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Economic Scenarios for an Asset and Liability Management Study of a Pension Fund

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Abstract

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In this thesis we investigate the possible ways of generating macroeconomic scenarios that serve as input for an asset and liability management (ALM) study of a pension fund. First we discuss the different type of models that can be used and we conclude that the vector autoregressive (VAR) model is the most suited. We estimate a VAR model ourselves extended with an affine term structure model of interest rates, in this way there are no arbitrage opportunities. The simulations from our model are used in an ALM study for a typical Dutch pension fund. Different time periods of historical data are being considered and we find that these have a great impact on our estimation results and corresponding scenarios. At last the term structure model is extended for longer maturities, but this extension does not lead to a better model fit.

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Chapter 1

Introduction

1.1 Background

Financial institutions seek to anticipate the future risks they face which are inherent to their business. These risks mainly depend on macroeconomic factors. In order to be prepared for adverse future situations it is vital to have insight in the macroeconomic scenarios that may take place. Especially scenarios that put the company in serious financial distress are of interest. Depending on scenarios, an institution will line up its policies, make investment choices, and allocate capital for potential losses. Well known examples are pension funds who base their investment policies on asset and liability management studies that use a large number of different future scenarios as input.

An asset and liability study can be described as a process with the goal to get insights in the mutual dependence and future development of the assets and liabilities of an organization. For a pension fund the asset side consist of an investment portfolio which has a market value at a certain time. The liabilities side consist among others of the build up pension rights of the participants that have to be paid out in the future. Ideally the assets and liabilities develop in such a way that their ratio stays the same. So if the investments would decreases with a certain percentage, the liabilities should decrease with the same percentage. Unfortunately this is not true in real life and in an asset and liability study the possible developments of assets and liabilities are being modeled for different macroeconomic scenarios.

One of the problems of macroeconomic scenarios are possible arbitrage opportunities. An arbitrage opportunity is the possibility that the same financial product has two different values at the same point in time. For example if we have interest rates scenarios for a pension fund we can derive a bond price from these interest rates. If we model bond returns in the same scenario set we also can derive bond prices from the bond returns. When the two bond prices do not have the same value for all scenarios at all points in time there are arbitrage opportunities. These arbitrage opportunities are not realistic and therefore we do not want our scenarios to exploit them. Although we do not perform option valuations in our thesis it could be of interest to calculate some embedded options in our asset and liability model and in order to do so there should be no arbitrage opportunities.

1.2 Research Questions and literature review

The goal of this thesis is to develop and describe the most suited model to generate macroeconomic scenarios. Additionally we want our scenarios to be arbitrage free, and together these two objectives are summarized in our main research question: *Which model is most suited to generate arbitrage free macroeconomic scenarios.*

There has been done more research on this topic. Hoevenaars et al. (2003) summarizes the different type of models that can be used for generating economic scenarios and we will start with discussing these models. The vector autoregressive (VAR) model is widely used, for instance by ORTEC. In its standard form, this model is based on the assumption of normality of the macroeconomic factors. Another type of model is the cascade model of Wilkie (1995) that imposes a structure on the interdependence of certain variables. Several other models have been used, each having their advantages and disadvantages. In line with Hoevenaars et al. (2003) we will conclude that the VAR model is most appropriate since it is an intuitive linear model which incorporates long term dynamics. Hoevenaars et al. (2003) estimate a VAR model for bond and stock returns themselves and illustrate how the scenarios are used in an ALM study. Also the effects on the predictability of their model by adding other macroeconomic variables is being reviewed. Also Steehouwer (2005) has done an extensive study on the subject of macroeconomics scenarios. He presents stylized facts about the empirical behavior of macroeconomic variables in the Netherlands, Europe and the United States. Furthermore he tests existing VAR scenario models and introduces his own frequency domain VAR scenario framework. But one drawback of his framework is that it exhibits arbitrage opportunities.

To obtain arbitrage free scenarios we should be able to value all assets and cash flows from our scenarios. This valuation can be done with the so called deflator method where a deflator, also called a stochastic discount factor or pricing kernel, is used for discounting. Jarvis et al. (2001) discuss the deflator method and compare it to valuation with risk-neutral simulation where a risk free rate is used for discounting. They conclude that both methods result in the same values for the assets and cash flows, but the deflator method has the advantage that real-world probabilities can be used which are more realistic compared to risk-neutral probabilities. Ang and Piazessi (2003) use the deflator, or pricing kernel, to extend their VAR model with an affine term structure of interest rates. In this manner they are able to value all assets and cash flows. Another application of the deflator is done by Cochrane and Piazzesi (2005), they attempt to decompose the yield curve into expected interest rate and risk premium components so that the risk factors become apparent. To do so the term structure of bond yield curves is derived by means of the pricing kernel such that they are affine in the state variables of the VAR model. Here affine means a linear function of the state variables.

Hoevenaars (2008) also combines the VAR model with an affine term structure model of interest rates is such a way that there are no arbitrate opportunities. He uses the model to generate macroeconomic scenarios that serve as input for an ALM model and his model will be our starting point. We will introduce the model and estimate it on approximately the same data as Hoevenaars (2008) in order to compare our estimation results and term structure model fit. After that we can use this model to generate scenarios for stock returns, inflation, interest rates and bond returns.

When we have our scenarios the next question is how these scenarios can be used in an asset and liability management study. We use our scenarios as input for an asset and liability study of a typical Dutch pension fund and this leads us to our second research question: *How can we use economic scenarios in a pension fund ALM study and what are the results for a typical Dutch pension fund.*

Starting point is again Hoevenaars (2008) who performs an simulation approach to asset and liability management where economic scenarios for stock returns, inflation, interest rates and bond returns serve as input. Hoevenaars (2008) examines different dynamic asset allocation strategies in his pension fund ALM model and besides output based on the future solvency position and indexation result he also calculates embedded surplus, deficit and indexation options. In our thesis the focus is on the traditional ALM output and we will not calculate the embedded options. Boender et al. (2007) also perform an ALM study for a pension fund where they use a VAR model for the macroeconomic scenarios with additionally a Nelson Sieghel model for the interest rates. But by using the Nelson Sieghel model for generating interest rate scenarios their model is not arbitrage free. They apply the scenario approach as well and investigate the consequences for the ALM results of using three different pension plans. The three pension plans have a different characteristics such as asset allocation and age distribution.

We want our own ALM model to be consistent with an average Dutch pension fund in 2010. Therefore we are going to make use of the age distribution for the total population in 2010 according to the Dutch Central Bureau of Statistics (CBS). The CBS also provides us with information about the participation rate of the Dutch working force and the life expectancy of the Dutch population. From the Dutch Central Bank (DNB) we obtain among others the average funding ratio in September 2010 of Dutch pension funds and we will implement all these characteristics of a typical Dutch pension fund in our model.

We started off with scenarios based on the same historical data as Hoevenaars (2008), which are monthly observations from 1973 till 2005. Our first extension of the model of Hoevenaars (2008) will be the addition of the historical data of the time period between 2006 and 2010. Secondly we will look at the consequences for our scenarios and ALM output of estimating our model on historical data between 1986 and 2010. This leads to the research question: *What are the consequences of using other historical data for the estimation of our model.*

The added historical data includes the credit crisis from 2008 and very low interest rates from 2010. As a result our estimated parameters will change and we find that especially the correlation between stock and bond returns turns out to be quite different for the different data sets. Furthermore we will encounter some problems with the low starting values of the interest rates. The low interest rates in 2010 will serve as starting values for our scenarios and cause some negative interest rate scenarios. Due to the dependence in our model between the inflation and the interest rates, we also encounter some negative inflation scenarios. This issue of negative scenarios that follow from a VAR model is also described in Steehouwer (2005). He proposes to use a truncated VAR model, but in his research is shown that this approach does not solve the issue entirely. We will discuss the approach but the application of the model to our data set is a topic for further research.

Our final attribution to the existing literature is the extension of the term structure model of interest rates and will answer our final research question: *Will adding a 15 year yield in our term structure model result in a better model fit.*

In the benchmark case of Hoevenaars (2008) our interest rate term structure model has the maturities of 1 month till 10 years. In that case we discount all cash flows in our ALM model that are taking place after more than 10 years in the future with the 10 year rate. This is off course not very realistic and it would be more ideal to use a more appropriate rate of higher maturity. Also a better model fit and a better shape of the term structure model are a motivation for this extension. We extend the term structure model in two ways up until a maturity of 15 years. For the first one we end up with a slightly better fit for the longer maturities. The second extension does not lead to a better fit.

1.3 Thesis outline

In chapter 2 we will give a short introduction about ALM and economic scenarios. Also the advantages of using simulations will be mentioned. Finally the different models that can be used to generate economic scenarios will be discussed, in line with Hoevenaars et al. (2003) we conclude that the vector autoregressive models is most suited. We want our scenarios to be arbitrage free and in chapter 3 the literature about arbitrage opportunities within scenarios is treated. The extended VAR model of Hoevenaars (2008) meets the requirements and in the next chapter we will estimate this model ourselves. We start with estimating the model on approximately the same historical data as Hoevenaars (2008) in chapter 4. In this way we can check our estimation results and model fit. We describe the model in detail, but also the estimation and simulation procedure. Next we will present the data and discuss the estimation results and term structure model fit.

After that we are going to perform an ALM study in chapter 5 for a typical Dutch pension fund where the economic scenarios serve as input. We introduce an average defined benefit pension plan from Kakes and Broeders (2006) and explain the assumptions we made about the age distribution, age expectancy, active and passive participants, wage developments, valuation and indexation of the liabilities and finally the investment policy. At last we will present our simulation results, where the uncertainty of our model outcomes turns out to be quite big. In chapter 6 we look at the consequences of estimating our model on different historical data. We find that using different time periods of the same historical data has a big impact on the estimation results and the corresponding scenarios. Furthermore we investigate whether extending our interest rate term structure model will yield a better model fit and for our first extension we will find some positive results for longer maturities. Finally we will conclude with a summary of our thesis and the presentation of our findings considering our ALM model, using different historical data and the extension of our term structure model. One of our recommendations will be to be cautious with generating interest rate scenarios with a VAR model when interest rates starting values are low. We will also recommend to be careful with selecting the time window of the historical data.

Chapter 2

ALM & Economic scenarios

In this chapter we start in section 2.1 with explaining what an asset and liability management problem is and where it can be used. Next we discuss in section 2.2 the concept of an economic scenario and after that we will clarify in section 2.3 why we make use of simulations. At last we will describe in section 2.4 the models that can be used to generate these economic scenarios. Here we conclude that the VAR model is the most appropriate model.

2.1 Asset and Liability Management

As we already explained in the introduction, an asset and liability study can be described as a process with the goal to get insights in the mutual dependence and future development of the assets and liabilities of an organization. There are ALM type of problems for commercial banks, insurance companies and pension funds, but in general each ALM problem can be divided into three parts. From Steehouwer (2005) we have the following description.

The first part consist of identifying the objectives and constraints of all stakeholders. For a pension fund ALM problem for example the stakeholders can consist of the participants on the one hand and the plan sponsor on the other hand. There are also indirect stakeholders such as pension regulators and pension accountants. Objective for the sponsor can be to profit from low pension contributions to the plan resulting from high portfolio returns. Constraints for the sponsor can be the extent the pension investment risk and other pension risk drivers are allowed to affect their own Profit & Loss account (P & L) and Balance Sheet under the rules of for instance IFRS (International Financial Reporting Standards). The participants can be split up in a group of active members (employees) and inactive members (beneficiaries). Objective of the active member are for instance the wish to pay as less premium as possible. Inactive members on the other hand have the primary objective to receive a certain level of pension payments each month. Typically the objectives and constraints of all stakeholders will be conflicting with each other.

Secondly an ALM problem consist out of one or more policy instruments which the decision maker (for example a pension fund board or a insurance company board) can use to meet the objectives and constraints as best as possible. For the board of a pension fund these policy instruments are for example the investment policy (strategic asset allocation), funding policy or indexation policy.

The third component of the ALM problems are the risk and return factors. This can be specific factors such as the mortality rate of the participants of a pension fund or changes in a countries tax regulation for life insurance policy holders. But the most important factors are the macroeconomic variables such as interest rates, inflation and equity returns.

2.2 Economic scenarios

To model the macroeconomic variables from the third component an economic scenario generator is used in ALM. Generating economic scenarios is also called scenario analysis, stochastic simulation or Monte Carlo simulation and can be seen as a possible future evolution of all relevant uncertain macroeconomic variables. These stochastic simulations include bear and bull market conditions as well as normal market conditions. And scenario generation techniques ensure that cross-relationships between historical series are accounted for in the future. Figure 2.1 also from Steehouwer (2005) shows how scenario analysis and ALM are related. Not one but a large number of scenarios for the macroeconomic variables are generated. Together with the strategic policy under consideration these scenarios are fed into a model which states all relations between policy instruments, scenario variables and relevant output measures with respect to the objectives of the stakeholders. Then the output of the model can for instance be the future evolution of the solvency ratio of an insurance company for each of the economic scenarios.

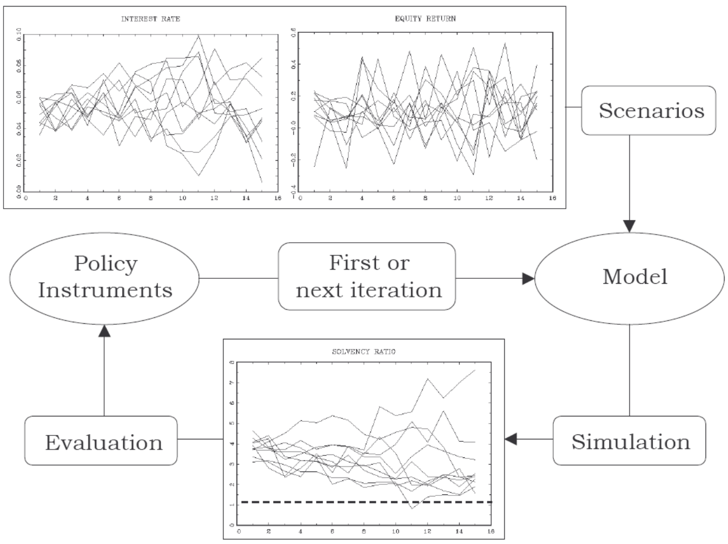


Figure 2.1: Scenario analysis and ALM

Different risk and return measures can be calculated from the scenarios. An example of a risk measure is the probability that the solvency ratio drops below 100 % and an example of a return measure is for instance the expected equity return for the next fifteen years. After calculating the relevant risk and return measures corresponding to a certain policy, the decision makers (pensions fund/ insurance company board) have to decide whether for example the risk falls withing their maximum risk tolerance. If this is not the case, alternative (less risky) policies should be analyzed using the same scenarios. By means of trial and error the (pension fund/ insurance company) board can decide which policy is the best in terms of the objectives and constraints.

2.3 Scenario analysis vs. analytical (optimization) models

Besides the scenario approach to ALM there are also other approaches such as analytical optimization models, but there are in general three reasons why the scenario analysis is preferred over the alternatives. The first reason is the flexibility of scenario analysis, in comparison to analytical optimization models it can deal with much more complex interactions and relations within and between the components of an ALM problem as described before. This point is best illustrated by the following statement of Kingsland (1982) from a classic ALM paper:

The dynamic behavior of a pension plan is clearly dominated by rules and methodology which are discontinuous and nonlinear functions of its financial condition. The task of developing a closed-form solution to evaluate the potential state of a pension plan following a series of stochastic investment and inflation experiences would be extremely difficult, if not impossible. To date, the only approach that has proven feasible is the application of Monte Carlo Simulation. Wherein an investment and inflation scenario is generated by random draws based on the expected probability distribution of year to year investment and inflation behavior. In order to develop an accurate assessment of the range of potential uncertainties, it is necessary to repeat this simulation process by generating dozens or hundreds possible scenarios, consistent with statistical expectations.

Although the stochastic optimization models have developed enormously since 1982, the statement of Kingsland (1982) is to a certain degree still valid. A second reason why scenario analysis is preferred over analytical optimization models is that it gives a great opportunity of learning about the ALM problem and the effects of different policies. When a model just calculates the optimal solution this learning effect is much smaller. The third and final reason are the strong visual aspects of scenario analysis which makes it more understandable for people with less knowledge.

2.4 Models for generating economic scenarios

There are several ways to generate future scenarios and we will discuss them below. Hoevenaars et al. (2003) provides us with a nice overview of the different models.

2.4.1 Independent drawings from a Normal distribution

A first method would be to draw future period returns from a standard normal distribution. Where we take the variance of the returns and the correlation between the returns into account by multiply the drawings with a Choleski decomposition of an exogenously specified covariance matrix. The major advantage of this approach is it simplicity, however there are some drawbacks as well. The long-term dynamics are not captured in the scenarios. This means that cross-relationships between asset classes, and in the time dimension within an asset class are not properly incorporated. Furthermore, conditional information is not accounted for. This means the unconditional correlations only represent the relationship between two asset classes while the cross-relationship to other asset classes is not taken into account. Conditional relationships are typically important because they deal with cross-relationships between all asset classes that are included simultaneously.

2.4.2 Vector autoregressive (VAR) model

For predicting stock, bond or interest rate returns we could use single-equation time series models. Stock returns at time $t+1$ for instance, may be explained by current stock returns at time t. Then we could use another single-equation model for predicting bond returns. In this way there is no causality assumed between the bond and stock return series, or the other way around. When we are not totally confident about the fact that there is no causal relationship between two variables, we can model them together with additional other variables in a multi-equation time series model like a Vector Autoregressive (VAR) model. In this manner the time path of the bond returns can influence the time path of the stock returns and vice versa. Besides the mean and the variance of the variables, a VAR not only includes the correlation of the variables through time (autocorrelation), but also the correlation between variables ((cross) correlation). Moreover, the VAR captures the conditional long-term dynamics from the data. So in this way the correlation between stocks and bonds is also based on the state of the world of the other variables (inflation for instance) that are included in the VAR model. The advantage of a VAR model is that it is an intuitive linear model that describes a stochastic time series, but it also captures long-term dynamics. These long term dynamics include variances and cross correlations between different variables, autocorrelations within time series and cross correlations between shocks in the residuals of different variables. The disadvantage is that there should be enough historical data available, this could be a problem if we want to include alternative asset classes such as for instance real estate. And secondly the accuracy of the estimated parameters decreases in the number of variables increases.

2.4.3 Cascade approach

An alternative model to generate the scenarios could be the cascade approach, which was introduced by Wilkie (1987). According to Wilkie (1987) and Wilkie (1995) the number of cross effects in the VAR model are overdone and he his approach is based on the economic theory that suggests that inflation is the driving force for returns. Here all relationships are modeled as single-equation time series equations (Auto Regressive (AR) processes). Technically generating the scenarios is done in the same way as the multi-equation framework from the VAR model. Advantages are that less parameters have to be estimated compared with the VAR model and that it is easier to extend single-equation AR processes with stochastic volatility models such as G(ARCH) that capture time-varying variances, than it is to extend a VAR with stochastic volatility models. The biggest drawback is that the approach is based on the theory that inflation is the driving force of the economy. And the question is whether this theory is right, particularly for long run simulations. If the theory is incomplete several dynamics and (cross) relationships are not accounted for and in that case important information is lost.

2.4.4 SDE approach

Another alternative model would be the stochastic differential equation approach from Mulvey (1994) and Mulvey et al. (2000). Basically this is the same approach as the cascade approach only in continuous time instead of discrete time. Drawbacks are again the theory uncertainty and the fact that the estimation methods for the stochastic differential equations are far from straightforward.

2.4.5 Risk-neutral simulations

It is also possible to use risk-neutral simulations as scenarios. These risk-neutral simulations can be made with for instance a one factor Hull-White Black Scholes model that is explained in Hull (2005). It is a commonly-used technique for valuation and the simulations are for instance used for determine the payoffs of embedded insurance options. No term structure is needed since discounting is done with the risk free rate. Drawback is that we can not use the real world scenarios from the VAR model that are more consistent with historical time series and patterns.

2.4.6 Conclusion

We do not use the risk-neutral simulations since we want our model to be consistent with historical time series and patterns. If the theory behind the cascade and the stochastic differential equation is correct, these models are preferred over the VAR model. But if the economy turns out to be very different from the underlying approaches, the problem seems less severe for the VAR model. The VAR accounts for many conditional (cross-)dynamics and in line with the academic literature and most practical users we are going to use a VAR model to generate our economic scenarios. One of the problems with the simulation of economic scenarios based on historical time series are the possible arbitrage opportunities. These are possible for the VAR, cascade and SDE approach and in the next chapter we discuss the consequences and possible solutions for the VAR model.

Chapter 3

Arbitrage opportunities in VAR model simulations

One of the drawbacks of making simulations with a VAR model is that the simulations can exploit arbitrage opportunities. This is not realistic and in order to valuate embedded options simulations must be arbitrage free. In this chapter we start with explaining in section 3.1 what an arbitrage opportunity is. In order to rule out the arbitrage opportunities we should be able to valuate all assets from our simulations. In section 3.2 we describe two possible methods for the valuation of assets namely the deflator and risk-neutral method. Next we review in section 3.3 the literature on the deflator technique and how this technique can be used for no arbitrage restrictions in a VAR model. Finally we will introduce the arbitrage free VAR model of Hoevenaars (2008) in section 3.4. He extends his VAR model with an affine term structure model of interest rates which is derived with the deflator method. In the next chapter we are going to estimate this model ourselves.

3.1 Arbitrage opportunities

As we already in explained in the introduction, an arbitrage opportunity is the possibility that the same financial product has two different values at the same point in time. For example if we have interest rates scenarios for a pension fund we can derive a bond price from these interest rates. If we model bond returns in the same scenario set we also can derive bond prices from the bond returns. When the two bond prices do not have the same value for all scenarios at all points in time there are arbitrage opportunities.

Ingersoll (1987) defines an arbitrage opportunity to be a situation in which it is possible to buy a portfolio at price zero which pays out a positive amount with positive probability in the future, whereas the probability of negative pay outs equals zero. Here the investor has a "free lunch": something for nothing. If the arbitrage opportunity of the previous example should occur it would be possible for an investor to sell the bond with the higher price and at the same time buy the same bond with the lower price. In this way the investor has the so-called "free lunch", he will make money out of nothing.

These arbitrage opportunities are not realistic and therefore we do not want our scenarios to exploit them. Furthermore it could be of interest to value some embedded options in our asset and liability model and in order to do so there should be no arbitrage opportunities. This valuation of embedded options is not part of our thesis though. To rule out arbitrage opportunities we should be able to valuate all assets from our simulations and in the next section we will discuss two possible methods.

3.2 Valuing assets in VAR simulations

We should be able to value all the assets in the economic scenarios in such a way that there is no arbitrage. For this valuation a term structure of interest rates is needed that is necessary for discounting.

A possible model for the term structure of interest rates is the Nelson-Sieghel type of model from Diebold and Li (2006), disadvantage is that there are arbitrage opportunities within this model as was shown by Christensen et al. (2007). Although with restrictions Christensen et al. (2007) also show that the Nelson-Sieghel model can be arbitrage free, it is not arbitrage free if we combine this model with a VAR model for valuation of all assets.

Another approach of modeling the term structure of interest rates is with a stochastic discount factor (SDF), also called a deflator or pricing kernel. Campbell et al. (1997) apply the stochastic discount factor in an affine yield model in such a way that there is no arbitrage. This can also be combined with a VAR model is such a way that all asset can be valued under the condition that there is no arbitrage. This approach will be further explained below.

The alternative of using a VAR model with a SDF or pricing kernel model for the term structure of interest rates are risk-neutral simulations. This can be done for instance with a 1 factor Hull-White Black Scholes model that is explained in Hull (2005). It is a commonly-used technique for valuation and the simulations are for instance used for determine the payoffs of embedded insurance options. No term structure is needed since discounting is done with the risk free rate. Drawback is that we can not use the real world scenarios from the VAR model that are more consistent with historical time series and patterns.

3.3 Deflator method

Jarvis et al. (2001) discuss the deflator method for valuing future cash flows that are generated with stochastic models. According to Jarvis et al. (2001) the actuarial literature contains many stochastic models simulate indices and asset prices, but they do not place a market value on other cash flows. This is because the valuation first requires a suitable risk-adjusted rate at which to discount any set of cash flows and with the deflator this is possible. They compare the deflator valuation method where the cash flows are valued with real-world probabilities and deflators with the risk-neutral method where the risk-neutral probabilities are discounted with the risk free rate. Both methods will place the same values on the cash flows, but the deflator method has the advantage that real-world probabilities can be used which are more realistic compared to risk-neutral probabilities.

Smith and Wickens (2002) introduce a new stochastic discount factor model. The model is based on the use of multiple factors that are observable and modeling the joint distribution of excess returns and the factors using a multi-variate GARCH-in-mean process. They argue that in general single equation and VAR models are inappropriate as SDF model as they do not satisfy the no-arbitrage condition. Accordingly the VAR model is inadequate in modeling the risk premium. One of the main reasons is that the VAR model is not able to provide a suitable risk premium for the risk-averse investor. To model the risk premium conditional covariances are needed and the VAR model does not include these. But there are two exceptions, the first is the assumption of risk neutrality then there is no risk premium needed. And the second exception is that when the risk premium can be shown to be a linear (affine) function of the variables in the VAR model, then it can also be exempt and the VAR model can be a SDF model.

Ang and Piazessi (2003) use the deflator, or pricing kernel, to extend their VAR model with an affine term structure of interest rates assuming the no-arbitrage condition of Harrison and Kreps (1979). Their model is a Gaussian VAR(1) model and with the pricing kernel they can price all assets at fair value. They follow Campbell et al. (1997) when pricing zero coupon bonds with the pricing kernel. Cochrane and Piazzesi (2005) attempt to decompose the yield curve into expected interest rate and risk premium components so that the risk factors become apparent. To do so the term structure of bond yield curves is derived by means of the pricing kernel such that they are affine in the state variables of the VAR model.

3.4 Arbitrage free VAR simulations in ALM

Hoevenaars (2008) also extend a VAR model for inflation, equity returns an other macroeconomic variables with the pricing kernel to derive an affine term structure of interest rates. As in Ang and Piazessi (2003) and Cochrane and Piazzesi (2005) identification is based on the absence of arbitrage. Bond yields are affine in the state variables of the VAR and arbitrage opportunities are ruled out to the included bonds. In this framework the model also captures time-varying risk premia which are important for the time variation in bond premia. Hoevenaars (2008) uses the above model to generate scenarios for equity return, bond return, inflation and interest rate term structures that are input for an ALM model. Besides the classical ALM output such as probability distributions for the funding ratio and indexation quality he also calculates value-based ALM outcomes such as the embedded surplus, deficit and indexation options. Conducting such a value-based ALM analysis reveals value transfers between stakeholders. Due to the added pricing kernel it is possible to value the cash flows from which the embedded options are calculated. There is consistency between the traditional ALM output from the VAR scenarios and the valuation results that are obtained by applying the pricing kernel on those same scenarios.

We can conclude that there is a solution for the arbitrage opportunities that appear in simulations from an unrestricted VAR model. Using the deflator or stochastic discount factor we can extend the VAR model with an affine term structure of interest rates where identification is based upon the absence of arbitrage. In this way we can price all securities from the model and obtain consistent valuations of the real world scenario generated cash-flows. With this framework it is also possible to calculate embedded options in ALM for pension funds besides the traditional ALM outcomes such as the funding ratio and indexation quality.

Chapter 4

Arbitrage free Economic Scenarios

In this chapter we are going to estimate our own model. Our start will be the arbitrage-free VAR model with an affine term structure model from Hoevenaars (2008). We will estimate it on almost the same data as Hoevenaars (2008) in order to check our results. We will start with the explanation of our model in sections 4.1, 4.2 and 4.3 and after that we will explain the estimation procedure and the simulation procedure in section 4.4 and 4.5. In section 4.6 the stationarity of our model will be discussed and at last we will introduce the data in section 4.7 and discuss the estimation results and the fit of the term structure model.

4.1 VAR(1) model

Our model setup is along the lines of Hoevenaars (2008). We model the return dynamics by a first-order VAR,

$$z_{t+1} = \nu + Bz_t + \Sigma\xi_{t+1} \tag{4.1}$$

where $\xi_{t+1} \sim N(0, I)$ is a (5×1) vector and z_t is a (5×1) vector of state variables including the 1-month interest rate, 10-year zero coupon rate, price inflation, stock returns in excess of the 1-month interest rate and the corresponding dividend yield. The dividend yield is included to capture dynamics in the data following Campbell and Viceira (2002). ν is a (5×1) vector of the constant terms and B and Σ are both (5×5) matrices. B contains the VAR coefficients and Σ is the covariance matrix.

4.2 Pricing Kernel

We extend the VAR model with the pricing kernel in order to derive an affine term structure model of interest rates which is arbitrage free. In line with Cochrane and Piazzesi (2005) we use the no-arbitrage assumption to develop the affine term structure model and we relate the pricing kernel or stochastic discount factor with the state variables as follows:

$$-m_{t+1} = \delta_0 + \delta_1 z_t + \frac{1}{2}\lambda_t'\lambda_t + \lambda_t'\xi_{t+1} \tag{4.2}$$

where $-m_{t+1} = -\ln M_{t+1}$. M_{t+1} is the stochastic discount factor, and $\delta_0 + \delta_1 z_t$ is the short rate which is assumed to be affine in the state variables of the VAR. As we use monthly data we use the observable 1-month yield as the short rate, such that $y_t^{(1)} = \delta_0 + \delta_1 y_t$. To achieve consistency between the VAR and the short rate dynamics we let $\delta_0 = 0$ and $\delta_1' = (1, 0, 0, 0, 0)$. λ_t is a (5×1) vector with time-varying market prices of risk which are affine in the state variables:

$$\lambda_t = \lambda_0 + \Lambda_1 z_t. \tag{4.3}$$

The (5×5) matrix Λ_1 accounts for time-variation in the risk premia and conditional heteroskedasticity in the pricing kernel in (4.2). The stochastic discount factor (M_{t+1}) varies with the random variables in the VAR. The first component ($\delta_0 + \delta_1 z_t$) is the short rate ($y_t^{(1)}$) which is also included in the VAR. The other component ($\frac{1}{2}\lambda_t'\lambda_t + \lambda_t'\xi_{t+1}$) relates shocks in the state variables to the pricing kernel. A positive market price of risk λ_t leads in (4.2) to low values for the stochastic discount factor in states of the world where the values of the innovations (error terms) are high. These effects are smaller for lower market prices of risk.

4.3 Term structure model

We take equation (4.2) to be a nominal pricing kernel which prices all nominal assets in the economy. This means that the total gross return process R_{t+1} of any nominal asset satisfies

$$E_t(M_{t+1}R_{t+1}) = 1. \tag{4.4}$$

If p_t^n represents the price of an n-period zero coupon bond, then equation (4.4) allows bond prices to be computed recursively by

$$P_t^{n+1} = E_t(M_{t+1}P_{t+1}^n) \tag{4.5}$$

At the same time, the affine class of term structure models expresses bond prices as exponential affine functions of the state variables. More precisely, bond prices are given by

$$P_t^{(n)} = \exp(A_n + B_n' z_t), \tag{4.6}$$

and therefore we can write the log bond prices as a linear function of the state variables in the VAR as follows:

$$p_t^{(n)} = A_n + B_n' z_t \tag{4.7}$$

The scalar A_n and the (5×1) vector B_n are defined under the no-arbitrage condition. Of course, $p_t^{(0)} = 0$, so $A_0 = 0$ and $B_0 = 0$. For a one-period bond we then have:

$$p_t^{(1)} = \ln E_t(M_{t+1}) = -\delta_0 - \delta_1' z_t, \tag{4.8}$$

The price at time t of a $n+1$ period maturity bond satisfies

$$P_t^{n+1} = E_t(M_{t+1}P_{t+1}^n) \tag{4.9}$$

Thus we must have

$$\begin{aligned} \exp(A_{n+1} + B'_{n+1}z_t) &= E_t\left[\exp\left(-\delta_0 - \delta_1' z_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\xi_{t+1} + A_n + B_n' z_{t+1}\right)\right] \\ &= \exp\left(-\delta_0 - \delta_1' z_t - \frac{1}{2}\lambda_t'\lambda_t + A_n\right) E_t\left[\exp\left(-\lambda_t'\xi_{t+1} + B_n' z_{t+1}\right)\right]. \end{aligned}$$

this can finally be rewritten as

$$A_{n+1} + B'_{n+1}z_t = \left(-\delta_0 + A_n + B_n'\nu - B_n'\Sigma\lambda_0 + \frac{1}{2}B_n'\Sigma\Sigma' B_n\right) + (-\delta_1' + B_n' B - B_n'\Sigma\lambda_1)z_t$$

Matching coefficients, we obtain

$$\begin{aligned} B'_{n+1} &= -\delta_1' + B_n' B - B_n'\Sigma\Lambda_1 \\ A_{n+1} &= -\delta_0 + A_n + B_n'\nu - B_n'\Sigma\lambda_0 + \frac{1}{2}B_n'\Sigma\Sigma' B_n \end{aligned} \tag{4.10}$$

Log bond yields are also linear in the state variables due to the inverse relationship between bond prices and yields $p_t^{(n)} = -ny_t^{(n)}$, such that

$$y_t^{(n)} = -\frac{A_n}{n} - \frac{B_n'}{n}z_t. \tag{4.11}$$

This equation describes the whole term structure of nominal interest rates. The constant part of the risk premia λ_0 influences A_n, and the time-varying component Λ_1 influences B_n. As a consequence, λ_0 only affects the average level of the term structure of interest rates and the term spread. Λ_1 introduces time-variation in the term structure and term spreads. This modeling framework ensures that the term structure model in (4.11) and the VAR in (4.1) have identical implications for the 1-month interest rate and the 10-year zero coupon rate. The return dynamics of the VAR are reflected in and consistent with the modeled term structure of interest rates.

4.4 Estimation procedure

We conduct a two-step estimation procedure. In the first step, the VAR parameters (ν, B, and Σ) in (4.1) are estimated with Ordinary Least Squares (OLS), which yields the same results as estimating them with maximum likelihood. In the second step we estimate the risk premia parameters conditional on the VAR parameters, just as Hoevenaars (2008) we estimate $\Sigma\lambda_0$ and $\Sigma\lambda_1$ instead of λ_0 and λ_1. We do this by minimizing the sum of the squared differences between the fitted yields of the term structure model in (4.11) and the historical zero coupon yields. The A_n and B_n are from the equations in (4.10) and we can already fill in ν, B and Σ which we estimated in the first step. Besides the 10-year interest rate, we calibrate the model on 2,3 and 5-years zero yields. In order to generate a self-consistent model we ensure that the expected equity return implied by the VAR is the same as the one implied by the asset pricing equation. Since the dividend yield is a non-tradable asset we assume that the risk premium on the dividend yield is zero. Furthermore, following Hoevenaars (2008) we assume that the inflation risk premium is zero. The details of the risk premia estimation procedure are in appendix A.

4.5 Simulation

We can generate scenarios by forward iterating our VAR model in equation (4.1), a draw is done from the probability distribution of the error terms given the variables of last period, such that the historical correlations are taken into account, and the values of the variables for the next period are computed. Then the scenario set *nsim* (where *nsim* goes for example from 1 to 10000) can be created using:

$$z_{nsim,t+h} = (I_n + B + ... + B^{h-1})\nu + B^h z_t + \sum_{i=0}^{h-1} B^i \Sigma' \xi_{nsim,t+h-i} \tag{4.12}$$

We can implement long term views of for instance the inflation rate by adjusting the constant term ν of the VAR model. In that manner we can set our scenario mean of the inflation rate equal to the long term inflation target of the European Central Bank. For the equity return we follow the 'Commission Parameters[1]' and we lower the historical average of the equity risk premium by 1.5 %. This is in line with the maximum long term equity return average that Dutch pension funds are allowed to use in their simulations. For the dividend yield we use the historical averages. Equity return and inflation scenarios follow directly from the VAR model and the interest rate scenarios follow from the affine term structure model in (4.11). Returns on a rolling 10-year constant maturity zero coupon bond portfolio are calculated from the underlying bond prices that follow from (4.7) as:

$$p_{t+1}^{(n-1)} - p_t^{(n)}$$

4.6 Stationarity

When we want to estimate a VAR model, an important assumption is that the historical data is stationary. This means that the properties of the process such as the mean and the autocovariances are fixed and do not depend on time t (strictly speaking the process is covariance stationary under these conditions). Stationarity is a crucial assumption for being able to describe the stochastic behavior of some variable by a single model and to be able to estimate the parameters of such a model on one sample of data. Otherwise each point in time would require another model and only one observation would be available to estimate

[1]The commission Parameters advices the ministry of social affairs among others about the maximum equity risk premium pension funds should use in their ALM studies.

each of these models.

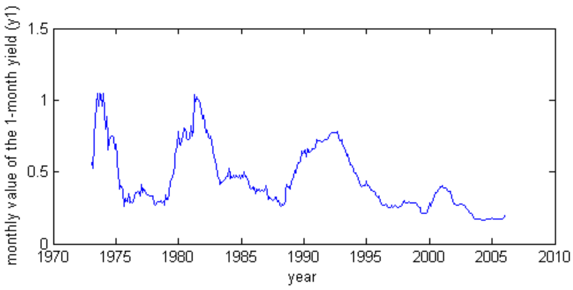
From Hamilton (1994) we get that the (covariance-) stationary condition for a VAR(p) model requires all roots $r_j, j = 1, ..., p$ of the characteristic (determinant equation)

$$|I_n - \lambda B_1 - \lambda^2 B_2 - ... - \lambda^p B_p| = 0 \tag{4.13}$$

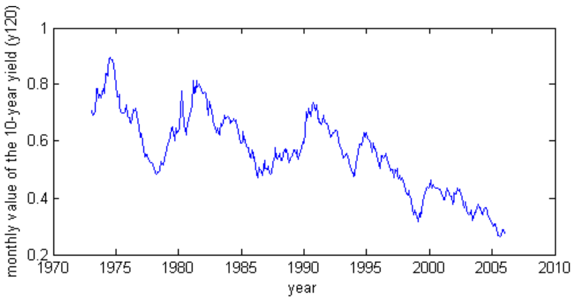
to lie outside the unit circle and therefore to have a modulus greater than one. This also holds as long as $|\lambda| < 1$, where λ are the eigenvalues of matrix B. So our VAR(1) process is covariance stationary if the eigenvalues of the matrix B have a modulus smaller than one. In the next section we will discuss the stationarity characteristics of our data.

4.7 Data and Estimation results

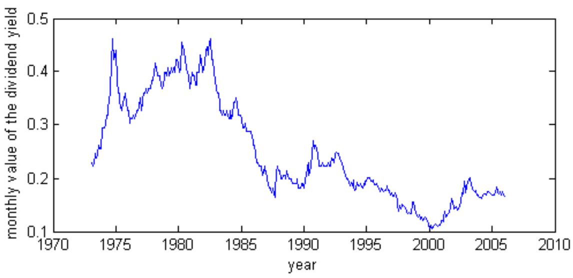
We have monthly data and all data start in January 1973 and end in December 2005. German zero coupon yields are from the site of Deutsche Bundesbank, and the German price inflation (non-seasonally adjusted) is from Datastream. For the stock returns we use the MSCI world Total return index from Bloomberg and the corresponding dividend yield. Below are the zero coupon yields and the dividend yield plotted, we see that the short rate is sometimes higher than the 10-year yield.



(a)



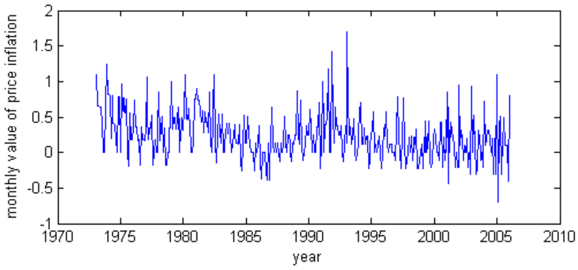
(b)



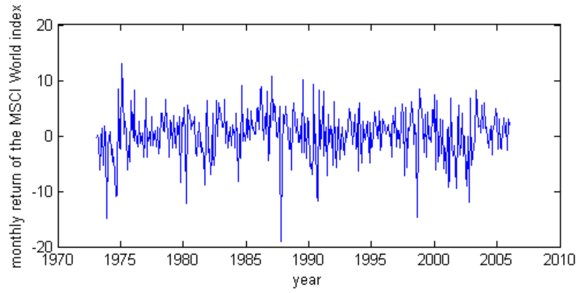
(c)

Figure 4.1: Monthly values of (a) 1-month yield (y^1), (b) 10 year zero coupon yield (y^{120}) and (c) yearly dividend yield corresponding to the MSCI world stock return index.

Next we also plot the monthly price inflation and stock returns in excess of the 1-month yield. We see that on monthly basis, negative inflation is not unusual. When we look at the stationarity conditions we see that the absolute value of the eigenvalues of matrix B are given by: 0.0931, 0.0931, 0.9509, 0.99 and 0.99. So they are all smaller than one and accordingly to the proposition of Hamilton (1994) this data is stationary.



(a)



(b)

Figure 4.2: Values of (a) monthly German price inflation (π) and (b) monthly MSCI world stock returns in excess of 1-month yield (x_s).

Table 4.1 gives the summary statistics of the data in panel (a) and in panel (b) are the parameter estimates of the VAR. Stock returns are explained by the dividend yield, the 10-year yield and inflation. Dividend yields rise if stock prices decline and a higher dividend yield predicts an increase of stock returns next period. The higher R^2 of inflation (0.17) reveals that this series is better explained than stock returns ($R^2 = 0.06$). Besides its own lag, inflation is explained by 1-month interest rates and the dividend yield. Furthermore inflation is an important driver for the other variables in our system. An increase in inflation predicts an increase in interest rates and dividend yield. On the contrary inflation is negatively related to next period stock returns.

Figure 4.3 shows the risk diversification properties between stocks and bonds. The correlation between a stock return portfolio and a bond return portfolio is calculated numerically by simulation and the correlation of 10000 bond and stock returns for each point in time is plotted in the upper graph. We see that that the correlation increase at a short horizon and reaches a maximum of 0.61 at a horizon of 77 months. After 77 months the correlation decreases again and goes to zero after 240 months. Furthermore we looked at the impulse response function of the 10-year yield which shows the cumulative effect of an one-standard deviation shock in the 10-year yield for the stock and bond return portfolios. The impulse response function is also obtained with numerical simulation, first we changed the ordering of the variables for the cholesky decompostion such that the 10-year yield affects the other variables. Next we set the starting values zero and also all error terms zero except for the first error term of the 10-year yield which is set equal to one standard deviation of the 10-year yield and which represents the shock. Now we look at the difference in percentages between the simulated return portfolios

Table 4.1: Summary statistics and VAR estimation results

Panel (a) provides summary statistics of the data, annualized means and standard deviations are provided for the entire sample (1973:01-2005:12). Panel (b) provides parameter estimates (B) of the VAR, the corresponding t-values are below. Panel (c) contains cross-correlations of the innovations with monthly standard deviations on the diagonal.

a) Summary statistics	(y^1)	(π)	(y^{120})	(x_s)	(dy)	
μ	5.59	2.88	6.96	3.69	3.04	
σ	2.77	1.13	1.76	14.68	1.20	
b) VAR estimates (B)	(y_t^1)	(π_t)	(y_t^{120})	$(x_{s,t})$	(dy_t)	R^2
y_{t+1}^1	0.96	0.01	0.05	−0.00	−0.01	0.97
	(71.08)	(1.13)	(1.95)	(−1.78)	(−1.31)	
π_{t+1}	0.43	0.10	0.03	0.00	0.10	0.17
	(3.84)	(1.90)	(0.12)	(0.99)	(1.83)	
y_{t+1}^{120}	0.01	0.01	0.98	0.00	−0.00	0.98
	(1.68)	(1.87)	(70.36)	(0.49)	(−0.40)	
$x_{s,t+1}$	−1.83	−1.71	−4.52	0.04	2.49	0.06
	(−1.18)	(−2.46)	(−1.55)	(0.86)	(3.32)	
dy_{t+1}	−0.01	0.02	0.06	−0.00	0.98	0.99
	(−0.42)	(2.66)	(1.86)	(−1.58)	(119.96)	
c) VAR estimates	(y^1)	(π)	(y^{120})	(x_s)	(dy)	
y^1	0.04					
π	−0.04	0.30				
y^{120}	0.07	0.13	0.02			
x_s	−0.04	0.01	−0.18	4.09		
dy	0.07	−0.03	0.16	−0.84	0.04	

with scenarios where all error terms are equal to zero and scenarios where all error terms are equal to zero except for the first error term of the 10-year yield. We see that if the interest rate goes up, the bond returns will decrease immediately. The stock returns also decrease only at a lower degree at the begin. The negative correlation between the error terms of the stock returns and the 10-year yield is -0.18 and the negative coefficient of the 10-year yield (-4.52) on next period stock returns reinforces this effect. After 77 months the impulse response function turns around which lead to a reduction of the correlation at longer investment horizons. We have to note that we did not provide confidence intervals for both figures and that there is some uncertainty since we simulate 20 years forward based on approximately 30 years of data.

For the estimation of the risk premia parameters we made the assumption that the risk premium is zero for the inflation and the dividend yield. Furthermore we assumed that the risk premia parameters for the equity return should be the same as in our VAR model in order to achieve consistency. In appendix A is shown how we estimated the risk premia parameters and how the assumptions are met. In figure B.1 in appendix B we can see that the term structure model in (4.11) fits the historical interest rates quite well. It provides a good fit in periods of high yields like the beginning of the eighties and nineties, and in periods of low yields after 2000.

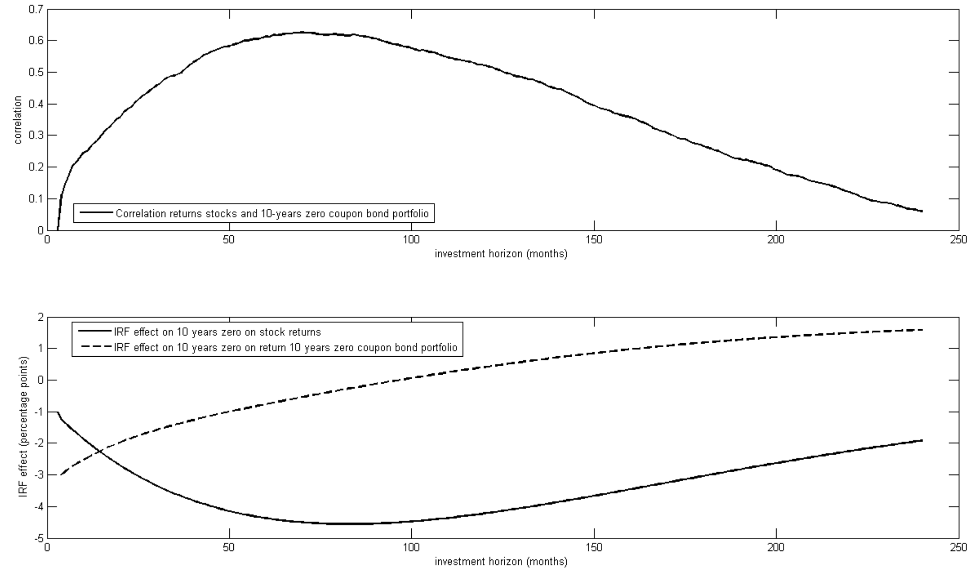


Figure 4.3: Impulse response analysis and correlation

Panel (b) of table 4.2 shows the mispricing of the fitted term structure of interest rates. The fit gets worse on the short end of the curve, and the 1 month yields is overestimated by 45 basis points on average. Panel (a) shows the estimated risk premia ($\Sigma\lambda_0$ and $\Sigma\lambda_1$). The estimates for $\Sigma\lambda_1$ indicate that risk premia for stocks and bonds are time-varying. Risk premia for stocks are identical to the VAR coefficients. This ensures the consistency between the VAR model and the term structure of interest rates. The risk premia on the dividend yield and the one-period inflation risk premium are assumed to be zero.

Table 4.2: Estimation results term structure model

Panel (a) shows risk premia ($\Sigma\lambda_0$ and $\Sigma\lambda_1$) and panel (b) gives the annualized means and standard deviations of historical mispricing of the nominal term structure of interest rates.

a) Risk Premia (L)	(y^1)	(π)	(y^{120})	(x_s)	(dy)	(λ_0)
y^1	0.02	0.00	0.01	0.01	−0.02	−0.02
π	0.00	0.00	0.00	0.00	0.00	0.00
y^{120}	0.02	0.01	−0.03	0.00	−0.00	−0.01
x_s	−1.83	−1.71	−4.52	0.04	2.49	0.33
dy	0.00	0.00	0.00	0.00	0.00	0.00
b) Mispricing term structure	(y^{12})	(y^{24})	(y^{36})	(y^{48})	(y^{60})	(y^{120})
μ	0.45	0.42	0.39	0.32	0.29	0.00
σ	0.62	0.53	0.49	0.41	0.36	0.00

Chapter 5

Pension Fund ALM Study

In the previous chapter we have obtained simulations for stock returns, bond returns, inflation and interest rates. These simulations are used as input for an ALM study which we are going to perform. There are many forms of pension plans, but for our study we are going to model a defined benefit pension plan which is the most common in the Netherlands. In section 5.1 this defined benefit plan is introduced and in sections 5.2 till 5.6 we will present the ALM model assumptions that we made. The simulation results are presented in section 5.7.

5.1 Average defined benefit pension plan

Our ALM framework represents a stand-alone funded defined benefit type of pension plan. From Kakes and Broeders (2006) we know that in a defined benefit plan the participants save collectively for a pension that is paid out as a life annuity from the age of retirement. We model an average-wage plan with the goal to fully index liabilities with price inflation. The participants acquire for each year of service 2% of their pensionable wage as accrued pension rights to be payed out from retirement. The final pension benefit depends on the average wage during the working period. In order to fund the accrued rights, yearly contributions are made by the participants. In our model we assume the contribution rate is a uniform rate of 20% of the pensionable wage. So all participants pay the same level of contributions for their accrued rights.

5.2 Age distribution

Our age distribution is based on the information of the total male population of the Netherlands accordingly to the Dutch Central Bureau of Statistics (CBS (CBS)). The exact population is known in 2009 and for the years 2010 till 2030 we make use of the prediction of the CBS (CBS). We do not use the mortality tables provided by the 'Actuarieel Genootschap' published in august 2010, these are also based on CBS data and follow purely from the historical data. The CBS (CBS) predictions also incorporates the long term views of experts. Although our historical data is from 1973-2005 we use the population predictions of the time period 2010-2030. This is because in the next chapter we will have based our simulations on historical data from 1973-2010 and in order to compare the two sets of simulations we use the same population predictions.

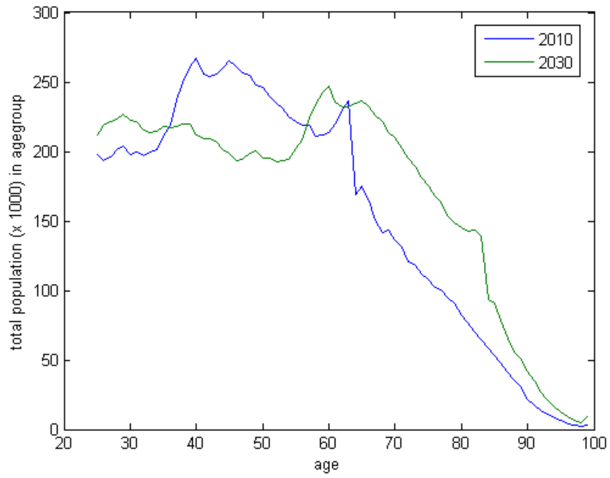


Figure 5.1: The total population of the Netherlands per age in 2010 and the prediction for 2030 according to the CBS

We assume that the participants have the age of 25 when they start their working life and retire at the age of 65. Furthermore we assume that all participants die when they reach the age of 100, since we only have the information up to the age 99. This is not an hard assumption because only a few hundred males of the total male population in the Netherlands will reach that age. Since we model for time periods of one year, we also assume that all participants are born on the first of January and will die at the 31th of December.

In figure 5.1 we see the male population in 2010 and in 2030. The peek of the population in 2010 around the age of 60 is a result of the baby boom after World War II and this group is close to retirement in 2010. This will put more pressure on the workforce since less active participants have to pay premium for a bigger group of retirees. Furthermore we see that the proportion of retirees over the workforce rises enormously if we compare the situation in 2030 and 2010.

In the real life situation the real development of the age distribution will change from the predictions that are made. But in our model we assume that real demographic development is equal to the prediction made. So the risk of changing demographics other than the predictions from the CBS are not taken into account.

5.3 Active and sleeping participants

Not all participants will retire at the age of 65, a large part of the population will actually retire at an earlier age. As a result not the entire population younger than 65 will be an 'active' participant and pay premium. The part of the population that retires at an age before 65 will be considered a 'sleeping' participant. The sleeping participants will not pay any premium from the day that they retire. They also will no longer build up any additional pension rights and at the age of 65 they will receive the pension payments that have been build up till the age of retirement.

In reality the percentage of the sleeping participants is different for every age group, but for our model we simplify the distribution. We assume that all participants start working at the age of 25 and that a group retires at the age of 50 and a group retires at the age of 60. The percentages are based on the data that is available at the CBS (CBS) about the working participation of the male population in the Netherlands. The resulting distribution is shown in table 5.1.

Table 5.1: Participation rate of dutch male population

age	participation rate
25 − 49	100%
50 − 59	85%
60 − 64	40%
65 − 100	0%

5.4 Wage development

During the working life of an individual his or her wage will increase. This is partly due to the compensation for price inflation and for the other part it is due to the career someone will make. We stated earlier that the goal of the pension fund is to fully index the pension payments with price inflation, the level at which the pension payments will be indexed depends on the funding ratio and will be further discussed in the section of the indexation policy. For the wage increase due to the career someone makes we assume a fixed percentage of 2 percent during the entire career.

5.5 Liabilities: Valuation and Indexation

By paying premium during their working life, participants build up pension rights that will be paid out from the age of 65. These payments represent the pension funds liabilities. According to Hoevenaars (2008) each year the liabilities change due to three factors: actuarial, interest rates and indexation.

5.5.1 Actuarial factors

The actuarial factors include the level and length of nominal cash flows of earned pension rights. Factors of this sort represent demographic trends, retirement age, job promotion, mortality rates and discharge. As was shown in the previous sections we take the aging op the population into account and make some assumptions about wage growth and labor participation. Other than those assumptions we do not take any further risks into account.

5.5.2 Valuation of nominal liabilities

Because the majority of the pension payments will take place in the future we need to discount these cash flows. Since the introduction of the Financial Assessment Framework in 2007, pension funds need to value their liabilities at the current market value and all cash flows need to be discounted at current market rates. In the previous chapter we modeled the interest rates for the maturities of 1 month till 10 years.

Table 5.2: Discounted value of a payment of one, t years ahead

years ahead	discounted value
1	$\frac{1}{(1+y^{12})^{1}}$
2	$\frac{1}{(1+y^{24})^{2}}$
$\vdots$	$\vdots$
10	$\frac{1}{(1+y^{120})^{10}}$
11	$\frac{1}{(1+y^{120})^{11}}$
$\vdots$	$\vdots$
57	$\frac{1}{(1+y^{120})^{57}}$

We have simulations of these rates at all time points between now and 2030 that follow from the affine term structure model in the previous section. For payments that take place longer than 10 years in the future we use the 10-years interest rate to discount. Changes in the interest rates we do take into account and these changes have an direct impact on the valuation of the liabilities. Given the interest rates from a certain scenario we valuate a payment of one t years ahead as is shown in table 5.2.

Now we have the discount rates, we only need the life expectancy of each age group at each point in time to calculate the value of nominal liabilities. We use the life expectancy for male individuals at each age for the period of 2010-2050 from the CBS (CBS). In order to simplify the calculation we take the nearest integer for each expectation. At 2010 the life expectancy in years of a male individual given the persons age now looks as follows:

Table 5.3: Expected lifetime (in years) of a male individual given current age in 2010

age	Expected lifetime
25	55
26	54
$\vdots$	$\vdots$
64	20
65	19
$\vdots$	$\vdots$
98	2
99	2

For each year from 2010 till 2030 we have this table and the life expectancy will rise a little bit along the time. For instance the life expectancy of a 25 year old male in 2030 is 57 in comparison to 55 in 2010. This trend is also shown graphically in figure 5.1. Since we assume that everyone will die when the age of 100 is reached, we change the life expectancy at the age of 99 and 98 accordingly. For the table of 2010 we thus change the life expectancy at the age of 99 from 2 to 1. Since the predictions of the life expectancy are equal to the actual average life time in our model the liabilities will not change due to differences between the real and predicted life time.

Now we have all the information to calculate the nominal value of the liabilities of a pension plan. We will first introduce some notation, ${}_kq_{x,t}$ is the probability that someone of age x at time t will die in k years. And ${}_kp_{x,t} = 1 - {}_kq_{x,t}$ is the probability that someone of age x at time t will survive k years. The discounted value of a payment of one in the next year is given by $v = \frac{1}{1+y^{12}}$. Now the value of a yearly payment of one that starts this year and will end when someone dies is given by:

$$\ddot{a}_{x,t} = \sum_{k=0}^{\infty} v^k {}_kp_{x,t}$$

And the value of a yearly payment of one starting n years from now until death is given by:

$${}_{n|}\ddot{a}_{x,t} = \ddot{a}_{x,t} - \ddot{a}_{x+n,t} = \sum_{k=0}^{\infty} v^k {}_kp_{x,t} - \sum_{k=0}^{n-1} v^k {}_kp_{x,t} = \sum_{k=n}^{\infty} v^k {}_kp_{x,t}$$

Next we define the total accrued rights of the active participants of age x at time t by:

$$TaccA_{x,t} = part_x \cdot \text{mean}(wage_x) \cdot agegroup_{t,x} \cdot 0.02 \cdot (x - 25)$$

So the total accrued rights of the active participants of age x at time t are given by the percentage of people that works at the age x ($part_x$) times the mean wage for someone of age x ($\text{mean}(wage_x)$) times the total size of the population of people have age x at time t ($agegroup_{t,x}$) times the total build up pension at the age x ($0.02 \cdot (x - 25)$). Now the total liabilities for the active participants are equal to:

$$L_t^{\text{act}} = \sum_{x=25}^{65} TaccA_{x,t} \cdot {}_{65-x|}\ddot{a}_{x,t}$$

Next we calculate the liabilities for the sleeping participants. For the ages $x = 50, 51, ..., 59$, participants have the following total accrued rights:

$$Tacc50_{x,t} = (1 - part_x) \cdot \text{mean}(wage_{50}) \cdot agegroup_{t,x} \cdot 0.02 \cdot 25$$

So the people that retire at the age of 50, only receive the pension rights that they have build up so far. For the ages $x = 60, 61, ..., 64$ we get a total accrued rights of:

$$Tacc60_{x,t} = (1 - part_x) \cdot \text{mean}(wage_{60}) \cdot agegroup_{t,x} \cdot 0.02 \cdot 35$$

Now the total liabilities for the sleeping participants equal:

$$L_t^{\text{sleep}} = \sum_{x=50}^{59} Tacc50_{x,t} \cdot {}_{65-x|}\ddot{a}_{x,t} + \sum_{x=60}^{64} Tacc60_{x,t} \cdot {}_{65-x|}\ddot{a}_{x,t}$$

Besides participants that are active and sleeping we also have retired participants. We assumed that everybody is retired at the age of 65, so all participants will receive pension payments from that age in our model. The pension payments are being paid over the average wage and this is for the retired participants equal to the sum of the average wage for the groups that retire at the age of 50, 60 and 65:

$$\begin{aligned} \text{mean}(wage_{ret}) &= (0.4 \cdot 0.02 \cdot 40 \cdot \text{mean}(wage_{65}) + 0.15 \cdot 0.02 \cdot 25 \cdot \text{mean}(wage_{50}) \\ &= +0.45 \cdot 0.02 \cdot 35 \cdot \text{mean}(wage_{60})) \end{aligned}$$

Now the liabilities for the retired group are simply the average wage times the agegroup size times the discount vector which includes the life expectancy for all age groups. Since all payments for the retired group will start next year, we discount accordingly and there are payments from next year till the expected lifetime for the relevant agegroups. The total liabilities for the retired group are now given by:

$$L_t^{\text{ret}} = \sum_{x=66}^{99} \text{mean}(wage_{ret}) \cdot agegroup_{t,x} \cdot \ddot{a}_{x,t}$$

Finally if we add up the liabilities for the active, sleeping and retired participants we obtain the total nominal liabilities for a pension fund at a given time t:

$$L_t^{\text{total}} = L_t^{\text{act}} + L_t^{\text{sleep}} + L_t^{\text{ret}}$$

5.5.3 Indexation policy

The third factor that influences the value of the liabilities is indexation. Since pensions are meant to provide a standard of living, the goal of pension funds is to index pension payments by inflation. In our ALM framework we index the pension rights with the price inflations that follow from the simulations we made with the VAR model in the previous chapter. Furthermore the indexation of the liabilities is conditional on the funding ratio of the pension fund. According to the Financial Assessment Framework, pension funds have to hand in a recovery plan when their funding ratio drops below the 105 percent. After the credit crisis in 2008, 340 of the 600 Dutch pension funds were obliged to deliver such a recovery plan to the Dutch central bank (DNB). The standard in these recovery plans is that below a funding ratio of 105 or 110 % there will be no indexation. Above that ratio only partial indexation is being rewarded and above some level of indexation the forgone indexation will be made up for. This has lead to the following indexation policy in our model:

When the nominal funding ratio is below 105, there will be no indexation by price inflation. For a funding ratio between 105 and 140, the liabilities will be indexed between 0 and 100 %. And pensions are fully indexed by price inflation otherwise, also foregone indexation when the nominal funding ratio was below 140 will be made up in this situation. The total indexation policy is shown in table 5.4 below:

Table 5.4: Indexation policy

funding ratio	indexation
> 140	full indexation and making up for forgone indexation
$105 - 140$	$0 - 100\%$ indexation (linear)
< 105	no indexation

5.6 Development of Assets

The asset side of the balance sheet is influenced by pension payments, contributions and investment returns. We use the simulations for stock and bond returns from the previous chapter as input for the investment returns. We will discuss the three components separately.

5.6.1 Pension payments

The pension payments are assumed to be well predictable and depend on the same actuarial factors that where described in the previous section. Again we take the aging of the population into account and make the same assumptions about wage growth and labor participation. Other than those assumptions we do not take any further risks into account.

5.6.2 Contributions

In order to pay for the build up pension rights, the participants pay a yearly premium. Following Hoevenaars (2008) we assume that a uniform rate of 20 % of the pensionable salary is charged to all participants. So every participant pays the same percentage of his pensionable income and younger participants pay a little more for their accrued rights. This is because younger participants have a bigger probability of dying before 65 and the pension fund can earn more investment returns on the same premium since the payments are further into the future.

5.6.3 Investment policy

As investment policy we take the calender rebalancing approach where the asset mix is rebalanced to the initial level at a fixed time interval. For the asset mix we looked at the 'pensioenthermometer'[1] where we find that the average pension fund has 30 % equity, 50 % bonds and 20 % other asset classes in their portfolio. Since we have only equities and bonds in our portfolio we assume a constant asset mix of 37,5 % equities and 62,5 % bonds and at the end of each year the portfolio will be rebalanced to obtain the same asset mix again at the beginning of the new year. Between two years monthly returns are used to calculate the investment return on the portfolio. We ignore the transaction costs that come along with the rebalancing and monthly stock and bond returns follow from the VAR model in the previous chapter.

Recent research by Bikker et al. (2009) has shown a relation between the average age and the equity exposure for Dutch pension funds. The higher the average age of their participants, the lower the average equity exposure. This negative age-dependent equity allocation may be interpreted as an application of the optimal life-cycle saving and investing theory which was introduced by Bodie et al. (1992). The effect is even stronger for larger pension funds and also the average age of the active participants has a much larger impact compared to the average age of all participants. But since we represent the average pension model of the Netherlands these effects are averaged out again and therefore are not taken into account in our model.

Combining all three components will give us the value of the assets at time t:

$$A_t = A_{t-1}(1 + R_t) + C_t - P_t$$

Where C_t represents the contributions of the active participants at time t and P_t represent the pension payments to the retired group. The contributions are calculated by taking the current price level (follows from the price inflation from the scenarios) times 20 % of the mean wage times the age group. Unlike the contributions, the pension payments are being paid at the current indexation level which is multiplied by the mean wage level of the retired participants and the corresponding age group.

[1] www.pensioenthermometer.nl displays the current situation for an average dutch pension fund according to Hewitt.

5.7 Simulation results

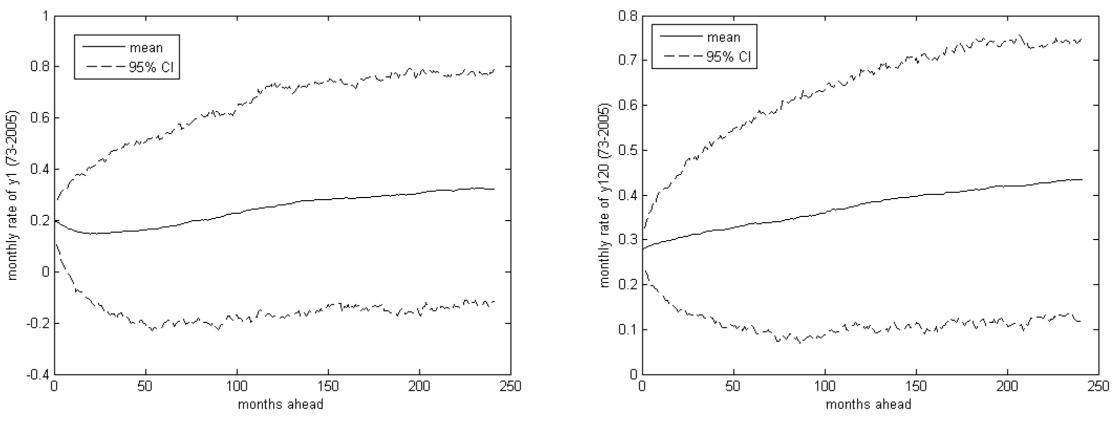
We combine the asset and the liability side of the balance sheet with a simulation approach. The risk and return dynamics of the scenarios are based on the model in chapter 4. With the simulation approach it is possible to deal with highly non-linear and company-specific decision rules in the contribution, indexation and investment policy.

In our ALM model we take the uncertainty into account of the investments, interest rates and inflation. All other uncertainty about changing regulation, demographics or wage development is taken away by fixing for instance the prediction of the population growth, wage growth and contribution rate. We perform 10000 simulations and we have a simulation period of 20 years. This is not a very long time window, considering that an individual will be working 40 years before retirement, but since our scenarios are based on historical data of approximately 30 years this is already quite a long period to look ahead.

With 10000 scenarios we will be able to say something about the future evolution of the funding ratio and the indexation and the corresponding probabilities. In the next sections we will discuss these simulation results.

5.7.1 Solvency position

First we look at the simulation results for the short rate and the 10 year zero coupon rate in figure 5.2.



(a) (b)

Figure 5.2: Plots of the mean and 95 percent confidence intervals of the simulations for respectively (a) the short rate (y^1) and (b) the 10 year zero coupon rate (y^{120}).

What strikes out the most are the possible negative short rate simulations. In real life these negative interest rates are not possible and in our model we assume the short rate to be zero if we have a negative simulation.

We furthermore observe that the inflation is also negative for some scenarios, but on a monthly time interval this is not very unusual. For the indexation policy we use the inflation of the last 12 months and if this has a negative value the indexation is equal to zero. Otherwise the pension rights will be worth less after indexation with negative inflation.

The initial funding ratio is set at 100 %, which is the average funding ratio of Dutch pension funds accordingly to the DNB (2010)[2]. The mean and median of the funding ratio start to increase immediately. This is due to the expected return of around 7 % on stocks and 4.5 % on bonds. Furthermore we see that the underbound of the 95 % confidence interval at first drops below the 100 % then rises a little to around 100 % and after 10 years it decrease a little again and goes below the 100 %. We notice that the confidence intervals are quite big and hence the uncertainty of our model outcomes is also quite big. This is among others a result of the big uncertainty that is incorporated in our scenarios about the interest rates, inflation, stock and bond returns. As time proceeds this uncertainty gets bigger, for instance after 20 years our 95 percent confidence interval of the funding ratio lies between 95 and 325. This means that accordingly to our model after 20 years in 95 out of the 100 cases the funding ratio will lie between 95 and 325, this is off course quite useless information for a pension fund board. A pension fund board would prefer more explicit information with less uncertainty and this uncertainty of the model outcomes is a drawback of our simulation analysis.

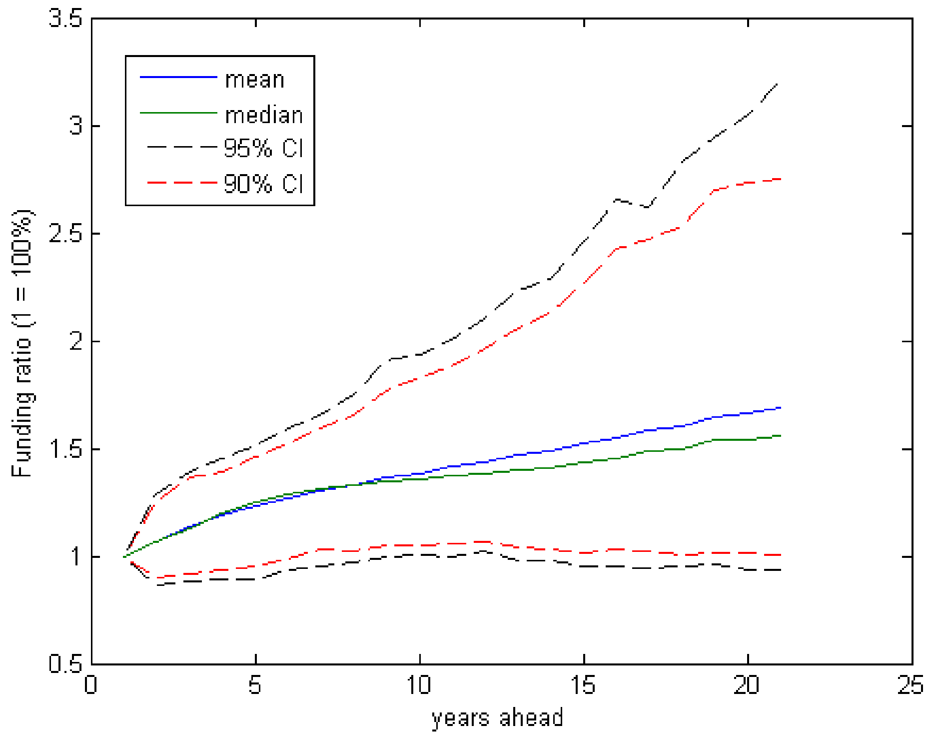


Figure 5.3: Simulation result for the funding ratio

[2]DNB Statistisch bulletin September 2010

In table 5.5 are the ALM results for the solvency position summarized. We see that the median and standard deviation of the funding ratio rises as time proceeds. The probability of underfunding is quite big in the first year, which is logical since we start at a funding ratio of 100 %. After the first year the probability of underfunding decreases and goes to 3 percent after ten years. After ten years the probability of underfunding rises again slightly. The mean funding ratio after one year in the 2.5 % worse scenarios is 83 and the biggest drop of the funding ratio in the first year is 42 percent.

Table 5.5: Solvency position

Summary statistics of probability distributions of variables from the solvency position where the initial funding ratio is 100 %. The selected output variables are the median of the funding ratio (FR) after 1, 10 and 20 years and corresponding standard deviations. The probability of under funding ($P(FR < 100)$) in the next 1, 10 or 20 years. The expected funding ratio after one year given it is in the 2.5 % worse scenarios ($CFRaR_{2.5\%,t+1}$). And the maximum drawdown (in percentage points) of the funding ratio in the next year ($\delta_{FR,t+1}$).

FR_{t+1}	1.06
FR_{t+10}	1.37
stdev FR_{t+10}	0.27
FR_{t+20}	1.59
stdev FR_{t+20}	0.59
$P(FR_{t+1} < 100)$	0.25
$P(FR_{t+10} < 100)$	0.03
$P(FR_{t+20} < 100)$	0.05
$CFRaR_{2.5\%,t+1}$	0.83
$\delta_{FR,t+1}$	0.42

5.7.2 Indexation quality

The initial funding ratio was set at 100 %. Because this initial funding ratio is below 105 %, there will be no indexation in the first year. The total indexation is given and we see that the median goes to 100 % percent after 20 years. Mean indexation level is around 95 % and underbound of the 95 % confidence interval goes to 75 % after 20 years.

We summarized the results in table 5.6. The median goes to 99 % after 20 years and the mean stays around 95 %. The standard deviation increases as the horizon becomes bigger, although not much. Also the probability that there is no full indexation after 10 years is 94 % and after 20 years 62 %. Finally we observe that the chance that the indexation is worse than 85 % after 20 years is not negligible with a probability of around 9 percent.

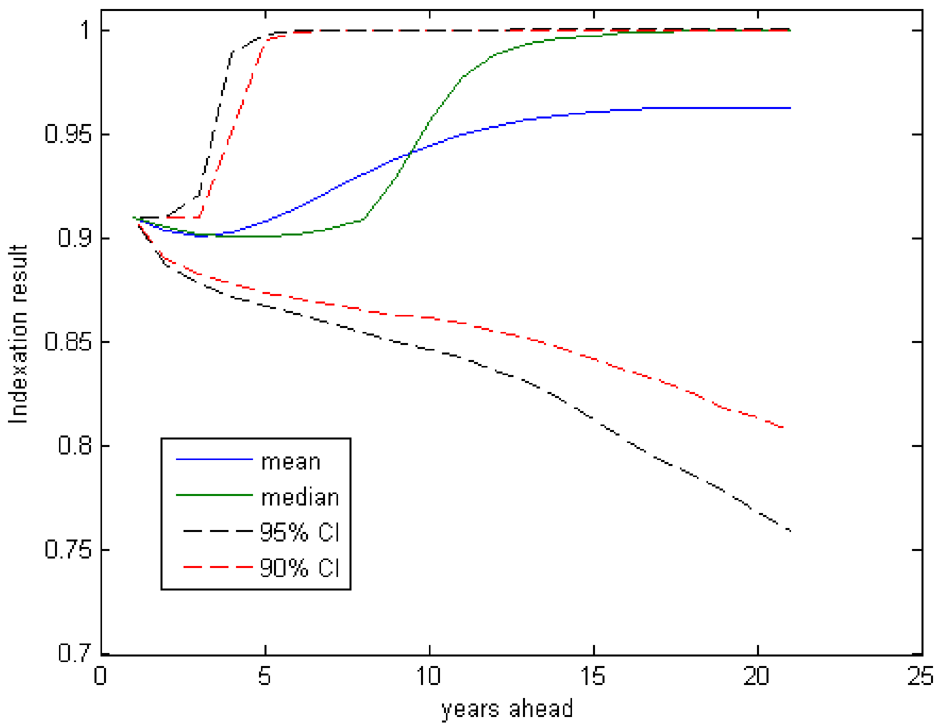


Figure 5.4: Simulation result for the indexation level

Table 5.6: Indexation quality

Summary statistics of probability distributions of variables from the indexation quality. The selected output variables are the median, mean and standard deviation of the indexation result (IR) after 1, 10 and 20 years. And the probability that the indexation result will drop below 100 % and 85 % respectively ($P(IR < 100)$ and $P(IR < 85)$).

median IR_{t+1}	0.90
mean IR_{t+1}	0.90
median IR_{t+10}	0.97
mean IR_{t+10}	0.95
stdev IR_{t+10}	0.05
median IR_{t+20}	0.99
mean IR_{t+20}	0.96
stdev IR_{t+20}	0.07
$P(IR_{t+10} < 100)$	0.94
$P(IR_{t+20} < 100)$	0.62
$P(IR_{t+10} < 85)$	0.03
$P(IR_{t+20} < 85)$	0.09

Chapter 6

Different time periods and extending the term structure model

In this chapter we look at effects of estimating our VAR model on different historical data. In chapter 4 we based our model on the same historical data as Hoevenaars (2008), namely monthly data from the period between 1973 and 2005. Here we add the historical data of the period between 2006 and 2010, we will start with estimating our model on the 1973-2010 data. After that we look at the consequences of estimating our VAR model on data between 1986 and 2010. We look at the difference in VAR estimates, correlations, impulse response functions and the fit of the affine term structure model. Furthermore we will look what the effects will be on the ALM output and on the corresponding scenarios. At last we will extend the affine term structure model with an 15 year zero coupon yield in two different ways and compare the fit with the benchmark term structure model from chapter 4.

6.1 Extend historical data with the 2006-2010 period

In figures 6.1 and 6.2 we can see the development of the short rate and the 10 year zero coupon yield of Germany in the period between 2006 and 2010.

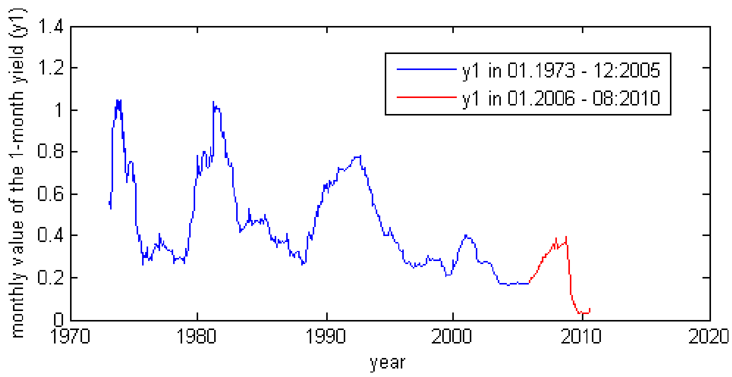


Figure 6.1: Monthly values of the German 1-month yield (y^1) in the time period of 01/1973 till 08/2010.

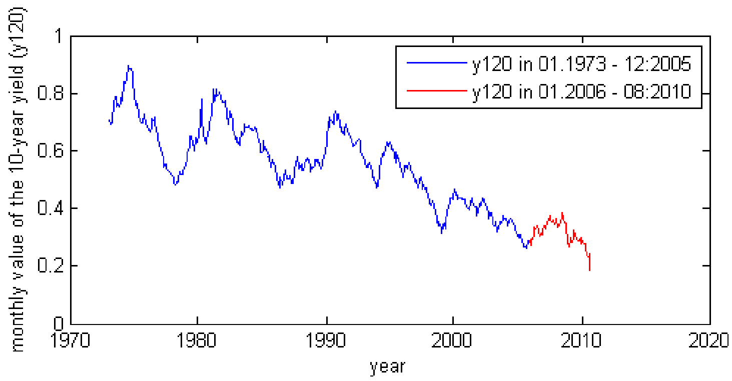


Figure 6.2: Monthly values of the German 10 year zero coupon yield (y^{120}) in the time period of 01/1973 till 08/2010.

From the year 2006 we see an upward sloping pattern till the credit crisis in 2008 and after that a huge dip in both rates. The dip in the short rate is more severe since the short rate is more volatile. The dip in the interest rates is partly due to the ECB policy of lowering the interest rates in order to give a positive boost to consumption in Europe. Also uncertainty on the stock market increased the demand for government bonds and lowered the yields. Finally the problems of some other European countries, for instance Greece, also increased the demand for the 'safe' German bonds and this resulted in the lowest yields in total observed time period between 1973 and 2010. These interest rates are used by pension funds in their ALM study to discount future cash flows of the pension rights. Low interest rates means lower discount rates and higher present values of the cash flows. As a result the liabilities have a much higher value and the funding ratio will go down. This effect is the main reason for the decreasing funding ratios of Dutch pension funds in 2010.

Besides the lower interest rates, we can see in figure 6.3 that the stock returns went down enormously during the credit crisis. Since all pension funds have a major part of their asset portfolio invested in stocks, this development had a great impact on the funding ratio as well. This is the main reason why the funding ratios of Dutch pension funds dropped dramatically end 2008.

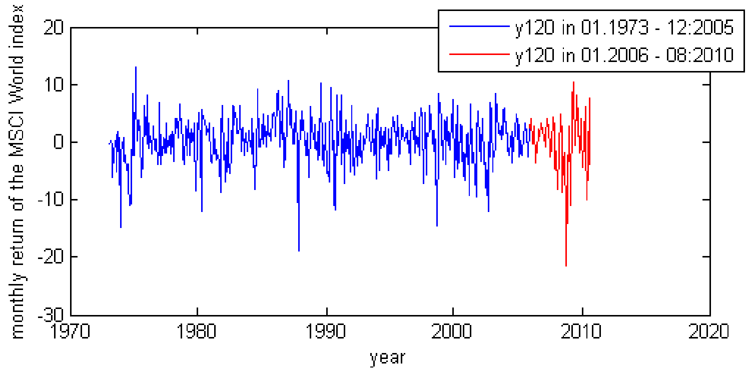


Figure 6.3: Monthly log return of the MSCI World stock index

Next we look at the VAR estimates which are shown in table D.1 in appendix D. In the summary statistics we can see that the means for all variables are slightly lower and the standard deviations are slightly higher, this is mainly due to the credit crisis in 2008. When we look at the VAR estimates we see that the biggest difference between the estimates based on the 1973-2005 sample and the is estimates based on the 1973-2010 sample is the coefficient of the 10 year yield on the stock returns. For the estimates based on the data period 1973 till 2010 it had a value of -4.52 and a t-value of -1.55. And for the period till 2010 a value of 0.35 with a t-value of 0.14, we see that the sign has changed and that the coefficient is less significant. Also the cross correlation between the stock return and the 10 year zero coupon yield is lower (-0.10 vs. -0.18). This different relation between the long term interest rate and the stock returns has also consequences for the long term correlation of bond and stock portfolios and the impulse response function as is shown in figure 6.4.

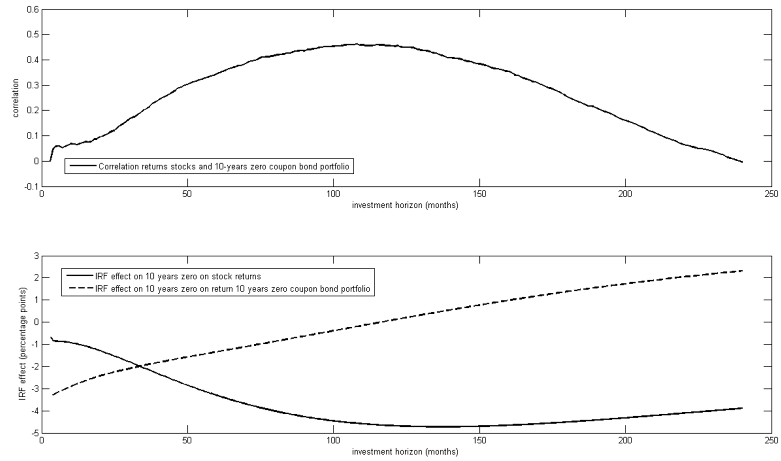


Figure 6.4: Impulse response analysis and correlation based on historical data for the time period 1973:01-2010:08

Compared to the figure based on the data between 1973 and 2005 the correlation increases at a slower pace and the maximum correlation after approximately 120 months (0.5) is lower. Also the moment (after 120 months) when the correlation starts to decrease again is later compared to the latter. When we look at the impulse response function we see that a shock in the 10 year interest rate initially results in a lower negative stock return compared to the 1973-2005 data, but it takes longer before the minimum is reached and the effect after 240 months is more negative compared to the latter.

We have to note that we did not provide confidence intervals for both figures and therefore some care should be taken with the conclusions. The correlations are calculated numerically by simulation and it is rather complicated to obtain confidence intervals. For 240 months ahead we simulate for each month 10000 bond and stock returns. Then we calculate for each month the corresponding correlation between the 10000 returns of stocks and bonds. The width of the confidence interval relies very much on the number of simulations that is used in this method. If we use 10000 simulations the confidence interval will be much smaller compared to the case where we would use 100 simulations to calculate this figure. Also for the impulse response function figure the same issue applies. All in all there is some uncertainty about these figure outcomes and this uncertainty should be kept in mind when interpreting the results.

At last we compare the risk premia parameters from table D.2 in appendix D and we see that there were not very dramatic changes here. Also the fit of the term structure model to the historical data is comparable and is also shown graphically in figure B.2 in appendix B.

6.1.1 Negative scenarios

Another thing that caught our attention are the low starting values for both the short rate and the 10 year zero coupon rate. These low starting values are responsible for some negative interest rate scenarios which we saw before in chapter 5. Since our initial interest rate values are close to zero and we generate our scenarios with Normally distributed error terms, it is inherent to our model that we obtain some of these negative scenarios. These negative interest rate scenarios are not very realistic because in the real world interest rates do not get below zero in most of the times.

One simple way of preventing interest rate scenarios from a VAR model to fall below zero is to replace the negative scenarios with a zero value. In this way we get a truncated Normal distribution. This is exactly our approach, disadvantages are the we obtain a rather undesirable probability distribution function and this approach changes the moments of the original distribution such as expected values, standard deviations and correlations.

Steehouwer (2005) introduces another way of truncating Normal distributions that deals with this disadvantages, in his approach the moments stay the same. An alternative could be to use a distribution with an absolute minimum value such as the lognormal, Snedecor's F or the Chi-Square distribution. According to Steehouwer (2005) there are some compelling reasons why one should prefer his truncated Normal distribution. First of all one stays as close as possible to the Normal distribution from which the theory of VAR models is derived. Secondly are the moments of the original distribution preserved in his approach. Also a generalization into a multivariate setting is easily made and finally there is more flexibility in choosing the truncation levels. In this thesis we will not implement one of these strategies that can be used when negative interest rates scenarios would occur. This is something that can be done in further research.

6.1.2 ALM output

Due to the lower interest rate scenarios, our mean bond return has also decreased. But for our ALM model we set the average return equal to our simulations of chapter 4. The stock returns actually increases due to the low interest rates and inflation and these are lowered a bit to have the same expected stock return as in chapter 4. Furthermore we replace negative interest rate and inflation scenarios with a zero value. In this way we do not use negative interest rates when discounting the liabilities. We use the inflation scenarios for the indexation of the pension rights and in case of a negative inflation scenario there will be no indexation in our model.

Now we look at the development of the funding ratio, in figure 6.5 are the results.

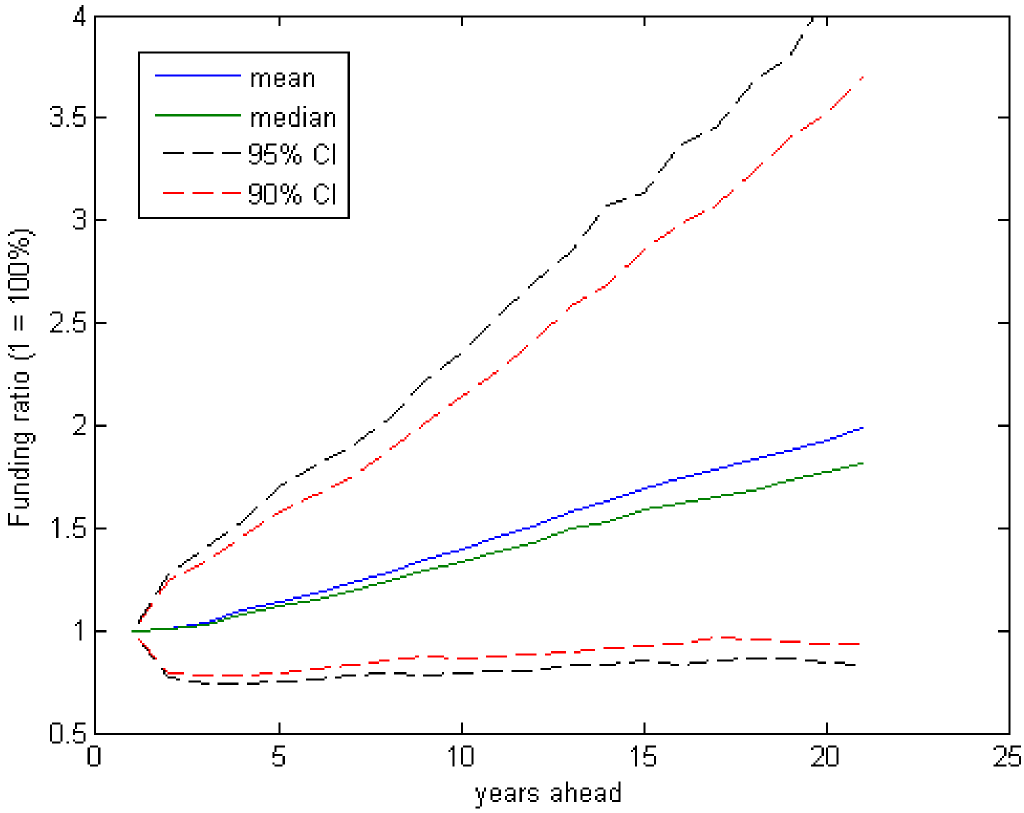


Figure 6.5: Simulation result for the funding ratio with the VAR model based on the 1973-2010 data.

We see that the standard deviation is higher, upward potential is higher, but also the probability of underfunding has increased compared to the figure of the previous chapter. This is due to the volatile data between 2005 and 2010 that is included in the estimation of the VAR model that produced the simulations. As a consequence this volatility is also visible in our scenarios. The upward potential is also due to the fact that there is no indexation of the liabilities in case of a negative inflation scenario, in this way the funding ratio can grow more. As a result also the confidence interval has a bigger range, which represent the grown uncertainty and volatility that was implemented in the 2006-2010 data. The results are summarized in table C.1 in appendix C. We see the same as in the funding ratio figure: higher standard deviations and more up- and downward potential.

Next we look at the indexation result, as already mentioned before we have a few negative inflation scenarios. For these scenarios we assume that there is no indexation, but when we calculate the indexation result it seems that in that case the indexation result is 100 % since the level of the liabilities is equal to the price level. In this way the indexation results do not say too much, but we look at them anyway. In appendix C is the indexation result plotted in figure C.1. As expected that indexation level is unrealistic high for the reason mentioned above. Also the results in the table C.2 are a too optimistic.

6.1.3 Conclusion

We can conclude that adding the historical data of the period between 2006 and 2010 influences our estimation results quite much. Especially the coefficients that include the equity return or interest rates differed from the coefficients found in chapter 4. This is off course partly due to the extreme events that occurred during the last 5 years. First of all we had the credit crisis in 2008 which caused some extra volatility in our data. Also the low interest rates had some consequences, the corresponding starting values were low and as a result of that we had a few negative interest rate scenarios. In the ALM output we saw a bigger confidence interval which resulted from the grown uncertainty that was incorporated in the historical data of the last 5 years.

6.2 Historical data period between 1986 and 2010

Now we are going to look at the effects of using a time interval from mid 1986 till 2010. One of the reasons for investigating this time interval is because we want to know whether excluding the high interest rates and inflation in the 70s will influence our estimation results and scenarios. We are aware of the fact that we have only 25 years of historical data in this case. This is not a very long time span, certainly not if we want to generate scenarios 20 years ahead.

In table D.3 in appendix D we can see the summary statistics and VAR estimation results based on the sample 1986:06 - 2010:08. The means for all variables are slightly lower compared to the 1973-2010 sample, this is also true for the standard deviations except for the stock returns. This is due to the credit crisis in 2008. Next we look at the VAR estimates and we see that for the inflation and stock return coefficients there are some differences compared to the coefficients based on the other historical data samples. For the inflation coefficients we see that the inflation is less explained by the dividend yield. The coefficient for the 10 years zero coupon yield and the previous inflation has become negative and the *t*-value is higher. Also the R^2 statistic has dropped from 0.17 to 0.09, which indicates that for this data sample the inflation is less explained by the model. For the stock returns it strikes out the most that the coefficient with the zero coupon yield now has the value 5.10 compared with -4.52 for the 1973-2005 sample, also the correlation is now 0.01 compared with -0.18 in the latter case. This is a huge difference and this also has far going consequences for the correlation and impulse response function figures. Furthermore we see that the stock return is explained more by the short rate and less by the inflation and dividend yield for the 1986:06 - 2010:09 sample. The R^2 statistic also decreased from 0.06 to 0.04.

In figure 6.6 are the consequences shown for the correlation and impulse response function figures.

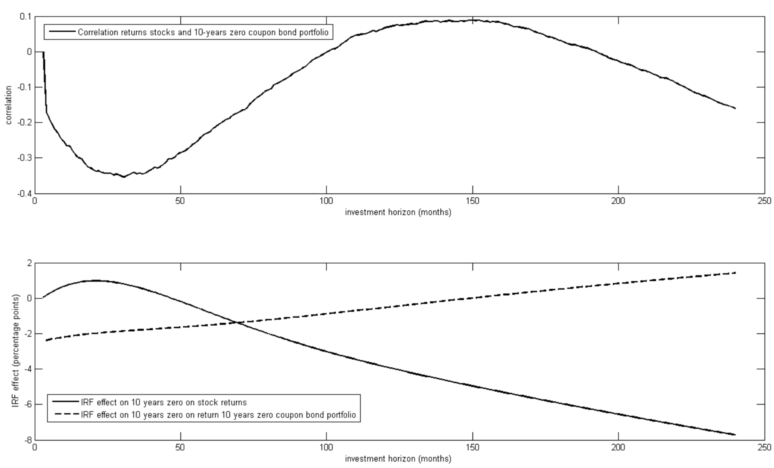


Figure 6.6: Impulse response analysis and correlation based on historical data for the time period 1986:06-2010:08

Instead of a positive correlation between a stock and a bond portfolio that we saw for the samples 1973-2005 and 1973-2010, we now have a negative correlation in the first 100 months. After approximately 35 months we reach a minimum of -0.35 and after 100 months the correlation becomes positive again to reach a maximum of approximately 0.08 after 150 months. After that it decreases again and drop below zero after 200 months. This is a huge difference with the previous two samples. It is mainly due to the change in the coefficient between stocks and the 10 year zero coupon yield which changed from -4.52 (-1.55) to 5.10 (1.38) and the difference in the cross correlation between these two variables (0.01 vs -0.18). This example shows that the VAR model and the corresponding correlation and impulse response figure are very sensitive for the data on which the model is estimated.

We want to state again that we do not provide confidence intervals for both figures for the reasons explained in the previous section. As a result some uncertainty about the correlation and impulse response function figure should be taken into account. This also concerns the conclusions made above.

6.2.1 ALM output

In the scenarios that were generated based on the historical data between 1986 and 2010 we also did encounter some negative scenarios. Since the high interest rate and inflation data is not included in the data here, the mean of the interest rates and inflation went down a bit and as a result we had some more negative scenarios here compared to the previous section. Again we replaced all negative scenarios with zero values. Also the bond and stock return expected values are adjusted to the same level as in chapter 4. Now we are going to take a look at the development of the funding ratio in figure 6.7.

We see that the mean and median at first decrease a little which was not the case in the previous section. Furthermore we observe a convex shape of the mean of the funding ratio, so as time proceeds the mean starts to increase faster. Also the lower bound of the confidence intervals lie lower compared to the 1973-2010 ALM model. This means that 5 and 10 percent worse scenarios of this model result in a far worse funding ratio of the pension fund compared to the model based on the 1973-2010 data. In table C.3 of appendix C are the results summarized.

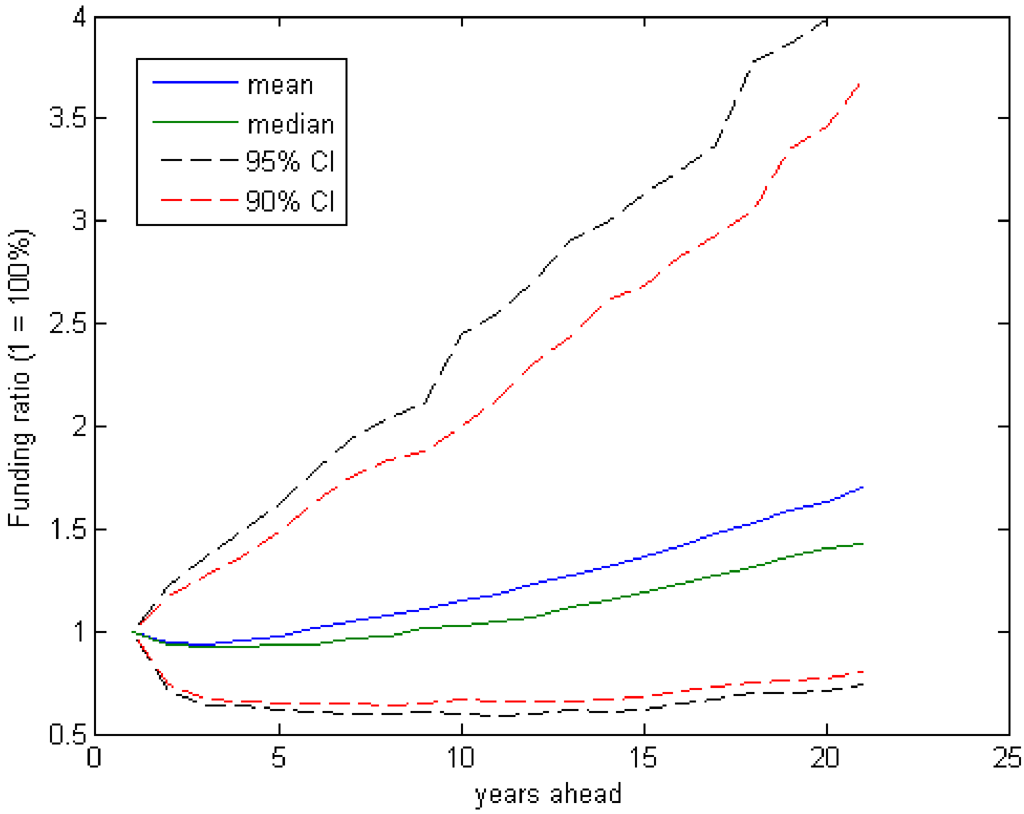


Figure 6.7: Simulation result for the funding ratio with the VAR model based on the 1986-2010 data.

6.2.2 Conclusion

We already concluded in the previous section that low starting values causes problems with interest rate and inflation scenarios. Also estimating the VAR model on data between 1986 and 2010 has these problems and the interest rates become even more negative due to the excluded high interest rates from the 70s. Another difference is the correlation between stock and bond portfolios which is totally different from the VAR models estimated on the 1973-2005 and 1973-2010 data. Although we should state that these figures incorporate some uncertainty which makes our findings less conclusive. When we use the scenarios as input for our ALM model we obtain a funding ratio mean that increases more as time proceeds, something we did not observe for the other two models. In the end we can conclude that using different time periods of historical data did influence our estimation results and the behavior of the corresponding scenarios. Another conclusion that we can make is that one should be very cautious with generating interest rate scenarios with a VAR model when the starting values are low.

6.3 Extending the term structure model

Till now we used the 10 year interest rate from our term structure model to discount all cash flows in our ALM model that are taking place after more than 10 years in the future. This is off course not very realistic and it would be more ideal to use a more appropriate rate of higher maturity. Also the dynamics and fit to the historical data could benefit from an extension to a higher maturity. Unfortunately there is not too much historical data available of interest rates with a maturity higher than 10 years, since there were few traded bonds with long maturity 25 years ago. But for our German zero coupon yields there is data available till a 15 year maturity from mid 1986 and this is the reason why we are investigating this time period. We have to note that this is a relatively short historical time period to estimate our term structure model on.

We want to improve the term structure model which we discussed in chapter 4. In order to do so we want to extend the model with a 15 year zero coupon yield, such that it covers the term structure from 1 month till 15 years instead of 1 month till 10 years. In figure 6.8 below are the 10 and 15 years zero coupon yields plotted.

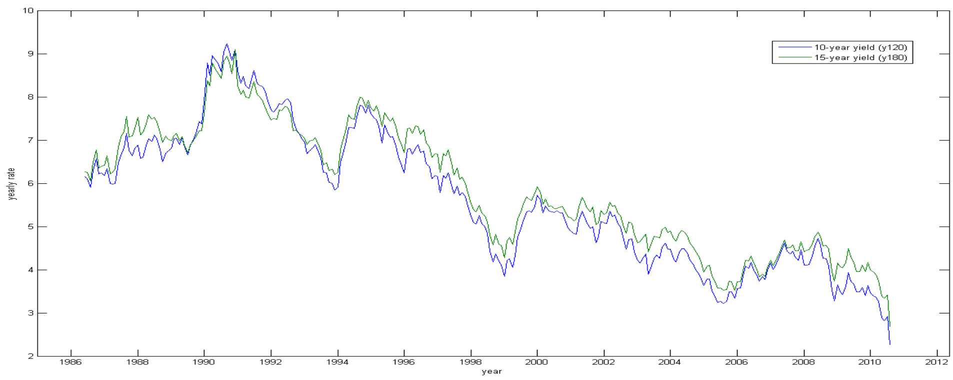


Figure 6.8: 10 versus 15-year yield in the time period 1986:06- 1010:08

Although the differences are not that big we can see that the 15 year yield is almost everywhere above that 10 year yield. Only in the period of high yields in the early nineties it lies below the 10 year yield. It is therefore less volatile and it could contribute to better dynamics of our term structure model. For longer maturities this could mean that our term structure model becomes less volatile.

We are going to compare three cases, the first one consist of the benchmark case with a term structure from 1 month till 10 years. In the second case we add the 15 years yield in the calibration of the model. And in the third case we replace the 10 year yield in the VAR model by the 15 year yield. Here we estimate the VAR model again and after that we estimate the term structure model in such a way that it is consistent with the VAR model. Here we mean with consistency that the short rate (1 month maturity) and the long rate (10 or 15 year maturity) from our VAR model are always equal to both of these rates from our term structure model. For the short rate this is true by definition as was explained in chapter 4.

For the long rate we must calibrated the optimization process in such a way that the long rate from our term structure model exactly equals the historical long rate. If this is the case then both rates have the same implications for our VAR and term structure model and in this way both models are consistent.

6.3.1 Benchmark case

We first estimate the term structure model in the same way as we did in chapter 4. The estimation procedure is further explained in appendix A. The VAR estimates are already discussed in the previous section and do not have a really big impact on our term structure model. In table D.4 of appendix D are the estimated risk premia parameters and the annualized means and standard deviations of the historical mispricing of the nominal term structure. We see that all the means and standard deviations are lower compared to the mispricing term structure of the 1973-2005 and 1973-2010 samples. This probably due to the fact that there is less data to calibrate on and therefore less opportunity to misprice, and the interest rates are more stable (lower standard deviation) in the period 1986-2010. Although we tried to penalize the mispricing for the 10 year yield, we were not able to get a zero mean and standard deviation. In this way the term structure model is not totally consistent with the VAR model, since both models do not have the exact same implications for the 10 year zero coupon yield.

In figure 6.9 below we have plotted the total fitted term structure for the maturities 1 till 120 months for the period 1986:06-2010:08. The x-axis, which is the left horizontal axis, represents the maturity. The y-axis represents the time, from June 1986 till August 2010. On the z-axis we find the monthly value of the yield.

To make things a bit more clear we plotted the yields with maturities 1 and 120 months separately in figure 6.10. Our term structure model is constructed in such a way that for the short rate (1-month maturity) the term structure model is equivalent to the historical data. We can recognize the pattern of the short in figure 6.9, for a maturity of 1 month. Secondly our model is calibrated in such a way that for the 10 year yield (120-month maturity) the estimation error is almost zero and one can also recognize, although a bit harder, the pattern of the 10 year rate (120-month maturity) in figure 6.9 for a maturity of 120 months. Between the 1-month short rate and the 10 year rate the term structure model is fitted with a pricing kernel as was explained in chapter 4. This pricing kernel smooths in a way the term structure between these both rates and is besides on the 1-month and 10 year rate also calibrated on the 2,3 and 5 year rate.

When we look at figure 6.9 again we can see that the shape of the term structure looks pretty plausible, except for the light curvature at the end of the curve towards the 10 year yield. This is due to the smoothing of the pricing kernel which we used to estimate the term structure model. In the next section we will investigate whether adding a 15 year yield will reduce this effect.

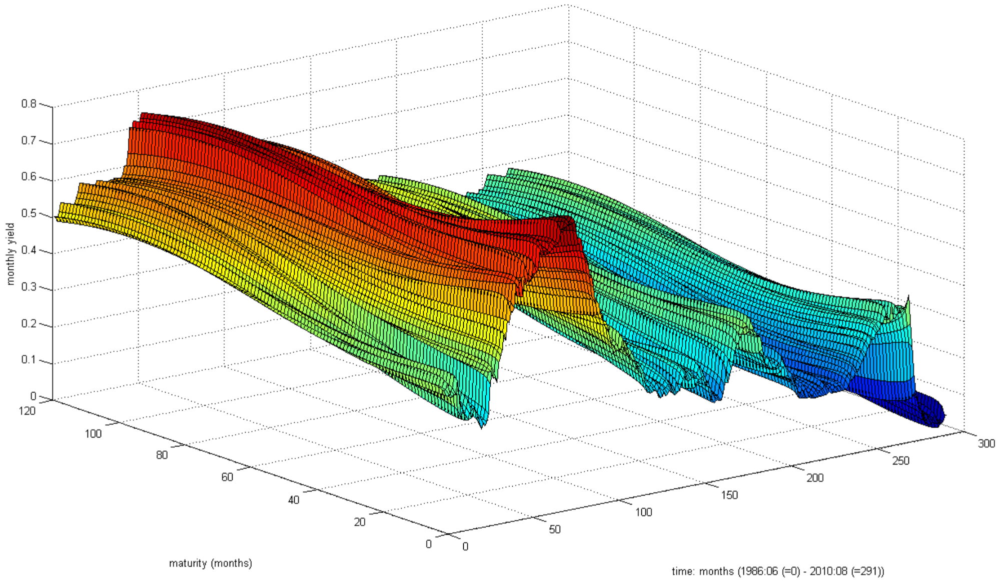


Figure 6.9: Benchmark term structure calibrated on the 2, 3, 5 and 10 year rate. The fitted model is plotted for all months and maturities of 1 till 120 months.

6.3.2 15-year yield added in calibration

To improve the dynamics and fit of the term structure model we will add the 15 year yield in the calibration. In the benchmark case we calibrate besides the 1-month and 10 year yield on the 2,3 and 5 year yield and here we add the 15 year yield. In table D.5 of appendix D are the estimated risk premia parameters and the annualized means and standard deviations of the historical mispricing of the nominal term structure. If we compare these with the benchmark case we see that the fit is slightly worse. Also the consistency with the VAR model is slightly worse, since the mean and the standard deviation of the historical difference are higher for the 10 year yield. The term structure model is consistent with the VAR model if it has the same implications for the 1-month and 10 year yield, which are also included in the VAR model. For the shorter maturities we see that the mispricing in table D.5 is also slightly higher compared to the benchmark case, so for the fitting part we can conclude that adding the 15 year yield does not contribute to a better fit on the short end of the curve.

Next we look at figure 6.11 where the term structure is plotted for the maturities of 1 till 180 months. Hence, here our term structure model has 60 months of extra maturity. We see that the curvature of the benchmark case has reduced and the fit seems to have straighted out a bit and this shape looks at bit more realistic. In real life we observe more term structures of interest rates that have this shape compared to the shape we saw for the benchmark case. But on the other hand we also observe that we have some negative interest rates at the short end of the curve in 2010. These negative rates are again the result of the smoothing of the pricing kernel.

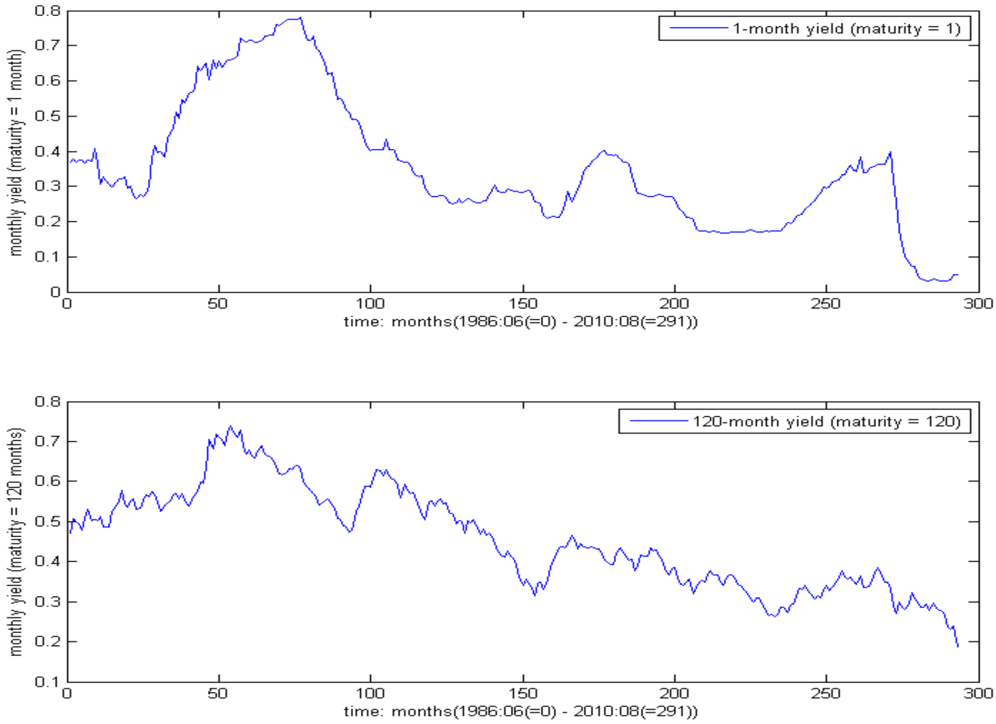


Figure 6.10: Monthly yields from the historical data between 1986:06 and 2010:08 for the maturities of 1 month plotted above and of 120 months plotted below.

6.3.3 15-year yield added in VAR

In this last case we replaced the 10 year yield in the VAR model by the 15 year zero coupon yield. In order to achieve consistency between the VAR model and the term structure model we now want the mean difference and standard deviation to be zero for the 15 year yield. Furthermore we calibrate the model on the 2, 3, 5 and 10 year yield. The estimation results and historical mispricing term structure are in table D.7 in appendix D. We see that the fit is even worse compared to the previous case. And a quick conclusion would be that this extension of the term structure does not contribute to a better fit of the term structure model.

Finally we look at the shape of the term structure model fit in figure 6.12, we see a pretty big curve towards the 15 year yield where the term structure model is calibrated on. This is not very realistic since in the whole term structure the 15 year yield lies below the 10 year yield, while in real life this is not the case. Furthermore this term structure shape is not a shape we will observe in real life and therefore worse compared to the shape we saw for the previous extension of the model

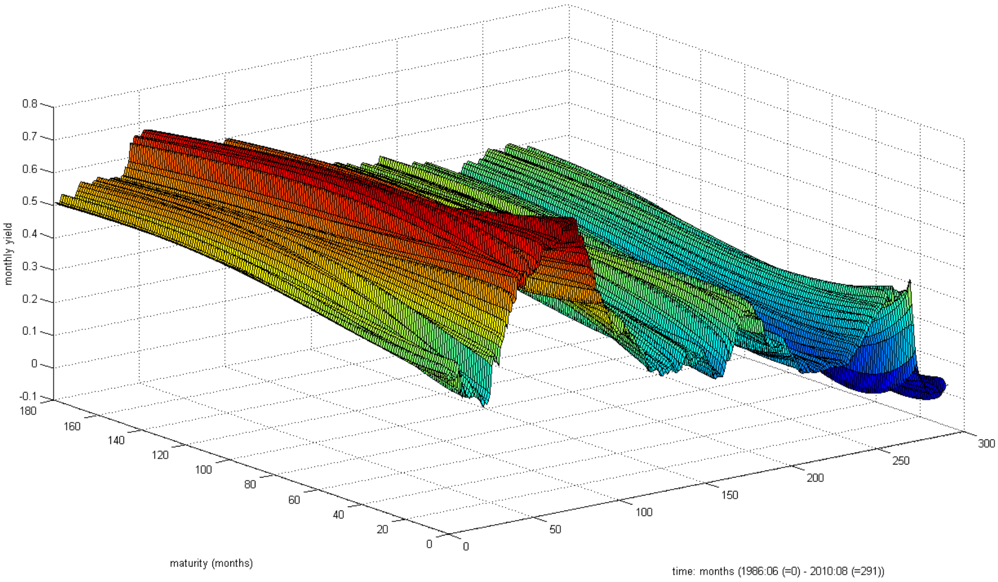


Figure 6.11: Term structure model calibrated on besides the 2, 3, 5 and 10 year rate also the 15 year rate, in order to be consistent with the VAR model the mispricing for the 10 year rate is almost zero. The fitted model is plotted for all months and maturities of 1 till 180 months.

6.3.4 Total model fit

We already observed in the previous sections that the addition of a 15 year yield did not lead to a better model fit accordingly to the model fit statistics introduced earlier. To show the total model fit graphically we made a plot of the absolute differences between fitted and historical yields for all maturities. We made the plot for all 3 cases and for both the maturities of 1 till 120 months and 1 till 180 months. For the benchmark model we use the fitted 10 year rate to compare with the 11 till 15 year historical rates. The results are in figure 6.13, but before we start interpreting them we will give a short explanation about the plots. There was data available for the maturities of 1 month, 6 months and 1 till 15 years. Just as for the figures from the previous sections the x-axis represents the maturity and the y-axis represents the time, from June 1986 till August 2010. For each of these maturities and for each point of time the absolute differences are plotted on the z axis. We will further explain it on the basis of figure 6.13 (a). For the maturity of 1 month we can see that the difference is zero for all points in time, this is due to the fact that the 1-month rate in our term structure model is equal to the 1-month rate of the historical data by definition. We can see that this holds for all the subfigures. In figure 6.13 (a) also the difference for the maturity of 120 months or 10 years is almost zero everywhere. This is because we calibrated our model here in such a way that the 10 year rate of our term structure model equals the historical 10 year rate. This was done by assigning a high penalty in the optimization process which is discussed more extensively in appendix A. Between the 1-month and the 10 year maturity we see that there are higher values on the z axis and these represent the absolute differences between the fitted and the historical yields. So the higher the value in the graph, the higher the difference between the fitted and historical yields. And bigger differences mean a worse model fit. In

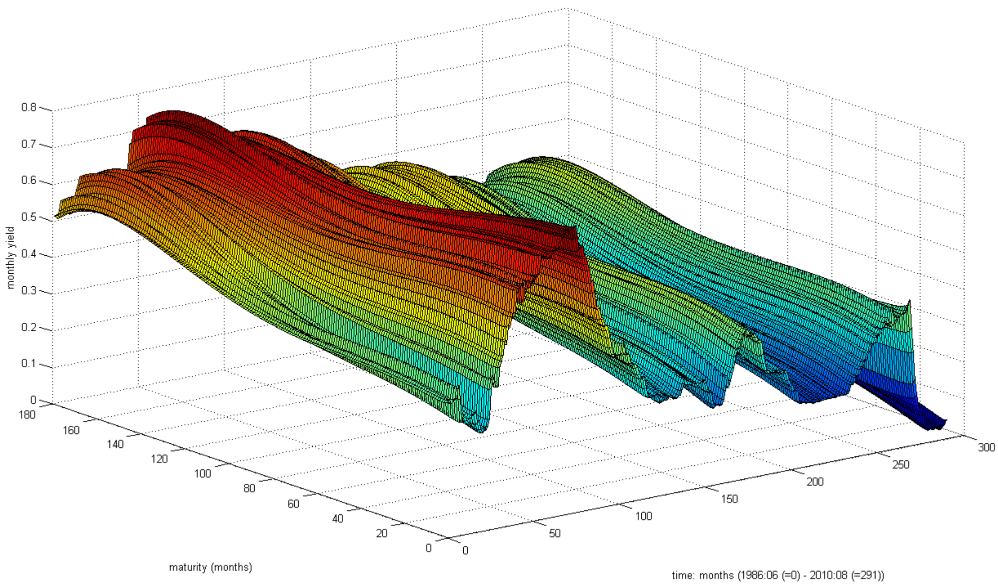
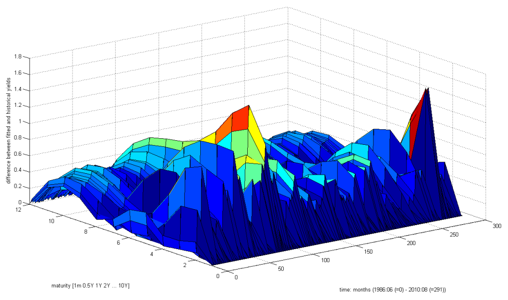


Figure 6.12: Term structure model calibrated on besides the 2, 3, 5 and 10 year rate also the 15 year rate. Here the mispricing for the 15 year rate is almost zero, since the 15 year rate has replaced the 10 year rate in the VAR model and we want consistency between these two models. The fitted model is plotted for all months and maturities of 1 till 180 months.

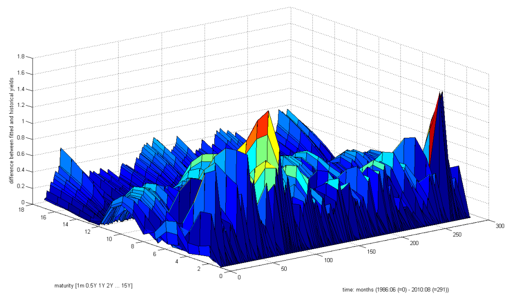
order to compare all plots with each other we equalized the range of all the axis.

Next we are going to evaluate the results from figure 6.13, we start with the subfigures (a) and (b). These two plots represent the fit of the benchmark model which has been calibrated on the 2, 3, 5 and 10 year rate. Subfigure (a) is the plot for the maturities from 1 till 120 months and subfigure (b) is the plot for the maturities from 1 till 180 months. The benchmark model only spans the maturities of 1 till 120 months and for differences between fitted and historical yields for the maturities between 11 and 15 years we compare the historical yields with the 10 year fitted yield. In order to be consistent with the VAR model the penalty assigned for the difference of the 10 year rate was set extra high in the optimization. For both figures we can see that for the 10 year rate the differences are indeed almost zero everywhere. The next thing that strikes out is the curvature towards the maturity of 120 months, this is the same curvature we already saw in figure 6.9. As a result we see that the fit for the 7, 8 and 9 year rate suffers from the high penalty on the 10 year rate. At last we look at subfigure (b) to see how the fit looks like for the higher maturities of 11 till 15 years. The fit seems reasonably well and this is as expected since we saw in figure 6.8 that the 10 and 15 year rate did not differ that much.

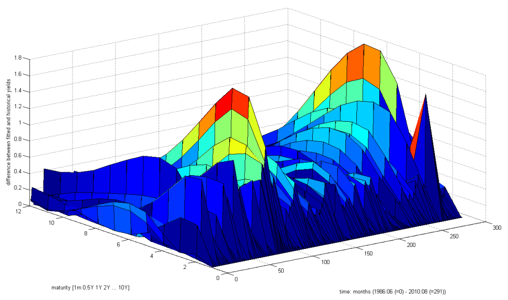
In subfigures (c) and (d) of figure 6.13 are the results of our first extension. Here we added the 15 year rate in the calibration but we still wanted the mispricing for 10 year rate to be zero. In this way our term structure model is still consistent with the VAR model. The left subfigure (c) is the plot for the maturities from 1 till 120 months and the right subfigure (b)



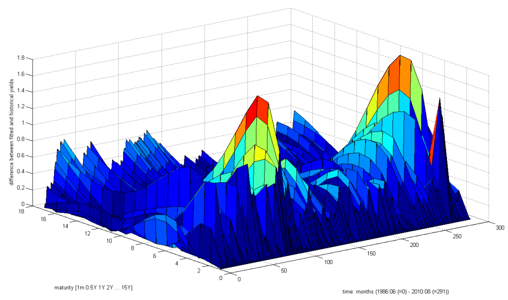
(a)



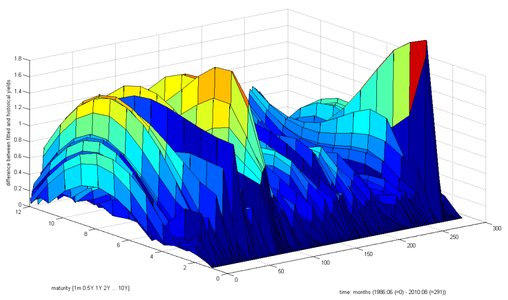
(b)



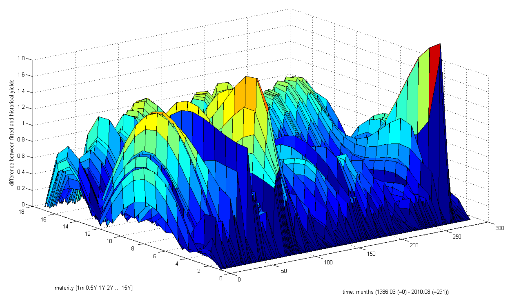
(c)



(d)



(e)



(f)

Figure 6.13: 3d plots of differences between fitted and historical yields for (a) benchmark case for the maturities of 1-120 months, (b) benchmark case for the maturities of 1-180 months, (c) Added 15Y yield in calibration for the maturities of 1-120 months, (d) Added 15Y yield in calibration for the maturities of 1-180 months, (e) 15Y in VAR for the maturities of 1-120 months and (f) 15Y in VAR for the maturities of 1-180 months.

is the plot for the maturities from 1 till 180 months. We observe that the difference between the fitted and historical 10 year yields are not exactly zero everywhere, this is the result of adding the 15 year rate in the calibration. Also the fit on the short end of the curve, so for shorter maturities, is worse compared to the benchmark case. But on the other hand the fit for the maturities between 5 and 10 years is a little bit better. At last we look at the fit for the maturities between 10 and 15 years. For the early 90s the fit is actually worse compared to the benchmark case, this seems strange since we add the 15 year rate in the calibration here but the reason is that this was the time period where the 15 year rate lied below the 10 year rate as can be seen in figure 6.8. After the first years the fit is slightly better for this model compared to the benchmark case, although the differences are not that big. So for the higher maturities the fit slightly better here and there, but that is at the cost of the fit for the shorter maturities and the mispricing of the 10 year rate and hence the consistency with the VAR model.

Finally we look at our second and last extension of the term structure model. Here we replaced the 10 year rate in the VAR model with the 15 year rate. We calibrated our term structure model again on the 2, 3, 5, 10 and 15 year rate, but in order to be consistent with the VAR model the mispricing of the 15 year rate instead of the 10 year rate should be zero here. In subfigures (e) and (f) are the model fit results, again (e) is the plot for the maturities of 1 till 120 months and (f) is the plot for the maturities of 1 till 180 months. We see immediately in subfigure (e) that the fit is far worse compared to the previous two cases. In subfigure (f) we see differences between the fitted and historical 15 year rate is indeed close to zero, but that goes on the costs of the fit of the lower maturities. We can conclude here that adding the 15 year rate does not lead to a better historical fit of the term structure model. A possible explanation is that the range between 1 month and 15 years is one and a half times bigger than the range between 1 month and 10 years and the pricing kernel is not capable of fitting the model properly for the larger range.

6.3.5 Conclusion

In this section we investigated two extensions of the term structure model. In the first extension we added the 15 year yield in the calibration and just as in the benchmark case we calibrated the optimization process in such a way that our term structure model had the same implications as the VAR model for the 10 year rate. In this way both models are consistent with each other. In the second extension we replaced the 10 year rate in the VAR model with the 15 year rate, and here we calibrated the optimization process again in such a way that it is consistent with the 15 year rate of the VAR model. In terms of the fit to the historical data our first extension showed some positive results. For longer maturities the fit was slightly better, but for shorter maturities it was worse. Also the difference with the 10 year rate from the VAR model was not exactly zero so this extension was not 100 percent consistent with the VAR model. For the second extension the fit was far worse and we can conclude that this extension did not lead to a better term structure model.

Not only the fit to the historical data is important for a term structure model, but also the shape of the term structure needs attention. We want a shape that we also observe in real life for interest rate term structures, so without extreme curvatures. In this perspective our first extension did show some good results, by adding the 15 year rate the curvature towards the 10 year rate did reduce. Off course this is a rather qualitative judgement which is not supported by statistical test results. We have to note that the assigning of the weights in the optimization process to the different yields on which the model is calibrated does have a big influence to the outcome of the parameters and the corresponding model fit. For our research this has been a process of trial and error and it might be that we did not cover a certain weighting which could have led to a better model fit for both extensions.

To our knowledge there is not much other research done on this topic of extending an affine term structure model. Reasons could be the fact that these type of models do not exist that long, the scarce yield data with longer maturities or that the objectives of other researches do not ask for this extension. An exception is the article of Cochrane and Piazzesi (2008), they extend the model from their 2005 article for longer maturities. Although their term structure model is not being used for the same purpose as our model it could be interesting to compare the results. Their goal is to decompose the yield curve into expected interest rate and risk premium components. It is not an explicit research goal in their article to extend their model with longer maturities. But where they used historical yields with a maturity of 1 till 5 years in the original article, they add in their 2008 article a data set with a maturity of 1 till 15 years. They compared different combinations and concluded that their original data set with maturities of 1 till 5 years captured all the information of the other data set with maturities of 1 till 15 years. So the data with longer maturities did not improved their model. This finding supports our results in a way that our extension did not improve our model much either.

Chapter 7

Summary and Conclusion

We started our research with the goal to find out what is the best way to generate macroeconomic scenarios. These scenarios could serve as input for various types of asset and liability management problems of different financial institutions, and in this thesis we focused on the consequences in an asset and liability management study of an average Dutch pension fund.

In chapter 2 we gave a short introduction of asset and liability management problems in general and their relation with macroeconomic scenarios. We described the different models that can be used for generating these scenarios and concluded that a vector autoregressive (VAR) model is most suited. The VAR model is preferred because it is an intuitive linear model which incorporates long term dynamics. One of the problems with VAR model simulations are possible arbitrage opportunities. These are not realistic and with present arbitrage opportunities we are not able to valuate all assets. As a result it would not be possible to value embedded options and although we do not perform these valuations in our thesis this is an important requirement for a model that generates economic scenarios. In chapter 3 we reviewed the literature about this topic and one of the solutions was to extend the VAR model with an affine term structure model of interest rates. Hoevenaars (2008) used such a type of model to generate economic scenarios for an asset and liability study of a pension fund and this model was our starting point. An alternative could have been to use risk-neutral simulations, but with this approach our scenarios would not have been consistent with historical time series and patterns.

Next we estimated the model of Hoevenaars (2008) in chapter 4. In order to check our results we used almost the same data as in Hoevenaars (2008). Our estimation results and term structure model fit turned out to be almost similar to those of Hoevenaars (2008) from which we concluded that our estimation procedure was right. We continued in chapter 5 where we used our scenarios as input for an ALM study of a Dutch pension fund. We made the set up of our ALM model in such a way that it represented an average Dutch pension fund. The simulation results showed us the possible evolution of the funding ratio and the indexation result. In the worst case scenarios the funding ratio dropped well below the 100 percent, but the most positive scenarios gave rise to enormous surpluses. So the range of the confidence interval of our funding ratio was quite big and hence the model outcomes contained much uncertainty. We have to noted that the model assumptions all have a big influence on the eventual model outcomes and therefore some sensitivity analysis on the model assumptions should be considered in further research. These model assumptions also influence the model uncertainty and corresponding confidence interval of the outcomes. Another critical topic that needs some attention is the time span of the historical data, in our research we had only 30 years of historical data available on which the model was estimated and with this model we generated scenarios for the next 20 years. Ideally our historical data would have contained a longer time period.

In chapter 6 we did some further research on whether using an other time period of historical data would influence our estimation and simulation results. In chapter 4 our model was estimated on historical data of the time period between 1973 and 2005 and here we extended our historical data set with the data of the time period between 2006 and 2010. In this time period the data of the credit crisis in 2008 is included and also very low interest rates of the year 2010 are part of the dataset. Our model coefficients changed indeed, especially the coefficients that included the stock returns or the 10 year zero coupon yield. As a consequence also the correlation between stock and bond portfolios was influenced by the different data. Another thing that striked out where some negative interest rate scenarios that we did encounter. These were the result of the low starting values from our 2010 data. In our model we replaced the negative scenarios with zero values, but a topic for further research could be the application of the truncated VAR method of Steehouwer (2005) to our model. An alternative to deal with the negative scenarios could be the use of an absolute value distribution such as the lognormal or chi squared distribution. At last we saw in our ALM model that the confidence interval was even larger compared to our model in chapter 5, this was due to the increased standard deviations of the variables during the time period between 2006 and 2010. We also investigated the consequences of using historical data between 1986 and 2010 and for these data we found even greater differences in the estimated VAR parameters. Also the correlation figure changed dramatically. Here we observed a negative correlation for the first few months, where for the latter two figures we only observed positive correlations. We have to noted though that we did not calculated the corresponding confidence intervals of the correlation figures and therefore some care should be taken with the conclusions.

The last topic we investigated was the extension of our term structure model. Initially our term structure reached from the maturity of 1 month till the maturity of 10 years. This had a consequence for our asset and liability model, namely all cash flows that where taking place longer than 10 years into the future were discounted with the 10 year interest rate. Where an interest rate with higher maturity would be more realistic. Also a better model fit and a better shape of the term structure model were a motivation for this extension. An important issue here is data availability, we only had access to historical interest rate data from the year 1986 with a maturity of 15 years. We extended the term structure model in two ways. In the first case we calibrated the model besides on the 2, 3, 5 and 10 year rate also on the 15 year rate. In the second case we included the 15 year rate instead of the 10 year rate in the VAR model and after that we calibrated the term structure model on the 2, 3, 5, 10 and 15 year rate. For the first extension we saw a slightly better fit for the longer maturities, but for the shorter maturities the fit was worse. Also the term structure shape looked a bit more to the shape we would normally see in real life, but this was a rather qualitative observation and not supported by statistical test results. On the other hand this extension was not totally consistent with the VAR model. Unfortunately the second extension did not result in a better fit. We were not able to draw a definite conclusion on whether the extension is better compared to the initial model, an advantage is the better fit for longer maturities and term structure shape, but the disadvantages are the worse fit for the shorter maturities and the fact that this model is less consistent with the VAR model.

All in all we can conclude that the VAR model extended with a term structure model of interest rates is perfectly suited for generating arbitrage free scenarios. The most important input for the model is off course the historical data and we would like to emphasize that the data selection is of high importance. We found that different time periods of historical data have a great influence on the parameter estimates and corresponding scenarios. One should also be cautious with low starting values of the interest rates, since these can lead to negative scenarios. At last we found some results indicating that adding higher maturity data could improve the term structure model, but further research is needed to draw some definite conclusion on this topic.

Bibliography

Ang, A. and M. Piazessi (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics 50*, 745–787.

Ang, A., M. Piazessi, and M. Wei (2006). What does the yield curve tell us about gdp growth? *Journal of Econometrics 131*, 359–403.

Bikker, J.A., D.W.G.A. Broeders, D.A. Hollanders, and E.H.M. Ponds (2009). Pension funds' asset allocation and participant age: a test of the life-cycle model. DNB Working Papers 223.

Bodie, Z., R.C. Merton, and W.F. Samuelson (1992). Labor sypply flexibility and portfolio choice in a life cycle model. *Journal of Economic Dynamics and Control 16*, 427–449.

Boender, G., C. Dert, F. Heemskerk, and H. Hoek (2007). *Handbook of Asset and Liability Management, Volume 2*, Chapter 18: A Scenario Approach of ALM. Elsevier B.V.

Campbell, J.Y., A.W. Lo, and A.C. MacKinlay (1997). *The Econometrics of Financial Markets*. New Jersey: Princeton.

Campbell, J.Y. and L.M. Viceira (2002). *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford: Oxford University Press.

CBS. Statistics from statline.

Christensen, J., F.X. Diebold, and G. Rudebusch (2007). The affine arbitrage-free class of nelson-siegel term structure models. FRB of San Fransisco Working Paper No. 2007-20.

Cochrane, J. and M. Piazzesi (2005). Bond risk premia. *American economic review 95*, 138–160.

Cochrane, J. and M. Piazzesi (2008). Decomposing the yield curve.

Diebold, F.X. and C. Li (2006). Forecasting the term structure of goverment bond yields. *Journal of Econometrics 130*, 337–664.

DNB (2010). Statistisch bulletin september 2010.

Hamilton, J.D. (1994). *Time Series Analysis*. New Jersey: Princeton.

Harrison, J.M. and D.M. Kreps (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory 20*, 381–405.

Hoevenaars, R.P.M.M. (2008). Strategic asset allocation and asset liability management. Phd. thesis, University of Maastricht.

Hoevenaars, R.P.M.M., R.D.J. Molenaar, and T.B.M. Steenkamp (2003). *Asset Liability Management Tools*, Chapter 11: Simulation for the long run.

Hull, J.C. (2005). *Options, Futures and Other Derivatives.* New Jersey: Prentice Hall.

Ingersoll, J.E. (1987). *Theory of Financial Decision Making.* New Jersey: Rowan and Littlefield.

Jarvis, S., F. Southall, and E. Varnell (2001, Februari). Modern valuation techniques. Presented to the Staple Inn Actuarial Society.

Kakes, J. and D.W.G.A. Broeders (2006). De houdbaarheid van het nederlandse pensionstelsel. Occasional Studies 6, DNB.

Kingsland, L. (1982). Combining financial and actuarial risk: Simulation analysis. projecting the financial condition of a pension plan using simulation analysis. *Journal of Finance 37*, 577–586.

Mulvey, J.M. (1994). An asset-liability investment system. *Interfaces 24*, 22–33.

Mulvey, J.M., G. Gould, and C. Morgan (2000). An asset and liability management system for towers perrin-tillinghast. *Interfaces 30*, 96–114.

Smith, P.N. and M.R. Wickens (2002, March). Asset pricing with observable stochastic discount factors. Discussion papers, University of York, Department of Economics.

Steehouwer, H. (2005). *Macroeconomic Scenarios and Reality.* Rotterdam: Optima Grafische Communicatie.

Wilkie, A.D. (1987). A stochastic investment model for actuarial use. *Transactions of Actuaries 39*, 391–403.

Wilkie, A.D. (1995). More on a stochastic asset model for actuarial use. presented to the Institute Actuaries and Faculty of Actuaries, London.

Appendix A

Risk premia estimation

We apply the same estimation methodology as explained in the appendix of Ang et al. (2006). For the estimation of the risk premia parameters we made the assumption that the risk premium is zero for the inflation and the dividend yield. Furthermore we assumed that the risk premia parameters for the equity return should be the same as in our VAR model in order to achieve consistency. These assumptions have the consequence that a few parameters are fixed in $\Sigma\lambda_0$ and $\Sigma\lambda_1$ and only 12 parameters have to be estimated. We want to estimate a y vector which contains the 12 parameters that are not fixed in $\Sigma\lambda_0$ and $\Sigma\lambda_1$. The $\Sigma\lambda_0$ and $\Sigma\lambda_1$ matrices with the parameters that are fixed and the one that we want to estimate now look as follows:

$$\Sigma\lambda_1 = \begin{pmatrix} y(1) & y(2) & y(3) & y(4) & y(5) \\ 0 & 0 & 0 & 0 & 0 \\ y(6) & y(7) & y(8) & y(9) & y(10) \\ \text{-1.83} & \text{-1.71} & \text{-4.52} & 0.04 & 2.49 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$\Sigma\lambda_0 = \begin{pmatrix} y(11) \\ 0 \\ y(12) \\ 0.33 \\ 0 \end{pmatrix}$$

Next we let the matlab function $fminunc$ search for the values of y that solve:

$$\min_{\Sigma\lambda_0,\Sigma\lambda_1} \sum_{t=1}^{T} \sum^{N} (\hat{y}_t^{(n)} - y_t^{(n)})^2 \quad \text{for } N = 24, 36, 60 \text{ and } 120. \tag{A.1}$$

Where $\hat{y}_t^{(n)}$ are the fitted yields of the term structure model in (4.11) and $y_t^{(n)}$ are the historical zero coupon yields. The observed factors, the short rate and the 10 year zero coupon yield, follow directly from the yields $y_t^{(1)}$ and $y_t^{(120)}$. Hence these are yields to be considered to be measured without any observation error, and in order to achieve that we have a higher weight on the difference for 120 months in the minimization. The other yields (with maturities 24, 36 and 60 months) are functions of $y_t^{(1)}$, $y_t^{(120)}$, inflation, equity return and the dividend yield. And these yields will have some measurement error. In figure B.1 in appendix B the fit for the different maturities is shown graphically.

Appendix B

Term structure model fits

Here are all the figures shown of the historical fit of the affine term structure model in equation 4.11 for five points on the term structure: 1-month (y^1), 2 year (y^24), 3 year (y^36), 5 year (y^60) and 10 year (y^120) zero coupon interest rates. For the extended term structure model the 15 year (y^180) zero coupon rate is added. Solid lines represent the actual series. Dashed lines denote the fitted series.

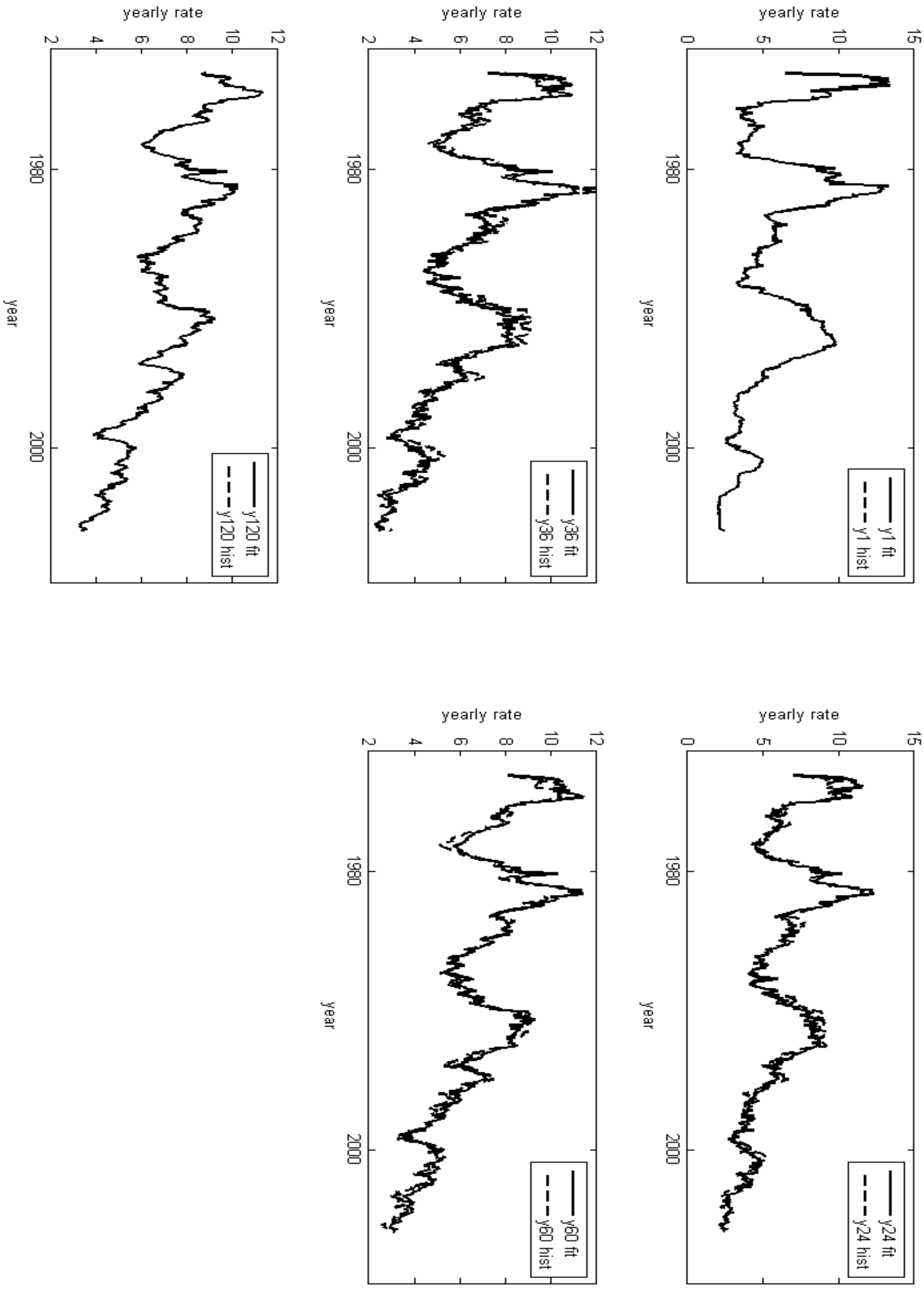


Figure B.1: Historical fit of term structure model for the 1973-2005 sample

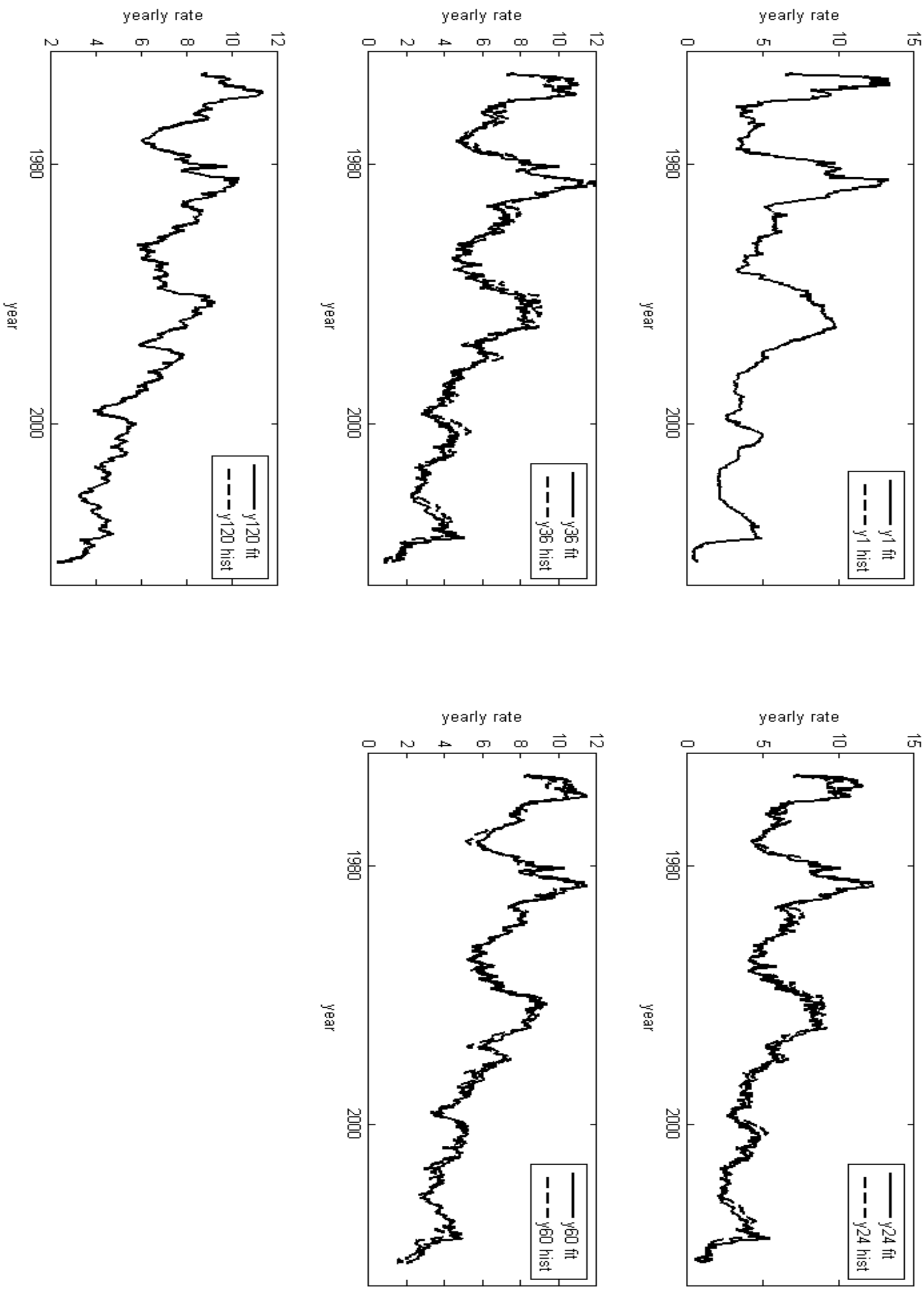


Figure B.2: Historical fit of term structure model for the 1973-2010 sample

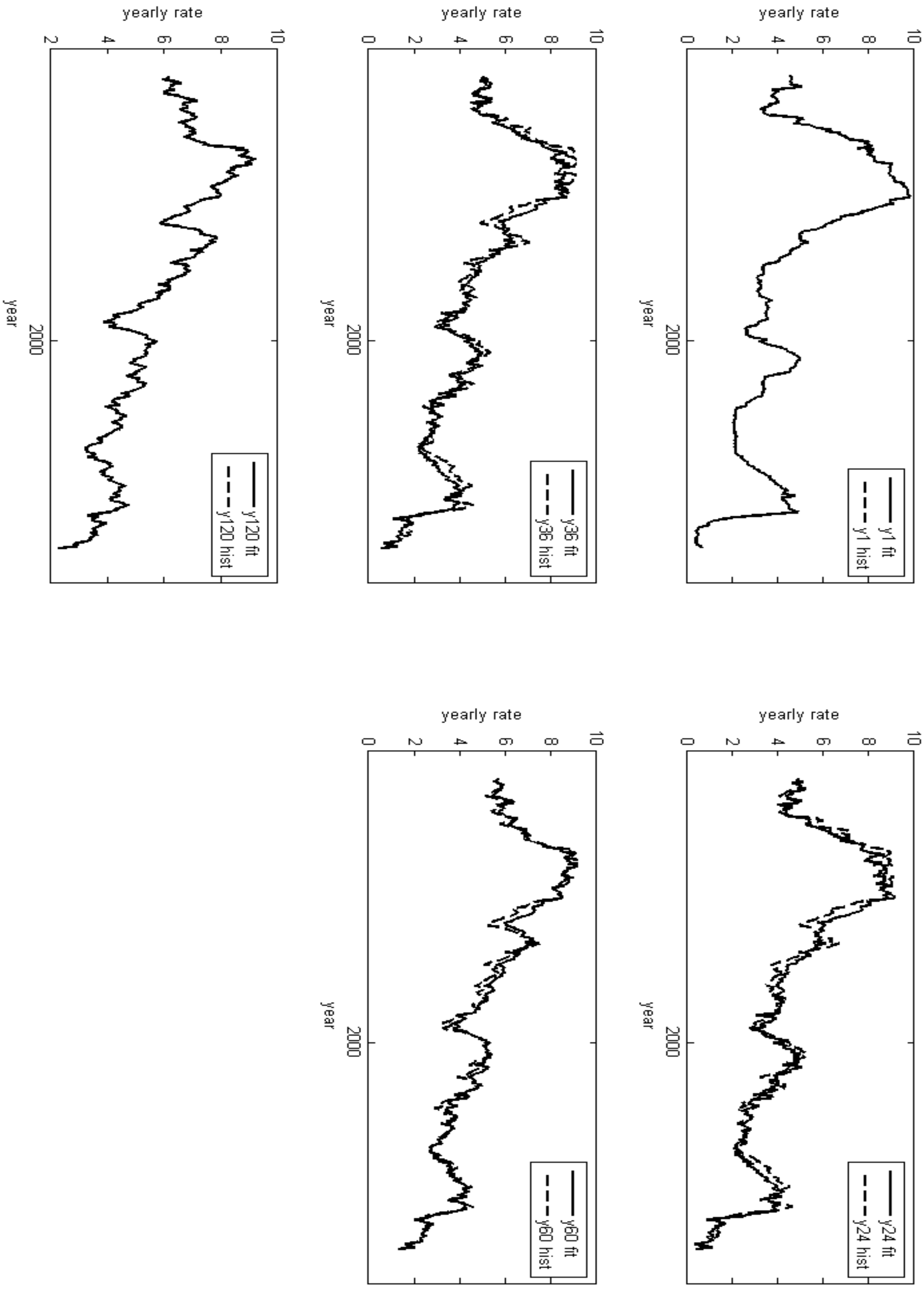


Figure B.3: Historical fit of term structure model for the 1986-2010 sample

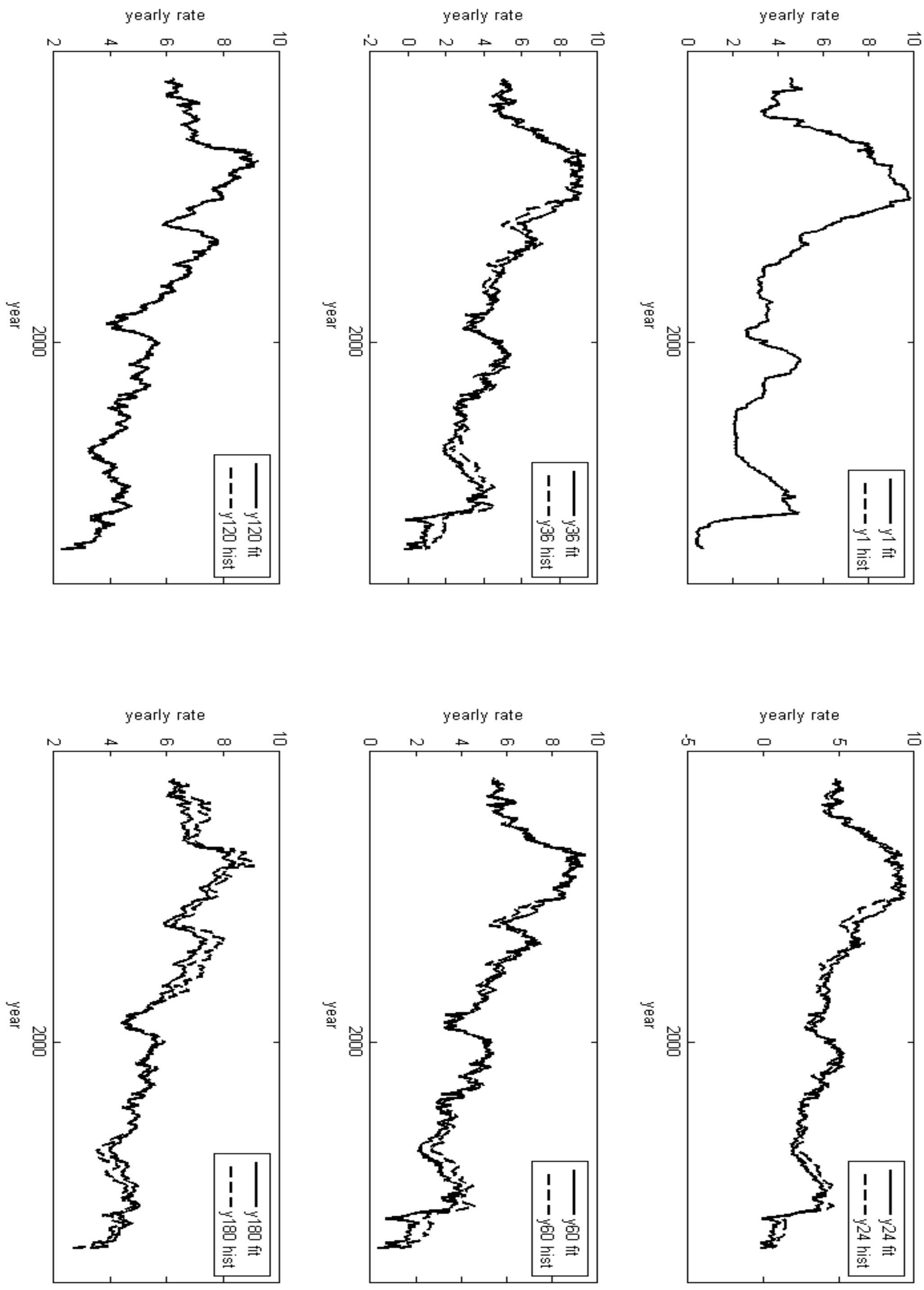


Figure B.4: Historical fit of term structure model for the 1986-2010 sample, where also the 15 year zero coupon rate is used in the calibration of the model

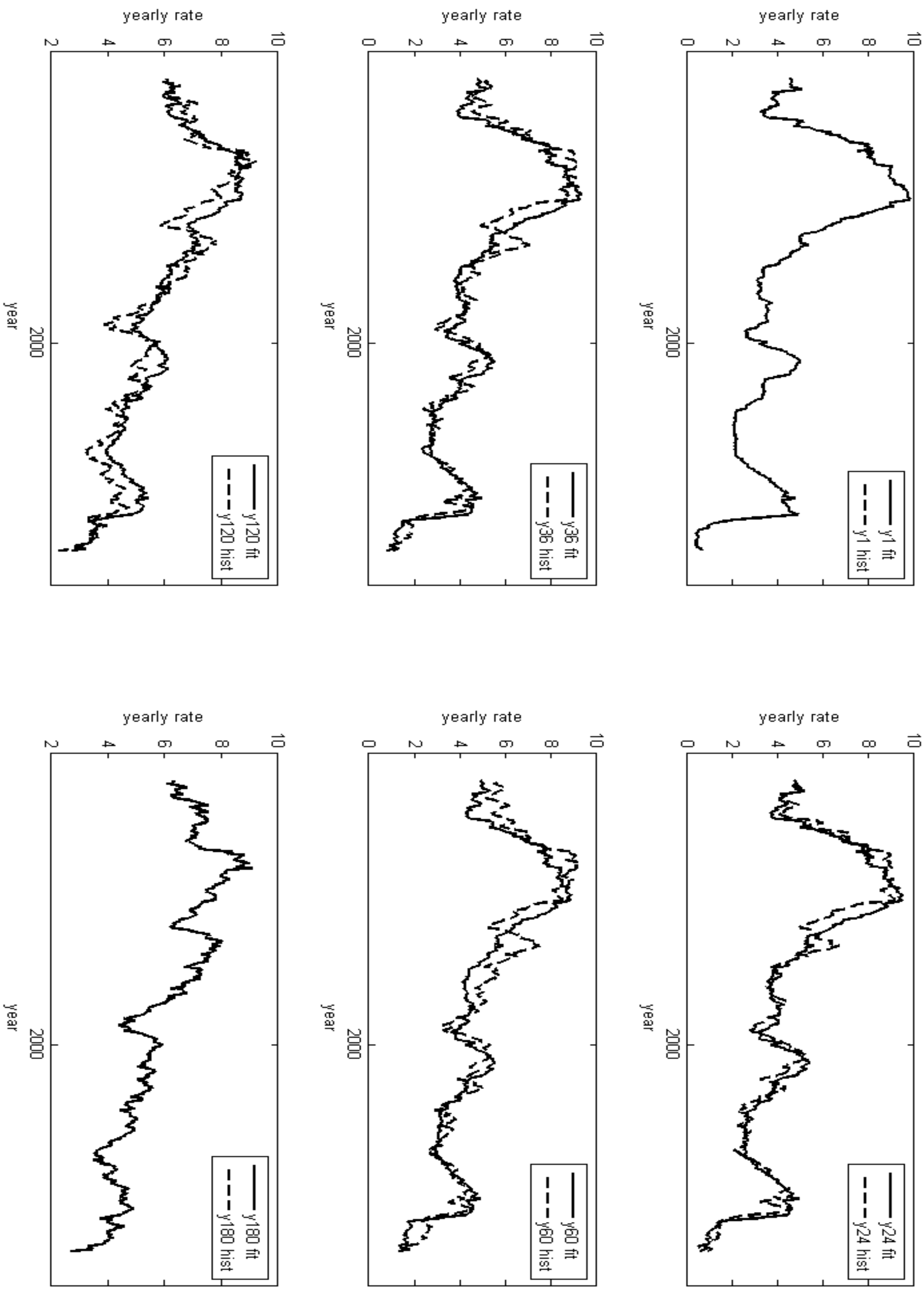


Figure B.5: Historical fit of term structure model for the 1986-2010 sample, where the 10 year yield in the VAR model is replaced by the 15 year zero coupon yield

Appendix C

ALM output

Here are all ALM output for the simulations based on the historical data periods 1973-2010 and 1986-2010.

Table C.1: Solvency position

Summary statistics of probability distributions of variables from the solvency position where the initial funding ratio is 100 % and the VAR model is estimated on historical data between 1973 and 2010. The selected output variables are the median of the funding ratio (FR) after 1, 10 and 20 years and corresponding standard deviations. The probability of under funding ($P(FR < 100)$) in the next 1, 10 or 20 years. The expected funding ratio after one year given it is in the 2.5 % worse scenarios ($CFRaR_{2.5\%,t+1}$). And the maximum drawdown (in percentage points) of the funding ratio in the next year ($\delta_{FR,t+1}$).

FR_{t+1}	1.01
FR_{t+10}	1.38
stdev FR_{t+10}	0.43
FR_{t+20}	1.81
stdev FR_{t+20}	0.88
$P(FR_{t+1} < 100)$	0.48
$P(FR_{t+10} < 100)$	0.12
$P(FR_{t+20} < 100)$	0.07
$CFRaR_{2.5\%,t+1}$	0.74
$\delta_{FR,t+1}$	0.48

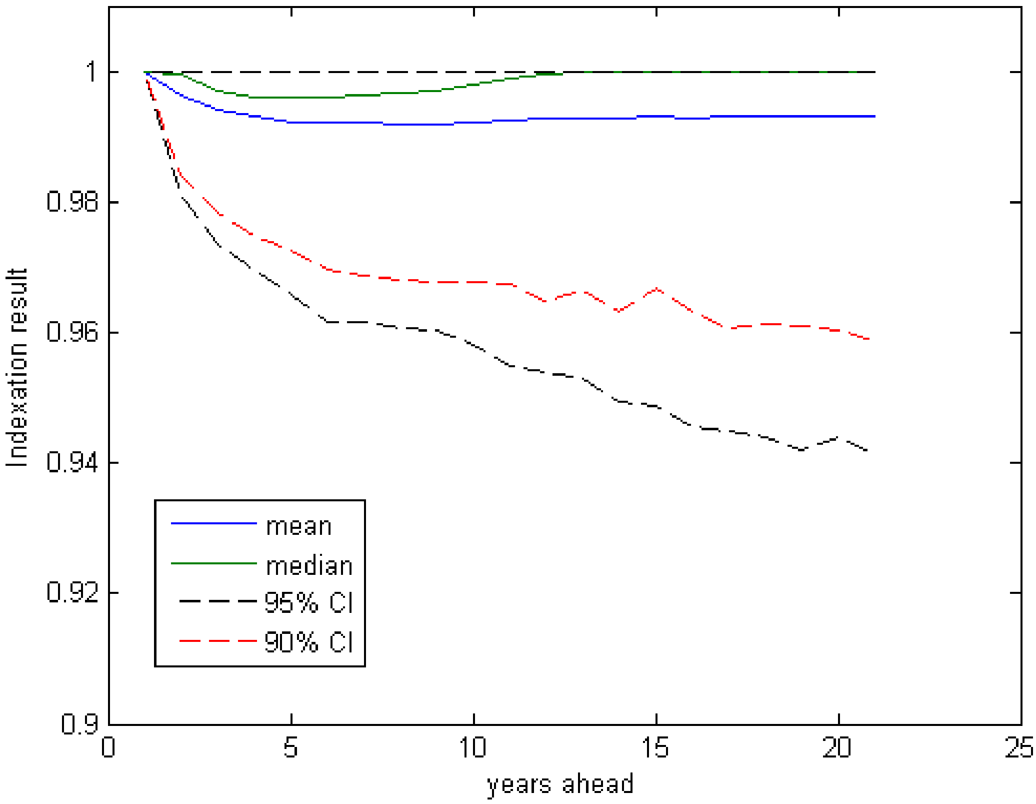


Figure C.1: Simulation result for the indexation level with the VAR model based on historical data between 1973 and 2010.

Table C.2: Indexation quality

Summary statistics of probability distributions of variables from the indexation quality. The selected output variables are the median, mean and standard deviation of the indexation result (IR) after 1, 10 and 20 years. And the probability that the indexation result will drop below 100 % and 85 % respectively ($P(IR < 100)$ and $P(IR < 85)$).

median IR_{t+1}	0.99
mean IR_{t+1}	0.99
median IR_{t+10}	0.99
mean IR_{t+10}	0.99
stdev IR_{t+10}	0.01
median IR_{t+20}	1.00
mean IR_{t+20}	0.99
stdev IR_{t+20}	0.02
$P(IR_{t+10} < 100)$	0.68
$P(IR_{t+20} < 100)$	0.44
$P(IR_{t+10} < 85)$	0.00
$P(IR_{t+20} < 85)$	0.00

Table C.3: Solvency position

Summary statistics of probability distributions of variables from the solvency position where the initial funding ratio is 100 % and the VAR model is estimated on historical data between 1986 and 2010. The selected output variables are the median of the funding ratio (FR) after 1, 10 and 20 years and corresponding standard deviations. The probability of under funding ($P(FR < 100)$) in the next 1, 10 or 20 years. The expected funding ratio after one year given it is in the 2.5 % worse scenarios ($CFRaR_{2.5\%,t+1}$). And the maximum drawdown (in percentage points) of the funding ratio in the next year ($\delta_{FR,t+1}$).

FR_{t+1}	0.94
FR_{t+10}	1.04
stdev FR_{t+10}	0.52
FR_{t+20}	1.42
stdev FR_{t+20}	0.91
$P(FR_{t+1} < 100)$	0.68
$P(FR_{t+10} < 100)$	0.44
$P(FR_{t+20} < 100)$	0.17
$CFRaR_{2.5\%,t+1}$	0.68
$\delta_{FR,t+1}$	0.53

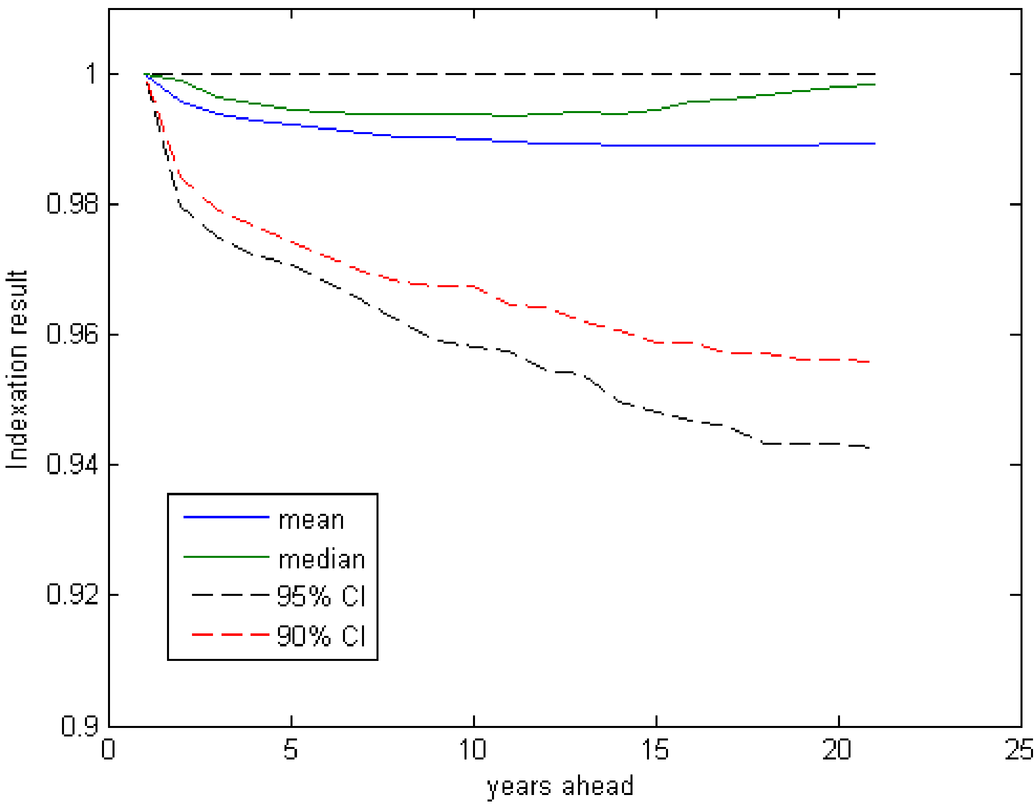


Figure C.2: Simulation result for the indexation level with the VAR model based on historical data between 1986 and 2010.

Table C.4: Indexation quality

Summary statistics of probability distributions of variables from the indexation quality where the VAR model is estimated on historical data between 1986 and 2010. The selected output variables are the median, mean and standard deviation of the indexation result (IR) after 1, 10 and 20 years. And the probability that the indexation result will drop below 100 % and 85 % respectively ($P(IR < 100)$ and $P(IR < 85)$).	
median IR_{t+1}	0.99
mean IR_{t+1}	0.99
median IR_{t+10}	0.99
mean IR_{t+10}	0.98
stdev IR_{t+10}	0.01
median IR_{t+20}	0.99
mean IR_{t+20}	0.98
stdev IR_{t+20}	0.02
$P(IR_{t+10} < 100)$	0.76
$P(IR_{t+20} < 100)$	0.64
$P(IR_{t+10} < 85)$	0.00
$P(IR_{t+20} < 85)$	0.00

Appendix D

VAR and Risk Premia tables

Table D.1: Summary statistics and VAR estimation results for the sample 1973:01-2010:08

Panel (a) provides summary statistics of the data, annualized means and standard deviations are provided for the sample 1973:01-2010:08. Panel (b) provides parameter estimates (B) of the VAR, the corresponding t-values are below. Panel (c) contains cross-correlations of the innovations with monthly standard deviations on the diagonal.

a) Summary statistics	(y^1)	(π)	(y^{120})	(x_s)	(dy)	
μ	5.20	2.74	6.56	2.72	2.98	
σ	2.82	1.14	1.95	15.40	1.14	
b) VAR estimates (B)	(y^1_t)	(π_t)	(y^{120}_t)	$(x_{s,t})$	(dy_t)	R^2
y^1_{t+1}	0.96	0.01	0.06	−0.00	−0.01	0.98
	(77.49)	(1.38)	(2.73)	(−1.33)	(−2.22)	
π_{t+1}	0.44	0.03	0.05	0.00	0.09	0.15
	(3.97)	(0.66)	(0.31)	(1.27)	(1.86)	
y^{120}_{t+1}	0.01	0.01	0.98	0.00	−0.00	0.98
	(1.83)	(1.84)	(87.36)	(1.45)	(−0.81)	
$x_{s,t+1}$	−3.27	−1.22	0.35	0.12	1.63	0.05
	(−2.09)	(−1.82)	(0.14)	(2.58)	(2.26)	
dy_{t+1}	0.01	0.01	0.02	−0.00	0.98	0.99
	(0.57)	(2.08)	(0.83)	(−2.96)	(129.60)	
c) VAR estimates	(y^1)	(π)	(y^{120})	(x_s)	(dy)	
y^1	0.03					
π	−0.01	0.30				
y^{120}	0.09	0.14	0.02			
x_s	−0.02	0.03	−0.10	4.33		
dy	0.07	−0.02	0.11	−0.85	0.05	

Table D.2: Estimation results term structure model for the sample 1973:01-2010:08

Panel (a) shows risk premia ($\Sigma\lambda_0$ and $\Sigma\lambda_1$) and panel (b) gives the annualized means and standard deviations of historical mispricing of the nominal term structure of interest rates.

a) Risk Premia (L)	(y^1)	(π)	(y^{120})	(x_s)	(dy)	(λ_0)
y^1	0.02	0.01	0.01	0.01	−0.01	−0.00
π	0.00	0.00	0.00	0.00	0.00	0.00
y^{120}	0.02	0.01	−0.03	0.00	−0.00	−0.01
x_s	−3.27	−1.22	0.35	0.12	1.63	0.40
dy	0.00	0.00	0.00	0.00	0.00	0.00
b) Mispricing term structure	(y^{12})	(y^{24})	(y^{36})	(y^{48})	(y^{60})	(y^{120})
μ	0.43	0.44	0.41	0.33	0.30	0.00
σ	0.63	0.57	0.52	0.41	0.30	0.00

Table D.3: Summary statistics and VAR estimation results for the sample 1986:06-2010:08

Panel (a) provides summary statistics of the data, annualized means and standard deviations are provided for the sample 1986:06-2010:08. Panel (b) provides parameter estimates (B) of the VAR, the corresponding t-values are below. Panel (c) contains cross-correlations of the innovations with monthly standard deviations on the diagonal.

a) Summary statistics	(y^1)	(π)	(y^{120})	(x_s)	(dy)	
μ	4.31	1.94	5.62	2.42	2.25	
σ	2.31	1.11	1.59	15.99	0.49	
b) VAR estimates (B)	(y^1_t)	(π_t)	(y^{120}_t)	($x_{s,t}$)	(dy_t)	R^2
y^1_{t+1}	0.98	0.00	0.04	−0.00	−0.03	0.99
	(95.46)	(0.25)	(2.84)	(−0.10)	(−4.03)	
π_{t+1}	0.67	−0.13	−0.27	0.01	−0.01	0.09
	(3.83)	(−2.27)	(−1.11)	(1.37)	(−0.10)	
y^{120}_{t+1}	0.01	0.00	0.99	0.00	−0.01	0.98
	(1.04)	(0.01)	(73.21)	(1.96)	(−1.29)	
$x_{s,t+1}$	−5.92	−0.18	5.10	0.13	2.82	0.04
	(−2.26)	(−0.21)	(1.38)	(2.11)	(1.58)	
dy_{t+1}	0.04	−0.00	−0.03	−0.00	0.96	0.95
	(2.07)	(−0.68)	(−1.13)	(−2.64)	(67.10)	
c) VAR estimates	(y^1)	(π)	(y^{120})	(x_s)	(dy)	
y^1	0.02					
π	0.01	0.30				
y^{120}	0.07	0.11	0.02			
x_s	0.01	0.06	0.01	4.52		
dy	0.02	−0.05	0.02	−0.89	0.04	

Table D.4: Estimation results term structure model for the sample 1986:06-2010:08

Panel (a) shows risk premia ($\Sigma\lambda_0$ and $\Sigma\lambda_1$) and panel (b) gives the annualized means and standard deviations of historical mispricing of the nominal term structure of interest rates.

a) Risk Premia (L)	(y^1)	(π)	(y^{120})	(x_s)	(dy)	(λ_0)
y^1	0.01	0.01	0.00	−0.00	−0.02	−0.01
π	0.00	0.00	0.00	0.00	0.00	0.00
y^{120}	0.02	0.01	−0.02	0.00	−0.01	−0.01
x_s	−5.92	−0.18	5.10	0.13	2.82	1.36
dy	0.00	0.00	0.00	0.00	0.00	0.00
b) Mispricing term structure	(y^{12})	(y^{24})	(y^{36})	(y^{48})	(y^{60})	(y^{120})
μ	0.36	0.35	0.33	0.26	0.22	0.02
σ	0.46	0.45	0.41	0.33	0.29	0.03

Table D.5: Estimation results term structure model for the sample 1986:06-2010:08, where the 15-year yield is added in the calibration

Panel (a) shows risk premia ($\Sigma\lambda_0$ and $\Sigma\lambda_1$) and panel (b) gives the annualized means and standard deviations of historical mispricing of the nominal term structure of interest rates.

a) Risk Premia (L)	(y^1)	(π)	(y^{120})	(x_s)	(dy)		(λ_0)
y^1	0.00	0.00	−0.00	−0.00	−0.01		−0.01
π	0.00	0.00	0.00	0.00	0.00		0.00
y^{120}	0.01	0.01	−0.01	−0.00	−0.01		−0.02
x_s	−5.92	−0.18	5.10	0.13	2.82		1.36
dy	0.00	0.00	0.00	0.00	0.00		0.00
b) Mispricing term structure	(y^{12})	(y^{24})	(y^{36})	(y^{48})	(y^{60})	(y^{120})	(y^{180})
μ	0.35	0.41	0.42	0.39	0.34	0.08	0.31
σ	0.45	0.52	0.53	0.49	0.43	0.09	0.37

Table D.6: Summary statistics and VAR estimation results for the sample 1986:06-2010:08, where the 15-year yield has replaced the 10-year yield in the VAR model

Panel (a) provides summary statistics of the data, annualized means and standard deviations are provided for the entire sample (1986:06-2010:08). Panel (b) provides parameter estimates (B) of the VAR, the corresponding t-values are below. Panel (c) contains cross-correlations of the innovations with monthly standard deviations on the diagonal.

a) Summary statistics	(y^1)	(π)	(y^{180})	(x_s)	(dy)	
μ	4.31	1.94	5.88	2.42	2.25	
σ	2.31	1.11	1.46	15.99	0.49	
b) VAR estimates (B)	(y^1_t)	(π_t)	(y^{180}_t)	$(x_{s,t})$	(dy_t)	R^2
y^1_{t+1}	0.99	0.00	0.03	0.00	−0.03	0.99
	(107.30)	(0.20)	(1.99)	(0.01)	(−4.00)	
π_{t+1}	0.65	−0.13	−0.27	0.01	−0.02	0.09
	(4.12)	(−2.22)	(−1.12)	(1.30)	(−0.17)	
y^{180}_{t+1}	0.01	−0.00	0.99	0.00	−0.01	0.98
	(1.43)	(−0.24)	(77.18)	(1.31)	(−0.99)	
$x_{s,t+1}$	−5.57	−0.17	5.26	0.12	2.77	0.04
	(−2.39)	(−0.19)	(1.49)	(2.10)	(1.56)	
dy_{t+1}	0.04	−0.00	−0.04	−0.00	0.96	0.95
	(2.30)	(−0.71)	(−1.34)	(−2.62)	(67.42)	
c) VAR estimates	(y^1)	(π)	(y^{180})	(x_s)	(dy)	
y^1	0.02					
π	0.00	0.31				
y^{180}	0.04	0.08	0.02			
x_s	0.01	0.07	−0.02	4.51		
dy	0.02	−0.05	0.05	−0.90	0.04	

Table D.7: Estimation results term structure model for the sample 1986:06-2010:08, where the 15-year yield has replaced the 10-year yield in the VAR model

Panel (a) shows risk premia ($\Sigma\lambda_0$ and $\Sigma\lambda_1$) and panel (b) gives the annualized means and standard deviations of historical mispricing of the nominal term structure of interest rates.

a) Risk Premia (L)	(y^1)	(π)	(y^{180})	(x_s)	(dy)		(λ_0)
y^1	0.00	−0.00	0.03	−0.00	−0.02		0.01
π	0.00	0.00	0.00	0.00	0.00		0.00
y^{180}	0.02	0.01	−0.03	−0.00	−0.01		0.00
x_s	−5.57	−0.17	5.26	0.12	2.77		1.36
dy	0.00	0.00	0.00	0.00	0.00		0.00
b) Mispricing term structure	(y^{12})	(y^{24})	(y^{36})	(y^{48})	(y^{60})	(y^{120})	(y^{180})
μ	0.34	0.43	0.50	0.56	0.60	0.47	0.04
σ	0.48	0.59	0.65	0.69	0.73	0.55	0.06