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The Relation between Demography and Economics with Special Reference to the Role of Mandatory Funded and Unfunded Old-Age Pensions

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The relation between demography and economics with special reference to the role of mandatory funded and unfunded old-age pensions

by

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Abstract

The relation between demography and economics with special reference to the role of funded and unfunded old-age pensions

In this paper we consider the influence of the demography on the dynamic equilibrium of an economy. More precisely, we focus on mandatory pensions, as in most Western countries except USA and Switzerland the role of 'the third pension-pillar', based on voluntary savings, is relatively minor. Our first main result is that the interest rate in a two-pillar system (PAYG and mandatory pensions) is increasing with the birth rate. This finding would give a theoretical underpinning for the actual conjecture that real long-term interest rates over the world tend to fall in line with falling birth rates. The second result is that the mix between mandatory funded and unfunded systems may be seen as an endogenous result of the system. If the birth rate is rising, the role of the unfunded system declines or even vanishes, while inversely with falling birth rate the role of the unfunded system appears to grow. A third important result is that average utility or welfare increases with increasing birth rates, while at the same time inequality between workers and retired, measured by the benefit ratio, appears to reduce. A final essential result is that a pension system built on a funded and an unfunded component does better in terms of average welfare than a system which consists of a funded system only, except for very low birth rates where a mandatory funded system only yields higher average welfare.

Keywords: demography, funded pensions, unfunded pensions, social security, interest rate
JEL codes: H55, H75, J1, J26

1.Introduction

In the last years there is a lot of discussion on the sustainability of old-age pensions. This discussion deals both with funded and unfunded pensions. The discussion is by no means merely academic, but has a dramatic relevance for most European countries, which cope with a severe ageing of the population. The old-age-dependency ratio is increasing rapidly. This is due to a severe decline in the birth rate and a simultaneous increase in longevity. In the fifties there was about one person over 65 to eight workers in the Netherlands, at present the ratio is about one to four and in the forties of this centuries it will stabilize at about one over two. A similar or even more dramatic evolution is found in most European countries, like e.g. Germany, Italy, Spain and Russia, let alone China and Japan.

Most national pension systems are a mix of three pillars or tiers. There is a huge variation between countries with respect to the composition of that mix. Evidence is given by Börsch-Supan (2004), which we reproduce below. (see also Bovenberg and Nijman (2009), OECD (2011)).

The first pillar consists of a mandatory state old-age pension which is run on a pay-as-you-go (PAYG) – basis. Given the demographic changes, it seems evident that it will be no longer possible to maintain in the future the present level of unfunded Pay-As-You-Go (PAYG)-systems, on which many state old-age pensions are based nowadays.

The second pension pillar is based on the funded or Capital-Reserve (CR)-system and is mandatory for most or all employees. The functioning of this system depends on the level of the interest rate on the capital invested. The third pillar is built up by voluntary savings. In countries with a well –established first and second pillar, the third pillar is relatively small and declining. In most OECD –countries the size of the third pillar is small, except for the USA, the UK, and Switzerland. Actually, this kind of pension saving is more or less restricted to entrepreneurs and independents, who have to save for their own pensions.

Pension systems in various countries

	The Netherlands	Germany	France	Italy	Spain	Switzer-land	UK	US
	% of total retirement benefits							
PAYG public pensions	50	85	79	74	92	42	65	45
Occupational pensions	40	5	6	1	4	32	25	13
Personal pensions	10	10	15	25	4	26	10	42

Source: Börsch-Supan (2004)

In this paper we consider economies of 'the European type' in which we assume for convenience that there is no third pillar and that the whole population is subject to a mandatory pension system, consisting of a combination of a PAYG- system and a mandatory occupational pension on a Capital reserve basis. This is in contrast with most contributions in the literature that assume that funded pensions are only generated by voluntary savings. That is, there is no mandatory saving and pension contributions may vary with age and person. Indeed there is no difference between voluntary savings and mandatory savings, when we consider a stylized demography with only a working and a retired generation. If there is more than one working generation involved, say the cohorts [20,44] and [45,65], the mandatory system forces one premium and one contribution rate upon the two cohorts, while a system of voluntary savings would yield cohort-specific rates, where in all likelihood the older workers would like to save more and to pay more social security contribution than their younger co-workers are prepared to. The main question on which we attempt to shed light in this paper is the question on how differences in demography affect the absolute and the relative size of the two pillars. The main difference with the literature is that it tacitly assumes that pension saving is done through a third- pillar system where individuals decide for themselves, while in a second –pillar system the trade unions in collective labor contracts negotiate on the distribution of gross wages over available income and postponed income. The outcome depends among other factors on the level of PAYG- pensions, provided by the state or firms. In our study we assume for simplicity that all PAYG-pensions are state pensions, although we are well aware of the fact that in some countries some sectors are providing pensions on a pay –as –you –go –basis as well.

We construct a model of an economy with a rather detailed demography, in which part of the old-age pension is financed by a pay-as-you-system and part by a mandatory firm- based pension system. The basic question is what will be the likely outcome for the total pension and what will be the relative contribution of the two systems to the total. This is a relevant question for many countries where systems seem to succumb under demographic pressures. As we want to reflect reality we choose in this paper for demography with 100 age cohorts. The problem is that decisions on mandatory pensions are mostly taken by the body of active workers, being part of a collective labor contract that defines the division of wage costs between net wage, and pension premium, while the electorate as a whole, that is including active workers and retired, decides about the social security system. Hence, the outcome that is optimal for one body is mostly non-optimal for the other. The result will be a compromise, as described later on. This is also one of the main differences with most of the literature, where it is tacitly assumed that all workers decide on their savings individually.

We find that the answer to that question is highly demography- dependent. We shall see that the interest rate in this model is also demography-dependent. This hangs on the fact that most savings are life-cycle savings, aimed at the realization of an old-age pension during retirement. In a non-stochastic world with perfect foresight precautionary savings do not play a role, while savings for bequests can be seen as a generalized case where savings at the end of life are not zero but positive. The absence of random influences also implies that we cannot distinguish between Defined Benefit systems and Defined Contribution systems.

In the last decade pension funds, insurance companies and other institutional investors have become the main players in financial markets. For instance, recent estimates (e.g. Conference Board (2009)) are that about 80% of the public stocks in the USA are in hands of those institutional investors[1], compared to 19 % around 1970. Their capital is

[1] Marleen Groesbeek Janssen writes in a recent paper (2011): In 'The New Capitalists' Davis, Lukomnik and Pitt---Watson (2006) show that in 1970 just a small group of wealthy individuals controlled corporations in the United States. And they really controlled them, as the financial institutions representing small investors owned only 19 percent of stock. Thirty years later, according to the American Conference Board, institutional investors owned 61.4 percent in the thousand largest public companies in the United States. In 2009 that percentage was 73. That same picture applies for continental Europe. In 1995 institutional investors owned 22 percent of the listed companies in the Netherlands. That percentage increased to 84 in 2009. The majority of the stocks of Dutch listed companies are owned by non –

mainly stemming from pension savings. This is also the message of Boeri et al. (2006). It follows that the price of capital, i.e., the interest rate, cannot be seen anymore as an exogenous variable but depends on the demographic structure. More precisely, an older population generally will have a greater pension reserve per head than a younger population and consequently the fund's capital supply curve with population ageing will shift to the right, except if the population becomes very old. Simultaneously, as the number of jobs per head of the population decreases when the population is ageing, the demand for capital per head will fall as well. In modern societies an overwhelming part of capital supply stems from pension funds, looking for investments. Hence, the performance of the funded system in terms of benefit paid out per contribution of €1.- will depend on the demography as well.

In this paper we will investigate the effect of the demography on the economy by means of a comparative-static analysis. More precisely, in this paper we will restrict ourselves on the effects of the birth rate on the economy. In the same manner we might investigate the effect of changing mortality rates, but we will not do so in this paper in order to keep the present paper at a reasonable length. If we talk about the economy, the relevant endogenous economic variables we have in mind are wages, the interest rate, social security taxes, mandatory pension fund contributions and benefits and the mix between the first and second pillar. We assume that the level of the pension fund premium is determined by the active population, while the contribution by workers to the PAYG-system is determined by the electorate as a whole, that is, workers and retired. The equilibrium is then the result of a game between two parties, viz. the workers' population on one hand and the electorate as a whole, part of which are the working population. We shall assume a stationary demography and for a first analysis we assume no technical growth.

In the literature we find various approaches to the relation between demography and economy. It is beyond the scope of this paper to consider the hundreds of articles writ-

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Dutch investors; on the other hand Dutch institutional investors have a global diversified portfolio. As half of the pension funds capital is invested in shares of listed companies and saving for a pension is mandatory for all Dutch employees we can without a doubt say that the average Dutch person is an international shareholder …..

ten. Basically, we may distinguish between more theoretical and more applied papers. In the theoretical analyses like Samuelson (1954), Aaron (1966), Casamatta, Cremer and Pestieau (2000), Galasso (2008) , Galasso and Profeta (2004), there is mostly a two- or three-period overlapping generation population and the demographic variables like birth and mortality are not differentiated. In the more applied papers simulations are performed on real populations with many age cohorts (e.g. the seminal Auerbach, Kotlikoff (1987), Miles (1999), Poterba (2001), Beetsma and Bovenberg (2009), Lee and Mason (2010)) in order to predict the development of the pension system for specific economies. Simulation models with a few generations are found in Mateos-Planas (2008) ,Gonzalez-Eiras and Niepelt(2008), Krueger and Ludwig(2007). For more realistic worlds analytical results are difficult to find, and we have to rely on model simulations. However, also in those papers, which are mainly dealing with non-stationary demographies and transition paths of specific countries (e.g.USA), we found no systematic analysis of the impact of (changes in) demographic variables on the economic variables mentioned within the framework of a comparative static analysis. One of the complexities, which is ignored in much of the literature, is that the 'first pillar', social security on a PAYG-basis, and the 'second pillar' on a mandatory funded basis are completely different systems, although at first sight they are yielding similar results in the end .Both systems yield benefits for the old and are fed by premiums/contributions/taxes by the young. Some authors argue that unfunded social security should be replaced by mandatory funded pensions, because the old-age dependency ratio in economies with low or even negative population growth will make social security benefits much more expensive than the equivalent benefits provided by a funded system. (see. e.g. Feldstein (1997) or Sinn(2000)). Such recommendations presume that pension systems can be structured at will. However, if we take into account fixed individual preferences and technology, then our model indicates that equilibrium is reached at only one unique composition of the two pension-pillars. The specific composition depends on the demographic process. If we attach some value to the notion of a dynamic equilibrium as a situation the economy tries to approximate under *ceteris paribus* conditions, this should make us rather pessimistic on the possibilities to change our pension structures at will. There is not much room for political choices. Rather we have to take the equilibrium outcomes for granted. Permanent changes will depend on changes in the demography, changes in individual preferences or changes in technology.

In this paper we consider a closed economy where old-age pensions are partly funded and partly unfunded. Such economies will have a stationary equilibrium, where the choice and the size of both pension systems will be endogenously determined by equilibrium conditions. We are interested in how this equilibrium looks like. By means of comparative static analysis we will especially focus on the influence of the birth rate. We compare dynamic stationary equilibria that are generated by different birth rates, including rates corresponding with shrinking populations. The demographic structure is detailed such that it gives a more or less realistic description of a human modern demography, involving a hundred years of maximum lifetime. Then we investigate how the relevant variables change with the birth rate. Except for the realistic demographic structure, there are some rather novel features in this paper that are worth to be noted. First, we investigate the relationship between the interest rate and demography, where the interest rate is considered to be endogenous. Second, we assume that the pension system itself is endogenous. This refers both to the chosen mix between first- and second pillar and to the level of the contributions and benefits in both systems. Third, we model the social decision process with respect to the two pension systems as finding a Nash-equilibrium where the two players are the workers' population on one hand and the electorate as a whole on the other hand.

Our main conclusions from extensive simulations are the following. The interest rate is (among others) a function of the birth rate. The second conclusion is that the benefit- (or replacement-) ratio tends to increase with an increasing population growth rate. A third observation is that in a two-pillar system the role of the first (PAYG) pillar seems to reduce when the population growth rate increases. Fourthly, in this model average population welfare increases with the population growth rate. A final essential result is that a pension system built on a funded *and* an unfunded component does better in terms of average welfare than a system in which there is no social security but a funded system only, except for very low birth rates where a funded system only appears to provide higher average utility than a two-pillar system.

Although our model is a very simplified version of reality, these conclusions seem to reflect the tendencies that we see in our real world of declining birth rates. The real long-term interest rate seems to fall, the benefit ratio seems to fall and the role of the social security PAYG-system seems to increase nowadays.

In Section 2 we sketch the demography. In Section 3 we describe the capital and labour market. In Section 4 we consider an economy with only a funded system and describe how an equilibrium is reached. In section 5 we consider the same problem in a setting where both the funded and unfunded system are functioning side by side. In Section 6 we evaluate our results and we draw some conclusions. We did some sensitivity analyses for different values of some other basic parameters than the birth rate. We sometimes refer to them but the results are not presented in this paper. However, all those results are available from the authors by request.

2. The demographic structure.

As the age composition is crucial for the analysis of the pension problem we start with the demographic model. We assume for convenience that all demographic events, being born and dying, take place at the beginning of the year, so to speak on January 1, 00:01 a.m.

Let $N_t = (N_{0t}, \ldots, N_{mt})$ stand for the population composition where N_{it} stands for the number of citizens in the age cohort *i* at time *t*.

The dynamics of the demography are described by the following linear model:

$$
\begin{aligned}
N_{0,t+1} &= \beta_1 N_{1,t} + \ldots + \beta_m N_{m,t} \\
N_{1,t+1} &= \mu_0 N_{0,t} \\
N_{2,t+1} &= \mu_1 N_{1,t} \\
&. \\
&. \\
&. \\
N_{M,t+1} &= \mu_{M-1} N_{M-1,t}
\end{aligned}
\qquad (2.1)
$$

where *M=100.*

More concisely we write

$$N_{t+1} = \begin{bmatrix} 0 & \beta' \\ M & 0 \end{bmatrix} N_t \qquad (2.2)$$

The first equation is the birth equation with age-specific birth rates *β*. In our numerical examples we assume a specific period of *fertility*, which starts at age *SF*(Start Fertility) and ends at age *EF.* We may think for instance at the age period [25,35]. During that fertility period the birth rate per individual is *β*, while before and after the period of fertility the birth rate equals zero. The average number of children for an individual is then *β* (*EF-SF*) . Taking a fertility period of 10 years and assuming one parent households it implies that the average number of children per parent is 1 if *β* is 0.1. If, as usual, parents are living in couples then the average number of children per household is about 2. Notice that we do not differentiate according to gender. It follows that the reproduction rate in this stylized example is such that *β* (*EF-SF*) =1. For *β* is 0.05, we get the Chinese one-child per couple situation, while *β* is 0.30 stands for the six-children family. We notice that this birth pattern is stylized, however, we have three fertility parameters *SF, EF* and *β*, which make it possible to study the impact of different fertility patterns.
The other equations describe mortality with age-specific survival rates μ. For simplicity, without becoming too unrealistic, we describe the mortality pattern by three parameters: we assume that individuals start dying at the age *SD* (Start Dying) and that the maximum age is reached at age *ED.* In each mortality-prone age cohort a constant fraction μ, for instance 97%, is surviving per year. For the last cohort with age *ED* (=M) the survival rate equals 0. For simplicity we consider a discrete model, where all births and deaths are assumed to take place at the beginning of the year, say, at January 1, before the population is counted.

Solving the equation (2.2) above, we write the equation for short as

$$N_{t+1} = AN_t$$

It is well-known that this system has a positive steady state solution of the type $N_{it} = C.p_i.\lambda^t$, where C stands for a positive constant, the vector *p* for a positive vector, the components of which add up to one, and where λ is a positive scalar. The vector *p* describes the age distribution of the population in the steady state. The scalar λ is the (largest) positive eigenvalue of the system. If λ>1, then the population is growing and if λ<1 the population will shrink over time. However, in the steady state the age distribution vector *p* will remain stable over time. The population growth rate is $\nu = \lambda - 1$. The total population at time *t* is N_t and there holds trivially $N_{t+1} = \lambda.N_t$.

The age distribution looks as follows. Let the fraction of just born be p_0, then the fraction of one-year-old is $p_1 = p_0.\lambda^{-1}$, the fraction of 20-years old is $p_{20} = p_0.\lambda^{-20}$, and the fraction of 64-years old is $p_{64} = p_0.\lambda^{-64}$. If we assume that mortality starts at the age of 65 , that is SD=65 and ends at ED=100, then $p_{65} = p_0.\lambda^{-65}.\mu, \ldots, p_{100} = p_0.\lambda^{-100}.\mu^{36}$. The fraction p_0 is determined by the condition that all age fractions add up to one.

A specially interesting magnitude is the so-called old-age dependency ratio or its inverse which we will use in this paper. The (inverse) old-age dependency ratio is defined as

$$\frac{\text{population between 20 and 64}}{\text{population over 64}}$$

We denote this ratio by $C(\lambda,\mu)$. In terms of the demographic model we get

$$C(\lambda,\mu) = \frac{p_0.\lambda^{-20} + \ldots + p_0.\lambda^{-64}}{p_0.\lambda^{-65}.\mu + \ldots + p_0.\lambda^{-100}.\mu^{36}} \tag{2.3}$$

The population between 20 and 64 is assumed to be active; the retirement age is set at 65. More generally, we could replace those ages by variables SW (Start-Working) and EW (End-Working). Notice that in this example EW+1=SD, that is, individuals start dying in their first retirement year.

In general the inverse dependency ratio (2.3) is

$$C(\lambda,\mu) = \frac{p_0.\lambda^{-SW} + \ldots + p_0.\lambda^{-EW}}{p_0.\lambda^{-(EW+1)}.\mu + \ldots + p_0.\lambda^{-ED}.\mu^{(ED-EW)}} \tag{2.4}$$

We take the survival rate μ at 0.97 and set the initial working age at 20. It is well-known that the inverse old-age dependency ratios in OECD-countries were about 8.0 in the fifties of the previous century and that they are nowadays about 4.0, while they will steadily fall and stabilize at about 2.0 in the fifties of this century. We denote for later use the fraction of active workers by P_A and the fraction of retired by P_R. Notice that $C(\lambda,\mu)=P_A/P_R$.

To get some feeling with this demographic structure we present the following table 1.

Here Table 1.

Table 1. Some demographic structures, varying the birth rate and retirement age.

The tables give some insight into the effect of the birth rate. We start at a birth rate of 0.5 of one- child policy, being considerably less than the reproduction rate 1.0. The corresponding population growth rate is then negative at -0.019, while the old-age (65) dependency ratio is 1. If the retirement rate is 60, there are even more old-aged than active workers, the inverse dependency ratio being 0.73. The median age of the working population is 47, while the median of the total population (retired included, children excluded) lies at 65. Consider now a birth rate of 0.3 (or its equivalent in the real world of about 6 children per female) and we get a positive growth rate of 4%. The old-age inverse dependency ratio increases steeply to 9.05, which means that that there are about 9 active on one old-aged. Median age of the working population is 33 while the median age of the total population (retired included, children excluded) lies at 35. We see that the main outcome, the old-age -dependency ratio, is extremely sensitive for changes in the birth rate. The sensitivity with respect to mortality and fertility periods is somewhat less. This change is realized without any change in the mortality pattern. This makes clear that, although the decrease in mortality and the resulting ageing affects the dependency ratio negatively, the main force behind a fall in the dependency ratio is the fall in fertility. As we will see in the following pages the birth rate will have a strong effect on the economy.

We calculated the same tables (not presented here) with fertility periods 20-30 and 30-40 respectively. There we see that shifting the fertility period has significant effects as well. The same holds for a change of the survival rate from 0.97 to 0.98.

3. The two pension pillars.

Following the usual setup of national pension systems, we distinguish between the first pillar, based on a PAYG-system, and the second pillar, based on a mandatory CR-system. The first system, also called the first pillar, represents national old-age pensions like the American OASDI or Dutch AOW. The structure and volume of such a system, the level of contributions, benefits and eligibility, is determined by Parliament, that is, by the electorate as a whole. The second system of mandatory occupational systems, the second pillar, stands model for the funded pension systems linked to firms or industrial sectors. They are carried out either by specific pension funds or by insurance companies.The level of those pensions is negotiated between employers and trade-unions in collective labour contracts, while the influence of retired workers is mostly negligible. We shall assume no stochastic components in the fund revenues nor inflation. This implies that the funds are offering Defined Benefits.

a. The first pillar.

In the first pillar the pension system follows a pay-as-you-go schedule. Each worker pays a contribution $\sigma_1.w$ at the beginning of the year, where σ_1 stands for the contribution rate and w for the annual uniform wage rate. The retired receive an annual benefit b_1. Since the system must be in balance, the sum of contributions received has to equal the sum of benefits paid out. The balance equation is

$$\sigma_1.w.[p_0.\lambda^{-20} + ... + p_0.\lambda^{-64}] = b_1.[p_0.\lambda^{-65}.\mu + ... + p_0.\lambda^{-100}.\mu^{36}] \qquad (0.1)$$

from which we get using (2.4)

$$b_1 = \sigma_1.w.C(\lambda, \mu) \tag{0.2}$$

It follows that the premium percentage σ_1 to ensure a benefit $b_1 = \pi_1.w$ is $\frac{\pi_1}{C(\lambda,\mu)} = \sigma_1$, where π_1 stands for the gross replacement rate. Given the behaviour of $C(\lambda,\mu)$ as found from Table 1, the population growth rate λ is the determining factor for the benefit ratio. We may interpret the factor $C(\lambda,\mu)$ as the benefit per $ contribution paid. From Table 1 we see that for a growth rate of 4% and a retirement age of 65 this factor is about 9.05, while for a growth rate of about -2.26% the population shrinks and $C(\lambda,\mu)$ becomes very small at 1.21. This is the reason that the PAYG-system becomes so uncomfortable for ageing or even shrinking populations, where $\lambda < 1$. The other side of the medal is that PAYG-systems are very comfortable for fast-growing populations.

b. The second pillar.

The second pillar is more complex to describe. It hangs on two equalities. First, for each individual pension policy there is a fund balance equation such that the discounted expected cash flow of premiums and pension payments equals zero. This yields a relation between the premium contribution σ_2 and the resulting old-age pension b_2. However, this relation depends critically, in a way to be made exact below, on the prevailing rate of interest *r*. Now there are two ways to bring the interest rate into the picture. The first one is to assume a small open economy where *r* can be assumed to be exogenously fixed by the world capital market. However, such a stand cannot be assumed for the USA or other large economies. They are neither open nor closed in the above sense. Actually, the world market for capital has to be seen nowadays as *one* market , where the rate of interest is generated as an equilibrium price that equates supply of and demand for capital and where American capital supply may be matched by European demand for capital or vice versa.

As in most countries the main suppliers of capital tend to become institutional investors representing directly or indirectly the interest of pension funds and pension insurers, we assume for simplicity in the context of this model that the only suppliers of capital are pension funds. This is certainly not true in the literal sense. However, if we see investment funds, hedge funds, etc. as intermediaries of pension funds, and entrepreneurs as temporary investors who disinvest during old age to cover their old-age consumption, the difference between pension funds and direct investors is not that big. The eventual savings motive of both is to earn money during the time of investment and to consume the savings after retirement. Pension funds are the largest institutional investors in global financial markets (see also footnote 1 and Boeri et al. (2006)). This evolution of the last decades makes it necessary to integrate pension funds as a major player in economic theory. The main driving force behind this growth is the ageing of society, increasing longevity and what we would like to call the retirement motive. Individuals are saving during their active period in order to finance living during their retirement.

Abstracting from all possible institutional and phenomenal variations we shall assume in this first analysis that pension funds are the only supplier of capital. Assuming for the moment that the fund contribution rate σ_2 is given, we have three unknowns : b_2, w, r. This implies that we need three equations, viz. the actuarial fund balance equation, a production function and the capital market equilibrium equation in order to determine the three unknowns : b_2, w, r.

The fund balance equation

Let the fund contribution by the workers be $\sigma_2 w$ at the beginning of the year, where σ_2 stands for the contribution rate. Since the system must be in balance, the discounted sum of contributions has to equal the present value of benefits paid out during an individual's lifetime. The balance equation at age 20 when the pension contract starts is

$$\sigma_2.w.[1+...+(1+r)^{-44}] = b_2.[(1+r)^{-45}.\mu+...+(1+r)^{-80}.\mu^{36}] \quad (0.3)$$

where r is the prevailing interest rate.

When multiplying both sides by $(1+r)^{-20}$ and using (2.4), we see that we may rewrite the equation as

$$b_2 = \sigma_2.w.C(1+r,\mu) \quad (0.4)$$

The equations (3.2) and (3.4) look very similar. Actually, the factors in λ are replaced by factors $(1+r)$. We may interpret the factor $C(1+r,\mu)$ as the benefit per $ contribution paid under the capital-reserve system.

The capital demand equation.

We assume all firms to have the same production function operating under perfect competition. Let us assume that production is linearly homogeneous according to a Cobb-Douglas production function $Y = F(L,K) = L^{1-\alpha}.K^{\alpha}$, where we chose a suitable money unit. The production per labour unit is $y = f(k) = k^{\alpha}$, where k is capital per labour unit. The firm maximizes profit with respect to k, that is

$$\max_k f(k) - w - (r+\delta)k$$

where r is the interest rate and δ is the rate of capital cost per year. This cost factor δ consists of the usual margin between lending and borrowing rate, the depreciation cost of capital, but also of innovation cost, maintenance cost and the annual cost of acquiring new capital. In short, the cost of capital for the firm is higher, and mostly considerably higher, than the interest rate associated with savings accounts. We see that annual capital demand is found from the equation

$$f'(k) = r + \delta \tag{0.5}$$

This is the capital demand function for capital per worker.
Due to the linear homogeneity there holds

$$f(k) = w + (r+\delta)k \tag{0.6}$$

Equilibrium on the capital market
Next, there has to hold equilibrium on the capital market. The pension funds are the suppliers of capital and the firms demand the capital. For a given σ_2, equation of capital supply and demand yields an equilibrium rate of interest.

First, let us look at the demand side. In case of a Cobb-Douglas production function, where α stands for the capital elasticity, the annual capital demand is found from the equation

$$f'(k) = \alpha k^{\alpha-1} = r + \delta \tag{0.7}$$

where marginal capital productivity is set equal to capital cost ($r+\delta$). Solving for *k* we denote the demand for capital per working place by

$$k = \left(\frac{\alpha}{r+\delta}\right)^{1/1-\alpha} \overset{def}{==} g(r+\delta) \tag{3.7a}$$

Demand falls with rising interest rate. Using linear homogeneity we get

$$w = k^{\alpha} - k.(r+\delta) \tag{0.8}$$

Notice that the wage rate depends via the two terms on ($r+\delta$).

Hence, for given *r* and σ_2 using (3.7) and (3.8) we find a unique benefit level $b_2 = b_2(r,\sigma_2) = \sigma_2.w.C(1+r,\mu)$ from (3.4). However, the problem is that *r* and σ_2 are not fixed yet. We assume σ_2 to be known for the moment; in the next section we will make σ_2 endogenous as well. We can determine the market interest rate *r* as follows.

We have an equilibrium interest rate at the point where demand for capital equals capital supply. Capital demand is described by (3.7a) .

The supply of capital is an aggregate. Each cohort has its own capital reserve built up. Aggregate capital is found by adding the capital reserves of all the age cohorts. We calculate the capital reserve built up for each age cohort separately. Averaging those cohort reserves with their population shares we get the average capital supply per individual.

The individual capital reserve profile per age cohort over life looks like a parabola with a maximum at 65, which intersects the age axis at 20 (SW) and 100 (ED). The reserves of children (<20) in this context are set at zero.

Let us denote the reserve of cohort *age* as $R(age, r)$, then the aggregate supply of capital per head of the population is found by population-share-weighted summation of the cohort reserves. For a given interest level *r* the total capital reserve per head of population, denoted by $\bar{k}$, is

$$\bar{k} = \sum_{age=20}^{100} p_{age}.R(age,r) = \sigma_2.w.[\sum_{age=20}^{64} p_{age}\sum_{i=20}^{age}(1+r)^{age-i} + C(1+r,\mu)\sum_{age=65}^{100} p_{age}\sum_{i=age}^{100}\mu^{i-age+1}(1+r)^{-(age-i)}] \quad (0.9)$$

where we substituted (3.4).

The capital supply per active worker (<65), that is per workplace, denoted by $\tilde{k}$, is then found by division by the population share P_A of the active population to be

or for short

$$\tilde{k}(r) = \frac{1}{P_A}\bar{k}(r) \quad (0.10)$$

Where $\bar{k}(r)$ is implicitly defined, and where *w* is also a function of *r* according to (3.8). We notice that capital supply also depends on the age distribution. If the population ages up to a certain point, the reserves per head will increase, since the median age in the fund increases, and older workers have saved more. However, for very old populations the reserves are again decreasing.

Since capital supply is proportional to σ_2, we can find for a given value of σ_2 a solution *r* of the equation

$$k(r) = g(r+\delta) = \tilde{k}(r) \quad (0.11)$$

We can also work inversely. For a given *r* we can find the corresponding σ_2 by solving the equation (3.12), yielding

$$\sigma_2 = \frac{g(r+\delta).P_A}{w(r+\delta).[\sum_{age=20}^{64} p_{age}\sum_{20}^{age}(1+r)^{i-20} + C(1+r)\sum_{age=65}^{100} p_{age}\sum_{i=age}^{100}\mu^{i-age-1}(1+r)^{-(100-i)}]} \quad (0.12)$$

For each value of *r* we find one contribution rate σ_2, for which capital demand equals capital supply. It also follows that σ_2 determines the interest rate under *ceteris paribus* conditions.

Here Table 2

Table 2. Premium funded system for full employment for varying birth rates and interest rates.

It is obvious that no simple analytical explicit solutions can be given. In table2 we present numerical values for σ_2, computed by (3.13), for various interest rates with δ=0.05 and δ=0.10.

The underlying parameter values are capital productivity α=0.25 , an annual survival rate $\mu = 0.97$ from the age of 65, and a fertility period from age 25 to 35.

We observe that, contrary the common belief, the interest rate may become negative if δ>0. This is due to the fact that workers are not saving their savings in cash, even if there would be no inflation. They are willing to pay a certain amount for asset management. Negative interest rates have occurred in Switzerland and in 2011 negative interest rates have been accepted by large companies for being able to store their liquidities at first-rate banks like RABO.

According to the outcomes in Table 3 we see, for example, that a prevailing interest rate of 1% a depreciation rate of 5% and a birth rate of 0.10, the premium would be 12,57 %. More generally, the interest rate falls with a rising savings rate for relevant parameter values. This relation between r and σ_2 appears to be monotonic. If the birth rate increases, the equilibrium contribution rate σ_2 has to increase as well. This is caused by the fact that a growing population needs more capital investment for the new workplaces. This has to be financed by an increasing fund. Finally, we look at the effect of depreciation cost. They lead to a falling interest rate and a falling savings rate.

However, in this model there is still one parameter undecided. That is the savings rate σ_2 under a funded system. How much do individuals want to spend on their old age? This will be looked after in the next section. In a two –pillar system the question is still more complex, what are equilibrium values for σ_2 *and* the first-pillar contribution rate σ_1 ? The latter question will be postponed to Section 5.

4. Making the decisions on capital reserves in a 'second-pillar' only system.

Now we approach the main question in this paper. How are the contribution rates σ_1 for the first pillar and σ_2 for the second pillar determined? For, if we know that, we have explained the genesis and size of the two pillars and as a side- result the level of interest rate. There is a certain hierarchy between the two systems. A modern world without a second –pillar, being the main or only supplier of capital, is inconceivable. It would imply the absence of a capital stock and hence zero production. On the contrary, a world without a first pillar may be undesirable, but is perfectly conceivable.

In this section we will consider the situation where there is only a funded pension system. In the next section we consider the situation where the two pillars may exist side by side.

Thus far we have taken the contribution rate σ_2 as exogenously fixed. In reality this is the key parameter to be determined by society. The decision on the contribution rate σ_2 and the ensuing benefit level b_2 is assumed to be the outcome of a democratic decision of the active workers. It is not a state decision, hence it is no business for the whole electorate. The retired do not have a vote in this. The employers have no interest whatsoever in how employees divide their earnings between present and future consumption[2]. Given the fact that older workers will lay more weight on their future pension than younger workers, it is obvious that older cohorts will be in favour of a higher contribution rate σ_2 than younger workers. The pivotal age cohort is here the cohort of the median worker M_{WORK}. The median worker faces a remaining lifetime with survival chances $\overbrace{1,...,1}^{64-M_{WORK}+1}, \overbrace{\mu,\mu^2,...,\mu^{36}}^{36YEARS}$,where μ stands for the retired's annual survival rate, taken constant for convenience. The corresponding income stream is

$$\overbrace{(1-\sigma_2)w,...,(1-\sigma_2)w,}^{64-M_{WORK}+1} \; \overbrace{\sigma_2 w.(\mu.C(\mu,1+r),...,\sigma_2 w.(\mu^{36}.C(\mu,1+r)}^{36YEARS}$$

[2] This would be different if employers would make promises on indexation, but in this context inflation and productivity growth have been excluded.

We assume that each individual has a subjective time discount function $e^{-\rho t}$, where ρ stands for the time discount parameter. We will take for our numerical simulations the values $\rho = 0.05$ or $\rho = 0.10$.

Individuals have an instantaneous utility function $U(y)$ where y stands for net disposable income. The utility flow of an individual of age M_{WORK} is $\overbrace{U(1-\sigma_2)w,...,U(1-\sigma_2)w,}^{(64-M_{WORK}+1)YEARS}\ \overbrace{\mu.U(\sigma_2.w.(C(\mu,1+r))),...,\mu^{36}U(\sigma_2.w(r+\delta).(C(\mu,1+r))}^{36YEARS}$

Now we may calculate the discounted value of the utility flow of the median worker. We have

$$\bar{U}(M_{WORK}) = \frac{1}{Tp\mu}[\sum_{t=M_{WORK}}^{64} (e^{-\rho})^{t-M_{WORK}}.U((1-\sigma_2)w) + \sum_{t=65}^{100}(e^{-\rho})^{t-M_{WORK}}\mu^{t-64}.U(w(r+\delta).(\sigma_2.C(1+r,\mu))]$$

where $$Tp\mu = \sum_{t=M_{WORK}}^{64} (e^{-\rho})^{t-M_{WORK}} + \sum_{t=65}^{100} (e^{-\rho})^{t-M_{WORK}}\mu^{t-64} \qquad (0.13)$$

In (4.1) the utility of the remaining life period is split up in a part before and after retirement. The difficulty when we try to maximize (4.1) with respect to σ_2 is that the interest rate r and the wage rate w will be affected by a change in σ_2 due to the equality between demand and supply on the capital market.

An analytical solution appears impossible. However, a numerical solution is feasible.

We calculate for each combination (β, r, σ_2) in Table 3 the corresponding $\bar{U}(M_{WORK})$ according to (3.18), where we lay a 0.1 –grid on r. Since σ_2 is found to be monotonic in r, a one-dimensional search process can be used.

Here Table 3

Table 3, System with only Funded Pensions and no Social Security

In Table 3 we present equilibrium values for various values of the birth rate. In the first panel we assume a time preference discount of 5 %. The utility function is taken to be a

CRRA-function (Arrow (1965), Pratt(1964)) with a constant relative risk aversion equal to $\gamma = 3.0$. The utility function is $U(y) = 1 - y^{1-\gamma}$.

Let us now consider table[3] 4 with $\rho = 0.05, \delta = 0.05$. We see a considerable effect of the birth rate. The rate of interest increases monotonically with the population growth rate λ, where the point elasticity with respect to λ is about 2. It is really very surprising that the birth rate has such a rather large effect on the interest rate. The interest rate equals – 2.75 % for a shrinking population with an average fertility of 1 child per couple, while the interest rate becomes 4 % for a high fertility of 6 children per couple. The funded premium percentage falls from 47% at 1 child per couple to about 9 % when fertility is 6 children per couple. The benefit ratio, that is the ratio between workers' net wage and the benefit of the retired is not monotonously increasing but tends to one if the population becomes more fertile.

An interesting question is how labour income is redistributed by the system over workers and retired. This cannot be seen immediately from the benefit ratio, since the population shares vary with the birth rate. We see that for low fertility 60% of the labour income is retained by the working population, while 40 % is redistributed over the retirees. For high fertility levels the part assigned to retirees becomes about 10%, even if the benefit ratio increases considerably. This is, of course, partly due to the fact that the population share of the retirees is much lower in a fast growing and hence younger population than in an old and shrinking population.

A second point of interest is the following. In the society considered the workers and retirees together are the owners of the pension reserve, which equals the stock of capital. So to say they may be seen as the shareholders in society. The question is then which percentage of the shares may be assigned to the workers and which percentage to the retirees. We find from the first panel that for an old and shrinking population the capital share of workers equals about 56%, and that the workers' share increases to about 71 % for a young and fast-growing population. Evidently, the final criterion is average social instantaneous utility, say, $\bar{U} = p_{<65}.U(netwage) + p_{\geq 65}.U(pension)$.

We see that the population's utility increases with rising birth rates from -2.06 to -1.08.

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[3] Tables for $(\rho = 0.10, \delta = 0.05), (\rho = 0.05, \delta = 0.10), (\rho = 0.10, \delta = 0.10)$ are available on request.

When the time discount rises to $\rho = 10\%$ we see similar phenomena, where pensions are lower and net wages higher, as expected. This is to be expected as a higher ρ implies that the future is weighed less heavily than in the case, where $\rho = 5\%$. The inequality between workers and retirees increases, as is obvious from the benefit ratio for shrinking populations which falls from 0.66 for $\rho = 5\%$ to 0.48 for $\rho = 10\%$. For fast growing populations the difference tends to become small. The same holds for the average utility. However, the workers' share in labour income is much greater.
The effect of the depreciation rate doubling to 10 % is rather drastic. Interest rates start at 11.5% and fall with the increasing birth rate to 8.25%. This behaviour of interest ,however, is not monotonous. The benefit ratio is much lower than in the case of $\delta = 5\%$. Although the funded premium percentage is insignificant, the pension is not completely nil, because the interest ratio is so high. Utility increases , but is much below the utility levels reached in the case of δ= 5% .
It is a signal that the role of capital cost is very important. The share of labour income assigned to workers is above 80 % throughout. However, the share of workers in the capital stock is rather meagre and for reasonable growth rates stays below 50%.
The effects of the depreciation costs δ are considerable. If δ = 10%, the premiums are higher, while net wages and pensions become lower than in the corresponding case of δ = 5 %.

5. *Decision making in a two –pillar system.*

The two-pillar system differs from the one-pillar system considered before in at least three important aspects. First, the contributions in the PAYG-system are redistributed immediately and do not add to the capital stock. Second, the benefit of the PAYG-system is $C(\lambda,\mu)$ per dollar contribution, while the benefit from the funded system is $C(1+r,\mu)$ per dollar contribution.

The third difference is decisions on either system are taken by different decision bodies. The group that decides on the structure and size of occupational pensions is not identical with the group that decides on the structure and size of the social security system or in this context the old-age state pension. On one hand the occupational pensions are negotiated between trade unions and employer unions in collective bargaining. The outcome is mandatory to all workers in the relevant sector, firm or industry. Typically, the retired workers have no say in these negotiations. We call this decision body the 'workers' for short. On the other hand, the social security system is fixed by law, and it is the electorate as a whole, including the retired, which decides about social security.

Although the workers are a sub-group of the electorate, the two groups have different interests at stake. The electorate is older than the group of workers and hence the electorate as a whole puts more weight on their pension situation than the workers.

According to (3.2) contributions and benefits in the first pillar are linked by

$$b_1 = \sigma_1.w(r+\delta).C(\lambda,\mu) \tag{0.14}$$

Analogously to (4.1) the average utility of cohort *age* over the remaining lifetime is

$$\bar{U}(age) = \frac{1}{Tp}[\sum_{t=age}^{64} e^{-\rho(t-age)}.U((1-\sigma_1-\sigma_2)w(r+\delta)) + \sum_{t=65}^{100} e^{-\rho(t-age)}\mu^{t-64}.U(w(r+\delta).(\sigma_1 C(1+\lambda,\mu)+\sigma_2.C(1+r,$$

(0.15)

where $Tp\mu = \sum_{t=M_{POP}}^{64} (e^{-\rho})^{t-M_{WORK}} + \sum_{t=65}^{100} (e^{-\rho})^{t-M_{WORK}} \mu^{t-64}$.

It follows that when *age* increases the weight of the period of retirement becomes heavier in the weighted sum above and the weight of the remaining working period lower. It follows that with increasing *age,* individuals, when optimizing their lifetime utility, tend to increase the values of σ_1 and σ_2 they consider optimal. This gives again clearly a pivotal role to the *median* cohort.

However, as already said, things are a bit complicated because the populations that decide about σ_1 and σ_2 are not the same. In most western democracies social security is decided by lawmakers, who depend for their election on the whole electorate, that is the workers *and* the retired. Decisions over the funded collective pensions are made by the workers only.

It follows then that the median cohort of the whole population, say at age M_{POP}, decides about σ_1, while the median cohort of the active labour force, say M_{WORK}, decides about the level of σ_2.

We shall assume then that σ_1 and σ_2 are determined as the solution of two equations:

$$\begin{aligned} \frac{\partial \bar{U}(M_{POP};\sigma_2)}{\partial \sigma_1} = 0 \\ \frac{\partial \bar{U}(M_{WORK};\sigma_1)}{\partial \sigma_2} = 0 \end{aligned} \tag{0.16}$$

where the electorate optimizes σ_1 , assuming σ_2 as given, and where the workers optimize σ_2 , assuming σ_1 to be given. The solution is a kind of Nash-equilibrium where the two players are the work force and the whole population. They partly overlap, but they are not the same. The power of both parties is clearly determined by the demography.

The solution may be found either by direct computations or by iteration. The results for the two-pillar system are presented in Table 5.

Evidently, the results of Table 5 may be compared with those in Table 4, but we have to keep in mind that the results in Table 5 are not optimal for either of the two parties, workers and retirees, but that they represent a Nash equilibrium between both parties, it is a compromise. Moreover, the admissible space is enlarged, because parties can realize mixed solutions, where part of the pension is in the first pillar, and part stems from the second pillar. [4]

Let us now again consider the case δ = 5 %, $\rho = 5\%$ in more detail.

Here Table 4

Table 4. System with Funded Pensions and Social Security Pensions

The interest rate again varies with the birth rate but less conspicuously as in the case of a second pillar only. The sum of the premium contributions varies from over 56.6% for a shrinking population to 12.5% for a fast- growing population[5]. Moreover, for the shrinking population the accent lays on the state pension, while for the fast-growing population the funded system is by far the most important system, The net benefit ratio hovers around 1.1, implying that the retiree's net income is higher than the net income of the worker. The share of the active population in labour income is for shrinking populations in the order of 0.5 and rises for increasing populations to around 80%. The same tendency is found with respect to the nation's capital stock. Average utility is higher than in the case of a funded system only, except for vary low birth rates where a second-pillar only system does definitely better. We present average utility as a function of the birth rate in Figure 1. On the horizontal axis we have the birth rate and on the vertical axis the average utility under the two systems.

[4] In these computations we have set an upper limit of $\sigma_1 = 50\%$.

[5] The first line must taken with a grain of salt, since the upper limit $\sigma_1 = 50\%$ is hit.

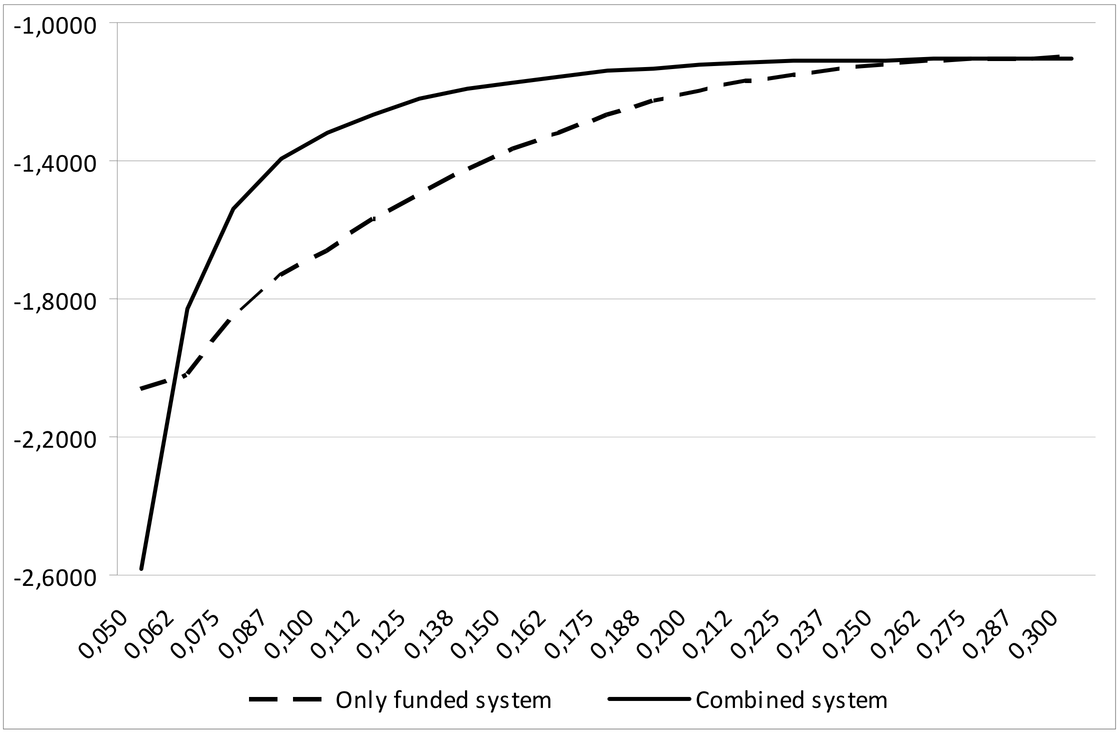

Figure 1a. Behavior of average utility as a function of the birth rate in the two systems , $\rho = 5\%, \delta = 5\%$.

If the time preference discount rate is equal to 10 %, changes are not much. The only point of difference is that for fast-growing economies the role of the social security system vanishes completely. This implies that the natural outcome of the compromise between the labour force and the electorate as a whole mostly yields better outcomes in terms of utility as a world without a pay-as-you-go state pension.

The effect of an increase of the capital costs δ from 5% to 10%, while $\rho = 5\%$ is remarkable. It is found that the utility level is remarkably diminished and this is due to the increase in the cost of capital. Secondly, since the PAYG-system has no capital costs the PAYG-system becomes more attractive than in the case of δ = 5%. This also explains why even for very lower birth rates it still remains advisable to have a mix of both systems, contrary to the situation of δ = 5%, where for very low birth rates a funded only –system does better than a mixed system.

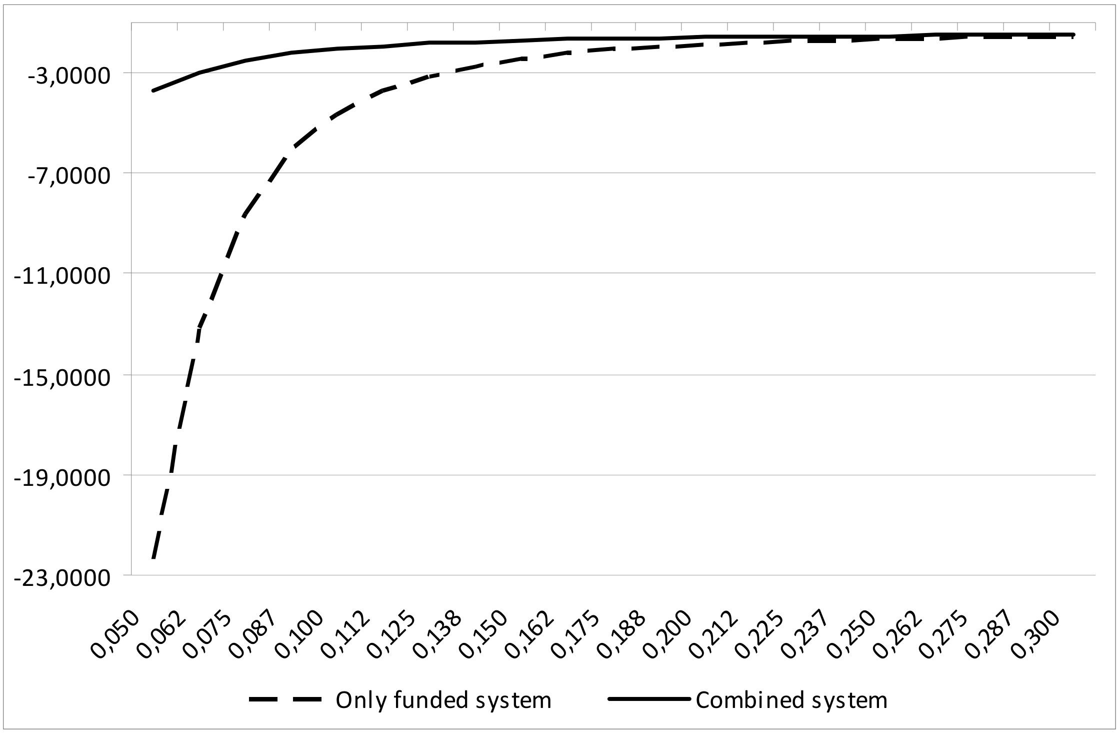

Figure 1b. Behavior of average utility as a function of the birth rate in the two systems , $\rho = 5\%, \delta = 10\%$.

6.Discussion and conclusions.

We do not pretend that the model sketched above gives an exact portrayal of the real economy. It assumes a stationary demography, while in reality the demography is changing all the time.. We did not assume technological growth, inflation or uncertainties, and we ruled out unemployment. We also ignored the education costs of children. It would be easy to list another series of other simplifying assumptions. Finally, in the present paper we made the assumption that the only capital supplier to firms are pension funds which either directly or via institutional investors channel their reserves. Although this is in essence the life-cycle saving assumption, the explicit assumption that savings are mainly channelled by pension funds or via institutional investors ,seems rather novel in the literature. Although it is not wholly true, actual tendencies during the last decades show that pension funds and insurers have become by far the first player on the capital market and that this tendency is still actual. The usual way in economic theory is to work with simplified models that focus on the core question. Here the core question is what the effects are when institutionalized pension arrangements become the main suppliers on the capital market. In order to simplify things, we assumed that pension funds were the only suppliers, although this is not 100% correct. However, if there are effects with 100% supplier-ship, we may safely assume that similar effects are realistic when the actual supplier-ship is only 80 %. Against this background we may safely conclude that the introduction of pension funds, both on a funded and unfunded basis, leads to new insights in macro-economic theory. It reveals that not only population growth, but also the specific age distribution has a strong impact on the level of savings, the resulting interest rate and the choice with respect to a mix of the two pension systems. Moreover it determines wage levels and the capital invested per workplace. Numerical calculations on the basis of our model specification show that there is a unique equilibrium found within the relevant parameter range. In this paper we concentrated our attention on the effect of the birth rate and our first main result is that the interest rate in a two-pillar system depends among others on the birth rate and that for the usual range the interest rate increases with the birth rate. This finding would give a theoretical underpinning for the actual conjecture that real long-term interest rates over the world tend to fall. According to this model this fall might be explained by the steady decline of birth rates in the developed world. The second result is that the mix between funded and unfunded systems may be seen as an endogenous result of the system. If the birth rate rises, the role of the unfunded system declines or even vanishes, while inversely with falling birth rate the role of the unfunded system appears to grow.

A third important result is that average utility or welfare increases with increasing birth rates, while at the same time inequality between workers and retired, measured by the benefit ratio, appears to reduce. A fourth essential result is that a pension system built on a funded and an unfunded component does mostly better in terms of average welfare than a system which consists of a funded system only.

If the way in which and how far pensions are collectively insured has such an important effect on the economy as a whole, it seems desirable that this point should be firmly incorporated in macro-economic theory. This first pape , as already said, abstracts from a great number of details which are important in the real world. Our plan is, as usual in economic analysis, to investigate the effect under more realistic assumptions in future research. For the time being we think it worthwhile to make the first preliminary outcomes already known.

References

Henry Aaron (1966), The Social Insurance Paradox Henry Aaron (1966), *The Canadian Journal of Economics and Political Science / Revue canadienne d'Economique et de Science politique*, 32, , 371-374

Arrow, K.J.,1965, "The theory of risk aversion," in Aspects of the Theory of Risk Bearing, by Yrjo Jahnsson in Saatio, Helsinki. Reprinted in: *Essays in the Theory of Risk Bearing*, Markham Publ. Co., Chicago, 1971, 90–109.

Auerbach A., L.Kotlikoff (1987),*Dynamic fiscal policy* , Cambridge University Press.

Barr , N. , P. Diamond, 2006, The Economics of Pensions, *Oxford Review of Economic Policy*, vol. 22, no. 1,15-39

R. M. W. J. Beetsma and A. L. Bovenberg, Pensions and Intergenerational Risk-sharing in General Equilibrium, *Economica* (2009) 76, 364–386

Boeri, T., Bovenberg, A., Coeuré, B. and Roberts, A. 2006. *Dealing with the New Giants: Rethinking the Role of Pension Funds.* London, Centre for Economic Policy Research. http://www.cepr.org/pubs/books/P182.asp

Börsch-Supan, A, (2004), Mind the Gap: The Effectiveness of Incentives to Boost Retirement Saving in Europe, Discussion Paper no. 52-04, Mannheim Research Institute for the Economics of Aging.

Boersch-Supan A.H.,A.Ludwig (2010). "Old Europe ages: reforms and reform backlashes", NBER Working Paper 15744, http://www.nber.org/papers/w15744

Bovenberg A.L. and T. Nijman(2009), "Developments in pension reform: the case of Dutch stand-alone collective pension schemes ",*International Tax and Public Finance,* Vol. 16, No. 4, pp. 443-467.

Casamatta, G, Cremer, H, & Pestieau, P 2000, 'The Political Economy of Social Security', *Scandinavian Journal of Economics*, 102, 3, pp. 503-522

Conference Board (2010) *The 2010 Institutional Investment Report Trends in Asset Allocation and Portfolio Composition.*

Davis, S., Lukomnik J., Pitt--Watson, D. (2006), *The New Capitalists: How citizen investors are reshaping the corporate agenda.* Harvard Business School Press. Boston.

Diamond, P. A. (2002), *Social Security Reform*, Oxford and New York, Oxford University Press.

Diamond, P. A. (2004), 'Social Security', *American Economic Review*, **94**(1), 1–24.

Feldstein M. (1985), The Optimal Level of Social Security Benefits, *The Quarterly Journal of Economics* Vol. 100, No. 2 (May, 1985), pp. 303-320

Feldstein (1997), *Transition to a fully funded pension system: five economic issues*, NBER .w.p.6149

Galasso V. (2008), 'Postponing retirement: the political effect of aging', *Journal of Public Economics* 92 (2008) 2157–2169

Galasso, V., Profeta, P., 2004. 'Lessons for an aging society: the political sustainability of social security systems'. *Economic Policy* 19, 63–115.

Gonzalez-Eiras M., D.Niepelt, The future of social security, *Journal of Monetary Economics*, Volume 55, Issue 2, March 2008, Pages 197-218

Groesbeek Janssen, M. (2011) , *Corporate citizenship in a civil economy*, report presented at a conference of the Institute for New Economic Thinking Sovereignty and Institutional Design in the Global Age: The Global Market and the Nation States ,Bretton-Woods.

Krueger D., A.Ludwig, (2007) , 'On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare', *Journal of Monetary Economics* 54,49–87,54, 2007

Lee, R., A Mason (2010), 'Some Macroeconomic Aspects of Global Population Aging,'*Demography*, Volume 47, Supplement, 2010, p. S151-172

Mateos-Planas X. (2008) "A quantitative theory of social security without commitment" *Journal of Public Economics*, 62, 652-671, 2008.

Miles, David, 1999. "Modelling the Impact of Demographic Change upon the Economy," ***Economic Journal***, , vol. 109(452), pages 1-36

OECD (2011), *Pensions at a Glance* 2011, RETIREMENT-INCOME SYSTEMS IN OECD AND G20 COUNTRIES.

Poterba , J.M.,2001. "Demographic Structure and Asset Returns"., *The Review of Economics and Statistics*, Vol. 83, No. 4 ,pp. 565-584

Pratt, J. W., "Risk aversion in the small and in the large," *Econometrica* 32, January–April 1964, 122–136. i

Samuelson, P. A. (1954), 'An Exact Consumption–Loan Model of Interest with or without the Social Contrivance of Money', *Journal of Political Economy*, **66**(6), 467–82; reprinted in N. Barr (ed.) (2001), *Economic Theory and the Welfare State*, Edward Elgar Library in Critical Writings in Economics, Vol. II, 63–78.

Sinn, Hans-Werner. "Why A Funded Pension System Is Needed And Why It Is Not Needed," *International Tax and Public Finance*, 2000, v7(4/5, 389-410.

Table 1. Some demographic structures, varying the birth rate and retirement age.

Birth rate	Grows rate	Retirement at 60 median			Retirement at 65 Median			Retirement at 70 median		
		Work-ers	Popu-lation	Inverse Depend-ency ratio	Work-ers	Popu-lation	Inverse Depend-ency ratio	Work ers	Popu-lation	Inverse Depend-ency ratio
Survival rate 0.97, Start fertility at 25, End fertility at 35										
0.050	-0.019	43	65	0.73	47	65	1.00	50	65	1.40
0.062	-0.012	42	60	0.94	45	60	1.30	48	60	1.81
0.075	-0.006	41	57	1.17	44	57	1.61	46	57	2.25
0.087	-0.001	40	54	1.41	42	54	1.94	45	54	2.71
0.100	0.003	39	51	1.65	41	51	2.28	43	51	3.20
0.112	0.007	38	49	1.89	40	49	2.63	42	49	3.71
0.125	0.010	37	47	2.14	39	47	2.99	41	47	4.25
0.138	0.014	37	45	2.40	39	45	3.36	40	45	4.81
0.150	0.016	36	44	2.66	38	44	3.75	39	44	5.39
0.162	0.019	36	43	2.93	37	43	4.14	39	43	5.99
0.175	0.022	35	42	3.20	37	42	4.55	38	42	6.61
0.188	0.024	35	41	3.47	36	41	4.96	37	41	7.25
0.200	0.026	35	40	3.75	36	40	5.38	37	40	7.91
0.212	0.028	34	39	4.03	35	39	5.81	36	39	8.59
0.225	0.030	34	38	4.32	35	38	6.25	36	38	9.28
0.237	0.032	34	38	4.61	35	38	6.70	36	38	10.00
0.250	0.034	33	37	4.90	34	37	7.15	35	37	10.73
0.262	0.035	33	37	5.19	34	37	7.62	35	37	11.48
0.275	0.037	33	36	5.49	34	36	8.09	34	36	12.25
0.287	0.038	33	36	5.79	33	36	8.56	34	36	13.03
0.300	0.040	32	35	6.10	33	35	9.05	34	35	13.83

Table 1

Table 2. Premium funded system for full employment for varying birth rates and interest rates.

Birth rate	Interest rate -0.02	-0.01	0.00	0.01	0.02	0.03	0.04
	Depreciation rate 0.05						
0.050	0.3046	0.1876	0.1216	0.0809	0.0547	0.0372	0.0254
0.062	0.3439	0.2135	0.1395	0.0936	0.0638	0.0437	0.0301
0.075	0.3785	0.2366	0.1556	0.1052	0.0721	0.0498	0.0345
0.087	0.4094	0.2574	0.1704	0.1158	0.0799	0.0555	0.0386
0.100	0.4375	0.2765	0.1839	0.1257	0.0871	0.0608	0.0425
0.112	0.4632	0.2940	0.1965	0.1349	0.0940	0.0659	0.0463
0.125	0.4869	0.3103	0.2082	0.1436	0.1004	0.0707	0.0499
0.138	0.5089	0.3255	0.2193	0.1518	0.1066	0.0753	0.0534
0.150	0.5295	0.3398	0.2296	0.1595	0.1124	0.0798	0.0567
0.162	0.5488	0.3532	0.2395	0.1669	0.1180	0.0840	0.0599
0.175	0.5670	0.3659	0.2488	0.1739	0.1233	0.0881	0.0630
0.188	0.5841	0.3780	0.2577	0.1806	0.1285	0.0920	0.0660
0.200	0.6004	0.3894	0.2661	0.1871	0.1334	0.0958	0.0689
0.212	0.6159	0.4003	0.2742	0.1932	0.1382	0.0995	0.0718
0.225	0.6306	0.4108	0.2820	0.1991	0.1427	0.1030	0.0745
0.237	0.6447	0.4207	0.2895	0.2049	0.1472	0.1065	0.0772
0.250	0.6582	0.4303	0.2966	0.2104	0.1515	0.1098	0.0798
0.262	0.6711	0.4395	0.3035	0.2157	0.1556	0.1131	0.0823
0.275	0.6835	0.4484	0.3102	0.2208	0.1596	0.1162	0.0848
0.287	0.6954	0.4569	0.3166	0.2258	0.1635	0.1193	0.0872
0.300	0.7069	0.4651	0.3228	0.2306	0.1673	0.1223	0.0896

	Depreciation rate 0.10						
0.050	0.1142	0.0834	0.0608	0.0441	0.0319	0.0229	0.0163
0.062	0.1290	0.0949	0.0698	0.0511	0.0372	0.0269	0.0193
0.075	0.1419	0.1052	0.0778	0.0574	0.0421	0.0306	0.0222
0.087	0.1535	0.1144	0.0852	0.0632	0.0466	0.0341	0.0248
0.100	0.1641	0.1229	0.0920	0.0686	0.0508	0.0374	0.0274
0.112	0.1737	0.1307	0.0982	0.0736	0.0548	0.0405	0.0298
0.125	0.1826	0.1379	0.1041	0.0783	0.0586	0.0435	0.0321
0.138	0.1909	0.1447	0.1096	0.0828	0.0622	0.0464	0.0343
0.150	0.1986	0.1510	0.1148	0.0870	0.0656	0.0491	0.0365
0.162	0.2058	0.1570	0.1197	0.0910	0.0688	0.0517	0.0385
0.175	0.2126	0.1626	0.1244	0.0949	0.0720	0.0542	0.0405
0.188	0.2191	0.1680	0.1288	0.0985	0.0749	0.0566	0.0424
0.200	0.2252	0.1731	0.1331	0.1020	0.0778	0.0590	0.0443
0.212	0.2310	0.1779	0.1371	0.1054	0.0806	0.0612	0.0461
0.225	0.2365	0.1826	0.1410	0.1086	0.0833	0.0634	0.0479
0.237	0.2418	0.1870	0.1447	0.1117	0.0858	0.0655	0.0496
0.250	0.2468	0.1913	0.1483	0.1147	0.0883	0.0676	0.0513

0.262	0.2517	0.1953	0.1518	0.1176	0.0908	0.0696	0.0529
0.275	0.2563	0.1993	0.1551	0.1204	0.0931	0.0715	0.0545
0.287	0.2608	0.2031	0.1583	0.1232	0.0954	0.0734	0.0561
0.300	0.2651	0.2067	0.1614	0.1258	0.0976	0.0752	0.0576

Table 3, System with only Funded Pensions and no Social Security

Birth rate	Interest rate	Premium funded pension	Benefit ratio	Gross wage	% net wage	Pension	workers' share income	workers' share capital	maximum utility value	Average utility value
Depreciation rate 0.05										
Time preference parameter 0.05										
0.050	−0.0275	0.4667	0.6626	1.6736	0.5333	0.5913	0.6012	0.5570	−1.7107	−2.0581
0.062	−0.0225	0.3925	0.5875	1.5653	0.6075	0.5587	0.6882	0.5886	−1.6340	−2.0193
0.075	−0.0200	0.3785	0.6070	1.5206	0.6215	0.5737	0.7261	0.6187	−1.5750	−1.8549
0.087	−0.0175	0.3621	0.6204	1.4805	0.6379	0.5859	0.7573	0.6426	−1.4997	−1.7314
0.100	−0.0125	0.3082	0.5855	1.4116	0.6918	0.5718	0.7953	0.6553	−1.4497	−1.6624
0.112	−0.0100	0.2940	0.6004	1.3815	0.7060	0.5856	0.8140	0.6715	−1.4027	−1.5654
0.125	−0.0050	0.2531	0.5878	1.3283	0.7469	0.5832	0.8357	0.6782	−1.3583	−1.4980
0.138	−0.0025	0.2412	0.6054	1.3046	0.7588	0.5992	0.8475	0.6895	−1.3347	−1.4248
0.150	0.0025	0.2092	0.6071	1.2618	0.7908	0.6057	0.8606	0.6922	−1.2941	−1.3669
0.162	0.0100	0.1669	0.6087	1.2069	0.8331	0.6120	0.8719	0.6866	−1.2567	−1.3160
0.175	0.0150	0.1463	0.6285	1.1751	0.8537	0.6305	0.8786	0.6864	−1.2358	−1.2679
0.188	0.0200	0.1285	0.6533	1.1464	0.8715	0.6527	0.8836	0.6851	−1.2043	−1.2275
0.200	0.0250	0.1130	0.6826	1.1204	0.8870	0.6783	0.8875	0.6828	−1.1873	−1.1945
0.212	0.0275	0.1080	0.7132	1.1082	0.8920	0.7050	0.8907	0.6868	−1.1641	−1.1684
0.225	0.0300	0.1030	0.7446	1.0965	0.8970	0.7324	0.8936	0.6901	−1.1521	−1.1483
0.237	0.0325	0.0982	0.7770	1.0853	0.9018	0.7605	0.8961	0.6927	−1.1416	−1.1330
0.250	0.0350	0.0936	0.8105	1.0746	0.9064	0.7894	0.8982	0.6947	−1.1283	−1.1216
0.262	0.0350	0.0965	0.8380	1.0746	0.9035	0.8136	0.9009	0.7035	−1.1215	−1.1130
0.275	0.0375	0.0918	0.8726	1.0642	0.9082	0.8434	0.9026	0.7046	−1.1155	−1.1072
0.287	0.0375	0.0943	0.8995	1.0642	0.9057	0.8670	0.9049	0.7124	−1.1088	−1.1029
0.300	0.0400	0.0896	0.9353	1.0543	0.9104	0.8978	0.9063	0.7127	−1.1052	−1.1008

Table 4. System with Funded Pensions and Social Security Pensions

Birth rate	Interest rate	Premium funded pension	Premium Social Security pension	Benefit ratio	Gross wage	% net wage	Pension	workers' share income	workers' share capital	Average utility value
Depreciation rate 0.05										
Time preference parameter 0.05										
0.050	0.0150	0.0664	0.5000	1.7139	1.1751	0.4336	0.8732	0.3682	0.4345	−2.5812
0.062	0.0100	0.0936	0.3600	1.3750	1.2069	0.5464	0.9067	0.4853	0.4941	−1.8282
0.075	0.0075	0.1158	0.2700	1.2290	1.2241	0.6142	0.9240	0.5670	0.5391	−1.5400
0.087	0.0075	0.1273	0.2100	1.1450	1.2241	0.6627	0.9288	0.6283	0.5709	−1.3969
0.100	0.0100	0.1257	0.1700	1.0915	1.2069	0.7043	0.9277	0.6758	0.5909	−1.3162
0.112	0.0125	0.1231	0.1450	1.0821	1.1906	0.7319	0.9429	0.7083	0.6075	−1.2641
0.125	0.0125	0.1312	0.1200	1.0640	1.1906	0.7488	0.9486	0.7376	0.6286	−1.2214
0.138	0.0150	0.1270	0.1000	1.0380	1.1751	0.7730	0.9429	0.7642	0.6401	−1.1921
0.150	0.0175	0.1226	0.0900	1.0562	1.1604	0.7874	0.9650	0.7802	0.6499	−1.1717
0.162	0.0200	0.1180	0.0800	1.0654	1.1464	0.8020	0.9795	0.7955	0.6580	−1.1556
0.175	0.0200	0.1233	0.0700	1.0723	1.1464	0.8067	0.9916	0.8092	0.6722	−1.1419
0.188	0.0225	0.1181	0.0600	1.0626	1.1331	0.8219	0.9895	0.8236	0.6780	−1.1310
0.200	0.0250	0.1130	0.0550	1.0836	1.1204	0.8320	1.0100	0.8324	0.6828	−1.1242
0.212	0.0250	0.1172	0.0450	1.0616	1.1204	0.8378	0.9965	0.8456	0.6940	−1.1162
0.225	0.0275	0.1117	0.0400	1.0711	1.1082	0.8483	1.0069	0.8537	0.6972	−1.1117
0.237	0.0275	0.1154	0.0400	1.1225	1.1082	0.8446	1.0506	0.8565	0.7069	−1.1109
0.250	0.0300	0.1098	0.0350	1.1252	1.0965	0.8552	1.0551	0.8641	0.7090	−1.1079
0.262	0.0325	0.1044	0.0300	1.1245	1.0853	0.8656	1.0564	0.8713	0.7106	−1.1056
0.275	0.0325	0.1074	0.0250	1.1159	1.0853	0.8676	1.0508	0.8787	0.7187	−1.1034
0.287	0.0350	0.1020	0.0250	1.1621	1.0746	0.8730	1.0901	0.8805	0.7194	−1.1055
0.300	0.0350	0.1046	0.0200	1.1449	1.0746	0.8754	1.0769	0.8876	0.7267	−1.1035