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Generational Accounts for Pension Plan Valuation

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Contents

Chapter 1

Introduction

During the last few years, the age at which a Dutch citizen is entitled to state retirement pension (AOW) was under discussion. The state retirement pension is financed with a pay-as-you-go system, meaning that the current tax payers pay for the AOW of the elderly. With the ageing population, the AOW costs are rising while the working population is diminishing. In December 2007, the government asked a committee under the leadership of Peter Bakker to come up with an advice to increase the labour participation to make head against these rising costs. One of their advices was to increase the age at which one is entitled to AOW from 65 to 67 years. At that time, the proposal was not adopted by the government.

Because of the worsened economic situation the discussion of increasing the AOW age has started over again in 2009. The necessity to economize led to the decision that the AOW age indeed will be increased, a decision that will save the government billions of euros. The current statutory retirement age is equal to the AOW age of 65 and the level of pension benefits is based on this age. Therefore it seems inevitable that an increase of the AOW age leads to a change in the employment based pension system.

While the government wants to decrease the maximum accrual rate, the details of the pension plan are the responsibility of the two sides of industry. Currently the different parties of interest are exchanging views about the changes in the employment based pension system, the question what will happen to the pension plans is however far from answered.

1.1 Research question

The situation described above forms the basis of this research project. The goal is to study a number of possible changes in the employment based pension system as a reaction to the increase in AOW age. The changes range from changing nothing to increasing the statutory retirement age along with the AOW age and decreasing the accrual rate.

To compare the alternatives, a pension fund representing an average Dutch fund is constructed. The different changes in the pension plan are implemented in this basic model and the consequences are evaluated by using Monte Carlo simulation. Different scenarios are generated to asses the different strategies in terms of expected values (e.g. the expected funding ratio) and probabilities (e.g. the probability that no indexation occurs).

Besides the simulation experiments described above, the different strategies will also be evaluated by using the theory of risk neutral pricing. The theory of risk neutral pricing is used to price derivatives such as options in the financial markets. Instead of evaluating decisions on expected values and probabilities, risk neutral pricing allows strategies to be compared based on the economic value of the decisions made. That is, on the value the market currently attaches to future cash flows.

Risk neutral simulation allows measuring the impact of policy changes on different age cohorts. Our whole pension system is based on solidarity. Some people die early and therefore do not benefit from contributions made during their working period, while other people live longer than average and receive more than they contributed. In the same way there is solidarity between generations; when funding rates are low, pension premiums may rise such that the elderly can still receive their pensions, while it is possible that future generations pay lower premiums again because of high funding rates. By using risk neutral valuation these intergenerational value transfers can be quantified and changes in solidarity can be displayed.

Using both the normal and the risk neutral simulation described above, we will try to answer the following research question: *What are the effects of changing the statutory retirement age and the accrual rate in the employment based pension system, and which change in the system is the most favourable?* The favourability of each system is assessed by means of the stability of the funding ratio, the level of premiums charged, the real retirement age and the change in value transfers between the different age groups.

1.2 Thesis outline

This thesis is concerned with the Dutch pension system. A general description of pensions and pension forms in The Netherlands will be given in Chapter 2. Special attention is paid to the recent developments concerning the state retirement pension. Besides giving a general overview we will also consider solidarity as the basis of our pension system.

The recent developments in the AOW will probably lead to changes in the employment based pensions. These changes will be studied in the context of an average Dutch pension fund. The construction of this fund is the subject of Chapter 3. All relevant assumptions will be explained and the process of contributions, benefits and changing liabilities will be described.

Chapter 4 introduces four changes in the employment based pension system as a reaction to the changes in the state retirement pensions. Assuming that every participants wants to receive the same level of benefits after the change compared to the old situation, all proposed scenarios imply changes in the benefit payments to the retirees and participation rate of the employees. The accompanying technical discussion can be found in Appendix A.

Changing participation rates and changing benefit schemes both affect many aspects of the pension fund. The consequences will be studied by means of simulation. In Chapter 5 the principle of simulation will be introduced and applied to our specific model and the four alternatives.

While in Chapter 5 the traditional simulation approach is adopted, in Chapter 6 an other less well-known method for evaluating an uncertain future is introduced. By using generational accounts and option theory, solidarity between generations can be displayed and changes in this solidarity due to changes in the pension system become visible. Chapter 6 focusus on the construction and use of these generational accounts and the general idea of market consistent valuation. The technical part of the discussion can be found in Appendices B and C. The results of applying this theory to our pension fund model are given in Chapter 7.

Finally in Chapter 8 the main findings are summarized and conclusions will be stated. Also some suggestions for future research will be given.

Chapter 2

Dutch Pension System

In this chapter a global overview of the Dutch pension system will given. The Dutch pension system is based on three pillars that together constitute the old age provision. Each pillar will be described separately. Special attention will be paid to the state retirement pension system and the current developments. Finally the importance of solidarity will be discussed.

2.1 First pillar: State retirement pension

The first and largest pillar is the state retirement pension (AOW) which is provided by the government. The state retirement pension was introduced in 1957 by minister Suurhoff to provide a minimum base income to all people aged 65 or above. For every year a person lives in The Netherlands between age 15 and 65, 2% of the state pension is accumulated. The funding of the AOW was based directly on solidarity between generations, or intergenerational solidarity. In case of the AOW the solidarity is reflected in a pay-as-you-go system, meaning that the current active workforce pays for the AOW benefits of the current seniors.

Already in the 1980s questions arose about the sustainability of the system. A birth increase after World War II, the so called baby boom, and an increasing life expectancy predicted the ageing of the Dutch population. While in a pay-as-you-go system a rising proportion of retirees over the active workforce implies that fewer people have to share the increasing costs of the state retirement pensions, it was feared that the AOW was to become unpayable. A state investigation led by Drees jr led to the conclusion that the AOW could be maintained, but some changes in the system were recommended. The changes never occurred, but the discussion was temporarily soothed.

In 1997 the pay-as-you-system was adapted to the increased life expectancy and ageing of the population. The increasing grey pressure led to rising costs which until then were completely for the account of the active workforce. To make sure the premiums paid by the employees remained acceptable, the AOW premium paid by employees was maximised. The remaining costs were financed by the government from the general fund.

Due to the rising governmental contribution, the AOW became a point of discussion again during the parliamentary elections in 2006. Different political parties came up with plans to decrease the total costs of the state retirement pension and in December 2007 the government asked a committee under the leadership of Peter Bakker to come up with an advice to increase the labour participation to make head against the rising costs. One of their advices was to increase the age at which one is entitled to AOW from 65 to 67 years. At that time, the proposal was not adopted by the government.

In 2009 the remaining life expectancy of someone aged 65 has increased to 19 years, compared to 15 years at the time the AOW was introduced[1]. Due to this changing life expectancy, ageing of the population and a decreasing labour participation, the number of people in the active workforce for each pensioner decreased from 6 to 4. Since the life expectancy is anticipated to increase even further, this number is expected to decrease to two actives for each pensioner in 30 years. The current rise in governmental contribution thus will continue, but the worsened economic situation promotes the necessity to economize. These developments together led to the decision of increasing the AOW age, a decision that will save the government billions of euros.

2.1.1 Current developments

October 15th 2009 the government coalition came to an agreement concerning the AOW age, the corresponding amendment of the law was published in December. The main changes read as follows: The age at which one is entitled to AOW increases from 65 to 67. People near their retirement are spared by a transitional measure; in 2020 the AOW-age will be increased to 66 years, in 2025 a second increase to age 67 will take place. It this way people born up to and including 1954 will not be affected by the new law and still receive their AOW from age 65 onwards, people born from 1955 to 1959 are entitled to AOW from age 66 and all other generations receive their state retirement pension starting at age 67.

Exceptions apply for people with a long working history and people with so called heavy jobs. Employees who have worked for at least three days a weak during 42 years will still be able to retire and receive their AOW at age 65, however in that case their AOW benefits are decreased for the entire remaining lifetime. The heavy work exception applies for workers who performed heavy jobs for 30 years. After that time their employers need to offer them less demanding work, or the employer has to offer financial aid such that the employee can retire at age 65. Which jobs fall within the categorie heavy is not determined yet.

2.2 Second pillar: Employment based pension

Even though the first pillar is still responsible for the major part of the old age provision, the relative importance of the second pillar is growing. The second pillar consists of the employment based pensions, that is pensions of which the payments depend on the individual employment history. The employment based pension is meant as a supplement to the AOW and therefore there is no need to accrue pension over the entire income. The AOW level is taken into account when calculating the pension accrual by means of the statutory offset. This statutory offset is subtracted from the wage to determine the pensionable base, that is the part of the wage on which pension is accrued. The offset depends on the specific pension contract but is bounded by law.

[1] Advice of the Council of State about the amendment of the old age pension act. http://docs.minszw.nl.

In the employment based pension system we distinguish different pension types. The most well known are the old age pension and surviving relatives pension. They account for about two third and one fourth of the total pension payments respectively (Van Rooij, Siegmann, and Vlaar (2005)). The old age pension is paid to the participant from the retirement age until death, the surviving relatives pension provides a yearly payment to the partner of a participant who passed away. Other pension forms that constitute the last part of the total pension payments are for instance the orphans pension and the disablement pension.

Recall that the first pillar is funded by a pay-as-you-system, the second pillar pensions are funded by a capital funding system. Here the current participants save for their own pension. The level of benefit payments at retirement depends on the underlying pension scheme and the pension contract. The specific content of a pension contract for a firm or a branch of industry is determined by the two sides of industry. Three pension schemes that are used in the Netherlands are defined benefit, defined contribution and collective defined contribution schemes. Based on Kakes and Broeders (2006) each will be discussed shortly.

Defined benefit

In 2008 almost 90% of the employees was accruing pension in a defined benefit scheme (DNB (2009)). In a defined benefit scheme there is an agreement between the employer and employee about the level of the benefit payment from retirement until death. We consider two different defined benefit schemes.

Average pay scheme

In an average pay scheme every year a certain percentage of the pensionable base is accrued as pension rights to be payed from retirement. The ultimate pension benefit depends on the average wage during the working period. To pay for the accrual, yearly contributions are made by the employer, employee or both. In general a uniform contribution rate is charged, that is each employee pays the same percentage of its gross wage for the accrual. By charging a uniform contribution rate all active participants collectively save for their second pillar pension. Different agreements can be made concerning indexation of the accrual. With unconditional indexation pension rights increase in line with price or wage inflation, this means that the purchasing power at retirement is guaranteed. More common are agreements with conditional indexation. In that case pension rights increase with inflation, provided that the fund has sufficient recourses. Indexation is paid from the contribution income and investment returns, therefore with unconditional indexation the inflation risk is carried by the active participants of the fund and the employer. In case of conditional indexation the pensioners also share in the risk since their benefits can be cut.

Final pay scheme

We saw that in an average pay scheme the level of pension benefits depends on the career average salary; in a final pay scheme the total accrual depends on the last earned wage. Basing the pension level on the final salary means that every time a participant gets a raise, the pension accrual needs to be increased accordingly. The premium needed for this increase is called backservice. Just as in an average wage plan generally a uniform contribution rate is charged to cover for the new accrual and backservice. Since the accrued pension is increased with the salary, by definition the accrued pension in a final pay plan is indexed unconditionally.

Defined contribution

While in a defined benefit scheme there is an agreement about the pension level and all active participants save for their old age provision together, a defined contribution scheme does not guarantee a certain payment at retirement and all participants save individually. Instead of fixing the benefit payments, a prespecified proportion of the pension base is invested each year. At retirement the total value of the investments is used for buying an annuity. The investment risk is completely for the participant, this in contrast to a defined benefit scheme where a nominal benefit is guaranteed.

Collective defined contribution

The collective defined contribution scheme is a relatively new scheme. In this system a fixed predefined premium is paid to the pension fund, just as in the defined contribution case. However, unlike in a traditional defined contribution plan the premium is not individually but belongs to all participants collectively. This total premium is translated into conditional pension entitlements for the individual participants. The ultimate benefit payments depend on the assets of the fund.

2.2.1 Current developments

Even though the two sides of industry are responsible for the specific content of a employment based pension agreement, they are restricted by the fiscal policy of the government. In accordance with the increase of the AOW age, the government coalition proposed an amendment of the law concerning the second pillar pensions [2].

Currently the statutory retirement age equals 65 years; this means that pension payments normally start at age 65. If one wants to retire early and receive benefits before this age, the accrued pension is recalculated actuarially. By law, the maximum accrual rate in defined benefit schemes and the maximum premium in defined contribution schemes are limited based on the statutory retirement age. The maximum yearly accrual rate for an average pay scheme currently equals 2.25% of the pensionable base yielding a maximum accrual of 90% of the career average wage after 40 years op employment.

At this time the statutory retirement age corresponds to the AOW age and with increasing the AOW age the government also aims at increasing the retirement age to 67 years. With an increase in the retirement age, the accrual period of old age pension is also prolonged and hence the maximum accrual rate can be decreased to obtain the same level of pension benefits at retirement. To achieve an old age pension of 90% of the career average wage at age 67, the accrual rate can be decreased to $\frac{90}{42} \approx 2.15\%$ of the pensionable base. Similarly, the maximum accrual rate in a final pay scheme can be decreased from 2 to 1.9%.

[2] Amendment of the Old Age Pension Act, Law Income Tax 2001 and Law Wage Tax 1964 in view of increasement of the age at which one is entitled to old age pension. Ministry of Social Affairs and Employment, http://home.szw.nl.

In contrast to the increase in the AOW age, the decrease of the maximum accrual rate will take place at once in 2010.

2.3 Third pillar: Private savings

The third pillar consists of all private savings meant for the old age provision. These are all individual and independent of the pension fund. Private savings will not be taken into account.

2.4 Solidarity

Knowing the basics of the Dutch pension system we return to solidarity; the sharing of risk between participants. Both the first and second pillar of the Dutch pension system as described above are largely based on the concept of solidarity. The sharing of risk in general can be described as the transfer of income from one group to another. We distinguish two types of solidarity namely intragenerational and intergenerational solidarity. Intragenerational solidarity is concerned with uncertainty within a generation. An example is mortality risk: some people die early and therefore do not profit from contributions made during their working period, while other people live longer than average and receive more than they contributed. Intergenerational solidarity relates to the solidarity between different generations and thus the income transfers from young to old participants and vice versa.

We saw that intergenerational solidarity is expressed in the first pillar directly by means of the pay-as-you-go system where active members pay for the pensioners; intergenerational solidarity in employment based pensions may seem less apparent. Earlier we mentioned that in a defined benefit scheme participants pay for their own pension collectively since in general a uniform contribution rate is charged. In Chapter 3 it will be explained that this uniform contribution rate makes sure that young participants pay too much compared to the value of their accrual while old participant pay too little. Income is thus transfered from young to old participants, implying that the uniform contribution rate promotes solidarity between the active participants. Another example that contributes to solidarity is conditional indexation. By conditional indexation the pensioners hand over part of their purchasing power in case of a insufficient funding status of the fund to stimulate the recovery. Here income can thus be transfered from old to young generations.

In a defined contribution scheme every participant does pay for its own pension individually. Where in a defined benefit scheme the investment and mortality risk were carried by all participants collectively, in a defined contribution scheme all risk is for account of the individual employee. Solidarity is thus far less important here.

Since solidarity is the basis of the pension system, the system needs to be supported by all generations to maintain sustainable. An advantage of solidarity is that risks are shared among the participants, on the other hand this risk sharing may harm a certain group temporarily. An example here can again be found in the uniform contribution rate. Since young participants contribute too much compared to their accrual, they might find it profitable to quit the contract and enter again when they have grown older. To overcome this problem participation in a pension plan is in general mandatory by law.

Even though mandatory participation contributes to the sustainability of the pension system, for a specific generation there needs to be a certain equilibrium between the contributions and benefits resulting from the solidarity. This equilibrium can be disrupted by a change in the pension system, for instance by causing one generation to pay more while not compensating later on.

In this thesis we focus on changes in the second pillar pension system based on the proposed amendment of the law. The consequences of changing the current pension system first will be discussed in general terms after which special attention will be given to the effects on the solidarity between generations. The whole discussion will be based on an average pension fund, this fund will be constructed in the next chapter.

Chapter 3

Pension Fund Model

In the Netherlands there are many different pension funds, each with their own unique characteristics. A change in the system can have different consequences for different funds and since it is impossible to study all funds, in this chapter an average pension fund will be constructed which will be used as a base model for our analysis. Later on this base model will be extended for evaluating the changes in the employment based pension system.

3.1 Pension scheme

The average pension fund characteristics as described in this section are based on pension fund statistics of the Dutch Central Bank, (DNB (2009)), and the Pensionthermometer constructed by Hewitt[1]. In constructing the average plan we take into account the number of pension plans and ignore the number of participants. As over half of the Dutch pension plans, our model is based on a defined benefit plan with an average earning scheme. The entrance age of the pension fund equals 25 years and both the statutory retirement age and the AOW age are equal to 65 years. The accrual rate acc is assumed to equal 2% of the pensionable salary[2], meaning that 2% of the premium base is guaranteed as nominal pension from retirement until death. No pension accrues over the statutory offset since that part of the wage respresents the AOW benefit payments. Calculating the statutory offset f as $f = \frac{10}{8} \times$ (AOW payment for a single person) now amounts to a maximum accrual at retirement of 80% of the career average wage.

The aim of the pension fund is to increase the accrued pension rights in line with the collective wage increases, whether or not the increase will actually take place depends on the funding status of the fund. In a situation of underfunding, indexation will be cut and hence the value of the accrued pension will decrease in real terms. This implies a decrease in purchasing power of the current and future pensioners.

In our model we only consider the old age pension which accounts for about 70% of all pension payments (CBS (2009)). Ignoring for instance the surviving relatives pension, disability pension and orphan's pension may seem constraining but they have no important role in our analysis. While surviving relatives and orphan's pension depend on the time of death

[1] http://www.pensioenthermometer.nl/.

[2] pensionable salary = premium base = gross wage - statutory offset.

of the participant and disability pension on the moment of disability, changing the retirement age does not affect these pension forms.

3.2 Participants

The age distribution of the participants of the fund is based on the total Dutch population. The population structure in 2009 is known as given by the Dutch Central Bureau of Statistics (CBS (2009)), but for future years the population structure is uncertain. However, the CBS has made a forecast of the Dutch population up to 2050 and we assume that the number of 24 year olds equals this forecast in all future years. Using the CBS forecast of mortality rates for men in the period $2010-2050$ and the number of 24 year olds in all future years, we construct the complete demographics for the period 2010 up to and including 2035. Note that we only use the mortality rates for men and thus implicitly assume that all participants are male. This is because of computational simplicity in calculating and tracking the liabilities of the fund. The age groups $pop_{x,i}$ of the pension fund participants are distributed according to this Dutch population forecast, with a total of 10000 participants in 2009. Here $pop_{x,i}$ denotes the number of participants aged x at time i.

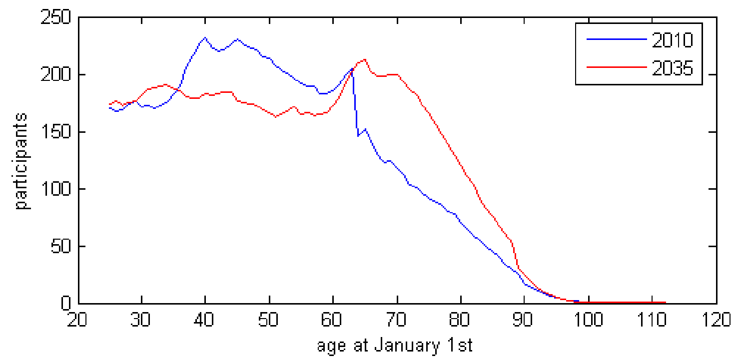

Figure 3.1: Pension fund participants in 2010 and 2035

Figure 3.1 shows the total pension fund population in both 2010 and 2035. In 2010 the baby boom after World War II is clearly visible, that is the generations born from 1946 until roughly 10 years afterward. In 2010 this large generation is close to retirement and hence the proportion of retirees over the current workforce will rise. While there are 3.6 people in the workforce for every retiree in 2010, this figure will decline to 2 in 2035. This ageing of the population is also visible in the figure.

Even though it is just a forecast and in reality the population will probably evolve in a different way, we assume that in our model the participant file behaves exactly as forecasted. We thus do not take demographic uncertainty into account. Since our model will evaluate time periods of one year, for simplicity it is assumed that all participants are born at the first of January and die at the end of December.

Not all members of the workforce are active participants. The participation rate of age group x, $part_x$, is derived from labour participation data of the CBS (2009). In reality the labour participation differs per year and cohort but for simplicity this distribution is discretized and rescaled. Furthermore changes in the participation rate over time are not taken into account. This leads to the following participation rates:

age	participation rate (in %)
25 - 49	100
50 - 54	88.6
55 - 59	72.2
60 - 64	26.6
65+	0

3.3 Economic data

Earlier we stated that the aim of the pension fund is to increase the accrued pension rights in line with the collective wage increases. We assume that each year the increase equals the price inflation π, yielding a real wage growth of zero. We set $\pi = 1.96\%$ in line with the inflation target set by the European Central Bank for the euro area.

Besides the collective wage growth equal to inflation, there are also age dependent wage increases. These wage increases are in general the largest in the beginning of someones career and decline towards zero when an employee approaches retirement. The maximum accrual rate for a final pay scheme is 2% compared to 2.25% for an average pay plan, implying that the career average wage should be about $\frac{2}{2.25} = 89\%$ of the final pay. This roughly coincides with Gortzak (2008) who states that an accrual of 80% in an average pay plan corresponds to about 70% in a final pay plan. Based on these two assumptions the following age dependent salary increases are implemented in the model.

age	individual wage increase (in %)
25-39	3
40-44	2
45-49	1
≥ 50	0

3.4 Pension policy and model equations

Knowing some of the basic structure of the economy and pension fund we can now define the assets and liabilities of the fund and the pension fund policy regarding benefits, contributions and investments. Benefits, contributions and investments are the three steering instruments of a pension fund for influencing the funding rate, which is defined as assets over liabilities. All of these will be explained in this section.

3.4.1 Liabilities

By accruing pension, employees ensure themselves of an income during their retirement. These pension payments are paid by the pension fund and hence constitute the liabilities of the fund. Every year liabilities increase due to new pension accrual of the active participants and decrease because of benefit payments made to the participants. Because many payments are taking place in the future, an appropriate provision has to be determined by means of the discounted value.

Ever since the introduction of the Financial Assessment Framework in 2007, pension funds have to value their liabilities using the most recent interest rate term structure (irts) that is derived from market prices. An interest rate term structure is a collection of interest rates corresponding to different maturities; therefore pension payments to be paid out one year from now are discounted at a different rate than benefits paid in 20 years. While predicting

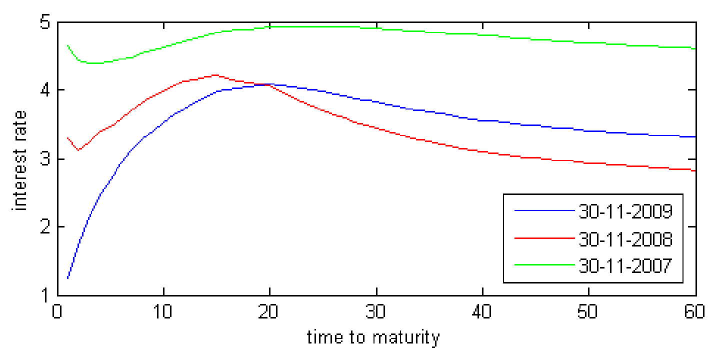

Figure 3.2: Nominal interest rate term structure

the term structure is subject to major uncertainty and very sensitive to the starting situation, see for instance Hulshoff (2009), assuming a constant irts also brings the problem of defining a realistic rate. The Dutch Central Bank publishes an up to date yield curve every month; Figure 3.2 displays the irts at the end of November in three subsequent years. Besides a parallel shift also a change in shape has occurred which shows that the time of measurement is very important. Since there is no clear reason for choosing one irts over an other, it is decided that the interest rate term structure will not be used in our model. Before the introduction of the irts a fixed actuarial nominal interest rate r^{nom} of 4% was used for discounting and this will also be used in our analysis.

Besides a discount rate also mortality rates are needed for valuing the liabilities. We use the mortality rate forecast for men in the period $2010 - 2050$ just as in the construction of our participant file. We denote the probability that a person aged x in year i will die within t years by ${}_tq_{x,i}$. Using the same mortality rates in constructing the file and valuing the liabilities implies that liabilities do not change because of differences between real and predicted mortality.

The value of the total nominal liabilities of the pension fund at time i, L_i^{nom}, is now given by:

$$L_i^{nom} = \sum_{x=25}^{65} tot\_acc_{x,i} \ \ {}_{65-x|}\ddot{a}_{x,i} + \sum_{x=66}^{112} tot\_acc_{x,i} \ \ \ddot{a}_{x,i}$$

Here $tot\_acc_{x,i}$ equals the total accrual of all people aged x at time i, and ${}_{m|}\ddot{a}_{x,i}$ is the present value of a yearly payment of one euro starting m years from now until death[3].

Note that we defined the nominal liabilities; that is, the liabilities when no indexation would be granted. Still, the aim of the pension fund is to index liabilities in line with inflation. We therefore define the real liabilities L_i as the liabilities of the fund when full indexation would always be granted. In that case the real interest rate is used for the discounting. Given $r^{nom} = 0.04$ and $\pi = 0.0196$, the real discount rate equals $r = (1+r^{nom})(1+\pi) = 0.02$. Since the pension fund is aiming for a pension in real terms and the participants are interested in the real value of their pension, in the remaining all variables and amounts are given in real terms unless stated otherwise.

Earlier we mentioned that every year the liabilities change because of new accrual and the payment of benefits. Recall that we assumed an accrual rate of $acc = 0.02$, meaning that each year 2% of the premium base $base_x$ of the active participants is accrued as new pension rights. The discounted value of the new accrual of all people aged x in year i, $new\_acc_{x,i}$ thus equals

$$new\_acc_{x,i} = acc \;\; pop_{x,i} \;\; part_x \;\; base_x \;\; {}_{65-x|}\ddot{a}_{x,i}.$$

The total benefit payments depend on the number of pensioners, the total accrual of the retirees and the level of indexation. With full indexation, the purchasing power of the retirees remains constant. When indexation is cut, real benefit payments decrease. The level of indexation is determined by the indexation policy of the pension fund.

Indexation policy

As stated before, our model is based on an index linked pension where the aim is to increase accrued pension rights in line with the rise in collectively agreed wages. The actual level of indexation depends on the funding status of the fund. The real funding rate is defined as the value of the assets of the fund over the value of the value real liabilities ($FR_i = \frac{A_i}{L_i}$), whereas the nominal funding rate equals assets over nominal liabilities ($FR_i^{nom} = \frac{A_i}{L_i^{nom}}$).

In our model a real funding rate of one corresponds to a nominal funding rate of 1.36 while a nominal funding rate of one implies a real funding rate of 0.73. This is in line with the actual Dutch situation as described in Kakes and Broeders (2006), where a real rate of one corresponds to a nominal rate of 1.38 when inflation equals 2%. In case the nominal funding rate drops below 100%, assets fall short for covering nominal liabilities and hence there is no room for indexation. With a real funding rate of 100%, assets are sufficient to cover real liabilities so full indexation can be granted. This results in the following indexation policy:

Real funding rate (in %)	Indexation
> 125	full indexation with catch-up indexation
$100 - 125$	full indexation
$75 - 100$	partial indexation (linear)
< 75	no indexation

[3] ${}_{m|}\ddot{a}_{x,i} = \ddot{a}_{x,i} - \ddot{a}_{x+m,i} = \sum_{k=0}^{\infty} v^k \; {}_kp_{x,i} - \sum_{k=0}^{m-1} v^k \; {}_kp_{x,i} = \sum_{k=m}^{\infty} v^k \; {}_kp_{x,i}$, with ${}_tp_{x,i} = 1 - {}_t\, q_{x,i}$ and $v = \frac{1}{1+r^{nom}}$.

Following Van Rooij et al. (2005), catch-up indexation is granted in case there is an indexation deficit caused by previously missed indexations. The indexation deficit is defined as $\frac{\prod^k(1+\pi)}{\prod^k(1+ind_k)} - 1$ where ind_i denotes the actual indexation in year i. Full catch-up takes place provided that the funding rate does not fall below 1.25. Note that only the liabilities are indexed, there is no compensation for missed indexation on the benefits that are already paid.

The concept of conditional indexation contributes to the intergenerational solidarity. In economic good times all participants benefit, in bad times all participants have to give in. However, young participants still have the possibility of a value recovery while retirees loose actual value. There is thus a possible value transfer from old to young participants.

3.4.2 Assets

While liabilities increase because of new accrual, the pension fund also needs premium income. A uniform contribution rate is charged to all participants, so everyone pays the same percentage of his pensionable base as premium. In Chapter 2 the importance of intergenerational solidarity in the pension system was explained and the uniform premium is a major contributor to this solidarity. Since ${}_{65-x|}\ddot{a}_{x,i}$ is increasing with x, charging the same premium to all participants makes that young generations are paying more than the value of their new accrual while the older participants pay less than the value of their accrual. After all, the younger the participant the larger the probability that the participant dies before retirement and the longer the premium payments can generate return. The level of the pension premium is determined by the contribution policy of the fund.

Contribution policy

The break-even premium is the premium that just covers the value of the new nominal accrual and administration costs; in our model administration costs are ignored. As explained in Van Rooij et al. (2005), the break-even premium gives a good indication of the intergenerational fairness of the premiums charged. Charging a premium far above the break-even premium makes that the expected benefits of an individual do no longer outweigh the individual costs. Note that the premium is based on nominal liabilities while the aim of the pension fund is in terms of real liabilities. The difference is expected to be paid from the excess return above the risk-free interest rate instead of contributions, but in practice this will not always be possible. Therefore the contribution policy is also contingent on the funding rate of the fund.

With a real funding rate exceeding one, assets are sufficient to grant indexation without extra premiums being charged. With a real funding rate below one only partial indexation is granted and participants lose value in real terms. Earlier we noted that the indexation policy hurts retirees more than active participants because they do not have the possibility of recovery. By charging a higher premium with a low funding rate, active participants make an extra contribution to the recovery of the fund in bad times. In economic good times they are compensated by a premium below the break-even premium.

This leads to the following contribution policy in our model:

Real funding rate (in %)	Premium (% of premium base)
> 200	negative premium
$140 - 200$	no premium
$125 - 140$	linear reduction on break-even premium
$100 - 125$	break-even premium
< 100	annual increase of 2.5%-points to a maximum of 35%

To limit the premium volatility we add the constraint that if the real funding rate is below 125%, the premium charged is not allowed to change more than 2.5%point per year.

Investment policy

Pension funds invest their assets in different products. Currently the average Dutch pension fund invests about 39% of the assets in stocks, 43% in bonds, 11% in real estate and the remaining 7% in commodities, hedge funds and cash[4]. Bonds and stocks thus constitute the major part of the investment mix. Following for instance Van Rooij et al. (2005), other products are not taken into account in our model and the percentage of wealth invested in stocks α equals 50%. We assume that the pension fund uses a periodic rebalancing strategy where the asset mix is rebalanced at the end of each year. This means that at the beginning of each year the proportion of stocks equals α again. Transaction costs are ignored.

The total wealth to be invested at time $t-1$ equals $A_{t-1} + C_t - P_t$. Here A_{t-1} denote the assets of the fund at time $t-1$ before benefit payments and contributions, C_t are the contributions for the accrual in the period $t-1$ to t and P_t are the benefit payments to retirees from time $t-1$ to t. Both benefit payments and contributions are paid at the beginning of the year. The value of the assets at time t now equals

$$A_t = (A_{t-1} + C_t - P_t)\left(\alpha(\frac{S_t}{S_{t-1}}) + (1-\alpha)(\frac{B_t}{B_{t-1}})\right) \tag{3.1}$$

Where S_t and B_t are the prices of one unit of stock and bond at time t respectively.

The return on the two traded assets depends on the model underlying the assets. We assume that the risk free bond B_t and stock S_t evolve according to the Black Scholes model. This model is given by

$$\begin{aligned} dB_t &= B_t \tilde{r} dt & B_0 > 0 \text{ given} \\ dS_t &= S_t(\tilde{\mu} dt + \sigma dW_t) & S_0 > 0 \text{ given} \end{aligned}$$

The first equation describes the price evolution of the bond where B_0 denotes the value at the current time and $\tilde{r}$ denotes the continuously compounded real risk free interest rate. The second equation describes the stock price process of one share of stock. Here $\tilde{\mu}$ and σ denote the real continuously compounded drift and volatility and W is a standard Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$. The annual real risk free interest rate equals the real discount rate used in calculating the liabilities, that is $r = 0.02$, and the

[4]http://www.pensioenthermometer.nl/.

annual drift of the stock price process is assumed to equal $\mu = 0.059$[5].

Solving the Black Scholes model yields that the returns on the assets are given by

$$\begin{aligned}\frac{B_t}{B_{t-1}} &= \exp(\tilde{r}) \\ \frac{S_t}{S_{t-1}} &= \exp\left((\tilde{\mu} - \frac{1}{2}\sigma^2) + \sigma(W_t - W_{t-1})\right)\end{aligned}$$

where $W_t - W_{t-1} \sim N(0,1)$. Technical details can be found in Appendix C.1.

3.5 Model summary

In this chapter we described the average pension fund model that will be used for our analysis. In our model the participant file of the fund represents the Dutch population and evolves according to the population forecast. In this way all demographic uncertainty is eliminated. We use a defined benefit average pay scheme with a pension policy based on the real funding rate. The real funding rate of the fund depends on the value of the assets and the real liabilities. The value of the liabilities depends on the conditional indexation; the value of the assets on the contributions and investment returns. The pension fund invests the assets of the fund in bonds and stocks and rebalances its position on a yearly basis.

The different aspects described above all are of interest to the different parties involved in the pension process, but the different parties have different concerns. While active participants want a low contribution rate and a sufficient pension level once they retire, deferred members and retirees mainly are concerned with their pension being inflation proof. The regulator, in the Netherlands that is the Dutch Central Bank, wants the policy to comply with the Pension Act and the pension board wants the funding rate to be sufficient to cover current en future liabilities. Note that the interests are not only short term but also long term; assets need to be sufficient to cover liabilities not only now but also in the future when the current active participants are retired.

The long term interests and uncertainty in the future have to be taken into account in evaluating the fund. This is accomplished by means of Monte Carlo simulation as will be discussed in Chapter 5. However, first in Chapter 4 we will describe some different possible changes in the employment based pension system as a reaction to the proposed changes in the state retirement pension.

[5] The real drift μ corresponds to a nominal yearly drift of 0.08. Furthermore $e^{\tilde{r}} = 1 + r$ and $e^{\tilde{\mu}} = 1 + \mu$. Comparable parameters are used in Cui, De Jong, and Ponds (2009) and Wijbenga (2009) who validated them with the S&P500.

Chapter 4

Employment Based Pension Alternatives

Up to now we considered an average Dutch pension fund in the context of the current employment based pension system. One of the main characteristics of the employment based system is the statutory retirement age of 65 years, equal to the current AOW age. With the plans of the government to raise the AOW age stepwise to 67 years, changes in the employment based system are likely to follow. After a short repetition of the governmental plans as described in Chapter 2, four alternative scenarios for the employment based pensions will be discussed in this chapter.

4.1 Governmental plans

As mentioned in Chapter 2, the age at which one is entitled to AOW will increase from 65 to 67 years in two steps. The first increase takes place in 2020 where the AOW age is increased to 66 years, the second increase to 67 years will take place in 2025. It this way people near their retirement are spared since the generations born up to and including 1954 will not be affected by the new law and still receive their AOW at age 65. People born from 1955 to 1959 are entitled to AOW from age 66 and all other generations receive their state retirement pension starting at age 67.

As a result of the changing AOW age, also the statutory retirement age in the employment based pension system will be increased to 67 years. This increase will take place at once in 2020, the moment that the AOW age is increased to age 66. With the increase of the retirement age the maximum accrual rates will be decreased from 2.25 to 2.15% in a average pay plan and from 2 to 1.9% in a final pay scheme. Note that since in our model we use an accrual rate of 2%, the accrual rate need not be altered by law.

The scenarios for the employment based pension system as described in this chapter are based on these plans. From now on we assume that the AOW age is increased according to plan and that the level of the AOW benefits and the statutory offset do not change in real terms. Furthermore all scenarios are based on the assumption that regardless of the specific changes in the system, everyone wants to retain the same level of yearly pension payments once retired. That is in each year benefits are paid to a retiree, the total level of individual

AOW and employment based pension payments are the same as they would have been in case the AOW age was not increased and the pension system was not changed at all. People who stopped working before age 65 still want to receive benefits from age 65; employees who worked until the statutory retirement age still want to receive benefits from the moment they retire. The implications of these assumptions will become more clear in the discussion of the scenarios.

In discussing the different scenarios we aim for developing some general intuition on the differences between the changes and the basic consequences for the exit age; the discussions are therefore rather general and only supported graphically. For a technical discussion we refer to Appendix A. In section 4.6 the results for the exit age will be discussed for our specific average pension fund model.

4.2 First scenario: 65; 2%

In the first scenario we consider, no changes are made concerning the employment based pension system; the statutory retirement age remains 65 years and the accrual rate still equals 2% of the pension base. We now have the following situation

old situation

year of birth	AOW age	retirement age
all	65	65

new situation

year of birth	AOW age	retirement age
$\leq$ 1954	65	65
1955 - 1959	66	65
$\geq$ 1960	67	65

A participant of the pension fund aged 55 or below in 2010 thus has an AOW deficit between ages 65 and 66 or between ages 65 and 67 compared to the old situation. We assumed that everyone wants to retain the same level of yearly pension payments once retired and to achieve this goal the exit age of these participants, defined as the age at which one ends is working engagement, has to increase. The size of the increase depends on the year of birth of the participant and the exit age in the old situation.

Recall from Chapter 3 that we discretized the participation rate into four steps. These steps can be interpreted as the possible exit ages, implying that in the old situation a participant ended its active engagement at age 50, 55, 60 or 65. The calculation of the additional years of service will be discussed separately for the situations of exiting before and exiting at the statutory retirement age.

Early exit

Consider the case where a participants stops working before age 65. There is an AOW deficit for one or two years starting at age 65; with increasing the working period one can fill this gap with the value of the new pension accrual. Figure 4.1 displays this situation for someone born after 1959. The left figure shows the situation when the AOW age is increased but the exit age stays the same. This exit age is depicted by the red vertical line. The old age pension benefits start at age 65 and are represented by the dark blue block, AOW payments start at age 67 and are represented by the light blue square. The red block is the AOW deficit. The right figure shows the situation where the exit age is increased. During the extra working period the value of the payments in the yellow block are accrued, that is the value of the missed AOW benefits.

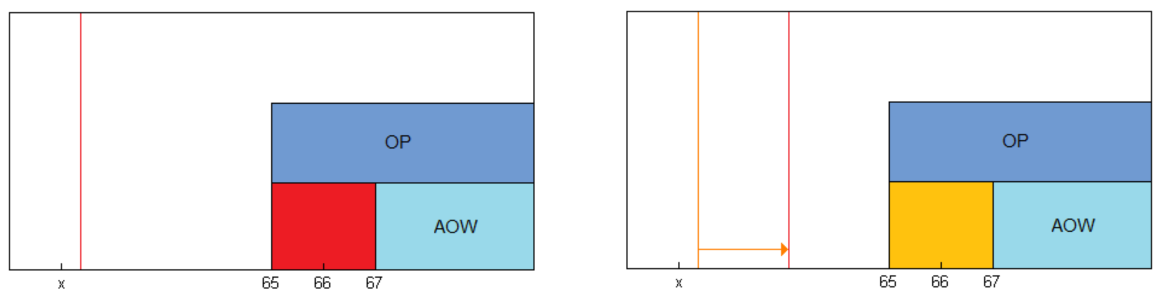

Figure 4.1: Early exit

The increase in the years of employment can be calculated by the present value of life annuities. The deficit caused by the increase of the AOW age corresponds to a temporary life annuity, the old age pension accrued during the extra years of service is paid from age 65 to death and thus is a deferred whole life annuity. The additional years of service are now calculated in such a way that the value of the new accrual equals the value of the deficit. For a technical discussion of the calculations we refer to Appendix A.

While we described the situation for someone born after 1959, a similar method of reasoning can be applied to people born between 1955 and 1959. In this case one has to make up for only one year of AOW and hence the exit age has to increase less.

Finally one additional assumption is made, namely that once someone has ended its working agreement he will not start working again. This assumption has implications for people born between 1955 and 1959 with an old exit age of 50 years. Even though they have an AOW deficit of one year, in 2010 they already have stopped working and they will thus not extend their career to make up for the deficit. Their exit age will remain 50 years.

Exit at age 65

Now we turn to the case where the employee originally stopped working at the statutory retirement age of 65. These employees also have an AOW gap which can be filled by extending the working period. The situation differs from to case with an early exit in that the employee is entitled to his old age pension at his original exit age. Besides accruing new pension rights, extending the working period thus also implies deferring the old age pension payments. Deferring payments in general leads to higher annual payments but since the goal was to obtain the same payment from retirement as in the old situation, the value of the deferred old age pension can be used for the AOW deficit. Since both the new accrual and part of the accrued old age pension are used to fill the gap, the increase in exit age is smaller than in case of an early exit.

Figure 4.2 shows this new situation for employees born after 1959. The left part shows the situation where the employee does not change its behaviour, again the red part represents the AOW deficit. In the right figure, part of the already accrued old age pension is deferred such that from a certain age the total benefits are equal in the old as well as in the new situation; this is given by the blue part. Note that AOW and old age benefits do not start at age 65 anymore. As stated before, people who originally retired at the statutory retirement age want benefit payments from the moment they retire and hence their exit age also has to increase. During this extra period of employment new old age pension is accrued, this is represented by the yellow rectangle. The exit age is thus determined in such a way that the value of the new accrual is just enough to cover benefit payments from the moment the employees stops working until the previously accrued old age pension and AOW benefits start to pay.

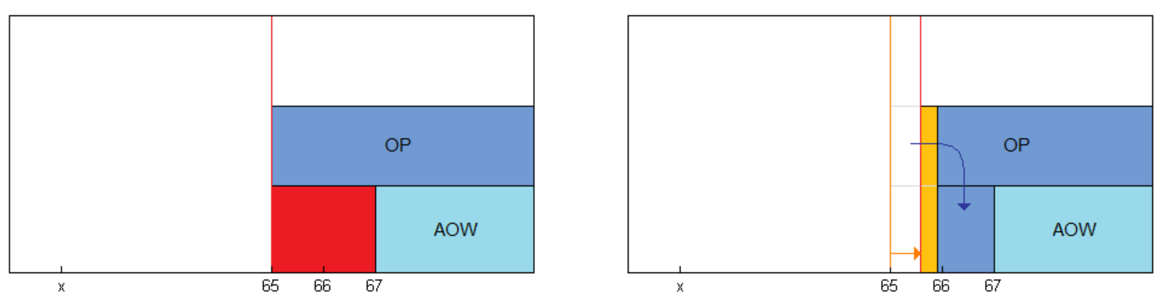

Figure 4.2: Exit at age 65

The situation with an old retirement age of 65 is more complex than with an early exit, but again the exit age can be calculated using annuities. For the calculations we again refer to Appendix A.

4.3 Second scenario: 65; 1.9%

One of the goals of the government was to increase the labour participation to make head against the rising costs. To achieve this not only the age at which one is entitled to AOW will be increased, but also the maximum accrual rates will be decreased. In an average pay scheme the maximum accrual rate will be decreased to 2.15% of the pensionable base in the year 2020. With a decreased accrual rate people have to work for a longer period of time to achieve the same old age pension.

In our model the accrual rate needs not be altered by law; this results in the situation as described in scenario one. There we found that changing the AOW age without changing the second pillar pension system already leads to an increased participation rate by itself. This change however will turn out to be much smaller than the increase in the AOW age. Decreasing the accrual rate would increase the participation rate further and this is the case we consider in the second scenario. In this scenario the retirement age again does not change, but the accrual rate will decrease from 2% to 1.9% from 2020 onwards. The consequences of the increased AOW age and the decreased accrual rate again will be discussed seperately for the case of an early exit as well as an axit at age 65.

Early exit

The situation with an early exit for a someone born after 1959 is given in Figure 4.3. The situation resembles scenario one but there is one important difference: besides an AOW deficit, there can also be a deficit in the accrued old age pension. In the figure the old age pension deficit is displayed by the upper rectangle. The deficit results from the years of employment after 2020; there 0.1% of old age pension accrual is missed each year compared to the old situation. As an example consider an employee born in 1970 with an original exit age of 55. When in 2020 the accrual rate is decreased, he already had 25 years of service. During this period he had an accrual rate of 2%. Age 55 corresponds to the year 2025, for the years of service between 2020 and 2025 his accrual rate equals 1.9%. His old age pension deficit is due to these last five years. Participants that stop working before 2020 do not have an OP deficit, people born after 1995 start to accrue pension after 2020 and thus face the maximum deficit.

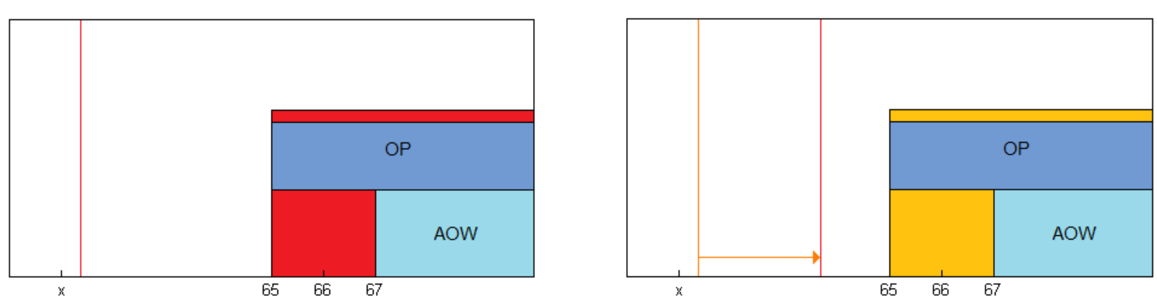

Figure 4.3: Early exit

Since we want the same level of benefits from retirement as in the situation were the pension system is not changed, just as the AOW deficit the OP deficit has to be filled by means of new accrual during additional years of service. The accrual rate during this extra working period again depends on the year of accrual. The new accrual represents a deferred whole life annuity, just as the old age pension deficit, while the AOW deficit is represented by a temporary life annuity. The additional years of service are determined in such a way that the value of the new accrual equals the value of the total deficit.

Exit at age 65

The case where an employee originally stopped working at age 65 is also very similar to that in scenario one. The difference again is that there can be an old age pension deficit on top of the AOW deficit. The value obtained by deferring part of the old age pension now is used not only for a temporary life annuity as compensation for the AOW, but also for a whole life annuity for compensating the old age pension deficit. As before the exit age is increased to make sure that pension payments start at the moment of retirement. The situation is displayed in Figure 4.4.

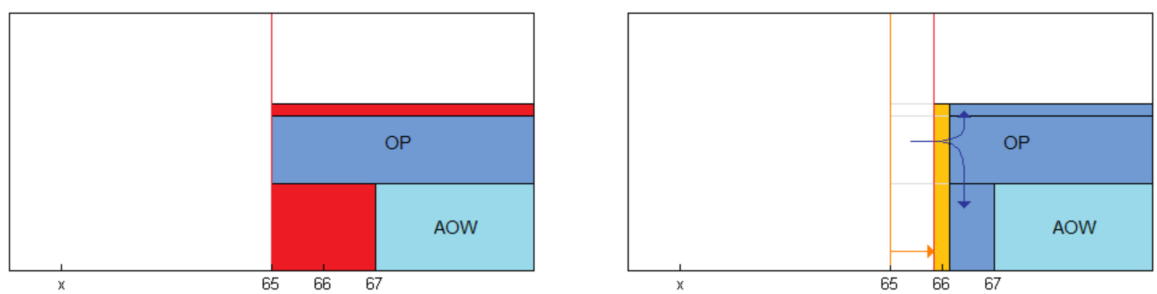

Figure 4.4: Exit at age 65

The calculations for the additional years of service for both the early exit as well as the case with an original exit age of 65 can be found in Appendix A.

4.4 Third scenario: 67; 2%

In the previous two scenarios we retained the original statutory retirement age of 65. However recall that the government wants to increase the statutory retirement age to 67 years in order that this age is in accordance with the AOW age again. If this plan is indeed implemented, accruing pension that is paid from age 65 onwards is not supported fiscally anymore. The reversal rule that states that pension accrual is taxed at the moment one receives the pension benefits instead of the moment it is accrued does not apply anymore and accruing pension becomes more expensive.

According to the plans, the increase in the statutory retirement age has to take place in one step in 2020, the year the AOW age is increased from 65 to 66. In this new situation it is still possible to have pension payments start at age 65, but this is then achieved by forwarding part of the pension that originally pays from age 67. The statutory retirement age is increased at once because of administrative costs and implementory easy for the pension funds.

In this third scenario we keep the accrual rate constant at 2% and increase the statutory retirement age to 67 in accordance with the plans. This means that we have the following situation:

year of birth	AOW age	retirement age
$\leq$ 1954	65	65
1955 - 1959	66	67
$\geq$ 1960	67	67

Consequences of this change in the system are as before described for the case of an early exit as well as for an original exit at age 65.

Early exit

An increase in the statutory retirement age in 2020 implies that for all pension accrued after this year benefit payments start at age 67. In Figure 4.5 the purple block represents the pension accrual after 2020, we denote this accrual by $OP^{\geq 20}$. All pension accrued before the decrease in 2020 starts to pay at age 65 and is denoted by $OP^{<20}$.

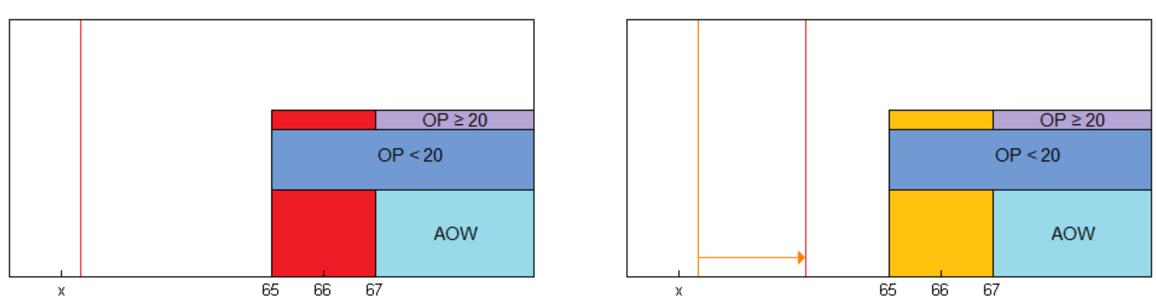

Figure 4.5: Exit at age 65

The upper red block represents the deficit in the old age pension compared to the old situation due to the change in the system. Note that instead of a whole life annuity as we had in scenario two when the accrual was lowered, the old age deficit now corresponds to a temporary life annuity. Filling the gap is again accomplished by extending the working period, so that the value of the additional pension accrual can be used used for a temporary life annuity compensating both the old age and the AOW deficit.

Exit at age 65

With an original exit age of 65 as given in Figure 4.6, part of $OP^{<20}$ is deferred and the value is used for compensating part of the old age pension and AOW shortage. The difference between the original exit age and the new age at which pension benefits start is overcome by increasing the working period. During these additional years pension is accrued of which the value is represented by the yellow block.

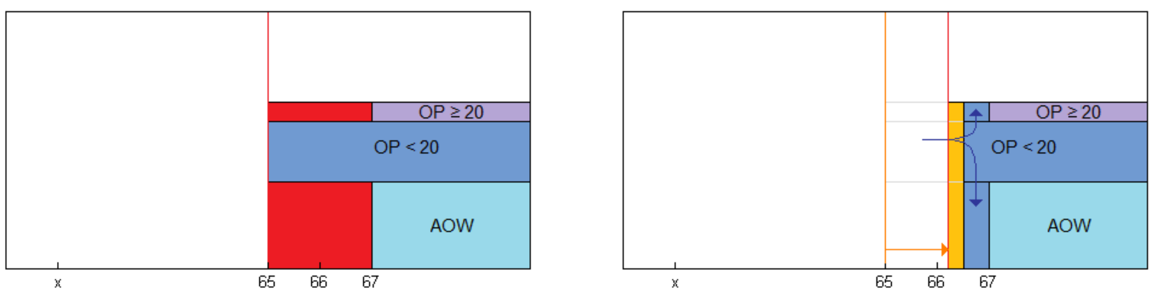

Figure 4.6: Exit at age 65

4.5 Fourth scenario: 67; 1.9%

While previously we only altered the accrual rate or the statutory retirement age, in this last scenario all changes proposed by the government are adopted in the pension plan. This means that in 2020 the retirement age is increased to age 67 and at the same time the accrual rate is decreased to 2.15%.

Early exit

Note that scenario four combines scenarios two and three. This implies that we now have an temporary old age deficit between ages 65 and 67 due to the statutory retirement age as in scenario three, and a whole life deficit due to the accrual rate as in scenario two. This is also apparent in Figure 4.7. Again, by increasing the working period one accrues new pension that can be used to fill the gap.

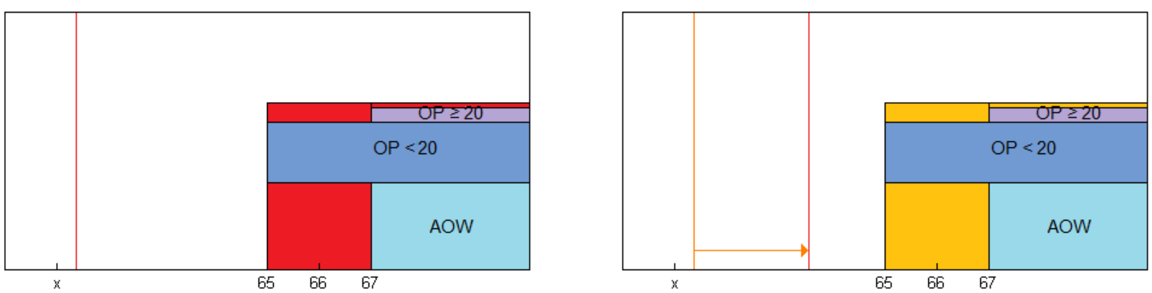

Figure 4.7: Exit at age 65

Exit at age 65

The case of an original exit age of 65 also resembles the scenarios described before; as shown in Figure 4.8 deferring part of $OP^{<20}$ and increasing the exit age results in pension payments at the original level from the moment the participants exits. Again with the new accrual one has to take into account whether the accrual takes place before or after 2020.

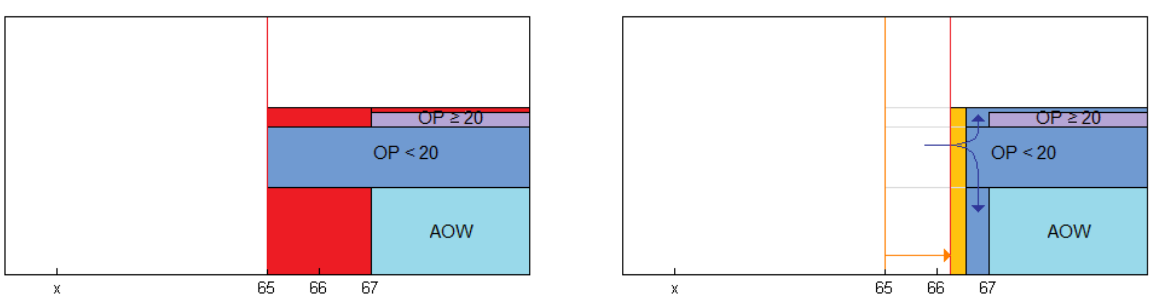

Figure 4.8: Exit at age 65

We have seen that even though the scenarios get more complicated, the same basic principles of deferring value and working longer underlie all scenarios. This also holds for finding the number of years one has to extend its working period; calculations get more difficult but similar techniques are used. All calculations for the additional years of service can be found in Appendix B.

4.6 Exit age and the average pension fund

We have described four scenarios based on the proposals of the Dutch government for changes in the employment based pension system. Until 2020 all scenarios are equivalent in that they adopt an accrual rate of 2% and a statutory retirement age of 65. From 2020 onwards the scenarios differed in these two points in the following way:

		Retirement age	
		65	67
accrual rate	2%	1	3
(after 2020)	1.9%	2	4

In all scenarios one has to work longer for compensating for the AOW and possible old age pension deficit, the size of this period depends on the original exit age and the specific scenario. The additional years of service for our average pension fund are displayed in Figure 4.9.

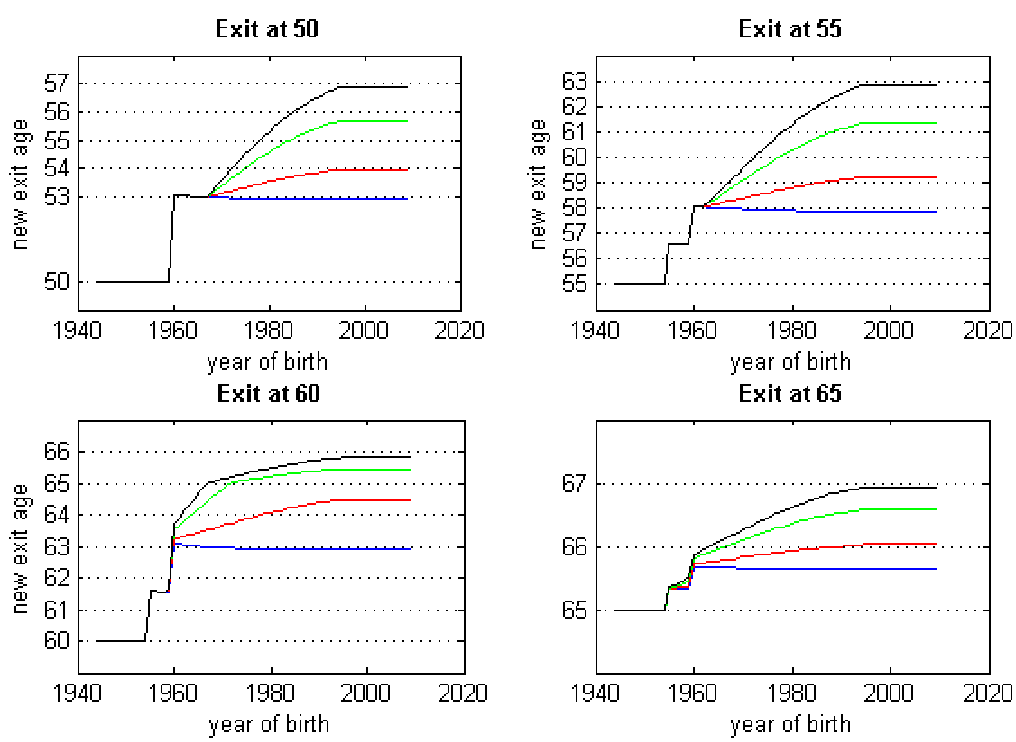

Figure 4.9: New exit ages for the scenarios (1=blue, 2=red, 3=green, 4=black)

The figure shows two jumps in the exit age; at the years of birth 1955 and 1960. Participants born between 1955 and 1959 face an AOW deficit of one year, all people born after that of two years. All those participants have to work longer, except those born between 1955 and 1959 with an original exit age of 50. In 2010 they have already exited and we assumed they do not enter the active workforce again. These people thus retain their AOW deficit.

Even though the scenarios are the same until 2020, differences in the model already occur earlier; in 2010 people start anticipating on the future changes and people who plan to exit early already lengthen their working period. In case of an early exit there are no differences between the scenarios for some generations. This is because for these generations all additional years of service are scheduled before 2020 and thus the same pension rules apply.

For scenario one we see that besides the two increases described before, the exit age stays relatively stable. This is because here only the AOW deficit needs to be filled and this deficit is the same for all generations. The slight decrease in exit age over time is due to the increasing life expectancy; the present value of one euro of pension increases with the future lifetime and therefore less accrual is needed for the temporary life annuity.

For scenarios two to four we see that the increase in exit age is the largest for scenario four where the pension accrual is restricted most. Furthermore increasing the retirement age has a larger effect than decreasing the accrual rate. Unlike scenario one, here the exit age is not stable for all generations with a two year AOW deficit but only for generations born after 1995. This is due to the old age pension deficit which size depends on the years of accrual before 2020. Participants born in 1995 enter the workforce in 2020 and thus completely accrue their pension under the new system. Until that time part of the pension accrual takes place under the old less restricted system.

One should notice that even though the change in the employment based pension system takes place in one step, the exit age increases gradually. Therefore there is no need to use multiple steps as is planned for the AOW age increase. The jumps we encounter are mainly due to the increase in the AOW age and not to the changes in the statutory retirement age. A stepwise increase would lead to a little smaller increase in exit age for people who are in the workforce between 2020 and 2025. They would then have 5 years with accrual based on a statutory retirement age of 66 instead of 67. Raising the retirement age stepwise along with the AOW age further just results in much more complex administration for the pension funds since there will be three different retirement ages with three different payment periods.

4.7 Break-even premium

We have seen that participation rates increase from the beginning of the simulation period, this increase in labour participation also affects the break-even premium from 2010 onwards as is shown in Figure 4.10. The dotted line represents the break-even premium in the base scenario, this premium shows an upward trend which can be explained as follows: Recall that the uniform contribution rate relies on solidarity in the sense that young participants pay more than the value of their accrual while older participants contribute less than they accrue. Increasing the number of young participants in the fund thus suppresses the premium,

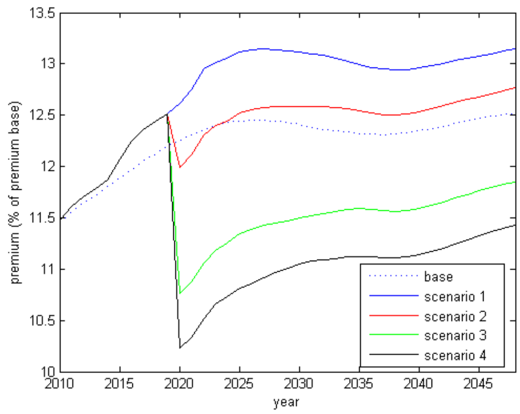

Figure 4.10: Break-even premium

while more older participants make premiums rise. Over time the average age of the active participants changes because of changing demographics; in the near future the ageing of the population causes an increase in this average age. Since accrual becomes more valuable if a person approaches retirement, the break-even premium increases over time. This process is enhanced by the increasing life expectancy.

In scenario one, part of the participants face an AOW deficit. The accompanying increase in participation rate results in a higher average age of the active participants since people stay active for an extended period while no additional young people enter the fund. This results in an increase in the break-even premium compared to the base scenario.

In scenario two the accrual rate is decreased from 2% to 1.9% in 2020. The lower accrual rate results in a higher participation rate for people age 50 and above and thus a higher average age than in scenario one and the base scenario. As explained before this has a positive effect on the premium. The decrease in the accrual rate itself has a negative effect on the premium since the value of the new accrual decreases for each individual. This explains the drop in break-even premium in 2020. Ultimately, the effect of the ageing participant file outweighs the effect of the decreased accrual rate as the break-even premium stabalizes at a higher rate than in the base scenario.

In scenario three it is not the accrual rate that counteracts the effect of the later exit, but now the value of the new accrual is suppressed because the newly accrued pension only starts to pay at age 67 instead of 65. Even though the exit age in scenario three is higher than in scenario two, there is a drop in the break-even premium in 2020. This implies that increasing the retirement age by two years has a stronger effect than lowering the accrual rate. In scenario four both the lower accrual rate as well as the higher retirement age have a negative effect on the premium, causing an even lower break-even premium. Both scenarios result in a break-even premium below the base level.

The changing participation rate and break-even premium have an effect on contributions, assets, liabilities and indexation. These consequences are the subject of Chapter 5.

Chapter 5

Simulation Results

In the previous two chapters we described the average pension fund model, which will be used as the basis of our analysis, and four possible changes in the employment based pension system as a reaction to changes in the state retirement system. In this chapter we will evaluate the pension fund using Monte Carlo simulation. Before discussing the results for our specific model, a short introduction to simulation will be given.

5.1 Simulation

As mentioned before, a crucial factor in relation to pensions is uncertainty. Pension funds have to deal with both short and long term consequences of their decisions and policy; all these consequences depend on an uncertain future. The risks to which a pension fund is exposed are related to different aspects ranging from interest risk, changing regulation, demographics, inflation, wage development and investment returns. By fixing demographics and certain economic variables we limited the uncertainty in our average pension fund model to investment risk. Many aspects of a pension fund are related to the investment returns: high returns lead to high funding rates resulting in low contributions and high indexation levels, low returns make contributions rise and indexations being cut.

We assumed that a Black Scholes model is underlying the stock price process meaning that the stock returns are normally distributed. This is the source of uncertainty in investments in our model. By simulating future stock returns for several years, future funding rates, contributions and benefit payments can be generated. One possible future however does not tell us anything about the pension fund, therefore we repeat the process 20,000 times. This is the principle of Monte Carlo simulation. Monte Carlo simulation in general is used in situations where one single simulation is not representative due to uncertainty and where the uncertainty of the variables can be represented by a probability distribution. Both apply in our case.

We use a simulation period of 25 years, that is the years 2010 - 2035, with time steps of one year. One can argue that this period is too short since there are already 40 years between the moment one enters the active workforce and retirement and since new generations enter the model each year. However, as explained in Hoevenaars (2008), simulating too far into the future is also arguable. A model is just a simplification of reality and the assumptions become less realistic if the horizon increases. With a policy horizon of 25 years both short as well as medium and long term implications are considered.

Unlike one single simulation, the 20,000 possible futures do give information about the sustainability of the pension policy in the long run. This information regarding for instance the funding rate, premium and indexation is in general given in terms of probability distributions and expected values. The next section describes these results for our specific pension fund and the four scenarios. For each scenario the same 20,000 futures are used so that we can compare them directly.

5.2 Simulation and the average pension fund model

The simulation results for the base scenario are graphically depicted in Figure 5.1.

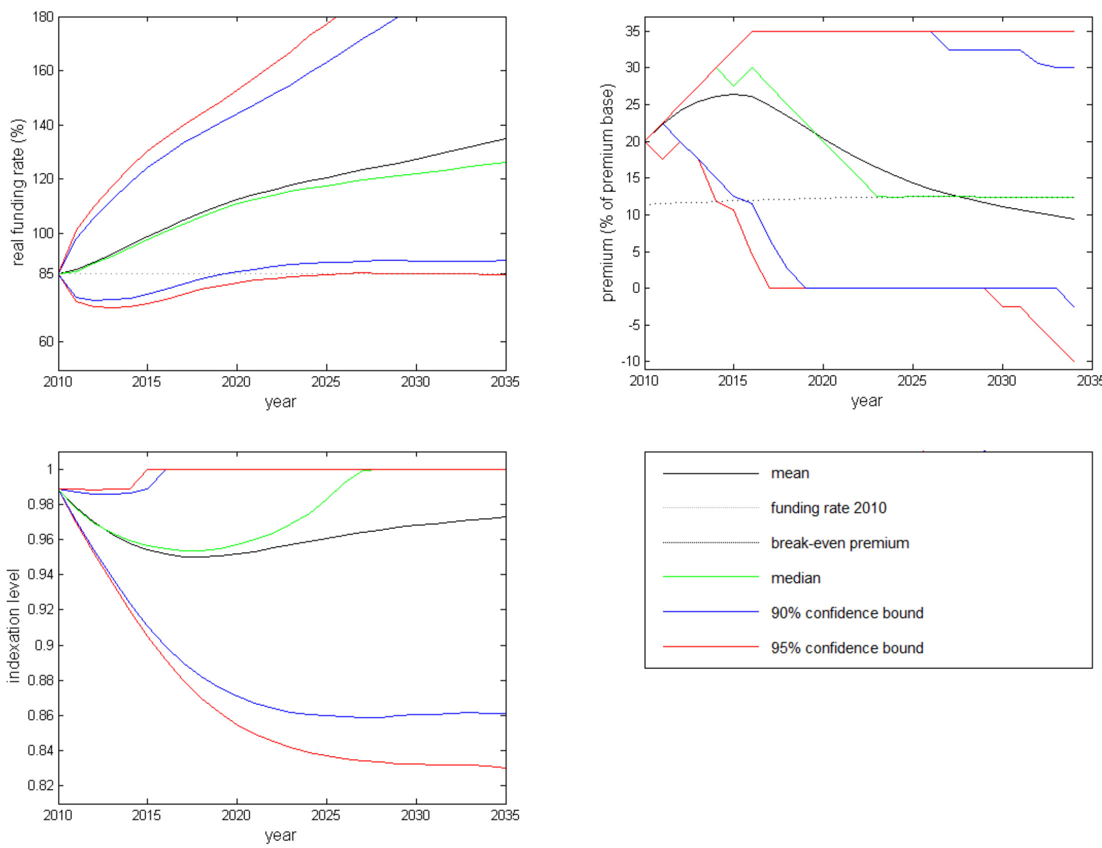

Figure 5.1: Funding rate, contribution rate and cumulative indexation level - average pension fund

The initial real funding rate (January 2010) is set to 85%, in our model this corresponds to a nominal funding rate of 116%. This is slightly higher than the average nominal funding rate of 111% on January 1st 2010 as given in the Pensionthermometer of Hewitt. Since the initial funding rate is below 100%, only partial indexation is granted. This results in an indexation deficit in 2010. We assume that until 2010 there was no indexation deficit. The initial contribution rate is set at 20% of the pensionable base, this is higher than the break-even premium. This decision is based on the status of underfunding in the late past which led to an increase in pension premiums.

On average, contributions rise initially. This is due to our contribution policy that is conditional on the real funding rate. For the same reason indexation is cut. Both the rising contribution level and the indexation cut contribute to the rise in real funding rate. Averaged over the 20,000 simulations, the real funding rate has risen to 100% in approximately six years, at that time the contribution rate starts to decline towards the break-even premium and the total indexation level stabalizes.

During the period 2010-2035 on average the contribution level increases initially to make up for the funding shortage, after which the premium declines again. At the end of the evaluation period, the mean contribution level is below the break-even premium while the median equals this level. With a mean contribution of 9.5 and a standard deviation of 10.7, the distribution of the contribution level has a large dispersion. In 2.87% of the simulations, the premium in 2035 equals the maximum premium of 35% of the premium base. The median of the average premium over the whole evaluation period equals 18.36% compared to an average break-even premium of 12.17%.

Note that the total indexation level given by $\frac{\prod^k(1+\pi)}{\prod^k(1+ind_k)}$ is displayed, and not the yearly indexation level. A total indexation of one means that there is no indexation deficit and all previous indexation cuts are made up for. The total indexation level increases only after catch-up indexation is granted. After 17 years, the indexation deficit is recovered in half of the simulations, after 25 years, full indexation is reached in 61% of the simulations. In 5 out of 100 simulations the indexation level at 2035 is below 86% of the cumulative end value of total indexation.

The mean real funding rate shows an upward trend with a mean funding rate of 134.98% in 2035. In over 97.5% of the scenarios the funding rate increases compared to the starting situation and 86 out of 100 times the fund recovers from its postion of underfunding and achieves a funding rate of at least 100%.

The results described above are summarized in Table 5.1 together with the results for the four alternative scenarios. Figure 5.2 gives a graphical summary of these results.

	base	scenario 1 (2%, 65)	scenario 2 (1.9%, 65)	scenario 3 (2%, 67)	scenario 4 (1.9%, 67)
Real funding rate					
2035					
median	126.62	125.56	126.49	127.67	128.79
mean	134.98	133.29	134.53	136.28	137.63
standard deviation	38.62	37.07	37.73	38.84	39.60
P($FR_{2035} \geq 85\%$)	0.9761	0.9759	0.9778	0.9795	0.9809
P($FR_{2035} \geq 100\%$)	0.8611	0.8560	0.8639	0.8723	0.8792
Contribution level					
2035					
median	12.34	13.02	12.56	10.84	9.61
mean	9.52	10.18	9.53	8.50	7.90
standard deviation	10.70	10.67	10.45	10.09	9.87
break-even premium (bep)	12.34	13.02	12.56	11.57	11.07
average					
median	18.36	18.68	18.43	17.91	17.63
mean	17.84	18.07	17.78	17.29	17.03
Total indexation level					
2035					
median	1.0000	1.0000	1.0000	1.0000	1.0000
mean	0.9724	0.9722	0.9739	0.9762	0.9777
standard deviation	0.0499	0.0493	0.0476	0.0451	0.0434
P(no indexation deficit)	0.6143	0.6029	0.6181	0.6371	0.6499
P(indexation level < 85%)	0.0395	0.0379	0.0331	0.0276	0.0248
5% confidence level	0.8579	0.8619	0.8668	0.8739	0.8786

Table 5.1: Simulation results

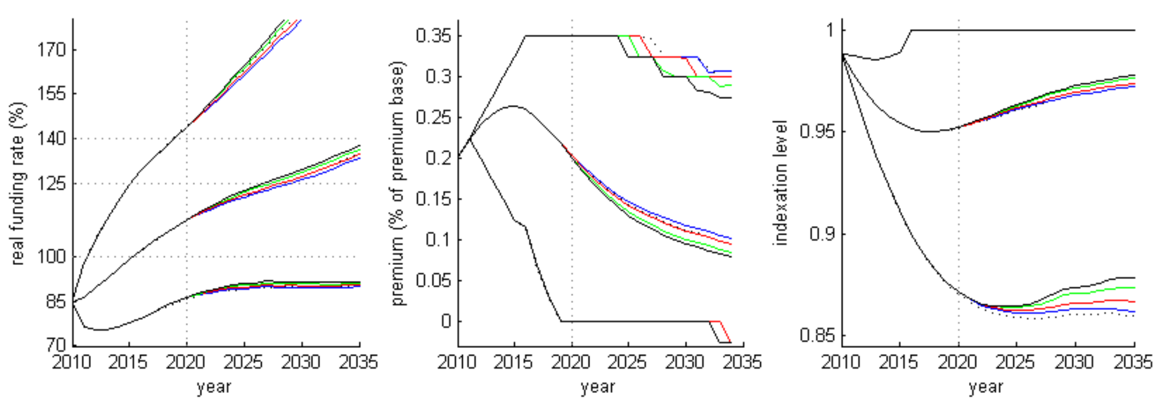

Figure 5.2: mean level and 90% confidence bounds (base=dotted, 1=blue, 2=red, 3=green, 4=black

5.3 Simulation and the four alternatives

When looking at the funding rate, premium and indexation level, the effects of changing the second pillar pension system as proposed compared to the current situation are quite small.

In Chapter 4 we saw that the break-even premium increases between 2010 and 2020 in all scenarios due to the increase in exit age. Nevertheless, the premium in all scenarios is equal in the first years. This results from the initial status of underfunding of the pension fund in combination with our premium policy. The premium level does not depend on the break-even premium until the funding status of the fund is sufficient again; until that time the premium increases by 2.5% point per year. Once a real funding rate of 100% is achieved the premium starts to move towards the break-even premium, yet the decrease is limited in that the premium may not change by more than 2.5%point per year unless the real funding rate exceeds 125%. In our model this takes at least three years, but on average it already takes until 2020 to return to the initial premium level of 20%. The effect of the increase in break-even premium on the premium charged before 2020 due to a change in exit age is thus negligible.

Charging the same premium while the break-even premium has increased means that the excess premium, that is the premium above the break-even level, decreases. A lower excess premium slows down the recovery of the funding rate. On the other hand more people paying a premium above the break-even premium, as is the case if the exit age increases, implies a faster recovery of the funding rate. Those two effects counteract, yielding a minimal change in the funding rate across the scenarios until 2020.

From 2020 onwards both the value of new accrual and the level of yearly benefit payments as well as the break-even premium are affected by the change in the pension accrual. At this point in time differences across the scenarios become visible.

In scenario one we did not change the retirement age or the accrual rate. The increase in exit age to overcome the AOW deficit was limited to about 8 months in case of an original exit age of 65 and to about 3 years in case of an early exit. The higher average age resulted in a increase in break-even premium of 0.7%-point in 2035 compared to the current situation. This increase in is reflected in a higher average contribution level in the whole 2020-2035 period. As before, the change in premium is limited by our policy implying that in case the premium level is still above the break-even premium, we charge the same premium in scenario one as in the base scenario. Again a lower excess premium has a negative effect on the funding rate and therefore also on the indexation level. The change in participant file, and thereby premiums charged and benefits paid, in general has a negative effect on the funding rate and contribution level in scenario one. Also the probability of full indexation in 2035 decreases slightly, but this is accompanied by a small increase in the 95% confidence level. Scenario two is comparable with scenario one, with the difference that the break-even premium is closer to that of the base scenario. Compared to the base scenario we find a slight increase in the median contribution rate and the average indexation level. All results are however very small.

In Chapter 4 we described a sharp drop in the break-even premium in scenarios three and four in 2020 due to the restricted accrual. This drop is not directly reflected in the average pension premium charged. In 2020 the median premium charged is still above the break-even premium due to the funding status of the fund and the 2.5% decrease limitation. In all these scenarios, the drop in break-even premium contributes to the funding status of the fund instead of a lower premium directly; the difference between the premium charged and the premium needed increases and the ratio of assets over liabilities improves. The rise in funding ratio on its turn does lead to a decrease in premium charged, but also to an increase in indexation.

In total the funding rate and indexation level improve in scenario three and four compared to the base scenario. The contribution level declines. The effect is the largest for scenario four where the accrual was limited most. Over the whole simulation period the median premium charged in scenario four is only 0.7%point lower than in the base scenario, but in 2035 the difference in median premium charged equals 2.7%point. The increase in funding rate due to the additional premium income over the premium needed results for most scenarios in a linear reduction on the break-even premium and catch-up indexation. In 2035 the average funding rate in scenario four has increased by 2.6%point and the probability of full indexation has increased with 3.5% point to 65%. Note however that alongside with an increase in the funding rate level also the variance has increased.

Overall we can conclude that some changes in the funding rate, indexation level and premiums charged occur due to the change in the system, but the differences are small. Even though the system changes, the value of the already accrued pension remains intact. The real value of the liabilities depends on the level of indexation and thus the funding status of the fund. The funding status is, among others, influenced by the contribution level. Taking only the pension result into account and ignoring the time of exit, scenarios with a retirement age of 65 are unfavourable compared to scenarios with a retirement age of 67. The best pension results are achieved in the most restricted scenario.

5.3.1 Initial funding rate

As mentioned before we used an initial real funding rate of 85%, corresponding to a nominal rate of 116%. The average nominal funding rate of Dutch pension funds at January 1st 2010 however equaled 111%. Furthermore we saw that the premium level greatly depends on the initial funding status. To get some insight into the effect of the initial funding rate on the simulation results, we repeated the simulations with an initial real funding rate of 80% and 100% for the base scenario and scenarios three and four. These correspond to nominal funding rates of 110% and 137% respectively.

First consider the situation with an initial funding rate of 80% compared to the situation with 85% as described above. In both cases we start with a situation of real underfunding, leading to strong premium increases in the beginning of the simulation period. Because the maximum increase in premium is limited, the period to recover from the funding shortfall is extended. On average it now takes 7 years to reach a funding level of 100%. The extended recovery period is reflected in the average premium level; after 6 years the average premium charged in the 80% funding rate case equals 28.75% against 26.10% in the 85% case. At the same time the mean cumulative indexation level is 2%point lower. After the funding rate is sufficient again, premiums decrease and the indexation level recovers. Still, in 2035 results in the 80% case are worse than in the 85% case, with a higher premium charged, lower funding rates and a lower cumulative indexation level. Overall, lowering the initial funding rate results in a slower recovery but similar trends in contributions, indexation and the funding rate over time.

With an initial real funding rate of 100% the situation changes. Instead of charging a premium of 20% of the pensionable base in 2010, we now start our simulation with the break-even premium since we are not in a situation of real underfunding anymore. The sharp increase in premium in the beginning of the simulation period does not occur anymore and the drop in break-even premium in 2020 is not absorbed by the premium policy restriction. Instead it is reflected in an immediate decrease in the median premium charged. In 2035 the mean funding rate is 1.8%point higher in the base scenario compared to the 85% case and 1.6% in the alternatives. The increased funding rate also has a positive effect on the indexation level, yielding an increase in the 95% confidence level of 1.5% in the base scenario. The relative changes resulting from the different initial funding rates are the largest in the base scenario and the smallest in scenario three, implying that the base scenario is most sensitive to the initial funding rate.

5.4 Classical simulation and market consistent valuation

The results described so far are classical simulation results where changes in the pension fund are evaluated by means of probability distributions and expected values. The simulation results show the change in expectation and uncertainty in variables like the contribution rate and indexation level, but they do not give any information about who carries the risks and what value these risks have for the stakeholders. To overcome this problem Chapman, Gordon, and Speed (2001) described an alternative approach where the uncertain future is assessed in terms of economic value at the current time. With this method of market consistent valuation one can determine the current value for the pension contract in general and for the different stakeholders.

Market consistent valuation can also be applied to different age cohorts of participants. This is achieved by using so called generational accounts. A generational account of a specific generation describes the difference between all contributions paid and all benefits received by a certain age cohort. The value of the generational account thus determines the value of the pension contract for a specific generation.

A change in the pension system does not change the total value of the fund, but it can result in changes in the different generational accounts. Comparing the value of the uncertain future cashflows for a specific generation under two different policies now gives insight in who gains and who loses from a change in the system in value terms. Applying the technique of market consistent valuation to generational accounts thus reveals value transfer between generations and hence provides information about the impact of a policy change on intergenerational solidarity. This can be a valuable supplement to the classical simulation results that provide information about the fund in general.

The construction and use of generational accounts and the technique of market consistent valuation will be discussed in Chapter 6.

Chapter 6

Market-Consistent Valuation

As mentioned in Chapter 3, the surplusses and deficits that arise due to uncertainty in the market are shared among the different generations by means of adjustment of the contribution rate and benefit level. This intergenerational risk-sharing leads to value transfers from one generation to another. Besides market uncertainty also policy changes can lead to redistribution of value; some groups of current or future participants will be harmed others will profit. To make sure that groups that are harmed will not opt out, mandatory participation in a pension fund is needed. However, even though this mandatory participation ensures the sustainability of the system, it is important to have insight into the consequences of a policy change on the different groups of interest. One method to show who gains and who looses from a change in policy is by looking at the change in the generational accounts of the different age cohorts. The method is first used in Kortleve and Ponds (2006b) and Hoevenaars and Ponds (2006) to examine the effects of a change in the investment and contribution policy and a switch from a defined benefit to a defined contribution plan. We will use the generational accounts technique for evaluating the effects of policy changes concerning the statutory retirement age. The construction of generational accounts will be discussed in section 6.1.

6.1 Generational account

At any point in time, the value of a pension fund equals the value of its assets. This value consists of the value of the accrued liabilities plus the funding residue. This implies that at $t = 0$ (in our case 1-1-2010)

$$A_0 = L_0 + R_0 \tag{6.1}$$

and in general $A_t = L_t + R_t$. Here
A_t = total assets at time t
L_t = total accrued real liabilities at time t
R_t = total funding residue at time t (can be positive as well as negative)

Next period ($t = 1$, 1-1-2011), the assets of the fund have generated return, contributions are received and benefits are paid by the fund. Furthermore, liabilities have changed due to extra accrual, benefit payments and indexation[1]. This process repeats itself every year and

[1] In reality liabilities will also change due to a difference between the real mortality and the mortality rates used to calculate the liabilities. However as noted in Chapter 3 we assumed the participant file evolves according to the forecast and hence the real mortality equals the forecast mortality.

the balance sheet of the pension fund changes accordingly. At time i the balance sheet in present value terms at $t = 0$ equals

$$A_0 + \sum_{j=1}^{i} V(C_j) - \sum_{j=1}^{i} V(P_j) = V(L_i) + V(R_i) \tag{6.2}$$

On the asset side of the balance $V(C_i)$ and $V(P_j)$ denote the present values of the contributions and benefit payments during the period $i-1$ to i; note that at $t = 0$ these are uncertain future cashflows. Furthermore the initial assets A_0 have generated return, however by definition its present value equals the initial assets. The present value of the real liabilities $V(L_i)$ consists of the initial real liabilities minus the liabilities written of due to benefit payments up to time i, increased with the present value of additional real accrual due to extra years of service and finally corrected for cuts in indexation. The present value of the residue is again the closing entry of the balance.

Combining 6.1 and 6.2 and collecting terms gives

$$(V(L_i) - L_0) - \sum_{j=1}^{i} V(P_j) + \sum_{j=1}^{i} V(C_j) + (V(R_i) - R_0) = 0 \tag{6.3}$$

This expression shows the a change in the liabilities is backed by contributions and possibly a change in the funding residue. Hoevenaars and Ponds (2006) refer to this as the zero-sum feature of a pension fund. At any time the total value of the fund equals the market value of the pension fund assets; changing the policy of the fund does not change the total economic value.

The participants of the fund have a claim on the assets of the fund. The value of the assets, liabilities and residue can thus be redistributed to all participants. Let $A^y = \sum^y A_t^y$, $L_y = \sum^y L_t^y$ and $R_y = \sum^y R_t^y$ where
A_t^y = claim on assets at time t of cohort born in the year y
L_t^y = real liabilities at time t of cohort born in the year y
R_t^y = claim on residue at time t of cohort born in the year y
For each age cohort the balance sheet at $t = i$ now reads

$$A_i^y = L_i^y + R_i^y$$

The total liabilities of the fund correspond to the total present value of the future benefit payments. The benefit payments depend on the accrued pension, which implies that the liabilities per age cohort can be specified according to the accrual of each cohort. The relative distribution of the real liabilities per age cohort is displayed in Figure 6.1. For redistributing the residue we need to specify a closure rule that specifies the claim on the residue for each cohort. The closure rule we apply is that the claim on the residue is proportional to the value of the real liabilities of the age cohort, that is $R_t^y = \frac{L_t^y}{L_t} R_t$.

While the balance sheet of a specific age cohort at a certain time is similar to that of the pension fund as given in Equation (6.1), Equation (6.2) does not hold for each generation separately. This is due to the intergenerational risk-sharing. Recall that by charging a uniform premium young generations pay more than the value of their accrual while older generations

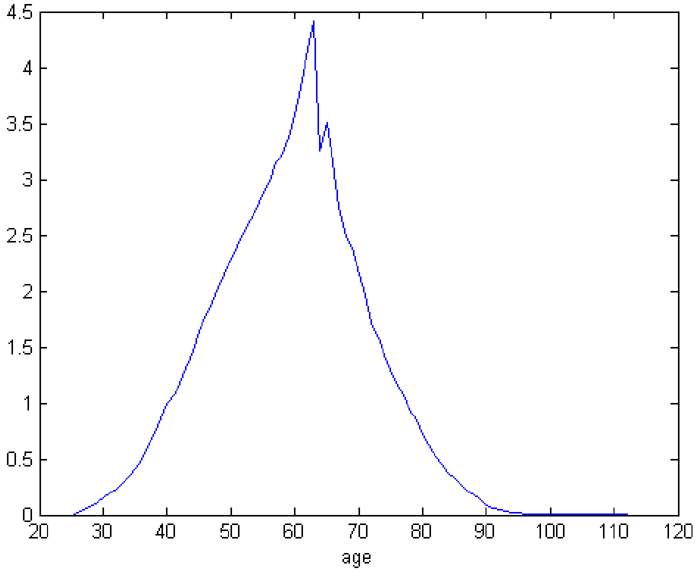

Figure 6.1: Relative distribution of real liabilities (2010)

pay less, value thus transfers from young to old participants. Therefore, when looking at a finite time period, the change in value of the liabilities and hence the residue corresponding to young age cohorts will in general be smaller than the present value of their contributions. Following Hoevenaars (2008) we define a generational account of a specific age cohort as the difference between the value of the benefits that are to be received by the cohort and the contributions that are to be paid. The change in economic value in the generational account ΔGA_i^y of cohort y from $t = 0$ to $t = i$ is given by

$$\Delta GA_i^y = (V(L_i^y) - L_0^y) - \sum_{j=1}^{i} V(C_j^y) + \sum_{j=1}^{i} V(P_j^y) + (V(R_i^y) - R_0^y) \tag{6.4}$$

Note that this equation follows from rewriting 6.3 for a specific age cohort and thus $\sum^y \Delta GA_i^y \equiv \Delta GA_i = 0$. As explained above, for a specific generation ΔGA_i^y will not equal zero in general.

6.1.1 Fairness of a pension agreement

Since by using generational accounts we have some knowledge about the redistribution of value, it seems tempting to make statements about the fairness of a pension contract. However, as stated in Hoevenaars and Ponds (2006), it is important to note that calculating the present value of future cashflows does not imply anything about the fairness of the pension agreement itself. A fair pension contract is a contract where each generations receives as much as it contributes. In our model only the future 25 years are taken into account while contributions and benefits during the past and far future are ignored. To be able to make statements about whether a pension agreement is fair for the current and future members of the fund, one needs to evaluate all cashflows to and from a generation. This implies that the whole lifespan of different generations needs to be considered resulting in an evaluation period of over 100 years. Such a long simulation period can not be justified since it is not possible to make realistic assumptions for such a long horizon.

6.1.2 Policy changes

While generational accounts are not suitable for assessing the fairness of a pension contract, they are appropriate for evaluating value transfers due to policy changes. As mentioned before policy changes can lead to redistribution of value; this also holds for our policy changes concerning the statutory retirement age. To asses the different alternatives, it is important to have insight into the consequences of the policy change on the different groups of interest. The consequences for the age groups can be quantified by the change in the generational accounts of the different age cohorts.

Let ΔGA_i^* correspond to the original policy and ΔGA_i^k to policy alternative k. For the fund in general a policy change will not lead to a change in economic value since $\Delta GA_i^k - \Delta GA_i^* = 0$. For the individual participants or age groups the change in generational account is given by

$$\begin{aligned} \Delta GA_i^{y,k} - \Delta GA_i^{y,*} &= (V(L_i^{y,k}) - V(L_i^{y,*})) - (\sum_{j=1}^{i} V(C_j^{y,k}) - \sum_{j=1}^{i} V(C_j^{y,*})) \\ &\quad + (\sum_{j=1}^{i} V(P_j^{y,k}) - \sum_{j=1}^{i} V(P_j^{y,*})) + (V(R_i^{y,k}) - V(R_i^{y,*})) \end{aligned}$$

To expression shows that there are four ways in which the value of a pension contract can increase or decrease for a participant due to a policy change. Namely by changes in the liabilities, contributions, benefit payments or the claim on the residue. In general $\Delta GA_i^{y,k} - \Delta GA_i^{y,*}$ does not equal zero.

Besides the change in generational account per age cohort, we can also measure the total redistribution of value of the policy change. Hoevenaars (2008) defines this total generational transfer as

$$GT_i^{*k} = \frac{\sum^x |\Delta GA_i^{y,k} - \Delta GA_i^{y,*}|}{2}$$

6.2 Present Value

So far we considered the idea and use of generational accounting, a technique based completely on the present value of the future cashflows. The method for calculating these present values is the subject of this section. The contribution rate and level of indexation (and thus the benefits and real liabilities) depend on the funding rate of the pension fund; the funding rate is dependent on the return on the assets. Since the return on assets is stochastic, all future cash flows are uncertain. Calculating the present value of these uncertain cashflows can be achieved by methods that are also used in pricing derivatives. Before describing such a method we start with trying to develop some intuition of why these methods can be used by explaining the similarities between a pension agreement and a financial derivative.

6.2.1 Embedded options

Several methods are available for pricing financial derivatives such as options. Maybe a pension contract seems different from a derivative, but the uncertain cashflows that result from the pension contract can actually be seen as embedded options written between the fund and the participants or between the different age cohorts.

Hoevenaars, Kocken, and Ponds (2009) define an indexation option and a contribution option. The conditional indexation policy implies that the pension fund can cut indexation in case of an insufficient financial position. The participants of the fund are obliged to give up part of their benefits and thus conditional indexation can be thought of as the participants writing a put option to the fund. The contribution option is the option of the fund to increase or decrease the contribution rate depending on the funding status. Rising the contribution in a status of underfunding corresponds to buying a put option from the participants while lowering the contribution rate implies selling a call option.

An other way of translating a pension contract into options is by means of an option on the residue. Suppose that the total value of the contributions from a participant to the fund exceeds the value of the benefit payments, in this case the participant leaves the difference in value in the pension fund. The participant is obliged to leave the value behind and thus this can be interpreted as the participant selling a put option to the fund. Similar, if the benefit payments exceed the contributions the difference is a gain to the participant. This situation corresponds to buying a call option from the fund. By buying a call and selling a put, the value to the participant can be positive as well as negative. Note that the value left in the fund by one participant is a gain to an other participant. The option thus also can be described as buying a call from and selling a put to other generations; the call option protects the participants own pension rights while the put option makes sure that rising potential is shared with the other participants.

Above we described different embedded options in a pension agreement. The cashflows from the pension fund thus can also be generated by a series op options and since identical cashflows should have identical present values, knowing how to value options implies knowing how to value the pension contract. One way to calculate the present value of an option is by discounting the cashflow resulting from the option by an appropriate discount rate. Using Monte Carlo simulation possible scenarios for the uncertain cashflows of the option can be generated. Discounting all cash flows and taking the expectation yields the correct option value. Valuing the options embedded in the pension contract is thus equivalent to discounting the cashflows resulting from the pension contract.

Instead of valuing the pension contract itself we want to value the change in the generational account for a specific age cohort. Recall from equation 6.4 that the change in value in the generational account was defined as

$$\underbrace{\Delta GA_i^y}_{\text{generational account option}} = \underbrace{(V(L_i^y) - L_0^y) - \sum_{j=1}^{i} V(C_j^y) + \sum_{j=1}^{i} V(P_j^y)}_{\text{net benefit option}} + \underbrace{(V(R_i^y) - R_0^y)}_{\text{residue option}}$$

Similar as we can rewrite a pension contract in embedded options, ΔGA_i^y can be described as the sum of a net benefit option and a net residue option. The net benefit option gives the value of the change in liabilities (that is future benefits) and benefit payments during the evaluation period minus the value of the contributions. The net residue option describes the difference between the uncertain claim on the future residue and the certain claim on the current residue. Again the values of the options and thus value of the change in generational account can be determined by discounting the corresponding cashflows and taking the expectation. Note that we already generated these cashflows by monte carlo simulation as described in Chapter 5.

Now we know that calculating the present value of the cashflows in the different scenarios yields the correct prices of the embedded options and thus of the change in generational account, the question arises how this discounting is accomplished.

6.2.2 Discounting

Valuing uncertain cashflows relies on the assumption that the market is complete; in a complete market every derivative can be perfectly replicated by the existing assets. Since there are no wage indexed financial products, assuming a real wage growth of zero avoids the problem of pricing in an incomplete market (Hoevenaars (2008)). Furthermore, by stating that identical cashflows have identical present values we assumed that the market is arbitrage free. In an arbitrage free market two methods for discounting future cashflows are risk neutral valuation and pricing by deflators. Below a short outline of the two methods is given.

Risk neutral valuation

In risk neutral pricing one uses the risk free interest rate to discount expected cashflows from an asset to determine its price. In the real world however the expected returns on derivatives depends on their risk: investors want to be compensated for the risk they incur with risky assets and therefore demand a higher return. As we do not correct for this effect of risk in the discount rate, we adjust the probabilities of future outcomes into so called risk neutral probabilities. The risk neutral probabilities constitute the risk neutral measure $\mathcal{Q}$ under which the present value of each asset equals the value of the expected future cashflows discounted at the risk free rate.

In the context of our simulation study risk neutral valuation implies that first the stock price process under the risk neutral measure needs to be derived. For the Black Scholes model the stock price process under $\mathcal{Q}$ can be found in Appendix C.2. Second instead of generating scenarios under the real measure as we did in Chapter 5, scenarios need to be generated under the alternative measure. The present values of all resulting cashflows are found by discounting by the risk free rate. Taking the weighted average of the present values over the different scenarios now yields the correct market consistent price of the uncertain future cashflows.

Deflators

In risk neutral pricing we use the same discount factor regardless of the state of the world and the riskiness of the assets. Furthermore we adopted a risk neutral probability measure instead of the real probability measure. In contrast, the deflator method uses the real probabilities and thus the real expected returns on the assets instead of the risk free rate. The future uncertain cashflows are discounted by using a stochastic discount factor: the deflator. The deflator depends on the state of the world; participants are risk averse and therefore different values may be attached to a certain cashflow in case of economic good times compared to bad times. When using deflators, economic favourable scenarios with high returns get assigned a lower weight in the discounting process than scenarios with low stock returns. The riskiness of the assets is thus taken into account in the discount rate instead of in the probabilities. Since deflators depend on the state of the world they depend on the scenario generated, nevertheless they are independent of the derivative on which it is applied. This means that the same deflator can be used for the different cashflows in a single scenario. In Appendix B the methods are explained more elaborately in case of a simple binomial model. Furthermore it is shown that both methods are equivalent in that they generate the same prices. For a extensive discussion of deflators in general we refer to Jarvis, Southall, and Varnell (2001).

While the constant discount factor used in risk neutral valuation is much simpler than the stochastic deflator, risk neutral pricing also has a major disadvantage in simulation studies. As stated before, implementing risk neutral pricing in our simulation study implies generating scenarios under $\mathcal{Q}$ apart from the scenarios already generated under the real measure. In the deflator approach the real scenarios that we already needed for analyzing for instance the funding level can be used. Since in our model one simulation run takes several hours, the deflator method can save computational time. For this reason we chose to implement the deflator approach in our MATLAB model.

For the Black Scholes model the deflator equals

$$DF^{\mathcal{P}} = \exp\{-\lambda W_t - (\frac{1}{2}\lambda^2 + \tilde{r})t\}$$

with $\lambda = \frac{\tilde{\mu}-\tilde{r}}{\sigma}$. The derivation of the deflator can be found in Appendix C.2. The present value of a financial derivative now is found by taking the weighted average over the discounted cashflows where a different discount rate is specified for each scenario and each point in time.

By using the theory of deflators and embedded options, we constructed the generational accounts in our pension models. The generational accounts and the generational transfers resulting from a change in the system are the subject of Chapter 7.

Chapter 7

Market-Consistent Simulation

In Chapter 5 we described some classical simulation results. We found small changes in the funding rate, indexation and premium level due to the proposed changes in the second pillar pension system. In this chapter we will complete our simulation results with the change in generational accounts. These changes quantify the generational transfers that result from the changes in the system. We start with a description of the embedded options in our average pension fund model; thereafter we will turn to the changes in the alternative scenarios.

7.1 Average pension fund model

Recall that we defined the change in economic value in the generational account of cohort y from $t = 0$ to $t = i$ by

$$\underbrace{\Delta GA_i^y}_{\text{generational account option}} = \underbrace{(V(L_i^y) - L_0^y) - \sum_{j=1}^{i} V(C_j^y) + \sum_{j=1}^{i} V(P_j^y)}_{\text{net benefit option}} + \underbrace{(V(R_i^y) - R_0^y)}_{\text{residue option}}$$

The net benefit option (BO) can be separated in three parts: the change in the value of the liabilities, the value of the contributions made to the fund and the value of the benefits received from the fund. The residue option (RO) is constituted by the change in the value of the claim on the residue. The generational account and the separate embedded options in our average pension fund are graphically displayed in Figure 7.1. We will discuss all elements shortly. From now on, if we refer to people with a certain age we aim at the age in 2010.

Employees pay contributions up to age 65, therefore the contribution option is zero for people aged 65 or above. People born at the beginning of the simulation period enter the workforce in 2035, their first contributions thus take place just outside the evaluation period. The contribution option increases with the number of years premium is paid; the option is maximum for employees aged 40 in 2010 who pay premium during the whole simulation period. Their option value equals 2,79% of the total real liabilities in 2010. People aged 41 to 64 exit the workforce before 2035, during the simulation period this group both pays contributions and starts to receive benefits.

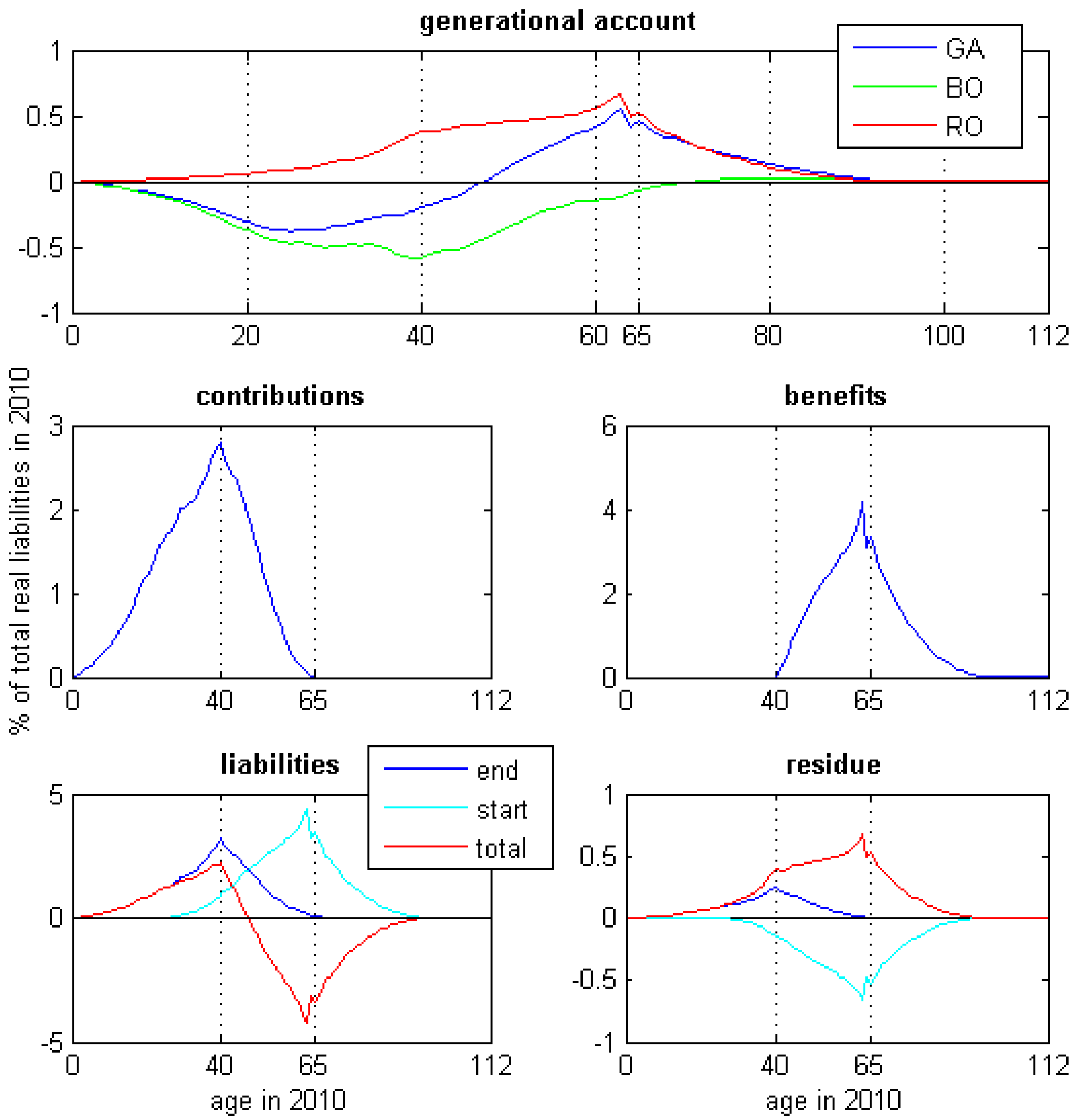

Figure 7.1: Embedded options

The benefit option is positive for all employees who receive benefits during the evaluation period. While all people aged 65 or above in 2010 receive benefits each year, the benefit option does decrease with age. This is due to the composition of the Dutch population as we have shown in Figure 3.1 and the increasing life expectancy. Currently the baby boom generation, with a peak at age 63, is close to retirement. This group has the largest benefit option with a value of 4.16% of the total real liabilities. Both contributions and benefit payments during the simulation period are reflected in the liabilities.

Recall that the net benefit option gives the value of the change in liabilities and benefit payments, minus the value of the contributions during the evaluation period. For all participants under age 72 the net benefit option is negative, meaning that they hand in value when looking at contributions and benefits compared to the corresponding change in liabilities. In the beginning of the simulation period active participants pay contributions above the break-even level to restore the funding deficit. Furthermore all participants deal with indexation cuts. Ultimately catch-up indexation may take place and contributions may fall below the break-even premium, but this occurs in good economic times. By using the deflator approach these cashflows are less valuable than the cuts and premium increases in hard economic times.

All participants of the pension plan have a claim on the residue of the fund; the initial status of real underfunding makes sure that this claim is negative in 2010. While the contribution and indexation policy were aimed at a recovery of the funding rate, the value of the residue increases resulting in a positive residue option for all participants.

The positive residue option compensates for the negative net benefit option. Ultimately the generational account is negative for all people below age 47 and positive for all people above this age. The difference between young and old generations is largely explained by the uniform contribution rate. As explained before this contribution system makes sure that young employees pay more than the value of their new accrual while older employees pay less.

7.1.1 Total residue option

By looking at the generational account for each age cohort we get some insight into the generational transfers between generations. As presented in Hoevenaars (2008), another measure of the transfers between current and future participants is the residue option for the pension fund as a whole. In measuring this total residue option we thus do not look at the separate age cohorts but at all participants at once.

Recall that at any point in time, the total funding residue can be positive as well as negative. If the funding residue is positive we have a surplus, if the residue is negative we have a deficit. Since $R_i = A_i - L_i = \max(0, A_i - L_i) + \min(0, A_i - L_i)$, the residue option can be written as

$$V(R_i) = V(\max(0, A_i - L_i)) - V(\max(0, L_i - A_i)) = V(S_i) - V(D_i)$$

The residue option can thus be split in a surplus option $V(S_i)$ and a deficit option $V(D_i)$. The surplus option can be seen as the participants buying a call option from the fund because any surplus within the fund belongs to the participants. Similarly, the deficit option can be seen as the participants selling a put option to the fund since any deficit is also at the expense of the participants. Both options have a strike price equal to the value of the real liabilities.

As explained in Hoevenaars (2008), the surplus option represents the upward potential of the residue while the deficit options represents the downside risk. The larger the difference between both options, the larger the imbalance between value transfers from current to future generations. In the average pension fund model we find a surplus option of 22.4% and a deficit option of 1.8% of the real liabilities in 2010. The large surplus option and small deficit option are due to the current status of underfunding combined with our pension policy that aims at a real funding rate of over 100%. We start with a negative residue of 20% of the real liabilities; by adjusting both contributions as well as the indexation level we have a high level of risk absorption for current generations, resulting in a transfer of upward potential to future generations.

Knowing the composition of the generational account and the options in the average pension fund model, we now turn to the change in the embedded options and the generational account due to the proposed policy changes.

7.2 Changes in generational account

We have seen that any change in the system in 2020 has an effect on the exit age, premium charged, indexation level and the value of new accrual. Therefore the value of the different embedded options will change in the four scenarios. We will discuss all components separately, starting with the benefit and contribution options.

7.2.1 Benefit and contribution option

The contribution option depends strongly on the total contributions made by a certain age cohort. The total contribution level changes due to changes in the premium charged and differences in the participation rate. Previously we noticed that over the scenarios the exit age and thus participation rate increases while the contribution rate decreases. The effects of these changes on the contribution option are opposing: a decrease in premium causes a decrease in contributions, an increase in exit age causes an increase in contributions.

The benefit option changes because of indexation differences and changes in the benefit payments due to the extra payments to overcome the AOW deficit. We saw that over the scenarios the indexation level improves. The effects of indexation and exit age are also opposing: an improved indexation situation results in higher benefits, an increased exit age means a smaller period where the AOW deficit has to be compensated and thus results in lower total benefits.

Recall that we have the following situation:

age in 2010	AOW age
≥ 56	65
51 - 55	66
≤ 50	67

		retirement age	
		65	67
accrual rate	2%	scenario 1	scenario 3
(after 2020)	1.9%	scenario 2	scenario 4

Figure 7.2 shows the changes in the benefit and contribution options in the four different alternatives compared to the base scenario.

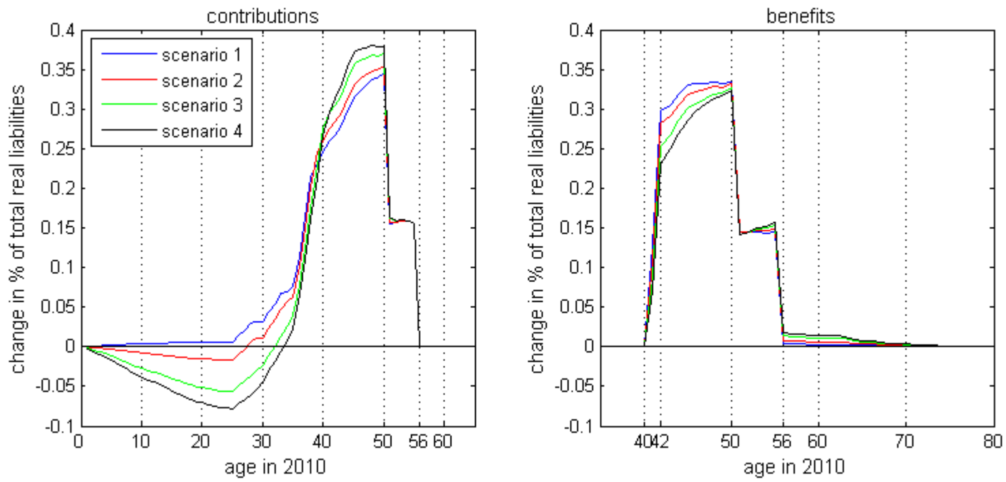

Figure 7.2: Change in benefit and contribution option

We start with the participants under age 25. This group has a maximum age of 50 at the end of our simulation period, implying that no one has left the workforce. Therefore the effect of an increase in exit age is not visible yet. Recall from Table 5.1 that we found an increase in contribution rate in scenario one. This results in an increase in contribution option. For scenarios two to four the contribution option decreases. While the greater part of this decrease in scenario three and four can be explained by the decrease in premium charged, another factor has to take part in the change. After all, we also find a decrease in scenario two where the mean and median contribution levels where comparable to the base scenario. This missing factor is the decrease in the standard deviation of the premium charged. Since premium decreases in bad economic times are more valuable than premium increases in good times, the value of the contribution option decreases.

Neither participants up to age 40 do retire during the evaluation period, but part of the participants do end their active career. This means that besides the effect of a changing contribution rate also the effect of increasing the working period becomes visible. This is reflected in an increase in contribution option over the different age cohorts. The effect is best explained in scenario one where the increase in exit age is similar for all age cohorts between age 25 and 40. For people aged 26 to 30, the group with an initial exit age of 50 lengthens it working period with approximately three years. Of these three years only one year falls within the simulation period for people aged 26 and only two years for people aged 27. This implies that the contribution option increases from age 26 to 28, after which it stabilizes because the whole increase in exit age is covered by the simulation period. Similar effects are apparent between ages 31 and 35, and between ages 36 and 40, where also the effects of an original exit age of 55 and 60 are included respectively. Since over 60% of the remaining active participants exits at age 60, the increase is strongest between age 36 and 40. In scenarios two to four the change is more gradual since the exit age keeps increasing over time as was explained in Chapter 4.

For ages 41 to 50, also the increase in working period for the group that originally exited at age 65 is taken into account. Where first the effect of a lower premium dominated resulting in a largest decrease in contribution option for scenario four, the effect of an increase in exit age now prevails leading to the largest increase for the same scenario. Even though the whole increase in exit age is processed in the contribution option, the option value keeps increasing between ages 44 and 50. This is explained by differences in age cohort size and the timing of the extra working period. Extending the working period in times of high premiums, as is the case for people aged 50, leads to larger option values than extending the working period when low contributions are charged. Furthermore, the average deflator decreases over time, causing benefit payments in the near future to be on average more valuable than payments in the far future.

While the change in exit age resulted in the largest contribution option in scenario four, the exit age also results in the smallest increase in benefit option for this scenario. By increasing the exit age, one shortens the period with an AOW deficit and hence smaller benefit payments are required. Between ages 42 and 50 the differences in exit age between the scenarios as displayed in Figure 4.9 are clearly visible. The increase in option value between ages 42 and 50 can be again be explained by the timing of the payment and the differences in cohort size.

Participants aged 51 to 55 have an AOW deficit of one year, which is reflected in both an increase in both the contribution as well as the benefit option. Differences between the scenarios are very small due to the small increases in exit age and the timing of the additional payments. Additional payments take place in the part of the simulation period where there are no clear differences between the scenarios yet.

For all people aged 56 or above the employment situation does not change; there is no AOW deficit and they exit at their original exit age. The contribution option thus may only change because of changes in the premium charged. In Chapter 5 however we found that even though the break-even premium changes from 2010 onwards, the change in premium charged was negligible until 2020. At that time, this whole generation has retired. The change in the contribution option is therefore also negligible. The benefit payments is positive in all alternatives, with the smallest increase in scenario one and the largest increase in scenario four. These increases are due to the improved indexation position, yet changes are very small.

Knowing the change in the contribution and benefit options in the different scenarios we now turn to the liabilities and residue.

7.2.2 Liability and residue option

Figure 7.3 displays the change in liability and residue option in the different scenarios. The change in liabilities is strongly related to the change in contributions and benefits. For the cohorts aged 25 or below, the change in liabilities in scenario one compared to the base scenario is negligible. Pension is accrued based on the same retirement age and accrual rate and hence differences can only occur due to changes in indexation. In Chapter 5 however we concluded that the difference in indexation level between both scenarios is very small. For the other scenarios the value of the accrual decreases since either the accrual rate is decreased or the retirement age is increased. Between ages 25 and 40 the same steps are visible as we

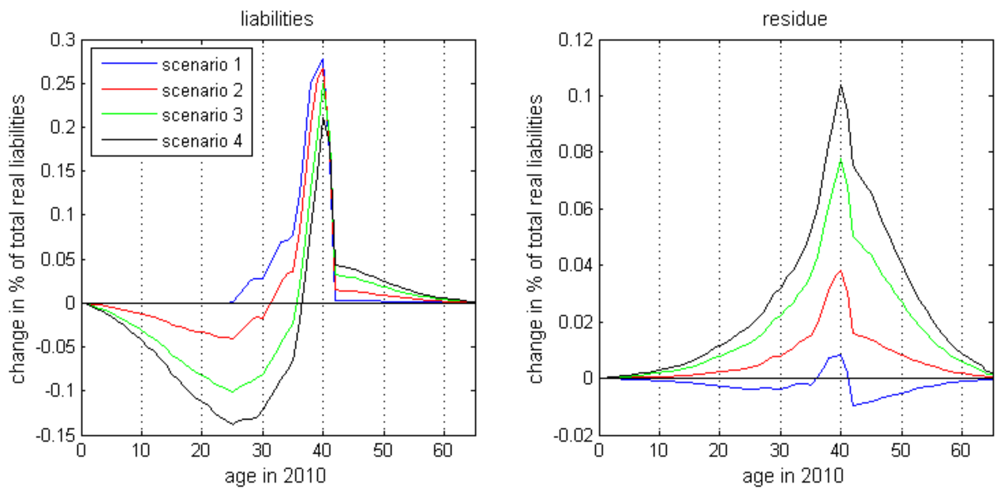

Figure 7.3: Change in liability and residue option

described in the contribution option.

Note that the decrease in liabilities is much larger than the decrease in contribution option while the increase in liabilities is much smaller than the increase in contribution option. Part of this difference is caused by the uniform contribution rate in which young people pay too much compared to the value of their accrual. A second factor contributing to the difference is the deviation between the premium charged and the break-even premium; a premium above the break-even premium is reflected in the contribution option without being compensated for in the pension accrual.

Recall that our whole model is based on the assumption that after a change in the system everyone wants to retain the same level of pension benefits as they would currently have. This means that between ages 65 and 67 benefit payments are higher in order to overcome the AOW deficit while after age 67 pension benefits are the same as before except for differences in indexation. The benefit payments at age 65 and 66 are thus import for changes in the benefit and liability options. For someone aged 40 both benefit payments are just outside the evaluation period, resulting in the largest increase in liabilities. For people aged 42 or above all payments occur during the evaluation period. This implies that the positive liability option for these cohorts is due to the improved indexation position as we found in Chapter 5.

The value of the residue option depends on the funding status and the liabilities of the fund in 2035. In the base scenario we found a total residue option of 20.5% of real liabilities in 2010, consisting of a surplus option of 22.4% and a deficit option of 1.8%. In scenario one, the surplus option decreases by 0.1%point and the deficit option increases by 0.1%point. There is a slight transfer of risk to future generations compared to the base scenario due to a decrease in risk absorption by the current generations. In scenarios two to four we find an opposite effect; surplus options increase while deficit options decrease resulting in an increase in the total residue option and a transfer of upward potential to future generations. The effect is largest for scenario four with a surplus option of 23.6% and a deficit option of 1.3%, in this scenario we also found the largest improvement in funding rate.

Since the claim of a cohort on the residue is proportional to the value of the real liabilities belonging to the cohort, the increase in residue option is largest for the cohort aged 40. Those participants have accrued almost their complete pension at the end of the evaluation period while they do not receive any benefits. Younger generations still have to accrue pension for some years and older generations already received benefit payments, both resulting in a lower share in the total liabilities.

7.2.3 Change in generational account

The embedded options described above together are responsible for the generational accounts. Adding the change in liability, residue and benefit option and subtracting the contribution option gives the change in generational account option compared to the base scenario as displayed in Figure 7.4.

In all scenarios we find an increase in generational account option for participants aged 56 or above. These age cohorts are not directly affected by the increase in AOW age or changes in the employment based pension system since they all retire before the changes take place. Their generational account option however does change because of changes in indexation level. In Chapter 5 we found an improvement in the cumulative indexation level in all scenarios; this improvement was reflected in both an increase in the mean indexation level as well as in increase in the 5% confidence level. The resulting positive change in the net benefit option, stimulated by a positive residue option in scenarios two to four, leads to a positive generational account option.

Participants aged 51 to 55 in 2010 face an AOW deficit of one year. Their extra years of service take place between 2010 and 2025; the period where on average contributions where high and indexation was low in order to achieve a sufficient funding status again. For this group the change in contribution option exceeds the change in benefit option; the increase in liabilities due to the improved indexation position partly compensates for this effect. In scenario one and two this still amounts to a negative net benefit option, but in scenarios three and four the net benefit option is just above zero. Combined with the residue option these cohorts are as well off as in the base case in scenario two, and better of in scenarios three and four.

In the previous section we found that the residue option increases most in the scenario where the accrual was restricted most; the net benefit option shows an opposite effect. Scenarios three and four result in a negative net benefit option for all participants with a two year AOW deficit. As described before this is mainly due to the use of an uniform contribution rate and the high contribution rates. All participants lose value due to the decrease in break-even premium which is not reflected directly in the premium charged. The increase in residue option however leads to a positive generational account option for all people aged 37 or above.

In scenarios one and two we find a different result. Where in scenarios three and four the decrease in break-even premium is not directly reflected in a decrease in premium charged, in scenarios one and two the increase in break-even premium does not directly result in an increase in premium. There is a smaller mismatch between the value of new accrual and con-

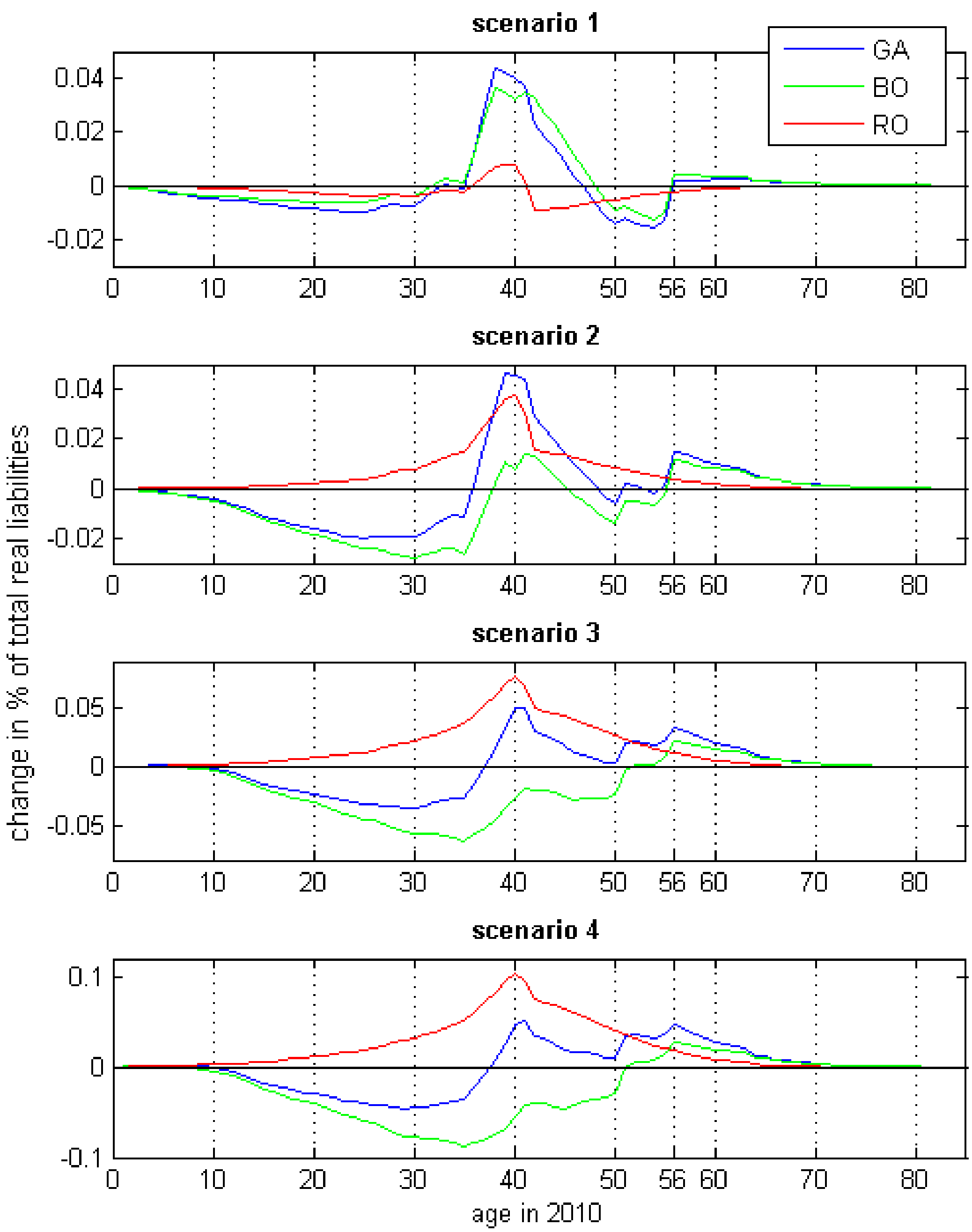

Figure 7.4: Change in generational account

tributions, resulting in a smaller decrease in net benefit option for the young generations and even a positive net benefit options for currently middle aged generations. This improvement in net benefit option compared to scenarios three and four is counteracted by the change in residue option, leading to a positive change in generational account option for generations aged 35 to 46 and 36 to 48 in scenarios one and two respectively.

In Chapter 5 we found that the more restricted the pension accrual, the better the pension results regarding indexation, contribution level and funding rate. In that case we have a higher level of risk absorption by the participants of the fund as was also reflected in the surplus and deficit options. It also appears that the changes in generational account are the largest in these most restricted scenarios. However, for each alternative the change in generational account is very small.

Recall that the total generational transfer measures the total redistribution of value of the policy change. In scenario four we find a total intergenerational transfer over the 25 year simulation period of 0.82% of real liabilities in 2010, which corresponds to a transfer of 1.12% of nominal liabilities. The size of this transfer is comparable to the transfer resulting from a change in investment mix from 50-50 to 30% stock and 70% bonds in our base scenario.

7.3 Initial funding rate

In Chapter 5 we described the effect of the initial funding rate on the simulation results. We found that lowering the initial real funding rate from 85 to 80% results in a slower recovery but similar trends in contributions, indexation and the funding rate over time. The same holds for the generational account. Again people aged 47 or above have a positive generational account while younger generations hand in value. The higher contribution level and worsened indexation lead to a larger contribution option and smaller liability and benefit option. The decrease in net benefit option is counteracted by an increase in the residue option. This option increases due to the lower initial residue and our premium and contribution policy that are aimed at a recovery of the funding rate. Besides differences in the size of the option values, similar trends thus occur.

With an initial funding rate of 100% the generational account in the base scenario does change more. In Chapter 5 we noted that we start our simulation period with charging the break-even premium instead of 20% of the pensionable base. We do not start with a situation of real underfunding anymore, which causes lower contribution rates and a higher indexation levels over the whole simulation period compared to the 85% funding case. The lower contributions result in a lower value of the contribution option; the option value for a 40 year old decreases from 2.8 to 2.1% of total real liabilities. The impact of the indexation on the benefits is smaller with an increase of the benefit option from 4.1 to 4.2%. While in the 80 and 85% funding cases we found a net benefit option below zero for all active participants, this now only holds for people below 41 years of age.

The high risk absorption in the 85% case was aimed at increasing the funding rate while it was hard for the situation to deteriorate. With a sufficient initial funding status the upward potential decreases and the downward risk increases over the simulation period. The value of the surplus and deficit option equal 8.3 and 3.2% of the total real liabilities in 2010 in the 100% funding case, compared to 22.4 and 1.8% in the original base scenario. The decrease in total residue option is reflected in a smaller residue option for all age groups.

As before, the generational account is negative for young current and future participants and positive for older participants. The transition point however decreases from 47 to 34. Furthermore participants aged 60 or above now have a generational account option close to zero, while in the 85% funding case the generational account option was maximum for people aged 63 in 2010.

The changes in the embedded options due to a change in the system also depend on the initial funding rate. Young participants still hand in value while older generations still benefit, but the individual increases change. In the 85% funding case we found that the participants aged around 40 in 2010 benefit most, followed by the participants who are not affected by the changes in the system because they already retired in 2020. The age cohorts between those two groups are harmed by the high premiums charged at the time they extend their working period to overcome their AOW and OP deficit. We concluded that with the sufficient initial funding status lower premiums were charged and this is beneficial to these cohorts. Their generational account option increases more in the 100% than in the 85% case.

The total generational transfer over the 25 year simulation period increases slightly in case of a higher initial funding rate. Table 7.3 shows the generational transfers as percentage of the real liabilities in the different scenarios. Note again that all transfers are small. These

	$FR_{2010} = 0.8$	$FR_{2010} = 0.85$	$FR_{2010} = 1.0$
scenario 1		0.30	
scenario 2		0.41	
scenario 3	0.58	0.62	0.87
scenario 4	0.74	0.82	1.06

limited generational transfers imply that even though the transfers are smallest in scenarios one and two, all policy changes are acceptable if one wants to limit intergenerational transfers resulting from a change in the system as much as possible.

Chapter 8

Conclusion

8.1 Summary and conclusion

In this thesis we focused on possible changes in the employment based pension system as a reaction to an increase in AOW age from 65 to 67. Four possible changes where described and examined in the context of an average Dutch pension fund. The four scenarios are based on the current governmental plans regarding the second pillar pensions; an increase of the statutory retirement age from 65 to 67 years in 2020 accompanied by a decrease in the maximum accrual rate.

We assumed that every participant wants to retain the same level of pension benefits after a change in the system as he would have had in the current situation. The increase in AOW age thus results in a deficit from age 65 until the AOW age. Furthermore, increasing the statutory retirement age or lowering the accrual amounts to an additional old age pension deficit. Both deficits can be compensated for by increasing the period of employment. The increase in working period to overcome the deficit depends on the year of birth and the specific change in the system. The more restricted the accrual, the longer the time needed to overcome the deficit. In our model we find that an increase in retirement age has a stronger effect on the exit age than a decrease in accrual rate. The largest increase results from the most restricted case where both the retirement age and the accrual rate are altered.

It is important to note that changing the system does not directly result in an improvement of the funding position; the value of the previously accrued pension remains the same. Liabilities do change because of changes in participation rate and the resulting changes in contributions and benefits. Even though the increase in exit age influences the break-even premium from 2010 onwards, the effects of a change in the system are minimal up to 2020, the year the AOW age is first changed. From 2020 onwards differences between the alternatives are observed, but changes in funding rate, indexation level and premium charged due to the change in the system remain small. This is mainly due to our pension policy that is aimed at reaching a real funding rate of 100% and the high level of risk absorption within the fund. The best pension results are achieved in the situation where the accrual is restricted most. Here the average contribution level is the smallest while both the cumulative indexation level and average real funding rate are the largest at the end of the evaluation period. Scenarios with a retirement age of 65 are unfavourable compared to the base scenario and the scenarios with a retirement age of 67.

Apart from the funding rate, indexation level and contribution rate, we also looked at the generational account of each cohort. The change in generational account option reflects the transfer of value between generations due to the change in the system. A positive option change means that the generation benefits from the change in the system, a negative change means that the generation hands in value. Overall, young generations hand in value while older participants benefit. The total generational transfer is however limited in all scenarios, with a maximum of 0.8% of the total real liabilities in 2010 for the most restricted scenario. The small changes in generational account option imply that even after the system changes there remains an equilibrium between the contributions made to the fund and the value received from the fund.

The different parties involved in the pension system have different interests. Participants want to retire as soon as possible while receiving high benefits and paying low premiums, retirees want their accrual fully indexed, the pension fund wants a sufficient funding rate to cover all liabilities and the government wants to increase the participation rate to receive as much tax income as possible. Furthermore it is of everyones interest to avoid intergenerational transfers resulting from a system change as much as possible. Based on the pension results and the change in generational account options we recommend to increase the statutory retirement age to 67. We saw that in our model the pension results are the best in the most restricted scenario, but this is accompanied by the largest value transfers and increase in exit age. The differences between both scenarios with a retirement age of 67 are small due to the high levels of risk absorption, implying that both represent acceptable policy changes. Ultimately the choice between these two alternatives, that is staying with the original accrual rate or lowering the accrual rate, can be based on the desired increase in exit age.

Finally note that while a model is just is simplification of reality, the results described above are greatly dependent on the model setup and assumptions underlying the model. Our model has a high level of risk absorption due to the contribution and indexation policy, leading to only small differences between the scenarios. Changing these policies might lead to different simulation results. In the following section we will discuss some of the assumptions we used and give some suggestions for further research.

8.2 Further research

There are several points of improvement in our model, one of them is the indexation policy of the fund. We used an indexation policy where in case indexation has been cut in the past, the possibility of catch-up indexation exists to restore the purchasing power of the participant. Similar as in for example Van Rooij et al. (2005), this catch-up applies to all current liabilities. An implication of this kind of catch-up indexation is that active participants may be compensated too much since they are not only compensated for the liabilities at the time indexation was cut, but also for the accrual after that time. This can lead to a situation where a pension plan with conditional indexation results in higher benefits than the same plan with unconditional indexation. In practice this is in general not the case and therefore an alternative indexation scheme where the cumulative indexation level with unconditional indexation never exceeds the indexation level with unconditional indexation might be more appropriate. More information about such an indexation scheme can be found in for instance Klein Haneveld, Streutker, and Van der Vlerk (2007).

While the indexation policy is part of the construction of our model, also improvements can be made concerning the model input. We have seen that the composition of the participant file influences the premium level through the uniform contribution rate and also has an effect on the intergenerational solidarity. The composition of the participant file is determined by the assumed mortality and the participation rate; both can be subjects of further study. First, we assumed that the participant file evolves according to the CBS forecast of mortality rates based on the 2006-2050 forecast. However, recently the CBS came up with a new mortality table in which mortality rates have declined, leading to a increase in the required provision for pension liabilities. Using this new table will have a negative effect on the initial funding rate of the pension fund. Secondly we assumed that regardless of the specific changes in the system, everyone wants to retain the same level of yearly pension payments once retired. We have seen that this implies that participants who end their active working period before age 65 have to work longer to overcome their pension deficit then people who worked until the statutory retirement age. However, it may be well possible that this group changes their working period less than the group who retired at age 65 and that all participants only compensate part of their deficit. Euwals et al. (2008) describe results from different studies about the influence of increasing the retirement age on the participation rate. They note that the literature about ex post studies on the effects of a changing retirement age is rather limited and that the results from all researches are inconclusive, with effects ranging from 8 to 50%. The authors therefore conclude that overall it appears difficult to give a good estimate of the effect. Further research may lead to more funded assumptions about the changes in exit age and therefore give a more reliable view on the effects of a changing retirement age.

Besides the model assumptions described above, another point of improvement is that our model does not comply with the Dutch regulation as described in the Dutch Pension Act. There are three main points where we differ from the regulations: first as described in Chapter 3, liabilities have to be valued using an interest rate term structure instead of a fixed discount rate. Second, the Dutch Central Bank requires that with a confidence level of 97.5 %, the assets of the fund exceed the provision for pension liabilities by at least five percent within a period of one year. Third, the Pension Act allows the benefit level to vary over time, but with the restriction that the lowest benefit payment may not be less than 75% of the highest payment. Our construction is thus only possible if the old age pension level after age 65 is three times as large as the AOW level, but this restriction is not taken into account in the model. To give a more practical view on the discussion at hand, it is advised to take these regulations into account.

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Appendix A

Additional Years of Service

In Chapter 4 we discussed different scenarios for changes in the employment based pension system. In all cases the AOW and old age pension deficit were filled by using the value of deferring part of the old age pension and by the value of the new accrual due to an increase in the years of service. The number of additional working years can be derived using annuities, in this appendix these derivations are given for the four scenarios. We will start however with some notation used in the derivations.

We assume the extra years of service are determined at the old exit age. At this age we will value the deficit, new accrual and deferred pension by using annuities. The following present values of annuities for someone aged z at year i are used:

${}_{m\|}\ddot{a}_{z,i:\overline{n}\|}$	value of one unit of cash paid for n years, starting m years from now ${}_{m\|}\ddot{a}_{z,i:\overline{n}\|} = \sum_{k=m}^{m+n-1} v^k \; {}_kp_{z,i} \rightarrow$ deferred temporary life annuity
${}_{m\|}\ddot{a}_{z,i}$	value of one unit of cash paid each year until death, starting m years from now ${}_{m\|}\ddot{a}_{z,i} = \sum_{k=m}^{\infty} v^k \; {}_kp_{z,i} \rightarrow$ deferred whole life annuity
$\ddot{a}_{z,i}$	value of one unit of cash paid each year until death, starting immediately $\ddot{a}_{z,i} = \sum_{k=0}^{\infty} v^k \; {}_kp_{z,i} \rightarrow$immediate whole life annuity

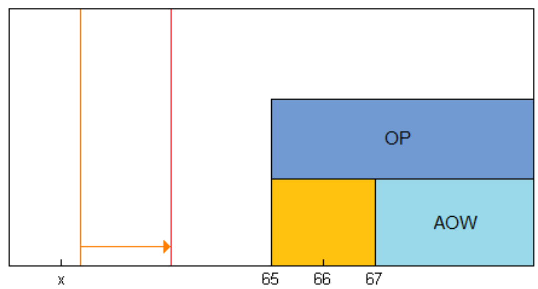
Here ${}_tp_{z,i} = 1 - {}_tq_{z,i}$ where ${}_tq_{z,i}$ is the probability that a person aged z in year i will die within t years and $v = \frac{1}{1+r}$. In the context of our scenarios we define:

z	original exit age of the participant, $z \in \{50, 55, 60, 65\}$
i	year of the original exit of the participant
m	number of years until age 65, $m \in \{15, 10, 5, 0\}$
n	number of years AOW payments are missed $n = 1$ if born between 1955 and 1959, $n = 2$ if born after 1959.
AOW	yearly AOW benefit
OP	yearly old age pension in case both pension pillars remain unchanged
$add_{z,i}$	additional years of employment for someone with an old exit age z in year i

We now turn to the four scenarios.

A.1 Scenario one: 65; 2%

In this scenario nothing changes in the employment based pension system compared to the old situation; the accrual rate still equals 2% and the retirement age remains 65 years. Recall Figure A.1 from section 4.2.



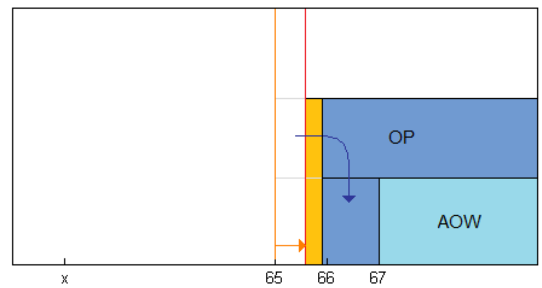


Figure A.1: Left: Early exit - Right: Exit at age 65

Early exit

The left part of the figure shows the situation of an early exit. The missed AOW is a deferred temporary life annuity, starting at age 65 and ending at the moment the real AOW payments start. The value of this deficit at the old exit age thus equals $AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n}|}$. In case of a person born in 1970 exiting at age 55 we thus have $AOW\ {}_{10|}\ddot{a}_{55,2025:\overline{2}|}$.

The old age pension accrued during the extra years of service is paid from age 65 until death and thus is a deferred whole life annuity. The value of the newly accruable pension at the old exit age equals $\underbrace{add_{z,i}\ base_z\ acc}_{new\ accrual}\ {}_{m|}\ddot{a}_{z,i}$.

The value of the new accrual should equal the value of the AOW deficit. The number of extra years of employment thus equals

$$add_{z,i} = \frac{AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n}|}}{base_z\ acc\ {}_{m|}\ddot{a}_{z,i}} \tag{A.1}$$

Exit at age 65

The right part of Figure A.1 shows the situation of an original exit age of 65. The value of the AOW deficit calculated at age 65 equals $AOW\ {}_{add65,i|}\ddot{a}_{65,i:\overline{n-add_{65,i}}|}$. Note that the value of the deficit depends on $add_{65,i}$.

The deficit is filled with new accrual and deferred old age pension. The new accrual represented by the yellow block has a value of $add_{65,i}\ base_{65}\ acc\ \ddot{a}_{65,i}$.

The vertical dark blue rectangle represents the part of the old age pension that is exchanged for a temporary benefit payment to make up for part of the AOW deficit. The value corresponding to this part equals $OP\ (\ddot{a}_{65,i} -_{add_{65,i}} \ddot{a}_{65,i})$.

Equating the value of the AOW deficit to the new accrual plus the deferred pension gives:

$$AOW\ _{add65,i|}\ddot{a}_{65,i:\overline{n-add_{65,i}|}} = add_{65,i}\ base_{65}\ acc\ \ddot{a}_{65,i} + OP(\ddot{a}_{65,i} -_{add_{65,i}} \ddot{a}_{65,i}) \qquad \text{(A.2)}$$

Solving for $add_{65,i}$ now gives the increase in exit ages for each year of birth.

A.2 Scenario two: 65; 1.9%

In this scenario the statutory retirement age remains 65 years, but the accrual rate is decreased to 1.9% in 2020. We denote this new accrual rate by acc_2. In accruing old age pension, the number of years with the low accrual rate equals $\max\{0, i - 2020\}$. The average pension base during this period for someone with exit age z at year i is given by

$$\widetilde{base}_{z,i} = \frac{\sum_{k=0}^{\max\{0,i-2020\}-1} base_{z-k-1}}{\max\{0, i - 2020\}}$$

Recall Figure A.2 from section 4.3.

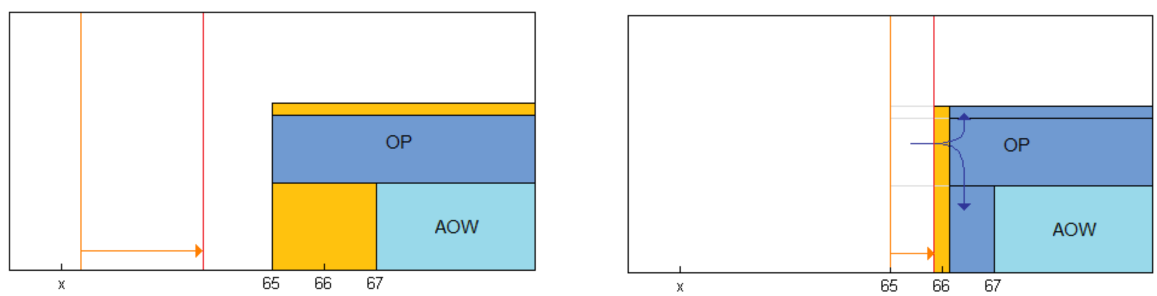


Figure A.2: Left: Early exit - Right: Exit at age 65

Early exit

In case of an exit age smaller than 65, the AOW deficit in scenario two equals that in scenario one. That is $AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n|}}$.

The old age pension deficit depends on the number of years with a low accrual and the corresponding pensionable base. The value of the old age pension deficit equals
$\underbrace{(acc - acc2)\ \max\{0, i - 2020\}\ \widetilde{base}_{z,i}}_{missed\ accrual}\ {}_{m|}\ddot{a}_{z,i}$

Adding the values of the AOW and old age pension deficit now yields the value of the total deficit:

$$AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n|}} + (acc - acc2)\ \max\{0, i - 2020\}\ \widetilde{base}_{z,i}\ {}_{m|}\ddot{a}_{z,i} \tag{A.3}$$

In calculating the value of the new accrual due to the additional years of service we have to take into account that the accrual rate changes in 2020. In all years 1.9% of the pensionable base is accrued, but for all years before 2020 an additional 0.1% has to be added. The number of years of additional service before the year 2020 is given by $\max\{0, \min\{add_{z,i}, 2020 - i\}\}$. The total value of the additional accrual thus equals

$$(add_{z,i}\ acc_2 + (acc - acc_2) \max\{0, \min\{add_{z,i}, 2020 - i\}\})\ base_z\ {}_{m|}\ddot{a}_{z,i} \tag{A.4}$$

Equating (A.3) and (A.4) and solving for $add_{z,i}$ again gives the increase in the number of years of employment due to the changes in the pension system.

Exit at age 65

Also with an original exit age of 65, the AOW deficit in scenarios one and two are valued in the same way. Namely by $AOW\ {}_{add65,i|}\ddot{a}_{65,i:\overline{n-add_{65,i}}|}$.

The value of the old age pension deficit is similar to that in the early exit case, the only difference is the time until the first payment: $(acc-acc2)\ max\{0,i-2020\}\ \widetilde{base}_{65,i}\ {}_{add_{65,i}|}\ddot{a}_{65,i}$.

Adding the two deficits now gives a total value of the deficit of

$$AOW\ {}_{add65,i|}\ddot{a}_{65,i:\overline{n-add_{65,i}}|} + (acc-acc2)\ max\{0,i-2020\}\ \widetilde{base}_{65,i}\ {}_{add_{65,i}|}\ddot{a}_{65,i} \tag{A.5}$$

The value of the new accrual is also similar to that of the early exit case; for all years of additional service we have an accrual rate of 1.9 % and for all years before 2020 an additional 0.1 % is accrued. The value of the new accrual thus equals

$$(add_{65,i}\ acc_2 + (acc-acc_2)\max\{0,\min\{add_{65,i}, 2020-i\}\})\ base_{65}\ \ddot{a}_{65,i} \tag{A.6}$$

Recall that OP denotes the yearly old age pension under the old pension system, that is with an accrual rate of 2% for al years. Due to the lower accrual rate from 2020 in scenario two, the old age pension benefits can be lower. This has to be taken into account in valuing the part of the old age pension that is deferred. The old age pension at age 65 under the new pension system equals the pension under the old system minus the missed accrual. That is $OP-(acc-acc2)\ \max\{0,i-2020\}\ \widetilde{base}_{z,i}$. The value part of the deferred part of this pension thus equals

$$(OP-(acc-acc2)\ \max\{0,i-2020\}\ \widetilde{base}_{z,i})(\ddot{a}_{65,i} - {}_{add_{65,i}}\ \ddot{a}_{65,i}) \tag{A.7}$$

The total value of the new accrual and deferring part of the old age pension is thus given by adding (A.6) and (A.7). Equating this to the total deficit as given by Equation (A.5) and solving for $add_{65,i}$ nog gives the additional years one has to work.

A.3 Third scenario: 67; 2 %

Scenario three is an extension of scenario one. The accrual rate equals 2% for all years, but in this scenario the statutory retirement age is increased to age 67 in the year 2020. This implies that all pension accrued before 2020 pays from age 65 onwards but benefit payments corresponding to the pension accrued after 2020 only start at age 67.

Recall Figure A.3 from section 4.4.

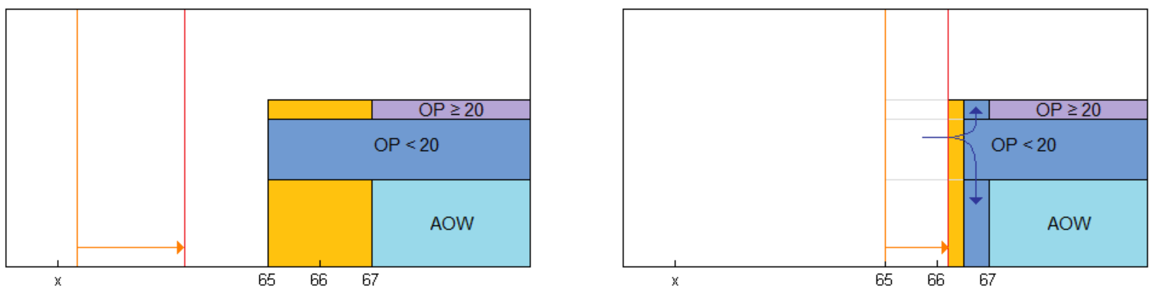


Figure A.3: Left: Early exit - Right: Exit at age 65

Early exit

The AOW deficit is the same as in the previous scenarios and thus has a value of $AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n}|}$.

The old age pension deficit depends on the number of additional working years from 2020 onwards and the corresponding pensionable base. Recall from scenario two that this period is given by $\max\{0, i-2020\}$ and that the corresponding average pension base during this period is given by $\widetilde{base}_{z,i}$. The value of the old age pension deficit now equals $\underbrace{acc\ \max\{0, i-2020\}\ \widetilde{base}_{z,i}}_{missed\ accrual}\ {}_{m|}\ddot{a}_{z,i:\overline{2}|}$. The value of the total deficit thus equals

$$AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n}|} + acc\ \max\{0, i-2020\}\ \widetilde{base}_{z,i}\ {}_{m|}\ddot{a}_{z,i:\overline{2}|} \tag{A.8}$$

In calculating the value of the new accrual one also has to take into account in which year the extra accrual takes place. In every additional working year pension is accrued that is paid from age 67 onwards, but for accrual during the years before 2020 the corresponding pension is also paid at age 65 and 66. Recall that the number of years of additional service before 2020 is given by $\max\{0, \min\{add_{z,i}, 2020-i\}\}$. The value of the total additional accrual is then given by

$$acc\ base_z\ \left(add_{z,i}\ {}_{m+2|}\ddot{a}_{z,i} + \max\{0, \min\{add_{z,i}, 2020-i\}\}\ {}_{m|}\ddot{a}_{z,i:\overline{2}|}\right) \tag{A.9}$$

The value of the new accrual again has to equal the value of the deficit; equating (A.8) and (A.9) and solving for $add_{z,i}$ yields the extra years of service needed to achieve this.

Exit at age 65

The AOW deficit in case of an original exit age of 65 is also valued similar as in the previous scenarios namely by $AOW \; {}_{add65,i|}\ddot{a}_{65,i:\overline{n-add_{65,i}|}}$.

Just as the AOW deficit, the value of the old age pension deficit decreases if one works after age 65. This is because the time until the first payment decreases by $add_{65,i}$ years. The value of the deficit in this situation is thus given by $acc \; \max\{0, i-2020\} \; \widetilde{base}_{z,i} \; {}_{add_{65,i}|}\ddot{a}_{65,i:\overline{2-add_{65,i}|}}$, yielding a total deficit of

$$AOW \; {}_{add65,i|}\ddot{a}_{65,i:\overline{n-add_{65,i}|}} + acc \; \max\{0, i-2020\} \; \widetilde{base}_{z,i} \; {}_{add_{65,i}|}\ddot{a}_{65,i:\overline{2-add_{65,i}|}} \tag{A.10}$$

The new accrual due to the extra years of service is similar to that of an early exit case, resulting in value of $acc \; base_{65} \; (add_{65,i} \; {}_{2|}\ddot{a}_{65,i} + \max\{0, \min\{add_{65,i}, 2020-i\}\} \; \ddot{a}_{65,i:\overline{2|}})$.

The benefit payments that were originally paid during the first $add_{65,i}$ years from age 65 are now deferred because during these years the participant still has an income from employment. The size of the benefit payments to be deferred depends on the old age pension accrued before 2020, $OP^{<20}$.

Before we derived that the missed accrual for ages 65 and 66 equals $acc \; \max\{0, i-2020\} \; \widetilde{base}_{z,i}$ implying that $OP^{<20} = OP - acc \; \max\{0, i-2020\} \; \widetilde{base}_{z,i}$. The value of the deferred old age pension is now given by $OP^{<20}\ddot{a}_{65,i:\overline{add_{65,i}|}}$

Adding the value of the new accrual and the deferred pension gives a value of

$$acc \; base_{65} \; (add_{65,i} \; {}_{2|}\ddot{a}_{65,i} + \max\{0, \min\{add_{65,i}, 2020-i\}\} \; \ddot{a}_{65,i:\overline{2|}}) + OP^{<20}\ddot{a}_{65,i:\overline{add_{65,i}|}} \tag{A.11}$$

Equating (A.10) and (A.11) and solving for the additional years of service now gives the increase in the working period due to the change in statutory retirement age.

A.4 Fourth scenario: 67; 1.9%

Scenario four is a combination of scenarios two and three; while in scenario two the accrual rate is decreased and in scenario three the retirement age is raised, now both occur simultaneously. This implies that before 2020 pension is accrued based on an accrual rate of 2%, the corresponding pension pays from age 65 onwards. Benefit payments corresponding to the pension accrued after 2020 only start at age 67 and are based on an accrual rate of 1.9%.

Recall Figure A.4 from section 4.5.

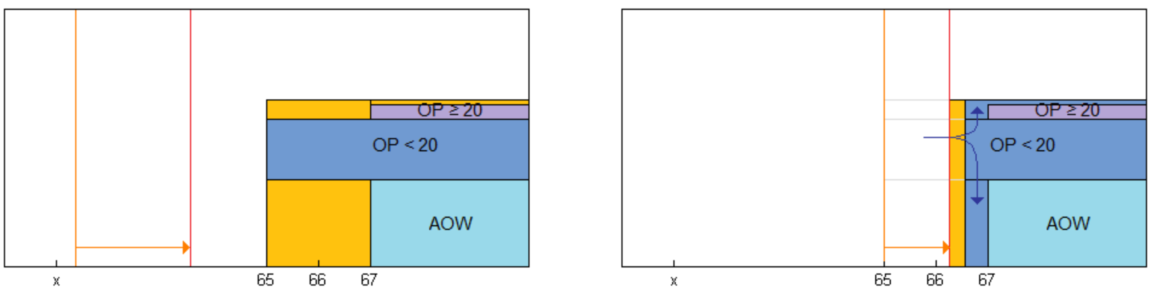


Figure A.4: Left: Early exit - Right: Exit at age 65

Early exit

The AOW deficit is the same as in all previous scenarios and thus has a value of $AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n}|}$.

The old age pension deficit in scenario four consists of two parts: the deficit resulting from an increased retirement age (upper left yellow rectangle in Figure A.4) and the deficit resulting from a lower accrual rate (upper right yellow rectangle)[1].

The deficit resulting from an increased retirement age is equal to that in scenario three. Recall that there we stated that the old age pension deficit depended on the number of additional working years from 2020 onwards and the corresponding pensionable base and that the value of the old age pension deficit equaled $acc\ \max\{0, i-2020\}\ \widetilde{base}_{z,i}\ {}_{m|}\ddot{a}_{z,i:\overline{2}|}$.

The deficit resulting from the decreased accrual rate is similar to that in scenario two, with the difference that the corresponding pension payments start at age 67 instead of age 65. The value of this deficit now equals $(acc - acc2)\ \max\{0, i-2020\}\ \widetilde{base}_{z,i}\ {}_{m+2|}\ddot{a}_{z,i}$.

The value of the total deficit thus equals

$$AOW\ {}_{m|}\ddot{a}_{z,i:\overline{n}|} + acc\ \max\{0, i-2020\}\ \widetilde{base}_{z,i}\ {}_{m|}\ddot{a}_{z,i:\overline{2}|} + (acc - acc2)\ \max\{0, i-2020\}\ \widetilde{base}_{z,i}\ {}_{m+2|}\ddot{a}_{z,i} \tag{A.12}$$

[1]Technically this distinction is not correct since part of the deficit we now consider as due to the retirement age is actually also due to the decreased accrual rate. The current distinction is just for simplicity in the calculations.

For the value of the new accrual one again has to take into account in which year the extra accrual takes place. In every additional working year pension is accrued that is paid from age 67 onwards with a rate of 1.9%; accrual during the years before 2020 takes places at a rate of 2% and the corresponding pension is also paid at age 65 and 66. Combining (A.4) and (A.9) yields that the value of the total additional accrual is given by

$$acc_2 \; base_z \; (add_{z,i} \; {}_{m+2|}\ddot{a}_{z,i} + \max\{0, \min\{add_{z,i}, 2020 - i\}\}(\; {}_{m|}\ddot{a}_{z,i:\overline{2}|} + \frac{(acc - acc_2)}{acc_2} \; {}_{m|}\ddot{a}_{z,i})) \tag{A.13}$$

Equating the value of the deficit and the new accrual, (A.12) and (A.13), and solving for $add_{z,i}$ again yields the extra years of service.

Exit at age 65

The AOW deficit in case of an original exit age of 65 is also valued similar as in the previous scenarios namely by

$$AOW \; {}_{add65,i|}\ddot{a}_{65,i:\overline{n-add_{65,i}}|} \tag{A.14}$$

Just as in the early exit case that we make a distinction between the deficit due to the increased retirement age and due to the decreased accrual rate. With increasing the working period the deficit due to an increased retirement age decreases because of an increase in the time until the first payment. The value of the deficit is again the same as in scenario three:

$$acc \; \max\{0, i - 2020\} \; \widetilde{base}_{z,i} \; {}_{add_{65,i}|}\ddot{a}_{65,i:\overline{2-add_{65,i}}|} \tag{A.15}$$

The deficit due to the decreased accrual rate does not change compared to the early exit situation since this concerns the period after age 67. This part of the deficit is thus given by

$$(acc - acc2) \; \max\{0, i - 2020\} \; \widetilde{base}_{65,i} \; {}_{2|}\ddot{a}_{65,i} \tag{A.16}$$

The value of the deficit is filled with new accrual due to the extra years of service and by deferring the pension payments that were originally paid during the first $add_{65,i}$ years from age 65. The value of the new accrual is similar to that of an early exit case, namely

$$acc_2 \; base_{65} \; (add_{65,i} \; {}_{2|}\ddot{a}_{65,i} + \max\{0, \min\{add_{65,i}, 2020 - i\}\}(\; \ddot{a}_{65,i:\overline{2}|} + \frac{(acc - acc_2)}{acc_2} \; \ddot{a}_{65,i})) \tag{A.17}$$

The size of the benefit payments to be deferred depends on the old age pension accrued before 2020. In this scenario $OP^{<20}$ is the same as defined in scenario three. That is $OP^{<20} = OP - acc \; \max\{0, i - 2020\} \; \widetilde{base}_{z,i}$ with associated value

$$OP^{<20}\ddot{a}_{65,i:\overline{add_{65,i}}|} \tag{A.18}$$

Solving $(A.14) + (A.15) + (A.16) = (A.17) + (A.18)$ for the additional years of service now gives the increase in the working period due to the change in statutory retirement age and the decreased accrual rate.

Appendix B

Discounting in the Binomial Model

Different methods for valuing financial derivatives are available, two of which are risk neutral pricing and pricing using deflators. Both methods are equivalent in the sense that they produce the same prices for the derivatives. In this appendix we will show this equivalence relation in a simple binomial case based on the discussion of the two methods in Hibbert, Morrison, and Turnball (2006). In the binomial model, just as in the Black Scholes model, the market consists of a risky stock and a risk free bond. At $t = 0$ the stock is worth S while the bond value equals B. There are only two possible future states: at $t = 1$ the stock value has increased to uS with probability p or decreased to dS with probability $1 - p$. The bond is risk free and has value $(1 + r)B$ in both future states. We assume the market is complete and arbitrage free, therefore $d < 1 + r < u$.

B.1 Risk neutral pricing

Consider a derivative A that pays A_{up} if the stock market goes up and A_{down} if the market goes down. Since the market is complete, the payout of the derivative can be replicated by the payout of a portfolio consisting of stocks and the bonds. That is $A = \alpha S + \beta B$, $A_{up} = \alpha uS + \beta(1 + r)B$ and $A_{down} = \alpha dS + \beta(1 + r)B$.

Solving for α and β yields $\alpha = \frac{A_{up}-A_{down}}{(u-d)S}$ and $\beta = \frac{A_{up}-\alpha uS}{(1+r)B}$. Inserting in the expression for A now yields that the price of the derivative at $t = 0$ is given by

$$A = q\frac{1}{1+r}A_{up} + (1-q)\frac{1}{1+r}A_{down} \tag{B.1}$$

with $q = \frac{(1+r)-d}{u-d}$.

Equation (B.1) shows that in a world where the probability of a stock going up equals q instead of p and the probability of the stock going down thus equals $1 - q$, the return on each derivative equals the risk free rate. This also holds for the stock itself. This alternative probability measure is called the risk neutral measure since investors earn the risk free rate regardless of the riskiness of the underlying asset. The probability q is called the risk neutral probability. Note that the price of the derivative is independent of the real world probability p.

The risk neutral pricing principle can be extended to more realistic and continuous time models. The general principle stays the same: the expected payoff of an asset is calculated under a risk neutral probability measure $\mathcal{Q}$ instead of the real probability measure $\mathcal{P}$ and we discount at the risk free rate. As shown in Appendix C.2, under $\mathcal{Q}$ the stock price process in the Black Scholes model is given by $dS_t = S_t\ rdt + S_t\ \sigma dW_t$. In the Black Scholes model we can thus value a derivative by simulation possible future cashflows of the derivative under the risk neutral measure (that is where the stock price process has drift r instead of μ), discounting these cashflows by the risk free rate and taking the expectation.

B.2 Pricing with deflators

In risk neutral pricing we used an equivalent probability measure to price derivatives implying that the probabilities in the basic model need to be adjusted. In some simulation studies the real probabilities are used for assessing for instance the funding rate of the fund, applying risk neutral pricing here thus implies that the simulation has to be performed twice; both under $\mathcal{P}$ and $\mathcal{Q}$. To overcome this problem we can use an alternative discounting approach, namely by using deflators.

Recall the price of a derivative as given in Equation (B.1). Instead of pricing using the risk neutral probabilities q and discounting with the risk free rate we want to rewrite the equation in such a way that we are pricing under $\mathcal{P}$ again. Rewriting and inserting q gives

$$\begin{aligned}
A &= \frac{1}{1+r}(q\frac{p}{p}A_{up} + (1-q)\frac{1-p}{1-p}A_{down}) \\
&= p\frac{1}{1+r}\frac{q}{p}A_{up} + (1-p)\frac{1}{1+r}\frac{1-q}{1-p}A_{down} \\
&= p\frac{(1+r)-d}{p(u-d)(1+r)}A_{up} + (1-p)\frac{u-(1+r)}{(u-d)(1-p)(1+r)}A_{down} \\
&= pD_{up}A_{up} + (1-p)D_{down}A_{down}
\end{aligned}$$

It appears that the value of the derivative can be determined using the real probability p, but that the discount factor now depends on the state of the world (D_{up} and D_{down} depend on u and d). This state dependent discount factor is the deflator. Note that the deflator does not depend on the cashflow being valued, but only on the model. This means that the same deflator can always be used independent of the riskiness of the derivative or stock. Both the deflator method and risk neutral pricing result in the same price for a derivative.

Just as with risk neutral pricing, the deflator method can be generalized to more complex and realistic models. In all cases the real world probabilities will be used with a stochastic discount rate. While the derivation of the deflator in the binomial model is quite straightforward, for more realistic models implementing the deflator method can be hard. For the Black Scholes model the derivation of the deflator can be found in Appendix C.2.

Appendix C

Black Scholes

The Black Scholes model is given by

$$
\begin{aligned}
dB_t &= B_t \tilde{r} dt \qquad\qquad B_0 > 0 \text{ given} && \text{(C.1)}\\
dS_t &= S_t\ \tilde{\mu} dt + S_t\ \sigma dW_t \qquad S_0 > 0 \text{ given} && \text{(C.2)}
\end{aligned}
$$

We assume there is a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$ underlying the stock price process where $\mathcal{F}_t = \sigma(S_s, 0 \leq s \leq t)$.

C.1 Returns

Solving Equation (C.1) gives $B_t = B_0 \exp(\tilde{r}t)$. The one period return on a bond $\frac{B_t}{B_{t-1}}$ is thus given by $\frac{B_t}{B_{t-1}} = \exp(\tilde{r})$.

To derive the return on the stock driven by (C.2) we define $f(t,x) = \ln(x), x > 0$. Then $D_0\ f(t,x) = \frac{\partial f}{\partial t}(t,x) = 0$, $D_1\ f(t,x) = \frac{\partial f}{\partial x}(t,x) = \frac{1}{x}$ and $D_{1,1}\ f(t,x) = \frac{\partial^2 f}{\partial x^2}(t,x) = -\frac{1}{x^2}$. Applying Ito's Lemma yields:

$$
\begin{aligned}
f(t,S_t) &= f(t,S_0) + \int_0^t D_0\ f(t,x)dt + \int_0^t D_1\ f(t,S_u)dS_u + \int_0^t \frac{1}{2} D_{1,1}\ f(t,S_u)d[S,S]_u \\
&= f(t,S_0) + 0 + \int_0^t \frac{1}{S_u}\ dS_u - \frac{1}{2}\int_0^t \frac{1}{S_u^2} d[S,S]_u \\
&\overset{C.2}{=} f(t,S_0) + \int_0^t \frac{1}{S_u}\ S_u\ \tilde{\mu} du + \int_0^t \frac{1}{S_u} S_u\ \sigma dW_u - \frac{1}{2}\int_0^t \frac{1}{S_u^2}\sigma^2 S_u^2 \underbrace{d[W,W]_u}_{u} \\
&= f(t,S_0) + \tilde{\mu} t + \sigma W_t - \frac{1}{2}\sigma^2 t \\
\ln(S_t) - \ln(S_0) &= (\tilde{\mu} - \frac{1}{2}\sigma^2)t + \sigma W_t \\
\frac{S_t}{S_0} &= \exp((\tilde{\mu} - \frac{1}{2}\sigma^2)t + \sigma W_t) && \text{(C.3)}
\end{aligned}
$$

Now the one period return on the stock is given by $\frac{S_t}{S_{t-1}} = \exp((\tilde{\mu} - \frac{1}{2}\sigma^2) + \sigma(W_t - W_{t-1}))$, where the increments $W_t - W_{t-1} \sim N(0,1)$.

C.2 Black Scholes deflator

The Black Scholes model above was defined on the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$. In this section we will derive the stock price process under an equivalent risk neutral measure Q. Under the risk neutral measure the present value of each asset equals the value of the expected future cashflows discounted at the risk free interest rate. Using this equivalent measure, the Black Scholes deflator will be constructed.

We start our derivation with a version of Girsanov's theorem as given in Bingham and Kiesel (1998) (Theorem 5.8.1):

Theorem - Girsanov:
For $0 \leq t \leq T$ let λ_t be a measurable and adapted process with $\int_0^T \lambda_t^2 dt < \infty$ and $E(\exp\{\frac{1}{2}\int_0^T \lambda_s^2 ds < \infty\})$. Define $L_t = \exp\{-\int_0^t \lambda_s dW_s - \frac{1}{2}\int_0^t \lambda_s^2 ds\}$ and $d\tilde{W}_t = dW_t + \lambda dt$. Then under the equivalent measure $\tilde{\mathcal{P}}$ with Radon-Nikodym derivative $\frac{d\tilde{\mathcal{P}}}{d\mathcal{P}}|_{\mathcal{F}_t} = L_t$, the process $\tilde{W}$ is a Brownian motion.

Define $d\tilde{W}_t = dW_t + \lambda_t dt$ with $\tilde{W}_0 = 0$. By the theorem $\tilde{W}$ is a Brownian motion under $\mathcal{Q}$. The stock price process under $\mathcal{Q}$ is thus given by $dS_t = S_t\ \tilde{\mu} dt + S_t\ \sigma(d\tilde{W}_t - \lambda_t dt)$ or equivalently $S_t = S_0 \exp((\tilde{\mu} - \frac{1}{2}\sigma^2 - \lambda_t\sigma)t + \sigma\tilde{W}_t)$.

Under both probability measures, the value of the expected stock price at time t $(0 \leq t \leq T)$ discounted to time 0 should equal S_0, that is $E^{\mathcal{Q}}(S_t\ DF^{\mathcal{Q}}|\mathcal{F}_0) = E^{\mathcal{P}}(S_t\ DF^{\mathcal{P}}|\mathcal{F}_0) = S_0$ where DF denotes the appropriate discount factor. The risk free discount rate under $\mathcal{Q}$ equals $DF^{\mathcal{Q}} = \exp(-\tilde{r}t)$. Now:

$$\begin{aligned}
E^{\mathcal{Q}}(S_t\ DF^{\mathcal{Q}}|\mathcal{F}_0) &= S_0 E^{\mathcal{Q}}[\exp((\tilde{\mu} - \frac{1}{2}\sigma^2 - \lambda_t\sigma)t + \sigma\tilde{W}_t)\ \exp(-\tilde{r}t)|\mathcal{F}_0] \\
&= S_0 \exp((\tilde{\mu} - \frac{1}{2}\sigma^2 - \lambda_t\sigma - \tilde{r})t) E^{\mathcal{Q}}[\exp(\sigma\tilde{W}_t)|\mathcal{F}_0] \\
&\overset{\text{mgf}}{=} S_0 \exp((\tilde{\mu} - \frac{1}{2}\sigma^2 - \lambda_t\sigma - \tilde{r})t) \exp(\frac{1}{2}\sigma^2 t) \\
&= S_0 \underbrace{\exp((\tilde{\mu} - \lambda_t\sigma - \tilde{r})t)}_{\text{should equal 1}} \\
\Rightarrow \exp((\tilde{\mu} - \lambda_t\sigma - \tilde{r})t) &= 1 \\
\lambda_t = \lambda &= \frac{\tilde{\mu} - \tilde{r}}{\sigma}
\end{aligned}$$

The third equality follows from the moment generating function of a normal distribution: $\tilde{W}_t - \tilde{W}_0 = \tilde{W}_t \sim N(0, \sqrt{t})$ so $E^{\mathcal{Q}}[\exp(\sigma\tilde{W}_t)] = \exp(0\sigma + \frac{1}{2}(\sqrt{t})^2\sigma^2) = \exp(\frac{1}{2}\sigma^2 t)$.
Since $\lambda = \frac{\tilde{\mu} - \tilde{r}}{\sigma}$ it now follows that under $\mathcal{Q}$, $dS_t = S_t\ \tilde{r} dt + S_t\ \sigma d\tilde{W}_t$.

While we know the discount factor under the risk neutral measure, the discount factor under $\mathcal{P}$ is still unknown. Using the Radon-Nikodym derivative $\frac{d\mathcal{Q}}{d\mathcal{P}}$ we know

$$E^{\mathcal{Q}}(X) = \int_\Omega X d\mathcal{Q} = \int_\Omega X \frac{d\mathcal{Q}}{d\mathcal{P}} d\mathcal{P} = E^{\mathcal{P}}(X \frac{d\mathcal{Q}}{d\mathcal{P}})$$

Hence

$$E^{\mathcal{P}}(S_t\ DF^{\mathcal{P}}|\mathcal{F}_0) = E^{\mathcal{Q}}(S_t\ DF^{\mathcal{Q}}|\mathcal{F}_0) = E^{\mathcal{P}}(S_t\ DF^{\mathcal{Q}}\frac{d\mathcal{Q}}{d\mathcal{P}}|\mathcal{F}_0) = E^{\mathcal{P}}(S_t \exp(-\tilde{r}t)\frac{d\mathcal{Q}}{d\mathcal{P}}|\mathcal{F}_0)$$

The stochastic discount factor at time t given by $\exp(-\tilde{r}t)\frac{d\mathcal{Q}}{d\mathcal{P}}|_{\mathcal{F}_t}$ is the deflator for the stock price process under $\mathcal{P}$. Recall from Girsanov's theorem that the Radon-Nikodym derivative was defined as $\frac{d\tilde{\mathcal{P}}}{d\mathcal{P}}|_{\mathcal{F}_t} = L_t$ with $L_t = \exp\{-\int_0^t \lambda_s dW_s - \frac{1}{2}\int_0^t \lambda_s^2 ds\}$. Since under $\mathcal{Q}$ we derived $\lambda_t = \lambda,\ 0 \leq t \leq T$, we now have

$$\begin{aligned} L_t &= \frac{d\mathcal{Q}}{d\mathcal{P}}|_{\mathcal{F}_t} \\ &= \exp\{-\int_0^t \lambda dW_s - \frac{1}{2}\int_0^t \lambda^2 ds\} \\ &= \exp\{-\lambda W_t - \frac{1}{2}\lambda^2 t\} \end{aligned}$$

The Black Scholes deflator thus equals $DF^{\mathcal{P}} = \exp\{-\lambda W_t - (\frac{1}{2}\lambda^2 + \tilde{r})t\}$ with $\lambda = \frac{\tilde{\mu}-\tilde{r}}{\sigma}$

Recall that we started with the statement that under both probability measures, the value of the expected stock price at time t discounted to time 0 should equal S_0. Under $\mathcal{Q}$ we used the risk free interest rate to show that this holds, and since we know the deflator we can also verify this under $\mathcal{P}$. Using the deflator as discount factor, the value of the expected stock price at time t discounted to time 0 indeed equals S_0:

$$\begin{aligned} E^{\mathcal{P}}(S_t\ DF^{\mathcal{P}}|\mathcal{F}_0) &= E^{\mathcal{P}}(S_t\ \exp\{-\lambda W_t - (\frac{1}{2}\lambda^2 + \tilde{r})t\}|\mathcal{F}_0) \\ &= E^{\mathcal{P}}(S_0 \exp((\tilde{\mu} - \frac{1}{2}\sigma^2)t + \sigma W_t)\ \exp(-\lambda W_t - (\frac{1}{2}\lambda^2 + \tilde{r})t)|\mathcal{F}_0) \\ &= S_0 \exp((\tilde{\mu} - \tilde{r})t - \frac{1}{2}(\sigma^2 - \lambda^2)t) E^{\mathcal{P}}(\exp((\sigma - \lambda)W_t)|\mathcal{F}_0) \\ &\overset{\text{mgf}}{=} S_0 \exp((\tilde{\mu} - \tilde{r})t - \frac{1}{2}(\sigma^2 - \lambda^2)t) \exp(\frac{1}{2}(\sigma - \lambda)^2 W_t) \\ &= S_0 \end{aligned}$$

Finally note that the expected value of the deflator equals the risk free discount rate:

$$E^{\mathcal{P}}(\exp(-\lambda W_t - (\frac{1}{2}\lambda^2 + \tilde{r})t)|\mathcal{F}_0) = \exp(-(\frac{1}{2}\lambda^2 + \tilde{r})t)E(\exp(-\lambda W_t) \overset{\text{mgf}}{=} \exp(-\tilde{r}t)$$

This property guarantees that a given non stochastic cashflow will be valued correctly.