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**Exploring the Pension Decumulation  
Phase by the Use of Annuities**



# Exploring the pension decumulation phase by the use of annuities

By  
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Master thesis Finance for the Pensions, Aging and Retirement track

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## Abstract

In this paper, various annuity types and the welfare implications for defined contribution schemes in the Dutch pension system are investigated. The optimal annuity decision between three basic annuity types is explored, and the main finding is that the inclusion of variable annuities (where the payoff depends on equity market performance) to the asset menu substantially increases welfare under a wide range of assumptions. The welfare effects of inflation protected annuities in the presence of variable annuities are limited. This is not because the variable annuity moves along with unexpected inflation. Instead, the compounding equity returns safeguard the investor from unexpected inflation in most instances. The main findings have implications for the current Dutch pension system design, where variable annuities should play a bigger role than they currently do. Furthermore, the often ascribed advantages of inflation protected annuities in retirement are not found in the model. The recommendation that the government should issue inflation linked bonds to provide a market for real annuities is not supported by the quantitative model.

## Table of Contents

Preface .....	4
<b>1. Introduction .....</b>	<b>5</b>
<b>2. Literature review .....</b>	<b>7</b>
2.1 Annuities .....	8
<b>3. Dutch framework .....</b>	<b>10</b>
3.1 Practical relevance .....	13
<b>4. Financial model, annuities and preferences .....</b>	<b>14</b>
4.1 Financial Model .....	14
4.2 Constructing Annuity income .....	15
4.2.1 Old age insurance, AOW .....	17
4.3 Preferences .....	17
4.3.1 Adding longevity risk .....	18
4.3.2 Presentation of results .....	18
<b>5 Results and sensitivity analysis: .....</b>	<b>19</b>
5.1 Deterministic longevity .....	19
5.2 The base case model: stochastic longevity .....	20
5.3 Changing risk aversion .....	22
5.3.1 Low risk aversion ( $\gamma = 2$ ) .....	23
5.3.2 High risk aversion ( $\gamma = 10$ ) .....	24
5.3.3 Optimal allocation for different $\gamma$ .....	24
5.4 No old-age pension .....	25
5.5 Higher AIR (AIR = 10%) .....	27
5.6 The rich & the poor .....	28
5.7 The 5% worst outcomes .....	29
5.8 Sensitivity in parameters of the financial market .....	30
5.9 Expanding the utility framework by habit formation .....	31
<b>6. Discussion of results, recommendations and future guidance .....</b>	<b>34</b>
6.1 Recommendations .....	34
6.2 Directions for further research .....	35
<b>7. Conclusion .....</b>	<b>37</b>
<b>References .....</b>	<b>39</b>
<b>Appendix .....</b>	<b>41</b>
A Financial market model details .....	41
A.2 Parameters .....	42
B Adding stochastic longevity risk to the model .....	43
C Fluctuations in annuity income (real or nominal) .....	44

D Certainty equivalent consumption .....	44
D.2 Relative Certainty equivalent Consumption .....	44
E Unadjusted parameters by Draper (2014) .....	45
F Average price index for average and extreme outcomes. ....	46

## Preface

As part of the Netspar track 2013/2014 Finance I took the opportunity to write my thesis on a pension topic. As I preferred combining the academic experience with a useful practical experience I was very happy that APG offered me a chance to write my thesis as an intern. I have learned a lot about pensions during my master, but especially during my time at APG. I would like to thank my colleagues who made my internship a meaningful experience. I would like to thank my company supervisors Ruben Laros and Maurits Aben, who have boosted my enthusiasm and helped me whenever needed. I am also very grateful to my academic supervisor, Bas Werker, as he provided me with very useful feedback and accurate comments. Lastly I would like to thank Albert-Jan Hummel, for sharing his enthusiasm on economics during the many coffee breaks.

## 1. Introduction

The distribution of risk in pension plans has been an ongoing discussion over the last decades. Developments in regulatory requirements and demographic trends have accelerated the shift from collective to more individual schemes in which risks are increasingly born by the participant/individual. This shift fits in the Dutch societal trend of more individualism, where even the term ‘participation society’ was used in the 2013 king’s speech.<sup>1</sup> More responsibility for individuals who can take care of themselves, and an increasing amount of individual pension schemes would fit in this trend.

As more people save individually instead of collectively for their pension, the demand for individual pension solutions in the form of annuities is expected to increase both within the Netherlands, and internationally (Rusconi, 2008). The most basic solution to convert pension savings into pension income is the nominal life-long annuity, which provides the retiree with a fixed nominal payment until death. In fact, current legal rules in the Netherlands require insurers and PPI’s (premie pensioeninstellingen) to fully convert the pension capital at retirement into an annuity (Verbond van Verzekeraars, 2012). However, apart from high-low (hoog-laag pensioen) constructions which allows to frontload or backload some annuity income, there is little flexibility to customize this requirement to the needs and preferences of the individual (Dietvorst, Hooghiemstra, Nijman and Oerlemans, 2010). Among those potential disadvantages are the inflexibility of a life-long annuity, unfavorable pricing and bequest motives. Furthermore, the annuity menu in the Netherlands is incomplete and as a consequence the retiree can not get protection against inflation risk and is unable to get equity exposure via annuities in retirement (Dietvorst et al., 2010). This thesis will examine if welfare can be gained in the decumulation phase by expanding the annuity menu in the Dutch framework. This topic has gotten political attention, and the consultant LCP commissioned by the Ministry of Social Affairs and Employment (Ministerie van Sociale zaken en Werkgelegenheid, 2014) explored conversion risk for defined contribution schemes and analyzed risk exposure during retirement. This topic is thus relevant from both an academic and societal perspective.

Yaari (1965) grandfathered the literature on annuities as he showed that in an economy with complete markets, no bequest motive, and fairly-priced annuities, it is optimal to fully annuitize retirement wealth. Fundamental to this result is that annuities reallocate wealth from states in which the agent is no longer alive, and derives no utility from consumption, to states in which the individual is still alive. Since the influential paper by Yaari (1965), the optimal annuity decision has gotten substantial attention and research has extended to various annuity types and risk factors. The annuity decision is important because the decision is irreversible, as insurance companies do not repurchase annuity products or at a very high price due to adverse selection (Kojien, Nijman and Werker, 2009). By choosing a certain annuity combination the consumer locks in financial market exposure, as he is unable to trade his annuity. For instance, if the retiree is completely invested in a nominal annuity, the real purchasing power of the pension might erode due to inflation. Alternatively, the retiree foregoes the equity risk premium if only a nominal annuity is available. Given the longer time spend in retirement due to rising longevity, this might add up to a sizeable amount. However, not only during but also towards retirement problems might arise. If interest rate risk is not managed towards retirement, the retiree might face a much lower pension income if interest rates unexpectedly drop, as happened amid the 2008 financial crisis (Ministerie SZW, 2013). Lower interest rates increase the price of an annuity and hence lower the available pension income.

Limiting possibilities in the asset menu for retirees can be welfare reducing as we have seen above that it can expose the consumer to unwanted risks. The current Dutch legal requirements might not be optimal and this thesis explores different annuitization strategies that could better match the goals of retirees in individual schemes. Hence the main research question is:

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<sup>1</sup> <http://www.rijksoverheid.nl/documenten-en-publicaties/toespraken/2013/09/17/troonrede-2013.html>

*Does including variable and real annuities to the asset menu in the decumulation phase of defined contribution schemes add welfare?*

To do so, this thesis will be divided in five parts. First of all, in section 2 the literature on consumption behavior in retirement and the optimal annuitization decision are discussed to get a sense of the needs of individuals and the past work that has been done with regard to the optimal annuity decision. In the second part at section 3, the Dutch framework will be analyzed and the different pillars are examined and the way they address risk. In the third part (section 4) a financial model will be developed and the annuity menu will be determined. The fourth part (section 5) will consist of discussing results for different extensions and assumptions in the model, including an addition to the standard-of-living. The last section will conclude.

## 2. Literature review

This section will be divided in three subsections. First, I will define the purpose of pension from an individual and public-policy perspective. Second, I will discuss consumption preferences in retirement. The third part will explain the attractiveness of annuities.

### 2.1 Purpose of pension

Both from an individual and a public-policy perspective a pension serves multiple purposes. Barr and Diamond (2008) recognize individual, public-policy and insurance objectives. First of all, a pension allows people to transform human capital in financial capital to consume later. In this way one can postpone income and smooth consumption over time. Second of all, a pension can serve as poverty relief. This is beneficial from a public-policy perspective because citizens will not have to appeal for social security by the government. Lastly, from an individual insurance perspective, a retiree wants to hedge the risk of depleting his financial wealth too quickly, otherwise he will have to adjust his consumption pattern significantly.

### 2.2 Consumption preferences

The theory of consumption smoothing is subject to academic debate, as for example Hamermesh (1984) documented that households in the USA reduce their consumption expenditures substantially around the age of retirement. The prediction of life-cycle models that individuals smooth the marginal utility of consumption if there is a foreseeable drop in income at retirement is not found, a phenomenon called the consumption retirement puzzle. Instead, the drop in income is mitigated by a drop in spending.

However, the debate is not settled and a whole set of augmentations to the standard model have been used to explain the drop in spending, such as subjective retirement expectations (Haider and Stephens, 2007), cessation of work-related expenses and substitution to home production (Hurd and Rohwedder, 2003) or a marital bargaining approach for which wives save more to support a longer expected retirement period (Lundberg, Startz and Stillman, 2001). On the other hand, Banks, Blundell and Tanner (1998) claim that labor-market participation partially explains the drop in consumption but in the end conclude that only the systematic arrival of unexpected adverse information can reconcile the fall in consumption with the life-cycle hypothesis.

Blake (2010) challenges the idea that retirees smooth consumption over time and claims that it conveys a u-shaped pattern. In the beginning of retirement there is active life with more expenditure, after which a period of inactive retirement follows and expenditure drops. Finally, medical care and treatment become the dominant expenditure and total expenditure goes up again.

Davidoff et al. (2005) extend the utility framework by incorporating habit formation, or a standard of living. A habit in the utility function means that the level of current consumption is compared to past consumption. This might fit better with reality as a standard-of-living provides a relative mechanism to evaluate consumption. Habit formation will be discussed in section 5.9.

Soede (2012) has done extensive research for changing incomes and needs for the elderly in the Netherlands. He related the income of the old and the young over the last 20 years and found that the income of the elderly has relatively increased compared to the young. In addition, their income has increased more than social security and allowances for the unemployed and the **incapacitated**. Furthermore, the old-age pension (AOW) has increased in real terms, changes in fiscal rules have had a positive effect on the disposable income and the participation in supplementary pension schemes has increased. The capital position for the elderly thus has increased, but the return (income) on their capital has decreased. According to Soede (2012), there are few people that work until the age of 65, because of arrangements which allow for early retirement. The differences in income of the elderly therefore fluctuate a lot. After the age of 65 the old-age pension becomes effective and this is a big boost for income and purchasing power. The needs of the Dutch elderly change over the years. They spend less on clothing and more on nutrition. Most remarkable is that retirees spend less on development, leisure and transportation. Soede (2012) does not find evidence that retirees indeed spend more in the early

stage of retirement, contrasting the claim by Blake (2010). Lastly, Soede (2012) addresses the perceived income for retired and working agents. He finds that age, separately, is a driving factor on the need for income, and has is negatively related to age. Furthermore, the composition of the household largely influences the need for income.

## 2.3 Annuities

Given the pension goals stated in section 2.1, life contingent annuities<sup>2</sup> are very useful products. A life contingent annuity provides the retiree with an income stream in money or unit terms till death. It allows to transform capital into a lifelong income and allows the participant to smooth consumption. Furthermore, it protects the annuitant from outliving his financial resources, as the claim is lifelong. This serves the government too, as it will reduce the chance that the agent will have to rely (and appeal for resources) on the government in retirement.

Annuities allow to pool longevity risk, and remove the idiosyncratic risk of outliving one's assets (Dietvorst et al., 2011). Furthermore, the annuity earns a 'mortality credit' in addition to the return of the underlying asset. The assets are pooled among a group and the individual forfeits his assets on decease, allowing for a higher return for the survivors of the group. In this way the annuity reallocates wealth from states in which the agent is no longer alive, and derives no utility from consumption, to states in which the individual is still alive. This is the classical Yaari (1965) result, where he showed that full annuitization is optimal in absence of a bequest motive, fairly priced annuities and a complete annuity market. Davidoff, Brown and Diamond (2005) extended this result and showed that it holds under more general conditions, namely that annuities need not be fairly priced. They showed that the necessary condition is that the annuity pays a rate of return greater than the otherwise matching financial asset, net of administrative costs.

Charupat and Milevsky (2002) show that in absence of inflation risk and with constant risk premia and constant interest rates the optimal portfolio of nominal, inflation-linked and variable annuities coincides with the optimal allocation before retirement. Browne et al. (2003) argue that the illiquidity of an annuity reduces the demand for annuities as it does not allow to rebalance the portfolio over time.

Brown et al. (2001) focus their attention mainly on welfare effects for inflation-linked annuities and variable annuities (for which the payoff depends on an underlying equity portfolio). Their analysis, based on empirical data, suggests that the inclusion of inflation-linked annuities does not add welfare by a large amount in the presence of variable annuities. In large this is caused by a favorable equity risk premium, as opposed to an inflation hedge by equities. Their estimates suggests a negative correlation between unexpected inflation and equity returns. The result that inflation protected annuities are not welfare enhancing is sensitive to the risk aversion level of the investor and the serial dependence of the inflation rate. This issue has also been addressed in a portfolio management context. Brière and Ombretta (2009) suggest that inflation expectations have become more stable after 2003 and hence, from a portfolio diversification perspective, have become less attractive. Naturally, the value of an inflation-linked bond will increase if it hedges a more imminent risk. If unexpected inflation is volatile, it will provide more value. Whether this result holds for a retiree will be discussed in this thesis.

In practice, people tend to avoid annuities, contrasting the view of forward looking utility maximizing agents, a phenomenon called the 'annuity puzzle'. Many authors have tried to solve this puzzle by various explanations. Mitchell, Poterba, Warshawsky, and Brown (1999) argue that the standard explanation is that annuity markets are small due to adverse selection and thus the price of annuities is too high. However, Mitchell et al. (1999) show that adverse selection costs have decreased over the last decade.

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<sup>2</sup> In this thesis, I will consider life-contingent annuities only (later to be called 'annuity'). For clarification purposes I have used life-contingent, as a temporary annuity does not possess the desirable feature of hedging the risk for an individual to outlive his resources.

I will assume that annuities are fairly priced, which might not be realistic as individuals with additional information of their own life expectancy might increase their annuity portfolio because they expect to live longer and hence receive benefits for an extended period of time. This in turn pushes up the price of annuities, because individuals who will expect to live shorter than average will decrease their annuity portfolio. This is a classical result of asymmetric information.

Other explanations for the annuity puzzle are bequest motives and health shocks which might affect the optimal fraction of wealth to be annuitized. For instance, Lockwood (2012) argues that bequest motives and health care costs shifts the optimal annuity allocation completely. However, Peijnenburg, Nijman and Werker (2011) find that even under health care costs and a bequest motive, full annuitization is still optimal, because the agent can reinvest nominal annuity income in the equity market.

Another argument which has gotten notable attention in the literature is the framing of the decision. Brown, Kling, Mullainathan and Wrobel (2008) find that people tend to choose annuities if it is presented in a consumption frame. However, if annuities are presented in an investment frame, people prefer non-annuitized assets. They claim that the investment frame is dominant in financial planning, and hence people prefer non-annuitized assets.

Koijen et al. (2009) research welfare implications for inadequate annuity menus. Furthermore, they find that an investor can benefit from incorporating term structure variables and information on risk premia to improve the annuity choice.

I will not try to solve or give an explanation for the annuity puzzle but focus on the optimal annuity decision in absence of a bequest motive and health care costs. I will assume that the agent will annuitize his wealth at a single point in time and relate this to the current Dutch framework, which will be discussed in the next section.

### 3. Dutch framework

In this section, I will explain the Dutch institutional framework by showing different characteristics of the three main pillars. Recommendations for improvement of the Dutch system will be discussed and lastly I explain the practical relevance of this thesis and how my research might provide recommendations for improvement of the current system.

#### 3.1 The three pillars

The Dutch framework consists of three pillars. The first pillar is the AOW, the old-age pension. It is financed by a pay-as-you-go (PAYG) system. The AOW has provided a real income stream in the past, which means that payouts have been corrected for (wage) inflation. Rights are accumulated for every year you live in the Netherlands. A single person receives 70% of the minimum wage, and a married person or cohabiter receives 50% of minimum wage.

The second pillar consists mainly of collective occupational schemes and can be executed by pension funds or insurers. The pension funds are typically defined benefit (DB) schemes, but also defined contribution (DC) or hybrid forms are around. For the typical DB scheme, rights are accrued by paying premiums and benefits are adjusted for inflation if the assets over the liabilities (the funding ratio), is high enough. The pension fund invests the premiums for its members. In the second pillar there are some legal boundaries imposed by the pension law:

- It requires that benefits are paid lifelong as protection of longevity risk for the employee.
- It is not allowed to frontload pension money or receive a lump sum.

However, there are a few exceptions:

- A payment can be temporary if a consumer want to retire early, as this can bridge the gap between early retirement and reaching the eligible AOW age.
- The level of benefit can be frontloaded (backloaded) by the so called High-Low (Hoog-laag) pension. This allows the retiree to receive a higher income in the early (or later) years in retirement. However, the highest benefit may not exceed 50% of the minimum level. This arrangement is intended to provide some flexibility for consumption in the early years of retirement. A requisite is that all the adjustments are actuarial fair. In other words, no value can be transferred ex-ante by using these flexibilities.

In the 2<sup>nd</sup> pillar, annuities for which the payoff depends directly on an underlying asset, is not allowed. Hence, variable annuities which pay out in units are excluded.

All in all, for the typical mandatory DB-scheme the retiree ends up with a nominal annuity with a rather complicated call option for (conditional) indexation depending on the funding ratio of the fund (Brown and Nijman, 2011).

The third pillar consists of voluntary pension schemes executed by insurers or banks, for people who are not able to use the second pillar, such as the self-employed, or people who still have a 'pension' gap. For instance, some people cannot use their tax benefits completely in the 2<sup>nd</sup> pillar so they resolve this by purchasing an individual product in the 3<sup>rd</sup> pillar. Product offering is limited in the 3<sup>rd</sup> pillar, as only nominal annuities are available. However, this is not caused by legal boundaries as payments are allowed to vary in the 3<sup>rd</sup> pillar. Essential is that the actual payout, after the product has been purchased, cannot be altered by either of the parties involved in the contract. Variable annuities or inflation linked annuities are allowed because the determination of the payout depends on realized inflation and realized equity returns. Table 1 summarizes the different characteristics of the pillars.

<b>Pillar</b>	<b>1<sup>st</sup> Pillar</b>	<b>2<sup>nd</sup> Pillar</b>	<b>3<sup>rd</sup> Pillar</b>
<b>Type of pension</b>	State pension	Collective mandatory pension	Individual voluntary pension
<b>Payout</b>	Real payout	Conditional Real payout No pay out in units (no variable annuities)	Nominal payout
<b>Sponsor</b>	Government	Employee/Employer	Individual

**Table 1: Characteristics of the different pillars in the Netherlands**

The composition of pension income is not the same for everybody. For instance, entrepreneurs or the self-employed without personnel (ZZP) do not have access to 2nd pillar products. In fact, the number of people who are switching between being employed or self-employed has increased over the years, and the number of companies making use of self-employed without personnel has grown too (UWV, 2014)<sup>3</sup>. It is therefore not self-evident that people will receive the largest part of their pension income from the mandatory 2nd pillar schemes. As heterogeneity in being self-employed or employee increases, people are exposed to different risks stemming from the different risk characteristics of the pillars. We will examine the different risk exposures for both typical schemes in accumulation and decumulation below and see how it affects the individual or the collective. Note that I have chosen a general representation of the differences between the two pillars.

<b>Type of risk</b>	<b>Defined Contribution (insurer, individual) 3<sup>rd</sup> Pillar</b>	<b>Defined Benefit (pension fund, collective) 2<sup>nd</sup> pillar</b>
Equity Risk	Born by individual participant, up and downside	Equity risk is shared across the fund. If funding ratio falls (below threshold), no indexation, increasing premiums or cutting benefits (in extremis).
Inflation risk	Born by individual, asset mix should provide (long term) inflation protection to preserve pension assets	Inflation risk is shared across the fund by asset mix. Whether rights are indexed for inflation depends on the funding ratio
Interest rate risk	Born by individual, this can create conversion risk but duration matching towards retirement mitigates this problem.	Interest rate risk is shared across the fund. A drop in interest rates increases the value of the fixed income assets and increases the value of the liabilities. Typically this reduces the funding ratio as the duration of the liabilities is greater than the duration of the assets.
Macro-longevity risk	Born by the individual. If life expectancy goes up, the price of the annuity will go up. The income that can be generated with the capital goes down.	Born by the cohorts, higher life expectancy increases the value of the liabilities and lowers the funding ratio of the fund. However, automatic mechanisms are discussed to mitigate this problems by adjusting the age at which participants can retire.

**Table 2: Risk exposures for the typical individual and collective schemes in the accumulation phase**

In the accumulation phase the individual scheme bears all the risk by himself, both down and upside. This can result in larger fluctuations in potential outcomes, as van der Lecq and van der Wurff (2011) documented. In a pension fund, these risks are shared between and among generations. Pension funds have mandatory participation and this allows the fund to share risks between generations. This is called intergenerational risk-sharing and Gollier (2008) found that this increases returns for participants substantially. The potential benefits

<sup>3</sup>[http://www.uwv.nl/OverUWV/perscentrum/persberichten/2014/Steeds\\_meer\\_bedrijven\\_maken\\_gebruik\\_van\\_ZZPers.aspx](http://www.uwv.nl/OverUWV/perscentrum/persberichten/2014/Steeds_meer_bedrijven_maken_gebruik_van_ZZPers.aspx)

of risk sharing will not be examined in this thesis, but it is useful to create intuition for the way risks are absorbed in the typical scheme.

Before moving to the decumulation phase let us think about the transitional phase for which two problems emerge for individual schemes. How to deal with interest rate risk and how to deal with equity risk? A retiree does not want to find out his pension capital has halved due to a stock market crisis. Neither does the retiree want to be confronted with a sudden drop in interest rates, which increases the cost of nominal annuities. If the return on money decreases, future payments will become more costly, lowering the monthly benefits of the annuity to the retiree. Common practice in the Netherlands is the use of life-cycling, which decreases the risk of the portfolio towards retirement. More specifically, it reduces the equity exposure gradually to zero and it constructs a duration matching portfolio to mitigate interest rate risk. But this solution creates its own problems. In the decumulation phase, the retiree is now exposed to several risks which he has locked in to mitigate conversion. As he cannot buy a real annuity, the nominal annuity will be exposed to inflation risk. Nor is the investor able to get exposure to equity risk (in annuitized form) and has to forego the equity risk premium. Table 3 exhibits the differences in the scheme for the decumulation phase:

Type of risk	Defined Contribution (insurer, individual) 3 <sup>rd</sup> Pillar	Defined Benefit (pension fund, collective) 2 <sup>nd</sup> pillar
Equity Risk	Born by individual participant, up and downside, although no variable annuity exists currently.	Equity risk is shared across the fund. If funding ratio falls (below threshold), no indexation or increasing premiums.
Inflation risk	Born by individual. Annuity is in nominal terms, with no prospect of inflation indexation due to the lack of inflation linked asset market in the Netherlands.	Inflation risk is shared across the fund by the asset mix. If funding ratio is sufficient inflation can be indexed.
Interest rate risk	As the annuity is locked in, interest rate risk is no longer an issue.	Interest rate risk is shared across the fund. A drop in interest rates increases the value of the fixed income assets and increases the value of the liabilities. Typically this reduces the funding ratio as the duration of the liabilities is greater than the duration of the assets. Via the funding ratio, retirees are exposed to interest rate risk in retirement.
Macro-longevity risk	Born by the insurer (equity holders) but premium paid by individual.	Born by the group, pay no (less) premium because the risk is shared among and between generations by the buffers of the fund.

**Table 3: Risk exposures for the typical individual and collective schemes in the decumulation phase**

The participant in the pension fund has a nominal payout in the decumulation phase but the asset mix provides optional inflation indexation. He will keep some risk budget during retirement to maintain the purchasing power of his pension income. But the most obvious difference is that the individual has no direct exposure to conversion risk in the collective scheme. Naturally, there are some effects for the fund as a whole which can cause the funding ratio to drop. There are other mechanisms to redistribute value in case of bad financial market performance, such as increasing premiums or foregoing rights indexation. Lastly, macro-longevity risk is shared differently. As the insurer at conversion will ask a premium to hedge this risk, the retiree will pay a premium. For the pension fund this risk is usually shared among the group, so there is no premium involved. Whether this is indeed beneficial depends on the pricing of the product and the way longevity develops.

### 3.2 Recommendations for the Dutch system.

Some authors have done recommendations and proposed changes for the Dutch system. For instance, Dietvorst et al. (2011) recommends making real annuities the new standard instead of nominal annuities. In order to achieve this, the government should issue inflation linked bonds. Second, they propose to use variable annuities

to allow for risk taking during an increasing retirement period. Furthermore, they propose to allow partial lump-sums, decrease the separation between the accumulation and decumulation phase (as to retain risk-budget during retirement) and to improve product information on annuities. Brown and Nijman (2011) recommend setting a minimum replacement rate of 50% and allowing more flexibility above that. They advise a bigger part of pension wealth be explicitly inflation protected if markets exists. Lastly they touch upon conversion risk and the “duty of care” (zorgplicht) legislation. The AFM (Authority of Financial Markets) has published guidelines to manage conversion from capital to income. It states that equity risk has to be reduced to zero around conversion and that the duration of the portfolio should be equal to the to be acquired annuity, as to manage interest rate risk. Brown et al. (2011) argue that these standards may not be appropriate, and instead ‘risk tolerance’ around conversion should be considered as a benchmark.

### **3.3 Practical relevance**

This thesis is practically relevant to the Dutch framework because it analyzes qualitative recommendations in a quantitative framework. As inflation linked and variable annuities are not offered currently we can examine if there is welfare to be gained from including this products to the asset menu. Although the shift from collective to individual schemes has yet to materialize, we should be prepared and know how to address the different risk exposures properly.

Now that we have seen the Dutch framework and differences between individual and collective schemes, we move on to the financial market model, the annuity menu and the preferences of the individual.

## 4 Financial model, annuities and preferences

In this section, we discuss the financial market model. Financial market parameters are important, because they determine the relative (un)attractiveness of risks such as inflation risk, interest rate risk and equity market risk. From this financial market model, which generates a term structure of interest rates, an equity market index and a price index, an annuity menu is constructed. Three basic annuity types are considered: a nominal, real and variable annuity. Furthermore, the state pension AOW is discussed. In the last section of this chapter the preferences of the individual are discussed. A Constant Relative Risk Aversion (CRRA) utility function is considered, in line with Koijen et al. (2009).

### 4.1 Financial Model

Earlier, we have seen the importance of the funding ratio to determine payouts in pension funds. However, calculating the value of liabilities can be a subjective task. As the accumulated rights of members are in the distant future, one has to come up with a reasonable discount rate to calculate the present value of the liabilities. As the term structure of interest rates becomes unreliable due to illiquidity for maturities greater than 30 years, determining the discount rate becomes a subjective matter. In this way, the discount rate for the liabilities becomes an accounting concept, instead of a practical one, where you can observe and determine the exact value of your liabilities by market prices. As pension funds determine member premiums and (indexation) rights based on their funding ratio, there exists a need to use consistent economic scenarios and to limit expected portfolio return and inflation development. Otherwise, the abovementioned accounting principle might be abused by the pension board to represent the status of the fund too optimistically, and yield to the myopic desire of participants to increase their rights. The Commission Parameters was entrusted with this task, and they imposed maximum expected returns and generated a uniform scenario set. Their advice can be summarized as follows:

Category	Geometric mean	Standard deviation	Note
<i>Minimum expectation</i>			
Price inflation	2,0%		
Wage inflation	2,5%		
<i>Maximum expectation</i>			
Listed Equity	7,0%	20%	
Government Bonds	2,5%	8%	In accordance with forward market rates
Alternative investments	7,5%	25%	
Credits	3%	8%	In accordance with forward market rates and credit risk of assets
Unlisted real estate	6%	15%	
Commodities	5%	20%	

**Table 4: Summary of maximum expectations from the advice of the Commission Parameters**

All the other asset categories are modelled as the return on equity, but I will not consider other asset categories. Wage inflation in the model is simply the price inflation with a 0.5 percentage point increase, but I will instead use price inflation in my model.

The financial market model that is used is based on the Koijen et al. (2010) model, estimated on US data. Draper (2014) estimated the model on Dutch data and calibrated the model to make it consistent with the expectations of the Commission Parameters. I will shortly introduce the model in the appendix, but the actual simulations are generated by the Tilburg Finance Tool, created by Prof. dr. Bas Werker. The used parameters and a brief overview of the model are reported in the appendix. The state variables are derived from swap curves by the Commission Parameters at 31-12-2013. All in all, the model contains stochastic inflation risk, interest rate risk and equity risk.

## 4.2 Constructing Annuity income

The focus of this thesis is on the decumulation phase of (individual) defined contribution schemes and the analysis begins when capital is converted to income, abstracting from conversion risk. In the literature review I have elaborated on the theoretical justification for annuities, and as the requirement of full annuitization is embedded in the Dutch law this will be my starting point. I will consider single premium life-contingent annuities with nominal, real and variable payouts. Single premium means that the annuities are purchased by a single deposit at retirement, abstracting from the accumulation phase and gradual annuitization strategies. I use notation in line with Peijnenburg et al. (2011):

The annuity income is:

$$Y = PR_0A^{-1} \quad (1)$$

A denotes the annuity factor.  $PR_0$  denotes the single premium. I will assume an actuarially fair annuity, so that no additional premium is paid for additional costs or risks for the insurer such as adverse selection or macro-longevity risk. The single premium is thus equal to the expected benefits of the annuity.

In my basic model, micro longevity risk is absent, so there is no uncertainty about the time of death. For a given T (the known age of death) the annuity factor depends on the term structure of interest rates at time zero.  $R_0^{(t)}$  is the yield on a zero coupon bond maturing at time t. Thus the annuity factor A is:

$$A = \sum_{t=1}^T \exp(-tR_0^{(t)}) \quad (2)$$

Which is the present value of all the payments until the known age of death, T.

In case of a nominal annuity  $R_0^{(t)}$  denotes a nominal zero coupon bond maturing at time t. In case of a real annuity,  $R_0^{(t)}$  denotes a real zero coupon bond maturing at time t. The real annuity gives a constant payout in real terms, and thus is corrected for realized inflation. As stated above, the annuities are assumed to be actuarially fair, acknowledging that this is a favorable representation of the inflation-linked and variable annuity due to the inflation risk premium and possible additional management fees (and transaction costs) for managing equity portfolios, a difference which the Commission Parameters sets at 10 basis points. Especially if a party would currently offer inflation protected annuities in the Netherlands it might be exposed to several risks which would justify a premium. For instance, if it would use non-domestic inflation linked bonds it would be exposed to basis risk. That is, even though we share the same currency with Germany and France, inflation might be different for other reasons such as supply side factors.

For the variable annuity things are slightly different. The annuity factor now depends on two factors. The assumed interest rate AIR, but also on the realized return of the underlying equity index  $S_u$ . The annuity factor now becomes:

$$A = \sum_{t=1}^T \exp(-tAIR) \quad (3)$$

The value of the nominal, fixed and variable annuities are the same at time  $t=0$  when no premium is paid on the annuities. However, the expected payout differs. If there is a positive risk premium, as is the case in my model, the expected value of the variable annuity is larger. This comes at the cost of running the risk of a lower payout.

The income now varies over time<sup>4</sup> and the payout in every period depends on the performance of the underlying stock index  $S_u$ :

$$Y_t = PR_0 A^{-1} \prod_{u=1}^t \left( \frac{1+S_u}{1+AIR} \right) \quad (4)$$

The chosen AIR determines the pay-off structure of the annuity. If it is higher than the expected rate of return of the underlying portfolio it will have (on average) a decreasing profile, if it is lower, vice versa. The easiest way of understanding is by imagining that the insurance company can pay-out 100 units of stocks over a period of 20 year. If you would think this units do not return anything (so you assume the AIR is 0%) you receive 5 units per year without any return if your expectation materializes. If this expectation is not correct and instead the stock unit returns 5% per annum in real terms, the payout in year 20 would correspond to 13.26 units today, an increase of 165%.<sup>5</sup> In the example the realized return is higher than the assumed rate of return and thus has an increasing profile. It also works the other way around. If you keep too little units for the end of the period, and the return is not as high as you assumed it to be, your pay out will decrease over time. This result holds in the general simulated case. A higher AIR than the expected return on the portfolio will generate a decreasing pay out stream over time on average.

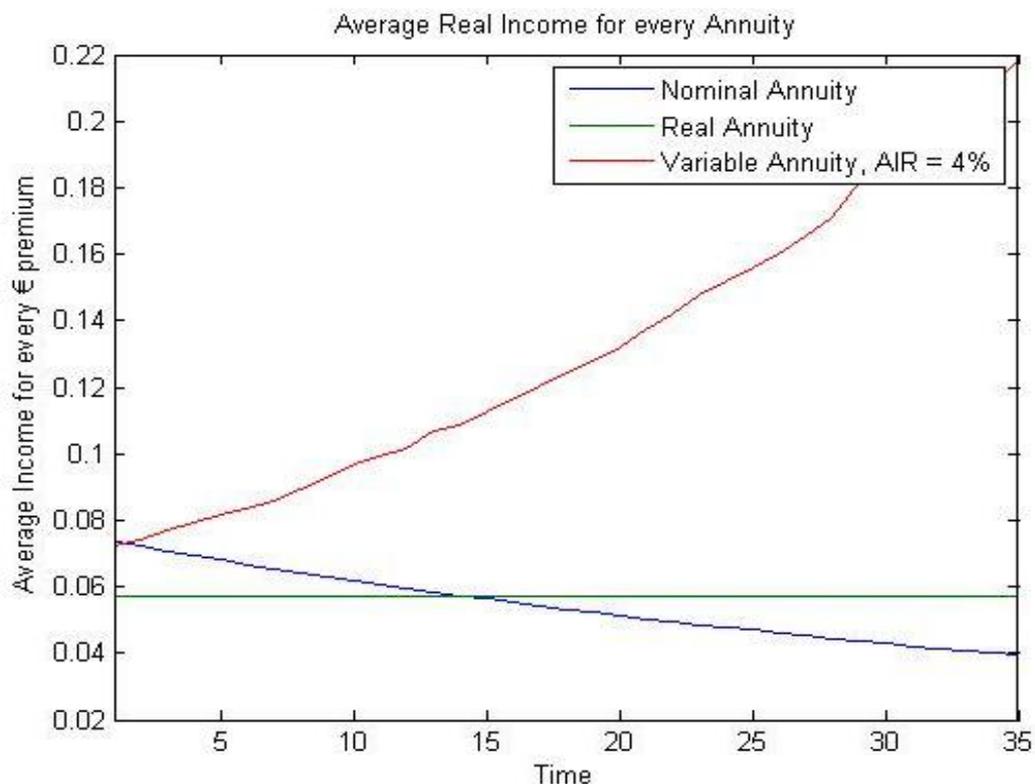


Figure 1: Average real annuity income for the three annuity types (1000 simulations).

As can be seen in figure 1, the real annuity (green line) –by definition– has a flat payoff pattern in real terms. The nominal annuity (blue line) has a flat payoff pattern in nominal terms, but in real terms, decreases over time as

<sup>4</sup> Depending on the interpretation of consumption the nominal and real consumption can also vary over time. For details see appendix C.

<sup>5</sup>  $1.05^{20} = 2.65$

it is deflated by the price index. As the chosen AIR is 4%, the variable annuity shows, on average, an increasing payoff pattern over time. We already see that the compounding effect of the variable annuity is strong.

#### 4.2.1 Old age insurance, AOW

The Netherlands offer old-age insurance in the form of a monthly pay-out conditional on the number of years you have lived in the Netherlands during age 15 and 65<sup>6</sup>. The AOW is linked to wage-growth. As my parameters are set to use price inflation, I will deviate from the Commission Parameters advice to use an expected wage inflation of 2.5%. Instead I will assume the AOW to be linked to price inflation, which is 2% in expectation. This decreases the attractiveness of the AOW somewhat.

According to the Sociale Verzekeringsbank (SVB)<sup>7</sup> the AOW, including holiday allowance, is approximately 13.200 euros per year (before taxes).

Therefore, AOW income  $YAOW$  at time  $t$  is:

$$YAOW_t = AOW * \Pi_t \quad (5)$$

Which is the starting value of the AOW, adjusted by the price index  $\Pi_t$  at time  $t$ . In the past, the AOW has been corrected for inflation, I assume this will continue in my model

Recall (1):

$$Y = PR_0 A^{-1}$$

$PR_0$ , is the single premium and is calculated by using the median income (€33,000) and subtracting the franchise of AOW income (13.200) and using an average premium of 15% over this income. After which I work backwards by reducing this premium by 2% inflation and an assumed return of 5% over the invested premium for the years 25 to 65. Under this assumptions I find the average available premium at age 65 to be € 230,000. In section 5.6 I will test for robustness of this scaling parameter.

The total real income depends on the income of the old-age pension the AOW and the income generated by the annuities, adjusted for the price index. Furthermore, I will assume a tax rate of 30% for the base case scenario over the entire pension income. Lastly, consumption will be evaluated in real terms, and thus the variable and the nominal annuity must be deflated by the price index  $\Pi_t$ .

#### 4.3 Preferences

In order to evaluate the consumption streams for different annuity products a time-separable, constant relative risk aversion utility function is used. As stated before, in the basic model the time of death is deterministic. The agent maximizes expected utility over real consumption  $C_t$ :

$$V = E \left( \sum_{t=1}^T \beta^{t-1} \frac{C_t^{1-\gamma}}{1-\gamma} \right) \quad (6)$$

Where  $\beta$  is the time preference discount factor,  $\gamma$  is the risk aversion coefficient. For ease of exposition I will assume that the agent consumes all his wealth in every period, so the agent does not save from annuity income. I will refrain from background risk or health care shocks, so the agent does not need liquidity to absorb these shocks.

<sup>6</sup> This will change to the years 16-66.

<sup>7</sup> ([http://www.svb.nl/int/nl/aow/hoogete\\_aow/bedragen](http://www.svb.nl/int/nl/aow/hoogete_aow/bedragen))

### 4.3.1 Adding longevity risk

To elaborate the model I include micro-longevity risk. To do so I retrieve the one-year death probabilities from the actuarial society (actuarieel genootschap). I will use the male one-year death probabilities, assuming that if the survival probabilities are combined this would benefit females, as they live longer. Consider an (male) individual of age  $x$  whose probability of not surviving another year at year  $t$  is denoted by:  $q_{x,t}^{(m)}$ . The chance of this individual to survive another year will then be:

$$p_{x,t}^{(m)} = 1 - q_{x,t}^{(m)} \quad (7)$$

The age  $x$  is set at 65, so now the survival probabilities can be incorporated in the value function.

$$V = E \left( \sum_{t=1}^T \beta^{t-1} (\prod_{s=1}^t p_s) \frac{C_t^{1-\gamma}}{1-\gamma} \right) \quad (8)$$

Here  $\beta$ , the time discount factor, is set at 0.98, slightly higher than Pang and Zahwaksy (2010) as their financial market model has higher interest rates which influences the decision between consuming today and in the future.  $p_s$  is the one-year survival probability. One can interpret this as the product of the one year survival probabilities. For instance, the  $\tau$ -year survival probability:

$$\tau p_{65,t}^{(m)} = p_{65,t}^{(m)} * p_{65+1,t+1}^{(m)} * \dots * p_{65+\tau-1,t+\tau-1}^{(m)} \quad (9)$$

In my model I will assume absence of macro longevity risk. That is, I will assume that the survival probabilities  $p_s$  are constant over time. Hence I use the survivor probabilities from age 65 in time onwards. I use the probabilities of the age cohort which are 65 in 2014. I assume a maximum age of 100, so the maximum period for any individual to be alive in retirement is 35 years. As micro-longevity is added, the annuity factors should be adjusted as the time of death becomes stochastic. See appendix B for the adjustments.

### 4.3.2 Presentation of results

For positive levels of gamma the expected utility does not have intuitive values nor is it easily comparable. The results become negative and hence are not easily interpretable. That is why a certainty equivalent consumption strategy is better to compare results. The certainty equivalent consumption determines the amount of (real) consumption independent of states and time and provides a good comparison number. For technical details, see appendix D. By calculating the certainty equivalent consumption we can show a representation of the 'monetarized' utility level. A riskless consumption level that represents the same utility as the risky strategy followed. This allows us to compare varying strategies in real certain consumption terms.

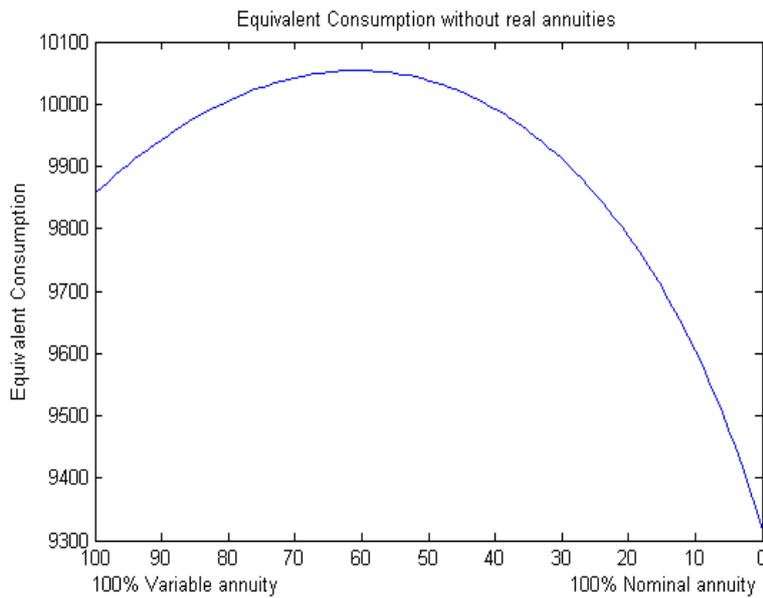
Because the real annuity is not available in the Netherlands I will start by showing the certainty equivalent consumption between a situation where the retiree is completely invested in the nominal annuity, and a situation where the retiree is fully invested in the variable annuity. This is closest to reality in the current Dutch situation, where real annuities are absent and variable annuities are allowed but not offered in the 3rd pillar.

In this chapter we have introduced the financial market model with the parameter set from the Commission Parameters. The income of the retiree consists of four parts: the AOW and a mixture of nominal, real and variable annuities. The preferences of the individual are represented by a CRRA utility function with time discounting and longevity risk. In the next section results are discussed.

## 5 Results and sensitivity analysis:

In this section results for the constructed annuity income are presented. The simplest model starts with a certain time of death. Afterwards, the base case model is developed with stochastic longevity. From this point several sensitivities are tested to see if the optimal allocation differs. The risk appetite of the individual will be adjusted, the AOW will be excluded, the AIR is adjusted and we will test for different single premium values. As the optimal portfolio contains a big allocation to the variable annuity, the 5% worst outcomes will be examined. As a robustness check I will check the results for the unadjusted financial market model by Draper (2014). The last subsection expands the utility function of the individual by including a standard-of-living. In this utility function the agent relates consumption to past consumption.

### 5.1 Deterministic longevity



Variable	Value
$\beta$	0.98
$T$	20 years
$\gamma$	5
AOW	€13,200
Single premium	€230,000
AIR	4%
Tax rate	30%

Table 5: Assumptions for figure 2

Figure 2: Equivalent consumption in absence of real annuities for deterministic longevity.

In the basic model, equity exposure via the variable annuity increases the certainty equivalent consumption. In fact, every other composition than the default of 100% nominal annuities outperforms the benchmark strategy. To see whether the inclusion of real annuities adds welfare, I optimize over the three weights:

Type of income	Nominal + Variable Annuity	All three annuities	Nominal and Real
Nominal annuity	39%	39%	100%
Real annuity	-	0%	0%
Variable annuity	61%	61%	-
Certainty Consumption Equivalent	€10,054	€10,054	€9,316

Table 6: Optimal allocations and certainty consumption equivalents for complete and incomplete annuity menus.

It appears to be somewhat odd that the real annuity does not seem to add welfare in this model, under the base case assumptions mentioned above. This is probably in part due to the AOW income, which already provides a real income stream and thus an (unexpected) inflation hedge. Likewise, the time discount factor plays a role, which devalues consumption in the future over consumption today. Naturally this is compensated for by the return on the money, but not enough in this case. By investing 39% in the nominal annuity and 61% in the variable annuity, the optimal is reached, with a €738 welfare gain.

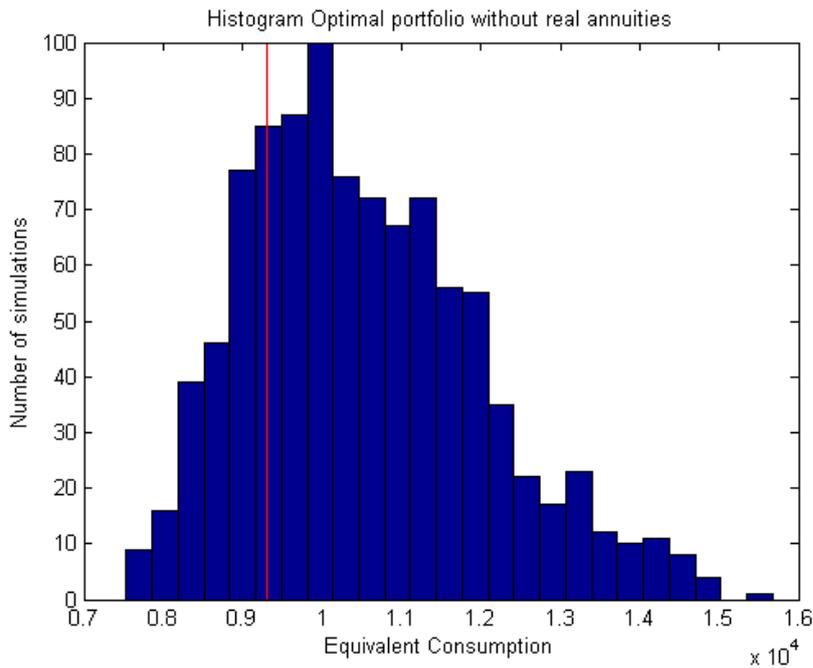
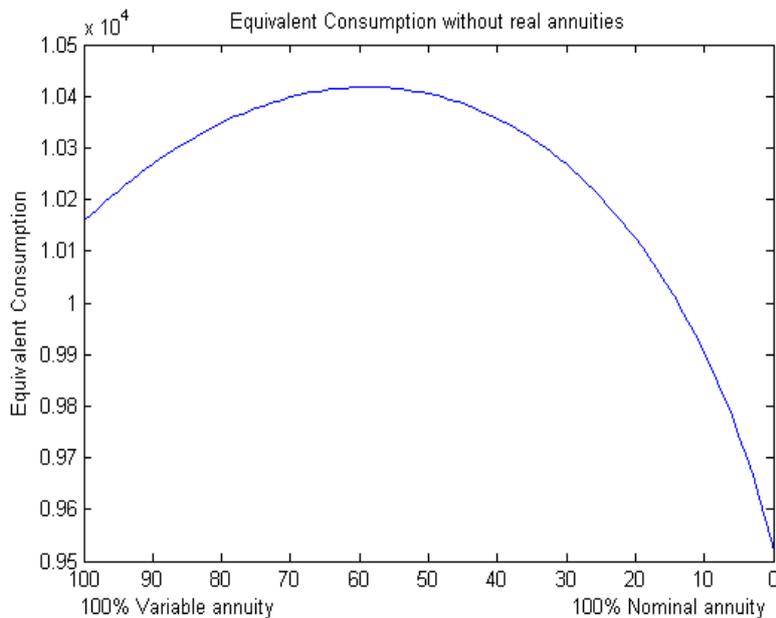


Figure 3: Histogram of 1000 simulations in the deterministic model. The vertical red line depicts the average equivalent consumption where the retiree is 100% invested in the nominal annuity.

In figure 3 the histogram of equivalent wealth for the optimal portfolio compared to the base portfolio is depicted. In about 23% of the cases, the base case portfolio outperforms the optimal portfolio with 61% invested in variable annuities and 39% in nominal annuities. The distribution of outcomes for the optimal portfolio has positive skew (more simulations to the right of the median) and is somewhat flat. This is not very surprising since the allocation to the variable annuity is quite large.

## 5.2 The base case model: stochastic longevity

As in the simple model I will first examine the nominal and variable annuity, and afterwards add the real annuity as to compare the difference.



Variable	Value
$\beta$	0.98
T	Stochastic male
$\gamma$	5
AOW	€12,300
Extra Pension	€230,000
AIR	4%
Tax rate	30%

Table 7: List of assumptions for figure 4.

Figure 4: Equivalent consumption in absence of real annuities for stochastic longevity.

Here we see the equivalent consumption on the y-axis and the weights for the nominal and the variable annuities on the x-axis. The most southeastern point in the graph depicts the situation when a retiree is 100% invested in nominal annuities, and the certainty equivalent wealth here is €9,521. As we can see, the highest equivalent consumption is situated somewhere left of the middle. We now add real annuities to the asset menu from which a retiree can choose. This results in the following optimum:

Type of income	Nominal + Variable Annuity	All three annuities	Nominal + Real annuity
Nominal fixed annuity	41%	41%	31%
Real annuity	-	0%	69%
Variable annuity	59%	59%	-
Certainty Consumption Equivalent	€10,417	€10,417	€9,545

Table 8: Optimal allocations and certainty consumption equivalents for complete and incomplete annuity menus.

As in the basic model, variable annuities add to welfare. The optimal portfolio hardly changes compared to the deterministic setting, as the weights change by 2 percentage points. In this case, the optimal portfolio adds approximately €900 certainty equivalent consumption compared to a situation where the agent is 100% invested in a nominal annuity. However, real annuities do not add welfare. This result is surprising as I expected the agent to rebalance more towards the real annuity. In the current situation the real annuity is actuarially fair, and hence no premium is paid for the annuity. To see whether real annuities still don't add value if the variable annuity is removed from the menu we see that this is not the case:

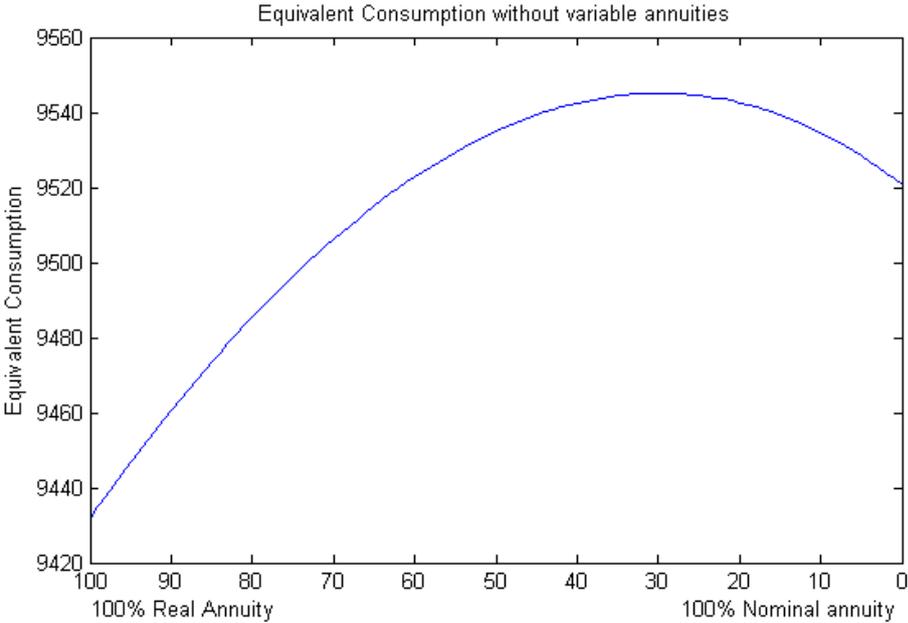


Figure 5: Equivalent consumption in absence of variable annuities with stochastic longevity.

However, the difference is marginal, amounting to approximately € 20. This might have to do with the old age pension AOW, which already provides an inflation hedge. Sensitivity to the state pension will be discussed in section 5.4. Figure 1 showed that the nominal annuity provides a bigger real income in the early years of retirement. As the consumer is impatient and devalues consumption in the future over consumption today, it leans towards more instant gratification in the standard framework. In fact, allocating 100% towards the real annuity lowers the certainty equivalent consumption by approximately € 90.

The variable annuity which reaps the equity risk premium is preferred over the inflation protection of the real annuity. This effect plays a bigger role for longer maturities, as the return difference compounds. Recall from

figure 1 that on average the inflation protected annuity has a higher payout than the nominal annuity in year 14. For these years, the nominal annuity is more attractive, as it provides a higher real income. After 14 years, the average income of the variable annuity is 90% larger than the real annuity. Besides this, inflation can be lower than expected (and what was paid for) when the annuity was negotiated, which increases the attractiveness of the nominal annuity as the real payout is larger than expected. In this way, the real annuity bears both the downside and upside risk of inflation. Another explanation could be that inflation is not a big risk for the consumer, and hence the real annuity provides no value. This of course depends on the financial market model.

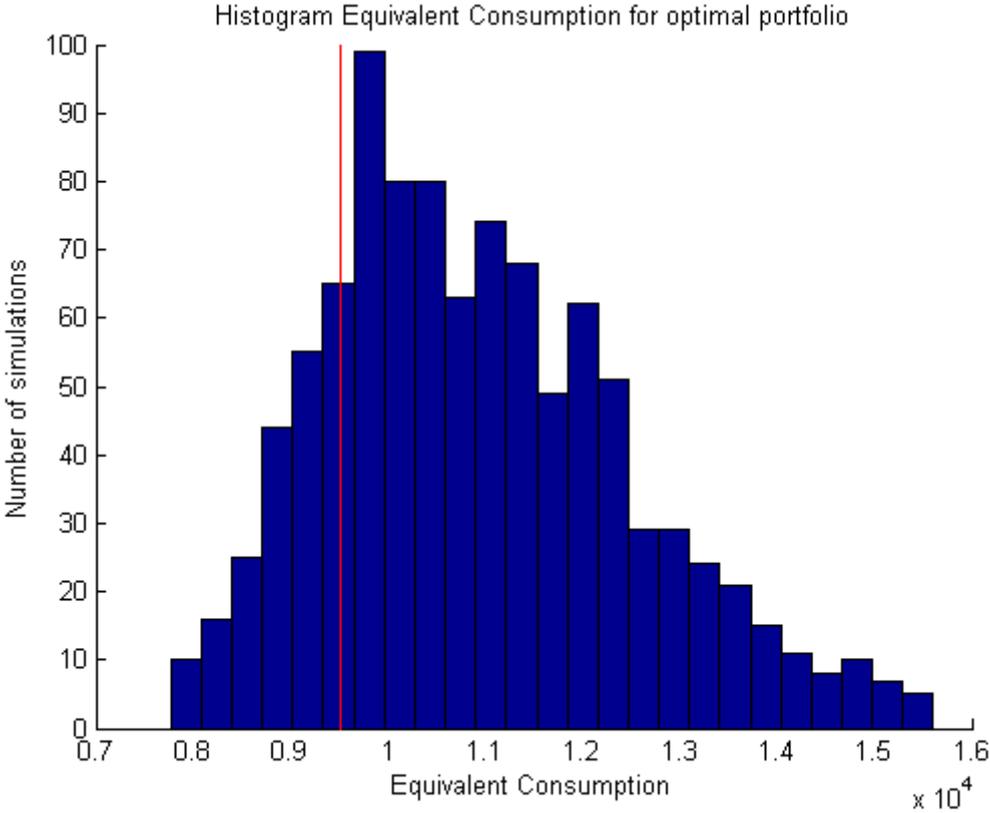


Figure 6: Histogram of 1000 simulations for stochastic longevity. The vertical red line depicts the average equivalent consumption where the retiree is 100% invested in the nominal annuity.

If we look at the distribution of the certainty equivalent wealth, we see that in approximately 19% of the instances the retiree is worse off than the position in which he is fully invested in nominal annuities. The red line depicts the average equivalent consumption for the base case scenario with a 100% allocation to the nominal annuity.

### 5.3 Changing risk aversion

To see if my general result still holds under different parameters of risk aversion I will vary the risk appetite of the investor. In general, a risk aversion level of 5 seems appropriate and consistent with literature (see for instance Peijnenburg et al. (2011) or Pang et al. (2010)), but for more risk loving (low  $\gamma$ ) or more risk averse (high  $\gamma$ ) investors the results might differ.

### 5.3.1 Low risk aversion ( $\gamma = 2$ )

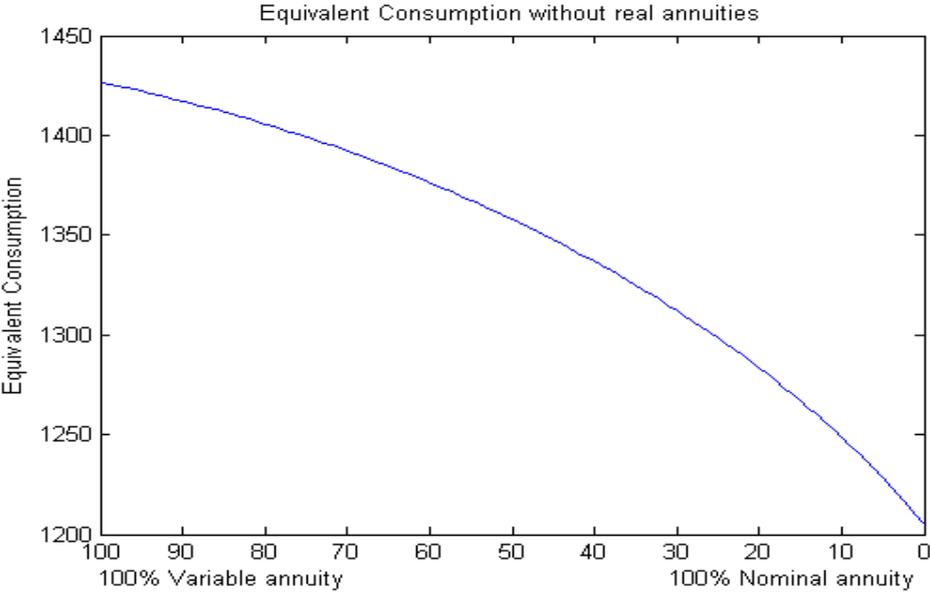


Figure 7: Certainty equivalent consumption for a low risk aversion agent in absence of real annuities.

With low risk aversion we see that the investor completely moves towards variable annuities. The old age pension acts as a riskless real income stream and the agents shifts the allocation completely towards the variable annuity. The certainty equivalent consumption cannot be compared with the original model, as the risk aversion parameter has changed, however, it can be compared towards the situation with a 100% allocation in the nominal annuity. It adds approximately €220, but in relative terms it is larger than in the previous case, as it is a relative gain of 18%<sup>8</sup>. The real annuity does still not increase utility for the agent, which is not a surprise, as real annuities should appeal more to risk averse than risk loving investors. The following optimum is found:

Type of income	Nominal + Variable annuity	All three annuities	Nominal + Real annuity
Nominal fixed annuity	0%	0%	100%
Real annuity	-	0%	-
Variable annuity	100%	100%	-
Certainty Consumption Equivalent	€1,426	€1,426	€1,205

Table 9: Optimal allocations and certainty consumption equivalents for the complete and incomplete annuity menus for low risk aversion agents.

<sup>8</sup> Certainty equivalent consumption can be compared relatively, by applying:  $\left(\frac{U_0}{U_c}\right)^{\frac{1}{1-\gamma}}$ . For full details see the appendix D.

### 5.3.2 High risk aversion ( $\gamma = 10$ )

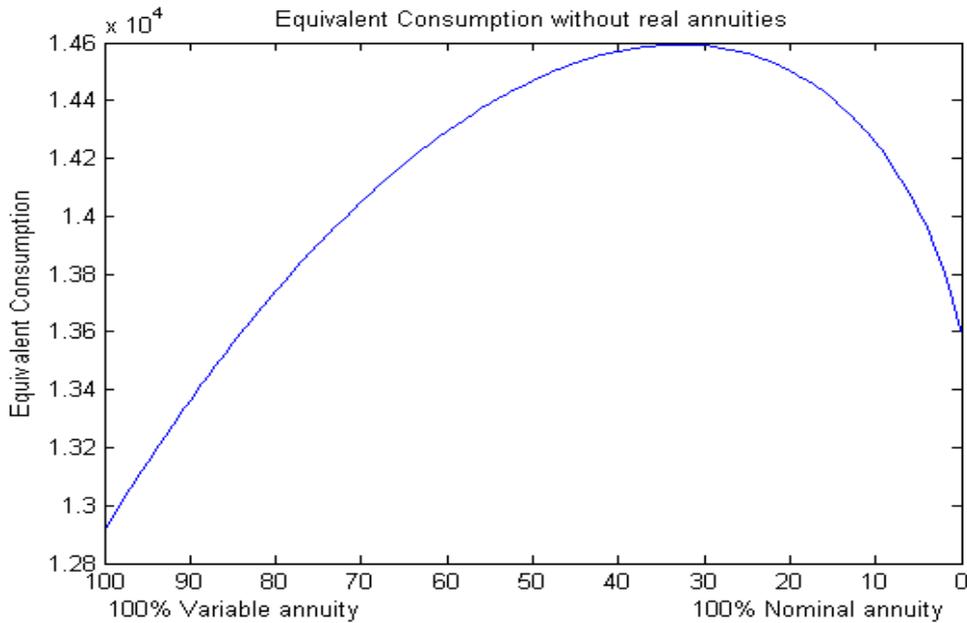


Figure 8: Certainty equivalent consumption for a risk averse agent in absence of real annuities.

For risk averse investors, with a high  $\gamma$ , the allocation does shift towards a less risky position, as it decreases the position in the variable annuity with 26% compared to the base case scenario with a  $\gamma$  of 5. This does once again not come as a surprise, because risk averse investors penalize downside fluctuations in income which are more probably if the allocation to the variable annuity increases. Optimizing over the three annuity types shows that the real annuity enters the optimal portfolio:

Type of income	Nominal + Variable annuity	All three annuities	Nominal + Real
Nominal fixed annuity	67%	49%	35%
Real annuity	-	20%	65%
Variable annuity	33%	31%	-
Certainty Consumption Equivalent	€14,593	€14,622	€13,952

Table 10: Optimal allocations and certainty consumption equivalents for the complete and incomplete annuity menu for risk averse agents.

As shown in the table above, adding real annuities in this example adds expected utility for the agent with a certainty equivalent consumption of €29. The relative certainty equivalent shows that it only adds a meagre 0.02%. The composition of the optimal portfolio still is rather tilted towards the variable annuity, with a 31% allocation. One would expect the risk averse investors to move more rapidly towards the real annuity, and that the welfare gains would be larger. This result is in line with Brown et al. (2001), who find that the real annuity does not add so much value due to the strong equity return underlying the variable annuity. However, if we withdraw the variable annuity from the asset menu, the allocation to the real annuity increases substantially.

### 5.3.3 Optimal allocation for different $\gamma$

If we now examine the optimal portfolio composition over  $\gamma$  I find the following:

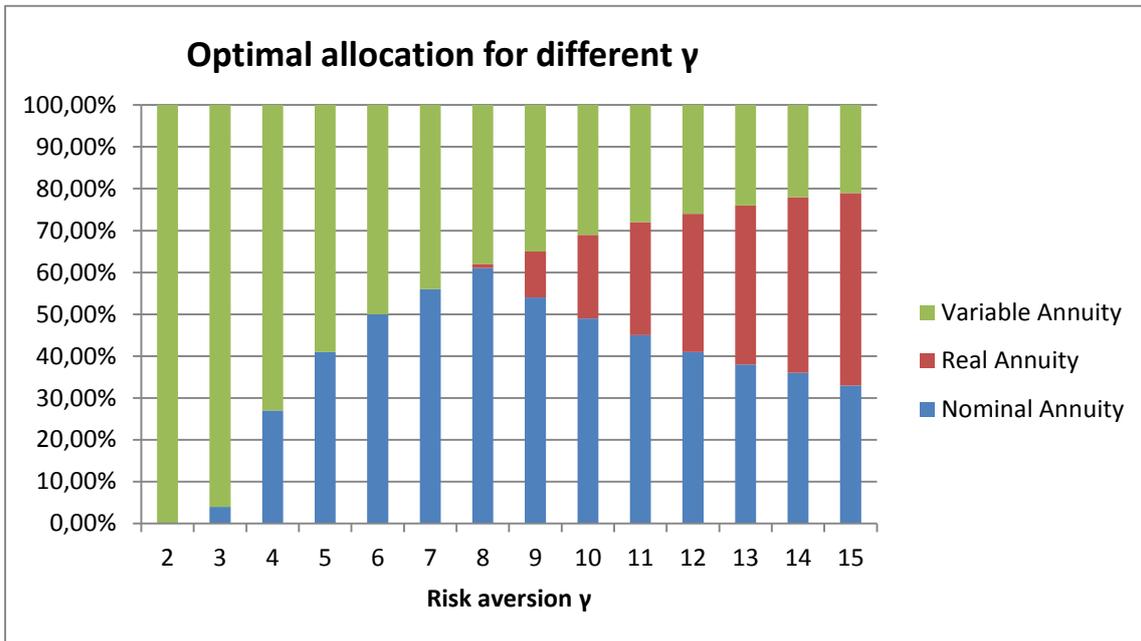


Figure 9: Optimal portfolio composition for varying levels of  $\gamma$ .

Here it becomes clear that the allocation for risk averse investors shifts towards real annuities from a risk aversion level of 8 and onwards. Only for very high  $\gamma$  levels does the optimal allocation shift more towards real annuities. For the risk loving and moderately risk averse investors, a combination of nominal and variable annuities is optimal.

### 5.4 No old-age pension

AOW rights are built up for every month a Dutch person has lived in the Netherlands. Some people might have been abroad for most of their adulthood, but still enjoy retirement in the Netherlands. Or the PAYG financed system might become too costly and has to be abandoned in the future. Analyzing the optimal portfolio in absence of the AOW might thus be useful. The AOW can be thought of as an initial position in an inflation protected annuity, as it provides a real income stream as part of social security.

Type of income	Nominal + Variable annuity	All three annuities	Nominal + Real annuity
Nominal fixed annuity	67%	47%	36%
Real annuity	-	21%	64%
Variable annuity	33%	32%	-
Certainty Consumption Equivalent	€5,328	€5,356	€4,873

Table 11: Optimal portfolio composition and certainty consumption equivalents for a situation without state pension (AOW). Note that we are back to  $\gamma = 5$  level.

In this case the difference is still very small, with an equivalent consumption gain of €28. A combination of nominal and variable annuities once again shows to be adequate. However, in absence of variable annuities, the real annuities do make a difference. The certainty equivalent consumption difference compared to the base case scenario is around €300, a relative loss of 1.6%, as can be seen below:

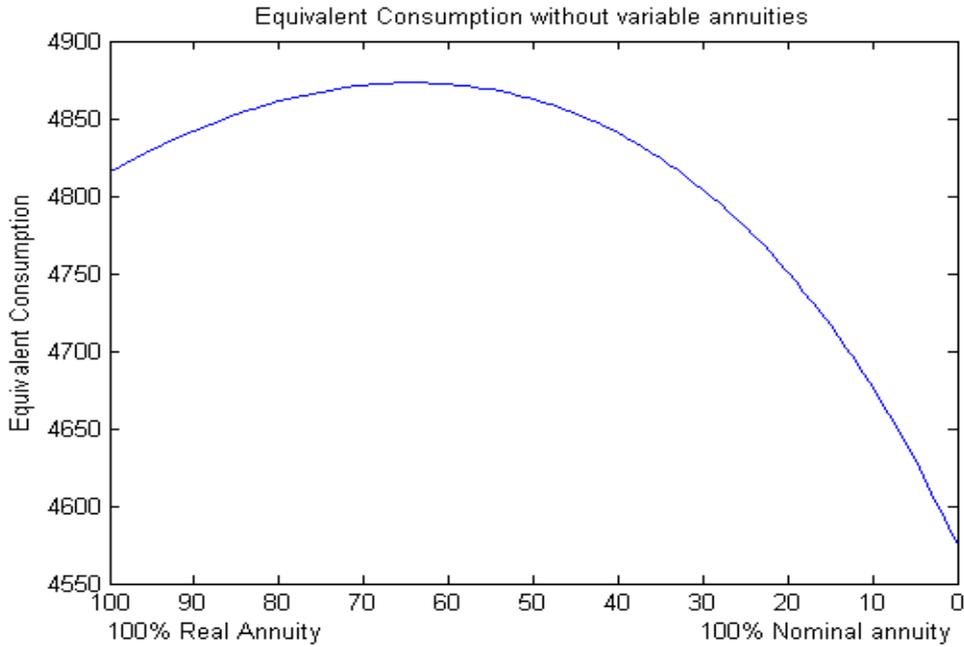


Figure 10: Certainty equivalent consumption without state pension (AOW) and in absence of variable annuities.

In this instance, the investor values the protection from (unexpected) inflation by the real annuity because he no longer receives inflation protection from the state pension. If we now look at the optimal portfolio in absence of real annuities there is more sensitivity in outcomes. Allocating too optimistically to the variable annuity can cause big welfare losses:

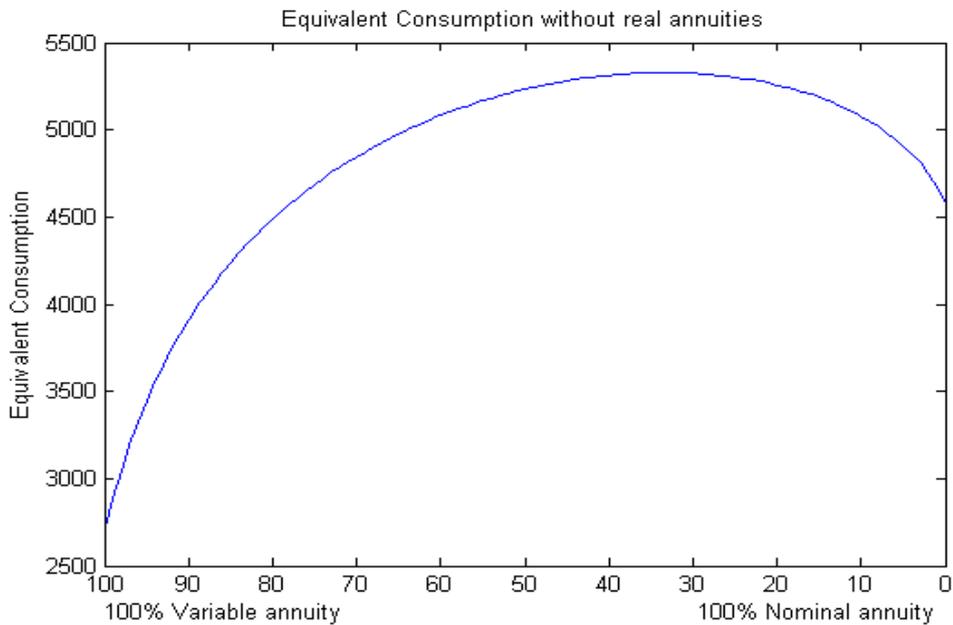


Figure 101: Certainty equivalent consumption without state pension (AOW) and in absence of real annuities.

Setting the original allocation, without AOW, at 61% variable annuities and 39% nominal annuities, reduces the certainty equivalent consumption with approximately €350, or a relative 1.7% loss. The size of the AOW does matter for the optimal portfolio and for welfare. To further analyze the dynamics of old-age pension I optimize over different yearly AOW payments:

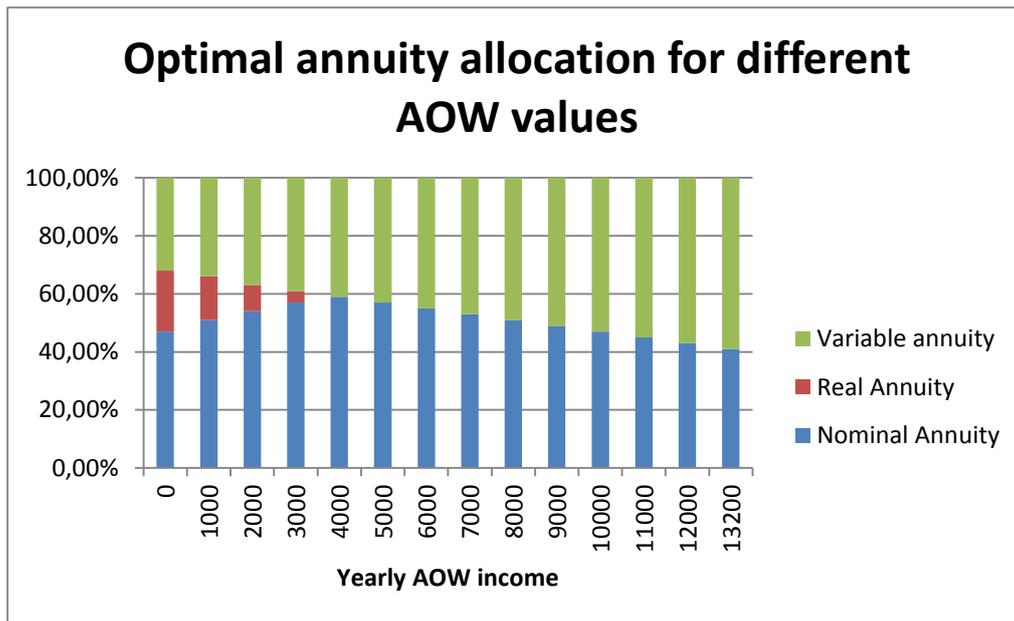


Figure 11: Optimal portfolio composition for varying levels of state pension (AOW).

We observe the same pattern as with higher risk aversion. The variable annuity, due to its compounding returns, is able to safeguard the investor from inflation and the addition of the real annuity has only a marginal welfare effect.

### 5.5 Higher AIR (AIR = 10%)

As the variable annuity, by the effect of compounding returns in the long ends of the simulations seems to protect the agent against losses in purchasing power we can adjust the AIR, which determines the amount of units available for the retiree, and thereby setting the payoff structure in expectation. I will choose an AIR of 10%, which is slightly unrealistic, but useful to see if this makes a difference.

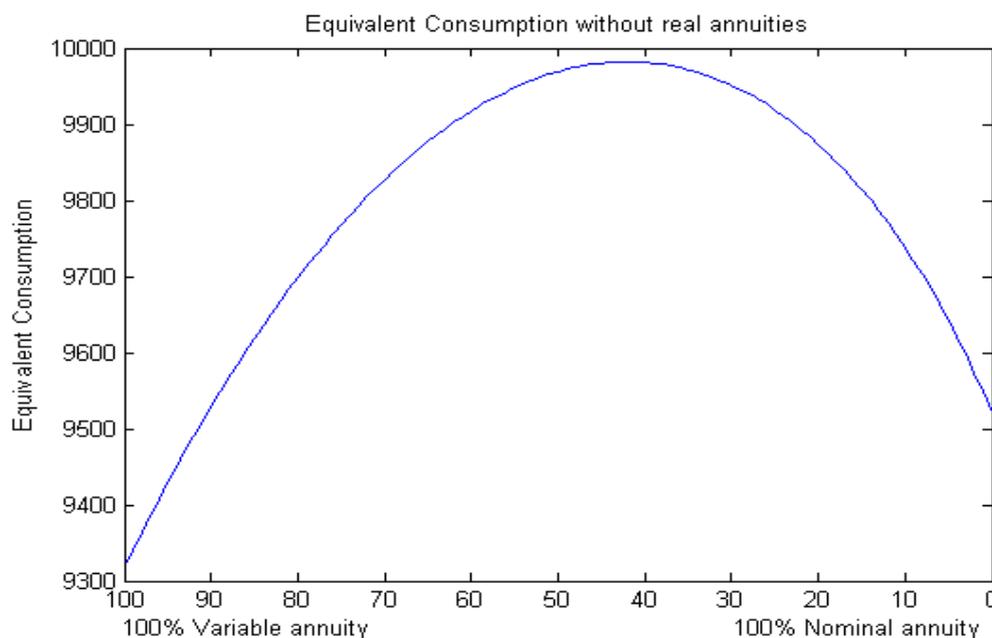


Figure 12: Certainty equivalent consumption with a lower assumed interest rate (AIR) and in absence of real annuities.

Type of income	Nominal + Variable annuity	All three annuities	Nominal + Real annuity
Nominal fixed annuity	58%	25%	31%
Real annuity	-	32%	69%
Variable annuity	42%	43%	-
Certainty Consumption Equivalent	€9,982	€10,007	€9,545

Table 12: Optimal portfolio composition and certainty consumption equivalents for a situation with a higher assumed interest rate (AIR).

Compared to the base case scenario, which had a certainty equivalent consumption level of €10,417 we see that in the optimum a higher AIR decreases welfare. The compounding returns are not as beneficial as the income generated from the variable annuity is now, on average, decreasing. We indeed see that a third of the single premium is invested in the real annuity, although the welfare effects remain negligible. In the simulation distribution, the lowered allocation to the variable annuity in combination with the higher AIR becomes visible:

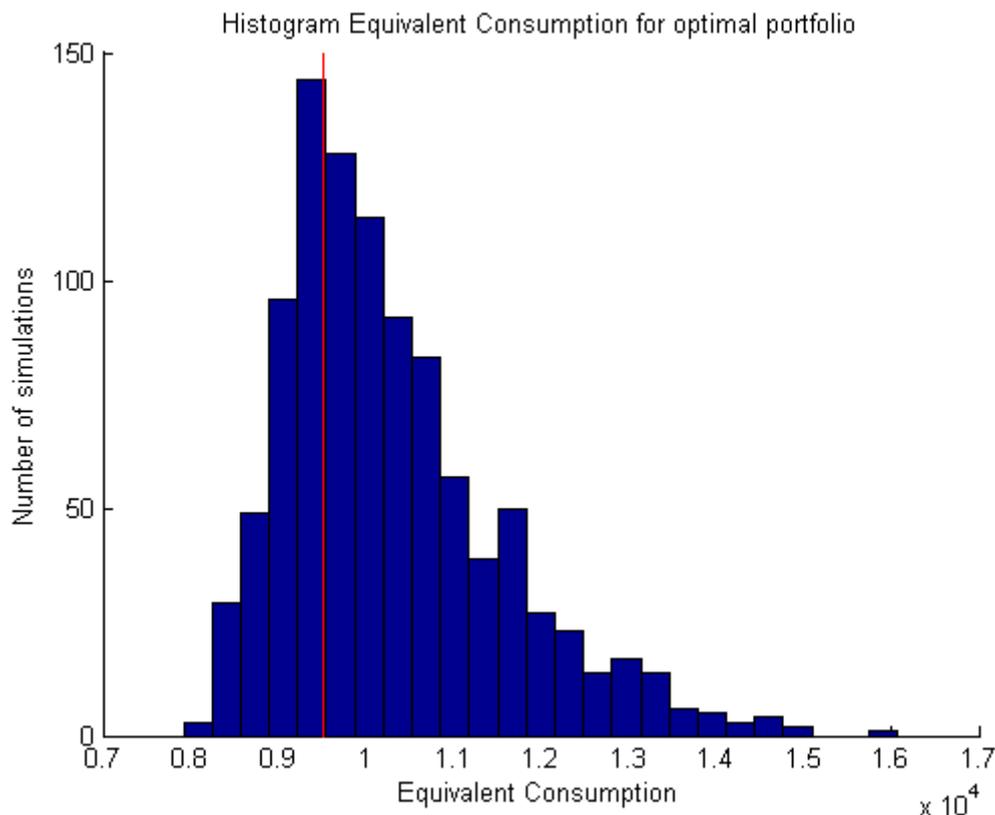


Figure 13: Equivalent consumption histogram for the optimal portfolio. The red line depicts the average equivalent consumption for the base case scenario where the retiree is 100% invested in the nominal annuity. Note that the X-axis is multiplied by  $10^4$ .

We see that in 32% of the cases, the retiree is worse off compared to the 100% nominal annuity situation. The positive skew has reduced compared to the base case scenario and the distribution has become thinner.

## 5.6 The rich & the poor

In the optimum the real annuity does not add value for the retiree. This might in part be caused by the size of the single premium compared to the AOW. To examine this I compare the situation of a big single premium (€460,000, 2x the base case value) to a small single premium (€115,000, 0.5x the base case value).

	<b>Rich</b>	<b>Poor</b>
Variable	Value	Value
$\beta$	0.98	0.98
T	Stochastic male	Stochastic male
$\gamma$	5	5
AOW	€12,300	€12,300
Single Premium	€460,000	€230,000
AIR	4%	4%
Tax rate	30%	30%

Table 13: Assumptions for the big and small single premium.

<b>Type of income</b>	The Rich	The Poor
Nominal fixed annuity	54%	17%
Real annuity	-	-
Variable annuity	46%	83%
Relative Certainty Equivalent Consumption	1.0199	1.0194

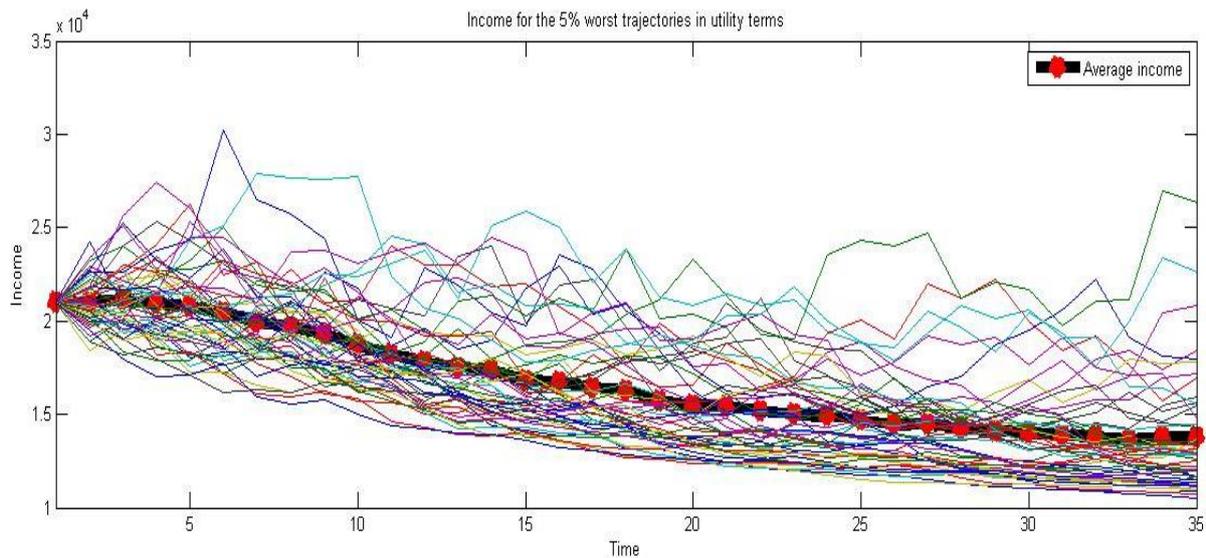
Table 14: Optimal allocations for the rich and the poor and relative certainty equivalent consumption effects.

Here I report the relative certainty equivalent consumption, as the initial amounts are not comparable for the bigger and smaller single premium. The result is, at first, counterintuitive. If the single premium becomes smaller, the allocation towards variable annuities increases. In fact, for single premium values smaller than €86,000 the agent prefers to be completely invested in the variable annuity.

I have depicted the relative certainty equivalent consumption results for the 'rich' and the 'poor' in table 14. For the rich the difference is 1.99% compared to the situation where only nominal annuities are available. For the poor the difference is 1.94%. These results can be compared, but the smaller the single premium gets, the bigger the impact of the (free) AOW becomes and the relative difference becomes smaller. Koijen et al. (2009) treat social security as a pre-imposed position in an inflation linked annuity. In this respect, the results do make sense. If the single premium becomes smaller, the relative position in the real annuity (AOW) increases. As to offset the pre-imposed real annuity position the investor wants to rebalance its portfolio relatively towards the risky position, investing a larger chunk of the available money in the variable annuity.

### 5.7 The 5% worst outcomes

We have seen that the optimal portfolio comprises an allocation of 61% to the variable annuity, and 39% to the nominal annuity. However, you only retire once in your life, so very disappointing outcomes must be taken into account. These bad outcomes already play a role in the standard framework, but to calibrate the results I will take a look at the 5% worst outcomes. In figure 14 you can see the real income trajectories for the 5% worst utility cases:



**Figure 14: Real income trajectories over time for the 5% worst cases in utility terms. The red dotted black line indicates the average of the trajectories.**

On average, after 20 years the retiree has lost 25% of initial real income. This is driven by two factors. First of all, average inflation is higher, as the price index in the worst case scenario has doubled over 20 years on average compared to a 50% increase on average for the total simulation. See appendix F for the average difference. Second of all, the equity returns disappoint (even in nominal terms) and hence the payoff from the variable annuity is much lower than the average of the complete simulation. In these instances, the old-age insurance is crucial, as it provides the retiree with a real income stream. After 20 years, the average contribution of the old-age insurance to income has increased from 45% to 60% of total real income. This underlines the important aspect of inflation protection in extreme cases, as both inflation can be higher than expected and equity returns can disappoint. The AOW is a beneficial insurance in this instances, as it provides a real consumption floor if both high inflation and bad equity performance materialize.

Bad states of the world can occur and it will never be pleasant for a consumer. The old-age insurance provides a real income floor and the retiree in my model prefers the (on average) higher consumption of the variable annuity over the (smaller) risk of very large negative shocks which reduces the consumption of the agent.

### 5.8 Sensitivity in parameters of the financial market

As stated in section 4.1, Draper (2014) adjusted his estimated financial market model to make it consistent with the expectations of the Commission Parameters. More specifically, the equity risk premium and equity volatility has been increased, and the parameter that governs the risk premium on bonds has been elevated to get an ultimate forward rate of 3.9%. Besides this, the parameter that governs long term inflation has been raised. We should keep in mind that one of the goals of the Commission was to create consistency between the valuation of assets and liabilities for pension funds, as this provides a more accurate representation of the funding status of the fund. Another important task was to decrease dependency of the term structure of interest rates with the expectation of risky asset returns. I will provide a table with optimal allocations for the reported sensitivities in this paper. I will leave the AIR unchanged to ease comparison.

Assumptions	Nominal Annuity	Real Annuity	Variable annuity
Base case	66%	0%	34%
Low risk aversion (2)	21%	0%	79%
High risk aversion (10)	62%	19%	19%
No AOW	60%	21%	19%
Rich ( $PR_0 = 460.000$ )	73%	0%	27%
Poor ( $PR_0 = 115.000$ )	53%	-	47%

**Table 15: Optimal portfolio composition for the unadjusted Draper (2014) financial market model with complete annuity menu.**

Although we see a clear shift away from the variable annuity, in the optimum it still gets a large allocation. Furthermore, the inclusion of real annuities in presence of variable annuities still does not add significant value. However, in absence of variable annuities, the investor allocates a big part of the single premium to real annuities. With the uncalibrated model by Draper (2014), the certainty equivalent wealth distribution for the optimal portfolio has less positive skew and is more normally distributed. In part, this is due to the lower variable annuity allocation but also the unchanged AIR of 4% plays a role. All in all, the attractiveness of the variable annuity decreases, but the finding that variable annuities add to welfare, and the conclusion that real annuities do not add significant value still stands.

### 5.9 Expanding the utility framework by habit formation

A Constant Relative Risk Aversion (CRRA) utility function might not be the most accurate representation of the preferences of the individual. It is used quite often in the economic literature as it possesses desirable properties such as additive separability. Nevertheless there might be some flaws in assuming a CRRA utility function. A millionaire who loses all of his money in the stock market surely will feel less good about living from social security compared to a long-term unemployed, who has gotten used to this expenditure pattern. In economic jargon this is called habit formation. It means that it is not just the level of consumption that matters, but also the level relative to past consumption. There are two methods for habit formation; the difference and the ratio approach. The main problem with the difference method is that it can take on negative values, if the consumption is lower than the habit. As a consequence, complications will arise in the results. Therefore I will focus on the ratio approach by Fuhrer (2000) as it addresses these practical issues.

If only last period consumption is taken into account, one can adjust the consumption term in the utility function by:

$$\hat{C}_t = \frac{C_t}{C_{t-1}^\theta} = \frac{C_t}{C_{t-1}} C_{t-1}^{1-\theta} \quad (10)$$

Here,  $\theta$  denotes the strength of the habit. If  $\theta$  is 0,  $\hat{C}_t$  becomes  $C_t$  and we are back in the old CRRA utility function. At the other extreme, where  $\theta$  is 1, current consumption is completely compared to the consumption in the last period. Utility is no longer time-separable as today's consumption influences the future reference level for consumption.

However, a one-period memory might be too shortsighted, especially for evaluating the habits of retirees, who might have stronger habits than younger people.

Therefore taking more periods in to account seems more natural:

$$h_t = \rho h_{t-1} + (1 - \rho) C_{t-1} \quad (11)$$

Here,  $\rho$  can be thought of as the memory of the habit. If  $\rho$  is 0, current consumption is only compared to the last period and we are back in specification (10), which only has a memory of one period. If  $\rho$  is 1, it will compare current consumption completely to the habit. This can be incorporated in to the consumption term in the utility function by replacing  $C_{t-1}$  by  $h_t$ :

$$\hat{C}_t = \frac{C_t}{h_t^\theta} = \frac{C_t}{h_t} h_t^{1-\theta} \quad (12)$$

Now we have three parameters which govern the habit:

- 1) The parameter  $\theta$  indexes the importance of the habit formation in the utility function
- 2) The parameter  $\rho$  indexes the persistence or memory in the habit formation reference level.
- 3) The parameter  $h_0$  which determines the starting value of the habit

Here, the habit is denoted in real terms. This is somewhat controversial as people are known to be prone to money illusion. That is, empirical evidence has shown that people have trouble making the distinction between nominal and real variables and hence people might hang on more to a nominal habit than to a real habit.

Davidoff et al. (2005) explain that the agent now has two additional effects compared to the standard separable utility case:

- a) The effect of the present standard-of-living on present marginal utility
- b) The effect of present consumption on future period's utility through subsequent standards of living.

Davidoff et. al (2005) do numerical calculations along two dimensions, time preference and  $\gamma$ . For the time separable case this can be interpreted as the risk aversion coefficient, for the habit formation utility function it can not.

They show that with a high initial standard-of-living (0.5 times the single premium), the optimal consumption becomes frontloaded and the agent optimally wants to consume a large fraction of his wealth in the early stages of retirement. For an agent with a low habit compared to his resources, the optimal consumption pattern conveys an upward sloping pattern, as it allows to sustain more consumption every year than the consumer is used too. Even though Davidoff et al. (2005) analyze this in a framework to see how it affects optimal annuitization levels I find similar patterns.

For consumers with a relatively high standard of living compared to their single premium euro amount ( $h_0 = \text{€}25,000$ ), the agent prefers to frontload consumption by investing a large part in the nominal annuity, as this has a higher real payout in the beginning of retirement. Consumers with a small habit compared to their single premium, prefer to invest in the real annuity, as this allows them to gradually increase consumption over retirement. Below, I report the optimal portfolio composition for different parameter levels for the starting habit.

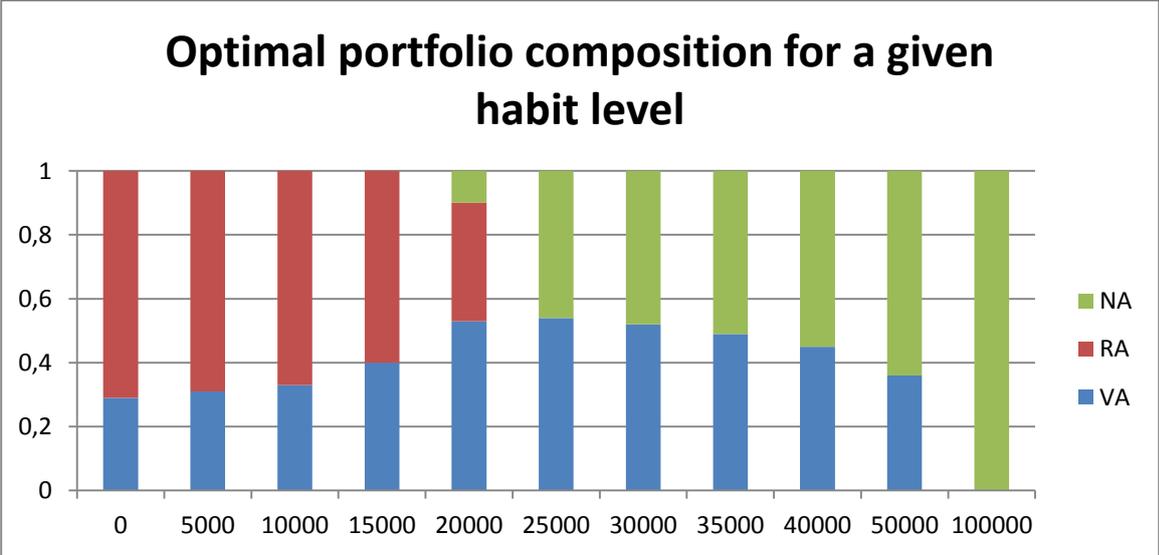


Figure 15: Optimal portfolio composition for a given habit level. Note:  $\theta$  is 1 and  $\rho$  is 0.5, and the scale is not linear.

For low initial levels of the habit, the consumer prefers safety by investing in the real annuity with a low variable annuity allocation. For larger, obtainable levels of the habit, the consumer increases the allocation to the risky variable annuity. If the habit becomes too high (and thus unobtainable), it will prefer frontloading the consumption and increase the allocation to the nominal annuity. Furthermore I find the following relationships:

Parameter	Nominal Annuity	Real Annuity	Variable Annuity
$\theta$	-	+	-
$\rho$	-	-	+
$h$	+	-	+ -

Table 16: Relationships for the parameters in my model to the allocation of the various annuity types.

For  $\theta$ , indicating the importance of habit formation in the utility function, the allocation to the real annuity increases if the importance of the habit increases. For  $\rho$ , which displays the memory of the habit, the allocation towards the real and nominal annuity decreases if the memory increases. As the consumption is related more and more towards the last period, the allocation to the variable annuity increases. Lastly, for  $h$ , the effect is ambiguous. For a small habit relative to the single premium, the consumer prefers the real annuity. Once the habit becomes larger relatively to the single premium, the agent starts allocating more to the variable annuity, and later to the nominal annuity, frontloading consumption. As the consumers' standard becomes too large, he will completely allocate towards the nominal annuity.

However, in my model the agent is constrained in its consumption decision, as I assumed that the agent consumes all his wealth in every period. We do see that the consumer will prefer investing in real annuities if the standard-of-living, or habit, compared to his single premium is low. In this example, the agent prefers to increase consumption steadily instead of aiming at more instant gratification as we have seen in the CRRA utility function. The agent here prefers allocating to real annuities.

If the retiree has a high standard-of-living going in to retirement, but has saved too little, the investor will try to maintain the standard by frontloading consumption and allocating to the nominal annuity. This situation is not uncommon in the Netherlands as many participants face a 'pension gap', a negative difference between the targeted 70% of last earned income and the actual income. In this situation, the optimal allocation hardly differs from the base case in the CRRA utility function.

## 6. Discussion of results, recommendations and future guidance

This section will discuss the main results and provide recommendations. The main finding is that adding risk exposure in retirement is welfare enhancing. The second finding is that the welfare effects for real annuities is limited in the presence of variable annuities. Lastly, I agree with the recommendation by Brown et al. (2011) to adjust the risk management requirements around conversion. Furthermore, direction for further research will be provided. This can be done along three dimensions. First, the asset menu of the underlying of the variable annuity can be expanded. Second, the use of lump sum payments can be investigated, alongside bequest motives or health care risk could be incorporated. Third, the annuity menu can be expanded to hedge certain risks or to derive income in alternative ways.

### 6.1 Recommendations

The inclusion of variable annuities is crucial and welfare enhancing among a very broad list of sensitivity checks including:

- Higher risk aversion parameter  $\gamma$  (10)
- Exclusion of the old age pension AOW
- Setting a higher AIR

The following relationships hold for the optimal percentage of variable annuities:

Input variable	Correlation to optimal VA allocation
$\gamma$	-
AIR > 4%	-
AOW	+
Single Premium	-

Table 17: Found relationship between various parameters and the correlation to the optimal variable annuity allocation.

For the risk aversion parameter  $\gamma$  this is obvious, as it determines how risk averse a consumer is. The variable annuity is linked to equity market performance which is most volatile, so that higher risk aversion leads to a lower optimal variable annuity allocation.

As explained in section 4.2, the chosen AIR determines the expected average payoff pattern for the variable annuity. In this thesis, I have refrained from optimizing over the weights and the AIR, but if the AIR is increased the optimal allocation towards variable annuities decreases. This is due to the compounding effect of the return in the tail of the simulations. If it is set higher, the retiree prefers allocating more towards the nominal and real annuity.

The AOW acts as a riskless real income stream till death, and therefore it exhibits a positive correlation towards the percentage of variable annuities in the optimum. This result is similar to life-cycle models where the wage acts as a riskless cash flow and therefore shifts the financial capital towards more risky assets.

Lastly, there is a negative relationship between the size of the single premium and the optimal allocation towards variable annuities. As noted earlier, this can be thought of as a rebalancing act to offset the pre-imposed position in an inflation protected annuity (AOW).

Even for very risk averse individuals ( $\gamma = 10$ ), the optimal allocation to the variable annuity is still 33%.

As a last sensitivity check I have used the unadjusted model by Draper (2014) to see if it affects my results. It indeed does to the extent that the allocation towards variable annuities is lowered, probably due to the lower equity risk premium in the adjusted model. However, it still adds welfare considerably in all cases.

This leads to my first recommendation:

**1) Include variable annuities to the asset menu as it adds to individual welfare in retirement.**

This conclusion is in line with Dietvorst et. al (2011) and Brown et al. (2011).

In this thesis, I have researched the role of real annuities and their welfare implications. In most instances, including the real annuity added no, or almost no value for the individual. Only in the case of no AOW or high risk aversion in conjunction with the absence of variable annuities did the real annuity add significant value. This endorses Brown et al. (2001) whom conclude that variable annuities are beneficial due to the equity risk premium and not so much by providing a natural inflation hedge, as the real annuity does.

Contrary to the recommendation based on qualitative analysis by Dietvorst et al. (2011), I do not find big additional value for inflation protected (real) annuities. Even though the inflation linked annuity was presented quite favorably, by refraining from adding an unexpected inflation premium which would (probably) decrease the attractiveness for this type of product in the model. Naturally, the attractiveness of inflation-linked annuities resides in the way inflation is modelled. The used financial model is based on estimations for the “the great moderation”, a period in which macro-economic variables have stabilized. Inflation has come down significantly and has become less volatile. However, even when using the unadjusted estimations by Draper (2014) with a lower equity risk premium, the real annuity does not add significant value. The compounding returns of the variable annuity in the unadjusted model still provide sufficient (un)expected inflation protection. The welfare effects of the real annuity are limited. Hence, my second recommendation:

**2) Including real annuities to the asset menu in presence of variable annuities is not important in retirement.**

As the optimal portfolio contains a sizeable allocation to variable annuities under a wide range of assumptions, framing the appropriate standard is crucial. Naturally, inherent to individual products is that investors are free to choose the conditions of their annuity. However, consumers should be confronted with the potential loss of purchasing power in a consumption frame as Brown et al. (2011) suggest. Deciding on the appropriate risk budget prior to retiring allows the conversion to be smooth. An investor need not cut back entirely on equity exposure during retirement as we have seen it is beneficial for almost every individual. The knife cuts both ways; the investor can reap the equity risk premium prior to and during retirement. Therefore I agree with the recommendation by Brown et al. (2011) that the current guideline for individual schemes is not appropriate:

**3) The current Dutch guideline for managing conversion risk for individual schemes should be adjusted to allow for maintaining equity exposure in to decumulation**

## 6.2 Directions for further research

This thesis has explored three simple annuity types and found beneficial effects for variable annuities. However, the framework can be extended among multiple dimensions:

- 1) The underlying of the variable annuity was a portfolio that consists of 100% equities, and there was no ability to rebalance. Adding more financial assets to be included in the variable annuity menu might increase the Sharpe ratio of the product. Furthermore, the portfolio could take time-varying volatility into account. For instance, in times of market distress, the volatility of equities increases. Given that the risk aversion of the consumer is constant, the optimal allocation to equities should decrease, because it entails more risk. This has been a trend in the UK, where total return funds target volatility by assuming a constant Sharpe ratio. A managed fund (actively or passively) could adjust the optimal portfolio for

the consumer in this way. This would elaborate on research Browne et al. (2003) who claim that the inability to rebalance the portfolio induces a liquidity risk premium for annuities.

- 2) In my model, I have not allowed for the possibility to use a lump-sum or to save out of annuity income and I have excluded the accumulation phase. Adding a bequest motive, background risk or stochastic health care cost might alter the optimal portfolio. This moves more in the direction of Peijnenburg et al. (2011). More flexibility in consumption and saving decisions can elaborate on the habit formation framework I have explored.
- 3) The annuity market can be extended by using impaired life annuities, reverse-mortgage annuities, equity-linked annuities with guarantee or phased annuities. All of these annuity types try to eliminate certain disadvantages of annuities (such as illiquidity in case of health care cost) while maintaining the advantages. Furthermore, designing appropriate defaults for individuals could be researched. The inability of individuals to make sound financial decisions means the default has to be selected properly.

## 7. Conclusion

As more schemes become individually based, this thesis tried to answer the question how the individual can improve his pension outcomes with annuities and how a benevolent government could improve welfare of its citizens by changing the legislation and allowing for more flexibility. I have considered the optimal annuity choice in a world with stochastic interest rates, inflation, equity risk and micro-longevity. The Dutch framework has been discussed, and the main characteristics of the three pillars. The investor had one single premium from which he could choose out of a combination of three simple annuity types: a nominal, inflation-linked and variable annuity. The preferences of the individual were represented by a CRRA utility function. In the base case scenario the optimal portfolio contained an allocation of 41% to the nominal annuity, 0% to the real annuity and 59% to the variable annuity. Contrary to my expectations, the real annuity did not add significant value if the variable annuity is present in the model. I expected the inflation hedge to be appealing for the investor, this turned out not to be the case.

I have deviated from the base case scenario along various lines. I found that low risk aversion shifts the allocation completely towards the variable annuity. High risk aversion decreased the allocation to the variable annuity, as expected. In the framework, removing the state-pension AOW, which can be thought of as a government financed pre-imposed allocation to the real annuity, was not detrimental to my results but did alter the optimal portfolio. Without AOW, the additional certainty consumption equivalent of adding real annuities was still negligible. Furthermore, I have tested whether the size of the single premium (capital at retirement) changed the optimal allocation. If the single premium becomes smaller, the optimal allocation towards variable annuities increases. This is caused by the pre-imposed real annuity position, as the relative size of the AOW becomes bigger as the single premium becomes smaller. The consumer optimally wants to offset this pre-imposed real annuity position by investing a larger part in the variable annuity. However, it seems not to be the case that the variable annuity acts as a natural inflation hedge. It is more due to the beneficial effect of the (compounding) equity risk premium that safeguards the investor from erosion of his purchasing power by unexpected inflation. This emerged from looking at the worst simulations, where both inflation was higher than average and equity returns disappointed. In this instances, the AOW protected the retiree by providing a real consumption floor in light of high inflation and bad equity market performance.

To check the robustness of my results from the scenario set of the Commission Parameters, I have used the unadjusted financial market model by Draper (2014). This reduced the attractiveness of the variable annuity to some extent due to a lower equity risk premium, but the main findings of the attractiveness of the variable annuity and the unattractiveness of the real annuity remained the same. Lastly, I incorporated habit formation in the utility function, which relates the level of current consumption to past consumption, and found that the size of the habit is of great importance to the optimal allocation. A low initial habit allows the retiree to increase consumption over time, while a bigger habit increases the incentive to frontload consumption by allocating towards the nominal annuity.

This analysis leads me to conclude that that adding risk exposure in retirement is beneficial to the retiree. The second finding is that the welfare effects for real annuities is limited in the presence of variable annuities, opposing the claim of Dietvorst et al (2010). Lastly, I agree with the recommendation by Brown et al. (2011) to adjust the risk management requirements around conversion, so that the consumer can reap risk premia prior to and during retirement.

In this paper, I have abstracted from bequest motives and health care risks, which increases the need for liquidity and might lower the optimal annuitization level. Furthermore, I have refrained from flexibility in varying the benefits, such as high-low constructions, which provide more flexibility to the annuitant. Lastly, I have not incorporated possibilities to gradually annuitize.

As a direction for further research I provide three dimensions to expand the framework. First, the asset menu can be expanded to create improved portfolio on the risk characteristics of the client. Second, the use of lump

sum payments can be investigated, alongside bequest motives or health care risk. Third, the annuity menu could be expanded to hedge health care risk or provide additional income to the retiree by using a reverse-mortgage annuity. All in all, we have seen that expanding the annuity asset menu for retirees improves welfare by a considerable amount.

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## Appendix

### A Financial market model details

$$r_t = \delta_{0r} + \delta'_{1r}X_t \quad (13)$$

And for the instantaneous expected inflation:

$$\pi_t = \delta_{0\pi} + \delta'_{1\pi}X_t \quad (14)$$

The state variables follow a mean-reverting process around zero:

$$dX_t = -KX_t dt + \Sigma'_X dZ_t \quad (15)$$

$$K \text{ is } 2 \times 2 \text{ and } \Sigma'_X = I_{2 \times 2} 0_{2 \times 2}$$

The expected inflation determines the price index  $\Pi$ :

$$\frac{d\Pi_t}{\Pi_t} = \pi_t dt + \sigma'_{\Pi} dZ_t \quad \sigma_{\Pi} \in R^4 \text{ and } \Pi_0 = 1 \quad (16)$$

The stock index  $S$  develops according to

$$\frac{dS_t}{S_t} = (R_t + \eta_S) dt + \sigma'_S dZ_t \quad \sigma_S \in R^4 \text{ and } S_0 = 1 \quad (17)$$

Here,  $\eta_S$  denotes the equity risk premium.  $R$  denotes the nominal instantaneous interest rate, which is defined as:

$$R_t = R_0 + R'_1 X_t \quad (18)$$

Lastly, the stochastic discount factor  $\phi_t^N$  is modelled:

$$\frac{d\phi_t^N}{\phi_t^N} = -R_t dt - \Lambda'_t dZ_t \quad (19)$$

the time-varying price of risk  $\Lambda$  affine in the state variables  $X$

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t \quad \text{and } \Lambda_t, \Lambda_0 \in R^4 \text{ and } \Lambda_1 \text{ } 4 \times 2 \quad (20)$$

Important to note is that there is no risk premium for unexpected inflation, as unexpected inflation risk cannot be identified on the basis of data on the nominal side of the economy alone (Kojien et al., 2010). Hence, the third row of  $\Lambda_1$  contains zeros only:

$$\begin{bmatrix} \Lambda_{1(1,1)} & \Lambda_{1(1,2)} \\ \Lambda_{1(2,1)} & \Lambda_{1(2,2)} \\ 0 & 0 \\ \Lambda_{1(4,1)} & \Lambda_{1(4,2)} \end{bmatrix} \quad (21)$$

As the Tilburg Finance Tool provides four points of the nominal and real term structure I interpolate linearly between the 1, 5, 10 and 30 year interest rates. For  $R_t$   $t > 30$  I assume  $R_t$  to be  $R_{30}$ .

## A.1 Parameters

Expected inflation  $\pi_t = \delta_{0\pi} + \delta'_{1\pi} X_t$

$\delta_{0\pi}$	1.98%
$\delta_{1\pi(1)}$	-0.63%
$\delta_{2\pi(2)}$	0.14%

Nominal interest rate  $R_t = R_0 + R'_{1} X_t$

$R_0$	2.40%
$R_{1(1)}$	-1.48%
$R_{1(2)}$	0.53%

Process real interest rate and expected inflation  $dX_t = -KX_t dt + \Sigma'_x dZ_t$

$\kappa_{11}$	0.08
$\kappa_{22}$	0.35
$\kappa_{21}$	0.19

Realized inflation process  $\frac{d\Pi_t}{\Pi_t} = \pi_t dt + \sigma'_{\Pi} dZ_t$

$\sigma_{\Pi(1)}$	0.02%
$\sigma_{\Pi(2)}$	-0.01%
$\sigma_{\Pi(3)}$	0.61%

Stock return process  $\frac{dS_t}{S_t} = (R_t + \eta_S)dt + \sigma'_S dZ_t$

$\eta_S$	6.57%
$\sigma_{S(1)}$	-0.53%
$\sigma_{S(2)}$	-0.76 %
$\sigma_{S(3)}$	-2.11%
$\sigma_{S(4)}$	17.69%

Price of risk  $\Lambda_t = \Lambda_0 + \Lambda_1 X_t$

$\Lambda_{0(1)}$	0.242
$\Lambda_{0(2)}$	-0.46
$\Lambda_{1(1,1)}$	0.149
$\Lambda_{1(1,2)}$	-0.381
$\Lambda_{1(2,1)}$	0.089
$\Lambda_{1(2,2)}$	-0.083

State variables

$X_1$	2.09
$X_2$	1.01

## B Adding stochastic longevity risk to the model

The annuity income still is the same as in (1):

$$Y = PR_0 A^{-1}$$

The nominal annuity factor adjusts to:

$$A = \sum_{t=1}^T \exp(-tR_0^{(t)}) \prod_{s=1}^t p_s \quad (22)$$

And the variable annuity factor is also amended for micro-longevity:

$$A = \sum_{t=1}^T \exp(-tAIR) \prod_{s=1}^t p_s \quad (23)$$

And the annuity income  $Y_t$  for the variable annuity at time t:

$$Y_t = PR_0 A^{-1} \prod_{u=1}^t \left( \frac{1+R_u^A}{1+AIR} \right) \quad (24)$$

## C Fluctuations in annuity income (real or nominal)

If one were to evaluate the nominal annuity in real terms the annuity income would be:

$$Y_t = \frac{PR_0 A^{-1}}{\prod_{u=1}^t \Pi_u} \quad (25)$$

Where  $\Pi_u$  is the price index.

If one were to evaluate the real annuity in nominal terms the annuity income would be:

$$Y_t = PR_0 A^{-1} \prod_{u=1}^t \Pi_u \quad (26)$$

## D Certainty equivalent consumption

Let us consider a constant consumption strategy that yields utility:

$$E[U] = E\left(\sum_{t=1}^T \beta^{t-1} \frac{\bar{c}^{1-y}}{1-y}\right) \quad (27)$$

$$= E\left(\frac{\bar{c}^{1-y}}{1-y} \sum_{t=1}^T \beta^{t-1}\right) \quad (28)$$

$$= \left(\frac{\bar{c}^{1-y}}{1-y} \sum_{t=1}^T \beta^{t-1} \prod_{s=1}^t p_s\right) \quad (29)$$

If we know the utility of some strategy

$$U_0 = \left(\frac{\bar{c}_0^{1-y}}{1-y} \sum_{t=1}^T \beta^{t-1} \prod_{s=1}^t p_s\right) \quad (30)$$

We can calculate the equivalent consumption  $\bar{c}_0$  by solving for  $\bar{c}_0$ .

$$\bar{c}_0 = \left(\frac{U_0(1-y)}{\sum_{t=1}^T \beta^{t-1} \prod_{s=1}^t p_s}\right)^{\frac{1}{1-y}} \quad (31)$$

### D.1 Relative Certainty equivalent Consumption

Instead of reporting an absolute money difference, one can calculate the relative difference by calculating a utility level  $U_b$ :

$$U_b = \left( \frac{\bar{C}_b^{1-y}}{1-y} \sum_{t=1}^T \beta^{t-1} \prod_{s=1}^t p_s \right) \quad (32)$$

We can solve again for  $\bar{C}_b$ :

$$\bar{C}_b = \left( \frac{U_b(1-y)}{\sum_{t=1}^T \beta^{t-1} \prod_{s=1}^t p_s} \right)^{\frac{1}{1-y}} \quad (33)$$

Now we can simplify the expression by:

$$\frac{\bar{C}_b}{\bar{C}_0} = \frac{\left( \frac{U_b(1-y)}{\sum_{t=1}^T \beta^{t-1} \prod_{s=1}^t p_s} \right)^{\frac{1}{1-y}}}{\left( \frac{U_0(1-y)}{\sum_{t=1}^T \beta^{t-1} \prod_{s=1}^t p_s} \right)^{\frac{1}{1-y}}} \quad (34)$$

And use:

$$\frac{\bar{C}_b}{\bar{C}_0} = \left( \frac{U_b}{U_0} \right)^{\frac{1}{1-y}} \quad (35)$$

#### E Unadjusted parameters by Draper (2014)

$\delta_{0\pi}$	1.98%
$\eta_S$	6.57%
$\sigma_{S(4)}$	16.59%
$\lambda_{0(1)}$	0.403

## F Average price index for average and extreme outcomes.

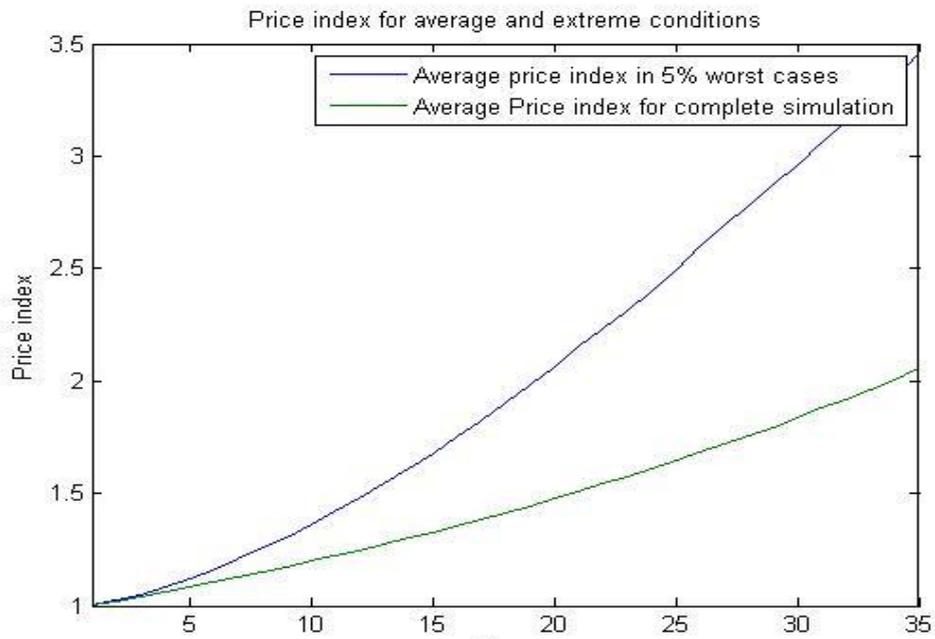


Figure 16: Average price index for the 5% worst outcomes in utility terms and the average price index development for the complete simulation.