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The Impact of Illiquidity on the Capital Requirements of Insurers Facing Long-Term Liabilities in a Risk-Based Environment

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The impact of illiquidity on the capital requirements of insurers facing long-term liabilities in a risk-based environment

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Abstract

This thesis analyzes the impact of applying the matching adjustment as described in Solvency II to a stylized insurance company. To this end, this thesis uses a one-factor Cox-Ingersoll-Ross model in which the second factor is related to illiquidity. The average estimated illiquidity premium equals approximately 26bps on German Pfandbriefe and 102bps on the iBoxx € Corporates A 3-5 bond index. The impact on the present value of the liabilities ranges from 4-7%, depending on the height of the matching adjustment. This thesis also shows that an incorrect estimation of the illiquidity premium can pose severe risks to the policy holder in terms of lower premium payments.

Keywords: illiquidity, matching adjustment, annuities, Solvency II, long-term investments, insurance

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2 Introduction

The past years have shown a great deal of turbulence on the financial markets, of which a great part is believed to be attributable to illiquidity, as explained for instance by Brunnermeier (2009). Briefly said, liquidity refers to the ease of buying and selling assets in a market. Cash is often considered the most liquid asset, as for instance justified by the general equilibrium model by Starr (2003). Assets like corporate bonds or collateralized debt obligations are often seen as less liquid. In return for taking this liquidity risk, one can argue that investors require a risk premium on illiquid assets. This idea is supported by empirical research, which finds that the spread on risky assets relative to default-free bonds is often found to be higher than can be explained historical default rates alone (Ericsson and Renault (2006)). The remainder of this spread is then attributed to illiquidity. Other possible explanations for this higher yield could be due to regulatory and legal restrictions, incorrect estimation of the price of risk, default probabilities or other factors. Liquidity shocks can cause big problems to institutions which are funded with short-term money: in case liquidity reduces, the supply of short-term money drops and it is hard to sell off illiquid assets to cover the liabilities. A more thorough analysis on illiquidity is given in Section 3.

Because insurance companies, in particular their annuity portfolios, have a highly predictive nature and are focused on the long-term, they will not face the risk of illiquidity shocks because they can hold illiquid fixed-income assets until maturity. In this way, it would be possible to "capture" the illiquidity premium. This thesis focuses on the impact of illiquid assets on the capital requirements of these insurance companies. In particular, I pay attention to the current draft of the Solvency II regulations (EIOPA (2013)), which proposes the so-called "matching adjustment" allowing insurers to add a liquidity spread to the discount rate used for the valuation of their liabilities.

This thesis is written during an internship at Ernst & Young Actuarissen B.V. of which the main activities are supporting companies, pension funds, insurance companies, banks, asset managers and individuals with solving issues related to insurance services, risk management and employee benefits. Due to the new Solvency II regulations investing in illiquid assets becomes more interesting for insurance companies and as a result many parties in the market try to deal with related issues. As this matching adjustment can have a significant impact on the financial state of insurers, it is important to carefully analyze the possible consequences of this matching adjustment.

Correctly measuring the illiquidity premium is not straight-forward, since illiquidity in unobservable and one has to use of proxies instead. By means of two different methods described in Sections 3.1 and 3.3, I estimate an illiquidity premium on German Pfandbriefe (covered bonds) and the iBoxx Overall 3-5 years (corporate bond index). The results from this empirical exercise, as presented in Section 7.2.1 and 7.2.2 are used to assess the impact on a stylized insurance company, as described in Sections 5 and 6.

Using a stylized insurance company that has a portfolio of annuities and a stochastic process for the interest rate and illiquidity, this thesis analyzes the impact of illiquidity on the insurer, in particular the following questions are treated:

- *What is the impact on the capital requirements of insurers with long-term liabilities when they discount their liabilities with an illiquidity premium added to the default-free rate?*

Application of the matching adjustment results in a 3%-7% reduction in the value of the liabilities, as shown in Figure 7.9, which results in a higher net asset value. Because the solvency capital requirements are related to the net asset value, the solvency capital requirements increase, as descibed in Section 7.5. However, the decrease in the present value of the liabilities is higher than the increase of the SCR resulting in an increased free-surplus as seen in Figure 7.14. In the long-run, the capital requirements converge.

- *What are the differences between the impact on the regalutory capital according to Solvency II and the internal model?*

In Section 7.5, the illiquidity premium is assumed to be free of default-risk. This, in combination with a cash-flow matching strategy and fixed nominal liabilities, excludes the possibility that the insurer is unable to pay its policy holders. Hence, the capital requirements according to the internal model in the form of the VaR and tVaR equal zero and are therefore lower than the SCR prescribed by Solvency II.

- *Does the matching create incentives for insurancers?*

As shown by Chen, Lesmond, and Wei (2007); Han and Zhou (2011), there is a positive correlation between illiquidity and credit risk, i.e. assets with a lower credit rating are often less liquid. This positive correlation also comes back in the example calculations of the matching adjustments by EIOPA in Section 7.3. Therefore, there is an incentive for insurers to take more risk than they would normally do. To limit this behaviour, Solvency II requires the assets to be of investment-grade quality, as described in Section 4.

- *Are there risks resulting from adding an illiquidity premium to the default-free discount rate?*

In case the illiquidity premium exists, is estimated correctly and the insurer follows a cash-flow matching investment strategy, there is no exposure to illiquidity shocks because the illiquid assets are held to maturity (see Sections 7.4 and 7.5). This is under the assumption of deterministic liabilities. However, an incorrect estimation of the liquidity premium results in severe risk for the policy holder and insurer. Section 7.6, shows that in about 25% of the scenarios, the policy holder is faces at least one default resulting in a reduction of the payments of 20% in the period after the default.

This thesis starts with a review on the literature on illiquidity. Thereafter Solvency II regulations are discussed, followed by the underlying model for the liquid and illiquid bonds and the insurance company. The following chapter discusses the results and this thesis ends with a conclusion and recommendations for further research.

3 Illiquidity

Illiquidity in financial markets is a complex topic that receives much attention in the academic literature. This section provides a broad overview of different types of illiquidity, illiquidity in different asset classes, markets, time periods and ways of measuring illiquidity.

In general, the academic literature makes a distinction between two types of liquidity: market liquidity and trader's funding liquidity. Market liquidity refers to the ease of trading and trader's funding liquidity to the availability of funds for trading. These two types of illiquidity are not unrelated: traders provide market liquidity. The extent to which they can do so depends on their funding. The research by Pedersen and Brunnermeier (2007) shows empirically that in times of crises, a reduction in market liquidity and trader's funding liquidity enforce each other, leading to an illiquidity spiral. This thesis discusses market liquidity.

Certain situations might force an investor to sell assets. In case the investor has illiquid assets in his portfolio, he is exposed to the risk that he might need to sell this asset for a lower price than what he would have obtained in a frictionless market. Amihud and Mendelson (1986) follow this argumentation and conclude that to compensate for this liquidity risk, the investors require an additional return compared to liquid assets. In later research, Lo, Mamaysky, and Wang (2004) extend this rationale by stating that illiquidity also limits the frequency of trading which in turn prevents investors to hedge continuously. To compensate for this, investors require a lower price and hence a higher yield for an illiquid asset compared to a liquid asset.

Probably the most famous recent example of the effects of liquidity risk is the case of the collapse of Lehman Brothers in September 2008. They were funded using short-term money and invested is in illiquid long-term investments. When the supply of short-term money dried up, Lehman Brothers had great difficulties in selling their illiquid long-term investments. As a result financial obligations could not be met and the bank collapsed. A good overview of the collapse of Lehman Brothers is given by Swedberg (2010).

The losses an investor possibly incurs due to illiquidity, are referred to as illiquidity costs. To go deeper into the components of this illiquidity costs, one can look at *market micro-structure models*, which attempt to model the trading mechanism on a trade-by-trade basis.

Due to the complexity and multidimensional character of illiquidity, it is difficult to provide a comprehensive definition. Kyle (1985) identifies three dimensions of liquidity:

- tightness - the cost of turning around a position in a short period of time,
- depth - the size of an order required to change the price of an asset by a given amount and
- resilience - the speed with which prices recover from a random, information shock.

Amihud and Mendelson (2006) provide an overview of the components that affect liquidity costs:

- Price-impact costs: reflects the price concession that a buyer or a seller of a security makes when trading. For small orders this is limited to the bid-ask spread. In case of a larger order than the depth of the market, the order does not occur at the best quoted price.
- Search and delay costs: costs related to searching and waiting for a better quote, e.g. the opportunity costs and the risk of waiting for a better quote; the price of an asset could go up.
- Direct trading costs: including exchange fees, taxes and brokerage commission.

Overall, liquidity is a complex phenomena with different dimensions and costs associated with it.

3.1 Corporate Bonds

One of the asset classes for which there is a consensus that illiquidity influences the yields is the class of corporate bonds. From historical data, one can observe that corporate bonds usually have higher yields and hence lower prices than safer investments like US Treasury Bills[1]. The difference in yield between the default-free bonds and corporate bonds, the spread, has attracted much attention in the scientific literature. Merton (1974) developed a model to explain the components of the yields of corporate bonds. He argues that the yield consists of three parts:

1. default-free rate,
2. expected losses and
3. credit-risk premium.

The expected losses are the average expected losses in case the issuer of the bond defaults. The credit risk premium is the premium an investor requires for taking credit risk compared to investing in a default free bond. Since Merton (1974), much research has focused on the credit risk component of the spread. However, many academic studies show that the yield spread cannot be fully explained by the credit-risk premium and expected losses: there is some residual spread, which is then often assumed to be attributable to illiquidity.

Estimates of the liquidity premium widely vary, but there seems to be some sort of consensus that a liquidity premium exists for corporate bonds. One of the determinants of the size of the spread is the credit rating of the bonds: the

[1] However, recent events regarding the U.S. debt ceiling have shown a default on US Treasury Bills should not be excluded. On 15-10-2014 Fitch Ratings writes: "Although Fitch continues to believe that the debt ceiling will be raised soon, the political brinkmanship and reduced financing flexibility could increase the risk of a U.S. default.", see http://www.fitchratings.com/creditdesk/press_releases/detail.cfm?pr_id=805079 &origin=home&cm_sp=homepage-_-Featured_Content_Archive-_-Fitch%20Places %20United%20States'%20'AAA'%20on%20Rating%20Watch%20Negative.

higher the credit rating, the higher the liquidity premium. For instance Chen et al. (2007) find a premium of 8-25bp for AAA-rated bonds to a premium of 59-459bp of B-rated bonds. Their sample consists of 4000 US corporate bonds and they conduct their research using a regression-based approach and using the bid-ask spread and zero-return days as proxies for illiquidity, for the period 1993-2003. Han and Zhou (2011) find a spread of 32bps on AAA-rated bonds and 60bps on BBB-rated bonds, also in the case of US corporate bonds relative to the US Treasury rates for the period 2001-2007.

Dick-Nielsen, Feldhütter, and Lando (2012) investigate bond illiquidity in the years 2005-2009. In the onset of the sub-prime crisis, corporate spreads increased significantly. The increase is relatively slow for high-grade bonds and stronger for speculative grade bonds. Furthermore, bonds become less liquid during times of crises.

Bao, Pan, and Wang (2011) use a proxy for illiquidity which makes use of the fact that the lack of liquidity of an asset has a transitory effect on the price of the asset, the resilience. These transitory effects result in negatively serially autocorrelated prices. The negative autocorrelation between prices is therefore argued to be a suitable proxy for illiquidity. Using transaction data, they find that illiquidity:

- increases in time to maturity,
- is higher for bonds with smaller issuance,
- is increasing with age (on-the-run versus off-the-run),
- is not adequately captured by the bid-ask spread and
- accounts for a great part of the variation in yield-spread.

Characteristics as issued amount, coupon, listed age, missing prices, yield volatility, number of contributors and yield dispersion can be used as proxies for illiquidity, Houweling, Mentink, and Vorst (2005) reject the null-hypothesis for seven out of eight proxies that illiquidity is not priced in the market for corporate Euro bonds. Their estimates of the liquidity premia range from 9 to 24 basis points.

One can estimate the illiquidity premium on corporate bonds by creating a synthetically default-free bond from corporate bonds using credit default swaps (Hibbert, Kirchner, Kretzschmar, and Li (2009b)). The residual spread over a default-free rate is then assumed to be due to illiquidity:

$$l_t = \mathrm{Corporate\ Bond\ Yield_t} - \mathrm{Default\ Free_t} - \mathrm{CDS\ spread_t}. \tag{3.1}$$

In the ideal situation, one should therefore collect data on corporate bonds and then find the corresponding CDS spreads. Due to limited access to data, this method is not feasible for this thesis. Therefore, I use an index of corporate bonds and a CDS index, which contains credit default swaps similar bonds as contained by the corporate bond index, as also applied by Hibbert et al. (2009b).

The index of corporate bonds I use is the iBoxx € Corporates A 3-5, which contains investment grade corporate bonds. As an approximation for the corresponding credit default swap spreads, I use the iTraxx CDS 5 year index which consists of 125 equally-weighted credit default swaps on investment grade European corporate entities, including financials, non-financials and high volume. Since the credit default swaps are not fully in line with the bonds in the index in terms of underlying companies and times to maturity, there is a slight mismatch. As a default-free rate I use the ECB AAA curve, with 4 years to maturity. Using this data, the liquidity premium with Equation 3.1. The CDS-premium is assumed to be attributable to default-risk of the underlying assets, however counterparty risk and liquidity of the CDS itself could also play a role, as found by Bühler and Trapp (2009); Tang and Yan (2007).

3.2 Government bonds

Government bonds are in general assumed to be more liquid than corporate bonds. This is confirmed by empirical research, in which lower liquidity premiums are found compared to corporate bonds. The liquidity premium is estimated in the range of 2.5bp to 55bp for US Treasury notes. High illiquidity premia are for instance found by Warga (1992), who looks at US Treasury data from the years 1976-1976 and finds an illiquidity premium of 55bps between on-the-run and off-the-run bonds.

More recent research as for instance by Elton and Green (1998) finds a much smaller illiquidity premium, in this case around 2.5bps. A possible explanation is that the US Treasury market has become more liquid over the years.

Looking at European data, Codogno, Favero, and Missale (2003) show that the yield differentials on bonds in the Eurozone are mainly due to credit risk, and not due to illiquidity. This is show by means of a regression-based approach. Proxies as bid-ask spreads, volume and issuance are not found to have any influence. However, as they note themselves, they were not very successful at allowing for structural changes in the market and the fact that the proxies did not seem to have influence could be due to a data availability problem.

I have not found any empirical research on a possible illiquidity premium on German government Bunds. In the remainder of these thesis these Bunds are assumed to be fully-liquid and default-free.

3.3 Covered Bonds

Covered bonds are debt securities issued by financial institutions. Their cash flows originate for instance from a pool of mortgages or other loans. A difference with asset-backed securities is that the collateral remains on the issuer's balance sheet.

An example of such covered bonds are German Pfandbriefe. As for instance noted by Mastroeni (2002), the turn-over rate of the Pfandbrief is about half of the Government Bonds, a proxy of a lower liquidity for the Pfandbriefe. The liquidity premium on German Pfandbrief is estimated to be around 20bp by Koziol and Sauerbier (2007), using German government bonds as perfectly liquid securities. The higher yield for the Pfandbriefe relative to government securities is then attributed to illiquidity. The Pfandbriefe are considered safe investments because[2]:

- no default since 1901,
- stringent regulations,
- Pfandbrief investors have a preferential claim on the cover assets in case of insolvency and
- the collateral assets remain on bank's balance sheet.

To calculate an illiquidity premium, I assume both the bonds and the Pfandbriefe are equal and only differ in terms of liquidity. The liquidity premium at time t on a Pfandbrief with then equals

$$l_t = \mathrm{YieldPfandBrief_t} - \mathrm{YieldBund_t}. \tag{3.2}$$

More technical details can be found in Section 7.2.1 where I estimate a liquidity premium on German Pfandbriefe. The estimated illiquidity on German Pfandbriefe is used as the basis for analyzing the impact of illiquid assets on a stylized insurance company.

[2] Germany: Technical Note on the Future of German Mortgage-Backed Covered Bond (Pfandbrief) and Securitization Markets, IMF Country Report 11/369; July 2011

4 Solvency II

The goal of Solvency II is create one set of regulations to which all European insurance companies will have to comply. Currently each country has its own regulation with regard to how insurers have to discount their liabilities: for instance the Netherlands uses the ECB AAA curve with UFR extrapolation and the United Kingdom allows insurers to use an illiquidity premium when discounting, see Patel and Daykin (2010). One of the features of Solvency II is a shift towards market consistent valuation. The consequence for insurers is that their liabilities have to be discounted at the default-free rate, which results in a significant increase in the value of their liabilities in the current interest-rate environment. Many parties in the industry argue that this market consistent valuation is not "fair", because liabilities of insurers typically consist of long-term highly predictable cash flows and they therefore should not suffer from the consequences of (temporary) illiquidity shocks in the market. Insurers can simply hold the bonds until maturity and therefore do not face the risk of having to sell their assets at a discount in the market. Instead, they are argue they said to profit from the illiquidity premium. Therefore they argue that their liabilities should not be discounted at the default-free rate, but at the discount rate plus an illiquidity premium.

EIOPA (European Insurance and Occupational Pensions Authority) is an independent advisory body to the European Parliament, the Council of the European Union and the European Commission[3]. Their main objective is to safeguard the stability and effectiveness of the financial system .

The European Commission has requested EIOPA to provide quantitative input into the finalization of the Commission's final proposal for Solvency II, called Quantitative Impact Study, or QIS 5 (EIOPA (2010b)). The Long-Term Guarantee Assessments (EIOPA (2013)) or LTGA, published by EIOPA on 14-06-2013, has as main objective testing various approaches towards those Solvency II measures related to long-term guarantees provided by life insurers and, to the extent relevant, by non-life insurers. The main advises are:

- "Classical" matching adjustment - implement, see Section 4.1.
- "Extended" matching adjustment - exclude because of concerns regarding policy holder protection, competition and supervisability, see Section 4.2.
- Counter-cyclical premium - "Crisis measure to complement market observations when these are determined to be temporarily unreliable or unfit for the prudential purpose, due to spread-related crisis situations in financial markets." EIOPA advises to replace this by the volatility balancer (EIOPA (2013)).
- Extrapolation - convergence of long-term interest rate to a certain rate (Ultimate Forward Rate), EIOPA advises to implement this measure with a longer convergence period (e.g. 40 years) instead of 10 years for the Euro.

[3]http://europa.eu/legislation_summaries/internal_market/single_market_services/financial_services_general_framework/mi0070_en.htm

4.1 "Classical" Matching Adjustment

The "classical" matching adjustment allows insurers to add an illiquidity premium to the discount rate for their long-term liabilities. To apply the matching adjustment, both the assets and liabilities need to satisfy certain criteria. The requirements for the liabilities are (EIOPA (2010b)):

- The only underwriting risks connected to the contracts are longevity risk and expense risk.
- The undertaking does not bear any risk in case of any form of surrender.
- The premiums have already been paid and no incoming cash-flows are allowed for in the technical provisions of the contracts.

However, this idea is not without controversy, as one can find in "CEIPOS' Advice for Level 2 Implementation Measures on Solvency II: Technical Provisions - Article 86 b - Risk-free interest rate term structure" (CEIOPS (2009)):

"3.34 The inclusion of the illiquidity premium would lead to a significant decrease of technical provisions and would lower inappropriately the level of protection of policyholders. Among other reasons (see Annex A), there are concerns that the decreased value of technical provisions would not be sufficient to meet the insurance obligations because undertakings may not be able to earn the illiquidity premium in a risk-free manner in practice."

Technical results of the "Long-Term Guarantee Assessment" can be found from page 89 onwards of EIOPA (2013). The main results are:

- the credit-quality of the assets should be sufficient (investment grade),
- immaterial mortality risk should be permissible and
- separate management of the portfolio is necessary.

Also, LTGA states that a negative matching adjustment is permissible.

To calculate the matchign adjustmetn, first a fundamental spread is calculated for the asset as the maximum of 75% of the long-term average fundamental spread and the sum of the cost of downgrade and probability of default:

$$\text{Fundamental Spread} = \max\left(75\%\ \text{LTA},\ \text{Cost of DG} + \text{Probability of Default}\right). \tag{4.1}$$

The matching adjustment is then calculated as the excess of the observed spread and the fundamental spread:

$$\text{MA} = \text{Observed Spread} - \text{Fundamental Spread}. \tag{4.2}$$

EIOPA performed an analysis on applying the matching adjustment and summarized their results in a spreadsheet[4]. They analyzed the spreads of corporate

[4] See 20130125_EIOPA MA2b Fundamental spreads for corporates EUR.xls

bonds in different industry sectors for different credit ratings and different maturities. Section 7.3 summarizes results of this spreadsheet. In the remainder of this thesis, matching adjustment refers to the "classical" matching adjustment.

4.2 "Extended" Matching Adjustment

Whereas the "classical" matching adjustment is strict about the requirements for the liabilities in order to apply the adjustment, the "extended" matching adjustment is more flexible: the liabilities are allowed to be exposed to lapse risk, disability-morbidity risk and it is allowed that the contracts receive future premiums. Of course, exposure to immaterial mortality risk is allowed, just as in the "classical" version of the matching adjustment. To compensate for the additional risks, the application of the matching adjustment is limited, see EIOPA (2013). The main findings are:

- Negative impact on policyholder protection, less likely that liabilities are met without forced asset sale
- Negative incentives insurers: encourages to take inappropriate levels of liquidity risk and creates incentives to invest in too risky assets to gain a higher matching premium
- More assumptions and estimations, increases complexity, extra risks. This means it also becomes more complex for a supervisor to supervise adequately.

EIOPA's technical advise on the "extended" matching adjustment: exclude from LTG package.

4.3 Standard Formula

Insurers have the choice to either make their own internal model, or to use a standard formula to calculate their Solvency II solvency capital requirements (SCR). This subsection explains the basis and the assumptions behind this formula, for as far they are relevant to this thesis. The total SCR, consists of six individual risk components (Wilson-Bilik and Zimmerman (2011)):

- market risk
- counterparty default risk,
- life underwriting risk,
- health underwriting risk,
- non-life and
- operational risk.

The total SCR then is calculated by the following formula

$$SCR_{overall} = \sqrt{\sum_{i,j} \rho_{ij} \times SCR_i \times SCR_j} \tag{4.3}$$

where SCR_i is the solvency capital requirement of risk component i and ρ_{ij} is the correlation between SCR_i and SCR_j. The market risk component is of most important to this thesis. This market risk component involves six sub-risks:

- interest rate risk,
- equity risk,
- property risk,
- spread risk,
- currency risk and
- concentration risk.

The shocks applied to the interest rates for QIS5 are shown op page 31 of EIOPA (2010a), on p353 the correlations between the market risk factors can be found.

According to SCR.5.15, interest rate exists for all assets and liabilities for which the net asset value is sensitive to changes in the term structure. The Solvency requirements define two extreme scenarios, one for an increasing interest rate and one for a decreasing interest rate. For both scenarios, different shocks are applied to rates for different maturities. The shocks are stated in the Appendix in Table B.1. For example, to calculate the 15-year interest rate in case of the upward scenario: $r_{15}^{up} = r_{15} \times (1+0.33)$, where r_{15} is the current observed 15-year rate. Furthermore, absolute changes in the interest rates in both the up and down scenario should be at least one percentage point. In case the unstressed interest rate is below 1%, the interest rate in the downward scenario should assumed to be equal to zero (SCR.5.24).

4.4 Internal Model

As an alternative to the standard formula provided by Solvency II, I also employ an internal model in the form of the Value-at-Risk and tail-Value-at-Risk measures. These measures are defined as (Artzner, Delbaen, Eber, and Heath (1999)):

- Value at risk: given a confidence level $\alpha \in (0, 1)$, the VaR of the portfolio at confidence level α, is given by the smallest number l such that the probability that the net asset value $A_t - L_t$ is small than l is at most $(1 - \alpha)$. Mathematically:

$$\mathrm{VaR}_\alpha (A_t - L_t) = \inf \{ l \in \mathbb{R} : \ P(-(A_t - L_t) > l) \leq 1 - \alpha \} . \qquad (4.4)$$

- Tail Value at Risk: given a random variable $A_t - L_t$ which is the payoff of a portfolio at some future time and given a parameter $0 < \alpha < 1$ then the tail value at risk is defined by

$$tVaR_\alpha (A_t - L_t) = E [A_t - L_t | \mathrm{VaR}_\alpha (A_t - L_t)] . \qquad (4.5)$$

These risk measures are used in this thesis to calculate the capital requirements such that a certain level of α is satisfied.

5 The Market

This section describes the underlying assumptions of the bond-market dynamics. Moreover the presence of arbitrage and default-risk are treated.

5.1 Bonds

A term-structure describes the relationship between interest rates or bond yields and different maturities. For simulating bond prices, one therefore needs to assume a model for the term-structure. Here, I use the model proposed by Cox, Ingersoll Jr, and Ross (1985) model which is an extension of the Vasicek (1977) model. In addition to the Vasicek model, the CIR model excludes negative interest rates. For the prices of liquid bonds, I use the one-factor CIR model, which is driven by only the short-rate r_t. As discussed before, illiquid bonds are relatively cheaper than liquid bonds, that is, they have a higher yield and hence a lower price. Following Grinblatt (2001); Kempf and Uhrig-Homburg (2000), I use a two-factor CIR model in which the second factor is a state-variable related to illiquidity l_t. The yield for the illiquid bond, is then the sum of the short-rate and the illiquidity yield. The two state-variables are driven by the following mean-reversion diffusion processes:

$$\begin{aligned} dr_t &= \kappa_1(\theta_1 - r_t)dt + \sigma_1\sqrt{r_t}dW^{\mathbb{P}}_{1,t} \\ dl_t &= \kappa_2(\theta_2 - l_t)dt + \sigma_2\sqrt{l_t}dW^{\mathbb{P}}_{2,t} \end{aligned} \tag{5.1}$$

These are mean-reversion processes with long-term average θ_i. The parameterκ_i can be interpreted as the speed of mean-reversion and the volatility factors $\sigma_1\sqrt{r_t}$ and $\sigma_2\sqrt{l_t}$ exclude negative value of the state variables. As one can observe, as the state variable converges to zero, the volatility also converges to zero. Hence, the state variable will only be driven by the drift term, resulting in the state variable reverting to its mean again. Moreover, the volatility is increasing in the value of the state-process, the. $W^{\mathbb{P}}_{i,t}$ for $i = 1, 2$ are two independent standard Brownian motions under the real-world measure. The parameters for the processes for the state variables are assumed to be constant over time and strictly positive. As shown by Cox et al. (1985), $2k_i\theta_i > \sigma_i^2$ excludes a value of zero for state variable i.

Assuming prices of risk λ_i, Cox et al. (1985) also show that Equation 5.1 holds under an equivalent martingale measure:

$$\begin{aligned} dr_t &= \bar{\kappa}_1(\bar{\theta}_1 - x_{1,t})dt + \sigma_i\sqrt{x_{1,t}}dW^{\mathbb{Q}}_{1,t} \\ dl_t &= \bar{\kappa}_2(\bar{\theta}_2 - x_{2,t})dt + \sigma_2\sqrt{x_{2,t}}dW^{\mathbb{Q}}_{2,t} \end{aligned} \tag{5.2}$$

with $\bar{\kappa}_i = \kappa_i + \lambda_i$ and $\bar{\theta}_i = \frac{\kappa_i\theta_i}{\kappa_i+\lambda_i}$ for $i = 1, 2$. For this model, the price of the illiquid bond is given by

$$P^I(r_t, l_t, t, T) = p_2(l_t, t, T) \times p_1(r_t, t, T). \tag{5.3}$$

which is the product of the discount factors for both state variables. Due to the multiplicative character of the 2-factor CIR model, the price of the liquid bond is only based on the state-factor related to interest rate risk:

$$P^L(r, t, T) = p_1(r_t, t, T). \tag{5.4}$$

Furthermore we have

$$\begin{aligned} p_1(r_t, t, T) &= A_1(r_t, t, T)e^{-B_i(t,T)r_t} \\ p_2(l_t, t, T) &= A_2(t, T)e^{-B_i(t,T)l_t} \end{aligned} \tag{5.5}$$

with

$$A_i(t, T) = \left[\frac{2\gamma_i e^{(\gamma_i+\bar{\kappa})(T-t)/2}}{(\gamma_i + \bar{\kappa})(e^{\gamma_i(T-t)} - 1) + 2\gamma_i}\right]^{2\bar{\kappa}_i\bar{\theta}_i/\sigma_i^2} \tag{5.6}$$

$$B_i(t, T) = \frac{2(e^{\gamma_i(T-t)} - 1)}{(\gamma_i + \bar{\kappa})(e^{\gamma_i(T-t)} - 1) + 2\gamma_i} \tag{5.7}$$

$$\gamma_i = \sqrt{\bar{\kappa}^2 + 2\sigma_i^2}. \tag{5.8}$$

The yield of the two-factor CIR model equals

$$R_t = r_t + l_t. \tag{5.9}$$

Appendix A shows the derivation of this pricing formula in an arbitrage-free way. However, when including the extra state variable for illiquidity, this means there will not be liquid bond anymore: simultaneously allowing an illiquid and liquid bond in this setting constitutes to arbitrage, because there are two bonds, with the same payoff but different prices. By selling the more expensive liquid bond and buying the cheaper illiquid bond, the investor could lock-in a guaranteed profit. More on this in Section 5.2.

As Cox et al. (1985) explains, bond prices are an increasing function of the market price of risk, mainly because a high value of λ_i indicates a high covariance of interest rate with wealth. Hence with large λ_i it is more likely that bond prices will be higher when wealth is low, and hence has greater marginal utility.

5.2 Arbitrage

To see that this market is not free of arbitrage, let us first recap the definition of arbitrage Schumacher (2012). Let V_t be the value of a self-financing portfolio at time $t \geq 0$ with $V_0 = 0$. This portfolio constitutes to arbitrage if at some time $T > 0$ the conditions

$$\begin{aligned} P(V_T \geq 0) = 1 \\ P(V_T > 0) > 0 \end{aligned} \tag{5.10}$$

hold. In other words, an arbitrage is an investment strategy which requires zero initial investment, generates non-negative cash flows with a positive probability that at least one of the cash flows is positive.

In our setting, an arbitrage opportunity can be created by shorting the more expensive liquid bond and buying the cheaper illiquid bond. The holdings in the bonds such that $V_0 = 0$ is given by

$$V_0 = \phi_0' Y_0 = \left[\begin{array}{cc} \phi_0^L & \phi_0^I \end{array}\right] \left[\begin{array}{c} p_1(r_0, 0, T) \\ p_1(r_0, 0, T) \times p_2(l_0, 0, T) \end{array}\right] = 0 \tag{5.11}$$

by shorting one liquid bond, the portfolio holdings are given by $\phi_0' = \left[\begin{array}{cc} -1 & \frac{1}{p_2(l_0,0,T)} \end{array}\right]'$. At time T both bonds expire and hence the payoff is $V_T = \phi_0' Y_T = \phi_0' \left[\begin{array}{c} 1 \\ 1 \end{array}\right] = -1 + \frac{1}{p_2(l_0,0,T)} \geq 0 \iff \frac{1}{p_2(l_0,0,T)} \geq 1 \iff p_2(l_0, 0, T) \leq 1$. Because $p_2(l_0, 0, T) \geq 0$, the condition $P(V_T \geq 0) = 1$ is satisfied. Because there is a positive probability that the state variable for illiquidity is positive, or even with probability 1 in case $2k_2\theta_2 > \sigma_2^2$, the second condition $P(V_T > 0) > 0$ is also satisfied, and hence this portfolio allows for an arbitrage strategy. One ad hoc solution to exclude arbitrage is by excluding short positions in bonds.

Having a model with arbitrage opportunities, is from a traditional financial point of view not a desirable property. However, I have not been able to find a model which allows for the a model with both a liquid and illiquid bond and excludes arbitrage at the same time. However, the presence of arbitrage does not cause any issues for the analyses in this thesis, since the insurer only has long positions in the bonds due to the cash-flow matching strategy. In practice, limits on short-selling of the bonds or other market frictions might reduce or eliminate the arbitrage opportunities in reality and prevent the insurance company to execute such strategy.

5.3 Default-Risk

Until here, the assumption has been that the yield of the illiquid bonds is the sum of the default-free rate and an illiquidity yield. This section however assumes that the liquidity premium is actually a compensation for default-risk. The market still assumes that the illiquidity premium exists and is default-free. Therefore the dynamics presented in the previous section still hold. As shown by Duffie and Singleton (1999), a defaultable bond can be priced as if it were default-free by replacing the short-term interest rate with the default-adjusted short-rate:

$$R_t = r_t + (1 + \zeta) h_t, \tag{5.12}$$

in other words, the illiquidity premium is now a compensation for default risk. h_t is the risk-neutral probability of default at time t and $\zeta = 0.8$[5] the fixed recovery

[5] Approximately the recovery rate of Senior Bonds in 2007, see https://www.moodys.com/sites/products/DefaultResearch/2007000000474979.pdf page 9

rate. Hence I still use the dynamics as described in the previous section, but make the assumption that

$$h_t = \frac{l_t}{1-\zeta} \tag{5.13}$$

is the time-varying default-rate. In case of a default, the value of the portfolio reduces by $1-\zeta = 1-.80 = 20\%$. This shock is distributed over all future years equally, by reducing the value of bonds with all different times to maturities with this same percentage. The random variable indicating a default at time t is X_t :

$$X_t \sim \text{Bern}\,(h_t) \tag{5.14}$$

where Bern stands for a Bernoulli random variable which equals 1 in case of a default and 0 otherwise. As shown by Duffie and Singleton (1999) this is a close approximation for the continuous time case. Hence, with these adjustments the portfolio of the insurer is now exposed to default-risk.

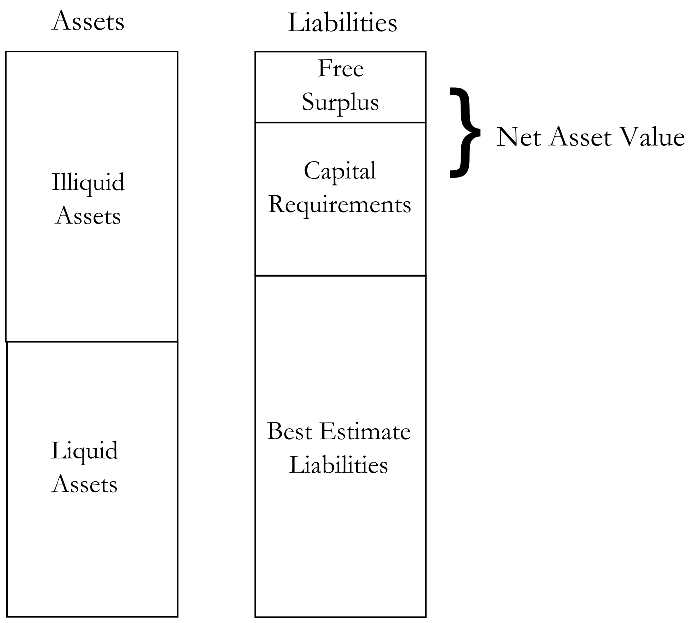

Figure 6.1: Balance sheet of stylized insurance company

6 The Insurer

This section describes the stylized insurance company which is used to analyze the impact of the matching adjustment on. The balance sheet of this simplified insurance company is shown in Figure 6.1 :

The best estimate liabilities, is the sum of the discounted expected future cash flows resulting from the insurance contracts . The net asset value, which equals the assets minus the liabilities, contains the capital requirement: the capital the insurer has to hold to compensate for fluctuations on the financial markets. The free-surplus can be distributed to the shareholders by means of dividend.

The liabilities of the insurer consist of the promises made to the policyholders. The stylized insurance portfolio I use consists of direct annuity contracts, that pay out €1,000 at the end of each year starting at the age of 55 until the policy holder dies, or reaches the age of 95. In total, the insurance company has 1,000 of these contracts, all held by males of the age of 55 at $t = 0$. The insurance company has perfect foresight with respect to when the policy holders will die and hence the nominal payments are already fully known ex ante. The annuity payments are calculated using the "AG Prognosetafel 2012-2062", the most recent mortality table published by the Dutch Actuarial Association. Moreover, all participants have already paid their premiums at the start of the time period. Figure 6.2 shows the nominal cash flow in each year. As expected, the cash flows to the participants are reducing over the years due to mortality of the policy holders.

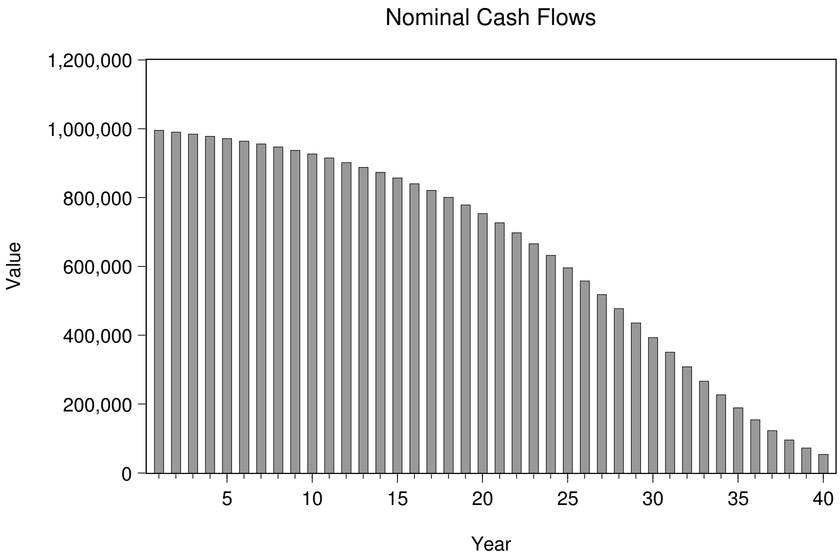

Figure 6.2: Future cash flows of the insurance portfolio, consisting of 1,000 males aged 55. The insurance product is a direct annuity, paying €1,000 at the end of each year as long as the policy holder is alive, with a maximum of 40 years.

The insurer has the following investment possibilities:

- liquid default-free bonds (zero-coupon) and
- illiquid bonds, using the calibration results from the German Pfandbriefe data (zero-coupon).

In case the insurer uses cash-flow matching, the initial value of the assets is set equal to the present value of the liabilities without applying the matching adjustment.

7 Results

This section provides the results of the calibration of the default-free rate, estimates of illiquidity premia, the impact on the insurance company and potential risks for the policy holder.

7.1 Default-Free Rate

A truly default-free rate is unobservable, since all investments carry some form of risk. Hence, in practice one has to resort to a proxy for the default-free rate. Examples of assets often assumed to be default-free are short-term US Treasury Notes or the overnight swap rates.

The CIR-model is calibrated using the short-rate. The ECB AAA yields, which is composed of bonds issued by European countries with a AAA rating, seems an obvious choice. However, only historical data since 2004 is available. Therefore, the choice is made to use data on German government bonds since the end of 1993. Both yields are plotted in Figure 7.1. As one can see, from 2004 until the end of 2008, there is hardly any spread between the ECB AAA and the German ond yield-curve. In the period after, the German Bond yield is lower which would imply that either the German bonds are safer or more liquid. In any case, the bigger dataset of the German bonds is beneficial for the calibration process and therefore the German government bonds with a 1-year maturity are used to calibrate the default-free rate. Comparable research that estimates the illiquidity premium on German Pfandbriefe also uses the German Bunds as default-free and fully liquid bonds, see for instance Kempf and Uhrig-Homburg (2000); Kempf, Korn, and Uhrig-Homburg (2012); Koziol and Sauerbier (2007).

	German Government Bund Yields			
	1-Year	5-Year	10-Year	15-Year
Mean (bps)	269.0	367.3	432.0	366.3
Median (bps)	311.7	377.9	420.6	376.1
Maximum (bps)	593.0	738.3	775.5	547.7
Minimum (bps)	-7.4	24.0	116.5	161.9
Std. Dev. (bps)	148.3	154.7	146.6	113.5
Skewness	0.0	-0.2	-0.4	0.3
Kurtosis	2.7	1.8	2.1	2.6
Observations	4,708	5,069	5,069	2,033
First Observation	10-01-1995	10-08-1993	10-08-1993	16-02-2001

Table 7.1: Summary statistics yields (bps) 1, 5, 10 and 15 years German Bunds, starting at different dates until 07-08-2013.

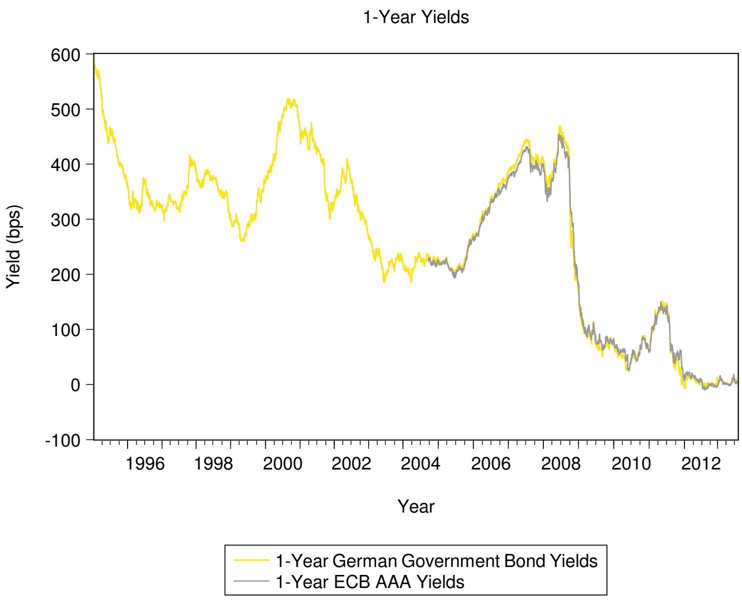

Figure 7.1: Daily yields (bps) on the German 1-year government bonds starting at 10-1-1995 until 7-8-2013 and 1-year ECB AAA yields starting at 6-9-2004 until 7-8-2013 on a daily basis.

Table 7.1 shows an overview of the summary statistics of yields of German government bonds with different maturities. The average yields are increasing with time to maturity. The average 15-year yields are lower than for shorter maturities, but this is because the sample period for this series only starts in 2001. As Figure 7.2 shows, the yields are higher for higher maturities.

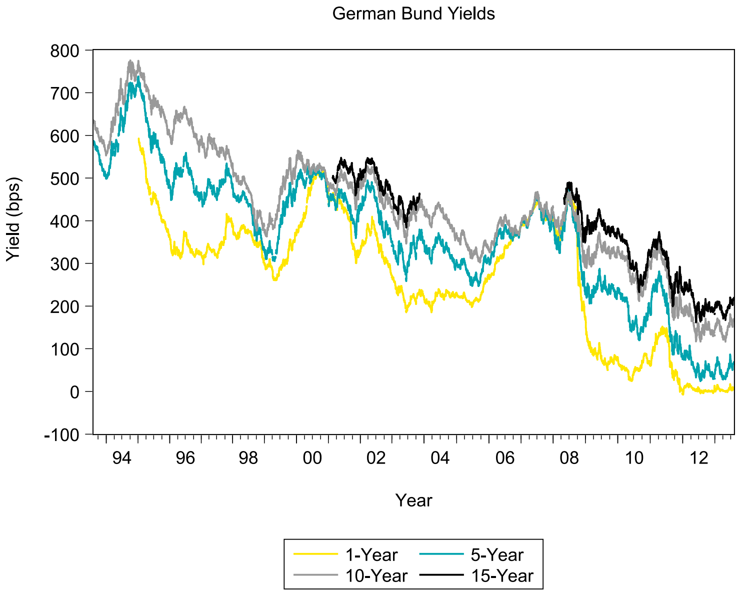

Figure 7.2: 1, 5 and 10-year German government bond yields.

Table 7.2 shows the results of the calibration procedure, after filtering out negative yields. For technical details on the calibration procedure I refer to Appendix D. The volatility is comparable to Kempf and Uhrig-Homburg (2000), the long-term mean is slightly lower and the speed of mean-reversion is about half of what they find. However, their sample period ranges from 1992 to 1994 which could account for the differences.

	Short-rate
Speed of mean-reversion	0.227
Long-term mean	0.041
Volatility	0.058
Log-likelihood	1.213

Table 7.2: Results of the calibration of the CIR using the 1-year German Bunds as short-rate.

The second step in the calibration procedure is fitting the implied yield-curve of the risk-neutral process to the observed yield curve by means of the market price of interest rate risk λ_1. This market price of risk determines whether the yields for long-term maturities are higher or lower than the average short rate. In case the (here the 5, 10 and 15-year) rate is higher than the average short-rate the market price of risk is positive and hence λ_1 is negative. This market price of interest rate risk can be estimated by minimizing the squared error between the price of the bond implied by the using the short-rate and the observed price of the actual 5, 10 and 15-year bond for each day. Table 7.3 shows the result of the fixed market price of interest rate risk that minimizes the squared-error between the yields implied by the model and the yields observed in the market.

Calibration Market Price of Interest Rate Risk	
λ_1	-0.095
Number of observations	14,846

Table 7.3: Results of calibration of the market price of interest rate risk using 5, 10 and 15-year German government bonds.

Figure 7.3 shows an example of the risk-neutral yield curve and estimated real-world yield-curve after taking the market price of risk into account. As one can see, the spread between the observed curve and the risk-neutral yield-curve is increasing over time to maturity, due to the negative λ_1, implying that investors require a higher risk premium on bonds with a longer maturity.

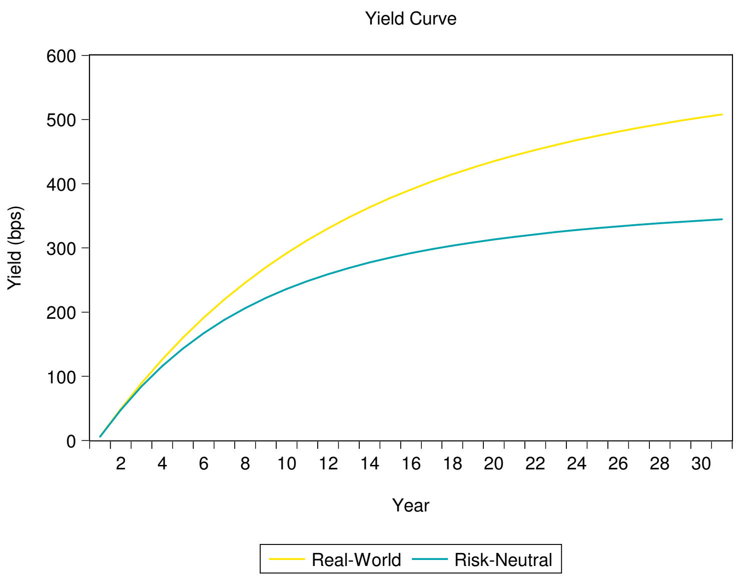

Figure 7.3: Example yield-curve (bps) using the results obtained in the calibration described in this section and using the short-rate of 07-08-2013, equal to 0.06%.

Appendix C.1 shows the estimation of the market price of interest rate risk on a daily basis.

7.2 Estimating an Illiquidity Premium

7.2.1 Direct Method

This section estimates an illiquidity premium using two assets, namely German Pfandbriefe (covered bonds) and German government bonds. The yield spread between the two is assumed to be the illiquidity spread, as described in Section 3.3. Figures 7.2 and 7.4 show respectively the bond yields and the Pfandbrief yields for maturities of 1, 5, 10 and 15 years. As one can observe, the yields on both assets are almost identical. Since the start of the dataset the yields have on average been decreasing and are currently at historically low levels. Also, rates for longer maturities are in general higher than for shorter maturities.

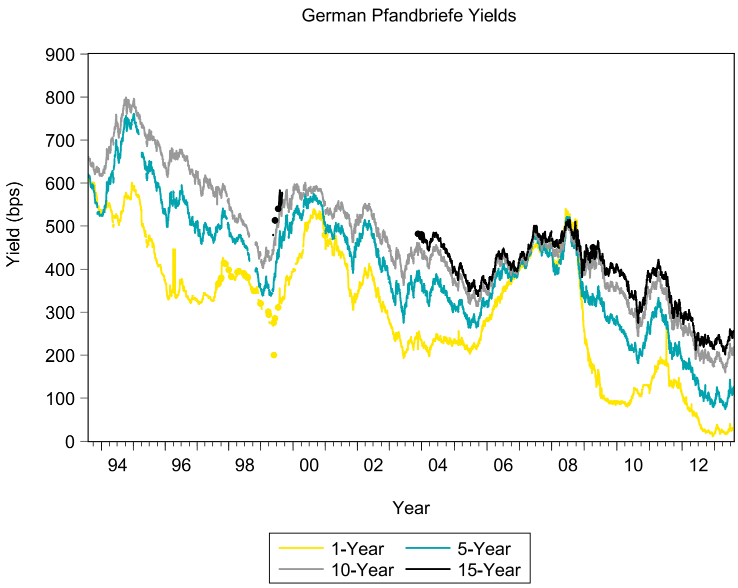

Figure 7.4: 1, 5 and 10-year German AAA Pfandbrief yields (bps).

	Liquidity Premium			
	1-Year	5-Year	10-Year	15-Year
Mean (bps)	26.4	38.3	35.8	42.4
Median (bps)	14.4	33.5	32.6	41.3
Maximum (bps)	261.7	115.9	108.6	106.2
Minimum (bps)	-11.9	-9.9	-8.5	12.9
Std. Dev. (bps)	28.6	23.8	19.6	14.1
Skewness	2.1	1.1	1.1	0.9
Kurtosis	8.8	3.9	4.1	4.8
Observations	4437	4938	1327	4970

Table 7.4: Summary statistics of illiquidity premia for Pfandbriefe (bps).

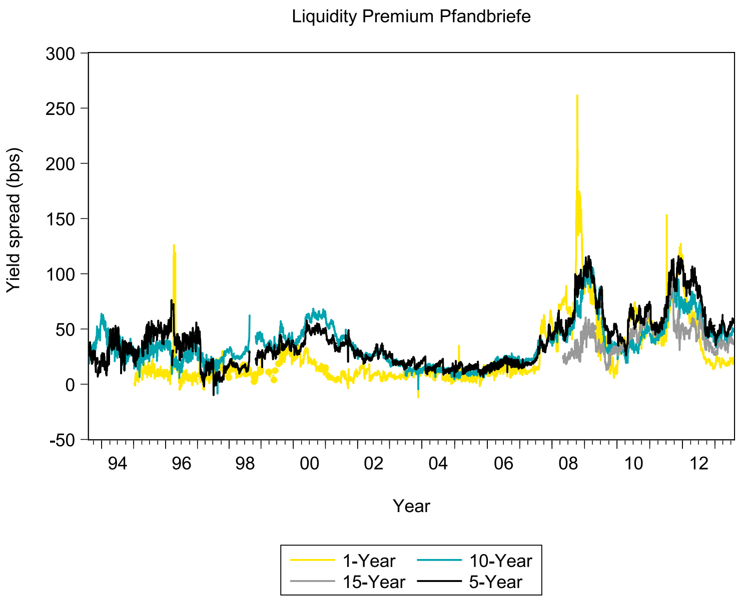

Figure 7.5: 1, 5 and 10-year liquidity premium (bps).

Figure 7.5 shows the difference between the two yields, which in this case is assumed to be attributable to illiquidity. The 1-year illiquidity premium shows higher spikes than for higher maturities. A very distinctive peak is observable in the end of 2008 during the onset of the credit crunch. The illiquidity premia for 5, 10 and 15 years do not exhibit clear spikes as the 1-year premium, but do show significant increases during the credit crunch and Euro crisis. The results are comparable to the results found by Hibbert et al. (2009b), who find a liquidity premium of around 90bps-130bps on the EUR AAA Composite bond index at the height of the credit crunch. They did not find a distinctive peak for the 1-year illiquidity premium, which might be due to the use of different covered bonds. They also find negative illiquidity premia in the period before.

Table 7.4 shows the summary statistics for the liquidity premia on Pfandbriefe with a maturity of 1, 5, 10 and 15 years. The average illiquidity premium is approximately 26bps, 38bps, 42bps and 36bps respectively for the 1, 5, 10 and 15-years to maturity. A striking feature is that the standard deviation is decreasing in maturity, indicating a more volatile liquidity premium for shorter maturities. Also, the 15-year illiquidity premium is on average lower than the 10-year illiquidity premium. The liquidity premium is only calculated for date on which data on both the Pfandbriefe and Bund yields were available. Moreover, negative values for the Bund yields have been excluded.

Table 7.5 shows the results of the calibration of the liquidity premia for the Pfandbriefe using the 1-year to maturity data. The estimated long-term mean (27bps) is slightly higher than observed in Table 7.4, this is due to the exclusion of negative values in the calibration procedure. The parameters are estimated using maximum likelihood as described in more detail in Appendix D.2.

	Illiquidity Premium
Speed of mean-reversion	8.793
Long-term mean	0.0027
Volatility	0.167
Log-likelihood	1.756

Table 7.5: Results of the calibration of the CIR process using the liquidity premium estimated using the negative CDS basis approach.

In the calibration of the second factor of the CIR model, I first subtract the yield from the 5,10 or 15-year German government bonds from the corresponding Pfandbriefe yields. To this data series, I apply the same calibration method as for the German Bunds: first using maximum likelihood to estimate the mean, speed of mean-reversion and volatility using the 1-year yields and then finding the market price of liquidity risk by minimizing the sum of squared errors, as described in Appendix D. The results of the calibration of the market price of liquidity risk is shown in Table 7.6.

Calibration Market Price of Liquidity Risk	
λ_2	-0.116
Number of observations	11,754

Table 7.6: Results of calibration of the market price of interest rate risk using 1-year German government bonds.

Figure 7.6 shows an example of the risk-neutral yield curve and estimated real-world, similar to Figure 7.3, but now with the yield-curve of illiquid bonds added to it. As one can see, the liquidity yield is relatively constant over time close to its long-term mean. But the decrease in illiquidity premium between 10 and 15-years cannot be captured by CIR-model, as a result, the best fit

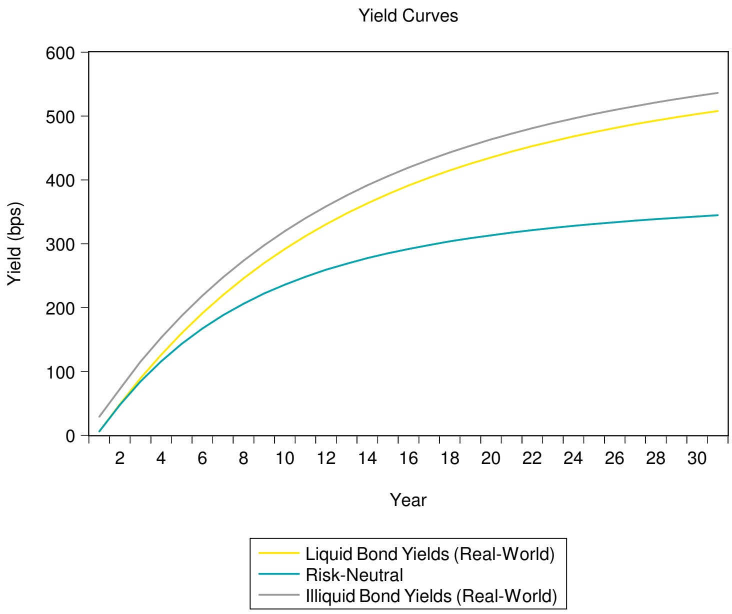

Figure 7.6: Example yield-curve (bps) using the results obtained in the calibration described in this section and using the short-rate of 07-08-2013, equal to 0.06%, now with illiquidity premium.

is a liquidity premium which is relatively constant over the different yield to maturities. Similar as for the market price of interest rate risk, a negative λ_2 implies a positive market price of illiquidity risk. Due to the relatively high amount of available data, estimates on the illiquidity premium in this section are used in the analysis of illiquidity on the stylized insurance company.

7.2.2 Negative CDS Basis Approach

Alternatively, the negative CDS basis approach can be used to estimate an illiquidity premium for corporate bonds, this method is described in Section 3.1. Data is obtained for 4-year ECB AAA yields (default-free), iBoxx € Corporates A 3-5 years (corporate bond index) and the iTraxx Eur CDSI Gen 5Y Corp (CDS index). The sample period starts on 22-06-2004 and ends on 06-08-2013. For the yields on the corporate bonds, daily data is only available from 16-04-2008 onwards. before that. only weekly data is available. As one can see in Figure 7.7, the spread between the ECB AAA yield and the iBoxx yield are very small from the beginning of the sample period until the end of 2007, with the highest yields around the fall of Lehman Brothers in September 2008. This increase in spread also corresponds to increasing swap spreads. These higher swap spreads and bigger yield spread for the corporate bond continue until the end of the sample period.

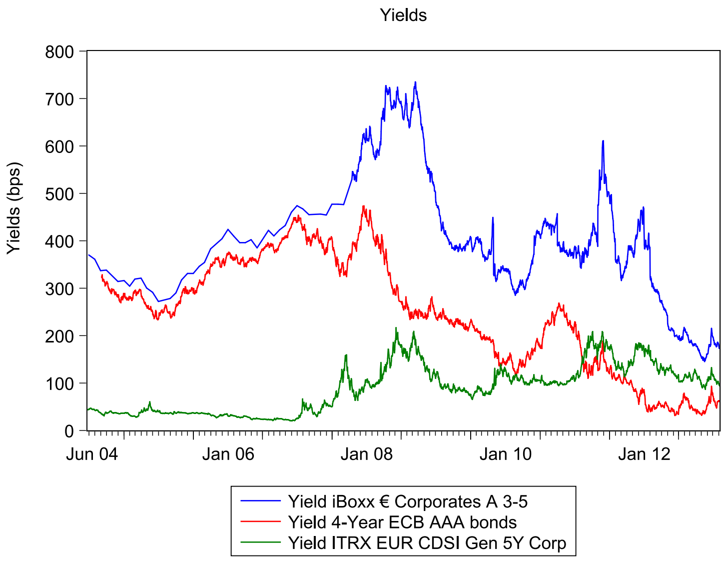

Figure 7.7: Yields on the iBoxx € Corporates A 3-5 corporate bond index, iTraxx CDS 5 year index and the ECB AAA 4-year yields (bps).

Figure 7.8 shows the liquidity premium estimated using Equation 3.1. Before the credit crisis, the liquidity premium fluctuates around zero. Also negative values occur, possibly explained by low liquidity of the CDS market, as argued by Bühler and Trapp (2009). Other possible explanations are the mismatch between the CDS index and the corporate bond index, or even other factors influencing the spreads which are not considered here. From the end of 2007 onwards, the liquidity premium is increasing with a peak around the fall of Lehman Brothers and during the Euro crisis.

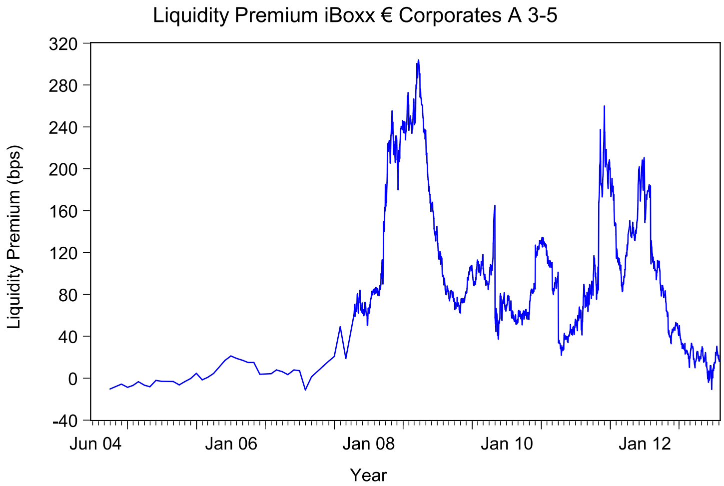

Figure 7.8: Liquidity premium (bps) using the negative CDS basis approach for the period 22-06-2004 until 06-08-2013.

Table 7.7 shows the summary statistics for the time series of the liquidity premium. For the total time period, the average liquidity premium is estimated to be around 102bps. For the period 01-2000 - 12-2001, Koziol and Sauerbier (2007) find a premium of 20bps, a value comparable to the average liquidity premium before the start of the credit crunch. The results are comparable to the results found by Hibbert et al. (2009b), who find a liquidity premium of around 300bps at the height of the credit crunch. They also find negative illiquidity premia in the period before.

Liquidity Premium	
Mean	102.1
Median	84.8
Maximum	303.9
Minimum	-11.3
Std. Dev.	67.6
Skewness	0.9
Kurtosis	3.1
Observations	1393

Table 7.7: Summary statistics liquidity premium (bps) calculated by subtracting the ECB AAA rate and CDS spread from the yields of the iBoxx € Corporates A 3-5 fund for the period of 22-06-2004 and ends on 06-08-2013. For the period prior to 2008, only weekly data is available.

Because only daily data is available from 2008 onwards, and the period thereafter is a uniquely turbulent one, I have decided to not use the illiquidity premium estimated in this section for analyzing the impact on the stylized insurance company.

7.3 Example EIOPA

EIOPA performed an analysis on applying the matching adjustment and summarized their results in a spreadsheet[6], as described in Section 4.1. This section provides an overview of these results.

Table 7.8 shows the average matching adjustment for the years 2004, 2009, 2011 and the average over these years. In line with results from Section 3.1, the illiquidity premium is higher for bonds with a lower credit rating. Also, in the year 2004, negative matching adjustments are observed. As also noted in EIOPA (2013), a consequence of this methodology is that the matching adjustment can turn negative in periods when assets are over-valued compared to their credit-risk, as in the run-up to the credit crunch. As sla (2012) argue, this should not be a problem, since in those cases the increase in the present value of the liabilities is compensated for by an increase in the value of the assets as well.

Average MA per Credit Rating				
	2004	2009	2011	Average
AAA	5.7	52.7	87.3	48.6
AA	-10.1	57.5	124.9	57.4
A	-38.1	72.8	173.8	69.5

Table 7.8: Average matching adjustment (bps) for corporate bonds in the years 20004, 2009 and 2011 and the average over the years for the credit ratings AAA, AA and A.

Table 7.9 shows the average matching adjustment for 2004, 2009, 2011 and the averages for different times to maturity. The matching adjustment for bonds with a 15+ residual time to maturity is lower then for the 10-15 years time to maturity. This is in line with the results I obtained in Section 7.2.1. Just as in the previous table, we see that the matching adjustment has been increasing over time.

Average MA for Different Residual Maturities				
	2004	2009	2011	Average
<5	-24.8	54.4	112.0	47.2
[5,10)	-11.5	69.7	143.0	67.1
[10,15)	-14.6	63.6	171.4	73.5
>15	-29.6	68.1	133.5	57.3

Table 7.9: Average matching adjustment (bps) for corporate bonds in the years 20004, 2009 and 2011 and the average over the years for the for different time to maturities.

[6] See 20130125_EIOPA MA2b Fundamental spreads for corporates EUR.xls

7.4 Impact on Liabilities

This section presents the results of the analysis of the matching adjustment on the stylized insurance company described in Section 6. The main assumptions are the following:

- The dynamics of the matching adjustment are based on the empirical estimates of the illiquidity premium on German Pfandbriefe and using example calculations by EIOPA.
- The insurer uses a cash flow matching strategy. i.e. the initial bond portfolio is chosen such that the cash flows for future years are matched. The insurer invests fully in illiquid assets such that the present value of the policy payments in each year is equal the the present value of the liabilities without applying the matching adjustment. Because the illiquid bonds are slightly cheaper than their liquid counterpart, the cash flows resulting from these illiquid bonds is slightly higher than the amount that needs to be paid to the policy holders.
- The insurer fully invests in illiquid assets.
- Any residual cash flows are invested in 1-year liquid bonds.

As seen in Section 7.2.1, the estimated illiquidity premium of the German Pfandbriefe is on average between 26bps and 38bps depending on the time to maturity. These values are slightly lower than the averages calculated by EIOPA as seen in Section 7.3.

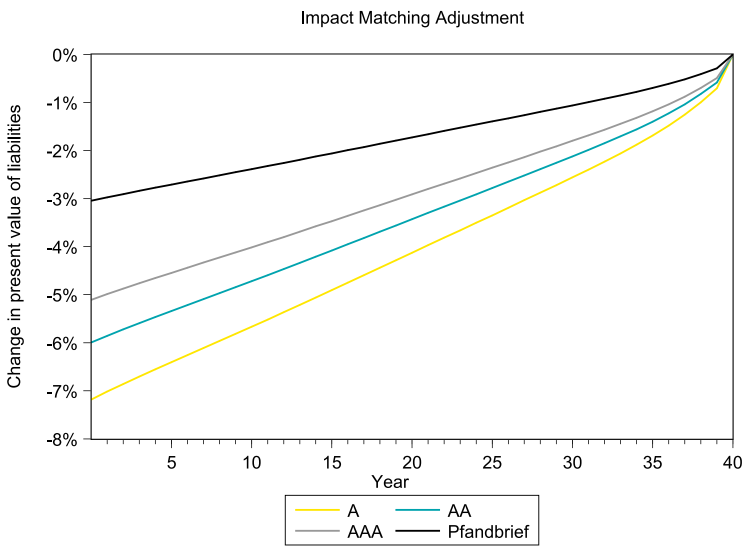

Figure 7.9: Average impact for different sizes of the matching adjustment.

Figure 7.9 shows that the impact of the matching adjustment ranges from approximately 3% to 7% at $t = 0$. These values can be approximated by multiplying the duration (about 10 at the start) of the insurance portfolio by the percentage impact on the discount curve. This means that if the insurer invested in 2011 in A-rated illiquid bonds, the impact of the matching adjustment would approximately be $10 * 1.738\% = 17.38\%$, hence in times of crises the matching adjustment can have a higher impact than implied by the average impact. Over time the impact of the matching adjustment reduces, because the duration of the liabilities also reduces.

The development of the average value of the assets and liabilities after applying the matching adjustment is shown in Figure 7.10. Because the liabilities are reduced in terms of present value because of the matching adjustment, the insurer ends up with residual assets of approximately €2.8M.

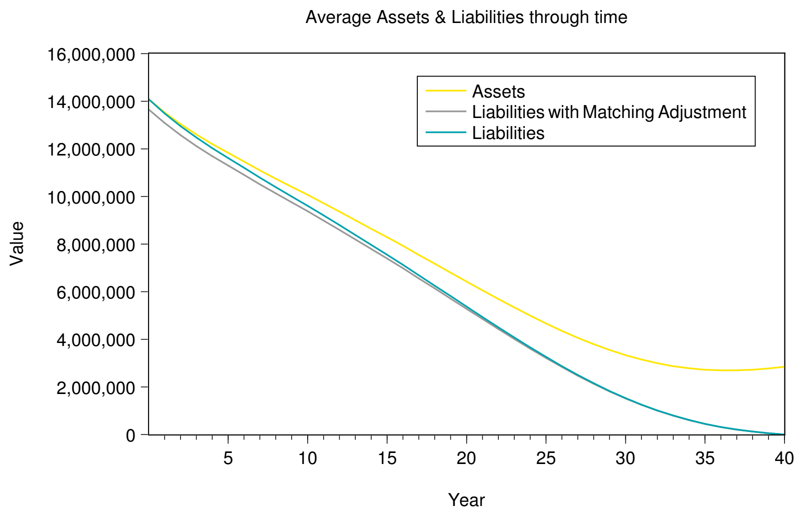

Figure 7.10: Development of the average value of the assets and liabilities through time, with and without applying the matching adjustment.

The development of the net asset value, the difference between the value of the assets and liabilities is shown in Figures 7.11 and 7.12.

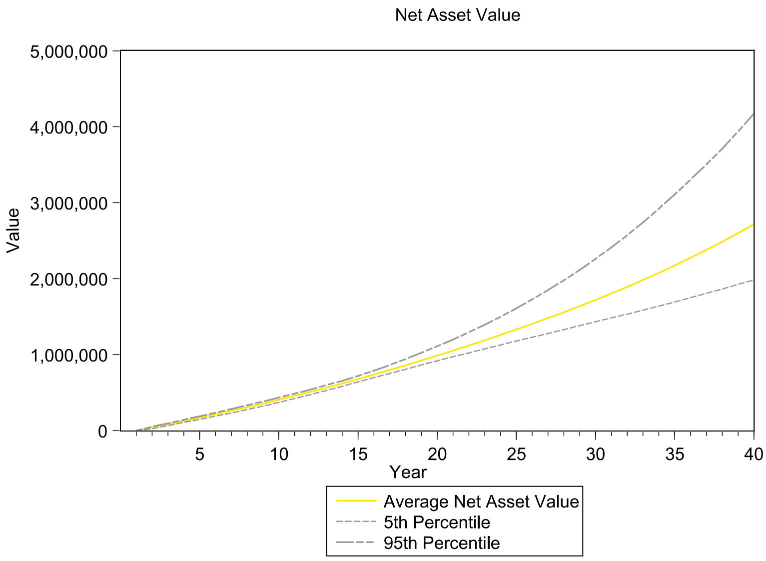

Figure 7.11: The 5th, 95th percentile and the mean of the net asset value through time without matching adjustment.

The net asset value equals zero in case of no matching adjustment, which is one of the assumptions made. In case of the matching adjustment, there is a positive net asset value. In both cases, the net asset value in all scenarios converges to the same value. This is because the investment strategy is the same in both cases and there are no liabilities in $t = 40$.

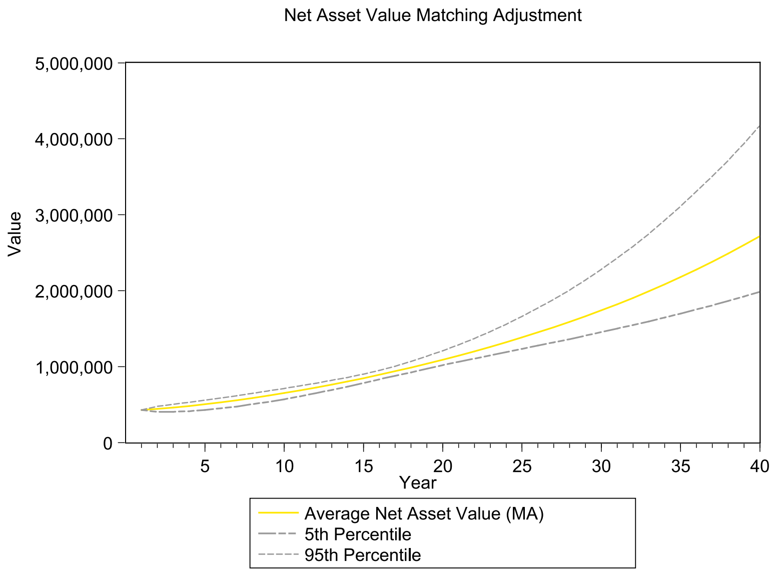

Figure 7.12: The 5th, 95th percentile and the mean of the net asset value through time with matching adjustment.

7.5 Impact on Capital Requirements

As described in Section 4, the solvency capital requirements are based on changes in the net asset value. The previous section has shown that this value is higher after application of the matching adjustment. Figure 7.13 shows the impact of applying the matching adjustment to the solvency required capital. As expected, after applying the matching adjustment the SCR increases. Over time, the values converge and in year 40 are exactly the same over all scenarios.

Measures as the tVaR and VaR are equal to zero throughout the whole period, which is due to the cash-flow matching strategy: the probability that the insurer is unable to pay its policy holders is equal to zero, because in this section the illiquid bonds are default-free.

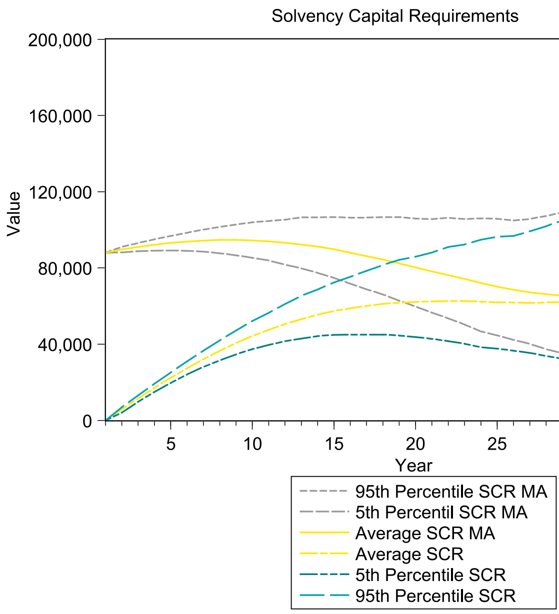

Figure 7.13: Development of the SCR through time with and without applying the matching adjustment.

A higher SCR implies that the insurer needs to hold more capital, as shown in Figure 7.13. However, the reduction in the present value of the liabilities is greater than the increase in SCR and hence the free surplus increases, as seen in Figure 7.14. Since the VaR is always equal to zero, the free capital is the highest in case of using this measure (or the tVaR).

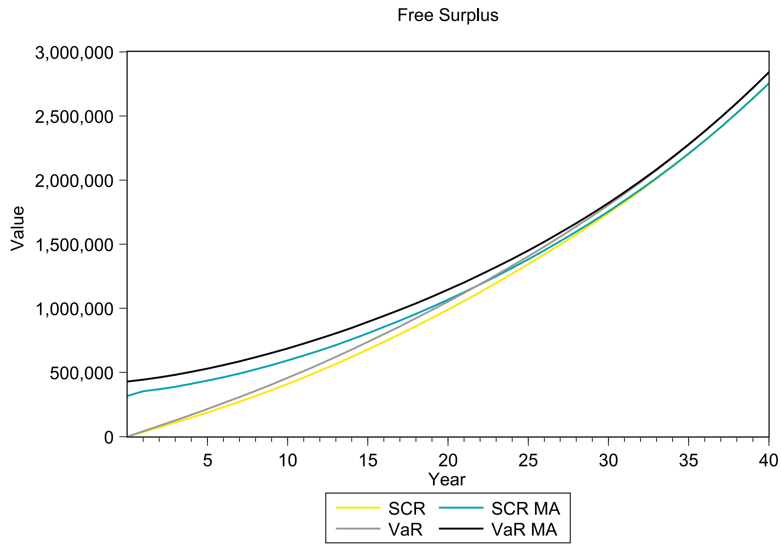

Figure 7.14: Free surplus in case of SCR and VaR and with and without matching adjustment.

7.6 Default-Risk

One of the main assumptions in the analyses in this work has been that the residual spread after taking credit risk into account is due to illiquidity and that holding illiquid bonds until maturity does not generate any additional risk-exposure. There is however a possibility that the yield-spread is not due to illiquidity but credit-risk or that the liquidity premium is incorrectly estimated. To get an idea of the extra risks in case of an incorrect estimation of the illiquidity premium, this section assumes the illiquidity premium does not exist but is fully attributable to default risk, as described in Section 5.3. The insurer does not know this and offers the annuity for a price based on a matching adjustment.

The following assumptions are made:

- cash-flow matching,
- the insurer fully invests in illiquid Pfandbriefe,
- the reduction in liabilities is either given to policy holder or distributed as dividend to shareholder; in other words the cash flows are exactly matched,
- in case of a default, the recovery rate equals 80% and all future payments are reduced by 20%, i.e. all risk is for the policy holders and the insurer will not go bankrupt.

Figure 7.15 shows the payments given the policy holder is alive. The 90th percentile still receives the full promised payments, in fact in approximately 75% of the scenarios the policy holder receives 100% of the promised payouts, as can be seen in Figure 7.16. The lower percentiles show significant risks on lower payments, due to incorrect estimations of the illiquidity premium.

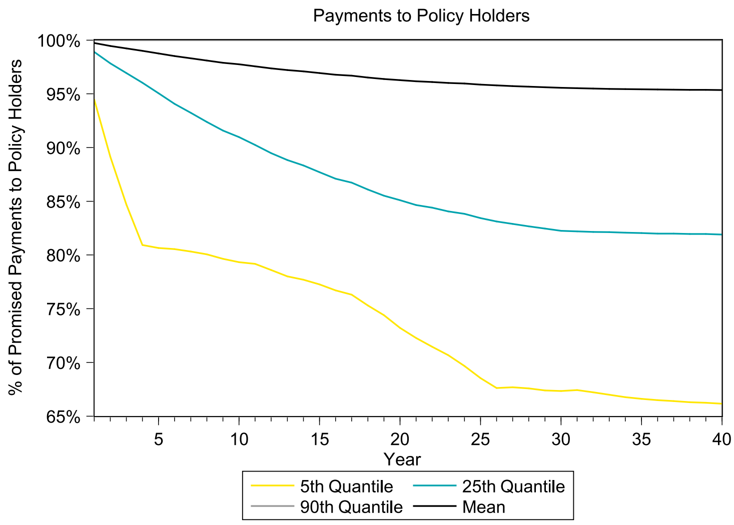

Figure 7.15: Statistics on the realized percentage of the promised payments to the policyholders conditional on being alive.

The cumulative number of defaults over the 40-years period can be found in Figure 7.16. In about 75% of the scenarios no defaults occur, in the 25% scenarios however up to 5 defaults occur causing lower payments to the policy holders. This data is also summarized in Table 7.10.

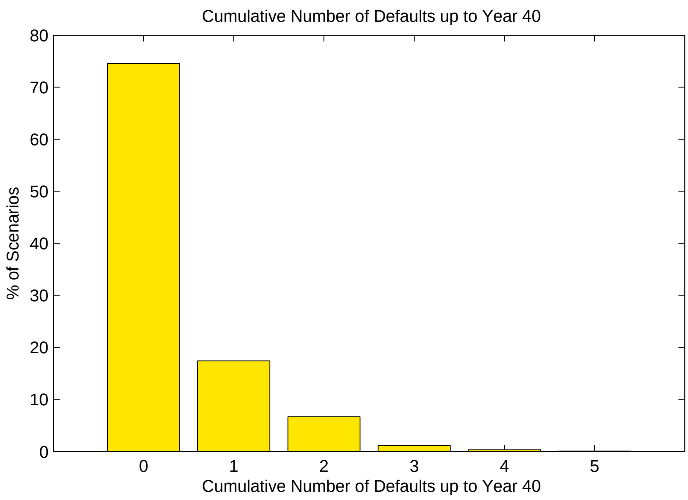

Figure 7.16: Cumulative number of defaults for the 40-year period.

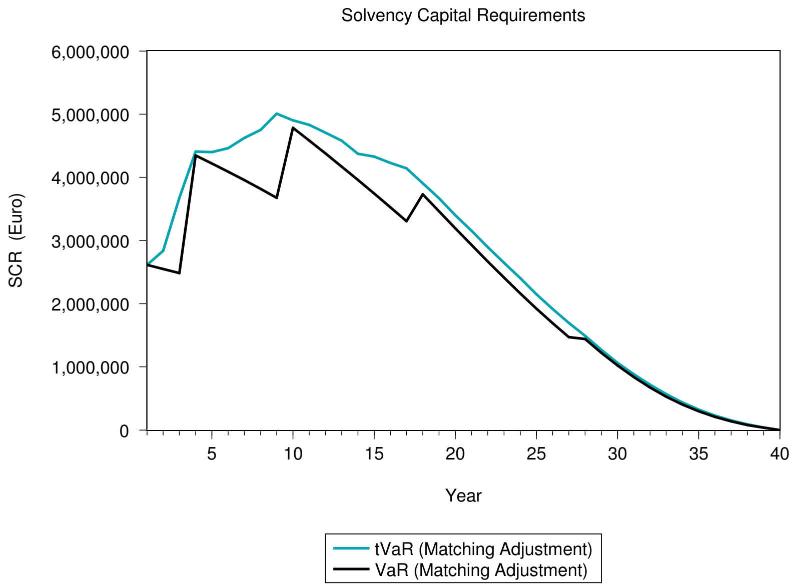

Figure 7.17: SCR in case the illiquidity premium is actually a default premium, according to the 99.5% VaR and tVaR.

	Cumulative number of defaults after 40 years					
	0	1	2	3	4	5
Percentage of scenarios	74.5 %	17.4 %	6.6 %	1.14 %	0.28%	0.04 %

Table 7.10: Number of defaults over the 40-year period.

Table 7.17 shows the capital requirements in case the 99.5% VaR and tVaR measures are used. As one can see, these are very high compared to the capital requirements in Figure 7.13.

8 Conclusion & Recommendations

In this thesis I estimate an illiquidity premium by assuming the German Bunds as default-free and fully liquid and then comparing the yield with German Pfandbriefe. The yield-spread is assumed to be attributable to illiquidity. The average liquidity premium I estimate equals 27bps, which is higher than Kempf and Uhrig-Homburg (2000) who find a long-term average of 18 bps, but use a different time period. Illiquidity has seem to be significantly higher since the start of the credit crisis in 2007. I find the illiquidity premium to be increasing from 1 to 10 years to maturity but reducing again for 15-years to maturity (see Section 7.2.1).

Using the negative CDS basis approach I assume the illiquidity premium equals the residual spread after subtracting the ECB 4Y AAA and iTraxx EUR CDSI 5Y swap premia from the iBoxx € Corporates A 3-5 corporate bond index. The illiquidity spread I find ranges from 0 to 300bps (Section 7.2.2). These results are comparable to Hibbert, Kirchner, and Kretzschmar (2009a).

The impact on the present value of a stylized portfolio of insurance contracts with the application of the matching adjustment is significant. My own estimates using German Pfandbriefe yields a reduction in the present value of the liabilities of approximately 3%. Using examples of calculations of the matching adjustment by EIOPA yield a reduction to 5%-7% at $t = 0$ (7.5). Over time, this impact reduces as the duration of the portfolio also reduces.

The capital requirements as prescribed by Solvency II state that there should be a buffer to compensate for losses in the net asset value after applying a predefined shock to the yield-curve. After applying the matching adjustment, the present value of the liabilities decreases, while the value of the assets remains the same. As a result the net asset value increases which in turn results in higher capital requirements in case of applying the matching adjustment. As time progresses, these capital requirements converge. On the other hand, the free-surplus, the difference between the net assets and the capital requirements also increases in case of applying the matching adjustment (Section 7.4). Hence, the application of the matching adjustment offers three options to the insurer:

- distribution of free surplus to the shareholder in the form of dividend,
- distribution of free surplus to the policy holder, in the form of lower prices for life-insurance products,
- increase of the solvency ratio, in case the insurer is underfunded.

The internal model, in the form of the VaR and tVaR do not require any capital to be held, since here the illiquidity premium is assumed to be default-free.

As shown by Chen et al. (2007); Han and Zhou (2011), there is a positive correlation between illiquidity and credit risk, i.e. assets with a lower credit rating are often less liquid. This positive correlation also comes back in the example calculations of the matching adjustments by EIOPA in Section 7.3. Therefore, there is an incentive for insurers to take more risk than they would normally do. To limit this behaviour, Solvency II requires the assets to be of investment-grade quality, as described in Section 4.

However, this is not without risk. As seen in this thesis, there is in general agreement in the scientific literature that an illiquidity premium exists for certain assets. There is however no consensus on how to accurately estimate and model illiquidity. Hence, estimating the illiquidity premium in an incorrect way can expose the insurer or policy holder to severe risks. Default-risk is investigated in this thesis, by assuming the illiquidity spread is fully due to default-risk. As Section 5.3 shows, in about 25% of the scenarios, the policy holder is facing a reduction of at least 20% of his promised payouts. Also, the capital requirements of the internal model would be very high if the illiquidity premium is actually a default-premium as shown by Figure 7.17. This shows that an incorrect calculation of the illiquidity premium poses severe risks in for the policy holder or insurer and results in too low solvency capital requirements.

In further research, it would be interesting to relax certain assumptions. The assumptions that an insurer is not forced to sell off any assets using a cash-flow matching strategy holds, because the liabilities are assumed to be deterministic. In reality this is not the case and there is still some uncertainty in the value of the liabilities which could result in more forced sales of assets.

In my opinion, the two-factor CIR model is not the best possible way to model illiquidity, because the changes in liquidity are not caputered adequately, as seen in Appendix C. A better way of modelling illiquidity could be for instance by means of look-back options as in the work by Koziol and Sauerbier (2007). Models which allow for jumps could al be of interest.

From the results in Appendix C one can conclude a fixed market price of risk for both interest rate risk and liquidity risk seems to vary heavily through time. Therefore one might assume a stochastic market price of risk, both for interest rate and liquidity risk in future research.

A possible effect on the market of an increased interest is illiquid assets is that they become more liquid which would reduce the illiquidity premium. An investigation of the impact on the market is however outside of the scope of this thesis.

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Appendix

A Derivation multi-factor CIR model

The one-factor Cox-Ingersoll-Ross model can be generalized to include multiple factors. The factor of interest to this thesis is a factor of illiquidity, as in Grinblatt (2001); Kempf and Uhrig-Homburg (2000). This section provides an arbitrage-free derivation of the bond price for a in a market with multiple state variables. Following the conventional state-space notation Schumacher (2012), the market dynamics are represented by

$$dX_t = \mu_X(t, X_t)dt + \sigma_X(t, X_t)dW_t \tag{A.1}$$

$$Y_t = \pi_Y(t, X_t)$$

The vector Y_t contains the prices of the assets at time t. The vector X_t contains the n state variables which evolve according to the following square-root process, under the real-world measure:

$$dx_{it} = \kappa_i(\theta_i - x_{it})dr + \sigma_i\sqrt{x_{it}}dW_t^{\mathbb{P}}, \quad i = 1, \dots, n. \tag{A.2}$$

To obtain a closed-form solution, the assumption is made that the Brownian motions are independent of each other. In case of simulations, this assumption can be relaxed easily. For notational convenience, the subscript t indicating time dependence is suppressed. As found in Schumacher (2012), the drift and volatility function of an asset with price P are given by

$$\mu_P = \frac{\partial \pi_P}{\partial t} + \sum_{i=1}^{n} \frac{\partial \pi_P}{\partial x_i}(\mu_X)_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 \pi_P}{\partial x_i \partial x_j}(\sigma_X \sigma_X')_{ij} \tag{A.3}$$

$$\sigma_P = \frac{\partial \pi_P}{\partial x}\sigma_X \tag{A.4}$$

This model is free of arbitrage if and only if there exists a k-vector valued function $\lambda = \lambda(t, x)$ and a scalar function $r = r(t, x)$ such that:

$$\mu_Y - r\pi_Y = \sigma_Y \lambda. \tag{A.5}$$

λ_i is the market price of risk factor i. Since there is only one asset in the market, the zero-coupon bond with maturity T, the vector Y only contains one element. Another assumption required to make a closed-form solution possible is to assume $\lambda(t, x) = \begin{bmatrix} \lambda_1\sqrt{x_1} \\ \vdots \\ \lambda_n\sqrt{x_n} \end{bmatrix}$. Writing equation A.3 is a simpler form, where the subscripts denote the partial derivative :

$$\mu_P = P_t + \sum_{i=1}^{n} P_{x_i}(\mu_X)_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} P_{x_{ij}}(\sigma_X \sigma_X')_{ij} \tag{A.6}$$

$$\sigma_P = \begin{bmatrix} P_{x_1} & \dots & P_{x_n} \end{bmatrix} \begin{bmatrix} \sigma_1\sqrt{x_1} & & \\ & \ddots & \\ & & \sigma_n\sqrt{x_n} \end{bmatrix}. \tag{A.7}$$

Plugging this into A.5 yields

$$P_t+\sum_{i=1}^{n} P_{x_i}(\mu_X)_i+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} P_{x_{ij}}(\sigma_X\sigma_X')_{ij}-rP = \begin{bmatrix} P\sigma_1\sqrt{x_1} & \dots & P\sigma_n\sqrt{x_n} \end{bmatrix} \begin{bmatrix} \lambda_1 x_1 \\ \vdots \\ \lambda_n x_n \end{bmatrix} \tag{A.8}$$

with boundary condition

$$P(T,T) = 1. \tag{A.9}$$

For details on solving this partial differential equation, see Cox et al. (1985). The price at time t of the zero-coupon bond with maturity T is then given by

$$P(t,T) = \prod_{i=1}^{n} p_i(t,T) \tag{A.10}$$

where

$$p_i(t,T) = A_i(t,T)e^{-B_i(t,T)x_{it}} \tag{A.11}$$

and

$$P(t,T) = \prod_{i=1}^{n} p_i(t,T) \tag{A.12}$$

where

$$p_i(t,T) = A_i(t,T)e^{-B_i(t,T)y_{it}} \tag{A.13}$$

and

$$A_i(t,T) = \left[\frac{2\gamma_i e^{(\gamma_i+k_i+\lambda_i)(T-t)/2}}{(\gamma_i+\kappa_i+\lambda_i)(e^{\gamma_i(T-t)}-1)+2\gamma_i}\right]^{2k_i\theta_i/\sigma_i^2} \tag{A.14}$$

$$B_i(t,T) = \frac{2(e^{\gamma_i(T-t)}-1)}{(\gamma_i+\kappa_i+\lambda_i)(e^{\gamma_i(T-t)}-1)+2\gamma_i} \tag{A.15}$$

$$\gamma_i = \sqrt{(\kappa_i+\lambda_i)^2+2\sigma_i^2} \tag{A.16}$$

$$A_i(t,T) = \left[\frac{2\gamma_i e^{(\gamma_i+k_i+\lambda_i)(T-t)/2}}{(\gamma_i+\kappa_i+\lambda_i)(e^{\gamma_i(T-t)}-1)+2\gamma_i}\right]^{2k_i\theta_i/\sigma_i^2} \tag{A.17}$$

$$B_i(t,T) = \frac{2(e^{\gamma_i(T-t)}-1)}{(\gamma_i+\kappa_i+\lambda_i)(e^{\gamma_i(T-t)}-1)+2\gamma_i} \tag{A.18}$$

$$\gamma_i = \sqrt{(\kappa_i+\lambda_i)^2+2\sigma_i^2} \tag{A.19}$$

B Solvency II

Maturity	Upward shock	Downward shock
0.25	70%	-75%
0.5	70%	-75%
1	70%	-75%
2	70%	-65%
3	64%	-56%
4	59%	-50%
5	55%	-46%
6	52%	-42%
7	49%	-39%
8	47%	-36%
9	44%	-33%
10	42%	-31%
11	39%	-30%
12	37%	-29%
13	35%	-28%
14	34%	-28%
15	33%	-27%
16	31%	-28%
17	30%	-28%
18	29%	-28%
19	27%	-29%
20	26%	-29%
90	20%	-20%

Table B.1: Upward and downward shock applied to the term structure to calculate the SCR for interest rate risk.

C Robustness

C.1 Daily Market Price of Risk

By assuming the market prices of risk are constant, the market price of interest rate risk which minimizes the total sum of squared error of the bond prices implied by the model and the observed prices equals: $\lambda_1 = -0.095$. If I estimate the market price of interest rate risk per observation, one can see in Figure C.1 that the market price of risk is decreasing over time ($\lambda_{1,t}$ is increasing). A positive $\lambda_{1,t}$ occurs when the yields on longer maturity bonds are lower than the estimated long-term mean of the short-rate.

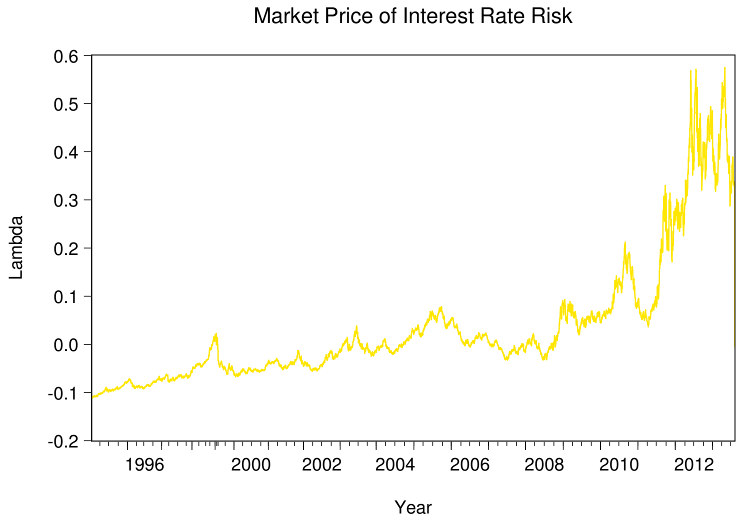

Figure C.1: Daily market price of interest rate risk ($\lambda_{1,t}$).

Figure C.2 shows the observed yields and the yields implied by the CIR model using a fixed market price of interest rate risk. In the first part of the sample period $\lambda_{1,t} < -0.095$, resulting in lower implied yields than observed yields. In the second part of the sample, starting in approximately 2008, this is the other way around.

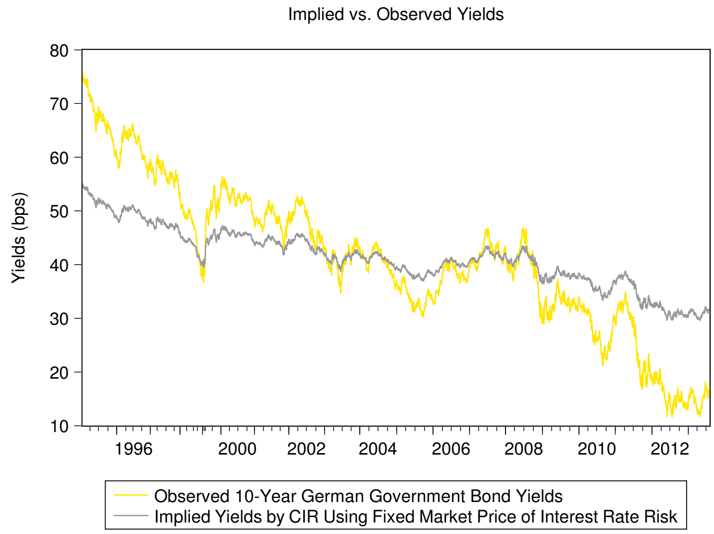

Figure C.2: Observed yields on 10-year German government bonds versus the yields implied by the CIR model while keeping the market price of interest rate fixed.

Figure C.3 shows the market price of liquidity risk through time.

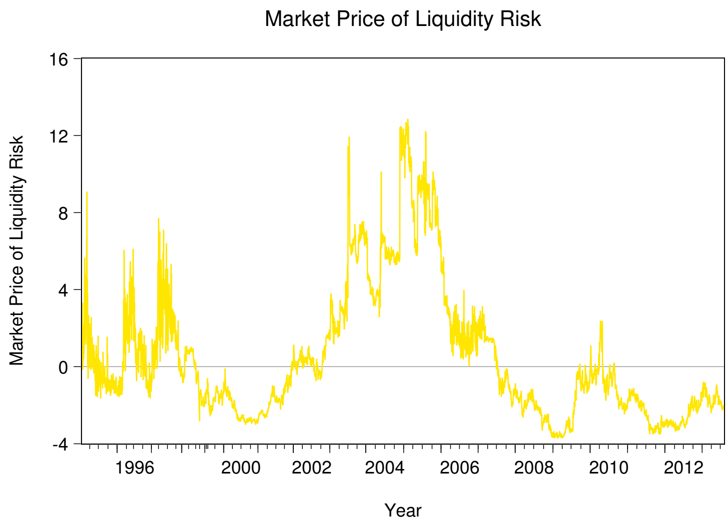

Figure C.3: Market price of liquidity risk ($\lambda_{2,t}$) on a daily basis using 5,10 and 15-year Pfandbriefe yields.

Figure C.4 shows the market price of liquidity risk through time. As one can observe, the yields implied by the CIR and the optimal single market price of risk differ significantly from the observed illiquidity yields, i.e. the dynamics of illiquidity do not seem to be captured well by the state variable for illiquidity.

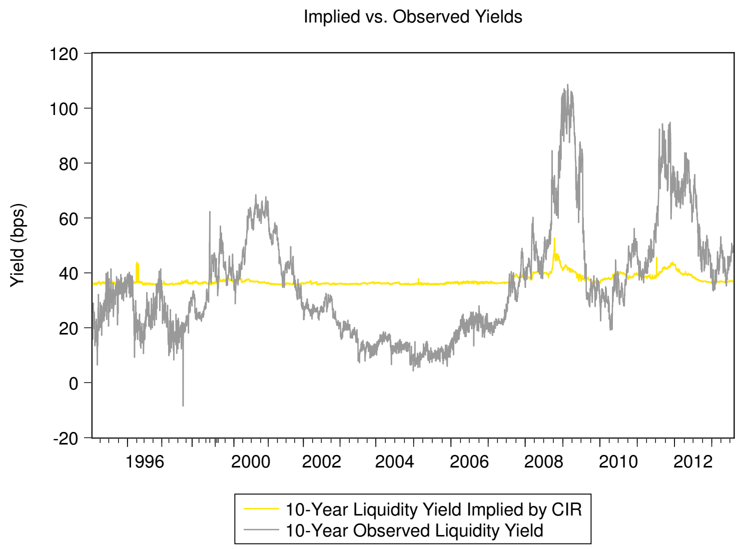

Figure C.4: Implied verus observed illiquidity yield, using the λ_2 which minimizes the total sum of squared errors.

D Calibration & Implementation

This section describes the calibration and implementation methods used for the models in this thesis for the calibration of the 2-factor CIR-model. The steps I take can be summarized as follows:

- calibrate κ_1, θ_1 and σ_1 using the previously described method, using 1-year German government bonds as short rate,
- calibrate the market price of risk by choosing λ_1 such that the sum of squared errors between the CIR-model and the observed yields, using data on 5, 10 and 15-year German government bonds,
- calibrate κ_2, θ_2 and σ_2 in the same way, but using the residual spread on 1-year Pfandbriefe relative to the 1-year German government bonds and
- calibrate the market price of liquidity risk by choosing λ_2 such that the sum of squared errors between the CIR-model and the observed 5, 10 and 15-year Pfandbriefe yields is minimized.

The results are presented in Section 7.

D.1 Discretization

The CIR model is formulated in a continuous time setting, but computers can only work with discrete processes. Therefore the continuous time models need to be discretized in order to approximate these continuous equations. The most simple scheme is the *Euler scheme*. Applying this to Equation 5.1 yields:

$$\hat{x}_{i,(t+\Delta t)} = \hat{x}_{i,t} + \bar{\kappa}_i(\bar{\theta}_i - \hat{x}_{i,t})\Delta t + \epsilon\sqrt{\hat{x}_{i,t}}Z\sqrt{\Delta t} \tag{D.1}$$

where $Z \sim N(0,1)$. One immediate problem is that the discretized process $\hat{x}(t)$ can become negative while the process itself is always positive. To solve this a fix can be applied to the scheme:

$$\hat{x}_{i,(t+\Delta t)} = \hat{x}_{i,t} + \bar{\kappa}_i(\bar{\theta}_i - \hat{x}_{i,t})\Delta t + \epsilon\sqrt{\hat{x}^{+}_{i,t}}Z\sqrt{\Delta t} \tag{D.2}$$

where , where $x_{t_{i+1}}$ is either r_{t_i+1} or l_{t_i+1}. This is the so-called *full-truncation scheme* by Lord, Koekkoek, and Dijk (2010), who conclude that this is the preferred method for simulating a CIR model because of the low bias and RMSE relative to other methods. The parameters for the simulation are shown in Table D.1.

Parameter	Value
Time step (Δt)	0.004
Number of years	40
Number of simulations	5,000

Table D.1: Settings simulation

D.2 Calibration

For the calibration of the Cox-Ingersoll-Ross model, I rely on the maximum likelihood method and implementation in MATLAB as described by Kladivko (2007). The likelihood function parameter estimation for a series with N observations is given by

$$L(\Theta) = \prod_{j=1}^{N-1} p(x_{i,t_{j+1}} | x_{i,t_j}; \Theta, \Delta t) \tag{D.3}$$

where $x_{i,t_{j+1}}$ is either r_{t_i+1} or l_{t_i+1} depending on which factor is being calibrated at time t_{i+1}. For computational convenience, the log-likelihood function is used:

$$\ln(L(\Theta)) = \sum_{j=1}^{N-1} \log(p(x_{i,t_{j+1}} | x_{i,t}; \Theta, \Delta t)). \tag{D.4}$$

Because log is an increasing function, maximizing the log-likelihood function also maximizes the likelihood function. Using the density of $x_{t+\Delta t}$, one can the derive the log-likelihood function:

$$\log(L(\Theta)) =$$
$$(N-1)\log(c) + \sum_{j=1}^{N-1} \left\{ u_{t_j} - v_{t_{j+1}} + 0.5q\log\left(\frac{v_{t_{j+1}}}{u_{t_j}}\right) + \log\left(I_q\left(2\sqrt{u_{t_j} v_{t_{j+1}}}\right)\right)\right\} \tag{D.5}$$

where $u_t = cx_{i,t_j}e^{-\kappa\Delta t}$, $v_{t_{j+1}} = cx_{i,t_{j+1}}$, $c = \frac{2\kappa_i}{\sigma_i^2(1-e^{-\kappa_i\Delta t})}$ and $I_q\left(2\sqrt{(uv)}\right)$ is the modified Bessel function of the first kind and of order q. The maximum likelihood estimates of $\hat{\Theta}$ are obtained by maximizing over its parameter space:

$$\hat{\Theta} = \arg\max_{\Theta} \log L(\Theta). \tag{D.6}$$

Equation D.6 is maximized in MATLAB using `fminsearch`, which is an implementation of Nelder-Mead simplex method.

As a starting point, the optimization procedure needs initial estimates for the parameters. These initial parameters are obtained by applying Ordinary Least Squares (OLS) to a discretized version of the CIR process which can be used in an OLS-estimation. The initial values for the parameters are then used in the `fminsearch` in MATLAB. For more details I refer to the original paper.

The market price of risk is determined such that

$$\tilde{\lambda}_1 = \arg\min_{\lambda_1} \sum_{j=1}^{N} \left(P^L(r_j, t, T, \lambda_1) - P_j^L\right)^2$$

where P_j is the observed market price and $P^l(r_j, t, T, \lambda_1)$ is the bond price implied by the model using the short-rate and time to maturity corresponding to P_j, the observed and assumed to be fully liquid bond. The market price of liquidity risk is then estimated by

$$\tilde{\lambda}_2 = \arg\min_{\lambda_2} \sum_{j=1}^{N} \left(P^I(r_j, l_j, t, T, \lambda_1, \lambda_2) - P_j^I \right)^2 \tag{D.7}$$

where $P^I(r_j, l_j, t, T, \lambda_1, \lambda_2)$ is the implied price of the illiquid bond as in 5.3, using the r_j, l_j and time to maturity corresponding to P_j^I, the observed price for the illiquid bond.