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Comparison between Nominal and Real Indexation Systems of Dutch Pension Fund

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Comparison Between Nominal and Real Indexation Systems of Dutch Pension Fund

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Abstract

This thesis compares the nominal and real indexation mechanisms in the Dutch pension system. To measure the asset and liability of the pension fund, a term structure is needed. We adopt the Vasicek model to estimate the current term structure. Then, we simulate the future stochastic term structures and wage index as well as the return on the assets. Using these simulation results as inputs, we establish a stylized pension fund and investigate how the pension assets, liabilities, and pension rights of the retirees evolve over time under both the nominal indexation system and the real indexation system. Furthermore, we compare the replacement rates and pension assets under both systems to see whether one system is superior to the other.

Keywords: nominal and real indexation, stylized pension fund, funding ratio, pension rights

Contents

1 Introduction

At the end of 2012, Dutch pension funds in total had 1 002.5 billion EURO assets which was 167% of Dutch GDP. The amount of total assets held by Dutch pension funds was 710 billion in 2008, 745 billion in 2009, 802 billion in 2010, and 872 billion in 2011[1]. In fact, since 2001, the pension fund assets enjoy an annual growth rate of 9.2% (DNB & Towers Watson). The large asset size and the fact that pension fund performance has a direct effect on retiree welfare require high attention on the pension system from both the academic and the political side. The recent financial crisis, the historically low interest rate in the latest decade, and population aging also put the Dutch pension system at stake. Many pension funds are facing solvency problems. At the end of 2008, the funding ratio of most of the pension funds fell below the threshold of 105% [8]. Underfunding will not only impede intergenerational fairness but also increase the discontinuity risks [3]. The Labor Foundation react to this by revising the regulatory framework and proposing a new pension agreement – the real indexation mechanism[2].

Nowadays, there are many debates about the fairness and rationality of the current nominal indexation mechanism. When the current FTK was introduced, it showed strong affinity with solvency [7], but later its weaknesses emerged. As shown by van Rooij, et al.[12], from both the transparency and economic perspective, the indexation ambition of the pension funds is problematic. They also gave some policy alternatives of the current regulation framework. Van Stalborch [13] also showed that the current FTK is not sustainable and fair across generations. Besides, as argued by many people [9], under the nominal indexation system, retirees can only enjoy the benefits of wage increase when the funding ratio is above a certain level. This is very discouraging for the young generation. After several decades, when they get retired, no one can promise them some pension rights as a certain proportion of wage at that time. This threatens the ambition of the Dutch pension fund to provide insurance against wage inflation [1]. In fact, one reason of the existence of defined benefit pension fund is to provide insurance against wage or price inflation and thus against standard-of-living risk after retirement [11]. The proposed 'soft real rights' under the real indexation mechanism intends to provide an inflation-proof right for pension fund participants. Under the real indexation mechanism, the pension rights are unconditionally indexed to wage inflation, although adjustments have to be made over a ten-year period when the assets and liabilities of the pension fund do not match. Thus, shocks in the

[1]DNB, T8.1.1 Summary balance sheet of pension funds

[2]The Social and Economic Council of the Netherlands(SER)

financial markets are smoothed and they will only hit the actual pension income of retirees after ten years [9]. Nijman, et al.[9] show that the smoothing of the new system generates a life cycle feature in that younger participants are more exposed to financial shocks than the old. They also derive a market consistent discount curve as the nominal interest rate minus expected inflation plus a horizon-dependent risk premium. Assuming constant inflation, S.M. van Stalborch [13] found that the value transfer between generations when switching from the current contract to a soft real contract depends on the length of the adjustment periods and fund specific characters like the duration of the fund liabilities. The real system is very appealing in the sense that it provides the participants with an insurance against the wage and price increase when they get retired. However, whether the new real agreement is superior to the nominal one is still not clear.

In this paper, we are going to do some simulations for both the nominal and the real agreements and investigate the strong and weak points of both mechanisms. First of all, I will introduce the current Dutch pension system and the two indexation mechanisms in Section 2. Subsequently, in Section 3, we are going to estimate the term structure with a Vasicek model using Germany Federal Security rate as a proxy for the risk free interest rate. In addition to the one-factor Vasicek model, we also estimate a two-factor Vasicek model with inflation and real interest rate as the two factors. The two-factor Vasicek model provides with better simulation results as shown in Section 4. Using these simulated short term interest rates and the estimated parameters in Section 3, we are able to simulate the long term stochastic term structure, which will be used in the simulation of pension fund liability in Section 5. We assume the pension fund holds an asset mix of zero-coupon bonds and stocks. Return on the zero-coupon bonds is derived from the stochastic term structure and return on stocks is simulated using a VAR model. Since pension rights need to be indexed to wage inflation and price inflation under the real system, we also need to simulate the evolvement of these two series which are obtained by two auto-regressive models. In Section 5, we establish a stylized pension fund mimicking the Dutch demography structure. We establish one model for each indexation system and study how the pension assets, pension liabilities, and replacement rate evolves over time. We then group those variables by wage index and by total return index to compare these two systems in different inflation scenarios and in different types of financial markets. We find that all generations benefit from a bull financial market while the effects of inflation on pension assets, replacement rates are not very significant. The pension fund assets, replacement rates are high when asset return is high, ceteris paribus. Besides, compared to the nominal system, the real system transfer value to the old

generation. In the early years, replacement rates under the real system are higher compared to those under the nominal system but the differences become smaller as time pass by. The replacement rates are actually converging under the two systems. So the current retirees and the relative old generations under the real system are better off and the young generations that just enter into the job market are indifference between those two systems in terms of replacement rate. The value of assets under the nominal system is larger and is getting even larger as time pass by. So, the nominal pension system is transferring value to the young generation indefinitely. Who will eventually get the value is ambiguous.

2 Dutch Pension system

The Dutch pension system consists of three pillars. The first pillar is the state pension which is the foundation of the pension benefits. The amount of the state pension is linked to the minimum wage. This pillar is financed as a pay-as-you-go system. What the retirees get is paid by the current workers. The second pillar is the collective pension system. It is a fully funded system. The employer and employee pay contributions and those contributions will be invested in a pension fund. There are more than 90% of the total employees belonging to a certain pension fund. The third pillar is also fully funded. It is formed by individual pension products. The individual pension plan is one example.[3]

The Financial Assessment Framework (FTK) is part of the Pension Act. The current FTK came into effect from 1 January 2007. Under the current FTK, the indexation of pension rights is conditional based on the nominal funding ratio which is the ratio of nominal assets to nominal liabilities. Since the current framework met some threats recently, the Labour Foundation proposed some adjustments on the current framework and designed a new agreement to maintain the sustainability of the Dutch pension system. The new agreement is the so called Pension Accord Spring 2010[4]. In the new agreement, indexation is applied unconditionally. When there is a shock resulting in lower assets than liabilities, 10 deductions will be made in the following 10 years to bring the funding ratio back to 100%. Similarly, if there is a positive shock, 10 additions will be made. On July 12, 2013, the Council of Ministers approved the new agreement. Recently, after consultations, the government has announced that it will come up with a "middle way" consisting of elements of both earlier proposed nominal and real contract[5]. Right now, the legislative process remains to be completed. According to the schedule, the new pension contracts will come into effect by early 2015[6].

[3]The Dutch Pension System: an overview of the key aspects. From Pension Federatie

[4]The Social and Economic Council of the Netherlands(SER)

[5]News from IPE, 04 Oct 2013

[6]News from Pension Federatie

3 Estimating the Term Structure of Interest Rate

3.1 Introduction to Vasicek Model

The risk-free interest rate is very fundamental and important in financial markets. There are extensive studies on this topic and numerous models have been put forward to describe the evolution of the interest rate like the Vasicek model[14], Cox Ingersoll Ross model[5] and Hull-White model[6]. Empirical studies like Chan, et al.[4] compared different kinds of continuous-time models of the short-term risk-free rate using GMM and found that models allowing for volatility change of interest rate to be highly related to the level of the risk-free rate are better in capturing the dynamics of the short-term interest rate. Here, I use the Vasicek model to estimate the term structure of interest rate and then use the predicted yield to calculate the pension liabilities. The reason to choose the Vasicek model over others is that the Vasicek model is the fundamental model and it is easy to implement. Recent studies also show that Vasicek model provides a good description of the yield curve, see [10] and [15]. The basic Vasicek model is a one-factor short-rate model which can be specified in continuous time by the following stochastic differential equation[7]:

$$dr_t = a(b - r_t)dt + \sigma dW_t, \tag{1}$$

where W_t is a Brownian motion and b is the long run nominal interest rate and r_t is the current interest rate. This model exhibits mean reversion of interest rate and $a \leq 0$ measures the speed of mean reversion. However, empirically, we only have data observed discretely. In fact, the data we are going to use in this thesis are all monthly data. To facilitate the empirical study, we express this model in discrete time. This method is adapted by many researchers such as K. C. Chan, et al.. Following them, I estimate the parameters of the continuous-time model using the following discrete-time econometric specification where the short-term interest rate is given by an auto-regressive model:

$$r_{t+1} = \overline{r} + \phi(r_t - \overline{r}) + \epsilon^r_{t+1}, \quad \epsilon^r_{t+1} \overset{i.i.d}{\sim} (0, \sigma^2_r), \quad |\phi| < 1 \tag{2}$$

where $\overline{r}$ is the average short-term interest rate and r_t is the short-term interest rate at time t. Taking expectation on both sides of equation (2), we have

$$E_t(r_{t+1}) = \overline{r} + \phi(r_t - \overline{r}) = r_t + (1 - \phi)(\overline{r} - r_t). \tag{3}$$

[7]Financial models chapter 4. P121

and by recursive substitution,

$$E_t(r_{t+\tau}) = \overline{r} + \phi^{\tau}(r_t - \overline{r}). \tag{4}$$

The term structure is then obtained using the expectation hypothesis:

$$\begin{aligned}
\exp(TR_t) &= \prod_{\tau=0}^{T-1} \exp(E_t(r_{t+\tau})) \\
&\text{so,} \\
R_t &= \frac{1}{T}\sum_{\tau=0}^{T-1} E_t(r_{t+\tau}) \\
&= \frac{1}{T}(T\overline{r} + (1+\phi+\phi^2+\cdots+\phi^{T-1})(r_t - \overline{r})) \\
&= \overline{r} + \frac{1-\phi^T}{T(1-\phi)}(r_t - \overline{r}) \\
&= \overline{r} + b_T(r - \overline{r}), \quad \text{with} \quad b_T \equiv \frac{1-\phi^T}{T(1-\phi)}.
\end{aligned} \tag{5}$$

This is the case when there is no risk premium. When considering long term interest rate, a risk premium must be taken into account. The long term interest rate is a function of the parameters ϕ, the current interest rate r_t, the average interest rate $\overline{r}$, time to maturity T, risk premium λ, and other parameters that are function of α and β. The following equation describes their relation[8].

$$\begin{aligned}
R_t^T = \overline{r} + b_T(r_t - \overline{r}) + \gamma_r^T = \overline{r} + \frac{1-\phi^T}{T(1-\phi)}(r_t - \overline{r}) + \lambda_r\sigma_r\frac{1}{T}\sum_{\tau=0}^{T-1} B_\tau \\
\text{with} \quad B_T = T \times b_T.
\end{aligned} \tag{6}$$

The one-factor Vasicek model is simple to implement. But the fact that it only has one state variable also restricts the shape of the term structure. This one-factor model can be easily extended to a multiple-factor model. Here, we investigates the two-factor Vasicek model which takes both risk-free real interest rate and inflation into consideration and allows for a non-monotonic term structure. Similar to the one-factor model, a discrete time model is constructed to get parameter estimates.

[8]see details in appendix

The one-period ahead inflation is given by an autoregressive model:

$$\pi_{t+1} = \overline{\pi} + \psi(\pi_t - \overline{\pi}) + \varepsilon^{\pi}_{t+1}; \tag{7}$$

$$\varepsilon^{\pi}_{t+1} \overset{i.i.d}{\sim} (0, \sigma^2_{\pi}), \quad |\psi| < 1 \tag{8}$$

The real interest rate is also described by an autoregressive model as in equation (12). But we don't have real interest rate at hand. To get the real interest rate, we impose the Fisher hypothesis, which states

$$r^r_t = r_t - E_t(\pi_{t+1}) \tag{9}$$

where $E_t(\pi_{t+1})$ is the expected inflation given by the autoregressive model in equation (13). Then we use another AR(1) model to estimate the process of real interest rate. With the parameters estimated from those models, we can estimate the long-term interest rate. The long-term interest rate in a two-factor Vasicek model is given by the following equation[9]:

$$\tilde{R^T_t} = \overline{r}^r + b_T(r^r_t - \overline{r}^r) + \gamma^T_r + \overline{\pi} + c_T(\pi_t - \overline{\pi}) + \gamma^T_{\pi}$$

where

$$\begin{aligned} b_T &= \frac{1-\phi^T}{T(1-\phi)}, \quad \gamma^T_r = \lambda_r \sigma_r \frac{1}{T} \sum_{\tau=0}^{T-1} B_\tau, \\ c_T &= \frac{\psi(1-\psi^T)}{T(1-\psi)}, \quad \gamma^T_{\pi} = \lambda_{\pi} \sigma_{\pi} \frac{1}{T} \sum_{\tau=0}^{T-1} C_\tau \end{aligned} \tag{10}$$

and

$$C_\tau = T c_\tau, \quad B_\tau = T b_\tau.$$

3.2 Data Description

Two series of data are used in estimating the term structure-inflation data and risk free rate. Here I use the listed Germany Federal securities (method by Svensson) with residual maturity of 0.5 years as the proxy for the risk free interest rate[10].

[9] see details in appendix

[10] The Svensson model is a parametric model that specifies a functional form for the spot rate as a function of time to maturity and several other parameters [11].

$$z(T) = \beta_0 + \beta_1 \left[\frac{1 - e^{-T/\tau_1}}{T/\tau_1} \right] + \beta_2 \left[\frac{1 - e^{-T/\tau_1}}{T/\tau_1} - e^{-T/\tau_1} \right] + \beta_3 \left[\frac{1 - e^{-T/\tau_2}}{T/\tau_2} - e^{-T/\tau_2} \right]$$

where $z(T)$ is the spot rate, T is the time to maturity and β_i, τ_i are parameters.

Inflation is derived from Calendar and Seasonally adjusted Germany Consumer Price Index. Both the inflation series and the risk free rate are monthly data and are downloaded from Deutsche Bundesbank[12]. The risk free rate series is available from 1972:09 to 2013:03 and the CPI data is available from 1948:06 to 2013:03. But during the first several years, the CPI data followed very different kind of process compared to now. To be consistent with the risk-free rate data, I only use data after 1972:09. Since the interest rate is annualized semi-annual data, we also calculate the annualized semi-annual inflation from the CPI data. The following equation is used to calculate the semi-annual inflation rate.

$$\text{infl}_t = 2 \times \ln \frac{\text{CPI}_t}{\text{CPI}_{t-6}} \tag{11}$$

The following table gives the statistical summarization of the two series. We can see clearly from the table both the two interest rates and inflation dropped in the subsample. This is mainly due to the high inflation and high interest rate during recessions in the early 1970s and early 1980s. Nominal interest rate was even double-digital during those periods. Although the full sample contain more information, its historically high inflation and interest rate makes it less representative.

	Period 1972:09-2013:03			
Variable	Mean	Medium	Std. Dev	No. of Obs.
Nominal Risk-free Rate	4.97%	4.48%	2.93%	487
Inflation	2.66%	2.20%	1.94%	487
Real Risk-free Rate	2.31%	2.38%	2.13%	487
	Period 1990:01-2013:03			
Nominal Risk-free Rate	3.73%	3.39%	2.54%	279
Inflation	1.90%	1.76%	1.16%	279
Real Risk-free Rate	1.88%	1.96%	2.10%	279

Table 1: Annualized Risk Free Rate and Inflation

[12]http://www.bundesbank.de/Navigation/EN/Statistics/statistics.html

3.3 Results

3.3.1 Regression Results

The parameters of the nominal interest rate model and inflation model are estimated by OLS from the following discrete-time systems of equations, respectively:

$$r_{t+1} = \alpha + \beta r_t + \epsilon^r_{t+1}; \quad \epsilon^r_{t+1} \overset{i.i.d}{\sim} (0, \sigma_r^2). \tag{12}$$

$$\pi_{t+1} = \iota + \kappa \pi_t + \epsilon^\pi_{t+1}; \quad \epsilon^\pi_{t+1} \overset{i.i.d}{\sim} (0, \sigma_\pi^2). \tag{13}$$

The parameters of the real interest rate model are also estimated by OLS using a auto-regressive regression similar to the one in (12). The parameters of the inflation model are estimated from (13). Here, the regression results of those models are reported in Table 2. The real and nominal interest rate in the subsample period of year 1990 to 2013 display higher speed of mean-reverting. However, the average nominal interest rate predicted by the AR(1) model is negative, which is not convincing in the long-period. So, the validity of the short nominal interest rate predicted by the one-factor Vasicek model is doubtful. This is also one reason to switch to the two-factor Vasicek model.

3.3.2 Risk Premium and Long-term Structure

Using the above data, the two AR(1) models as in equation (12) and (13) can be estimated. Since we are using monthly available data to estimate future semi-annual interest rate, we need to make some adjustment in the parameters in equation (6) and (7). The next period semi-annual interest rate is actually the 6-month later interest rate in our data. So, we should have $\phi = \beta^6$ and $\psi = \kappa^6$. Now, the only parameter unknown in the long term interest rate is the risk premium. The risk premium is obtained by minimizing the squared sum of the difference between the estimated interest rate and the corresponding observed interest rate and then calibrating with the observed interest rate. There are different ways to calculate the risk premium regarding the information we are going to use and we compare two possibilities here. In the first one, we use all the information at hand and also look at both historical and current term structure. The risk premium is given by

$$\hat{\lambda} = argmin \sum_t \sum_T (\hat{R_t^T} - R_t^T)^2, \tag{14}$$

where $\hat{R_t^T}$ is the estimated interest rate for maturity T at time t and R_t^T is the observed interest rate for maturity T at time t. This method has the advantage that it makes

	1972:09-2013:03		
	π	r^{real}	r
β or κ	0.949***	0.871***	0.946***
	(0.014)	(0.023)	(0.014)
Constant	0.001**	0.003***	0.002**
	(0.000)	(0.001)	(0.001)
R-squared	0.910	0.756	0.900
No.Obs	486	486	486
	1990:01-2013:03		
β or κ	0.873***	0.962***	0.995***
	(0.029)	(0.017)	(0.005)
Constant	0.001***	0.000	-0.000
	(0.000)	(0.000)	(-0.000)
R-squared	0.761	0.923	0.992
No.Obs	279	279	279
* $p < 0.05$,	**$p < 0.01$,	*** $p < 0.001$	

Table 2: Regression Results of Inflation, Nominal Interest Rate and Real Interest Rate using AR(1) model. Numbers in the brackets are standard errors.

full use of all the information and will take into account some extreme cases that happened in the past and maybe will happen in the future. It might be much more representative for a long time period.

In the second one, we only use the current benchmark term structure and the current estimated term structure. The risk premium is given by the following formula:

$$\hat{\lambda} = argmin \sum_{T} (\hat{R_c^T} - R_c^T)^2, \tag{15}$$

where $\hat{R_c^T}$ is the current estimated interest rate with maturity T and R_c^T is the current observed interest rate with maturity T. This method has the advantage that it gives the highest weight to the most recent information and will produce a term structure much similar to the benchmark term structure. In the following sections, we will denote all models using risk premiums estimated from the first method as model (a) and those using risk premiums estimated from the second method as model (b).

	$\hat{\lambda} = argmin \sum_t \sum_T (\hat{R_t^T} - R_t^T)^2$	
	One-factor Model	Two-factor Model
λ_r	19.5937	-
λ_r^{real}	-	1.2449
λ_π	-	0.7796
σ_r	0.2273	-
σ_r^{real}	-	0.6397
σ_π	-	0.2826
$\rho_{r,\pi}$	-	0.167
$\overline{r}$	3.73	-
$\overline{r^{real}}$	-	1.2764
$\overline{\pi}$	-	1.8540
ϕ	0.9715	0.7911
ψ	-	0.4428
	$\hat{\lambda} = argmin \sum_T (\hat{R_c^T} - R_c^T)^2$	
	One-factor Model	Two-factor Model
λ_r	55.1934	-
λ_r^{real}	-	1.1302
λ_π	-	-11.5094
σ_r	0.2273	-
σ_r^{real}	-	0.6397
σ_π	-	0.2826
$\rho_{r,\pi}$	-	0.167
$\overline{r}$	3.73	-
$\overline{r^{real}}$	-	1.2764
$\overline{\pi}$	-	1.8540
ϕ	0.9715	0.7911
ψ	-	0.4428

Table 3: Risk Premium Estimation Results

The estimation results of both models are shown in table 3. The estimated inflation risk premium using method (b) is negative, indicating the nominal interest rate in the long-run might be lower than the real interest rate. This is easily seen in figure 2. The sum of average inflation and the inflation risk premium in the long-run is in fact negative, which is not reasonable in the long run. This is one reason we are in favor of model (a) than model (b). The estimated term structures are shown in figure 1 to figure 4. In the one-factor Vasicek mode the estimated term structure is an increasing function of time to maturity as shown in figure 1 and figure 2. The estimated term structure is more similar to the observed term structure if we use the second method. We see the estimated term structure intersect with the observed one several times in figure 2. This is because the second method gives full weight to the current term structure and we minimize the differences between this estimated term structure and the observed one while the first method also gives equal weight to historical term structures. The two-factor Vasicek model gives us different estimation results. In figure 4, we can see that the nominal interest rate first decreases and then increases. Besides, we can see from those four figures, when we use method (b) to estimate the risk premium, the interest rate is much lower than the one when we using method (a) to estimate the term structure. This is because in the current year, the interest rate and the observed term structure are much lower than the historical ones.

4 Simulation

4.1 Simulated Short Rate and Inflation

4.1.1 Simulation Using One-Factor Model

In this part, we simulate the annualized 6-month treasury security rate and semi-annual inflation rate based on the model we specified. In the previous part, we estimated the two series in discrete time and specified them as two AR(1) processes. The error terms in the two models are assumed to be normally distributed. Using residuals from the two estimated models, we can get the variance-covariance matrix of the two series as a proxy for that of the error terms. The variance-covariance matrix for the full sample period 1972:09-2013:03 is $10^{-5} \times \begin{bmatrix} 8.447 & 0.069 \\ 0.069 & 0.840 \end{bmatrix}$ and the variance-covariance matrix for the sub-sample 1990:01-2013:03 is $10^{-5} \times \begin{bmatrix} 0.517 & 0.070 \\ 0.070 & 0.799 \end{bmatrix}$. Assume the error terms from these two models are jointly normally distributed with the above variance-covariance matrix, we simulate the future interest rate and semi-annual inflation.

Figure 5 shows the simulation results when we use the model estimated with data from 1972:09 to 2013:03 and simulated for 30 years. The numbers on the horizontal lines are the number of periods (months) we simulated and the numbers on the vertical line are the corresponding rate. The left figure shows one simulated path of the annualized 6-month Treasury Security Rate and 6-month Inflation Rate. The right figure shows the results when we simulate for 500 times. The red lines in the right figure are the average annualized 6-month Treasury Security Rate and 6-month Inflation Rate of 500 simulations. The blue lines are one standard deviation apart from the mean. As shown in the right figure, when we increase the number of simulations to 500, the average inflation rate/interest rate will converge to their long-run mean. The simulated average inflation in the last period is 2.46% and average interest rate is 4.5%. Their standard deviations are 1.8% and 2.85%, respectively. Correlation between the first simulated paths is 20.25%.

Figure 6 shows the simulation results when we use model estimated with subsample data from 1990:01 to 2013:03. Compared to the results in figure 5, the annualized 6-month Treasury Security Rate is now decreasing over time. This is due to the reason that when using subsample data, the constant term in the model is negative, resulting in negative long-run mean of interest rate. This phenomenon is owing to the interest rate drop after the financial crisis which may not be very representative.

Based on the simulation results, estimation using the full sample data is much more plausible. The simulated average inflation in the last period is 1.90% and average interest rate is -1.68%. Their standard deviations are 1.32% and 2.32%, respectively.

4.1.2 Simulation Using Two-Factor Model

The simulation results using the one-factor model is not very convincing. The one using the full sample data gives us low correlation between the two series and the one using sub sample data gives us negative short rate. So, we turn to the two-factor model. We first simulate the real 6-month interest rate and 6-month inflation. Then we calculate the nominal interest rate as the sum of these two series.

The real interest rate is calculated using historical nominal interest rate and inflation. In our case, since the interest rate is the annualized 6-month interest rate and inflation is annualized 6-month inflation rate, real interest rate should be $r_t^{real} = r_t - E_T(\pi_{t+1})$ where $E_t(\pi_{t+1})$ is the calculated as the fitted value of inflation from the estimated AR(1) model. As shown in figure 7. Compared to the first simulation results, using this method give us much lower interest rate. And the correlation between the first simulation of the two series is 51.55%, much higher than the previous one.

The simulated interest rate using the subsample data is much more reasonable than the previous one when we adopt this approach. The correlation between the first simulated paths is 54.46% and the long run interest rate remains positive as shown in figure 8.

4.2 Simulated Term Structure

As shown in the previous part, the two-factor model has better simulation results for the short rate and inflation. So, in this part, we use the simulated results of the two-factor model as the basic rate to simulate the stochastic term structure. We discussed in the Estimation part that there are two ways to estimate the risk premium. For method (a), $\hat{\lambda} = argmin \sum_T (\hat{R_c^T} - R_c^T)^2$ and for method (b), $\hat{\lambda} = argmin \sum_t \sum_T (\hat{R_t^T} - R_t^T)^2$. The estimated inflation premium and interest rate premium for model (a) are 0.78 and 1.24, respectively, and for model (b) they are -11.51 and 1.13, respectively. Using these estimated results and formula (10), we simulate the stochastic term structure for the two models. They are shown in figure 9. Similar to the current estimated term structure, if we use historical term structure in estimating the risk premium, interest rate is monotonically increasing with time to

maturity and long-term interest rate is much higher. The average interest rate after 30 years is around 6.5%. However, if risk premium is estimated only from the current data, in the first several years, interest rate is decreasing with time to maturity and long-term interest rate is much lower . The average interest rate is only 3.5% after 30 years as shown in figure 9(b).

4.3 Simulated Stock Return

In this part, I analyze the return characteristics of pension fund investment vehicles -stocks and bonds. Since pension funds invest globally, I use the global return index instead of local ones. The indices used are listed below where "E" indicates the corresponding index is in Euro and "$" indicates the corresponding index is in US dollar. The stock return index and exchange rate are monthly data provided by Thomson Reuters Datastream and the risk free rate is from Deutsche Bundesbank[13]:

- Stocks: S&P GLOBAL 1200 - TOT RETURN IND (SPGLOBL(RI)) ($)
- Exchange Rate: US $ TO EURO (WMR&DS) - EXCHANGE RATE
- Risk Free rate: Germany Federal securities (method by Svensson) with residual maturity of 0.5 years (E)

From the stock total return index, we can calculate the annualized 6-month stock return in local currency ($) using formula $r_{tl} = \ln(\text{Index}_t/\text{Index}_{t-6}) \times 2$. And the return in Euro can be derived from $r_t = (1 + r_{tl})(1 + r_{tx}) - 1$ where r_{tl} is return in the local currency at time t and r_{tx} the return from local currency appreciation (depreciation).

These series of data are used in simulating stock return. We use a VAR(1) model

	Period 1990:01-2013:03			
Asset Class	Mean	Medium	Std. Dev	No. of Obs.
Stocks	7.46%	9.898%	21.42%	270

Table 4: Return Characteristics of Asset Mix

to model the return on stocks. In addition to the stock return, the risk free rate and

	Dependent Variables		
Lag dependent variable	Stock	Risk-free rate	Inflation
Stocks	0,874	0,002	0,004
	(0,031)	(0,000)	(0,001)
Risk-free rate	0,747	0,995	0,066
	(0,438)	(0,007)	(0,017)
Inflation	−3,085	−0,004	0,772
	(0,954)	(0,015)	(0,037)
Constant	0,041	−0,000	0,002
	(0,017)	(0,000)	(0,001)

Table 5: Regression Results of VAR model

inflation also enter into the model. Estimation results of the VAR model are in table 5. The numbers in brackets are the standard errors.

The simulated stock return is shown in figure 10. On average, investments on stocks have an annual return of 10%. The red line in figure 10 shows the average stock return and the blue lines are the one standard devation.

4.4 Simulated Bond Return

The liability side of a pension fund are the benefits needed to be paid to retirees. For an employee who just enters into the job market, the benefits will come about 40 years later and last as long as the employee is alive. Thus, the liability of a pension fund has a very long duration and is influenced by mortality risk, inflation risk, interest rate risk, and other macro-economic risks. To match the long duration of liabilities, a pension fund usually has high exposure to bonds. Also, bonds can provide relatively stable cash flow and lower return volatility relative to stocks. In this part, we are going to simulate the bond return which is a main driving factor of the total return on the asset.

Bond returns are derived from the stochastic term structure. Assume a pension fund only invests in 10 year nominal zero-coupon bonds. At each year, bonds with remaining maturity of 9 years will be sold and new bonds with maturity 10 years will

[13]http://www.bundesbank.de/Navigation/EN/Statistics/statistics.html

be bought. The bond return is given by

$$\begin{aligned} r_{b,t} &= \ln(\tfrac{P_{9,t+1}}{P_{10,t}}) = \ln(P_{9,t+1}) - \ln(P_{10,t}) \\ &= \ln(e^{-9 \cdot r_{t+1,9}}) - \ln(e^{-10 \cdot r_{t,10}}) \\ &= 10 r_{t,10} - 9 r_{t+1,9} \end{aligned} \tag{16}$$

where $r_{t+1,9}$ and $r_{t,10}$ are yield with maturity 9 years at time $t+1$ and yield with maturity 9 years at time t from term structure model, respectively. Since there are two simulated results for the stochastic term structure, there are also two results for the simulated bond return. They are shown in figure 12. As expected, on average, the simulated return from model (a) is higher than that from model (b).

As seen in the introduction part, a typical pension fund invests roughly 65% of their assets in bonds and 35% in stocks. So, in our stylized pension fund, we also follow this asset distribution ratio. The asset return is the weighted average return on stocks and bonds. That is

$$r_t = \omega_s \times r_{s,t} + \omega_b \times r_{b,t}, \tag{17}$$

where $r_{s,t}$ is the simulated stock return and ω_s, ω_b are the weights of stocks and bonds in the total assets, respectively. The results are shown in figure 13. The medium asset return from model (a) is around 8% and return from model (b) is around 6.5%.

4.5 Simulated Wage Inflation

The wage index used in this paper is the Netherlands Gross Hourly Earnings (Total, including remuneration) provided by Thomson Reuters Datastream. It is monthly data adjusted for the price index but not seasonally adjusted. It is available from January of 1990. The average annualized 6-month wage inflation is 2.34%. We also use a AR(1) model to simulate the wage inflation. The simulated wage inflation is in figure 11.

5 Model

5.1 Stylized Pension Fund

We establish a stylized pension fund in this part and use the simulated results from the previous part to investigate how the pension fund's assets, liabilities and other relevant variables evolve over time under both the nominal indexation mechanism and the real indexation mechanism. From the population size data, we can calculate the dependency ratio of the Netherlands and then establish a stylized pension fund that has a similar dependency ratio. In 2010, the population size of those who are under 25 years old is around 4.94 million and for those 25 to 64 is around 9.10 million and for those above 65 years old is around 2.54 million.[14][15] The aged dependency ratio is around 27.91% and the children dependency ratio is around 54.32%. So, we establish a styled pension fund that has 7 generations with age difference 10 from age 10 to age 70. Each generation starts at an equal size of 100 000. In our model, the two dependency ratios are 25% and 50%, respectively.

The number of survivors of each generation evolves according to the Netherlands 2009 both-sexes life table downloaded from the Human Mortality Database. Denote the probability of death between age x and $x+1$ for generation i by q_x^i, then the one year survival probability is $1-q_x^i$. Total number of survivals in year t ($2013 \leq t \leq 2013+T_{max}$, where $T_{max}=110$) for generation i is $N_i^t = \prod_{2013}^{t} 100\,000(1-q_{t-2013+i}^i)$. The claims of the pension system participants are built up linearly for each generation, the liability for each generation should be multiplied by a factor $f_i^t = 0.025 \times (min(age_t^i, 65) - 25)$. Liability of pension fund is not only depending on the number of survivors and their pension right claims, but also depends on how much nominal benefits or entitlements each participant has. Based on the different policies to determine these benefits or entitlements, there are roughly two types of indexation mechanisms. The first one is the nominal indexation with immediate adjustment mechanism and the other one is the real indexation with smooth adjustment mechanism.

5.2 Nominal Indexation System

Under the nominal indexation system, indexation is decided annually on the basis of the nominal funding ratio which is the ratio of the pension asset and pension

[14]The exact numbers are 4 941 244 , 9 095 417 and 2 538 328, respectively.

[15]Data are from Human Mortality Database. http://www.mortality.org/

liability. The value of the pension liability is computed by discounting the accrued benefit obligations at the nominal interest rate and the pension asset includes the total return of the initial assets and the contributions made to the pension system. New contributions will increase the pension asset and paid out pension benefits will reduce the pension assets. Indexation is given according to a policy ladder. There is no indexation when the funding ratio is below 105% and when the funding ratio is above 120%, benefits and entitlements are fully indexed to wages. When the funding ratio is in between these two ratios, linear interpolation will be applied. The indexation decision can be given by formula (18).

$$I_t = I_{t-1} \times \begin{cases} 1 + \pi_t^{full}, & \text{if } FR_{t-1} \geq 120\% \\ 1 + \frac{FR_{t-1} - 105\%}{120\% - 105\%} \pi_t^{full}, & \text{if } 120\% > FR_{t-1} \geq 105\% \\ 1, & \text{if } FR_{t-1} < 105\% \end{cases} \tag{18}$$

The total liability in 2013 is given by:

$$L_{2013} = \sum_i f_{i,2013} \times \sum_{t=2013}^{T} N_{i,t} \times D_{i,t} \times 1 \times (1 + r_{2013,t-2013})^{2013-t} \tag{19}$$

where i stands for generation i and t is the future year t and T stands for maturity. And the projected liability in year s ($s > 2013$) under each scenario is given by:

$$L_s = I_s \times \sum_i f_{i,s} \times \sum_{t=s}^{T} N_{i,t} \times D_{i,t} \times (1 + r_{s,t-s})^{s-t} \tag{20}$$

where the meaning of the symbols is as follows:

- I_s is the indexation decision in year s under a certain scenario which is given by equation (18) in the nominal indexation mechanism. Its corresponding part in the real term is the pension right LB;
- $D_{i,t}$ is a dummy variable that takes the value 1 if generation i is retired in year t and 0 otherwise;
- $N_{i,t}$ is the number of survivors for generation i at the beginning of year t;
- $f_{i,s}$ is the multiplier given by $f_{i,s} = 0.025 \times (min(\text{age}_{i,s}, 65) - 25)$;
- c is the contribution rate which is a percentage of wage;

- WI_t is the wage index at the beginning of year t;
- A_t is the pension asset at the beginning of year t;
- L_s is the liability at the beginning of year s;
- FI_t is the nominal funding ratio at the beginning of year t;
- P_t is the total pension payment made at the beginning of year t;
- C_t is the total contribution received at the end of year t;
- ${}_sp_t$ is the s year survival rate at the beginning of year t;
- $r_{s,t-s}$ is the annual interest rate in year s for maturity $t-s$;
- $N25_t$ is the total number of survivals at the beginning of year t for those 25 years old in 2013.

Using data from Dutch life table and the estimated term structure, the total liability in 2013 of model (a) is 2 038 100. This is more than 5.6 times the total income produced by the population in 2013. For model (b), the total liability is 2 943 100.

Assets of the stylized pension fund come from two sources. Returns on the initial assets and the future contribution of the pensioners. Contribution rate is set such that the initial generational account for those who just enter into the pension system (the 25-year old) equals to zero when discounting their future benefits and contributions at the estimated 2013 term structure. That is

$$\sum_{t=1}^{40} \mathrm{c} \times N25_t \times \mathrm{WI}_t \times \text{discount factor}_t = \sum_{t=41}^{110} N25_t \times \mathrm{WI}_t \times \text{discount factor}_t. \quad (21)$$

Assuming benefits are 1 unit per year after retirement and contribution is a fixed percent of wage, the contribution rate is 19.44% for model (a) and 51.27% for model (b). The initial asset is set such that the initial funding ratio equals to 100%. So, the initial asset is equal to the projected liability in year 2013. $A_0 = L_{2013} = 2\,038\,100$ for model (a) and 2 943 100 for model (b). These numbers show that the initial assets and contribution rates are very sensitive to interest rate. As in model (b), the term structure is much more flatter than in model (a). Using these low interest rate to discount promised pension rights ends up with very high liability and higher contribution rate. Of course, a contribution that is more than half of the total salary is not

realistic and this is another reason we are in favor of model (a) than model (b).

I simulate the funding ratio for 30 years. At the beginning of year 2013, the initial funding ratio is 100%, therefore, there is no indexation in year 2014. At the beginning of year 2014, the initial assets generate return and contribution is made and benefit is paid. Funding ratio in year 2014 is calculated. Based on the realized funding ratio in year 2014, indexation decision is made for year 2015 and thereafter. This process is carried out for each scenario and for all the remaining time periods till the end. At each time point, liability L_t is the present value of the projected future liabilities and asset is given by

$$A_t = (A_{t-1} - P_{t-1})(1 + r_{t_1}) + C_{t-1} \tag{22}$$

where $C_t = \sum_i (1 - D_{i,t+1}) \cdot N_{i,t} \cdot {}_1p_t \cdot \mathrm{WI}_t$ and $P_t = \sum_i D_{i,t} \cdot N_{i,t} \cdot I_t$ are the contributions and payments made at time t. Assets at the beginning of year 2013 are set equal to the liability.

$$A_{2013} = L_{2013} \tag{23}$$

The average simulated funding ratio is given by Figure 14.

If we only look at the average funding ratio, the situation is very optimistic. We are on average having funding ratio above 1 and pensioners will not have to worry about their pension rights after retirement. But when we turn to the quantiles of the funding ratio, problems appear. From figure 15 we can see that in model (a), when long term interest rate is high, and the contribution rate is lower, there are 5% of the cases our pension fund is under funded. In model (b), when the long term interest rate is low and contribution rate is higher, although the funding ratio in year 30 is above 1, we still have 5% of the cases the pension fund is underfunded before year 2020. Besides, we know the contribution rate is more than 50% which is not realistic. So, in the following sections, we will only study situation when the risk premiums are estimated from model (a).

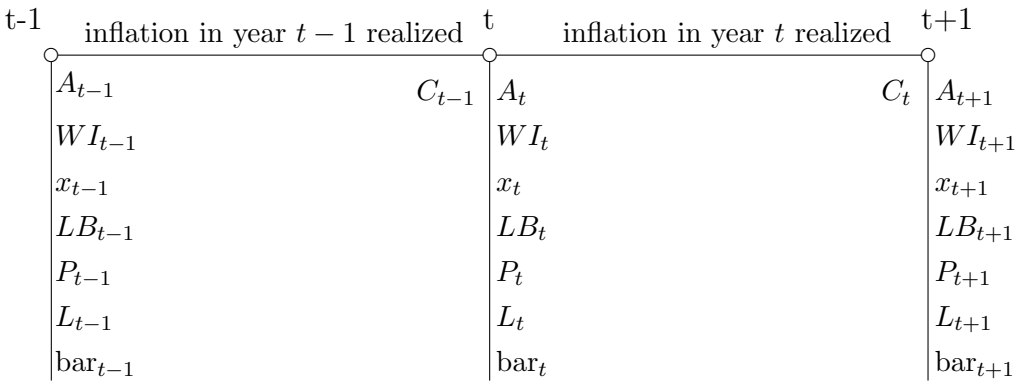
The simulated average assets and liabilities of the stylized pension fund are shown in figure 16. In model (a), the average pension fund assets become more than four times larger after 30 years while the total liabilities increase less than three times. In model (b), the pension fund starts with more assets and assets also increase faster. Assets increase for more than five times and liabilities only increases more than two times.

5.3 Real Indexation System

The alternative indexation system is the real liability with smooth adjustment indexation mechanism. In this system, the liability is discounted at risk-free real interest rate plus the risk premium reflecting the uncertainty in actual payment. Liability is fully indexed against wage inflation and price inflation. In our model, the initial asset is set equal to the fully indexed liability and contribution rate is calculated in the same way as in the nominal indexation mechanism. Based on the data and simulation results, we have $A_{2013} = 2\,256\,000$ and $c = 26.86\%$. In order to compare the two systems, we adjust the contribution rates c in the real system equal to the one in the nominal system ($c = 19.44\%$). The following information is about the meaning of the symbols that are going to be used later.

- x_t is the adjustment rate determined at the beginning of year t;
- LB_t is the pension right in Euro per retiree at the beginning of year t;
- $\mu_{t,s}$ is the adjusted interest rate at time t for maturity s;
- L_t is the adjusted liability of pension fund at the beginning of year t;
- L_t^{non} is the liability before adjustment at the beginning of year t;
- π_t^{full} is the full wage inflation in year t.

The following time line illustrates when the asset, liability, adjustments are determined and when the contribution and payments are made. In general, pension payments are made at the beginning of the year and contributions are received at the end of the year. The contribution rate is determined ex ante. Also, the pension right for each retiree in 2013 is predetermined to be 1 Euro. At the beginning of year t, the inflation during the year $t-1$ is realized. Based on that, total contribution for year $t-1$ is determined. The increase in asset at time t is the return on $A_{t-1} - P_{t-1}$ during year $t-1$ plus the contribution. As inflation is realized, wage index WI_t is also determined. Adjustment x_t is set to make the liability at time t –L_t equal to the asset A_t. Adjustment is also reflected in the immediate payment P_t and pension rights LB_t. Then, the adjustments that will be made in the following 10 years (bar) is updated to take the new adjustment x_t into account. In the following time line, all the variables below x_t will take the new adjustment x_t into account and all the variables above x_t are realized before x_t is determined.



At the beginning of 2013, assume the pension payment for each retiree is normalized to 1 Euro. The total projected liability is the sum of the discounted pension payment made to all the generations. For each generation, the annual payment in year $2013+s$ is the pension right of each retiree times the total number of retirees of that generation at time $2013+s$, which is given by formula (24).

$$\begin{aligned} & LB_{2013} \cdot f_{i,2013} \cdot N_{i,2013} \cdot_s p_{i,2013} \cdot D_{i,2013} \\ =& f_{i,2013} \cdot N_{i,2013} \cdot_s p_{i,2013} \cdot D_{i,2013} \end{aligned} \tag{24}$$

We need to discount those payments back and sum across generations. So the total liability in 2013 is

$$L_{2013} = \sum_i f_{i,2013} \cdot N_{i,2013} \sum_{s=0}^{T} ({}_sp_{i,2013} \cdot D_{i,2013+s}(1+\mu_{2013,s})^{-s}) \tag{25}$$

where T is the total number of periods. In our stylized pension fund, the youngest generation is 10-year old at this moment and the maximum age is 110, thus T is 100 in our model. Based on our data, the estimated liability at the beginning of year 2013 is 2,246,00 which is higher than that one in the nominal system.
By assumption, the initial funding ratio equals to 100%, so

$$A_{2013} = L_{2013} \tag{26}$$

To simplify the calculation, we introduce one variable 'bar' to denote the adjustments need to be made in the next 10 years.
Since the initial funding ratio is 100%, there is no adjustment. We have

$$\text{bar}_{2013}(j) = 0, j = 1, 2, \cdots, 10 \tag{27}$$

From 2014, there may be unbalance between asset and liability for the pension fund. If it is faced with a setback, 10 reductions will be made in the following 10 years. If it is faced with a positive shock, 10 additions will be made. The numbers on the bar denote the net adjustments to be made in the following 10 years.

$$\text{bar}_t(j) = \begin{cases} (1 + \text{bar}_{t-1}(j+1))(1+x_t) - 1, & j = 1, \cdots, 9 \\ x_t, & j = 10 \end{cases} \tag{28}$$

The pension payment per retiree is determined based on those adjustments and also on the realized full wage inflation. Retirees expect an increase in their pension rights when facing with a positive inflation.

$$LB_t = (1 + \text{bar}_t(1)) \cdot LB_{t-1} \cdot (1 + \pi_{t-1}^{full}) \tag{29}$$

The asset of the pension fund develops according to equation (30). Since payment is made at the beginning of a certain year, only the remaining part $A_{t-1} - P_{t-1}$ will enjoy a return during that year.

$$A_t = (A_{t-1} - P_{t-1}) \times (1 + r_{t-1}) + C_{t-1} \tag{30}$$

where r_{t-1} is the return on the asset during the year $t-1$ and C_{t-1} and $P_t - 1$ are the contributions and payments made in year $t-1$. Contribution is a percentage of the total income of all the workers which will be affected by the contribution rate c, total number of workers, and wage index WI. Since it is made at the end of a year, inflation during that year will be realized and will be reflected in the wage index.

$$\begin{aligned} C_t &= c \sum_i N_{i,t} \cdot_1 p_t(1 - D_{i,t+1}) \cdot WI_t \cdot (1 + \pi_t^{full}) \\ &= c \sum_i N_{i,t+1}(1 - D_{i,t+1}) \cdot WI_{t+1} \end{aligned} \tag{31}$$

where $WI_t = WI_{t-1} \times (1 + \pi_t^{full})$ is the wage index at time t. Payments are made at the beginning of a certain year and should take adjustments of that year into consideration. They are the adjusted pension right for each retiree times the total number of retirees.

$$P_t = LB_t \cdot \sum_i N_{i,t} \cdot D_{i,t} \tag{32}$$

Suppose we are at the beginning of year t, we want to calculate the liability at that moment. At that moment, we only know the numbers on the bars from the previous

period bar_{t-1} and new adjustments haven't been determined. But we can consider the liability as a function of the unknown adjustment x_t and make the adjusted liability equal to the asset to solve for x_t. The liability is the present value of all the future payments taking adjustments made in the previous years and the current year into account. The first payments will be paid out immediately and the second one will be paid out 1 year later, etc. The following formula is the present value of the payments in each year for generation i.

$$
\begin{aligned}
&\text{Year} \quad t+0 \\
&\qquad PL^i_{t+0} = N_{i,t} \cdot D_{i,t} \cdot f_{i,t} \cdot LB_t \\
&\text{Year} \quad t+1 \\
&\qquad PL^i_{t+1} = N_{i,t} \cdot_1 p_{i,t} \cdot D_{i,t+1} \cdot f_{i,t} \cdot (1+\mu_{t,1})^{-1} \cdot (1+bar_t(2)) \cdot LB_t \\
&\text{Year} \quad t+2 \\
&\qquad PL^i_{t+2} = N_{i,t} \cdot_2 p_{i,t} \cdot D_{i,t+1} \cdot f_{i,t} \cdot (1+\mu_{t,1})^{-1} \cdot (1+bar_t(2)) \cdot (1+bar_t(3)) \cdot LB_t \\
&\qquad\quad \cdots \\
&\qquad\qquad \cdots \\
&\text{Year} \quad t+9 \\
&\qquad PL^i_{t+9} = N_{i,t} \cdot_9 p_{i,t} \cdot D_{i,t+9} \cdot f_{i,t} \cdot (1+\mu_{t,9})^{-9} \cdot \prod_{j=2}^{j=10} (1+bar_t(j)) \cdot LB_t \\
&\text{Year} \quad t+k, k \geq 10 \\
&\qquad PL^i_{t+k} = N_{i,t} \cdot_k p_{i,t} \cdot D_{i,t+k} \cdot f_{i,t} \cdot (1+\mu_{t,k})^{-k} \cdot \prod_{j=2}^{j=10} (1+bar_t(j)) \cdot LB_t
\end{aligned}
\tag{33}
$$

The total adjusted liability at the beginning of year t is the sum of these projected liabilities for all generations and the adjustment made at time t is x_t, which is determined such that the liability after adjustment is equal to the value of the asset:

$$
L_t = \sum_i \sum_{s=0}^{T-(t-2013)} PL^i_{t+s} = A_t \tag{34}
$$

To simulate the funding ratio in this case, we also need to know the risk premium. According to Bovenberg, Nijman, and Werker, the premium is horizon dependent and is also function of the risk free rate, inflation, the asset allocation, the stock premium,

and the adjustment speed. It is as follows:

$$\mu_{t,s} = r^r_{t,s} + \omega\lambda(1 - \rho\frac{1-\rho^s}{s(1-\rho)}) \tag{35}$$

where the meaning of the symbols is as follows:

- $\mu_{t,s}$: discount rate for maturity s
- $r^r_{t,s}$: real discount rate for maturity s
- ω: fraction of assets invested in stocks, 0.35 in our model
- λ: risk premium for stocks (we set it equal to 3%)
- ρ: parameter indicating speed of adjustment, 0.1 in our model

The adjusted term structure is shown in figure 21. It is on average lower than the term structure in the nominal scheme given that the average inflation is higher than the risk premium in equation (35). The simulated funding ratio is indeed always equal to one as shown in figure 22. The average asset in this system increases for less than four times, which is slightly lower than in the nominal system.

Figure 24 shows the average adjustment made at the beginning of each year to make the pension liabilities match the pension assets. The adjustments are on average negative, indicating not full indexation as it promised. One reason is that we adjust the contribution rate and make it lower than the model predicted contribution rate. Figure 25 and 26 show the nominal and real level of benefit for each retiree as time passes. From figure 25 we can say that the nominal pension right is on average increasing over these 30 years. Even in the worst cases, nominal pension rights will increase from 1 Euro to around 2.2 Euro. The median of the pension rights is increasing monotonically during these 30 years to 3.7 Euro. Figure 26 is the real level of benefits after adjusting for inflation. It is the difference between the level of benefit and the expected inflation. From the figure we can see that the median of pension rights increases from 1 Euro to around 1.3 after 30 years. But it is not a monotonic function of time. There are still some drops during these years. The retirees on average benefit even after controlling for inflation. But there are still some extreme cases (5%) in which the retirees will get nothing from the pension system.

Figure 27 shows the total payment the pension system needs to pay out each year. From this figure we see jumps every 10 years. This is due to the fact that we have an age gap of 10 years between the generations. Every other 10 years, a new generation will be retired and begin to claim pension rights from the pension system. During the 10 year gaps, there are increasing and decreasing moments, which show the effect of inflation and adjustments.

Previously, we take the 3% as the premium of stock return. We also want to know the results if we use model based estimates of the stock return premium λ. It is the difference between the average stock return and the average risk free rate. From table 3 and table 10 we can calculate this difference which is 4.33%. When we use this estimator, we find similar results. The adjusted term structure is steeper than it used to be. The interest rate is much higher in the latter case. The rests are more or less the same. The results are shown in figure 28 to figure 32.

5.4 Comparison

In the nominal indexation system, the average funding ratio is increasing over time, which means the pension participants cannot fully benefit from the high return on the asset. If we compare this system with the real system in terms of pensioner's pension rights, the real system is better than the nominal system. To make the comparison more interesting, we would like to add some features from the real indexation system to the nominal indexation system, such that the pensioners under the nominal system can also benefit from a "rich" pension fund. To achieve this goal, we add another ladder to the indexation. Now the indexation decision becomes as follows:

$$I_t = I_{t-1} \times \begin{cases} 1 + 2 \times \pi_t^{full}, & \text{if } FR_{t-1} \geq 150\% \\ 1 + (1 + \frac{FR_{t-1} - 135\%}{150\% - 135\%}) \times \pi_t^{full}, & \text{if } 150\% > FR_{t-1} \geq 135\% \\ 1 + \pi_t^{full}, & \text{if } 135\% > FR_{t-1} \geq 120\% \\ 1 + \frac{FR_{t-1} - 105\%}{120\% - 105\%} \pi_t^{full}, & \text{if } 120\% > FR_{t-1} \geq 105\% \\ 1, & \text{if } FR_{t-1} < 105\% \end{cases} \tag{36}$$

Under this policy ladder, the indexation in the nominal system is not bounded by one anymore. When the funding ratio is higher than 120%, pension rights increase compared to the previous year would be higher than the full wage index. Under this policy ladder, if the funding ratio is still increasing over time and sufficiently high after some years, the replacement rate could be larger than one even if replacement rate in the beginning several years is below one. That implies a "speed up" effect in

the real pension rights/replacement rates when funding ratio passes the critical value of 120%.

In addition to the high funding ratio, the initial assets in the nominal system are lower than those in the real system. In order to make a fair comparison, we also increase the initial assets of the nominal system to the same level of the real system. With this adjustment, the initial funding ratio is 111% instead of 100% and the average funding ratio after 30 years is much lower than that of the original model. As shown in figure 14, the average funding ratio in the 30th year is around 134% instead of over 200% in the original model. Also, the funding ratio increasing speed is slower. After 8 years, the average funding ratio increases from 111% to 120% and in the remaining 22 years, it only increases by 14%. We see some fluctuations in the figure. The main reason is that, after 5 years, 15 years, 25 years, there will be one more generation getting retired. Before retirement, their projected liability is increasing over time because they are building up their pension rights before getting retired. After retirement, the only effect is the decreasing number of survivals. So the projected liability for that generation will be decreasing. The kink points are at the year they get retired. Figure 33 shows the quantiles of the funding ratio. Although there seems to be no problem from the average funding ratio graph, we still see some under funding cases from the quantiles of the funding ratio. The 5% quantile of the log funding ratio is -0.1, indicating a nominal funding ratio of 90.5%, much lower than the critical value of 105%.

The nominal pension rights of each retiree in the nominal system is actually the indexation in that period. So, the replacement rate is the ratio of the indexation decision over the wage index. Figure 34 show the quantiles of the replacement rate. Starting from 100%, the median replacement rate first decreases to around 85% after 15 years and then slowly increases to around 89% after 30 years. The 5% quantile of the replacement rate is decreasing over time to around 49% after 30 years. Although a median replacement rate below 100% indicates the retirees still cannot fully benefit from the wage inflation, compared to the replacement rates in the original model, the retirees are better off. From this figure we also see a "speed up" effect. When the funding ratio is above a certain level, the replacement rate will slowly increase.

To further compare the two systems, we look at the replacement rates and assets of the two systems in year 10, year 20, and year 30. Since we have 500 simulation paths, we have 500 observations of each variable in each year. We look at those cross-section data to compare the two systems. When we regress the variable under the nominal system on the corresponding variable under the real system, the coefficients and confidence intervals of the slope are reported in table 6. There is no constant term

in the regression. The coefficient is equivalent to the slope of the trend line if we plot the data in a two-dimension graph with the x-axis the variable under the real system and y-axis the corresponding variable under the nominal system. We see that the slope of the replacement rates are below one and increase over time. This indicates replacement rates in the real system are higher than in the nominal system but the differences between the two systems are decreasing. One explanation could be under the nominal system, the nominal funding ratios in the first decade are on average below 120%, so pension participants can not fully benefit from the wage inflation. This is also implied by figure 34. In the first decades, the replacement rate decreased by over 10%. However, in the real system, although there is small adjustment every year, the replacement rate remain relatively high. In the first decades, the median replacement rate only decreased by about 3% as shown in figure 26. So, the gap between these two systems in terms of replacement rate is enlarging in the first decade. But after the first decade, the average funding ratio in the nominal system is over 120% and pension participant can actually have higher indexation than the full wage inflation. This is similar to a positive adjustment in the real system. With average funding ratio over 120%, the average "adjustment" in the nominal system is also positive. So, replacement rate in the nominal system is catching up with that in the real system as implied by the slope of the trend lines. The slope of the assets are all above one and increasing over time. This indicates the nominal system has more assets than the real system and the differences are getting larger. This is mainly due to the asset accumulation in the first decades under the nominal system since return on assets are not distributed to the retirees as pension rights. Although the two systems are starting with equal amount of assets, the nominal system managed to accumulate more asset by distributing less to the retirees in the first decade. After the first decade, the nominal system begin to distribute more to the retirees. But they can still benefit from the high return on the higher amount of asset since pension payment is a relatively small part of asset compared to return on the total asset.

We also want to know the effect of inflation and asset return on the replacement rates and pension assets under both systems. So, we further group data into four groups according to the full wage index and total return index, respectively. Group decisions are made according to wage index or total return index quantiles. If the index is under the 25% quantile, data go to the low group and they go to the medium low group if the index is between 25% quantile and 50% quantile. Then the medium high group contain index between 50% quantile and 75% quantile and the remaining data go to the high group. Figure 35 to 40 show the comparison results when we group data according to wage index. Similar to the results in table 6, we observe the

Variables	Slope	Confidence Interval
Replacement Rate after 10 years	0.883	(0.877, 0.889)
Replacement Rate after 20 years	0.898	(0.889, 0.906)
Replacement Rate after 30 years	0.978	(0.965, 0.992)
Asset after 10 years	1.105	(1.104, 1.106)
Asset after 20 years	1.225	(1.219, 1.231)
Asset after 30 years	1.399	(1.386, 1.413)

Table 6: Slopes of trend lines. The independent variables are the variables under the real system and the dependent variables are the corresponding variables under the nominal system. There is no constant term in the regression.

slope of the trend lines are getting closer to one over time regardless which group the data fall into. Results observed in the full sample can also be applied to the subsample grouped by wage index. Besides, the differences among the four groups are very small. Inflation doesn't play a big role in our model.

Similarly, we also group data according to asset return. They are shown in figure 41 to 46. We observe similar slope changes over time as in table 6 despite of which group the data falling into. However, quite different from the results when the groups are classified by wage index, the differences among groups are very obvious here. First, both replacement rate and pension asset increase with total return index. We observe from figure 41 that the replacement rates after 30 years in the high return group vary from 90% to over 350% and from around 80% to 250% under the nominal system. But in the low return group, the range of the replacement rate under the real system is 30% to 110% under the real system and 30% to 100% under the nominal system. Similarly, assets in the high return group increase by 5 to 15 times after 30 years under the real system and 5 to 25 times under the nominal system. But those in the low return group increase less than 5 times under both systems. When total returns are high, pension assets under both systems will benefit from the bull financial market and increases at higher speed. Since asset return is the weighted average of bond return and stock return, we expect higher bond return or stock return when asset return is high. However, as shown in figure 35 to figure 40, effects of inflation on assets or replacement rates are not very big, since inflation is positively related to the nominal term structure and bond return is derived directly from the term structure, we would expect the higher asset return is mainly due to positive shock in the stock market. Stock market return doesn't have a direct effect on the liability

side of the pension fund. So, when asset return is high, the nominal systems will have higher funding ratio and in return the system will pay more to the retirees. This outweighs the effect of higher wage inflation and result in higher replacement rate. Under the real system, higher asset would imply positive adjustment and thus higher replacement as well. Second, the distinctions between the real system and the nominal system is the largest in the high return group. The trend lines in the high return group in figure 41 to 46 are more apart from the diagonal lines. High return magnify the differences between the two systems because although we add one more step of policy ladder to the nominal indexation system, we still have an indexation cap of 2 in nominal system with 3-step policy ladders. When the asset return is sufficiently high, the funding ratio could be above 150%. As shown in figure 33, the 75% quantiles of the log funding ratio is above 0.4 after 30 years, implying a nominal funding ratio of 149%. In these situations, the "adjustment" of pension rights under the nominal system are at most 200% of the full wage inflation but under the real system, there is no upper bound and the real system can distribute out all the surplus to the pension fund participants. Thus, when the asset return is high, the differences between the two systems will be large.

6 Conclusion

In this thesis, we compare the two proposed pension agreements - the nominal indexation with immediate adjustment mechanism and the real indexation with smooth adjustment mechanism. To do that, we use a two-factor Vasicek model to estimate the term structure which will be used in calculating the discounted liability of the pension funds and also to simulate the stochastic term structure in the future. Then, we use several auto-regressive model to simulate the future term structure, the asset return of pension fund, and the full wage inflation. After that, we establish a stylized pension fund that has similar dependency ratio as the real Dutch demographic situation to see how the funding ratio, pension assets, replacement rates of retirees evolve over time under both systems. Finally, we compare the replacement rates and assets of pension fund for both systems under different levels of inflation and total return.

The main findings of this thesis are as follows. First, the replacement rates under the real indexation system are higher than those under the nominal indexation system and the differences are getting smaller over time. So, the current retirees and the relative old generations are benefiting from the real indexation system and for the young generations, they are indifference between the two systems in terms of replacement rate. This is because in the first decade, the average funding ratio in the nominal indexation system is below 120% and indexation is therefore below one. Thus pension participants in the nominal indexation system cannot fully benefit from wage inflation. In the real system, all the surpluses or deficits are distributed to the pension participants. In the following two decades, average nominal funding ratio in the nominal indexation system is above 120% and the third step in the policy ladder will give extra indexation to the full wage inflation as funding ratio pass 120%. So, the pension rights distributed to the retirees are "speeding up" and the nominal system is "catching up" with the real system, resulting in smaller differences in terms of replacement rate. As for pension asset, the nominal indexation system has more assets than the real indexation system and the differences are increasing over time. This is because in the first decade, the nominal system managed to accumulate more assets by distributing less and effect of the return on the huge amount of assets is relatively large compared to the pension payment made each year. So, although in the following decades, pension payments increased in the nominal system, this cannot fully offset the return on the high amount of assets and assets in the nominal indexation system remain much more than those in the real indexation system. More asset in the nominal system indicating the value is transferred from the current generations to the future generations indefinitely. Whether that value is ended up at the hand of

the future pension participants is doubtful.

Second, we find that the effect of inflation on the replacement rates and pension assets is not significant. Inflation affects both the asset return and the nominal pension rights. When inflation is high, we expect to have higher interest rate and thus higher bond return and higher asset return. Also, since the pension rights are indexed to full wage inflation, when inflation is high, we expect to have higher indexation of nominal pension rights thus higher pension payment in both systems. The higher bond return increases pension assets and higher pension payment decreases pension assets. As for replacement rate, both the nominal pension rights and wage indexation increases, thus the effect of inflation on replacement rate is ambiguous.

Third, we find that both systems are benefiting from positive shocks to the financial market and the differences between two systems are large when asset return is high. When asset return is high, replacement rates and pension assets increase in both systems. We argue that high return on stocks increases assets in both systems without affecting the projected liabilities, which results in higher pension rights and replacement rates. Higher return on assets also increases funding ratio in the nominal indexation system. Since there is an indexation cap in the nominal system when funding ratio is above 150%, the pension fund under the nominal indexation system is not distributing out all the surplus it has to the pension participants as it does in the real system. So, the differences between the two systems are large when asset return is high.

There are some shortcomings of this thesis. First, we assume a uniform premium over time. This is neither realistic nor fair as shown by Bovenberg [2]. In the future research, we would like to see what will happen if we adjust the contribution rate annually in order to have a fair contract. Second, we would like to compare the option values of the two contracts to the pension participants in a risk neutral world. Third, we ignore the recovery plan in the nominal indexation system when the funding ratio is below the minimum required funding ratio of 105%. Under the nominal indexation system, the pension fund need to draw up a recovery plan if the funding ratio is below 105%. After three years, if the funding ratio is still below the critical value, pension funds will cut pension rights to recover. We would like to model the recovery plan in the future research.

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7 Appendix A

7.1 One-Factor Vasicek With Risk Premium

Equation (5) gives us the expression for long-term interest rate when there is no risk premium. If we take risk premium into account, we will have

$$R_t^T = \overline{r} + b_T(r_t - \overline{r}) + \gamma_T^r. \tag{37}$$

Now that γ_T^r is unknown. Here is how we obtain γ_T^r.
We know the holding period return of a zero coupon bond with maturity T from period t to period $t+1$ without risk premium is

$$\begin{aligned}
H_{t+1}^{(T)} &= \ln(P_{t+1}^{(T-1)}) - \ln(P_t^{(T)}) \\
&= \ln(\exp(-(T-1)R_{t+1}^{T-1})) - \ln(\exp(-TR_t^T)) \\
&= -(T-1)R_{t+1}^{T-1} + TR_t^T \\
&= T\overline{r} + B_T(r-\overline{r}) - (T-1)\overline{r} - B_{T-1}(r_{t+1}-\overline{r}) \quad (\text{using} \quad (5)) \\
&= \overline{r} + B_T(r-\overline{r}) - B_{T-1}r_{t+1} + B_{T-1}\overline{r} \\
&= \overline{r} + B_T(r-\overline{r}) - B_{T-1}\phi(r-\overline{r}) - B_{T-1}(\overline{r}+\epsilon_{t+1}^r) + B_{T-1}\overline{r} \quad (\text{using} \quad (2)) \\
&= \overline{r} + B_T(r-\overline{r}) - B_{T-1}\phi(r-\overline{r}) - B_{T-1}\epsilon_{t+1}^r \\
&= \overline{r} + (B_T - \phi B_{T-1})(r-\overline{r}) - B_{T-1}\epsilon_{t+1}^r \\
&= \overline{r} + (Tb_T - \phi(T-1)b_{T-1})(r-\overline{r}) - B_{T-1}\epsilon_{t+1}^r \\
&= \overline{r} + (\frac{1-\phi^T}{1-\phi} - \phi\frac{1-\phi^{T-1}}{1-\phi})(r-\overline{r}) - B_{T-1}\epsilon_{t+1}^r \\
&= \overline{r} + (r-\overline{r}) - B_{T-1}\epsilon_{t+1}^r \\
&= r_t - B_{T-1}\epsilon_{t+1}^r.
\end{aligned} \tag{38}$$

When taking risk premium into consideration,

$$H_{t+1}^T = r_t - B_{T-1}\epsilon_{t+1}^r + \Phi_t^T, \tag{39}$$

with Φ_t^T the risk premium with maturity T.$\Phi_t^1 = 0$ and $\Phi_t^T = \lambda_r B_{T-1}\sigma_r$. Compare this equation with step (3) of equation (38) and equation (37), we can conclude that

$$\Phi_t^T = -(T-1)\gamma_{T-1}^r + T\gamma_T^r. \tag{40}$$

With iteration, we have

$$\begin{aligned} T\lambda_T^r &= \Phi_t^1 + \Phi_t^2 + \cdots + \Phi_t^T \\ &= \lambda_r\sigma_r(B_0 + B_1 + \cdots + B_{T-1}) \\ &= \lambda_r\sigma_r \sum_{\tau=0}^{T-1} B_\tau \end{aligned} \tag{41}$$

So, $\lambda_T^r = \lambda_r\sigma_r(\frac{1}{T}\sum_{\tau=0}^{T-1} B_\tau)$.

7.2 Two-Factor Vasicek With Risk Premium

From equation (7), the expectation of future inflation can be expressed as follows:

$$E_t(\pi_{t+\tau}) = \overline{\pi} + \psi^\tau(\pi_t - \overline{\pi}). \tag{42}$$

Hence, the average of future inflation is

$$\begin{aligned} \frac{1}{T}\sum_{\tau=0}^{T-1} E_t(\pi_{t+\tau+1}) &= \overline{\pi} + \frac{\psi}{T(1-\psi)}(\pi_t - \overline{\pi}) \\ &= \overline{\pi} + c_T(\pi_t - \overline{\pi}), \\ \text{with} \quad c_T &= \frac{\psi}{T(1-\psi)}. \end{aligned} \tag{43}$$

Under Fisher Hypothesis, the nominal interest rate ($\tilde{r_t}$) is the sum of the real interest rate (r_t) and expected inflation. Since we can express the real interest rate as in equation (6). Combine (6) and equation (43), we have

$$\tilde{R_t} = \overline{r_t} + b_T(r_t - \overline{r}) + \overline{\pi} + c_T(\pi_t - \overline{\pi}). \tag{44}$$

Risk premium of long-term real interest rate and inflation are obtained in the same way as in the one-factor model.Similar to equation (41), we have $\lambda_T^r = \lambda_r\sigma_r(\frac{1}{T}\sum_{\tau=0}^{T-1} B_\tau)$ and $\lambda_T^\pi = \lambda_\pi\sigma_\pi(\frac{1}{T}\sum_{\tau=0}^{T-1} C_\tau)$.

8 Figures

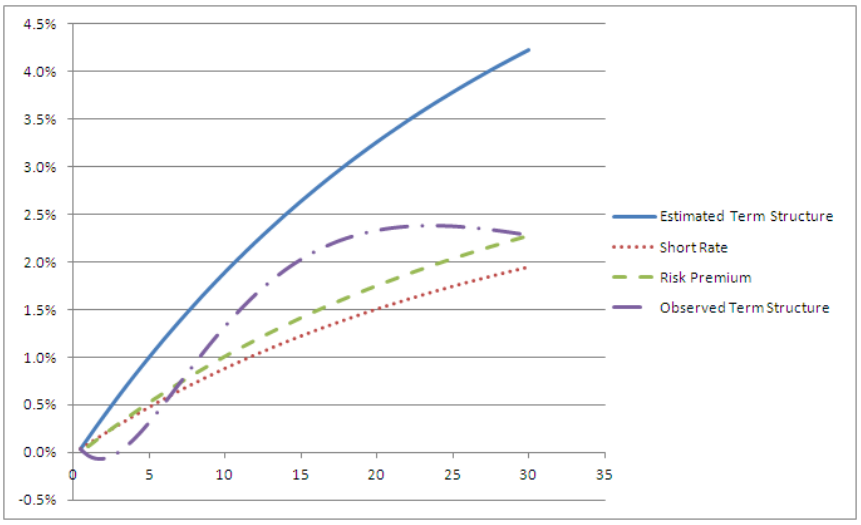


Figure 1: Estimated Current Term Structure Using One-Factor Model and risk premium from model (a)

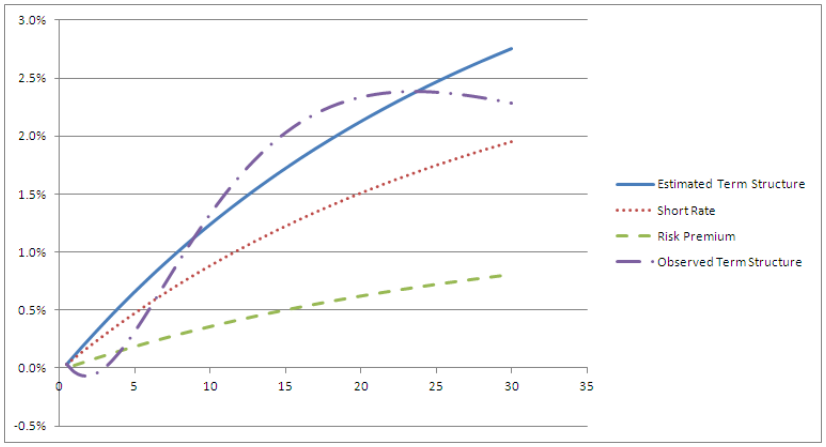


Figure 2: Estimated Current Term Structure Using One-Factor Model and risk premium from model (b)

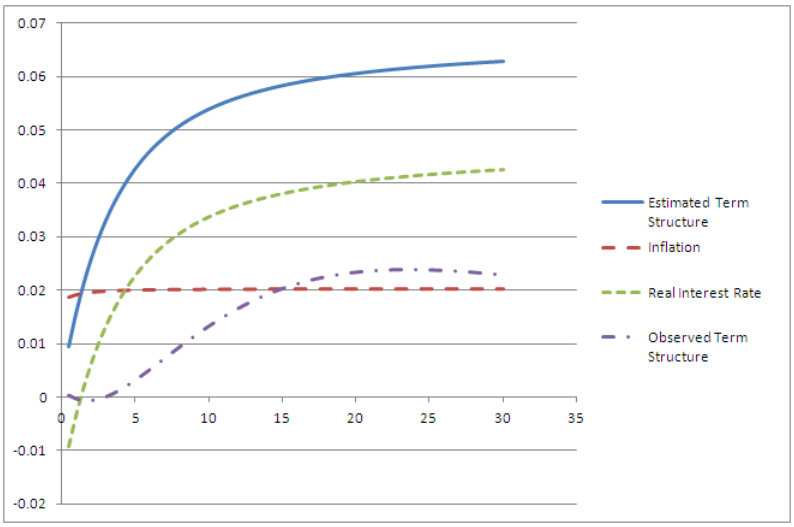


Figure 3: Estimated Current Term Structure Using Two-Factor Model and risk premium from model (a)

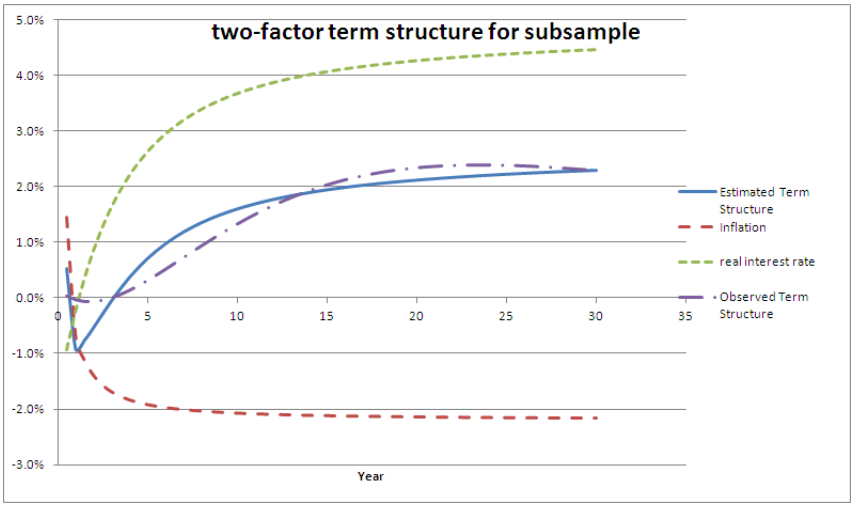
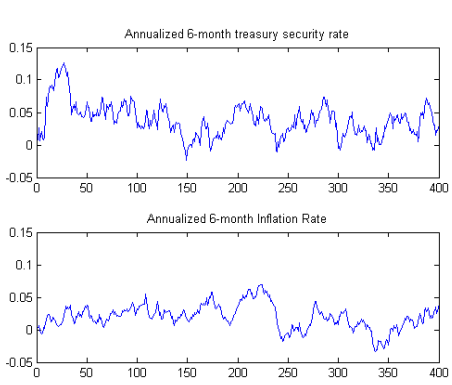
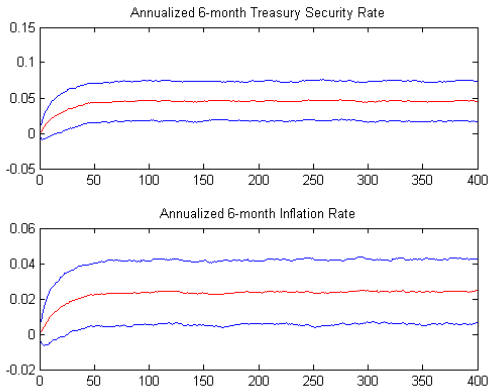


Figure 4: Estimated Current Term Structure Using Two-Factor Model and risk premium from model (b)

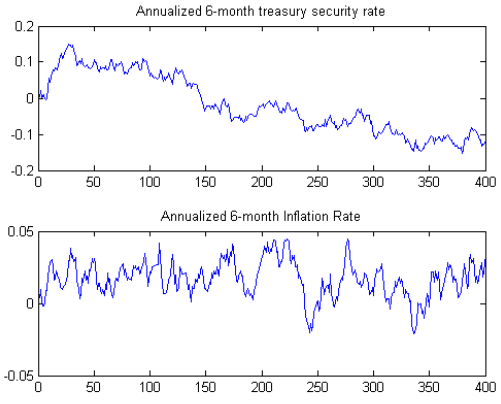


(a) One Path

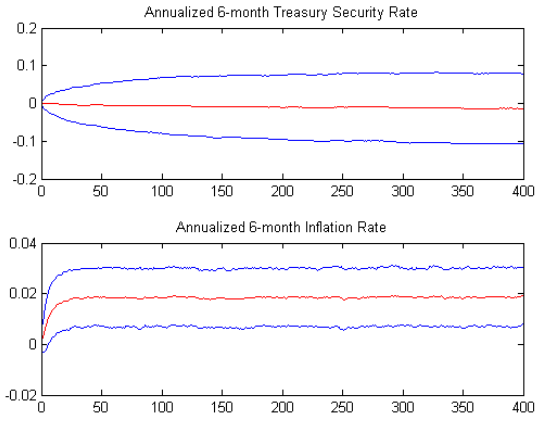


(b) Average Rate

Figure 5: Simulation of annualized 6-month Treasury Security Rate and 6-month Inflation Rate based on specified 1-factor model using data from 1972:09 to 2013:03

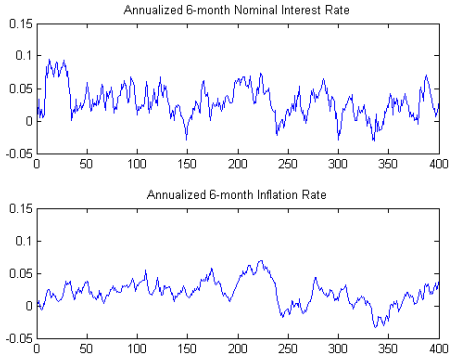


(a) One Path

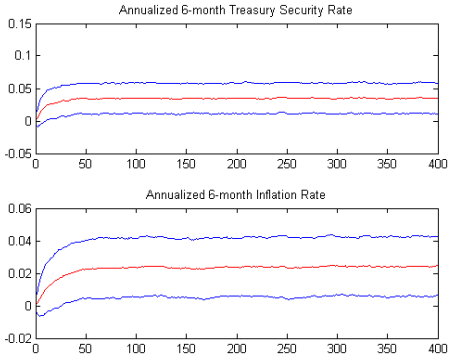


(b) Average Rate

Figure 6: Simulation of annualized 6-month Treasury Security Rate and 6-month Inflation Rate based on specified 1-factor model using data from 1990:01 to 2013:03

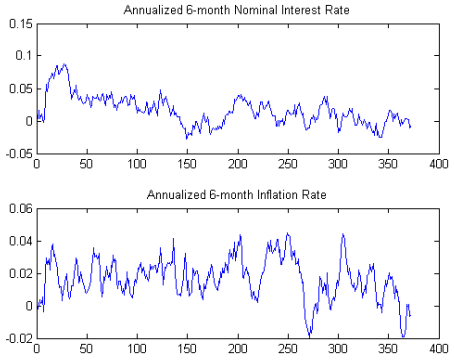


(a) One Path

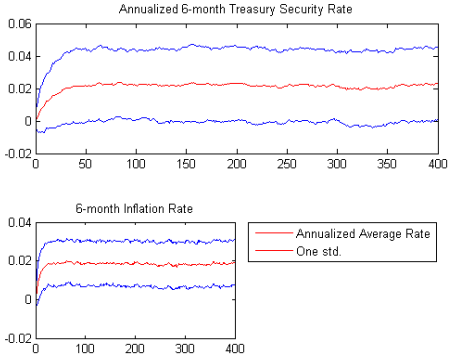


(b) Average Rate

Figure 7: Simulation of annualized 6-month Treasury Security Rate and 6-month Inflation Rate based on specified 2-factor model using data from 1972:09 to 2013:03

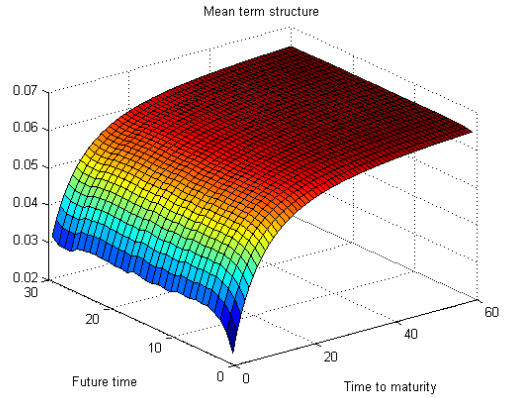


(a) One Path

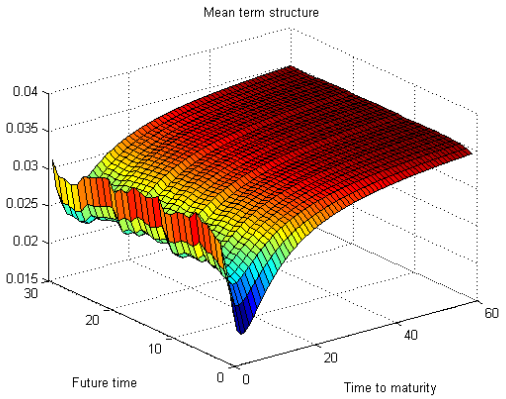


(b) Average Rate

Figure 8: Simulation of annualized 6-month Treasury Security Rate and 6-month Inflation Rate based on specified 2-factor model using data from 1990:01 to 2013:03



(a) Mean of the Estimated Stochastic Term Structure using risk premium from model (a). Future time (in years) on the x-axis is the time when the term structure is estimated and time to maturity on the y-axix is measured in peirods (semi-annual in our case). 60 means the corresponding time to maturity is 30 years.



(b) Mean of the Estimated Stochastic Term Structure using risk premium from model (b) Future time (in years) on the x-axis is the time when the term structure is estimated and time to maturity on the y-axix is measured in peirods (semi-annual in our case). 60 means the corresponding time to maturity is 30 years.

Figure 9: Simulated Term Structure

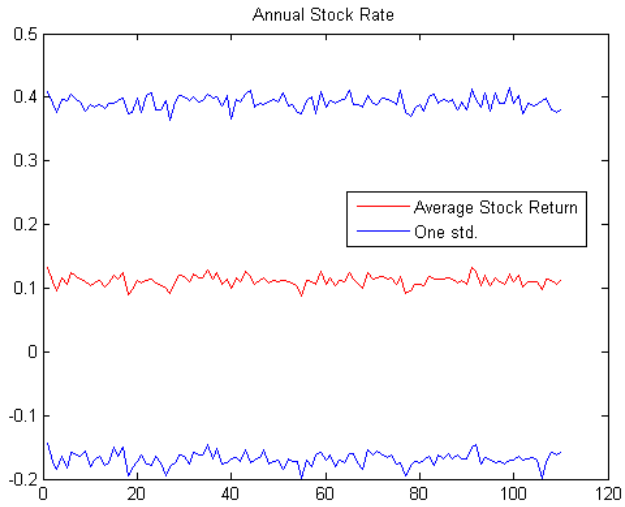
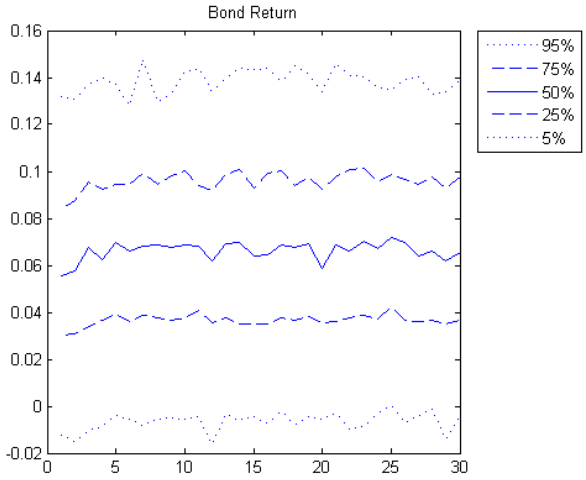
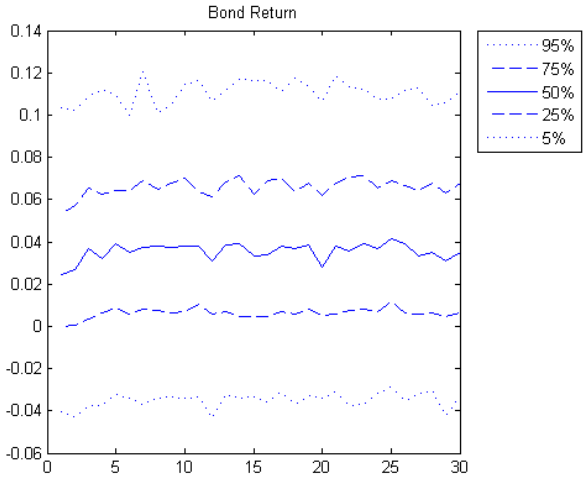


Figure 10: Simulated Stock Return using VAR model. The red line is the average stock return and the blue lines are the one standard deviation. Numbers on the x-axis is the number of years in the future.

Figure 11: Simulated Real Wage Inflation using AR(1) model. The average annualized real wage inflation is 2.34%. This figure shows the quantiles of the simulated real wage inflation

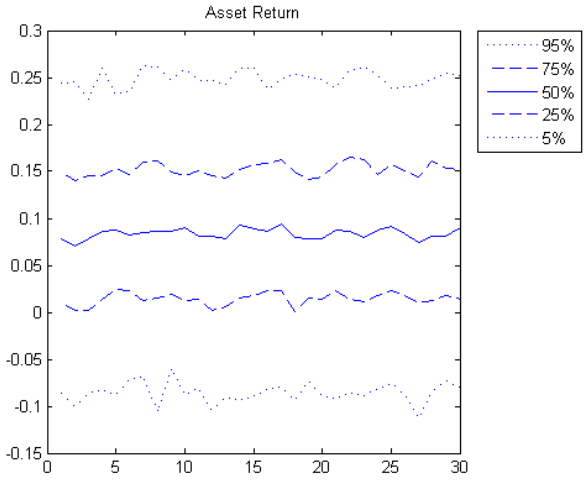


(a) Simulated Bond Return from model (a)

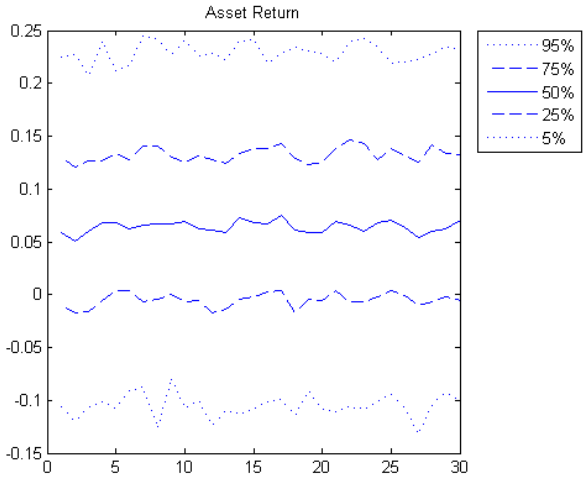


(b) Simulated Bond Return from model (b)

Figure 12: Simulated Bond Return. The top figure is the quantiles of the simulated bond return from model (a) and the bottom figure is the quantiles of the simulated bond return from model (b).



(a) Simulated Asset Return frommodel (a)



(b) Simulated Asset Return from model (b)

Figure 13: Simulated Asset Return. The top figure is the quantiles of the simulated asset return from model (a) and the bottom figure is the quantiles of the simulated asset return from model (b).

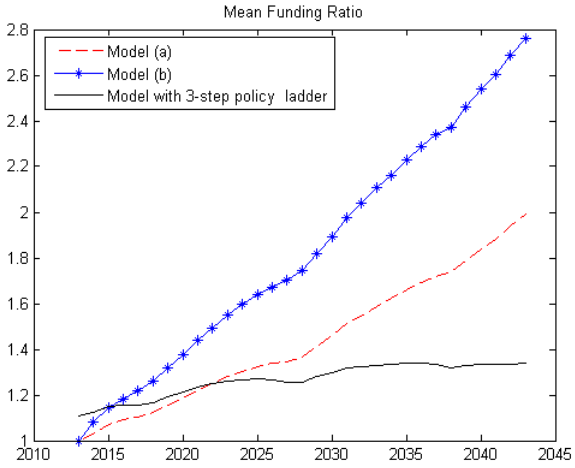
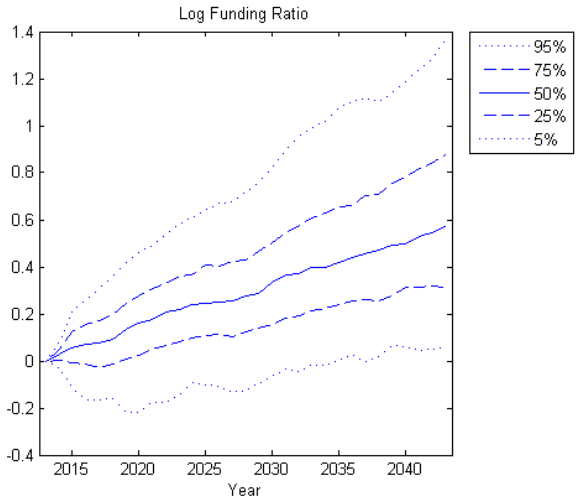
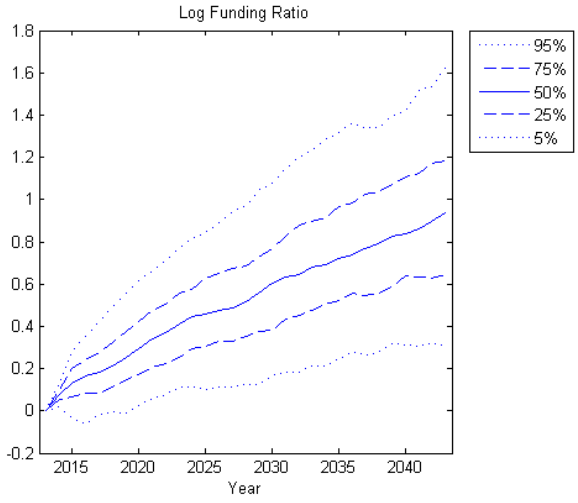


Figure 14: Mean Funding Ratio in nominal systems. As shown in the legend, the blue line with stars shows the average funding ratio in model (b), the red dash line shows the average funding ratio in model (a) and the black line shows the average funding ratio in nominal systems with 3-step policy ladder.



(a) Log Funding Ratio of Model (a) under nominal indexation system



(b) Log Funding Ratio of Model (b) under nominal indexation system

Figure 15: Quantiles of Log Funding Ratio. We take the logarithm of the funding ratio and display the 95%, 75%, 50%, 25%, and 5% quantiles of the logarithm of the funding ratio.

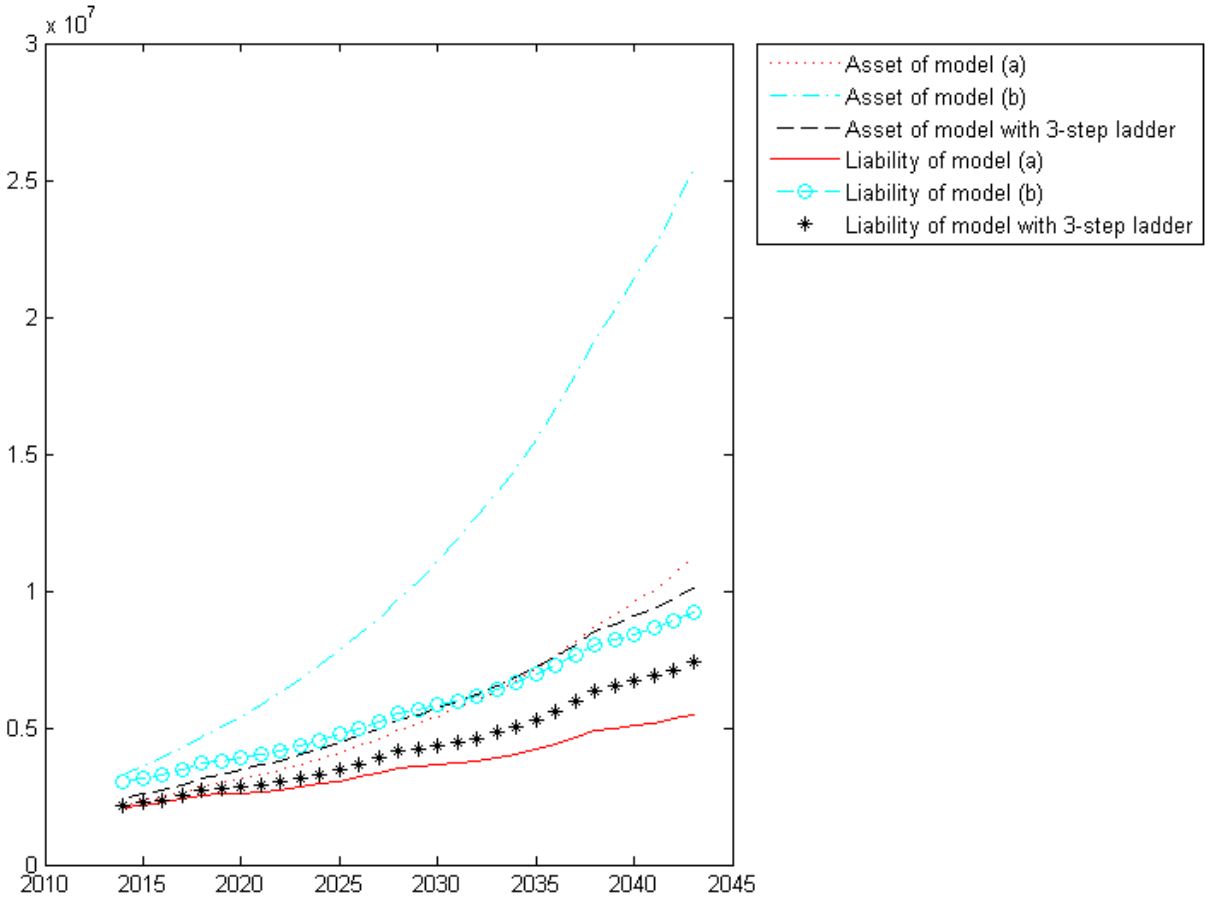


Figure 16: Assets and Liabilities of pension funds under nominal indexation system. The red lines show the assets and liabilities under model (a) and the blue lines show assets and liabilities under model (b) and the black lines show assets and liabilities under model with 3-step policy ladders

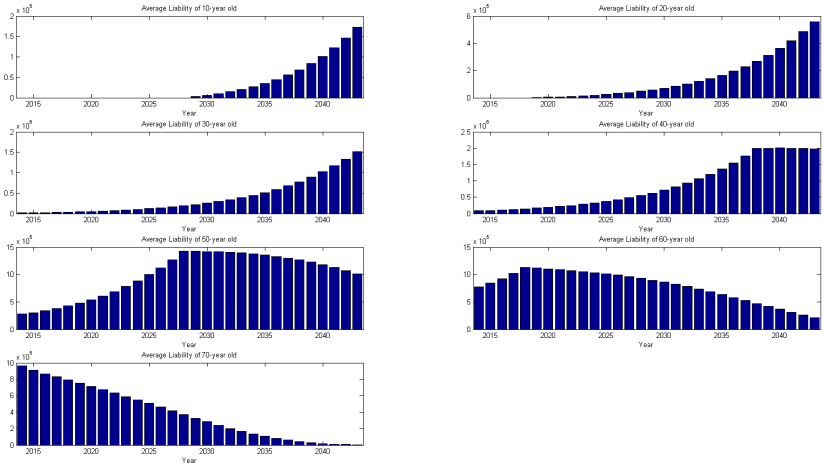


Figure 17: Projected Liability of different age group for model (a) under nominal indexation system

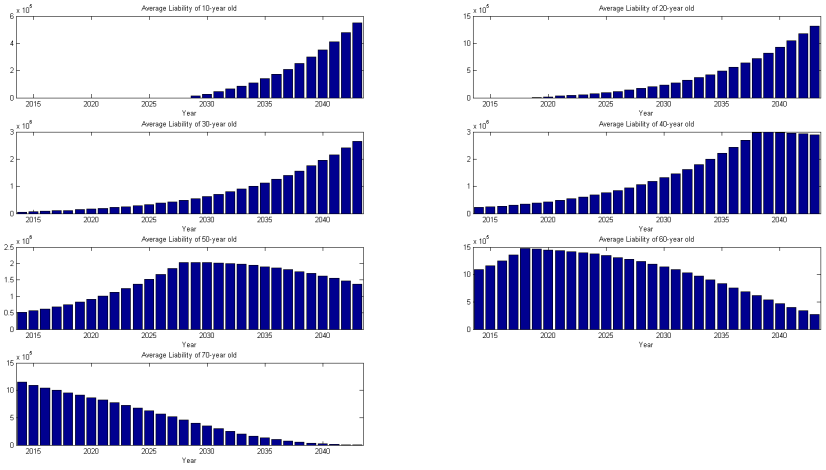


Figure 18: Projected Liability of different age group for model(b) under nominal indexation system

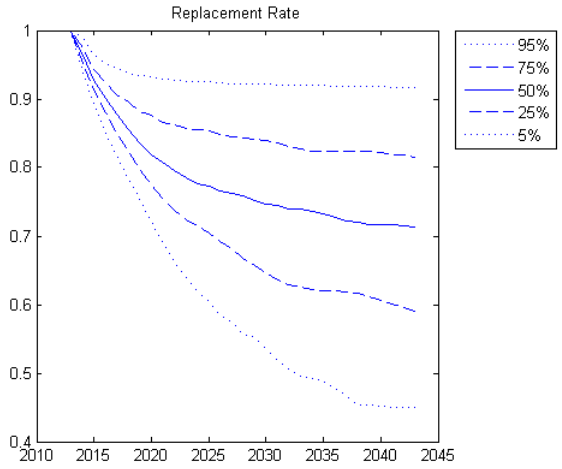


Figure 19: Replacement Rate for model (a) under nominal indexation system

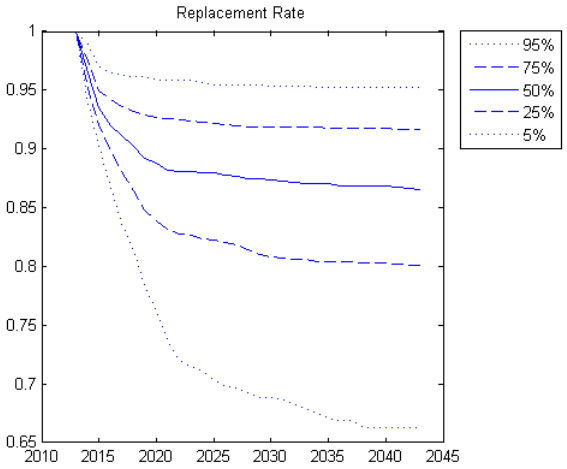


Figure 20: Replacement Rate for model (b) under nominal indexation system

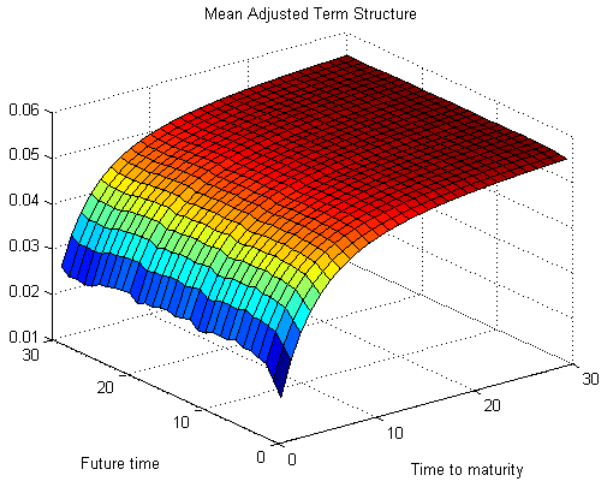


Figure 21: Risk Premium Adjusted Term Structure using Bovenberg-Nijman-Werker Formula

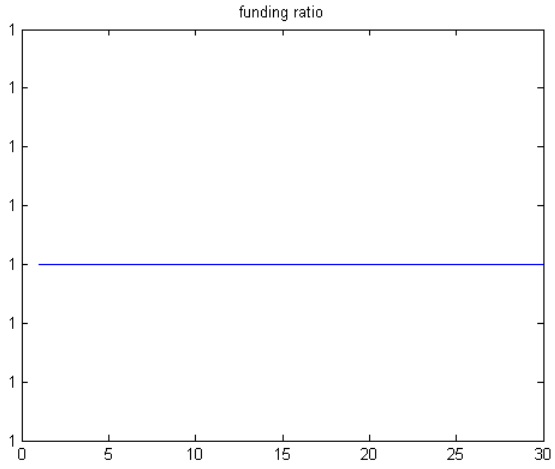


Figure 22: Adjusted Funding Ratio under real indexation system

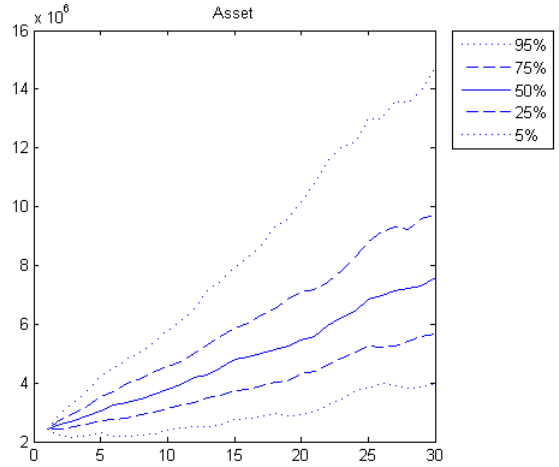


Figure 23: Quantile of Assets under real indexation system

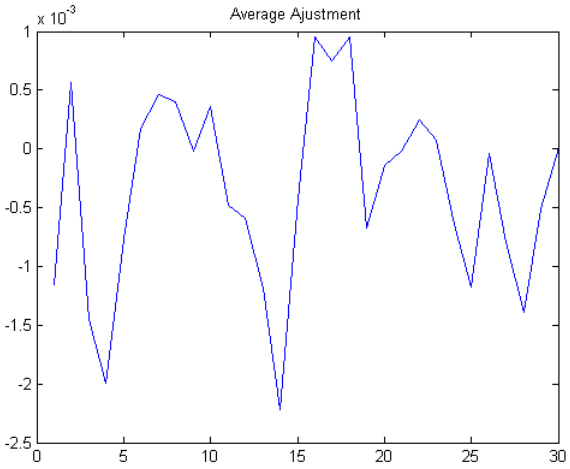


Figure 24: Average adjustment for each year under real indexation system

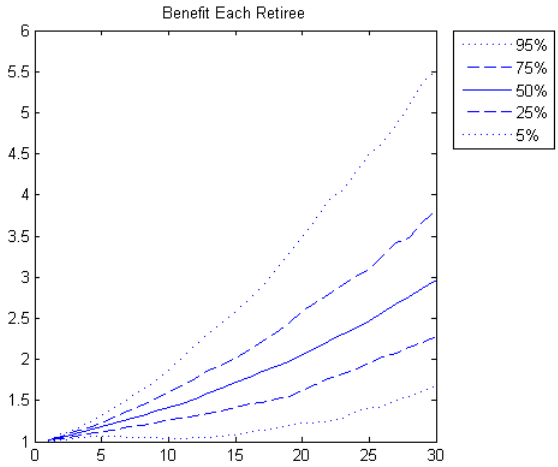


Figure 25: Quantile of pension rights each retiree under real indexation system

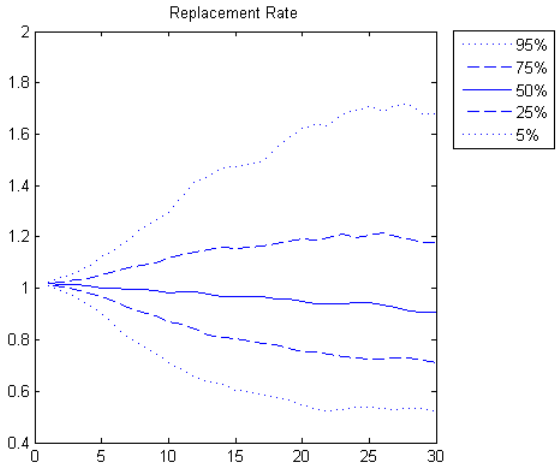


Figure 26: Quantile of Replacement Rate under real indexation system

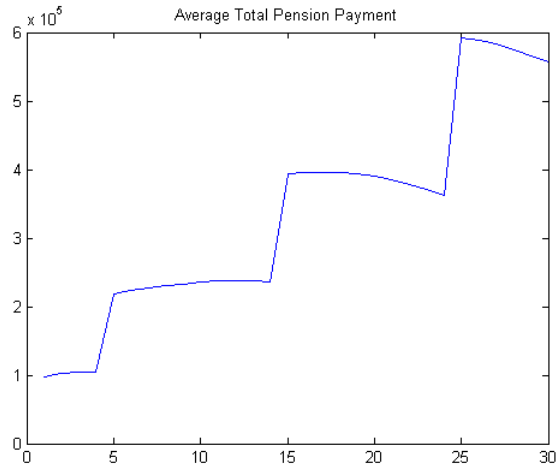


Figure 27: Total Pension Payment for each year under real indexation system

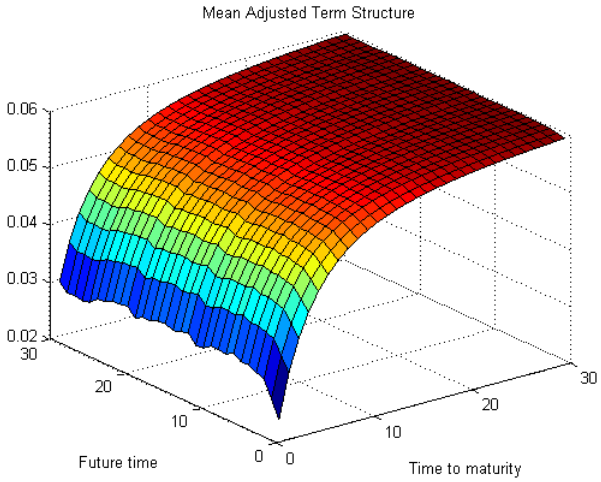


Figure 28: Adjusted Term Structure using model based parameters

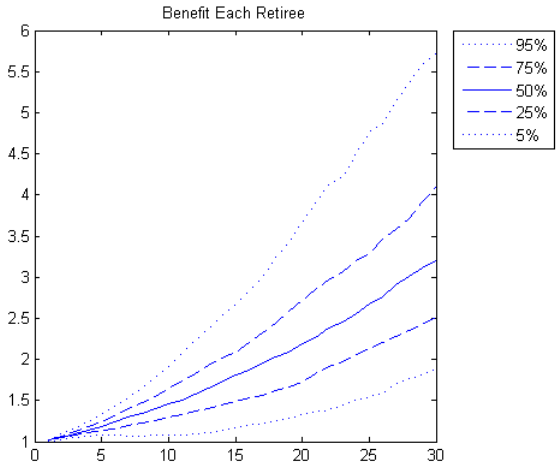


Figure 29: Nominal pension rights of each retiree using model based adjusted term structure

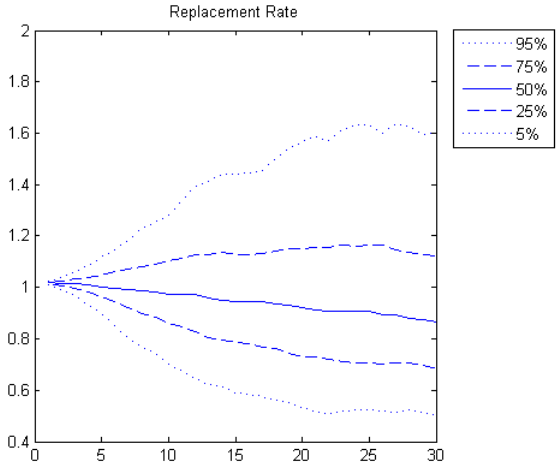


Figure 30: Replacement Rate under real indexation system using model based adjusted term structure

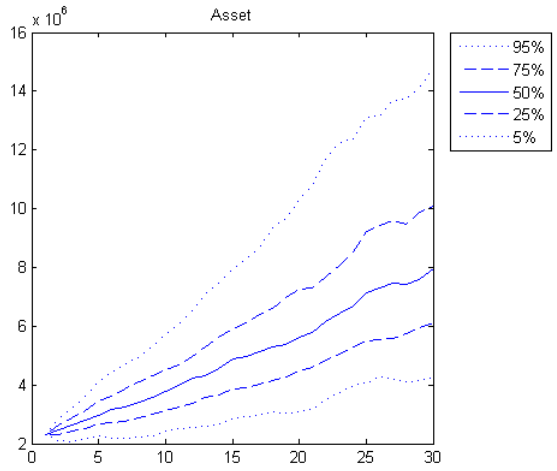


Figure 31: Assets under real indexation system using model based adjusted term structure

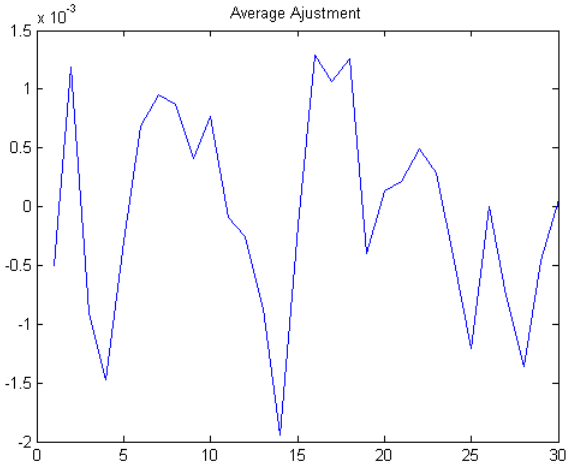


Figure 32: Average adjustments under real indexation system using model based adjusted term structure

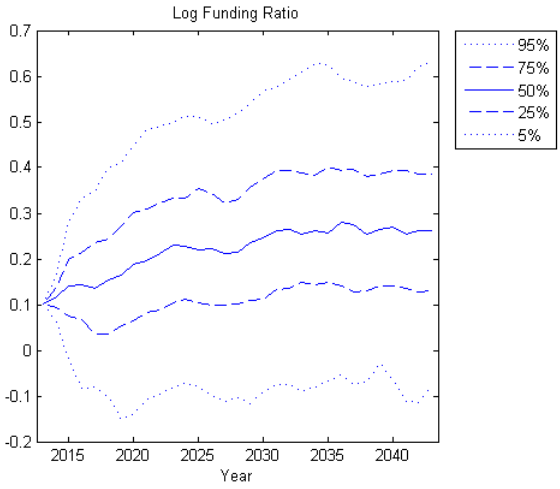


Figure 33: Log Funding Ratio in nominal system with 3-step ladders

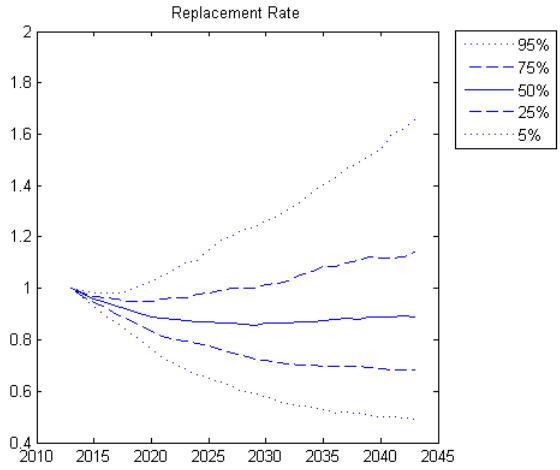


Figure 34: Replacement Rate in nominal system with 3-step ladders

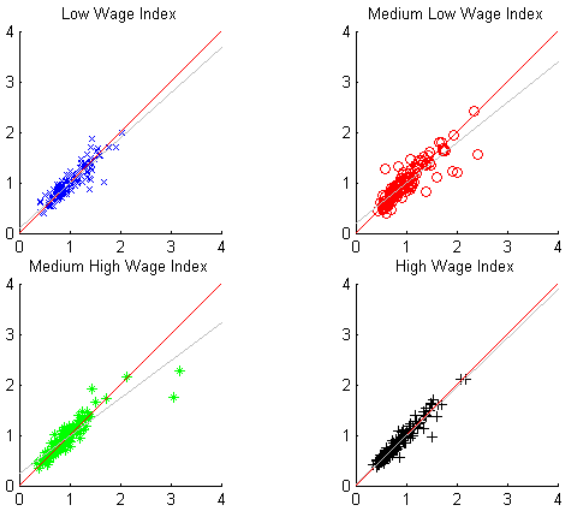


Figure 35: Comparison of Replacement Rate after 30 years grouped by wage index. The horizontal line shows the replacement rate under the real system and the vertical line shows the replacement rate under the nominal system.

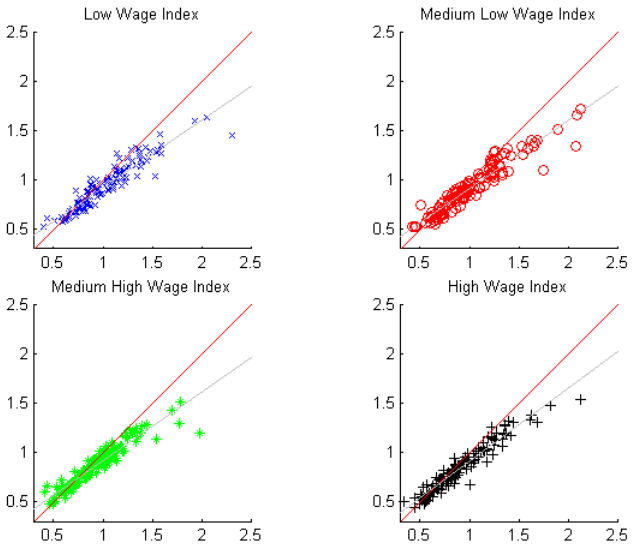


Figure 36: Comparison of Replacement Rate after 20 years grouped by wage index. The horizontal line shows the replacement rate under the real system and the vertical line shows the replacement rate under the nominal system.

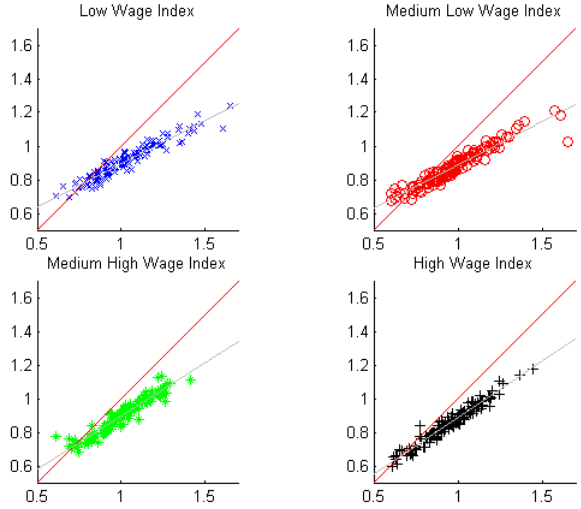


Figure 37: Comparison of Replacement Rate after 10 years grouped by wage index. The horizontal line shows the replacement rate under the real system and the vertical line shows the replacement rate under the nominal system.

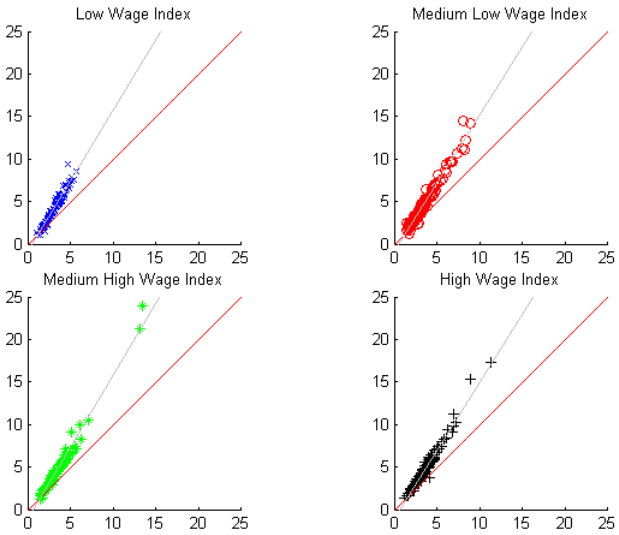


Figure 38: Comparison of Assets after 30 years grouped by wage index. The horizontal line shows the relative asset with respect to the initial asset under the real system and the vertical line shows the relative asset with respect to the initial asset under the nominal system. For instance, 3 means the asset after 30 years is 3 times the initial asset.

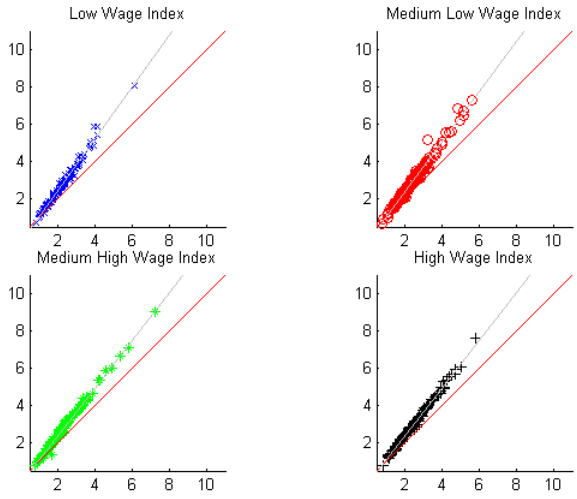


Figure 39: Comparison of Assets after 20 years grouped by wage index. The horizontal line shows the relative asset with respect to the initial asset under the real system and the vertical line shows the relative asset with respect to the initial asset under the nominal system.

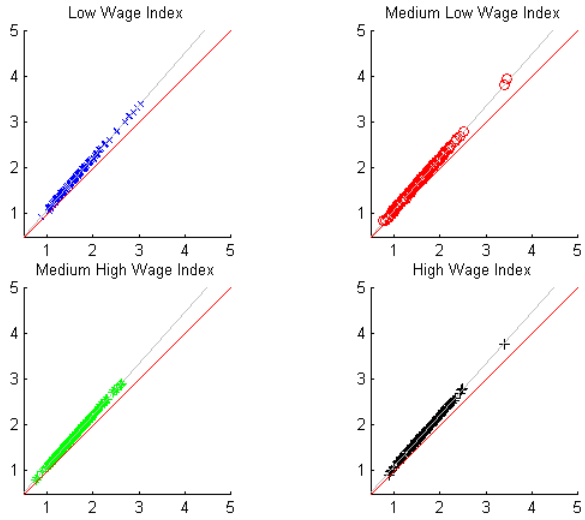


Figure 40: Comparison of Assets after 10 years grouped by wage index. The horizontal line shows the relative asset with respect to the initial asset under the real system and the vertical line shows the relative asset with respect to the initial asset under the nominal system. For instance, 3 means the asset after 10 years is 3 times the initial asset.

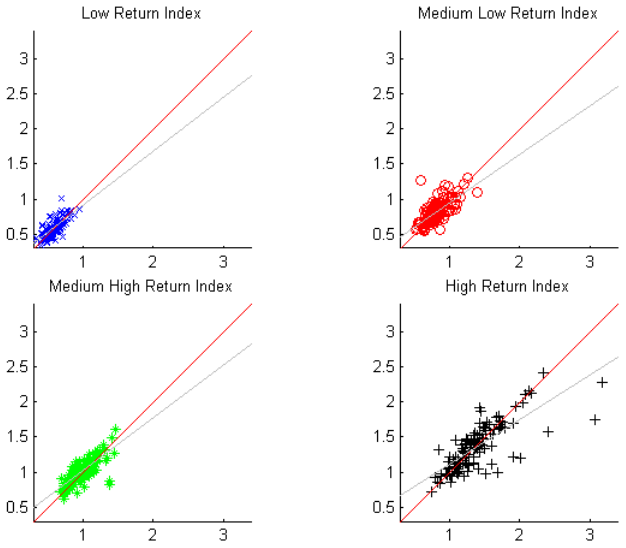


Figure 41: Comparison of Replacement Rate after 30 years grouped by total return index. The horizontal line shows the replacement rate under the real system and the vertical line shows the replacement rate under the nominal system.

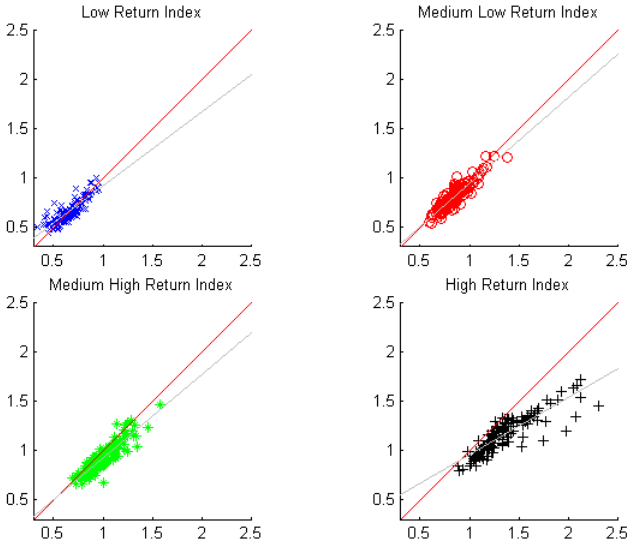


Figure 42: Comparison of Replacement Rate after 20 years grouped by total return index. The horizontal line shows the replacement rate under the real system and the vertical line shows the replacement rate under the nominal system.

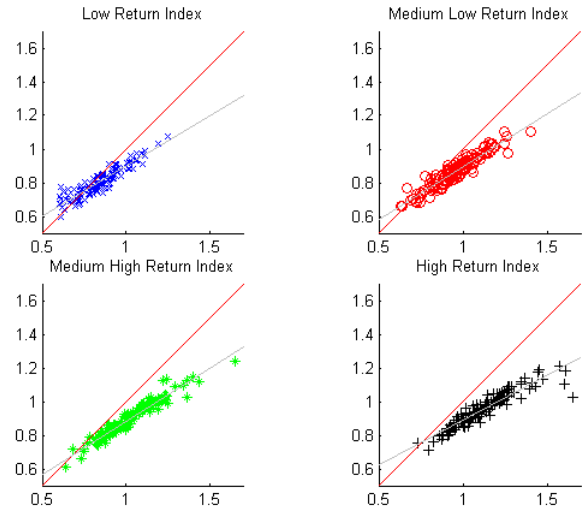


Figure 43: Comparison of Replacement Rate after 10 years grouped by total return index. The horizontal line shows the replacement rate under the real system and the vertical line shows the replacement rate under the nominal system.

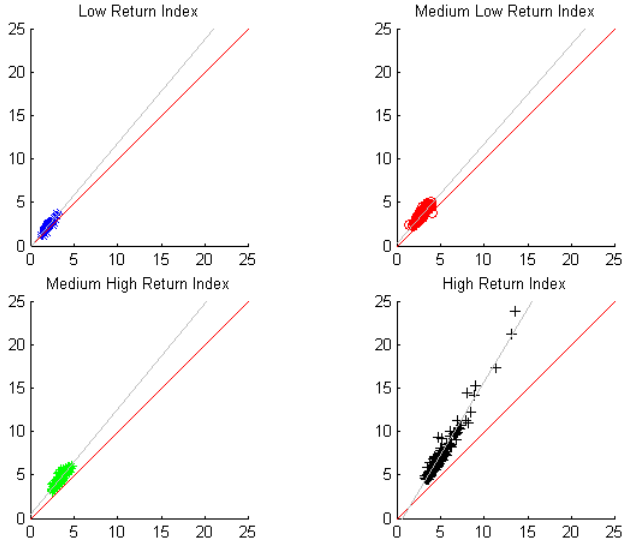


Figure 44: Comparison of Assets after 30 years grouped by total return index. The horizontal line shows the relative asset with respect to the initial asset under the real system and the vertical line shows the relative asset with respect to the initial asset under the nominal system. For instance, 3 means the asset after 30 years is 3 times the initial asset.

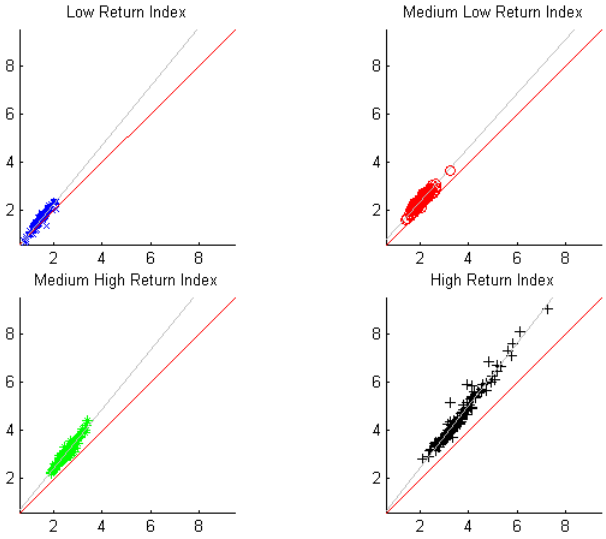


Figure 45: Comparison of Assets after 20 years grouped by total return index. The horizontal line shows the relative asset with respect to the initial asset under the real system and the vertical line shows the relative asset with respect to the initial asset under the nominal system.

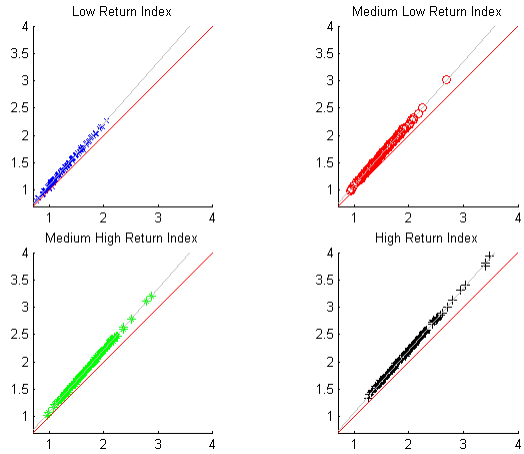


Figure 46: Comparison of Assets after 10 years grouped by total return index. The horizontal line shows the relative asset with respect to the initial asset under the real system and the vertical line shows the relative asset with respect to the initial asset under the nominal system.