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# Financial and demographic risks of personal pension accounts

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# Financial and Demographic Risks of Personal Pension accounts

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# 1 Introduction

Developments like low interest rates, the financial crisis of 2008, and an aging population have put the Dutch pension system under serious pressure. Recently, therefore, the Dutch government has given the go-ahead for a national pension dialogue ("Nationale pensioendialoog"). Core subjects of this 'dialogue' are freedom of choice, solidarity, collectivity, and responsibility.

In the context of this dialogue and the debate on the current pension system in general, the social partners, represented in the Social Economic Council (SER)<sup>1</sup>, have published an advice which recommends further investigation of a so-called *Personal Pension with collective Risk sharing (PPR)*<sup>2</sup> as an alternative occupational pension system (SER, 2015). The PPR is considered to be an interesting option by the SER (2015), that is yet insufficiently specified and insufficiently investigated. Lans Bovenberg and Theo Nijman have advocated this reform of the Dutch occupational pension system (Bovenberg & Nijman, 2014, 2015). In a PPR, every participant has a personal pension account, i.e. is the owner of a certain amount of capital that is invested for the purpose of providing lifelong payments after retirement. Contrary to many other collective pension schemes, in a PPR demographic and financial risks are not shared intergenerationally, but only intragenerationally.

According to Bovenberg and Nijman (2014, 2015), there are many advantages of a PPR. For instance, a PPR allows for transparent communication towards the participants, each of whom owns a certain amount of capital which is closely related to the pension contributions he or she has paid. Moreover, a PPR allows for a *life-cycle* in the risk profile. Presumably, at a younger age one is willing to take more investment risks than at a higher age when it is harder to adjust capital accumulation and/or decumulation to shocks. Such a life-cycle investment strategy is only possible within a collective contract in the implicit sense that the pension fund may adjust its overall investment strategy to the relative cohort sizes of its participants; a

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<sup>1</sup> *Sociaal-Economische Raad*

<sup>2</sup> *Persoonlijk pensioen(vermogen) met risicodeling*

pension fund with many young participants is probably willing to take more risks than one with many retired participants. Also, by use of an open adjustment mechanism to shocks, a collective contract can ensure that the risks of the old are lower than those of the young. However, it is not possible in a collective contract to provide younger cohorts with a particular transparent *investment* profile which is different from that of the older cohorts. Both of these advantages also play a role in diminishing the cause for intergenerational conflicts.

The question rises whether these and other advantages of a PPR weigh up against the additional risks that are involved in participating in a system of personal pension accounts. Indeed, the absence of intergenerational risk sharing may lead to more transparency and to more flexibility, but it is hard for a retired generation to accommodate to unexpected *financial* and *demographic* shocks if they cannot make use of the incomes of the younger workers and vice versa. The goal of the present study is to give a quantitative evaluation of the PPR which takes these kinds of risks (financial and demographic) into account.

This quantitative evaluation will be based on a simulation study. For the purpose of simulating a variety of pension contracts, a scenario-set with financial and demographic uncertainty will be combined with an asset/liability management (ALM) model. A financial scenario generator for the Dutch economy made by the Dutch Bureau for Economic Policy Analysis<sup>3</sup>(CPB) (Draper, 2014) has recently been extended with demographic scenarios by (Muns, 2015b) of the CPB (see Section 4). The cash flows of a pension fund will be simulated by appropriate adjustments of an already existing asset/liability management model (ALM model) in MATLAB which is also provided by the CPB and based on Draper, Ewijk, Lever, and Mehlkopf (2014) (see Section 5).

From the perspective of a participant of a pension scheme, two quantifiable elements of his or her pension result are important (assuming a fixed level of contributions). First, what is the level of the pension benefits that he or she can expect to receive during retirement? In particular, the median

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<sup>3</sup>*Centraal Planbureau.*

*replacement ratio*, which is the level of the pension benefits (in real terms) as a percentage of the average wage (in real terms) earned prior to retirement, is of interest.

Second, the uncertainty of this level of pension benefits is relevant for the pension participant. Note that there are, with respect to the *outcome* of a pension contract, at least two kinds of uncertainty that are of interest.

The first kind of uncertainty is the uncertainty in the average level of the pension benefits over the pension period. So, someone expecting median yearly pension payouts worth 80% of the earned wage also wants to know what deviations of this level can reasonably be expected. Under the assumption of risk aversion, this person would prefer a less uncertain pension result to a more uncertain pension result, e.g. he or she would prefer the 'bandwidth' (see below) of the replacement ratios being about 70 – 90% to this bandwidth being 60 – 100%.

The second kind of uncertainty lies in the extent to which, given the level of the pension benefits someone receives in year  $t$ , the pension benefits in year  $t + 1$  can be expected to deviate from this level. People generally do not like to make serious adjustments in their consumption patterns on a yearly basis, so they prefer a more stable pension over a less stable pension. In this paper, therefore, the uncertainty will mainly be measured by the 90% confidence interval of the pension result over the pension period and by the 90% confidence interval of the yearly relative changes in the level of the pension benefits. The 90%-confidence interval of a quantity is constructed by omitting the bottom 5% and top 5% results of that quantity in the simulations. I will frequently use the term *bandwidth* instead of the 90%-confidence interval.

The evaluation of the PPR will therefore be based on these two elements, the median replacement ratio and the uncertainty (both over the entire pension period as well as the variations from year to year). I will assume that people are risk averse, i.e. given an equal median pension result people prefer less uncertainty over more uncertainty in their pension result. Similarly, given the same level of uncertainty, people prefer a higher median pension result. Without a utility function which determines the utility for

every combination of median pension result and its uncertainty, a comprehensive comparison between two contracts is only possible if one of the two components is equal between the two.

As was mentioned above, the present thesis will take two *sources* of uncertainty into account: *financial* and *demographic* uncertainty. With respect to the risks of a PPR, there has already been a study to the financial risks of personal pension accounts (Lever & Michielsen, 2015), but not to the demographic risks.<sup>4</sup> An important demographic risk for the PPR is *macro-longevity risk*, the risk that mortality rates for a cohort turn out lower (or higher) than anticipated, such that not enough (too much) capital has been reserved for that cohort. Demographic risks other than macro-longevity risk, such as changes in migration and fertility, will be ignored, mainly for simplicity but also because these risk factors do not really have an impact on a PPR. Indeed, fertility only affects the size of cohorts and is in particular relevant for contracts where risks are shared intergenerationally, i.e. for contracts in which the retired generations rely on a sufficient size of the working population. Similarly, migration only affects the size of a cohort and is therefore less relevant than mortality. So, the risks of a PPR that will be taken into account are the financial and (macro-)longevity risks.

In this thesis, I will try to set up a PPR which has approximately the same median pension result as a standardized collective contract, whilst minimizing the uncertainty involved. The set-up of the PPR will vary in the adjustment mechanism to shocks, in the discount rates, and in the life-cycle investment strategy, which will only involve stocks and 5-year bonds. Let's call the resulting set-up of the PPR, which closely matches the median pension result of the collective contract and has relatively low uncertainty, the "Life-cycle PPR". Then the two following questions will be answered in this thesis.

1. Is the Life-cycle PPR, with respect to the uncertainty of the pension result, an improvement over the standardized collective contract?

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<sup>4</sup>Lever and Michielsen (2015) speak about 'individual' contracts instead of personal pension accounts. Throughout the present paper, only the term personal pension accounts (and the acronym PPR) will be used in order to avoid confusion.

2. How do the two sources of risks for the Life-cycle PPR, i.e. the financial and the macro-longevity risk, compare to each other?

Note that if the first question is given a negative answer, this is no proof that the collective contract is in general better than the PPR relative to the elements of pension result and uncertainty evaluated in this thesis. For one, the set-up of the PPR is limited in the present project: the asset mix is limited to stocks and 5-year bonds such that there is, of particular importance, no interest rate risk insurance. Moreover, even given the limited set-up it will be impossible to simulate all possible life-cycles of stocks and bonds and all adjustment mechanisms to shocks. Also, no attempt will be made to match the uncertainty level of the collective contract while having a higher median pension result, which would also be a possible way for a PPR to beat the collective contract. The second question, comparing the financial risks to the macro-longevity risk, is relevant for at least two reasons. First, as the inclusion of macro-longevity risk for the analysis of pension schemes is new for the CPB, it is interesting to see how much it matters compared to only financial risks. Second, if the PPR turns out to involve a high level of uncertainty, additional measures for intergenerational risk sharing may be considered, which possibly involve sharing only one of the two risks.

## Structure

The structure of the thesis will be as follows. First, in Section 2 a short explanation of the Dutch pension system will be given. Then, the PPR system as it is proposed by Bovenberg and Nijman (2014) will be presented in more detail (Section 3). The fourth section will discuss the scenario set and its underlying model. Section 5 describes the ALM model. The contracts that have been simulated and the results are described and presented in Section 6. The thesis ends with a conclusion (Section 7).

## 2 Pensions in the Netherlands

This section serves as a short, and certainly not exhaustive, non-technical introduction to pensions in general and to the pension system in the Netherlands in particular.

### 2.1 Pensions: Basic terminology

Basically, there are two types of pension schemes. First, in a *Defined Benefit* (DB) pension plan the benefit payments in the pension period are predetermined, usually as a percentage of the pensionable income. Contributions may be adjusted in order to ensure that the desired benefit payments can be paid. Quite possibly, for every participant a lifelong annuity is bought at the age of retirement in a DB scheme. In the case of an occupation pension, then, the employer is responsible that a certain level of the benefits will be realized and he therefore bears the associated financial and demographic risks. In a sense a pension in such an occupational DB scheme can be regarded as a level of deferred salary promised by the employer to its employees.

Second, in a *Defined Contribution* (DC) pension plan, contrary to DB, the level of the contributions in the working period is fixed, and the level of the benefits depends on the realized investment returns and relevant developments in the demography. In a DC scheme the risks are borne by the participants of the pension plan.

The *funding ratio* of a pension fund is the value of its assets divided by the value of its liabilities. If the funding or solvency ratio equals one (or 100%), then the assets are exactly sufficient for meeting the liabilities. A pension fund usually has a particular policy, whether or not imposed by government regulations, that determines how the contributions, the benefit payments, and the investment strategy are adjusted in response to different values of the funding ratio.

Risk sharing can be organized both among and within generations, referred to as *intergenerational* and *intragenerational* risk sharing respectively. So in practice there is often intergenerational risk sharing, particularly in collective DB schemes, when contributions of the current working population

are adjusted in order to ensure that a pension fund can fulfill its obligations towards the current retired generation. Intergenerational risk sharing in a collective DC (CDC) scheme will be realized by adjusting the level of the benefit payments in some way. *Idiosyncratic longevity risk*, the 'risk' for an individual to become old and consequently to depend on pension payouts for a longer time, is a good example of a risk that is often shared both inter- and intragenerationally, if participants are guaranteed lifelong benefit payments.

*Indexation* of the pension benefits occurs when their level increases over the years in accordance with the price or wage inflation or some other appropriate *index*. By indexing the benefit payments, the pension will provide a constant purchasing power relative to the index of choice.

The *franchise level* is the wage level under which no pension contributions are paid. This is usually related to the first pillar pension, which will be explained below.

## 2.2 The Netherlands: Three Pillars

In the Netherlands pensions are organized in a system of three pillars.

### **The first pillar**

The first pillar is an unfunded pension provided by the government to all residents after they have reached the state pension age, called the AOW age after the name of the law.<sup>5</sup> The goal of the AOW is to prevent poverty of the elderly by ensuring that all retired citizens have a basic income. For this purpose the benefits are directly related to the minimum wage. In principle, all citizens that are older than the pension age receive the same level of AOW benefits, depending only on the number of years that one has resided in the Netherlands.

The state pension age was fixed at 65 years, but is now gradually being increased to 66 in 2018 and 67 years in 2021 with an increasing pace. Thereafter, the AOW age will be linked to developments of the life expectancy. An

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<sup>5</sup>De Algemene Ouderdomswet

estimate of the AOW age of a citizen born at 1 January 1990 is about 71,5 years if the life expectancy rises in accordance with the expectations.

### **The second pillar**

The second pillar consists of occupational pensions, the total assets of which amount to about 160% of the Dutch GDP (Draper et al., 2014). For most employees, paying pension contributions is mandatory, such that in practice more than 90% of the employees participate in an occupational pension scheme. The pensions are managed by pension funds, who invest the capital on the financial markets.

The retirement age for occupational pensions is currently 67, although in practice workers may choose to retire at 65 in return for a lower pension. In the majority of the cases, the pension accumulation is defined by a yearly accrual rate of at most 1.875%, as a percentage of the average earned wage or a lower percentage of the final earned wage, where this rate is higher (lower) for those retiring at 67 (65). After a career of 40 years one can then expect to receive pension benefits at a level of 75% of the average earned wage. I.e. the pension benefits are related to a targeted replacement ratio. So with this accrual rate the scheme takes the form of a collective DB scheme. The risks are then borne by the social partners, although recently there has been a shift towards schemes that are more like DC and provide conditional annuities. As an alternative to DB and DC, this is sometimes referred to as 'defined ambition' (DA). Because of insufficient funding ratios, pension funds have therefore recently been forced to take measures both at the contribution level (i.e. increasing contribution rates) as well as at the benefit level (i.e. not indexing benefits or in some extreme cases even cutting benefit levels).

Pension funds in the second pillar are supervised and regulated by the Dutch central bank (DNB)<sup>6</sup> and the Dutch Authority for the Financial Markets (AFM)<sup>7</sup>. The first assesses the financial health of pension funds, whereas the latter monitors the behaviour of financial institutions towards their customers. In particular, they are subject to the FTK, the Financial

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<sup>6</sup>De Nederlandsche Bank

<sup>7</sup>De Autoriteit Financiële Markten

Assessment Framework<sup>8</sup>. According to regulations, pension funds have to hold certain capital buffers, must take measures when the funding ratio falls outside of a desirable range, have to determine the cost-effective contribution rates, and so forth.

### **The third pillar**

Other pension products that are individual and voluntary form the third pillar. This pillar is in particular relevant for the self-employed ('zelfstandigen zonder personeel') and for workers that do not participate in a collective second pillar pension scheme. In order to ensure that they can continue their pre-retirement consumption level to a certain extent, these workers rely on products provided by the private sector. For example, such a pension product can take the form of a lifelong nominal annuity that one buys from an insurance company. In that case the insurance company bears the financial and longevity risks. In the Netherlands, the third pillar is by far the smallest of the three pillars.

## **3 Personal pension account with risk sharing**

This section will describe the Personal Pension with Risk sharing (PPR), an alternative to the current pension system in the second pillar (see Section 2), as it is proposed by Bovenberg and Nijman (2014, 2015). The PPR is characterized by three features which correspond to three main functions of a pension and which also correspond to the three elements of the acronym 'PPR'. Those functions are, respectively, (i) the investment function, (ii) the pay-out function, and (iii) the function of insurance and risk-sharing.

First, the PPR is *personal* because every participant of a PPR is the owner of a personal investment account.<sup>9</sup> This feature relates to the investment function. So, the investment account 'belongs' to the person paying

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<sup>8</sup>Financieel Toetskader

<sup>9</sup>But, as will be clarified through the pay-out function, the ownership of the investment account is limited in the sense that the person cannot decide to withdraw money to his liking.

contributions instead of to the insurance company or the pension fund gathering the contribution payments. Growth of the assets on this account is due to contributions and financial returns on the assets. In addition, there may be changes in the account due to risk sharing. After retirement, the account is depleted by pay-outs. What is important to note is that the value of a PPR can be communicated as the value of the personal investment account. In addition, the expected level of benefit payments may be communicated, but there will be no guaranteed annuity after retirement, as is the case in collective DB schemes.

Note that because the pension account is now personal, it is not necessary that pension funds have a uniform risk profile. In particular, a life-cycle investment strategy will be possible in a PPR. This probably implies that the assets will be invested more risky in the working period than in the retirement period. Possibly, in the PPR the investment strategy and risk management may be personalized even further, such that it varies per individual instead of only per cohort.

Secondly, a PPR is supposed to be a *pension*, such that the financial assets will be used for the purpose of providing a lifelong income stream during the retirement phase. That is, a PPR is not simply a savings account, because a savings account would fail to fulfill the pay-out function of a pension. Asset-liability management (ALM) is applied at the cohort (or possibly individual) level in order to determine the adequate level of the benefit payments during retirement that rises with a particular index. The decumulation rule determines which level of the benefits is selected. It is determined by a range of factors, in particular the current level of the assets, by the expected mortality rates (implying the expected biometric returns, see below), and the discount rates. The higher the discount rates and the expected mortality rates, the higher the decumulation rate that is allowed, because then a higher growth of the capital is expected and less has to be reserved for future annual payments. How this decumulation rule is determined in the ALM model used in this study, will be explained in more detail in Section 5.

This means that shocks in the returns during the retirement period

will be absorbed by adjusting the level of the benefit payments. This can be done either immediately and completely or gradually depending on what is desirable. The advantage of the first option is that shocks are immediately translated into an annuity level that is expected to be sustainable in the future such that benefits on the short term are not cut softly at the expense of benefits on the longer term (the cuts of which have to be higher if done gradually). The advantage of the latter is that it limits the year-to-year volatility of the payments, and it thereby does not demand a lot of flexibility in the year-to-year consumption behavior of the pensioner.

The third feature of the PPR is related to the risk sharing and insurance function. Note that the account of a PPR should have value at the moment a person dies, since the purpose of the PPR is to provide a life-long stream of income and there should always be capital available for the possibility that the person survives one or more years. In order to share longevity risk, therefore, the assets of the accounts of the deceased will be disseminated over the accounts of their surviving peers (cohort members), the so-called *biometric* returns. Note that this comes at the price of a biometric return of  $-100\%$  at the moment of death.

The question remains what happens to the money that is left after all members of a cohort have died. Due to the nature of the simulations below, in which this never occurs, I will leave the discussion of that question to other researchers and policy-makers.

## 4 Scenarios

In order to simulate the cash flows of a pension scheme,  $N$  scenarios are generated on the basis of Muns (2015b, 2015a). The model of Muns (2015b, 2015a) generates demographic, financial, and productivity risk factors which vary over the time horizon  $t = 2015, \dots, T$ . It is inspired by the analysis of Broer (2010). Many technical details concerning, for instance, the estimation techniques and the estimated parameters will be omitted.<sup>10</sup> I will describe

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<sup>10</sup>The reader interested in such details is referred to Muns (2015b); Broer (2010) and for the financial model in particular to Muns (2015a); Draper (2014); Kojien, Nijman, and

the model in the extent to which it concerns the variables relevant for the ALM simulations in this paper.

## 4.1 Demography: mortality

The scenario set contains a number of demographic risk factors, but the only demographic risk factor used in this project is the mortality rate. Let  $q_x^t$  be the probability that a person of age  $x$  in year  $t$  dies before the start of year  $t + 1$ . Note that the mortality rates are not differentiated on the basis of factors like gender, lifestyle, and so forth. These mortality rates have a scenario dimension, i.e. there will be values  $q_{x,i}^t$  for ages  $x = 1, \dots, 121$ , periods  $t = 2015, \dots, T$ , and scenarios  $i = 1, \dots, N$ .

### The mortality model for low ages

The model approach will be based on a Lee-Carter model (Lee & Carter, 1992). That is:

$$\log q_{x,i}^t = \alpha_x + \beta_x \cdot f_{t,i}^{(m)} + \epsilon_{x,t,i}, \quad (1)$$

$$f_{t,i}^{(m)} = \lambda_0 + \lambda_1 \cdot f_{t-1,i}^{(m)} + \eta_{t,i}, \quad (2)$$

will be the model for  $x = 1, \dots, 90$ , with mutually independent disturbances  $\epsilon_{x,t,i} \sim N(0, \sigma_{x,\epsilon}^2)$  and  $\eta_{t,i} \sim N(0, \sigma_\eta^2)$ . Note that the *mortality process*  $f_{t,i}^{(m)}$  is independent of the age  $x$ , while the parameters  $\alpha_x$ ,  $\beta_x$ , and  $\sigma_{x,\epsilon}$  are dependent on the age  $x$ , but fixed over  $t$ , determining how responsive the log mortality rate at age  $x$  is to the mortality process (by  $\beta_x$ ) and how variable it is (by  $\sigma_{x,\epsilon}$ ). The parameters governing the mortality process (i.e.  $\lambda_0$ ,  $\lambda_1$ , and  $\sigma_\eta$ , the standard error of the disturbance of the mortality process) are all fixed over  $x$  and  $t$ .<sup>11</sup>

In order to ensure that the model is identified and that covariances of the aggregate model shocks take the form of a correlation matrix, there are

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Werker (2010).

<sup>11</sup>Note that contrary to the original model from Lee and Carter (1992),  $q_{x,i}^t$  is *not* taken as the averaged death rate over the ages  $x - 1, x, x + 1$ .

some restrictions imposed on the model, that is  $\sigma_\eta^2 = 1$ , and

$$\sum_{t=0}^T f_{t,i}^{(m)} = 0 \text{ and} \quad (3)$$

$$f_{0,i}^{(m)} > f_{T,i}^{(m)} \text{ (sign),} \quad (4)$$

for all scenarios  $i$ .

### The mortality model for high ages

Note that the process described above is applied only to ages lower than 91. So for the older ages  $x = 91, \dots, 121$ , another modelling strategy is used which follows Kannisto (1992) as is advised by the Dutch Royal Actuarial Association<sup>12</sup> (AG) (AG, 2014). In this model, the logit of the mortality intensity  $\tilde{q}_{x,i}^t := -\log(1 - q_{x,i}^t)$  is considered, that is:

$$\mu_{x,i}^t = \text{logit}(\tilde{q}_{x,i}^t) \quad (5)$$

$$= \log\left(\frac{\tilde{q}_{x,i}^t}{1 - \tilde{q}_{x,i}^t}\right). \quad (6)$$

The logit mortality intensities for  $x = 80, \dots, 90$  are 'observed' (i.e. generated by the Lee-Carter model for low ages) for every scenario  $i$  and period  $t$ . Then, the following linear regression is estimated on the basis of these ten observed logit mortality intensities:

$$\mu_{x,i}^t = a_{t,i} \cdot x + b_{t,i} + \varepsilon_{x,t,i}, \quad (7)$$

where  $\varepsilon_{x,t,i} \sim (0, \sigma_{t,i}^2)$ . This then leads to the parameter estimates  $\hat{a}_{t,i}$ ,  $\hat{b}_{t,i}$ , and  $\hat{\sigma}_{t,i}$ . So nota bene, contrary to the model for mortality rates for the ages lower than 91, the parameters underlying this model are estimated within each scenario  $i$  and period  $t$  separately. The logit mortality intensities for  $x = 91, \dots, 121$  are then determined by independently drawing  $\varepsilon_{x,t,i} \sim N(0, \hat{\sigma}_{t,i}^2)$

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<sup>12</sup>*Koninklijk Actuarieel Genootschap.*

for all  $t$  and  $i$ , and by then setting the logit mortality intensities as follows:

$$\mu_{x,i}^t = \hat{a}_{t,i} \cdot x + \hat{b}_{t,i} + \varepsilon_{x,t,i}, \quad (8)$$

which can easily be transformed to the corresponding mortality probabilities.

Given the simulated mortality probabilities, the corresponding survival probabilities can also be computed. So if  $q_{x,i}^t$  is the probability that a person with age  $x$  in year  $t$  dies before the end of that year, then  $p_{x,i}^t := 1 - q_{x,i}^t$  is the probability that an  $x$ -year old person survives the entire year  $t$ . Moreover,

$${}_s p_{x,i}^t = \prod_{k=0}^{s-1} p_{x,i}^{t+k} \quad (9)$$

gives the probability that an  $x$ -year old person alive at the beginning of year  $t$  will also be alive at the beginning of year  $t + s$  (note that  $p_{x,i}^t = {}_1 p_{x,i}^t$ ). Then, for instance,  ${}_x p_{25,i}^{t-x+25}$  is the probability that someone with birth year  $t - x$  will reach the age  $x$  in year  $t$  conditional on that he was alive at the age of 25, the age at which all people start working in the pension simulations.

### Deterministic mortality rates

For the purpose of isolating the financial risks, there will be a scenario set in which the mortality rates are deterministic. These are determined by the median over the stochastic mortality rates generated by the process described in this section, so

$$q_{x,i}^t = \text{median} (q_{x,1}^t, \dots, q_{x,i}^t, \dots, q_{x,N}^t), \quad (10)$$

for every age  $x$  and year  $t$ . As will become clear in Section 5, this deterministic mortality set will also be used for predicting the future mortality rates for the purpose of choosing an adequate level of pension benefits.

For the purpose of illustrating of what this deterministic mortality set looks like, Figures 1-2 show the deterministic mortality rates over the 21st

century for a variety of ages. The mortality rates for 50, 70, and 90 year olds are expected to decline quite spectacularly according to Figure 1 and 2, even though the rate of decline seems also to decline, in particular for the mortality rate of 50 and 70 year olds. However, the pattern in Figure 2, showing the mortality rates for 110 year olds in the deterministic set, is completely different, being less smooth than the other graphs, but most strikingly the mortality rate is increasing here over time. The relative increase from about 0.505 to 0.525 is, however, not so dramatic as the rates of decline observed in the other figures.

This phenomenon of slightly increasing mortality rates exists in the

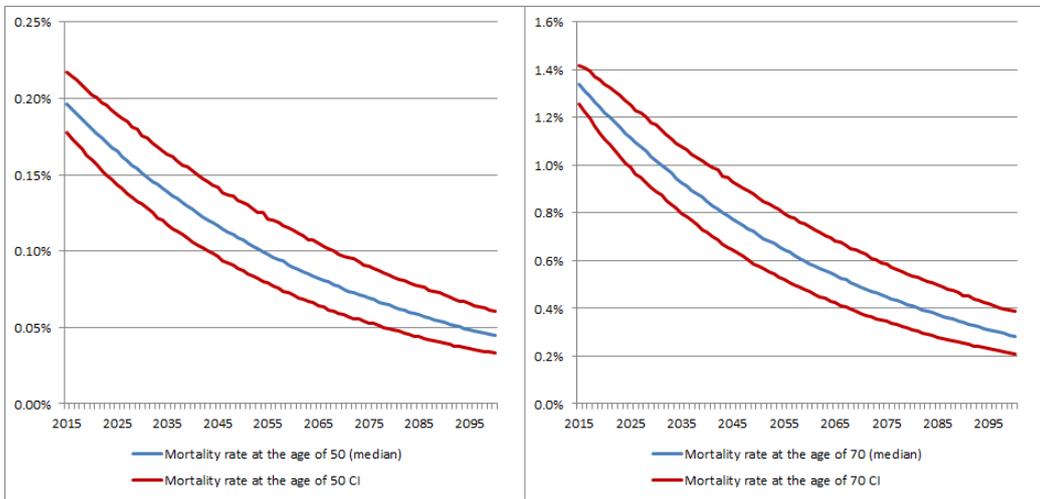


Figure 1: Bandwidth and median mortality rate of a 50 (left) and of a 70 year old (right).

scenario set for the ages  $x = 105, \dots, 121$ . It must be caused by the Kannisto procedure for the older ages (91+) that was described above, with the core equation being (7). That is, changes in the regression coefficients  $\hat{a}_{t,i}, \hat{b}_{t,i}$  over time, due to changing patterns (over  $t$ ) in mortality rates of the ages  $x = 80, \dots, 90$  on which the extrapolations are based, must be the cause of this phenomenon. One may expect increasing mortality rates for higher ages, perhaps due to a process called rectangularization of the survival curve (Nusselder & Mackenbach, 1996). However, as this model extrapolates based on simulated mortality rates of ages until 90 and does not use actual data on

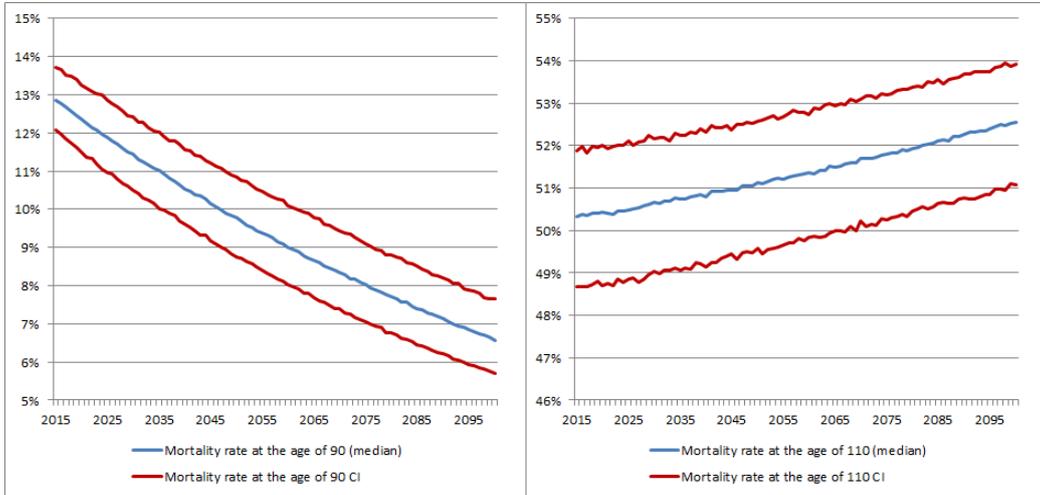


Figure 2: Bandwidth and median mortality rate of a 90 (left) and of a 110 year old (right).

mortality rates of higher ages it may very well be a weakness of the Kannisto procedure and hence a limitation of the scenario set that is produced by it. But as the sizes of cohorts older than 105 are extremely small and the rate of increasing mortality rates is only marginal for those ages, the implications of this weakness are negligible.<sup>13</sup>

The corresponding median (and bandwidth with uncertainty) survival probabilities for a 25 year old in 2015, i.e.  ${}_x p_{25}^{2015-x+25}$  as defined in (9), are then presented in Figure 3, where the age  $x$  is given in the horizontal axis. So, almost half of the current 25 year olds will die in their nineties if the probabilities in Figure 3 are accurate predictions. Also note that if the pension age for this cohort would be 70, then more than nine out of ten are expected to reach that phase of their lives.

As the uncertainty of  ${}_x p_{25}^{2015-x+25}$  is hard to read off from Figure 3, the size of the 90% confidence interval is plotted in Figure 4. So, the survival rate given that one is 25 in 2015 is most uncertain around the ages 90 – 95,

<sup>13</sup>One could argue that the *uncertainty* of the mortality rates is the most important for the present study rather than what the long-term trends are. However, the Kannisto procedure also determines how the uncertainty of the old-age mortality is simulated via the estimation of  $\hat{\sigma}_{t,i}$  and hence another (perhaps more appropriate) model may have more or less uncertainty in the old-age mortality rates.

the 90% level being about 10%. Also note that the uncertainty drops rapidly after that, from  $x = 105$  on the probability of becoming that old given that one is now 25, is in absolute percentage terms not so uncertain.

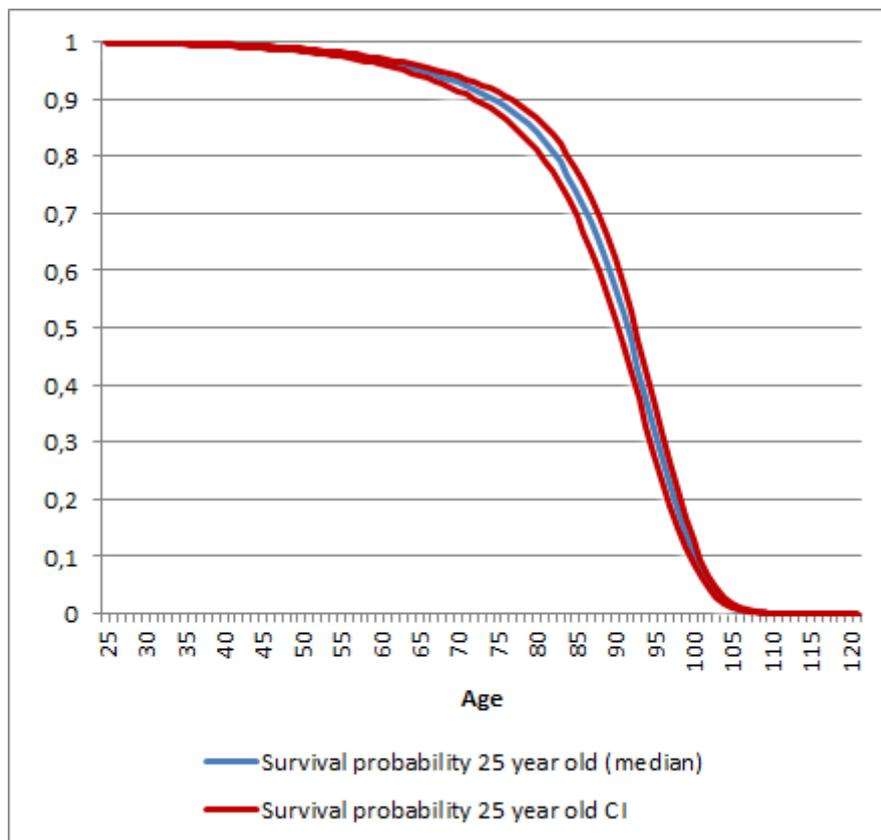


Figure 3: Bandwidth and median probabilities of reaching certain ages conditional on being 25 years old in 2015.

## 4.2 Financial returns and inflation

The financial component of the scenario set produces yearly values for a number of risk factors relevant for a pension fund. The price inflation ( $\pi_{t,i}$ ), the bond returns ( $R_{t,i}^b$ ), and the stock returns ( $R_{t,i}^s$ ) are the financial risk factors

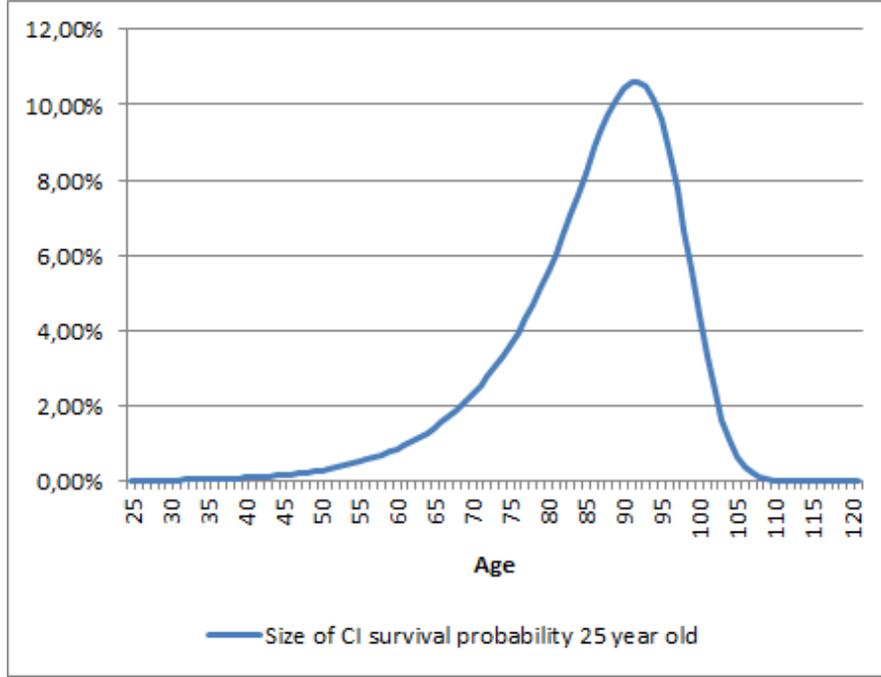


Figure 4: Distance between the 90% confidence bounds of  ${}_x p_{25}^{2015-x+25}$  which are fully presented in Figure 3.

for a pension fund that will be taken into account.<sup>14</sup> The model used for the generation of these financial risk factors is based on Kojien et al. (2010) and further refined by Draper (2014) and Muns (2015b, 2015a).

The following two-factor model models "[t]he term structure of continuously compounded nominal swap interest rates on (yields)  $y_t(\tau)$ " (Muns, 2015b):

$$y_t(\tau) = -\frac{1}{\tau} (A(\tau) + \mathbf{B}(\tau)' \mathbf{X}_{t,i}) + \xi_{t,\tau}, \quad (11)$$

where  $\mathbf{X}_{t,i}$  consists of two unobserved  $AR(1)$  stochastic processes  $\mathbf{X}_{t,i} = (X_{t,1,i}, X_{t,2,i})$  which are based on the term structure of nominal interest rates. The 2- and 5-year yields are taken as given without measurement error. The

<sup>14</sup>Quite possibly, the discount rates a pension fund uses to compute its funding ratio could be based on the nominal interest rate curve which is also generated in the scenarios. As will be explained in Section 5, in the simulations in this essay the discount rates are set independent of the nominal interest rates.

slopes  $A(\tau)$  and  $\mathbf{B}(\tau)$  and the two factors are then estimated (more detail is given in Muns, 2015b). These factors then form the basis of the processes of the inflation and the bond and stock returns. Let  $r_t$  be the instantaneous real interest rate,  $\pi_t$  the instantaneous expected inflation,  $\Pi_t$  the price index,  $S_t$  the stock index, and  $\phi_t^n$  the nominal stochastic discount factor. The following equations then form the core of this capital model (Draper, 2014):

$$r_t = \delta_{0r} + \boldsymbol{\delta}'_{1r} \mathbf{X}_t, \quad (12)$$

$$\pi_t = \delta_{0\pi} + \boldsymbol{\delta}'_{1\pi} \mathbf{X}_t, \quad (13)$$

$$d\mathbf{X}_t = -\mathbf{K} \cdot \mathbf{X}_t dt + \boldsymbol{\Sigma}'_{\mathbf{X}} d\mathbf{Z}_t, \quad (14)$$

$$\frac{d\Pi_t}{\Pi_t} = \pi_t dt + \boldsymbol{\sigma}'_{\Pi} d\mathbf{Z}_t, \quad (15)$$

$$\frac{dS_t}{S_t} = (\mathbf{R}_t \cdot \boldsymbol{\eta}_S) dt + \boldsymbol{\sigma}'_S d\mathbf{Z}_t, \quad (16)$$

$$\frac{d\phi_t^n}{\phi_t^n} = -\mathbf{R}_t dt - \boldsymbol{\lambda}'_t d\mathbf{Z}_t, \quad (17)$$

where  $\mathbf{Z}_t$  is the source of the uncertainty in the financial market, being a four-dimensional vector of independent Brownian motions,  $R_t$  is the nominal instantaneous interest rate,  $\boldsymbol{\eta}_S$  is the equity risk premium,  $\boldsymbol{\lambda}_t$  is the time-varying price of risk which depends on  $\mathbf{X}_t$  via  $\boldsymbol{\lambda}_t = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{X}_t$ . Moreover,  $S_0 = \Pi_0 = 1$ , and  $\boldsymbol{\Sigma}'_{\mathbf{X}} = [\mathbf{I}_{2 \times 2} \quad \mathbf{0}_{2 \times 2}]$ . If one defines

$$\mathbf{R}_0 = \delta_{0r} + \delta_{0\pi} - \boldsymbol{\sigma}'_{\Pi} \boldsymbol{\Lambda}_0, \text{ and} \quad (18)$$

$$\mathbf{R}_1 = (\boldsymbol{\delta}'_{1r} + \boldsymbol{\delta}'_{1\pi} - \boldsymbol{\sigma}'_{\Pi} \boldsymbol{\Lambda}_1), \quad (19)$$

then it can be shown that the following holds for the nominal rate:

$$R_t = \mathbf{R}_0 + \mathbf{R}_1' \mathbf{X}_t. \quad (20)$$

The financial variables in (12-17) are continuous. For the simulation of the pension fund, however, the discretized yearly values are required. For this purpose the model is written as an Ornstein-Uhlenbeck process, of which the details are provided by Muns (2015a).

Of this scenario set, then, the geometric average stock return is 5.60%, the geometric average bond return is 3.09%, and the price inflation's geometric average is 1.85%. These numbers are useful to compare with other scenario sets used for simulating the cash flows of pension funds. A scenario set with, for instance, higher expected stock returns, will obviously have higher average replacement ratios.

### 4.3 Productivity

According to Broer (2010), the most important risk factor for pension schemes is productivity risk. In particular in pay-as-you-go or collective DB schemes the productivity level of the working force is an important factor determining the extent to which the desired level of the pension benefits can be realized.

In this thesis the productivity risk mainly applies because of the ambition of all contracts to index the level of the nominal pension benefits to the nominal wage inflation. Then ironically, the risk in the personal pension accounts in this essay is that this wage inflation index cannot be met in case of high productivity gains. The relevant variable modeled by Muns (2015b) is then the real wage inflation,  $\pi_{t,i}^{Rw}$ . The nominal wage inflation, which will be used to compute pension benefits in real terms, depends on the real wage inflation and the price inflation by  $\pi_{t,i}^w = (1 + \pi_{t,i}^{Rw}) \cdot (1 + \pi_{t,i}) - 1$ .

Let  $f_{t,i}^{(p)}$  be a latent productivity factor driving the productivity growth of a selection of 14 countries and let  $\pi_{t,i}^P$  be the real productivity growth of the Netherlands (more details of how this factor and the productivity growth is modeled are given by Muns, 2015b). Then, the change in real wage growth  $\Delta\pi_{t,i}^{Rw} / \pi_{t,i}^{Rw}$  is modeled by a regression on a constant, on  $[f_{t-1,i}^{(p)} + f_{t,i}^{(p)}]$ , on  $\pi_{t-4,i}^{(p)}, \dots, \pi_{t,i}^{(p)}$  and on an error term  $\epsilon_{t,i}^R \sim N(0, \hat{\sigma}_{Rw}^2)$ . The estimation is based on the data starting from the year  $t = 1986$  until 2013 (so including the 1982, ..., 1985 values where necessary). The geometric average nominal wage inflation of the resulting scenarios is 2.44%.

## 4.4 Aggregation of the model

The model also takes correlations between the three blocks (demography, finance, and productivity) into account. This is done by estimating a correlation matrix between the factors  $f_{t,i}^{(m)}$ ,  $f_{t,1,i}^{(f)}$ ,  $f_{t,2,i}^{(f)}$ ,  $f_{t,i}^{(p)}$  (where  $f_{1,t}^{(f)} = X_{t,1,i}$  and  $f_{2,t}^{(f)} := X_{t,2,i}$  are the factors of the nominal term structure) and some other factors which are of less relevance to the simulations in this essay (e.g. fertility). These correlations are then used for the purpose of simulating the disturbances of these factors. Details are given by Muns (2015b).

## 5 ALM Model

This section describes the ALM model used for simulating the cash flows of a PPR. It is partly based on Michielsen (2015).

### 5.1 Model Inputs

#### Financial and demographic scenarios

For the purpose of isolating the mortality and financial risks, three scenario-sets are used in this study, all (partly) generated by the model of Muns (2015b) described in Section 4.

1. The first, standard scenario-set captures all risk factors relevant for our study - i.e., both mortality and financial uncertainty - and is simply the complete scenario-set described in Section 4.
2. In the scenario-set which isolates financial risks, all mortality rates are determined by (10), while the financial variables are based on the simulation described in Section 4. So as will become clear below, the *projected* mortality and survival rates are then exactly equal to the realized mortality rates in each scenario of this set, such that there is no macro-longevity risk.
3. In the scenario-set which isolates mortality risk, the mortality rates are based on Section 4 and the financial variables are deterministic based

on the geometric average values over all scenarios.

## Population

A stylized population is used for the simulations in this thesis. All cohorts have size 100,000 at the age of 25, the age at which everyone starts working until retirement. For ages older than 25, the cohort sizes depend on the relevant mortality rates. So, let the number of living individuals at time  $t$  of the cohort that has birth year  $c$  in scenario  $i$  be given by  $\ell_{c,i}^t$ . Then it follows from the above that  $\ell_{c,i}^{c+25} = 100,000$  and the other cohort sizes are computed by  $\forall t, \ell_{c,i}^{t+1} = \ell_{c,i}^t \cdot (1 - q_{t-c,i}^t)$  with age  $x = t - c$ , the age of the cohort with birth year  $c$  at time  $t$ , such that the scenario-dependent mortality rate determines the size of the cohorts in each scenario.

Note that by construction the population and cohort sizes are continuous variables instead of discrete variables. As the sizes of the cohorts become very small at higher ages, this is an unrealistic way of modeling the cohorts. Some uncertainty that follows from the fact that a discrete number of individuals die out of a cohort every year instead of the continuous number  $\ell_{c,i}^t \cdot q_{t-c,i}^t$  is therefore not taken into account here.

All individuals are fully employed in the period before the pension age. The wage of a worker finding him- or herself in scenario  $i$  and in year  $t$  is given by  $W_i^t$ , independent of the age  $x$ . So note that there is no career profile in which older (more experienced) workers earn more than the younger ones in the same period  $t$ . In the first year  $t = 0$  then,  $W_i^0 = \text{€}35,000$  in every scenario for working cohorts. Thereafter, wages rise yearly with the wage inflation such that  $W_i^{t+1} = W_i^t \cdot \pi_{t,i}^w$ . That is, the wage increase from year  $t - 1$  to year  $t$  only depends on the scenario-dependent  $\pi_{t,i}^w$ . The franchise level starts in 2015 at  $F_i^0 = \text{€}12,642$  and similarly rises according to the wage inflation:  $F_i^{t+1} = F_i^t \cdot \pi_{t,i}^w$ . The pensionable income therefore starts at  $W_i^0 - F_i^0 = \text{€}22,358$  and is similarly related to the wage inflation.

In the collective contract it is assumed that in the years before the simulation starts, rights have been accrued at 2% of the pensionable income and the funding ratio at the start is 128%, corresponding to the funding ratio

that the collective contract simulated below realizes in the long term. The idea behind this is that it allows for intergenerational risk sharing from the beginning of the simulation. Then there is no need for waiting a hundred years or so before the pension fund has accumulated enough money.

### Pension policy

For simplicity, the pension age is set fixed at  $x_R = 70$  years old.<sup>15</sup>

All contracts in this study will be DC contracts with uniform contribution rate  $p_{DC}$ , such that the contribution payments of all individuals in period  $t$  are determined by  $p_{DC} \cdot (W_i^t - F_i^t)$ .

For the purpose of selecting the adequate level of the pension benefits, there will be values for  $E_t(C_{c,i}^x)$  which, for each cohort  $c$ , gives the total nominal value of the benefit payments which, from the perspective of the pension fund at time  $t$ , is expected to be paid to the cohort at the ages  $x = x_R, \dots, 121$ . This will be based on the current capital available, the expected survival probabilities, and the discount rates. Moreover, the selected adjustment mechanism to shocks may be relevant in what the values of  $E_t(C_{c,i}^x)$  are. This will become clear below.

In order to determine the 'funding ratio' of the pension fund in each year  $t$ , which is the value of the assets at time  $t$  divided by the present value of the liabilities at time  $t$ , the present value of the 'liabilities' (that is the present value of  $E_t(C_{c,i}^x)$ ) has to be computed.<sup>16</sup> For this purpose, discount rates at time  $t$  with maturity  $h$  have to be determined, i.e.  $r_{t,i}^h$ , by the pension policy. The contracts simulated in this study will have constant discount rates  $r$  independent of  $t$ ,  $h$ , and  $i$ . The reason for this is that a constant discount rate results in a pension result which is stable, i.e. the *median* replacement ratios and in particular the *median* relative yearly changes are relatively constant

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<sup>15</sup>An AOW age of about 70 is an estimate of what the new regulations will implicate for people that are 25 in the year 2015, which is based on on-line computation tools such as <http://www.aowleeftijdberekenen.nl>.

<sup>16</sup>Note that in a PPR, there are no true liabilities and neither is there a true funding ratio. However, it is useful to assume a certain level of liabilities and to compute the corresponding funding ratio, for the purpose of determining the adequate level of the pension benefits.

over the pension period if an appropriate constant discount rate is selected.<sup>17</sup> This works mainly because all simulated PPR contracts use a constant investment strategy over the pension period.

The target funding ratio for all contracts is 100% and if the actual funding ratio deviates from the target funding ratio, the rights will be adjusted in the first step of the simulation towards improving the funding ratio, as will be explained below in Section 5.2. Of this adjustment mechanism several things may be specified. First, the parameter of recovery  $m$  may be selected, where  $m = 1$  indicates that all rights are adjusted equally and immediately towards recovery of the funding ratio, and a higher  $m$  implies a more gradual adjustment of rights, i.e. a smoother path of adjustment at the cost of relatively high adjustments in the future. Secondly, if there is such a smoothing path the adjustment may be linear or asymptotic. This is illustrated in Figure 5 for the example where  $m = 10$  and the funding ratio falls 10% short of the target for the maturities until 20. Then, linear AFS implies that the nearest rights are cut 1%, then 2%, et cetera until 10%, while an asymptotic AFS smoothly reaches the cut of 10% in the long term. The values depicted in Figure 5 are relative to the other maturities, but are in practice adjusted by a constant such that the target funding ratio is realized (see also equation (23) in Section 5.2). Finally, the mechanism of adjustment may be either 'open' (future right accruals are affected, may be selected in the case of a collective contract) or 'closed' (only accrued rights are adjusted, standard in a PPR).

To determine an appropriate level of the rights, the future biometric returns must be estimated, which are determined by the future mortality rates. As the mortality rates may be scenario-dependent, computations are made with expected mortality rates as the mortality rates in the determin-

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<sup>17</sup>Simulations that have been done with the nominal interest curve determining the discount rates yield high expected and median fluctuations of the pension results, both negative and positive, depending on the age and the set-up of the particular simulated contract. Arguably, the fact that I use these flat discount rates is unrealistic in the sense that in practice pension funds have to abide strict regulations concerning the discount rates they use. But, with a new occupational pension system such as the PPR, also the regulations with respect to discount rates are likely to change and it seems better to select discount rates which contribute to a stable pension.

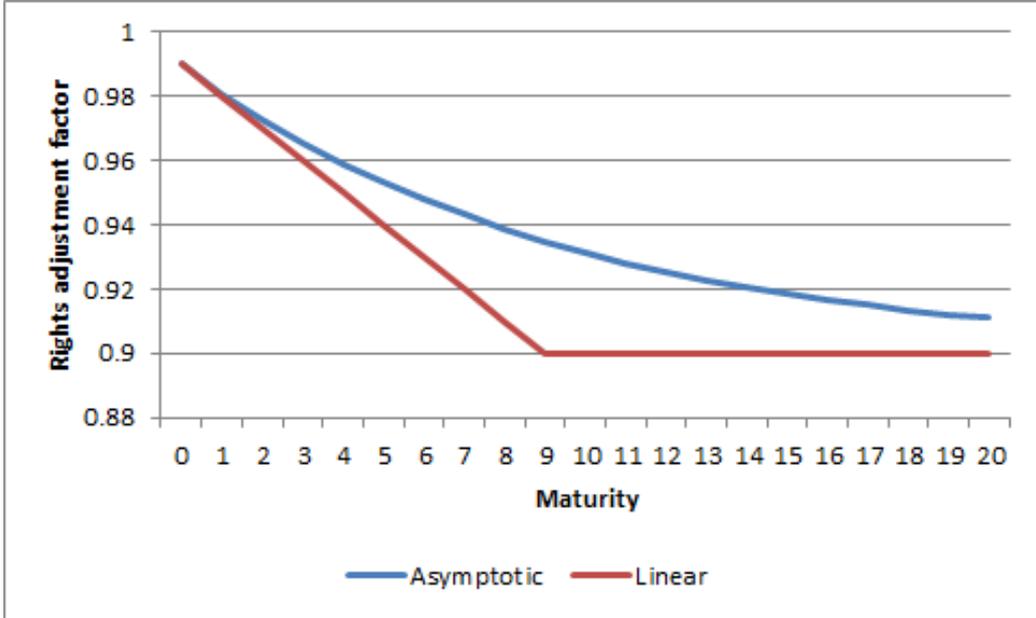


Figure 5: Factors of rights adjustment for a linear and an asymptotic adjustment to shocks, in the case that  $m = 10$  and the funding ratio is 10 percentage points below its target.

istic scenario-set (see Section 4), which are determined at  $t = 0$ .

This raises the question whether, as the years go by, the mortality table with which future payments are estimated has to be adjusted in the light of the realized mortality and survival rates in the given scenario, which may diverge from the expected mortality table based on the median over all scenarios. As a first attempt, the MATLAB model now allows for this in a very simple way. Then in the year  $t$  and scenario  $i$ , the logit survival probabilities of the future are adjusted to the extent to which the logit survival probabilities realized in year  $t$  diverge from the ones expected at  $t = 0$ .

A more sophisticated approach of updating the projected mortality rates would be to use the parameter estimates  $\hat{\alpha}_x$  and  $\hat{\beta}_x$  which underly the scenario set and estimate the structural mortality process  $f_{t,i}^{(m)}$  based on all mortality rates for  $x < 91$ . Another approach would then be taken for the older ages.

In the PPR a life-cycle investment policy can be chosen, such that

$w_{t,c}^{stocks}$  and  $w_{t,c}^{bonds}$  are the weight of the capital  $K_{c,i}^t$  invested in stocks and bonds, respectively. This investment policy will be taken to be a function of the age of the cohort,  $t - c$ . In the collective contract, only a constant investment strategy  $w^{stocks}$  and  $w^{bonds}$  is chosen in the simulations of this thesis such that  $w^{stocks} = w^{bonds} = 0.5$ .<sup>18</sup>

## 5.2 Simulation

The cash flows of the pension fund are simulated for every particular scenario  $i$  over the time horizon  $t = 1, \dots, T$ . For every year  $t$ , four steps are executed (Michielsens, 2015).

### 5.2.1 Step 1: Rights adjustment.

The financial returns of period  $t - 1$  have been realized such that the value of the assets  $K_{c,i}^t$  is given (see Step 4), and the current population size  $\ell_{c,i}^t$  is now also given, which may deviate from  $E_{t-1}(\ell_{c,i}^t)$ , the population size which was expected in the previous period. The rights  $E_{t-1}(C_{c,i}^x)$  that were determined in period  $t - 1$  have to be adjusted if the funding ratio is not equal to the target funding ratio (100%). This adjustment will lead to the rights  $E_t^{adj}(C_{c,i}^x)$ .

In the case of a PPR the funding ratio,  $FR_{c,i}^t$ , must be computed for all cohorts alive at time  $t$ . In a collective scheme, there is just one funding ratio at time  $t$ ,  $FR_i^t$ , that covers the projected cash flows of all cohorts at the same time. It is computed by dividing the assets by the liabilities which are discounted by the discount rates.

There will be rights adjustment factors,  $RA_{c,i}^{t,h}$ , depending on the maturities  $h$  of the rights, cohort-dependent for the PPR (such that the corresponding age  $t + h - c \in [x_R, 121]$ ) and a cohort-independent factor  $RA_{c,i}^{t,h} = RA_i^{t,h}$  for the collective contract. All corresponding rights will be

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<sup>18</sup>In practice, of course, a pension fund may base its investment strategy on characteristics of its participants, say the ratio of the pensioned to the working people in the fund.

adjusted through being multiplied by this factor plus one:

$$E_t^{adj}(C_{c,i}^x) = E_{t-1}(C_{c,i}^x) \cdot \left(1 + RA_{c,i}^{t,h}\right), \quad (21)$$

where  $x = t + h - c$  is the age corresponding to rights with maturity  $h$  at time  $t$  of cohort  $c$ , computed for all  $h = c + x_R - t, \dots, c + 121 - t$  with  $h \geq 0$ .

How  $RA_{c,i}^{t,h}$  is determined, depends on the pension policy (see Section 5.1), that is on the parameter of rights adjustment  $m$  and on whether the adjustment mechanism to shocks (AFS) is open or closed. Then, for the open AFS which only applies for the collective contract,

$$RA_i^{t,h,open} = \frac{FR_i^t - FR^{target}}{m} - 1, \quad (22)$$

for all maturities  $h$ . This is called an Open AFS because the rights adjustments made at time  $t$  are only  $1/m$ -th of what is necessary to reach the target funding ratio. That is, in the following years, more adjustments will have to be made and these will also affect future right accruals. So, all rights that will be accrued in the years from  $t$ , including those of cohorts that haven't even started working, will be influenced by the deviation of the funding ratio in year  $t$ .

Note that (22) implies an *asymptotic* adjustment mechanism, such that the measures that will have to be taken in the periods after  $t$  (do not confuse this with the particular rights of these periods) recover another  $1/m$ -th of the funding ratio deficit or surplus that is encountered in those periods. Assuming that no new shocks occur, this process will ensure that in the limit the funding ratio is completely recovered. Only in a *linear* adjustment mechanism the term  $m$  would be adequately referred to as 'recovery period', because then after a shock the funding ratio will be recovered toward the target ratio after exactly  $m$  periods conditional on the assumption that no new shocks occur in those periods. In the simulations in this essay, no linear adjustment mechanisms are used.

In case of a Closed AFS, which will standardly be used for the PPR, rights adjustments made at time  $t$  (which may affect all future rights which

have already been accrued at this point) will recover the funding ratio to its target immediately. Then for all  $h \geq 0$ ,

$$\begin{aligned}
RA_{c,i}^{t,h,closed} &= \frac{\sum_{x=\max\{t-c,x_R\}}^{121} [E_{t-1}(C_{c,i}^x)/(1+r)^{c+x-t}]}{\sum_{x=\max\{t-c,x_R\}}^{121} [\kappa_m(c+x-t+1) \cdot E_{t-1}(C_{c,i}^x)/(1+r)^{c+x-t}]} \\
&\quad \times \kappa_m(h) \cdot \frac{FR_i^t - FR^{target}}{FR^{target}}, \tag{23}
\end{aligned}$$

provides the rights adjustment factor for the rights with maturity  $h$  (corresponding with the age  $x = t+h-c$ ). The factor  $\kappa_m(h) := \left(1 - \left(1 - \frac{1}{m}\right)^{h+1}\right)$  determines the adjustment factor for the rights of maturity  $h$  which increases with  $h$ . As all adjustments made should suffice for recovering the funding ratio, the first fraction in (23) ensures that the weighted average of the values of  $RA_{c,i}^{t,h,closed}$ , weighted by the present value of the corresponding rights, is equal to  $\frac{FR_i^t - FR^{target}}{FR^{target}}$ . A similar equation can be given for the collective contract, in which the summation in the first term of the right-hand side of (23) ranges over over all cohorts and in which  $RA$  is not cohort-specific but refers to the entire fund.

## 5.2.2 Step 2: Right accruals

The way new rights are quantified on the basis of the contributions in the year  $t$  is the same for a PPR as for a CDC contract.

For the working cohorts (such that  $c + x_R > t$ ) the total pension contributions in year  $t$  are given by  $\ell_{c,i}^t \cdot p_{DC} \cdot (W_i^t - F_i^t)$ . These are then transformed into additional rights  $E_t^{accr}(C_{c,i}^x)$  for those cohorts  $c$ . So note that these additional rights will be determined after the already accumulated rights have been adjusted in the previous step.

Rights will be accrued in an actuarially fair manner.<sup>19</sup> That is, the

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<sup>19</sup>Which is contrary to the average pension premium system (*doorsneesystematiek*) common in the Netherlands, in which all right accruals are independent of one's age and do

same nominal level of contributions leads to relatively high accrual of rights for the younger workers because their capital can be invested for a longer time than the capital of cohorts which are closer to their pension age.

For every working cohort  $c$  it is computed what the present value is of promising a person of cohort  $c$  a yearly payment of € 1 for the duration of his pension period. For this purpose a value of  ${}_s\hat{p}_{t-c}^t$ , the expected probability that a member of cohort  $c$  alive at time  $t$  will still be alive at the beginning of the year  $t+s$ , has to be computed for all  $s$  such that  $t+s \in \{c+x_R, \dots, c+121\}$ . Indeed, the promise of paying someone € 1 in the future is proportional to the probability that he or she is still alive at that time. Of course this expected survival rate will be based on the expected mortality rates, which are either updated yearly according to some rule (in which they depend on the scenario  $i$ ), or which stay the same during the simulation. Then the corresponding present value is given by

$$PV_{t,c}^{\text{€}1} = \sum_{s=c+x_R-t}^{c+121-t} {}_s\hat{p}_{t-c,i}^t \cdot \frac{1}{(r_{i,t}^s)^s}. \quad (24)$$

Hence the amount of euros the pension fund can promise to pay yearly on the basis of the contributions added in year  $t$  is equal to the total accumulated contributions per  $c$  in  $t$  divided by this present value; this number will then be the newly accrued rights  $E_t^{accr}(C_{c,i}^x)$  for all  $c, i$  and pension ages  $x$ .

### 5.2.3 Step 3: Contribution and benefit payments.

In this step the asset levels  $K_{c,i}^t$  are changed according to the contributions and benefits in this period. So this means that the contributions  $\ell_{c,i}^t \cdot p_{DC} \cdot (W_i^t - F_i^t)$  are added to the capital of the cohorts which are working and thus are in the accumulation phase. For the cohorts in the decumulation phase, i.e. the cohorts that are pensioned but not yet extinct, benefit payments are made. Based on the rights which apply to  $t$ , that is, based on

$$E_t(C_{c,i}^x) = E_t^{adj}(C_{c,i}^x) + E_t^{accr}(C_{c,i}^x), \quad (25)$$

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not take the investment horizons into account.

for which  $t = c + x$ , each individual of the cohort will be paid  $CP_{c,i}^x = E_t(C_{c,i}^x) / \ell_{c,i}^t$ .

#### 5.2.4 Step 4: Realization of financial shocks.

The returns on stocks and bonds, i.e.  $R_{t,i}^{stocks}$  and  $R_{t,i}^{bonds}$  respectively, will change the value of what is left of the capital  $K_{c,i}^t$ . So, then, the new capital per cohort will be given by

$$K_{c,i}^{t+1} = K_{c,i}^t \cdot (w_{t,c}^{bonds} \cdot (1 + R_{t,i}^{bonds}) + w_{t,c}^{stocks} \cdot (1 + R_{t,i}^{stocks})), \quad (26)$$

as input for Steps 1-4 in period  $t + 1$ .

#### 5.2.5 Outcome variables

After Steps 1-4 have been executed for the entire time horizon, the outcome variables are computed. The cohort of interest in all contracts is the one born in 1990, which starts working in 2015. For every year in the pension period, the bandwidth and the median of the replacement ratio and of the year-to-year variability are computed. Moreover, the statistics of the weighted average of the replacement ratios, weighted by the survival probabilities are computed. In the case of the PPR, also the median and the bandwidth of the real value of the pension account is computed for both the working and the retirement period.

## 6 Contracts and results

All simulated pension contracts are DC contracts, in the sense that all workers pay a fixed percentage,  $p_{DC} \cdot 100\%$ , of their earned wage as a premium for the pension fund, with  $p_{DC} = 0.18$ .

In the *Benchmark PPR* contract, there is a constant life-cycle investment strategy in which 50% of the capital is invested in stocks and the other 50% is invested in 5-year bonds, i.e.  $w_{t,c}^{stocks} = w_{t,c}^{bonds} = 0.5$  for all  $t$  and  $c$ . In its basic form, benefit levels are immediately adjusted to financial and

demographic shocks in the PPR contracts.

There will be one CDC contract, with an *Open* AFS and parameter of adjustment  $m = 10$  (see Section 5.2). The statistics for the weighted average replacement ratios in Table 1 show that the CDC with an open AFS is superior to one with a closed AFS. Moreover, an open AFS corresponds to the current collective contracts in the Netherlands. In the CDC contract, the same investment strategy as in the Benchmark PPR is used, with 50% of the assets invested in stocks, and the other 50% of the assets invested in bonds.

Discount rates of the CDC contract and of the Benchmark PPR are set at 2.29% for all maturities, which corresponds to the expected real return on capital given a 50/50 investment strategy. This realizes the goal of these contracts to provide stable pension benefits that rise yearly with the nominal wage inflation.

For the purpose of illustration and as a first step in developing an appropriate life-cycle for the PPR, there will be an *Extreme PPR* contract. In this contract, during the working period, 100% of the capital is invested in stocks, and during retirement the capital is for 100% invested in 5-year bonds. In Extreme PPR, the discount rate is set at 0.82% for all maturities, corresponding to the expected real return given the investment strategy in the pension period.

Figures 6 and 12 show the outcome variables for the Benchmark PPR and the Extreme PPR. From Figure 6 it is clear that the Extreme PPR leads to a much more uncertain pension result than the Benchmark PPR due to the aggressive investment strategy in the working period, but with a higher median and mean in return for that (see also Table 1). By the conservative investment strategy in the pension period, however, the Extreme PPR succeeds in significantly limiting the year-to-year variability in the pension payouts compared to the Benchmark PPR, as is also depicted in Figure 6. Then, Figure 12 shows how the pension capital develops for these two contracts. Indeed, in the Extreme PPR one usually has more capital on one's account, but the bandwidth of this result is very uncertain. So, the Extreme PPR is an improvement over the Benchmark PPR with respect to the ex-

pected pension result and with respect to the year-to-year variability of the pension payouts, however the higher median comes at the cost of a lot more uncertainty in the overall pension result.<sup>20</sup>

The *Life-cycle PPR* is the result of a trial-and-error procedure, with

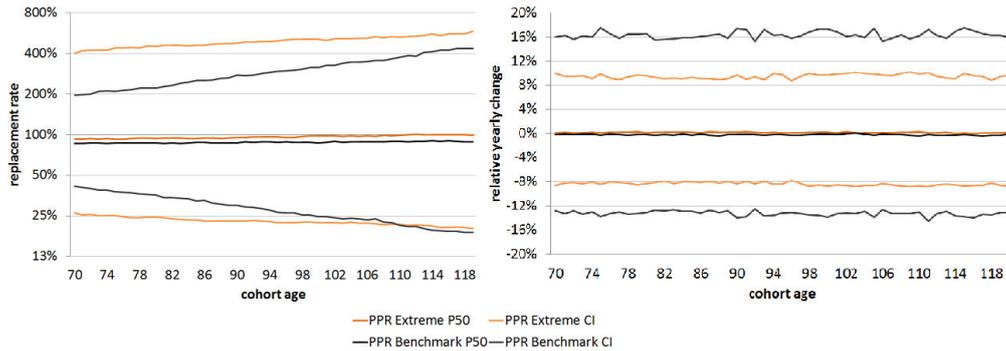


Figure 6: Bandwidth of the replacement ratios (left) and of the relative yearly changes (right) over the pension period: PPR Benchmark vs PPR Extreme.

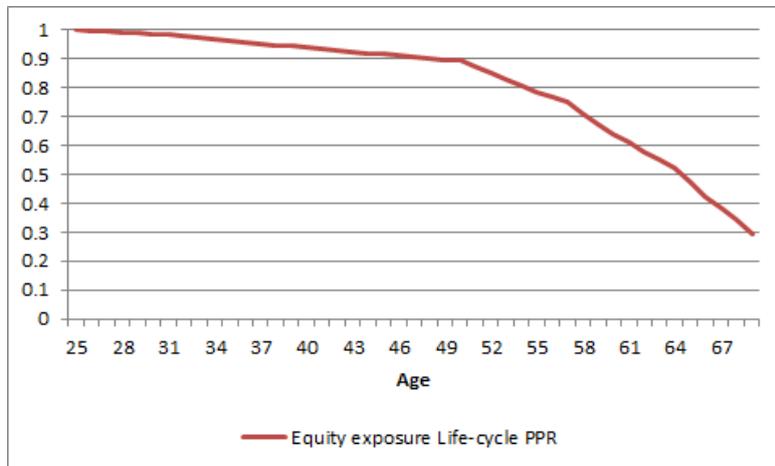


Figure 7: Exposure to equity in the Life-cycle PPR during the working period.

<sup>20</sup>Note that because neither the medians nor the uncertainty level is matched between PPR Benchmark and PPR Extreme, no comprehensive comparison can be given. As noted, PPR Extreme only serves as a simple illustration of what happens when there is a very basic life-cycle investment strategy in the PPR and should not be regarded as a set-up of the PPR that is seriously proposed.

the goal of realizing the same median pension result as the CDC Open contract. The exposure to equity over the working period is given in Figure 7; at an increasing rate of decline, less equity exposure is taken during the working period. During the pension period, the fraction of capital invested in stocks is held constant at 25%. The discount rate is set at 1.47%, which is deliberately taken smaller than the expected real return such that the median path of the replacement ratios approximates that of the CDC Open. That Life-cycle PPR is successful in matching the medians of CDC Open can be seen in Figure 8. This allows for a comparison between PPR and CDC based on the uncertainty.

For ages higher than about 85, the ex ante bandwidth of the replacement ratios is larger in CDC Open than in Life-cycle PPR as is given in Figure 8. For the younger ages, there is more uncertainty in the case of the Life-cycle PPR. Interestingly, though, the yearly variability in the level of the pension benefits, as shown in Figure 8 is consistently higher in the case of the Life-cycle PPR. Both these phenomena can be explained by the fact that the collective contract uses an open adjustment mechanism with recovery parameter  $m = 10$ . That is, contrary to the life-cycle PPR which does not use a recovery period, in CDC Open, a cut (addition) in the pension benefits will be followed by more cuts (additions) in the years to come. Therefore variations in the pension benefits are likely to be of the same direction in subsequent years in the collective contract, while this is not the case in this particular set-up of the PPR.

Table 1 shows that the range of the weighted average replacement ratios of CDC Open is slightly smaller than that of the Life-cycle PPR, i.e. there is more uncertainty in the Life-cycle PPR with respect to the weighted average pension result. If this seems surprising given Figure 8, note that the earlier ages, in which the bandwidth of CDC Open is smaller than that of Life-cycle PPR, are assigned the highest weights because the probability of surviving until that age is obviously higher than the probability of becoming even older. However, the differences in Table 1 are not huge, with a couple percentage points in the lower bounds, and only 13.7 percentage points in the upper 99%-bound, which is not impressive relative to these levels being

	CDC open	CDC closed	Life-cycle	Benchmark	Extreme
Mean	125.7%	121.4%	127.1%	105.5%	150.1%
P99	548.7%	518.2%	562.4%	324%	812.7%
P97.5	394.7%	377.2%	409.9%	262.5%	580%
P95	301.7%	290.5%	310.7%	223.7%	452.5%
Median	93.5%	90.4%	93.6%	88.0%	94.4%
P5	36.6%	34.6%	33.7%	37.6%	24.6%
P2.5	32.0%	30.4%	29.2%	33.2%	19.8%
P1	26.9%	25.5%	24.8%	28.3%	15.7%

Table 1: Statistics for the weighted average of the replacement ratios, CDC Open, CDC closed, Life-cycle PPR, Benchmark PPR, and Extreme PPR

about 550%. This leaves open the possibility that a change of the life-cycle might improve the performance of the PPR closer towards that of the CDC Open. Moreover, Table 1 shows a slight advantage for the Life-cycle PPR in terms of the median and and in particular the mean weighted average replacement ratios.

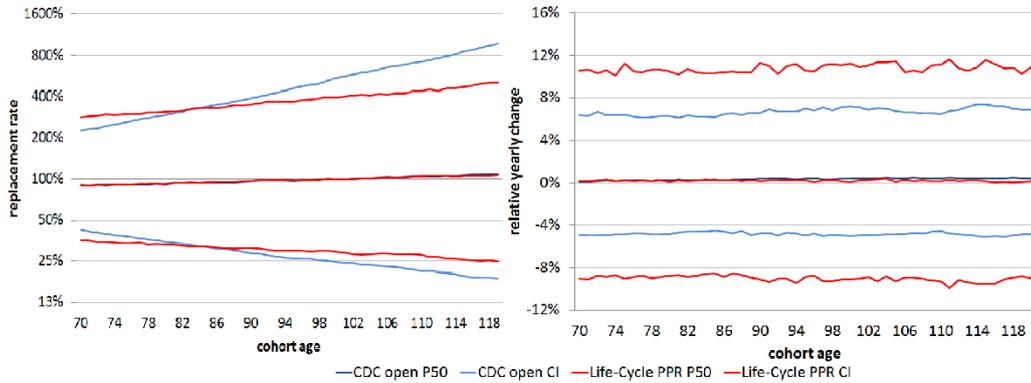


Figure 8: Bandwidth of the replacement ratios (left) and the yearly changes over the pension period: Life-cycle PPR versus CDC Open.

In Figure 9 the results are plotted for CDC Open and for Life-cycle PPR with the additional measure of the parameter of adjustment being  $m = 2$ . The bandwidth of the replacement ratios for this particular PPR set-up is, after the age of 85, closer to that of the CDC Open than of the Life-cycle PPR with no smoothing in the adjustment mechanism in Figure

8. Note that the median replacement ratio of Life-cycle PPR with  $m = 2$  is increasing faster than CDC Open. The reason for this is that on average the real pension benefit levels are slightly increased on a yearly basis, just as happens in Figure 8, but as this happens more gradually with a gradual adjustment mechanism these benefits are transferred to the higher ages, leading to a higher median and more uncertainty there. Also note that reserving more capital for later instead of immediately adjusting the pension benefit levels allows for more returns on that capital, which may partly explain the rising pattern of the pension benefits. Also the yearly variability is limited by the measure of  $m = 2$  compared to  $m = 1$  (Life-cycle PPR), but this only holds for ages until about 100, after which the bandwidth starts to increase. These effects will be more pronounced for a higher  $m$ , such that even though the relative yearly changes may be limited by the factor of adjustment in the early phase of the pension period, this uncertainty is always transferred to the higher ages for that same cohort.

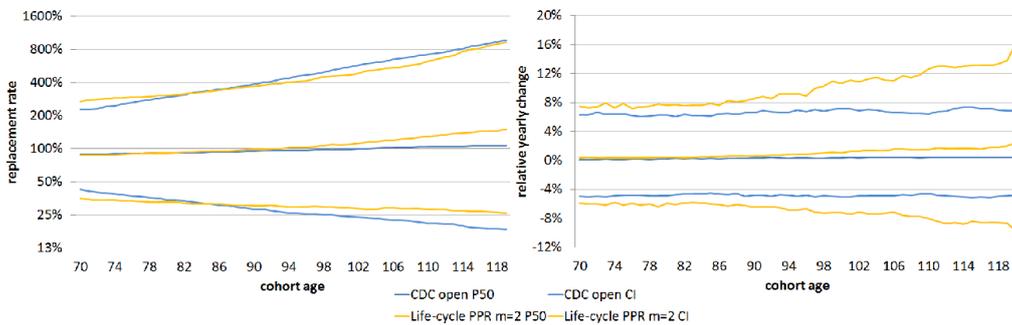


Figure 9: Bandwidth of the replacement ratios (left) and the yearly changes over the pension period: Life-cycle PPR with  $m = 2$  versus CDC Open.

## Financial versus Macro-longevity Risk

The second question posed in the introduction was, how macro-longevity risk in a PPR compares against the financial risks. Figures 10, 11, and 13 show the results for the PPR benchmark contract in the scenario covering all risks which is also used for the results above and the two scenario-sets which isolate either financial or mortality risk. Lets first compare the two scenario-

sets which isolate one of the risks. In the scenario-set with only financial risks, the bandwidth is strikingly wider compared to the scenario set with only mortality risk. In Figure 10, the maximum size of the 90%-bandwidths of the replacement ratio in the case of mortality risk is about 34%, while in the case of financial risk it is even more than 400%. The relative yearly changes due to mortality risk are very small compared to the shocks in pension benefits due to financial risk, as can clearly be seen in Figure 11. Also, the development of the pension capital is almost perfectly predictable in a world without financial risk, being close to €400k at retirement, but with only financial risk (and without mortality risk) the corresponding real value of one's pension account is likely to be anywhere between 160k and 1200k (Figure 13). Also at the higher ages during retirement, where mortality becomes a factor of interest, the uncertainty due to mortality risk is quite small. The curves for the simulations that include all risks are hardly distinguish-

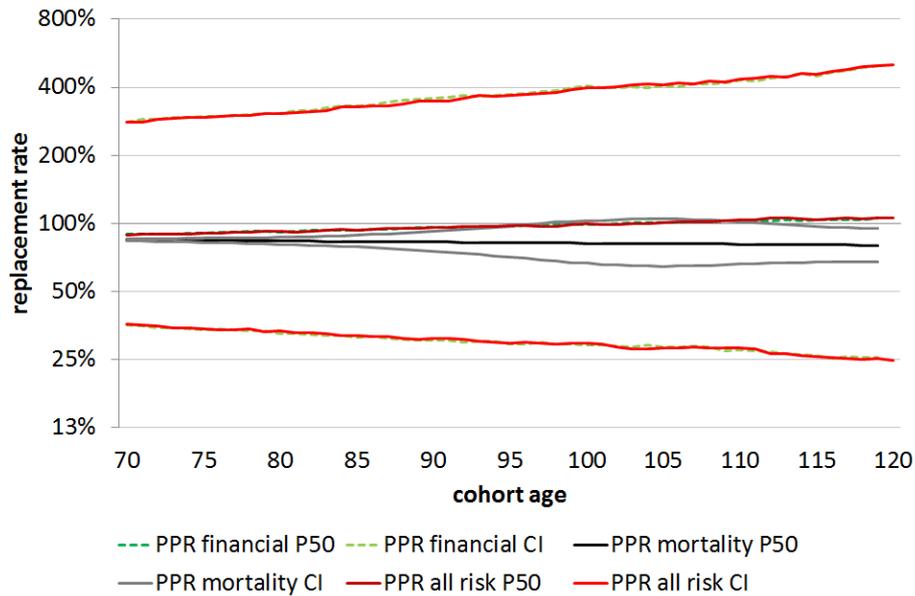


Figure 10: The bandwidth of replacement rates over the pension period for the life-cycle PPR contract: all vs mortality vs financial risks.

able from those of the scenario-set with only financial risk in Figures 10, 11, and 13. Notice how, for instance, the highest absolute relative yearly changes of the pension benefits due to mortality risk in Figure 11 around the

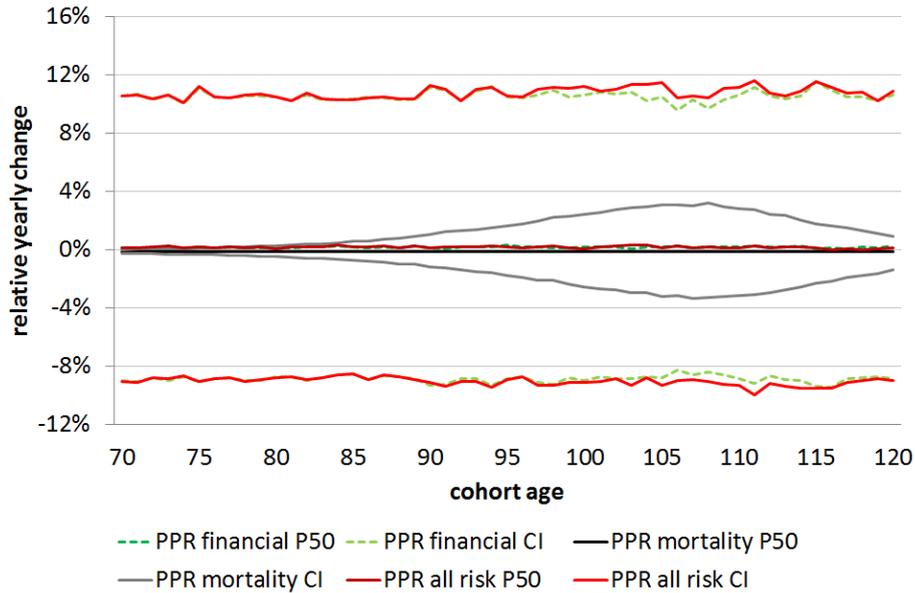


Figure 11: Relative yearly pension benefit changes of the life-cycle PPR contract: all vs mortality vs financial risks

ages 106-109 do not have a strong correlate in the curve which combines the financial and mortality risks.

One would expect that the simulations which include both the financial as well as the mortality risks, show a clear increase in risks compared to the financial risk only results. But it turns out that this addition of mortality risk produces little change. On the basis of the scenario-set used in this paper and in particular its mortality model, therefore, mortality risk does not seem to be a serious issue for a PPR. Using an updating rule for the mortality table with which future payments are estimated (which was briefly discussed in Section 5) is therefore not top priority.

## 7 Conclusion

All conclusions that can be drawn from this study are relative to the scenario-set that was used.

No evidence has been delivered that a PPR is able to beat a collective

contract with an open AFS in terms of having lower risks with the same median pension results. The Life-cycle PPR in this essay is slightly riskier in terms of the bandwidth of the weighted average pension result. Moreover, the year-to-year variability of a PPR is quite a bit higher than that of a collective contract. The use of a non-immediate mechanism of adjustment to shocks is not very promising (but this also should not be discarded right away), since this shifts part of the year-to-year variability and the uncertainty in the replacement ratios from the early ages of retirement to the older ages. However, it is at least conceivable that a more sophisticated life-cycle of stocks and bonds could beat the PPR. There may also be chance for the PPR if other asset classes are allowed for, such as products which insure interest rate risk.

On the basis of the scenarios and the simulations in this study, the financial risks are more important than the mortality risks for a PPR in terms of their effects on the bandwidth of the replacement ratios over the pension period and on the year-to-year variability of pension benefits. Suppose that the PPR will be implemented into the second pillar, but with the intention to limit the year-to-year variability through additional measures for intergenerational risk sharing. Then this suggests that in particular the sharing of the financial risks is worth looking into, while the sharing of macro-longevity risk seems a less promising approach.

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## A Development of the PPR capital: Figures

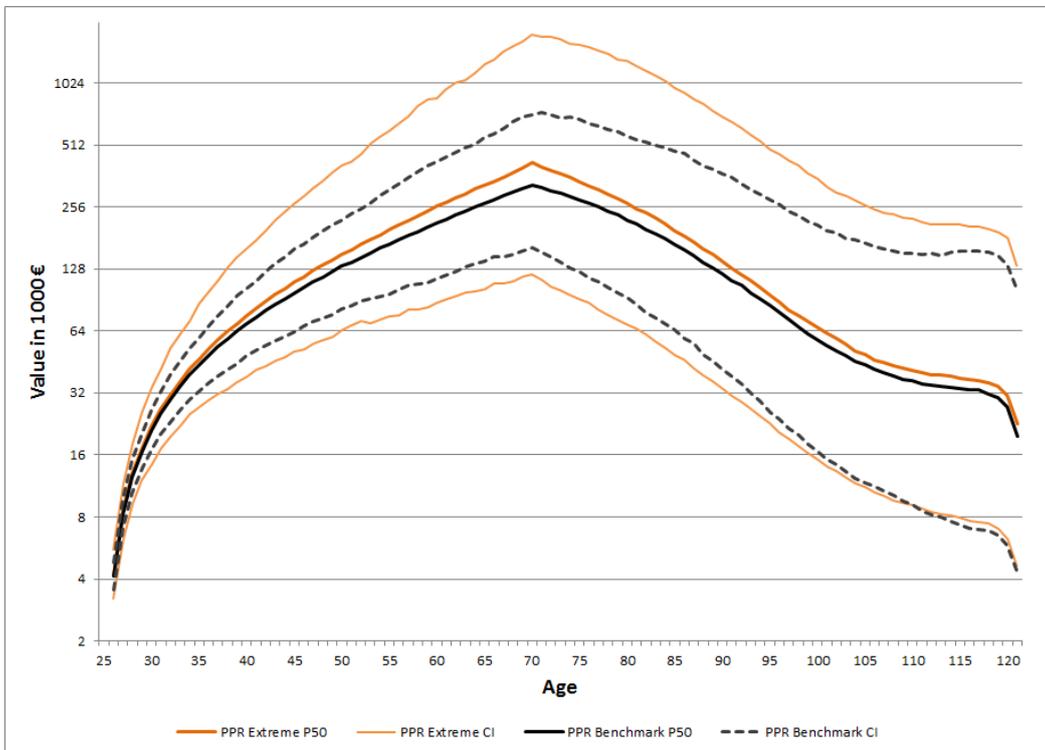


Figure 12: Bandwidth of the pension capital: PPR Benchmark (red) vs PPR Extreme.

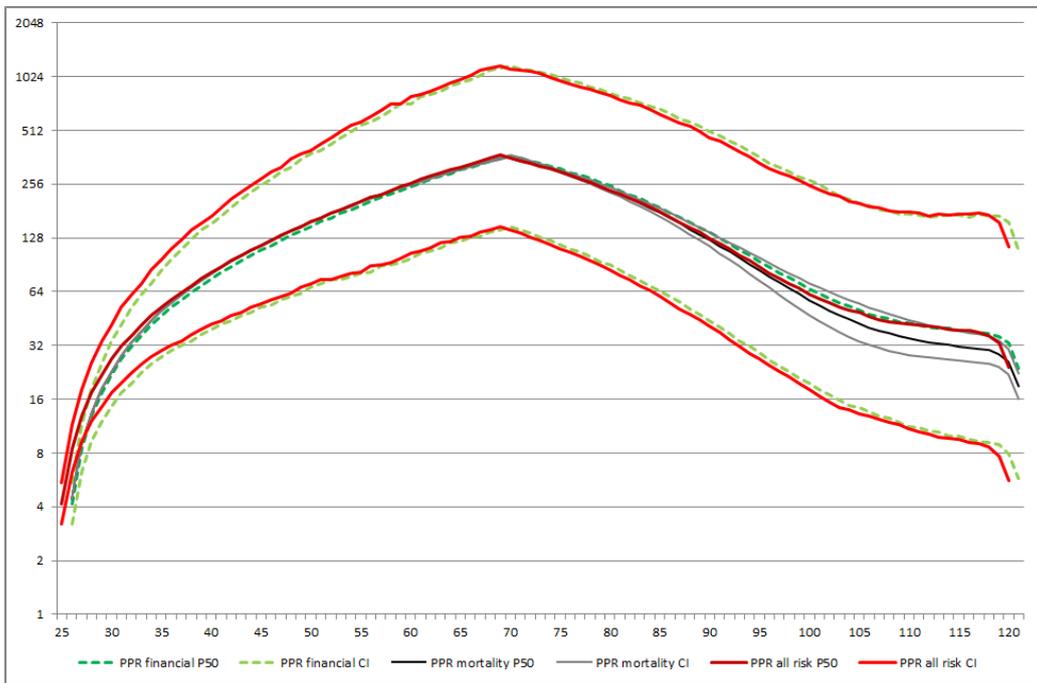


Figure 13: The bandwidth of the pension capital over the pension period for the PPR Benchmark contract: all (red) vs financial (green) vs mortality risks.