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Optimal Investment Strategy for Pension Funds in the New Dutch Pension Contract

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Optimal Investment Strategy for Pension Funds in the New Dutch Pension Contract

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Abstract

The situation in which pension funds have to operate has changed. Some developments in the market led to drops in the funding ratios of Dutch pension funds. Pension funds were too much exposed to financial shocks in the market, which is not desirable. Therefore, it is of upmost importance for pension funds to choose an investment strategy that meets their needs best. In this thesis, it is studied what the optimal investment strategy of a pension fund is. First, it is determined what the goal and the preferences of a Dutch pension fund and its stakeholders are. On the basis of these goal and preferences utility functions are specified. By maximizing these utility functions, three investment strategies turned out to be optimal, namely the constant mix investment strategy, the constant proportion portfolio insurance investment strategy and the option based portfolio insurance investment strategy. These strategies are tested by using an ALM model. Results show that the option based portfolio insurance investment strategy that locks profits fits the preferences of Dutch pension funds best and is therefore considered to be the optimal investment strategy for a pension fund in the new Dutch pension contract.

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Chapter 1

Introduction

During the global financial crisis, the funding ratios of many Dutch pension funds dropped. This was mainly caused by decreasing interest rates and bad returns on the stock market. Another factor that led to drops in funding ratios as well is the aging of the population. Pension funds were more exposed to shocks in the stock market than was affordable. Because of the drops in funding ratios, it turned out that the investment strategies of pension funds do not provide enough downside protection. Nieuwland [2010] states that the investment policy of most Dutch pension funds is a constant mix investment strategy, which means that the proportion invested in risk seeking assets remains constant over time.

Pension funds invest in risk seeking assets because these assets have a higher return than the risk mitigating (or riskless) assets in expectation. Pension funds want to be compensated for the risk they take. Some risks are rewarded by risk premiums. By taking these risks, pension funds can reach a higher return on their investments and can compensate their participants for inflation. This is called indexation. Indexation is very important to participants, for maintaining purchasing power. Providing indexation is one of the ambitions of most pension funds. Now we know that the positive side of taking risk is a higher expected return. However, taking risk also leads to a return that is more volatile. And therefore the funding ratio of pension funds becomes more volatile as well.

The following figure shows that a pension fund should define a policy that strikes the right balance between the ambition, risk on investment and costs for the social partners (employers and employees):

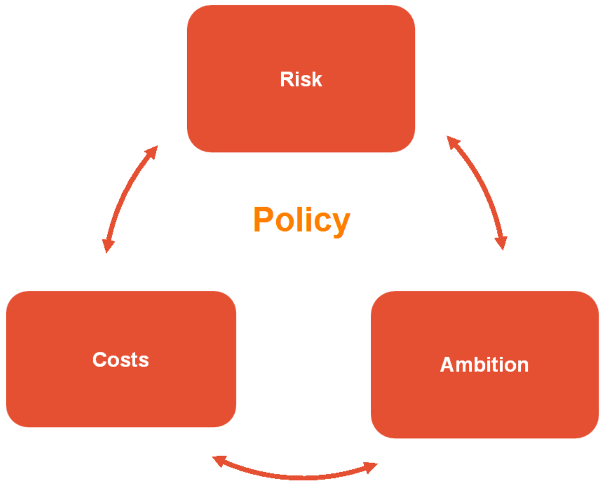


Figure 2.1. What a pension fund should take into consideration

The three components of the figure can be summarized as follows: by increasing the costs for the social partners (i.e. the contributions) the ambition level can be increased or the risk that is taken on investments can be reduced (because less return is needed). However, high costs are not desirable and note that it will not happen in practice that more contribution is asked just to provide indexation. In case the risk on investments is increased, the ambition level can be increased but on the other hand, ceteris paribus, there is a probability that costs have to be increased in case of bad stock returns. Again, that is not desirable. Increasing the ambition level requires to take more risk on investments or to ask higher contributions, but high contributions should be avoided and again, contributions will not be raised just to provide indexation. Therefore, the pension fund should define a policy to serve its needs best.

One of the elements of the pension funds' policy is the investment strategy. In this thesis, the optimal investment strategy for a pension fund will be studied. We will research this question by taking into account elements of the new Dutch pension contract.

In this thesis it is assumed that the pension fund can invest in a portfolio of risk seeking assets and in a portfolio of risk mitigating assets only, as is done in the Hoogovens pension fund [As, van, 2007]. The portfolio of risk seeking assets is called the 'return portfolio' and has a maximum Sharpe ratio.[1] The determination of the elements of this portfolio is beyond the scope of this thesis. The portfolio of risk mitigating assets is called the 'matching portfolio' and is assumed to hedge the liabilities. In this thesis it is assumed that the pension fund can divide its assets over the return portfolio and the matching portfolio.

We will consider what the optimal investment strategy for a pension fund is. This will be tested by using Monte Carlo simulations. In that way we are able to perform an asset liability management (ALM) study. An ALM study can be used to gain insight in the development of the

[1]The Sharpe ratio of a portfolio is defined as the excess return (the return of the portfolio minus the risk free rate) divided by the volatility of the portfolio returns.

future balance sheet of the pension fund. An ALM study considers what happens in expectation and it also measures the risk in case an adjustment in one of the policies is made. In that way, different investment strategies can be compared such that we can measure which investment strategy would fit the needs of a pension fund the best.

In order to define the optimal investment strategy for a Dutch pension fund, first it should be determined what the goal of a Dutch pension fund is and what the preferences of the pension funds' stakeholders are. This will be addressed in chapter 2.

In the subsequent chapter, the current situation for pension funds will be described to explain the circumstances in which pension funds have to operate nowadays.

Then in chapter 4, different investment strategies will be explained. The constant mix strategy will be explained, which is currently used by most Dutch pension funds. Then two portfolio insurance investment strategies will be explained. Portfolio insurance investment strategies become more popular because these strategies aim to insure the pension funds against adverse shocks in the market. One of the two portfolio insurance investment strategies that will be explained is the option based portfolio insurance strategy. This strategy uses options to protect the pension fund against downside risk. The other portfolio insurance investment strategy that will be explained is the constant proportion portfolio insurance strategy, which is such that the amount invested in the return portfolio depends on the funding ratio of the pension fund.

In chapter 5, the model that we made to test the different investment strategies is described. As already explained, the analysis will be done by Monte Carlo simulations. In this chapter the financial market assumptions are explained, information about the participants is presented and the policy of the pension fund is described.

The goal and preferences of the pension fund and its stakeholders from chapter 2 can be used to define a utility function. This will be explained in chapter 6. In this chapter, the utility functions will be maximized to define the optimal investment strategies.

In chapter 7, the investment strategies that follow from the optimal investment strategies (defined in chapter 6) and that are tested in this thesis will be elaborated on.

In the subsequent chapter it will be tested whether the derived optimal investment strategies are indeed optimal for a Dutch pension fund.

Because the optimal investment strategy of pension funds can depend on the initial funding ratio, in chapter 9 we will perform a sensitivity analysis.

Chapter 10 concludes this thesis. Recommendations for further research will be provided as well.

Chapter 2

The goal and preferences of a Dutch pension fund and its stakeholders

2.1 Introduction

A pension fund has to make sure that every participant receives sufficient pension benefits during retirement. The level of these pension benefits depends on what is accrued by every participant. In order to make sure that every participant receives pension benefits during retirement, the pension fund has to take care of the pension contributions of its participants. These pension contributions will be invested in the market. Pension funds usually invest part of its assets in the return portfolio in order to gain a return higher than the riskless rate such that indexation can be provided.

Every pension fund has made a policy to serve his participants as best as it can. A pension fund aims to give indexation and wants to avoid cuts because then it is attractive for individuals to participate in the pension fund. This is not always possible however. Assets have to be available to provide the indexation. Sometimes, not enough assets are available to pay the pension liabilities and cuts have to be implemented.

The goal and preferences of a pension fund and its stakeholders will be defined in this chapter. They can be translated into Key Performance Indicators (KPIs). These KPIs will be used to formulate a utility function in chapter 6.

2.2 Key Performance Indicators

The KPIs of a pension fund represent the goals and preferences of that pension fund and its stakeholders. These KPIs are measurements that give an expression about how successful a pension fund is. The following KPIs will be used in this thesis:

- Provide an attractive pension
- Be able to pay the liabilities
- Provide a compensation for inflation
- Minimize the probability to need to announce cuts in benefits
- Ask stable and attractive pension contributions from the participants
- Take acceptable risks that are rewarded

Provide an attractive pension

One of the KPIs of a Dutch pension fund is to provide an attractive pension. It should be attractive to individuals to participate in the pension fund. Because there are old and young participants in a pension fund, risk sharing takes place. The risk sharing does not only take place between these groups, but also within these groups, as some people live longer than others.

Be able to pay the liabilities

The pension fund should be solvent. In order to be solvent, a pension fund has to make sure that it can meet its financial obligations now and in the future (i.e. their liabilities). This will be achieved if the pension fund has enough assets to pay its liabilities, which means that the funding ratio should be adequate.

Provide a compensation for inflation

A pension fund aims to provide an indexed pension, because it wants to give a compensation for inflation. In that way, participants of the pension fund can remain their purchasing power. Because the pension fund has this expensive aim, it has to invest in risky assets. Investing in only risk mitigating assets does in general not provide enough return to finance the indexation. Currently in the Netherlands, indexation is conditional on the level of the funding ratio. For this KPI a high funding ratio is needed as well, as full indexation can be given then.

Minimize the probability to need to announce cuts in benefits

Another KPI of a Dutch pension fund is that the probability of having cuts is as small as possible. If the funding ratio of the pension fund is sufficient, cuts are not needed to meet the requirements of being a solvent pension fund. In 2014, 29 pension funds had to implement cuts and 200.000 participants were hit by these cuts. They were very unhappy to note that their pension rights were reduced. This number of pension funds that had to implement cuts and number of participants that were hit are much lower than they were one year before, because in 2013 at least 68 pension funds had to implement cuts and even 5.6 million participants were hit.

Ask stable and attractive pension contributions from the participants

From the first KPIs it follows that a pension fund should try to reach a high funding ratio. A possible way to reach a high funding ratio is to increase the contributions. However, the participants do not prefer to pay a large amount of money to the pension fund. Then the pension fund is not attractive anymore. Therefore, an additional KPI of the pension fund is to have the contributions for the participants at a stable and relatively low level. In the Netherlands, pension funds are obliged to ask a cost covering contribution. They are also allowed to ask a contribution that is higher than this cost covering contribution.

Take acceptable risks that are rewarded

Lastly, the funding ratio of a pension fund can be increased in expectation if the pension fund takes a lot of risk, as this, in expectation, leads to a higher return. However, if the pension fund takes much risk, its funding ratio may drop fast in bad scenarios. Then, cuts have to be implemented and that is not desirable. Therefore, the last KPI states that not too much risk should be taken, and the risk that is taken should be rewarded.

The conclusion of these KPIs is that a pension fund should have a high funding ratio that allows for providing full indexation and does not require any benefit cuts. However, the pension fund should not be too expensive and not too risky.

Chapter 3

Dutch pension contract

3.1 Introduction

Since the global financial crisis has started in 2008, the funding ratios of many Dutch pension funds decreased drastically as can be seen in figure 3.1.

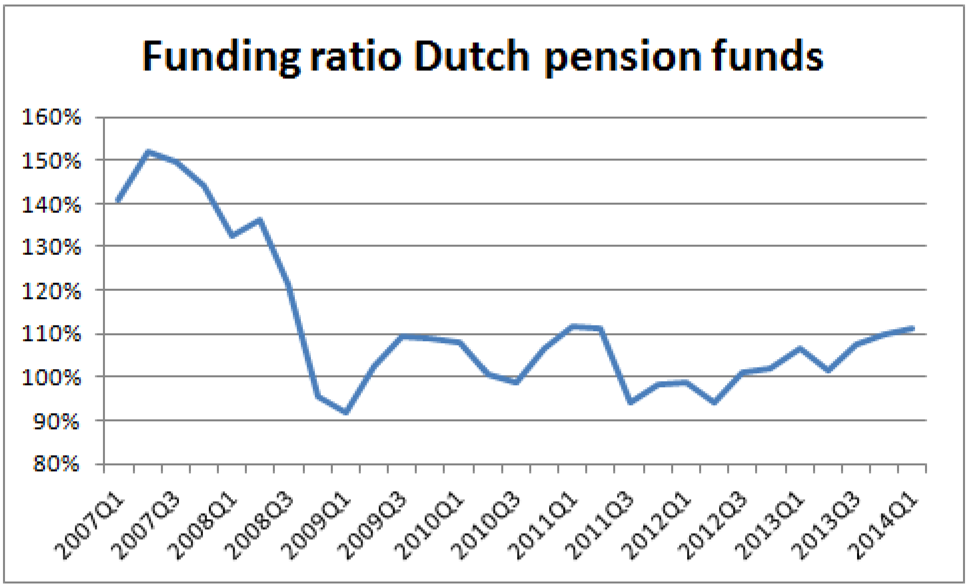


Figure 3.1 Funding ratio of Dutch pension funds from the first quarter of 2007 until the first quarter of 2014 (Source: DNB [2014b])

Because of these large drops in funding ratios, the government started to think about a new Dutch pension contract in order to make sure that pension funds are better protected against shocks in the future. This new pension contract is planned to be (partly) implemented in January 2015. The new pension contract will be stricter than the current Dutch pension contract.

In this chapter we will first explain the current situation for Dutch pension funds. We will do this by describing the current rules for pension funds and by describing the problems that pension funds have to deal with. Then we will explain the elements that will (probably) be implemented as of January 1, 2015.

3.2 Current situation

The Dutch pension system consists of three pillars:

First pillar
The first pillar consists of payments made by the government to everyone in the Netherlands who has reached the retirement age, and is called AOW (which is an abbreviation for 'Algemene Ouderdomswet'). AOW is meant to prevent poverty amongst the elderly. Therefore, AOW is an amount of money that is paid to everybody who is older than the retirement age, conditional on living in the Netherlands since the age of 15. If a person had been living in a foreign country for a couple of years, the AOW amount will be less.

Second pillar
The second pillar is an additional pension which is related to work. In most of the sectors workers are obliged to pay a pension contribution to save for retirement. This pillar is relatively large in the Netherlands which is a strong point of the Dutch pension system. Second pillar products are provided by both pension funds and insurance companies.

Third pillar
The third pillar is voluntary. If persons think they do not accrue enough money for retirement they could buy an extra pension product, for instance an annuity. The third pillar is mostly used by entrepreneurs as they often do not have a collectively organized pension. Third pillar products are provided by insurance companies.

In this thesis, we will focus on the second pillar only as this pillar is the pillar where pension funds come into play.

In contrast to the first pillar, which is a Pay As You Go (PAYG) system which means that the current working population pays for the current retirees, the second pillar is a capital covered system. This means that every participant of the pension fund pays contribution to the pension fund to save for his or her own pension payments. These pension contributions are a specified percentage of salary and are most of the times partly paid by the employer and partly paid by the employee. The size of the pension contributions is restricted by the government. The policy of Dutch pension funds is to ask a contribution that is equal to a constant percentage of salary (in Dutch: doorsneepremie). This is illustrated in the following figure:

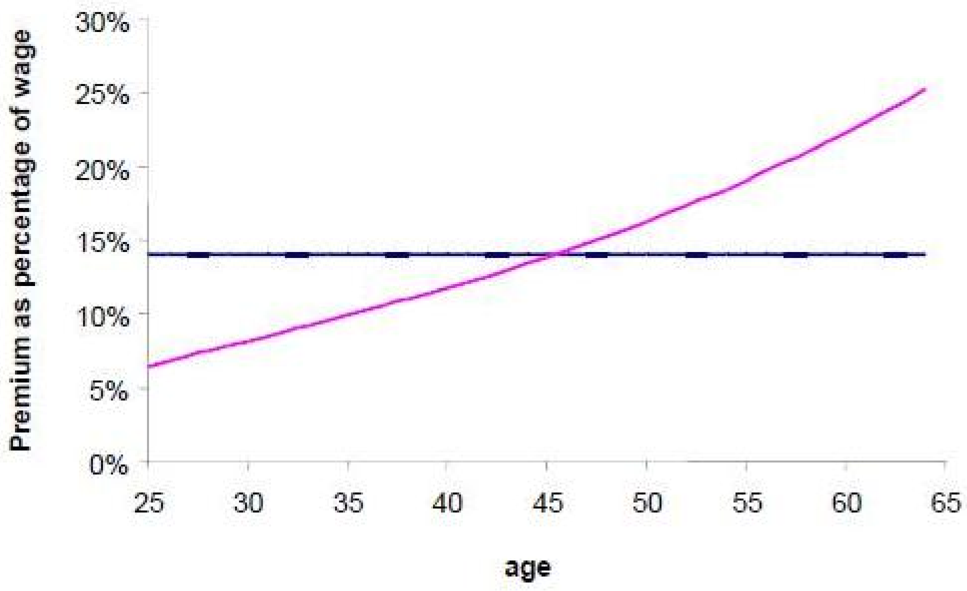


Figure 3.2 Contribution levels

In figure 3.2 the blue line is the constant percentage of the salary that is contributed to the pension fund and the pink line is the actuarially neutral value of the pension that is accrued at every age. It can be seen that in an actuarially point of view young participants pay too much and old participants pay too little to the pension fund compared to what they accrue.

On January 1, 2007, the FTK (which is an abbreviation for 'Financiële Toetsingskader') was introduced, which is the prudential legislation for pension funds. Due to the FTK, pension funds have to restrict themselves to some set of rules described in the pension act [Pensioenwet, 2006]. The requirements of the FTK are supervised by the Dutch Central Bank (in Dutch: De Nederlandsche Bank, DNB). The FTK states that pension funds should have a funding ratio that is at least equal to the minimum required funding ratio, which is about 105%. If this is not the case, the pension fund should submit a recovery plan to DNB in which it describes how to recover within three years. If at the end of this period the pension fund has not recovered, the pension fund has to cut pension benefits. Moreover, the pension fund is required to hold a solvency buffer because the probability of having a funding ratio less than 100% in one year from now should be less than 2.5%. The more risk a pension fund takes, the higher this solvency buffer is. If the funding ratio of a pension fund is less than this required funding ratio, the pension fund has a reserve deficit and then it has to submit a plan to DNB that describes how to recover within 15 years.

The FTK also requires that assets and liabilities are valued at market value. Before the FTK was implemented, liabilities were calculated based on a fixed discount rate. This led to the fact that the liabilities did not change in case the interest rate changed. This will be explained more extensively in section 5.2.1.

In the Netherlands, a Defined Benefit (DB) system is mostly used. In a DB system, pension benefits are known in advance and are 'guaranteed'. Guaranteed is between quotation marks as it can happen that the fund becomes underfunded and then there is a possibility that cuts have to be announced as a last option to recover. The amount of benefits received is related to the salary earned during the working period. In that way, participants of the pension fund can (partly) remain their purchasing power after retirement. In earlier years, the benefits were a certain percentage of the final salary a participant earned. Nowadays, benefits are often a certain percentage of the average salary of the participant. In a DB scheme, the pension fund aims to give indexation. Participants of the Dutch DB systems always thought to have pension payments that are guaranteed during retirement. However, recently some pension funds had to cut benefits because their funding ratio was not high enough according to the rules of the FTK. Although the cuts were announced in advance, the participants of these pension funds were deeply disappointed.

Nowadays, Collective Defined Contribution (CDC) schemes are becoming more popular. In a CDC system, the pension contributions are fixed and the pension benefits depend on the funding ratio of the pension fund. In that way, pension benefits are high if the returns on the investments of the pension fund are high and pension benefits are low if the returns on the investments of the pension fund are low. This results in the fact that the participants of the pension fund bear the risk of the investments, in contrast to the policy in a DB scheme. Of course, this is attractive for the employers, as they do not have to invest in the pension fund in case of bad returns. However, risk averse participants of a pension fund want to have guarantees so they do not appreciate it if the fund decides to change from DB to CDC. In this thesis we will focus on DB schemes.

3.3 Problems

Last years, some developments have led to problems for Dutch pension funds. We will explain the following developments: low interest rates, aging of the population and bad returns on investments.

Low interest rates

Last years, interest rates decreased. This can be seen in the following figure which represents the 5 year and 15 year interest rates since 2004.

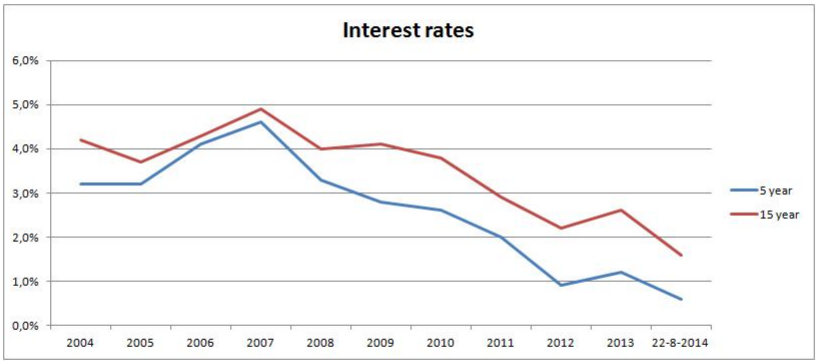


Figure 3.3 5 year and 15 year interest rates

At the moment, interest rates are historically low. As liabilities are discounted by the interest rate, liabilities are higher than when we have high interest rates. Furthermore, returns on assets that are invested in the matching portfolio are lower as well. These two effects lead to a decrease in funding ratio.

Aging of the population

Aging of the population is one of the developments that leads to problems for pension funds, because of three different features. The first feature is that life expectancies of people increased last years. This follows from data of Statistics Netherlands (in Dutch: Centraal Bureau voor de Statistiek, CBS).

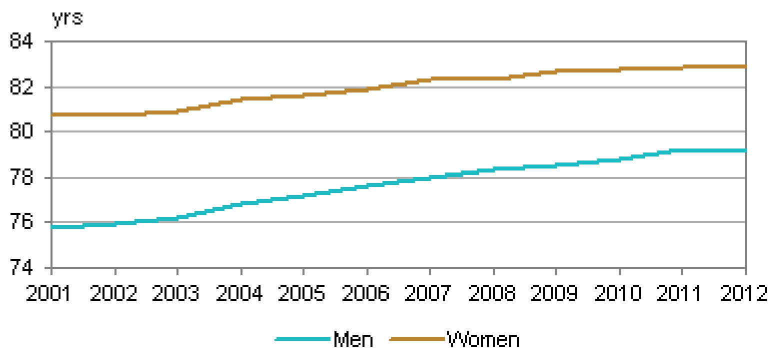


Figure 3.3 Life expectancies at birth (Source: CBS [2012])

As can be seen in figure 3.3, the life expectancies at birth of both men and women increased during last years. Because of the increase in life expectancy, people are more years in retirement compared to years ago and therefore pension funds have to pay more pension, which increases the liabilities.

From CBS [2014] it follows that the ratio of the labor force aged between 50 and 60 over the labor force aged between 30 and 40 increased for 15 years now. Last year (2013), the ratio was even bigger than one, which means that there were more working people in the age category $50-60$ than in the age category $30-40$. Because of this increase in the average age of the working population, pensions become more expensive as well. In the Netherlands, every participant of a pension fund pays the same percentage of the pension base as contribution. As we have seen in the previous section, this leads to the fact that young participants pay too much in an actuarially point of view, while old participants pay in an actuarially point of view too little for their pension accrual. Now we have that there are more old workers than young workers, which means that there are more participants that pay too less than participants that pay too much. This is a problem for pension funds as well.

Thirdly, because of the aging of participants, the number of active participants compared to passive participants (working participants compared to retired participants) decreases as well. Less solidarity is possible, because there are less participants that are able to bear the risk.

Bad returns on investments

In 2008, the global financial crisis started. This led to adverse shocks in the stock market. Pension funds invest a part of their assets in the stock market because they strive for high returns. Stock returns are higher than the risk free rate in expectation, however the volatility of stock returns is higher as well. If returns are high, pension funds are able to provide indexation to the participants. However, because stock markets were very bad last years, returns on the investments of pension funds were bad as well. This led to a drop in funding ratios.

3.4 New Dutch pension contract

At June 25, 2014, the bill for the adjustment of the FTK was made publicly available by the Rijksoverheid. Because the House of Representatives has not treat this bill, the precise interpretation of the adjustments of the FTK is not clear yet. We will state the elements of this bill that are most relevant for this thesis, which are the policies concerning indexation and cuts.

Indexation policy

The indexation policy will be stricter in the new FTK compared to the old FTK. Pension funds are obliged to base their calculations on the 'policy funding ratio'. The policy funding ratio is defined as the average of the funding ratios of the last 12 months. The development of the policy funding ratio is smoother than the funding ratio.

The lower bound for providing a compensation for inflation becomes a policy funding ratio of 110%. Above this level, it is allowed to provide some indexation conditional on the requirement that the pension fund can provide this indexation in the future as well. Therefore, there will not be a strict upper bound for providing full indexation anymore.[1] The level of indexation that will be provided is calculated by calculating the nominal liabilities without indexation and discount the indexation cash flows by the expected geometric return on equities. Although there is no

[1] Note that indexation is restricted by fiscal bounds.

strict upper bound, this method will result in an upper bound for providing full indexation that is equal to a policy funding ratio of approximately 130%.

Next to the indexation, recovery indexation can be provided if the policy funding ratio is above the funding ratio that is needed for providing full indexation. Recovery indexation is a compensation for missed inflation or cuts in earlier years. Not all assets are available for the recovery indexation. Only one-tenth of the assets that are not needed for the funding ratio that is needed for full indexation is available for the recovery indexation.

Cut policy

In the new Dutch pension contract, pension funds immediately have to react when their policy funding ratio is below the required funding ratio. Pension funds need to announce cuts if their policy funding ratio plus the capacity for recovery (in Dutch: herstelcapaciteit) for ten years is below the required funding ratio (RF). The RF is dependent on the risk that is taken and so it is different for different pension funds. The cut that has to be implemented is one-tenth of the difference between the RF and the current policy funding ratio plus the capacity for recovery for ten years.

This cut policy can lead to large periods of underfunding. (A pension fund is called underfunded if the policy funding ratio is below the minimum required funding ratio (MRF).) Therefore, another rule is set that says that if the pension fund is underfunded for five years, cuts have to be announced such that the funding ratio is equal to the MRF. This last cut can be spread over a maximum period of ten years.

Chapter 4

Investment strategies

4.1 Introduction

In recent years it turned out that pension funds are more vulnerable to financial shocks than desired. Results from the past already showed that the portfolio of a pension fund is often not protected against financial shocks, which led to the need of announcing cuts in benefits. For example, due to the aging of the population, pension funds had to take more risk in their investments in order to be able to provide, in expectation, the promised pension benefits and to realize the ambition of providing indexation to their participants in expectation. As said, this made the pension funds more vulnerable to financial shocks and recovery became harder. The Dutch parliament asked the committee Frijns to investigate the investment policy and risk management of pension funds. On the basis of this research, the committee recommended among other things to focus on the risk that is taken rather than on the ambition level and to adjust the investment policy on the basis of the specific characteristics of the pension fund. Therefore, pension funds might be interested in insuring their portfolio against the financial shocks, which might be reached by choosing another investment strategy.

There are many different ways of investing. Although the policy for the indexation of the pension payments and the policy for the cuts of pension rights are often dynamic (they depend on the funding ratio of the pension fund), the investment policy is not. It might be a good idea to consider dynamic investment in order to improve the performance of a pension fund.

In this chapter, we will explain different kind of investment strategies that are described in Perold and Sharpe [1988]. We will begin by explaining the constant mix investment strategy. This strategy is used by many pension funds nowadays. Then we will introduce the concept of portfolio insurance investment strategies. In that section, we will explain two different portfolio insurance investment strategies, namely the constant proportion portfolio insurance strategy and the option based portfolio insurance strategy.

4.2 Constant mix investment strategy

The constant mix investment strategy is such that a constant proportion of the portfolio is invested in the return portfolio at any point in time. For this strategy we need to rebalance the portfolio,[1] because markets change and so the fraction invested in the return portfolio changes as well if no rebalancing would take place. The performance of the constant mix investment strategy is shown in the following example:

We assume we have an investor who is able to invest € 100 and for this example we assume he invests 70% in the return portfolio and 30% in the matching portfolio, so he invests € 70 in the return portfolio and € 30 in the matching portfolio. The initial return portfolio index is assumed to be equal to 100 and the riskless rate of return is assumed to be 4%.

After one year, if the market has performed well and the return portfolio index is worth 110, the new portfolio value can be calculated by taking the sum of the new value of the return portfolio, $70\% \cdot € \ 110 = € \ 77$, and the new value of the matching portfolio, $€ \ 30 \cdot 1.04 = € \ 31.2$, which results in a new portfolio value of € 108.2. Now the proportion invested in the return portfolio is equal to $\frac{77}{108.2} = 71.16\%$. In order to rebalance the portfolio such that the initial mix is obtained, the investor has to sell $77 - 70\% \cdot 108.2 = € \ 1.26$ of the return portfolio and invest this amount in the matching portfolio.

However, it can be the case that the market performs badly and that the return portfolio index is worth 90 after one year. The new value of the assets of the pension fund becomes $70\% \cdot € \ 90 = € \ 63$ plus $€ \ 30 \cdot 1.04 = € \ 31.2$ which adds up to € 94.2. Now in order to rebalance the total portfolio such that the initial mix is obtained, the investor has to buy $70\% \cdot 94.2 - 63 = € \ 2.94$ of the return portfolio and sell the same amount of the matching portfolio. Then the initial mix is obtained again.

From the example it follows that this type of strategy buys shares of the return portfolio when they fall, and sells shares of the return portfolio when they rise.

As this strategy always aims to invest a constant proportion in the return portfolio, there is no downside protection in bad markets.

[1]Rebalancing can be done in many different ways. An example is to rebalance at certain points in time, for example every day, month, year, 5 years etc. Another example is to rebalance at every time the value of the stocks has changed by a certain percentage compared to the previous rebalancing moment. The determination of a rebalancing strategy is beyond the scope of this thesis. For now we will assume that rebalancing takes place every year.

4.3 Portfolio insurance strategies

As explained in the previous chapter, pension funds will be very dissatisfied if their funding ratio is below a specified floor. Then, they have to announce cuts to the participants of the fund, which they do not like according to the in chapter 2 specified KPIs. Therefore, they want to be sure to have a funding ratio that is at least equal to the floor. This cannot be obtained by investing via the previously explained investment strategy, as this investment strategy does not guarantee a minimum funding ratio.

It would be interesting for the pension fund to have some kind of insurance, to be sure to reach the minimum funding ratio they need in order to be an attractive pension fund. An insurance is not for free, so having a guaranteed funding ratio might reduce the potential of having gains as well.

A portfolio of stocks can be hedged against bad shocks by portfolio insurance. For example, a pension fund can invest in the return portfolio and buy a put option on the return portfolio as well (or actually, a pension fund can invest such that it replicates the buying of puts on this portfolio). In that way, the portfolio value will never be lower than the strike price of the put option.

In this section, we will explain two different portfolio insurance investment strategies. The strategies we will explain are the option based portfolio insurance strategy and the constant proportion portfolio insurance strategy. These investment strategies guarantee that the portfolio value at any point in time is greater than or equal to the discounted value of the specified minimum funding ratio, i.e. the floor. Furthermore, they keep the potential of having gains in rising markets.

4.3.1 Option based portfolio insurance investment strategy

The option based portfolio insurance (OBPI) investment strategy has the aim to insure a portfolio. This strategy relies on derivatives and put and call options are mostly used. First a horizon and a floor which the pension fund wants to reach at that horizon should be specified. A typical OBPI strategy is such that it gives the same payoff as a portfolio composed of call options and shares of the matching portfolio, in which the shares of the matching portfolio have the same value as the present value of the floor and the rest is invested in the calls. Or, equivalently because of the put-call parity explained in Appendix A, the portfolio is composed of shares of the return portfolio and put options, in which the strike price of the put options is equal to the value of the floor at horizon.

Here we will give an example of the performance of an OBPI investment strategy:

We assume we have an investor that is able to invest €100 and has a floor of €80, which are the liabilities in one year from now multiplied by the MRF in this example. We assume the interest rate to be 0 and the return portfolio index is 100 again. Now the investor has to invest €80 in the matching portfolio, in order to be sure to have a total asset value in one year from now that is at least equal to the value of the floor. The money that remains can be invested in call options. The price of a call option is calculated by using the Black Scholes formula, which is explained in Appendix A. The payoff of the call option in one year from now (C_{t+1}) is equal to

$$C_{t+1} = \max[A_{t+1} - k \cdot L_{t+1}, 0], \tag{4.1}$$

where A_{t+1} and L_{t+1} are the value of the total assets and the liabilities in one year from now, respectively, and k is equal to the MRF. So $k \cdot L_{t+1}$ is the strike price of this option. In this way, there is a good downside protection as the investor is sure to have a funding ratio that is at least equal to the MRF in one year from now and there is some potential for gains.

4.3.2 Constant proportion portfolio insurance investment strategy

A constant proportion investment strategy is such that the fraction invested in the return portfolio depends on the value of the total assets and the value of the floor. The value of the assets minus the value of the floor is called the cushion and the value invested in the return portfolio is equal to the cushion times a multiplier. If this multiplier is greater than one, the strategy is called a constant proportion portfolio insurance (CPPI) investment strategy. For now we assume the floor to be equal to the MRF times the liabilities. CPPI investment strategies are designed to be optimal if trades are continuously made. However, this is not possible in practice.

This strategy is mathematically described by

$$E_t = m \cdot (A_t - k \cdot L_t), \tag{4.2}$$

where E_t is the exposure to the return portfolio given in euros, m is the multiplier, A_t is the value of the assets of the pension fund at time t, k is the MRF and L_t are the liabilities at time t. Note that the present value of the floor must initially be less than the total initial assets and note that, without constraints, this might result in investing more than the total assets in the return portfolio. However, here we assume that no more than the value of the total assets can be invested in the return portfolio. Furthermore, we assume that the amount invested in the return portfolio cannot be negative. So the formula for the exposure to the return portfolio given in euros can be extended to

$$E_t = \min[\max[m \cdot (A_t - k \cdot L_t), 0], A_t], \tag{4.3}$$

and the fraction invested in the return portfolio is given by

$$\pi_t = \frac{E_t}{A_t} = \min[\max[m \cdot (1 - k\frac{L_t}{A_t}), 0], 1]. \tag{4.4}$$

If the value of the return portfolio rises, the fraction invested in the return portfolio will be greater. On the other hand, if the value of the return portfolio falls, less will be invested in the return portfolio. So CPPI investment strategies sell shares of the return portfolio as they fall and buy shares of the return portfolio as they rise. The total value of the assets of the pension fund can fall below the floor only in case there is a very fast drop in the return portfolio, such that the pension fund is not able to rebalance its portfolio in time. Perold and Sharpe [1988] show that the maximum drop in the market before the floor is reached is equal to $\frac{1}{m}$.

Now we will give an example of the performance of the CPPI investment strategy. Although the CPPI investment strategy is designed such that it is an optimal investment strategy if trading takes place continuously, in practice this is not possible and therefore discrete trading should be implemented. In this example, rebalancing takes place once a year.

We assume we have an investor that is able to invest €100 and has a floor of €80, which are the liabilities multiplied by the MRF. The investor is assumed to use a multiplier of 2 as is used in Perold and Sharpe [1988]. Here, we assume the interest rate to be 0 and the return portfolio index is 100 again. Now we can calculate the investment in the return portfolio which was given by equation (4.3). The exposure to stocks is equal to $\min[\max[2 \cdot (100 - 80), 0], 1] =$ € 40 and so the investor invests €40 in the return portfolio and €60 in the matching portfolio.

After one year, if the market has performed well and the return portfolio index is worth 110, the new portfolio value can be calculated by taking the sum of the new value of the return portfolio (40% · € 110 = € 44) and the value of the matching portfolio (€60), which results in a new portfolio value of € 104. According to the strategy, the investor should invest $\min[\max[2 \cdot (104 - 80), 0], 1] =$ € 48 in the return portfolio now. Therefore, in order to rebalance, the investor has to invest €4 extra in the return portfolio and sell shares of the matching portfolio to obtain this amount of money.

However, it can also be the case that the market performs badly such that the return portfolio index is worth 90 after one year. The new portfolio value of the portfolio becomes 40% · € 90 = € 36 plus € 60 which adds up to € 96. According to the strategy, the investor should invest $\min[\max(2 \cdot (96 - 80), 0), 1] = 32$ in the return portfolio now. Therefore, in order to rebalance, the investor has to sell €4 of the return portfolio and invest this money in the matching portfolio.

From the example it follows that this strategy buys shares from the return portfolio when they fall, and sells shares from the return portfolio when they rise. Because shares of the return portfolio will be sold if they fall, there is a good downside protection.

Chapter 5

The model

5.1 Introduction

In this chapter, a description of the pension fund that we used for the simulations will be given. The assumptions are based on the assumptions in Schothuis [2014] and are updated and improved.

In the next section, the financial market assumptions are explained. First of all, short term interest rates will be modeled for every point in time. These interest rates will be used to form term structures. Next, the level of the return portfolio and the matching portfolio will be determined for each future point in time. The level of the return portfolio is assumed to be stochastic, while the level of the matching portfolio is not and is assumed to follow the same trend as the one-year interest rate. Furthermore, inflation is assumed to be stochastic.

In section 5.3 the participants and their wages will be described. For this, data from Statistics Netherlands is used. By using this data, the composition of the pension fund is determined. Furthermore, assumptions on the wages that are earned by the participants are described.

Then, the policy of the pension fund is explained. We made assumptions about the pension that is accrued and about the contributions that are paid by the participants. Furthermore we made assumptions about the (recovery) indexation policy and cut policy. These assumptions are mainly based on the new Dutch pension contract. Then we made assumptions about the development of the liabilities of the pension fund and lastly we made assumptions about the development of the funding ratio over time.

5.2 Financial market assumptions

In order to gain insight in the development of the assets and liabilities of a pension fund, financial market assumptions have to be made. In this section, we will give descriptions of the development of short term interest rates and about the term structures that follow from these short term interest rates, about the development of the return portfolio and the matching portfolio and about the development of the inflation level.

5.2.1 Interest rates

Before 2007, the liabilities of Dutch pension funds were calculated based on 'actuarial valuation', which means that a constant discount rate (4%) was used to calculate the liabilities. So a change in the interest rate did not change the liabilities of a pension fund. However, the value of the assets was changed by a change in the interest rate. For example, a decrease in the interest rate increased the value of bonds, such that the funding ratio of the pension fund increased if the pension fund had invested in bonds.

Nowadays, the liabilities of a Dutch pension fund are calculated based on 'market valuation', which means that the current term structure is used to calculate the liabilities. So now a change in the interest rate does not only lead to a change in the value of the assets but it also leads to a change in the liabilities. For example, a decrease in the interest rate leads to an increase in the value of the assets as we have seen, but on the other hand it leads to an increase in the liabilities as well. The problem is that the liabilities increase much faster than the value of the assets in this case. This is because the liabilities can be seen as a short portfolio in long term bonds with a longer duration than the duration of the assets, and because assets are not fully invested in bonds. Now a decrease in the interest rate leads to a drop in funding ratio!

Due to the phenomenon described above, interest rates and term structures are very important for a pension fund. A term structure is constructed by using forward rates. However, the market of forward rates is not liquid enough for longer maturities, which leads to the fact that forward rates for longer maturities are less applicable to form a term structure. Therefore, it is decided to adjust the term structure for pension funds from September 30, 2012 onwards according to DNB [2012]. In this document of DNB it is stated that the term structure for a pension fund should converge to the ultimate forward rate (UFR) for longer maturities. So this is because for longer maturities, there are too less observations available in the market to define the term structure. Therefore, if these observations are used to form the term structure, the liabilities would become more volatile. Because of the implementation of the UFR, the volatility of the liabilities is reduced and therefore the pension fund becomes more stable. For shorter maturities the swap market is liquid enough to construct the first part of the term structure. Currently it is the case that a constant UFR is used to which the term structure converges. In the new Dutch pension contract the assumption about the converging of the term structure of interest rates is adjusted as we will explain below.

Now in order to calculate the term structure of interest rates, we first have to determine the instantaneous nominal interest rates. From these instantaneous nominal interest rates, the interest rates at every time t for all maturities can be calculated. Then, we follow the extrapolation method of the new Dutch pension contract which leads to the term structure of interest rates.

Here, we will use the short term interest rate model of Vasicek [1977] to model the instantaneous nominal interest rate. Vasicek assumes that interest rates are mean-reverting, which means that if the current interest rate is above the long term mean, the drift term is negative and if the current interest rate is below the long term mean, the drift term is positive. He proposed the following stochastic discount factor (SDE) for the development of interest rates:

$$dr_t = a_r(\bar{r} - r_t)dt + \sigma_r dW_t^r, \tag{5.1}$$

where r_t is the instantaneous nominal interest rate in year t, a_r is the parameter for the speed of mean reversion, $\bar{r}$ is the long term mean of the interest rate, σ_r is the volatility of the interest rate and dW_t^r is a Brownian motion.[1] In the simulations, the following values for the parameters are used: for $\bar{r}$ we use 2.2%. In the current term structure of interest rates we see that for the long term, short term interest rates converge to 2.2%. For a_r we use a value of 0.5 and for σ_r we choose 0.5%. With these values, the simulated interest rates fit the current interest rates the best. Furthermore, for the initial interest rate we choose 0.5%. This is the current Euribor one-year interest rate.

The instantaneous nominal interest rate will be used for the determination of the interest rate at time t for horizon T. First, as Vasicek [1977] has shown, the price of a zero coupon bond is given by

$$P_0(T) = exp\left(-\left[\left(\bar{r} - \frac{\sigma_r^2}{2a_r^2}\right)T + \left(r_0 - \bar{r} + \frac{\sigma_r^2}{a_r^2}\right)\frac{1 - e^{-a_r T}}{a_r} - \frac{\sigma_r^2}{2a_r^2}\frac{1 - e^{-2a_r T}}{2a_r}\right]\right), \tag{5.2}$$

where r_0 is the current interest rate. With the use of formula (5.2) we are able to calculate the price of a zero coupon bond for every maturity T from now on. The price of a bond with maturity T at a general time $t \leq T$ can be obtained by replacing r_0 by r_t and T by $T - t$. However, since we need the interest rate at every time t for horizon T (which is in fact for maturity $T + t$), we only need to replace r_0 by r_t.

Now we know the price of a zero coupon bond for every time t for every horizon T and by using the fact that the price of a bond can be given by $P_t(T)$ =exp$(-Y_t(T) \cdot T)$, where $Y_t(T)$ is the

[1] A Brownian motion has the following properties: $W_0^r = 0$, increments are independent and $W_t^r - W_s^r\ N(0, t-s)$ for $t \geq s \geq 0$.

yield at time t for horizon T, we come to the following expression for the yield:

$$Y_t(T) = \bar{r} - \frac{\sigma_r^2}{2a_r^2} + \left(r_t - b + \frac{\sigma_r^2}{a_r^2}\right)\frac{1-e^{-a_rT}}{a_rT} - \frac{\sigma_r^2}{2a_r^2}\frac{1-e^{-2a_rT}}{2a_rT}. \tag{5.3}$$

For the final determination of the term structure of interest rates we follow the paper of Langejan et al. [2014]. It says that for horizons up until 20 years the market for zero coupon bonds is liquid enough. Furthermore the method says we need to calculate forward rates. A forward rate is a rate that is decided upon at time t and specifies the rate that has to be received for a zero coupon bond starting at time $t+m$ with maturity $t+m+k$ $(m, k \geq 0)$. The continuous forward rate (f_c) at time t starting at time $t+m$ with maturity $t+m+k$ can be calculated by

$$f_c(t, m, m+k) = \frac{Y_t(m+k)(m+k) - Y_t(m)m}{k}. \tag{5.4}$$

For horizons bigger than 20 years an extrapolation method will be used. The horizon of 20 years will therefore be called 'first smoothing point'. First, the UFR for every time t has to be calculated. This UFR is the average of the 20-years forward rates over the last 120 months. However, as we determine interest rates per year in this thesis, we will use an UFR that is the average of the 20-years forward rates over the last 10 years. The formula for the UFR at time t is given by

$$UFR(t) = \frac{1}{10}\sum_{i=t-10}^{t} f_c(i, 20, 21). \tag{5.5}$$

In this thesis we need to know the UFR of last year, which is assumed to be 3.9% as this was assumed in Langejan et al. [2014]. We assume that for the last ten years, $f_c(t, 20, 21)$ was equal to 3.9%. The UFR will be rounded to one decimal. Furthermore, we need that $UFR_c(t) = log(1 + UFR(t))$.

Then, every day a Last Liquid Forward Rate (LLFR) has to be estimated. The LLFR is a weighted moving overage of forward rates after the first smoothing point. Because market information after the first smoothing point is taken into account, there is less interest rate sensitivity around the first smoothing point and furthermore, changing rates with long maturities are partly taken into account. Only if a change is structural, it influences the shape of the curve. In this thesis we will estimate this LLFR on a yearly basis. The LLFR will be estimated as follows:

$$LLFR(t) = \alpha LLFR(t-1) + (1-\alpha)w\left(f_c(t, 20, 25) + \frac{1}{2}f_c(t, 20, 30) + \frac{1}{4}f_c(t, 20, 40) + \frac{1}{8}f_c(t, 20, 50)\right), \tag{5.6}$$

where it is assumed that $\alpha = \frac{1}{2}$, $w = \frac{8}{15}$ and $LLFR(0)$ is assumed to be equal to $UFR_c(0)$ because this is done in Langejan et al. [2014] as well.

Then we have the input for the extrapolation. First the forward rates after the first smoothing point will be extrapolated. This will be done via the following formula:

$$f_c(t, 20, 20 + l) = UFR_c(t) + (LLFR(t) - UFR_c(t))B(l), \tag{5.7}$$

where $B(l)$ is given by $B(a, l) = \frac{1-e^{-al}}{al}$, where $a = 0.10$ according to Langejan et al. [2014]. The function $B(l)$ is decreasing in l which means that for longer maturities, the LLFR has less influence on the 20-years forward rate.

Secondly, the yields after the first smoothing point can be extrapolated. This will be done via:

$$Y_t^*(20 + l) = \frac{20Y_t(20) + l \cdot f_c(t, 20, 20 + l)}{20 + l}. \tag{5.8}$$

So the term structure is equal to $Y_t(T)$ until the first smoothing point and equal to $Y_t^*(T)$ after the first smoothing point. In the remainder of this thesis, the interest rate at time t for horizon T that follows from the term structure is given by $R_t(T)$.

5.2.2 Assets

As already notified, the pension fund is assumed to be able to invest in the return portfolio and in the matching portfolio. Here, the behavior of the return portfolio and the matching portfolio are described.

Behavior of the return portfolio

Black and Scholes [1973] introduced a model to describe the behavior of assets over time. According to this model, the development of the asset price follows a geometric Brownian motion over time. In that way, stock prices never fall below the level of 0. So here the development in the price of the return portfolio is assumed to follow a geometric Brownian motion. The price of the return portfolio at time t (S_t) is described by the following SDE:

$$dS_t = \mu_t S_t dt + \sigma S_t dW_t^S, \tag{5.9}$$

where μ_t is the expected return on the return portfolio at time t which can be seen as the return on the matching portfolio at time t plus an equity premium which is a compensation for taking risk, σ is the volatility of the stock and dW_t^S is a Brownian motion. This Brownian motion is assumed to be uncorrelated to the Brownian motion of the interest rate. This means that it is assumed that there is no correlation between the return portfolio and the instantaneous nominal interest rate. In this thesis, the expected return on the return portfolio μ_t is assumed to be equal to the return on the matching portfolio which was defined in the previous section ($R_t(1)$) plus an equity premium λ which is equal to 4.8%. With this equity premium the expected return on the return portfolio becomes close to 7% which is the recommended parameter for the return on the return portfolio. This is recommended by the Parameter Committee [Langejan et al., 2014]. They also recommend to assume the volatility of the return portfolio to be 20%, so we assume σ to be 20%.

Behavior of the matching portfolio

The matching portfolio is assumed to have the one-year interest rate as return at time t, which is given by $R_t(1)$. The price of this matching portfolio at time t (B_t) is then described by the following SDE:

$$dB_t = R_t(1)B_t dt, \tag{5.10}$$

where $R_t(1)$ is the one-year interest rate at time t. This SDE does not have a stochastic part. It is advised by this Parameter Committee to let the return on the matching portfolio follow the short term interest rate.

5.2.3 Price inflation

In this thesis we assume price inflation to be stochastic. It is assumed that price inflation follows a mean-reverting process, as well as the short term interest rate does. This is assumed because of the findings in the paper of Lee and Wu [2002]. The SDE for price inflation is given by:

$$dI_t = a_I(\bar{I} - I_t)dt + \sigma_I dW_t^I, \tag{5.11}$$

where I_t is the level of inflation at time t, a_I is the speed of mean reversion, $\bar{I}$ is the long term mean of inflation, σ_I is the volatility of the inflation and dW_t^I is another Brownian motion. As the European Central Bank strives to maintain price stability and price stability is given by a price inflation level of less than, but close to 2% [DNB, 2014a] the long term mean of inflation ($\bar{I}$) is assumed to be 2%. Furthermore, data from last years inflation level show a volatility of about 0.5%, so σ_I is assumed to be 0.5%. Furthermore, the speed of mean reversion (a_I) is assumed to be 0.5. Lastly, the initial value of the price index is assumed to be 1.03%, because this is the average of the price index levels of the year 2014 up until August [Worldwide inflation data, 2014].

5.3 Participants and their wages

In this section, the composition of the pension fund, assumptions about the wages and assumptions about the pension accrual will be explained.

5.3.1 Composition of the fund

In this thesis, it is assumed that every participant starts working at the age of 25. Although the retirement age will increase in the future, at the moment it is close to 65. Therefore, we assume the retirement age to be 65. However, not every participant will reach this age, due to accidents, illnesses etc. Therefore, mortality rates need to be taken into account. One-year mortality rates are available at the website of Statistics Netherlands for every age for both men and women. Here, we will use the average of the mortality rates of men and women to calculate the mortality rate for the participants in the pension fund. These average mortality rates for every age x (q_x) can be found in appendix B. It is assumed that the maximum age is equal to 100, so there are no people alive from the age of 101 onwards.

In this thesis, we assume that in the initial situation the pension fund has participants of every age from 25 until 100. The number of participants of every age can be calculated by using the average mortality rates, simply by taking the product of the average mortality rates. Note that this means that the number of participants at a certain age is less than 1 because the number of 25 years old participants is normalized to 1. These numbers are not rounded. The number of participants of age x (N_x) is given by:

$$N_x = \prod_{i=1}^{x} p_x, \tag{5.12}$$

where p_x is the survival probability given by $p_x = 1 - q_x$. Every year, one participant of age 25 is assumed to enter the fund. Because the number of 25 years old participants is assumed to be 1, the number of participants N_x can also be seen as the part of one participant that is expected to be alive at age x and so it can be used as the probability to reach the age of x conditional on being alive at the age of 25. So in this thesis N_x is used as the number of participants that is alive at age x and it is used as the probability to be alive at age x given that the age of 25 is reached.

It is assumed that mortality rates stay constant over the simulation period. This means that it is assumed that no macro longevity risk is present. Micro longevity risk is assumed to be diversified due to the number of simulations.

5.3.2 Wages

In the report of committee Goudswaard (2009), two different wage profiles are distinguished, namely a flat and an increasing wage profile. A flat wage profile assumes that the wage of participants stays constant over time. An increasing wage profile assumes that the wage of participants increases over time due to developments in career.

In this thesis a wage profile is assumed that includes increases in wage due to career patterns and due to a compensation for wage inflation. This wage profile is based on the policy of the tax authority. The career pattern is as follows: participants' wages increase by 3% yearly in the first decennium of the working life (age 26-35), by 2% in the second decennium of the working life (age 36-45), by 1% in the third decennium of the working life (age 46-55), and stays constant in the rest of the career. Next to the increases in wage due to the career pattern it is assumed that wages are corrected for wage inflation every year. Wage inflation is assumed to be 2.5% which is in line with the Goudswaard report. Furthermore, the wage of a 25 years old participant at time 1 is normalized to 1. Then we come to the following expression for the wage W for age x at time t:

$$W_t^x = \begin{cases} 0 & \text{for } x < 25 \\ 1.03^{x-25} \cdot 1.025^{t-1} & \text{for } 25 \leq x \leq 35 \\ 1.03^{10} \cdot 1.02^{x-35} \cdot 1.025^{t-1} & \text{for } 35 < x \leq 45 \\ 1.03^{10} \cdot 1.02^{10} \cdot 1.01^{x-45} \cdot 1.025^{t-1} & \text{for } 45 < x \leq 55 \\ 1.03^{10} \cdot 1.02^{10} \cdot 1.01^{10} \cdot 1.00^{x-55} \cdot 1.025^{t-1} & \text{for } 55 < x \leq 64 \\ 0 & \text{for } x \geq 65 \end{cases} \tag{5.13}$$

Note that we have to write the wage inflation to the power $t-1$ here because of the normalization that the wage of a 25 year old participant at time 1 is equal to 1.

5.3.3 Pension accrual

Goudswaard et al. [2009] states that the target level of the replacement ratio is 70%. The replacement ratio is defined here as the ratio of the pension payments over the last wage earned before retirement. This replacement ratio should be reached due to the income from the first and second pillar. Nowadays, people realize that the target of 70% is too high and the target is decreased.

A participant accrues pension over the 'pension base', which is defined as the pensionable income minus the offset in order to correct for the income in the first pillar. Here, it is assumed that there is no income from the first pillar and the offset is equal to zero, such that pension is accrued over the whole pensionable income. In this way, these effects cancel each other. The pensionable income is assumed to be the wage.

Every year, a participant of a Dutch pension fund is assumed to accrue 1.875% of the current wage. A participant is assumed to work for 40 years. After 40 years, the participant has accrued 75% of his average wage, which corresponds to a replacement ratio of 63.63% (see appendix C). This replacement ratio is lower than 70%.

Because on average people live longer than years ago, pension is becoming more expensive.

That is not appreciated, so therefore it is decided that less pension can be accrued and the retirement age will increase in the next years and will be linked to life expectancy. In case the retirement age is higher, there will be more years in which pension is accrued and less years in which pension is paid on average.

5.4 Policy of the pension fund

In this section the policy of the pension fund used in the simulations will be described. We will first discuss the contribution policy and the indexation policy. Then we will explain the development of the liabilities and lastly, we will explain the development of the funding ratio over time.

5.4.1 Contributions

In this thesis, we only consider old age pension. This means that we do not consider partner pension and we do not take into account probabilities of disability etc. We assume that the contributions that have to be paid in a year by the active participants are equal to the current value of the rights the active participants accrue in that certain year plus some extra premium to meet the conditions of the required capital. Hereby extra contribution such as for example contributions to finance the costs the pension fund has to make for transactions etc. and contributions to finance the indexation are beyond the scope of this thesis.

We assume that every participant pays the same percentage of the salary to the pension fund. This was already explained in section 3.2. We have seen that, from an actuarial point of view, this means that a participant pays too much when he is young and too little when he is old. A young participant pays too much because the current value of the accrued pension in a certain year is lower than the contribution paid in that year. For an old participant the opposite holds, the current value of the accrued pension in a certain year is higher than the contribution paid in that year.

Now in order to calculate the contribution that should be asked from participants, we will first calculate the wage earned by the total population at time t. This is simply the wage at time t of a participant with age x times the number of participants alive at that age, summed over the period in life in which people work (age 25 until 64):

$$W_t = \sum_{x=25}^{64} W_t^x \cdot N_x. \tag{5.14}$$

Then we need to calculate the pension that is accrued in year t. As the accrual rate is assumed to be 1.875%, it means that every participant accrues 1.875% of his current wage to be received at each age he is alive during future retirement. This has to be multiplied by the probability that he is alive at that age. For this probability we can use N_x as this can be seen as the probability that an age cohort is alive at age x given that they reached the age of 25. These payments in the future have to be discounted to calculate the current value. So the actuarial value of accrued pension (AP) for an age cohort of x years old at time t is given by

$$AP_t^x = \sum_{i=65-x}^{100-x} \frac{1.875\% \cdot W_t^x \cdot N_{x+i}}{(1+R_t(i))^i}. \tag{5.15}$$

This leads to the fact that the actuarial value of the total accrued pension at time t is given by

$$AP_t = \sum_{x=25}^{64} AP_t^x. \tag{5.16}$$

Thus now we have the actuarial value of the total accrued pension at time t and the total wage earned at time t. If the contributions would only be such that accrued pension is covered, the contribution that has to be paid by the active participants at time t can be calculated by:

$$C_t = \frac{AP_t}{W_t}. \tag{5.17}$$

Here, C_t is the percentage of the pensionable income that has to be paid to the pension fund.

In this thesis, we assume the extra contribution to be 20% of the contributions that we just described. Therefore the previous defined contribution level has to be multiplied by 1.20 such that the total contribution is given by:

$$C_t = \frac{AP_t}{W_t} \cdot 1.20. \tag{5.18}$$

This leads to a contribution level of approximately 23% of the pensionable income.

5.4.2 Indexation policy

A pension fund can adjust the accrued pension. For example, if the funding ratio of the pension fund is high, the pension fund can provide indexation. The percentage at which accrued pension is adjusted depends on the indexation that is given, the recovery indexation which is a compensation for missed inflation and the cuts that have to be implemented in a certain year. Every year, the indexation, recovery indexation and cuts that will be implemented depend on the policy funding ratio of last year. The determination of the policy funding ratio is explained at the end of this chapter.

Indexation

The indexation policy that is used to model the pension fund is a 'Staffel' policy. It is assumed that if the policy funding ratio is below a certain lower bound, no indexation will be given. If the policy funding ratio is above a certain upper bound, full indexation is assumed to be given. Furthermore, if the policy funding ratio is between these two bounds, it is assumed that indexation will be given linearly related to the funding ratio. Although this is not the exact way of the new Dutch pension contract which was explained in section 3.4, this adaption will hardly have an influence on the results. Here, the lower threshold for giving indexation is equal to a policy funding ratio of 110%. This is the lower bound of the new Dutch pension contract. If the policy funding ratio is below 110%, no indexation will be given. Furthermore I assume that full indexation is given when the policy funding ratio is at a level of 130%. In that case, pension rights are corrected for full price inflation. For levels of the policy funding ratio between 110% and 130%, indexation is given linearly related to the funding ratio. The part of price inflation that is indexed for every policy funding ratio is shown in figure 5.4.2:

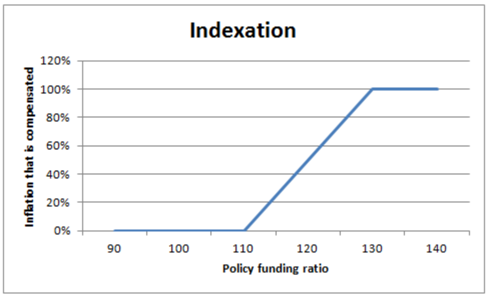


Figure 5.4.2 Part of price inflation that is indexed

Then we come to the following equation for the level of indexation that is given at time t:

$$i_t = \min[\max[\frac{F_t^{policy} - 110\%}{130\% - 110\%}, 0], 1] \cdot I_t, \tag{5.19}$$

where F_t^{policy} is the policy funding ratio that will be described below, 110% is the lower bound for indexation, 130% is the upper bound for indexation and I_t is the price inflation at time t.

Recovery indexation

The pension fund is assumed to provide recovery indexation, which is a compensation for missed indexation, if the funding ratio allows this. Providing recovery indexation is only possible in case the pension fund is very healthy. Here it is assumed that recovery indexation can be provided if the policy funding ratio is above 130%. However, if the pension fund did not index the pension accrual for a few years, it could be the case that the amount of recovery indexation at time t turns out to be relatively big if the pension fund wants to compensate for all missed inflation. In order to avoid this situation, the pension fund should spend only one tenth of the money it does not need to obtain a funding ratio of 130%. Then the formula for the recovery indexation becomes:

$$i_t^r = \max[\frac{1}{10}(\frac{F_t^{policy}}{130\%} - 1), 0]. \tag{5.20}$$

where i_t^r is the recovery indexation at time t, F_t^{policy} is the policy funding ratio at time t and 130% is the threshold whether or not to provide recovery indexation. Note that recovery indexation is given as a compensation for missed inflation, so it is given only in case the pension fund did not provide full indexation before and/or had to cut in benefits before.

Cuts

According to the rules of the new Dutch pension contract, cuts have to be announced as soon as the capacity to recover is inadequate to have a policy funding ratio equal to the RF within 10 years. Then, one-tenth of the difference between the RF and the policy funding ratio plus the capacity to recover has to be cut. This cut is called a small cut in the remainder of this thesis. Note that the RF is dependent on the risk that is taken, i.e. on the fraction of assets that is invested in the return portfolio. Here, the following values for the RF will be used.

Fraction invested in return portfolio	RF
0%	105.0%
1%-20%	112.5%
21%-40%	120.0%
41%-60%	127.5%
61%-80%	135.0%
81%-100%	142.5%

Table 5.4 RF dependent on the fraction invested in the return portfolio

Furthermore, if the pension fund is underfunded for more than 5 years, the pension fund should increase cuts in order to have a funding ratio that is at least equal to 105%. This last cut can be spread over maximally 10 years. For the ease of notation we will assume that this cut will not be spread. The last cut is called a big cut and is mathematically described by:

$$bc_t = \frac{F_t^{end}}{105\%} - 1, \tag{5.21}$$

where bc_t is the big cut at time t, F_t^{end} is the funding ratio at the end of year t and 105% is the MRF. Note that here the funding ratio at the end of the year is used while for the other calculations the policy funding ratio is used. This is in line with the new Dutch pension contract. Moreover, the total amount of cuts in a year (c_t) is the sum of the small cut in that year and the big cut in that year.

5.4.3 Liabilities

As we already have seen, every year a participant accrues 1.875% of the current wage to be paid out every year during future retirement. In year X, the pension rights are the pension rights that were already accrued in year $X-1$ plus the newly accrued pension, possibly increased by indexation and recovery indexation or decreased by a cut. For a retiree, the pension rights also change over time. Retirees do not accrue new pension because they are already retired, but their pension rights change by indexation and cuts as well. So the pension rights for active participants of age x at time t (PR_t^x) can be described by

$$PR_t^x = (PR_{t-1}^{x-1} + 1,875\% \cdot W_t^x \cdot N_x)(1+i_t)(1+i_t^r)(1+c_t), \tag{5.22}$$

and for retired participants of age x the pension rights at time t can be described by

$$PR_t^x = PR_{t-1}^{x-1}(1+i_t)(1+i_t^r)(1+c_t). \tag{5.23}$$

Note that these last two formulas are in general the same because the wage for retirees is equal to zero.

As the accrued pension rights depend on the accrued pension rights one year earlier, we have to know the initial situation. It is assumed that every participant of the pension fund has accrued pension over their working life so far and that every participant worked since the age of 25. Furthermore, it is assumed that pension rights were compensated for price inflation, where the price inflation over the last years is assumed to be equal to the long term average of 2%. In this way, pension rights that the participants have accrued so far can be determined easily. As an example we will calculate the initial pension rights of the participants in the age cohort of age 27. At time 1, this age cohort accrues 1.875% of his wage (W_1^{27}). At time 0, he has accrued pension over his wage W_0^{26} which can be seen as $\frac{W_1^{26}}{1+2.5\%}$ because the wage inflation is 2.5%. This amount is indexed by 2% so at age 26 he has accrued $1.875\% \cdot W_1^{26} \cdot \frac{1.02}{1.025}$. Following the same steps, it can be calculated that this participant has accrued $1.875\% \cdot W_1^{25} \cdot (\frac{1.02}{1.025})^2$ when he was 25, which is indexed every year. Then we come to the following formula for the initial pension rights for each age cohort:

$$PR_1^x = \sum_{i=1}^{min(x,65)} 1.875\% \cdot W_1^i \cdot N_i \cdot (\frac{1.02}{1.025})^{(x-i)} \cdot 1.02^{max(x-65,0)}. \tag{5.24}$$

Now we know the pension rights, we are able to calculate the liabilities of the pension fund. The liabilities of the pension fund are determined by the sum of the liabilities for each age cohort. For an age cohort that is already retired this is simply an annuity of the pension rights starting immediately. For an age cohort of age x that is not yet retired, this is an annuity of the pension rights accrued so far starting in $65-x$ years from now. So the liabilities for each age cohort can be calculated by the current value of this annuity times the probability that the age cohort is alive at retirement, as payments will be made conditional on being alive. The discount factor that will be used is again the term structure at time t. The liabilities for participants aged x at time t are then given by

$$L_t^x = \sum_{i=max(65-x,1)}^{100-x} \frac{PR_t^x \cdot N_{x+i}}{(1+R_t(i))^i}. \tag{5.25}$$

The total liabilities for the pension fund at time t can be calculated by adding the liabilities of all age cohorts:

$$L_t^{begin} = \sum_{x=25}^{100} L_t^x. \tag{5.26}$$

This variable is called L_t^{begin} as it represents the liabilities at the beginning of the year.

5.4.4 Annual progress

Now I will explain the development of the funding ratio over time. We assume the pension fund to go through different stages. Because we consider yearly steps, we assume the stages to take place once a year.

Stage 1

In the first stage the pension benefits (payments) are paid to the retirees. In this stage the contributions of the active participants are collected as well. This changes the level of the assets. This stage is assumed to take place at the beginning of the year, so the assets at the beginning of the year can be described by

$$A_t^{begin} = A_{t-1}^{end} + Contributions_t - Payments_t. \tag{5.27}$$

The liabilities at the first stage can be calculated by using equation (5.26).

So in the first stage we are able to calculate the assets and liabilities of the pension fund at the beginning of every year. This leads to the funding ratio at the beginning of the year:

$$F_t^{begin} = \frac{A_t^{begin}}{L_t^{begin}}. \tag{5.28}$$

We assume that the first value of the assets (A_1^{begin}) is such that the initial funding ratio, which is the first funding ratio at the beginning of the year, is 110%, so $A_1^{begin} = 1.10 \cdot L_1^{begin}$.

Stage 2

Once we know the value of the assets and liabilities at the beginning of the year, we can go trough to the second stage. The point in time of this stage is seen as the end of the period, so here it is seen as the end of the year. Again, the value of the assets changes and now it changes by the return the pension fund gains on its investment. Let π_t be the part of assets at the beginning of year t that is invested in the return portfolio such that $(1 - \pi_t)$ is invested in the matching portfolio. Then the return on the investments (r_t^i) is described by

$$r_t^i = \pi_t(\mu_t + \sigma \cdot N(0,1)) + (1 - \pi_t)R_t(1). \tag{5.29}$$

This leads to the following equation for the assets at the end of the year:

$$A_t^{end} = A_t^{begin}(1 + r_t^i). \tag{5.30}$$

To calculate the total liabilities of the pension fund at the end of the year, we need to increase the liabilities of (5.26) by the one-year interest rate:

$$L_t^{end} = L_t \cdot (1 + R_t(1)). \tag{5.31}$$

Now we know the value of the assets and the liabilities at the end of the year, we are able to calculate the funding ratio of the pension fund at the end of the year:

$$F_t^{end} = \frac{A_t^{end}}{L_t^{end}}. \tag{5.32}$$

Stage 3

On the basis of the funding ratio of stage 2 of this year and last year, we are able to calculate the policy funding ratio. The policy funding ratio of a certain year is defined as the average of all monthly funding ratios in that year. However, since we only calculate funding ratios on a yearly basis, we have to make some assumptions. We assume that the funding ratio grows linearly from the funding ratio at the end of last year to the funding ratio at the end of this year. For example, if last years funding ratio was 110% and this years funding ratio is 120%, the funding ratio increased by 10% this year. We assume that the funding ratio increased $\frac{1}{12} \cdot 10\%$ every month. To calculate the policy funding ratio, the average of these monthly funding ratios is taken. In this example, the policy funding ratio would be 115.83%. The policy funding ratio is called F_t^{policy}.

By using the policy funding ratio, we are able to determine the indexation, recovery indexation and cuts in the next year as described in section 5.4.2. The indexation, recovery indexation and cuts that are calculated, influence the payments and liabilities of the first stage of next year.

To compare different kind of strategies, Monte Carlo simulation is used. Every scenario considers a time span of 50 years and the number of scenarios is equal to 1000.

Chapter 6

Utility specification

6.1 Introduction

In this chapter, utility functions that belong to the goal and preferences of a pension fund and its stakeholders will be specified. From these utility functions, the optimal investment strategies will be determined. A utility function is a function which measures the amount of utility you have, or, in other words, how happy you are with some amount of 'goods'. The 'goods' can be wealth, consumption, money, etc. Different (investment) policies of a Dutch pension fund can be measured and compared by using utility functions. Note that one single value of utility does not say anything. However, it is possible to compare different values of utility to say something about which strategy would be optimal.

As people are usually in the opinion of 'the more, the better', the utility of an individual increases with the amount of goods he has. Therefore, utility functions are upward sloping. Furthermore, one can imagine that an individual that is relatively poor is more happy with one euro of extra income per month than an individual that is relatively rich. This means that the utility function at a low level of goods is steeper than at a high level of goods. Therefore, utility functions are usually concave.[1]

6.2 Utility functions

Now, for a Dutch pension fund, I will specify a concave utility function, where the 'good' is the funding ratio. This is the most important parameter for a pension fund as we have seen in chapter 2. The above specified properties also hold for a pension fund, as a pension fund is happier with a higher funding ratio ('the more, the better') and is happier with an increase in

[1] Cvitanic and Zapatero [2004]

the funding ratio when the funding ratio is at a relatively low level than when the funding ratio is already at a relatively high level.

So in this chapter, utility functions are defined over the funding ratio at the end of the year. For the ease of notation, we will write F_t instead of F_t^{end} in this chapter.

Of course, a concave utility function can be specified in many different ways. However, the 'Constant Relative Risk Aversion (CRRA)' utility function, also known as the power utility function, is the one that is mostly used according to Wakker [2008]. This utility function assumes that investors are risk averse. Some people are more risk averse than others. Risk aversion is specified by the parameter γ, where the higher γ, the higher ones aversion to risk. The CRRA utility function is defined as:

$$U(F_t) = \frac{F_t^{1-\gamma}}{1-\gamma}. \tag{6.1}$$

Here U is the obtained utility, F_t is the funding ratio at the end of year t and γ is the risk aversion parameter.

There are some extensions to the CRRA type of utility function. Those extensions use the CRRA utility function and add a constraint. Because the utility function defined in (6.1) does not have any constraints, we call this equation the utility function for the unconstrained investor. One constraint that can be added to this utility function is the constraint that a pension fund wants to achieve a specified 'floor' (k) at any time in the future. If the funding ratio reaches a level that is below the floor, the pension fund has a very low utility. This is mathematically described by:

$$U(F_t) = \begin{cases} -\infty & \text{if } F_t < k \\ \frac{F_t^{1-\gamma}}{1-\gamma} & \text{if } F_t \geq k. \end{cases} \tag{6.2}$$

For a pension fund, the value of the floor will be a funding ratio. Then the interpretation of the utility function is that the pension fund is very dissatisfied if its funding ratio is below the funding ratio k.

Another type of variation on the CRRA utility function is to add a constraint that assumes the pension fund to have a satiation level (k_+). Then it is assumed that the pension fund does not obtain a higher utility from a funding ratio higher than the satiation level. This satiation level can be called an upper bound, as at the satiation level the highest utility is obtained. Here we assume again that if the funding ratio reaches a level that is below the floor, the pension fund becomes very unhappy. If the funding ratio of the pension fund reaches a level that is above the satiation level, the pension fund will not become more satisfied than it is at the level of satiation. This is mathematically described by:

$$U(F_t) = \begin{cases} -\infty & \text{if } F_t < k \\ \frac{F_t^{1-\gamma}}{1-\gamma} & \text{if } k \leq F_t \leq k_+ \\ \frac{k_+^{1-\gamma}}{1-\gamma} & \text{if } F_t > k_+. \end{cases} \tag{6.3}$$

Lastly, I will specify the type of variation on the CRRA utility function that is about the surplus, i.e. the funding ratio minus the level of the floor. This utility function is not specified for funding ratios that are less than the level of the floor, so for funding ratios less than the floor we assume the pension fund to have a very low utility, i.e. minus infinity. Actually, the utility function is shifted to the right. This variation is mathematically described by:

$$U(F_t) = \begin{cases} -\infty & \text{if } F_t < k \\ \frac{(F_t-k)^{1-\gamma}}{1-\gamma} & \text{if } F_t \geq k. \end{cases} \tag{6.4}$$

This utility function can be extended to the case of the satiation level as well:

$$U(F_t) = \begin{cases} -\infty & \text{if } F_t < k \\ \frac{(F_t-k)^{1-\gamma}}{1-\gamma} & \text{if } k \leq F_t \leq k_+ \\ \frac{(k_+-k)^{1-\gamma}}{1-\gamma} & \text{if } F_t > k_+. \end{cases} \tag{6.5}$$

Now in order to find an optimal investment policy for a pension fund, we can maximize these utility functions. This is possible because the preferences of the pension fund are incorporated in the utility functions. In chapter 2 it was stated that a tradeoff has to be made between the ambition, costs and risk.

In the utility functions the ambition is taken into account, as the higher the funding ratio, the higher the utility level and this utility level will be maximized.

Furthermore, risk is incorporated in the utility function as well, since the level of the utility depends on the risk aversion. Some pension funds are more risk averse than others. A grey pension fund for example, in which the ratio between old individuals and young individuals is relatively high, is more risk averse than a pension fund in which this ratio is lower, because the grey pension fund has to pay the pension benefits earlier than the other pension fund, which means that it has less time to recover from a downward shock [Jagannathan and Kocherlakota, 1996].

Currently in the Netherlands, pension funds are obliged to ask contribution levels that are at least equal to the cost covered contribution level. As explained in the previous chapter, it is assumed that the contributions will be set such that they cover the new accrued pension rights every year, plus some extra contribution. It is not likely that more than this contribution level will be asked, because then the contributions will become very large and this is not attractive.

It is not likely that less than this level will be asked, because the pension fund should be very healthy then, and this is currently not the case.[2] In this thesis we assume the contribution levels to be constant, so they do not have to be taken into account in the utility functions.

6.3 Optimization of utility functions

As already said, for the ease of notation, we will use F_t instead of F_t^{end} in this chapter. Furthermore, because we will only consider the end of the year in this section, we will write A_t instead of A_t^{end} and L_t instead of L_t^{end}.

The principle of expected utility maximization states that a rational investor should take the investment strategy that maximizes his or her expected utility. In this thesis it will be assumed that assets follow a geometric Brownian motion process, as we had already seen in the previous chapter. Constant opportunities are assumed, which means that it is assumed that the equity premium (λ) and the volatility of the assets in the return portfolio stay constant over time.

In the previous chapter, we have seen that the value of the liabilities of the pension fund depends on the value of the assets, because the value of the assets decides whether to implement cuts or to give (recovery) indexation. However, in this chapter we need to assume that liabilities increase by the one-year interest rate ($R_t(1)$) and by wage inflation every year. The liabilities were assumed to be non-stochastic. Furthermore, here we assume to have a Stochastic Discount Factor (SDF) Z_t to discount cash flows. The three GBMs are given by:

$$\begin{aligned} dA_t &= [R_t(1) + \pi \cdot \lambda] A_t dt + \pi \sigma A_t dW_t \\ dL_t &= R_t(1) \cdot 1.025 \cdot L_t dt \\ dZ_t &= -R_t(1) Z_t dt - \theta Z_t dW_t. \end{aligned} \tag{6.6}$$

Here, θ is defined as $\theta = \frac{\lambda}{\sigma}$. Note that with these three GBMs we assume the contributions received in a certain year and the pension payments paid to the retirees in that year to be equal to each other, because we assume the assets to grow with the return on investments here. We assume that the stochastic term in the SDF is the same as in the assets. Furthermore, we also assume that no indexation, recovery indexation and cuts take place. Therefore, the results of this section are just an approximation for the optimal investment policies under the different utility functions. However, as we will see, the solutions approximate the optimal investment strategies quite well.

[2] DNB [2007]

The solutions of the three GBMs are:

$$\begin{aligned} A_T &= A_0 \cdot \exp((R_t(1) + \pi\lambda - \tfrac{1}{2}(\pi\sigma)^2)T + \pi\sigma W_T) \\ L_T &= L_0 \cdot \exp(R_t(1) \cdot 1.025 \cdot T) \\ Z_T &= Z_0 \cdot \exp(-(R_t(1) + \tfrac{1}{2}\theta^2)T - \theta W_T), \end{aligned} \tag{6.7}$$

where A_0, L_0 and Z_0 are the initial values of the assets, liabilities and SDF, respectively.

With the use of these solutions, I will maximize the utility functions which were described in the previous section.

Unconstrained investor

Let me first maximize the utility function of the unconstrained investor, i.e. the utility function to be optimized is given by equation (6.1). The utility function should be optimized with respect to the level of the assets, as the level of the liabilities cannot be influenced. The level of the assets at time t (A_t) are all payoffs that can be obtained starting from the initial level of the assets (A_0) following some dynamic investment strategy. By assuming that the market is complete, the utility function has to be optimized subject to the constraint $E[A_tZ_t] = A_0$. By using Lagrange and using the fact that $F_t = \frac{A_t}{L_t}$, we come to the following:

$$\begin{aligned} & \underset{A_t}{max} && E[U(F_t)] \ s.t.\ E[A_tZ_t] = A_0 \\ \Leftrightarrow\ & \underset{A_t}{max} && \tfrac{F_t^{1-\gamma}}{1-\gamma} - \lambda(A_tZ_t - A_0) \\ & \underset{A_t}{FOC} && F_t^{-\gamma}\tfrac{1}{L_t} - \lambda Z_t = 0 \\ & \Leftrightarrow && F_t = \zeta^{-\frac{1}{\gamma}}(Z_tL_t)^{-\frac{1}{\gamma}} \\ & \Leftrightarrow && A_t = \zeta^{-\frac{1}{\gamma}}(Z_t)^{-\frac{1}{\gamma}}(L_t)^{1-\frac{1}{\gamma}} \\ & \Leftrightarrow && A_t = \zeta^{-\frac{1}{\gamma}} exp((\tfrac{1}{\gamma}(R_t(1) + \tfrac{1}{2}\theta^2) + (1 - \tfrac{1}{\gamma}) \cdot 1.025 \cdot R_t(1))t + \tfrac{\theta}{\gamma}W_t). \end{aligned} \tag{6.8}$$

In the remainder of this section I will refer to F_t as $F_t^u = \zeta^{-\frac{1}{\gamma}}(Z_tL_t)^{-\frac{1}{\gamma}}$ which is defined as the funding ratio at time t for the unconstrained investor.

As we already know that $A_t = A_0 \cdot \exp((R_t(1) + \pi \cdot \lambda - \frac{1}{2}(\pi\sigma)^2)t + \pi\sigma W_t)$, it has to hold that

$$\frac{\theta}{\gamma} = \pi\sigma, \tag{6.9}$$

from which we can derive the optimal weights for the unconstrained investor:

$$\pi_u = \frac{1}{\gamma} \cdot \frac{\lambda}{\sigma^2}. \tag{6.10}$$

As constant opportunities are assumed, the proportion invested in stocks stays constant. This corresponds to the constant mix investment strategy, explained in subsection 4.2.

Utility function with a floor

Secondly, I will optimize the utility function with the floor given by equation (6.2) subject to the constraint $E[A_tZ_t] = A_0$. By using Lagrange, we get:

$$\begin{array}{ll} \underset{A_t}{max} & E[U(F_t)]\ s.t.\ E[A_tZ_t] = A_0\ and\ F_t \geq k \\ \underset{A_t}{max} & \frac{F_t^{1-\gamma}}{1-\gamma} - \nu(A_tZ_t - A_0) - \eta(F_t - k) \\ \underset{A_t}{FOC} & F_t^{-\gamma}\frac{1}{L_t} - \nu Z_t = 0\ or\ F_t = k \quad (*) \\ \Leftrightarrow & F_t = max[\nu^{-\frac{1}{\gamma}}(Z_tL_t)^{-\frac{1}{\gamma}}, k] \\ \Leftrightarrow & F_t = max[hF_t^u, k]. \end{array} \tag{6.11}$$

(*): The Kuhn-Tucker conditions say that $\eta(F_t - k)$ should be zero. So either η is equal to zero, or $F_t - k$ is equal to zero.

In the equation above, $h = (\frac{\nu}{\zeta})^{-\frac{1}{\gamma}}$ and $F_t^u = (\zeta Z_tL_t)^{-\frac{1}{\gamma}}$ as we have seen before. This strategy corresponds to the OBPI strategy which is explained in subsection 4.3.1, because we can write F_t as $F_t = k+\max[hF_t^u - k, 0]$, which can be seen as investing in the matching portfolio in order to obtain enough to reach the value of the floor and invest the rest in calls. This leads to:

$$\begin{array}{rll} & F_t^{OBPI} & = \max[hF_t^u, k] \\ \Leftrightarrow & A_t^{OBPI} & = \max[hA_t^u, kL_t] \\ & & = \max[hA_t^u - kL_t, 0] + kL_t \\ \Leftrightarrow & A_0^{OBPI} & = hA_0^uN(d_{1,t}) - kL_0N(d_{2,t}) + kL_0 \end{array} \tag{6.12}$$

which holds due to the Black Scholes formula which is explained in Appendix A. Here, $d_{1,t}$ and $d_{2,t}$ are given by

$$d_{1,t} = \frac{log(hA_0^u/(k \cdot L_0)) + (R_t(1) + \sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_{2,t} = d_{1,t} - \sigma\sqrt{T}. \tag{6.13}$$

Then we have

$$\begin{array}{rl} A_0^{OBPI} & = hA_0^uN(d_{1,t}) - kL_0N(d_{2,t}) + kL_0 \\ & = hA_0^uN(d_{1,t}) + kL_0(1 - N(d_{2,t})) \\ & = hA_0^uN(d_{1,t}) + kL_0N(-d_{2,t}) \\ \text{and so } hA_0^uN(d_{1,t}) & = A_0^{OBPI} - kL_0N(-d_{2,t}). \end{array} \tag{6.14}$$

This leads to

$$\begin{array}{rl} A_t^{OBPI}\pi_t^{OBPI}\sigma & = kL_tN(-d_{2,t})\sigma_L + hA_t^uN(d_{1,t})\sigma_u \\ & = hA_t^uN(d_{1,t})\sigma_u \\ & = (A_t^{OBPI} - kL_tN(-d_{2,t}))\sigma_u \\ \Leftrightarrow\ \pi_t^{OBPI} & = \left(1 - \frac{kN(-d_{2,t})}{F_t}\right)\pi_u \\ & = \left(1 - \frac{kN(-d_{2,t})}{F_t}\right)\frac{\lambda}{\gamma\sigma^2}. \end{array} \tag{6.15}$$

Utility function with a floor and satiation level

Then I will optimize the utility function with the floor and the satiation level given by equation (6.3) subject to the constraint $E[A_t Z_t] = A_0$. Here again we have to make sure that the funding ratio is indeed above the floor k at maturity and now we also have that the pension fund does not obtain a higher utility in case the funding ratio becomes above the satiation level k_+. In theory, one of the possibilities that would be appropriate is to buy puts with a strike price of k and to sell calls with a strike price of k_+. If the level of the assets decreases so much that the funding ratio of the pension fund becomes below the floor k, the put options can be exercised and if the level of the assets increases so much that the funding ratio of the pension fund is above the satiation level, the call options will be exercised by the buyer. The same results will be obtained if the pension fund invests in the risk free asset in order to have a guaranteed funding ratio at the end of the period and buys call options from the remaining money. It can then sell call options as well to incorporate the satiation level.

This strategy corresponds to another OBPI strategy. Thus, the funding ratio should be the maximum of hF_t^u and the floor k minus $hF_t^u - k_+$ in case that amount is positive:

$$
\begin{aligned}
& & F_t^{OBPI} &= \max[hF_t^u, k] - \max[hF_t^u - k_+, 0] \\
\Leftrightarrow & & A_t^{OBPI} &= \max[hA_t^u, kL_t] - \max[hA_t^u - k_+L_t, 0] \\
& & &= \max[hA_t^u - kL_t, 0] - \max[hA_t^u - k_+L_t, 0] + kL_t \\
\Leftrightarrow & & A_0^{OBPI} &= hA_0^u N(d_{1,t}) - kL_0 N(d_{2,t}) + kL_0 - (hA_0^u N(d_{1,t}^+) - k_+ L_0 N(d_{2,t}^+)) \\
& & &= hA_0^u N((d_{1,t}) - N(d_{1,t}^+)) + kL_0(N(-d_{2,t}) + N(d_{2,t}^+)) \\
and & & hA_0^u N((d_{1,t}) - N(d_{1,t}^+)) &= A_0^{OBPI} - kL_0(N(-d_{2,t}) + N(d_{2,t}^+)).
\end{aligned}
\tag{6.16}
$$

Here $d_{1,t}$ and $d_{2,t}$ are given by (6.13) and $d_{1,t}^+$ and $d_{2,t}^+$ are given by

$$
d_{1,t}^+ = \frac{log(hA_0^u/(k_+ \cdot L_0)) + (R_t(1) + \sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_{2,t}^+ = d_{1,t}^+ - \sigma\sqrt{T}. \tag{6.17}
$$

Then we have that

$$
\begin{aligned}
A_t^{OBPI}\pi_t\sigma &= kL_t(N(-d_{2,t}) + N(d_{2,t}^+))\sigma_L + hA_t^u(N(d_{1,t}) - N(d_{1,t}^+))\sigma_u \\
&= hA_t^u(N(d_{1,t}) - N(d_{1,t}^+))\sigma_u \\
&= (A_t^u - kL_t(N(-d_{2,t}) + N(d_{2,t}^+)))\sigma_u \\
\Leftrightarrow \quad \pi_t &= (1 - \tfrac{k}{F_t}(N(-d_{2,t}) + N(d_{2,t}^+)))\pi_u \\
&= (1 - \tfrac{k}{F_t}(N(-d_{2,t}) + N(d_{2,t}^+)))\tfrac{\lambda}{\gamma\sigma^2}.
\end{aligned}
\tag{6.18}
$$

Utility function over the surplus

Now, I will maximize the utility function over the surplus given by equation (6.4) subject to the constraint $E[A_tZ_t] = A_0$. Then, again using Lagrange, the optimization is stated as follows:

$$\begin{array}{ll} \underset{A_t}{max} & E[U(F_t)] \ s.t. \ E[A_tZ_t] = A_0 \\ \underset{A_t}{max} & \frac{(F_t-k)^{1-\gamma}}{1-\gamma} - \nu^*(A_tZ_t - A_0) \\ \underset{A_t}{FOC} & (F_t-k)^{-\gamma}\frac{1}{L_t} - \nu^* Z_t = 0 \\ \Leftrightarrow & F_t = k + (\nu^*)^{-\frac{1}{\gamma}}(Z_tL_t)^{-\frac{1}{\gamma}} \\ \Leftrightarrow & F_t = k + h^*F_t^u. \end{array} \tag{6.19}$$

In the equation above we have $h^* = (\frac{\nu^*}{\zeta})^{-\frac{1}{\gamma}}$ and $F_t^u = (\zeta Z_t L_t)^{-\frac{1}{\gamma}}$ as we have seen before. Then it follows that

$$F_t - k = h^*F_t^u \qquad \text{and} \qquad h^*A_t^u = A_t - kL_t \tag{6.20}$$

which leads to

$$\begin{array}{rl} A_t\pi_t^*\sigma & = kL_t\sigma_L + h^*A_t^u\sigma_u \\ & = h^*A_t^u\sigma_u \\ & = (A_t - kL_t)\sigma_u \\ \Leftrightarrow \quad \pi_t^* & = \left(1 - \frac{k}{F_t}\right)\pi_u \\ & = \left(1 - \frac{k}{F_t}\right)\frac{\lambda}{\gamma\sigma^2}. \end{array} \tag{6.21}$$

This corresponds to the CPPI strategy in an ALM setting which is explained in subsection 4.3.2. Here, $\left(1 - \frac{k}{F_t}\right)$ is the risk budget.

Utility function over the surplus with a satiation level

Lastly, I will maximize the utility function that is given by equation (6.5) subject to the constraint $E[A_tZ_t] = A_0$. As this utility function is about the surplus, it is not defined for levels of the funding ratio less than k. To incorporate the satiation level, the pension fund can invest such that it replicates the selling of calls just as we have seen in the OBPI investment strategies. Then we come to the following formula for the funding ratio:

$$F_t = k + h^*F_t^u - \max[k + h^*F_t^u - k_+, 0]. \tag{6.22}$$

This leads to

$$\begin{array}{lrl} & F_t & = k + h^*F_t^u - \max[k + h^*F_t^u - k_+, 0] \\ \Leftrightarrow & A_t & = kL_t + h^*A_t^u - \max[h^*A_t^u - (k_+ - k)L_t, 0] \\ \Leftrightarrow & A_0 & = kL_0 + h^*A_0^u - h^*A_0^uN(d_{1,t}^+) + (k_+ - k)L_0N(d_{2,t}^+) \\ & & = h^*A_0^u(1 - N(d_{1,t}^+)) + L_0(k + (k_+ - k)N(d_{2,t}^+)) \\ & & = h^*A_0^uN(-d_{1,t}) + L_0(k + (k_+ - k)N(d_{2,t})) \\ and & h^*A_0^uN(-d_{1,t}) & = A_0 - L_0(k + (k_+ - k)N(d_{2,t})) \end{array} \tag{6.23}$$

and so we have that

$$\begin{aligned}
A_t\pi_t^*\sigma &= L_t(k+(k_+-k)N(d_{2,t}))\sigma_L + h^*A_t^uN(-d_{1,t})\sigma_u \\
&= h^*A_t^uN(-d_{1,t})\sigma_u \\
&= (A_t - L_t(k+(k_+-k)N(d_{2,t})))\sigma_u \\
\Leftrightarrow \pi_t^*\sigma &= (1-\tfrac{(k+(k_+-k)N(d_{2,t}))}{F_t})\sigma_u \\
\Leftrightarrow \quad \pi_t^* &= (1-\tfrac{(k+(k_+-k)N(d_{2,t}))}{F_t})\pi_u \\
&= \left(1-\tfrac{(k+(k_+-k)N(d_{2,t}))}{F_t}\right)\tfrac{\lambda}{\gamma\sigma^2}.
\end{aligned} \tag{6.24}$$

This corresponds to a combination of the CPPI strategy and the OBPI strategy in an ALM setting. Here, $\left(1-\frac{(k+(k_+-k)N(d_{2,t}))}{F_t}\right)$ is the risk budget.

Note that in this chapter some assumptions were made that are not applicable to the pension fund of this thesis. For the assumed pension fund, the value of the assets changes every year by the payments to the retirees and the contributions received. The liabilities possibly change by indexation, recovery indexation or cuts. Furthermore, the stochastic discount factor is different from the method described in this chapter. The assumption that the market is complete is not applicable as well, as there are for example almost no zero coupon bonds with a maturity of 70 years in the market.

Chapter 7

Considered strategies

7.1 Introduction

In the previous chapter we have seen different kind of utility functions for a pension fund. We have seen that if a pension fund has an unconstrained utility function then the constant mix investment strategy is optimal under some assumptions. However, if a pension fund has a floor in the utility function, the pension fund really wants to avoid funding ratios below that floor. Therefore, it would be useful to insure themselves against these events. Portfolio insurance strategies might be useful and we indeed have seen that these strategies are optimal investment strategies under some assumptions. We have seen that if the constrained utility function of the pension fund is defined over the funding ratio then OBPI is optimal, and if the pension only gains utility over the surplus, which is defined as the funding ratio minus the floor, then CPPI turned out to be optimal.

In this chapter, the investment strategies that we have taken into consideration are explained. The optimal investment strategy of the unconstrained investor is the constant mix investment strategy. Because this investment strategy is the strategy that is currently mostly used by pension funds, we will compare the other investment strategies with this investment strategy. We will not only consider the optimal constant mix investment strategy but we will also consider other constant mix investment strategies, to make a better comparison.

In the next section, the constant mix investment strategy will be explained. Then the OBPI investment strategies that are considered will be explained and lastly the CPPI investment strategies will be explained.

7.2 Constant mix

As we already explained in chapter 4, constant mix strategies are such that a constant proportion of the assets is invested in the return portfolio and the rest is invested in the matching portfolio. If the market changes, the proportion invested in the return portfolio changes as well. If the proportion has changed by the end of the year, the portfolio will be rebalanced. In the previous chapter, we have seen that a mix that is such that $\pi_u = \frac{\lambda}{\gamma\sigma^2}$ is optimal. We already explained that we assumed σ to be 20% and the equity premium (λ) to be 4.8%. Furthermore, values for the risk aversion typically lie between zero and ten. Here, we assume γ to be equal to 3 as this is the moderate risk aversion according to Horneff et al. [2006]. If we then fill the formula we have that the optimal investment in the return portfolio is $\pi_u = \frac{\lambda}{\gamma\sigma^2} = \frac{1}{3}\frac{4.80\%}{(20\%)^2} = 40\%$ of the assets at the beginning of the year.

The optimal investment strategy for this investor seems to be to invest 40% of the assets at the beginning of the year in the return portfolio and 60% of the assets at the beginning of the year in the matching portfolio. This corresponds to the average pension fund.[1] For this, the notation (40%/60%) will be used from now on.

As already said in the introduction, we will compare the portfolio insurance investment strategies with the constant mix strategies. Therefore it is useful to consider more than one constant mix investment strategy. In this thesis, we will look at the constant mix (0%/100%), (25%/75%), (50%/50%), (75%/25%), (100%/0%) investment strategies as well in order to compare the portfolio insurance investment strategies.

7.3 OBPI

In this thesis we will consider three OBPI investment strategies. The first one only assumes to have a floor and the second one assumes, next to a floor, also that profits that are made will be locked.[2] The third OBPI investment strategy assumes a satiation level next to the floor.

In this thesis, we will only use call options. Note that the same results can be obtained by using put options. This is due to the put-call parity as explained in Appendix A. Furthermore, we assume the fraction invested in the return portfolio for the options to be equal to the optimal constant mix fraction, so we assume it to be equal to 40%.

[1] Pensioenkijker [2008]

[2] Overhaus [2007] explains that for a CPPI investment strategy an investor can lock in gains it has made from upside movements of the market. That is what we will consider in this thesis as well. Next to a CPPI with lock investment strategy, we will also consider an OBPI with lock investment strategy.

7.3.1 OBPI without lock

For the OBPI investment strategy with a floor we assume that the pension fund invests part of its assets at the beginning of the year in the matching portfolio, such that it is sure that it has a funding ratio of at least 105% at the end of the year. The remaining money will be invested in call options with a strike price that is equal to 1.05 times the liabilities at the end of the year. In that way, there is upside potential and the funding ratio will always be at least 105%.

However, if the pension fund has a funding ratio at the beginning of the year that is less than or equal to 105%, no options can be bought. This is possible for example if the funding ratio of the year before was 105% and the contributions received at the beginning of the year were less than the payments that had to be made to the retirees. Then, it is assumed that the pension fund fully invests its assets in the matching portfolio.

7.3.2 OBPI with lock

The OBPI with lock investment strategy is assumed to follow the same strategy as the OBPI investment strategy without lock. The only difference is that as soon as the funding ratio reaches a level of 135%, the pension fund wants to lock this profit and wants to reduce the risk it takes. Then, more money will be invested in the matching portfolio, just as much as is needed to have a funding ratio of 130% at the end of the year. The money that is left will be used to buy call options with a strike price that is equal to 1.30 times the value of the liabilities at the end of the year.

Here it can also be the case that the funding ratio drops below the level of 135% after it had been above this level. Then, the floor will be set equal to 105% again.

If the funding ratio even drops below the level of 105%, the floor will be set equal to 100%. Then there is more potential for upward gains.

7.3.3 OBPI with satiation

The OBPI with satiation investment strategy is such that as much call options with a strike price of k times the value of the liabilities at the end of the year will be bought as call options with a strike price of k_+ times the value of the liabilities at the end of the year will be sold. So the pension fund is assumed to invest in a package of call options, such that buying one package means to buy a call option with a strike price of k times the value of the liabilities at the end of the year and to sell one call option with a strike price of k_+ times the value of the liabilities at the end of the year.

The satiation level k_+ is assumed to be equal to 135%. Then full indexation can be given and there is some space for providing recovery indexation in case that is needed.

Again, the pension fund invests part of its assets at the beginning of the year in the matching portfolio, such that it is sure that it has a funding ratio of at least 105% at the end of the year. The remaining assets will be used to buy the packages of call options. When we compare this strategy with the OBPI without lock investment strategy, the price of a package of call options is less than the price of one call option.

Again, if the pension fund has a funding ratio at the beginning of the year that is less than or equal to 105%, no packages can be bought. Then it is assumed that the pension fund fully invests its money in the matching portfolio.

7.4 CPPI

In this thesis, we will research three different CPPI investment strategies. The first one is a CPPI investment strategy that assumes to have a floor. The second CPPI investment strategy is such that it locks profits it has made. The third one is the strategy that incorporates a satiation level such that it sells upside potential by selling call options. As CPPI investment strategies are designed to be optimal if rebalancing is done continuously, annually rebalancing would probably not be optimal. However, continuously rebalancing is not possible in practice. Therefore, we will consider these three described strategies both when rebalancing takes place once a year and when rebalancing takes place once a month.

7.4.1 CPPI without lock

For the investor that has a utility function that is defined over the surplus we have seen that the optimal proportion invested in the return portfolio is equal to $\pi_t^* = \left(1 - \frac{k}{F_t}\right)\frac{\lambda}{\gamma\sigma^2}$, where F_t was defined as the funding ratio at the end of the year. If we compare this to the CPPI investment strategy described in section 4.3.2, we see that the formulas correspond to each other. For the value of the floor we have $1.05 \cdot L_t^{end}$ here and for the value of the multiplier, we have $\frac{\lambda}{\gamma\sigma^2}$, which is equal to 40% as we have seen in the previous section. However, as Perold and Sharpe [1988] describe, constant proportion investment strategies are indeed of this type, but to make sure that the strategy is a CPPI investment strategy, it should be such that the multiplier is greater than one. Therefore, we will use another multiplier.

Perold and Sharpe [1988] explain that the multiplier m should be such that the market can fall by at most $1/m$ per period. Here we will explain the determination of the multiplier m for the yearly rebalancing, and for the monthly rebalancing we will use the same multiplier. So here it should be such that the market can fall by at most $1/m$ per year. If the market falls by more than $1/m$ in a year, there is no time to rebalance in time and the value of the assets reaches a level below the floor. If the return portfolio falls by $\frac{1}{m}$, this corresponds to a return of $1-\frac{1}{m}$ on the return portfolio. Here m will be defined slightly different, just in order to keep notations easy.

We know that the yearly return on the return portfolio is equal to $\mu_t + \sigma N(0,1)$. To calculate the value of the assets that are invested in the return portfolio at the end of the year, we have to multiply the assets that were invested in the return portfolio at the beginning of the year with $1 + R_t(1) + \lambda + \sigma N(0,1)$, where λ is the equity premium. This is greater than or equal to $1 + R_t(1) + \lambda + \sigma\Phi^{-1}(0.001)$ with a certainty of 99.9%, where Φ^{-1} is the inverse normal distribution. Now we are able to determine our multiplier m:

$$\begin{aligned} 1 + R_t(1) + \lambda + \sigma\Phi^{-1}(0.001) &= \tfrac{1+R_t(1)+\lambda+\sigma\Phi^{-1}(0.001)}{1+R_t(1)}(1+R_t(1)) \\ &= \left(1 + \tfrac{\lambda+\sigma\Phi^{-1}(0.001)}{1+R_t(1)}\right)(1+R_t(1)) \\ &\geq \left(1 - \tfrac{1}{m_t}\right)(1+R_t(1)) \\ \Leftrightarrow \quad \tfrac{1}{m_t} &= -\tfrac{\lambda+\sigma\Phi^{-1}(0.001)}{1+R_t(1)} \\ \Leftrightarrow \quad m_t &= -\tfrac{1+R_t(1)}{\lambda+\sigma\Phi^{-1}(0.001)}. \end{aligned} \tag{7.1}$$

If we fill out this formula with 2.2% for $R_t(1)$, 4.8% for λ and 20% for σ, we obtain a multiplier of 1.79.

In order to calculate the fraction of the assets at the beginning of the year to be invested in the return portfolio in order to reach a funding ratio of at least 105%, we need:

$$\begin{aligned} A_t^{begin}[\pi_t(1+r_t^i) + (1-\pi_t)(1+R_t(1))] &\geq A_t^{begin}[\pi_t(1-\tfrac{1}{m_t})(1+R_t(1)) + (1-\pi_t)(1+R_t(1))] \\ &= 1.05 \cdot L_t^{end}. \end{aligned} \tag{7.2}$$

If we solve this for π_t, we indeed see that

$$\pi_t = m_t\left(1 - 1.05\frac{L_t^{begin}}{A_t^{begin}}\right) = m_t\left(1 - \frac{1.05}{F_t^{begin}}\right), \tag{7.3}$$

would be the optimal investment strategy, which corresponds to the result of the previous chapter. Note that here we have F_t^{begin} instead of F_t^{end}.

In the next figure, the fraction invested in the return portfolio against the funding ratio at the beginning of the year is illustrated:

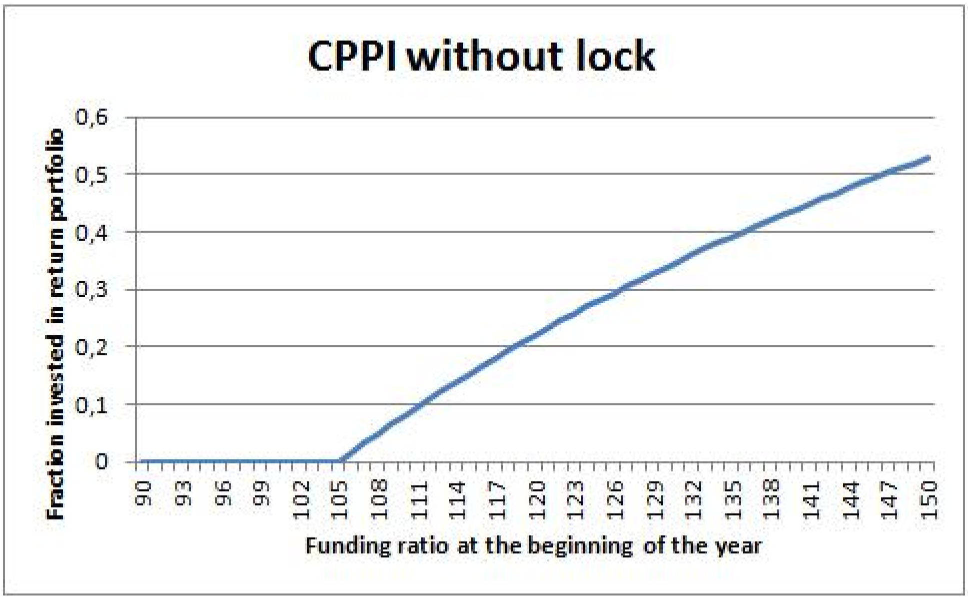


Figure 7.4.1 Fraction invested in the return portfolio plotted against the funding ratio at the beginning of the year

For the monthly rebalancing, the same multiplier will be used. The only difference is that the fraction invested in the return portfolio needs to be adjusted every month. For this, we will use equation (7.3), where A_t^{begin} will be replaced by the level of the assets in the previous month.

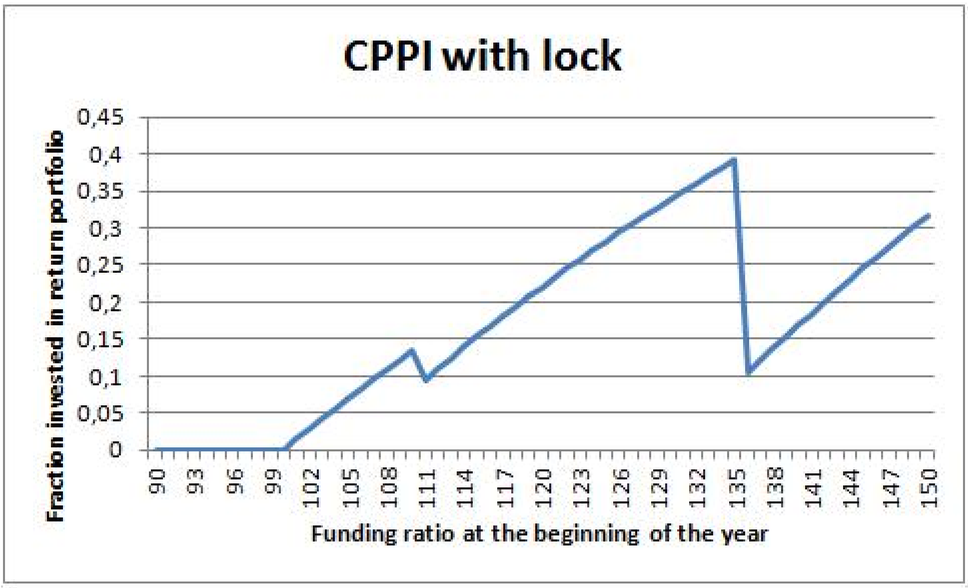
7.4.2 CPPI with lock

The second CPPI investment strategy is such that it locks in profits that are already made. In this thesis, we will assume the following for the CPPI with lock investment strategy:

If the funding ratio is between 110% and 135% at the beginning of the year, the same strategy as the CPPI without lock investment strategy will be used.

If the funding ratio is below 110% at the beginning of the year, the floor and the multiplier will be adjusted. The floor will assumed to be 100% of the liabilities at the end of the year. In that way, the cushion becomes larger which allows to invest more in the return portfolio. This increases the upside potential. However, the pension fund has to make sure that the funding ratio does not decrease further. Therefore, we will adjust the multiplier. Now we want to have more certainty and we want the return on the investments to be greater than $(1 - \frac{1}{m})$ with a certainty of 99.99%. This leads to a multiplier of about 1.47.

If the funding ratio becomes above 135% at the beginning of the year, the pension fund can lock in the profit it has made. Then, again the floor and the multiplier will be adjusted. The floor will assumed to be 130% of the liabilities at the end of the year. In that way, the fraction of the assets invested in the return portfolio will decrease drastically. However, this leads to the fact that less risk is taken and the probability that the funding ratio stays at this high level is increased. If this strategy would be applied from the level of 130% onwards, no investment in the return portfolio would be taken if the funding ratio is at this level. Then there would be no potential for any gains which is not desirable. Furthermore, the level of the multiplier is adjusted. Because we have a buffer, the multiplier can be increased. We want that the return on the investments is greater than $(1 - \frac{1}{m})$ with a certainty of 99% which leads to a multiplier of approximately 2.45. In the next graph, the fraction invested in the return portfolio against the funding ratio can be seen:



Graph 7.4.2 Fraction invested in the return portfolio plotted against the funding ratio

In the figure we see that if the funding ratio of the pension fund at the beginning of the year is below 110%, the pension fund increases its fraction invested in the return portfolio, because it wants to have upside potential. Once the level of 110% is reached, the fraction invested in the return portfolio decreases because the pension fund wants to lock the profit it has made.

For the monthly rebalancing, the multiplier and floor will be adjusted every month in the same way as for the yearly rebalancing. Furthermore, the fraction invested in the return portfolio needs to be adjusted every month.

7.4.3 CPPI with satiation level

For a pension fund that has a utility function defined over the surplus and has a satiation level, we know that the pension fund does not gain a higher utility level from having a funding ratio above the satiation level. Therefore, it might be useful for that pension fund to sell call options with a strike price equal to the satiation level. Then if the funding ratio is below the satiation level, the call option will not be exercised so that does not give any problems, and if the funding ratio is above the satiation level, the call option will be exercised and the funding ratio is equal to the satiation level which is good enough for that pension fund.

The advantage of selling call options is that the pension fund will receive the price of the call option at the beginning of the year anyway. The price of the sold call option is assumed to be invested in the matching portfolio here. Note that the poorer the pension fund is, the less expensive the call options are.

Here, we again assume the satiation level to be a funding ratio of 135%. Then full indexation can be given and there is some space for providing recovery indexation in case that is needed.

For the monthly rebalancing, the fraction invested in the return portfolio will be adjusted every month, just as for the other CPPI investment strategies. Furthermore, the call option is based on the fraction invested in the return portfolio of the first month. The option still has a maturity of one year, so during the year nothing happens with the option. At the end of the year, the option will be exercised or not.

Chapter 8

Results

8.1 Introduction

In this chapter, we will show the results which we obtained by using the model described in chapter 5. We will show results for the different constant mix investment strategies, the OBPI investment strategies and the CPPI investment strategies that were described in chapter 7. For each of these strategies, some elements will be taken into consideration in order to measure and compare the strategies. Next to analyzing these elements, we will show a graph per investment strategy in order to show the development of the expected funding ratio over time. In these graphs, the development of the expected funding ratio at the end of the year, the expected policy funding ratio, the median of the funding ratio at the end of the year and its 95% confidence interval are displayed. In a 95% confidence interval, we are able to see between which bounds the funding ratio will be at that moment in time with a certainty of 95%. In the last section, we will compare the results.

First, we will describe the elements that are used to make the comparison:

Median funding ratio
As we have seen in chapter 2, the funding ratio is an important element for a pension fund. Therefore, this is the first element we consider in order to compare different strategies. Because the expected path of the funding ratio is not symmetrically distributed, we will take the median into account instead of the mean. In the tables below, we will show the median of the funding ratio at the end of the simulation period, so after 50 years.

Standard deviation funding ratio
We know that we are able to obtain a high funding ratio in expectation if we take much risk on investments. However, we have seen in chapter 2 that not too much risk should be taken. If too much risk is taken, the standard deviation of the funding ratio would be high, which is

not desirable. However, because the distribution of the funding ratio is not symmetrically, we cannot use the standard deviation as it is done normally. Because a standard rule says that 34% of the observations lies between the mean and the mean minus one times the standard deviation, we will take the median at the end of the simulation period minus the $16th$ percentile at the end of the simulation period as the standard deviation here. In the remainder of this thesis, this will be called the standard deviation.

Percentage above MRF

It is important for a pension fund that the funding ratio is above the MRF. If the funding ratio is below the MRF for more than five years, additional cuts have to be taken and that should be avoided according to the KPIs. This element gives the percentage of simulations for which the funding ratio is above the MRF.

Percentage above RF

For a pension fund it is important to have a funding ratio that is above the RF. This element gives the percentage of simulations for which the funding ratio is above the RF. Note that the RF depends on the risk that is taken as was explained in section 5.4.2.

Mean purchasing power

In chapter 2 we have seen that indexation is of upmost importance for the participants of a pension fund. When indexation is given, participants can (partly) remain their purchasing power during retirement and then it is attractive for individuals to participate in the pension fund. Therefore we will take this element into consideration as well. The purchasing power is defined by the given indexation over the total indexation. Here we will look at the expected purchasing power at the end of the simulation period.

$2.5th$ **percentile purchasing power**

Because indexation is so important for participants, we take the $2.5th$ percentile of the purchasing power into account which is defined as the $2.5th$ percentile of the given indexation over the total indexation. In that way, we are able to take the worst scenarios into account. Again, we will consider the $2.5th$ percentile at the end of the simulation period.

Mean number of small cuts

In chapter 2 we have seen that participants of a pension fund are disappointed in case their pension benefits have to be cut. Therefore, it is important to minimize the number of cuts. Small cuts are defined as the cuts that have to be implemented because the capacity to recover is not high enough. We will look at the mean of all small cuts that needed to take place during the whole simulation period.

Mean number of big cuts

Big cuts are defined as the cuts that have to be implemented because the pension fund was underfunded for more than 5 years. Obviously, it is important to minimize the number of these cuts as well. Again, we will look at the mean of all big cuts that needed to take place during the whole simulation period.

Average percentage invested in return portfolio
For the constant mix investment strategies, the fraction invested in the return portfolio stays constant over time. However, for the CPPI this fraction changes over time. When options are involved, which is the case in the OBPI investment strategies and in the CPPI with satiation level investment strategy, the fraction invested in the return portfolio influences the price of the call option and the payoff. For the CPPI investment strategy with satiation, the fraction invested in the return portfolio is determined just as in the other CPPI investment strategies and for the OBPI investment strategies, this fraction is kept constant as explained in the previous chapter. This constant fraction is set equal to the optimal fraction of the constant mix investment strategy, so is set equal to 40%. For the OBPI investment strategies, we will not show this parameter as it cannot be interpreted as the average percentage invested in the return portfolio.

8.2 Results constant mix

In table 8.2.1 we will show the results of the constant mix (40%/60%) investment strategy.

Constant mix	(40%/60%)
Median funding ratio	151.31%
Standard deviation funding ratio	31.09%
Funding ratio above MRF	90.43%
Funding ratio above RF	70.96%
Purchasing power	84.60%
2.5th percentile purchasing power	37.53%
Mean number of small cuts	2.267
Mean number of big cuts	0.457
Average percentage invested in return portfolio	40.00%

Table 8.2.1 Results of the constant mix (40%/60%) investment strategy

In table 8.2.1 we see that with the constant mix (40%/60%) investment strategy, the pension fund is expected to have a funding ratio of 151.31% in 50 years from now. We see that in almost 10% of all scenarios, this strategy is not able to provide a funding ratio that is equal to the MRF. Therefore, we see that the pension fund has to implement a big cut in many scenarios. This strategy shows an average purchasing power of 84.60%, which is not bad. However, the 2.5th percentile of the purchasing power is just 37.53%, which is very low.

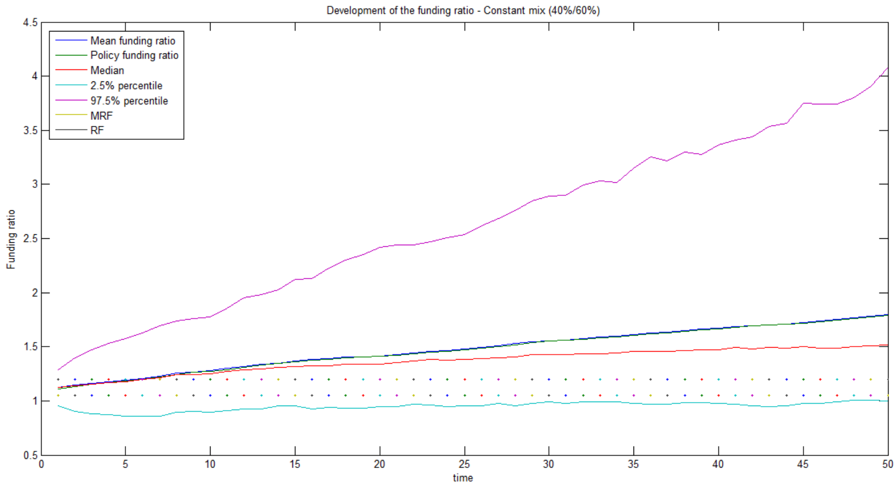


Figure 8.2 Development of the funding ratio of the constant mix (40%/60%) investment strategy

In figure 8.2 the development of the funding ratio of the constant mix (40%/60%) investment strategy can be seen, together with the development of the policy funding ratio, the median of the funding ratio and the 95% confidence interval. We see that this investment strategy does not provide any downside protection, as the worst performing 2.5% of the scenarios show a funding ratio of less than about 100%.

We already explained that the constant mix investment strategy is the strategy that is currently mostly used by pension funds. Now we have seen that this strategy indeed does not provide any downside protection as was experienced by many pension funds in the past.

Because we will compare the results of the portfolio insurance strategies with the constant mix strategies, we will consider the (0%/100%), (25%/75%), (50%/50%), (75%/25%) and (100%/0%) as well. The results are described in table 8.2.2:

Constant mix	(0/100)	(25/75)	(50/50)	(75/25)	(100/0)
Median funding ratio	117.52%	131.32%	169.67%	227.39%	279.14%
Standard deviation funding ratio	2.02%	13.98%	47.06%	99.14%	149.21%
Funding ratio above MRF	99.99%	94.51%	89.00%	86.43%	84.38%
Funding ratio above RF	99.99%	64.78%	64.69%	64.80%	63.97%
Purchasing power	50.20%	77.92%	86.00%	85.93%	83.38%
2.5th percentile purchasing power	42.93%	43.55%	33.58%	23.36%	13.83%
Mean number of small cuts	0.006	3.317	3.467	3.525	3.683
Mean number of big cuts	0	0.168	0.540	0.735	0.900
Average percentage invested in return portfolio	0.00%	25.00%	50.00%	75.00%	100.00%

Table 8.2.2 Results of the constant mix (0%/100%), (25%/75%), (50%/50%), (75%/25%) and (100%/0%) investment strategies

We see that the higher the fraction invested in the return portfolio, the higher the median of the funding ratio but also the higher the standard deviation of the funding ratio. We see that the numbers of small and big cuts that need to be taken increase with the fraction invested in the return portfolio as well, because of the risk that is taken.

The highest purchasing power that is obtained with the constant mix investment strategies is in case half of the assets is invested in the return portfolio and the rest is invested in the matching portfolio. In case more risk is taken, the expected purchasing power slightly decreases, while in case less risk is taken, the expected purchasing power significantly decreases. Furthermore, we see that the highest percentage for the $2.5th$ percentile of the purchasing power is obtained when 25% of the assets is invested in the return portfolio and the rest is invested in the matching portfolio. We see that this percentile is decreasing much in case more risk is taken. In case all assets are invested in the return portfolio, the $2.5th$ percentile is even 13.83%, which is very low. If that scenario would happen in reality, participants of the pension fund are not able to remain their purchasing power during (future) retirement.

8.3 Results OBPI

Now we will show the results for the portfolio insurance investment strategies, where we begin with the OBPI investment strategies. As explained in the previous chapter, we expect the portfolio insurance investment strategies to give downside protection because they are designed to do so, and so that is what we hope to see in the results. In this section we give results for the three different OBPI investment strategies. Because the pension fund is assumed to invest in the matching portfolio in order to have a funding ratio that is at least equal to the MRF at the end of the year, we assume the RF to be equal to the MRF and so the probability that the funding ratio is above the MRF is the same as the probability that the funding ratio is above the RF for the OBPI investment strategies.

8.3.1 OBPI without lock

In table 8.3.1 the results for the OBPI without lock investment strategy are given.

OBPI without lock	
Median funding ratio	137.58%
Standard deviation funding ratio	28.55%
Funding ratio above MRF	94.13%
Funding ratio above RF	94.13%
1 Purchasing power	76.33%
2.5th percentile purchasing power	39.46%
Mean number of small cuts	2.485
Mean number of big cuts	0.091

Table 8.3.1 Results of the OBPI without lock investment strategy

If we compare the results from table 8.3.1 with the results of the optimal constant mix (40%/60%) investment strategy (table 8.2.1), we see that both the median and the standard deviation of the funding ratio are lower. Furthermore, in expectation less indexation can be given. Furthermore, we see that the number of small cuts is higher. This is because there is less capacity to recover. On the other hand, the number of big cuts is lower and we see that the 2.5th percentile of the purchasing power is slightly higher. The probabilities that the funding ratio is above the MRF and the RF are higher.

In figure 8.3.1 it can be seen that this investment strategy is not able to provide the portfolio insurance, i.e. that the funding ratio is above the MRF with a certainty of 97.5%. We see that the 2.5th percentile is above the MRF for the simulation period, except for the first few years. From the table it follows that more than 94% of the simulations show a funding ratio that is above the MRF.

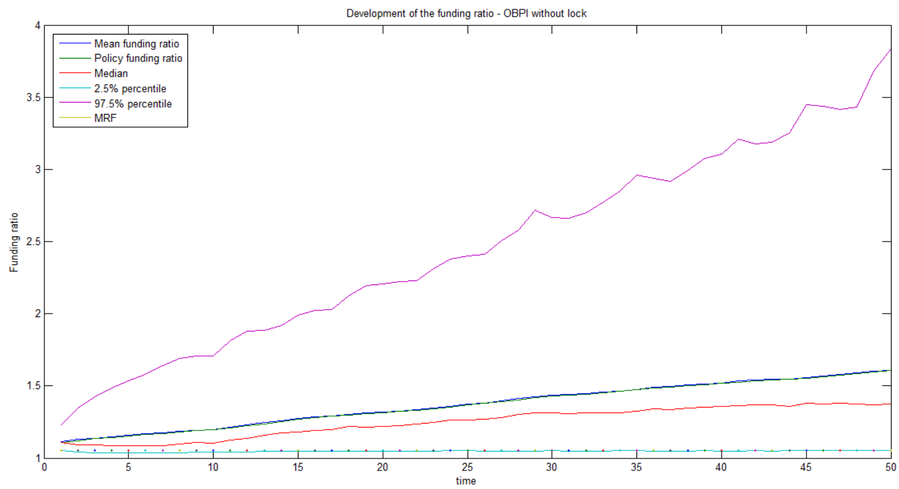


Figure 8.3.1 Development of the funding ratio of the OBPI without lock investment strategy

8.3.2 OBPI with lock

The results for the OBPI with lock investment strategy are given in table 8.3.2.

OBPI with lock	
Median funding ratio	137.92%
Standard deviation funding ratio	27.87%
Funding ratio above MRF	95.24%
Funding ratio above RF	95.24%
Purchasing power	78.11%
2.5*th* percentile purchasing power	40.30%
Mean number of small cuts	0.610
Mean number of big cuts	0.205

Table 8.3.2 Results of the OBPI with lock investment strategy

If we compare this strategy to the OBPI without lock investment strategy we see that this strategy has a slightly higher median of the funding ratio at the end of the period. Furthermore, the standard deviation is a bit lower, which is positive. The purchasing power and its 2.5*th* percentile are slightly higher and so are the probabilities that the funding ratio is above the MRF and RF. Furthermore, we see that the number of small cuts is reduced significantly. However, we see that the number of big cuts more than doubled, which is not desirable.

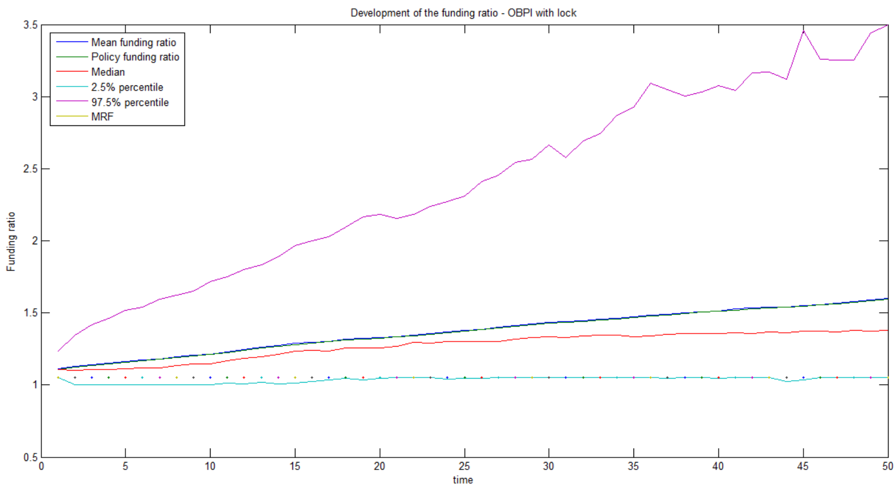


Figure 8.3.2 Development of the funding ratio of the OBPI without lock investment strategy

In figure 8.3.2 we see that this strategy is not able to guarantee a funding ratio above the MRF with a certainty of 97.5% as well. There is a little bit less upside potential than in the OBPI without lock investment strategy, but this is reasonable as a price has to be paid for locking in the profits.

8.3.3 OBPI with satiation

The results for the OBPI investment strategy where the pension fund is assumed to have a satiation level are given in table 8.3.3.

OBPI with satiation	
Median funding ratio	133.39%
Standard deviation funding ratio	25.33%
Funding ratio above MRF	94.53%
Funding ratio above RF	94.53%
Purchasing power	73.70%
2.5th percentile purchasing power	40.18%
Mean number of small cuts	2.300
Mean number of big cuts	0.089

Table 8.3.3 Results of the OBPI with satiation investment strategy

This investment strategy gives a lower median and a lower standard deviation of the funding ratio than the other two OBPI investment strategies. The purchasing power decreased, which is a negative effect. We see that the 2.5th percentile is not higher than in case of the OBPI with lock investment strategy, so the participants are not compensated for the decrease in the purchasing power. Furthermore, the number of both small and big cuts are less than the OBPI without lock investment strategy. If we compare it to the OBPI with lock investment strategy, the number of small cuts is much higher but the number of big cuts is much lower. Lastly, we see that also this strategy is not able to guarantee a funding ratio of 105% with a certainty of 97.5%.

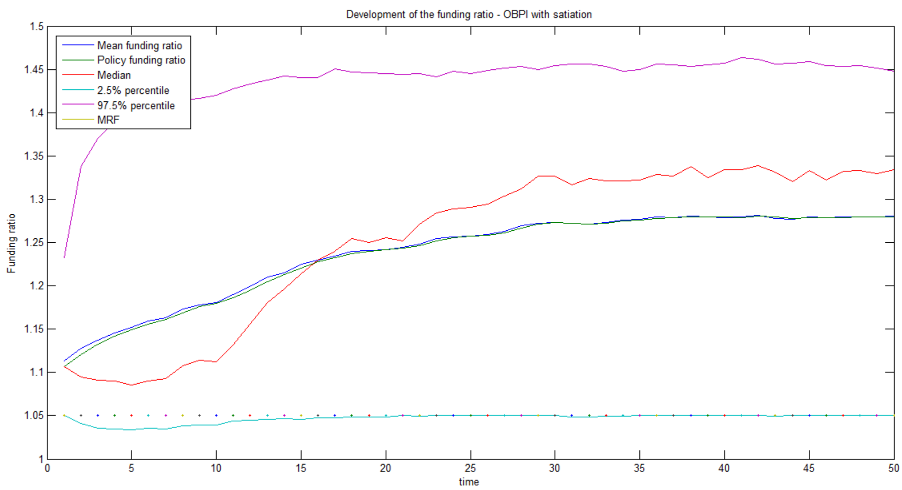


Figure 8.3.3 Development of the funding ratio of the OBPI with satiation investment strategy

From figure 8.3.3 it follows that the median of the funding ratio lies above the expected funding ratio, which is in contrast to what we have seen before. This means that the distribution of the funding ratio is skewed to the right. This is explained by the fact that call options are sold.

8.4 Results CPPI

In this section we will give the results for the second portfolio insurance strategy, namely for the CPPI investment strategies. We will first give the results for the CPPI investment strategy that does not lock profits, then for the CPPI investment strategy that locks profits and lastly for the CPPI investment strategy that sells call options because it has a satiation level. We will give the results for both the cases where rebalancing takes place once a year and once a month.

8.4.1 CPPI without lock

In table 8.4.1 we see the results for the first CPPI investment strategy.

CPPI without lock	Annually rebalanced	Monthly rebalanced
Median funding ratio	124.76%	125.36%
Standard deviation funding ratio	11.34%	12.50%
Funding ratio above MRF	99.67%	99.94%
Funding ratio above RF	67.55%	61.89%
Purchasing power	71.36%	74.19%
$2.5th$ percentile purchasing power	42.52%	42.60%
Mean number of small cuts	3.282	2.722
Mean number of big cuts	0.004	0
Average percentage invested in return portfolio	27.14%	31.80%

Table 8.4.1 Results of the CPPI without lock investment strategy

We see that the average fraction invested in the return portfolio is 27.14% for the yearly rebalancing and 31.80% for the monthly rebalancing. So for the optimal CPPI investment strategy less risk is taken on average than for the optimal constant mix investment strategy. This is because we start with a relatively low initial funding ratio, so the amount invested in the return portfolio is relatively low.

We see that the median of the funding ratio is less than in the case of the constant mix (40%/60%) investment strategy, and the standard deviation is significantly reduced as well. This reduced standard deviation is important for participants of a pension fund to take into consideration, because they do not like a high standard deviation. We see that the expected purchasing power is lower, while the $2.5th$ percentile of the purchasing power is higher. The striking element of this strategy is that the average number of small cuts is higher. This is due to the fact that we have less capacity to recover if the funding ratio is close to the MRF, because less risk is taken then. Note that the number of big cuts is much lower and is even equal to zero in the case of monthly rebalancing. Overall, we see that monthly rebalancing gives a higher variance and the probability that the funding ratio is above the RF is lower, but for the other elements the monthly rebalancing gives better results than when rebalancing takes place once a year.

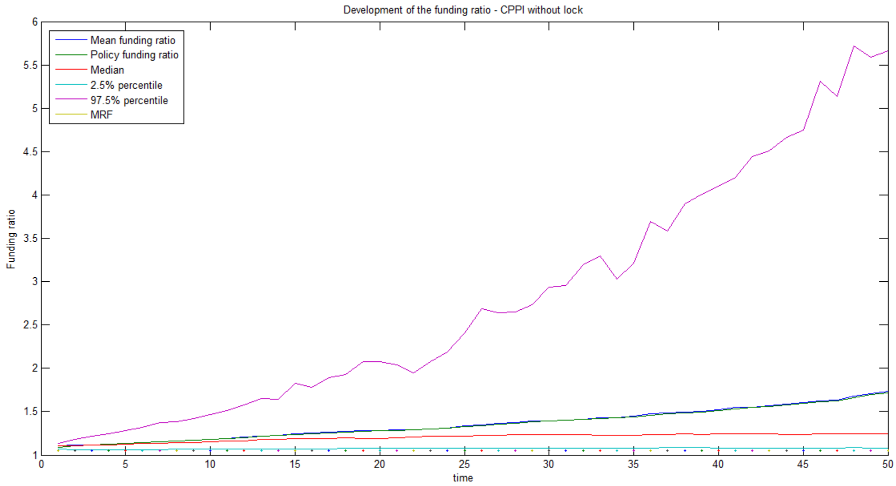


Figure 8.4.1 Development of the funding ratio of the CPPI without lock investment strategy - yearly rebalancing

In figure 8.4.1 we see that this investment strategy provides a much better downside protection than the considered constant mix investment strategy. Furthermore, this strategy does not limit the potential to make gains, at the end of the simulation period, the 97.5th percentile is even higher than for the constant mix (40%/60%) investment strategy.

8.4.2 CPPI with lock

In the next table we see the results for the CPPI investment strategy that locks a profit of 130% at the moment the funding ratio is at least 135% and takes more risk compared to the CPPI without lock investment strategy when the pension fund has a funding ratio between 100% and 110%.

CPPI with lock	Annually rebalanced	Monthly rebalanced
Median funding ratio	125.48%	128.27%
Standard deviation funding ratio	12.14%	15.01%
Funding ratio above MRF	99.57%	99.45%
Funding ratio above RF	69.03%	63.19%
Purchasing power	71.60%	73.99%
2.5th percentile purchasing power	42.84%	42.59%
Mean number of small cuts	2.837	2.346
Mean number of big cuts	0.004	0.011
Average percentage invested in return portfolio	22.75%	24.46%

Table 8.4.2 Results of the CPPI with lock investment strategy

Now we will compare the results from this strategy with the results from the CPPI without lock investment strategy. From table 8.4.2 it follows that this strategy gives a higher median of the funding ratio at the end of the simulation period. However, the standard deviation is bigger as well. Due to the locks, the results are more spread and this leads to a higher standard deviation. Furthermore we see that the probability that the funding ratio is above the MRF is lower while the probability that the funding ratio is above the RF is higher. For the purchasing power we see that the CPPI with lock investment strategy provides a higher purchasing power than the CPPI without lock investment strategy if the portfolio is rebalanced on an annually basis. However, when the rebalancing takes place once a month, the purchasing power is increased compared to annually rebalancing, but is not higher than the purchasing power of the CPPI without lock investment strategy. The mean of the number of small cuts is lower, but we see that more big cuts are needed in case of monthly rebalancing. Furthermore, when we compare the monthly rebalancing to the annually rebalancing, we see that the monthly rebalancing gives a higher median, a higher purchasing power and a lower number of small cuts, but for the other elements, the annually rebalancing shows better results. Lastly, we see that the average fraction invested in the return portfolio is lower for this strategy but this is what we expected as this strategy locks in profits and reduces the risk once the level of the specified profit is reached.

In figure 8.4.2 we see that this strategy reduces the potential for upward gains, as the 97.5th percentile is lower than in the case of the CPPI without lock investment strategy. Again, this is what we expect because the fraction invested in the return portfolio is reduced in case the profit level is reached.

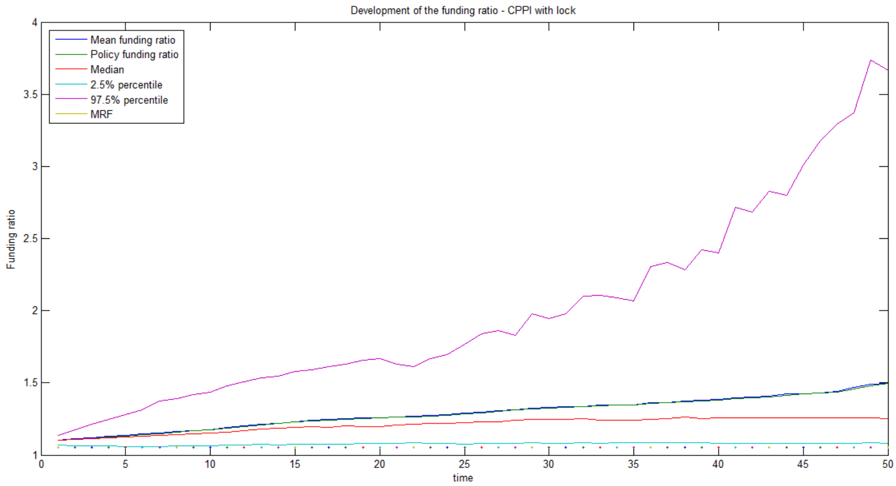


Figure 8.4.2 Development of the funding ratio of the CPPI with lock investment strategy - yearly rebalancing

8.4.3 CPPI with satiation

Lastly we will show the results for the last CPPI investment strategy, namely the investment strategy where it is assumed that pension funds have a satiation level and sell call options.

CPPI with satiation	Annually rebalanced	Monthly rebalanced
Median funding ratio	125.88%	123.97%
Standard deviation funding ratio	12.41%	11.07%
Funding ratio above MRF	99.75%	99.93%
Funding ratio above RF	69.46%	63.79%
Purchasing power	69.73%	69.83%
$2.5th$ percentile purchasing power	42.83%	42.38%
Mean number of small cuts	2.813	2.575
Mean number of big cuts	0.002	0
Average percentage invested in return portfolio	23.81%	25.97%

Table 8.4.3 Results of the CPPI with satiation investment strategy

From table 8.4.3 it follows that this CPPI investment strategy has a slightly higher median for the annually rebalancing and a lower median for the monthly rebalancing compared to the other CPPI investment strategies. Furthermore we see that the standard deviation corresponds to this, as for the annually rebalancing the standard deviation of this strategy is the highest and for the monthly rebalancing the standard deviation is the lowest compared to the other CPPI investment strategies. Furthermore we see that the probability that the funding ratio is above the MRF and above the RF is about the same or slightly higher, which is positive. We also see that the purchasing power is much lower. This can be explained by the fact that upside potential is sold and so there is less room for recovery indexation. The $2.5th$ percentile of the purchasing power does not differ that much from the results of the other CPPI investment strategies. Furthermore, in this strategy, the least number of big cuts are needed.

In the following figure we see that this strategy is able to guarantee a funding ratio of 105% with a certainty of 97.5% as well. We see that the 97.5th percentile of the funding ratio is much lower than for the other two CPPI investment strategies, but this is obvious as this strategy sells its upside potential. Furthermore we see that for this strategy, the distribution of the funding ratio is almost symmetrically in the end.

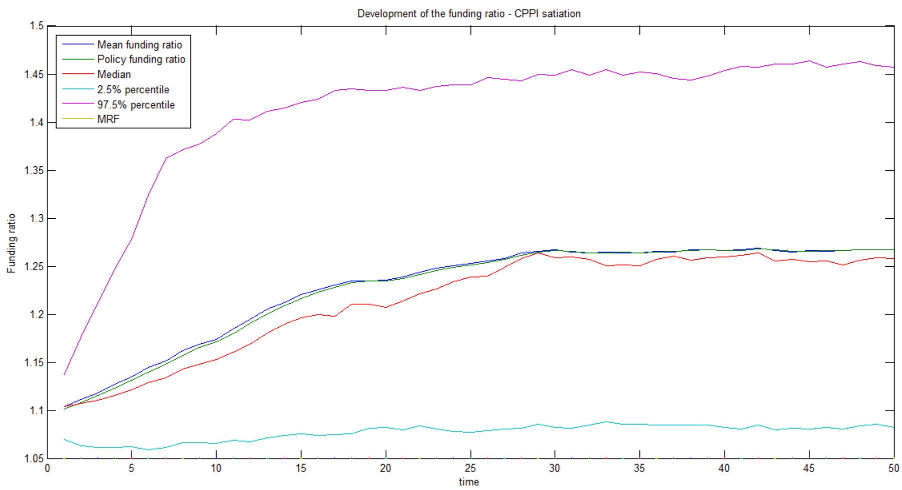


Figure 8.4.3 Development of the funding ratio of the CPPI with satiation investment strategy - yearly rebalancing

8.5 Comparison of the results

Now we have all important information of all strategies. We want to compare the strategies in order to say which strategy would be the best for a pension fund. As explained, by means of a utility function, different strategies can be compared. However, since we defined utility functions where the funding ratio was the only parameter in chapter 6, we cannot use these utility function as a method to compare the strategies. This would namely imply that the higher the funding ratio the better, while other elements (such as the purchasing power, number of cuts etc.) are important for a pension fund as well. Therefore, we will define the following utility function in order to compare the different strategies now.

$$\begin{aligned} U^*(strategy) = \; & \alpha_1 E(F_T^{median}) + \alpha_2 std(F_T) + \alpha_3 E(Funding\ ratio\ above\ MRF) \\ & + \alpha_4 E(Funding\ ratio\ above\ RF) + \alpha_5 E(Purchasing\ power) \\ & + \alpha_6 E(Lowerbound\ purchasing\ power) \\ & + \alpha_7 E(\#\ small\ cuts) + \alpha_8 E(\#\ big\ cuts). \end{aligned} \tag{8.1}$$

So the utility of an investment strategy is a factor (α_1) times the median of the funding ratio at the end of the period (time T), plus a factor (α_2) times the standard deviation of the funding ratio at the end of the period, plus a factor (α_3) times the percentage of simulations that gave a funding ratio that was higher than the MRF, plus a factor (α_4) times the percentage of simulations that gave a funding ratio that was higher than the RF, plus a factor (α_5) times the average purchasing power that is maintained, plus a factor (α_6) times the $2.5th$ percentile of the purchasing power, plus a factor (α_7) times the average number of small cuts, plus a factor (α_8) times the average number of big cuts. As explained in the introduction of this chapter, all KPIs that were defined in chapter 2 are represented. If the purchasing power is high, compensation for inflation is geven and so the pension fund is attractive. If the funding ratio is high, the pension fund is able to pay its liabilities. The KPI that says to minimize the probability of cuts is represented in this utility function as well. And lastly, because the standard deviation is implemented in the utility function, the last KPI that says to take acceptable risks that are rewarded is represented as well.

We assume α_2, α_7 and α_8 to be negative as the standard deviation and the average number of cuts are negative effects. Here we assume α_8 to be bigger than α_7 as a big cuts means that the pension fund is underfunded for more than 5 years and a big cut is a more negative effect than just a small cut. We assume the other factors to be positive, as for these elements it holds that 'the more, the better'. We assume α_3 to be bigger than α_4 because it is more important for a pension fund to be above the MRF than above the RF.

In this thesis, we assume the following values for the factors. The value of α_5 and α_6 are higher than the values of the other factors, because it is really important for a pension fund that their participants can remain their purchasing power.

α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
1	-1	1	0.5	10	10	-0.5	-1

Table 8.5 Values for the factors of the utility function

Now we are able to calculate the utilities of the pension fund for each of the strategies. These utilities are displayed in figure 8.5. For completeness, we added all considered constant mix strategies as well. The numbers can be found in Appendix E.

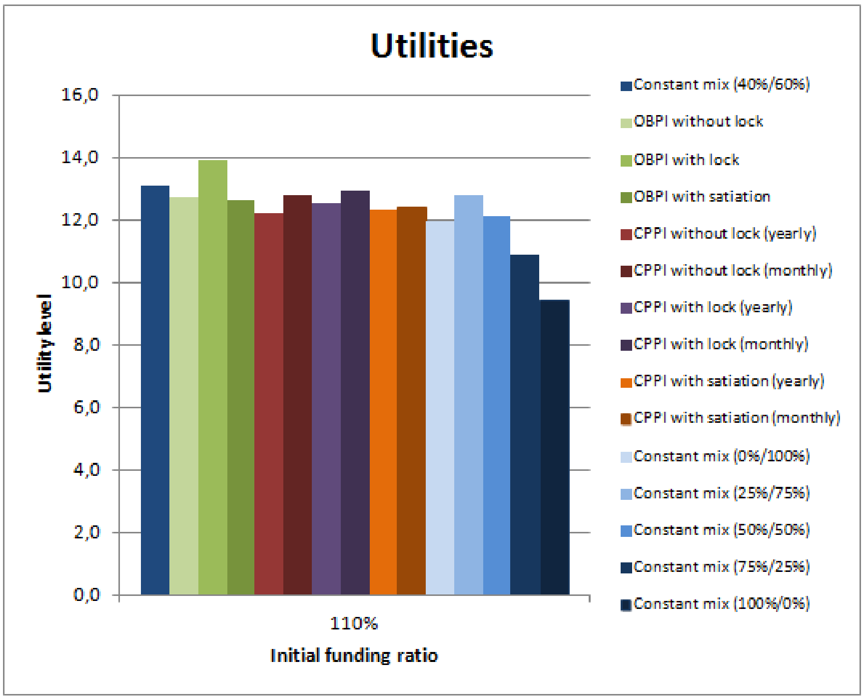


Figure 8.5 Utilities of all considered investment strategies

From figure 8.5 it follows that the theoretically optimal constant mix investment strategy ((40%/60%)) is indeed the optimal strategy if we compare it to the other constant mix investment strategies. However, we also see that we can obtain a higher utility level in case we invest via the OBPI with lock investment strategy.

The highest utility level can be obtained by investing via the OBPI with lock investment strategy. This is the only strategy that gives a higher return than the constant mix (40%/60%) investment strategy. This strategy provides a relatively good purchasing power and not many cuts.

The utilities from the CPPI without and with lock investment strategies where rebalancing takes place every month come close to the utility of the constant mix (40%/60%) as well. Note that the CPPI with satiation investment strategy gives a lower utility level. This is mainly due to the low purchasing power.

So to conclude, we see that the OBPI with lock investment strategy is of added value for a Dutch pension fund. This strategy gives a higher utility level than all other strategies. Note that for the OBPI investment strategies and for the CPPI with satiation investment strategy we had to make the assumption that options are available on every strike price we want and that the market is such that the options we want to buy are available in the market and that it is possible to sell the options we want to sell.

Now in the next chapter we will investigate what the impact of the initial funding ratio is on the results.

Chapter 9

Sensitivity analysis

9.1 Introduction

In the previous chapter, the initial funding ratio was assumed to be 110%. We have seen that the OBPI with lock investment strategy performed better than the optimal constant mix investment strategy. However, one can imagine that the initial funding ratio might influence the performance of the different strategies. Therefore, in this chapter, we will perform a sensitivity analysis by changing the initial funding ratio and see whether the results change drastically. We will show the results for initial funding ratios of 105%, 110%, 115%, 120% and 125% in this chapter.

For all different investment strategies we considered, we will show the tables with results as we did in the previous chapter for each of the initial funding ratios. These tables can be found in Appendix D. We will analyze these tables in the next section.

In the last section of this chapter, we will compare the utilities the pension fund obtains for the different strategies under the different initial funding ratios.

9.2 Analysis of the results

For all tables in Appendix D, we see about the same effects. Therefore, we will first describe the effects that happen in general and after that, we will explain per table whether there are outstanding effects.

General effects

As can be expected, the higher the initial funding ratio, the higher the median of the funding ratio at the end of the period. This is the case for all different investment strategies, however for some strategies the median increases faster than for other strategies in case the initial funding ratio increases, as will be explained below.

Furthermore, we see that the standard deviation is higher for higher initial funding ratios. This is because the standard deviation here is defined by the difference of the median and the $16th$ percentile of the funding ratio. If we subtract these two, we are able to calculate the $16th$ percentile and we see that this is higher for higher initial funding ratios as well.

From the tables it follows that the percentage of the simulations for which the funding ratio is above the MRF and the RF are higher for a higher initial funding ratio. This is what can be expected because if you start with a higher funding ratio, there is a bigger probability you stay at this high funding ratio.

Then we see that the higher the initial funding ratio, the higher the purchasing power and its $2.5th$ percentile are. For these elements it also holds that for some strategies the elements increase faster than for other strategies. We will elaborate on that below.

For the number of both small and big cuts it holds that the higher the initial funding ratio, the less (small and big) cuts are needed to be taken.

Lastly, for the CPPI investment strategies it holds that the higher the initial funding ratio, the bigger the average percentage invested in the return portfolio is. This is easily explained as these strategies are designed to invest more in the return portfolio when the funding ratio is high.

Now we will discuss the results of the sensitivity analysis per investment strategy of which the results differ from the general results.

Constant mix investment strategies
The results of the sensitivity analysis for the constant mix investment strategies do not differ from the general results. We see that for the constant mix (0%/100%) investment strategy, the median of the funding ratio at the end of the period is higher for a higher initial funding ratio, however, the difference is very small. Furthermore, we see that the standard deviation stays approximately constant as well. This is because more indexation will be given if there is a higher initial funding ratio. We see that the higher the fraction invested in the return portfolio is, the more influence the initial funding ratio has on the median and the standard deviation.

For the purchasing power, we see that the lower the fraction invested in the return portfolio is, the more the expected purchasing power at the end of the simulation period increases when the initial funding ratio is increased. For example, for the constant mix (0%/100%) investment strategy, the difference in purchasing power between the initial funding ratios of 125% and 105% is almost 10%. This is because for this strategy, the expected purchasing power is relatively low. This difference is about 3% for the constant mix (100%/0%) investment strategy because there the expected funding ratio for a low initial funding ratio is already high.

OBPI investment strategies

The results of the sensitivity analysis for the OBPI investment strategies do not differ from the general results in principle. However, we see that the difference between the median of the funding ratio at the end of the simulation period in case the initial funding ratio is 125% and in case the initial funding ratio is 105%, is smaller for the OBPI with lock investment strategy than for the OBPI without lock investment strategy. For the OBPI with satiation investment strategy this difference is even smaller. The same effect holds for the standard deviation as well. This is because for the OBPI without lock investment strategy there is more upside potential. The OBPI with satiation investment strategy has the least upside potential.

CPPI investment strategies

Lastly, also the results of the sensitivity analysis for the CPPI investment strategies do not differ from the general results. We see that the median of the funding ratio at the end of the period for the CPPI with satiation investment strategy increases less than for the other two CPPI investment strategies in case of a higher initial funding ratio. This is because the satiation level stays the same. The standard deviation does not increase that fast as well and even stays almost constant when rebalancing takes place on a monthly basis. We see that the number of small cuts is quite high for these strategies in case of a low initial funding ratio. This is because less is invested in the return portfolio and therefore there is almost no capacity to recover. Lastly, we see that the mean number of big cuts does not decrease for funding ratios higher than 110% for the CPPI without lock investment strategy. This is because too much risk is taken and the pension fund is not able to rebalance in time. Indeed, when we look at the CPPI without lock investment strategy where rebalancing takes place every month, we see that the pension fund does not have to announce big cuts for these initial funding ratios.

9.3 Utilities

Now we have all results for the considered strategies and for the different initial funding ratios, we are able to calculate the corresponding utility functions. These are displayed in figure 9.3. In Appendix E the numbers are represented.

In case the initial funding ratio is equal to 105% or 110%, we see that the constant mix (40%/60%) is optimal compared to the other constant mix investment strategies. If the initial funding ratio is 115%, the utility level of the constant mix (40%/60%) investment strategy is the same as the utility level for the constant mix (50%/50%) investment strategy, and for the higher initial funding ratios, the constant mix (25%/75%) investment strategy gives a higher utility level. So for an initial funding ratio of 120% and 125%, the constant mix (40%/60%)

investment strategy is not optimal anymore. This means that if we only consider the constant mix investment strategies, these results imply that the pension fund should better become a bit more risk averse and invest less in the return portfolio if it has a higher initial funding ratio. Furthermore, we see that the investment strategy that invest all assets in the return portfolio gives the lowest utilities for all considered initial funding ratios.

If we compare the utilities for the OBPI investment strategies, we see that the OBPI with lock investment strategy gives the highest utility level for all initial funding ratios considered. This means that under this strategy it is optimal to lock the profits that are made.

For the CPPI investment strategies, the CPPI with lock investment strategy that rebalances monthly provides a utility level that is at least as high as the utility levels of the other CPPI investment strategies. So for this strategy it is optimal to lock the profits that are made as well while rebalancing takes place every month.

For all considered initial funding ratios, we see that the OBPI with lock investment strategy gives the highest utility level compared to all other considered strategies. Furthermore, we see that the higher the initial funding ratio is, the better the CPPI investment strategies perform relative to the others.

If the initial funding ratio is equal to 105%, no portfolio insurance investment strategy performs better than the constant mix (40%/60%), (25%/75%) and (50%/50%) investment strategies except for the OBPI with lock investment strategy. This changes if the initial funding ratio changes. For an initial funding ratio of 110%, the third best strategy is the CPPI with lock (monthly) investment strategy. This strategy is even the second best investment strategy for an initial funding ratio of 115% and 120%. However, for an initial funding ratio of 125%, the second best strategy is the constant mix (25%/75%) investment strategy.

So we see that the OBPI with lock strategy adds value to a Dutch pension fund as this gives the highest utility for all initial funding ratios considered. The CPPI with lock investment strategy is the second best investment strategy for the initial funding ratios of 115% and 120%. For lower initial funding ratios, the CPPI investment strategies invest too less in the return portfolio. This decreases the potential for gains and this decreases the capacity to recover so that cuts have to be taken. This implies that the purchasing power is decreased, which is of upmost importance for the participants of pension funds. For higher initial funding ratios, the CPPI investment strategies perform relatively worse as well. This can be explained by the fact that too much is invested in the return portfolio then, which increases the risk that is taken. Furthermore, in our simulations, the expected purchasing power of the CPPI investment strategies is lower than the expected purchasing power of the constant mix (25%/75%) strategy, which is the main reason that those strategies are less preferred.

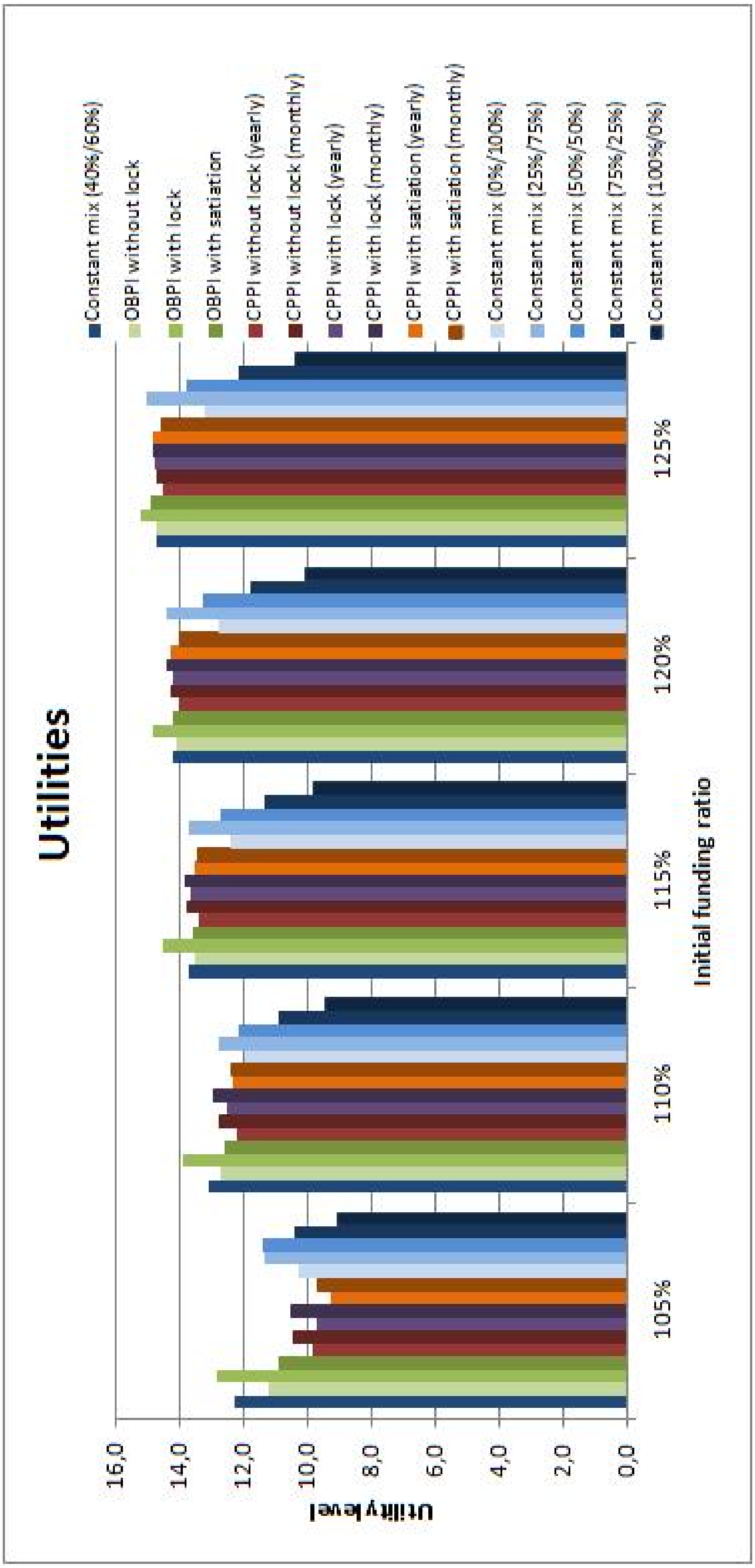


Figure 9.3 Utility levels for all considered investment strategies and for all different initial funding ratios

Chapter 10

Conclusions and recommendations

10.1 Conclusions

The situation in which pension funds have to operate has changed. Some of the developments in the market led to drops in the funding ratios of Dutch pension funds. We have seen that the developments that influenced the Dutch pension funds the most are the low interest rates, the aging of the population and bad returns on investments.

During recent years, pension funds were too vulnerable to shocks in the market which led to the fact that pension funds where not stable anymore, nominal rights even had to be cut. Therefore, the government decided that a new pension contract should be introduced. Due to this new pension contract, pension funds should be better protected against shocks in the market. The new Dutch pension contract is not decided on yet. However, the bill is already made publicly available. Therefore, we tried to incorporate the elements of this bill in this thesis, in order to make it applicable for next years.

The Dutch pension funds were too much exposed to the shocks in the market. This is not desirable. Therefore, it is of upmost importance for pension funds to choose an investment strategy that meets their needs best. In this thesis, it is studied what the optimal investment strategy of a Dutch pension fund is, where elements of the new Dutch pension contract are taken into account.

The first question that had to be answered is what the goal and the preferences of a pension fund and its stakeholders are. On the basis of these goal and preferences, we are able to determine whether an investment strategy is optimal. We have seen that this led to the assumption that the funding ratio is a very important parameter for a pension fund. Therefore, on the basis of this parameter, we have determined several utility functions that could be applicable to a Dutch pension fund. By means of optimizing these utility functions, we came to three different optimal investment strategies for a Dutch pension fund. These three investment strategies are the constant mix strategy, the constant proportion portfolio insurance strategy and the option based portfolio insurance strategy.

In this thesis it is assumed that the pension fund can invest in a return portfolio and in a matching portfolio, which can be interpreted as a portfolio to earn high returns and a portfolio that matches the liabilities, respectively. If a pension fund would only invest in the matching portfolio, the pension fund would not be able to give a compensation for inflation to its participants. Then, the participants cannot remain their purchasing power.

The first investment strategy that seems optimal is the constant mix investment strategy. This is the strategy that is currently mostly used by Dutch pension funds. With this strategy, the pension funds invest a constant proportion of their total assets in the return portfolio. We considered this strategy for six different fractions to invest in the return portfolio, namely for the optimal fraction and for five different reference fractions.

The second investment strategy that seems optimal is the option based portfolio insurance strategy. This strategy uses options to provide downside protection. We considered three versions of this strategy. The first version invests in the matching portfolio in order to be sure to have a funding ratio at the end of the year equal to the MRF and buys call options to have some upside potential. The second version has the same strategy but locks profits that were made and the last version sells upside potential because it assumes a satiation level.

The last investment strategy considered is the constant proportion portfolio insurance strategy. For this strategy, the fraction of assets invested in the return portfolio depends on the funding ratio of the pension fund. Again, we considered three versions. The first version is such that the fraction invested in the return portfolio just depends on the funding ratio. The second version is assumed to lock profits and the last version is assumed to have a satiation level, so it can sell its upside potential. These three version are considered when rebalancing takes place once a year and when rebalancing takes place once a month.

To measure and compare the different investment strategies, another utility function needed to be specified, as the utility functions that were specified before were only based on the funding ratio. In this last utility function, the goal and preferences of a Dutch pension fund and its stakeholders that were described in the second chapter are represented. In this way, we are able to make a good comparison between the different strategies.

The conclusion of this thesis is that portfolio insurance investment strategies add value to a Dutch pension fund. The OBPI with lock investment strategy provided the highest utility levels for all initial funding ratios considered which means that we consider this strategy as the best strategy for a Dutch pension fund. Furthermore, for some initial funding ratios, the CPPI investment strategies showed a good performance as well. However, if the initial funding ratio is low, this strategy has too little upside potential. And, for the highest considered initial funding ratio, the CPPI investment strategies performed worse, relatively. This is mainly because it is not able to give a purchasing power that is as high as the OBPI with lock investment strategy.

The optimal investment strategy for Dutch pension funds in the new Dutch pension contract is thus the OBPI with lock investment strategy. This strategy gives the highest utility for all initial funding ratios. Furthermore, we saw that the CPPI with lock investment strategy with monthly rebalancing adds value for a Dutch pension contract as well, although the difference in utility between this strategy and the constant mix investment strategies is very low.

10.2 Recommendations

In this thesis, we had to make many assumptions. For example, we made assumptions about the parameters that define the development of the interest rates, the return portfolio and the inflation. Furthermore, we also made assumptions on the policy of the pension fund and about the participants and their wages. These assumptions could influence the results we obtained. Because of all these assumptions, a suggestion for further research is to do a sensitivity analysis on these assumptions.

Furthermore, in chapter 7 we defined several investment strategies. However, one can think about much more investment strategies that could be optimal. For example, for the strategies with lock, the levels at which profits are locked can be changed. We recommend to investigate more investment strategies in further research. In that way, even more optimal investment strategies can be obtained.

In chapter 2 we have determined the goal and preferences of a Dutch pension fund and its stakeholders. However, we do not know the exact preferences, for example whether they prefer providing indexation over announcing cuts. Therefore, one of the recommendations for further research is to interview pension funds' board members and its stakeholders. On the basis of the results of those interviews, one would be able to gain insight in the risk preferences and ambition levels of the board members and the stakeholders of the pension fund. On the basis of these insights, the factors that are used in chapter 8 can be determined. In this way, the utility function can be defined based on the preferences of the pension funds' board members and its stakeholders.

As we have seen before, old participants of a pension fund are usually more risk averse than young participants. Therefore, the preferences might not be homogeneous, in a sense that the preferences of a pension fund that has old participants are not the same as the preferences of a pension fund that has young participants. This could be taken into account as well. For example, utility functions could be defined in which the parameters are dependent on age cohorts.

Another element that we suggest to take into account during further research is that sellers of options are often compensated for the risk they take by selling the options, which is called the volatility risk premium. According to Rennison and Pedersen [2012], this volatility risk premium causes the implied volatility, which is the volatility that is used for pricing the options, to exceed the realized volatility on average. This is not considered in this thesis.

Furthermore, it would be good to take transaction costs into account. In general, there are much more transaction costs if rebalancing is done more frequently. We did not implement transaction costs in this thesis.

Lastly, for further research, we recommend to calculate the required funding ratio more precisely than we did in this thesis. This would give a better representation of reality.

Appendix A

Black Scholes model

Black and Scholes [1973] derived a model for the valuation of options. They would like to valuate a call option which gives the right to buy the stock S, which follows a geometric Brownian motion, at a single point in time T (so we are looking at an European option here) for a price of K. The payoff of this call option is equal to $[S_T - K]^+$. The result they derived in this paper is that the price of this call option is given by

$$C_t = S_0 N(d_1) - Ke^{-r(T-t)} N(d_2), \tag{A.1}$$

where

C_t is the price of the call at time t

S_0 is the value of the underlying stock at time 0

T is the time of maturity

$T - t$ is the time to maturity

$N(\cdot)$ is the cumulative standard normal distribution

K is the strike price

r is the risk-free interest rate

and d_1 and d_2 are given by

$$d_1 = \frac{log(S_0/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \qquad d_2 = d_1 - \sigma\sqrt{T-t}$$

where σ is the volatility of the stock which is assumed to be constant over time.

As the price of the stock plus a put option on that stock gives the same payoff as an investment in fixed income that is equal to K at the time of maturity plus a call option on the same stock, it has to hold that

$$P_t + S = Ke^{-r(T-t)} + C_t, \tag{A.2}$$

which is also known as the put-call parity. Here, P_t is the price of the put at time t.

Appendix B

Mortality rates

Age	Death Probability Men	Death Probability Women	Average Death Probability	Age	Death Probability Men	Death Probability Women	Average Death Probability
25	0.052%	0.018%	0.035%	46	0.158%	0.138%	0.148%
26	0.035%	0.021%	0.028%	47	0.191%	0.179%	0.185%
27	0.049%	0.014%	0.032%	48	0.199%	0.172%	0.186%
28	0.042%	0.015%	0.029%	49	0.225%	0.196%	0.211%
29	0.047%	0.030%	0.039%	50	0.275%	0.206%	0.241%
30	0.054%	0.027%	0.041%	51	0.284%	0.240%	0.262%
31	0.044%	0.035%	0.040%	52	0.318%	0.235%	0.277%
32	0.063%	0.031%	0.047%	53	0.363%	0.312%	0.338%
33	0.077%	0.039%	0.058%	54	0.383%	0.303%	0.343%
34	0.062%	0.040%	0.051%	55	0.458%	0.333%	0.396%
35	0.067%	0.037%	0.052%	56	0.473%	0.359%	0.416%
36	0.062%	0.035%	0.049%	57	0.567%	0.412%	0.490%
37	0.066%	0.051%	0.059%	58	0.609%	0.481%	0.545%
38	0.069%	0.061%	0.065%	59	0.680%	0.491%	0.586%
39	0.082%	0.055%	0.069%	60	0.711%	0.544%	0.628%
40	0.100%	0.069%	0.085%	61	0.781%	0.598%	0.690%
41	0.109%	0.082%	0.096%	62	0.888%	0.587%	0.738%
42	0.117%	0.086%	0.102%	63	0.957%	0.685%	0.821%
43	0.114%	0.093%	0.104%	64	1.076%	0.693%	0.885%
44	0.128%	0.103%	0.116%	65	1.171%	0.852%	1.012%
45	0.141%	0.114%	0.128%	66	1.278%	0.852%	1.065%

Age	Death Probability Men	Death Probability Women	Average Death Probability	Age	Death Probability Men	Death Probability Women	Average Death Probability
67	1.441%	0.917%	1.179%	84	9.552%	6.420%	7.986%
68	1.611%	1.031%	1.321%	85	10.863%	7.122%	8.993%
69	1.729%	1.085%	1.407%	86	11.441%	8.389%	9.915%
70	1.993%	1.165%	1.579%	87	13.256%	9.608%	11.432%
71	2.123%	1.367%	1.745%	88	14.134%	10.691%	12.413%
72	2.351%	1.527%	1.939%	89	15.474%	11.705%	13.590%
73	2.635%	1.503%	2.069%	90	18.230%	13.826%	16.028%
74	2.884%	1.686%	2.285%	91	20.290%	15.742%	18.016%
75	3.221%	2.007%	2.614%	92	21.451%	18.162%	19.807%
76	3.594%	2.293%	2.944%	93	23.509%	18.852%	21.181%
77	4.067%	2.475%	3.271%	94	26.948%	21.775%	24.362%
78	4.761%	2.855%	3.808%	95	29.572%	23.271%	26.422%
79	5.382%	3.249%	4.316%	96	31.075%	25.899%	28.487%
80	5.994%	3.717%	4.856%	97	30.539%	28.030%	29.285%
81	6.490%	4.339%	5.415%	98	37.866%	30.407%	34.137%
82	7.807%	4.870%	6.339%	99	37.524%	36.187%	36.856%
83	8.247%	5.483%	6.865%	100	100.000%	100.000%	100.000%

Table B.1 Death probabilities of men, women and the average of these numbers

In this table, the death probabilities of men, women and the average of these numbers are given. The death probability is the probability that a person aged x dies within the next year. So for example, a man who is 50 years old is assumed to die within one year with probability 0.275. This data is from Statistics Netherlands. Note that the death probabilities at age 100 are set equal to 100% as this is the assumed maximum age.

Appendix C

Pension accrual

Age	Wage	Pension accrual	Age	Wage	Pension accrual
25	1.00000	0.01875	46	1.65461	0.03102
26	1.03000	0.01931	47	1.67115	0.03133
27	1.06090	0.01989	48	1.68787	0.03165
28	1.09273	0.02049	49	1.70475	0.03196
29	1.12551	0.02110	50	1.72179	0.03228
30	1.15927	0.02174	51	1.73901	0.03261
31	1.19405	0.02239	52	1.75640	0.03293
32	1.22987	0.02306	53	1.77396	0.03326
33	1.26677	0.02375	54	1.79170	0.03359
34	1.30477	0.02446	55	1.80962	0.03393
35	1.34392	0.02520	56	1.80962	0.03393
36	1.37079	0.02570	57	1.80962	0.03393
37	1.39821	0.02622	58	1.80962	0.03393
38	1.42617	0.02674	59	1.80962	0.03393
39	1.45470	0.02728	60	1.80962	0.03393
40	1.48379	0.02782	61	1.80962	0.03393
41	1.51347	0.02838	62	1.80962	0.03393
42	1.54374	0.02895	63	1.80962	0.03393
43	1.57461	0.02952	64	1.80962	0.03393
44	1.60610	0.03011			
45	1.63823	0.03072	Total accrual		1.15153

Table C.1 Pension accrual

Here, the pension accrual is calculated by taking the product of the accrual rate (1.875%) and the wage. Note that here we had to make the assumption that wage inflation is equal to price inflation and that full indexation is given every year. This leads to a replacement ratio of 63.63% ($\frac{1.15153}{1.80962} = 0.63633$).

Appendix D

Results for sensitivity analysis

D.1 Constant mix

Constant mix (40%/60%) **investment strategy**

	Initial funding ratio				
Constant mix (40%/60%)	105%	110%	115%	120%	125%
Median funding ratio	148.58%	151.31%	154.53%	158.05%	161.05%
Standard deviation funding ratio	29.24%	31.09%	33.36%	35.77%	37.94%
Funding ratio above MRF	88.60%	90.43%	91.73%	92.83%	93.73%
Funding ratio above RF	68.24%	70.96%	73.59%	76.18%	78.88%
Purchasing power	82.77%	84.60%	86.16%	87.59%	88.88%
2.5 percentile purchasing power	35.91%	37.53%	39.08%	40.69%	42.34%
Mean number of small cuts	2.837	2.267	1.916	1.632	1.447
Mean number of big cuts	0.593	0.457	0.386	0.321	0.279
Average percentage invested in return portfolio	40.00%	40.00%	40.00%	40.00%	40.00%

Table D.1.1 Results of the constant mix (40%/60%) investment strategy for different initial funding ratios

Constant mix (0%/100%) investment strategy

	Initial funding ratio				
Constant mix (0%/100%)	105%	110%	115%	120%	125%
Median funding ratio	117.48%	117.52%	117.57%	117.60%	117.64%
Standard deviation funding ratio	2.04%	2.02%	2.09%	2.07%	2.05%
Funding ratio above MRF	95.87%	99.99%	100.00%	100.00%	100.00%
Funding ratio above RF	95.87%	99.99%	100.00%	100.00%	100.00%
Purchasing power	47.88%	50.20%	52.40%	54.61%	56.83%
2.5 percentile purchasing power	40.99%	42.93%	44.77%	46.65%	48.55%
Mean number of small cuts	2.179	0.006	0.000	0.000	0.000
Mean number of big cuts	0.118	0.000	0.000	0.000	0.000
Average percentage invested in return portfolio	0.00%	0.00%	0.00%	0.00%	0.00%

Table D.1.2 Results of the constant mix (0%/100%) investment strategy for different initial funding ratios

Constant mix (25%/75%) investment strategy

	Initial funding ratio				
Constant mix (25%/75%)	105%	110%	115%	120%	125%
Median funding ratio	130.42%	131.32%	132.10%	133.49%	134.82%
Standard deviation funding ratio	13.61%	13.98%	14.53%	15.26%	16.34%
Funding ratio above MRF	92.31%	94.51%	95.84%	96.62%	97.17%
Funding ratio above RF	61.58%	64.78%	68.20%	72.11%	75.93%
Purchasing power	75.09%	77.92%	80.41%	82.69%	84.76%
2.5 percentile purchasing power	41.61%	43.55%	45.51%	47.30%	49.14%
Mean number of small cuts	4.973	3.317	2.477	2.003	1.671
Mean number of big cuts	0.274	0.168	0.112	0.098	0.079
Average percentage invested in return portfolio	25.00%	25.00%	25.00%	25.00%	25.00%

Table D.1.3 Results of the constant mix (25%/75%) investment strategy for different initial funding ratios

Constant mix (50%/50%) investment strategy

	Initial funding ratio				
Constant mix (50%/50%)	105%	110%	115%	120%	125%
Median funding ratio	165.33%	169.67%	173.96%	180.39%	185.64%
Standard deviation funding ratio	43.38%	47.06%	50.48%	55.02%	58.68%
Funding ratio above MRF	87.43%	89.00%	90.24%	91.40%	92.35%
Funding ratio above RF	62.29%	64.69%	67.04%	69.56%	72.11%
Purchasing power	84.52%	86.00%	87.26%	88.43%	89.51%
2.5 percentile purchasing power	32.17%	33.58%	35.05%	36.53%	38.02%
Mean number of small cuts	4.078	3.467	3.045	2.670	2.399
Mean number of big cuts	0.623	0.540	0.456	0.396	0.340
Average percentage invested in return portfolio	50.00%	50.00%	50.00%	50.00%	50.00%

Table D.1.4 Results of the constant mix (50%/50%) investment strategy for different initial funding ratios

Constant mix (75%/25%) investment strategy

	Initial funding ratio				
Constant mix (75%/25%)	105%	110%	115%	120%	125%
Median funding ratio	217.16%	227.39%	238.77%	247.04%	257.90%
Standard deviation funding ratio	90.38%	99.14%	109.21%	116.28%	126.56%
Funding ratio above MRF	85.32%	86.43%	87.56%	88.45%	89.31%
Funding ratio above RF	63.07%	64.80%	66.66%	68.45%	70.33%
Purchasing power	84.85%	85.93%	86.90%	87.74%	88.53%
2.5 percentile purchasing power	22.22%	23.36%	23.95%	25.53%	26.44%
Mean number of small cuts	3.900	3.525	3.185	2.939	2.724
Mean number of big cuts	0.806	0.735	0.699	0.629	0.571
Average percentage invested in return portfolio	75.00%	75.00%	75.00%	75.00%	75.00%

Table D.1.5 Results of the constant mix (75%/25%) investment strategy for different initial funding ratios

Constant mix (100%/0%) investment strategy

	Initial funding ratio				
Constant mix (100%/0%)	105%	110%	115%	120%	125%
Median funding ratio	264.14%	279.14%	289.60%	303.87%	318.17%
Standard deviation funding ratio	135.47%	149.21%	158.98%	171.93%	184.64%
Funding ratio above MRF	83.38%	84.38%	85.25%	86.10%	86.75%
Funding ratio above RF	62.56%	63.97%	65.40%	66.78%	68.17%
Purchasing power	82.50%	83.38%	84.18%	84.84%	85.60%
2.5 percentile purchasing power	13.27%	13.83%	14.36%	14.93%	15.46%
Mean number of small cuts	4.000	3.683	3.459	3.261	3.103
Mean number of big cuts	0.987	0.900	0.843	0.796	0.757
Average percentage invested in return portfolio	100.00%	100.00%	100.00%	100.00%	100.00%

Table D.1.6 Results of the constant mix (100%/0%) investment strategy for different initial funding ratios

D.2 OBPI

OBPI without lock investment strategy

	Initial funding ratio				
OBPI without lock	105%	110%	115%	120%	125%
Median funding ratio	132.75%	137.58%	142.18%	146.96%	152.59%
Standard deviation funding ratio	24.73%	28.55%	32.03%	35.71%	40.87%
Funding ratio above MRF	90.92%	94.13%	95.18%	96.05%	96.74%
Funding ratio above RF	90.92%	94.13%	95.18%	96.05%	96.74%
Purchasing power	71.39%	76.33%	79.22%	81.88%	84.15%
2.5 percentile purchasing power	37.88%	39.46%	40.48%	42.04%	43.69%
Mean number of small cuts	3.975	2.485	1.969	1.300	1.298
Mean number of big cuts	0.202	0.091	0.063	0.048	0.034

Table D.2.1 Results of the OBPI without lock investment strategy for different initial funding ratios

OBPI with lock investment strategy

	Initial funding ratio				
OBPI with lock	105%	110%	115%	120%	125%
Median funding ratio	135.24%	137.92%	140.51%	144.76%	147.65%
Standard deviation funding ratio	26.45%	27.87%	29.30%	32.29%	33.47%
Funding ratio above MRF	92.58%	95.24%	96.27%	96.79%	97.25%
Funding ratio above RF	92.58%	95.24%	96.27%	96.79%	97.25%
Purchasing power	73.78%	78.11%	80.69%	83.00%	85.07%
2.5 percentile purchasing power	38.60%	40.30%	42.37%	42.48%	44.12%
Mean number of small cuts	1.073	0.610	0.455	0.375	0.318
Mean number of big cuts	0.375	0.205	0.150	0.122	0.112

Table D.2.2 Results of the OBPI with lock investment strategy for different initial funding ratios

OBPI with satiation investment strategy

	Initial funding ratio				
OBPI with satiation	105%	110%	115%	120%	125%
Median funding ratio	130.83%	133.39%	134.60%	135.04%	135.53%
Standard deviation funding ratio	23.44%	25.33%	26.51%	26.34%	26.34%
Funding ratio above MRF	91.25%	94.53%	95.78%	96.57%	97.28%
Funding ratio above RF	91.25%	94.53%	95.78%	96.57%	97.28%
Purchasing power	67.53%	73.70%	77.42%	80.55%	83.21%
2.5 percentile purchasing power	38.15%	40.18%	42.37%	43.61%	45.63%
Mean number of small cuts	3.849	2.300	1.756	1.368	1.067
Mean number of big cuts	0.201	0.089	0.061	0.043	0.026

Table D.2.3 Results of the OBPI with satiation investment strategy for different initial funding ratios

D.3 CPPI

CPPI without lock investment strategy

	Initial funding ratio				
CPPI without lock (annually)	105%	110%	115%	120%	125%
Median funding ratio	123.48%	124.76%	126.40%	127.91%	129.19%
Standard deviation funding ratio	10.73%	11.34%	12.83%	14.18%	15.06%
Funding ratio above MRF	99.38%	99.67%	99.71%	99.69%	99.68%
Funding ratio above RF	61.84%	67.55%	73.49%	75.39%	77.70%
Purchasing power	67.61%	71.36%	74.81%	77.43%	79.56%
2.5 percentile purchasing power	41.44%	42.52%	44.40%	46.41%	47.79%
Mean number of small cuts	6.994	3.282	2.044	1.790	1.597
Mean number of big cuts	0.016	0.004	0.004	0.005	0.006
Average percentage invested in return portfolio	24.16%	27.14%	30.84%	34.38%	38.03%

Table D.3.1a Results of the CPPI without lock investment strategy for different initial funding ratios when re-balancing takes place every year

	Initial funding ratio				
CPPI without lock (monthly)	105%	110%	115%	120%	125%
Median funding ratio	124.47%	125.36%	127.80%	128.87%	131.72%
Standard deviation funding ratio	11.82%	12.50%	14.71%	15.60%	18.10%
Funding ratio above MRF	99.43%	99.94%	99.97%	99.98%	99.98%
Funding ratio above RF	56.43%	61.89%	67.77%	71.26%	74.32%
Purchasing power	70.69%	74.19%	77.11%	79.31%	81.20%
2.5 percentile purchasing power	40.83%	42.60%	43.49%	44.86%	46.13%
Mean number of small cuts	6.158	2.722	1.593	1.323	1.138
Mean number of big cuts	0.007	0	0	0	0
Average percentage invested in return portfolio	28.51%	31.80%	35.49%	38.88%	42.22%

Table D.3.1b Results of the CPPI without lock investment strategy for different initial funding ratios when re-balancing takes place every month

CPPI with lock investment strategy

	Initial funding ratio				
CPPI with lock (annually)	105%	110%	115%	120%	125%
Median funding ratio	124.74%	125.48%	127.38%	129.69%	130.91%
Standard deviation funding ratio	11.74%	12.14%	13.66%	15.71%	16.52%
Funding ratio above MRF	97.00%	99.57%	99.65%	99.71%	99.73%
Funding ratio above RF	63.00%	69.03%	75.02%	77.05%	80.07%
Purchasing power	67.80%	71.60%	75.32%	78.29%	80.76%
2.5 percentile purchasing power	41.00%	42.84%	45.17%	46.39%	47.83%
Mean number of small cuts	7.021	2.837	1.844	1.546	1.312
Mean number of big cuts	0.055	0.004	0.004	0.001	0.003
Average percentage invested in return portfolio	20.67%	22.75%	25.74%	28.65%	31.58%

Table D.3.2a Results of the CPPI with lock investment strategy for different initial funding ratios when rebalancing takes place every year

	Initial funding ratio				
CPPI with lock (monthly)	105%	110%	115%	120%	125%
Median funding ratio	126.28%	128.27%	129.97%	132.78%	133.70%
Standard deviation funding ratio	13.30%	15.01%	16.55%	19.14%	19.81%
Funding ratio above MRF	96.18%	99.45%	99.74%	99.77%	99.80%
Funding ratio above RF	57.71%	63.19%	68.95%	72.88%	75.91%
Purchasing power	70.10%	73.99%	77.04%	79.68%	81.72%
2.5 percentile purchasing power	40.56%	42.59%	43.89%	45.07%	46.07%
Mean number of small cuts	5.652	2.346	1.456	1.186	0.989
Mean number of big cuts	0.091	0.011	0.002	0.001	0
Average percentage invested in return portfolio	22.44%	24.46%	27.17%	29.75%	31.98%

Table D.3.2b Results of the CPPI with lock investment strategy for different initial funding ratios when rebalancing takes place month

CPPI with satiation investment strategy

	Initial funding ratio				
CPPI with satiation (annually)	105%	110%	115%	120%	125%
Median funding ratio	125.08%	125.88%	126.86%	127.96%	129.07%
Standard deviation funding ratio	11.86%	12.41%	13.17%	14.14%	14.72%
Funding ratio above MRF	97.23%	99.75%	99.79%	99.81%	99.82%
Funding ratio above RF	63.81%	69.46%	75.55%	77.93%	80.76%
Purchasing power	64.21%	69.73%	73.98%	77.43%	80.30%
2.5 percentile purchasing power	41.41%	42.83%	44.91%	47.47%	48.41%
Mean number of small cuts	7.278	2.813	1.813	1.493	1.240
Mean number of big cuts	0.078	0.002	0.002	0.002	0.002
Average percentage invested in return portfolio	21.12%	23.81%	26.12%	28.32%	30.45%

Table D.3.3a Results of the CPPI with satiation investment strategy for different initial funding ratios when rebalancing takes place every year

	Initial funding ratio				
CPPI with satiation (monthly)	105%	110%	115%	120%	125%
Median funding ratio	123.50%	123.97%	125.24%	126.07%	127.73%
Standard deviation funding ratio	10.81%	11.07%	12.12%	12.83%	14.12%
Funding ratio above MRF	98.31%	99.93%	99.96%	99.97%	99.98%
Funding ratio above RF	57.70%	63.79%	69.53%	72.99%	76.24%
Purchasing power	64.94%	69.83%	73.58%	76.50%	79.06%
2.5 percentile purchasing power	40.71%	42.38%	43.50%	44.80%	46.37%
Mean number of small cuts	6.539	2.575	1.603	1.305	1.101
Mean number of big cuts	0.006	0	0	0	0
Average percentage invested in return portfolio	23.66%	25.97%	28.17%	30.09%	31.97%

Table D.3.3b Results of the CPPI with satiation investment strategy for different initial funding ratios when rebalancing takes place every month

Appendix E

Utilities

Strategy	Utility
Constant mix (40%/60%)	13.1
OBPI without lock	12.7
OBPI with lock	13.9
OBPI with satiation	12.6
CPPI without lock (yearly)	12.2
CPPI without lock (monthly)	12.8
CPPI with lock (yearly)	12.5
CPPI with lock (monthly)	12.9
CPPI with satiation (yearly)	12.3
CPPI with satiation (monthly)	12.4
Constant mix (0%/100%)	12.0
Constant mix (25%/75%)	12.8
Constant mix (50%/50%)	12.1
Constant mix (75%/25%)	10.9
Constant mix (100%/0%)	9.4

Table E.1 Utility levels for the different strategies

	Initial funding ratio				
Strategy	105%	110%	115%	120%	125%
Constant mix (40%/60%)	12.3	13.1	13.7	14.2	14.7
OBPI without lock	11.2	12.7	13.5	14.7	15.2
OBPI with lock	12.8	13.9	14.5	14.1	14.7
OBPI with satiation	10.9	12.6	13.6	14.8	15.2
CPPI without lock (yearly)	9.8	12.2	13.4	14.0	14.5
CPPI without lock (monthly)	10.5	12.8	13.7	14.2	14.7
CPPI with lock (yearly)	9.7	12.5	13.6	14.2	14.7
CPPI with lock (monthly)	10.5	12.9	13.8	14.4	14.8
CPPI with satiation (yearly)	9.3	12.3	13.5	14.3	14.8
CPPI with satiation (monthly)	9.7	12.4	13.4	14.0	14.5
Constant mix (0%/100%)	10.3	12.0	12.4	12.8	13.2
Constant mix (25%/75%)	11.3	12.8	13.7	14.4	15.0
Constant mix (50%/50%)	11.4	12.1	12.7	13.3	13.8
Constant mix (75%/25%)	10.4	10.9	11.3	11.8	12.1
Constant mix (100%/0%)	9.0	9.4	9.8	10.1	10.3

Table E.2 Utility level for the different strategies for the different initial funding ratios

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