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PROEFSCHRIFT

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To my family and friends

Preface

This dissertation is a collection of three essays coming out of my research as a Ph.D. candidate at the Department of Finance, Tilburg University. Chapter 1 gives a brief overview of the topics and describes the hedge fund industry. Chapter 2 is titled "Being Locked Up Hurts", which is coauthored by Frans de Roon and Jenke ter Horst. It investigates the portfolio implication of a hedge fund lockup period. Chapter 3 is titled "A Random Walk by Fund of Funds Managers?", also coauthored by Frans de Roon and Jenke ter Horst. It examines the investment preference by fund of hedge funds managers. Chapter 4 is titled "Long-Term Tactical Asset Allocation". A tactical asset allocation program, which takes advantages of short-term investment opportunities such as market momentum trading, can be constrained to match the desired long-term asset allocation.

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My parents and sisters play a great role in encouraging and supporting me all those years when I live five thousand miles away from them. They are always optimistic and confident in all that I do. I don't know how to describe my gratitude and appreciation for their understanding. Finally, I want to thank all my friends, for being my friends and not letting time or distance diminish our friendship.

Research is never easy. There is no guarantee that you will ever see the moment of truth for every piece of your work. The road ahead tends to be winding and endlessly long. Fortunately, you will never walk alone. It is always a pleasure to be surrounded by people with passion and vision. With great admiration and gratitude to those people, I hereby present my dissertation.

Jinqiang Guo

Utrecht, April 2012

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Chapter 1

Introduction

Since the early 1990s, hedge funds have gained popularity among both private and institutional investors due to their flexible investment strategies, a better incentive alignment scheme for fund managers (featuring clauses such as incentive fees, hurdle rates and high-water mark), limited regulation, etc. They quickly caught the attention from academic researchers, witnessed by an increasingly large body of literature on hedge fund research. The introduction begins with a brief overview of the hedge fund industry in the first section. Subsequent sections describe the motivation of two hedge fund research papers from my dissertation. The final section describes the motivation of my research in the portfolio optimization with information variables and constraints.

1.1 Hedge Fund Industry

A hedge fund is a private investment vehicle that is mainly accessible to accredited investors such as institutional investors or high net worth individuals. It is labeled "alternative investment", although many hedge funds invest in traditional asset classes only. It has become an important player in global financial markets. Events such as the near-collapse of Long Term Capital Management in 1998 and the sudden debacle of Amaranth Advisors in 2006 often involve huge losses of investors' capital, and cause potential panic or contagion in financial markets. Nevertheless, hedge fund returns exhibit low volatility and weak correlation with returns from traditional assets. As a result, many investors consider hedge funds as an attractive investment vehicle. The growth of the hedge fund industry has accelerated over the last twenty years, in terms of assets under management and the number of funds.

Hedge funds have many features distinguishing themselves from other investment funds such as mutual funds or private equity funds. Like mutual funds, most hedge funds are structured as open-end funds, accepting investing capitals from investors on an ongoing basis. However, hedge funds can impose restrictions such as a lockup period or redemption period, which give hedge fund managers more flexibility, at the expense of illiquidity for investors. Most of U.S.-domiciled hedge funds are structured as Limited Partnerships (LPs) while offshore funds are usually structured as corporations. Limited number of investors under the limited partnership structure implies that minimum investments in hedge funds are relatively high for U.S. domiciled hedge funds. When a hedge fund is structured as a limited partnership, the fund manager is a general partner. A fund manager may invest a substantial portion of his or her own wealth in the fund.

The hedge fund industry is lightly regulated and is characterized by a lack of transparency. In U.S., when certain conditions set by SEC are met, hedge funds do not have to register with SEC. In addition, hedge fund managers are not required to periodically report to the public about their performance or portfolio holdings. In fact, they are often forbidden to reveal such information in the media. About half of hedge funds are registered offshore, and they are referred to as offshore funds.

Hedge fund managers receive a performance fee in proportion to the profitability of the fund's investments, typically 20%, in addition to a management fee of 1% to 2% based on assets under management. Many hedge funds include hurdle rates and high-water mark clauses, such that whether fund managers can actually collect the performance fee depends on the high-water mark and hurdle rates. A hurdle rate is a benchmark that a fund must achieve in order to charge a performance fee. A high-water mark states that a fund manager must recover any previous loss before he or she can collect the performance fee. The hurdle rate usually is the same for all investors in the same fund, while the high-water mark can vary.

Hedge funds have a high degree of flexibility in investment strategies and choices of instruments. They can invest in options, futures, other derivative products and structured products, some of which are considered too risky by traditional institutional investors and individual investors. In addition, use of leverage and short selling are allowed for hedge funds. Hedge fund strategies very often crucially rely on manager skills and investment opportunities available. Hedge funds can be categorized into several styles according to strategies they typically employ and main exposures they have. For example, Event Driven funds are referred to as hedge funds that specialize in betting on the price movement triggered by an event such as merger or acquisition of companies. Emerging Markets funds are hedge funds mainly investing in emerging markets around the world.

Unlike mutual funds that are subject to strict regulation and periodic reporting requirements, hedge funds report to a data vendor on a voluntary basis. Data-related biases such as survivorship bias, self-selection, and backfill bias create difficulties in evaluation of hedge funds. For instance, self-selection in hedge fund disclosure may bias hedge fund performance upwards, and the magnitude of bias is hard to estimate. In addition, the distribution of hedge fund returns tends to be nonnormal, with significant negative skewness and excessive kurtosis. Returns of some hedge funds exhibit strong autocorrelation due to stale prices of illiquid assets in some hedge fund portfolios.

1.2 Portfolio Implication of a Hedge Fund Lockup Period

While the illiquidity imposed on investors may help hedge fund managers, it certainly causes pains to investors. The illiquidity can take several forms. A fund may require a minimum size of investments in the fund by each investor, or be closed to all or new investors. On the exit side, a fund may impose a lockup period for new investments, or a gate provision for all investments.[1] A secondary market for hedge funds which allows investors to buy and sell shares in hedge funds can mitigate liquidity problem to some extent (Ramadorai (2012) describes the development of the hedge fund secondary market, Hedgebay. However, such a secondary market for trading hedge funds is still restrictive and is mainly for investing in closed hedge funds).[2] Before a mature secondary market for hedge funds becomes available, a hedge fund investor has to rely on alternative ways to mitigate the liquidity issue.

Chapter 2 takes a portfolio perspective and evaluates the portfolio implication of a hedge fund lockup period. An investor who holds a portfolio of risky assets can still rebalance liquid assets to take into account the inflexible investment in hedge funds or any other illiquid assets. In a one-period setup such as Mayer (1976), an investor who owns a house would adjust his holdings in liquid assets to hedge unexpected changes in the house value. The same thing holds in a multi-period setting. A lockup period for hedge funds restricts a multi-period investor's ability to rebalance his portfolio. Investors compensate for a hedge fund lockup period by making adjustments to their equity and bond holdings over the investment horizon. The hedge demand for stocks and bonds will take into account the cross-asset correlation across different periods. Nevertheless, a lockup period for hedge funds limits the asset allocation and creates negative impact on portfolio efficiency. Chapter 2 shows that adding hedge funds with a lockup period to the portfolio of stocks and bonds generates large, negative hedge demands for stocks, and hedge demands tend to vary over the investment horizon. The analysis suggests that an investor with a portfolio of stocks, bonds and hedge funds under both the unconditional strategy and conditional strategy is hurt by the presence of a hedge fund lockup period.

[1]A gate provision is a restriction placed on hedge fund investors limiting the amount of withdrawals from the fund during a certain period. The purpose of the provision is to prevent a run on the fund, as large amount of capital withdrawals by investors may force the manager to substantially reduce portfolio holdings, even though those positions are considered to have high risk-adjusted expected return. The hedge fund manager has the discretion to enforce the implementation of a gate, but such an event may be perceived as negative. This prevents the abuse of a gate provision by hedge fund managers.

[2]Typically, approval from the hedge fund manager is required when an investor transfers his hedge fund shares to other investors in order to complete a deal in Hedgebay.

1.3 Investment Preference by Fund of Funds Managers

Investing in hedge funds requires a high level of sophistication of investors who are able to analyze risks and returns of hedge fund investments. Not every investor has the required knowledge and skills to proactively manage exposures to hedge funds. A fund of hedge funds or fund of funds is a hedge fund that claims to have the required sophistication for investing in hedge funds on behalf of investors. A fund of hedge funds can achieve the economies of scale, give investors access to closed hedge funds, and exercise due diligence for investors. In addition, a fund of hedge funds is appealing to investors with a relatively lower net worth. Such investors may have no access to single-strategy hedge funds because of minimum investment capitals required by hedge fund managers. A minimum investment capital requirement also restricts larger investors who aim to hold a diversified portfolio of singe-strategy hedge funds. Investing via a fund of hedge funds which typically requires no or low minimum investments is one way to obtain exposures to a diversified portfolio of single-strategy hedge funds with different styles, at the expense of double fees paid by investors.[3]

The growing popularity of funds of hedge funds over the last ten years indicates a strong demand for funds of hedge funds by investors. It is expected that the growth of funds of funds will continue together with the hedge fund industry. Chapter 3 analyzes the investment preference by fund of funds managers. As portfolio holdings by funds of funds are not reported to hedge fund databases, the analysis infers the preference by fund of funds managers indirectly. If a single-strategy hedge fund has investments from funds of funds, it reports such information, together with fund characteristics and performance. The analysis suggests that single-strategy hedge funds added to the portfolio of funds of funds display some distinct characteristics such as a larger size or longer operational history, and more restrictive liquidity requirements. Moreover, they have much higher risk-adjusted returns or ex post alphas, lower tracking errors and higher information ratios, in both the short run and long run.

1.4 Portfolio Optimization, Information Variables and Investor Preference

Long-term investors such as pension funds typically separate strategic asset allocations from the short-term portfolio optimization. A strategic asset allocation should reflect the long-term return objective and risk tolerance of an investor. Under a strategic asset

[3] Many funds of hedge funds also invest in other funds of hedge funds, i.e. they are funds of "funds of funds". However, it is less obvious that there are additional benefits from investing in funds of funds of funds to justify triple fees.

allocation, an investor usually specifies allocations at the level of asset classes: domestic stocks, international stocks, government bonds, corporate bonds, etc. The allocation policy is subject to periodic reviews. Unless there are significant changes in the investor's long-term risk-return profile or in the long-term capital market equilibrium, the strategic asset allocation is kept stable. Once a strategic asset allocation is established, its implementation is relatively straightforward. Deviations in the portfolio weights from the strategic asset allocation will be reduced periodically by a beta management team.

A long-term investor expects to earn returns by taking systematic market risks. In the short-term, the investor may wish to add extra returns to the portfolio by alpha management. If the investor or the delegated portfolio manager has skills to consistently generate alpha, by either market timing or security selection skills, additional surplus will be built up. On the other hand, seeking alpha should not be done at the expense of the long-term objective. The investor may be worse off with alpha management if there are persistent, large deviations of the short-term asset allocation from the long-term strategic asset allocation. Chapter 4 evaluates a long-term tactical asset allocation strategy that takes advantages of perceived short-term opportunities in the capital market while benchmarked against the long-term strategic portfolio. The portfolio strategy makes the portfolio weights depend directly on changes in state variables, and uses an expanded asset menu in the Markowitz framework. The optimal portfolio consists of a benchmark portfolio that controls the active risk, and a pure overlay portfolio that adds active return to the whole portfolio. The analysis in Chapter 4 shows that the aggregate equity market momentum and other state variables, have significant power in predicting optimal asset allocation, resulting in positive outperformance. Since the average systematic risk exposures under the unconstrained tactical asset allocation are much higher than the benchmark portfolio, the investor may want to constrain the active portfolio strategy such that the tactical asset allocation's risk profile is matched with the strategic asset allocation in the long term.

Chapter 2

Being Locked Up Hurts

Abstract

A lockup period for hedge funds restricts a multi-period investor's ability to rebalance his portfolio and has non-trivial effects on the allocation decision and portfolio efficiency. Investors compensate for a hedge fund lockup period by making adjustments to their equity and bond holdings. Adding hedge funds with a lockup period to the portfolio of stocks and bonds generates large, negative hedge demands for stocks. More importantly, an investor with a portfolio of stocks, bonds and hedge funds under both the unconditional strategy and conditional strategy is hurt by the presence of a hedge fund lockup period. In an unconditional setting, we find a Sharpe ratio of 1.10 for the portfolio of stocks, bonds and hedge funds adjusted for stale pricing, with a three-month lockup period for hedge funds and monthly rebalancing of stocks and bonds. For the same portfolio, but without a hedge fund lockup period, we find a significantly higher Sharpe ratio of 1.42. The certainty equivalent is 1.9%, i.e. the investor is willing to pay 1.9% per year in order to move to the ideal situation of unlimited rebalancing or no lockup.

2.1 Introduction

An important issue for both practitioners and academic researchers in portfolio management is to solve a multi-period investment problem. The question is how to rebalance a portfolio before the investment horizon, which is often complicated by restrictions such as the inability to go short and the fact that some risky assets cannot be rebalanced easily. For instance, investments in hedge funds are often accompanied by a lockup period, during which investors cannot withdraw their money. There are other restrictions such as a redemption notice period and a low redemption frequency that make it difficult for an investor to get his money out of hedge funds. The implication of a hedge fund lockup period is best illustrated by the experience of hedge fund investors during the recent financial crisis beginning in July, 2007. Investors would like to liquidate their investments in hedge funds to avoid further losses or to meet liquidity needs elsewhere. But there is no escape if the hedge fund lockup period has not yet expired.

Over the past few years, institutional investors from many countries show strong interest in increasing allocations (in terms of both absolute dollar amounts and portfolio weights) to hedge funds, private equity and venture capital (Source: The 2007-2008 Russell Investments Survey on Alternative Investing). In this paper, we study the asset allocation problem for a multi-period investor whose investment opportunity set consists of the risk-free asset, stocks, bonds and hedge funds with a lockup period. While hedge funds are also subject to a redemption notice period and a redemption frequency constraint, both constraints can be viewed as a form of lockup. The method in our paper extends Brandt and Santa-Clara (2006) where the multi-period investment portfolio is solved in a static Markowitz framework. We show that a hedge fund lockup period can be incorporated into the multi-period asset allocation decision by an investor. In addition, we find that the lockup constraint considered in this paper is empirically highly relevant. Our empirical analysis shows that investing in hedge funds can improve portfolio outcomes under both the unconditional strategy and the conditional strategy. However, there are economically large costs of the hedge fund lockup restriction, even after controlling for stale pricing and higher moments of hedge fund returns. A hedge fund lockup period hurts the investor as it reduces the benefits of adding hedge funds to the portfolio of stocks and bonds.

This paper contributes to three strands of literature. The first strand is the literature on the portfolio choice and valuation of illiquid or nonmarketable assets (e.g. Mayer (1976), Longstaff (2001), Ang, Papanikolaou and Westerfield (2010)). Our paper naturally extends the single-period framework to a multi-period setting. Since investors often hold a portfolio of liquid and illiquid assets, it is interesting to investigate how they rebalance portfolio holdings of liquid assets in each period, taking into account illiquid holdings, to maximize the multi-period utility. Our paper also contributes to the hedge fund literature by evaluating the economic value of hedge fund investments from a portfolio perspective.[1] The existing literature typically evaluates hedge funds on the basis of a factor model or a benchmark model. We take a portfolio perspective and compute optimal allocations to different asset classes in a portfolio. The benefits of adding hedge funds to a portfolio are measured by incremental Sharpe ratios or certainty equivalents. Finally, our paper contributes to the literature of the share restrictions of hedge funds. Ang and Bollen (2008) model hedge fund lockups and notice periods as a real option.[2] In our paper,

[1] The evaluation of hedge fund performance has been studied in a number of papers, e.g. Agarwal and Naik (2004), Fung, Hsieh, Naik and Ramadorai (2008), Kosowski, Naik, and Teo (2005), Malkiel and Saha (2005).

[2] Aragon (2007), Liang and Park (2008) examine the liquidity premium that a hedge fund investor is

we derive the cost of a hedge fund lockup period, taking into account the possibility of rebalancing stocks and bonds in a portfolio of stocks, bonds and hedge funds. In other words, investors can at least partially hedge unexpected changes in hedge fund returns by adjusting holdings in stocks and bonds.

As in Brandt and Santa-Clara (2006), we solve a multi-period portfolio problem, exploiting the concepts of timing portfolios and conditional portfolios. In a multi-period setting, a timing portfolio is a strategy that invests in only risky assets in one period and in only the risk-free asset in all other periods. We incorporate a hedge fund lockup period into the portfolio analysis. If we assume that the investment horizon is equal to the length of the lockup period, there are no timing portfolios for hedge funds. Because once an investment in hedge funds is made, an investor has to hold on to it until the lockup restriction expires. A portfolio of stocks, bonds and hedge funds with a lockup period will certainly behave differently from a portfolio of the same assets without a hedge fund lockup period, in terms of optimal asset allocations over time, as well as portfolio performance.

The paper uses broad market indexes as the proxies for stocks, bonds and hedge funds (for the hedge fund proxy, we use the HFRIFOF composite index, an equal weighted composite of funds of hedge funds constructed by the HFR, Inc.) in the empirical analysis. The investment horizon is three months, and we rescale optimal portfolio weights such that the first period portfolio is the tangency portfolio. After the first period, an investor can move upwards along the efficient frontier, investing more in risky assets funded by a risk-free loan, or move downwards along the efficient frontier, investing in the risk-free asset as well as in risky assets. When there is no lockup period, the investor is relatively more aggressive in the first period as the total investments in risky assets are reduced in the second and third period. With the portfolio of stocks, bonds and funds of hedge funds, the investor takes long positions of 20% and 30% in the risk-free asset in the second and third month, respectively, implying that the portfolio is less risky over time. Hence, without a lockup period, the investor becomes more conservative as the investment horizon approaches. When there is a three-month lockup period, however, the optimal portfolio strategy is quite different. Allocations to stocks and bonds tend to increase over time, while allocations to hedge funds are stable because of a lockup period. To buy additional risky assets, the investor shorts the risk-free asset by about 10% and 20% in the second and third month, respectively, suggesting that the investor becomes more aggressive as the investment horizon approaches, which is the opposite to what would happen when there is no lockup period. Such different optimal portfolio strategies stress the importance of taking into account a hedge fund lockup period in the investment decision. In addition, we investigate the hedge demands for stocks and bonds arising from the inclusion of funds of hedge funds with a lockup period. Indeed, we find that investing in funds of hedge

expected to earn from investing in hedge funds with a lockup period.

funds with a three-month lockup period induces large, negative hedge demands for stocks in order to obtain the desired intertemporal equity exposure that cannot be obtained by rebalancing hedge funds due to the lockup constraint. For the portfolio of stocks, bonds and funds of hedge funds with a three-month lockup period under the unconditional strategy, the Markowitz demands for stocks are 27%, 26% and 19% in the first, second and third period, decreasing over time. For bonds, the Markowitz demands increase from 25% in the first month, to 33% in the second month and 44% in the third month. The inclusion of funds of hedge funds generates hedge demands of -18% , -12%, and -7% for stocks, and 5%, 3%, and 4% for bonds over the three months.

Our empirical analysis shows that both the unconditional strategy and the conditional strategy can be improved upon when adding hedge funds to the portfolio of stocks and bonds, but portfolio performance is hurt by a hedge fund lockup period. For instance, under the unconditional strategy, the annualized Sharpe ratio for the portfolio of stocks, bonds and funds of hedge funds with a three-month lockup period is 1.21, which is significantly higher, both economically and statistically, than the Sharpe ratio of 0.90 under the unconditional strategy of investing in stocks and bonds only. But if there is no lockup period, the portfolio Sharpe ratio with the three asset classes is 1.51, which is significantly higher than the reported Sharpe ratio of 1.21 for the portfolio of stocks, bonds and funds of hedge funds with a three-month lockup period. In terms of certainty equivalents, an investor is willing to pay as much as 1.85% per year in order to move from the portfolio of stocks and bonds to the portfolio of stocks, bonds and funds of hedge funds, with a three-month lockup period. Hence, the economic value of hedge fund investments is large to the investor. In addition, the investor with the portfolio of stocks, bonds and funds of hedge funds is willing to pay 1.78% per year in order to make the lockup period “disappear”. In other words, without the lockup period, the certainty equivalent of adding hedge funds to the portfolio of stocks and bonds will be much larger at 3.63% per year. A lockup period takes away some utility gains, but overall gains from the inclusion of hedge funds are still positive and large. Finally, adjusting stale pricing in hedge funds seems to reduce the diversification benefits of hedge funds, but the utility costs of a lockup period for hedge fund investments are still large.

The rest of the paper is organized as follows. Section 2.2 explains the methodology to derive optimal asset allocations for a multi-period investor facing a lockup period for hedge funds. Section 2.3 describes the data. Section 2.4 shows the empirical results under the unconditional strategy, while Section 2.5 presents the empirical results in the conditional framework. Section 2.6 addresses several issues of hedge fund investability. Finally, Section 2.7 concludes.

2.2 Asset Allocation with a Lockup Period

We consider the multi-period allocation problem for a risk-averse investor. The investor's portfolio consists of liquid assets and illiquid assets. Liquid assets include stocks, bonds, money market instruments, etc., while illiquid assets can be hedge funds, private equity and venture capital investments. We restrict our analysis to a portfolio of stocks, bonds, Treasury bills and hedge funds, but the same method is applicable when the portfolio includes other asset classes with different liquidity features. The investor can change allocations to liquid assets every period, but adjusting allocations to illiquid assets is difficult if not impossible. The form of illiquidity in this paper is restricted to the situation in which a lockup period is imposed for investments in hedge funds.

2.2.1 Multi-period Asset Allocation with Lockup Constraints

We first derive two-period asset allocations with lockup constraints before generalizing the method. There are K_1 liquid risky assets, and K_2 illiquid risky assets with a lockup period equal to L. For simplicity, the investment horizon is assumed to be the same as the lockup period. Consider the two-period quadratic utility optimization problem for an investor:

$$\max \ E_t \left[r^p_{t\to t+2} - \frac{\gamma}{2} \left(r^p_{t\to t+2} \right)^2 \right], \tag{2.1}$$

where $r^p_{t\to t+2}$ is the excess portfolio return over two periods and γ is the coefficient of risk aversion. Denote portfolio weights of liquid assets and illiquid assets at time t by $w_{z,t}$ and $w_{x,t}$, respectively. In addition, denote the one-period gross return on the risk-free asset at time t by R^f_t, and the gross returns of illiquid assets by R^x_{t+1}. The vector r_{t+1} contains one-period excess returns of liquid risky assets. The two-period excess return of the portfolio with only liquid assets is:

$$\begin{aligned} r^p_{t\to t+2} &= \left(R^f_t + w'_{z,t} r_{t+1} \right) \left(R^f_{t+1} + w'_{z,t+1} r_{t+2} \right) - R^f_t R^f_{t+1} \\ &= w'_{z,t} \left(R^f_{t+1} r_{t+1} \right) + w'_{z,t+1} \left(R^f_t r_{t+2} \right) + \left(w'_{z,t} r_{t+1} \right) \left(w'_{z,t+1} r_{t+2} \right) \\ &\approx w'_{z,t} \left(R^f_{t+1} r_{t+1} \right) + w'_{z,t+1} \left(R^f_t r_{t+2} \right). \end{aligned} \tag{2.2}$$

Because r_{t+1} and r_{t+2} are excess returns, the product $\left(w'_{z,t} r_{t+1} \right) \left(w'_{z,t+1} r_{t+2} \right)$ is very small at short horizons, so the excess portfolio return over two periods is approximately the sum of $w'_{z,t} \left(R^f_{t+1} r_{t+1} \right)$ and $w'_{z,t+1} \left(R^f_t r_{t+2} \right)$.

Brandt and Santa-Clara (2006) interpret $w'_{z,t} \left(R^f_{t+1} r_{t+1} \right)$ and $w'_{z,t+1} \left(R^f_t r_{t+2} \right)$ as "timing portfolios". First, $w'_{z,t} \left(R^f_{t+1} r_{t+1} \right)$ is the two-period excess return from investing in risky assets at time t and then investing in the risk-free asset at time $t+1$. Second, $w'_{z,t+1} \left(R^f_t r_{t+2} \right)$ is the two-period excess return from investing in the risk-free asset at time t and then investing in risky assets at time $t+1$.

When the portfolio includes illiquid assets with a two-period lockup, the two-period portfolio excess return takes the form of the following:

$$\begin{aligned} r^p_{t\to t+2} &= \left(R^f_t + w'_{z,t} r_{t+1}\right)\left(R^f_{t+1} + w'_{z,t+1} r_{t+2}\right) - R^f_t R^f_{t+1} + w'_{x,t} r^x_{t\to t+2} \\ &\approx w'_{z,t}\left(R^f_{t+1} r_{t+1}\right) + w'_{z,t+1}\left(R^f_t r_{t+2}\right) + w'_{x,t} r^x_{t\to t+2}, \end{aligned} \tag{2.3}$$

where $r^x_{t\to t+2}$ is a K_2 dimensional vector of excess returns of illiquid assets, and for each illiquid asset, $r^x_{i,t\to t+2} = R^x_{i,t+1} R^x_{i,t+2} - R^f_t R^f_{t+1}$ for $i = 1, 2, \cdots, K_2$. For the two-period investment in the illiquid asset i, one dollar will grow by $R^x_{i,t+1} R^x_{i,t+2}$ and after paying back the risk-free loan, the two-period excess return on the illiquid asset is $R^x_{i,t+1} R^x_{i,t+2} - R^f_t R^f_{t+1}$. There is no "timing" portfolio for illiquid assets since they are locked up for two periods.

The S dimensional vector of z_t is a set of state variables at time t. The portfolio weights are assumed to be linear in state variables. For liquid risky assets,

$$w_t = \beta_1 z_t \tag{2.4}$$

$$w_{t+1} = \beta_2 z_{t+1}, \tag{2.5}$$

where the matrices β_1 and β_2 both have a dimension of $K_1 \times S$. For illiquid risky assets, we have

$$w_{x,t} = \beta_x z_t, \tag{2.6}$$

where β_x is a $K_2 \times S$ matrix. Throughout this paper, the portfolio strategy using a constant as the only state variable is defined as the "unconditional strategy". If state variables include a constant and time-varying instruments, then the portfolio strategy is called the "conditional strategy". For the optimal portfolio under the conditional strategy, the investor can simply maximize the utility (2.1) after inserting (2.3) into the utility function. It is obvious that the unconditional strategy is a special case of the conditional strategy with the constant term being the only state variable.

The equations (2.4), (2.5) and (2.6) express portfolio weights as linear combinations of state variables, and the two-period portfolio excess return in (2.3) becomes

$$r^p_{t\to t+2} = (\beta_1 z_t)'\left(R^f_{t+1} r_{t+1}\right) + (\beta_2 z_{t+1})'\left(R^f_t r_{t+2}\right) + (\beta_x z_t)' r^x_{t\to t+2}. \tag{2.7}$$

Using some linear algebra, we find

$$(\beta_1 z_t)'\left(R^f_{t+1} r_{t+1}\right) = vec(\beta_1)'\left(R^f_{t+1} z_t \otimes r_{t+1}\right), \tag{2.8}$$

$$(\beta_2 z_{t+1})'\left(R^f_t r_{t+2}\right) = vec(\beta_2)'\left(R^f_t z_{t+1} \otimes r_{t+2}\right), \tag{2.9}$$

$$(\beta_x z_t)'\left(r^x_{t\to t+2}\right) = vec(\beta_x)'\left(z_t \otimes r^x_{t\to t+2}\right), \tag{2.10}$$

where $vec\left(\beta_j\right)$ is a vector that stacks the columns of the matrix β_j, $j = 1, 2, x$, and $\otimes$ is the Kronecker product. The investment menu becomes a set of scaled returns or expanded asset return space,

$$\tilde{r}_{t+1} = z_t \otimes r_{t+1}, \tag{2.11}$$

$$\tilde{r}_{t+2} = z_{t+1} \otimes r_{t+2}, \tag{2.12}$$

$$\tilde{r}^x_{t\to t+2} = z_t \otimes r^x_{t\to t+2}. \tag{2.13}$$

The investor's problem is to choose a set of parameters to maximize the multi-period quadratic utility:

$$\max_{\tilde{w}} E_t \left[\tilde{w}'\tilde{r}_{t\to t+2} - \frac{\gamma}{2}\tilde{w}'\tilde{r}_{t\to t+2}\tilde{r}'_{t\to t+2}\tilde{w}\right], \tag{2.14}$$

where parameters

$$\tilde{w} = \begin{pmatrix} vec\left(\beta_1\right) \\ vec\left(\beta_2\right) \\ vec\left(\beta_x\right) \end{pmatrix} \tag{2.15}$$

can be considered as the unconditional weights of expanded assets. The scaled returns of those expanded assets are defined as

$$\tilde{r}_{t\to t+2} = \begin{pmatrix} R^f_{t+1}\tilde{r}_{t+1} \\ R^f_t\tilde{r}_{t+2} \\ \tilde{r}^x_{t\to t+2} \end{pmatrix}. \tag{2.16}$$

Once the unconditional weights $\tilde{w}$ are derived, they can be inserted into equations (2.4), (2.5) and (2.6) to compute the portfolio weights of liquid and illiquid assets. The unconditional weights $\tilde{w}$ that maximize the conditional expected utility at all dates t should also maximize the unconditional expected utility. The optimization of the multi-period quadratic utility still makes use of the static Markowitz approach on the basis of the unconditional moments of scaled returns. The optimal unconditional weights are:

$$\tilde{w} = \gamma^{-1}E\left[\tilde{r}_{t\to t+2}\tilde{r}'_{t\to t+2}\right]^{-1}E\left[\tilde{r}_{t\to t+2}\right]. \tag{2.17}$$

The sample analogue of the population moments in equation (2.17) leads to a consistent estimate of the unconditional weights $\tilde{w}$, which is a vector of length $(2K_1S + K_2S)$. The optimal weights $\tilde{w}$ are with respect to scaled returns or expanded asset returns, but one can recover the optimal portfolio weights of K_1 risky assets at time t and $t+1$, $w_{z,t}$ and $w_{z,t+1}$, using the instrument values and the unconditional weights $\tilde{w}$ as follows:

$$w^i_{z,t} = \begin{pmatrix} \tilde{w}_{(i)} & \tilde{w}_{(i+K_1)} & \cdots & \tilde{w}_{(i+(S-1)K_1)} \end{pmatrix} z_t, \; i = 1, 2, \cdots, K_1, \tag{2.18}$$

$$w^i_{z,t+1} = \begin{pmatrix} \tilde{w}_{(i+K_1S)} & \tilde{w}_{(i+K_1+K_1S)} & \cdots & \tilde{w}_{(i+(S-1)K_1+K_1S)} \end{pmatrix} z_{t+1}. \tag{2.19}$$

For illiquid assets, the portfolio weights at time t can be derived in the same way as those of liquid risky assets,

$$w^i_{x,t} = \left(\tilde{w}_{(i+2K_1S)} \quad \tilde{w}_{(i+K_2+2K_1S)} \quad \cdots \quad \tilde{w}_{(i+(S-1)K_2+2K_1S)} \right) z_t, \; i = 1, 2, \cdots, K_2. \quad (2.20)$$

However, the static optimal portfolio weights in (2.17) do not give direct solutions to the portfolio weights of illiquid assets at time $t+1$. We can normalize the initial portfolio value to one, and the portfolio weight of illiquid asset i is the ratio of its value to the portfolio value at the beginning of time $t+1$.

We can generalize the method above to solve the L-period asset allocation problem with lockup constraints on certain risky assets. The optimal static portfolio weights of expanded assets are

$$\tilde{w} = \gamma^{-1} E\left[\tilde{r}_{t\to t+L}\tilde{r}'_{t\to t+L}\right]^{-1} E\left[\tilde{r}_{t\to t+L}\right], \quad (2.21)$$

where $\tilde{r}_{t\to t+L}$ is a set of returns of the timing portfolios of expanded liquid assets and L-period excess returns of illiquid assets scaled by the information set z_t.

The solution in (2.21) may produce negative weights for illiquid assets with a lockup period. In reality, while shorting stocks and bonds is relatively easy, shorting illiquid assets is either too costly or impossible. For instance, investors cannot short hedge funds or transfer their stakes in hedge funds to other investors. In this case, we should add nonnegative constraints on the portfolio weights of illiquid assets to the analysis.

2.2.2 Econometrics Issues

We estimate the set of portfolio weights in (2.21) by sample analogue. The construction of the estimated covariance matrix of $\tilde{w}$ and the test procedure follow the method by Britten-Jones (1999). Given a time-series sample of asset returns, the estimation of $\tilde{w}$ can be sensitive to the starting dates of the sample. Specifically, for a lockup period with the length L, we have L choices of starting dates or estimation windows, and the resulting L sets of the estimated $\tilde{w}$ are all consistent asymptotically. Following Jegadeesh and Titman (1993), and Rouwenhorst (1998), we consider L strategies that contribute equally to a composite portfolio. Specifically, at the start of each period, the composite portfolio consists of L sub-portfolios. Each sub-portfolio invests optimally according to one set of estimated $\tilde{w}$ on the basis of an estimation window. For example, suppose that the lockup period is two months and the sample data consists of ten-year monthly asset returns. We can estimate $\tilde{w}$ using two different estimation windows: the first window starts one month earlier than the second window in the data. The composite portfolio invests one half in the sub-portfolio according to the first set of estimated $\tilde{w}$ and one half in the sub-portfolio according to the second set of estimated $\tilde{w}$. The method is comparable to that in Jegadeesh and Titman (1993), and Rouwenhorst (1998). In those two papers, they report the monthly average return of K strategies for a K-month holding period in order to evaluate relative strength portfolios. In other words, this method uses overlapping portfolios instead of overlapping data.

2.2.3 Power Utility Function and GMM Estimation

The quadratic utility in expression (2.1) can be considered as a second-order approximation of the power utility function. The approximation is not a serious concern if asset returns are normally distributed. However, some risky assets may exhibit some non-normality such as a large, negative skewness or excess kurtosis. This is especially true for hedge funds pursuing a relative value strategy such as merger arbitrage. Extreme gains or losses are more likely for such hedge funds than what normality implies. Therefore, the quadratic utility maximization may result in asset allocations undesirable from the perspective of a power utility investor. To measure the effect of including the third and fourth moments on the asset allocation decision, portfolio characteristics and economic values of hedge fund investments, we also consider a higher order approximation of a power utility function, which is similar to the fourth-order approximation scheme in Brandt and Santa-Clara (2006) and Brandt et al. (2009). The optimal weights are implied in the first order conditions of the expected utility maximization based on the fourth-order approximation. Brandt and Santa-Clara (2006) suggest a guessing method to obtain optimal weights. Nevertheless, optimal weights and covariance matrices can be derived by the generalized method of moments (GMM, Hansen and Singleton (1982)). On the basis of the fourth-order approximation, we find that the results of asset allocations and portfolio characteristics are quantitatively similar to those under the second-order approximation.

2.2.4 Return Smoothing in Hedge Fund Returns

Hedge fund returns are often serially correlated. One explanation is that hedge funds may hold a large amount of illiquid assets. Hedge funds may use an estimated price or the last transaction price when calculating the value of those illiquid assets, creating lags in the evolution of net asset values. The resulting smoothness of returns implies that volatilities of hedge fund returns are underestimated and Sharpe ratios are overestimated. One way to deal with return smoothing is to calculate a smoothing-adjusted Sharpe ratio (e.g. Getmansky, Lo and Makarov (2004)). For the asset allocation purpose, using smoothing-adjusted returns as inputs for portfolio optimization makes it possible to evaluate the effect of stale pricing of some risky assets on the optimal asset allocation.

Following the discussion related to hedge fund return smoothing by Kat and Lu (2002), Getmansky et al. (2004), and Lhabitant (2006), we assume that the reported hedge fund return $\bar{R}_t^x$ is a weighted average of the funds' true returns over recent two periods, R_t^x and

R^x_{t-1}. Accordingly, the true, unobserved and unsmoothed returns R^x_t can be written as:

$$R^x_t = \frac{\bar{R}^x_t - \phi \bar{R}^x_{t-1}}{1-\phi} \tag{2.22}$$

Setting the unsmoothing parameter ϕ equal to the autocorrelation coefficient at the first lag will lead to an estimated series of unsmoothed returns that have the same mean as reported returns and no first-order autocorrelation. Essentially, the unsmoothing method increases the return volatility.

2.3 Data

For hedge fund returns, we obtain various fund of hedge funds indexes from Hedge Fund Research, Inc. (HFR, Inc.). A fund of funds or fund of hedge funds is a hedge fund that invests with multiple managers of hedge funds or managed accounts. Since a fund of hedge funds holds a diversified portfolio of hedge funds, it lowers the risk of investing with an individual hedge fund manager and gives access to hedge funds that are closed to new money (Nicholas (2004)). The length of the lockup period for a fund of funds depends on the liquidity of underlying individual hedge funds in fund of hedge funds portfolios. Some funds of hedge funds require no lockup periods, but a lockup period of three months up to one year is not uncommon. A U.S. single strategy or single manager hedge fund typically requires a one-year lockup period plus a notice period ranging from one month to three months. In contrast, less than 40 percent of funds of hedge funds require a lockup period, and among those that have a lockup period, about two thirds of them set a lockup period of six months or longer (Nicholas (2004)). The median (mean) lockup period is around three (four) months of all funds of hedge funds. The HFRI Fund of Funds composite index (HFRIFOF) is an equal weighted index that includes over 800 funds of hedge funds with at least USD 50 million under management. Monthly returns are net of all fees. HFR, Inc. also provides four equal weighted sub-indexes or strategy indexes according to the classification of fund of hedge funds strategies: Conservative, Diversified, Market Defensive, and Strategic. A fund of hedge funds is classified as "Conservative" if it tends to invest in funds with conservative strategies such as Equity Market Neutral, Fixed Income Arbitrage, etc. that exhibit low historical volatilities. A fund of hedge funds is "Diversified" if it invests with various strategies/managers and exhibits performance close to that of the HFRIFOF composite index. A "Market Defensive" fund of hedge funds invests in hedge funds with short-biased strategies and exhibits a low or negative correlation with equity market benchmarks. Finally, a "Strategic" fund of hedge funds tends to invest in hedge funds with more opportunistic strategies and exhibits a greater volatility relative to the HFRIFOF composite index. From CRSP, we obtain the value weighted NYSE equity index as the proxy for stocks, the 1-month Treasury bill as the proxy for the risk-free asset, and the Fama Bond Portfolio (U.S. Treasuries) with maturities greater than 10 years as the proxy for bonds. We obtain the state variable from CRSP. We include the market dividend-price ratio that is known to predict asset returns.[3] The market dividend-price ratio is based on the value weighted NYSE equity index, and it is calculated as the ratio of the sum of dividends over past twelve months to the NYSE index level. The market dividend-price ratio tends to move with the ups and downs of the U.S. stock market, so the long bull market in 1990s results in a downward trend in the market dividend-price ratio during this period. The relatively short history of the hedge fund data limits the empirical analysis to the sample period from January 1990 through December 2007. Table I gives summary statistics for asset returns and the state variable.

Over the sample period, the average return and volatility of stocks are 11.35% and 12.60%, respectively. Bonds have an average return of 8.48% and a volatility of 7.90%, but the Sharpe ratio of bonds is only slightly lower than that of stocks. The HFRIFOF composite index has a lower average return of 9.67% and a lower volatility of 5.46% compared to stocks, but its Sharpe ratio of 1.03 is almost twice as large as the Sharpe ratio of stocks or bonds. The HFRIFOF Conservative index has the lowest volatility among all fund of hedge funds indexes, consistent with the style classification. Although average returns and volatilities differ among four HFRIFOF strategy indexes, their Sharpe ratios are not too far away from each other. Unsmoothing fund of hedge funds returns leads to higher volatilities than the reported, original fund returns, while the mean returns remain the same. The unsmoothed HFRIFOF composite index has a volatility of 7.37%, which is one third higher than the original HFRIFOF composite index. As a result, there are large reductions in the Sharpe ratio of fund returns. For instance, the Sharpe ratios of the HFRIFOF Conservative index and its unsmoothed index are 1.32 and 0.97, respectively. Table I also reports the autocorrelations with lags up to six months and the results of the Ljung-Box test. For stocks and bonds, the autocorrelation is not too much an issue. For the reported HFRIFOF composite index and three out of four HFRIFOF strategy indexes, the Ljung-Box test rejects the null hypothesis that there are no autocorrelations up to order 6. The first-order autocorrelation is clearly visible for those indexes. Unsmoothing the hedge fund returns removes the first-order autocorrelation in the HFRIFOF indexes, and the Ljung-Box test for the unsmoothed hedge fund returns cannot reject the null hypothesis at any conventional significance level.

Table II gives the correlation matrix for risky asset returns, the market dividend-price ratio and the risk-free return. Stock returns and bond returns are weakly correlated as expected. The correlations of hedge fund returns with stock returns are moderate, while

[3]See Campbell (1987), Campbell and Shiller (1988a), (1988b), Campbell and Viceira, (1999), (2002), Cochrane (2008), Fama and French (1988), (1989), Keim and Stambaugh (1986), Hodrick (1992), and Lettau and Ludvigson (2005). Goyal and Welch (2008) and Campbell and Thompson (2008) include a comprehensive list of these variables along with some others as predictors used in predictability studies.

hedge funds and bonds are also weakly correlated, suggesting the potential of portfolio diversification by adding hedge funds to a portfolio of stocks and bonds. The coefficient of correlation between stocks and the HFRIFOF Market Defensive strategy index is close to zero, consistent with the definition of the index. Moreover, unsmoothing hedge fund returns seems to increase the return correlation between stocks or bonds and hedge funds. The coefficient of correlation between stocks and the unsmoothed HFRI composite index is 0.49, compared to a coefficient of correlation of 0.43 before unsmoothing. Finally, for most hedge fund indexes, their correlations with the market dividend-price ratio are stronger than the correlations of stocks and bonds with the market dividend-price ratio.

2.4 Empirical Results under the Unconditional Strategy

Section 2.4.1 starts by reporting optimal portfolio weights under the unconditional strategy with a three-month investment period. We are interested in changes in allocations to stocks and bonds when hedge funds are added to the portfolio, as well as changes in investment patterns over the three-month investment horizon. We rescale total demands for stocks, bonds and hedge funds such that the first-period portfolios are tangency portfolios, i.e. the sum of the portfolio weights of stocks and bonds (and hedge funds in a three-asset portfolio) in the first period is equal to 100%. Hence, after the first period, an investor makes two allocation decisions: how to allocate between risky assets and the risk-free asset, and how to allocate among different risky assets within the mix of risky assets.[4] Section 2.4.2 describes the characteristics of timing portfolios. In Section 2.4.3, we decompose the total demand for stocks and bonds in the portfolio of stocks, bonds and hedge funds with a three-month lockup period into a Markowitz or speculative demand and a hedge demand. Section 2.4.4 compares the portfolio performance under the unconditional strategy with a lockup period and without a lockup period. In addition, we test whether adding hedge funds improves the portfolio Sharpe ratio and calculate the utility cost of a hedge fund lockup period.

2.4.1 Optimal Portfolios

Table II reports optimal portfolios under the unconditional strategy with a three-month investment horizon. We estimate the covariance matrix for portfolio weights using GMM.

[4]Portfolio weights of stocks, bonds and hedge funds in Table II, Table IV, Table VI and Table VII are proportions of stocks, bonds and hedge funds to the total portfolio value of risky assets and the risk-free asset. Whenever the mix of risky assets is mentioned in the paper, it refers to a portfolio of risky assets, excluding the risk-free asset. The proportion of each risky asset to the mix of risky assets is not equivalent to the portfolio weight of the asset in the portfolio of all assets, unless the investment in the risk-free asset is zero.

The results under quadratic utility in Panel A of Table II show that portfolio weights vary in a systematic way over the investment horizon. To start out, in the portfolio of stocks and bonds, the allocations to stocks and bonds display distinct patterns over the investment horizon. Over the three months, the allocations to stocks decrease monotonically from 52% to 37%, while the allocations to bonds increase monotonically from 48% to 85%. Thus, an investor starts with a relatively risky portfolio and gradually adjusts his portfolio holdings in order to obtain a less risky portfolio by the end of the investment horizon. However, since the increase in the allocations to bonds is greater than the decrease in the allocations to stocks in the second and third month, the investor will invest more than 100% in stocks and bonds after the first month. Such a portfolio is considered to be more risky, but the increase in risk due to leverage in the second and third month is probably offset somewhat by the increasing allocations to less risky government bonds in the mix of risky assets.

Adding hedge funds to the portfolio of stocks and bonds changes the pattern of investments in stocks over the investment horizon, while the allocations to bonds remain monotonically increasing. The allocations to stocks after inclusion of the HFRIFOF composite with a three-month lockup period are 9%, 14% and 12% in the first, second and third month. In the first month, hedge funds have a weight of 62%. The number of shares invested in hedge funds is the same for all three periods and the portfolio weights of hedge funds will be around 62% in the second and third month. This implies that after the first period, the investor will invest more than 100% in risky assets funded by a risk-free loan. The investor purchases additional risky assets by taking short positions in the risk-free asset of about 10% and 20% in the second and third month, respectively. The investor becomes more aggressive over time, as he holds a leveraged portfolio after the first month. In addition, within the mix of risky assets, the proportion of bonds increases a bit in the second and third month, while there is a decrease in the proportion of hedge funds over the same periods. The proportion of stocks in the mix is relatively stable over time. Therefore, even though the investor has a short position in the risk-free asset to purchase additional risky assets in the second and third period, bond investments in the mix of risky assets also increase slightly. Overall, the investor seems to pursue a more aggressive portfolio strategy with increasing leverage to invest in risky assets as the investment horizon approaches.

When there is no lockup period, an investor is relatively more conservative over time as he increases the allocations to the risk-free asset in the second and third month. For the portfolio of stocks, bonds and the HFRIFOF composite, investments in the risk-free asset are about 20% and 30% in the second and third month, respectively. Hence, if the investor keeps investing in stocks, bonds and hedge funds in the same proportions as in the first period, the portfolio is going to be less risky. Interestingly, the investor also adjusts the proportions of stocks, bonds and hedge funds in the mix of risky assets over time. Within the mix of risky assets, the investor will increase the allocations to stocks and bonds, and reduce the allocations to hedge funds in the second and third month. For instance, in the third month, the allocations to stocks, bonds and hedge funds are 13%, 35% and 22%. This implies that the proportion of bond investments in the mix of risky assets is about 50% in the third month, which is sharply higher than the proportion of bond investments in the mix in the first period, 24%. Compared to a proportion of 76% in the first month, stocks plus hedge funds only account for about 50% in the mix of risky assets in the third month. Since bonds are relatively less risky than stocks and hedge funds, the adjustment of the mix of risky assets leads to a less risky portfolio. Together with the increasing allocations to the risk-free asset in the portfolio, the portfolio strategy is unambiguously less aggressive over time. To sum up, without a lockup period, the investor pursues a more conservative portfolio strategy as the investment horizon approaches. In contrast, the investor is more aggressive over the investment horizon when there is a three-month lockup period for hedge funds. Such contrasting investor behavior underscores the importance of taking into consideration a hedge fund lockup period in the multi-period portfolio decision.

Table II also reports the allocations to stocks, bonds and the unsmoothed HFRIFOF composite index, with and without a lockup period. Since the unsmoothed hedge fund returns have a higher volatility and higher correlations with stocks and bonds, compared to the original, reported hedge fund returns, it is not surprising to observe a reduction in the allocation to hedge funds in the portfolio of stocks, bonds and hedge funds. In addition, when there is no hedge fund lockup period, the investor somehow decides to compress the allocations to hedge funds over the investment horizon to offset the increased volatility of hedge fund returns after unsmoothing. The effect of a hedge fund lockup period on the optimal allocations to stocks and bonds is rather familiar: with a lockup period, an investor is more aggressive over time, with increasing leverage to fund additional investments in stocks and bonds after the first month. When there is no lockup period, an investor is more conservative as the investment horizon approaches, with more allocations to bonds in the mix of risky assets and increasing investments in the risk-free asset after the first month.

We repeat our portfolio analysis using the fourth-order approximation of a power utility function, which takes into account the higher moments of returns. The effect of higher return moments on the portfolio of stocks and bonds is negligible, as the difference between Panel A and Panel B indicates. Surprisingly, the four-order approximation does not substantially change the tangency portfolio of stocks, bonds and hedge funds, with or without a lockup. The allocations to hedge funds are lower in general when the higher moments are taken into account, but the difference is no more than 3% (5% if using unsmoothed returns). Furthermore, the overall effect on the allocations to stocks and bonds doesn't seem substantial. We will report the effect of higher moments on the portfolio characteristics in Section 2.4.4.

2.4.2 Timing Portfolios

The time-varying portfolio weights of risky assets in Table II is puzzling. They suggest that risky asset returns may not be independent over time. Table III reports the summary statistics of the timing portfolio returns of stocks, bonds and hedge funds, as well as the hedge fund returns locked for three months. We start with a discussion of the role of timing portfolios $w'_{z,t}\left(R^f_{t+1}r_{t+1}\right)$ and $w'_{z,t+1}\left(R^f_t r_{t+2}\right)$, which are the results of the approximation method, in generating the autocorrelations of risky asset returns in Table II. Essentially, the short-term interest rate may play a role in generating time-varying portfolio weights via autocorrelations (e.g. bond returns) of asset returns and associated cross-asset autocorrelations of returns. But the approximation approach per se has no effect on the time variation in portfolio weights. The timing portfolio returns are the product of risky asset returns and the gross short-term risk-free return. If we use a less precise (perhaps unwise) approximation of multi-period returns $w'_{z,t}r_{t+1}$ and $w'_{z,t+1}r_{t+2}$, it would lead to the coefficients of autocorrelations (including cross-asset autocorrelations) almost identical to Table III.[5] Hence, while the timing portfolios may have higher mean returns due to their compounding of gross risk-free returns, the time variation in portfolio weights can still be obtained using multi-period returns $w'_{z,t}r_{t+1}$ and $w'_{z,t+1}r_{t+2}$.

From Table III, for each asset over three months, the difference in the return-risk trade off is not large enough to generate the time-varying weights. In addition, the autocorrelations of the timing portfolios of stocks are quite small. In contrast, the autocorrelations of the timing portfolios of bonds and hedge funds are more visible. Moreover, the timing portfolios of stocks, bonds and hedge funds at different periods are (cross-asset) correlated, which may explain the time-varying portfolio weights of risky assets. For instance, in the two-asset portfolio of stocks and bonds, the timing portfolios of stocks at time t are positively correlated with the timing portfolios of bonds at time t, $t-1$, $t-2$, and negatively correlated with the timing portfolios of bonds at time $t+1$ and $t+2$. As a result, the total stock demand at month 1 is higher than the total stock demands at month 2 and month 3. For bonds, the total demand is lower in earlier periods because the timing portfolios of bonds in earlier periods have higher correlations with the timing portfolios of stocks and bonds in other periods.

In the three-asset portfolio without a hedge fund lockup, there are positive correlations between the timing portfolio of stocks at month 1 and the timing portfolios of hedge funds at all three months, and negative correlations between the timing portfolio of stocks and the lagged timing portfolios of hedge funds. Hence, we will expect increasing stock demands due to the cross-asset autocorrelation between stocks and hedge funds. The

[5] The difference is in the magnitude of the thousandth. Since we only report numbers up to the hundredth, the cross-asset autocorrelations based on r_{t+1} show hardly any difference from Table III.

increase partly offsets the decreasing stocks demands driven by the cross-asset autocorrelation between stocks and bonds described previously. With respect to bond allocations, the correlations between the timing portfolios of bonds and the timing portfolios of hedge funds and stocks are higher in earlier periods, which explains the increasing allocations to bonds over time in the three-asset portfolio.

2.4.3 Demand Decomposition: Markowitz vs. Hedge Demand

To further investigate changes in the investment patterns of stocks and bonds, we calculate the Markowitz (or pure speculative) demands and hedge demands for stocks and bonds in the three-asset portfolio with a hedge fund lockup period. Table IV shows the optimal demand for stocks and bonds as the combination of the Markowitz demand and the hedge demand, using either the HFRIFOF composite index or the unsmoothed HFRIFOF composite index as the proxy for hedge funds in the three-asset portfolio with a hedge fund lockup period. Specifically, the optimal demand for stocks and bonds in the three-asset portfolio takes the following form:[6]

$$w_z = \gamma^{-1}\Sigma_{zz}^{-1}\mu_z - \Sigma_{zz}^{-1}\Sigma_{z,x}w_{x,t}^{*} \qquad (2.23)$$

The w_z is the optimal demand for stocks and bonds over the investment horizon, Σ_{zz}^{-1} is the covariance matrix of the timing portfolios of stocks and bonds, μ_z is the vector of the expected returns of the timing portfolios of stocks and bonds, $\Sigma_{z,x}$ is the covariance matrix of the three-period excess returns of hedge funds and the timing portfolios of stocks and bonds, and $w_{x,t}^{*}$ is the optimal demand for hedge funds in the three-asset portfolio. The Markowitz demand for stocks and bonds in the three-asset portfolio, $\gamma^{-1}\Sigma_{zz}^{-1}\mu_z$, is simply the optimal demand for stocks and bonds in the portfolio of stocks and bonds for the investor with the same risk aversion. The hedge demand, $-\Sigma_{zz}^{-1}\Sigma_{z,x}w_{x,t}^{*}$, is the product of two determinants: the optimal demand for hedge funds at time t, $w_{x,t}^{*}$, and $\Sigma_{zz}^{-1}\Sigma_{z,x}$, which are slope coefficients from the regression of the three-month excess returns of hedge funds on a constant and the returns of the timing portfolios of stocks and bonds:

$$\begin{aligned} r_{t\to t+3}^{x} &= \alpha + b_{s,1}'(R_{t+1}^{f}R_{t+2}^{f}r_{t+1}^{s}) + b_{s,2}'(R_{t}^{f}R_{t+2}^{f}r_{t+2}^{s}) + b_{s,3}'(R_{t}^{f}R_{t+1}^{f}r_{t+3}^{s}) \\ &\quad + b_{b,1}'(R_{t+1}^{f}R_{t+2}^{f}r_{t+1}^{b}) + b_{b,2}'(R_{t}^{f}R_{t+2}^{f}r_{t+2}^{b}) + b_{b,3}'(R_{t}^{f}R_{t+1}^{f}r_{t+3}^{b}) + \varepsilon_t \qquad (2.24) \end{aligned}$$

For instance, the hedge demand for stocks in the first period is $-b_{s,1}w_{x,t}^{*}$. The hedge demands for stocks in other periods and those for bonds follow the same logic.

[6] Note that the optimal weights in equation (2.21) are based the quadratic utility or mean-second moment utility function. However, from optimal weights derived from the mean-second moment utility function to those derived from the mean-variance utility function is only a matter of rescaling the risk-aversion. Hence, we can get a mean-variance version of equation (2.21). See Britten-Jones (1999) for a description on the conversion of mean-second moment portfolios to mean-variance portfolios.

From Table IV, we find that in each month, the hedge demand is negative for stocks and positive for bonds. Furthermore, the hedge demand for stocks is the most negative in the beginning and increases over time, which results in a pattern of the optimal demand different from that of the Markowitz demand for stocks. For instance, adding the HFRIFOF composite to the portfolio of stocks and bonds gives rise to a smaller allocation to stocks relative to the Markowitz demand in the first month (9% vs. 27%). The Markowitz demand decreases to 26% in the second month, while the total demand increases to 14% due to an increase in the hedge demand. In the third month, the total demand for stocks decreases to 12%, as the increase in the hedge demand is more than offset by the decrease in the Markowitz demand. This is the reason that the total demand for stocks exhibits an inverted U-shape over time. For bond investments in the three-asset portfolio, the hedge demand is relatively small. Monthly changes in the hedge demand over the three-month horizon are not large enough to reverse the investment pattern of bonds.

We can explain the difference in the patterns and magnitudes of the hedge demands for stocks and bonds by examining the second determinant of the hedge demands, which is the set of slope coefficients from the equation (2.24). We look at the correlations between hedge funds and stocks or bonds in Table III to get a rough estimation of the magnitude and direction of the hedge demands. From Table III, the correlation between stocks and the HFRIFOF composite is 0.33, 0.22 and 0.08 in the first, second and third period, while the correlation between bonds and the HFRIFOF composite is negative but close to zero for all periods. In other words, hedge funds look more like stocks. To hedge the changes in the value of hedge funds, the investor can simply go short in stocks, and the hedge demand for stocks is $-b_{s,1}w^*_{x,t}$ for the first month. As bonds and hedge funds are weakly correlated, the hedge demand for bonds is relatively small.

Investing in hedge funds when there is a lockup period, basically leads to an exogenously given exposure to hedge funds after the first period, which induces hedge demands for stocks and bonds. The optimal investments in stocks and bonds in the three-asset portfolio are the sum of the Markowitz demand and the hedge demand. The Markowitz demand is the optimal demand for stocks and bonds when the investment menu includes stocks and bonds only. The hedge demand arises because the investor wants to hedge the changes in the value of hedge fund investments, which are locked up for three months. A negative hedge demand for stocks implies that the overall allocation to stocks will be lower than it would be in the portfolio of stocks and bonds.

2.4.4 Portfolio Efficiency and Certainty Equivalents

The above analysis has shown that taking into account the lockup period for hedge funds has important implications for the allocation decision. A question of considerable interest is whether hedge funds offer diversification benefits when they are added to the portfolio of stocks and bonds. Panel A of Table V reports the performance of the portfolios of stocks, bonds and hedge funds for an investor with a quadratic utility. In each case, a different hedge fund index is used as the proxy. The mean excess return and volatility of the two-asset portfolio are 7.05% and 7.83%, respectively. The Sharpe ratios of the three-asset portfolios are much higher than the Sharpe ratio of the two-asset portfolio. The portfolio of stocks, bonds and the HFRIFOF composite with a lockup has a mean excess return of 6.58%, a volatility of 5.44%, and a Sharpe ratio of 1.21. Replacing reported hedge fund returns with unsmoothed returns leads to a lower portfolio Sharpe ratio. The fourth-order approximation of the power utility in general leads to a lower mean return as well as a lower volatility, but we find that Sharpe ratios are similar when contrasting Panel A with Panel B.

The test of portfolio efficiency follows Jobson and Korkie (1982) and De Roon and Nijman (2001). Denote the sample Sharpe ratio for the benchmark portfolio r^p by $\hat{\theta}_p$, and the sample Sharpe ratio for the portfolio of test assets r and benchmark assets r^p, by $\hat{\theta}$. The Wald statistic of the Sharpe ratio test is:

$$\xi_W = T\left(\frac{\hat{\theta}^2 - \hat{\theta}_p^2}{1+\hat{\theta}_p^2}\right) \sim \chi^2_K \tag{2.25}$$

where T is the sample size and K is the degrees of freedom. The degrees of freedom are the difference in the number of parameters between the two portfolios. The Sharpe ratio takes into account only the mean and standard deviation of portfolio returns, and it is not a proper measure of performance if portfolio returns exhibit non-normality. Monthly returns of some hedge fund indexes seem to have excess kurtosis, especially for the HFRIFOF Diversified strategy index. However, for a portfolio of stocks, bonds and any hedge fund index, even though the skewness and excess kurtosis of portfolio excess returns are not zero, they are not too different from those implied by a normal distribution. The Jacque-Bera test (also reported in Table V for both Panel A and Panel B) does not reject the null hypothesis that the three-month portfolio excess returns are normally distributed, even if the portfolio invests in the HFRIFOF Diversified strategy index. One reason is that the three-asset portfolios also have long positions in bonds that have low kurtosis. The bottom line is that the first two moments of three-asset portfolio excess returns are sufficient to describe portfolio characteristics, and the Sharpe ratio test can be justified.

From the p-values of the Sharpe ratio test in Table V, the difference in Sharpe ratios between the two-asset portfolio and every three-asset portfolio with a three-month lockup period is statistically significant at the 1% significance level, suggesting that the two-asset portfolio can be significantly improved upon by adding hedge funds. Unsmoothing hedge fund returns and imposing a three-month lockup period create drags in portfolio performance, but the drags are not large enough to completely offset the diversification benefits of adding hedge funds to the portfolio of stocks and bonds.

An investor who ignores the existence of a hedge fund lockup period will get a wrong estimate of portfolio performance. If there would be no hedge fund lockup period, a portfolio of stocks, bonds and hedge funds would have a higher Sharpe ratio, relative to a portfolio of stocks, bonds and hedge funds with a lockup period of three months. As shown in Table V, the difference in Sharpe ratios between the three-asset portfolios with a hedge fund lockup period and the three-asset portfolios without a hedge fund lockup is large and statistically significant. For instance, the portfolio of stocks, bonds and the HFRIFOF composite with a lockup period has the Sharpe ratio of 1.21, but the Sharpe ratio is 1.51 if there is no lockup period. The difference is statistically significant at the 1% significance level. Similarly, for the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite index, the difference in Sharpe ratios is 0.32 (1.10 vs. 1.42) and statistically significant. Hence, overlooking the existence of a hedge fund lockup period may overestimate the diversification benefit of adding hedge funds to the portfolio of stocks and bonds.

We calculate the certainty equivalents for an investor with a quadratic utility function and a relative risk aversion of 10, as the difference in utilities. We report two certainty equivalents. The first certainty equivalent is the difference in utilities derived from the portfolio of stocks, bonds and hedge funds with a lockup period, and the portfolio of stocks and bonds. The certainty equivalent for the three-asset portfolio with a lockup period can be considered as the fee an investor is willing to pay in order to move from the two-asset portfolio to the portfolio of stocks, bonds and hedge funds. The portfolio of stocks, bonds and the HFRIFOF composite has a certainty equivalent of 1.85% per annum with a three-month lockup period, while the certainty equivalent of the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite index is 1.18% per annum with a three-month lockup. Adjusting stale pricing of hedge fund returns takes away some utility gains from investing in hedge funds. The second certainty equivalent is the utility cost of imposing a hedge fund lockup for the investor, calculated as the utility derived from a portfolio of stocks, bonds and hedge funds without a lockup minus the utility derived from a portfolio of stocks, bonds and hedge funds with a lockup. For instance, a three-month lockup period costs the investor 1.78% per annum, when considering the portfolio of stocks, bonds and the HFRIFOF composite. An alternative interpretation is that an investor with the portfolio of stocks, bonds and the HFRIFOF composite is willing to pay 1.78% per annum in order to move to the ideal situation of unrestricted rebalancing. Unsmoothing hedge fund returns, though affecting the overall utility gains from adding hedge funds to the portfolio of stocks and bonds, has little impact on the utility costs of a hedge fund lockup period to the investor. A three-month hedge fund lockup period costs 1.90% to the investor with the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite.

2.5 Empirical Analysis of the Conditional Strategy

This section reports optimal asset allocations and performance of various portfolios under the conditional strategy. We consider asset allocations conditional on one state variable, the market dividend-price ratio. We decompose the (average) total demand for stocks and bonds in the portfolio of stocks, bonds and hedge funds with a three-month lockup period, as the combination of the speculative demand (Markowitz demand) and the hedge demand. We test the difference in Sharpe ratios of the three-asset portfolio with a lockup period and the portfolio without a lockup period. Furthermore, we test whether using the conditional strategy improves the efficiency of the unconditional strategy. As before, we rescale optimal portfolio weights such that the portfolios in the first month are tangency portfolios.

2.5.1 Optimal Portfolios under the Conditional Strategy

Table VI reports optimal asset allocations under the conditional strategy. The state variable is standardized to have a zero mean and a volatility of one, such that the intercepts are average allocations over the sample period. In the portfolio of stocks and bonds, average allocations to stocks and bonds change with the passage of time. The average allocations to stocks are not too different across the three sub-periods (53%, 59% and 50%), while the average allocation to bonds is 47% in the first month and increases from 53% at month 2 to 65% at month 3. This implies that bonds become relatively important in the portfolio as the investment horizon approaches. In addition, for all three-asset portfolios, average allocations to bonds are positive and appear to be increasing over time, a pattern similar to what we found for the two-asset portfolio.

Average allocations to stocks and bonds in the portfolio of stocks, bonds and the HFRIFOF composite with a three-month hedge fund lockup period increase monotonically. The most important effect of a three-month lockup period on the asset allocation decision is quite similar to the findings under the unconditional strategy: when there is a lockup period, on average, the investor is going to purchase additional amount of risky assets in the second and third period, financed by a risk-free loan. If there is no lockup period, after the first month, the investor will shift some funds to the risk-free asset. Average allocations to the risk-free asset are 28% and 17% in the second and third month, respectively, when the HFRIFOF composite is added to the portfolio. Within the mix of risky assets, there are large decreases in the relative importance of hedge funds in the second month or the third month, by almost a half. Both bonds and stocks gain some importance in the mix of risky assets. We find similar investment patterns with respect to the average allocations to stocks and bonds in the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite.

Changes in the state variable lead to changes in portfolio weights under the conditional strategy. The sign of coefficients on the market dividend-price ratio in determining portfolio weights of stocks changes over time. For instance, for the two-asset portfolio, the change in the market dividend-price ratio is positively related to the allocations to stocks in the second and third month, but not in the first month. In contrast, the change in the market dividend-price ratio is always positively related to the allocations to stocks in all three-asset portfolios. In addition, the change in the market dividend-price ratio is always negatively associated with the change in allocations to bonds. The investor will reduce allocations to bonds and increase allocations to stocks when there is an increase in the market dividend-price ratio. With a hedge fund lockup period, the three-asset portfolios tend to be more sensitive to changes in the market dividend-price ratio than the three-asset portfolios without a lockup period, especially after the first month. One standard deviation increase in the market dividend-price ratio in the second month will lead to decreases in the allocations to bonds of 25% and 22%, for the portfolios of stocks, bonds and the HFRIFOF composite with and without a hedge fund lockup period, respectively. Meanwhile, there are increases in the allocations to stocks of 22% and 9% for the three-asset portfolios with and without a hedge fund lockup period.

Table VII shows the decomposition of the total demand for stocks and bonds as the combination of the Markowitz demand and the hedge demand. In the portfolio of stocks, bonds and the HFRIFOF composite, the average Markowitz demands for stocks are 25%, 28%, and 24% in the first, second and third month, respectively. Adding the HFRIFOF composite to the portfolio of stocks and bonds induces the average hedge demands for stocks in the first, second and third month of –13%, –4% and 1%, resulting in the total demands for stocks of 12%, 24%, and 25%, accordingly. The presence of the negative hedge demands for stocks is not surprising. Since the investment in hedge funds is locked up for three months, an offsetting position in stocks provides the hedge against unexpected changes in the value of hedge funds over the investment horizon. The inclusion of the HFRIFOF composite with a three-month lockup period generates negative average hedge demands for bonds. The hedge demands for bonds are large relative to the Markowitz demands, especially in the first and third month. The magnitude of hedge demands for stocks or bonds remains similar when unsmoothed HFRIFOF composite replaces the HFRIFOF composite in the portfolio.

2.5.2 Conditional Strategy vs. Unconditional Strategy

A comparison of the conditional strategy and unconditional strategy reveals some interesting results.[7] One similarity is the patterns of investments in bonds: the allocations to bonds increase monotonically over time in all portfolios under both the unconditional strategy and conditional strategy. Compared to the unconditional strategy, the conditional strategy seems to reduce the allocations to bonds in the portfolios of stocks, bonds and the HFRIFOF composite. The conditional strategy on average allocates more to stocks in every period in the two-asset portfolio and the three-asset portfolios with the HFRIFOF composite or the unsmoothed HFRIFOF composite. This reflects the advantages of portfolio rebalancing in responses to changing market conditions. It appears that the ability to adjust portfolio weights according to changes in the state variable induces an investor to allocate more aggressively to stocks. Finally, when the conditional strategy includes hedge funds, the average allocations to hedge funds tend to be greater compared to the allocations to hedge funds under the unconditional strategy. Together with the results related to stocks above, we conclude that the investor becomes more aggressive under the conditional strategy.

2.5.3 Portfolio Efficiency

Table VIII shows portfolio performance under the conditional strategy using different hedge fund indexes as the proxies for investments in hedge funds. It also reports certainty equivalents for three-asset portfolios. We perform the Jacque-Bera normality test on all three-period portfolio excess returns, and cannot reject the null hypothesis. Hence, the mean and standard deviation of portfolio excess returns provide sufficient information on portfolio performance. Three questions arise. First of all, is the two-asset portfolio under the conditional strategy mean-variance efficient or does adding hedge funds to the portfolio improve the portfolio efficiency? Second, are portfolios under the unconditional strategy mean-variance efficient, relative to portfolios under conditional strategy? Third, what difference does a three-month hedge fund lockup period make in terms of portfolio performance?

We can use the Sharpe ratio test to determine the portfolio efficiency of the two-asset portfolio against the three-asset portfolios under the conditional strategy, and the three-asset portfolios under the unconditional strategy against the three-asset portfolios under the conditional strategy. We report p-values from four Sharpe ratio tests. The Sharpe ratios of the portfolios of stocks, bonds and the HFRIFOF composite index under the conditional strategy with a lockup and without a lockup are 1.56 and 1.75, respectively. The p-value of 0.00 to the right of the Sharpe ratio of the three-asset portfolio with a

[7] All comparisons of results in Section 5.2 are made between the conditional strategy and the unconditional strategy.

lockup period is based on the Sharpe ratio test with the two-asset portfolio being the benchmark portfolio. The p-value of 0.15 next to the Sharpe ratio of the three-asset portfolio without a lockup period is based on the Sharpe ratio test of the difference in the Sharpe ratios of two three-asset portfolios, i.e. the portfolio with a lockup period vs. the portfolio without a lockup period. The p-values of 0.08 and 0.46 in the row of 'Con vs. Unconditional', below the Sharpe ratios of the three-asset portfolios with a lockup and without a lockup, are based on the Sharpe ratio test for the unconditional strategy vs. the conditional strategy.

For all cases, the difference in the Sharpe ratios of the three-asset portfolio with a three-month lockup period and the two-asset portfolio is significant at the 5% significance level. Hence, we conclude that the two-asset portfolio of stocks and bonds is not mean-variance efficient under the conditional strategy. An investor should add hedge funds to the portfolio even though there is a lockup period of three months. Taking into account the return smoothing of hedge funds will reduce the diversification benefit of hedge funds, but not sufficient to build a case against the inclusion of hedge funds.

The two p-values below the Sharpe ratios of three-asset portfolios come from the Sharpe ratio test for the unconditional strategy vs. the conditional strategy. For the three-asset portfolios without a lockup period and using any hedge fund index (reported or unsmoothed), the Sharpe ratios of the portfolios under the conditional strategy do not differ from those of the portfolios under the unconditional strategy at the 10% significance level. Therefore, even if the market dividend-price ratio predicts returns of stocks, bonds and hedge funds, and generates allocations different from those under the unconditional strategy, the investor does not significantly benefit from the conditional strategy. However, in terms of economic significance, the difference in portfolio Sharpe ratios under the unconditional strategy and the conditional strategy is quite large. The certainty equivalents are 1.85% for the three-asset portfolio of stocks, bonds and the HFRIFOF composite with a three-month lockup period under the unconditional strategy, and 2.46% under the conditional strategy. An annualized difference of 0.61% is quite large in economic terms.

To answer the last question, we perform the Sharpe ratio test for the three-asset portfolio with a three-month lockup period vs. the three-asset portfolio without a lockup period, in order to assess the effect of a three-month hedge fund lockup on portfolio performance. When the HFRIFOF composite and four HFRIFOF strategy indexes are considered as the proxies for hedge funds, the difference in the Sharpe ratios of the three-asset portfolios with a lockup period and the three-asset portfolios without a lockup period is not statistically significant for four out of five cases. The difference is significant at the 10% level only if the HFRIFOF Conservative index is used as the hedge fund proxy in the three-asset portfolio. Although the difference in Sharpe ratios is of little statistical significance, it is still large in terms of economic significance. We can also compare the certainty equivalents for portfolios with or without a lockup. The difference is in the range from 0.81% to 1.04% when the HFRIFOF indexes are used as the hedge fund proxy. Moreover, the difference is in the range from 0.84% to 1.58% when the unsmoothed HFRIFOF indexes are used as the hedge fund proxy. Hence, the utility cost of having a hedge fund lockup period is large for the investor under the conditional strategy.

2.6 Investability

This section addresses two issues concerning investability of hedge funds. The first issue is the relation between the investment horizon and the hedge fund lockup period. The time dimension of investability of all risky assets is rather easy to understand. Given the length of the investment horizon, shorter the lockup period, more flexible the investment becomes. Or given the length of the lockup period, a longer investment horizon should increase the flexibility of investment choices. We demonstrate this in Section 2.6.1 by adding a case where the investment horizon is longer than the hedge fund lockup period. A hedge fund index such as the HFRIFOF composite is a portfolio with several hundreds of funds of funds. Although its returns may represent returns for the 'average' fund of hedge funds, it is not an investable index, at least not for small investors whose wealth is not great enough to purchase all hedge funds in the index or to replicate the index by using stratified sampling. The statistical property of a hedge fund index can be quite different from individual hedge funds due to diversification. Section 2.6.2 repeats the analysis in Section 2.6.1 using an individual fund of hedge funds instead of the HFRIFOF composite index.

2.6.1 Investment Horizon vs. Lockup Period

Table IX reports the portfolio weights and characteristics for the three-asset portfolio of stocks, bonds and hedge funds (and hedge funds with unsmoothed returns), when the hedge fund lockup periods are one, two and three months. The investment horizon is three months in all cases. We've already discussed the cases where the lockup periods are three months and one month.[8] Both results are included in Table IX to ease the comparison.

The portfolio strategy when the hedge fund lockup period is shorter than the investment horizon becomes less flexible than the portfolio strategy when there is no lockup, but still more flexible than the portfolio strategy when there is a three-month lockup period. At the beginning of the second month, the investor can make additional allocations to hedge funds and adjust allocations to stocks and bonds accordingly. The investor can't short hedge funds. Hence, the initial investments in hedge fund at the beginning of the first month have to stay for at least two months. Subsequently, at the beginning of the

[8] Since the rebalancing frequency is monthly, a one-month lockup period is the no-lockup case.

third month, the investor can decide whether to reduce allocations to hedge funds that were invested in the first period. The additional opportunity to buy and sell investments in hedge funds adds flexibility to the investment strategy and should be favored by the investor.

The implementation of the investment strategy is as follows. The investor can structure the investments in hedge funds over three months into three portfolios. In the first month, the investor can invest in two portfolios of hedge funds: one held for three months and the other held only for two months. The investor can invest in both portfolios since they are not exclusive from each other. In the second month, the investor can decide if he wants to add another two-month investment in hedge funds. In the third month, the investor will withdraw the two-period hedge fund investment made in the first period, and keep the remaining two portfolios of hedge funds until the end of the investment horizon. The investment and rebalancing of stocks and bonds follow the same approach as before. Hence, the total investment menu over three-month investment horizon when the hedge fund lockup period is two months looks like the investment menu with a three-month hedge fund lockup period, except that two additional timing portfolios are added:[9]

$$\tilde{r}^x_{t\to t+3} = \begin{pmatrix} R^f_{t+2}\tilde{r}^x_{t\to t+2} \\ R^f_t\tilde{r}^x_{t+1\to t+3} \\ \tilde{r}^x_{t\to t+3} \end{pmatrix}. \tag{2.26}$$

We denote the optimal investments in the three portfolios of hedge funds with returns in the form of (2.26) by:

$$\tilde{w}^x = \begin{pmatrix} \tilde{w}^x_{1\to 2} \\ \tilde{w}^x_{2\to 3} \\ \tilde{w}^x_{1\to 3} \end{pmatrix}. \tag{2.27}$$

The portfolio optimization still follows the same approach described in Section 2.2. And as before, the non-negative constraints are always imposed on hedge funds to reflect the liquidity nature of a hedge lockup period. After the optimization over timing portfolios of stocks, bonds and hedge funds, the allocations to hedge funds in each period can be derived as follows:

$$\tilde{w}^x_a = \begin{pmatrix} \tilde{w}^x_{1\to 2} + \tilde{w}^x_{1\to 3} \\ \tilde{w}^x_{1\to 2} + \tilde{w}^x_{2\to 3} + \tilde{w}^x_{1\to 3} \\ \tilde{w}^x_{1\to 3} + \tilde{w}^x_{2\to 3} \end{pmatrix}. \tag{2.28}$$

Table IX suggests that the main difference in the portfolio strategies when hedge fund lockup periods are one month and two months comes from the allocations in the second

[9] Here only hedge fund returns are listed. For timing portfolio returns of stocks and bonds, the structure is the same as in (2.16).

month. The investor increases the allocations to hedge funds from 71% in the first month to 104% in the second month. As the average cross-asset autocorrelations between stocks and hedge funds are positive (see Table III), the stock takes a 10% hit in the second period due to the increase in the allocation to hedge funds relative to the portfolio without a hedge fund lockup (the third column with lockup = 1). The total allocations to all risky assets in the second period amount to 142%, which is the highest among all three portfolios. The percentage is down to 97% in the third period as the investor dumps about 57% of the initial investments in hedge funds in the first period, and increases the allocation to stocks in the third period by 8%. Hence, among all three strategies, the average allocation to hedge funds is the highest when the hedge fund lockup period is two months.

The increase in investment flexibility associated with a lower hedge fund lockup period is likely to improve the portfolio efficiency relative to a full-length lockup situation. The portfolio has a mean excess return of 8.40% and a volatility of 6.16%. The portfolio Sharpe ratio is 1.37, which is higher than the portfolio Sharpe ratio for the portfolio of stocks, bonds and hedge funds with a three-month lockup. The difference is 0.16 and statistically significant. In addition, there is a statistically and economically significant difference of 0.14 in the Sharpe ratios of the portfolios with a two-month lockup and one-month lockup. The certainty equivalent for the portfolio with a two-month lockup is 81 basis points relative to the portfolio with a three-month lockup period. If there is one way to shorten the lockup (e.g. the secondary market or a private contract with a third party to shift hedge fund exposures) from three months to two months, the investor is willing to pay 81 basis points to the relevant counterparty. The certainty equivalent is 96 basis points for the portfolio with a one-month lockup relative to the portfolio with a two-month lockup.

We repeat the analysis using the unsmoothed hedge funds returns. The allocations to hedge funds are lowered in general and more compressed. When there is a two-month hedge fund lockup period, the allocations to hedge funds and risky assets are the highest in the second month. The gap in the Sharpe ratio as well as the certainty equivalent is large when the portfolios with a three-month lockup period and a two-month lockup period are compared. Going from a full-length lockup to a two-month lockup means much more to the investor than shortening the lockup from two months to one month.

2.6.2 Hedge Fund Index vs. Individual Hedge Funds

We replace the hedge fund index with an individual fund of hedge funds and repeat our analysis. While the hedge fund index represents a well diversified portfolio of funds of hedge funds, an individual fund of hedge funds itself is a diversified portfolio of single-strategy hedge funds. If the investor decides to invest only a few funds of funds due to the wealth constraint, the analysis in Section 2.4 can be applied in the same way to examine the lockup effect. We choose a fund of funds whose characteristics do not deviate too much from the mean or median characteristics of a large sample of funds of funds (For an analysis of individual funds of funds, see De Roon, Guo, ter Horst (2010)).

The fund of hedge funds chosen is managed by Optima Fund Management LP (New York), under the fund name Optima Fund. The U.S. fund of hedge funds started to operate as of January 1989 and is classified as Equity style fund of funds. At the end of the sample period, the fund has a size of USD 380 millions and a management fee of 1.5%. The fund has no lockup period, but its quarterly redemption frequency acts like a quasi-lockup period. It has a mean excess return of 7.26%, a volatility of 7.69% and a Sharpe ratio of 0.94. Hence, it slightly underperformed the HFRIFOF composite on the basis of the Sharpe ratio. Unsmoothing the fund returns leads to a lower Sharpe ratio of 0.81, which suggests that stale pricing or smoothing is present in reporting Optima fund returns.

Table X reports the results. The portfolio strategy, relative to the corresponding strategy in Table IX, favors more allocations to bonds in the second period and the third period regardless of the lockup period. In addition, assuming a two-month lockup period, the additional investment in hedge funds in the second month is only 1% while the investor reduces the exposure to hedge funds from 69% to 20% in the third month. If there is no hedge fund lockup period, there are reductions of the allocations to hedge funds in both the second month and the third month by 30%. Overall, the Optima Fund is not favored as much as the HFRIFOF composite.

The higher volatility of the Optima Fund relative to the HFRIFOF composite leads to a higher portfolio volatility even though the investor reduces the exposures to hedge funds in the portfolio. The Sharpe ratios of the three-asset portfolios are still higher than the Sharpe ratio of the portfolio of stocks and bonds. However, the gap in the Sharpe ratios between two different three-asset portfolios are not as large as those in Table IX. The portfolio with a one-month hedge fund lockup period has a Sharpe ratio of 1.32, higher than the portfolio with a two-month lockup by 0.07. The latter portfolio in turn has a higher Sharpe ratio than the portfolio with a three-month lockup by 0.10. Not surprisingly, the certainty equivalents are also smaller when Optimal Fund is used as the hedge fund proxy. Shortening the hedge fund lockup period from two months to one month only implies a certainty equivalent of 32 basis points (and 18 basis points when unsmoothed hedge fund returns are used).

2.7 Conclusion

A lockup period is a realistic feature of investments in hedge funds, private equities and venture capital. This paper investigates the impact of a hedge fund lockup period on the asset allocation decision by an investor who re-adjusts his portfolio periodically. The presence of a hedge fund lockup period creates large, negative hedge demands for stocks. We extend the framework by Brandt and Santa-Clara (2006) to illustrate the effect of a hedge fund lockup period on multi-period asset allocations and portfolio efficiency. The empirical results suggest that the investor is better off by investing in a portfolios of stocks, bonds and hedge funds, relative to a portfolio of stocks and bonds only. More importantly, the presence of a lockup period has a nontrivial impact on portfolio efficiency. An investor may overstate the benefits from adding hedge funds to the portfolio when he overlooks the existence of a hedge fund lockup period. The results are robust after adjusting for return smoothing of hedge funds and taking into account the skewness and excess kurtosis of hedge fund returns.

2.A Tables

Table I: Summary Statistics
Panel A. Descriptive Statistics

This table gives summary statistics of risky assets, the risk-free asset, and the market dividend-price ratio from January 1990 to December 2007. The value weighted NYSE index is the proxy for stocks, and the Fama Bond Portfolio (U.S. Long-term Treasuries) with maturities greater than 10 years is the proxy for bonds. For hedge funds, various indexes are considered: HFRI Fund of Funds composite index (FOF), HFRIFOF Conservative strategy index (FOFC), HFRIFOF Diversified strategy index (FOFD), HFRIFOF Market Defensive strategy index (FOFM), and HFRIFOF Strategic strategy index (FOFS). The table also includes statistics for hedge fund indexes on the basis of unsmoothed returns: FOF (un), FOFC (un), FOFD (un), FOFM (un), and FOFS (un). The U.S. 30-day Treasury bill (Tbill) is the short-term risk-free asset. The market dividend-price ratio is the sum of dividends in past twelve months divided by the current NYSE index level. Means, standard deviations, maximums and minimums are expressed in percentages. Means, standard deviations and Sharpe ratios are annualized figures, while the remaining statistics are on a monthly basis. The AC(n) are autocorrelations of monthly asset returns and the market dividend-price ratio with lags equal to n. The p-values in the table come from the Ljung-Box test for autocorrelation with lags equal to 6.

	Mean	Stdev	Sharpe	Max	Min	Skew	Kurt	AC(1)	AC(2)	AC(3)	AC(4)	AC(5)	AC(6)	*p-value*
U.S. Stocks	11.35	12.60	0.58	10.65	-14.73	-0.53	1.35	0.00	-0.03	-0.05	-0.11	0.06	0.01	0.67
U.S. Long-term Government Bonds	8.48	7.90	0.56	7.21	-8.28	-0.43	0.85	0.07	-0.12	0.02	-0.01	-0.05	-0.02	0.56
HFRIFOF Composite	9.67	5.46	1.03	6.85	-7.47	-0.28	4.05	0.29	0.09	0.00	-0.06	-0.06	0.01	0.00
HFRIFOF Conservative	8.29	3.23	1.32	3.96	-3.88	-0.51	3.14	0.29	0.15	0.03	0.03	0.02	0.02	0.00
HFRIFOF Diversified	9.09	5.81	0.87	7.73	-7.75	-0.13	4.18	0.30	0.07	-0.03	-0.11	-0.11	-0.01	0.00
HFRIFOF Market Defensive	9.45	5.82	0.93	7.38	-5.42	0.15	1.23	0.11	-0.01	0.04	-0.09	0.03	0.01	0.51
HFRIFOF Strategic	12.73	8.64	1.01	9.47	-12.11	-0.39	3.83	0.27	0.08	0.04	0.01	-0.06	0.05	0.00
HFRIFOF Composite (Unsmoothed)	9.67	7.37	0.76	7.65	-10.46	-0.34	3.80	0.00	0.01	-0.01	-0.05	-0.05	0.03	0.97
HFRIFOF Conservative (Unsmoothed)	8.28	4.37	0.97	4.53	-5.65	-0.58	3.16	-0.02	0.08	-0.02	0.01	0.02	0.00	0.96
HFRIFOF Diversified (Unsmoothed)	9.10	7.90	0.64	8.74	-10.97	-0.22	3.82	0.01	0.00	-0.03	-0.09	-0.09	0.02	0.73
HFRIFOF Market Defensive (Unsmoothed)	9.44	6.53	0.83	7.82	-6.17	0.05	1.10	0.00	-0.03	0.05	-0.10	0.04	0.01	0.75
HFRIFOF Strategic (Unsmoothed)	12.78	11.39	0.77	11.60	-16.25	-0.35	3.50	0.00	0.00	0.02	0.02	-0.08	0.06	0.89
Short-term Treasury Bill	4.04	0.49		0.68	0.06	-0.19	-0.44	0.95	0.93	0.91	0.88	0.84	0.80	0.00
Dividend-Price Ratio	2.24	0.64		4.10	1.44	0.90	-0.04	0.98	0.96	0.94	0.92	0.91	0.89	0.00

Panel B. Correlation Matrix

This table displays the correlation matrix of risky asset returns, the lagged state variable, and the risk-free return from January 1990 to December 2007. The value weighted NYSE index is the proxy for stocks, and the Fama Bond Portfolio (U.S. Long-term Treasuries) with maturities greater than 10 years is the proxy for bonds. For hedge funds, various indexes are considered: HFRI Fund of Funds composite index (FOF), HFRIFOF Conservative strategy index (FOFC), HFRIFOF Diversified strategy index (FOFD), HFRIFOF Market Defensive strategy index (FOFM), and HFRIFOF Strategic strategy index (FOFS). The matrix includes hedge fund indexes on the basis of unsmoothed returns: FOF (un), FOFC (un), FOFD (un), FOFM (un), and FOFS (un). The U.S. 30-day Treasury bill (Tbill) is the short-term risk-free asset. The market dividend-price ratio (DP) is the sum of dividends in past twelve months divided by the current NYSE index level.

	Stock	Bonds	FOF	FOFC	FOFD	FOFM	FOFS	FOF(un)	FOFC(un)	FOFD(un)	FOFM(un)	FOFS(un)	Tbill	DP
Stock	1.00													
Bonds	0.05	1.00												
FOF	0.43	0.02	1.00											
FOFC	0.44	0.05	0.89	1.00										
FOFD	0.43	0.00	0.97	0.84	1.00									
FOFM	0.04	0.11	0.69	0.62	0.63	1.00								
FOFS	0.48	0.01	0.93	0.82	0.87	0.55	1.00							
FOF(un)	0.49	0.05	0.96	0.83	0.93	0.68	0.89	1.00						
FOFC(un)	0.50	0.06	0.86	0.96	0.79	0.62	0.80	0.87	1.00					
FOFD(un)	0.49	0.03	0.93	0.77	0.95	0.61	0.83	0.97	0.80	1.00				
FOFM(un)	0.05	0.11	0.68	0.61	0.61	0.99	0.54	0.69	0.62	0.61	1.00			
FOFS(un)	0.51	0.04	0.89	0.77	0.82	0.54	0.96	0.92	0.81	0.85	0.55	1.00		
Tbill	0.04	0.06	0.08	0.15	0.06	0.17	0.08	0.05	0.10	0.04	0.15	0.06	1.00	
DP	0.08	0.05	0.12	0.13	0.10	0.07	0.19	0.10	0.11	0.09	0.06	0.16	0.34	1.00

Table II: Optimal Portfolios under the Unconditional Strategy
(Investment Horizon: Three months)

This table reports the optimal portfolios under the unconditional strategy (rescaled such that the first-month portfolios are tangency portfolios). The investment horizon is three months. Column 2 to 3 report the portfolio weights and their t-statistics (in brackets) of the portfolio of stocks and bonds. Column 4 to 7 show the portfolio weights and their t-statistics (in brackets) of the portfolio of stocks, bonds and the HFRIFOF composite index (FOF). Column 8 to 11 report the portfolio weights and their t-statistics (in brackets) of the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite index, FOF (un). The lockup period is three months, if applicable. The results in panel A and B are based on the qudratic utility (mean-second moment utility) and the fourth order approximation of the power utility, respectively.

	Stocks and Bonds		Stocks, Bonds and FOF				Stocks, Bonds and FOF(un)			
			with a lockup		without a lockup		with a lockup		without a lockup	
Column	2	3	4	5	6	7	8	9	10	11
A. Quadratic Utility										
Stocks										
Month 1	0.52	(1.70)	0.09	(0.40)	0.01	(0.10)	0.14	(0.59)	0.08	(0.43)
Month 2	0.50	(1.58)	0.14	(0.69)	0.16	(0.90)	0.18	(0.79)	0.16	(0.67)
Month 3	0.37	(1.10)	0.12	(0.59)	0.13	(0.77)	0.12	(0.51)	0.16	(0.70)
Bonds										
Month 1	0.48	(0.88)	0.29	(0.96)	0.24	(1.01)	0.33	(0.93)	0.34	(1.05)
Month 2	0.65	(1.27)	0.37	(1.33)	0.28	(1.46)	0.43	(1.31)	0.37	(1.37)
Month 3	0.85	(1.77)	0.48	(1.81)	0.35	(1.76)	0.54	(1.72)	0.45	(1.62)
Hedge Funds										
Month 1			0.62	(3.00)	0.76	(2.91)	0.53	(2.33)	0.58	(2.02)
Month 2					0.36	(1.44)			0.47	(1.59)
Month 3					0.22	(1.05)			0.24	(0.97)
B. Fourth Order Approximation										
Stocks										
Month 1	0.53	(1.72)	0.11	(0.57)	0.02	(0.21)	0.19	(0.83)	0.11	(0.66)
Month 2	0.49	(1.63)	0.12	(0.61)	0.15	(1.07)	0.17	(0.79)	0.15	(0.68)
Month 3	0.37	(1.23)	0.14	(0.78)	0.14	(0.94)	0.14	(0.66)	0.18	(0.88)
Bonds										
Month 1	0.47	(0.91)	0.30	(1.00)	0.23	(1.12)	0.33	(0.99)	0.36	(1.26)
Month 2	0.62	(1.32)	0.34	(1.34)	0.27	(1.53)	0.40	(1.30)	0.36	(1.39)
Month 3	0.85	(1.85)	0.49	(2.01)	0.34	(1.97)	0.53	(1.87)	0.43	(1.78)
Hedge Funds										
Month 1			0.59	(3.09)	0.75	(3.30)	0.48	(2.33)	0.53	(2.21)
Month 2					0.34	(1.55)			0.42	(1.77)
Month 3					0.19	(1.10)			0.20	(0.90)

Table III: Timing Portfolios

This table describes the characteristics of timing portfolio returns of stocks (St), bonds (Bt) and hedge funds (Ft), where $t = 1, 2, 3$. The three-month excess return of locked up hedge funds is denoted as $F_{1=>3}$. The unsmoothed timing portfolio returns of hedge funds are listed as ($FUNt$) and the unsmoothed three-month excess return of locked up hedge funds is denoted as $FUN_{1=>3}$ in the table.

	S1	S2	S3	B1	B2	B3	F1	F2	F3	F1=>3	FUN1	FUN2	FUN3	FUN1=>3
A. Return Characteristics														
Mean	7.74	7.89	7.82	4.32	4.77	4.77	5.82	5.75	5.68	5.84	5.90	5.72	5.65	5.78
Stdev	12.70	12.59	12.60	7.91	7.90	7.90	5.45	5.47	5.47	6.64	7.29	7.34	7.34	7.50
Sharpe Ratio	0.61	0.63	0.62	0.55	0.60	0.60	1.07	1.05	1.04	0.88	0.81	0.78	0.77	0.77
B. Correlation Matrix														
S1	1.00													
S2	-0.01	1.00												
S3	-0.03	0.00	1.00											
B1	0.06	0.03	0.11	1.00										
B2	-0.10	0.03	0.03	0.07	1.00									
B3	-0.07	-0.11	0.03	-0.12	0.06	1.00								
F1	0.43	-0.13	-0.14	0.03	-0.09	-0.09	1.00							
F2	0.15	0.43	-0.13	-0.08	0.02	-0.09	0.30	1.00						
F3	0.13	0.15	0.43	0.01	-0.08	0.02	0.09	0.30	1.00					
F1=>3	0.33	0.22	0.08	-0.02	-0.07	-0.07	0.66	0.76	0.67	1.00				
FUN1	0.49	-0.10	-0.12	0.06	-0.07	-0.08	0.96	0.29	0.11	0.64	1.00			
FUN2	0.02	0.49	-0.09	-0.09	0.05	-0.07	0.01	0.96	0.29	0.60	0.01	1.00		
FUN3	0.09	0.03	0.49	0.03	-0.09	0.05	0.00	0.02	0.96	0.47	0.02	0.01	1.00	
FUN1=>3	0.33	0.24	0.16	0.00	-0.06	-0.05	0.54	0.71	0.78	0.97	0.58	0.58	0.60	1.00

Table IV: Demand Decomposition under the Unconditional Strategy
(Investment Horizon: Three months)

This table displays, for the three-asset portfolios (rescaled such that the first-month portfolios are tangency portfolios) under the unconditional strategy with a three-month hedge fund lockup period, the decomposition of the total demand for stocks and bonds into two parts: a Markowitz demand or speculative demand and a hedge demand. The t-statistics of portfolio weights are in brackets. The top and bottom half report the decomposition results for the portfolio of stocks, bonds and the HFRIFOF composite index (FOF) and the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite index (FOF (un)), respectively.

	Markowitz Demand		Hedge Demand		Total Demand	
A. Stocks, Bonds and FOF						
Stocks						
Month 1	0.27	(1.70)	-0.18	(2.91)	0.09	(0.40)
Month 2	0.26	(1.58)	-0.12	(1.94)	0.14	(0.69)
Month 3	0.19	(1.10)	-0.07	(1.06)	0.12	(0.59)
Bonds						
Month 1	0.25	(0.88)	0.05	(0.64)	0.29	(0.96)
Month 2	0.33	(1.27)	0.03	(0.88)	0.37	(1.33)
Month 3	0.44	(1.77)	0.04	(0.43)	0.48	(1.81)
B. Stocks, Bonds and FOF(un)						
Stocks						
Month 1	0.32	(1.70)	-0.18	(3.08)	0.14	(0.59)
Month 2	0.31	(1.58)	-0.13	(2.22)	0.18	(0.79)
Month 3	0.23	(1.10)	-0.11	(1.72)	0.12	(0.51)
Bonds						
Month 1	0.30	(0.88)	0.03	(0.59)	0.33	(0.93)
Month 2	0.40	(1.27)	0.03	(1.05)	0.43	(1.31)
Month 3	0.53	(1.77)	0.01	(0.39)	0.54	(1.72)

Table V: Portfolio Performance under the Unconditional Strategy
(Investment Horizon: Three months)

This table reports performance of various portfolios under the unconditional strategy (rescaled such that the first-month portfolios are tangency portfolios). There are ten hedge fund indexes that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report the p-values (in brackets) of Sharpe ratio tests. The benchmark portfolio for the three-asset portfolio with a lockup is the two-asset portfolio, and the three-asset portfolio with a lockup is the benchmark portfolio for the three-asset portfolio with no lockup. The certainty equivalent for the three-asset portfolio with a lockup is calculated as the difference in the utilities of the three-asset portfolio with a lockup and the two-asset portfolio, for the investor with a mean-second moment utility function (or four-order approximation of the power utility in Panel B) and a risk aversion of 10. For the same investor, the certainty equivalent for the three-asset portfolio without a lockup is the difference in utilities of the three-asset portfolio without a lockup and the three-asset portfolio with a lockup (this is the utility cost of the lockup).

Panel A. Portfolio Characteristics (quadratic utility)

	Two-Asset Portfolio	Three-Asset with Lockup		Three-Asset NO Lockup		Three-Asset with Lockup		Three-Asset NO Lockup	
A.	*Stocks and Bonds*	*Stocks, Bonds and FOF*				*Stocks, Bonds and FOF(un)*			
Mean excess return	7.05%	6.58%		7.11%		6.53%		8.95%	
Std return	7.83%	5.44%		4.69%		5.94%		6.31%	
Sharpe ratio	0.90	1.21	(0.00)	1.51	(0.00)	1.10	(0.01)	1.42	(0.00)
p-value (normality)	(0.66)	(0.80)		(0.81)		(0.73)		(0.70)	
Certainty equivalent		1.85%		1.78%		1.18%		1.90%	
B.	*Stocks and Bonds*	*Stocks, Bonds and FOFC*				*Stocks, Bonds and FOFC(un)*			
Mean excess return	7.05%	5.12%		5.29%		5.15%		7.13%	
Std return	7.83%	3.72%		3.30%		4.17%		4.72%	
Sharpe ratio	0.90	1.37	(0.00)	1.60	(0.02)	1.23	(0.00)	1.51	(0.01)
p-value (normality)	(0.66)	(0.98)		(0.78)		(0.98)		(0.75)	
Certainty equivalent		2.81%		1.30%		1.97%		1.65%	
C.	*Stocks and Bonds*	*Stocks, Bonds and FOFD*				*Stocks, Bonds and FOFD(un)*			
Mean excess return	7.05%	6.31%		6.33%		6.33%		8.08%	
Std return	7.83%	5.53%		4.64%		6.08%		6.39%	
Sharpe ratio	0.90	1.14	(0.01)	1.36	(0.02)	1.04	(0.04)	1.26	(0.03)
p-value (normality)	(0.66)	(0.74)		(0.83)		(0.68)		(0.74)	
Certainty equivalent		1.44%		1.36%		0.83%		1.38%	
D.	*Stocks and Bonds*	*Stocks, Bonds and FOFM*				*Stocks, Bonds and FOFM(un)*			
Mean excess return	7.05%	6.67%		7.42%		6.68%		7.79%	
Std return	7.83%	5.13%		4.88%		5.30%		5.22%	
Sharpe ratio	0.90	1.30	(0.00)	1.52	(0.02)	1.26	(0.00)	1.49	(0.02)
p-value (normality)	(0.66)	(0.71)		(0.86)		(0.72)		(0.85)	
Certainty equivalent		2.34%		1.30%		2.10%		1.37%	
E.	*Stocks and Bonds*	*Stocks, Bonds and FOFS*				*Stocks, Bonds and FOFS(un)*			
Mean excess return	7.05%	8.11%		9.16%		7.72%		10.54%	
Std return	7.83%	6.84%		6.45%		7.07%		7.89%	
Sharpe ratio	0.90	1.19	(0.00)	1.42	(0.02)	1.09	(0.02)	1.33	(0.02)
p-value (normality)	(0.66)	(0.79)		(0.70)		(0.88)		(0.80)	
Certainty equivalent		1.70%		1.35%		1.14%		1.41%	

Panel B. Portfolio Characteristics (fourth-order approximation of the power utility)

	Two-Asset Portfolio	Three-Asset with Lockup		Three-Asset NO Lockup		Three-Asset with Lockup		Three-Asset NO Lockup	
A.	*Stocks and Bonds*	*Stocks, Bonds and FOF*				*Stocks, Bonds and FOF(un)*			
Mean excess return	6.96%	6.44%		7.12%		6.30%		8.83%	
Std return	7.73%	5.35%		4.72%		5.75%		6.26%	
Sharpe ratio	0.90	1.20	(0.00)	1.51	(0.00)	1.09	(0.02)	1.41	(0.00)
p-value (normality)	(0.62)	(0.68)		(0.80)		(0.69)		(0.78)	
Certainty equivalent		2.37%		2.76%		1.47%		2.83%	
B.	*Stocks and Bonds*	*Stocks, Bonds and FOFC*				*Stocks, Bonds and FOFC(un)*			
Mean excess return	6.96%	5.17%		5.53%		5.14%		7.56%	
Std return	7.73%	3.78%		3.47%		4.19%		5.03%	
Sharpe ratio	0.90	1.37	(0.00)	1.59	(0.02)	1.22	(0.00)	1.50	(0.01)
p-value (normality)	(0.62)	(0.89)		(0.80)		(0.92)		(0.83)	
Certainty equivalent		3.56%		2.16%		2.36%		2.69%	
C.	*Stocks and Bonds*	*Stocks, Bonds and FOFD*				*Stocks, Bonds and FOFD(un)*			
Mean excess return	6.96%	6.18%		6.40%		6.16%		8.11%	
Std return	7.73%	5.44%		4.71%		5.93%		6.44%	
Sharpe ratio	0.90	1.14	(0.01)	1.36	(0.02)	1.04	(0.04)	1.26	(0.03)
p-value (normality)	(0.62)	(0.59)		(0.80)		(0.60)		(0.80)	
Certainty equivalent		1.80%		2.11%		1.03%		1.99%	
D.	*Stocks and Bonds*	*Stocks, Bonds and FOFM*				*Stocks, Bonds and FOFM(un)*			
Mean excess return	6.96%	6.51%		7.54%		6.49%		7.95%	
Std return	7.73%	5.03%		4.97%		5.18%		5.35%	
Sharpe ratio	0.90	1.29	(0.00)	1.51	(0.02)	1.25	(0.00)	1.48	(0.02)
p-value (normality)	(0.62)	(0.84)		(0.91)		(0.85)		(0.91)	
Certainty equivalent		3.05%		2.24%		2.72%		2.34%	
E.	*Stocks and Bonds*	*Stocks, Bonds and FOFS*				*Stocks, Bonds and FOFS(un)*			
Mean excess return	6.96%	7.85%		9.06%		7.30%		10.24%	
Std return	7.73%	6.64%		6.42%		6.70%		7.71%	
Sharpe ratio	0.90	1.18	(0.00)	1.41	(0.02)	1.09	(0.02)	1.33	(0.02)
p-value (normality)	(0.62)	(0.81)		(0.76)		(0.84)		(0.86)	
Certainty equivalent		2.27%		2.04%		1.50%		2.06%	

Table VI: Optimal Portfolios under the Conditional Strategy
(Investment Horizon: Three months; State Variable: Market Dividend-Price Ratio)

This table reports the portfolio policies under the conditional strategy (rescaled such that the first-month portfolios are tangency portfolios). Column 3 to 4 report the intercepts and coefficients of the state variable for the portfolio of stocks and bonds. Column 5 to 8 show the intercepts and coefficients of the state variable for the portfolios of stocks, bonds and the HFRIFOF composite with and without a hedge fund lockup period. Column 9 to 12 show the intercepts and the coefficients of the state variable for the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite with and without a hedge fund lockup period. The t-statistics of intercepts and coefficients are in brackets.

		Stocks and Bonds		Stocks, Bonds and FOF				Stocks, Bonds and FOF(un)			
				with a lockup		without a lockup		with a lockup		without a lockup	
Column		3	4	5	6	7	8	9	10	11	12
Stocks											
Month 1	Constant	0.53	(1.37)	0.12	(0.51)	0.02	(0.76)	0.17	(0.59)	0.10	(0.99)
	DP ratio	-0.06	(1.00)	0.04	(0.76)	0.05	(0.61)	0.01	(0.77)	0.02	(0.42)
Month 2	Constant	0.59	(1.49)	0.24	(0.99)	0.25	(1.34)	0.30	(1.02)	0.30	(1.18)
	DP ratio	0.16	(1.31)	0.22	(1.00)	0.09	(0.50)	0.24	(1.05)	0.13	(0.60)
Month 3	Constant	0.50	(1.20)	0.25	(1.03)	0.21	(1.27)	0.27	(0.97)	0.27	(1.24)
	DP ratio	0.20	(1.16)	0.25	(1.13)	0.12	(0.72)	0.27	(1.28)	0.15	(0.83)
Bonds											
Month 1	Constant	0.47	(0.77)	0.11	(0.29)	0.08	(0.28)	0.11	(0.27)	0.15	(0.37)
	DP ratio	-0.25	(1.28)	-0.36	(0.79)	-0.18	(0.54)	-0.40	(0.77)	-0.28	(0.55)
Month 2	Constant	0.53	(0.86)	0.19	(0.56)	0.17	(0.67)	0.24	(0.58)	0.24	(0.63)
	DP ratio	-0.24	(1.40)	-0.25	(0.51)	-0.22	(0.59)	-0.23	(0.46)	-0.26	(0.47)
Month 3	Constant	0.65	(1.09)	0.22	(0.62)	0.19	(0.72)	0.24	(0.56)	0.24	(0.60)
	DP ratio	-0.29	(1.11)	-0.36	(0.77)	-0.25	(0.70)	-0.34	(0.64)	-0.31	(0.59)
Hedge Funds											
Month 1	Constant			0.77	(2.99)	0.90	(1.94)	0.72	(2.50)	0.75	(1.55)
	DP ratio			0.44	(1.68)	0.24	(0.48)	0.44	(1.59)	0.31	(0.63)
Month 2	Constant					0.30	(1.15)			0.52	(1.16)
	DP ratio					0.23	(0.42)			0.44	(0.79)
Month 3	Constant					0.43	(1.39)			0.53	(1.46)
	DP ratio					0.41	(0.77)			0.57	(1.01)

Table VII: Demand Decomposition under the Conditional Strategy
(Investment Horizon: Three months; State Variable: Market Dividend-Price Ratio)

This table displays, for the three-asset portfolios (rescaled such that the first-month portfolios are tangency portfolios) under the conditional strategy with a three-month hedge fund lockup period, the decomposition of the total demand for stocks and bonds into two parts: a Markowitz demand or speculative demand and a hedge demand. The t-statistics of portfolio weights are in brackets. The top and bottom half report the decomposition results for the portfolio of stocks, bonds and the HFRIFOF composite index (FOF) and the portfolio of stocks, bonds and the unsmoothed HFRIFOF composite index (FOF (un)), respectively.

	Markowitz Demand		Hedge Demand		Total Demand	
A. Stocks, Bonds and FOF						
Stocks						
Month 1	0.25	(1.37)	-0.13	(2.26)	0.12	(0.51)
Month 2	0.28	(1.49)	-0.04	(1.41)	0.24	(0.99)
Month 3	0.24	(1.20)	0.01	(0.63)	0.25	(1.03)
Bonds						
Month 1	0.23	(0.77)	-0.12	(0.55)	0.11	(0.29)
Month 2	0.26	(0.86)	-0.06	(0.71)	0.19	(0.56)
Month 3	0.32	(1.09)	-0.10	(0.39)	0.22	(0.62)
B. Stocks, Bonds and FOF(un)						
Stocks						
Month 1	0.31	(1.37)	-0.14	(2.43)	0.17	(0.59)
Month 2	0.35	(1.49)	-0.05	(1.67)	0.30	(1.02)
Month 3	0.30	(1.20)	-0.02	(1.24)	0.27	(0.97)
Bonds						
Month 1	0.28	(0.77)	-0.16	(0.50)	0.11	(0.27)
Month 2	0.31	(0.86)	-0.07	(0.94)	0.24	(0.58)
Month 3	0.38	(1.09)	-0.14	(0.51)	0.24	(0.56)

Table VIII: Portfolio Performance under the Conditional Strategy
(Investment Horizon: Three months; State Variable: Market Dividend-Price Ratio)

This table reports performance of portfolios under the conditional strategy (rescaled such that the first-month portfolios are tangency portfolios). There are ten hedge fund indexes that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report the p-values (in brackets) of Sharpe ratio tests. For each three-asset portfolio, two p-values are reported using different benchmark portfolios. In the 'Sharpe ratio' row, the benchmark portfolio for the three-asset portfolio with a lockup is the two-asset portfolio, and the three-asset portfolio with a lockup is the benchmark portfolio for the three-asset portfolio with no lockup. In the 'Con vs. Unconditional' row, the test portfolio and the benchmark portfolio are the conditional portfolio and the corresponding unconditional portfolio. The certainty equivalent follows the same definition as in Table V.

	Two-Asset Portfolio	Three-Asset with Lockup		Three-Asset NO Lockup		Three-Asset with Lockup		Three-Asset NO Lockup	
A.	*Stocks and Bonds*	*Stocks, Bonds and FOF*				*Stocks, Bonds and FOF(un)*			
Mean excess return	10.00%	8.93%		7.53%		9.44%		10.55%	
Std return	8.76%	5.71%		4.31%		6.49%		6.35%	
Sharpe ratio	1.14	1.56	(0.00)	1.75	(0.15)	1.45	(0.00)	1.65	(0.11)
p-value (normality)	(0.88)	(0.40)		(0.75)		(0.64)		(0.75)	
Con vs. Unconditional		(0.08)		(0.46)		(0.09)		(0.45)	
Certainty equivalent		2.46%		1.01%		1.82%		1.15%	
B.	*Stocks and Bonds*	*Stocks, Bonds and FOFC*				*Stocks, Bonds and FOFC(un)*			
Mean excess return	10.00%	6.19%		6.47%		6.54%		9.63%	
Std return	8.76%	3.65%		3.42%		4.16%		5.18%	
Sharpe ratio	1.14	1.69	(0.00)	1.89	(0.11)	1.57	(0.00)	1.86	(0.03)
p-value (normality)	(0.88)	(0.60)		(0.72)		(0.64)		(0.82)	
Con vs. Unconditional		(0.10)		(0.27)		(0.09)		(0.15)	
Certainty equivalent		3.16%		1.04%		2.46%		1.58%	
C.	*Stocks and Bonds*	*Stocks, Bonds and FOFD*				*Stocks, Bonds and FOFD(un)*			
Mean excess return	10.00%	8.47%		6.84%		9.01%		10.15%	
Std return	8.76%	5.82%		4.20%		6.65%		6.61%	
Sharpe ratio	1.14	1.45	(0.00)	1.62	(0.18)	1.35	(0.03)	1.52	(0.18)
p-value (normality)	(0.88)	(0.55)		(0.80)		(0.56)		(0.65)	
Con vs. Unconditional		(0.14)		(0.38)		(0.16)		(0.39)	
Certainty equivalent		1.85%		0.99%		1.24%		1.03%	
D.	*Stocks and Bonds*	*Stocks, Bonds and FOFM*				*Stocks, Bonds and FOFM(un)*			
Mean excess return	10.00%	8.50%		7.89%		8.66%		8.54%	
Std return	8.76%	5.33%		4.51%		5.56%		4.99%	
Sharpe ratio	1.14	1.59	(0.00)	1.75	(0.22)	1.55	(0.00)	1.71	(0.23)
p-value (normality)	(0.88)	(0.62)		(0.81)		(0.66)		(0.79)	
Con vs. Unconditional		(0.15)		(0.47)		(0.15)		(0.52)	
Certainty equivalent		2.62%		0.84%		2.41%		0.84%	
E.	*Stocks and Bonds*	*Stocks, Bonds and FOFS*				*Stocks, Bonds and FOFS(un)*			
Mean excess return	10.00%	11.89%		10.89%		12.25%		12.92%	
Std return	8.76%	7.80%		6.51%		8.45%		8.05%	
Sharpe ratio	1.14	1.52	(0.00)	1.67	(0.25)	1.45	(0.00)	1.60	(0.23)
p-value (normality)	(0.88)	(0.55)		(0.52)		(0.75)		(0.65)	
Con vs. Unconditional		(0.09)		(0.40)		(0.09)		(0.36)	
Certainty equivalent		2.23%		0.81%		1.79%		0.86%	

Table IX: Investment Horizon vs. Lockup Period

This table reports the optimal portfolios under the unconditional strategy (rescaled such that the first-month portfolios are tangency portfolios). The investment horizon is three months. Panel A reports portfolio strategies (t-statistics in brackets) for the portfolio of stocks, bonds and hedge funds (or hedge funds with unsmoothed returns, FOF (un)) when the hedge fund lockup periods are one, two and three months. Panel B describes characteristics of each portfolio. The Sharpe ratio tests use the portfolio to the left of the test portfolio as the benchmark portfolio when the lockup is shorter than three months. The two-asset portfolio in Table V is the benchmark portfolio against which the portfolio in Table IX with a three-month lockup period is tested. The certainty equivalents of the portfolios when the lockup is shorter than three months are calculated as the utility derived from portfolio x minus the utility derived from the portfolio to the left of the portfolio x. When the lockup is three months, the certainty equivalent is the difference in utilities between the three-asset portfolio in Table IX and the two-asset portfolio in Table V.

	Stocks, Bonds, FOF						Stocks, Bonds, FOF(un)					
	Lockup=3		Lockup=2		Lockup=1		Lockup=3		Lockup=2		Lockup=1	
A. Portfolio Strategy												
Stocks												
Month 1	0.09	(0.40)	0.00	(0.13)	0.01	(0.10)	0.14	(0.59)	0.12	(0.59)	0.08	(0.43)
Month 2	0.14	(0.69)	0.05	(0.21)	0.16	(0.90)	0.18	(0.79)	0.01	(0.05)	0.16	(0.67)
Month 3	0.12	(0.59)	0.13	(0.60)	0.13	(0.77)	0.12	(0.51)	0.20	(0.88)	0.16	(0.70)
Bonds												
Month 1	0.29	(0.96)	0.29	(1.00)	0.24	(1.01)	0.33	(0.93)	0.42	(1.28)	0.34	(1.05)
Month 2	0.37	(1.33)	0.33	(1.32)	0.28	(1.46)	0.43	(1.31)	0.37	(1.40)	0.37	(1.37)
Month 3	0.48	(1.81)	0.36	(1.58)	0.35	(1.76)	0.54	(1.72)	0.42	(1.56)	0.45	(1.62)
Hedge Funds												
Month 1	0.62	(3.00)	0.71	(1.15)	0.76	(2.91)	0.53	(2.33)	0.46	(1.20)	0.58	(2.02)
Month 2			1.04	(1.73)	0.36	(1.44)			0.76	(1.80)	0.47	(1.59)
Month 3			0.47	(0.74)	0.22	(1.05)			0.30	(0.35)	0.24	(0.97)
B. Portfolio Characteristics												
Mean excess return	6.58%		8.40%		7.11%		6.53%		8.58%		8.95%	
Std return	5.44%		6.16%		4.69%		5.94%		6.34%		6.31%	
Sharpe ratio	1.21	(0.00)	1.37	(0.07)	1.51	(0.02)	1.10	(0.01)	1.36	(0.01)	1.42	(0.10)
p-value (normality)	(0.80)		(0.99)		(0.81)		(0.73)		(0.55)		(0.70)	
Certainty equivalent	1.85%		0.81%		0.96%		1.18%		1.52%		0.38%	

Table X: Hedge Fund Index vs. Individual Hedge Fund

This table reports the optimal portfolios under the unconditional strategy (rescaled such that the first-month portfolios are tangency portfolios). The investment horizon is three months. Panel A reports portfolio strategies (t-statistics in brackets) for the portfolio of stocks, bonds and Optima fund of funds (or Optima fund of funds with unsmoothed returns, FOF (un)) when the hedge fund lockup periods are one, two and three months. Panel B describes characteristics of each portfolio. The Sharpe ratio tests use the portfolio to the left of the test portfolio as the benchmark portfolio when the lockup is shorter than three months. The two-asset portfolio in Table V is the benchmark portfolio against which the portfolio in Table X with a three-month lockup period is tested. The certainty equivalents of the portfolios when the lockup is shorter than three months are calculated as the utility derived from portfolio x minus the utility derived from the portfolio to the left of the portfolio x. When the lockup is three months, the certainty equivalent is the difference in utilities between the three-asset portfolio in Table X and the two-asset portfolio in Table V.

	Stocks, Bonds, FOF						Stocks, Bonds, FOF(un)					
	Lockup=3		Lockup=2		Lockup=1		Lockup=3		Lockup=2		Lockup=1	
A. Portfolio Strategy												
Stocks												
Month 1	0.09	(0.33)	0.10	(0.47)	0.06	(0.34)	0.14	(0.48)	0.21	(0.79)	0.15	(0.64)
Month 2	0.11	(0.48)	0.08	(0.15)	0.18	(0.63)	0.16	(0.63)	0.12	(0.27)	0.19	(0.57)
Month 3	0.06	(0.20)	0.28	(0.81)	0.20	(0.68)	0.06	(0.20)	0.29	(0.73)	0.23	(0.62)
Bonds												
Month 1	0.32	(0.79)	0.22	(0.56)	0.19	(0.54)	0.34	(0.78)	0.28	(0.63)	0.24	(0.58)
Month 2	0.44	(1.21)	0.40	(1.21)	0.38	(1.27)	0.47	(1.22)	0.44	(1.19)	0.44	(1.24)
Month 3	0.61	(1.80)	0.53	(1.63)	0.50	(1.83)	0.63	(1.75)	0.58	(1.62)	0.56	(1.71)
Hedge Funds												
Month 1	0.60	(3.11)	0.68	(1.03)	0.75	(2.20)	0.53	(2.66)	0.51	(1.02)	0.61	(1.75)
Month 2			0.69	(1.19)	0.42	(1.40)			0.62	(1.37)	0.46	(1.57)
Month 3			0.20	(0.45)	0.11	(0.29)			0.14	(0.45)	0.12	(0.37)
B. Portfolio Characteristics												
Mean excess return	7.77%		8.48%		8.08%		7.59%		8.85%		8.77%	
Std return	6.78%		6.79%		6.12%		6.95%		7.30%		6.98%	
Sharpe ratio	1.15	(0.01)	1.25	(0.15)	1.32	(0.13)	1.09	(0.02)	1.21	(0.12)	1.25	(0.24)
p-value (normality)	(0.91)		(1.00)		(1.00)		(0.95)		(1.00)		(1.00)	
Certainty equivalent	1.40%		0.68%		0.32%		1.07%		0.77%		0.18%	

Chapter 3

A Random Walk by Fund of Funds Managers?

Abstract

This paper investigates whether hedge fund of funds managers invest in single-strategy hedge funds in a random fashion. By examining the underlying single-strategy hedge funds from which a fund of funds can select, we find that single-strategy hedge funds added to the portfolio of funds of funds display some distinct characteristics: (1) they tend to be larger in size, have longer operational history and are more likely to operate as offshore hedge funds; (2) they have higher incentive fees and are more likely to have a high watermark clause; (3) they tend to have greater minimum initial investment requirements and a higher proportion of closed funds; (4) their managers have more years of investment experience and (5) they have a much higher risk-adjusted return or ex post alpha, a lower tracking error and a higher information ratio, in both the short run and long run. More importantly, a probit analysis of fund inclusion confirms the importance of several fund features and manager characteristics mentioned above, with the ex post alpha and the tracking error being the key factors to the inclusion decision. In addition, there is a concave relation between the probability of fund inclusion and the minimum initial investment such that the probability increases with the minimum initial investment at a decreasing rate. Finally, the importance of those characteristics to the inclusion decision varies by fund styles.

3.1 Introduction

A fund of hedge funds or fund of funds (FOF) is a hedge fund that invests in a multiple of single-strategy or single-manager hedge funds. It is often argued that funds of funds tend to under-perform the underlying single-strategy hedge fund universe, in terms of risk-adjusted returns net of fees. The reasons can be: (1) double fees reduce reported returns of funds of funds and (2) less survivorship bias in funds of funds data (Fung and Hsieh (2000), Brown, Goetzmann, and Liang (2004)). One is tempted to conclude that investing through funds of funds is inferior to directly holding a portfolio of single-strategy hedge funds. However, despite the apparent under-performance reported, funds of funds grow at an exceptional pace. In 2004, a total of 1159 funds of funds are included in the combined database of HFR, CISDM and TASS (Fung, Hsieh, Naik and Ramadorai (2008)). In 2008, there are over 2700 funds of funds in the HFR database alone. Moreover, assets under management by funds of funds have been growing at a much higher rate than the global hedge fund industry. By the second quarter of 2008, the assets under management of the global hedge fund industry have reached \$1.9 trillion (Source: Hedge Fund Research Inc.), and around 45% of that is invested in funds of funds. According to HFR, the compound annual growth rate of assets under management by funds of funds is 32.4% from 2000 to 2008, while the growth rate is 18.5% for the global hedge funds during the same period. Figure I shows the rising importance of funds of funds in the global hedge fund industry. Such rapid growth in funds of funds reflects on the one hand the fact that there is increasing availability of funds of funds open to retail investors, and on the other hand the attractiveness of funds of funds to the institutional investors and high-net-worth investors who want to hold a diversified portfolio of hedge funds.

This paper aims to investigate characteristics and performance of hedge funds. Specifically, the question of interest is whether there are systematic differences between hedge funds with fund of funds ownership and hedge funds without fund of funds ownership. If every fund of fund manager just randomly picks a portfolio of hedge funds from the hedge fund universe, differences in fund features and performance between hedge funds selected and hedge funds not selected should be small. However, FOF investors expect FOF managers to perform necessary due diligence, an area in which FOF managers claim to have expertise and resources. The first step of the due diligence process is to carefully select hedge funds with a consistent performance record, sound investment process and philosophy, adequate resources and staff, and strong risk management. After a fund is included in an FOF portfolio, due diligence by FOF managers involves monitoring the underlying hedge fund operation and removing any fund that has performed poorly and is expected to deliver disappointing performance in the future.

Our paper contributes to a growing body of hedge fund literature. It is the first paper, to our knowledge, that examines differences in fund characteristics and performance between hedge funds with FOF ownership and hedge funds without FOF ownership. Our results will shed light on investment decision-making by FOF managers. This paper screens out important factors relevant to fund inclusion decision by FOF managers. An FOF manager may select hedge funds on the basis of historical performance, fund size or operational history, and liquidity restrictions of hedge funds. In addition, he or she may apply a different set of selection criteria across different styles of hedge funds. Our analysis also covers the relation between manager characters and the probability of fund inclusion. Given the private nature of the hedge fund industry, finding a trustworthy hedge fund manager is the keystone to the success of a fund of funds.

Our empirical results show that there are significant differences in several fund characteristics between hedge funds with FOF ownership (FOF-Held funds) and hedge funds without any FOF ownership (NO-FOF-Held funds). On average, FOF-Held funds are larger in size, have a longer operational history or age, have a higher proportion of offshore hedge funds, and exhibit lower portfolio turnover. The mean fund sizes of FOF-Held funds and NO-FOF-Held funds are \$324.54 million and \$79.42 million, respectively. The difference in age is about 1.5 years between the two groups of hedge funds. The proportions of hedge funds having an auditor and a legal counsel are significantly higher for FOF-Held funds, so part of the due diligence process is carried out at the level of single-strategy hedge funds. Moreover, FOF-Held funds have higher performance fees, as well as a greater chance of including a high watermark clause, while the difference in management fees appears insignificant. With respect to the lockup length and the advance notice period, the differences between NO-FOF-Held funds and FOF-Held funds are rather small and insignificant. The required minimum initial investment in FOF-Held funds is quite large, with a mean of \$1.2 million. For NO-FOF-Held funds, the mean minimum initial investment is \$0.6 million, which is significantly lower at the 1% significance level. Finally, FOF-Held fund managers seem to have more years of investment experience than NO-FOF-Held fund managers. The proportions of team managed funds, CFA charterholder managers and male managers do not differ between the two groups of funds.

Our probit analysis shows that the probability of fund inclusion is increasing with the historical alpha, fund size and age. A higher tracking error seems to depress the fund inclusion probability. Moreover, the minimum initial investment bears a concave relation to the probability of fund inclusion, such that the probability is increasing with the minimum initial investment, but at a decreasing rate. A hedge fund that is closed to all investors sees a better chance of appearing in an FOF portfolio. Finally, a fund with a high watermark or a higher performance fee is associated with a higher inclusion probability. Although the probit analysis indicates the importance of several key factors that affect the fund inclusion probability, they are more important in the Equity style and Arbitrage style hedge funds.

The rest of the paper is organized as follows. Section 3.2 describes the data. Section 3.3 examines the differences in characteristics and performance between FOF-Held funds and NO-FOF-Held funds, while Section 3.4 presents the probit analysis of fund inclusion. Finally, Section 3.5 concludes.

3.2 Data

We obtain data of hedge funds from Morningstar. The sample period covers January 1994 until May 2009. Most data vendors divide hedge funds into two broad groups: single-strategy hedge funds and funds of funds. The single-strategy hedge funds, often referred as single-manager hedge funds, invest in various asset classes, and their portfolios consist of stocks, bonds, convertible securities, derivatives and several unconventional asset classes. The name 'single-strategy' or 'single-manager' is a bit misleading, as a hedge fund manager from a single-strategy hedge fund may pursue different investment strategies at the same time, and there can be a team of managers in a 'single-manager' hedge fund. In contrast, funds of funds (FOFs) invest with a multiple of hedge fund managers, and the underlying assets in FOF portfolios are single-strategy hedge funds. By holding a portfolio of single-strategy hedge funds, FOFs provide investors with some level of risk diversification and economy of scale in due diligence process. As a result, FOF investors have to pay two layers of fees, one at the level of single-strategy hedge funds included in FOF portfolios, and the other at the level of FOFs. Our analysis focuses on living single-strategy hedge funds using USD as the base currency to report net asset values, returns and so on.[1] At the end of May 2009, there are 2899 living single-strategy hedge funds and 2102 dead single-strategy hedge funds. For funds of funds, the Morningstar database includes 4576 FOFs with about one third of them being dead or defunct. The number of living FOFs using USD as the base currency is 1492. The classification of hedge fund styles varies from one data provider to another. Morningstar classifies five grand categories of single-strategy hedge funds: Equity, Arbitrage, Event, Global, and Multi-Strategy. Each grand category can be further grouped into several sub-categories. For instance, the Arbitrage category has Convertible Arbitrage, Equity Arbitrage, Debt Arbitrage and several other sub-styles pursuing an arbitrage strategy. For funds of funds, Morningstar categorizes seven fund of funds styles: Equity, Debt, Event, Derivative, Non-Directional, Multi-Strategy, and Other. It does not further divide each style into sub-categories.

Table I gives a snapshot of the Morningstar hedge funds. Panel A reports the summary statistics of living funds of funds. The equal weighted portfolio of living funds of funds has an annualized mean return and a standard deviation of 7.42% and 5.83%, respectively. The annual Sharpe ratio of 0.63 is great, as the Sharpe ratios of the NYSE value weighted equity index and the Ibbotson Associates long-term U.S. government bonds are 0.31 and 0.55 during the same period. Across different fund of funds styles, variations in mean returns, volatilities and Sharpe ratios are large. Equity funds of funds have a relatively high volatility and a modest mean return, resulting in a low Sharpe ratio compared to other fund of funds styles. From Panel B, the equal weighted portfolio of all living single-

[1]The Morningstar database includes 7538 single-strategy hedge funds, among which living funds account for 4472. The number of living single-strategy hedge funds using USD as the base currency is 2899.

strategy hedge funds has a mean return and a standard deviation of 13.01% and 6.57%, respectively. The portfolio Sharpe ratio is 1.41, which is twice as large as funds of funds since single-strategy funds have a much higher mean return and only a slightly higher standard deviation. Among five grand styles of hedge funds, Equity has the highest mean return of 14.28%, but its standard deviation is also the greatest among all fund styles. The mean return and standard deviation of monthly returns of Arbitrage hedge funds are the lowest, with a mean of 10.04% and a volatility of 3.63%. Arbitrage, Event and Multi-Strategy hedge fund styles show negative skewness and fat tails in their returns, which is consistent with previous empirical findings concerning statistical properties of hedge fund returns. An interesting observation is that although several style groups of single-strategy hedge funds have a large excess kurtosis, the equal weighted portfolio of all single-strategy hedge funds has a much smaller excess kurtosis.

For hedge funds from the Morningstar database, there is an indicator variable that shows if currently a single-strategy hedge fund has FOF ownership at the end of May 2009. Panel C and Panel D report summary statistics for the living single-strategy hedge funds with FOF ownership, and the single-strategy hedge funds without FOF ownership. There are 1185 single-strategy hedge funds that have FOF ownership, and 1714 single-strategy hedge funds that are identified as funds without FOF ownership. The equal weighted FOF-Held portfolio of all styles has a mean return of 13.61% and a volatility of 6.11%. The mean and volatility of the equal weighted NO-FOF-Held portfolio are 12.45% and 7.17%, respectively. Accordingly, the Sharpe ratio of the FOF-Held portfolio is higher than that of the NO-FOF-Held portfolio. Using the Sharpe ratio as a performance measure, we find that FOF-Held funds show better results in four out of five styles. Several style groups of FOF-Held funds show larger excess kurtosis, and greater negative skewness in returns, than the NO-FOF-Held funds from the same styles.

3.3 Difference in Fund Characteristics and Performance

The empirical analysis starts with examining the style preference by funds of funds in Section 3.3.1 and the difference in fund characteristics of FOF-Held funds, NO-FOF-Held funds, and FOFs in Section 3.3.2. Finally, Section 3.3.3 reports long-term and short-term alphas, tracking errors and information ratios of FOF-Held funds, NO-FOF-Held funds and funds of funds.

3.3.1 Style Preference

FOFs may choose to invest in certain hedge fund styles according to the expertise of FOF managers or the fund mandate. The style preference by FOF managers can be examined by comparing the distribution of hedge fund styles for single-strategy hedge funds with FOF ownership to the distribution of hedge funds styles for single-strategy hedge funds without FOF ownership. According to Table II, the Equity category accounts for 46.67% of all FOF-Held funds, which is significantly lower than its share of all NO-FOF-Held funds, 52.39%. Within the Equity category, U.S. Equity, Emerging Market Equity and Global Equity are dominant sub-styles in both FOF-Held funds and NO-FOF-Held funds. However, there is no significant difference between FOF-Held funds and NO-FOF-Held funds in each sub-style's presence. The Global category is the second largest grand style in terms of percentage shares. It has a share of 23.63% of all FOF-Held funds and 22.29% of all NO-FOF-Held funds, and the difference in the two shares is not statistically significant. Furthermore, FOF-Held funds tend to have more exposure to the Arbitrage strategy, with about 13.25% of funds falling into the Arbitrage style, compared to 9.16% of NO-FOF-Held funds investing in Arbitrage funds. The difference is statistically significant at the 1% level, but it is mainly driven by the difference in the shares of the Debt Arbitrage sub-style of FOF-Held funds and NO-FOF-Held funds. The remaining two grand styles, Event funds and Multi-Strategy funds, account for about 16% in both FOF-Held funds and NO-FOF-Held funds. The share of Event funds for FOF-Held funds is significantly higher than that for NO-FOF-Held funds by 2.63%. Finally, for the Multi-Strategy style, its share is significantly higher for NO-FOF-Held funds.

3.3.2 Hedge Fund Characteristics

Table III reports characteristics of FOF-Held funds, NO-FOF-Held funds and funds of funds. Hedge fund characteristics are categorized into basic features, due diligence proxy, fees and incentives variables, and liquidity restrictions. For a small subset of data, Morningstar provides information on hedge fund managers, such as gender, professional credentials, investment experience, etc., which are also included in Table III.

Basic Features

The mean and median size of FOF-Held funds are $324.54 millions and $75.89 millions, respectively. For NO-FOF-Held funds, the mean size is $79.42 millions and the median size is $13 millions. Funds of funds have a mean size of $257.55 millions and a median size of $63.33 million. The mean ages of FOF-Held funds and NO-FOF-Held funds are 7.79 years and 6.37 years, respectively, while the mean age of FOFs is 6.9 years. Offshore funds account for 45% of all FOF-Held funds, 34% of NO-FOF-Held funds, and 40% of funds of funds. Hence, compared to NO-FOF-Held funds, FOF-Held funds tend to be larger in size, have a longer operational history and have more offshore hedge funds in the group. The differences in all three features between FOF-Held funds and NO-FOF-Held funds are large and statistically significant at the 1% significance level. FOFs have a smaller size, younger age and a smaller proportion of offshore funds than FOF-Held funds, but they are larger, older and have a greater proportion of offshore funds than NO-FOF-Held funds.

In addition, about 20% of FOF-Held funds, NO-FOF-Held funds and FOFs also report the average annual portfolio turnover. Among funds that report portfolio turnover, FOF-Held funds have a mean annual portfolio turnover of 270.16%, and NO-FOF-Held funds have a mean annual portfolio turnover of 347.87%. The difference is not statistically significant, however. In contrast, at the 1% significance level, FOFs have a significantly lower mean annual portfolio turnover than both FOF-Held funds and NO-FOF-Held funds.

Due Diligence

The process of due diligence involves examination of a hedge fund over its historical returns, operation risk, financial statements, fee structures and any other aspects important to the success of a hedge fund. Hedge fund investors have to bear costs of such due diligence process and very often are deterred by lack of resources and prohibitively high costs of due diligence. The practice of having an auditor and a legal counsel in place is supposed to get some due diligence jobs done at the level of hedge funds.[2] An auditor, for instance, can reduce operation risks of investing in hedge funds and give investors some assurance of the accuracy and quality of hedge fund financial statements or other material financial information. The expenses related to auditors and legal counsels are ultimately paid by investors as a part of management fees. Nevertheless, for potential investors, a hedge fund with an auditor and a legal counsel may give them some assurance and save them time and resources. Moreover, hedge funds, in an attempt to remain flexible in fund operation and investment, often seek to get exemption from registering with an appropriate regulator. Under security regulations in U.S., hedge funds need not register with the federal government agency or the state regulator if they meet certain requirements set out by regulators.[3] Most offshore hedge funds are set up in a tax heaven with light regulation, and often find their way to avoid regulation by U.S. federal or state regulators.[4] Having more flexibility may be beneficial to fund managers and fund investors, but the dark-side

[2]It is advisable to have a legal counsel in place to deal with tax code issues and legal matters, although regulations may not require hedge funds to appoint a legal counsel. Similarly, an auditor assists hedge fund managers to conform to relevant regulatory requirements for accounting and financial reporting (See Lhabitant (2006)).

[3]In U.S. there are exemptions, exceptions or safe haven rules that allow hedge funds and their advisers to circumvent the registration requirements from the Security Act (Regulation D and Regulation S), the Security Exchange Act (Section 12(g)), the Investment Company Act (Section 3(c)(1) and 3(c)(7)), and the Investment Advisers Act (Section 203(b)(3)). In July 2004, the SEC approved Rule 203(b)(3)-2 under the Investment Advisers Act that would virtually require most US-based hedge funds to register with the SEC by the 1 February 2006 deadline. However, the new registration rule was vacated after the SEC lost the court case initiated by several hedge funds (Source: Lhabitant (2006)).

[4]Offshore hedge funds are hardly affected by U.S. regulations, and they domicile in countries or jurisdictions with a low tax level and less onerous regulations with respect to hedge funds. According to Lhabitant (2006), among registered hedge fund advisers, 88% of them are U.S. based advisers.

is the potential, higher risk of fraudulent behaviors by hedge fund managers (although there is no evidence that registered investment advisers are less likely to commit fraud in U.S.).

The proportion of FOF-Held funds having an auditor is 76%, which is 11% higher than that of NO-FOF-Held funds and 12% higher than that of FOFs. In addition, the proportions of FOF-Held funds and NO-FOF-Held funds having a legal counsel are 70% and 61%, respectively. The proportion of FOFs having a legal counsel is significantly lower than both FOF-Held funds and NO-FOF-Held funds. It seems that a hedge fund having an auditor or a legal counsel is attractive to FOF managers in a significant way. Furthermore, FOFs are more likely to register with either the Federal or a state regulator. There are 46% of FOFs that register as investment advisors, but only 36% and 40% of FOF-Held funds and NO-FOF-Held funds choose to do so. Relative to NO-FOF-Held funds, the lower percentage of FOF-Held funds registering with the U.S. federal or a state regulator may simply the result of more offshore funds in this group.

Fees and Incentives

The fee structure in the hedge fund industry typically consists of a management fee and a performance fee or incentive fee. The management fee has little or no incentive effect on fund managers' performance, and normally is about 1%-2% of the asset under management. The high-powered incentive comes from the profit-sharing compensation scheme of performance fees, ranging from 10% up to 50% of total profits generated by hedge fund managers for a certain period. In addition, in order to avoid double dipping and excessive risk taking by hedge fund managers, a high watermark clause is common in the hedge fund industry. It requires that hedge fund managers make up any past losses in order to receive incentive fees.

It is not surprising to observe a similar level of mean management fees for FOF-Held funds and NO-FOF-Held funds, and the difference is statistically insignificant. The management fees charged by FOFs are significantly lower than those charged by either FOF-Held funds or NO-FOF-Held funds. The mean performance fee for FOF-Held funds is 19.33%, about 0.46% higher than that for NO-FOF-Held funds. The difference in the mean performance fees for the two groups is statistically significant at the 1% level. FOFs have a much lower mean performance fee of 8.59% than either group of single-strategy hedge funds. Finally, the proportion of FOF-Held funds having a high watermark clause is 86%, while there are only 79% of NO-FOF-Held funds that adopt a high watermark. For FOFs, the proportion of funds having a high watermark clause is 65%. To summarize, hedge funds in FOF portfolios tend to have greater performance incentives and hedge funds with a high watermark clause are more appealing to FOFs.

Manager Characteristics

Assessing hedge fund managers is one of the most important checking points of the hedge fund investment process. In addition to examining the track record by hedge fund managers, hedge fund investors often make quite some efforts in assessing a hedge fund manager's character and reputation in the industry. Some studies indicate that manager characters can affect risk taking and performance of hedge funds (e.g. Boyson (2002), Li, Zhang and Zhao (2005)).

The first interesting question about fund managers is whether a hedge fund is run by a team of fund managers or by an individual manager. A team may bring in expertise in different areas and additional networks for investment or attracting capitals. On the other hand, different managers may have different investment philosophy that gives rise to conflicts and slows down decision-making process. Approximately 43% of FOF-Held funds and 46% of NO-FOF-Held funds are team managed, although the difference is not significant. A team-managed fund is even more common in FOFs, with a proportion of 53% of FOFs using a team of fund managers. Among those funds run by individual managers, male fund managers account for more than 90%. Although female fund managers are rather common in mutual funds, they have not yet found their way in the hedge fund industry. There is little research on the male fund managers' predominance in hedge funds. Relatively high opportunity costs of a hedge fund career to a female in terms of family life is one of the surmises.

Among hedge fund managers in FOF-Held funds, 10% of them are CFA charterholders. The percentage of CFA charterholders in NO-FOF-Held funds is 7%, but the difference between the two is statistically insignificant. The most striking difference between FOF-Held fund managers and NO-FOF-Held fund managers is the investment experience. On average, FOF-Held fund managers have 7.51 years investment experience, 1.1 years greater than NO-FOF-Held fund managers. The difference is statistically significant at the 1% level.

Liquidity Restrictions

One of the common characteristics of alternative investments is illiquidity to investors. Some hedge funds, after reaching the perceived optimal size, decide not to take new capitals from any investors or new investors. There are hedge funds that impose a lockup period during which investors are not allowed to liquidate their investments in hedge funds. In addition, investors need to notify hedge fund managers well in advance, and may only redeem once a week, a month, a quarter, or even worse, a year. Hence, even after the expiry of a lockup period, the advance notice period and redemption frequency restriction may still come into play, effectively preventing liquidation by investors at will. Furthermore, the minimal size of the initial investment is set to be large, especially when a hedge fund operates as a partnership and only takes a limited number of investors. Such a minimum size requirement effectively excludes investors with a small portfolio. There is some empirical evidence suggesting that those liquidity restrictions can affect performance of hedge funds (Aragon (2007), Liang and Park (2008)).

FOF-Held funds and NO-FOF-Held funds have roughly the same median of advance notice days. The mean advance notice days is about 40 for FOF-Held funds, only slightly shorter than the mean advance notice days for NO-FOF-Held funds. FOFs are more restrictive in that they require 57 advance notice days on average. The proportions of funds closed to all investors are 12% for FOF-Held funds, and 8% for NO-FOF-Held funds. The proportions of funds closed to new investors are 12% and 8%, for FOF-Held funds and NO-FOF-Held funds, respectively. FOFs have roughly the same proportions of funds closed to all investors and funds closed to new investors as those in FOF-Held funds. Furthermore, it seems that the two variables are highly correlated. A fund that decides not to take any new investors also closes its door to additional capital contributions from existing investors, although two decisions may be made at different dates.

There are about 47% of FOF-Held funds and 50% of NO-FOF-Held funds that impose a lockup period. The difference is statistically insignificant. Among those funds with non-zero lockup periods, the average lockup periods for FOF-Held funds and NO-FOF-Held funds are close to 12 months with the difference being insignificant. Hence, in terms of lockup restrictions, FOF-Held funds are not too different from NO-FOF-Held funds. In contrast, FOFs are slightly less restrictive, with 40% of funds imposing a lockup period.

Although there are hedge funds that require only a small amount of threshold investments, the average threshold is quite high. The average minimal size of the initial investment required is $1.18 million for FOF-Held funds and $0.62 million for NO-FOF-Held funds. The difference in the mean is statistically significant at the 1% level. In addition, the median minimal initial investment required by FOF-Held funds is four times as large as that by NO-FOF-Held funds. A fund of funds typically invests in 10 to 30 single-strategy hedge funds, and a $1.18 million minimal size of initial investment is only problematic for very small FOFs (the mean and median of assets under management by FOFs are $257.55 million and $63.33 millions). It is interesting to observe an average minimum initial investment of $0.87 million for FOF investors. Such a threshold is certainly high enough to prevent many small investors from investing in funds of funds.

Finally, the redemption frequency for hedge funds is clustered in monthly frequency and quarterly frequency for both FOF-Held funds and NO-FOF-Held funds. About 44% of FOF-Held funds allow monthly redemption, and 42% of FOF-Held funds permit quarterly redemption. For NO-FOF-Held funds, monthly redemption and quarterly redemption account for 50% and 38%, respectively. Hedge funds with redemption frequencies other than monthly or quarterly are relatively few. For instance, only 5% of FOF-Held funds and 3% of NO-FOF-Held funds allow investors to withdraw their money once per year.

A redemption fee is sometimes required, but the mean redemption fee is not significantly different from each other between FOF-Held funds and NO-FOF-Held funds. It is somewhat surprising to observe that FOFs are more likely to require annual redemption as well as quarterly redemption than both FOF-Held funds and NO-FOF-Held funds. It seems that FOFs are less flexible as far as redemption is concerned, than single-strategy hedge funds.

3.3.3 Alpha and Tracking Error

We estimate historical alphas and tracking errors from the seven-factor model by Fung and Hsieh (2001) for each individual hedge fund, and report the equal weighted averages of alphas, tracking errors and information ratios for single-strategy hedge funds and funds of funds. The tracking error is defined as the standard deviation of the alpha, while the information ratio is defined as the mean alpha divided by tracking error. The sample periods for long-term performance and short-term performance are from January 1994 to December 2008 and from January 2006 to December 2008, respectively. Panel A in Table IV shows the equal weighted averages of long-term and short-term alphas, tracking errors and information ratios for FOF-Held funds and NO-FOF-Held funds. The average long-term alpha for FOF-Held funds is 8.44%, and NO-FOF-Held funds have a lower mean alpha of 6.30%. The difference in the mean alphas of FOF-Held funds and NO-FOF-Held funds is statistically significant at the 1% level. The difference of 2% per year in alpha is an economically large wedge. In addition, for every category, FOF-Held funds always dominate NO-FOF-Held funds in terms of mean alphas, and the difference is significant at the 1% level for the Equity and Global hedge funds. Higher long-term alphas of FOF-Held funds do not result in higher tracking errors. In fact, the equal weighted average of the long-term tracking errors of FOF-Held funds is lower than that of NO-FOF-Held funds by 68 basis points. As a result, FOF-Held funds have a greater mean long-term information ratio than NO-FOF-Held funds by 0.26. What is more, FOF-Held funds have better short-term performance than NO-FOF-Held funds. The mean alpha, tracking error and information ratio are 6.87%, 10.03% and 0.79 for FOF-Held funds. NO-FOF-Held funds have a mean alpha of 4.37% and a mean tracking error of 10.78%. Hence, the better performance of FOF-Held funds exists for every style group in both the short run and the long run. It is interesting to observe that the short-term alpha and information ratio of hedge funds are lower than the long-term counterparts, which is consistent with the capacity constraint hypothesis by Fung et al. (2008) and the model prediction by Berk and Green (2004).[5] The increased money flows into hedge funds create a problem

[5]The decreasing alpha over time is also consistent with the hypothesis that the growing hedge fund industry has attracted new entrants without skills. For instance, Deuskar, Pollet, Wang and Zheng (2010) suggest that mutual fund managers with bad track records are more likly to completely switch to hedge funds. Moreover, their performance in hedge funds further indicates that they do not have skills.

for hedge fund managers: investment opportunities are limited and the profitability of investment strategies may decrease if the optimal scale for existing strategies has been reached.[6] Furthermore, the capacity constraint appears to hit NO-FOF-Held funds harder, as the gap between the long-term and short-term performance is greater for NO-FOF-Held funds. The short-term information ratio of NO-FOF-Held funds is 0.43, lower than the long-term information ratio by one-third. In contrast, the difference between the long-term and short-term information ratio of FOF-Held funds is rather small.

Panel B shows the equal weighted averages of alphas, tracking errors and information ratios of funds of funds. The average long-term alpha and tracking error of FOFs are 2.67% and 6.24%, respectively. There is some variation in alphas and tracking errors across seven FOF style groups, but their information ratios are quite similar. A comparison of the long-term alphas and the short-term alphas of various FOF styles indicates that the capacity constraint seems to affect the Equity, Debt and Event funds of funds, but not the other four styles. Panel C reports the results of testing the difference in the long-term/short-term performance between single-strategy hedge funds and funds of funds. Both FOF-Held funds and NO-FOF-Held funds have higher mean alphas, tracking errors as well as information ratios than FOFs. The differences are significant in most cases except for the test of the short-term information ratios of NO-FOF-Held funds and FOFs. Therefore, funds of funds tend to under-perform single-strategy hedge funds in terms of risk-adjusted returns. Such a result is consistent with findings by Fung and Hsieh (2000), Brown, Goetzmann, and Liang (2004) who suggested that double fees in funds of funds can contribute to the performance differential.

3.4 Probit Analysis of Fund Inclusion

A question of considerable interest is the probability of a hedge fund having an FOF investor at some point in time. A natural candidate of econometric models to describe the probability that a hedge fund has FOF ownership is a binary choice model:

$$\begin{aligned} y_i^\star &= \beta' x_i + \varepsilon_i, \qquad \varepsilon_i \sim NID\,(0,1) \\ y_i &= 1 \text{ if } y_i^\star > 0 \\ &= 0 \text{ if } y_i^\star \leq 0 \end{aligned}$$

where the error terms are independent of all explanatory variables. The dependent variable in the regression is the unobserved latent variable, while the explanatory variables are characteristics of single-strategy hedge funds. The indicator variable equals one if a hedge fund has FOF ownership, and zero otherwise. The set of hedge fund characteris-

[6]According to Wang and Zheng (2008), total hedge fund assets have grown more than twenty folds from 1994 to 2007.

tics affecting the probability of a single-strategy hedge fund appearing in FOF portfolios includes long-term and short-term alphas and tracking errors, basic features of a hedge fund such as size, age, an offshore dummy, due diligence variables using an auditor dummy and a legal counsel dummy, managerial incentive variables and fee variables, and liquidity restriction variables. The coefficients in the probit model do not share the same interpretation as those in a linear regression model. Signs of the coefficients can be interpreted as the direction of the change in the probability caused by a change in one of the explanatory variables. However, the magnitude of the change in the probability depends on the levels of the explanatory variables. We can use the average values of the explanatory variables to get the probability density, and interpret the marginal effect of each variable on the probability.

Table V displays the results of probit analysis of hedge fund inclusion by FOFs. The indicator variable equals one if a hedge fund has FOF ownership in May 2009. The table reports several columns of results where the difference is the number of independent variables included in each model. Some of the independent variables are likely to be correlated and including all of them may obscure the interpretation of estimation results. Since only funds still operating as of May 2009 are included in the analysis, we interpret each coefficient as the effect of a hedge fund variable on the probability of a hedge fund inclusion in an FOF portfolio in May 2009.

Model 1 includes important features of single-strategy hedge funds such as historical abnormal performance, tracking error, log fund size, fund age, offshore dummy, a high watermark dummy, and fee structure variables. From the test statistics of the likelihood ratio test against the intercept model, it is obvious that hedge fund variables jointly explain the fund selection outcome significantly better than the intercept model. Hedge funds with better historical abnormal performance seem to be able to win the heart of FOFs, as the coefficient of the alpha is significantly greater than zero at the 1% level. To estimate the effects of the explanatory variables on the probability of fund inclusion, we use the average values of the explanatory variables to compute the probability density, which returns a probability density of 0.40. Hence, a 1% increase in the recent alpha (monthly) would increase the probability by $0.40 \times 0.19 = 0.08$. Meanwhile, for a given level of the alpha, a higher tracking error discourages investments by fund of funds. The coefficient of the tracking error is significantly negative. A 1% increase in the recent tracking error would decrease the probability by $0.40 \times (-0.07) = -0.03$. In addition, fund size, fund age and whether a fund is from a fund family are positively related to the probability of a hedge fund being included in an FOF portfolio. As to the fee structure, arrangements such as a high watermark and a performance fee have incentive effects on hedge fund managers, and it is not surprising to observe that FOFs prefer hedge funds that have greater incentives to out-perform. Nevertheless, while the high watermark is significant at the 1% level, the performance fee has no significant impact under this particular specification. For the management fee, it has significant effect on the probability of fund inclusion at the 10% level.

Model 2 includes only three due diligence variables: an auditor dummy, a legal counsel dummy and a registered investment advisor dummy. Having an auditor and a legal counsel has some positive effect on the inclusion probability, but only the auditor dummy is statistically significant. The effect of the registered investment dummy is negative but statistically insignificant. Registration under the Investment Advisers Act entails many requirements on disclosure, compliance procedures, record keeping, performance fees, and on-site examinations by the SEC (Source: Lhabitant (2006)). A greater regulation level normally increases costs of operating a hedge fund, and restricts investment strategies. If fund of funds managers have confidence in their abilities to screen and monitor single-strategy hedge fund managers, they will not consider registering with the SEC by single-strategy hedge funds as an advantage. Model 3 only considers the impact of liquidity restrictions in hedge funds on the probability of fund inclusion by FOFs. The advance notice period (in months) variable and its square show no statistical significance. In addition, whether a hedge fund is closed to all investors for money inflows is not important for fund inclusion. Although some argue that one of the advantages of FOFs is their ability to invest in hedge funds closed to new money, it does not appear to be a significant factor for fund inclusion. The coefficient of the lockup period (in months) is negative, indicating a smaller probability of fund inclusion if a fund has a longer lockup period. Nevertheless, at the 10% level, the coefficient of the lockup is not statistically different from zero. The coefficient of the minimum initial investment is positive and statistically significant, but the square of the minimum initial investment has a negative coefficient, also statistically significant. This implies a concave relation between the probability of fund inclusion and the minimum initial investment. The probability is increasing with the minimum initial investment at a decreasing rate. Although a high minimum initial investment is considered by many as barriers to invest in hedge funds, it appears that such a restriction is welcomed by FOFs. Finally, the redemption frequency (in months) appears to be insignificant under this model specification.

Model 4 examines the effect of manager characteristics on the probability of fund inclusion. Both the gender and investment experience of a fund manager appear to be insignificant. The CFA charterholder dummy is positively associated with the probability of fund inclusion, and significant at the 10% level. Nevertheless, the model 4 does not explain the probability of fund inclusion better than the intercept model in a significant way. Model 5 gives a specification of the probit analysis including all hedge fund features (but excluding manager characteristics) as explanatory variables. The coefficients of the alpha and tracking error are of opposite signs and statistically significant. Fund size, fund age, auditor dummy, management fee, minimum initial investment and its square have coefficients of the same signs as those in the reduced models, and they are statistically significant. In addition, the closed to all dummy and performance fee become statistically significant. The coefficient of the closed to all is positive, while the performance fee is also positively associated with the probability of fund inclusion.

Finally, Model 6 gives a full specification of the probit analysis, including manager characteristics as well as all hedge fund variables in Model 5. The problem with Model 6 is that the number of observations is reduced sharply since the number of managers providing manager characteristics information is quite small and the specification will exclude funds with team-management. Therefore, the analysis is restricted to a small set of the sample data. Adding manager characteristics to the analysis leads to insignificant coefficients of all due diligence variables and fee structure variables. In addition, the concave relation between the minimum initial investments and the probability of fund inclusion is still valid, but neither of the coefficients is statistically significant in Model 6. In fact, it appears that the only significant factor is the alpha with an expected positive sign. All other hedge fund variables and manager characteristics variables are not different from zero at the 10% significance level.

The results in Table V utilize all available hedge fund information for each model specification. Because not all hedge funds report all information to a database, those results are not based on a common sample. We also perform probit analysis on a common sample and report results in Table VI. To examine the contribution of each set of variables to the explanatory power, we start with a basic model and add one set of variables to the previous model. According to Table VI, adding due diligence variables to the analysis increases R square by 2%, while fee structure variables seem to have little marginal impact on the model fit. Furthermore, liquidity restriction variables increases R square by another 3%, as shown in Model 4. Most variables remain the same sign and statistical significance as additional variables are included in the analysis (also compared to those in Table V).

Overall, the analysis in this section suggests that a hedge fund's historical performance is a very important factor to the inclusion of a hedge fund in an FOF portfolio. The higher the alpha, and the lower the tracking error, the greater the probability of fund inclusion. In addition, FOFs prefer hedge funds with a larger size, a longer operational history, and greater minimum initial investments. Even though an auditor and a legal counsel may serve to reduce operational risks and increase credibility of hedge fund managers, only the auditor's presence increases the probability of fund inclusion. Brown, Fraser and Liang (2008) suggest that FOFs have the economy of scale in the process of fund selection or screening and monitoring once the investment is made. Therefore, FOFs can perform due diligence on their own instead of relying on an auditor and a legal counsel.

It is rather straightforward to interpret FOFs' preference for hedge funds with a larger size. A greater fund size is positively associated with the survival probability of a hedge fund, holding everything else constant (Baquero, ter Horst and Verbeek (2005), Malkiel and Saha (2005)). From Table III, hedge funds selected by FOFs are larger in size, in terms of either the mean or median, than those not selected by FOFs. This already indicates a preference for larger hedge funds by FOFs. On the condition that a hedge fund is still operating as of May 2009, a larger size is associated with a greater probability of fund inclusion. Similarly, a greater minimum initial investment positively contributes to a higher probability of fund inclusion. Usually, both a long lockup period and a high minimum initial investment can be considered as liquidity restrictions. While a lockup period reduces the probability of fund inclusion, minimum initial investments have the opposite impact. A high minimum initial investment may be a problem for high-net-worth individuals or institutional investors with a small allocation to alternative investments, but it is not an impediment to FOFs due to relatively large assets under management pooled from investors. In fact, it seems that FOFs are more likely to add funds with high minimum initial investments.

Table VII presents results of probit analysis of five hedge fund styles. It is possible that some fund features are important factors affecting the probability of fund inclusion for one hedge fund style, while the same fund features are less relevant for inclusion of hedge funds from a different style group. The analysis is based on one model specification, neglecting variables such as advance notice period, advance notice square and manager variables. Those variables appear to be insignificant under model specifications in Table V.[7] The first noticeable result is the explanatory power of the model across five fund styles. The likelihood ratio test statistics show that the model does a pretty good job in describing the relation between the explanatory variables and the probability of fund inclusion for every fund style group. At the 10% significance level, we can reject the null hypothesis that the intercept model is as good as the model with fund variables.

For the Equity style, the coefficients of the alpha and tracking error have expected signs and both are statistically significant. In addition, the coefficients of fund size and auditor dummy are positive and statistically significant. The concave relation between the minimum initial investment and the probability of fund inclusion is also evident for Equity funds. This concave relation is also visible and statistically significant for Arbitrage funds. What is more, the alpha is positively associated with the probability of inclusion of Arbitrage funds. None of other fund variables are statistically significant. For Event funds, the ex post alpha and tracking error do not bear a significant relation to the probability of fund inclusion. The only significant variables are fund age, the registered investment adviser dummy and closed to all dummy. If an Event fund is closed to all investors for capital inflows, it is more likely to appear in an FOF portfolio. Both fund size and fund age of Global funds are significant variables in the probit analysis, so a large size and a longer operational history increase the probability of fund inclusion. Just like

[7] Including all fund features and manager variables in the model is not a problem when the number of observations is large. Unfortunately, for the Event and Multi-Strategy that have only a dozen of observations of all fund features and manager variables, a model of too many explanatory variables leads to badly estimated coefficients due to a close to singular or badly scaled covariance matrix.

Event funds, a Global fund that is close to all investors has a greater chance of being included in an FOF portfolio. Having an auditor in a Global fund seems to reduce the probability of FOF inclusion, while the effect is positive for other style funds. For Global funds, the concave relation between the minimum initial investment and fund inclusion probability is still found, but only the coefficient of the level term is significant. Finally, the coefficient of the auditor dummy appears statistically significant in the probit analysis of Multi-Strategy funds.

3.5 Conclusion

This paper gives a unique examination of single-strategy hedge funds in an attempt to understand the difference between hedge funds with FOF ownership and hedge funds without FOF ownership at the end of May 2009. Our analysis shows that hedge funds with FOF investors are larger in size, have a longer operational history, and are more likely to hire an auditor or a legal counsel and use a high watermark. In addition, the average minimum initial investment in these funds is higher. Their managers have more investment experience, and have a greater proportion of CFA charterholders. The raw returns and risk-adjusted returns are higher for FOF-Held funds. The probit analysis confirms that the alpha and tracking error are important factors to the probability of fund inclusion. Furthermore, there are some sizable differences across hedge fund styles in the outcomes of the probit analysis of fund inclusion.

3.A Tables and Figures

Table I: Summary Statistics of Morningstar Hedge Funds

This table reports summary statistics for the Morningstar hedge funds by category or style. The Morningstar Inc. publishes the Morningstar category classifications for funds of funds and single-strategy hedge funds. Panel A gives summary statistics for all living funds of funds, while Panel B show results for living single-strategy hedge funds. Panel C and Panel D report summary statistics for FOF-Held funds and NO-FOF-Held funds. The 'FOF-Held Funds' is the equal weighted portfolio of living single-strategy hedge funds with FOF ownership. The 'NO-FOF-Held Funds' refers to the equal weighted portfolio of living single-strategy hedge funds without any FOF ownership. Mean, median, standard deviation and Sharpe ratio are annualized. Kurt is the excess kurtosis for hedge fund returns. The sample period is from January 1994 to December 2008.

	Number	Mean	Median	Stdev	Sharpe	Min	Max	Skew	Kurt
A. Funds of Funds									
Equity	479	7.63	9.15	7.55	0.52	-7.69	9.21	-0.38	3.32
Debt	71	7.85	9.20	7.38	0.56	-13.26	6.76	-1.42	10.05
Event	87	6.27	9.54	5.00	0.51	-6.65	3.26	-2.13	6.88
Derivative	114	9.17	7.60	6.60	0.83	-4.42	5.89	0.04	0.02
Non-Directional	62	8.19	8.94	3.77	1.18	-4.47	3.54	-1.26	4.65
Multi-Strategy	501	7.09	8.14	5.24	0.64	-6.45	4.42	-1.16	4.03
Other	178	6.78	7.84	5.50	0.55	-6.63	4.05	-1.18	3.79
All Styles	1492	7.42	8.63	5.83	0.63	-6.59	6.01	-0.80	3.32
B. Single-Strategy Hedge Funds									
Equity	1451	14.28	17.39	9.70	1.09	-8.85	9.92	-0.38	1.41
Arbitrage	314	10.04	10.93	3.63	1.74	-5.05	3.26	-1.90	8.61
Event	202	11.96	15.95	6.07	1.36	-7.24	4.85	-1.42	4.60
Global	662	12.84	13.08	8.38	1.09	-5.02	9.19	0.35	0.41
Multi-Strategy	270	12.58	14.05	4.52	1.96	-4.83	4.00	-1.14	4.22
All styles	2899	13.01	13.75	6.57	1.41	-5.48	5.99	-0.21	0.82
C. FOF-Held Funds									
Equity	553	14.92	18.17	8.95	1.25	-7.81	10.37	-0.22	1.72
Arbitrage	157	10.58	11.69	4.41	1.55	-6.82	4.08	-2.38	11.28
Event	101	12.51	16.04	6.37	1.38	-8.39	5.64	-1.34	5.03
Global	280	13.84	12.02	9.19	1.10	-5.82	10.18	0.27	0.27
Multi-Strategy	94	12.93	14.79	4.15	2.22	-5.16	3.57	-1.37	4.85
All Styles	1185	13.61	13.27	6.11	1.62	-5.05	5.88	-0.10	0.79
D. NO-FOF-Held Funds									
Equity	898	13.78	17.42	10.45	0.96	-9.88	9.50	-0.45	1.21
Arbitrage	157	9.36	9.41	3.46	1.63	-3.49	3.79	-0.54	3.21
Event	101	11.24	15.72	5.87	1.28	-6.81	4.42	-1.28	3.68
Global	382	11.72	11.15	8.15	0.98	-5.92	10.03	0.55	1.26
Multi-Strategy	176	12.55	12.05	5.69	1.55	-5.49	7.61	-0.17	3.10
All Styles	1714	12.45	13.97	7.17	1.22	-6.10	6.11	-0.29	0.76

Table II: Style Preference of Funds of Funds

This table displays the distribution of investment strategies/styles for single-strategy hedge funds. 'FOF-Held' is the group of single-strategy hedge funds with FOF ownership. 'NO-FOF-Held' refers to the single-strategy hedge funds without any FOF ownership. The Morningstar Inc. publishes the Morningstar Category classifications for single-strategy hedge funds. Diff is the difference in the proportion of each fund style between FOF-Held funds and NO-FOF-Held funds. The last column reports the p-value from the two-proportion z-test (two-sided test) of Diff, assuming unequal variance.

Morningstar Category	FOF-Held		NO-FOF-Held			
	Number	Percentage	Number	Percentage	Diff	p-value
Equtiy						
HF U.S. Equity	204	17.22	320	18.67	-1.45	0.31
HF U.S. Small Cap Equity	50	4.22	61	3.56	0.66	0.37
HF Developed Asia Equity	45	3.80	86	5.02	-1.22	0.11
HF Europe Equity	43	3.63	81	4.73	-1.10	0.14
HF Emerging Market Equity	103	8.69	173	10.09	-1.40	0.20
HF Global Equity	98	8.27	169	9.86	-1.59	0.14
HF Short Equity	10	0.84	8	0.47	0.38	0.23
Subtotal	553	46.67	898	52.39	-5.73	0.00
Arbitrage						
HF Convertible Arbitrage	27	2.28	30	1.75	0.53	0.33
HF Equity Arbitrage	67	5.65	92	5.37	0.29	0.74
HF Debt Arbitrage	63	5.32	35	2.04	3.27	0.00
Subtotal	157	13.25	157	9.16	4.09	0.00
Event						
HF Corporate Actions	60	5.06	52	3.03	2.03	0.01
HF Distressed Securities	41	3.46	49	2.86	0.60	0.37
Subtotal	101	8.52	101	5.89	2.63	0.01
Global						
HF Global Debt	57	4.81	77	4.49	0.32	0.69
HF Global Trend	135	11.39	183	10.68	0.72	0.55
HF Global Non-Trend	88	7.43	122	7.12	0.31	0.75
Subtotal	280	23.63	382	22.29	1.34	0.40
Multi-Strategy						
HF Multi-strategy	94	7.93	176	10.27	-2.34	0.03
Grand Total	1185		1714			

Table III: Characteristic Difference Among 'FOF-Held' hedge funds, 'NO-FOF-Held' hedge funds and Funds of Funds

This table categorizes a variety of hedge fund characteristics and compares those characteristics among three groups. Panel A reports basic features, due diligence variables, fees and manager characteristics while Panel B displays liquidity restrictions of hedge funds and funds of funds. Fund size and minimum initial investment are in millions (USD). Fund age and investment experience are in years. Diff is the difference in the mean of fund characteristics. The sample period is from January 1994 to May 2009.

Panel A. Basic Features, Due Diligence, Fees and Managers

Variables	FOF-Held			NO-FOF-Held			FOFs								
	Mean	Median	Stdev	Mean	Median	Stdev	Mean	Median	Stdev	Diff	p-value	Diff	p-value	Diff	p-value
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)-(4)		(1)-(7)		(4)-(7)	
Basic Features															
Fund Size	324.54	75.89	2138.10	79.42	13.00	208.20	257.55	63.33	651.31	245.12	0.00	66.99	0.30	-178.13	0.00
Fund Age	7.79	6.42	5.04	6.37	5.17	4.67	6.90	5.76	4.83	1.42	0.00	0.89	0.00	-0.53	0.00
Offshore Vehicle (Yes=1)	0.45	0.00	0.50	0.34	0.00	0.47	0.40	0.00	0.49	0.11	0.00	0.05	0.01	-0.06	0.00
Fund Family (Yes=1)	0.83	1.00	0.37	0.79	1.00	0.41	0.92	1.00	0.28	0.04	0.00	-0.08	0.00	-0.12	0.00
Avg Annual Portfolio Turnover	270.16	150.00	405.30	347.87	100.00	987.82	27.69	15.00	164.95	-77.72	0.20	242.47	0.00	320.19	0.00
Due Diligence															
Auditor (Yes=1)	0.76	1.00	0.43	0.65	1.00	0.48	0.64	1.00	0.48	0.11	0.00	0.12	0.00	0.01	0.65
Legal Counsel (Yes=1)	0.70	1.00	0.46	0.61	1.00	0.49	0.53	1.00	0.50	0.10	0.00	0.17	0.00	0.08	0.00
Registered Inv. Advisor (Yes=1)	0.36	0.00	0.48	0.40	0.00	0.49	0.46	0.00	0.50	-0.04	0.10	-0.10	0.00	-0.06	0.01
Fees															
High Watermark (Yes=1)	0.86	1.00	0.35	0.79	0.79	0.41	0.65	1.00	0.48	0.06	0.00	0.21	0.00	0.15	0.00
Management Fee (%)	1.55	1.50	0.65	1.52	1.50	0.60	1.31	1.25	0.56	0.03	0.22	0.24	0.00	0.21	0.00
Performance Fee (%)	19.33	20.00	4.28	18.87	20.00	4.89	8.59	10.00	5.86	0.46	0.01	10.74	0.00	10.28	0.00
Manager Character															
Team Management (Yes=1)	0.43	0.00	0.50	0.46	0.00	0.50	0.53	1.00	0.50	-0.03	0.24	-0.10	0.00	-0.07	0.00
Gender (Male=1)	0.95	1.00	0.21	0.93	1.00	0.25	0.94	1.00	0.24	0.02	0.40	0.01	0.64	-0.01	0.79
CFA Holder (Yes=1)	0.10	0.00	0.30	0.07	0.00	0.26	0.05	0.00	0.23	0.03	0.11	0.04	0.01	0.02	0.26
Investment Experience	7.51	6.04	4.80	6.41	5.42	4.63	6.83	6.09	4.53	1.10	0.00	0.68	0.03	-0.42	0.16

Panel B. Liquidity Restrictions

Variables	FOF-Held			NO-FOF-Held			FOFs								
	Mean	Median	Stdev	Mean	Median	Stdev	Mean	Median	Stdev	Diff	p-value	Diff	p-value	Diff	p-value
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)-(4)		(1)-(7)		(4)-(7)	
Liquidity Restrictions															
Advanced Notice Days	39.83	30.00	31.00	40.31	30.00	32.31	56.73	60.00	28.48	-0.48	0.73	-16.91	0.00	-16.43	0.00
Closed to All Investors (Yes=1)	0.12	0.00	0.32	0.08	0.00	0.28	0.12	0.00	0.32	0.04	0.00	0.00	0.97	-0.03	0.00
Closed to New Investors (Yes=1)	0.12	0.00	0.33	0.08	0.00	0.28	0.12	0.00	0.33	0.04	0.00	0.00	0.95	-0.04	0.00
Lockup Months	12.64	12.00	5.82	12.38	12.00	6.58	12.50	12.00	6.62	0.26	0.53	0.14	0.77	-0.12	0.80
Lockup Dummy (Yes=1)	0.47	0.00	0.50	0.50	0.00	0.50	0.40	0.00	0.49	-0.03	0.34	0.06	0.11	0.09	0.01
Minimum Initial Investment	1.18	1.00	2.13	0.62	0.25	1.23	0.87	0.25	3.61	0.56	0.00	0.31	0.00	-0.25	0.00
Redeem Daily (Yes=1)	0.03	0.00	0.17	0.02	0.00	0.14	0.01	0.00	0.09	0.01	0.24	0.02	0.00	0.01	0.01
Redeem weekly (Yes=1)	0.02	0.00	0.14	0.03	0.00	0.18	0.01	0.00	0.12	-0.01	0.08	0.01	0.33	0.02	0.01
Redeem Semi-Monthly (Yes=1)	0.01	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.06	0.01	0.01	0.00	0.55	0.00	0.05
Redeem Monthly (Yes=1)	0.44	0.00	0.50	0.50	0.00	0.50	0.35	0.00	0.48	-0.06	0.01	0.09	0.00	0.15	0.00
Redeem Quarterly (Yes=1)	0.42	0.00	0.49	0.38	0.00	0.48	0.47	0.00	0.50	0.05	0.03	-0.05	0.04	-0.09	0.00
Redeem Semi-Annually (Yes=1)	0.03	0.00	0.17	0.04	0.00	0.19	0.05	0.00	0.22	-0.01	0.26	-0.02	0.01	-0.01	0.14
Redeem Annually (Yes=1)	0.05	0.00	0.21	0.03	0.00	0.17	0.10	0.00	0.30	0.02	0.02	-0.05	0.00	-0.07	0.00
Redemption Fee (%)	0.85	0.00	1.62	0.78	0.00	1.79	0.81	0.00	1.40	0.07	0.44	0.03	0.70	-0.03	0.68

Table IV: Performance of Single-Strategy Hedge Funds and Funds of Funds

This table displays annualized alphas, tracking errors (both in percentages) and information ratios for single-strategy hedge funds and funds of funds. Panel A shows the long-term and short-term alphas, tracking errors and information ratios of FOF-Held hedge funds and NO-FOF-Held hedge funds, by hedge fund style. Both alphas and tracking errors are equal weighted averages. Fund alpha and tracking error are estimated from the seven-factor model from Fung and Hsieh (2001). Panel B shows alphas, tracking errors and information ratios of funds of funds. Panel C tests the difference in performance between single-strategy hedge funds and funds of funds. Only hedge funds with at least 24 months of historical returns are included in the regression analysis. The sample periods for long-term performance and short-term performance are from January 1994 to December 2008 and from January 2006 to December 2008.

Panel A. Single-Strategy Hedge Funds

	Equity	Arbitrage	Event	Global	Multi	All
A. Long-Term Alpha						
FOF-Held Funds	8.73	5.95	7.41	9.39	9.35	8.44
NO-FOF-Held Funds	6.06	4.87	6.08	6.99	7.85	6.30
Difference	2.67	1.08	1.33	2.39	1.51	2.14
p-value	0.00	0.31	0.13	0.00	0.40	0.00
B.Long-Term Tracking Error						
FOF-Held Funds	11.96	7.45	8.58	13.15	8.37	11.04
NO-FOF-Held Funds	12.66	8.07	8.38	13.26	9.34	11.72
Difference	-0.69	-0.61	0.20	-0.10	-0.97	-0.68
p-value	0.10	0.29	0.81	0.90	0.26	0.03
C.Long-Term Information Ratio						
FOF-Held Funds	0.76	0.76	0.79	0.87	0.90	0.88
NO-FOF-Held Funds	0.50	0.57	0.57	0.51	0.64	0.61
Difference	0.26	0.19	0.23	0.36	0.27	0.26
p-value	0.00	0.00	0.00	0.00	0.00	0.00
D. Short-Term Alpha						
FOF-Held Funds	6.61	5.50	4.59	8.72	7.89	6.87
NO-FOF-Held Funds	3.92	3.28	2.54	5.70	6.33	4.37
Difference	2.69	2.22	2.05	3.02	1.55	2.49
p-value	0.00	0.10	0.05	0.00	0.43	0.00
E.Short-Term Tracking Error						
FOF-Held Funds	10.61	7.27	7.09	12.20	8.37	10.03
NO-FOF-Held Funds	11.49	7.77	7.59	12.00	9.45	10.78
Difference	-0.88	-0.50	-0.50	0.20	-1.08	-0.74
p-value	0.03	0.40	0.48	0.80	0.21	0.01
F.Short-Term Information ratio						
FOF-Held Funds	0.60	0.66	0.65	0.81	0.82	0.79
NO-FOF-Held Funds	0.34	0.44	0.41	0.31	0.47	0.43
Difference	0.26	0.22	0.25	0.50	0.35	0.36
p-value	0.00	0.00	0.00	0.00	0.00	0.00

Panel B. Funds of Funds

	Equity	Debt	Event	Derivative	Non-Direction	Multi	Other	All
A. Long-Term								
Long-term Alpha	2.65	0.32	1.02	5.53	2.66	2.61	3.11	2.67
Tracking Error	6.67	5.42	4.61	8.53	6.16	5.64	6.72	6.24
Information ratio	0.41	0.38	0.36	0.43	0.43	0.49	0.50	0.46
B. Short-Term								
Short-term Alpha	1.98	-0.54	0.32	5.72	2.94	2.54	3.00	2.35
Tracking Error	6.67	5.38	4.79	8.58	6.36	5.96	6.82	6.38
Information ratio	0.26	0.25	0.22	0.32	0.31	0.41	0.44	0.37

Panel C. Single-Strategy Hedge Funds vs. Funds of Funds

	FOF-Held vs. FOF		NO-FOF-Held vs. FOF	
	Difference	p-value	Difference	p-value
A. Long-Term				
Alpha	5.77	0.00	3.63	0.00
Tracking Error	4.80	0.00	5.48	0.00
Information Ratio	0.42	0.00	0.15	0.00
B. Short-Term				
Alpha	4.52	0.00	2.03	0.00
Tracking Error	3.65	0.00	4.39	0.00
Information Ratio	0.42	0.00	0.06	0.16

Table V: Probit Analysis of Fund Inclusion

This table shows the results of the probit analysis of fund inclusion by FOFs, with an indicator variable that equals one if the single-strategy hedge fund has FOF ownership, and zero otherwise. The alpha and tracking error are estimated from the seven-factor model by Fung and Hsieh (2001), including hedge funds with an operational history longer than two years. Log fund size is in billions and both advance notice and redemption frequency are in months. Definitions of other explanatory variables follow Table III. The t-statistics are in parentheses. LR test statistics (p-values in parentheses) is from the likelihood ratio test against the intercept model.

	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
Constant	-0.79	(-4.46)	-0.31	(-3.62)	-0.30	(-4.14)	-0.35	(-1.38)	-1.25	(-4.41)	-0.63	(-0.68)
Alpha	0.19	(5.70)							0.15	(3.11)	0.44	(2.92)
Tracking Error	-0.07	(-4.38)							-0.09	(-4.14)	-0.08	(-1.33)
Fund Size	0.62	(5.35)							0.32	(2.47)	0.30	(0.91)
Fund Age	0.03	(3.82)							0.02	(2.13)	0.15	(1.10)
Offshore	0.01	(0.20)							-0.06	(-0.74)	-0.28	(-1.31)
Fund Family	0.13	(1.68)							0.00	(0.04)	-0.26	(-0.90)
Auditor			0.26	(2.42)					0.47	(3.14)	0.23	(0.52)
Legal Counsel			0.02	(0.21)					0.19	(1.41)	0.09	(0.21)
Registered Inv. Advisor			-0.10	(-1.58)					-0.12	(-1.51)	-0.14	(-0.74)
High Watermark	0.26	(3.06)							0.03	(0.23)	0.30	(0.97)
Management Fee	0.11	(1.79)							0.14	(1.75)	0.07	(0.30)
Performance Fee	0.01	(1.28)							0.02	(2.04)	-0.01	(-0.28)
Advance Notice					0.02	(0.25)			-0.04	(-0.46)	0.19	(0.73)
Advance Notice Square					0.00	(-0.31)			0.01	(0.80)	-0.02	(-0.46)
Closed to All					0.14	(1.51)			0.21	(1.93)	0.31	(1.07)
Lockup Months					-0.01	(-1.06)			0.00	(-0.24)	-0.03	(-1.65)
Minimum Initial					0.26	(6.95)			0.24	(5.22)	0.52	(1.58)
Minimum Initial Square					-0.01	(-5.03)			-0.01	(-4.22)	-0.08	(-1.12)
Redemption Frequency					0.02	(1.07)			-0.03	(-1.39)	0.02	(0.59)
Manager Gender							0.18	(0.73)			-0.09	(-0.21)
CFA Charterholder							0.26	(1.65)			0.34	(1.26)
Investment Experience							0.01	(1.23)			-0.12	(-0.86)
Sample size	1824		1590		1523		472		1128		224	
Pseudo R square	0.08		0.01		0.04		0.01		0.10		0.16	
McFadden R square	0.06		0.01		0.03		0.01		0.08		0.12	
LR test statistics	141.03	(0.00)	13.60	(0.00)	62.81	(0.00)	4.76	(0.19)	119.23	(0.00)	36.00	(0.03)

Table VI: Probit Analysis of Fund Inclusion
(Common Sample)

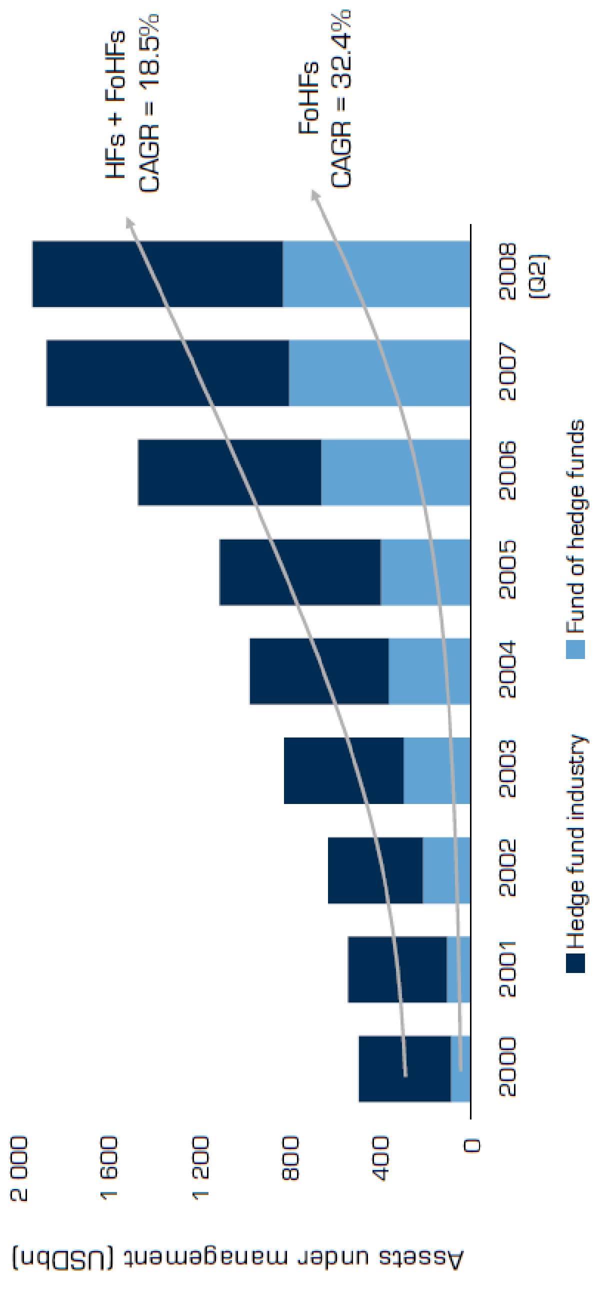
This table shows the results of the probit analysis of fund inclusion by FOFs, with an indicator variable that equals one if the single-strategy hedge fund has FOF ownership, and zero otherwise. The alpha and tracking error are estimated from the seven-factor model by Fung and Hsieh (2001), including hedge funds with an operational history longer than two years. Log fund size is in billions and both advance notice and redemption frequency are in months. Definitions of other explanatory variables follow Table III. The t-statistics are in parentheses. LR test statistics (p-values in parentheses) is from the likelihood ratio test against the intercept model. The analysis is based on the common sample such that each model includes the same set of single-strategy hedge funds.

	Model 1		Model 2		Model 3		Model 4	
Constant	-0.10	(-0.76)	-0.58	(-2.99)	-1.08	(-3.97)	-1.25	(-4.41)
Alpha	0.16	(3.42)	0.17	(3.51)	0.16	(3.28)	0.15	(3.11)
Tracking Error	-0.09	(-4.20)	-0.08	(-3.91)	-0.09	(-4.33)	-0.09	(-4.14)
Fund Size	0.46	(3.50)	0.45	(3.45)	0.44	(3.36)	0.32	(2.47)
Fund Age	0.01	(1.67)	0.02	(1.84)	0.02	(2.22)	0.02	(2.13)
Offshore	0.05	(0.69)	0.00	(-0.03)	-0.02	(-0.25)	-0.06	(-0.74)
Fund Family	0.09	(0.89)	0.07	(0.67)	0.06	(0.58)	0.00	(0.04)
Auditor			0.42	(2.86)	0.40	(2.71)	0.47	(3.14)
Legal Counsel			0.21	(1.59)	0.21	(1.61)	0.19	(1.41)
Registered Inv. Advisor			-0.16	(-2.05)	-0.15	(-1.81)	-0.12	(-1.51)
High Watermark					0.03	(0.20)	0.03	(0.23)
Management Fee					0.16	(1.96)	0.14	(1.75)
Performance Fee					0.01	(1.60)	0.02	(2.04)
Advance Notice							-0.04	(-0.46)
Advance Notice Square							0.01	(0.80)
Closed to All							0.21	(1.93)
Lockup Months							0.00	(-0.24)
Minimum Initial							0.24	(5.22)
Minimum Initial Square							-0.01	(-4.22)
Redemption Frequency							-0.03	(-1.39)
Sample size	1128		1128		1128		1128	
Pseudo R square	0.05		0.07		0.07		0.10	
McFadden R square	0.04		0.05		0.05		0.08	
LR test statistics	56.76	(0.00)	76.51	(0.00)	84.32	(0.00)	119.23	(0.00)

Table VII: Probit Analysis of Fund Inclusion by Style

This table shows the results of the probit analysis of fund inclusion by style, with an indicator variable that equals one if the single-strategy hedge fund has FOF ownership, and zero otherwise. The alpha and tracking error are estimated from the seven-factor model by Fung and Hsieh (2001), including hedge funds with an operational history longer than two years. Log fund size is in billions and both advance notice and redemption frequency are in months. Definitions of other explanatory variables follow Table III. The t-statistics are in parentheses. LR test statistics (p-values in parentheses) is from the likelihood ratio test against the intercept model.

	Equity		Arbitrage		Event		Global		Multi	
Constant	-0.80	(-3.29)	-0.42	(-0.81)	-0.79	(-1.18)	-0.08	(-0.24)	-2.06	(-2.49)
Alpha	0.13	(2.31)	0.28	(2.14)	0.28	(1.01)	0.15	(1.28)	0.27	(1.58)
Tracking Error	-0.10	(-3.62)	-0.01	(-0.12)	0.00	(-0.05)	-0.06	(-1.48)	-0.05	(-0.55)
Fund Size	0.51	(2.07)	0.48	(0.83)	-0.12	(-0.91)	0.78	(2.77)	0.18	(0.56)
Fund Age	0.01	(0.56)	-0.02	(-0.62)	0.06	(2.05)	0.03	(1.93)	0.04	(1.00)
Fund Family	0.16	(1.25)	0.07	(0.24)	0.02	(0.04)	-0.43	(-1.89)	0.24	(0.49)
Auditor	0.78	(4.62)	0.07	(0.25)	0.05	(0.11)	-0.36	(-1.89)	1.47	(2.52)
Registered Inv. Advisor	-0.16	(-1.54)	-0.05	(-0.24)	-0.52	(-1.88)	0.23	(1.33)	0.01	(0.02)
Closed to All	0.03	(0.23)	0.27	(0.91)	0.74	(1.76)	0.53	(2.18)	-0.09	(-0.28)
Minimum Initial	0.21	(3.21)	0.48	(2.50)	0.34	(1.39)	0.26	(2.30)	0.32	(1.11)
Minimum Initial Square	-0.01	(-2.36)	-0.06	(-2.05)	-0.02	(-0.55)	-0.01	(-0.95)	-0.02	(-0.62)
Sample size	668		182		122		268		94	
Pseudo R square	0.11		0.09		0.15		0.19		0.21	
McFadden R square	0.08		0.07		0.11		0.14		0.16	
LR test statistics	75.27	(0.00)	16.94	(0.08)	18.79	(0.04)	51.32	(0.00)	20.56	(0.02)



Source: Alternative Investment Solutions, Hedge Fund Research
CAGR: Compound Annual Growth Rate

Figure I. Growth of Assets under Management (2000 - 2008)

Chapter 4

Long-term Tactical Asset Allocation

Abstract

This paper evaluates a long-term tactical asset allocation strategy that takes advantages of perceived short-term opportunities in the capital market while benchmarked against the long-term strategic portfolio. The portfolio strategy makes the portfolio weights depend directly on changes in state variables, and uses an expanded asset menu in the Markowitz framework. The optimal portfolio consists of a benchmark portfolio that controls the active risk, and a pure overlay portfolio that adds active return to the whole portfolio. The empirical analysis shows that the equity market return momentum has significant predicting power for optimal portfolio weights. The information ratio of the optimal portfolio under the single-period strategy is 0.85. However, an investor may want to constrain the active portfolio strategy such that the tactical asset allocation's risk profile is matched with the strategic asset allocation in the long term. The certainty equivalent to a mean-variance investor with risk aversion of 10 is less than 10 basis points under the unconstrained portfolio strategy and 1.3% under the constrained portfolio strategy, both relative to passive benchmark indexing strategy. Hence, the investor is better off with the constrained active portfolio management.

4.1 Introduction

Despite increasing popularity of passive investment vehicles such as index mutual funds, exchange-traded funds (ETFs) and derivative instruments on the underlying indices, a significant number of stock and bond portfolios are actively managed in U.S. and other major finance centers. Many investment firms offer a spectrum of investment products from indexed portfolios and enhanced indexed portfolios to full-blown actively managed portfolios to suit each investor's need. An institutional investor typically follows a core-satellite approach under which the indexed and semi-active portfolios form the core holding and active managed portfolios stay around the core as satellites. The investor uses active portfolios to earn active returns while holding the indexed and semi-active portfolios to control the overall level of active risks. There are two main approaches of active management. The first approach is security selection that relies on skills in selecting under-valued or over-valued individual securities, mostly based on fundamental analysis and quantitative approach. The second approach involves intentional active decisions with respect to asset class timing (tactical asset allocation, TAA) and country/sector relative value strategy (global tactical asset allocation, GTAA). Both TAA and GTAA rely on some forecasting models to predict out-performing asset classes, sectors or countries, which drive asset allocation decisions. One popular (global) tactical asset allocation involves mean-variance optimization on the basis of forecasted expected returns and covariances. The difficulty associated with mean-variance optimization is the large sensitivity of portfolio weights to small changes in expected returns. A small estimation error in expected returns will create a large swing in portfolio weights and under-diversified portfolios (e.g. Michaud (1989)). One way to deal with the excess sensitivity problem is to use Black-Litterman model (Black and Litterman (1992), Litterman et al. (2003)) that combines market equilibrium with tactical views. The Black-Litterman model has some successes in creating a well-diversified portfolio, but it still counts on a forecasting model and subjective confidence assigned to investor views.

This paper applies and extends the conditional asset allocation method by Brandt and Santa-Clara (2006) to active portfolio management. The conditional portfolio strategy under Brandt and Santa-Clara (2006) does not rely on forecasting returns and covariances. Instead, the strategy makes portfolio weights dependent on a set of state variables. The method augments the asset space by using scaled returns (e.g. Cochrane (2005)) such that optimal solutions are still in the Markowitz paradigm. In addition, Brandt and Santa-Clara (2006) introduce the concept of timing portfolios, and use static Markowitz approach to solve a dynamic portfolio selection problem in a rather intuitive way. This paper builds on the insights from Cochrane (2005), Brandt and Santa-Clara (2006) and literature on active portfolio management to analyze active portfolio strategies. Instead of maximizing a portfolio's Sharpe ratio, an active manager chooses to maximize the information ratio if his performance is judged against a benchmark. The optimal portfolio is the sum of two portfolios: a benchmark portfolio and a pure overlay portfolio. The benchmark portfolio exactly tracks the active manager's benchmark to control overall active risks, while the pure overlay portfolio serves to earn active returns. The pure overlay portfolio is the optimal portfolio that maximizes the Sharpe ratio, without giving any consideration to the benchmark. The optimal portfolio has the highest active return possible at a given level of the tracking error.

Many studies suggest that on average active management is worse than a passive investment style after adjusted for fees and top performance does not persist.[1] While the statement is intuitive, it does not imply that active management has no value or active managers have no skills. Studies do show that the average active manager has a positive abnormal performance before fees, and active managers are able to capture the surplus, possibly due to the competition among investors for higher returns. Moreover, passively managed investment also incurs expenses, although they are lower than those charged by active managers. Therefore, the relevant benchmark for evaluating active managers should be fee-adjusted passive investment returns rather than indices or risk factors. From a normative perspective, active management serves to bring back deviations from market equilibrium or efficiency. From an empirical perspective, investors continue to invest with active managers in despite of the lack of strong out-performance evidence. Hedge funds, a special group of actively managed assets, attract increasingly large capital inflows in recent years, even though the majority of hedge funds are equity and fixed income funds. In addition, Berk and Green (2004) argue that the net return an investor earns from investing in actively managed funds reflects the skills of active managers. The lack of mutual fund performance persistence is simply the result of competitive financial markets.

This paper suggests one way to balance the long-term strategic asset allocations and the short-term tactical asset allocations. The strategic portfolio weights are set to reflect both the long-term capital market expectations and the investor's return and risk objectives, time horizon, liquidity preference and other constraints. A tactical asset allocation program creates tracking errors as a result of intended deviations from its benchmark. Even if a tactical asset allocation program is able to earn a higher risk-adjusted return, it may cause problems for the investor if average exposures to systematic risks under the tactical asset allocation differ sharply from the strategic asset allocations in the long run. This paper proposes one way to balance the desire to follow tactical asset allocations and the need to keep strategic asset allocations in check. The investor obtains higher welfare with the constrained active portfolio management than with the unconstrained one.

The empirical analysis in this paper uses the value weighted NYSE stock index and Ibbotson Associates government bonds as proxies for risky assets. Four state variables are used to predict portfolio weights: market dividend price ratio, short-term interest rate, default spread and aggregate stock market return momentum. The single-period conditional strategy creates an active portfolio consisting of a benchmark portfolio and a pure overlay portfolio. The pure overlay portfolio weights are linear in state variables,

[1] Sharpe (1991) argues that after adjusted for costs, average actively managed portfolios should underperform average passively managed portfolios. Gruber (1996), Jensen (1968), Malkiel (1995) and many other studies suggest that active mutual funds do not have stock-picking abilities. Barras, Scaillet and Wermers (2010), Chen, Jagadeesh and Wermers (2000), Daniel, Grinblatt, Titman, and Wermers (1997), Jones and Wermers (2011), Kosowski, Timmermann, and White (2006), Wermers (2000) support that value of active fund management and find persistence in superior alphas. Berk and Green (2004), Berk (2005) argue that the lack of performance persistence is the result of competition among investors.

such that any changes in state variables can cause changes in portfolio weights directly. The average allocations to stocks and bonds in the pure overlay portfolio are 15% and 58%, respectively. The pure overlay portfolio has the highest Sharpe ratio possible, but in isolation, the pure overlay portfolio may not create positive active returns. It is the combination of the benchmark portfolio and the pure overlay portfolio that will have the highest information ratio possible. The information ratio is 0.85, with a mean active return of 3.59% and a tracking error of 4.24%.[2]

I stress three main results from the empirical analysis. Firstly, the return momentum of the aggregate equity portfolio is a significant predictor under the conditional portfolio strategy, for both stocks and bonds. The momentum is defined as the cumulative return of the value weighted NYSE stocks over previous twelve months. Both coefficients of the return momentum in predicting stock and bond weights are statistically significant at the 5% level under the single-period conditional strategy. Holding everything else constant, a higher equity market return over the past twelve months predicts a higher allocation to stocks and a lower allocation to bonds for the coming month. One standard deviation of the aggregate return momentum will increase allocations to stocks by 7%, and decrease allocations to bonds by 33% in the pure overlay portfolio. In the univariate analysis, the equity market momentum based portfolio strategy generates the highest Sharpe ratio, outperforming the portfolio strategy based on other state variables such as dividend price ratio, short-term interest rate and default spread. The incremental portfolio Sharpe ratio/information ratio is 0.10 when the momentum is added to the set of state variables in determining optimal portfolio strategy. Hence, the economic significance of the return momentum in asset allocation decision is also present.

Secondly, the investor is not necessarily better off with the active portfolio strategy that maximizes the information ratio. The certainty equivalent to the mean-variance investor with risk aversion of 10 under the unconstrained active portfolio strategy is 9 bps. This means that the maximal amount of management fees paid to active portfolio managers is capped at 9 bps per year just for the investor to be indifferent between passive benchmark indexing and active portfolio management. The reason for a low investor welfare gain is that the unconstrained portfolio strategy gives too much exposure to risk assets on average, which leads to a portfolio volatility higher than the benchmark by 2.6%. Even though the tactical asset allocation is benchmarked against the strategic asset allocation, there is still a large mismatch between the actual risk exposures and the strategic asset allocation in the long run. An investor may choose to impose the constraint that the tactical asset allocation should converge to the strategic asset allocation in the long run. The restriction, however, negatively affects active performance as it leaves less

[2] According to Grinold and Kahn (2000), an information ratio of 0.50 will put the active manager to the top quintile. Moreover, the long-term information ratio of the best active managers using a GTAA strategy is between 0.50 and 1.00 in the study done by Carhart (Litterman et al. (2003)).

room for active portfolio management. The information losses of the constrained tactical asset allocations under the single-period strategy and multi-period strategy are 0.25 and 0.27, respectively. The active manager is not necessarily worse off with the constraint: he can potentially earn a higher management fee. Without imposing the constraint, the investor may opt for indexing if the active management fee is higher than 9 bps, leaving nothing for the manager. Even though the constrained portfolio gives a lower information ratio, the manager can charge a management fee as high as 1.32% per year.

Finally, a multi-period active portfolio strategy better exploits the time-series covariance of asset returns, and dominates a repeated single-period active portfolio strategy. While the average allocations under the multi-period strategy only slightly differ from those under the single-period strategy, it seems that the multi-period strategy is relatively more aggressive in reacting to changes in state variables, especially in the beginning of the investment horizon. Such aggressiveness leads to a mean active return of 5.09% and a tracking error of 5.05%, with an information ratio of 1.01. This implies that the multi-period strategy with the same target tracking error as the single-period strategy will generate a higher mean active return. The information loss, defined as the difference in the maximum information ratio achievable, for the single-period active portfolio is 0.16, relative to the multi-period active portfolio.

The empirical analysis also addresses the optimal level of tactical asset allocations and the sensitivity of active performance to benchmark choices. A decentralized tactical asset allocation where a stock and a bond manager run tactical asset allocation programs separately, is inferior to the centralized tactical asset allocation where all tactical decisions are placed in the hands of one active manager. A three-month decentralized tactical asset allocation has an information ratio of 0.88, with an information loss of 0.13. Finally, it turns out that active portfolio performance is invariant to choices of benchmarks. Choices of benchmarks will affect portfolio returns and volatilities, but not active performance.

The organization of the rest of the paper is as follows. I describe the portfolio optimization problem in the context of the active portfolio management and data in Section 4.2 and Section 4.3, respectively. In Section 4.4, I present empirical results under the single-period strategy and the multi-period strategy. Finally, Section 4.5 gives concluding remarks.

4.2 Method

4.2.1 Single-period Conditional Strategy

This paper defines the conditional strategy as the active portfolio strategy that uses time-varying instruments to dictate optimal portfolio weights. A risk-averse investor's portfolio consists of N risky assets and a risk-free asset. Denote the vector of portfolio weights on the risky assets at time t by w_t, the vector of gross returns of the risky assets by R_{t+1}, and the gross return of the risk-free asset by R_t^f. The portfolio excess return, denoted as r^p, is simply $w_t' r_{t+1}$, where $r_{t+1} = R_{t+1} - R_t^f$ is the vector of excess returns of the risky assets. The investor has finished long-term planning and determined the optimal asset mix in strategic asset allocations. In the short term, the investor attempts to exploit market opportunities and deliberately moves away from strategic portfolio weights temporarily. The investor can choose either an active portfolio manager to manage the mix of risky assets, or passively following an index portfolio. For the evaluation of active portfolio management, the active return is defined as the portfolio return in excess of the benchmark return, and the active risk or tracking error is defined as the volatility of active returns. The ratio of the active return to the tracking error is the information ratio. The active manager has a mean-variance function of active returns and aims to maximize the expected active return at any given level of the tracking error at time t:[3]

$$\max \ E(r_{t+1}^p - r_{t+1}^b) - \frac{\gamma}{2} Var\left(r_{t+1}^p - r_{t+1}^b\right), \tag{4.1}$$

where r_{t+1}^b is the benchmark portfolio return and γ is the coefficient of risk aversion of the manager.

The optimal portfolio weights are assumed to be linear in instruments z_t that include a constant and other time-varying state variables:

$$w_t = \alpha z_t. \tag{4.2}$$

The dimension of instruments z_t is $K \times 1$, and α is an $N \times K$ matrix. The instruments are normalized to have a zero mean such that the coefficients of the constant in the instrument set, or the intercept, are average allocations. The benchmark portfolio weights, denoted by w_b, are constant. To make it compatible, the dimension and the structure of w_b are the same as w_t:

$$w_b = b z_t. \tag{4.3}$$

where b is an $N \times K$ matrix with the first column being the benchmark portfolio weights for N risky assets and the remaining columns being zeros. Hence, the benchmark portfolio weights do not depend on state variables. The benchmark portfolio return r_{t+1}^b is equal to $(bz_t)' r_{t+1}$ and the active portfolio return r_{t+1}^p is equal to $(\alpha z_t)' r_{t+1}$.

The optimization problem as in expression (4.1) becomes

$$\max_{\alpha} E\left[(\alpha z_t - bz_t)' r_{t+1}\right] - \frac{\gamma}{2} Var\left[(\alpha z_t - bz_t)' r_{t+1}\right]. \tag{4.4}$$

[3]Equivalently, the optimization is to maximize the information ratio.

To derive the optimal portfolio weights from (4.4), first define the unconditional portfolio weights $\tilde{w}$ and the expanded set of excess returns or scaled excess returns $\tilde{r}_{t+1}$ as

$$\tilde{w} = vec(a) \tag{4.5}$$

$$\tilde{w}_b = vec(b), \tag{4.6}$$

$$\tilde{r}_{t+1} = z_t \otimes r_{t+1}, \tag{4.7}$$

where $vec(\alpha)$ is a vector that stacks the columns of the matrix α, $vec(b)$ is a vector that stacks the columns of the matrix b, and $\otimes$ is the Kronecker product. The scaled returns $\tilde{r}_{t+1}$ are the risky asset returns scaled by the state variables, or they are returns of the expanded asset menu. Portfolio weights $\tilde{w}$ are unconditional weights of the expanded assets and they are a set of parameters invariant to changes in instruments. We can use algebra to obtain $(\alpha z_t)' r_{t+1} = z_t' \alpha' r_{t+1} = vec(\alpha)'(z_t \otimes r_{t+1}) = \tilde{w}'\tilde{r}_{t+1}$ and similarly $(bz_t)' r_{t+1} = \tilde{w}_b' \tilde{r}_{t+1}$. Inserting those results into expression (4.4) leads to:

$$\max_{\tilde{w}} \; (\tilde{w} - \tilde{w}_b)' E(\tilde{r}_{t+1}) - \frac{\gamma}{2} (\tilde{w} - \tilde{w}_b)' Var(\tilde{r}_{t+1}) (\tilde{w} - \tilde{w}_b) \tag{4.8}$$

Therefore, to obtain optimal portfolio weights, the first step is to solve the unconditional portfolio weights $\tilde{w}$ under the conditional strategy, and the second step is to use the relation $w_t = \alpha z_t$. The unconditional portfolio weights come from the first-order condition of the optimization problem (4.8):

$$\tilde{w} = \gamma^{-1} Var[\tilde{r}_{t+1}]^{-1} E[\tilde{r}_{t+1}] + \tilde{w}_b \tag{4.9}$$

The optimal unconditional weights consist of two portfolios: $\gamma^{-1} Var[\tilde{r}_{t+1}]^{-1} E[\tilde{r}_{t+1}]$ and the unconditional benchmark portfolio weights $\tilde{w}_b$. The first portfolio is the Markowitz demand or the speculative demand by the investor. It is the optimal unconditional portfolio weights from the portfolio optimization problem when no benchmark is used:[4]

$$\max_{w_t^a} E(r_{t+1}^p) - \frac{\gamma}{2} Var\left(r_{t+1}^p\right), \tag{4.10}$$

Following the same method of derivation and algebra as in (4.2), (4.5) and (4.7), the optimal Markowitz demand is:

$$\tilde{w}_a = \gamma^{-1} Var[\tilde{r}_{t+1}]^{-1} E[\tilde{r}_{t+1}] \tag{4.11}$$

The optimal portfolio $\tilde{w}_a$ is the pure overlay portfolio that maximizes its Sharpe ratio.

The second portfolio in the right hand side of expression (4.9) is simply the benchmark portfolio and can be interpreted as the hedge demand to minimize the tracking error. It

[4]This is the optimization problem in Brandt and Santa-Clara (2006).

is straightforward to show that $\tilde{w}_b = Var[\tilde{r}_{t+1}]^{-1} Cov\left[\tilde{r}_{t+1}, r^b_{t+1}\right]$. If the benchmark return is the portfolio return of traded assets, an active manager can simply choose to hold the benchmark portfolio as the hedge portfolio. Hence, in the context of active portfolio management, the optimal portfolio strategy is to hold the benchmark portfolio and the pure overlay portfolio. The active manager uses the pure overlay portfolio to generate active returns and holds the benchmark portfolio to control the active risk.

4.2.2 Investor's Perspective

Previous subsection discussed the optimal portfolio strategy from the manager's perspective. The manager takes the benchmark portfolio as given and decides the trade-off between active returns and active risks. In this subsection, I will take the perspective from the investor. Suppose there are two sets of assets, A and B, with excess returns r_t^A and r_t^B. The set A contains a broader set of assets that can include both timing portfolios as well as stock selection abilities while the assets in B are stock and bond indices. If the investor would do the full optimization himself, the optimal portfolio would be given by

$$\begin{pmatrix} w_B \\ w_A \end{pmatrix} = \gamma^{-1} \begin{pmatrix} \Sigma_{BB} & \Sigma_{BA} \\ \Sigma_{AB} & \Sigma_{AA} \end{pmatrix}^{-1} \begin{pmatrix} \mu_B \\ \mu_A \end{pmatrix}. \tag{4.12}$$

Using partitioned inverses, this can be written as

$$\begin{aligned} w_B &= \gamma^{-1} \Omega_{ee}^{-1} \widetilde{\alpha}, \\ w_A &= \gamma^{-1} \Sigma_{AA}^{-1} \mu_A - \Sigma_{AA}^{-1} \Sigma_{AB} w_B, \end{aligned} \tag{4.13}$$

with

$$\begin{aligned} r_t^B &= \widetilde{\alpha} + B r_t^A + e_t, \\ \Omega_{ee} &= Var\left[e_t\right]. \end{aligned} \tag{4.14}$$

The interpretation of this is straightforward. First, from the regression, note that $\widetilde{\alpha} + e_t = r_t^B - B r_t^A$, which is the return on the optimally or minimum-variance hedged positions in B, using A as the hedge assets. Thus, if the investor knows that he is going to hedge the benchmarks B optimally with the assets in A, the expected returns are $\widetilde{\alpha}$ and the risk is given by the covariance matrix Ω_{ee}. This means that the investor chooses his positions in B assuming he will optimally hedge these positions. Next, the positions in A are two-fold: a pure investment position in A, plus the hedge demand to hedge the risk of B.

In the current setting, r_t^B is also a subset of r_t^A, implying that a perfect hedge is possible. This means that $\widetilde{\alpha} = 0$ and $\Omega_{ee} = 0$, implying that optimal $w_B = 0$ as well. Hence, if both the investor and the active manager have the same risk aversion, the optimal demand of the investor w_A equals the choice of the manager.

Utility losses can thus result from:

- unequal risk aversions: $\gamma_{Investor} \neq \gamma_{Manager}$,
- The benchmark is not equal to zero, $w_B \neq 0$,
- The investor chooses w_B based on Σ_{BB} and μ_B instead of Ω_{ee} and a while delegating to the fund manager.

4.2.3 Multi-period Conditional Strategy

It is likely that a portfolio manager with a tactical asset allocation program has a multi-period investment horizon. He can use the single-period solution period by period, but a multi-period approach may be more efficient as it exploits time-series covariance structures of asset returns. There are a large number of studies on asset return predictability that has implications for both strategic and tactical asset allocations (e.g. Campbell and Viceira (1999), (2002)). Therefore, this paper also extends the multi-period method by Brandt and Santa-Clara (2006) to the context of active portfolio management. For instance, for an active manager with a two-period horizon, the two-period excess return of the portfolio with N risky assets and a risk-free asset is:

$$\begin{aligned} r^p_{t\to t+2} &= \left(R^f_t + w'_t r_{t+1}\right)\left(R^f_{t+1} + w'_{t+1} r_{t+2}\right) - R^f_t R^f_{t+1} \\ &= w'_t\left(R^f_{t+1} r_{t+1}\right) + w'_{t+1}\left(R^f_t r_{t+2}\right) + \left(w'_t r_{t+1}\right)\left(w'_{t+1} r_{t+2}\right) \\ &\approx w'_t\left(R^f_{t+1} r_{t+1}\right) + w'_{t+1}\left(R^f_t r_{t+2}\right). \end{aligned} \tag{4.15}$$

Because r_{t+1} and r_{t+2} are excess returns, the product $\left(w'_t r_{t+1}\right)\left(w'_{t+1} r_{t+2}\right)$ is very small at short horizons, so the excess portfolio return over two periods is approximately the sum of $w'_t\left(R^f_{t+1} r_{t+1}\right)$ and $w'_{t+1}\left(R^f_t r_{t+2}\right)$. Brandt and Santa-Clara (2006) interpret $w'_t\left(R^f_{t+1} r_{t+1}\right)$ and $w'_{t+1}\left(R^f_t r_{t+2}\right)$ as "timing portfolios". First, $w'_t\left(R^f_{t+1} r_{t+1}\right)$ is the two-period excess return from investing in risky assets at time t and afterwards investing in the risk-free asset. Second, $w'_{t+1}\left(R^f_t r_{t+2}\right)$ is the two-period excess return from investing in the risk-free asset at time t and then investing in risky assets at time $t+1$.

The benchmarks in the first and second period, denoted by $w_{b,t}$ and $w_{b,t+1}$, need not be the same, but normally they are constant in the short run. The two-period excess return of the benchmark $r^b_{t\to t+2}$ is approximately the sum of $w'_{b,t}\left(R^f_{t+1} r_{t+1}\right)$ and $w'_{b,t+1}\left(R^f_t r_{t+2}\right)$. The manager's problem is to maximize the two-period information ratio,

$$\max \; E_t(r^p_{t\to t+2} - r^b_{t\to t+2}) - \frac{\gamma}{2} Var_t\left(r^p_{t\to t+2} - r^b_{t\to t+2}\right). \tag{4.16}$$

The difference between expression (4.1) and (4.16) is that the latter searches for the optimal portfolios over two periods. But the solution strategy under the two-period optimization problem follows its single-period counterpart. Both the optimal portfolio weights and benchmark portfolio weights are linear combinations of instruments and a set of unconditional portfolio weights in each period, as

$$w_t = \alpha_1 z_t, \tag{4.17}$$

$$w_{t+1} = \alpha_2 z_{t+1}, \tag{4.18}$$

$$w_{b,t} = b_1 z_t, \tag{4.19}$$

$$w_{b,t+1} = b_2 z_{t+1}. \tag{4.20}$$

The parameters α_1, α_2, b_1 and b_2 are $N \times K$ matrices. In addition, the first columns in b_1 and b_2 are benchmark portfolio weights in the first and the second period, respectively. All remaining columns in b_1 and b_2 are zeros such that benchmark portfolio weights are independent of state variables.

The investment menu becomes a set of timing portfolios and scaled returns. The optimal unconditional portfolio weights are:

$$\tilde{w} = \gamma^{-1} Var[\tilde{r}_{t \to t+2}]^{-1} E\left[\tilde{r}_{t \to t+2}\right] + \tilde{w}_b \tag{4.21}$$

where

$$\tilde{w} = \begin{pmatrix} vec(\alpha_1) \\ vec(\alpha_2) \end{pmatrix}, \tag{4.22}$$

$$\tilde{w}_b = \begin{pmatrix} vec(b_1) \\ vec(b_2) \end{pmatrix}, \tag{4.23}$$

$$\tilde{r}_{t \to t+2} = \begin{pmatrix} R^f_{t+1} \tilde{r}_{t+1} \\ R^f_t \tilde{r}_{t+2} \end{pmatrix}. \tag{4.24}$$

The scaled returns $\tilde{r}_{t+1}$ and $\tilde{r}_{t+2}$ are defined by the equation (4.7). Thus, the two-period portfolio has the same structure as the single-period portfolio. In each period, the optimal portfolio consists of a pure overlay portfolio and a benchmark portfolio. The pure overlay portfolios in two periods are speculative demands or Markowitz demands that have the goal to earn active returns, while the benchmark portfolios bring down the active risk. The combination of pure overlay portfolios and benchmark portfolios maximizes the two-period information ratio.

The solution to the two-period asset allocation problem above can be easily extended to a general setting of L periods:

$$\tilde{w} = \gamma^{-1} Var[\tilde{r}_{t \to t+L}]^{-1} E\left[\tilde{r}_{t \to t+L}\right] + \tilde{w}_b \tag{4.25}$$

4.2.4 Portfolio Turnover and Transaction Cost

An active portfolio requires more frequent trading than passively held benchmark portfolios, and results in higher portfolio turnovers and transaction costs, driven by the pure overlay portfolio. However, while an investor would normally hold underlying assets to form the benchmark portfolio, the implementation of the pure overlay portfolio often uses derivative instruments such as futures and forwards. Derivative instruments, relative to underlying cash instruments, may have a high liquidity and low transaction costs, and provide an easy way to use leverage. Disadvantages of derivative instruments include a finite life and lower liquidity for contracts with relatively remote maturities. This is not a problem for a pure overlay portfolio that makes use of short-term, highly liquid derivative contracts. Since the investment horizon in a tactical asset allocation is short, it is not necessary to trade contracts with a long maturity. The disadvantages of derivative instruments imply that holding underlying cash instruments may be a better option for long-term strategic portfolios or benchmark portfolios. An alternative cost-effective way to implement an active portfolio strategy is to use a low cost index mutual fund or an index ETF. Finally, unequal growth rates among different assets will cause drifts in an asset mix. After determining the optimal pure overlay portfolio, actual trading should take into account any drift in the benchmark portfolio.

Moreover, transaction costs can be taken into account in the portfolio optimization. For instance, in the single-period setting, a portfolio manager can adjust the optimization problem to achieve the optimal cost-adjusted information ratio:

$$\max_{\tilde{w}} \ (\tilde{w} - \tilde{w}_b)' E(\tilde{r}_{t+1}) - \frac{\gamma}{2} (\tilde{w} - \tilde{w}_b)' Var(\tilde{r}_{t+1}) (\tilde{w} - \tilde{w}_b) - \lambda c (\Delta \tilde{w}) \tag{4.26}$$

where λ represents the transaction cost aversion and $c(\cdot)$ is the transaction cost function. Analytical solutions are no longer possible unless the cost function is linear. But given a cost function, optimization can be easily programmed and implemented.

4.3 Data

We obtain historical returns of the value weighted NYSE stock index from CRSP and the index of medium-term Treasuries from Ibbotson Associates as proxies for stocks and bonds, respectively. We include four state variables: the short-term interest rate, the market dividend-price ratio, the default spread, and the momentum.[5] For the short-term interest rate, the U.S. 30-day Treasury bill is used. The market dividend-price ratio is based on the value weighted NYSE stock index, calculated as the ratio of the sum of

[5] The term spread is not included for parsimony, but the results with term spread as an additional predictor are also available upon request. Coefficients of the term spread are never signficiant at any conventional level, and adding this variable has little marginal impact on portfolio charactistics and active portfolio performance.

dividends over past twelve months to the current NYSE stock index level. The default spread is the difference between the Moody's BAA corporate bond yield and the AAA corporate bond yield. The equity market momentum is the return of the value weighted NYSE stock over previous twelve months. Many other state variables that potentially help predict asset returns are available, such as smooth earning-price ratio, consumption-wealth ratio, ROE, inflation, and potentially many others.[6]

The sample period is from January 1927 to December 2008. Table I describes the data. Mean, median, standard deviation and Sharpe ratio are annualized. The mean return and volatility of stocks are 10.77% and 18.57%, respectively, while the Sharpe ratio of stocks is 0.38. Compared to bond returns, stock returns have a greater mean as well as a greater volatility. Nevertheless, the Sharpe ratio of stocks is slightly lower than that of bonds. The government bonds have a mean return of 5.40% and a volatility of 4.44%. The Sharpe ratio of bonds is 0.39, not too different from that of stocks. The relative low coefficient of correlation between stock returns and bond returns indicates a good potential of portfolio diversification benefits.

The historical mean market dividend-price ratio is about 3.84%, with a maximum of 12.90% and a minimum of 1.44%. The correlation of the market dividend-price ratio with stocks or bonds is quite low. In addition, there is a low correlation of the short-term interest rate with stocks while the correlation of the short-term interest rate with bonds is moderate. The mean short-term interest rate is only lower than the medium-term government bond returns by 1.75%. Finally, the correlations among state variables are moderate, except for the correlation between the default spread and the dividend-price ratio, with a coefficient of correlation of 0.50.

4.4 Empirical Results

This paper assumes that the investor has finished his long-term financial planning and determined the long-term, strategic portfolio weights. The investor considers delegated portfolio management to an active manager. He only hires the manager if his utility is higher with active portfolio management after fees. Section 4.4.2 addresses the effect of active portfolio management on the investor welfare by calculating the certainty equivalent to the investor with various active portfolio strategies relative to the benchmark indexing. To evaluate active performance and avoid a large mismatch between the asset allocation and desired systematic risk exposures by the investor, a benchmark is used. The goal of the active manager is to earn an active return while keeping the tracking error as low as possible. Although the benchmark is not necessarily the same as the strategic portfolio,

[6] Goyal and Welch (2008), and Campbell and Thompson (2008) include a comprehensive list of these variables along with some others as predictors used in predictability studies, e.g. Campbell (1987), Campbell and Shiller (1988), Campbell and Viceira, (1999), (2002), Cochrane (2008).

it is intuitive to use strategic portfolio weights as the portfolio weights in the benchmark. Transaction costs are excluded in the analysis, but can be included once an active manager specifies the cost function.

4.4.1 Single-period Conditional Strategy

Table II reports the single-period optimal portfolios under the conditional strategy. The manager's risk aversion is 20. Note that the risk aversion with respect to the active risk is different from the risk aversion with respect to the standard deviation of portfolio returns. An active manager is supposed to be more risk averse when it comes to the active risk, as he can always invest passively in the benchmark portfolio. An active manager is judged against a benchmark. Hence, if he believes that the active return is hard to find, it is more difficult to convince him to take a large active risk. Panel A reports the portfolio weights of stocks and bonds for the benchmark portfolio, the pure overlay portfolio and the optimal portfolio for a manager with one-month investment horizon and a manager with one-quarter investment horizon. The optimal portfolio is the sum of the benchmark portfolio and the pure overlay portfolio. Panel B and Panel C describe the active portfolio performance and portfolio characteristics. All performance measures and portfolio measures are annualized.

Portfolio Strategy

For the portfolio with a one-month investment horizon, the benchmark portfolio invests 60% in stocks and 40% in bonds. Section 4.4.5 will discuss the sensitivity of portfolio outcomes to benchmark choices. For a tactical asset allocation with one-month investment horizon, the pure overlay portfolio on average (intercepts) invests 15% and 58% in stocks and bonds, respectively. The total portfolio weights of stocks and bonds are 75% and 98%, accordingly. Thus, the total portfolio represents a small deviation in stock positions and a large deviation in bond positions from those in the benchmark portfolio. Table II also reports the optimal portfolio strategy when the investment horizon is one quarter. The optimal portfolio on average allocates 74% to stocks and 88% to bonds.

One perspective to evaluate the effectiveness of the conditional strategy is to test whether the state variables predict portfolio weights in a statistically significant way. If they are useless, their slope coefficients will be small relative to the standard errors of estimates, such that it is not possible to reject the null hypothesis that the coefficients are jointly equal to zero. One alternative test procedure is to test the coefficients of individual state variables, but testing them jointly provides an overall assessment of the predictive power of state variables. Panel A reports the p-value of the test against the null hypothesis that all slope coefficients are jointly equal to zero. The p-value is zero under the one-month portfolio strategy, and 0.01 under the one-quarter portfolio strategy, indicating that the state variables jointly predict the pure overlay portfolio weights and thus the optimal portfolio weights, because the two portfolios have the same slope coefficients and standard errors of estimates. Many state variables are individually significant in predicting portfolio weights of stocks and bonds. For instance, under the one-month investment horizon, all state variables are individually significant in predicting changes in the allocation to stocks. Moreover, both the short-term interest rate and the stock market momentum are statistically significant in predicting bond allocations.

Additionally, it is interesting to examine the sign of coefficients, since they represent the direction of trading in pure overlay positions. Using quarterly data instead of monthly data does not affect the signs of coefficients, but allocation decision becomes relatively less sensitive to changes in the state variables. When the active manager observes an increase in the market dividend-price ratio over the last period, holding everything else constant, he would increase allocations to stocks and bonds in the optimal portfolio. An increase in the short-term interest rate would decrease allocations to stocks and bonds. This negative impact of the rate hike on equity markets and long-term government bonds is intuitively clear and observed in practice. An increase in default spread is very often the signal of a weakening economy or negative news in companies or banks, and it is associated with a decreasing allocation to stocks in the pure overlay portfolio. On the other hand, flight to quality by investors during turmoil times with increasing defaults in corporate bonds implies that government bonds will be a favorable asset class. The analysis also includes the return of the value weighted NYSE stocks over previous twelve months as a state variable, which is considered as a momentum indicator in the aggregate stock market. An increase in the momentum indicator is associated with an increase in allocations to stocks and a decrease in allocations to bonds. One standard deviation increase in the aggregate return momentum will increase allocations to stocks by 7%, and decrease allocations to bonds by 33% in the pure overlay portfolio or total portfolio.

Active Portfolio Management

The one-month optimal portfolio has an annualized mean active return of 3.59%, and a tracking error of 4.24%. The information ratio, a risk-adjusted active performance measure, is 0.85 for the total portfolio. For the one-quarter optimal portfolio, the average active return and tracking errors are 2.59% and 3.60%, respectively, resulting in an information ratio of 0.72. Note that the active performance measures and the portfolio efficiency measures are closely linked. Denote the excess portfolio return of the pure overlay portfolio by r^a_{t+1}. The excess portfolio return of the optimal portfolio, r^p_{t+1}, is the sum of the excess returns of the benchmark portfolio, r^b_{t+1}, and the pure overlay portfolio, r^a_{t+1}. The active return for a portfolio is the portfolio return excess of the benchmark portfolio return. Hence, the active return of the optimal portfolio, $r^p_{A,t+1}$, and the active return of the pure overlay portfolio, $r^a_{A,t+1}$, can be defined as:

$$r^p_{A,t+1} = r^p_{t+1} - r^b_{t+1} \quad (4.27)$$

$$r^a_{A,t+1} = r^a_{t+1} - r^b_{t+1} \quad (4.28)$$

Since $r^p_{t+1} = r^a_{t+1} + r^b_{t+1}$, the active return in equation (4.27) becomes $r^p_{A,t+1} = r^a_{t+1}$. As a result, the mean excess return, volatility and the Sharpe ratio of the pure overlay portfolio are the mean active return, tracking error and the information ratio of the optimal portfolio. The manager earns active return via the pure overlay portfolio, while holding the benchmark portfolio to reduce the mismatch between the total portfolio and the benchmark portfolio. As long as the pure overlay portfolio earns a positive mean return, the mean active return of the total portfolio is also positive. However, if the manager, who is still judged against the benchmark portfolio, chooses to hold only the pure overlay portfolio, he will have an active return $r^a_{A,t+1} = r^a_{t+1} - r^b_{t+1}$. A positive return of the pure overlay portfolio does not guarantee a positive active return $r^a_{A,t+1}$, because the pure overlay portfolio may not have a return higher than the benchmark portfolio return. This explains why the pure overlay portfolio, when treated in isolation, has a negative mean active return of -1.39% in Table II, and a tracking error of 9.98%. An active manager who focuses on Sharpe ratios and ignores the benchmark will find himself in a hot place at the year-end performance review.

The certainty equivalent to the manager for the benchmark portfolio is the manager's utility from holding the optimal portfolio.[7] Equivalently, it is the risk-adjusted active return to the manager by deviating from the benchmark. The certainty equivalent for the pure overlay portfolio is the difference between the utility of the active manager holding the pure overlay portfolio and the utility of the active manager holding the benchmark portfolio. Given the large tracking error and the negative active return associated with the pure overlay portfolio in isolation, it is not surprising to observe a large, negative certainty equivalent for the pure overlay portfolio, with a value of -11.35% per year for the one-month portfolio strategy. On the other hand, the benchmark portfolio has a certainty equivalent of 1.79%. The active manager is indifferent between holding the benchmark portfolio and holding the optimal portfolio if he is required to pay 1.79% in fees to hold the latter portfolio. Hence, 1.79% is the maximal amount of transaction costs that the manager can afford in implementing the active portfolio strategy. For the one-quarter portfolio strategy, The benchmark portfolio has a certainty equivalent of 1.29%, about 50 bps lower than its counterpart in the one-month portfolio.

[7] Certainty equivalents to the manager are calculated based on the manager's utility over active returns while certainty equivalents to the investor are based on the investor's utility over portfolio returns. Section 4.4.2 reports certainty equivalents to the investor.

Portfolio Characteristics

Panel C gives a snapshot of portfolio characteristics. If we start with the one-month portfolio strategy, the benchmark portfolio has a mean excess return and a volatility of 4.98% and 11.46%, respectively. The pure overlay portfolio has a low mean return of 3.59%, but its volatility is also relatively low at 4.24%, which are the mean active return and the tracking error of the total, optimal portfolio, respectively. The mean return of the optimal portfolio is the sum of the mean returns of the benchmark portfolio and the pure overlay portfolio, while the volatility of the optimal portfolio is lower than the sum of the volatilities of the benchmark portfolio and the overlay portfolio because the coefficient of correlation between the benchmark portfolio return and the pure overlay portfolio return is less than one. The Sharpe ratio of the optimal portfolio is 0.61, about 40% higher than the Sharpe ratio of 0.43 for the benchmark portfolio. Among the three portfolios, the pure overlay portfolio has the highest Sharpe ratio. This is not surprising as the pure overlay portfolio is the portfolio that maximizes the Sharpe ratio of the portfolio of stocks and bonds. A combination of the pure overlay portfolio and the benchmark portfolio will not have the highest Sharpe ratio, but it will lead to the greatest information ratio. Compared to the one-month optimal portfolio, the active one-quarter portfolio has a lower mean excess return and a higher volatility.

4.4.2 Long-term Systematic Risk Exposures Mismatch vs. Short-term Active Risk Minimization

Systematic Risk Match Portfolios

The optimal portfolios in Section 4.4.1, even though investing in the benchmark portfolio in order to avoid a large mismatch between the asset allocations and the desired systematic risk exposures, have one shortcoming. For the one-month portfolio, the average allocations to stocks and bonds are 75% and 98%. This implies that if an investor uses an active manager to implement the active portfolio program as described in Table II, he will end up with exposures to stocks and bonds different from the desired, strategic asset allocations or benchmark portfolios in the long-term. One way to limit the mismatch between the asset allocations and the desired long-term systematic risk exposures is to require that the average allocations to stocks and bonds in the pure overlay portfolio should be close to zero. Month by month, the optimal portfolio can deviate from the benchmark portfolio or the strategic asset allocations in the short-term, but the tactical asset allocation program in the long-term should have average allocations to stocks and bonds close to those in the benchmark portfolio. This can be accomplished by restricting the intercept term of the instruments used to determine tactical portfolio weights. The question is: how much the active performance and investor welfare are affected by imposing constraints on the average allocations to stocks and bonds?

Table III reports the optimal portfolios with the constraints that the average allocations to stocks and bonds in optimal portfolios are within a boundary. We name those portfolios 'Systematic Risk Match Portfolios'. Table III reports three systematic risk match optimal portfolios in the left panel. The 'Zero Avg Overlay' optimal portfolio is the optimal portfolio with the most restrictive constraint: the average stock and bond allocations in the pure overlay portfolio are zero. The average allocations to stocks and bonds in the optimal portfolio are 60% and 40%, same as those in the benchmark portfolio. The mean active return of the 'Zero Avg Overlay' optimal portfolio is 1.78%, which is substantially lower than the mean active return of 3.59% for the unrestricted optimal portfolio in Table II. The restriction also lowers the tracking error of the optimal portfolio from 4.24% to 2.94%. The net effect is that the information ratio of the 'Zero Avg Overlay' optimal portfolio is lower than that of the unrestricted optimal portfolio by 0.25. The '5% Avg Overlay' and the '10% Avg Overlay' are the optimal portfolios with the constraints that the average allocations to each risky asset class in the pure overlay portfolios are within in the ranges of 5% and 10%, respectively. One observation is that the optimal portfolios' mean active returns, tracking errors and information ratios, and the certainty equivalents for the benchmark portfolios are increasing with the ranges of allowed average deviations from the benchmark portfolio weights. The fewer restrictions on an active manager, the more room is there for active portfolio management. In addition, the mean excess returns, volatilities and Sharpe ratios of the optimal portfolios are also higher when the manager can deviate from the benchmark portfolio by a greater extent. However, differences among three Sharpe ratios are relatively small, with a 0.01 increment moving from the 'Zero Avg Overlay' optimal portfolio to the '10% Avg Overlay' optimal portfolio. In contrast, the information ratio of the 'Zero Avg Overlay' optimal portfolio is lower than that of the '10% Avg Overlay' optimal portfolio by 0.18. Therefore, the restrictions on average allocations to stocks and bonds have more pronounced impact on the active portfolio performance than they do on the portfolio efficiency.

Active Risk Minimization Portfolios

Putting limits on average allocations to stocks and bonds in active portfolios severely restricts an active manager's ability to earn active returns at a given level of the tracking error. To assess the loss of active returns as a result of imposing constraints on average allocations, an active manager with the same risk budget as the manager of systematic risk match portfolios can compute the costs associated with the allocation constraints. Specifically, Table III reports three 'Active Risk Minimization Portfolios'. The purpose of creating active risk minimization portfolios is to answer the following question: if an active manager has the same risk budget as one of the systematic risk match portfolio managers, how much active return can he generate when there are no restrictions on the average allocations to stocks and bonds? The tracking errors of the 'Zero Avg Overlay' optimal portfolio, the '5% Avg Overlay' optimal portfolio and the '10% Avg Overlay' optimal portfolio are 2.94%, 3.09% and 3.41%. The corresponding active risk minimization portfolios are obtained by maximizing the portfolio's information ratio or the utility function in expression (1), subject to the constraints that the tracking errors of the portfolios are 2.94%, 3.09% and 3.41%. There are no restrictions on average allocations to stocks and bonds for the active risk minimization portfolios.

The active risk minimization portfolio with a target tracking error of 2.94% has a mean active return of 2.48%, which is greater than the mean active return of 1.78% for the 'Zero Avg Overlay' optimal portfolio with the same tracking error. The difference in the mean active returns is 70 bps, considerably large relative to the mean active return of the 'Zero Avg Overlay' optimal portfolio. The 70 bps active return is the loss to the manager with 'Zero Avg Overlay' mandate. Such a loss is much lower when a greater degree of long-term systematic risk mismatch is allowed. The losses are 38 bps and 20 bps when average allocations to each risky asset are allowed to fluctuate in the ranges of 5% and 10%, respectively.

Figure I and Figure II display the allocations to stocks and bonds in the optimal portfolios of stocks and bonds, respectively. The 'Unrestricted' portfolio is the portfolio without any restrictions on the intercept or a target tracking error of the total portfolio. The 'Zero Overlay' portfolio is the portfolio with zero average allocations in the overlay portfolio such that the average allocations in the total optimal portfolio are the same as those in the benchmark portfolio. The 'Active Risk Minimization' portfolio is the optimal portfolio with the target tracking error equal to the tracking error of the 'Zero Overlay' total portfolio. Figure I indicates that the allocations to stocks in the 'Unrestricted' portfolio are in the range of 50% to 100% for most of the period, with a mean of 75% and a standard deviation of 13%. However, it is obvious that the allocations in stocks are often above 60%, creating a small mismatch between the long-term average allocation to stocks and the benchmark portfolio weight of stocks over the long-term. The 'Zero Overlay' portfolio reduces the systematic risk mismatch problem by forcing the average allocations to converge to the benchmark weight. The 'Active Risk Minimization' portfolio also reduces the systematic risk mismatch to some extent, as the average allocation to stocks in the portfolio is 70%. In contrast with the relatively smoothed pattern of stock allocations, there are large fluctuations in bond allocations over time, suggesting that bond investments are more sensitive to changes in the state variables. From Figure II, it can be observed that the allocations to bonds in the 'Unrestricted' portfolio are more than 100% in early years, reaching as high as 450% during the Great Depression. The standard deviation of bond allocations is 57%, while the average allocation exceeds the benchmark portfolio weight of bonds by 58%. The 'Zero Overlay' portfolio brings down the average allocations to bonds, but the fluctuation in bond allocations remains large. Finally, the

'Active Risk Minimization' portfolio has some successes in reducing the volatility of bond allocations, although the systematic risk mismatch is still quite large with this portfolio.

One remark on the active risk minimization portfolios is that they have the highest information ratio possible, equal to the information ratio of the unrestricted optimal portfolio. The restriction on the maximal tracking errors does not affect information ratios. An active manager simply moves down the active return spectrum when the target tracking error is lower. The difference between the maximum information ratio and the actual information ratio is called the information loss in active portfolio management. From an active portfolio manager's perspective, it is more efficient to implement the active risk minimization programs for a given target level of the tracking error, in order to avoid an information loss. However, those portfolios create the systematic risk mismatch problem for the investor, especially with respect to the exposure to bond risks. The average allocations to bonds represent 79%, 81% and 86% in the three active risk minimization portfolios, respectively. Therefore, there is a trade-off between the short-term active portfolio performance maximization and the long-term systematic risk match. For a given risk budget, an investor's degree of aversion to the long-term systematic risk mismatch would dictate the choice of the trade-off.

Portfolio Strategy and Investor Welfare

Unrestricted active portfolio strategy, which achieves the highest information ratio possible, creates a mismatch in systematic risk exposures for investors. We can quantify the effect of various active portfolio strategies on the welfare of the investor by calculating the certainty equivalent to the investor. We assume that the investor has a standard mean-variance utility over portfolio returns. He is only interested in investing via an active portfolio manager if that leads to a higher investor's utility. The certainty equivalent to the investor is defined as the difference in his utilities $U(w^A, \gamma^I)$ and $U(w^b, \gamma^I)$, where w^A represents active portfolio strategy, w^b is benchmark indexing strategy, and γ^I is risk aversion of the investor. This certainty equivalent can be interpreted as the maximal fee that the investor is willing to pay to the active portfolio manager. If the certainty equivalent is lower than the management fee charged, then the investor is better off with passive benchmark indexing.

Figure III plots the certainty equivalent to the investor with various degrees of risk aversion while fixing the active portfolio manager's risk aversion at 20. The investor can choose among four portfolio strategies: unconstrained active portfolio, active portfolio with zero average overlay, active portfolio with deviation from the benchmark is bounded by 5%, and active portfolio with maximal 10% deviation from the benchmark. Although information ratios and Sharpe ratios of unconstrained active portfolios (Table II) are quite impressive, Figure III suggests that the investor may not hire the active manager regardless of his risk aversion. The certainty equivalent is only 18 bps if the investor has risk aversion of 5, monotonically decreasing with risk aversion of the investor. The unconstrained active portfolio strategy would give too much exposure to risky assets and an increase in portfolio volatility is not compensated enough by an increase in portfolio returns. When constraints on average allocations to stocks and bonds are imposed in active portfolios, higher certainty equivalents are realized. For instance, the portfolio strategy with zero average overlay will give the investor with risk aversion of 10 utility gains of 1.32% relative to the passive benchmark indexing strategy.

A more restrictive constraint on average allocations will lead to a higher utility gain to the investor, while the effect goes to the opposite for the portfolio manager. The certainty equivalent to the active manager based on active returns or information ratio is the highest when the portfolio strategy is free of constraints on average allocations to stocks and bonds. Hence, unless the investor puts a specific mandate on average allocations to stocks and bonds or maximal portfolio volatility, the manager would not follow the optimal constrained portfolio strategy. Suppose that the portfolio manager would charge 1% management fee per year, then the investor, regardless of risk aversion, is better off with passive benchmark indexing rather than the unconstrained active portfolio strategy. A less risk averse investor may find it beneficial to invest in the active portfolio manager with zero average overlay targets, as long as the certainty equivalent is greater than management fees.

Figure IV plots the certainty equivalent to the investor with risk aversion of 10 while varying the active portfolio manager's risk aversion. Since the overlay portfolio is proportional to the manager's risk aversion, the graph depicts the sensitivity of investor welfare to the aggressiveness of the active portfolio strategy. If the active manager is a more aggressive one, the investor can suffer a lot as the portfolio return is too volatile. On the other hand, if the active manager is too conservative, the decrease in returns may outweigh the reduction in volatility. In any case, the unconstrained portfolio strategy does not benefit the investor too much, especially when the manager is too aggressive. From the investor's perspective, the constrained portfolio strategy with zero average overlay dominates the remaining three strategies.

4.4.3 Multi-period Conditional Strategy

Portfolio Strategy

One-period portfolio optimization on the basis of the Markowitz framework is very popular in practice. For a tactical asset allocation manager with a multi-period investment horizon, he could adopt the single-period asset allocation method to exploit the short-term opportunities period by period until the end of the investment horizon. However, a multi-period active portfolio strategy can better exploit time-series covariance structure of asset returns and use available information in a more efficient way.

The multi-period asset allocation analysis in this section assumes a three-month investment horizon and a risk-aversion of 20. Panel A in Table IV shows the optimal portfolio strategy when the benchmark portfolio consists of 60% in stocks and 40% in bonds in every month. Although it is possible to require that an active manager should follow a different benchmark month by month, we will show that the active portfolio strategy and active performance are invariant to the choice of benchmarks in Section 4.4.5. The average allocations to stocks in the first, second and third month in the pure overlay portfolio are 18%, 18% and 16%, slightly decreasing over time. On the other hand, the allocations to bonds increase from 58% in the first to 62% in the second month, and then decrease to 61% in the third month. The pure overlay portfolio, therefore, increases exposures to less risky bonds and decreases exposures to risky stocks as the investment horizon approaches. Such horizon effects on average allocations, albeit small in such a short investment horizon, may help an active manager to improve active performance. The p-value at the bottom of Panel A indicates that state variables are statistically significant in predicting portfolio weights. One observation is that the sensitivities of the pure overlay portfolio to changes in the state variables are different across the investment horizon, but the signs of coefficients are consistent over time.

It is interesting to examine differences in the portfolio strategy between the single-period and the multi-period asset allocations. Assuming an active manager has a three-month investment horizon, but instead of following the multi-period portfolio strategy, he decides to simply follow the single-period strategy in every month (monthly strategy, the left panel in Table II) or the single-period strategy using quarterly returns (quarterly strategy, the right panel in Table II). The average allocations to stocks and bonds in the pure overlay portfolio under the single-period, monthly portfolio strategy for three months are 15% and 58%, respectively. Meanwhile, the average allocations to stocks and bonds under the single-period, quarterly strategy are 14% and 48%, with a sum of 62% invested in the risky assets. The average allocations to stocks and bonds in the pure overlay portfolio under the multi-period portfolio strategy are 17% and 60%.[8] The sum of average allocations to stocks and bonds under the multi-period portfolio strategy, 77%, is greater than that under the single-period strategy. It seems that the multi-period portfolio strategy tends to yield a more aggressive pure overlay portfolio. One question arises: does a more aggressive multi-period portfolio strategy, relative to the single-period strategies, lead to better active performance?

Active Portfolio Performance and Portfolio Characteristics

Panel B in Table IV gives an overview of active portfolio performance under the multi-period portfolio strategy. The pure overlay portfolio itself has a negative mean active

[8]The average allocations to stocks or bonds in the three-month portfolio are the average of three intercepts in Panel A in Table IV.

return and a huge tracking error per annum. But when it is combined with the benchmark portfolio to form the optimal portfolio, the mean active return is 5.09%, which is 1.50% higher than the mean active return of the optimal portfolio under the single-period monthly strategy. The optimal portfolio under the multi-period strategy has a larger tracking error of 5.05%, compared to a tracking error of 4.24% under the single-period monthly strategy. The higher tracking error of the multi-period optimal portfolio is well compensated, as its information ratio of 1.01 is greater than that of the single-period optimal portfolio by 0.16. This implies that if an active manager with a risk budget of 4.24%, using the multi-period portfolio strategy instead of the single-period strategy will gain an additional mean active return of 69 bps (4.28% vs. 3.59%).[9] Furthermore, the certainty equivalent of the benchmark portfolio is 2.54% under the multi-period strategy. The utility gains from following the multi-period strategy instead of passively investing in the benchmark portfolio are non-trivial.

Panel C in Table IV shows that the optimal portfolio has a mean excess return of 10.13% and a volatility of 15%, with a Sharpe ratio of 0.68. All three measures are higher than their counterparts under the single-period optimal portfolio strategy with a one-month investment horizon. The efficiency gains come from the pure overlay portfolio, as the Sharpe ratios of the benchmark portfolios under the multi-period strategy and the single-period strategy are not too different. The multi-period pure overlay portfolio has a Sharpe ratio of 1.01, while the Sharpe ratio of the pure overlay portfolio under the single-period monthly strategy is 0.85.

Constrained Multi-period Portfolio Strategy

An active portfolio manager with a multi-period investment horizon still faces the trade-off between matching the optimal portfolio's long-term systematic risk exposure with strategic asset allocations and maximizing short-term active performance. The analysis of the trade-off for the single-period portfolio manager in Section 4.4.2 is also applicable here. When there are constraints such as zero average pure overlay of stocks and bonds, an active manager has less room to generate risk-adjusted active returns and suffers from an information loss. Nevertheless, following a constrained multi-period active portfolio strategy can still dominate holding the benchmark portfolio. One implementation issue in the three-month conditional strategy with the restrictions on average pure overlay portfolios involves the form of the restrictions. One is tempted to impose the constraints that the average allocations to each asset in the first, second and third month should be zero. However, in a multi-period setting, imposing the restriction that the average of the

[9] The active risk minimization portfolios in Table III show that the information ratio is unaffected when an active manager has different target tracking errors. This result is still valid in the multi-period asset allocations. A manager with 4.24% risk budget will generate a mean active return of 4.28% under the multi-period portfolio strategy.

average allocations to stocks or bonds in three months should be zero would lead to zero average allocations while allowing for different intercepts at different periods in the active portfolio. As long as the average of the average allocations to stocks or bonds over three months is zero, the pure overlay portfolio will have zero net exposure on average.

Table V shows the constrained multi-period portfolio strategy, active performance and portfolio characteristics. The 'Zero Avg Overlay' optimal portfolio allows different intercepts in each month for stocks and bonds, but the average of the three intercepts should be equal to the benchmark portfolio weights for stocks or bonds, i.e. the average of the three intercepts should be zero in the pure overlay portfolio. The constraint disturbs the bond allocations much more than it does to the stock allocations. The intercepts or the average allocations to bonds are 54%, 46% and 20% in the first, second and third month in the optimal portfolio. The mean active return is 2.98%, while the tracking error is 4.02%. Relative to the unconstrained portfolio, the information ratio of the constrained portfolio is lower by 0.27. The '5% Avg Overlay' and '10% Avg Overlay' allow for greater deviation from the benchmark portfolio weights for stocks and bonds. In both portfolios, the average allocations will hit the allowed upper bounds for stocks and bonds, implying that the long-term average allocations to risky assets will exceed 100%. Hence, both the tracking error and the portfolio volatility increase when the constraint on average allocations is less restrictive. The '10% Avg Overlay' optimal portfolio has an information ratio of 0.92, only lower than that of the unconstrained portfolio by 0.09.

4.4.4 Decentralized TAA vs. Centralized TAA

An investor with a considerably large portfolio often hires a slew of active managers, even within an asset class. Each manager is assigned a benchmark and a risk budget or target tracking error. For active management using security selection or fundamental analysis, multiple managers are necessary because no one manager can have all skills with respect to various asset classes, sectors and countries. Moreover, security selection is time consuming and one manager can only cover a limited number of securities. A tactical allocation or market timing strategy, on the other hand, requires a set of skills and information that are useful to several asset classes. The conditional strategies in this paper use the same set of macroeconomic variables and optimization approach to forecast optimal portfolio weights of stocks and bonds. If an investor decides that an equity manager and a bond manager should run tactical asset allocations separately, it will lead to an information loss since separately running tactical asset allocations for different asset classes fails to exploit the cross-section covariance of stock and bond returns.

Table VI reports the results from the decentralized TAA and the centralized TAA under the single-period monthly strategy and the multi-period strategy. A decentralized TAA is the tactical asset allocation program that assigns a stock manager for a stock

TAA and a bond manager for a bond TAA. A centralized TAA uses one active manager to manage a portfolio of stocks and bonds, with a benchmark of 60% in stocks and 40% in bonds. Additionally, the stock manager and the bond manager in the decentralized TAA are assigned 60% and 40% of total investment capitals, respectively. We re-scale risk aversions of the stock manager and the bond manager such that the target tracking error of the decentralized TAA is the same as the tracking error of the centralized TAA. Panel A shows that the average allocation to stocks in the stock TAA is 128% and the average allocation to bonds in the bond TAA is 220%. Multiplying each by its assigned weight of investment capitals will give portfolio weights to stocks and bonds in the decentralized TAA. The average allocations to stocks and bonds are 77% and 88% in the decentralized TAA. The mean active return of the decentralized TAA is the weighted average of the mean active returns of two managers. However, the active returns of the stock TAA and the bond TAA are not perfectly correlated (coefficient of correlation = 0.06), and the tracking error of the decentralized TAA is lower than the weighted average of the tracking errors of the stock TAA and the bond TAA. The mean active return of the decentralized TAA is lower than that of the centralized TAA by 15 bps. There is an information loss of 0.04 associated with the decentralized TAA, which is the difference in the information ratios of the centralized TAA and the decentralized TAA. Additionally, the decentralized TAA has a lower mean portfolio return and a higher portfolio volatility. The Sharpe ratio of the decentralized TAA is lower than the Sharpe ratio of the centralized TAA by 0.03.

In the multi-period tactical asset allocations, the average allocations to stocks in the decentralized TAA remain relatively stable over the investment horizon, but the average allocations to bonds are decreasing from 94% in the first month to 69% in the third month. The mean active return and the tracking error of the decentralized TAA are 4.45% and 5.05%, respectively. The centralized TAA has a mean active return of 5.09%, which is higher than that of the decentralized TAA by 64 bps. There are always some information losses and mean-variance efficiency losses of the decentralized TAA. To sum up, a tactical asset allocation program described in this paper should be run in the hands of one active manager (or management team) as a centralized active portfolio strategy to exploit the time-series and cross-section covariance structures of asset returns.

4.4.5 Benchmark Independence

One practical concern is the sensitivity of active portfolio strategies to benchmark choices. The conditional strategies in the paper, either in the single-period or multi-period context, have a nice property that the total optimal portfolio consists of a pure overlay portfolio and a benchmark portfolio. The pure overlay portfolio is independent of the benchmark portfolio, as it is simply a mean-variance optimization with respect to asset returns. Any difference in benchmarks will enter into the total portfolio via the benchmark portfolio, but not the pure overlay portfolio.

Table VII reports active performance and portfolio characteristics under the single-period portfolio strategy and the three-period portfolio strategy, with three different benchmarks. The asset mixes consist of 70/30, 50/50 and 30/70 in stocks and bonds for the benchmark portfolios. As the conditional strategy uses pure overlay portfolio to generate active returns and the benchmark portfolio to minimize active risks, changing benchmarks has no effect on active performance. All active performance measures remain the same regardless of benchmark choices. The portfolio characteristics, however, are sensitive to benchmarks. A benchmark portfolio with 70% instead of 30% in stocks represents higher systematic risk exposures to equities, and it should be accompanied by a higher mean excess return and a higher volatility. The Sharpe ratio is lower when the benchmark portfolio consists of a greater proportion invested in stocks.

4.4.6 Out-of-Sample Results

This section presents single-period portfolio results in an out-of-sample setting. A comparison of in-sample and out-of-sample empirical results gives indication of the estimation risk associated with portfolio strategies. The second half of the sample is used as the out-of-sample investment period, which begins in January 1967, and results are based on parameters estimated from sub-sample prior to the investment period. As the investment period evolves, parameters are re-estimated every twelve months based on the increased sub-sample by adding 12 most recent monthly returns each time. Table VIII reports portfolio weights of stocks and bonds averaged over the investment period from January 1967 to December 2008. Two active portfolio strategies are considered: unconstrained portfolio strategy and constrained portfolio strategy with zero average overlay. Active portfolio performance and portfolio characteristics are calculated and reported in the same table.

The average allocations to stocks and bonds in the pure overlay portfolio under the unconstrained portfolio strategy are 12% and 20%, respectively. The huge difference between in-sample and out-of-sample allocations to bonds (20% vs. 58%) is mainly caused by the excessively large allocations to bonds during the Great Depression captured in the full sample. In addition, the portfolio weight of bonds tends to be more volatile than that of stocks, both in-sample and out-of-sample. The constrained portfolio strategy does not reduces allocations to stocks and bonds to zero in the out-of-sample setting. The short exposure to stocks (-13%) offsets the long exposure to bonds (17%) in the pure overlay portfolio such that the exposure to risky assets is close to zero. Hence, one may expect a lower portfolio volatility of the constrained portfolio strategy than that of the unconstrained portfolio strategy.

Both the unconstrained and constrained portfolio strategies achieve an information ratio above 0.50. However, the information ratio of the constrained portfolio strategy is not far away from its in-sample counterpart of 0.60, while the unconstrained portfolio strategy has a much higher information ratio in-sample than out-of-sample (0.85 vs. 0.58). One concern would be a relative large tracking error for both strategies when it is compared to in-sample tracking errors, suggesting that estimation risk may be quite substantial. The tracking error of the unconstrained portfolio strategy is 9.08% out-of-sample, 4.84% higher than its tracking error in-sample. Putting zero average overlay as a constraint does not help to reduce the tracking error out-of-sample, but the active return seems to increase a lot.

The unconstrained portfolio strategy leads to a mean excess return of 8.91% and a volatility of 14.60%, both higher than their in-sample counterparts. It achieves a Sharpe ratio of 0.61, which is same as its Sharpe ratio in-sample. The constrained portfolio strategy with zero average overlay also has similar portfolio volatility compared to the in-sample portfolio volatility (11.91% vs. 11.88%), but its out-of-sample mean excess return is 1.23% higher than the in-sample mean excess return. When judged against the benchmark portfolio, the unconstrained portfolio strategy generates too much volatility. The certainty equivalent to the investor with risk aversion of 10 is 60 bps with the unconstrained portfolio strategy and 1.90% with the constrained portfolio strategy (zero average overlay).

Finally, Figure V shows the portfolio growth of one dollar initial investment over the investment period for the benchmark portfolio, unconstrained portfolio and constrained portfolio. The unconstrained portfolio strategy achieves substantial growth in value, at the expense of a high volatility. At the end of the investment period when financial markets became extremely volatile, the unconstrained portfolio lost 40% in value over one year. The constrained portfolio strategy with zero average overlay shows a similar path of growth and volatility as the benchmark portfolio, but the terminal value of the constrained portfolio is much higher due to a higher mean return.

4.5 Conclusion

This paper describes an active, benchmarked portfolio strategy that makes use of state variables to predict portfolio weights directly in an effort to maximize active portfolio performance relative to a benchmark. The implementation is relatively easy and uses the insights from the Markowitz solutions and the idea of scaled returns and timing portfolios (Cochrane (2005), Brandt and Santa-Clara (2006)). The empirical results in the paper suggest that the information ratios are quite impressive under the active portfolio strategies, especially under the multi-period conditional strategy. Most importantly, the analysis addresses the issue of matching tactical asset allocations with long-term strategic asset allocations, if an investor prefers to keep systematic risk exposures in check in the long-term while pursuing a short-term active portfolio strategy. The investor welfare is higher when the constraint on the target exposures is imposed under the active portfolio strategy. The manager is not necessarily worse off, as a higher management fee is more feasible to the investor under the constrained portfolio strategy.

4.A Tables and Figures

Table I: Summary Statistics and Correlation Matrix

Panel A reports summary statistics for asset returns and instruments. Stocks and Bonds are value weighted NYSE index and Ibboston Associates medium-term government bonds, respectively. The U.S. 30-day Treasury bill (T-Bill) is proxy for the risk-free rate, obtained from Ibboston Associate. The default spread is the yield difference between the Moody's BAA and AAA corporate bonds while the Momentum variable is the return of the value weighted NYSE stock over previous twelve months. Mean, median, standard deviation, minimum and maximum are in percentages. Mean, median, standard deviation and Sharpe ratio of stocks, bonds, and T-Bill rate are annualized. Panel B displays the correlation matrix. The sample period is from January 1927 to December 2008.

Panel A. Summary Statistics

	Mean	Median	Stdev	Min	Max	Sharpe
Stocks	10.77	14.63	18.57	-29.01	38.37	0.38
Bonds	5.40	3.28	4.44	-6.41	11.98	0.39
DP ratio	3.84	3.68	1.46	1.44	12.90	
T-Bill	3.65	3.29	0.88	-0.03	1.52	
Default Spread	1.13	0.89	0.71	0.32	5.64	
Momentum	12.11	13.14	21.05	-65.84	156.35	

Panel B. Correlation Matrix

	Stocks	Bonds	DP	T-Bill	Default	Mom
Stocks	1.00					
Bonds	0.09	1.00				
DP Ratio	0.07	0.00	1.00			
T-Bill	-0.02	0.19	-0.18	1.00		
Default Spread	0.05	0.08	0.50	-0.09	1.00	
Momentum	0.05	-0.09	-0.34	0.04	-0.28	1.00

Table II: Single-period Conditional Strategy

Panel A reports the single-period portfolio results under the conditional strategy which makes use of four state variables to determine the optimal asset allocations. All state variables are normalized. The investor's benchmark portfolio is the asset mix consisting of 60% in stocks and 40% in bonds, respectively. The pure overlay portfolio is the portfolio that maximizes the Sharpe ratio. The optimal portfolio is the sum of the benchmark portfolio and the pure overlay portfolio. Heteroskedasticity and autocorrelation consistent standard errors (Newey and West (1987)) are in brackets. In addition, it provides the p-value of the Wald test under the null hypothesis that all coefficients of the state variables are equal to zero. Panel B gives a snapshot of active portfolio performance. It shows annualized average active returns, tracking errors, and information ratios (p-values in brackets) for the pure overlay portfolio and the optimal portfolio. The certainty equivalent (p-values in brackets) to the manager is the annualized difference in utilities between the optimal portfolio/pure overlay portfolio and the benchmark portfolio. Panel C summarizes the characteristics of each portfolio, i.e. annualized mean excess returns, volatilities and Sharpe ratios (p-values in brackets). Mean excess returns, volatilities, active returns, tracking errors and certainty equivalents are in percentages.

		Investment Horizon: Monthly					Investment Horizon: Quarterly				
	State Variable	Benchmark	Pure Overlay		Optimal Portfolio		Benchmark	Pure Overlay		Optimal Portfolio	
A. Portfolio Strategy											
Stock	Intercept	0.60	0.15	(0.04)	0.75	(0.04)	0.60	0.14	(0.04)	0.74	(0.04)
	DP Ratio		0.08	(0.04)	0.08	(0.04)		0.07	(0.03)	0.07	(0.03)
	T-Bill		-0.09	(0.04)	-0.09	(0.04)		-0.07	(0.05)	-0.07	(0.05)
	Default Spread		-0.08	(0.03)	-0.08	(0.03)		-0.07	(0.02)	-0.07	(0.02)
	Momentum		0.07	(0.03)	0.07	(0.03)		0.06	(0.04)	0.06	(0.04)
Bond	Intercept	0.40	0.58	(0.21)	0.98	(0.21)	0.40	0.48	(0.19)	0.88	(0.19)
	DP Ratio		0.13	(0.23)	0.13	(0.23)		0.08	(0.21)	0.08	(0.21)
	T-Bill		-0.22	(0.11)	-0.22	(0.11)		-0.16	(0.11)	-0.16	(0.11)
	Default Spread		0.20	(0.19)	0.20	(0.19)		0.19	(0.18)	0.19	(0.18)
	Momentum		-0.33	(0.16)	-0.33	(0.16)		-0.12	(0.14)	-0.12	(0.14)
p-value			0.00					0.01			
B. Active Portfolio Management											
Mean Active Return			-1.39		3.59			-2.76		2.59	
Tracking Error			9.98		4.24			12.16		3.60	
Information Ratio			-0.14		0.85	(0.00)		-0.23		0.72	(0.00)
Certanity Equivalent			-11.35	(0.00)	1.79	(0.01)		-17.54	(0.00)	1.29	(0.20)
C. Portfolio Characteristics											
Mean Excess Return		4.98	3.59		8.56		5.35	2.59		7.94	
Portfolio Volatility		11.46	4.24		14.11		13.73	3.60		15.97	
Sharpe Ratio		0.43	0.85	(0.00)	0.61	(0.07)	0.39	0.72	(0.00)	0.50	(0.46)

Table III: Long-term Systematic Risk Mismatch vs. Short-term Active Risk Minimization

Panel A reports six optimal portfolios. The investment horizon is one-month. There are three systematic risk match portfolios: the 'Zero Avg Overlay' portfolio is the optimal portfolio with the constraint that the average stock and bond allocations in the pure overlay portfolio are zero. The '5% Avg Overlay' and '10% Avg Overlay' portfolio are the optimal portfolios with the constraints that the average stock and bond allocations in the pure overlay portfolios (in absolute value) are in the ranges of 5% and 10%, respectively. In addition, there are three optimal portfolios (active risk minimization portfolios) that maximize the manager's utility subject to the constraints that the maximum tracking errors (TE) are 2.34%, 2.49% and 2.85%, but imposing no overlay limits. Heteroskedasticity and autocorrelation consistent standard errors (Newey and West (1987)) are in brackets. In addition, Panel A reports the p-value of the Wald test under the null hypothesis that all coefficients of the state variables are equal to zero. Panel B gives a snapshot of active portfolio performance. The certainty equivalent (p-values in brackets) to the manager is the annualized difference in utilities between the optimal portfolio and the benchmark portfolio. Panel C summarizes the characteristics of each portfolio. Mean excess returns, volatilities, active returns, tracking errors and certainty equivalents are in percentages.

		Systematic Risk Match Portfolios						Active Risk Minimization Portfolios					
	State Variable	Zero Avg Overlay		5% Avg Overlay		10% Avg Overlay		Target TE=2.94		Target TE=3.09		Target TE=3.41	
A. Portfolio Strategy													
Stock	Intercept	0.60	(0.04)	0.65	(0.04)	0.70	(0.04)	0.70	(0.04)	0.70	(0.04)	0.71	(0.04)
	DP Ratio	0.08	(0.03)	0.08	(0.03)	0.09	(0.03)	0.06	(0.03)	0.06	(0.03)	0.07	(0.03)
	T-Bill	-0.07	(0.03)	-0.07	(0.03)	-0.08	(0.04)	-0.06	(0.04)	-0.06	(0.04)	-0.07	(0.04)
	Default Spread	-0.04	(0.03)	-0.06	(0.03)	-0.07	(0.03)	-0.06	(0.03)	-0.06	(0.03)	-0.06	(0.03)
	Momentum	0.06	(0.02)	0.06	(0.03)	0.07	(0.03)	0.05	(0.03)	0.05	(0.03)	0.05	(0.03)
Bond	Intercept	0.40	(0.20)	0.45	(0.20)	0.50	(0.20)	0.79	(0.20)	0.81	(0.21)	0.86	(0.21)
	DP Ratio	-0.12	(0.22)	-0.12	(0.22)	-0.12	(0.22)	0.08	(0.22)	0.08	(0.22)	0.10	(0.22)
	T-Bill	0.00	(0.11)	-0.01	(0.11)	-0.03	(0.11)	-0.15	(0.11)	-0.16	(0.11)	-0.17	(0.11)
	Default Spread	0.40	(0.19)	0.39	(0.19)	0.38	(0.19)	0.18	(0.19)	0.18	(0.19)	0.19	(0.19)
	Momentum	-0.37	(0.17)	-0.39	(0.17)	-0.41	(0.17)	-0.23	(0.16)	-0.24	(0.16)	-0.27	(0.16)
p-value		0.00		0.00		0.00		0.01		0.00		0.00	
B. Active Portfolio Management													
Mean Active Return		1.78		2.23		2.68		2.48		2.61		2.88	
Tracking Error		2.94		3.09		3.41		2.94		3.09		3.41	
Information Ratio		0.60	(0.00)	0.72	(0.00)	0.78	(0.00)	0.85	(0.00)	0.85	(0.00)	0.85	(0.00)
Certanity Equivalent		0.91	(0.37)	1.28	(0.17)	1.51	(0.07)	1.62	(0.05)	1.65	(0.05)	1.72	(0.04)
C. Portfolio Characteristics													
Mean Excess Return		6.76		7.20		7.65		7.46		7.58		7.86	
Portfolio Volatility		11.88		12.53		13.19		13.12		13.23		13.47	
Sharpe Ratio		0.57	(0.19)	0.57	(0.16)	0.58	(0.14)	0.57	(0.19)	0.57	(0.17)	0.58	(0.13)

Table IV: Multi-period Conditional Strategy

Panel A reports the optimal portfolios under the multi-period active portfolio strategy that makes use of four normalized state variables. The investment horizon is three-month. Heteroskedasticity and autocorrelation consistent standard errors (Newey and West (1987)) are in brackets. In addition, Panel A reports the p-value of the Wald test under the null hypothesis that all coefficients of the state variables are jointly equal to zero. Panel B gives a snapshot of active portfolio performance. It shows annualized average active returns, tracking errors, and information ratios (p-values in brackets) for the pure overlay portfolio and the optimal portfolio. The certainty equivalent (p-values in brackets) to the manager is the annualized difference in utilities between the optimal portfolio/pure overlay portfolio and the benchmark portfolio. Panel C summarizes the characteristics of each portfolio. Annualized mean excess returns, volatilities, active returns, tracking errors and certainty equivalents are in percentages.

			Investment Horizon: Three Months				
	Month	State Variable	Benchmark	Pure Overlay		Optimal Portfolio	
A. Portfolio Strategy							
Stock	First	Intercept	0.60	0.18	(0.08)	0.78	(0.08)
		DP Ratio		0.09	(0.09)	0.09	(0.09)
		T-Bill		-0.10	(0.08)	-0.10	(0.08)
		Default Spread		-0.11	(0.06)	-0.11	(0.06)
		Momentum		0.06	(0.06)	0.06	(0.06)
	Second	Intercept	0.60	0.18	(0.08)	0.78	(0.08)
		DP Ratio		0.08	(0.08)	0.08	(0.08)
		T-Bill		-0.09	(0.08)	-0.09	(0.08)
		Default Spread		-0.11	(0.06)	-0.11	(0.06)
		Momentum		0.08	(0.06)	0.08	(0.06)
	Third	Intercept	0.60	0.16	(0.08)	0.76	(0.08)
		DP Ratio		0.10	(0.08)	0.10	(0.08)
		T-Bill		-0.07	(0.07)	-0.07	(0.07)
		Default Spread		-0.09	(0.06)	-0.09	(0.06)
		Momentum		0.11	(0.06)	0.11	(0.06)
Bond	First	Intercept	0.40	0.58	(0.38)	0.98	(0.38)
		DP Ratio		0.18	(0.48)	0.18	(0.48)
		T-Bill		-0.35	(0.26)	-0.35	(0.26)
		Default Spread		0.29	(0.42)	0.28	(0.42)
		Momentum		-0.37	(0.32)	-0.37	(0.32)
	Second	Intercept	0.40	0.62	(0.39)	1.02	(0.39)
		DP Ratio		0.10	(0.49)	0.10	(0.49)
		T-Bill		-0.41	(0.27)	-0.41	(0.27)
		Default Spread		0.23	(0.43)	0.23	(0.43)
		Momentum		-0.32	(0.32)	-0.32	(0.32)
	Third	Intercept	0.40	0.61	(0.41)	1.01	(0.40)
		DP Ratio		0.07	(0.49)	0.07	(0.49)
		T-Bill		-0.38	(0.26)	-0.38	(0.26)
		Default Spread		0.03	(0.43)	0.03	(0.42)
		Momentum		-0.34	(0.32)	-0.34	(0.32)
p-value				0.00			
B. Active Portfolio Management							
Mean Active return				0.04		5.09	
Tracking Error				11.09		5.05	
Information Ratio				0.00		1.01	(0.00)
Certanity Equivalent				-12.44	(0.00)	2.54	(0.07)
C. Portfolio Characteristics							
Mean Excess Return			5.04	5.09		10.13	
Portfolio Volatility			12.18	5.05		15.00	
Sharpe Ratio			0.42	1.01	(0.00)	0.68	(0.56)

Table V: Long-term Systematic Risk Mismatch in Multi-period Portfolios

Panel A reports three optimal portfolios that reduce systematic risk mismatch under the multi-period active portfolio strategy. The investment horizon is three-month. There are three systematic risk match portfolios: the 'Zero Avg Overlay' portfolio is the optimal portfolio with the constraint that the average stock and bond allocations in the pure overlay portfolio are zero. The '5% Avg Overlay' and '10% Avg Overlay' portfolio are the optimal portfolios with the constraints that the average stock and bond allocations in the pure overlay portfolios are in the ranges of 5% and 10%, respectively. The investor's benchmark portfolio is the asset mix consisting of 60% in stocks and 40% in bonds. Panel B and Panel C report performance measures. Annualized mean excess returns, volatilities, active returns, tracking errors and certainty equivalents are in percentages.

			Investment Horizon: Three Months					
	Month	State Variable	Zero Avg Overlay		5% Avg Overlay		10% Avg Overlay	
A. Portfolio Strategy								
Stock	First	Intercept	0.59	(0.07)	0.64	(0.07)	0.70	(0.07)
		DP Ratio	0.08	(0.09)	0.08	(0.08)	0.08	(0.08)
		T-Bill	-0.12	(0.07)	-0.12	(0.07)	-0.12	(0.07)
		Default Spread	-0.07	(0.05)	-0.08	(0.05)	-0.10	(0.05)
		Momentum	0.04	(0.05)	0.05	(0.05)	0.06	(0.05)
	Second	Intercept	0.60	(0.07)	0.65	(0.07)	0.70	(0.07)
		DP Ratio	0.05	(0.08)	0.06	(0.08)	0.07	(0.08)
		T-Bill	-0.07	(0.07)	-0.08	(0.07)	-0.08	(0.07)
		Default Spread	-0.07	(0.05)	-0.08	(0.05)	-0.09	(0.06)
		Momentum	0.03	(0.05)	0.05	(0.05)	0.06	(0.06)
	Third	Intercept	0.61	(0.07)	0.65	(0.07)	0.70	(0.07)
		DP Ratio	0.10	(0.07)	0.10	(0.07)	0.10	(0.08)
		T-Bill	-0.06	(0.06)	-0.06	(0.06)	-0.06	(0.07)
		Default Spread	-0.05	(0.05)	-0.06	(0.06)	-0.07	(0.06)
		Momentum	0.10	(0.05)	0.11	(0.05)	0.12	(0.05)
Bond	First	Intercept	0.54	(0.35)	0.54	(0.36)	0.55	(0.36)
		DP Ratio	-0.02	(0.47)	-0.02	(0.47)	-0.04	(0.47)
		T-Bill	-0.13	(0.24)	-0.16	(0.24)	-0.18	(0.25)
		Default Spread	0.54	(0.41)	0.52	(0.41)	0.51	(0.42)
		Momentum	-0.37	(0.30)	-0.41	(0.30)	-0.43	(0.31)
	Second	Intercept	0.46	(0.36)	0.50	(0.36)	0.55	(0.37)
		DP Ratio	-0.13	(0.48)	-0.12	(0.48)	-0.12	(0.49)
		T-Bill	-0.14	(0.25)	-0.18	(0.25)	-0.20	(0.26)
		Default Spread	0.62	(0.42)	0.58	(0.42)	0.53	(0.43)
		Momentum	-0.49	(0.31)	-0.46	(0.31)	-0.44	(0.32)
	Third	Intercept	0.20	(0.38)	0.30	(0.38)	0.40	(0.39)
		DP Ratio	-0.22	(0.48)	-0.19	(0.48)	-0.19	(0.48)
		T-Bill	-0.11	(0.25)	-0.15	(0.25)	-0.18	(0.25)
		Default Spread	0.39	(0.44)	0.34	(0.43)	0.31	(0.43)
		Momentum	-0.43	(0.33)	-0.43	(0.32)	-0.44	(0.32)
p-value			0.00		0.00		0.00	
B. Active Portfolio Management								
Mean Active return			2.98		3.47		3.96	
Tracking Error			4.02		4.10		4.31	
Information Ratio			0.74	(0.00)	0.85	(0.00)	0.92	(0.00)
Certanity Equivalent			1.36	(0.55)	1.79	(0.37)	2.10	(0.24)
C. Portfolio Characteristics								
Mean Excess Return			8.02		8.52		9.00	
Portfolio Volatility			12.79		13.32		13.90	
Sharpe Ratio			0.63	(0.83)	0.64	(0.77)	0.65	(0.72)

Table VI: Decentralized TAA vs. Centralized TAA

Panel A reports the average allocations (intercepts) to stocks or / and bonds in optimal portfolios under the single-period conditional strategy. The investment horizon is one month. The optimal portfolio 'Stock TAA' is the portfolio with only stocks, without any consideration given to bond allocations. Similarly, the optimal portfolio 'Bond TAA' is the portfolio with only bonds as the single risky asset. The 'Decentralized TAA' is the weighted average of 'Stock TAA' and 'Bond TAA'. The weights given to 'Stock TAA' and 'Bond TAA' are 60% and 40%, respectively. The benchmark portfolio consists of 60% and 40% invested in stocks and bonds. The average allocations to stocks and bonds in the 'Decentralized TAA' are separated by a '/'. The optimal portfolio 'Centralized TAA' optimizes with stocks and bonds in a single portfolio (i.e. it is the optimal portfolio from Table II with one-month investment horizon). The average allocations to stocks and bonds in the 'Centralized TAA' are also separated by a '/'. Panel B reports active portfolio performance. The certainty equivalent is the annualized difference in utilities between the optimal portfolio and the benchmark portfolio. Panel C summarizes characteristics of each portfolio. Annualized mean excess returns, volatilities, active returns, tracking errors and certainty equivalents are in percentages. Returns and certainty equivalents for the 'Decentralized TAA' are the weighted averages of the 'Stock TAA' and the 'Bond TAA'. For the tracking error and volatility of the 'Decentralized TAA', the calculation takes into account the correlation between the 'Stock TAA' and the 'Bond TAA'. Panel D, E, and F are the multi-period counterparts of Panel A, B and C. The investment horizon is three-month. The p-values of information ratios, certainty equivalents and Sharpe ratios are in brackets.

	Single-Period Optimal Portfolios							
	Decentralized TAA						Centralized TAA	
	Stock TAA		Bond TAA		Overall		Overall	
A. Average Allocations								
Intercept	1.28		2.20		0.77 / 0.88		0.75 / 0.98	
B. Active Portfolio Management								
Mean Active Return	3.46		3.42		3.44		3.59	
Tracking Error	4.27		5.70		4.24		4.24	
Information Ratio	0.60	(0.00)	0.60	(0.00)	0.81	(0.00)	0.85	(0.00)
Certanity Equivalent BP	1.81	(0.04)	1.79	(0.05)	1.81	(0.01)	1.79	(0.01)
C. Portfolio Characteristics								
Mean Excess Return	10.59		5.17		8.42		8.56	
Portfolio Volatility	22.69		9.21		14.49		14.11	
Sharpe Ratio	0.47	(0.20)	0.56	(0.01)	0.58	(0.14)	0.61	(0.07)

	Month	Multi-Period Optimal Portfolios							
		Decentralized TAA						Centralized TAA	
		Stock TAA		Bond TAA		Overall		Overall	
D. Average Allocations									
Intercept	First	1.23		2.36		0.74 / 0.94		0.78 / 0.98	
	Second	1.31		2.12		0.79 / 0.85		0.78 / 1.02	
	Third	1.27		1.71		0.76 / 0.69		0.76 / 1.01	
E. Active Portfolio Management									
Mean Active Return		5.02		3.60		4.45		5.09	
Tracking Error		5.01		5.99		5.05		5.05	
Information Ratio		0.71	(0.00)	0.60	(0.00)	0.88	(0.00)	1.01	(0.00)
Certanity Equivalent BP		2.51	(0.08)	1.80	(0.20)	2.23	(0.12)	2.54	(0.07)
F. Portfolio Characteristics									
Mean Excess Return		12.12		5.34		9.41		10.13	
Portfolio Volatility		25.50		9.72		16.09		15.00	
Sharpe Ratio		0.48	(0.89)	0.55	(0.41)	0.58	(0.98)	0.68	(0.56)

Table VII: Benchmark Sensitivity

Panel A reports active portfolio performance under the single-period conditional strategy and the multi-period conditional strategy, both using four state variables. The investment horizons are one-month and three-month for the single-period and the multi-period portfolios, respectively. Both the single-period and the multi-period portfolios utilize three different benchmark portfolios. The benchmark portfolio '70/30' consists of 70% in stocks and 30% in bonds. The benchmark portfolios '50/50' and '30/70' have asset mixes of 50% in stocks and 50% in bonds, and 30% in stocks and 70% in bonds, accordingly. Panel B summarizes characteristics of each optimal portfolio. Annualized mean excess returns, volatilities, active returns, tracking errors and certainty equivalents are in percentages. The p-values of information ratios, certainty equivalents and Sharpe ratios are in brackets.

	Single-Period Optimal Portfolios						Three-Period Optimal Portfolios					
	70 / 30		50 / 50		30 / 70		70 / 30		50 / 50		30 / 70	
A. Active Portfolio Management												
Mean Active Return	3.59		3.59		3.59		5.09		5.09		5.09	
Tracking Error	4.24		4.24		4.24		5.05		5.05		5.05	
Information Ratio	0.85	(0.00)	0.85	(0.00)	0.85	(0.00)	1.01	(0.00)	1.01	(0.00)	1.01	(0.00)
Certanity Equivalent	1.79	(0.01)	1.79	(0.01)	1.79	(0.01)	2.54	(0.07)	2.54	(0.07)	2.54	(0.07)
B. Portfolio Characteristics												
Mean Excess Return	9.10		8.02		6.95		10.69		9.58		8.46	
Portfolio Volatility	15.74		12.55		9.76		16.73		13.35		10.42	
Sharpe Ratio	0.58	(0.15)	0.64	(0.02)	0.71	(0.00)	0.64	(0.81)	0.72	(0.35)	0.81	(0.04)

Table VIII: Out-of-Sample Results

Panel A reports out-of-sample mean portfolio weights and their standard deviation (in brackets) under the single-period conditional strategy. The investment horizon is one-month. The out of sample investment period starts in January 1967 and ends in December 2008. The parameter estimation uses sample returns prior to the investment period and is updated every 12 months going forward. Two portfolio strategies are reported: unconstrained portfolio strategy and constrained portfolio strategy with zero average overlay. Panel B and Panel C summarize active portfolio performance and portfolio characteristics. Annualized mean excess returns, volatilities, active returns, tracking errors and certainty equivalents are in percentages. The p-values of information ratios, certainty equivalents and Sharpe ratios are in brackets.

	Unconstrained					Zero Avg Overlay				
	Benchmark	Pure Overlay		Total		Benchmark	Pure Overlay		Total	
A. Portfolio Strategy										
Stock	0.60	0.12	(0.33)	0.72	(0.33)	0.60	-0.13	(0.29)	0.47	(0.29)
Bond	0.40	0.20	(1.06)	0.60	(1.06)	0.40	0.17	(1.07)	0.57	(1.07)
B. Active Portfolio Management										
Mean Active Return		1.64		5.27			0.72		4.36	
Tracking Error		11.73		9.08			13.90		8.64	
Information Ratio		0.14		0.58	(0.00)		0.05		0.50	(0.00)
C. Portfolio Characteristics										
Mean Excess Return	3.63	5.27		8.91		3.63	4.36		7.99	
Portfolio Volatility	9.64	9.08		14.60		9.64	8.64		11.91	
Sharpe Ratio	0.38	0.58	(0.42)	0.61	(0.02)	0.38	0.50	(0.79)	0.67	(0.00)

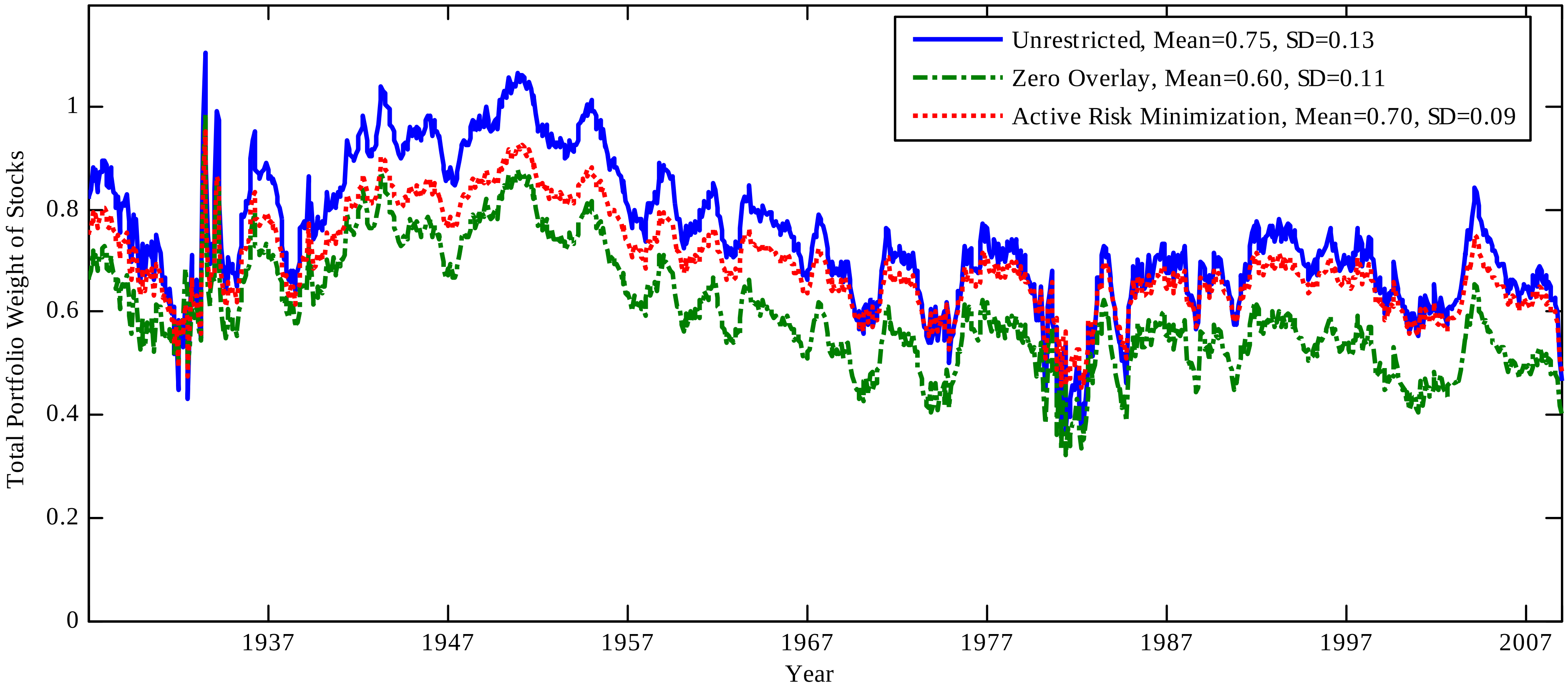


Figure I. Allocations to Stocks in the Single-period Optimal Portfolio

This figure shows the evolution of the portfolio weight of stocks in the single-period optimal portfolio from 1927 to 2008. In addition, the mean and the standard deviation of the portfolio weight are displayed. The 'Unrestricted' stock allocations are based on the optimal portfolio without any restrictions on the average overlay positions. The 'Zero Overlay' stock allocations impose the constraint that the average allocations to stocks and bonds are zero in the pure overlay portfolio. Finally, the 'Active Risk Minimization' stock allocations come from the optimal portfolio with the same target tracking error as the optimal portfolio that has zero average allocations to stocks and bonds in the pure overlay portfolio.

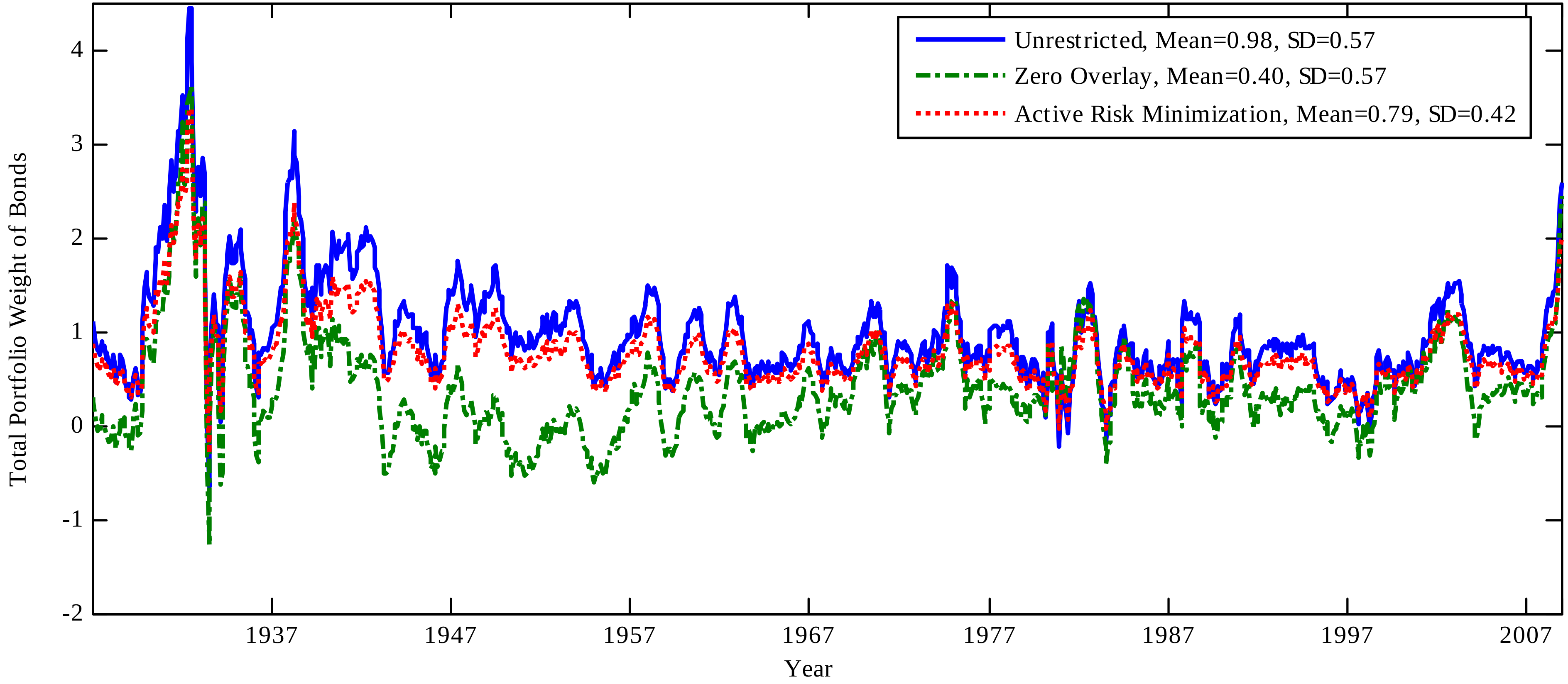


Figure II. Allocations to Bonds in the Single-period Optimal Portfolio

This figure shows the evolution of the portfolio weight of bonds in the single-period optimal portfolio from 1927 to 2008. In addition, the mean and the standard deviation of the portfolio weight are displayed. The 'Unrestricted' bond allocations are based on the optimal portfolio without any restrictions on the average overlay positions. The 'Zero Overlay' bond allocations impose the constraint that the average allocations to stocks and bonds are zero in the pure overlay portfolio. Finally, the 'Active Risk Minimization' bond allocations come from the optimal portfolio with the same target tracking error as the optimal portfolio that has zero average allocations to stocks and bonds in the pure overlay portfolio.

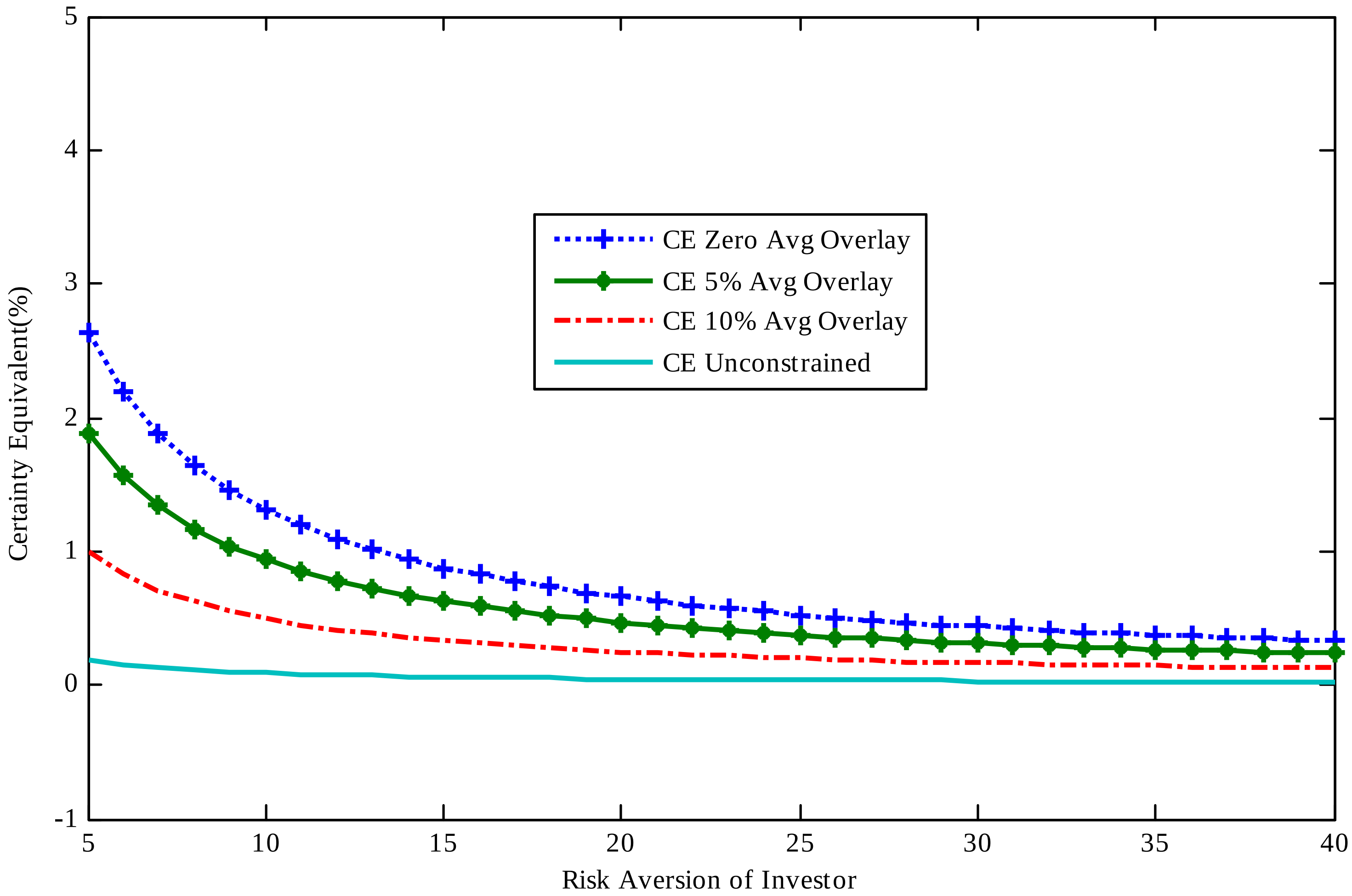


Figure III. Certainty Equivalent to the Investor (Fixing Risk Aversion of the Manager at 20)

This figure shows the certainty equivalent to the investor with various degrees of risk aversion, fixing the risk aversion of the portfolio manager at 20. The investor has a mean-variance utility over portfolio returns.

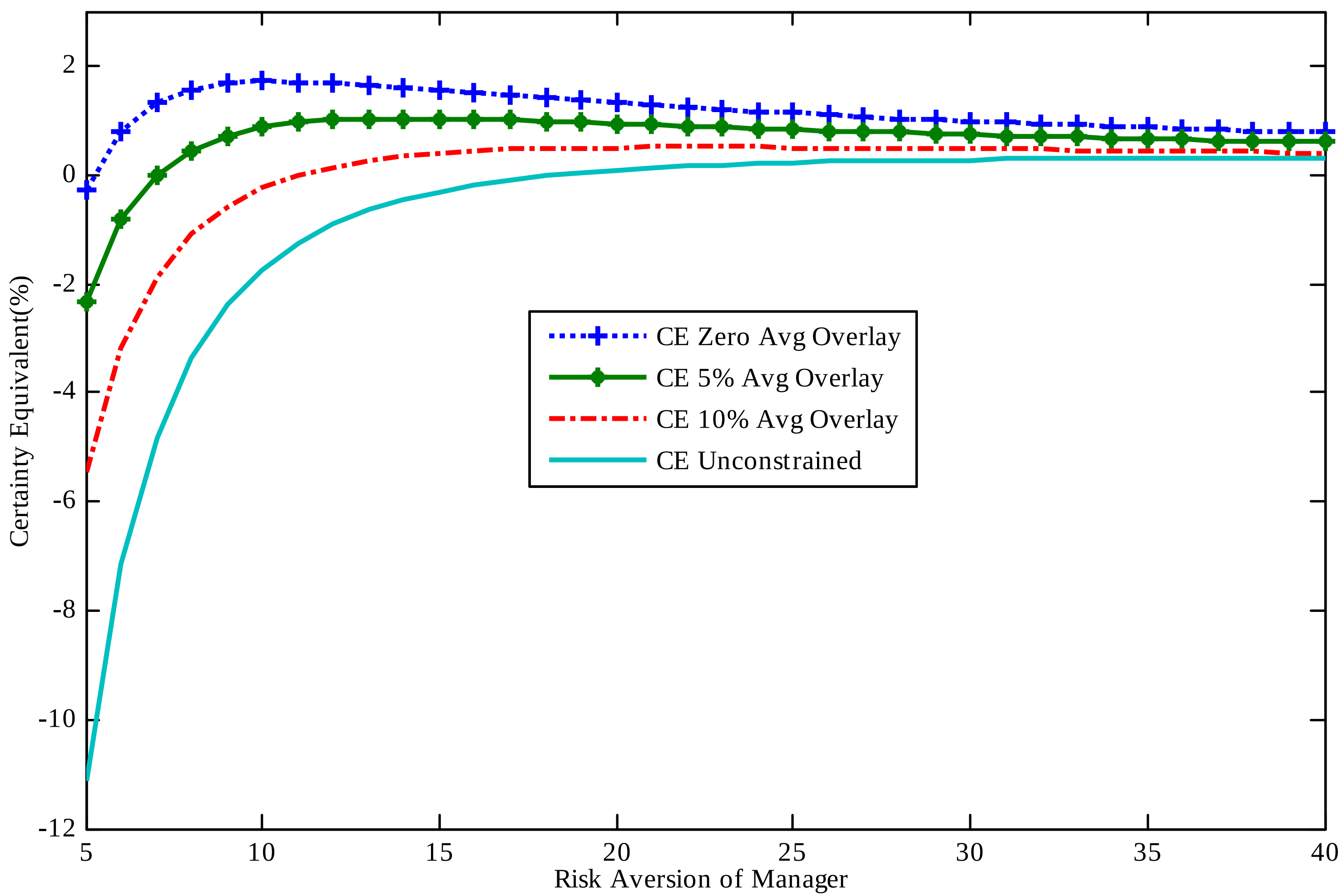


Figure IV. Certainty Equivalent to the Investor (Fixing Risk Aversion of the Investor at 10)

This figure shows the certainty equivalent to the investor with different degrees of risk aversion of the manager, fixing the risk aversion of the investor at 10. The investor has a mean-variance utility over portfolio returns.

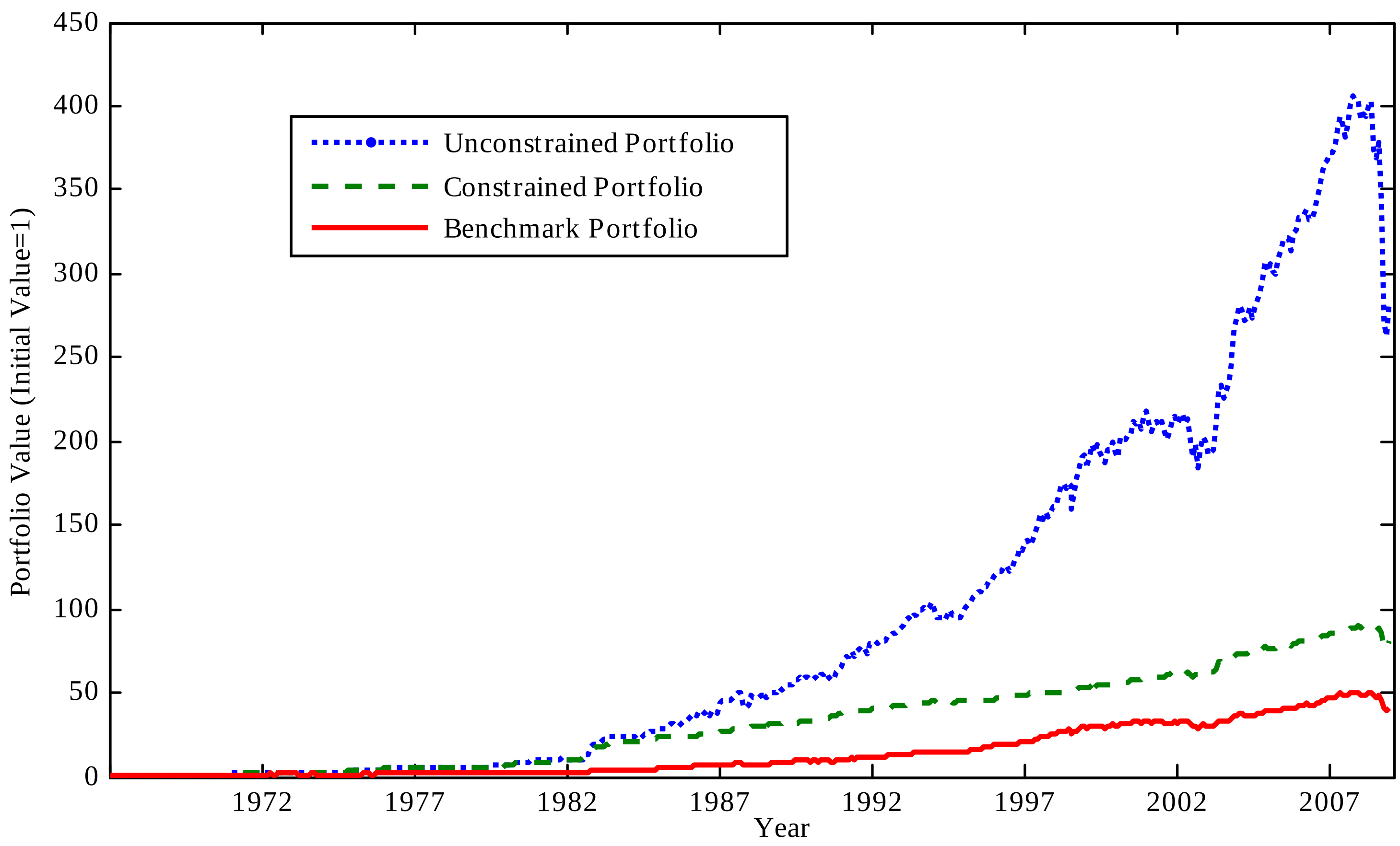


Figure V. Growth of Portfolio Value

This figure shows the out-of-sample growth of portfolio value over 42 years with an initial investment of one dollar. The constrained portfolio is the portfolio with zero average overlay.

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