Netspar

Network for Studies on Pensions, Aging and Retirement

Netspar THESES

Martin Hrachovec

Risk-Adjusted Returns under Leverage Constraints

Implications of the “Betting against Beta” Factor for Pension Funds

MSc Thesis 2013-009

TILBURG UNIVERSITY

SCHOOL OF ECONOMICS AND MANAGEMENT

MSc. Economics and Finance of Aging

MASTER THESIS

Risk-adjusted returns under leverage constraints – Implications of the "Betting against Beta" factor for pension funds

Author: Martin Hrachovec
ANR: 680247
Supervisors: Bertrand Melenberg
Rob van den Goorbergh
Michiel van Leuvensteijn
Second reader: Mitja Stadje
Academic year: 2012/2013

Declaration of Authorship

The author hereby declares that he compiled the thesis *„ Risk-adjusted returns under leverage constraints – Implications of the "Betting against Beta" factor for pension funds "* independently, using only the listed sources and literature.
The author grants to Tilburg University permission to reproduce and to distribute copies of this document in whole or in part.

Tilburg, August 23, 2013 ……...……..

Signature

Acknowledgements

Hereby I would like to thank my supervisors, Bertrand Melenberg, Rob van den Goorbergh and Michiel van Leuvensteijn for their patience, valuable recommendations and insightful advice throughout the process of writing this thesis. I would like to express my appreciation to employees of APG for their support and suggestions that contributed to the final outcome.

Abstract

Based on an OLG model with leverage constraints and margin requirements we use cross-sectional and time-series regression framework to analyze potential of a new factor premium in several asset classes. We confirm the presence of the so called "Betting Against Beta" premium in markets for US Treasury bonds, US corporate bonds, futures and equity indices in both developed and emerging markets. We explore long-term investment properties of the factors and find low but time-varying correlation structure among BAB factors in different asset classes. We note that before implementing the strategy the omitted transaction costs have to be considered as well as existing asset allocation strategies based on the fact that equity based BAB factors realize significant loadings on the recently popular minimum volatility indices.

JEL Classification G110, G120, G150, G230

Keywords BAB factor, CAPM, risk-adjusted excess returns, factor investing, leverage constraints, pension funds

Author's e-mail martin.hrachovec@gmail.com

Supervisor's e-mail b.melenberg@tilburguniversity.edu

Table of Contents

Acronyms

ACF	Auto-correlation Function
APT	Arbitrage Pricing Theory
BAB	Betting-Against-Beta
CAPM	Capital Asset Pricing Model
CML	Capital Market Line
EI	Equity Index
FF	Fama and French
HML	High-minus-Low factor (Value factor)
ICAPM	Intertemporal CAPM
MVF	Mean-Variance Frontier
OLG	Overlapping Generations
PACF	Partial Auto-correlation Function
SMB	Small-minus-Big factor (Size factor)
SML	Security Market Line
WML	Winners-minus-Losers factor (Momentum factor)

1. Introduction

> *"Models work when they are appropriate for the particular circumstance, but some of the best investment judgments over time have come when people recognized that models derived in other periods were broken or not directly relevant."*
>
> Abby Cohen Jones, American economist and financial analyst

After each downturn or crisis in the financial markets investors look for ways how to change and improve their strategies. New asset classes such as hedge funds, real estate or private equity that gained popularity as a reaction to the ".com bubble" proved during the recent crisis to have too much exposure to the traditional asset classes and so different approach to diversification surfaced (Bender et al., 2010). The so called factor investing has currently received substantial attention throughout the academic as well as the corporate field. It is based on capturing excess returns available in the financial markets, i.e. the "factor premiums" and diversification across these factor premiums instead of the asset classes. It can be argued that factor premium models are nothing new as the best known factor premium is the market premium from the original CAPM model as developed by Sharpe (1964) and Lintner (1965). Nevertheless, it has been only decades later that additional factors besides the market risk have been identified. Banz (1981), Fama and French (FF from now on) (1992, 1993) and Jegadeesh and Titman (1993) with Carhart (1997) introduced factors based on size (small vs. big), book-to-market value (value vs. growth) and return momentum (winners vs. losers) of equities respectively[1]. More importantly a whole investment strategy based on factor exposure is even younger since the paper by Ang et al. (2009) can be considered as the point of origin. Other recent innovations falling into the "smart beta" strategies include the minimum volatility approach and risk parity approach. In general they all tilt the investment towards one or more factor premiums. For more information on the smart beta strategies and risk parity in particular, including their pitfalls, see Blitz (2012, 2012a), Callan Associates (2010) and Asness, Frazzini, and Pedersen (2012).

In this study we want to confirm the existence of additional factor identified in a recent paper by Frazzini and Pedersen (2011) and extend the analysis to new asset classes and to deeper analysis and understanding of its properties, especially over long-term investment horizon. One of the basic premises of the CAPM is that investors choose assets with the highest expected return per unit of risk (Sharpe ratio) and then use leverage to move up the capital market line to satisfy their individual risk preference. In reality, some investors are constrained or averse to using leverage, whereas others are not or less so. However, the

[1] Earlier papers exist bringing evidence on the existence of value factor but the formalization and authorship is generally attributed to FF.

constrained agents invest in high-beta assets, because too much leverage is necessary when aiming at the same required returns via investing in portfolio based on low-beta assets. This behavior results in investors accepting lower returns from the riskier, high-beta assets, creating opportunity to capitalize on what is a relative flatness of the security market line. Using the overlapping generations (OLG) model introduced by Frazzini and Pedersen (2011) we assess the potential of the so called "Betting Against Beta" (BAB) factor across various asset classes including bonds, equity indices and futures using a cross-section and time-series regression framework. We then explore the impacts of changing market conditions and of alternative construction of the BAB factors on their performance. To add insight into the properties of the BAB factor returns the correlation structure across asset classes is analyzed as well as the performance over different investment horizons. Moreover, for equity indices we make first steps in analyzing the relations with minimum volatility strategy.

We find that the BAB factor premium is present across asset classes although its statistical significance in the futures domain is questionable and the positive returns in emerging markets are driven by exceptionally high returns in the early 1990s. As opposed to the assumptions of the model, the influence of liquidity conditions in financial markets proxied by the TED Spread differs by asset class and suggests low correlation between the individual BAB factors. This is confirmed by the subsequent analysis, albeit we report that the correlations increase in times of financial distress such as the recent crisis. Together with the fact that BAB factors capture to a large extent the same premium as minimum volatility indices we advise careful consideration when implementing the BAB approach dependent on the market conditions and asset management strategies already in place. Taking into account the leverage embedded in the BAB approach, the regulatory restrictions and possibility to include BAB factors with other strategies, US Treasury bonds emerge as a particularly interesting field for pension funds to benefit from the BAB premium.

The rest of the thesis is organized as follows: Chapter 2 introduces the theoretical foundations of the original CAPM and the factor investing, Chapter 3 deals with the results of previous research, the applied OLG model and introduction of the BAB factor and Chapter 4 describes the data set and empirical methodology applied. Chapter 5 summarizes the results of the analyses; Chapter 6 includes the discussion and implications for pension funds while Chapter 7 concludes.

2. Theoretical background

The formulation of the Capital Asset Pricing Model (CAPM) can be considered as the beginning of asset pricing theory. The importance of its contributions has been recognized by the Nobel Prize in Economic Sciences for professors Markowitz, Miller and Sharpe in 1990. Despite its imperfections and limitations, the CAPM and its modifications are still widely used both in the theoretical academic research as well as in practice. The aim of this chapter is to introduce the logic and theoretical foundations of the original CAPM and the first notable extension of Black (1972), who points to a two-factor CAPM in face of no risk-free asset and/or constrained borrowing.[2]

Rivaling equilibrium theories naturally exist, for instance the Arbitrage Pricing Theory (APT) pioneered by Ross (1976) is a notable example. The APT presents an equilibrium theory of security-expected returns that assumes the returns to stem from a factor model, thus sharing some common ground with the CAPM. On the one hand the restricting assumption of homogeneous anticipations of all investors is dropped and the model fits the real as well as theoretical data very well, thus making it unlikely to be rejected. On the other hand any testable predictions are virtually non-existent and the APT provides minimal insights into the relationships between expected returns and assets' attributes. Based on the availability of reasonable approximations, Sharpe summarizes why the CAPM was indeed the preferred model in the early empirical tests: *"So, although Capital Asset Pricing Models are clear about the major item to be measured, it is un-measurable. On the other hand, the Arbitrage Pricing Theory deals with items that might be easily measured but neglects to tell us what they are."* (Sharpe, 1984, p. 24).

In the second part the principles and cornerstones of previously mentioned factor investing are explained in more detail. This stream of literature is fairly recent and still quite limited, but we believe that it provides the necessary basis for understanding of the topic at hand. Conversely, given the extensive body of work on the CAPM and its development over time, we restrict ourselves to the basics and point the interested reader to the original papers for formal proofs when appropriate.

2.1. Mean-variance portfolio and the CAPM

Until the ground-breaking paper of Harry Markowitz (1952) the theory of portfolio formation was solely based on the maximization of discounted future returns (or rather expected returns in the face of uncertainty). This would naturally lead to investments into only one asset, the one with highest expected return and could never lead to an optimal solution of diversified portfolio. Based on the observed behavior towards diversification, Markowitz was the first to

[2] Other extensions that are less relevant for the topics discussed here are mentioned by Sharpe (1984, p. 22) and include e.g. Merton (1973) or Lintner (1969).

propose a one-period model in which *"... the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing."* (Markowitz, 1952, p. 77). Due to correlation of returns of individual assets, there is no single portfolio that offers both the highest expected return and the minimum variance of the return. Therefore, the rational investor has to choose at time $t-1$ from the available mean-variance efficient portfolios the one that provides the maximum expected return $E_{t-1}\left(R_{pt}\right)$ for a given level of portfolio return variance σ_{pt}^2 and that minimizes the variance given the required $E_{t-1}\left(R_{pt}\right)$.

In Section 3.6 of his paper on liquidity preference, Tobin (1958) derives the so called separation theorem for investor dividing his funds among cash and alternative monetary assets. It generally holds that under specific assumptions and in the presence of a risk-free asset (in Tobin's original example: cash), the efficient portfolio[3] of risky assets (Tobin: non-cash monetary assets) will have the same proportionate composition for all levels of risk. Hence the actual investment decision can be separated into two parts: finding the efficient risky portfolio (tangency portfolio) as a first step and allocating wealth to the risk-free asset and to the tangency portfolio. Therefore the separation theorem implies the composition of the tangency portfolio being independent of the share invested in the risky assets on total wealth.

The above mentioned papers, among others, concern the rules for behavior of individual agents and their portfolio selection. While contributions of Markowitz are representative of a "normative" or "prescriptive" theory, Tobin's work is the first in line of a "positive" or "descriptive" theory in this field. Sharpe (1964) and Lintner (1965) take the positive approach, asking "What would the market for capital assets be like if everyone acted according to mean and variance?" Hence they derive a theoretical model of market equilibrium of asset prices that is known as the CAPM[4]. Despite slight differences in their approach, both authors arrive at the same results. Using the approach of utility maximization of Markowitz, they identify a straightforward rule for a single market-clearing equilibrium for all available assets. This comes at a price of restricting assumptions, namely:

i) Homogeneous investors with joint probability distribution of the returns on all individual stocks;
ii) Investors are risk averse and maximize their expected utility of wealth at the end of next period;
iii) Trading occurs at a single purely competitive market with no taxes and no transaction costs;
iv) Common probability distribution of the possible returns is joint normal (or joint stable with a single characteristic exponent);

[3] Efficient in the sense of Markowitz as described above.
[4] This transition from micro analysis to market analysis of price formation for financial assets is what gained Sharpe the Nobel prize.

v) Unlimited short-selling of all assets, including the risk-free asset, i.e. investors can borrow or lend any amount at the risk-free rate of interest.

Following Sharpe (1964) we can illustrate that indeed *"in equilibrium, capital asset prices have adjusted so that the investor, ..., may obtain a higher expected rate of return on his holdings only by incurring additional risk."* (Sharpe, 1964, p. 425). Moreover, the source of risk is identified and the relation between (expected) return and risk can be used for capital asset pricing.

Based on the assumptions above, the investor's utility can be linked directly to the rate of return on his investments and to its risk measured by the standard deviation:

$$U = f\left(E(R), \sigma_R\right) \text{ with } \frac{\partial U}{\partial E(R)} > 0 \text{ and } \frac{\partial U}{\partial \sigma_R} < 0.$$

Considering all available risky assets and the probability distributions on their returns, the attainable set of investment opportunities can be depicted in the $E(R)$, σ_R plane.

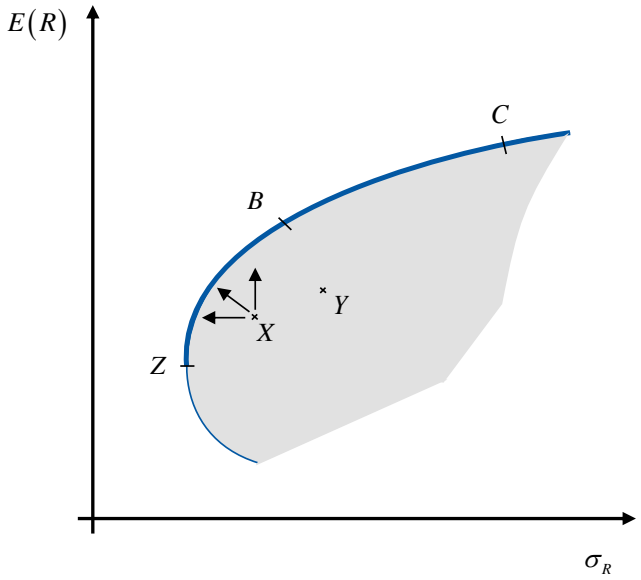


Figure 1: Investment opportunity set with Markowitz efficiency frontier
Source: Author's representation

To understand the mechanism behind the shape and derivation of the set we can illustrate the relation of two risky assets (or portfolios of assets) X and Y. Investing share of wealth α in asset X and the rest in asset Y the properties of resulting portfolio Z are:

$$E(R_Z) = \alpha E(R_X) + (1-\alpha) E(R_Y)$$

$$\sigma_{R_Z} = \sqrt{\alpha^2 \sigma_{R_X}^2 + (1-\alpha)^2 \sigma_{R_Y}^2 + \alpha(1-\alpha) 2 \rho_{XY} \sigma_{R_X} \sigma_{R_Y}}$$

where ρ_{XY} is the correlation coefficient between the expected returns on X and Y. Except the case of perfect correlation when $\rho_{ij} = 1$ the relationship between X and Y in the $E(R)$, σ_R plane is non-linear and there is a potential for diversification among the two assets or combinations of assets.

Then if all possible combinations of assets and portfolios of assets in the economy are considered, one can derive the investment set. The efficient part of the set (ZBC) is depicted in bold in Figure 1. Finding the efficient set of investment opportunities simplifies after including a risk-free asset F. Again building a hypothetical portfolio P by investing share α in F and remainder to risky asset X would lead to:

$$E(R_P) = \alpha E(R_F) + (1-\alpha) E(R_X)$$

$$\sigma_{R_P} = \sqrt{\alpha^2 \sigma_{R_F}^2 + (1-\alpha)^2 \sigma_{R_X}^2 + \alpha(1-\alpha) 2 \rho_{FX} \sigma_{R_F} \sigma_{R_X}} = \sqrt{(1-\alpha)^2 \sigma_{R_X}^2} = (1-\alpha)\sigma_{R_X}$$ due to the fact that $\sigma_{R_F} = 0$.

This implies a linear relationship as depicted in Figure 2 and each investor can move along straight lines from F to (and beyond due to the assumption of unlimited borrowing) any asset or portfolio in the investment set. However, from Figure 2 it is apparent that the efficient set is now the straight line FTT', also called the capital market line (CML). Therefore, all investors will want to hold combinations of F and T in proportions dependent on their risk preferences. Demand for only portfolio T will lead to adjustment of prices (and hence expected returns) until all the assets are included in at least one efficient portfolio lying on the capital market line.[5] If we then consider a single asset A that lies below the CML and is part of the efficient portfolio T, the curve representing all possible combinations of A and T must be tangent to the CML at point T.

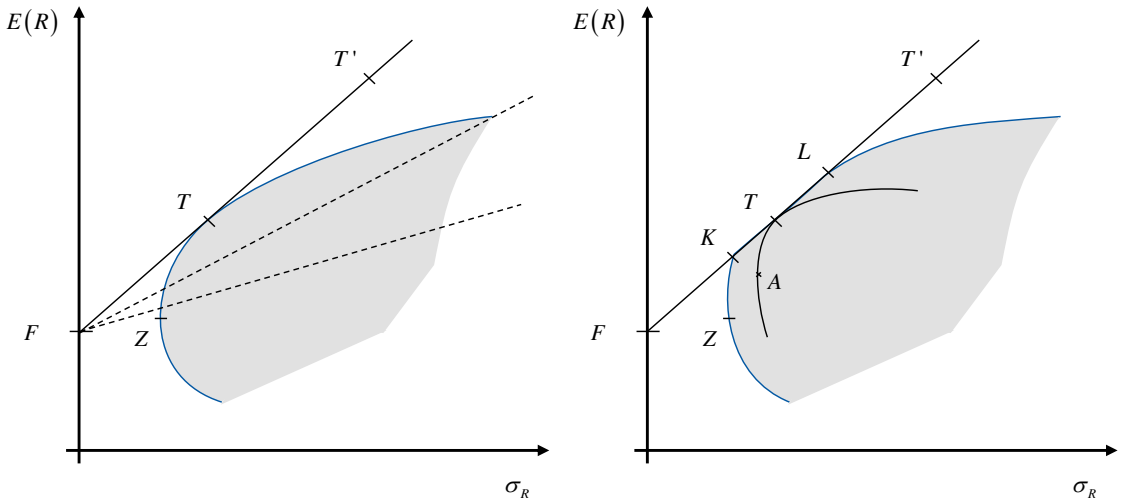


Figure 2: MVF with risk-free asset (left) and investment frontier after adjustment of prices (right)

Source: Author's representation

[5] Sharpe here contradicts the Tobin separation theorem saying that individual investors may not hold the same asset mix. However the portfolios on the CML, such as K and L, must be perfectly correlated which allows for derivation of relationship about individual asset prices and underlying risks.

From this fact Sharpe (1964, p. 438, footnote 22) derives relationship between the risk and (expected) return of assets A, F and efficient portfolio T in a form of:

$$\frac{\rho_{AT}\sigma_{R_A}}{\sigma_{R_T}} = -\left[\frac{R_F}{E(R_T)-R_F}\right]+\left[\frac{1}{E(R_T)-R_F}\right]E(R_A)$$

which rearranged gives the more familiar expression for expected return on asset A[6]:

$$E(R_A) = R_F + \left(E(R_T)-R_F\right)\frac{\rho_{AT}\sigma_{R_A}}{\sigma_{R_T}} \qquad (1)$$

where $\beta_{AT} = \frac{\rho_{AT}\sigma_{R_A}}{\sigma_{R_T}}$ can be interpreted as the sensitivity of asset A's returns to returns of the efficient portfolio T.

The same relationship was independently derived by Lintner (1965, p. 26). He uses a different approach to prove the separation theorem (p. 16-17 and Appendix) and based on the assumption of no perfect correlation among assets identifies only one point on the efficient Markowitz frontier as the tangency portfolio. As compared to Sharpe, who includes only positive holdings in risky assets, Lintner shows that the relationship holds also if short-selling is allowed. Another postulate of his paper is that holdings of asset A (respectively its share in the tangency portfolio) can be positive even if it has negative excess return over the risk-free rate. Further contributions to the CAPM theory are provided by Mossin (1966) who builds on Sharpe's early work but derives conclusions independently of Lintner.

Following the reasoning of Fama and French (FF) (2003, p. 4), we see that all investors hold the same portfolio T. But for markets to clear the prices of individual assets must level so that the weight of each asset in portfolio T is equal to its market value divided by the market value of all assets. Hence the weight of each asset in the tangency portfolio is its weight in the market portfolio of total investments, implying that the tangency portfolio is in fact the market portfolio. Based on this discussion, equation (1) and denoting $\beta_i = \beta_{iM} = \frac{\rho_{iM}\sigma_{R_i}}{\sigma_{R_M}} = \frac{Cov(R_i, R_M)}{\sigma^2_{R_M}}$ as the sensitivity of asset *i* with respect to the market portfolio, we can rewrite the relation into the widely used and cited CAPM risk-return equation:

$$E(R_i) = R_F + \beta_{iM}\left[E(R_M)-R_F\right] \Leftrightarrow \left[E(R_i)-R_F\right] = \beta_{iM}\left[E(R_M)-R_F\right] \qquad (2)$$

From equation (2) we infer the traditional interpretation of the excess return[7] on an individual asset being determined solely by the market excess return and the asset-specific sensitivity to

[6] Since an arbitrary efficient portfolio T was selected to derive this expression, Sharpe shows in footnote 26 that due to perfect correlation of returns of all efficient portfolios any of them can be selected without altering the resulting expression.

[7] Throughout the rest of the paper "excess return" refers to the return in excess of the appropriate risk-free rate unless stated otherwise.

the market risk measured by β_i.[8] The two representations of equation in (2) are then often used for a straightforward regression analysis using empirical data.

The so called Security Market Line (SML) expressed in equation (2) represents the equilibrium relation under the CAPM. We can also express the optimal allocation of investors or in other words the composition of tangency portfolio based on agents' utility function. It is possible to derive the optimal asset allocation vector $\mathbf{x}^*$ for general utility function, but if we assume the trade-off between risk and return to be linear and express the utility function of individual investor as $U = f\left(E_t\left(R_{p,t+1}\right), \sigma^2_{p,t}\right) = E_t\left(R_{p,t+1}\right) - \frac{\gamma}{2}\sigma^2_{p,t}$, where γ is the coefficient of risk-aversion that can differ among investors and the portfolio return can be expressed as $R_{p,t+1} = \mathbf{x_t'R_{t+1}} + \left(1 - \boldsymbol{\iota}'\mathbf{x_t}\right)R_f$, the analytical solutions become simple.

The maximization problem faced by the investor is now:

$$\max_{\mathbf{x_t}}\left(E_t\left(R_{p,t+1}\right) - \frac{\gamma}{2}\sigma^2_{p,t}\right) = \max_{\mathbf{x_t}}\left(\mathbf{x_t'}\left(E_t\left(\mathbf{R_{t+1}}\right) - R_{f,t+1}\boldsymbol{\iota}\right) - \frac{\gamma}{2}\mathbf{x_t'\Sigma_t x_t}\right)$$

where $\mathbf{x_t}$ is the vector of weights assigned to each risky asset, $\mathbf{R_{t+1}}$ is the vector of returns on the risky assets, $\boldsymbol{\iota}$ is a vector of ones, and $\mathbf{\Sigma_t}$ is the variance-covariance matrix. It should be noted that subtracting the risk-free return doesn't change the maximization problem. Taking the partial derivative and using the properties of matrix operations we derive the optimal solution:

$$\mathbf{x_t^*} = \frac{1}{\gamma}\mathbf{\Sigma_t^{-1}}\left(E_t\left(\mathbf{R_{t+1}}\right) - R_{f,t}\boldsymbol{\iota}\right) \qquad (3)$$

with the remaining $1 - \boldsymbol{\iota}'\mathbf{x_t^*}$ invested in the risk-free asset.

Expression (3) defines the vector of risky assets' weights based on the variance-covariance matrix, assets' excess returns and the agent-specific risk aversion coefficient. From this expression we can see that the separation theorem holds as portfolios of investors will only differ in total wealth invested in risky assets (and scaled by γ). The composition of investors' portfolios will be identical.

Besides the poor empirical performance of the original CAPM model which will be dealt with in Chapter 3, the main criticism from the early years was centered around the unrealistic assumption of unlimited borrowing at the risk-free rate. Black (1972) dismisses issues related to all the other assumptions of the original CAPM and focuses on the assumption of unlimited borrowing. He shows formally for a case with no riskless asset, that every efficient portfolio held by any investor in the economy will consist of a combination of two basic portfolios. As long as the two basic portfolios p and q have different $\beta's$, every

[8] Beta itself is the covariance risk of the individual asset in market portfolio measured relative to the weighted average of covariance risks of all assets (FF, 2003).

efficient portfolio can be constructed as a combination of *p* and *q*.[9] Moreover, basic portfolios can be transformed to have any values of β, including $\beta = 1$ and $\beta = 0$, while still consisting of *p* and *q*. Stemming from the facts that the market portfolio itself is a combination of *p* and *q*, and that weighted combinations of the two new basic portfolios must be efficient, these two transformed basic portfolios are the market portfolio (*M*) and the minimum-variance zero-Beta portfolio (*Z*). The return on such weighted efficient portfolio *K* and taking expectations can be then written as:

$$\begin{aligned} R_K &= \beta_K R_M + (1-\beta_K) R_Z \\ E(R_K) &= E(R_Z) + \beta_K \left(E(R_M) - E(R_Z)\right) \end{aligned} \tag{4}$$

and will hold not only for efficient portfolios but also for individual assets:

$$E(R_i) = E(R_Z) + \beta_i \left(E(R_M) - E(R_Z)\right) \tag{4'}$$

From the three equations above we can see that the only difference from the original CAPM is the risk-free rate of return being replaced by the expected return on minimum-variance zero-Beta portfolio.

The previous approach can be also applied to the case when a single risk-free asset is introduced, but no borrowing is allowed, i.e. $x_F \geq 0$. There will be investors that do not hold the risk-free asset due to their preferences and the previous results still apply to them, while the more risk-averse investors hold a combination of the risk-free asset and the tangency portfolio. The tangency portfolio itself is a combination of M and Z, therefore, the results of the case with no risk-free asset are altered only slightly. Most importantly the expected return on the minimum-variance zero-beta portfolio must exceed the risk-free rate. It should be noted that the market and minimum-variance zero-Beta portfolios M and Z are not identical for the two cases when it comes to their compositions and/or returns. The principle and implications however stay the same. Black (1972, p. 454) points out the main conclusion and extends the application of his model to more generalized cases: *"The expected return on a security continues to be linear function of its Beta. ... The general approach used in this section can be used to obtain similar results when every individual has a limit on the amount he can borrow that may be greater than zero"*. The critical assumption necessary for these relationships to hold is the possibility of unrestricted positions in the risky assets. The lean linear equations stated above are valid only when investors can take unlimited short and long positions in all risky assets. Black recognizes this assumption as similarly unrealistic to the unrestricted risk-free borrowing but argues that imposing restrictions on short selling will only strengthen the effect of limits on borrowing.

In spite of the recurring notion in academic literature of "Beta being dead" (Wallace (1980), Grinold (1993) and FF (2003) among others), CAPM remains to be widely used and

[9] For the proof see Black (1972, p. 447-450).

provides invaluable foundation for further research. As Rosenberg (1981, p. 5) argues, the practical application of the original CAPM model is not its main contribution. Should the CAPM be true there will be no place for investment research and active management of funds. But the creation of the CAPM and the ensuing discussions about its validity brought the attention to the topic and led to important frameworks that are very much alive as we shall see in Chapter 3.

2.2. Factor investing – New direction of research and asset allocation

To avoid any confusion we would like to first stress the difference between factor models and factor investing. Factor models are very common and the CAPM is just one of the representatives of this class. Any factor model can be in general written as:

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + ... + b_{iM}F_M + e_i \quad (5)$$

where R_i is an excess return on asset *i*, F_k is the value of *k*-th factor, e_i is the asset specific return and b_{ik} is the sensitivity of the return on asset *i* to factor *k* (sometimes also called factor loading). Through the traditional assumption of $E(e_i) = 0$ and $\text{cov}(e_i, e_j) = 0$ for $i \neq j$ we obtain returns on assets that are correlated only through the common factors.

While the factor model as described by (5) represents a return generating process, factor investing uses this information to create an efficient asset allocation strategy. Investment strategies in general rely on capturing various risk premiums.[10] That is true also for the traditional asset allocation; however, the focus of such a strategy is on the premium stemming from the asset classes as such. In case of the recent global crisis the diversification through geographical locations, industries, and stocks vs. bonds proved insufficient and alternative asset classes such as private equity or hedge funds that seemed to provide a solution turned out to have a high exposure to traditional equities and bonds. A different direction is therefore necessary. Multiple studies have documented that other sources of risk premiums are available in financial markets, especially for equities.[11] Factors such as Size, Value and Momentum were used to explain the realized returns in financial markets with varying success; however the improvement over the original one-factor CAPM model is significant. Recent developments suggest taking this knowledge one step further, resulting in a factor investing approach. That is, instead of the traditional diversification across dimensions such as asset classes, geographical regions or industrial sectors, factor investing strategy diversifies across various factor premiums. This is directly in line with the observation of Kees Koedijk

[10] Risk premium is defined as "the amount an investment is expected to earn for bearing a systematic risk" (Bender et al., 2010, p. 18).

[11] Asness, Moskowitz and Pedersen (2009) document the presence of some of these premiums in broad range of asset classes.

(Blitz et al., 2012, p. 6), who stresses the importance of understanding the underlying sources of risk behind these factors. Only once this is understood, investors can construct portfolios with an appropriate risk profile that will be stable in good times as well as in periods of increase volatility in markets. Figure 3 demonstrates the difference between traditional and factor investing portfolios. Although constructing factors that would capture risk premiums is nothing overly complicated, the lack of theoretical foundations for the empirically observed phenomenon might limit their general applicability and hence be the reason why factor investing came into light fairly recently.

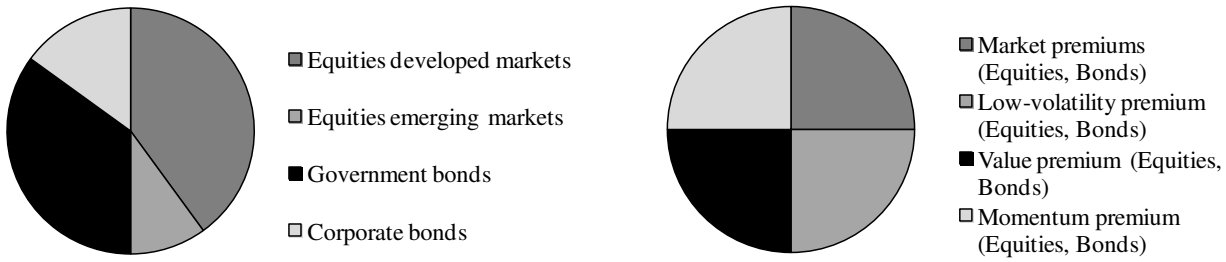


Figure 3: Composition of traditional portfolio (left) and factor-investing based portfolio (right)

Source: Adapted from Blitz et al. (2012, p. 11)

The first study to explicitly recommend additional exposure to risk premiums based on other characteristic than asset class was Ang et al. (2009). Historically it would be impossible to easily capture other premiums besides the asset class premium. But just as Sharpe (1991) praises the contribution of stock index futures to functioning of financial markets and of accuracy of CAPM predictions as a result, the specialized mutual funds which are readily available provide the chance to focus on alternative sources of risk and return. Building on these facts, Bender et al. (2010) define three types of risk premiums and suggest a way how to construct risk factors that exploit the Style and Strategy risk premiums from Table 1. They compare the performance of traditional 60/40 stock-bond portfolio with equally weighted allocations to the Style and Strategy risk factors to find that the factor investing approach outperforms the traditional portfolio in face of adverse events and offers more than three-times higher Sharpe ratio over the period May 1995 – May 2009.

Asset Class	Asset Class Risk Premium	Style Risk Premium	Strategy Risk Premium
Equity	Broad Equity Markets	Size	Merger Arbitrage
		Value	
		Momentum	
Fixed Income	Broad Fixed-Income Markets	Credit Spreads	Convertible Arbitrage
		High-Yield Spread	
		Term Structure Spread	
Currency	Broad Currency Markets		Carry Trade
			Momentum
			Value

Traditional premiums Main focus of thesis

Table 1: Risk premium types of different asset classes
Source: Bender et al. (2010, p. 18), Author's representation

Factor investing should be relevant for all institutional investors, including pension funds since it seems to offer a significant upgrade over the traditional approach during times of distress. Blitz (2012) assesses the potential of factor investing in equity markets using conservative expectations for future developments with both optimized and equally weighted portfolios [12] and concludes that *"... investors can substantially improve the risk/return characteristics of their strategic asset allocation by considering not only the classic equity premium, but other premiums present in the equity market"*. The real-life implementation of such strategies as well as the identification of systematic sources of return is thus an interesting direction for future research. In fact, this thesis should be considered under this class of research as the Betting Against Beta (BAB) factors introduced in Chapter 3.3 in the Fixed Income domain are equivalent to the Term Structure Spread and High-Yield Spread style premiums, using different and more generally applicable reasoning.

[12] Optimized portfolio invests strictly into factors while equally weighted portfolio includes market portfolio and three factors.

3. Literature review – Empirical results, extensions and implications

Testing and criticism of the CAPM is almost as old as the model itself. Even though its track record is not very impressive to say the least, Sharpe (1970, p. 2) argues that for model to be accepted it doesn't have to be a perfect fit, it merely has to confine with general logic and outperform its competitors. That could be regarded as a response to the first critiques; however the theory, model and counter-arguments have come a long way since then. In this chapter we will provide a brief summary of the findings concerning the performance of the original CAPM, its implications and relevant extensions, thus documenting the main development of the theory including its application. For more detailed summary and discussion of developments of the CAPM including extensive list of references see FF (2003).

3.1. The (mis)performance of Beta

As a market equilibrium model, the CAPM has been applied in several areas of research and financial industry such as evaluation of mutual funds' performance or assessing the equity cost of capital. Leaving aside the potential problem of unrealistic assumptions of the model itself, testing of its validity has other obstacles to overcome. Especially two aspects of empirical analysis have become standard in order to deal with imprecise estimates of beta for individual securities and with the covariance of residuals in cross sectional regressions. The former is solved by the grouping approach suggested by Blume (1970) where expected returns and betas of portfolios are weighted averages over individual stocks forming the portfolio. The latter issue is eliminated by the approach of Fama and MacBeth (1973) who applied monthly cross sectional regressions instead of average returns of portfolios on their betas. The resulting model can be written as:

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{pt} + \varepsilon_{pt} \qquad p = 1,...,P \quad t = 1,...,T \tag{6}$$

where R_{pt} is the realized return on portfolio p in month t, $\hat{\beta}_{pt}$ is the estimated beta of portfolio p in month t with respect to the market and P and T are the numbers of portfolios and calendar months in the sample respectively.

Estimating equation (6) serves as a test of two predictions: if the original CAPM holds, the mean value of the coefficient γ_{0t} will not be significantly different from the average risk-free rate over the period and similarly γ_{1t} will be the estimate of the expected market premium. The minimum expectation would have the estimate of the market premium to be positive and the relationship between expected return and market premium in linear fashion. Early evidence of Black, Jensen, and Scholes (1972), Blume and Friend (1973) and Fama and MacBeth (1973) confirms the linear relationship and positive values of γ_{1t} from equation (6).

However, the magnitude of γ_{1t} is on average lower than the observed excess return on the market portfolio and the hypothesis in such regressions, that the intercept is equal to the risk-free rate is consistently rejected.

These findings were confirmed by time-series estimations that stem from Jensen's attempt to measure the performance of mutual funds, introducing the so called Jensen's alpha (Jensen, 1967). Following Jensen's approach and the original CAPM settings, for each asset and portfolio the intercept term in the following regression must be zero:

$$R_{it} - R_{ft} = \alpha_i + \beta_i \left(R_{Mt} - R_{ft} \right) + \varepsilon_{it} \qquad (7)$$

Here the left-hand side is an excess return on an asset and beta is the sensitivity to the market excess return. As opposed to the cross sectional regressions where an average risk-free rate is widely used, here it is allowed to change every period. While Jensen (1967) doesn't find support for a positive impact of funds managers' security price predictions in terms of positive alpha, the evidence concerning portfolio returns is to the contrary. Given the evidence on γ_{jt}, estimates of the γ_{0t} coefficient from (6) are on average higher than the risk-free rate. Moreover, one important pattern arises from the time-series regressions in the form of (7) as described by Black, Jensen, and Scholes (1972) and Friend and Blume (1970). Namely there is an inverse relation between the intercept estimates and risk of the portfolio measured by its beta: high-beta portfolios are associated with negative alpha and low-beta portfolios with positive alpha. As a result the CML as implied by empirical data is flatter than assumed by the original CAPM.

3.2. Multifactor models and the CAPM

Another crucial implication, subject to testing in the 1970's is the postulate of the original CAPM that the market risk represented by beta is the only factor driving asset returns. One of the first tests was performed by Fama and MacBeth (1973) using the residual variances from regressions of returns on the market return as the potential force behind asset returns.[13] Black, Jensen and Scholes (1972) conclude that the original version of the model is not consistent with the data and propose a two-factor model on the grounds of Black's (1972) theory of restricted borrowing. Given the evidence on the relation between beta and return they specify the realized returns relationship as:

$$R_j = \left(1 - \beta_j\right) R_Z + \beta_j R_M + e_j \qquad (8)$$

that is, a linear relationship between the realized return on individual asset or portfolio with the realized market return and a realized return on a second factor, the so called "beta factor" since it is a function of the asset's beta. Although the authors do not provide explanations why the beta factor exists and acknowledge alternative theories leading to the real-life data

[13] Their tests of linearity of the CAPM relationship also fall into the multifactor CAPM category but the underlying motivation is different from the rest of the cases presented here.

observations, the beta factor was thereby established. The beta factor is not stationary over time but both cross sectional and time series analysis lead to the conclusion of statistical significance.

Cross-sectional regression tests in general can be set up by augmenting equation (6) by *J-1* new factors Z_{jp} leading to a new equation to be estimated:

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{pt} + \sum_{j=2}^{J} \gamma_{jt} Z_{jp,t-1} + \varepsilon_{pt} \qquad (9)$$

where the variables remain the same as in equation (6) and Z_{jp} are predetermined factors

Naturally if the original CAPM was the true model all time-series means of the slope coefficients γ_{jt} must not be significantly different from zero. The main issue faced by the researcher is finding the factors Z that would improve the performance of the original CAPM at a given significance level and hence invalidate it. FF (2003) note that even though cross-sectional as well as time-series regressions in fact test the efficiency of a selected market portfolio proxy (usually a value- or equally-weighted index) instead of CAPM itself, this issue can be raised with all models using proxies. The approach to time-series testing of multifactor models is more complicated, nevertheless the early papers using this technique (e.g. Stambaugh (1982)) did not rule out a variety of market portfolio proxies as efficient.

The search for significant underlying factors other than the market risk eventually yielded positive results. The first papers identifying additional return drivers came from Basu (1977) and Banz (1981) who find positive outperformance of stocks with high relative E/P ratio and small relative market capitalization, respectively. Since then, there has been extensive research into multiple possibilities of other factors having impact on performance of individual assets and portfolios. FF (1992) collect the evidence on the topic and extend the analyzed period to December 1990. They discuss the previously found relations between stock returns and factors including market beta, size, E/P, leverage, and book-to-market equity. Their ultimate results identify book-to-market equity as the most important factor which, in combination with the size factor of Banz, absorbs the relationship between average returns and the remaining factor candidates. Their cross-sectional analysis also confirms earlier findings about the market beta itself. Namely for the period 1963-1990 the relationship between beta and average return vanishes or becomes even slightly negative as opposed to the predictions of the CAPM for the time periods tested in earlier works. Over the 50 year period of 1941-1990 the relation is positive but very weak. Various explanations for the source of anomalies are at hand. One of the frequently debated possibilities is the empirical failure of the CAPM due to a bad proxy of the market portfolio. Roll (1977) provided the base argument, questioning the validity of any test of the CAPM based on the nonobservability of the market portfolio. Even though Stambaugh (1982) largely dismisses this concern, the application of the CAPM in practical matters such as cost of equity capital estimation will be biased. Roll and Ross (1994) present further evidence on how even slight departures of the

proxies from the real but unobservable market portfolio cause the CAPM to fail. An often used argument of data mining has been mostly countered by out-of-sample tests, at least for the value factor, while the behavioral stream of literature stresses irrational pricing that stems from investors' overreactions. Under the behavioral explanation the original CAPM in essence holds, but the historical underperformance is caused by irrational mispricing. Given the very simplistic structure of preferences, it is also plausible that investors do care about additional risk factors besides return and its variance and models more complicated than CAPM are necessary. Empirically selected factors in multifactor models can capture these risks, e.g. often discussed financial distress risk. However, it remains true that theoretical foundations of the factors are rather weak if not non-existent, which casts doubts on their generality. FF (1996) and Novak and Petr (2010) among others offer the discussion of the logic underlying the connections between the factors and portfolio returns focused on the rational risk pricing. Either way the multifactor models remain widely used and the rationale behind CAPM anomalies is open for discussion.

In their influential works, FF (1993, 1996) apply the time series approach to both stocks and bonds and provide a 3-factor model that became widely used. Building on the evidence in support of size and book-to-market factors they propose the excess return on portfolio to be related to the excess return on the market portfolio, the difference between returns on portfolios of small-cap stocks and large-cap stocks (small-minus-big, SMB) and the difference between returns on portfolios of high book-to-market stocks and low book-to-market stocks (high-minus-low, HML). For the precise construction of the returns used as explanatory variables see FF (1996, p. 58, Table I). Formally the expected excess return on portfolio *i* can be written as follows:

$$E(R_i) - R_f = \beta_{iM}\left(E(R_M) - R_f\right) + \beta_{iSize}E(SMB) + \beta_{iValue}E(HML) \qquad (10)$$

and time series regression for empirical estimation:

$$R_{it} - R_{ft} = \alpha_i + \beta_{iM}\left(R_{Mt} - R_{ft}\right) + \beta_{iSize}\left(SMB_t\right) + \beta_{iValue}\left(HML_t\right) + \varepsilon_{it} \qquad (10')$$

Results suggest that the model defined by (10) and (10') explains much of the variation in the cross-section of average stock returns. It performs very well with portfolios formed by several financial ratios causing a problem to the original CAPM. FF (1996, p. 57) go as far as calling the model represented by (10) an equilibrium pricing model that is a special 3-factor version of APT or of the ICAPM (not discussed here). One "anomaly" of the original CAPM cannot, however, be explained by the 3-factor model. The persistence of short-term returns first documented by Jegadeesh and Titman (1993) is in contradiction with the reversal in trend predicted by the model. The so called "momentum factor" was then added by Carhart (1997) to produce a 4-factor model. Similar to the two factors of FF, the momentum factor (winners-minus-losers, WML) is the difference between returns on diversified portfolios with high and low returns on previous 11 months lagged by one month. More details on the formation of WML factor can be found in Carhart (1997, p. 61, footnote 3). Factor models aren't critique-

prone or generally applicable to all settings as is documented by e.g. Huij and Verbeek (2007) or Novak and Petr (2010). Combined with the earlier notion of lacking theoretical fundamentals, their use and interpretation should be very cautious.

3.3. A new factor to the rescue

To conclude the discussion on performance and future use of CAPM, Fama and French say that: *"If market betas do not suffice to explain expected returns, the market portfolio is not efficient, the CAPM is dead in its tracks ..."* (FF, 2003, p. 13). The empirical under-performance of the original CAPM suggests strong support for such a statement. However, Fama took a shot at the CAPM even earlier, namely calling the beta dead and dismissing any relation between beta and average returns (Black, 1993). This prompted a fast response from Fischer Black in defense of the Sharpe-Lintner-Black (SLB) model (the original CAPM with borrowing restrictions) and especially of the concept of beta. Besides the fact that the predictions of SLB have not been conclusively rejected, if they were it would have even greater impact as Black (1993, p. 75) puts it: *"Beta is a valuable investment tool if the line is as steep as the CAPM predicts. It is even more valuable if the line is flat."* Moreover, Black recommends using the beta to construct a factor as a diversified portfolio that is long in low-beta stocks and short in high-beta stocks with the overall beta approximately zero. Empirical evidence on such portfolios is stronger than for the value and size factors and most importantly, work of Black (1972) provides the theoretical foundation for this factor.

In their recent work Frazzini and Pedersen (2011) introduced a dynamic model with leverage constraints that integrates multiple aspects from the theory outlined above. The following section is based heavily on their contributions as their model is further applied in the remainder of this thesis. The economic intuition behind their work is simple and straightforward: in an economy where (at least some) investors are leverage constrained, these constrained investors need to purchase assets riskier than would be optimal in order to reach their required expected return. Due to this "additional" demand for risky high-beta assets, the risk-adjusted returns on high-beta assets will in equilibrium be lower than the returns on safer securities. This is nothing else than the relative flatness of the security market line as discussed in the previous section and observed in the equity markets throughout history. Given the varying leverage constraints among investors as a very realistic assumption and the sound theoretical base, there should be a possibility to exploit this phenomenon for unconstrained investors.[14] One of the ways to do this is by investing in a so called "Betting against Beta" (BAB) factor. We adopt the model of an overlapping generations economy with agents $i = 1, ..., I$ being born every period t with initial investment wealth W_t^i. Consumption in the first period of life is not considered, therefore agents invest the full amount of their

[14] Unconstrained in a relative sense, meaning that the leverage constraint is not binding for the individual given his risk/return preferences.

investment wealth. There are stocks $s = 1,...,S$ available, paying dividends δ_t^s and having x^{*s} shares outstanding. From these stocks every young investor creates a portfolio of infinitely divisible shares $\mathbf{x}' = \left(x^1, x^2, ..., x^S\right)$, with the rest of his wealth invested at the risk-free rate r_f. Young agents maximize their utility given as:

$$\max \mathbf{x}'\left(E_t\left(\mathbf{P_{t+1}} + \boldsymbol{\delta}_{\mathbf{t+1}}\right) - \left(1 + r^f\right)\mathbf{P}_t\right) - \frac{\gamma^i}{2}\mathbf{x}'\boldsymbol{\Omega}_{\mathbf{t}}\mathbf{x} \qquad (11)$$

where $\mathbf{P_t}$ is the vector of stock prices, $\boldsymbol{\delta}_{\mathbf{t+1}}$ the vector of dividend streams between *t* and *t+1*, γ^i is the risk aversion coefficient of agent *i* and $\boldsymbol{\Omega}_{\mathbf{t}}$ is the variance-covariance matrix of the vector $\mathbf{P_{t+1}} + \boldsymbol{\delta}_{\mathbf{t+1}}$. Every investor is facing a budget constraint on his investment possibilities:

$$m_t^i \sum_s x^s P_t^s \le W_t^i \qquad (11')$$

The parameter m_t^i is the crucial difference of this model with respect to the original CAPM and to Black's version of no risk-free borrowing. This budget constraint states that each investor can allocate in stocks up to $\frac{1}{m_t^i}$-multiple of his investment wealth W_t^i. The parameter can in theory take on values ranging from zero to infinity, but the extreme cases are not likely to be observed and/or to result in a binding budget constraint. Values of $0 < m_t^i < 1$ signal that the investor can take on leverage, although limited as is the case when there are margin requirements. For example $m_t^i = 0.5$ means that investor *i* can invest up to twice the amount of his wealth; that is using leverage up to 100% or facing 50% margin. Conversely, values of $m_t^i > 1$ mean that the investor has to hold part of his wealth in cash: e.g. 10% mandatory "cash buffer" translates into $m_t^i = \frac{1}{(1-0.1)} = 1.\overline{11}$. To illustrate the connection with the previous theory, should $m_t^i = 1$ for all *i* and *t*, we arrive at the Black's (1972) version of CAPM without risk-free borrowing outlined at the end of Section 2.2.

Applying this setup, we now explore the implications of a competitive equilibrium by first constructing the Lagrangean of each individual *i*:

$$L = \mathbf{x^{i}}'\left(E_t\left(\mathbf{P_{t+1}} + \boldsymbol{\delta}_{\mathbf{t+1}}\right) - \left(1 + r^f\right)\mathbf{P}_t\right) - \frac{\gamma^i}{2}\mathbf{x^{i}}'\boldsymbol{\Omega}_{\mathbf{t}}\mathbf{x^{i}} - \psi_t^i\left(\mathbf{x^{i}}'\mathbf{P}_t - \frac{W_t^i}{m_t^i}\right)$$

where ψ_t^i is the lagrange multiplier of individual *i*'s budget constraint.

From this we can derive the first order condition and optimal asset allocation for each investor:

$$0 = E_t\left(\mathbf{P_{t+1}} + \boldsymbol{\delta}_{\mathbf{t+1}}\right) - \left(1 + r^f\right)\mathbf{P}_t - \gamma^i\boldsymbol{\Omega}_{\mathbf{t}}\mathbf{x^{i}} - \psi_t^i\mathbf{P}_t$$

$$\mathbf{x^{i}} = \frac{1}{\gamma^i}\boldsymbol{\Omega}^{-1}\left(E_t\left(\mathbf{P_{t+1}} + \boldsymbol{\delta}_{\mathbf{t+1}}\right) - \left(1 + r^f + \psi_t^i\right)\mathbf{P_t}\right)$$

Defining $\frac{1}{\gamma} = \sum_i \frac{1}{\gamma^i}$ as the aggregate risk aversion and $\psi_t = \sum_i \frac{\gamma}{\gamma^i}\psi_t^i$ as the weighted average Lagrange multiplier that measures how strict the budget constraint is for the whole economy, we can sum over all individual investors in the economy to get the vector of optimal positions for each asset:

$$\mathbf{x}^* = \frac{1}{\gamma}\mathbf{\Omega^{-1}}\left(E_t\left(\mathbf{P_{t+1}} + \mathbf{\delta_{t+1}}\right) - \left(1 + r^f + \psi_t\right)\mathbf{P_t}\right) \tag{12}$$

and also the vector of equilibrium prices:

$$\mathbf{P_t} = \frac{E_t\left(\mathbf{P_{t+1}} + \mathbf{\delta_{t+1}}\right) - \gamma\mathbf{\Omega x}^*}{1 + r^f + \psi_t} \tag{12'}$$

It should be noted that individual leverage constraints in the average ψ_t are not only weighted by the relative risk aversion of investors, as it may appear from the definition, but also by W_t^i and m_t^i that determine the value of ψ_t^i.

We can now express the equilibrium price of a single asset and its expected return that will eventually lead to a CAPM-like relation. As opposed to the derivation in Frazzini and Pedersen (2011, Appendix), we opt to include all the intermediate steps for a better clarity of the approach:

$$P_t^s = \frac{E_t\left(P_{t+1}^s + \delta_{t+1}^s\right) - \gamma\mathbf{e_s'\Omega x}^*}{1 + r_f + \psi_t}$$

can be rewritten into an expected return as:

$$1 + E_t\left(r_{t+1}^s\right) = \frac{E_t\left(P_{t+1}^s + \delta_{t+1}^s\right)}{P_t^s} = 1 + r_f + \psi_t + \frac{\gamma\mathbf{e_s'\Omega x}^*}{P_t^s}$$

$$E_t\left(r_{t+1}^s\right) = r_f + \psi_t + \gamma\frac{1}{P_t^s}\left[\mathbf{e_s'\Omega x}^*\right]$$

The term in the square bracket is nothing else but a conditional covariance of future prices (including dividends) of the stock *s* and the total market. The last term becomes:

$$\gamma\frac{1}{P_t^s}\left[e_s'\Omega x^*\right] = \gamma\frac{1}{P_t^s}\mathrm{cov}_t\left(P_{t+1}^s + \delta_{t+1}^s, \left[\mathbf{P_{t+1}} + \mathbf{\delta_{t+1}}\right]'\mathbf{x}^*\right) =$$

$$= \gamma\frac{1}{P_t^s}\mathrm{cov}_t\left(\frac{P_{t+1}^s + \delta_{t+1}^s}{P_t^s}P_t^s, \left(\left[\left(\mathbf{P_{t+1}} + \mathbf{\delta_{t+1}}\right)'\mathbf{x}^*\right]\left[\mathbf{P_t'x}^*\right]^{-1}\right)\mathbf{P_t'x}^*\right) = \gamma\frac{P_t^s}{P_t^s}\mathrm{cov}_t\left(1 + r_{t+1}^s, 1 + r_{t+1}^M\right)\mathbf{P_t'x}^*$$

$$= \gamma\,\mathrm{cov}_t\left(r_{t+1}^s, r_{t+1}^M\right)\mathbf{P_t'x}^*$$

Leading to a final expression for expected return on individual asset:

$$E_t\left(r_{t+1}^s\right) = r_f + \psi_t + \gamma\,\mathrm{cov}_t\left(r_{t+1}^s, r_{t+1}^M\right)\mathbf{P_t'x}^* \tag{13}$$

Using this expression for single-stock expected return we can obtain an expected return on the equilibrium market portfolio by applying appropriate weight to each equation and summing over all stocks:

$$w^s = \frac{x^{*s} P_t^s}{\sum_{j=1}^{S} x^{*j} P_t^j}$$

$$\sum_{s=1}^{S}\left(w^s E_t\left(r_{t+1}^s\right)\right) = \sum_{s=1}^{S}\left(w^s\left[r_f + \psi_t + \gamma \operatorname{cov}_t\left(r_{t+1}^s, r_{t+1}^M\right)\mathbf{P_t'x^*}\right]\right)$$

$$E_t\left(r_{t+1}^M\right) = r_f + \psi_t + \gamma \operatorname{var}\left(r_{t+1}^M\right)\mathbf{P_t'x^*} \tag{14}$$

Finally combining (13) and (14) leads to an equation connecting the return on single asset with the return on market portfolio:

$$E_t\left(r_{t+1}^s\right) = r_f + \psi_t + \frac{E_t\left(r_{t+1}^M\right) - r_f - \psi_t}{\operatorname{var}\left(r_{t+1}^M\right)} \operatorname{cov}\left(r_{t+1}^s, r_{t+1}^M\right)$$

$$E_t\left(r_{t+1}^s\right) = r_f + \psi_t + \beta_t^s\left(E_t\left(r_{t+1}^M\right) - r_f - \psi_t\right) \tag{15}$$

$$E_t\left(r_{t+1}^s\right) = r_f + \psi_t\left(1 - \beta_t^s\right) + \beta_t^s\left(E_t\left(r_{t+1}^M\right) - r_f\right) \tag{15'}$$

From equation (15') we see that the return on individual asset is dependent on the risk free rate, the traditional CAPM risk premium, but also on an additional term $\psi_t\left(1-\beta_t^s\right)$. Recalling the role of ψ_t as a measure of budget or liquidity constraints in the economy, the asset returns contain an extra term based on their own market risk and the strictness of liquidity constraints in the economy.

Following Figure 4 illustrates the theoretical background of constrained agents' preference for assets with excess risk over the tangency portfolio, hence implying the market portfolio is in fact to the northeast of T. This is true because agents without binding budget constraints will invest strictly in the tangency portfolio T, while agents for whom the constraint is binding will be forced to select riskier MV-efficient portfolios such as A or B because their CML is altered. Agent that can use limited amount of leverage can move along the CML (chain-dotted line) up to point T' by holding combination of T and the risk-free asset, but in order to reach the preferred risk-return combination A' he has to hold combination of the risk-free asset and portfolio A. Analogous situation holds for agent who is required to hold cash. He can choose combination of T and risk-free asset up to point T'', while investment into riskier portfolio B from the efficient frontier is necessary in order to reach preferred position B'. The market portfolio therefore consists of a combination of portfolios A, B and T (and potentially many more portfolios), but both the risk and return of the market portfolio must be strictly higher than for the tangency portfolio T.

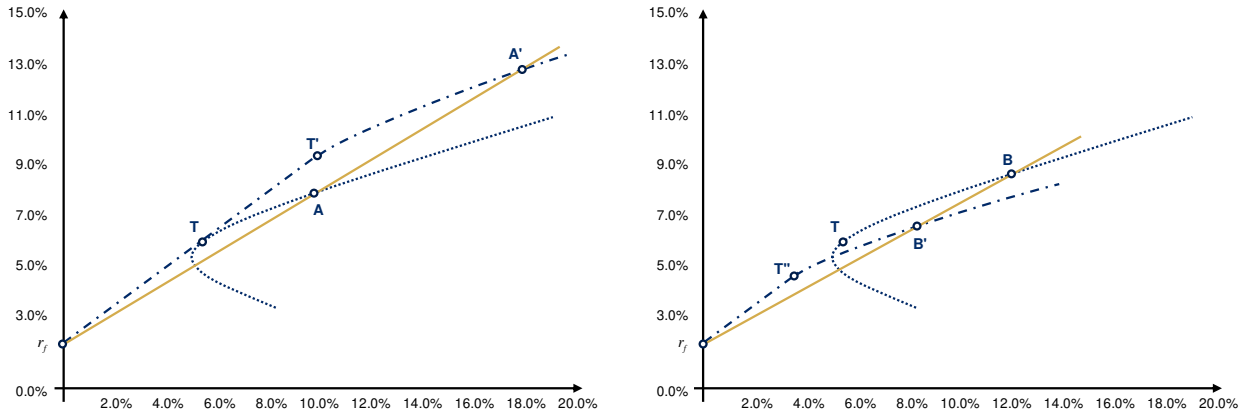


Figure 4: MV analysis of investors facing margin requirements (m<1, left) and need to hold cash (m>1, right)

Source: Adapted from Frazzini and Pedersen (2011, Appendix)

Equation (15) is very similar to the original CAPM but, not surprisingly, is in fact a more explicit representation of (4'), that is the Black's version of the model where $E(R_Z) = r_f + \psi_t$. From this equation we can derive some of the testable implications of the model that will be further explored in line with Frazzini and Pedersen (2011):

a) Expressing equation (15') as an excess return we can see that Jensen's alpha becomes $\alpha_t^s = \psi_t(1-\beta_t^s)$, that is decreasing in beta and increasing in ψ_t as tighter leverage constraints lowers the slope of the SML, while raising its intercept.
b) The Sharpe ratio will be increasing in beta for efficient portfolios with low betas until some $\beta_t^{s*} < 1$ and decreasing afterwards.

The implications of the model in the sense of a low beta leading to a high alpha can be exploited by constructing a self-financing zero-beta long-short portfolio that loads on assets with low beta and short-sells the riskier high beta assets. Frazzini and Pedersen (2011) call this the Betting Against Beta (BAB) factor and derive some interesting properties. Assume two portfolios, one constructed from the low-beta assets and the other one from high-beta assets. Let $\mathbf{w_L}$ be the relative weights of individual assets in the low-beta portfolio (sum of the weights is equal to one and high-beta assets have weights of zero), then the return on this portfolio is $r_{t+1}^L = \mathbf{w_L'} \mathbf{r_{t+1}}$. A high-beta counterpart is constructed in the same way and is denoted by a superscript H (in case of the weights vector a subscript H). Market betas of the portfolios are denoted by β_t^L and β_t^H, respectively, with naturally $\beta_t^L < \beta_t^H$. Then the BAB factor is constructed by going long in the low-beta portfolio which is financed by shorting the riskier high-beta portfolio. Both sides are leveraged (resp. deleveraged) to a beta of one and the return on the BAB factor is:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}\left(r_{t+1}^L - r^f\right) - \frac{1}{\beta_t^H}\left(r_{t+1}^H - r^f\right) \qquad (16)$$

There are two important properties of the BAB factor stemming from the model. Given its construction, we can plug in (15) to analyze the behavior of the expected return on the BAB factor:

$$E\left(r_{t+1}^{BAB}\right)=\frac{1}{\beta_t^L}\left(\left[r_f+\psi_t+\beta_t^L\left(E_t\left(r_{t+1}^M\right)-r_f-\psi_t\right)\right]-r^f\right)-\frac{1}{\beta_t^H}\left(\left[r_f+\psi_t+\beta_t^H\left(E_t\left(r_{t+1}^M\right)-r_f-\psi_t\right)\right]-r^f\right)$$

$$\Leftrightarrow E\left(r_{t+1}^{BAB}\right)=\frac{\psi_t}{\beta_t^L}+\left(E_t\left(r_{t+1}^M\right)-r_f-\psi_t\right)-\frac{\psi_t}{\beta_t^H}-\left(E_t\left(r_{t+1}^M\right)-r_f-\psi_t\right)$$

$$\Leftrightarrow E\left(r_{t+1}^{BAB}\right)=\frac{\beta_t^H-\beta_t^L}{\beta_t^H\beta_t^L}\psi_t>0 \qquad (17)$$

So firstly the expected return on BAB factor is positive and increasing in both the tightness of leverage constraints and the ex-ante beta spread of the two sides of the long-short portfolio.

Secondly, we want to consider the implications of changes caused by changing portfolio constraints represented by changes to m_t^k, the constraint of an individual. This can be understood as changing conditions on the financial markets, e.g. liquidity tightening or even the financial and credit crisis as experienced currently. After a shock to the constraint the agent might need to reduce the levered position or short more of the high beta assets leading to an increasing required return on the BAB factor and hence contemporaneous losses.

$$\frac{\partial r_t^{BAB}}{\partial m_t^k}\le 0 \quad \text{and} \quad \frac{\partial E\left(r_{t+1}^{BAB}\right)}{\partial m_t^k}\ge 0 \qquad (18)$$

Frazzini and Pedersen (2011, Appendix A) derive a formal proof of these properties but the intuition behind is that the increase in individual constraint raises the aggregate average Lagrange multiplier ψ_t and hence increases the required return of the BAB factor (as can be seen from equation (17)). Since prices move opposite to the required returns, the increase in m_t^k leads to a contemporaneous loss of the factor.

These theoretical expressions provide the basis for the empirical testing of the relations and the assessment of the potential of the introduced BAB factor. Many aspects have already been evaluated in the original paper, however, several areas remain open for investigation. We aim at providing deeper insights into the behavior of the BAB factor and its potential applications, especially for the long-term investors. In the remainder of the thesis we first confirm the initial findings on the relative flatness of the CML on an updated dataset[15] which should provide a base for positive and statistically significant returns on the BAB factor. The BAB factor realized positive excess returns across various developed stock markets with the exception of Austria, with the potential reaching up to 1.66% per month in the case of Canada (Frazzini and Pedersen, 2011). Since these numbers were obtained on the individual-stock basis we test whether the relation holds on more aggregate level and extend

[15] Updated version of the original paper was released during the period of writing the thesis, therefore the covered time periods of the datasets in certain asset classes (e.g. Treasury bonds) do not differ significantly.

the analysis to equity indices including the available emerging markets from the MSCI universe. In the next step we examine aspects relevant for strategy and policies: the potential AR properties of the BAB factor returns and impact of liquidity and funding conditions proxied by the TED Spread as well as behavior of the BAB factor returns throughout time and across asset classes in more detail. To do that, we examine the cumulative returns in individual asset classes over various investment horizons. Transaction costs are disregarded in our analysis but even though Huij and Verbeek (2007) find factor returns persistent after including the transaction costs we take a look at the performance of the BAB factor with less frequent update of its composition. In the last part we explore the relation of BAB factor returns in the equity indices asset class with the traditional factors and the currently popular Minimum Volatility strategies.

4. Data and methodology

4.1. Asset classes and sources of data

We consider multiple asset classes ranging from equity indices through Treasury and corporate bonds to three subsets of futures contracts. In this section we briefly introduce the datasets while the details of the sources of data and some descriptive statistics are provided in Appendix I.

To estimate the potential of the BAB factor for equities at an aggregate level, return index data on the main equity indices from developed as well as emerging market countries from MSCI universe were used. We obtained the daily time series from Datastream covering the period of January 1989 to June 2013 for developed countries and the shorter period of January 1993 to June 2013 for emerging markets. Data availability differs for individual countries especially for the latter, therefore more investment opportunities are added over time in the construction of the BAB factor. We construct simple daily returns from the obtained return index values. We use the returns in excess of the 1M US Treasury bill rate. The daily risk free rate was downloaded from the Kenneth French library. The market index proxies for stock indices and for equity index futures introduced later consist of GPD-weighted returns on available investment assets. The annual data for country-level GDP in current-prices USD were obtained from the World Bank and due to missing data for the last two years the 2011 values were used for 2012 and 2013 as well. Data on Global factor returns are obtained from the Kenneth French library but only run from July 1990 on a monthly basis. The global factor returns are chosen in order to compare the situation in developed and emerging markets since factor data for only developed markets are available at Andrea Frazzini's online data library and Frazzini and Pedersen (2011, p.14) report robustness of results to both local and global benchmarks.

The sample of US Treasury bonds was obtained from the CRSP annually updated file of Fama bond portfolios. It includes monthly return data on seven bond portfolios of different maturities, ranging from less than one year to more than 10 years. The time period covered is January 1952 to December 2012 and the returns of the maturity buckets are constructed as an equally weighted average returns on the individual available bonds in the appropriate maturity range. The market return is calculated as the simple average of the monthly maturity bucket returns. Data sample for Treasury bonds is very similar to the one used by Frazzini and Pedersen (2011) and therefore the results for the BAB factor in this asset class are expected to be similar to theirs.

We also investigate the US market for corporate credit. Two groups of aggregate corporate bond index returns are used. The first dataset consists of bond indices based on maturities ranging again from one to more than ten years that are split into four maturity buckets. The second dataset is based on the rating and comprises of eight portfolios ranging

from the top investment grade AAA to a speculative Ca-D class. The data originate from Barclay's Capital's "Bond.Hub" database, access to which was a courtesy of APG. The covered time period is from January 1973 through June 2013 but the exact periods for each aggregate index might slightly differ. As was the case for Treasury bonds, the market portfolio proxy is constructed as the simple average of the individual bucket returns.

Last but not least, we explore the class of futures contracts. In line with the previous findings and due to the high liquidity in the market we expect the potential of the BAB factor to be lower if not completely absent. The daily data for the analysis are extracted from Datastream and include the excess return index on commodities, settlement prices on various bond futures, and settlement prices of equity index futures. The market portfolio proxies are GDP-weighted average for equity index futures and simple average for international treasury bond futures. As Assness et al. (2012) point out, the returns calculated from futures prices are returns on self-financing position and include the financing rate. Therefore, given the nature of futures contracts, the return calculated from settlement prices is already an excess return so that there is no need for adjustments for the risk-free rate. Besides the data for individual commodities we also use the S&P GSCI™ Commodity Index excess return time series to serve as a proxy for the market portfolio in the analysis of commodities markets.

For the second group of time series tests and to analyze the influence of funding liquidity on the BAB returns, we obtain the data on the TED spread from Datastream. The data is only available for the period 1986-2013 which restricts our analysis. The TED spread is defined as the difference between the 3M LIBOR rate and the 3M US Treasury bills rate; and is regularly used in the academic research to approximate the funding conditions in financial markets and subsequently the funding constraints of the agents.

4.2. BAB factor methodology – Construction and time-series tests

Before proceeding to the methodology of the actual construction of the BAB factor we first outline the rather standard approach for testing the first hypothesis of the thesis, that is, if high beta is indeed associated with low alpha. Following the work of Black, Jensen, and Scholes (1972), we apply time series regressions to individual asset classes in order to explore the suspected relation between alpha and beta in the original CAPM for equity indices. To efficiently test this hypothesis, we first run the regression of each equity index excess returns on constant and market excess return for both developed and emerging equity indices market. Besides the full sample tests we also form two portfolios – high beta and low beta – of individual indices that allow for higher consistency of portfolio betas throughout the observed period. Since assigning the assets to the two portfolios based on their realized betas would introduce selection bias, the high and low beta portfolios for empirical analysis are created on an out-of-sample basis. Therefore we use a one-year rolling regression of individual index excess returns on the market excess returns to estimate the pre-ranking betas, while requiring at least 200 observations in order to include the index in one of the two portfolios. For

monthly data the rolling window is 3 years and we require at least 12 observations. Individual indices are then assigned to one of the two portfolios and take equal weight in the portfolio. Returns over next month are recorded for each portfolio and the composition of portfolios is then adjusted in the same manner each month. In the case of bonds the data obtained are already the returns on portfolios that are formed based on either maturity or rating, both of which can be considered a proxy for beta. For futures all individual assets are analyzed separately and two portfolios (high beta and low beta) are then considered as was the case for the equity indices.

We make use of adjustments employed by Frazzini and Pedersen (2011) in estimating the pre-ranking betas, namely employing lagged market returns and a constant shrinkage factor (w). Their estimation approach looks as follow:

$$r_t - r_t^f = \hat{\alpha} + \sum_{k=0}^{K} \hat{\beta}_k \left(r_{t-k}^M - r_{t-k}^f \right) + \hat{\varepsilon}$$

$$\hat{\beta}^{TS} = \sum_{k=0}^{K} \hat{\beta}_k$$

$$\hat{\beta}_i = w_i \hat{\beta}_i^{TS} + (1 - w_i) \hat{\beta}^{XS}$$

where $w = 0.6$ and $\beta^{XS} = 1$ as the cross-sectional mean are set arbitrarily by the authors for all the assets throughout the whole period under consideration. Lagged market returns should account for asynchronous trading and the authors decided to use $K = 5$ for daily data and $K = 1$ for data of monthly frequency. The role of the shrinkage factor lies in diminishing the effects of outliers. In our model the values are chosen as $w = 1.0$ and $K = 0$ for bonds while there is no reason to suspect a large influence of asynchronous trading at monthly frequency and outliers are not expected to play major role. For equity indices and futures we set $w = 0.6$ and $K = 1$. Since the choice of the shrinkage factor doesn't influence the relative ordering and therefore composition of the portfolios, its role for the first part of the analysis is negligible. However, we find that the performance of the BAB factors is heavily influenced by the choice of w and K, especially when daily data are used, hence invalidating the BAB potential from the robustness point of view. The inclusion of shrinkage factor in general reduces the realized excess returns on the BAB factor and can be considered as a conservative measure, but the substantial and unstable impact of additional lags for pre-ranking beta calculations is alarming. Details are reported in Section 5.1 and Appendix III. Once the daily returns on the portfolios are constructed we proceed with the time series analysis for each portfolio $p = 1, ..., N$ and focus on the relation of α_p and β_p. For each portfolio the estimated model is stemming from equation (7):

$$r_{pt} - r_{ft} = \alpha_p + \beta_p \left(r_{Mt} - r_{ft} \right) + \varepsilon_{pt}$$

By construction we have beta increasing in the portfolio number[16] and from the hypothesis we expect the alpha to be decreasing in beta. The market portfolio return $\left(r_{Mt}\right)$ for the equity indices asset class is the GDP-weighted return on available investment assets. We estimate only the simple one factor model for the full sample regressions.

The cornerstone of this thesis is the self-financing zero-beta portfolio called the BAB factor. We adopt the methodology from Frazzini and Pedersen (2011, p. 18-19) in order to create such a long-short combination of assets. For each asset- or country-specific dataset described above we use again the previously defined pre-ranking betas. The assets are ordered by pre-ranking beta and split instead of deciles into two portfolios: low beta and high beta. Weights of the assets in the two portfolios are determined by the ex-ante rank with higher weights assigned to the extreme values (highest weight for the lowest beta in the low beta portfolio and for the highest beta for the high beta portfolio, marginal weights for assets around the median of the ranking). Pre-ranking procedures and composition of the two portfolios are adjusted each month. Let us $\mathbf{v}$ denote the $n\times 1$ vector of beta-based ranks $v_i = rank\left(\beta_{it}\right)$ and $\bar{v} = \frac{\mathbf{1_n'}\mathbf{v}}{n}$ the average rank of all the assets with n being the number of assets and $\mathbf{1_n}$ the vector of ones of dimension $n\times 1$. Given these definitions, the vectors of individual asset weights resulting in low-beta and high-beta portfolios are:

$$\mathbf{w_L} = -\frac{1}{k}\min\left(0, \mathbf{v} - \bar{v}\right)$$

$$\mathbf{w_H} = \frac{1}{k}\max\left(0, \mathbf{v} - \bar{v}\right)$$

respectively, where k is the normalizing constant that assures the sum of weights in each portfolio to be equal to one, hence $k = \frac{\mathbf{1_n'}\left|\mathbf{v} - \bar{\mathbf{v}}\right|}{2}$. To make it easier to understand, in case of $n = 7$ (which is the case for Treasury bonds in most of the analyzed period) and after ordering the betas $\beta_1 < \beta_2 < \beta_3 < \beta_4 < \beta_6 < \beta_5 < \beta_7$ the variables take on following values

$$\boldsymbol{\beta} = \begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\\ \beta_6\\ \beta_7\end{bmatrix},\ \mathbf{v} = \begin{bmatrix}1\\ 2\\ 3\\ 4\\ 6\\ 5\\ 7\end{bmatrix},\ \bar{v} = \frac{\sum_{i=1}^{7}\mathbf{v}_i}{7} = 4,\ k = \frac{\sum_{i=1}^{7}\left|\mathbf{v_i} - \bar{v}\right|}{2} = 6,\ \mathbf{w_L} = \begin{bmatrix}1/2\\ 1/3\\ 1/6\\ 0\\ 0\\ 0\\ 0\end{bmatrix},\ \mathbf{w_H} = \begin{bmatrix}0\\ 0\\ 0\\ 0\\ 1/3\\ 1/6\\ 1/2\end{bmatrix}$$

[16] In maturity or rating for bonds and from low to high beta portfolios for equity indices and futures.

with resulting portfolio characteristics[17]:

$r_{t+1}^{L} = \mathbf{r_{t+1}'w_L}$, $r_{t+1}^{H} = \mathbf{r_{t+1}'w_H}$, $\beta_t^{L} = \boldsymbol{\beta}_\mathbf{t}'\mathbf{w_L}$ and $\beta_t^{H} = \boldsymbol{\beta}_\mathbf{t}'\mathbf{w_H}$.

Having constructed the two portfolios we continue by creating the BAB factor which is a long-short combination of the two portfolios – low beta portfolio is the long side while high beta portfolio is sold short. Each side is scaled by the portfolio beta prior to formation so that they have a beta of one and the resulting factor is therefore market-neutral. As introduced in the previous section the return of BAB factor is calculated as

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^{L}}\left(r_{t+1}^{L} - r^{f}\right) - \frac{1}{\beta_t^{H}}\left(r_{t+1}^{H} - r^{f}\right) \tag{19}$$

For the BAB factor we run the same time-series regression as for the full sample tests, then add Global FF factors (3-factor model) and finally the momentum factor (4-factor model) for equity indices to analyze how much of the excess risk-adjusted return of low beta equity indices can be attributed to the loading on factors proven and analyzed in previous research. For other asset classes we restrict ourselves to the one-factor model for BAB factor returns. Based on the theory we expect the BAB factor to yield positive excess returns, high alpha and a relatively good Sharpe ratio as compared to the individual portfolios.

Secondly, we focus on time series tests of the BAB excess returns. We explore the autoregressive properties of the BAB factor by plotting the autocorrelation (ACF) and partial autocorrelation functions (PACF) in order to determine any potential AR signature. To test the third hypothesis of our main model (shock to the constraint leads to an increasing required return on the BAB factor and hence contemporaneous losses: $\frac{\partial r_t^{BAB}}{\partial m_t^{k}} \leq 0$ and $\frac{\partial E\left(r_{t+1}^{BAB}\right)}{\partial m_t^{k}} \geq 0$) we use the TED spread as a proxy for funding conditions. First the monthly BAB returns are regressed only on the contemporaneous TED spread, while several control variables are included at a later stage. These control variables include the market return, ex-ante beta spread of the BAB factor and lagged values of the BAB returns up to the lag identified by the PACF. We expect the excess returns on the BAB factor to be lower in times of a high TED spread which would confirm the fact that worsening liquidity conditions lead to lower or even negative returns. We also separate the effects of the previous level of the TED spread and contemporaneous change in the liquidity constraint proxy. According to the hypothesis the previous high TED spread level should indicate higher future returns while the positive contemporaneous change, i.e. further worsening of the funding conditions, should influence the performance of the BAB factor negatively. Formally the two models estimated for all the asset classes separately can be written as:

$$r_t^{BAB} = \alpha + \beta_1 TED_t + \beta_2 r_t^{M} + \beta_3 BetaSpread_t + \sum_{l=1}^{L} \beta_{3+l} r_{t-l}^{BAB} + \varepsilon_t \tag{20}$$

[17] Time subscript *t* is omitted in the variable description and is only presented in the BAB factor definition as two subsequent periods are used.

where TED_t is the monthly average TED Spread, r_t^M is the market return, $BetaSpread_t$ is the ex-ante Beta spread of the two sides of the BAB factor in form of $\frac{\beta_t^H - \beta_t^L}{\beta_t^H \beta_t^L}$ and r_{t-l}^{BAB} are the lagged returns on the BAB factor and

$$r_t^{BAB} = \alpha + \beta_1 TED_{t-1} + \beta_2 \Delta TED_t + \beta_3 r_t^M + \beta_4 BetaSpread_t + \sum_{l=1}^{L} \beta_{4+l} r_{t-l}^{BAB} + \varepsilon_t \qquad (21)$$

where the TED Spread is now the average of previous month and the only new variable is the change in the TED spread values $\Delta TED_t = TED_t - TED_{t-1}$.

4.3. Additional tests of BAB factor properties

In the last part we explore the correlation structure of the returns on individual asset classes' BAB factors. Should the returns be positively correlated we can suspect the previous equations to reveal periods of favorable economical conditions for the BAB factors across the board. Conversely a negative relationship would provide traditional diversification opportunities in the new investment approach. Similar information could be incurred from plotting the cumulative returns on the BAB factor for each asset class. We consider different windows ranging from 1 year to the full period available. The 20-year period can be considered closest to the investment horizon of a pension fund.[18]

With the emergence of the so called "smart beta" investing we want to connect the BAB approach with "smart beta". Blitz (2012a) defines "smart beta" investing as *"passively following an index in which stock weights are not proportional to their market capitalizations, but based on some alternative weighting scheme"*. For the equity based asset classes we explore the connection with the currently popular Minimum Volatility approach, which is a representative of the "smart beta" investing. Given the similar basic principle of construction of these two investment strategies and their focus on low risk assets it might be possible that the success of BAB factor can be explained by the loading on the minimum volatility index. Since the smart beta strategies, including the minimum volatility portfolios, themselves exploit the Fama and French factors, Momentum factor and other factors, we include Size, Value and Momentum factors in our analysis as well. Using the FF Global factor returns we estimate the loadings on these factors in the familiar regression framework. We first include only the market return and the return on MSCI Global Minimum Volatility index obtained from Datastream, then add the returns on the Value and Size factors and finally include also the Momentum factor. The MSCI Global Minimum Volatility index is constructed on the basis of Barra risk model and optimization mechanism but in general gives exposure to the low-volatility premium.

[18] 50-year and longer periods would be of utmost interest, however data of such history is only available for Treasury bonds.

Formally the full equation looks as follows:

$$r_t^{BAB} = \alpha + \beta_M \left(r_t^M \right) + \beta_{MinVol} \left(r_t^{MinVol} \right) + \beta_{Size} \left(r_t^{SMB} \right) + \beta_{Value} \left(r_t^{HML} \right) + \beta_{Mom} \left(r_t^{WML} \right) + \varepsilon_t \qquad (22)$$

If all the success of the BAB factor can be attributed to the good performance of the Minimum Volatility strategies and to the loading on the previously identified factors the alpha of the regressions will be equal to zero. We expect some positive alpha together with positive signs for the MinVol coefficient. Traditionally the companies with high book-to-market ratio constitute the core of the minimum volatility strategies therefore the inclusion of the Value factor should lead to a decrease in the loading on the MinVol factor. On the other hand the small cap companies are expected to have low beta and hence have high weight in the long side of the BAB factor. This should lead to a positive and statistically significant coefficient for the Size factor in regression (22).

5. Empirical results

In this section we provide the results of models outlined in Section 4. First we report the findings on the relation of alpha and beta across asset classes and analyze the basic performance of the BAB factors. In Section 5.2. we focus on the empirical influence of the TED spread on the performance of the BAB factors and report the results of models defined at the very end of Section 4.2. Last part of this Section provides evidence on the issues described in Section 4.3. The results of additional tests are reported in the Appendix.

5.1. Evidence on relation of alpha and beta and first results of the BAB factor performance

US Treasury bonds

To confirm the relative flatness of the CML we run regression of monthly returns for the seven maturity buckets of the Fama bond portfolios on the market return, and then construct the BAB factor as explained in Section 4.2. This approach is justified by the fact that the short maturity empirically coincides with low beta and is indeed true in this case as well. Table 2 provides information on the US Treasury bonds asset class and the performance of the BAB factor is reported in the last column.

The table shows monthly portfolio returns in excess of the risk free rate. The test assets are the monthly returns on the Treasury Fama Bond Portfolios and the market return is the equal-weighted average of all available bonds returns. The BAB factor is a zero-beta, long-short portfolio. Each of the assets is assigned to either high-beta or low-beta portfolio where the lowest-beta asset has the largest weight in the low-beta portfolio and vice versa. The two portfolios are rescaled to have a beta of 1 and the BAB factor is long the low-beta portfolio and shorts the high-beta portfolio. Excess returns and alpha are in monthly percent, volatility and Sharpe ratio are annualized. T-statistics are shown below in parenthesis and significance at 5% level is indicated in bold.

	P1 low beta	P2	P3	P4	P5	P6	P7 high beta	BAB factor
Maturity	1-12 M	13-24 M	25-36M	37-48 M	49-60 M	61-120 M	>120 M	
Excess return (% m)	**0.05**	**0.09**	**0.11**	**0.13**	**0.13**	**0.16**	**0.25**	**0.13**
	(5.68)	(3.93)	(3.4)	(3.13)	(2.67)	(2.61)	(2.31)	(5.06)
Alpha (% m)	**0.03**	**0.03**	**0.02**	0.01	-0.01	**-0.02**	**-0.06**	**0.13**
	(5.42)	(2.86)	(1.72)	(0.86)	(-1.40)	(-2.19)	(-1.70)	(5.29)
Beta (ex ante)	0.14	0.45	0.75	0.99	1.21	1.45	2.18	0.00
Beta (realized)	0.16	0.48	0.76	0.97	1.16	1.44	2.11	-0.05
Volatility (% y)	0.80	2.06	3.16	3.97	4.70	5.77	9.28	2.27
Sharpe ratio (y)	0.73	0.50	0.43	0.40	0.34	0.33	0.32	0.66

Table 2: US Treasury bonds (1952-2012)

We see that the portfolio of short maturity bonds recorded some abnormal return $(\alpha = 0.05\% \text{ per month})$ while this abnormal return is declining in maturity and hence in beta. Also the Sharp ratio is declining monotonically in beta, making the shortest-term bonds the most attractive for unconstrained investor. Consistently with our expectations the BAB factor realizes a high positive abnormal return $(\alpha = 0.13\% \text{ per month})$ which is statistically significant at 5% level (t-statistic of 5.29). As the BAB factor is an alpha-capturing strategy it is expected to deliver higher alpha than any of the individual assets. Its realized annual Sharpe ratio of 0.66 is lower than for the shortest maturity bucket, but that doesn't mean the predictions of our model do not hold. The model only predicts highest Sharpe ratio for portfolio with $\beta < 1$.

Corporate bonds

Next asset class considered is the corporate bond indices. Again we use monthly returns and build the portfolios based on maturity with the BAB factor being constructed on the 3-year pre-ranking period and updated every month. Table 3 reports the results of the analysis and confirms that alpha is declining in beta. The Sharpe ratio is not strictly decreasing but remains the highest for the shortest maturity bonds. The BAB factor realizes a positive abnormal return of 0.12% per month (t-statistic of 4.73) and an annual Sharpe ratio similar to the longest maturity portfolio at 0.78. On the right-hand side of Table 3 we separate the credit risk exposure by hedging away the interest rate risk.

The table shows monthly portfolio returns in excess of the risk free rate. The test assets are the monthly returns on the Corporate Bond Portfolios based on maturity and the market return is the equal-weighted average of all the available bonds returns. The BAB factor is a zero-beta, long-short portfolio. Each of the assets is assigned to either high-beta or low-beta portfolio where the lowest-beta asset has the largest weight in the low-beta portfolio and vice versa. The two portfolios are rescaled to have a beta of 1 and the BAB factor is long the low-beta portfolio and shorts the high-beta portfolio. Excess returns and alpha are in monthly percent, volatility and Sharpe ratio are annualized. T-statistics are shown below in parenthesis and significance at 5% level is indicated in bold. The Hedged panel shows results for returns of returns with hedged interest rate exposure. The long position in corporate bonds is hedged by short position in Barclay's US government bond index based on a one year rolling regression. This gives a cleaner exposure to the credit risk.

	Unhedged				BAB	Hedged				BAB
	P1	P2	P3	P4	factor	P1	P2	P3	P4	factor
Maturity	1-3 Y	1-5 Y	5-10 Y	7-10 Y		1-3 Y	1-5 Y	5-10 Y	7-10 Y	
Excess return (% m)	**0.18**	**0.22**	**0.38**	**0.38**	**0.11**	**0.11**	**0.10**	0.09	0.08	**0.08**
	(5.13)	(4.54)	(3.70)	(3.83)	(4.77)	(4.76)	(3.66)	(1.10)	(1.06)	(2.96)
Alpha (% m)	**0.03**	0.01	**-0.04**	**-0.06**	**0.12**	**0.05**	**0.03**	**-0.03**	**-0.07**	**0.07**
	(2.27)	(0.49)	(-3.63)	(-4.15)	(4.73)	(4.90)	(2.78)	(-2.57)	(-3.95)	(2.32)
Beta (ex ante)	0.59	0.83	1.39	1.51	0.00	0.71	0.89	1.31	1.37	0.00
Beta (realized)	0.61	0.84	1.38	1.49	-0.02	0.61	0.78	1.33	1.39	0.13
Volatility (% y)	2.63	3.55	5.71	5.97	1.71	1.58	1.93	3.98	3.95	1.78
Sharpe ratio (y)	0.84	0.74	0.80	0.77	0.78	0.82	0.63	0.26	0.23	0.54

Table 3: Corporate bonds, maturity (1976-2013)

We hedge the long position in the corporate bond index by an appropriate short position in Barclay's US government bond index based on a one-year rolling window and take the new market portfolio as the equal-weighted average of the hedged returns. After removing the interest rate exposure there is no a priori reason to suspect the long maturity bonds to have the

highest beta anymore. The four original maturity buckets become "random portfolios" with uncertain credit risk profile. Results from the right panel of Table 3 however reveal that the beta and maturity are still interchangeable and that the model assumptions hold also for the pure credit BAB factor. Both the abnormal return $(\alpha = 0.07\%$ per month, t-statistic of $2.32)$ and Sharpe ratio (0.54) of the pure credit BAB are lower but this can be attributed to the subtraction of part of the risk through hedging.

We also explore the corporate bonds market sorted by ratings. The safest investment assets with the highest ranking Aaa now play role of the shortest maturity bonds and beta is expected to rise with the deterioration of the rating. We test eight portfolios of ratings from Aaa to Ca-D on the basis of monthly excess returns and construct the BAB factor in the same way as for the other bond classes. In Table 4 the high beta portfolios are again associated with negative alpha but the highest abnormal return and Sharpe ratio are recorded for the Ba rating. The BAB factor has a high positive alpha of 0.45% per month but also rather high volatility due to the exposure to the Caa and Ca-D bonds via the short side of the portfolio. The worse than expected performance of high rating bonds can be caused by the beginning of the observed period for which only the four highest ratings are available.

The table shows monthly portfolio returns in excess of the risk free rate. The test assets are the monthly returns on the Corporate Bond Portfolios based on rating and the market return is the equal-weighted average of all the available bonds returns. The BAB factor is a zero-beta, long-short portfolio. Each of the assets is assigned to either high-beta or low-beta portfolio where the lowest-beta asset has the largest weight in the low-beta portfolio and vice versa. The two portfolios are rescaled to have a beta of 1 and the BAB factor is long the low-beta portfolio and shorts the high-beta portfolio. Excess returns and alpha are in monthly percent, volatility and Sharpe ratio are annualized. T-statistics are shown below in parenthesis and significance at 5% level is indicated in bold.

	P1 low beta	P2	P3	P4	P5	P6	P7	P8 high beta	BAB factor
Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-D	
Excess return (% m)	**0.21**	**0.23**	**0.25**	**0.31**	**0.49**	**0.41**	0.31	0.80	**0.53**
	(2.24)	(2.55)	(2.59)	(3.11)	(4.81)	(3.07)	(1.48)	(1.54)	(2.56)
Alpha (% m)	0.02	0.04	0.03	0.08	**0.16**	-0.03	**-0.31**	**-0.49**	**0.45**
	(0.37)	(0.64)	(0.55)	(1.51)	(3.10)	(-0.48)	(-2.35)	(-1.73)	(2.18)
Beta (ex ante)	0.66	0.70	0.76	0.82	0.83	1.04	1.52	2.44	0.00
Beta (realized)	0.62	0.68	0.75	0.81	0.81	1.06	1.52	3.06	0.23
Volatility (% y)	7.00	6.97	7.30	7.63	6.69	8.69	13.88	27.99	15.07
Sharpe ratio (y)	0.35	0.40	0.41	0.49	0.88	0.56	0.27	0.34	0.42

Table 4: Corporate bonds, rating (1973-2013)

Therefore we run the same analysis on restricted period running from 1993 to 2013 for which the returns on all the portfolios are observed. Results are presented in Table 5 below. The abnormal returns of high rating bonds increase dramatically compared to the full sample, but they are still not decreasing monotonically in rating. Portfolio of bonds with Ba rating still demonstrates the highest annual Sharpe ratio of 0.83. The BAB factor records very high abnormal returns $(\alpha = 0.70\%$ with t-statistic $1.64)$ but the excess return on the BAB factor can be caused by positive loading on the market risk as the realized beta is 0.43. This is due to

the noise in measurement of the pre-ranking betas and can be seen through the difference of ex ante and realized betas of the individual rating portfolios, namely of portfolios P1 and P8 in Table 5 that have the highest weights in BAB construction.

To account for the different periods of data availability and its impact on the performance of individual portfolios, this table presents the same analysis for period 1993-2013 for which data on all rating levels are available.

	P1 low beta	P2	P3	P4	P5	P6	P7	P8 high beta	BAB factor
Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-D	
Excess return (% m)	**0.26**	**0.31**	**0.32**	**0.38**	**0.49**	**0.40**	0.41	0.80	**0.87**
	(3.36)	(3.47)	(3.00)	(3.44)	(3.74)	(2.38)	(1.54)	(1.54)	(2.05)
Alpha (% m)	**0.19**	**0.17**	0.11	**0.13**	**0.15**	-0.04	-0.22	**-0.49**	0.70
	(2.50)	(2.19)	(1.36)	(1.82)	(2.33)	(-0.50)	(-1.38)	(-1.73)	(1.64)
Beta (ex ante)	0.35	0.46	0.58	0.69	0.84	1.03	1.53	2.52	0.00
Beta (realized)	0.17	0.33	0.50	0.59	0.80	1.04	1.50	3.06	0.43
Volatility (% y)	4.18	4.78	5.73	6.02	7.11	9.05	14.33	27.99	21.18
Sharpe ratio (y)	0.74	0.77	0.67	0.76	0.83	0.53	0.34	0.34	0.49

Table 5: Corporate bonds, rating (1993-2013)

We try to explain the outperformance of the Ba rating bonds by considering the investment grade (Baa and better) and speculative grade bonds as separate markets due to e.g. restrictions on some investors prohibiting them to invest into the speculative grade assets or limits on such investments in investors' portfolios. Should the market separation theory hold, high alpha will be associated with low beta in each of the subgroups and BAB factor will provide investment potential to capture alpha. Table A1 in Appendix II provides the results of this analysis. While this holds for the speculative grade bonds, highest grade assets have negative abnormal returns in this regression. Given the very similar properties of the investment grade bonds and the absence of relative flatness of the CML, it is not surprising that BAB factor does not provide substantial investment upside. The suspected segregation of the markets was not confirmed and the reason for the good performance of the "borderline rating" portfolios (Baa and Ba) stems from different source.

Equity indices

Frazzini and Pedersen (2011) investigate in their original BAB paper the US and international equity markets at a single stock level and find positive returns on the BAB factor for all countries covered except for Austria.[19] Their international sample covers the nineteen countries from the MSCI developed countries universe. We estimate the potential of the BAB factor on more aggregate level taking the main equity indices as our test assets. To ensure investability of this strategy we consider returns in US dollars on thirteen developed countries' equity indices and extend the analysis to another separate asset class of emerging markets with sample of eleven equity indices from countries belonging to the MSCI Emerging markets universe. We use daily data to calculate descriptive statistics for each individual asset

[19] The statistical significance varies from country to country but the results are robust.

and then create the BAB factor updated each month based on one year rolling regression while requiring at least 200 observations for the index to be included in the BAB construction. To check the alpha-beta relation with regard to the GDP-weighted market portfolio, we analyze scatter plots (Figure A1 in the Appendix) expecting a negative slope of the trend line which is the case for both developed and emerging markets. To avoid the bias stemming from individual asset beta estimation we also create low-beta and high-beta portfolios from all the available indices in the class. Each of the two portfolios includes half of the assets sorted on beta and the returns are calculated on an equal-weighted basis. The confirmation of the low-beta portfolio realizing higher alpha can be found in Figure A3 together with the other asset classes.

In Table 6 we present the results for BAB factor for four different ways of constructing the BAB factor in both the developed and emerging markets. First four columns do not make use of the shrinkage factor (i.e. no correction for outliers in factor construction) while the Base and Asyn. columns represent a base case (no correction for asynchronous trading) and asynchronous case (lag of market return was used in pre-ranking beta estimation) respectively. Regardless of the construction methodology, the BAB factor provides significant excess return and high abnormal return in both groups of countries.

The table shows monthly returns on the BAB factor in excess of the risk free rate. The test assets are the daily returns on the equity indices in US dollars from Developed and Emerging countries and the market return is the GDP-weighted average of all the available index returns in the subgroup. The BAB factor is a zero-beta, long-short portfolio updated each month. Each of the assets is assigned to either high-beta or low-beta portfolio where the lowest-beta asset has the largest weight in the low-beta portfolio and vice versa. The two portfolios are rescaled to have a beta of 1 and the BAB factor is long the low-beta portfolio and shorts the high-beta portfolio. Excess returns and alphas are in monthly percent, volatility and Sharpe ratio are annualized. T-statistics are shown below in parenthesis and significance at 5% level is indicated in bold. Base case doesn't include the lag of BAB returns for ex-ante beta estimation while the Asynchronous trading case (Asyn.) uses the first lag of BAB returns for the pre-ranking of indices. The inclusion of shrinkage factor is indicated in the first row of the table.

	Developed USD		Emerging USD		Developed USD		Emerging USD	
	Base	Asyn.	Base	Asyn.	Base	Asyn.	Base	Asyn.
Shrinkage w=0.6	no	no	no	no	yes	yes	yes	yes
Excess return (% m)	**0.86**	**0.79**	**2.40**	**2.00**	**0.44**	**0.43**	**0.96**	**0.79**
	(3.20)	(3.88)	(3.71)	(3.26)	(2.11)	(2.45)	(2.26)	(1.95)
Alpha (% m)	**0.74**	**0.67**	**2.19**	**1.80**	**0.53**	**0.45**	**1.25**	**1.02**
	(2.78)	(3.34)	(3.40)	(2.94)	(2.56)	(2.59)	(3.02)	(2.55)
Beta (realized)	0.19	0.19	0.20	0.19	-0.14	-0.04	-0.28	-0.22
Volatility (% y)	19.51	14.64	37.26	35.45	14.94	12.64	24.46	23.42
Sharpe ratio (y)	0.51	0.62	0.74	0.65	0.33	0.39	0.45	0.39

Table 6: Equity indices (1974-2013)

The use of the shrinkage factor lowers both the excess return and the volatility of the returns but the annual Sharpe ratio decreases. It also means applying less leverage as the scaling factors for both the short and long sides of the portfolio are pushed towards one. When correcting for the asynchronous trading the effect is not as large but again leads to lower excess and abnormal returns as well as lower volatility. As opposed to the shrinkage factor, the final effect on Sharpe ratio can be either negative or positive. We opt for working with the

most conservative case employing both the shrinkage factor and the asynchronous trading in constructing the BAB factor. This leads to realized betas on the factor closest to zero with almost 0.5% of monthly abnormal return on the developed equity indices BAB factor and more than 1.0% of monthly alpha for the emerging markets. However the comparison with a simple 1/n, long-only strategy (AVG_Mkt in Table A2) suggests that the conservative BAB strategy is dominated in terms of both excess returns and Sharpe ratio. We will discuss this in more detail in Section 6.

We are further interested in analyzing to what extent the abnormal returns stem from the traditional factors such as Value and Size. From the daily returns we construct monthly return series of the BAB factor and explore its performance in multifactor model framework. We use Global factor returns from Kenneth French data library and the asset class-specific market returns (GDP-weighted average). This however limits our sample period to August 1990 – June 2013 and it should be noted that the global factor returns are based on 23 developed countries, putting the applicability to the emerging markets into question. Nevertheless, we assume that returns on factors in the emerging countries were same as in the developed counterpart. Table 7 shows the results of this exercise. Firstly we note that for this period we cannot reject the hypothesis of zero abnormal returns for the emerging market equity indices even without the factors included. Secondly the realized alpha declines monotonically with added factors but remains positive even for the 4-factor model. It seems that some part of the BAB factor potential persists even after controlling for loadings on other known factors. In order to properly confirm this finding we would need to construct factor returns for precisely the same area under consideration, especially for the emerging markets. The results hold also when the market returns are taken from the Global factor returns file although the explanatory power for emerging markets is not high (not reported here).

This table shows results of the time series regressions of the monthly BAB factor returns of the equity indices asset classes on their GDP-weighted market returns and Fama-French Global factors. In each asset class the first regression includes only the market excess return as explanatory variable to obtain a basis for the abnormal return in the shortened period. In the next steps Size and Value factors are added and finaly Momentum factor in the 4-factor model. Alpha is

	Equity Indices - Developed			Equity Indices - Emerging		
	1-factor	3-factor	4-factor	1-factor	3-factor	4-factor
Alpha (% m)	**0.345**	0.266	0.192	0.473	0.252	0.173
	(2.06)	(1.57)	(1.13)	(1.00)	(0.53)	(0.36)
Market return	-0.055	-0.038	0.003	**0.119**	**0.130**	**0.129**
	(-1.44)	(-1.01)	(0.08)	(1.80)	(1.95)	(1.88)
SMB (Size)		**14.234**	4.450		**41.863**	**42.931**
		(1.83)	(0.57)		(1.9)	(1.89)
HML (Value)		**0.167**	**0.186**		**0.498**	**0.618**
		(2.34)	(2.57)		(2.48)	(3.00)
WML (Momentum)			**10.454**			**22.197**
			(2.43)			(1.83)

Table 7: Abnormal returns in multifactor models

Futures

In the last part of this section we examine the performance of the BAB factor in the derivatives asset class, namely the futures on commodities, equity indices and international treasury bonds. We consider the futures classes separately and point out that only the commodities data are all in US dollars. The daily data on treasury bonds and both developed and emerging market equity index futures are in local currencies, making the implications for investment decision rather unclear. Performance of the alternatively constructed BAB factors is shown in Table 8, however we cannot reject the hypothesis of zero abnormal returns besides couple of exceptions. Moreover, the scatter plot for commodities in Figure 6 suggests that the CML is not flat as expected. This is not confirmed on the low-beta/high-beta portfolio level but Figure 7 leads to suggestion that the potential of the BAB factor will be hard to capture in the treasury bond futures and emerging market EI futures as the alpha for high-beta portfolio is higher than for the low-beta counterpart.

The two tables show monthly returns on the BAB factor in excess of the risk free rate. The test assets are the daily returns on the different futures in the sample in USD for commodities and in local currencies for equity indices and treasury bonds. The market returns are GSCI returns for commodities, GDP-weighted average return for equity indices and equal-weighted average return for treasuries. The BAB factor is a zero-beta, long-short portfolio updated each month. Each of the assets is assigned to either high-beta or low-beta portfolio on basis of the previous one-year beta where the lowest-beta asset has the largest weight in the low-beta portfolio and vice versa. The two portfolios are rescaled to have a beta of 1 and the BAB factor is long the low-beta portfolio and shorts the high-beta portfolio. The top table shows results for BAB factor constructed without the shrinkage factor in pre-ranking estimation and the bottom table employs the shrinkage factor w=0.6 to correct for outliers. Excess returns and alphas are in monthly percent, volatility and Sharpe ratio are annualized. T-statistics are shown below in parenthesis and significance at 5% level is indicated in bold. Base case doesn't include the lag of BAB returns for ex-ante beta estimation while the Asynchronous trading case (Asyn.) uses the first lag of BAB returns for the pre-ranking of indices.

	Commodities		Equity Ind. Developed		Equity Ind. Emerging		Country Bonds	
	Base	Asyn.	Base	Asyn.	Base	Asyn.	Base	Asyn.
Excess return (% m)	9.39	11.19	0.32	**0.46**	0.87	0.66	**0.35**	0.11
	(0.27)	(0.47)	(1.13)	(2.05)	(1.44)	(1.24)	(1.93)	(0.79)
Alpha (% m)	9.07	11.18	0.28	**0.40**	0.66	0.40	**0.31**	0.10
	(0.26)	(0.47)	(1.00)	(1.81)	(1.10)	(0.77)	(1.73)	(0.76)
Beta (realized)	1.28	0.05	0.08	0.13	0.20	0.24	0.37	0.04
Volatility (% y)	1930.94	1329.73	15.84	12.72	26.82	23.53	10.92	8.18
Sharpe ratio (y)	0.06	0.10	0.23	0.42	0.37	0.32	0.36	0.15

	Commodities		Equity Ind. Developed		Equity Ind. Emerging		Country Bonds	
	Base	Asyn.	Base	Asyn.	Base	Asyn.	Base	Asyn.
Excess return (% m)	0.10	0.34	0.21	0.38	0.34	0.30	-0.03	-0.03
	(0.21)	(0.7)	(0.80)	(1.64)	(0.68)	(0.64)	(-0.32)	(-0.30)
Alpha (% m)	0.24	0.47	0.27	0.39	0.47	0.36	0.03	0.04
	(0.56)	(1.04)	(1.06)	(1.68)	(0.96)	(0.78)	(0.41)	(0.50)
Beta (realized)	0.31	0.48	-0.15	-0.02	-0.13	-0.06	-0.60	-0.65
Volatility (% y)	27.11	27.29	14.57	13.20	21.80	20.57	5.40	5.45
Sharpe ratio (y)	0.04	0.14	0.16	0.33	0.18	0.17	-0.06	-0.06

Table 8: Futures: Commodities, equity indices, treasury bonds (1989-2013)

5.2. Effects of liquidity constraints and changing funding conditions

To test the predictions of our third proposition on the influence of the liquidity and funding conditions and the tightening thereof, we employ time series regression framework described by equations (20) and (21). We proceed as described in Section 4.2, running four models for each BAB factor. The dependent variable is a monthly return on the BAB factor that is constructed without the shrinkage and without the market lag for US Treasury and corporate bonds. For futures on treasury bonds we employ the lag to account for asynchronous trading and for remaining asset classes we use both the shrinkage factor and the lag. We first explore the autoregressive properties of the BAB factor return series in order to determine possible signature and an order of lags to be included as control variables in the extended models. Autocorrelation and partial autocorrelation functions (ACF, PACF respectively) are plotted in Figure A4 of Appendix, but there is no apparent pattern across the asset classes and Durbin-Watson statistics close to 2 confirm that autocorrelation should not be a problem. Nevertheless, due to the relatively high values for third lag in bond classes based on maturity we include three lags of the BAB return as control variables. To the best of author's knowledge there is no clear reason why such a quarterly effect should be present; and including only one or two lags of the BAB return as control variables doesn't alter the results (not reported here).

In the first specification of the time series regression including only the contemporaneous TED Spread we expect a negative sign since in times of funding illiquidity the high-beta assets fare relatively better than the low-beta assets. In the second specification with the lagged value of TED spread and the change in TED spread as explanatory variables we expect, in line with equation (18), that the lagged level will predict BAB returns positively while the sign of coefficient for change in TED spread will be negative. The theory tells us that the tightening of the constraints leads to contemporaneous loss for the BAB factor returns. Regarding the control variables in extended models we expect the Beta spread to have positive sign stemming from equation (17) and the lagged values of BAB return should perform accordingly to the sign identified by the respective PACF. Following Tables 9 and 10 report the results of the regression where for each asset class the two basic models are reported first, followed by the extended specifications. We find mixed results depending on particular asset class. Although statistical significance of the coefficients is seldom, we can infer some properties of the BAB factors. In the first specification bond asset classes based on maturity show positive relation of TED spread and the contemporaneous BAB return while corporate bonds sorted on rating, equity based asset classes and commodity futures report negative sign as expected.[20]

[20] After inclusion of control variables, however, corporate bonds sorted on rating realize positive coefficient for the contemporaneous TED spread while treasury bonds futures turn from positive to negative coefficient.

This table shows results of the time series regressions of the fixed income securities-based BAB factors on the funding conditions. For each asset class 4 regressions are carried out: first has only the contemporaneous level of TED Spread as explanatory variable and the second the lagged value of TED Spread and the contemporaneous change in TED Spread. In the next two regressions several control variables are included in the first two specifications respectively. These variables include the market return, beta spread as defined in the main body of the thesis and 3 lags of the BAB return. T-statistics are in parenthesis and statistical significance at 5% level is indicated in bold.

	Treasury Bonds				Corporate Bonds - Rating			Corp. Bonds - Maturity Unhedged				Corp. Bonds - Maturity Hedged				Futures - International Bonds			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
TED Spread	**0.175**		**0.155**		-0.591		0.046		0.064		0.077		0.000		0.039		0.020		-0.069	
	(1.95)		(1.71)		(-0.93)		(0.07)		(0.94)		(1.00)		(-0.004)		(0.53)		(0.07)		(-0.26)	
Lag. TED Spread		0.115		0.085		-0.022		0.432		0.081		0.101		0.005		0.043		-0.091		0.085
		(1.253)		(0.91)		(-0.04)		(0.64)		(1.14)		(1.29)		(0.07)		(0.58)		(-0.32)		(0.91)
Δ TED Spread		**0.637**		**0.621**		**-5.201**		**-3.647**		-0.057		-0.132		-0.038		-0.022		0.891		**0.621**
		(3.28)		(3.29)		(-3.81)		(-2.59)		(-0.38)		(-0.80)		(-0.24)		(-0.12)		(1.48)		(3.29)
Market return			**-0.107**	**-0.095**			**0.348**	0.234			-0.027	-0.042			**0.080**	0.073			**0.365**	**-0.095**
			(-2.90)	(-2.59)			(2.59)	(1.69)			(-1.00)	(-1.43)			(1.87)	(1.54)			(3.61)	(-2.59)
Beta Spread			0.000	0.000			0.002	**0.003**			0.001	0.001			0.001	0.001			0.001	-0.040
			(-1.03)	(-1.18)			(1.40)	(1.75)			(0.87)	(0.98)			(1.67)	(1.66)			(0.63)	(-1.18)
Lag1 BAB return			0.063	0.077			0.082	0.065			-0.039	-0.051			**-0.169**	**-0.173**			0.009	0.077
			(1.17)	(1.42)			(1.44)	(1.15)			(-0.71)	(-0.93)			(-3.06)	(-3.07)			(0.17)	(1.42)
Lag2 BAB return			**-0.104**	**-0.106**			**-0.297**	**-0.286**			-0.022	-0.009			0.040	0.045			-0.037	**-0.106**
			(-1.92)	(-1.97)			(-5.55)	(-5.38)			(-0.41)	(-0.15)			(0.73)	(0.79)			(-0.68)	(-1.97)
Lag3 BAB return			**0.180**	**0.190**			0.084	0.073			**0.204**	**0.200**			**0.176**	**0.177**			-0.022	**0.190**
			(3.31)	(3.52)			(1.48)	(1.30)			(3.74)	(3.67)			(3.16)	(3.17)			(-0.40)	(3.52)

Table 9: Influence of changing liquidity and funding conditions – Bonds and futures on international treasury bonds (1986-2013)

This table shows results of the time series regressions of the equity-based and commodities BAB factors on the funding conditions. For each asset class 4 regressions are carried out: first has only the contemporaneous level of TED Spread as explanatory variable and the second the lagged value of TED Spread and the contemporaneous change in TED Spread. In the next two regressions several control variables are included in the first two specifications respectively. These variables include the market return, beta spread as defined in the main body of the thesis and 3 lags of the BAB return. T-statistics are in parenthesis and statistical significance at 5% level is indicated in bold.

	Equity Indices - Developed				Equity Indices - Emerging				Futures - EID				Futures - EIE				Futures - Commodities			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
TED Spread	-0.420		-0.300		-1.553		-0.931		-0.418		-0.575		**-1.514**		-0.709		-1.826		-1.417	
	(-0.91)		(-0.65)		(-1.31)		(-0.76)		(-0.87)		(-1.14)		(-1.79)		(-0.62)		(-1.47)		(-1.64)	
Lag. TED Spread		-0.281		-0.186		-0.921		-0.485		-0.399		-0.501		-1.096		-1.146		-1.671		**-3.343**
		(-0.59)		(-0.39)		(-0.76)		(-0.39)		(-0.80)		(-0.96)		(-1.26)		(-1.29)		(-0.13)		(-2.69)
Δ TED Spread		-1.444		-1.215		**-6.361**		**-4.913**		-0.546		-1.025		**-4.298**		**-3.336**		-2.837		**-6.419**
		(-1.43)		(-1.15)		(-2.58)		(-1.92)		(-0.56)		(-1.06)		(-2.53)		(-1.91)		(-1.14)		(-2.71)
Market return			0.019	0.009			**0.140**	0.113			-0.064	-0.068			**0.627**	**0.170**			**0.195**	**-0.563**
			(0.41)	(0.19)			(2.09)	(1.66)			(-1.36)	(-1.43)			(6.91)	(2.42)			(2.89)	(-7.29)
Beta Spread			**0.044**	**0.044**			0.006	0.007			-0.010	-0.009			0.017	0.045			**0.047**	0.006
			(2.99)	(3.00)			(0.30)	(0.32)			(-0.54)	(-0.49)			(1.18)	(1.68)			(1.74)	(0.51)
L1 BAB return			-0.006	-0.002			**0.157**	**0.161**			-0.016	-0.014			-0.095	-0.086			-0.087	0.027
			(-0.11)	(-0.03)			(2.66)	(2.73)			(-0.27)	(-0.23)			(-1.37)	(-1.13)			(-1.13)	(0.49)
L2 BAB return			0.023	0.021			0.051	0.045			-0.061	-0.061			**0.252**	0.044			0.039	0.028
			(0.41)	(0.39)			(0.85)	(0.76)			(-1.04)	(-1.04)			(3.77)	(0.58)			(0.52)	(0.50)
L3 BAB return			-0.033	-0.024			0.008	0.008			0.090	0.092			-0.053	-0.056			-0.065	0.026
			(-0.60)	(-0.43)			(0.13)	(0.14)			(1.55)	(1.57)			(-0.78)	(-0.74)			(-0.87)	(0.46)

Table 10: Influence of changing liquidity and funding conditions – Equity indices, futures on equity indices and commodity futures (1986-2013)

For the second specification the Treasury bonds stand out with both coefficients positive. Corporate bonds conform to the predictions of the model with negative coefficient for the innovation in the TED Spread and positive relation of the lagged level of TED spread. In case of equity indices, commodities futures and equity indices futures both the coefficients have a negative sign. As Frazzini and Pedersen (2011, p. 27) point out, this would be in line with the model if high TED spread was representing the worsening of funding conditions instead of a tightness of investors' constraints.[21] The strictly positive relation of funding tightness proxies with the BAB returns of Treasury bonds could be explained by the unique position of Treasuries in the financial markets. The so called flight-to-quality during the periods of financial distress can be the underlying reason behind these unexpected results.

The coefficients of Beta spread control variable are positive in most cases, although exceptions do exist. In general results from this section suggest that the BAB factors do behave differently in changing economic conditions depending on asset class. Treasury bonds, corporate bonds and equity based assets can be distinguished as three groups that react to changes in TED Spread in different way.

5.3. Additional properties of the BAB factors and loadings on Minimum Volatility index

In the previous sections we have established the BAB factor as an alpha capturing strategy that provides positive average excess returns across asset classes. However, consistent responses of the various BAB factors to the changing liquidity constraints have not been found. To understand the possible connections between asset classes, we calculate the correlation structure of monthly BAB factor returns and explore the cumulative returns on BAB factors over time with varying investment horizon.

From theory we would expect the BAB returns to be positively correlated across asset classes through the changing conditions on financial markets. On the other hand the results of Section 5.2. suggest that it is not the case in reality. Correlation structures of all the estimated BAB factor returns are presented in Table 11. In the first table the correlations of monthly returns are calculated from all available data ranging from 1975 to 2013. Only values above the arbitrary level of 0.2 are among different bond classes, which is not surprising for the datasets sorted on maturity, but the negative correlation of Treasury bonds with corporate bonds sorted on rating may provide significant potential for diversification. It may, however, be the case that the corporate bonds indices may have inverse maturity or duration structure when compared to Treasury bonds. That is, Aaa rated bond index may have a long maturity and the maturity will decline when the rating worsens. Then the BAB factors of Treasury bonds and corporate bonds sorted on ratings would load on the opposite sides of the interest

[21] Positive change in the TED spread would then be interpreted as worsening conditions at an increasing pace.

rate risk which would lead to the negative correlation. Before exploiting this negative correlation the maturity and duration structure of the indices has to be taken into account.

Correlation structure of the monthly BAB factor returns over the full sample period 1975-2013 (time period covered differs per asset class, the mentioned longest period holds for Treasury bonds and Developed market equity indices). Correlation larger (absolute value) than 0.2 is indicated in bold .

	TB	CBR	CBMU	CBMH	EID	EIE	F_COM	F_EID	F_EIE	F_Bonds
TB	**1.00**									
CBR	**-0.24**	**1.00**								
CBMU	**0.60**	-0.13	**1.00**							
CBMH	**0.51**	-0.13	**0.81**	**1.00**						
EID	-0.02	-0.01	-0.02	-0.04	**1.00**					
EIE	0.09	-0.06	0.08	0.10	0.02	**1.00**				
F_COM	0.03	0.00	0.03	-0.02	-0.08	-0.02	**1.00**			
F_EID	-0.05	-0.06	-0.06	0.04	0.03	0.03	-0.02	**1.00**		
F_EIE	-0.01	0.07	-0.03	-0.03	0.02	0.19	0.03	0.15	**1.00**	
F_Bonds	-0.01	0.02	0.00	0.03	0.02	-0.04	0.03	**0.28**	0.05	**1.00**

Correlation structure of the monthly BAB factor returns over the full sample period 1995-2013 (time period covered differs per asset class, the mentioned longest period holds for Treasury bonds and Developed market equity indices). Correlation larger (absolute value) than 0.2 is indicated in bold.

	TB	CBR	CBMU	CBMH	EID	EIE	F_COM	F_EID	F_EIE	F_Bonds
TB	**1.00**									
CBR	**-0.34**	**1.00**								
CBMU	**0.76**	-0.14	**1.00**							
CBMH	**0.63**	-0.15	**0.83**	**1.00**						
EID	-0.14	0.09	-0.19	-0.20	**1.00**					
EIE	-0.06	0.02	-0.05	-0.08	0.03	**1.00**				
F_COM	-0.01	-0.03	0.04	0.05	-0.11	0.01	**1.00**			
F_EID	-0.05	0.02	-0.14	-0.10	0.11	0.02	-0.09	**1.00**		
F_EIE	0.02	-0.04	0.00	0.06	0.02	0.19	0.03	0.15	**1.00**	
F_Bonds	**-0.20**	0.11	-0.20	-0.17	-0.01	0.09	-0.04	0.08	0.05	**1.00**

Correlation structure of the monthly BAB factor returns over the full sample period 2007-2013 (time period covered differs per asset class, the mentioned longest period holds for Treasury bonds and Developed market equity indices). Correlation larger (absolute value) than 0.2 is indicated in bold.

	TB	CBR	CBMU	CBMH	EID	EIE	F_COM	F_EID	F_EIE	F_Bonds
TB	**1.00**									
CBR	**-0.46**	**1.00**								
CBMU	**0.71**	-0.16	**1.00**							
CBMH	**0.62**	-0.19	**0.85**	**1.00**						
EID	**-0.32**	**0.32**	**-0.31**	**-0.42**	**1.00**					
EIE	**-0.21**	0.10	**-0.31**	**-0.39**	**0.24**	**1.00**				
F_COM	0.16	0.13	**0.22**	0.19	**-0.23**	-0.16	**1.00**			
F_EID	-0.12	-0.11	**-0.25**	-0.17	-0.02	0.16	**-0.25**	**1.00**		
F_EIE	0.04	0.01	-0.03	0.08	-0.01	0.04	-0.02	**0.31**	**1.00**	
F_Bonds	**-0.30**	0.04	**-0.31**	**-0.22**	0.11	0.04	-0.06	0.10	-0.01	**1.00**

Table 11: Correlation structure of BAB returns over different periods

One of the explanations for such a low correlation of returns on the BAB factors across asset classes might lie in different levels of constraints in the various markets. If the leverage constraints across markets differ vastly and the conditions change unevenly, the realized returns on BAB factors will not be related. It is safe to assume that most investors use diversification and are present in most of the available markets, therefore the leverage or funding constraints should mainly origin from regulatory authorities. Including a proxy for regulatory requirements that lead to investor constraints could be then an interesting direction for further research. We hypothesize that in times of financial distress the budget constraints become binding in more markets for more investors leading to a co-movement in the BAB returns. We calculate correlations for BAB returns for two shorter periods, namely years 1995-2013 that includes the Asian crisis of 1997-1998, and then focus on the recent developments between years 2007 and 2013. The middle part of Table 11 shows virtually no change as compared to the full sample estimates and the possible positive (or negative) correlation in the crisis periods is offset by negative (positive) correlation in the periods of boom. Finally the bottom part of the table confirms our expectation of more correlation in times of financial distress. We can see that instead of positive correlation of the BAB factor returns due to the stricter conditions in financial markets in general the bonds are positively correlated with each other and negatively correlated with the equity indices where corporate bonds indices sorted on ratings should be considered as an equity based asset class due to the underlying risks of individual companies. This could mean that the conditions actually change differently in the individual markets and we would need an asset-specific proxy for funding conditions in Section 5.2. As it is, the returns on BAB factors have same correlation structure during the times of crises as the underlying assets.

In Figure A5 we plot cumulative returns on the BAB factors for different holding periods over time. At each month the realized cumulative returns over the previous 1, 5, 20 or more years is recorded. Once again it can be seen that no correlation is apparent from the graphs and that the realized cumulative returns are very volatile especially for the 5-year holding period. Since we are more interested in the longer-horizon performance it appears that in case of 20-year holding period[22] some level of stability can be found in the bonds asset class. When plotting the returns of bond portfolios sorted on maturity in more detail we find that the 20-year cumulative returns for US Treasury bonds fluctuate between about 20% and 60%, quite wide dispersion for institutional investors. Longer data series would be necessary to perform more tests but the volatility of cumulative returns even for longer investment horizons of 20 years signals no guarantee for good performance of the strategy.

Last, but not least, the Table 12 below reports the results of the analysis of connection of the BAB strategy with the Minimum Volatility approach. On a short sample of almost twenty three most recent years we confirm that the BAB returns of the equity based asset

[22] The composition of the factor in this case is still updated every month, holding period can be interchanged with investment horizon here.

classes load positively on the Minimum Volatility index. For the developed equity indices the abnormal return of 0.334% per month over the period under observation reduces to only 0.137% per month after including the Minimum Volatility index and drops down to mere 0.037% per month in the full model featuring Value, Size and Momentum factors. Impact of the addition of Minimum Volatility index into the analysis, namely the substantial reduction of alpha, is similar in equity indices of emerging markets and in the futures of developed equity indices. For futures and emerging markets the interpretation should be careful due to the previously mentioned issues of local currencies and the factor returns originating from developed countries only. From this analysis we infer that the long-only, "smart beta" Minimum Volatility strategy captures from main part the same abnormal returns in the equity markets as does our BAB approach.

This table shows results of the time series regressions of the monthly BAB factor returns of the equity based asset classes on the MSCI Global Minimum Volatility Index and Fama-French Global factors. In each asset class the first regression includes only the market excess return and excess return on the Minimum Volatility Index as explanatory variables, in the next steps Size and Value factors are added and finaly Momentum factor in the 5-factor model. Alpha is the intercept of the regression and in monthly percent. T-statistics are in parenthesis and statistical significance at 5% level is indicated in bold.

	Equity Indices - Developed				Equity Indices - Emerging				Futures - EID				Futures - EIE			
	1-factor	2-factor	4-factor	5-factor	1-factor	2-factor	4-factor	5-factor	1-factor	2-factor	4-factor	5-factor	1-factor	2-factor	4-factor	5-factor
Alpha (% m)	**0.334**	0.137	0.085	0.037	0.577	0.281	0.103	0.142	**0.377**	0.227	0.198	0.112	0.264	0.231	0.135	0.114
	(2.01)	(0.84)	(0.52)	(0.22)	(1.22)	(0.58)	(0.22)	(0.29)	(2.12)	(1.27)	(1.11)	(0.92)	(0.70)	(0.60)	(0.35)	(0.29)
Market return	-0.056	**-0.387**	**-0.412**	**-0.370**	0.099	**-0.398**	-0.393	-0.335	-0.034	**-0.289**	**-0.363**	**-0.319**	**0.243**	0.180	0.022	0.035
	(-1.51)	(-5.16)	(-5.11)	(-4.61)	(0.94)	(-1.81)	(-1.66)	(-1.4)	(-0.85)	(-3.52)	(-4.12)	(-3.56)	(3.05)	(1.18)	(0.13)	(0.21)
MinVol return		**0.491**	**0.539**	**0.543**		**0.736**	**0.780**	**0.630**		**0.377**	**0.487**	**0.492**		0.100	0.312	0.306
		(5.01)	(5.13)	(5.24)		(2.56)	(2.53)	(2.04)		(3.53)	(4.24)	(4.27)		(0.48)	(1.39)	(1.35)
SMB (Size)			**0.224**	0.128			**0.604**	**0.592**			**0.239**	**0.167**			**0.468**	**0.452**
			(2.94)	(1.66)			(2.70)	(2.59)			(2.87)	(1.96)			(2.63)	(2.44)
HML (Value)			0.051	0.058			0.314	**0.428**			-0.063	-0.038			-0.141	-0.127
			(0.71)	(0.80)			(1.49)	(1.96)			(-0.80)	(-0.47)			(-0.94)	(-0.81)
WML (Momentum)				**0.083**				0.166				**0.096**				0.028
				(2.03)				(1.36)				(2.11)				(0.32)

Table 12: Equity based asset class BAB factor returns and loading on the Minimum Volatility index and Fama-French factors (1990-2013)

6. Discussion and implications for pension funds

Investors worldwide look for new investment opportunities and strategies. The Betting Against Beta approach delivers significant abnormal returns across asset classes and is based on simple but sound theoretical framework. Several issues however exist with the implementation of real-life strategies based on the model. We have already reported the high volatility of cumulative BAB returns over 20-years investment horizon and unclear impact of changing liquidity conditions.

Another issue mentioned earlier is the impact of actual construction of the BAB factor. Even if we decide to include the shrinkage factor which is not strictly necessary, the number of lags used for the correction of asynchronous trading can play an important role as well. In Table 13 we report the results of BAB factors constructed using the shrinkage multiplier $w = 0.6$ and lags of market return in the pre-ranking beta construction ranging from 0 to 5. The impact on developed equity indices class is not substantial although Sharpe ratio fluctuates between 0.31 and 0.39 and monthly excess return can differ by up to 0.1% per month.

Equity Indices - Developed							
Lag	0	1	2	3	4	5	Avg_Mkt
Excess return (% m)	0.44	0.43	0.35	0.39	0.34	0.34	0.64
	(2.11)	(2.45)	(2.00)	(2.26)	(1.99)	(1.99)	(3.28)
Alpha (% m)	0.53	0.45	0.40	0.42	0.35	0.33	0.12
	(2.56)	(2.59)	(2.27)	(2.44)	(2.04)	(1.93)	(1.39)
Beta (realized)	-0.14	-0.04	-0.07	-0.05	-0.02	0.02	0.98
Volatility (% y)	14.94	12.64	12.69	12.42	12.31	12.46	14.31
Sharpe ratio (y)	0.33	0.39	0.32	0.36	0.31	0.32	0.51

Commodities							
Lag	0	1	2	3	4	5	Avg_Mkt
Excess return (% m)	0.10	0.34	0.68	0.61	0.19	0.24	0.22
	(0.21)	(0.70)	(1.31)	(1.13)	(0.35)	(0.43)	(0.97)
Alpha (% m)	0.24	0.47	0.79	0.72	0.30	0.33	0.07
	(0.56)	(1.04)	(1.63)	(1.39)	(0.57)	(0.63)	(0.49)
Beta (realized)	0.31	0.48	-0.45	-0.42	-0.41	-0.38	0.48
Volatility (% y)	27.11	27.29	28.95	30.50	30.48	31.01	12.88
Sharpe ratio (y)	0.04	0.14	0.27	0.23	0.07	0.09	0.19

Table 13: Impact of including more lags in ex-ante beta estimation on BAB factor performance (Shrinkage factor applied in all cases)

More profound differences exist in the highly liquid commodities futures. Depending on the number of lags the monthly excess return on the factor can take on values ranging from 0.10% to 0.68% and Sharpe ratio covers the band of 0.04to 0.27. Moreover, we can see from Table 13 that all the equity index BAB factors and most of their commodity counterparts are dominated in the sense of Sharpe ratio by simple long-only, 1/n strategies (Avg_Mkt column) that invest equal weight in all the available assets. On the other hand the very low or even negative correlations of the BAB factor returns provide opportunity to use basic diversification with individual BAB factor being considered as investment assets.

We have also seen in Section 5.3. that the BAB strategy should be carefully considered in case the investor already follows a Minimum Volatility strategy. Combination of the two without prior consideration might lead to unnecessary overlay costs. Sophisticated overlay management systems are usually employed by institutional investors to avoid such situations. What cannot be avoided are the transaction costs, in our case stemming from the need to update the BAB factor composition. Table A3 in the Appendix V shows performance of the equity indices BAB factors for lower frequency of portfolio rebalancing. Return numbers actually improve when we update the portfolio on yearly basis instead of each month, but when we extend the updating period to eighteen months all excess returns, abnormal returns and Sharpe ratios plummet for all the BAB factor specifications. The chance of deteriorating BAB factor performance in case of less frequent rebalancing increases with volatility of the assets in the class. For Treasury bonds that are very stable over time the problem of high transaction costs due to frequent rebalancing would then shift to the necessary rolling over of the positions in maturing bonds.

As was pointed out in the theoretical setup of the model, the BAB factor is a market neutral portfolio. Due to the unequal scaling of the two sides of the portfolio the factor is not dollar neutral though. The larger leveraged position in low alpha assets is financed by short selling the risk free asset while this short position cannot be fully covered from the short sale of deleveraged high-beta assets resulting in net short position in the risk-free asset. Due to the substantial leverage embedded in the strategy the implementation for pension funds might not be easy. Table A4 in the Appendix shows the levered and delevered investment positions including the net position in the risk-free rate. The amounts of leverage necessary differ substantially with the maximum reaching almost 300 percent for US Treasury bonds. Although investments into hedge funds and similar vehicles can achieve the required exposure, policy changes allowing the pension funds to lever and short-sell in the fixed income securities would make the BAB factor strategy more appealing as the performance of treasuries BAB factor has been stable over long horizons.

7. Conclusion

Recent developments in the financial markets call for novelty approaches regarding investment strategies. Under a realistic assumption of liquidity constrained investors one can achieve abnormal returns by focusing on low-beta assets because constrained individuals are willing to pay premium for riskier assets that provide higher return. Using an OLG model with liquidity-constrained agents we confirm this flatness of the Security Market Line for markets of US Treasury and corporate bonds as well as for equity indices in both developed and emerging markets. Futures asset class on the other hand gives mixed results. We have then assessed the potential of an alpha-capturing long-short strategy of Frazzini and Pedersen (2011) called the Betting Against Beta factor (BAB). These zero-beta portfolios that are long in the low-beta assets leveraged to beta of one and short in the high-beta assets deleveraged to beta of one provide significant excess returns across the above mentioned asset classes. In case of equity indices the positive excess returns persist also when controlled for value, size and momentum factors. We were, however, unable to identify consistent common responses to changing market conditions across the BAB factors. Tightening of the liquidity constraints seems to affect differently the Treasury bonds, corporate bonds and equity based asset classes. From this result we suspected low correlation of returns on the individual BAB factors which is confirmed later on. While it provides potential for diversification in portfolio constructed from multiple BAB factors we find that correlations increase in times of distress such as the current financial crisis.

Before implementing the approach for real-life investors several aspects have to be taken into account. In our analysis we disregard the transaction costs and rely on asset class specific market portfolios The cumulative returns over a period of 20 years are quite volatile and much longer time series of data would be necessary to quantify performance of the BAB factors over investment horizons relevant for pension funds, e.g. fifty or more years. The recent surge of corporate bonds sorted on ratings is worth further investigation namely including the maturity and duration structure. BAB factor in the Treasury bonds class is attractive for its relative stability and can on average add about 1.5% of abnormal annual return. On the other hand the positive results of equity indices in emerging markets are driven by the extremely positive performance in the early 1990s and last twenty years would record a loss for the factor. Two additional reasons exist for preferring the fixed income asset classes as a target market. The BAB factor captures same alpha as the Minimum Volatility strategy that is recently being employed among investors and the BAB factor would therefore provide limited upside should the Minimum Volatility be already in place. Second argument concerns the level of leverage included in the construction of the BAB factor. Since some institutional investors such as pension funds cannot take on leverage directly this strategy could be carried out through investments into hedge funds or similar vehicles. Such investments are under

increasing supervision and governed in the Netherlands by the Prudent Person Rule, however the regulations are not so strict for investments into treasury bonds.

The fundamental research can be extended to other asset classes such as single-stock level in emerging equity markets, and other derivatives besides futures. Moreover, all the work until today concerning the BAB factor has been so far carried out in the US dollars. Dutch pension funds face liabilities in Euros, therefore we would like to explore the potential of the BAB factors in Euros in the future. After confirming that the BAB factor performs well in Equity indices asset class even with annual instead of monthly rebalancing, more detailed research into embedded transaction costs of the strategy would be of major interest, assessing either the level of transaction costs that would fully cancel the upside potential of the strategy or finding the optimal rebalancing frequency depending on the level of transaction costs. So while the BAB factor strategy provides opportunities to capture alpha for unconstrained investors, more research has to be done before undertaking the strategy in real life.

References

1. Acharya, V. V. and L. H. Pedersen (2005): "Asset Pricing with Liquidity Risk", *Journal of Financial Economics, 77, 375-410*
2. Ang, A., W.N. Goetzman and S. M. Schaefer (2009): "Evaluation of Active Management of the Norwegian Government Pension Fund", *available at http://www.regjeringen.no/upload/FIN/Statens%20pensjonsfond/rapporter/AGS%20Report.pdf*
3. Asness, C., A. Frazzini and L. H. Pedersen (2012): "Leverage Aversion and Risk Parity", *Financial Analysts Journal, 68 (1), 47-59*
4. Asness, C., T. Moskowitz and L.H. Pedersen (2009): "Value and Momentum Everywhere", *AFA 2010 Atlanta Meetings Paper*
   Available at SSRN: http://ssrn.com/abstract=1363476
5. Banz, R. W. (1981): "The Relationship between Return and Market Value of Common Stocks", *Journal of Financial Economics, 9, 3-18*
6. Black, F. (1972): "Capital market equilibrium with restricted borrowing", *The Journal of Business, 45 (3), 444-455*
7. Black, F. (1993): "Beta and Return", *Journal of Portfolio Management, 20 (1), 8-18*
8. Black, F., M. C. Jensen and M. Scholes (1972): "The Capital Asset Pricing Model: Some Empirical Tests", *In Michael C. Jensen (ed.), Studies in the Theory of Capital Markets, Praeger Publishers Inc., New York, 79-121*
   Available at SSRN: http://ssrn.com/abstract=908569
9. Blitz, D. (2012): "Strategic Allocation to Premiums in the Equity Market", *The Journal of Index Investing, Spring 2012, 2 (4), 42-49*
10. Blitz, D. (2012a): "How Smart is "Smart Beta" investing?", *Robeco Whitepaper for Institutional investors, December 2012*
11. Blitz, D., J. Huij and T. Steenkamp (2012): "Factor Investing – Collected Robeco Articles", *Robeco 2012*
12. Carhart, M. (1997): "On Persistence of Mutual Fund Performance", *Journal of Finance, 52, 57-82*
13. Callan Associates (2010): "The Risk Parity Approach to Asset Allocation", *Callan Investments Institute Research, February 2010*
14. Fama, E. F. and K. R. French (1992): "The Cross-Section of Expected Stock Returns", *Journal of Finance, 47 (2), 427-465*
15. Fama, E. F. and K. R. French (1996): "Multifactor Explanations of Asset Pricing Anomalies", *Journal of Finance, 51, 51-84*

16. Fama, E. F. and K. R. French (2003): “The CAPM: Theory and Evidence”, *CRSP Working Paper, 550*
17. Fama, E. F. and J. D. MacBeth (1973): “Risk, Return, and Equilibrium: Empirical Tests”, *Journal of Political Economy, 81 (3), 607-636*
18. Friend, I. and M. Blume (1970): “Measurement of Portfolio Performance under Uncertainty”, *American Economic Review, 60 (4), 561-575*
19. Frazzini, A. and L. H. Pedersen (2011): “Betting Against Beta”, *Swiss Finance Institute Research Paper, No. 12-17*
Available at SSRN: http://ssrn.com/abstract=2049939
20. Grinold, R. C. (1993): ”Is Beta Dead Again?”, *Financial Analysts Journal, 49 (4), 28-34*
21. Huij X. and Y. Verbeek (2007): “On the Use of Multifactor Models to Evaluate Mutual Fund Performance”, *EFA 2006 Meetings Paper; ERIM Research Paper Series.*
Available at SSRN: http://ssrn.com/abstract=906723
22. Jegadeesh, N. and S. Titman (1993): “Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency”, *Journal of Finance, 48 (1), 65-91*
23. Jensen, M. C. (1968): “The Performance of Mutual Funds in the Period 1945-1964”, *Journal of Finance, 23 (2), 389-416*
24. Lintner, J. (1965): “The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets”, *Review of Economics and Statistics, 47 (1), 13-37*
25. Markowitz, H. M. (1952): “Portfolio Selection”, *Journal of Finance, 7 (1), 77-91*
26. Merton, R. C. (1973): “An Intertemporal Capital Asset Pricing Model”, *Econometrica, 41 (5), 867-887*
27. Moskowitz, T., Y. H. Ooi and L. H. Pedersen (2012): “Time Series Momentum”, *Journal of Financial Economics, 104 (2), 228-250*
28. Mossin, J. (1966): "Equilibrium in a Capital Asset Market", *Econometrica, 34 (4), 768–783*
29. Novak J. and P. Petr (2010): “CAPM Beta, Size, Book-to-Market, and Momentum in Realized Stock Returns”, *Czech Journal of Economics and Finance, 60 (5), 447-460*
30. Rosenberg, B. (1981): “The Capital Asset Pricing Model and the Market Model”, *Journal of Portfolio Management, 7 (2), 5-16*
31. Ross, S. A. (1976): “The Arbitrage Theory of Capital Asset Pricing”, *Journal of Economic Theory, 13, 341-360*
32. Sharpe, W. F. (1964): “Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk”, *Journal of Finance, 19 (3), 425-443*

33. Sharpe, W. F. (1970): “Portfolio Theory and Capital Markets”, *McGraw-Hill, Inc., Series in Finance, USA, 1970*
34. Sharpe, W. F. (1984): “Factor models, CAPMs and the ABT”, *Journal of Portfolio Management, 11 (1), 21-25*
35. Sharpe, W. F. (1991): “Capital Asset Prices with and without Negative Holdings”, *Journal of Finance, 46 (2), 489-509*
36. Tobin, J. (1958): “Liquidity Preference as Behavior Toward Risk”, *Review of Economic Studies, 25, 65-86*
37. Wallace, A. (1980): “Is Beta Dead”, *Institutional Investor, July, 23-30*

Appendix

Appendix I – Dataset overview

Fixed income securities

FIXED INCOME SECURITIES

Type	Source	Maturity	Datatype *	Freq. **	Period Start	Period End ***	Type	Source	Rating	Datatype *	Freq. **	Period Start	Period End
Treasury	CRSP	< 1 year	MEWPR	M	Jan. 1952	Dec. 2012	Corporate	Barclay's	Aaa	MTD TR	M	Dec. 1973	Apr. 2013
Bonds	Fama Bond	1-2 years	MEWPR	M	Jan. 1952	Dec. 2012	Bonds	Capital's	Aa	MTD TR	M	Dec. 1973	Apr. 2013
	Portfolios	2-3 years	MEWPR	M	Jan. 1952	Dec. 2012		Bond.Hub	A	MTD TR	M	Dec. 1973	Apr. 2013
		3-4 years	MEWPR	M	Jan. 1952	Dec. 2012			Baa	MTD TR	M	Dec. 1973	Apr. 2013
		4-5 years	MEWPR	M	Jan. 1952	Dec. 2012			Ba	MTD TR	M	Jul. 1983	Apr. 2013
		4-10 years	MEWPR	M	Jan. 1952	Dec. 2012			B	MTD TR	M	Jul. 1983	Apr. 2013
		> 10 years	MEWPR	M	Jan. 1952	Dec. 2012			Caa	MTD TR	M	Jul. 1983	Apr. 2013
									Ca-D	MTD TR	M	Jan. 1993	Apr. 2013
Credit	Barclay's	1-3 years	MTD TR	M	Jan. 1976	Apr. 2013							
Indices	Capital's	1-5 years	MTD TR	M	Jan. 1976	Apr. 2013							
	Bond.Hub	5-10 years	MTD TR	M	Dec. 1991	Apr. 2013							
		7-10 years	MTD TR	M	Aug. 1988	Apr. 2013							

* MEWPR = Monthly equal-weighted portfolio return; MTD TR = Month-to-date total return

** D = Daily; M = Monthly

*** Data missing for Treasury bonds of maturity > 10 years in the period August 1962 - December 1971

Equity Indices

DEVELOPED COUNTRIES

Country	Source	Currency	Code	Datatype *	Freq. **	Period Start	Period End
Australia	Datastream	Local	TOTMKAU	RI	D	Jan. 1974	Jun. 2013
Germany	Datastream	Local	TOTMKBD	RI	D	Jan. 1974	Jun. 2013
Canada	Datastream	Local	TOTMKCN	RI	D	Jan. 1974	Jun. 2013
Spain	Datastream	Local	TOTMKES	RI	D	Mar. 1987	Jun. 2013
France	Datastream	Local	TOTMKFR	RI	D	Jan. 1974	Jun. 2013
Hong Kong	Datastream	Local	TOTMKHK	RI	D	Jan. 1974	Jun. 2013
Italy	Datastream	Local	TOTMKIT	RI	D	Jan. 1974	Jun. 2013
Japan	Datastream	Local	TOTMKJP	RI	D	Jan. 1974	Jun. 2013
Netherlands	Datastream	Local	TOTMKNL	RI	D	Jan. 1974	Jun. 2013
Sweden	Datastream	Local	TOTMKSD	RI	D	Jan. 1982	Jun. 2013
Switzerland	Datastream	Local	TOTMKSW	RI	D	Jan. 1974	Jun. 2013
UK	Datastream	Local	TOTMKUK	RI	D	Jan. 1974	Jun. 2013
US	Datastream	Local	TOTMKUS	RI	D	Jan. 1974	Jun. 2013
Australia	Datastream	USD	TOTMAU$	RI	D	Jan. 1974	Jun. 2013
Germany	Datastream	USD	TOTMBD$	RI	D	Jan. 1974	Jun. 2013
Canada	Datastream	USD	TOTMCN$	RI	D	Jan. 1974	Jun. 2013
Spain	Datastream	USD	TOTMES$	RI	D	Mar. 1987	Jun. 2013
France	Datastream	USD	TOTMFR$	RI	D	Jan. 1974	Jun. 2013
Hong Kong	Datastream	USD	TOTMHK$	RI	D	Jan. 1974	Jun. 2013
Italy	Datastream	USD	TOTMIT$	RI	D	Jan. 1974	Jun. 2013
Japan	Datastream	USD	TOTMJP$	RI	D	Jan. 1974	Jun. 2013
Netherlands	Datastream	USD	TOTMNL$	RI	D	Jan. 1974	Jun. 2013
Sweden	Datastream	USD	TOTMSD$	RI	D	Jan. 1982	Jun. 2013
Switzerland	Datastream	USD	TOTMSW$	RI	D	Jan. 1974	Jun. 2013
UK	Datastream	USD	TOTMUK$	RI	D	Jan. 1974	Jun. 2013
US	Datastream	USD	TOTMKUS	RI	D	Jan. 1974	Jun. 2013

EMERGING MARKETS

Country	Source	Currency	Code	Datatype *	Freq. **	Period Start	Period End
Brazil	Datastream	Local	TOTMKBR	RI	D	Jul. 1994	Jun. 2013
Chile	Datastream	Local	TOTMKCL	RI	D	Jul. 1989	Jun. 2013
Colombia	Datastream	Local	TOTMKCB	RI	D	Mar. 1992	Jun. 2013
Mexico	Datastream	Local	TOTMKMX	RI	D	May 1989	Jun. 2013
Peru	Datastream	Local	TOTMKPE	RI	D	Jan. 1994	Jun. 2013
Czech Rep.	Datastream	Local	TOTMKCZ	RI	D	Nov. 1993	Jun. 2013
Egypt	Datastream	Local	TOTMKEY	RI	D	Oct. 1996	Jun. 2013
Hungary	Datastream	Local	TOTMKHN	RI	D	Jun. 1991	Jun. 2013
Morocco	Datastream	Local	TOTMKMC	RI	D	Apr. 1994	Jun. 2013
Poland	Datastream	Local	TOTMKPO	RI	D	Mar. 1994	Jun. 2013
Russia	Datastream	Local	TOTMKRS	RI	D	Feb. 1998	Jun. 2013
South Africa	Datastream	Local	TOTMKSA	RI	D	Jan. 1988	Jun. 2013
Turkey	Datastream	Local	TOTMKTK	RI	D	Jan. 1988	Jun. 2013
India	Datastream	Local	TOTMKIN	RI	D	Jan. 1990	Jun. 2013
Indonesia	Datastream	Local	TOTMKID	RI	D	Apr. 1990	Jun. 2013
South Koea	Datastream	Local	TOTMKKO	RI	D	Jan. 1988	Jun. 2013
Malaysia	Datastream	Local	TOTMKMY	RI	D	Jan. 1988	Jun. 2013
Phillipines	Datastream	Local	TOTMKPH	RI	D	Jan. 1988	Jun. 2013
Thailand	Datastream	Local	TOTMKTH	RI	D	Jan. 1988	Jun. 2013
Brazil	Datastream	USD	TOTMBR$	RI	D	Jul. 1994	Jun. 2013
Chile	Datastream	USD	TOTMCL$	RI	D	Jul. 1989	Jun. 2013
Mexico	Datastream	USD	TOTMMX$	RI	D	May 1989	Jun. 2013
Poland	Datastream	USD	TOTMPO$	RI	D	Mar. 1994	Jun. 2013
South Africa	Datastream	USD	TOTMSA$	RI	D	Jan. 1988	Jun. 2013
Turkey	Datastream	USD	TOTMTK$	RI	D	Jun. 1989	Jun. 2013
Indonesia	Datastream	USD	TOTMID$	RI	D	Apr. 1990	Jun. 2013
South Koea	Datastream	USD	TOTMKO$	RI	D	Jan. 1988	Jun. 2013
Malaysia	Datastream	USD	TOTMMY$	RI	D	Jan. 1988	Jun. 2013
Phillipines	Datastream	USD	TOTMPH$	RI	D	Nov. 1988	Jun. 2013
Thailand	Datastream	USD	TOTMTH$	RI	D	Jan. 1988	Jun. 2013

* RI = Return index

** D = Daily; M = Monthly

Futures

EQUITY INDICES - DEVELOPED COUNTRIES

Country	Source	Underlying Index	Code	Datatype *	Freq. **	Period Start	Period End
Australia	Datastream	SPI200	AAPCS00	SP	D	May 2000	Jun. 2013
Germany	Datastream	DAX	GDXCS00	SP	D	Dec. 1990	Jun. 2013
Spain	Datastream	IBEX35	MBXCS00	SP	D	Apr. 1992	Jun. 2013
Canada	Datastream	CAC40	FCXCS00	SP	D	Jan. 1999	Jun. 2013
Italy	Datastream	FTSE MIB	MSMCS00	SP	D	Mar. 2004	Jun. 2013
Japan	Datastream	TOPIX	JSXCS00	SP	D	Jan. 1989	Jun. 2013
Netherlands	Datastream	AEX	ETICS00	SP	D	Jan. 1989	Jun. 2013
Sweden	Datastream	OMXS30	OMFCS00	SP	D	Jan. 1990	Jun. 2013
Switzerland	Datastream	EUREX SMI	ZMICS00	SP	D	Nov. 1990	Jun. 2013
UK	Datastream	FTSE 100	LSXCS00	SP	D	Jan. 1989	Jun. 2013
USA	Datastream	S&P 500	ISPCS00	SP	D	Jan. 1989	Jun. 2013

EQUITY INDICES - EMERGING MARKETS

Country	Source	Underlying Index	Code	Datatype *	Freq. **	Period Start	Period End
Brazil	Datastream	BOVESPA	BMICS00	SP	D	Jan. 1998	Jun. 2013
Mexico	Datastream	IPC	MIECS00	SP	D	Jun. 1999	Jun. 2013
Poland	Datastream	WIG 20	WIGCS00	SP	D	Jan. 1998	Jun. 2013
Hungary	Datastream	BUX	BUXCS00	SP	D	Jan. 1998	Jun. 2013
Russia	Datastream	RTS 50	RTSCS00	SP	D	Aug. 2005	Jun. 2013
Turkey	Datastream	ISE 100	TRFCS00	SP	D	Nov. 2005	Jun. 2013
India	Datastream	CNX NIFTY	INICS00	SP	D	Jun. 2000	Jun. 2013
South Korea	Datastream	KOSPI 200	KKXCS00	SP	D	Jan. 1998	Jun. 2013
Taiwan	Datastream	TAIEX	TTXCS00	SP	D	Jul. 1998	Jun. 2013
Thailand	Datastream	SET 50	TSTCS00	SP	D	May 2006	Jun. 2013

COUNTRY BOND INDICES - DEVELOPED COUNTRIES

Country	Source	Maturity	Code	Datatype *	Freq. **	Period Start	Period End
Australia	Datastream	3 years	ATYCS00	SP	D	May 1988	Jun. 2013
Australia	Datastream	10 years	AGBCS00	SP	D	Jan. 1985	Jun. 2013
USA	Datastream	2 years	CTECS00	SP	D	Jun. 1990	Jun. 2013
USA	Datastream	5 years	CFVCS00	SP	D	May 1988	Jun. 2013
USA	Datastream	10 years	CTYCS00	SP	D	Jan. 1985	Jun. 2013
USA	Datastream	30 years	CUSCS00	SP	D	Jan. 1985	Jun. 2013
Japan	Datastream	10 years	JGBCS00	SP	D	Dec. 1986	Jun. 2013
UK	Datastream	10 years	LIGCS00	SP	D	Jan. 1985	Jun. 2013
Canada	Datastream	10 years	CDGCS00	SP	D	Sep. 1989	Jun. 2013
Spain	Datastream	10 years	MDACS00	SP	D	Apr. 1992	Jun. 2013
Germany	Datastream	2 years	GMBCS00	SP	D	Mar. 1997	Jun. 2013
Germany	Datastream	5 years	GGBCS00	SP	D	Oct. 1991	Jun. 2013
Germany	Datastream	10 years	GTBCS00	SP	D	Dec. 1990	Jun. 2013

* SP = Settlement price, total return is generated as described in the main body of the thesis

** D = Daily; M = Monthly

COMMODITIES

Country	Source	Code	Datatype *	Freq. **	Period Start	Period End
Aluminum	Datastream	GSIAEXR	ERI	D	Jan. 1991	Apr. 2013
Brent Oil	Datastream	GSBREXR	ERI	D	Jan. 1999	Apr. 2013
Crude Oil	Datastream	GSCLEXR	ERI	D	Jan. 1989	Apr. 2013
Biofuel	Datastream	SGBFEXR	ERI	D	Jan. 1995	Apr. 2013
Cocoa	Datastream	GSCCEXR	ERI	D	Jan. 1989	Apr. 2013
Coffee	Datastream	GSKCEXR	ERI	D	Jan. 1989	Apr. 2013
Copper	Datastream	GSICEXR	ERI	D	Jan. 1989	Apr. 2013
Corn	Datastream	GSCNEXR	ERI	D	Jan. 1989	Apr. 2013
Cotton	Datastream	GSCTEXR	ERI	D	Jan. 1989	Apr. 2013
Gold	Datastream	GSGCEXR	ERI	D	Jan. 1989	Apr. 2013
Lead	Datastream	GSILEXR	ERI	D	Jan. 1995	Apr. 2013
Nickel	Datastream	GSIKEXR	ERI	D	Jan. 1993	Apr. 2013
Platinum	Datastream	SGPLEXR	ERI	D	Jan. 1989	Apr. 2013
Silver	Datastream	GSSIEXR	ERI	D	Jan. 1989	Apr. 2013
Soybeans	Datastream	GSSOEXR	ERI	D	Jan. 1989	Apr. 2013
Sugar	Datastream	GSSBEXR	ERI	D	Jan. 1989	Apr. 2013
Tin	Datastream	SGISEXR	ERI	D	Mar. 2007	Apr. 2013
Zinc	Datastream	GSIZEXR	ERI	D	Jan. 1991	Apr. 2013
Gasoil	Datastream	GSGOEXR	ERI	D	Jan. 1999	Apr. 2013
Heating Oil	Datastream	GSHOEXR	ERI	D	Jan. 1989	Apr. 2013
Hogs	Datastream	GSLHEXR	ERI	D	Jan. 1989	Apr. 2013
Feeder Cattle	Datastream	GSLCEXR	ERI	D	Jan. 1989	Apr. 2013
Natural Gas	Datastream	GSNGEXR	ERI	D	Jan. 1994	Apr. 2013
Soy Oil	Datastream	SGBOEXR	ERI	D	Jan. 2005	Apr. 2013
Wheat	Datastream	GSWHEXR	ERI	D	Jan. 1989	Apr. 2013
GSCI Index	Datastream	GSCIEXR	ERI	D	Jan. 1989	Apr. 2013

* ERI = Excess return index

** D = Daily; M = Monthly

Appendix II – Relation of alpha and beta and performance of the BAB factor

Table A1 – Split by investment and speculative grade

The next table shows the analysis separately for the Investment grades and Speculative grades sub-groups. The market portfolios are now the simple average returns on the bonds in the sub-group.

Investment Grade, 1973-2013

	P1 low beta	P2	P3	P4 high beta	BAB factor
Rating	Aaa	Aa	A	Baa	
Excess return (% m)	**0.21**	**0.23**	**0.25**	**0.31**	0.04
	(2.24)	(2.55)	(2.59)	(3.11)	(1.12)
Alpha (% m)	-0.03	-0.01	-0.01	**0.05**	0.03
	(-1.35)	(-0.96)	(-0.58)	(1.82)	(0.93)
Beta (ex ante)	0.92	0.99	1.05	1.05	0.00
Beta (realized)	0.96	0.98	1.03	1.04	0.02
Volatility (% y)	7.00	6.97	7.30	7.63	2.56
Sharpe ratio (y)	0.35	0.40	0.41	0.49	0.18

Speculative Grade, 1983-2013

	P1 low beta	P2	P3	P4 high beta	BAB factor
Rating	Ba	B	Caa	Ca-D	
Excess return (% m)	**0.49**	**0.41**	0.31	0.80	**0.51**
	(4.81)	(3.07)	(1.48)	(1.54)	(2.75)
Alpha (% m)	**0.27**	0.09	-0.17	-0.23	**0.54**
	(4.74)	(1.64)	(-1.64)	(-0.99)	(2.87)
Beta (ex ante)	0.53	0.73	1.19	1.77	0.00
Beta (realized)	0.48	0.68	1.04	1.95	-0.06
Volatility (% y)	6.69	8.69	13.88	27.99	11.57
Sharpe ratio (y)	0.88	0.56	0.27	0.34	0.53

Figure A1 – Equity indices: Scatter-plot for individual indices' alpha vs. beta

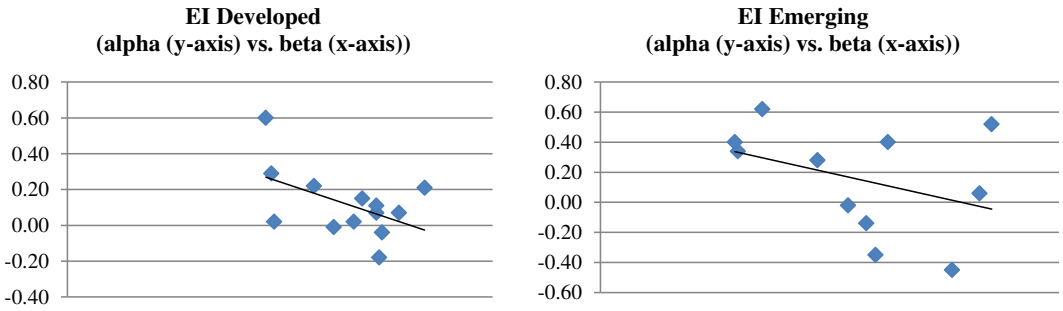


Figure A2 – Futures: Scatter-plot for individual assets' alpha vs. beta

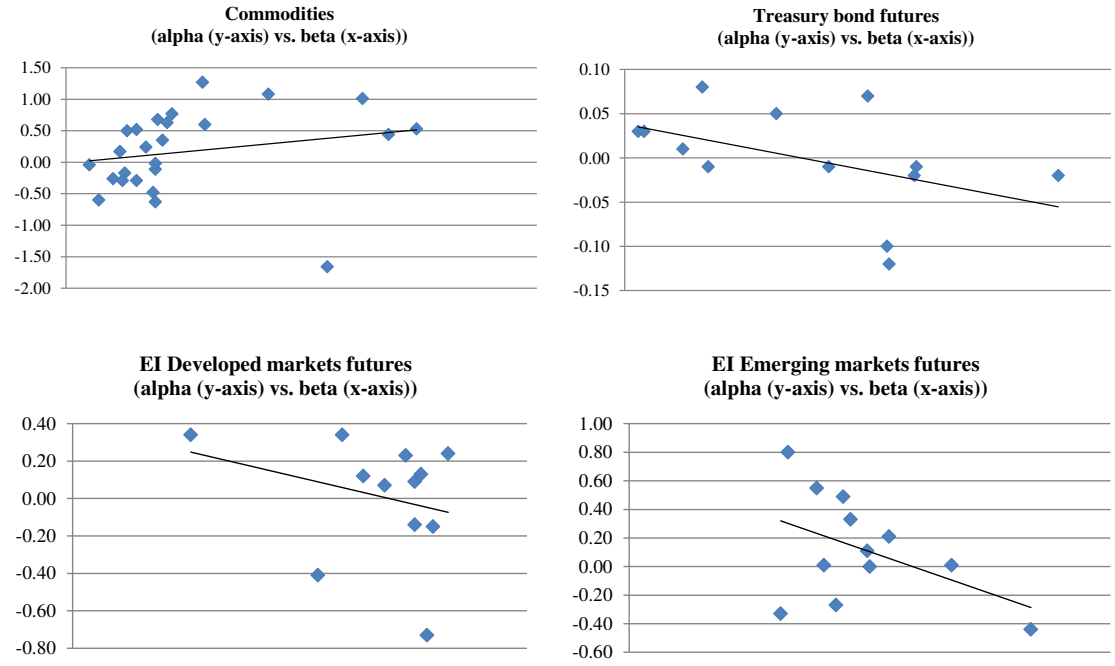


Table A2 – Full sample statistics of Equity indices

Following two tables show results of full sample regressions of the individual country indices and the equal-weighted "1/n" portfolio on the GDP-weighted market portfolio and descriptive statistics of the market portfolio for comparison:

Developed countries

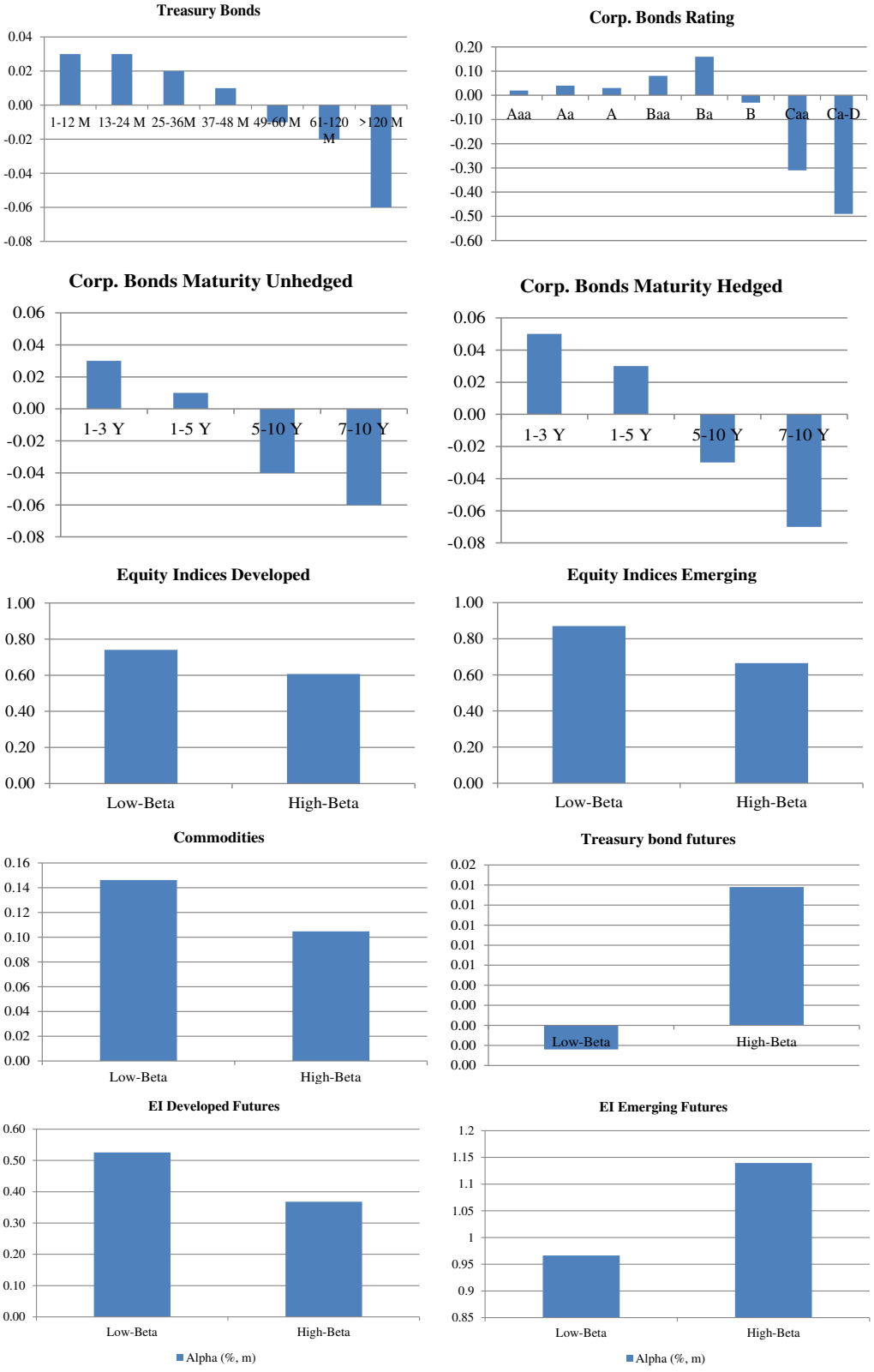
	AUS	GER	CAN	ESP	FRA	HKG	ITA	JPN	NED	SWE	SUI	UK	USA	Avg_Mkt	GDP_Mkt
Excess return (%, m)	0.69	0.38	0.47	0.54	0.66	1.04	0.55	0.15	0.56	0.98	0.39	0.74	0.56	0.59	0.47
t-stat	2.99	1.66	2.37	1.61	2.55	2.93	1.86	0.61	2.34	2.88	1.93	3.15	2.43	3.52	2.82
Alpha (%, m)	0.48	-0.08	0.07	0.14	0.16	0.7	0.08	-0.18	0.08	0.37	0.01	0.31	0.02	0.17	0
t-stat	2.21	-0.49	0.53	0.56	0.84	2.09	0.34	-0.82	0.48	1.42	0.08	1.72	0.12	2.22	
Beta (realized)	0.44	0.99	0.84	0.99	1.07	0.73	0.99	0.69	1.02	1.09	0.81	0.93	1.17	0.9	1
Volatility (%, y)	16.95	16.91	14.52	19.96	18.86	26.05	21.49	17.76	17.43	22.23	14.84	17.29	17.03	12.25	12.19
Sharpe ratio (y)	0.47	0.26	0.37	0.31	0.4	0.46	0.29	0.1	0.37	0.5	0.3	0.49	0.38	0.55	0.44

Emerging markets

	BRA	CHL	COL	MEX	PER	CZE	EGY	HUN	MOR	POL	RUS	RSA	TUR	IND	INO	KOR	MAL	PHI	THA	AVG_Mkt	GDP_Mkt
Excess return (%, m)	1.36	1.32	1.28	1.80	0.99	0.69	1.03	1.09	0.79	0.42	2.56	1.25	3.76	1.25	1.00	0.72	0.83	1.11	1.04	1.45	1.46
t-stat	2.67	5.14	4.29	4.91	2.90	1.60	2.05	2.31	3.56	0.78	2.68	3.99	5.50	2.65	1.72	1.47	2.38	3.07	2.21	7.27	6.05
Alpha (%, m)	0.02	0.72	0.84	0.62	0.48	-0.18	0.64	-0.10	0.71	-0.59	0.33	0.25	1.80	-0.02	-0.17	-0.94	0.14	0.48	-0.19	0.33	0
t-stat	0.05	3.07	2.97	2.07	1.52	-0.48	1.30	-0.25	3.21	-1.26	0.45	0.93	2.98	-0.04	-0.31	-2.29	0.44	1.39	-0.44	4.49	
Beta (realized)	1.16	0.44	0.35	0.83	0.46	0.72	0.33	0.89	0.07	0.90	1.77	0.69	1.35	0.95	0.87	1.14	0.47	0.43	0.85	0.77	1
Volatility (%, y)	25.82	14.68	16.05	20.95	17.58	22.39	23.87	25.86	11.37	27.43	43.71	18.48	40.28	26.65	32.71	28.91	20.43	21.34	27.82	11.74	14.17
Sharpe ratio (y)	0.60	1.03	0.91	0.98	0.64	0.35	0.49	0.48	0.80	0.17	0.67	0.78	1.07	0.54	0.35	0.29	0.46	0.6	0.43	1.42	1.18

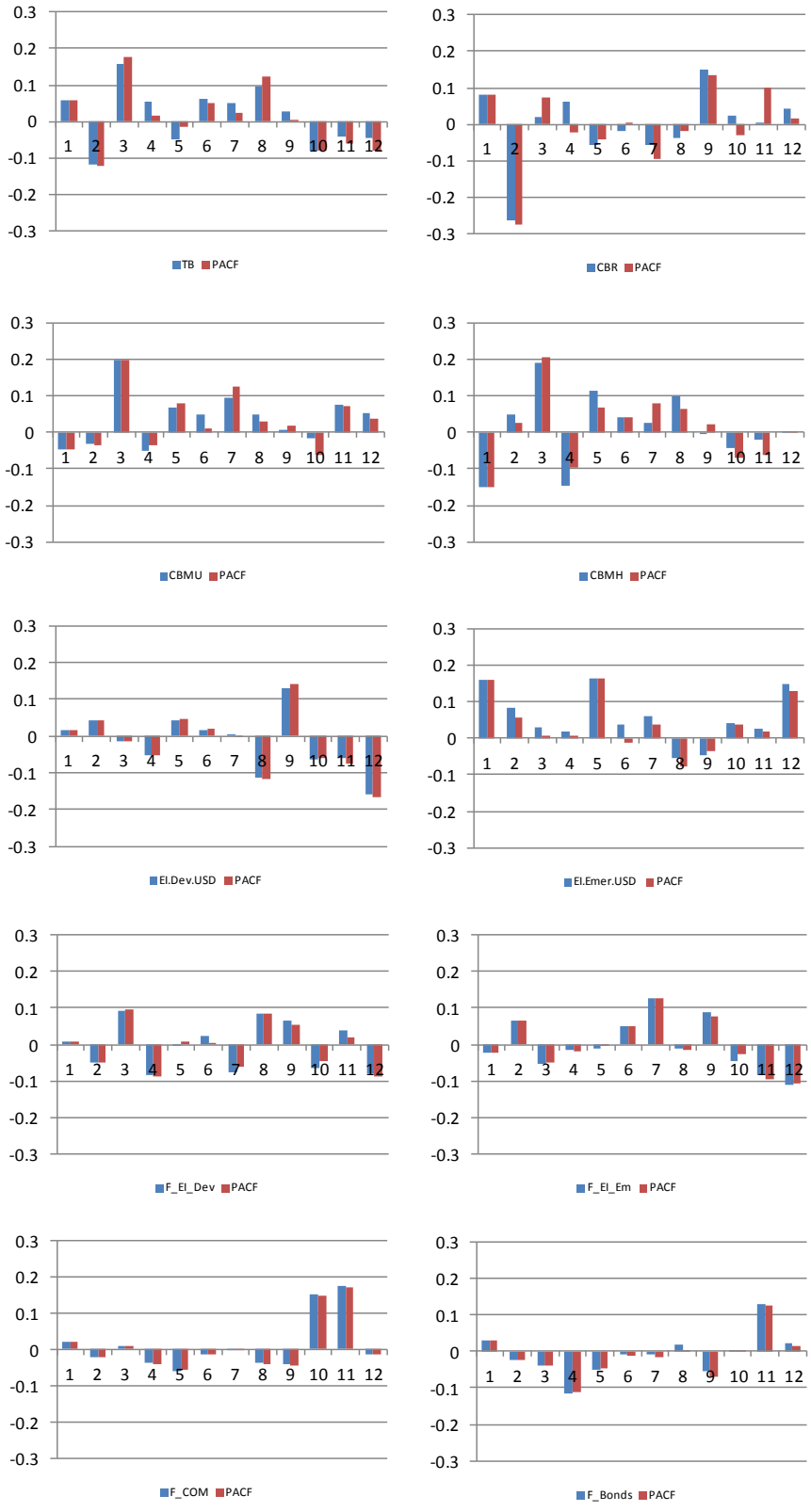
Figure A3 – Alphas of beta sorted portfolios

Graphs below plot the realized full sample alphas of one-factor regressions for the portfolios sorted on beta. For bonds it means ordering based on maturity or rating and for other asset classes two portfolios of low and high beta assets with equal weights of their constituents.



Appendix III – Time series tests of the influence of changing liquidity and funding conditions

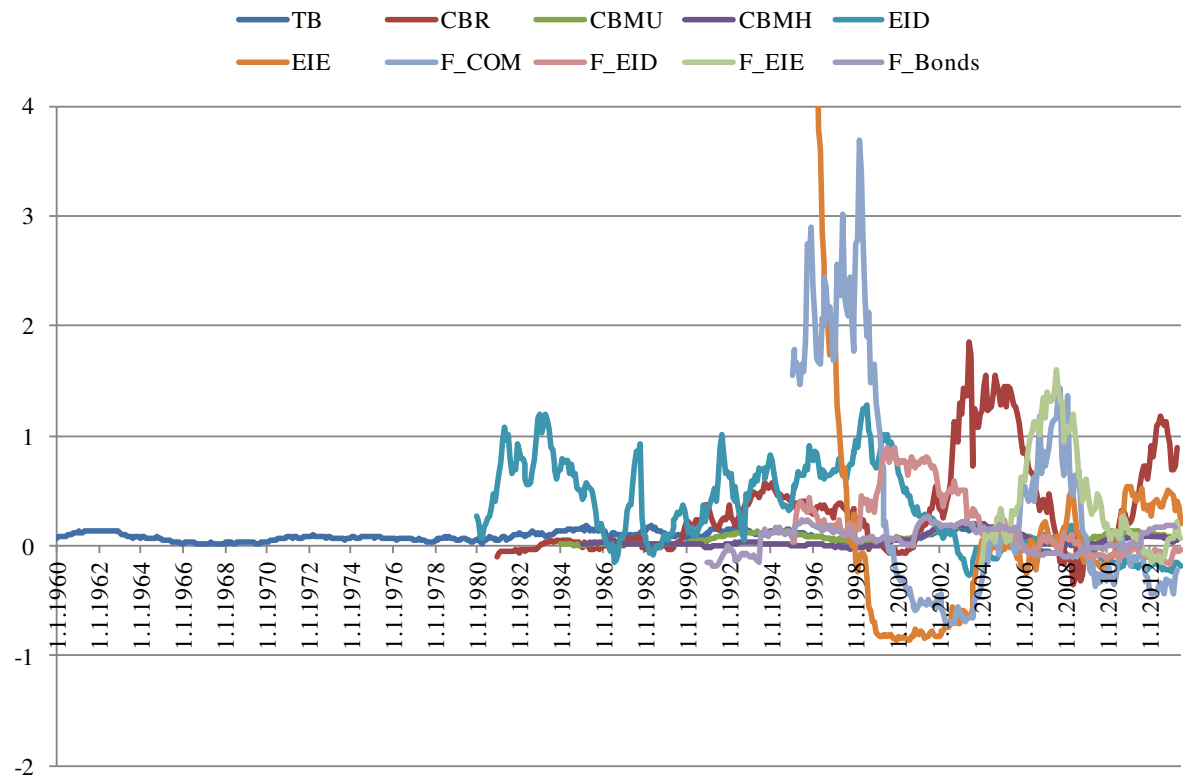
Figure A4 – ACF and PACF of the BAB factor returns for each of the individual asset classes. ACF is in blue and the name indicates the asset class while PACF is indicated in red.



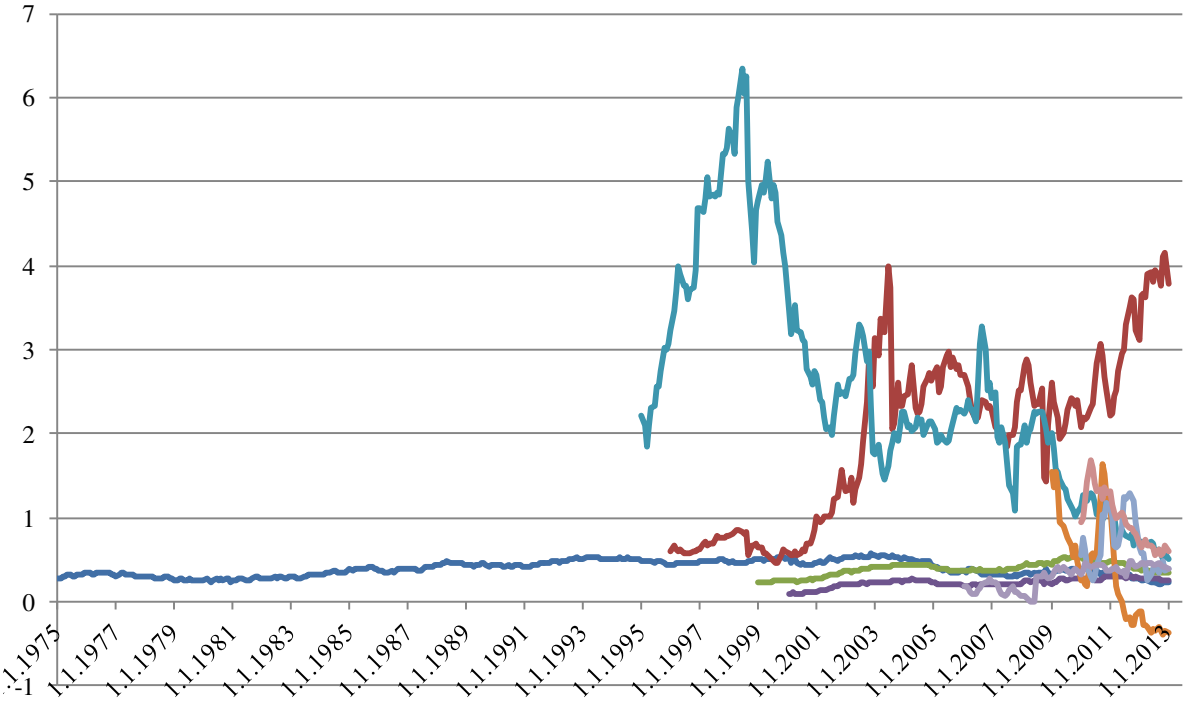
Appendix IV – Additional BAB tests – Cumulative returns on the BAB factors

Figure A5 – Cumulative returns on the BAB factors

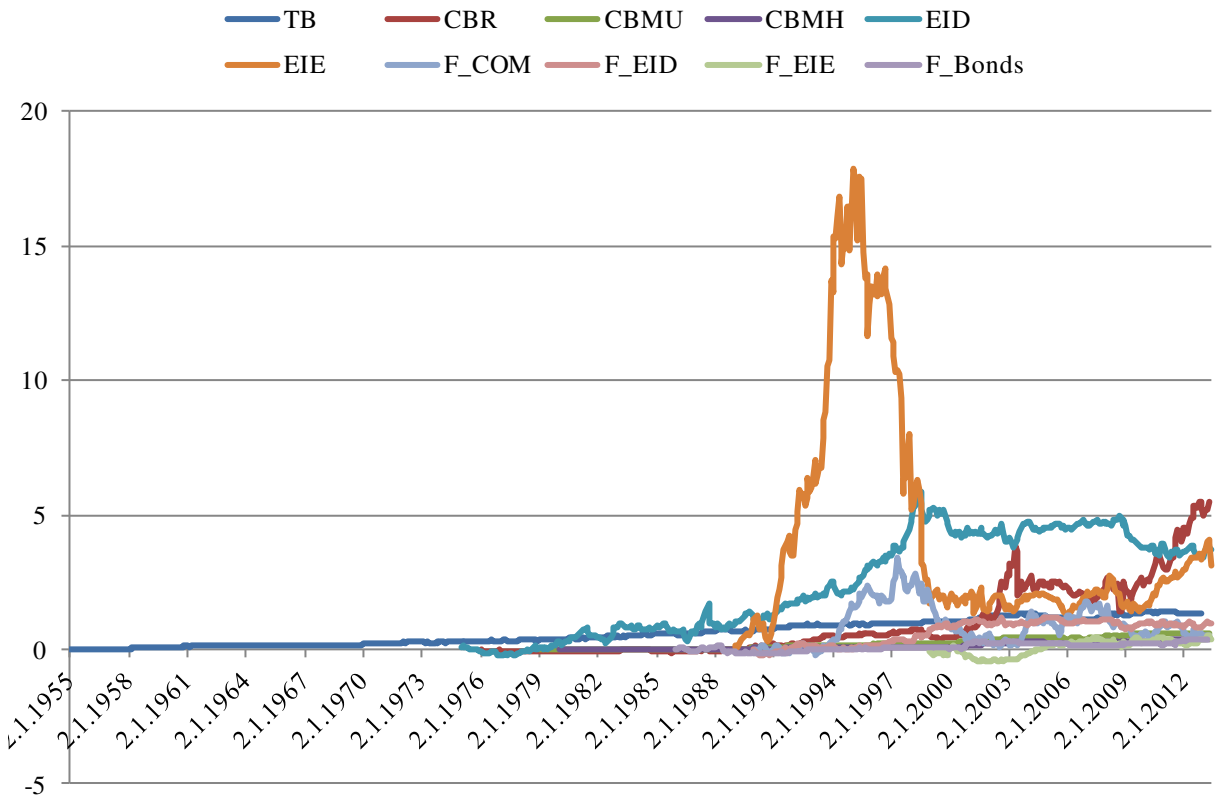
5-year cumulative return on the BAB factor for individual asset classes



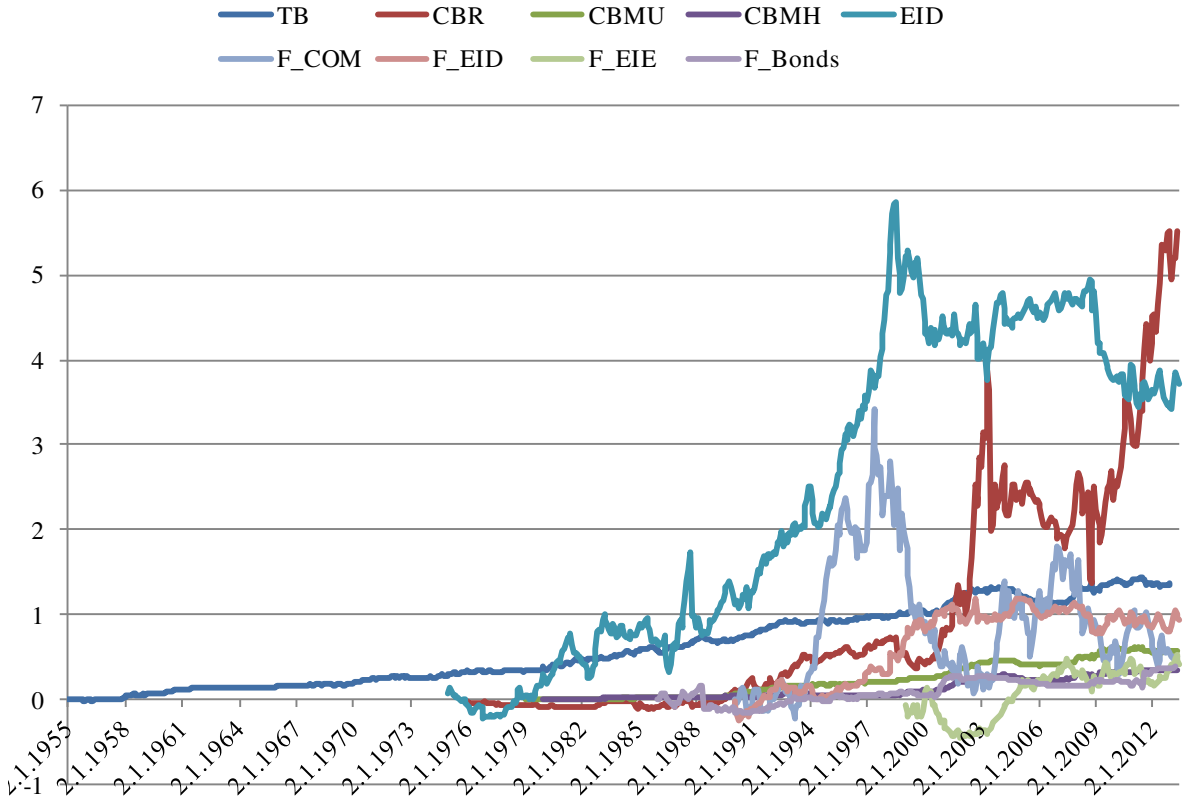
20-year cumulative return on the BAB factor for individual asset classes



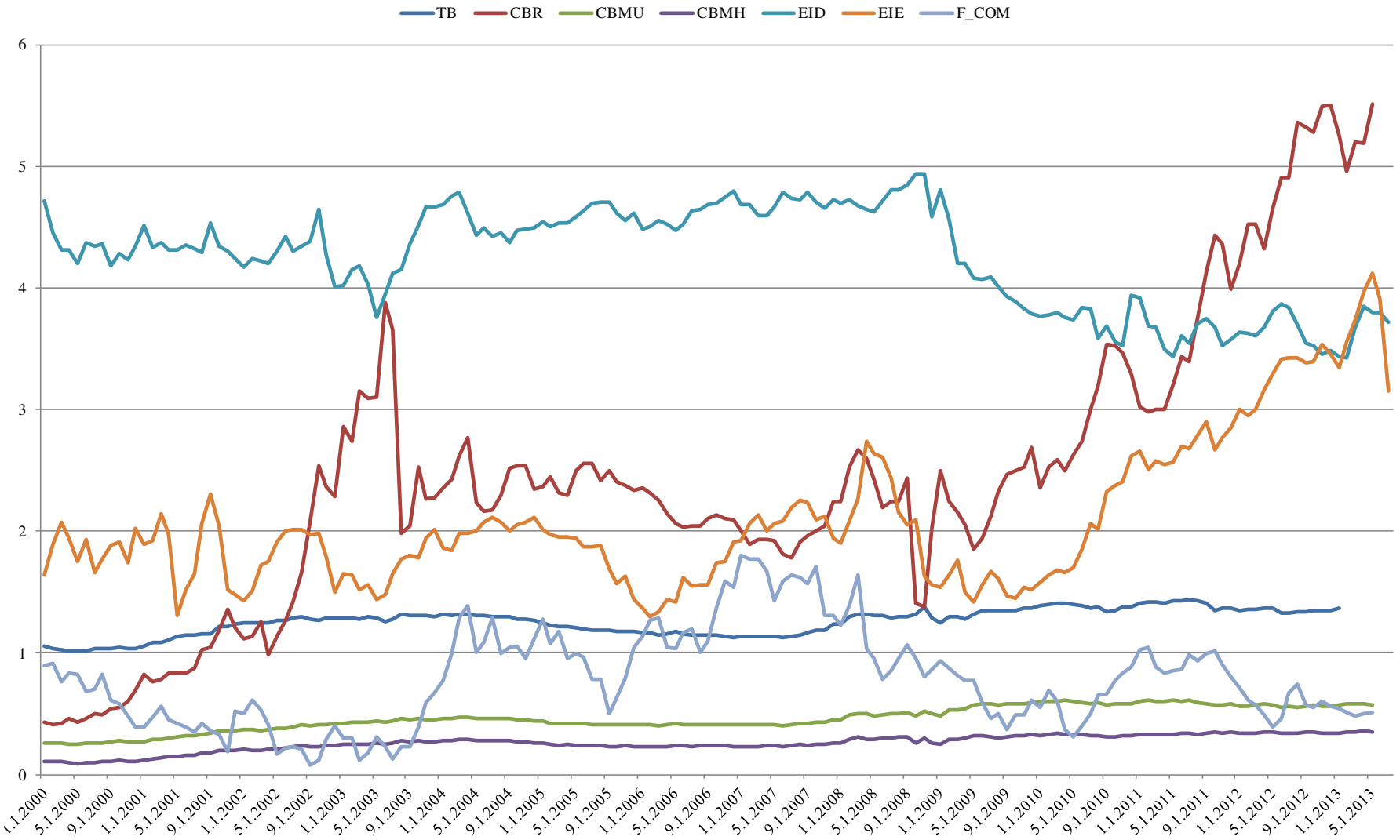
Total cumulative returns on the BAB factors (from the beginning of the period for which the data are available)



Total cumulative returns on the BAB factors excluding the Emerging markets equity indices



Detail of last 13 years of total cumulative returns on the BAB factors



Appendix V – Alternative construction of the BAB factor and infrequent updating of the BAB composition

Table A3 – Impact of extending the update window (case of Equity Indices)

The two tables show monthly returns on the BAB factor in excess of the risk free rate. The test assets are the daily returns on the equity indices in US dollars from Developed and Emerging countries and the market return is the GDP-weighted average of all the available index returns in the subgroup. The BAB factor is a zero-beta, long-short portfolio updated each year in the top table and every 18 months in the bottom table. Each of the assets is assigned to either high-beta or low-beta portfolio on basis of the previous one-year beta (top table) or three-year beta (bottom) where the lowest-beta asset has the largest weight in the low-beta portfolio and vice versa. The two portfolios are rescaled to have a beta of 1 and the BAB factor is long the low-beta portfolio and shorts the high-beta portfolio. Excess returns and alphas are in monthly percent, volatility and Sharpe ratio are annualized. T-statistics are shown below in parenthesis and significance at 5% level is indicated in bold. Base case doesn't include the lag of BAB returns for ex-ante beta estimation while the Asynchronous trading case (Asyn.) uses the first lag of BAB returns for the pre-ranking of indices. The inclusion of shrinkage factor is indicated in the first row of the table.

	Developed USD		Emerging USD		Developed USD		Emerging USD	
	Base	Asyn.	Base	Asyn.	Base	Asyn.	Base	Asyn.
Shrinkage w=0.6	no	no	no	no	yes	yes	yes	yes
Excess return (% m)	**1.24**	**1.06**	**2.25**	**2.02**	**0.73**	**0.68**	**1.06**	**1.02**
	(4.10)	(4.28)	(3.62)	(3.76)	(3.46)	(3.73)	(2.58)	(2.55)
Alpha (% m)	**1.07**	**0.86**	**1.98**	**1.72**	**0.76**	**0.64**	**1.32**	**1.15**
	(3.58)	(3.59)	(3.210)	(3.26)	(3.63)	(3.52)	(3.27)	(2.89)
Beta (realized)	0.30	0.35	0.25	0.27	-0.06	0.07	-0.24	-0.12
Volatility (% y)	21.50	17.57	35.50	30.69	14.93	13.00	23.50	22.83
Sharpe ratio (y)	0.66	0.69	0.73	0.75	0.56	0.60	0.52	0.51

	Developed USD		Emerging USD		Developed USD		Emerging USD	
	Base	Asyn.	Base	Asyn.	Base	Asyn.	Base	Asyn.
Shrinkage w=0.6	no	no	no	no	yes	yes	yes	yes
Excess return (% m)	**0.73**	**0.65**	0.88	0.69	**0.43**	**0.46**	0.35	0.20
	(3.00)	(3.33)	(1.15)	(1.17)	(2.09)	(2.57)	(0.74)	(0.47)
Alpha (% m)	**0.61**	**0.47**	0.45	0.40	**0.45**	**0.38**	0.47	0.30
	(2.54)	(2.52)	(0.60)	(0.68)	(2.20)	(2.13)	(1.00)	(0.71)
Beta (realized)	0.21	0.32	0.44	0.30	-0.04	0.15	-0.12	-0.10
Volatility (% y)	16.65	13.41	42.24	32.64	13.97	12.24	25.84	23.56
Sharpe ratio (y)	0.50	0.56	0.24	0.24	0.35	0.43	0.15	0.10

Table A4 – Long side and short side sizing of the BAB factor

Investment positions in the long and short sides of the BAB factor and the resulting net position in the risk-free asset

Asset class	Long side	Short side	Net Rf
US Treasury Bonds	2.966	0.584	-2.382
Corporate Bonds Rating	1.589	0.652	-0.936
Corporate Bonds Maturity Unhedged	1.616	0.724	-0.892
Corporate Bonds Maturity Hedged	1.390	0.778	-0.612
Equity Indices Developed	1.193	0.823	-0.370
Equity Indices Emerging	1.433	0.836	-0.597
Futures EID	1.093	0.795	-0.298
Futures EIE	1.212	0.762	-0.450
Futures Commodities	2.114	0.979	-1.135
Futures Treasury Bonds	1.708	0.677	-1.031