{\partial \frac{\gamma' k^2 * \sigma^2}{2}}{\partial k*C} = \frac{k*\partial\mu}{k*\partial C} - \frac{k\partial \frac{k*\gamma'\sigma^2}{2}}{k*\partial C} = \frac{\partial\mu}{\partial C} - \frac{\partial \frac{k*\gamma'\sigma^2}{2}}{\partial C} = 0$$

The first order condition of the original optimization problem with the choice variable, F_t, which is measured by k euro is

$$\frac{\partial E[U(C)]}{\partial C} = \frac{\partial \mu}{\partial C} - \frac{\partial \frac{\gamma \sigma^2}{2}}{\partial C} = 0$$

The optimization with the new measurement will share the first order condition of the original one if and only if:

$$\gamma' = \frac{\gamma}{k}$$

`Apply Proposition 1 to the initial optimization:`

Suppose the original measurement is 1 euro. Let's change it to a new measurement of $PV[B_{t+1}]$ euro. Then, we get

$$\max_{FR_t} E[(\frac{I^{pri}_{t+1}}{PV[B_{t+1}]})]$$

subject to

$$\frac{I^{pri}_{t+1}}{PV[B_{t+1}]} = \frac{G_{t+1} - W^{pub}_{t+1}}{PV[B_{t+1}]} - \frac{B_{t+1}}{PV[B_{t+1}]} + \frac{F_t(1+r_t)}{PV[B_{t+1}]} - \frac{F_{t+1}}{PV[B_{t+1}]}$$

Now, every value is measured by its relative size to the present value of public sector pension benefits. The transformed absolute risk aversion coefficient is $PV[B_{t+1}]\gamma$. Moreover, the present value of future benefits, $PV[B_{t+1}]$, is a deterministic value discounted by the risk free rate

$$PV[B_{t+1}] = \frac{B_{t+1}}{1+r_f}$$

In addition, the dynamics of funding contribution, F_t, and pension benefits, B_t, are assumed to be

$$F_{t+1} = F_t(1+\mu_g)$$

$$B_{t+1} = B_t(1+\mu_g) = B_1(1+\mu_g)^t$$

where B_1 is a predetermined value of pension benefits.

Hence, the transformed budget constraint is given by

$$\frac{I_{t+1}^{pri}}{PV[B_{t+1}]} = \frac{G_{t+1} - W_{t+1}^{pub}}{PV[B_{t+1}]} - \frac{B_{t+1}}{PV[B_{t+1}]} + \frac{F_t(1+r_t)}{PV[B_{t+1}]} - \frac{F_{t+1}}{PV[B_{t+1}]}$$
$$= \frac{G_{t+1} - W_{t+1}^{pub}}{PV[B_{t+1}]} - (1+r_f) + FR_t(1+r_t) - FR_t(1+\mu_g)$$
$$= \frac{G_{t+1} - W_{t+1}^{pub}}{PV[B_{t+1}]} + FR_t(r_t - \mu_g) - (1+r_f)$$

Finally the transformed optimization problem is

$$\max_{FR_t} E[\frac{I_{t+1}^{pri}}{PV[B_{t+1}]}] - \frac{PV[B_{t+1}]\gamma\sigma^2}{2}$$

subject to

$$\frac{I_{t+1}^{pri}}{PV[B_{t+1}]} = \frac{G_{t+1} - W_{t+1}^{pub}}{PV[B_{t+1}]} + FR_t(r_t - \mu_g) - (1+r_f)$$

Appendix C

Derivation of optimization results

Optimization problem:

$$\max_{FR_t} E[\frac{I_{t+1}^{pri}}{PV[B_{t+1}]}] - \frac{PV[B_{t+1}]\gamma\sigma^2}{2}$$

subject to

$$\frac{I_{t+1}^{pri}}{PV[B_{t+1}]} = \frac{(G_t - W_t^{pub})(1+g_t)}{PV[B_{t+1}]} + FR_t(r_t - \mu_g) - (1+r_f)$$

where σ is the standard deviation of $\frac{I_{t+1}^{pri}}{PV[B_{t+1}]}$.
First order condition:

$$\mu_r - \mu_g - \frac{PV[B_{t+1}]\gamma}{2}\frac{\partial\sigma^2}{\partial FR_t} = 0$$

$$\Rightarrow \mu_r - \mu_g - \tfrac{PV[B_{t+1}]\gamma}{2}\tfrac{\partial\sigma^2}{\partial FR_t} = \mu_r - \mu_g - \tfrac{PV[B_{t+1}]\gamma}{2}(2FR_t\sigma_r^2 + 2\tfrac{G_t - W_t^{pub}}{PV[B_{t+1}]}\sigma_g\sigma_r\rho_{g,r}) = 0$$

$$\Rightarrow \mu_r - \mu_g - PV[B_{t+1}]\gamma FR_t\sigma_r^2 - PV[B_{t+1}]\gamma\tfrac{G_t - W_t^{pub}}{PV[B_{t+1}]}\sigma_g\sigma_r\rho_{g,r} = 0$$

$$\Rightarrow FR_t = \tfrac{\mu_r - \mu_g - \gamma(G_t - W_t^{pub})\sigma_g\sigma_r\rho_{g,r}}{PV[B_{t+1}]\gamma\sigma_r^2}$$

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