Netspar Network for Studies on Pensions, Aging and Retirement

Roy Hoevenaars

Roderick Molenaar

Peter Schotman

Tom Steenkamp

Strategic asset allocation with liabilities: Beyond stocks and bonds

Discussion Papers 2005 – 006
February 2005

Strategic Asset Allocation with Liabilities: Beyond Stocks and Bonds

Roy Hoevenaars[a,b] Roderick Molenaar[b] Peter Schotman[a,d]
Tom Steenkamp[b,c]

This version: February 2005

Abstract: This paper considers the strategic asset allocation of long-term investors who face risky liabilities and who can invest in a large menu of asset classes including real estate, credits, commodities and hedge funds. We study two questions: (i) do the liabilities have an important impact on the optimal asset allocation? (ii) do alternative asset classes add value relative to stocks and bonds? We empirically examine these questions using a vector autogression for returns, liabilities and macro-economic state variables. We find that the costs of ignoring the liabilities in the asset allocation are substantial and increase with the investment horizon. Second, the augmented asset menu adds value from the perspective of hedging the liabilities.

Keywords: strategic asset allocation, asset liability management

JEL codes: G11, C32.

Affiliations: [a] Maastricht University, [b] ABP Investments, [c] Vrije Universiteit Amsterdam, [d] CEPR.

Corresponding author: R. Hoevenaars
Department of Quantitative Economics
Maastricht University
P.O. Box 616
6200 MD Maastricht
Netherlands
email: `r.hoevenaars@ke.unimaas.nl`

* We thank Michael Brandt, Rob van der Goorbergh, Ralph Koijen, Franz Palm, Luis Viceira, colleagues at ABP, and participants of the Erasmus Finance Day for helpful comments.

** The views expressed in this paper are those of the authors and do not necessarily reflect those of our employer and our colleagues.

1 Introduction

Many years ago Samuelson (1969) and Merton (1969, 1971) derived the conditions under which optimal portfolio decisions of long-term investors would not be different from those of short-term investors. One important condition is that the investment opportunity set remains constant over time. Among other things this implies that excess returns are not predictable. Interest in long-term portfolio management has been revived now that a growing body of empirical research has documented predictability for various asset returns. Excess stock returns appear related to valuation ratios like the dividend yield, price-earnings ratio, and also to inflation and interest rates.[1] Similarly, the term spread is a well-known predictor of excess bond returns.[2]

A growing number of studies explores the implications of the changing investment opportunity set for long-term investors.[3] As an insightful exploratory tool Campbell and Viceira (2005) introduce a "term structure of the risk-return trade-off". They use this term structure to show why time-varying expected returns lead to portfolios that depend on the investment horizon. They focus on a long-term investor who has a choice between stocks and long- and short-term bonds.

Typically these studies focus on an individual investor, who either is concerned about final wealth or who solves a life-cycle consumption problem.[4] In this paper we will focus on the strategic asset allocation problem of an institutional investor, especially a pension fund. We therefore extend the existing models to an asset and liability portfolio optimization framework and expand the investment universe with assets that are nowadays part of the pension fund investment portfolios. We explicitly include risky liabilities in the optimal portfolio choice. Liabilities are a predetermined portfolio component in the institutional investor's portfolio with a negative portfolio weight and with a return that is subject to real

[1] As a few references to this large literature we mention Barberis (2000) for work on the dividend yield, Campbell and Shiller (1988) for the price-earnings ratio, Lettau a Ludvigson (2004) for the consumption-wealth ratio. As the evidence is not uncontroversial we also refer to Goyal and Welch (2002) for a dissenting view.

[2] See for example the enormous amount of evidence against the expectations model of the term structure reviewed in Dai and Singleton (2002, 2003).

[3] We refer to Brennan, Schwartz and Lagnado (1996, 1997), Barberis (2000), Campbell, Chan and Viceira (2003), Wachter (2002), Brandt and Santa-Clara (2004), Brennan and Xia (2004) and especially Campbell and Viceira (2002).

[4] See Barberis (2000) for an example of end-of-period wealth. See Campbell, Chan and Viceira (2003) for an example with intermediate consumption.

interest rate risk and inflation risk. The optimal portfolio for an institutional investor can therefore be different from the portfolio of an individual investor. Assets that hedge against long-term liabilities risk are valuable components for an institutional investor.

Second, we study the risk characteristics of other assets than stocks, bonds and cash at various investment horizons. In this way we extend the term structure of the risk-return trade-off of Campbell and Viceira (2005) for assets like credits, commodities, real estate and hedge funds. We consider these assets in an "asset-only" context and also investigate if these asset categories are a hedge against inflation and real rate risk at different investment horizons. This gives insights into whether there is more than stocks and bonds in the universe that is interesting for strategic asset allocation. Which asset classes have a "term structure of risk" that is markedly different from that of stocks and bonds?

The remainder of this paper is organized as follows. Section 3 describes our model and presents the estimation results. Like Campbell and Viceira (2005) we consider a vector autoregression model to describe the time series properties of returns jointly with those of macro-economic variables. Since we include more assets the dimension of our VAR is larger. A large part of the section deals with parsimony of the model and with the problem of handling asset classes for which we have a short time series of returns. Section 3 explores the risk and hedging qualities of the asset classes at different horizons using the dynamics implied by the vector autoregression. In section 4 we will derive optimal portfolios both in an "asset-only" and an "asset-liability" context. We also use a certainty equivalent calculation to estimate the costs of sub-optimal asset allocations that ignore either liabilities or the alternative asset classes. Section 5 concludes.

2 Return dynamics

This section describes a vector autoregression for the return dynamics. The return dynamics extend Campbell and Viceira (2002, 2005) and Campbell, Chan and Viceira (2003) in two ways. First, we include more asset classes. Campbell, Chan and Viceira (2003) include stocks, bonds and real interest rates. We augment this set with credits and alternatives (i.e. listed real estate, commodities and hedge funds). We also add the credit spread as an additional state variable driving expected returns. Second, we introduce risky liabilities to the VAR. These liabilities are compensated for price inflation during the holding period (they are comparable to real coupon bonds (e.g. Treasury Inflation Protected Securities (TIPS))). The return on risky liabilities follows the return of long-term (in our case 17 years constant maturity) real bonds.

Below we will first describe the model, then the data, and finally estimation results.

2.1 Model

As in Campbell, Chan and Viceira (2003) we describe the dynamics of the relevant variables by a first-order VAR for quarterly data. Specifically, let

$$z_t = \begin{pmatrix} r_{tb,t} \\ s_t \\ x_t \end{pmatrix},$$

where r_{tb} represents the real return on the 3-month T-bill; x_t is a $(n+m)$-vector of excess returns of assets and liabilities, and s_t is a $(s \times 1)$ vector of other state variables. The vector of excess returns is split in two parts,

$$x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix},$$

where x_1 contains the quarterly excess returns, relative to the 3-month T-bill return (r_{tb}), on stocks (x_s) and bonds (x_b), and x_2 is an m-vector containing excess returns on credits (x_{cr}), commodities (x_{cm}), hedge funds (x_h), listed real estate (x_{lre}), and the liabilities (x_0). This gives $n = 2$ and $m = 5$. The variables in x_1 are the assets that are also included in the model of Campbell, Chan and Viceira (2003). The variables in x_2 are the additional asset classes.

The vector with state variables s_t contains $s = 4$ predictive variables: the nominal 3-months interest rate (r_{nom}), the dividend price ratio (dp), the term spread (spr) and the credit spread (cs).

Altogether the VAR contains $m + n + s + 1 = 12$ variables. For most time series, data are available quarterly for the period 1952:III until 2003:IV. The exception are many of the alternative asset classes in x_2, for which the historical data are available for a much shorter period. Because of the large dimension of the VAR, and due to the missing data for the early part of the sample, we can not obtain reliable estimates with an unrestricted VAR. We deal with this problem in two ways: (i) by imposing a number of restrictions and (ii) by making optimal use of the data information for estimating the dynamics of the series with shorter histories.

The restrictions on the VAR concern the vector x_2. The additional assets are assumed to provide no dynamic feedback to the basic assets and state variables. For the subset of variables,

$$y_t = \begin{pmatrix} r_{tb,t} \\ s_t \\ x_{1,t} \end{pmatrix}$$

we specify the subsystem unrestricted VAR

$$y_{t+1} = a + By_t + \epsilon_{t+1}, \tag{1}$$

where ϵ_{t+1} has mean zero and covariance matrix $\Sigma_{\epsilon\epsilon}$. For the variables in x_2 we use the model

$$x_{2,t+1} = c + D_0 y_{t+1} + D_1 y_t + Hx_{2,t} + \eta_{t+1}, \tag{2}$$

where D_0 and D_1 are unrestricted $(m \times (n + s + 1))$ matrices, and $H = \text{diag}(h_{11}, .., h_{mm})$ is a diagonal matrix. The diagonal form of H implies that $x_{2i,t}$ only affects the expected return of itself, but not of the other additional assets. The shocks η_t have zero mean and covariance matrix Ω. Contemporaneous covariances are captured by D_0. Without loss of generality we can therefore set the covariance of η_t and ϵ_t equal to zero.

Combining (1) and (2) the complete VAR can be written as

$$z_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 z_t + u_{t+1}, \tag{3}$$

where

$$\mathbf{\Phi}_0 = \begin{pmatrix} a \\ c + D_0 a \end{pmatrix}, \qquad \mathbf{\Phi}_1 = \begin{pmatrix} B & \mathbf{0} \\ D_1 + D_0 B & H \end{pmatrix},$$

and u_t has covariance matrix

$$\mathbf{\Sigma} = \begin{pmatrix} \Sigma_{\epsilon\epsilon} & \Sigma_{\epsilon\epsilon} D_0' \\ D_0 \Sigma_{\epsilon\epsilon} & \Omega + D_0 \Sigma_{\epsilon\epsilon} D_0' \end{pmatrix}$$

The form of (2), with the contemporaneous y_{t+1} among the regressors, facilitates efficient estimation of the covariances between shocks in y_t and x_{2t} when the number of observations in x_{2t} is smaller than in y_t. This approach is based on Stambaugh (1997) and makes optimally use of all information in both the long and short time series. Furthermore it ensures that the estimate of $\mathbf{\Sigma}$ is positive semi-definite. As in Campbell and Viceira (2005) we assume that the errors are homoskedastic.

2.2 Data

Our empirical analysis is based on quarterly US data. Most data series start in 1952:III; all series end in 2003:IV. However, data for commodities, hedge funds and listed real estate are only available for a shorter history. Commodities start in 1970:I, hedge funds start in 1994:II, listed real estate starts in 1970:II.

The 90-days T-bill, the 10-years constant maturity yield and the credit yield (i.e. Moody's Seasoned Baa Corporate Bond Yield) are from the FRED website.[5] In order to generate the yield and credit spread we obtain the zero yield data from Duffee (2002).[6] As these data are only available until 1998:04, we have extended the series using a similar approach for the data after 1998:04. For inflation we use the non-seasonally adjusted consumer price index for all urban consumers and all items also from the FRED website. Data on stock returns and the dividend price ratio are based on the S&P Composite and are from the "Irrational Exuberance" data of Shiller[7]. Credit returns are based on the Salomon Brothers long-term high-grade corporate bond index, and are obtained from Ibbotson (until 1994:IV) and Datastream. Hedge fund returns are based on the CSFB Tremont hedge fund price index. Commodity returns are based on the GSCI index, while the NAREIT series forms the (listed) real estate returns.

All return series are in logarithms. We construct the gross bond return series $r_{n,t+1}$ from 10 year constant maturity yields on US bonds using the approach described by Campbell, Lo and MacKinlay (1997) which is given in (4).

$$r_{n,t+1} = \tfrac{1}{4} y_{n-1,t+1} - D_{n,t}(y_{n-1,t+1} - y_{n,t}), \tag{4}$$

[5] http://research.stlouisfed.org/fred2/
[6] http://faculty.haas.berkeley.edu/duffee/affine.htm
[7] http://aida.econ.yale.edu/~shiller/data.htm

where n is the bond maturity, $y_{n,t} = \ln(1 + Y_{n,t})$ is the yield on the n-period maturity bond at time t and $D_{n,t}$ is the duration which is approximated by

$$D_{n,t} = \frac{1 - (1 + Y_{n,t})^{-n}}{1 - (1 + Y_{n,t})^{-1}}.$$

We approximate $y_{n-1,t+1}$ by $y_{n,t+1}$. Excess returns are constructed in excess of the logarithm of the 90-day T-bill return, $x_{b,t} = r_{n,t} - r_{tb,t-1}$.

The liability return series is derived from (5) and also based on the loglinear transformation (4),

$$r_{0,t+1} = \tfrac{1}{4} rr_{n,t+1} - D_{n,t}(rr_{n,t+1} - rr_{n,t}) + \pi_{t+1} \quad (5)$$

We assume that the pension fund pays unconditionally full indexation, therefore the liabilities should be discounted by the real interest rate. The $n-$period real yield, $rr_{n,t+1}$, is the (proprietary) Bridgewater 10 year US real interest rate. The liabilities are indexed by the price inflation π_{t+1} of the corresponding quarter; the duration is assumed to be 17 years ($D_{n,t} = 17$), which is the average duration of pension fund liabilities.

To describe the liabilities of a pension fund as a constant maturity (index-linked) bond we need to assume that the fund is in a stationary state. A sufficient condition for this to be true, is that the distribution of the age cohorts and the built-up pension rights per cohort are constant through time. Furthermore, we assume that the inflow from (cost-effective) contributions equals the net present value of the new liabilities and that it equals the current payments.

The first two terms (i.e. $\frac{1}{4} rr_{n,t+1} - D_{n,t}(rr_{n,t+1} - rr_{n,t})$) in (5) reflect the real interest risk, whereas the third term reflects the inflation risk (i.e. π_{t+1}). By definition TIPS would be the risk free asset (although the returns of TIPS are based on the lagged inflation, π_t). However, we have not included TIPS in the analysis due to their short existence and because the size of the market is not large enough to allow investors to hedge all there liabilities with them. So implicitly we assume an incomplete market with respect to inflation, which could be realistic in the case of pension liabilities.

Return series of illiquid assets are often characterized by their high returns, low volatility and low correlation with other series. Hedge funds are a good example in this context. Asness, Krail and Liew (2001) and Brooks and Kat (2001) note that hedge fund managers can smooth profits and losses in a particular month by spreading them over several months, hereby reducing the volatilities. Underestimation of volatility can make a return class more attractive in asset allocation than it actually is in reality. Geltner (1991, 1993) discusses the methodologies to unsmooth return series to make them comparable to the more liquid assets. He proposes the autocorrelation in returns as a measure of illiquidity. Geltner (1991, 1993) suggests to construct unsmoothed return series as

$$r_t^* = \frac{r_t - \rho r_{t-1}}{1-\rho} \tag{6}$$

where r_t is the original smoothed return series, ρ is the first order autocorrelation coefficient and r_t^* is the unsmoothed return series which will be used in the VAR.[8] Note that the unsmoothed series have the same mean as the smoothed series. We have applied this unsmoothing for hedge fund returns in the same way as Brooks and Kat (2001). As Posthuma and Van der Sluis (2003) show that the reported historical returns of hedge funds are on an annual basis 4.35 percent too high due to among others the back-fill bias, we have corrected the returns of the hedge fund series by subtracting an annual 4.35 percent from the published returns. Note that this adjustment only affects the average returns, but does not influence the risk properties.

Apart from the return series we include four other variables that drive long-term risks. The real T-bill return is defined as the difference between the nominal T-bill return and the price inflation. The log of the dividend price ratio of the S&P Composite is used. The yield spread is computed as the difference between the log 10-year zeros yield and the log 90-day T-bill. In addition, the difference between the log BAA yield and the log 10-years US Treasury is included as the credit spread.

These state variables are common in the literature. Several empirical studies suggest macro-economic variables that capture important dynamics in future returns: dividend price ratio (Campbell and Shiller (1988)), nominal short-term interest rate (Campbell (1987)), yield spread between short-term and long-term bonds (Campbell and Shiller (1991)). Cochrane and Piazessi (2002) find that a linear combination of forward rates predicts bond returns well, while Lettau and Ludvigson (2001) find that fluctuations in the

[8] Alternatively, if we would include the unsmoothed series, the VAR would take care of the unsmoothing. This would lead to the same long-term volatility, but it would seriously underestimate the short-term volatility. Unsmoothing produces more representative short term volatilities.

consumption-wealth ratio are strong predictors of stock returns. Campbell, Chan and Viceira (2003) and Campbell and Viceira (2005) include the short-term nominal interest rate, yield spread and dividend price ratio in the VAR. They find that the dividend price ratio helps predicting future stock returns. Furthermore, Campbell, Chan and Viceira (2003) find that shocks in the nominal short rate are strongly correlated with shocks in excess bond returns. In addition the yield spread is helpful in predicting future excess bond returns. Brandt and Santa-Clara (2004) study conditional asset allocation using the dividend-yield, term spread, default spread and the nominal T-bill rate.

Furthermore, Fama and French (1989) link state variables as the dividend yield, credit spread and yield spread to the business cycle. They argue that the risk premia for investing in both bonds, yield spread, and corporate bonds, credit spread, are high in contraction periods and low in expansion periods. The opposite applies to the dividend price ratio which is high in expansion periods and low in contraction periods. Since both the dividend yield and the credit spread adjust very slowly over time, they describe the long run business cycles. The yield spread, on the other hand, is less persistent and describes shorter business cycles. Moreover, Cochrane (2001) shows that the explanatory power of the price dividend ratio to stock returns is substantial for longer horizon returns. He considers stocks returns on a 1 year horizon to a 5 year horizon.

Table 1 reports summary statistics of the data. Due to the different starting dates, the statistics must be interpreted with some care. Credits have a higher return than duration equivalent bonds. This is reflected in the higher Sharpe ratio (0.19 versus 0.15); the mean return of commodities is similar to that of the stocks although the volatility is higher (18.49% vs. 15.89%), which results in a lower Sharpe ratio. Although listed real estate is often seen as equivalent to equity (see e.g. Froot (1995)) it has a lower return and a higher volatility than stocks, which results in a lower Sharpe ratio of 0.29. Hedge funds have the second highest Sharpe ratio (0.40) and thus remain one of the most attractive investment categories in our sample from a risk-return perspective.

2.3 Estimation results

Table 2 reports the parameter estimates of the subsystem VAR in (1) on the quarterly data 1952:03-2003:IV. Correlations and standard deviations are given in Table 3. The quarterly standard deviations are on the diagonal.

Excess stocks returns are explained by the nominal interest rate, dividend price ratio and the credit spread. These are the only variables which have an absolute t-value above 2. The negative correlation of shocks in the dividend price ratio and credit spread with shocks in stocks returns implies that a positive innovation in the credit spread or dividend price ratio has a negative effect on contemporaneous stock returns. The significant positive coefficients, however, predict that next period stock returns rise. In this way both the credit spread and the dividend price ratio imply mean reversion in stocks returns.

The return on Treasuries is related to the yield spread (t-value larger than 3). Although less significant, the nominal interest rate and stock returns also seem to capture some dynamics in expected returns. The nominal interest rate is a mean-reversion mechanism in bond returns, whereas the covariance structure of the term spread leads to a mean aversion part. The R^2 of 8% implies that Treasury returns are difficult to explain, even more difficult than stocks which have an R^2 of 10%. Nevertheless, a low R^2 on quarterly basis implies a higher R^2 on an annual basis. Moreover, Campbell and Thompson (2004) show that even a very small R^2 can be economically meaningful because it can lead to large improvements in portfolio performance.

The state variables serve as predictor variables. The coefficients of both the nominal interest rate (1.03) and the dividend price ratio (0.95) on their own lags indicate that these series are very persistent. The maximal eigenvalue of the coefficient matrix equals 0.977. The system is stable, but close to being integrate of order one. Although the credit spread is less persistent than the dividend price ratio, its autocorrelation coefficient (0.78) is higher than that of the yield spread (0.68). These results are in line with the reasoning of Fama and French (1989). They argue that the dividend yield and the credit spread describe long run business cycles, while the yield spread captures business cycles in the shorter run.

Since some of the state variables are very persistent, they might well have a unit root. As in the models of Brennan, Schwartz and Lagnado (1997), Campbell and Viceira (2002) and

Campbell, Chan and Viceira (2003) we do not adjust the estimates of the VAR for possible small sample biases related to near non-stationarity of some series (see e.g. Stambaugh (1999), Bekaert and Hodrick (2001) and Campbell and Yogo (2004)).

The return dynamics of the remaining asset classes are not described by the unrestricted VAR in (1). A serious concern when extending the number of asset classes is the accuracy of the coefficient estimates. The number of parameters in the VAR increases quadratically with the number of additional series. Therefore we model the additional asset classes in the separate model in (2). In order to improve the precision of the estimates further, we also impose restrictions on the coefficients and we make optimal use of the data information for estimating the dynamics of the series with shorter histories. By this approach we try to mitigate error maximization problems in the optimal portfolio choice. As we are typically interested in the risk properties and hedging portfolios we do not change the constant term. As a consequence the unconditional mean implied by the VAR is the unconditional mean in the historical data.

Table 4 shows the estimation results for x_2. The credits are well explained by bonds, its own lagged return, the change of the credit spread and the change of the long yield ($R^2 = 0.91$). Commodities have as much (or as little) predictability as stocks and bonds ($R^2 = 0.11$); the negative exposure to stocks confirms the findings of Gorton and Rouwenhorst (2004). The real estate series are rather well explained by contemporaneous bonds, stocks and term spreads. The exposure to stocks reflects the fact, that the real estate series are derived using the listed series. Hedge funds are only explained by the contemporaneous stocks, which is in line with Asness, Krail and Liew (2001). Finally the liabilities are mainly driven by real T-bills, bonds and the change in the long yield. The exposure to the change in the long yield reflects the higher duration of liabilities.

3 Risk and hedging at different horizons

This section discusses the long-run covariance structure of assets and liabilities.[9] We first consider the term structure of risk of all individual assets. Next we consider the covariances of stocks and bonds with the other asset classes, liabilities and inflation. Horizon effects in the correlations between asset classes follow from the return dynamics implied by the VAR model. The inflation hedging characteristics are derived using the nominal returns, while the liability hedging properties follow from the correlations of the real returns with the real liabilities.

3.1 Time diversification

The first set of implications of the VAR model concerns the time diversification properties. Figure 1 shows the annualized conditional standard deviation of cumulative excess holding period returns of all asset classes.

Results for stocks, bonds and T-bills are similar to Campbell and Viceira (2005). Stocks are less risky in the long run: the standard deviation equals 15% in the first quarter, 11% after 10 years and just below 9% after 25 years. The decrease in the annualized volatility is caused by two effects. As in Campbell and Viceira (2005) a positive shock in stock returns results in an immediate negative shock in dividend price ratio, and therefore lower expected future returns. In our model this effect is reinforced by the credit spread. A negative shock in the credit spread results in an immediate positive shock in stock returns, but the tightening of the credit spread today predicts a lower stock return in next period. This initial increase followed by a decrease in returns is the mean reversion and leads to time diversification.

We find similar mean reversion in bonds. A negative shock in the short rate induces a positive shock in bond returns, and subsequently predicts next period bond returns to decrease. In contrast, shocks in the term spread variable are positively related with current bond returns and they predict positive bond returns for the next period as well. As the

[9] Prudence is called for the interpretation risk properties at very long horizons. Our sample size induces us not to put too much emphasis on these characteristics, because estimation risks might accumulate. Of course, the risk characteristics and hedging properties that we derive in this section are implied by the way we have described the return dynamics of our economic environment.

short rate is more persistent than the spread, the mean reverting effect of the short rate dominates. As a consequence, annualized standard deviations of bond returns are much lower in the long run than in the short run: it decreases from 10% after one quarter to 6% after 25 years.

Credits show time diversification as well. Mean reversion is the result of the predictability of the returns from the credit spread and the high correlation with bonds (correlations are all above 90%, see also Figure 3). The coefficient estimates are significant and positive, while the shocks are negatively correlated. The short term volatility of credits is below that of bonds, which is due to a combination of lower duration of the credits and the negative correlation between credit spreads and yield changes.

In contrast to the mean reverting assets, investing in the 90-day T-bill is more risky in the long run due to the reinvestment risk. For longer investment horizons the risk of reinvesting in the 90-day T-bill approaches the one of a rolling investment in 10 year bonds (See e.g. Campbell and Viceira (2002)). Additionally, we observe persistence in the inflation process, meaning that inflation is a long-term risk factor.

Since table 4 shows that listed real estate is mainly driven by stocks (as it is partly included in most indices) and bonds, its volatility pattern is also a combination of that of the two asset categories: the decrease of the volatility over the horizon is less than that of the stocks.

No time diversification is observed in commodities. In the regression analysis we only found a significant effect of stocks and the real T-bill rate. Since stocks are mean-reverting and the T-bill exhibits mean-aversion, it is not surprising that their combined effect results in a flat term structure of risk.

Hedge fund returns show some mean reversion, which is due to the fact that is explained by stocks. Also the fact, that the return series of hedge funds are unsmoothed, will have an effect on the observed pattern.

Finally, the liability risk shows mean reversion as well. Liabilities are the sum of long term real bond returns plus inflation. The real bond returns exhibit mean reversion, whereas inflation does not. The total effect is a modestly downward sloping term structure of risk.

In summary, we find that the return dynamics imply mean reversion for stocks, bonds, credits, listed real estate and the liabilities. This means that a long term holding period investor can benefit from time diversification in these asset classes. The risk of hedge funds and commodities hardly exhibits horizon effects.

3.2 Risk diversification

In the previous section we considered diversification possibilities *within* an asset class. This section describes diversification possibilities *between* asset classes. We show the correlations between several asset classes over different horizons.

Figure 2 shows that there are substantial horizon effects in the correlation between real returns on stocks and other asset classes. The risk diversification with bonds is the strongest in the very short (around 25%) and the long run (it reduces to 47% for a 25-year horizon). For investors with a medium term investment horizon the correlation can be up to 67%. Risk diversification possibilities between stocks and credits are quite similar to those of bonds. The correlation is slightly higher than that of bonds at all horizons, from 33% in the short-term to 51% in the long-term.

Listed real estate is often seen as similar to equity. This is supported by the high correlation (57%) between stocks and listed real estate at short investment horizons. This correlation diminishes, however, with the investment horizon.

Like Campbell and Viceira (2005) we find that the correlation between stocks and the T-bill is high for short horizons (47%) but this reduces to 2.5% after 25 years. Horizon effects are much weaker in the correlation between stocks and hedge funds. The correlation moves from 55% for short horizons towards 33% in the long run. The magnitude of the short term correlation is in line with the findings in Brooks and Kat (2001).

Since the correlation between stocks and commodities is negative at all horizons, they have the best risk diversifying properties relative to stocks. It changes from -22% for a quarterly horizon, to -18% at a 25 year horizon. The negative correlations are in line with Gorton and Rouwenhorst (2004), who also find negative correlations for quarterly, annual and 5-year returns. They show that the negative correlation is due to the different behavior over the business cycle. Commodities are interesting as diversification opportunity both for short-term as well as long-term investors.

Horizon effects show that the correlation of stocks with bonds and credits first rise, but then reduce with the holding period, whereas the correlation with the T-bill, real estate and hedge funds steadily reduces across the horizon. This suggests that diversification is stronger at longer horizons.

Figure 3 shows that the risk diversification properties between bonds and the other asset categories change over the investment horizon as well. Commodities seem again to be the best diversifier, but hardly horizon dependent. The correlation is negative at all horizons and stays between -11% and -16%, which is confirmed by Gorton and Rouwenhorst (2004) and Ankrim and Hensel (1993).

The correlation of bonds with the T-bill has a U-shape. It starts high for short horizons at 50%, reduces to a low of 7.5% for the medium term and rises again at longer horizons. Most other asset classes exhibit a hump-shaped term structure of correlation. Correlations increase at short horizons and fall at medium and long horizons. As expected, bonds are highly correlated with credits at all horizons: correlation is always more then 90%.

Stocks and real estate show a very similar pattern. Froot (1995) explains that similar factors (e.g. productivity of capital and labor) drive both stocks and real estate and that lots of corporate assets are invested in real estate anyway. In this sense real estate does not seem like a very different asset class. Hedge funds have a low correlation for short horizons (18%), which increases to a maximum at 5 years (32%).

3.3 Inflation hedging qualities

Inflation risk is a well-known problem to long-term investors and typically in "asset-liability" management. As inflation reduces the value of nominal liabilities, pension funds generally have the ambition to index their pension payments by inflation. In our definition of the liabilities in (5) inflation risk is one of the two risk factors. From this perspective the institutional investor should try to adjust his asset mix to hedge the exposure to inflation risk. This section examines the potential of stocks, bonds and the alternatives as a hedge against inflation for different investment horizons. Since inflation is not explicitly included in the VAR, we construct its properties from the difference between the real T-bill return and the lagged nominal interest rate (i.e. $\pi_t = r_{nom,t} - r_{tb,t}$).

Figure 4 shows the correlation of *nominal* asset returns with inflation across investment horizons. The inflation hedging qualities of most assets change substantially with the horizon. These hedging properties seem to differ radically in the short, medium and long term. All asset classes are a better hedge against inflation in the long run than in the short run, while the hedging qualities differ between the assets.

Overall we find that the T-bill quickly catches up with inflation changes. As a consequence the T-bill seems the best inflation hedge at all horizons among the asset classes we considered (it even reaches a correlation of 0.97 after 7 years). This is achieved by rolling over 3-months T-bills, which ensures that the lagged inflation is incorporated. At longer horizons bonds and credits are good inflation hedges as well (correlation of 0.65 after 25 years), whereas the short term hedging qualities are poor due to the inverse relationship between yield changes and bond prices. The return loss due to yield increases has to be locked in at each rebalancing date before the inflation hedge can improve in the long run. The positive long term correlations are mainly due to the use of constant maturity bonds, whereas Campbell and Viceira (2005) show that holding bonds to maturity is akin to accumulating inflation risk. The negative short term hedging qualities of credits is also related to the negative relation between inflation and real economic growth. Therefore the credit spread widens in business cycles downturns, which leads to a negative return.

Stocks also turn out to be a good inflation hedge in the long run and a poor one in the short run, consistent with the large existing literature on this relation. Fama (1981) argues that inflation, acting as a proxy for real activity, leads to the negative short-term correlation. Increasing inflation would lead to lower real economic activity and this leads to lower stock returns. The positive inflation hedge potential in the long run could be explained by the effect that inflation has on the present value calculation of stock prices. Campbell and Shiller (1988a) distinguish two offsetting effects. First, inflation increases the discount rates which lowers stock prices. Second, inflation rises future dividends, which increases stocks prices. They argue that due to price rigidities in the short run, the net effect will be negative in the short run, but positive in the long run. The hedging qualities of both stocks and bonds are in line with the findings of Gorton and Rouwenhorst (2004).

Commodity prices move both in the short and the long term along with inflation, which makes them very attractive from an inflation hedge perspective. They have very stable inflation hedge qualities (correlation of 0.30 for investment horizons longer than 5 years) and for investment horizons up to 20 years they are the second best inflation hedge after the T-bill. Bodie (1983) already showed that the risk-return trade-off of a portfolio in an inflationary environment can be improved by inclusion of a portfolio of commodity futures to a portfolio consisting of stocks and bonds: the returns on the latter are negatively affected by inflation, whereas commodities tend to do well when there is unanticipated inflation, because commodity and consumer prices tend to move together. Gorton and Rouwenhorst (2004) also note that as "futures prices include information about foreseeable trends in commodity prices, they rise and fall with unexpected deviations from components of inflation".

Listed real estate behaves like stocks in the short run from an inflation hedge perspective, although stocks are a slightly better inflation hedge in the long run. This is again in line with the observation that listed real estate behaves like stocks. Hedge funds are better inflation hedges in the short run than most assets, but they still have a negative inflation hedge potential. As hedge fund returns are often seen as Libor plus an alpha component, the inflation hedge qualities may come from the Libor part of the return which moves with the lagged inflation, which results in a positive long term correlation.

3.4 Real interest rate hedging qualities

This section studies the potential of stocks, bonds and alternatives (in real terms) as a hedge against liability risk at different investment horizons. The liabilities of pension funds are the present value of future obligations, discounted at a real interest rate. Liability risk is associated with both the future obligations as well as the discount factor. Both inflation, affecting the future obligations, as well as the discount rate lead to liability risk. Our time series of liability returns accounts for both types of risks (see (5)).

Figure 5 shows that the liability hedge potentials of the asset classes change substantially with the investment horizon as well. Among the asset classes in our model nominal bonds provide the best liability hedge. They have a correlation of around 75% at an annual investment horizon. Due to cumulative inflation the correlation reduces to 35% on the long run. The hedging qualities of credits mimic those of the bonds. The mismatch between the quarterly inflation compensation of the liabilities and the expected long inflation implicitly in the long yields underlying the investment strategies becomes more severe at longer horizons. As a consequence, due to this cumulative inflation shocks the liability hedge potential of bonds and credits reduces across the investment horizon.

The term structure of risk for real estate is hump-shaped. In the short run the liability hedging correlation is around 12%; it reaches a maximum at the six year horizon with a correlation of 32%; it then falls to 20% at a 25 year horizon. We observe a similar hump-shaped correlation pattern for stocks: it reaches its maximum of 29% at around 10 years.

As before, commodities are very different. The hedge potential is limited at all horizons

Finally, the short real T-bill has a positive liability hedge potential of 10% correlation in the short run, but after 1 year it is already negative. It converges to -67% in the long run.

4 Strategic asset liability management

4.1 Model

In this section we present the optimal mean-variance portfolio choice for a buy-and-hold investor. We compare the optimal holdings of an "asset-only" investor who is only concerned about the real return of his investment with the optimal portfolio of an investor with risky liabilities. Campbell and Viceira (2002, 2005) solve the mean-variance problem for an "asset-only" investor with equity, bonds and cash as the financial instruments. In this section we add risky liabilities into their mean-variance framework.

Campbell and Viceira (2005) formulate the mean-variance problem for an investor with a horizon of k periods as

$$\max_{\alpha_t(k)} \ln \mathsf{E}_t \left[1 + R^{(k)}_{A,t+k}\right] - \tfrac{1}{2}\gamma\sigma^2_A(k) \tag{7}$$

where $R^{(k)}_{A,t+k}$ is the cumulative return of the asset portfolio from t to $t+k$; $\sigma^2_A(k)$ is the conditional variance of k-period cumulative log-returns; and $\alpha_t(k)$ is the set of weights in the asset mix. This formulation of the mean-variance problem is equivalent to maximizing power utility of wealth over a k-period horizon. The investor chooses his optimal portfolio at the beginning of the first period, and he does not rebalance his portfolio. Although this is a static framework, it enables the investor to benefit from time-diversification properties of the assets. To obtain a closed-from solution for the optimal portfolio choice, Campbell and Viceira (2002) approximate the expectation of the simple return in (7) to obtain

$$\max_{\alpha_t(k)} \mathsf{E}_t \left[r^{(k)}_{A,t+k} \right] + \tfrac{1}{2}\sigma^2_A(k) - \tfrac{1}{2}\gamma\sigma^2_A(k) \tag{8}$$

where lower case $r = \ln(1+R)$ and $r^{(k)}_{i,t+k} = \sum_{\ell=1}^{k} r_{i,t+\ell}$ for any asset i. Using log-linear approximations the logarithmic portfolio return is rewritten as

$$r^{(k)}_{A,t+k} = r^{(k)}_{tb,t+k} + \alpha'_t(k)\tilde{x}^{(k)}_{t+k} + \tfrac{1}{2}\alpha'_t(k)\sigma^2_x(k) - \tfrac{1}{2}\alpha'_t(k)\Sigma_{xx}(k)\alpha_t(k), \tag{9}$$

where $\tilde{x}_t$ is the vector of excess returns excluding the liabilities x_0 as they are not an investable asset, and where

$$\begin{aligned} \Sigma_{xx}(k) &= \mathrm{Var}(\tilde{x}^{(k)}_{t+k}) \\ \sigma^2_x(k) &= \mathrm{diag}(\Sigma_{xx}(k)), \end{aligned}$$

as in Campbell and Viceira (2005). Noting that the T-bill rate r_{tb} is not a riskfree return for a long-run investor, the portfolio variance is,

$$\sigma^2_A(k) = \sigma^2_{tb}(k) + \alpha'_t(k)\Sigma_{xx}(k)\alpha_t(k) + 2\alpha'_t(k)\sigma_{tb,x}(k) \tag{10}$$

where $\sigma_{tb,x}(k)$ is the vector of covariances of excess log-returns with the 3-month T-bill. Substituting (9) and (10) in the mean-variance problem (8) leads to a quadratic optimization problem with solution

$$\alpha_t(k) = \frac{1}{\gamma}\Sigma^{-1}_{xx}(k)\left(\mu_t(k) + \tfrac{1}{2}\sigma^2_x(k)\right) - (1-\frac{1}{\gamma})\Sigma^{-1}_{xx}(k)\sigma_{tb,x}(k), \tag{11}$$

where $\mu_t(k)$ is the vector of expected excess returns over a k-period horizon. As is well-known, the portfolio has two components: the speculative demand and hedging demand. Infinitely risk-averse investors ($\gamma \to \infty$) invest in the global minimum variance portfolio $-\Sigma^{-1}_{xx}(k)\sigma_{tb,x}(k)$. Explicit expressions for the quantities $\mu_t(k)$, $\Sigma_{xx}(k)$, and $\sigma_{tb,x}(k)$ are provided in the appendix.

We now turn to the portfolio choice of an "asset-liability" investor. Following Leibowitz, Kogelman and Bader (1994) we approach "asset-liability" management from a funding ratio return perspective. The funding ratio is defined as the ratio of assets over liabilities. The funding ratio log-return r_F is then defined as the return of the assets minus the return on the liabilities,

$$r_{F,t+k}^{(k)} = r_{A,t+k}^{(k)} - r_{L,t+k}^{(k)} \tag{12}$$

The difference between the "asset-only" investor and the "asset-liability" investor is the benchmark against which they measure their returns. The "asset-only" investor cares about financial returns in excess of inflation; the "asset-liability" investor considers returns in excess of the liabilities. Subtracting the 3-month T-bill benchmark form both assets and liabilities, and applying the log-linearization, the funding ratio return is equal to

$$r_{F,t+k}^{(k)} = \alpha_t'(k)\tilde{x}_{t+k}^{(k)} - x_{0,t+k}^{(k)} + \tfrac{1}{2}\alpha_t'(k)\sigma_x^2(k) - \tfrac{1}{2}\alpha_t'(k)\Sigma_{xx}(k)\alpha_t(k) \tag{13}$$

where x_0 are the excess returns (relative to the 3-month T-bill) of the liabilities defined in section 2.2. Since we solve the optimization problem in real terms, we implicitly assume that the liabilities are fully indexed by price inflation.

The variance of the funding ratio return, called the mismatch risk, is equal to

$$\sigma_F^2(k) = \sigma_0^2(k) + \alpha_t'(k)\Sigma_{xx}(k)\alpha_t(k) - 2\alpha_t'(k)\sigma_{0x}(k) \tag{14}$$

The optimization problem for the "asset-liability" investor is

$$\max_{\alpha_t(k)} \mathsf{E}_t\left[r_{F,t+k}^{(k)}\right] - \tfrac{1}{2}(\gamma - 1)\sigma_F^2(k) \tag{15}$$

from which we obtain the optimal portfolio as

$$\alpha_t(k) = \frac{1}{\gamma}\Sigma_{xx}^{-1}(k)\left(\mu_t(k) + \tfrac{1}{2}\sigma_x^2(k)\right) + (1 - \frac{1}{\gamma})\Sigma_{xx}^{-1}(k)\sigma_{0x}(k) \tag{16}$$

The speculative component of the "asset-liability" investor is the same as for the "asset-only" investor. The difference is in the hedging component of the portfolio. The best liability hedging portfolio correspond to minimizing the mismatch risk (14). The difference in sign between (11) and (16) is due to the short position in the liabilities instead of the long position in the T-bill. Liabilities are not an investable asset themselves. The "asset-liability" investor invests in the risky assets, but cannot invest in the risky benchmark. In a complete market the best liability hedging portfolio would consist of a portfolio of TIPS that perfectly match the risky liabilities.

4.2 Optimal portfolio choice

We solve the optimal portfolio choice based on the mean variance problem (7) for the "asset-liability" investor using (16). In order to make it comparable to the "asset-only" asset allocation we also compute the optimal portfolio in (11). We use the covariance matrix of cumulative returns, which changes with the investment horizon and the unconditional full sample mean of the asset returns.

Obviously, the coefficient of risk aversion, γ, depends on the risk attitude of the investor. For an investor with risky liabilities the initial funding ratio could influence the level of risk tolerance. If the initial funding ratio is far above one, the investor may not need to take any mismatch risk because the value of the assets are more than sufficient to meet the liabilities. On the other hand, he might still decide to take some risk to reduce contribution rates because he has a downside risk buffer anyway. If the funding ratio is below one, the investor could decide to take risk in exchange for a higher expected return to raise the asset value above the liabilities and extricate him from underfunding. Leibowitz and Hendriksson (1989) express the degree of risk aversion as a shortfall constraint that is determined by the initial funding ratio. In this approach the shortfall constraint requires that the funding ratio stays above some minimum acceptance level. Below we examine the optimal portfolio choice for different risk attitudes, γ =5, 10, 20, ∞.

Our first set of results pertains to the strategic asset allocation for an investor who is extremely risk averse ($\gamma \rightarrow \infty$). This results in the best liability hedging portfolio (LHP) and the global minimum variance (GMV) portfolio for the "asset-liability" and "asset-only" investor, respectively. These portfolios do not depend on expected returns. The results are shown in Table 5 for a 1, 5, 10 and 25 year investment horizon. At the 1-year horizon the GMV portfolio is entirely invested in the T-bill, exactly as in Campbell and Viceira (2005). At the 25-year horizon 15% of the asset mix is invested in other assets like stocks, credits and commodities. Here the results are different from Campbell and Viceira (2005), as the latter three asset classes drive bonds out of the GMV.

The best liability hedge portfolio is quite different. At the 1-year horizon the risk averse "asset-liability" investor chooses bonds (54%) and T-bills (60%) in his asset mix. Although in section 3.4 we found that the T-bill is not a very good real interest rate hedge, here it turns out that it still a good investment as it has low risks at all, and particularly at short horizons. Bonds were are the best real rate hedge, and therefore have a high weight in the LHP. For longer horizons we observe that the T-bill and bonds are replaced by other assets. Despite the bad long real rate hedge of the T-bill, it still has a high weight in the LHP, because of its better risk diversification properties at particularly longer horizons. Credits get a substantial weight (17% at a 25 year horizon) in the LHP, because they are the second best real rate hedge. They replace bonds to some part, because the risk diversification seems to win from the slightly better real rate hedge properties of bonds. Although commodities do not hedge against liability risk and have a high volatility, they have a positive weight in the portfolio at all horizons simply because they are a good risk diversifier to the other asset classes. Hedge funds and real estate are not in the LHP.

A less risk averse investor will deviate from the LHP or GMV portfolios to benefit from higher expected returns. In Table 6 we show the strategic asset allocation for an "asset-liability" and an "asset-only" investor for different degrees of risk aversion.

An "asset-liability" investor with a 1 year horizon typically invests in hedge funds, bonds and commodities. Hedge funds are in the optimal portfolio for their return enhancement qualities, at the cost of stocks and real estate, because hedge funds have a higher Sharpe ratio and a high correlation with stocks at the one-year horizon. Bonds are in the portfolio for their liability-hedge qualities and their low correlation with all other assets. Commodities are particularly interesting as a risk diversifier. Combined with the high Sharpe ratio of commodities this explains the substantial positive weight of this asset class. These effects become stronger the lower the risk aversion.

Risk diversification is a dominant investment motive at longer horizons. The mean reverting character of stocks results in increasing weights at longer horizons. In addition, credits replace bonds to some extent. As could be expected from the flat term structure of risk of commodities, their portfolio weight is stable over the investment horizon. The weight increases at lower levels of risk aversion, due to the high Sharpe ratio of commodities.

Listed real estate does not seem to add much in portfolio context, except at low levels of risk aversion.

An "asset-only" investor will choose a very different portfolio. The T-bill has a positive and high weight at a risk aversion of 20, because it has low risk. For lower levels of risk aversion the weight of T-bills reduces due to the low return expectations. The poor liability-hedge properties, which were dominant from the "asset-liability" perspective, are not an issue here. Also, stocks are much more attractive due to the high Sharpe ratio. The weights of fixed income securities (bonds and credits) has reduced substantially. They were the best real rate hedged, but again, that is no issue any longer. The "asset-only" investor moves out of fixed income due to their low expected returns and high correlation with stocks in the medium and long-term.

The difference between "asset-only" and "asset-liability" portfolios are much smaller for commodities, listed real estate and hedge funds. Listed real estate behaves like stocks, but with a lower Sharpe ratio. In line with Froot (1995) we find that listed real estate does not add much value to a well-diversified portfolio. Hedge funds and commodities remain attractive because of their return enhancement. In addition, commodities are also attractive for risk diversification within the asset mix.

In summary, we found that, due to the high correlation with real rates, fixed income securities are more interesting for an "asset-liability" investor than for an "asset-only" investor. The high correlation of fixed income with stocks in the medium and long-term and the higher Sharpe ratio of stocks make stocks more attractive for an "asset-only" investor. Although the low risk of T-bills makes them attractive for a highly risk averse "asset-only" investor, their poor liability-hedging qualities are dominant for the "asset-liability" investor. The portfolio weights of hedge funds, commodities and listed real estate are not very sensitive to the inclusion of liabilities. Commodities are not only interesting for their high Sharpe ratio, but also for their good risk diversifying qualities. Listed real estate does not seem to add much in an already diversified portfolio.

4.3 Economic evaluation

In this section we consider the costs of suboptimal portfolios, and whether these costs are associated with expected returns, with asset risk diversification or with the liability-hedge potential.

In practice, investors could adopt another asset allocation than in (11) or (16) for reasons of liquidity, reputation risk or legal constraints. Liquidity forms a restriction as the desired allocation to an asset class is not available in the market at realistic transaction costs. Reputation risk comes in as most investors are evaluated and compared to their peers and competitors, while legal constraints could follow from rules which restrict investments to specific classes (e.g. no hedge funds are allowed). An investor could be reluctant to invest in alternatives if its peers only invest in more traditional assets as stocks and bonds. In this case it is interesting to evaluate the economic benefits or losses from ignoring alternative assets.

We use the certainty equivalent to evaluate the economic loss of deviating from the optimal strategic asset allocation. We define the economic loss of holding some sub-optimal portfolio a_0 instead of the optimal portfolio by computing the percentage riskfree return the investor requires to be compensated for holding the sub-optimal portfolio a_0 instead of $\alpha_t(k)$ in (16). For the "asset-liability" investor the certainty equivalent is defined as the percentage with which the initial funding ratio should increase to compensate the investor for suboptimal investing. It is computed as the difference between the mean-variance utility of the two portfolios. Substituting (13) and (14) into (15) and subtracting the utility from an arbitrary portfolio gives

$$\begin{aligned} f_t(k) &= (\alpha_t(k) - a_0)' \left(\mu_t(k) + \tfrac{1}{2}\sigma_x^2(k)\right) - (1-\gamma)(\alpha_t(k) - a_0)'\sigma_{0x}(k) \\ &\quad -\tfrac{1}{2}\gamma \left(\alpha_t'(k)\Sigma_{xx}(k)\alpha_t(k) - a_0'\Sigma_{xx}(k)a_0\right) \end{aligned} \tag{17}$$

The three components on the right hand side attribute the certainty equivalent to compensations for return enhancement, for liability hedge, and for risk diversification. These components are both time and horizon dependent. The first term, $(\alpha_t(k) - a_0)' \left(\mu_t(k) + \frac{1}{2}\sigma_x^2\right)$, reflects the compensation for the difference in expected return by investing in the sub-optimal portfolio. The second term, $-(1-\gamma)(\alpha_t(k) - a_0)'\sigma_{0x}$, represents the compensation for a suboptimal liability hedge. The third term, $-\frac{1}{2}\gamma\left(\alpha_t'(k)\Sigma_{xx}\alpha_t(k) - a_0'\Sigma_{xx}a_0\right)$ accounts for diversification of risks among alternative asset classes.

We use (17) to answer the question whether there is more than just stocks and bonds for strategic asset allocation. We use the same certainty equivalence calculation to determine the economic loss from choosing the strategic asset allocation in an "asset-only" context, when the relevant criterion would be the "asset-liability" perspective with risky liabilities.

In the figures below we show the certainty equivalent as the percentage the initial funding ratio should rise in order to compensate the investor for suboptimal investing: $F_t(k) = 100(\exp(f_t(k)) - 1)$. In other words it is the monetary compensation the investor requires in dollar terms in order to put 100 dollars in a_0 instead of the optimal portfolio $\alpha_t(k)$.

Asset-only versus "asset-liability": does it really matter? Figure 6 indicates that it does. It shows the benefits for an investor with risky liabilities if he solves the strategic asset allocation in an liability context instead of an "asset-only" context. This is done by deriving $\alpha_t(k)$ as the optimal asset allocation from an "asset-liability" perspective as in (11), whereas a_0 is the optimal asset allocation from an "asset-only" perspective as in (16). The certainty equivalent is positive and increases with the level of risk aversion and the investment horizon. A more risk averse investor puts more emphasis on ignoring the liability hedging qualities typically at longer horizons, because otherwise the mismatch risk rises even further. The investor with a risk aversion of 20 requires a 2% higher initial funding ratio if his investment horizon is 1 year. With a 25 years horizon he requires 33% more to compensate for ignoring the liabilities in the asset allocation. With lower risk aversion ($\gamma = 5$) the compensation reduces to 5% at the 25 year horizon.

Figure 7 provides insights in the sources of the compensation: return enhancement, liability hedge or risk diversification. The compensation for missed liability hedge opportunities is substantial at all horizons and dominates the certainty equivalent. The loss to missed return opportunities is only relevant at longer horizons. In the "asset-liability" framework the investor explicitly maximizes the return of the asset mix in excess of the liabilities, whereas he maximizes the return on the asset mix in the "asset-only" context. The "asset-liability" investor is worse off, however, in terms of risk diversification of the asset mix itself. At short horizons the certainty equivalent is mainly attributed to the liability hedge part. At medium and longer horizons the attribution to the return enhancement part becomes important as well. However, the required compensation for lost return and liability hedge is partly undone by the better risk diversification in the "asset-only" portfolio.

Is there more in the investment universe than stocks and bonds? To answer this question we compare $\alpha_t(k)$ with a suboptimal portfolio that is restricted to T-bills, stocks and bonds only. Figure 8 indicates that at the 1-year horizon a risk averse ($\gamma = 20$) "asset-liability" investor requires a lump sum of 2.2 dollars for each 100 dollars of initial investment to be compensated for having to ignore credits and alternatives. The loss increases steadily with the horizon to 78 dollars at a 25 year investment horizon. Alternatives and credits have good liability hedge properties at medium and long investment horizons. The liability hedge component in (17) dominates the certainty equivalent attribution, especially since the extra return advantage of alternatives and credits is partly undone by their higher risk in the asset mix itself.

Figure 9 summarizes and combines the two questions above. The costs of ignoring the liabilities in the asset allocation are substantial and increase with the investment horizon. The cost of investment constraints, which exclude credits and alternatives from the portfolio choice, is even higher. We have seen that these asset classes are not only interesting from a return enhancement perspective, but also for their liability hedging qualities.

5 Conclusions

This paper has studied the implications of horizon effects in volatilities and correlations for stocks, bonds, T-bills, credits and alternatives (e.g. commodities, hedge funds and listed real estate) on strategic asset allocation. The long-term investor can benefit from both time diversification and cross-sectional risk diversification possibilities of these assets. We have shown how inflation hedging qualities and real interest rate hedge properties of the asset classes change with the investment horizon. This is particularly important for "asset-liability" management where the investor has risky liabilities.

In a vector autoregressive model for return dynamics, stocks, bonds, credits, listed real estate and the liabilities all exhibit mean reversion. Hardly any horizon effects are observed in hedge funds and commodities.

For both stocks and bonds we find that the correlations with alternatives first rise and subsequently fall with the horizon. Credits move closely with bonds. Although commodities do not exhibit time diversification properties, they are a good risk diversifier in the cross section, since the correlation with stocks and bonds is low and sometimes even negative.

The T-bill is the best inflation hedge at all horizons. At longer horizons bonds and credits are good inflation hedges as well, whereas the short term hedging qualities are poor due to the inverse relationship between yield changes and bond prices. Stocks also turn out to be a good inflation hedge only in the long. Both in the short and the long run commodity prices move along with inflation, which makes them very attractive from an inflation hedge perspective. Listed real estate behaves very much like stocks.

Risky liabilities are subject to real interest rate risk. We find that the real interest rate hedge potentials of the asset classes change substantially with the investment horizon as well. The best hedges are bonds, closely followed by credits. For listed real estate, stocks and hedge funds we find that the real rate hedging qualities have a hump-shaped term structure. The maximum correlation occurs at the 10-years horizon. Commodities have a positive, but low correlation with the real interest rate.

In the optimal portfolio choice we have found that credits and bonds are more interesting for an "asset-liability" than for an "asset-only" investor, due to the high correlation with real rates. The high correlation with stocks in the medium and long-term combined with the higher Sharpe ratio of stocks makes stocks more attractive than credits and bonds for an "asset-only" investor. Although the low risk of T-bills makes them attractive for an "asset-only" investor, its bad liability hedge qualities are dominant for the "asset-liability" investor. Furthermore, we found that the weights in hedge funds, commodities and listed real estate are quite insensitive against including the liabilities in the analysis. The high sharpe ratio makes hedge funds very attractive from a return enhancement perspective. Commodities are not only interesting for their high sharpe ratio, but also for their good risk diversifying qualities. Listed real estate does not seem to add much in an already diversified portfolio. They behave like stocks, but have a lower sharpe ratio.

Asset-only versus "asset-liability": does it really matter? We show that it does. The costs of ignoring the liabilities in the asset allocation are substantial and increase with the investment horizon. For short investment horizons this is mainly attributed to the liability hedge part. In addition, at medium and longer horizons the attribution to the return enhancement part becomes important as well. However, the required compensation for lost return and liability hedge is partly undone by the better risk diversification in the "asset-only" portfolio.

Is there more in the investment universe than stocks and bonds? There certainly is. Alternatives and credits are interesting from a liability hedge as well as a return enhancement perspective. They have good liability-hedge properties at medium and long investment horizons. The liability-hedge properties are the largest source of costs of suboptimal portfolios.

Appendix A Holding period risk and return

With z_t defined by the first order VAR in (3), we can forward substitute to obtain z_{t+j} as

$$z_{t+j} = \left(\sum_{i=0}^{j-1} \mathbf{\Phi}_1^i\right) \mathbf{\Phi}_0 + \mathbf{\Phi}_1^j z_t + \sum_{i=0}^{j-1} \mathbf{\Phi}_1^i u_{t+j-i} \tag{A1}$$

Therefore the j-period ahead forecast is

$$\hat{z}_{t+j|t} = \sum_{i=0}^{j-1} \mathbf{\Phi}_1^i \mathbf{\Phi}_0 + \mathbf{\Phi}_1^j z_t \tag{A2}$$

For cumulative holding period returns over k periods we need $Z_{t+k}^{(k)} = \sum_{j=1}^{k} z_{t+j}$, which has expectation

$$\hat{Z}_t^{(k)} = \sum_{j=1}^{k} \left(\sum_{i=0}^{j-1} \mathbf{\Phi}_1^i \mathbf{\Phi}_0 + \mathbf{\Phi}_1^j z_t\right) \tag{A3}$$

and forecast error

$$Z_{t+k}^{(k)} - \hat{Z}_t^{(k)} = \sum_{j=1}^{k} \sum_{i=0}^{j-1} \mathbf{\Phi}_1^i u_{t+j-i} \tag{A4}$$

The covariance matrix of the k-period errors follows as

$$\Sigma(k) = \sum_{j=1}^{k} \left(\left(\sum_{i=0}^{j-1} \mathbf{\Phi}_1^i\right) \Sigma \left(\sum_{i=0}^{j-1} \mathbf{\Phi}_1^i\right)'\right) \tag{A5}$$

For portfolio choice we are only interested in the risk properties of the $(n+m)$ asset classes. The $((n+m) \times (n+m+s+1))$ selection matrix

$$\mathbf{S} = \begin{pmatrix} \mathbf{0}_{n+m,s+1} & I_{n+m,n+m} \end{pmatrix} \tag{A6}$$

extracts the excess returns from the vector z. Expected excess returns are thus defined as

$$\mu_t(k) = \mathbf{S}\hat{Z}_t^{(k)} \tag{A7}$$

Similarly, the excess return covariance matrix is defined as

$$\Sigma_{xx}(k) = \mathbf{S}\Sigma(k)\mathbf{S}' \tag{A8}$$

The variance as in (A5) is based on excess return. In some applications, however, we are interested in the variance of the total (real) returns. To derive these variances we need the alternative $((n+m+1) \times (n+m+s+1))$ transformation matrix

$$\mathbf{T} = \begin{pmatrix} 1 & \mathbf{0}'_{s,1} & \mathbf{0}'_{n+m,1} \\ \iota_{n+m} & \mathbf{0}_{n+m,s} & I_{n+m,n+m} \end{pmatrix} \tag{A9}$$

References

Ankrim, E.M., and C.H. Hensel (1993), Commodities in Asset Allocation: A Real-Asset Alternative to Real Estate?, *Financial Analysts Journal*, 49, 20-29

Asness, C., R. Krail and J. Liew (2001), Do Hedge Funds Hedge, *Journal of Portfolio Management*, Fall, 6-19

Barberis, N. (2000), Investing for the Long Run when Returns are Predictable, *Journal of Finance* 55, 225-264

Bekaert, G., and R.J. Hodrick (2001), Expectations Hypotheses Tests, *Journal of Finance* 56, 1357-1394

Bodie Z. (1983), Commodity futures as a hedge against inflation, *Journal of Portfolio Management*, Spring 1983, 12-17

Brandt, M.W., and P. Santa-Clara (2004), Dynamic Portfolio Selection by Augmenting the Asset Space, NBER Working Paper no. 10372

Brennan, M.J., E.S. Schwartz and R. Lagnado (1997), Strategic Asset Allocation, *Journal of Economic Dynamics and Control* 21, 1377-1403

Brennan, M.J., and Y. Xia (2004), Persistence, Predictability, and Portfolio Planning, Working Paper Wharton School of University of Pennsylvania

Brennan, M.J., and Y. Xia (2002), Dynamic Asset Allocation under Inflation, *Journal of Finance* 57, 1201-1238

Brooks, C., and H.M. Kat (2001), The Statistical Properties of Hedge Fund Index Returns and Their Implications for Investors, Working Paper ISMA Centre.

Campbell, J.Y., (1987), Stock returns and the term structure. *Journal of Financial Economics* 18, 373-399.

Campbell, J.Y., Chan Y.L., and L.M. Viceira (2003), a Multivariate Model for Strategic Asset Allocation, *Journal of Financial Economics* 67, 41-80.

Campbell, J.Y., A.W. Lo and A.C. MacKinlay (1997), *The Econometrics of Financial Markets*, Princeton University Press.

Campbell, J.Y., and R.J. Shiller (1988a), Stock prices, earnings and expected dividends, *Journal of Finance* 43, 661-676.

Campbell J.Y., and R.J. Shiller (1988b), The dividend price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195-228.

Campbell, J.Y., and R.J. Shiller (1991), Yield spreads and interest rates: a bird's eye view, *Review of Economic Studies* 58, 495-514.

Campbell, J.Y., and S.B. Thompson (2004), Predicting the Equity Premium Out of Sample: Can Anything Beat the Historical Average?, working paper, Harvard university.

Campbell, J.Y., and L.M. Viceira (1999), Consumption and Portfolio Decisions when expected Returns are time varying, *Quarterly Journal of Economics*, May, 433-495

Campbell, J.Y., and L.M. Viceira (2002), *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*, Oxford University Press.

Campbell, J.Y., and L.M. Viceira (2005), The Term Structure of the Risk-Return Tradeoff, *Financial Analysts Journal* 61, January/February, 34-44 .

Campbell, J.Y., and M. Yogo (2004), Efficient tests of stock return predictability, working paper, Harvard University.

Cochrane, J.H. (2001), *Asset Pricing*, Princeton University Press.

Cochrane, J.H., and M. Piazessi (2002), Bond Risk Premia, NBER Working Paper, no. 9178.

Dai, Q., and K.J. Singleton (2002), Expectation puzzles, time-varying risk premia, and affine models of the term structure, *Journal of Financial Economics* 63, 415-441

Dai, Q., and K.J. Singleton (2003), Term Structure Modeling in Theory and Reality, *Review of Financial Studies* 16, 631-678

Detemple, J.B., R. Garcia and M. Rindisbacher (2003), A Monte Carlo Method for Optimal Portfolios, *Journal of Finance* 58, 401-446.

Duffee, G.R., (2002), Term Premia and Interest Rate Forecasts in Affine Models, *Journal of Finance* 57, 405-443.

Fama, E.F. (1981), Stock returns, real activity, and money, *American Economic Review* 71, 545-565.

Fama, E.F., and K.R. French (1989), Business Conditions and Expected Returns on Stocks and bonds, *Journal of Financial Economics* 25, 23-49.

Froot, K.A., (1995), Hedging Portfolios with Real Assets, *Journal of Portfolio Management*, Summer, 60-77.

Geltner, D., (1991), Smoothing in appraised-based returns, *Journal of Real Estate Finance and Economics* 4, 327-345.

Geltner, D., (1993), Estimating market values from appraised values without assuming an efficient market, *Journal of Real Estate Research* 8, 325-345.

Gorton, G., and K.G. Rouwenhorst (2004), Facts and Fantasies about Commodity Futures, NBER Working Paper no. 10595.

Goyal, A., and I. Welch, (2002), Predicting the Equity Premium with Dividend Ratios, NBER Working Paper no. 8788.

Hamilton, J.D., (1994), *Time Series Analysis*, Princeton University Press.

Kim, T.S., and E. Omberg (1996), Dynamic Nonmyopic Portfolio Behavior, *Review of Financial Studies* 9, 141-161.

Leibowitz, M.L., and R.D. Hendriksson (1989), Portfolio Optimization with Shortfall Constraints: A Confidence-Limit Approach to Managing Downside Risk, *Financial Analyst Journal*, March/April, 34-41.

Leibowitz, M.L., S. Kogelman and L.N. Bader (1994), Funding Ratio Return, *Journal of Portfolio Management*, Fall, 39-47.

Lettau, M., and S. Ludvigson (2001), Consumption, Aggregate Wealth, and Expected Stock Returns, *Journal of Finance* 51, 815-849.

Markowitz, H., (1952), Portfolio Selection, *Journal of Finance* 7, 77-91.

Merton, R.C., (1969), Lifetime Portfolio Selection Under Uncertainty: The Continuous Time Case, *Review of Economics and Statistics* 51, 247-257.

Merton, R.C., (1971), Optimum Consumption and Portfolio Rules in a Continuous-Time Model, *Journal of Economic Theory* 3, 373-413.

Perold, A.F., and W.F. Sharpe (1988), Dynamic Strategies for Asset Allocation, *Financial Analysts Journal*, January-February, 16-27.

Posthuma, N., and P.-J. Van der Sluis (2003), A reality check on hedge fund returns, Research memorandum / Vrije Universiteit, Faculty of Economics and Business Administration, 2003-17.

Samuelson, P., (1969), Lifetime Portfolio Selection by Dynamic Stochastic Programming, *Review of Economics and Statistics*, 51, 239-246.

Schotman, P., and M. Schweitzer (2000), Horizon sensitivity of the inflation hedge of stocks, *Journal of Empirical Finance* 7, 301-315.

Stambaugh, R. (1997), Analysing investments whose histories differ in length, *Journal of Financial Economics* 45, 285-331.

Stambaugh, R. (1999), Predictive Regressions, *Journal of Financial Economics* 54, 375-421.

Wachter, J., (2002), Optimal Consumption and Portfolio Allocation under Mean-Reverting Returns: An Exact Solution for Complete Markets, *Journal of Financial and Quantitative Analysis* 37, 63-91.

Table 1: Summary Statistics

The table reports summary statistics over the entire sample that a series is available. The starting quarter is given in the last column. The sample ends in 2003:IV. The average, standard deviations and Sharpe Ratio (SR) are annualized. The remaining statistics are on a quarterly basis. XK is the excess kurtosis. The mean log returns are adjusted by one-half their variance so that they reflect mean gross returns.

	Mean	Stdev	Sharpe	Min	Max	Skew	XK	Start
Excess returns								
Stocks (x_s)	7.04	15.89	0.44	-30.85	18.92	-0.92	1.80	1952:II
Bonds (x_b)	1.46	9.58	0.15	-18.56	18.26	0.26	2.29	1952:II
Credits (x_{cr})	1.67	8.75	0.19	-17.07	18.15	0.08	2.78	1952:II
Commodities (x_{cm})	6.77	18.49	0.37	-23.86	42.03	0.30	2.30	1970:I
Real Estate (x_{lre})	4.86	16.85	0.29	-30.58	28.98	-0.38	1.93	1972:II
Hedge Funds (x_h)	3.60	8.96	0.40	-13.29	15.11	0.15	2.83	1994:II
Liabilities (x_0)	3.12	6.89	0.45	-9.27	14.58	0.48	1.77	1970:II
State variables								
Real rate (r_{tb})	1.35	1.34		-1.58	2.59	0.08	1.32	1952:II
Dividend Yield (dp)	-3.43	0.39		-4.50	-2.76	-0.86	0.52	1952:II
Nominal rate (r_{nom})	5.11	1.36		0.15	3.55	0.93	1.12	1952:II
Term Spread (spr)	1.21	0.59		-2.84	3.92	-0.09	0.40	1952:II
Credit Spread (cs)	1.55	0.32		0.30	3.42	0.38	-0.45	1952:II

Table 2: VAR of core variables: Parameter estimates

The table reports parameter estimates of the VAR $y_{t+1} = a + By_t + \epsilon_{t+1}$ with variables: 3 month tbill, 10-year bonds, stocks, dividend yield, nominal 3 month tbill, yield spread, and credit spread. T-statistics are reported in parenthesis. The last column contains the R^2 and the p-value of the F-statistic of joint significance.

	$r_{tb,t}$	$x_{b,t}$	$x_{s,t}$	dp_t	$r_{nom,t}$	spr_t	cs_t	R^2/p
$r_{tb,t+1}$	0.34	0.01	0.00	-0.16	0.35	0.43	-0.56	0.24
	(4.71)	(1.21)	(0.23)	(1.31)	(3.57)	(2.34)	(1.49)	(0.00)
$x_{b,t+1}$	-0.18	-0.03	-0.07	-0.56	1.20	5.16	-1.82	0.08
	(0.33)	(0.40)	(1.68)	(0.59)	(1.58)	(3.52)	(0.62)	(0.02)
$x_{s,t+1}$	0.67	0.03	0.04	5.25	-3.54	-0.57	10.44	0.10
	(0.72)	(0.20)	(0.63)	(3.35)	(2.82)	(0.24)	(2.15)	(0.00)
dp_{t+1}	-0.01	-0.00	-0.00	0.95	0.03	0.02	-0.14	0.96
	(0.80)	(0.26)	(0.67)	(60.55)	(2.63)	(0.75)	(2.86)	(0.00)
$r_{nom,t+1}$	-0.01	0.01	0.00	-0.01	1.03	0.19	-0.43	0.89
	(0.54)	(1.59)	(1.63)	(0.30)	(27.90)	(2.70)	(3.01)	(0.00)
spr_{t+1}	0.02	-0.01	-0.00	0.03	-0.07	0.68	0.54	0.68
	(0.82)	(2.13)	(0.95)	(0.98)	(2.52)	(12.84)	(5.01)	(0.00)
cs_{t+1}	-0.01	-0.00	-0.00	-0.04	0.05	0.06	0.78	0.79
	(1.19)	(1.41)	(2.48)	(2.31)	(4.22)	(2.50)	(16.60)	(0.00)

Table 3: VAR of core variables: Error correlation matrix

The table reports the error covariance matrix $\Sigma_{\epsilon\epsilon}$ of the VAR $y_{t+1} = a + By_t + \epsilon_{t+1}$ with variables: 3 month tbill, 10 years bonds, stocks, dividend yield, nominal 3 month tbill, yield spread, and credit spread. Diagonal entries are standard deviations; off-diagonal entries are correlations.

	r_{tb}	x_b	x_s	dp	r_{nom}	spr	cs
r_{tb}	0.58	—	—	—	—	—	—
x_b	0.40	4.60	—	—	—	—	—
x_s	0.27	0.20	7.57	—	—	—	—
dp	-0.29	-0.24	-0.95	0.08	—	—	—
r_{nom}	-0.35	-0.66	-0.07	0.13	0.22	—	—
spr	0.17	0.15	-0.03	-0.03	-0.83	0.17	—
cs	0.14	0.61	-0.07	0.09	-0.28	-0.11	0.07

Table 4: Excess return regressions

The table reports parameter estimates for the excess returns of the assets in the subset x_2: credits (x_{cr}), commodities (x_{cm}), listed real estate (x_{lre}), hedge funds (x_h) and liabilities (x_0). For each asset we report the regression results after setting all insignificant coefficients in the general specification

$$x_{2,t+1} = c + D_0 y_{t+1} + D_1 y_t + H x_{2,t} + \eta_{t+1}$$

equal to zero and after reparameterization. Reparameterization involves the first differences of the variables $y^{10} = spr + r_{nom}$ and cs. Explanatory variables in y_t are the nominal T-bill rate (r_{nom}), the term spread (spr), the default spread (cs), and the dividend price ratio (dp). The last column reports the regression R^2 and the p-value of the F-statistic of the zero restrictions with respect to the general model.

	Contemporaneous						Lagged				
	r_{tb}	x_b	x_s	spr	Δy^{10}	Δcs	r_{tb}	x_b	spr	own	R^2/p
x_{cr}	—	0.53	0.04	—	-16.06	-11.22	—	0.16	—	-0.21	0.91
	—	(6.57)	(3.33)	—	(5.09)	(6.75)	—	(2.81)	—	(3.43)	0.22
x_{cm}	-2.94	—	-0.22	—	—	—	1.31	—	—	—	0.11
	(2.57)	—	(2.36)	—	—	—	(1.17)	—	—	—	0.86
x_{lre}	—	0.23	0.56	7.51	—	—	—	—	-3.02	—	0.46
	—	(2.16)	(8.26)	(2.78)	—	—	—	—	(1.11)	—	0.12
x_h	—	—	0.30	—	—	—	—	—	—	—	0.35
	—	—	(4.41)	—	—	—	—	—	—	—	0.98
x_0	-2.10	0.31	-0.06	—	-10.78	—	—	—	—	—	0.81
	(11.30)	(2.83)	(3.57)	—	(2.71)	—	—	—	—	—	0.05

Table 5: Optimal Mean-Variance Portfolios

The table shows the optimal holdings for a buy-and-hold investor for four investment horizons: 1, 5, 10, and 25 years. The left panel shows the weights of the global minimum variance portfolio for the "asset-only" problem, while the right panel shows the weigths of the liability hedging portfolio for the "asset-liability" problem.

	Global Minimum Variance				**Liability Hedge**			
Horizon (years)	1	5	10	25	1	5	10	25
T-bills (r_{tb})	1.06	0.99	0.93	0.85	0.60	0.47	0.35	0.20
Bonds (x_b)	0.00	0.06	0.07	0.03	0.54	0.61	0.62	0.49
Stocks (x_s)	-0.02	-0.04	-0.03	0.05	-0.11	-0.13	-0.12	0.08
Credits (x_{cr})	-0.05	-0.04	0.00	0.04	-0.04	0.01	0.10	0.17
Commodities (x_{cm})	0.01	0.02	0.02	0.03	0.03	0.03	0.04	0.07
Real estate (x_{lre})	0.00	0.00	0.00	0.00	-0.01	0.01	0.01	0.00
Hedge funds (x_h)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6: Optimal portfolio choice

The table shows the optimal holdings for two types of investor ("asset-only" versus "asset-liability") for four investment horizons (1, 5, 10, and 25 years) for three levels of risk aversion (γ = 5, 10, 20).

		Asset-only				**Asset-Liability**			
γ	Horizon (years)	1	5	10	25	1	5	10	25
5	T-bills (r_{tb})	-1.16	-1.01	-1.04	-1.54	-0.79	-0.59	-0.57	-1.02
	Bonds (x_b)	0.29	-0.46	-0.84	-0.56	-0.14	-0.90	-1.27	-0.92
	Stocks (x_s)	0.45	0.76	1.17	1.68	0.53	0.84	1.24	1.66
	Credits (x_{cr})	0.22	0.51	0.51	0.26	0.22	0.47	0.43	0.15
	Commodities (x_{cm})	0.58	0.57	0.55	0.51	0.57	0.55	0.54	0.48
	Real estate (x_{lre})	0.05	0.05	0.04	0.04	0.05	0.05	0.04	0.04
	Hedge funds (x_h)	0.56	0.58	0.60	0.62	0.56	0.58	0.60	0.62
10	T-bills (r_{tb})	-0.28	-0.27	-0.34	-0.67	0.14	0.20	0.18	-0.09
	Bonds (x_b)	0.42	0.08	-0.11	-0.04	-0.07	-0.42	-0.60	-0.45
	Stocks (x_s)	0.17	0.31	0.53	0.88	0.25	0.40	0.61	0.86
	Credits (x_{cr})	0.09	0.26	0.31	0.21	0.08	0.22	0.22	0.09
	Commodities (x_{cm})	0.30	0.30	0.30	0.29	0.29	0.28	0.28	0.26
	Real estate (x_{lre})	0.02	0.03	0.02	0.02	0.03	0.02	0.02	0.02
	Hedge funds (x_h)	0.28	0.29	0.30	0.31	0.28	0.29	0.30	0.31
20	T-bills (r_{tb})	0.16	0.10	0.00	-0.23	0.60	0.60	0.56	0.38
	Bonds (x_b)	0.48	0.34	0.25	0.23	-0.03	-0.18	-0.26	-0.21
	Stocks (x_s)	0.03	0.09	0.20	0.48	0.11	0.18	0.29	0.46
	Credits (x_{cr})	0.02	0.14	0.21	0.19	0.01	0.09	0.11	0.07
	Commodities (x_{cm})	0.16	0.17	0.17	0.18	0.15	0.15	0.15	0.14
	Real estate (x_{lre})	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01
	Hedge funds (x_h)	0.14	0.15	0.15	0.15	0.14	0.15	0.15	0.15

Table 7: Restricted Portfolios

The table shows the optimal holdings for two types of investor ("asset-only" versus "asset-liability") for four investment horizons (1, 5, 10, and 25 years) with risk aversion $\gamma = 20$, who is restricted to invest in only T-bills, stocks and bonds.

		Asset-only				**Asset-Liability**			
γ	Horizon (years)	1	5	10	25	1	5	10	25
20	T-bills (r_{tb})	0.48	0.41	0.31	0.07	0.91	0.89	0.83	0.64
	Bonds (x_b)	0.49	0.49	0.47	0.41	-0.03	-0.08	-0.14	-0.13
	Stocks (x_s)	0.03	0.10	0.22	0.52	0.12	0.20	0.30	0.49

Figure 1: Annualized volatilities

Annualized volatilities of real asset returns and liabilities for a buy and hold investor across different investment horizons (in quarters).

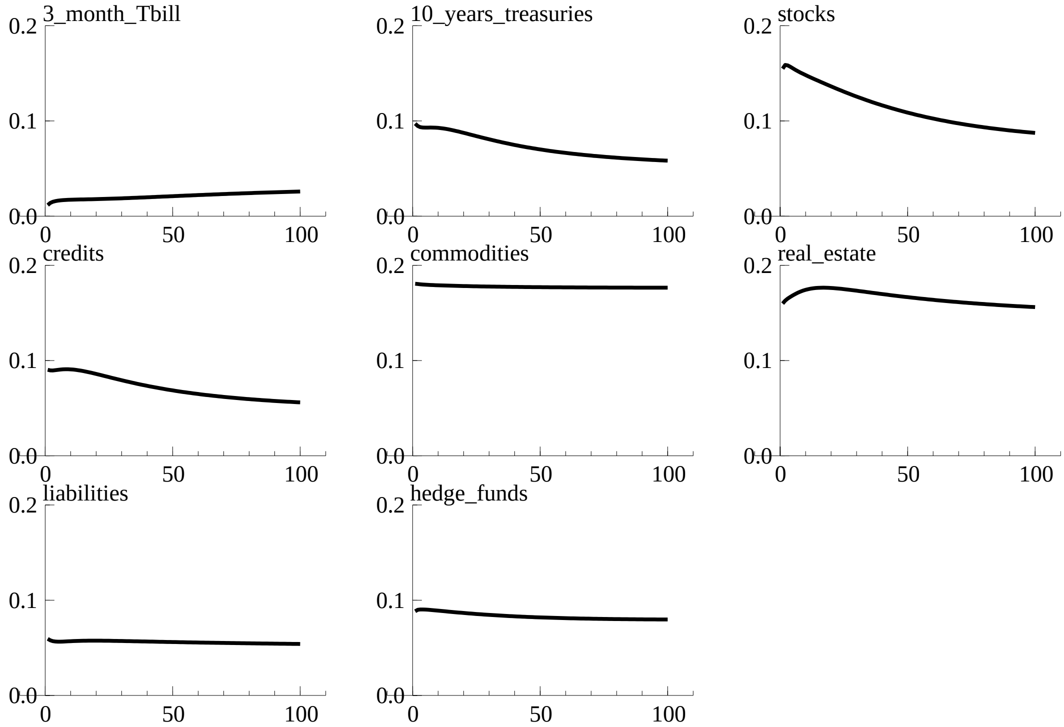

Figure 2: Correlations with stocks

Correlations of real stocks with other real assets for a buy and hold investor across different investment horizons (in quarters).

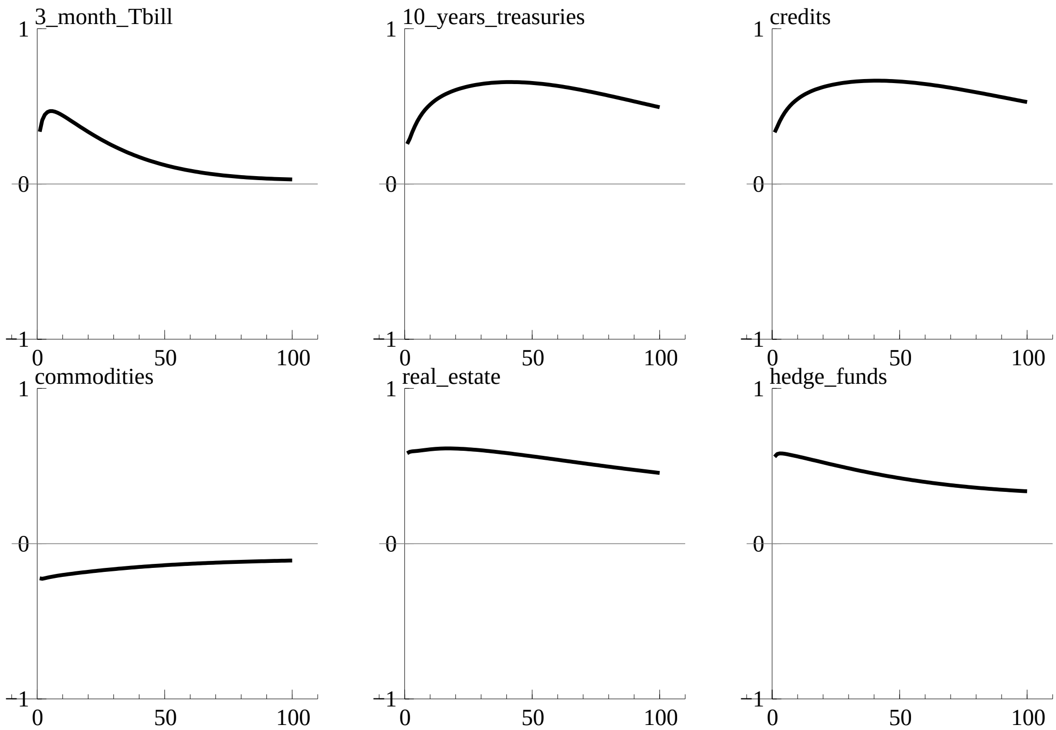

Figure 3: Correlations with bonds

Correlations of real bonds with other real assets for a buy and hold investor across different investment horizons (in quarters).

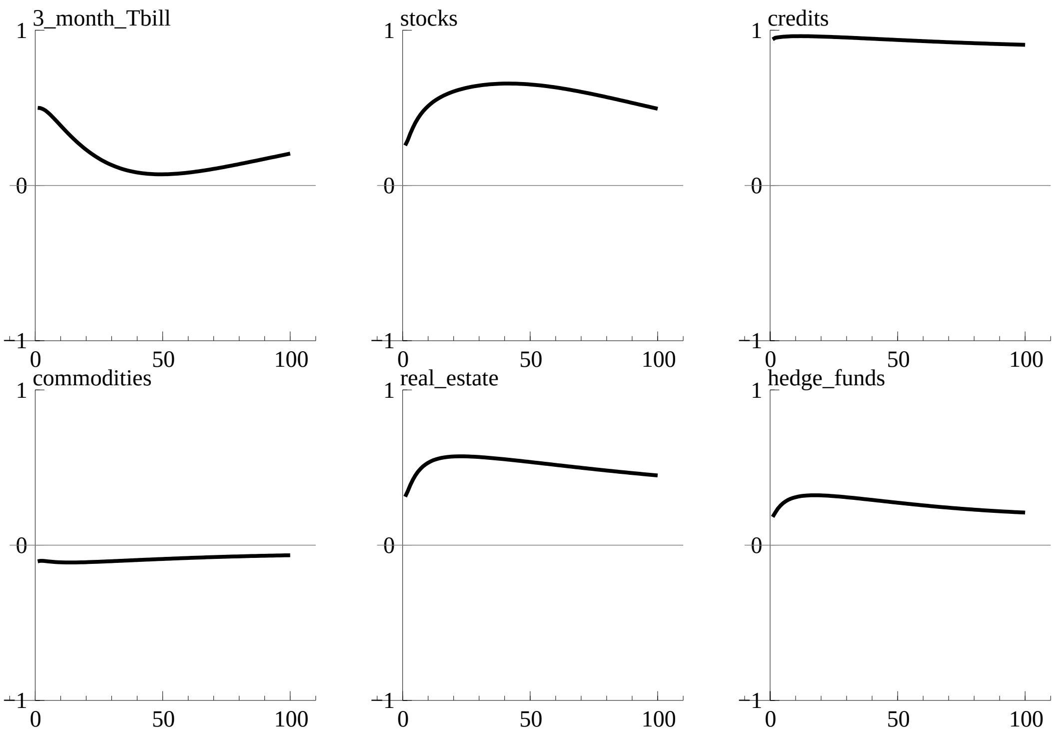

Figure 4: Inflation hedge properties

Inflation hedge properties of nominal assets for a buy and hold investor across different investment horizons (in quarters).

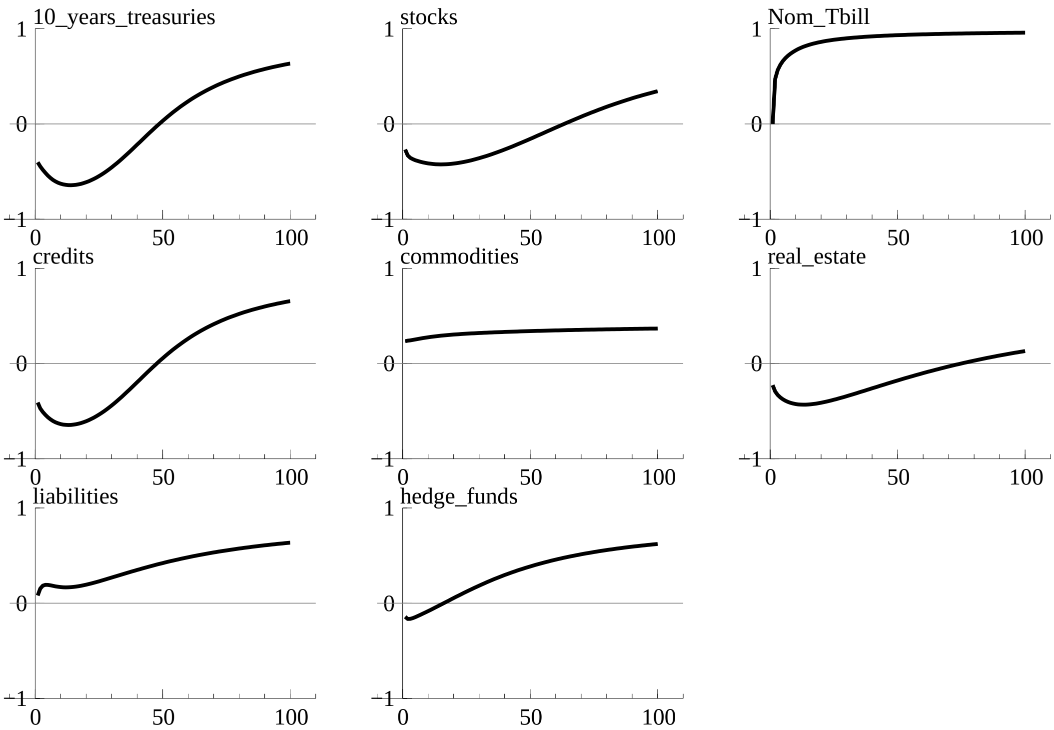

Figure 5: Real rate hedge properties

Liability (i.e. real rate) hedge properties of real assets for a buy and hold investor across different investment horizons (in quarters).

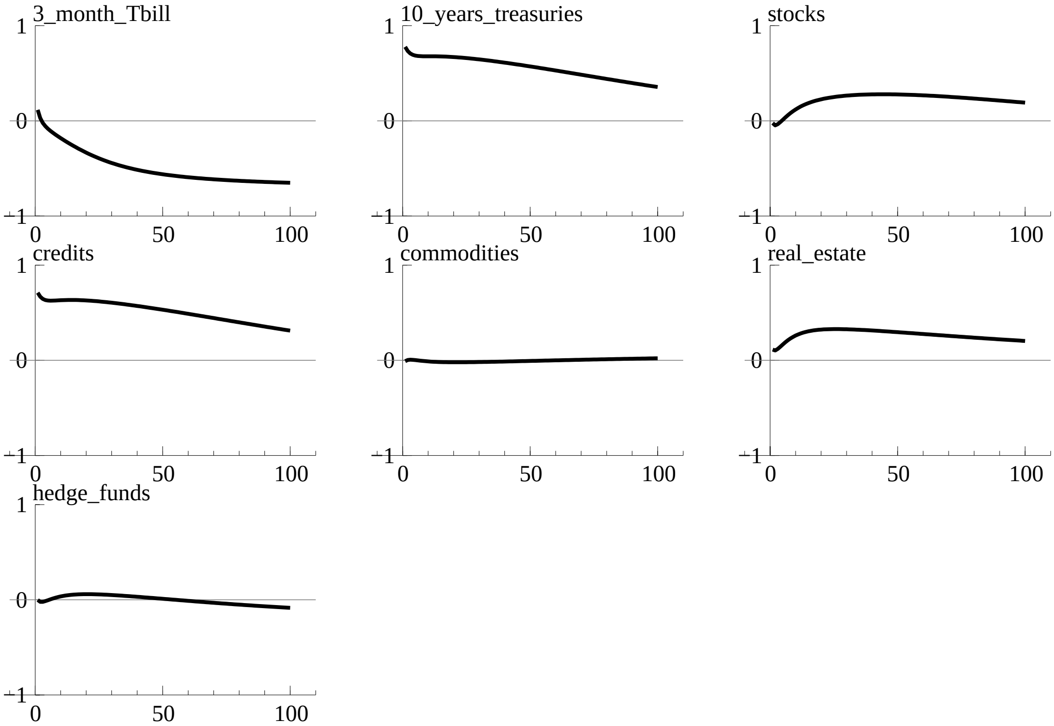

Figure 6: Certainty Equivalent: Ignorance of Liabilities

Certainty equivalent in simple terms for a k-holding period investor, which indicates the percentage increase in the intial funding ratio the investor with risky liabilities requires as a lump sum for investing in the optimal portfolio choice in an "asset-only" context, instead of the optimal portfolio choice in an "asset-liability" context. The log of the certainty equivalent is based on (17).

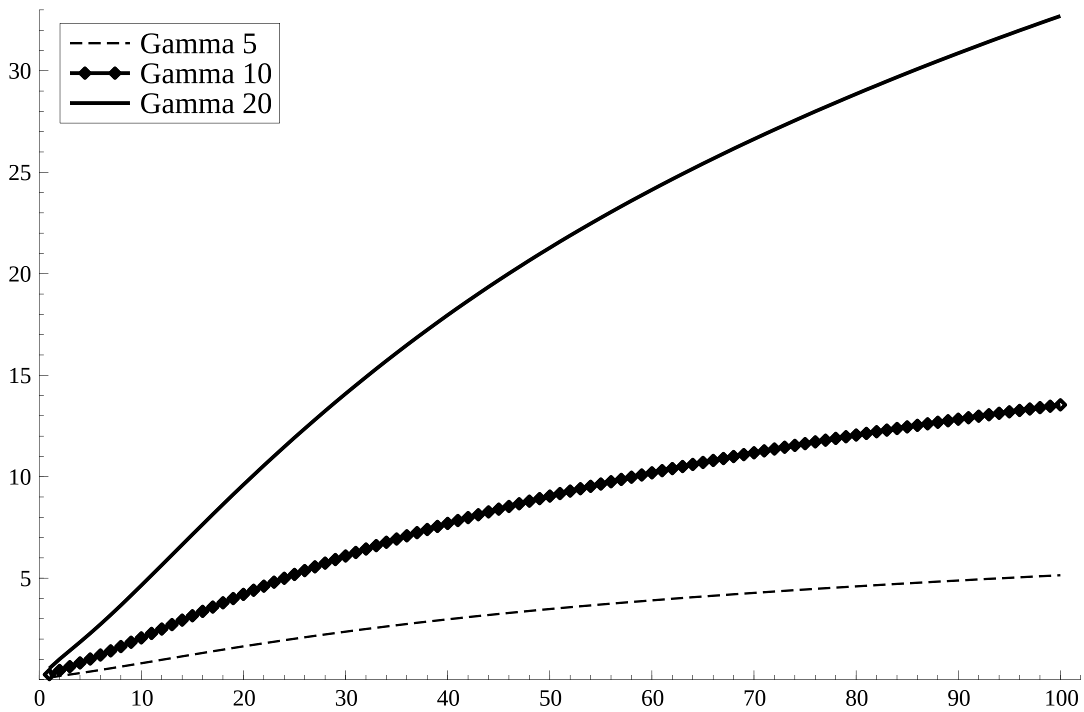

Figure 7: Certainty Equivalent Attribution: Ignorance of Liabilities

The log certainty equivalent is attributed to three parts: a return compensation, a liability hedge compensation and a risk diversification compensation. This graph is based on a k-holding period investor with risky liabilities and risk aversion 20 who chooses its optimal portfolio in an "asset-only" context, instead of in an "asset-liability" context. The log of the certainty equivalent is based on (17).

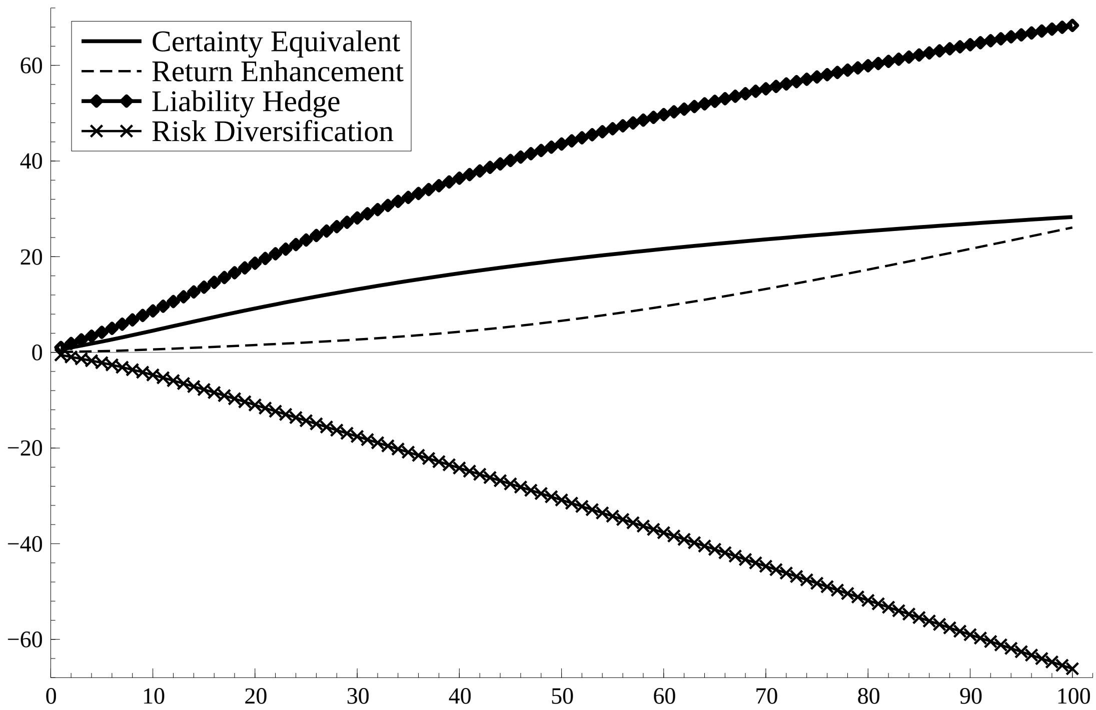

Figure 8: Certainty Equivalent Attribution: Ignorance of Alternatives and Credits

The log certainty equivalent is attributed to three parts: a return compensation, a liability hedge compensation and a risk diversification compensation. This graph is based on a k-holding period investor with risky liabilities and risk aversion 20 who chooses its optimal portfolio in an "asset-liability" context, but by only considering stocks and treasuries, instead of also credits, commodities, real estate and hedge funds. The log of the certainty equivalent is based on (17).

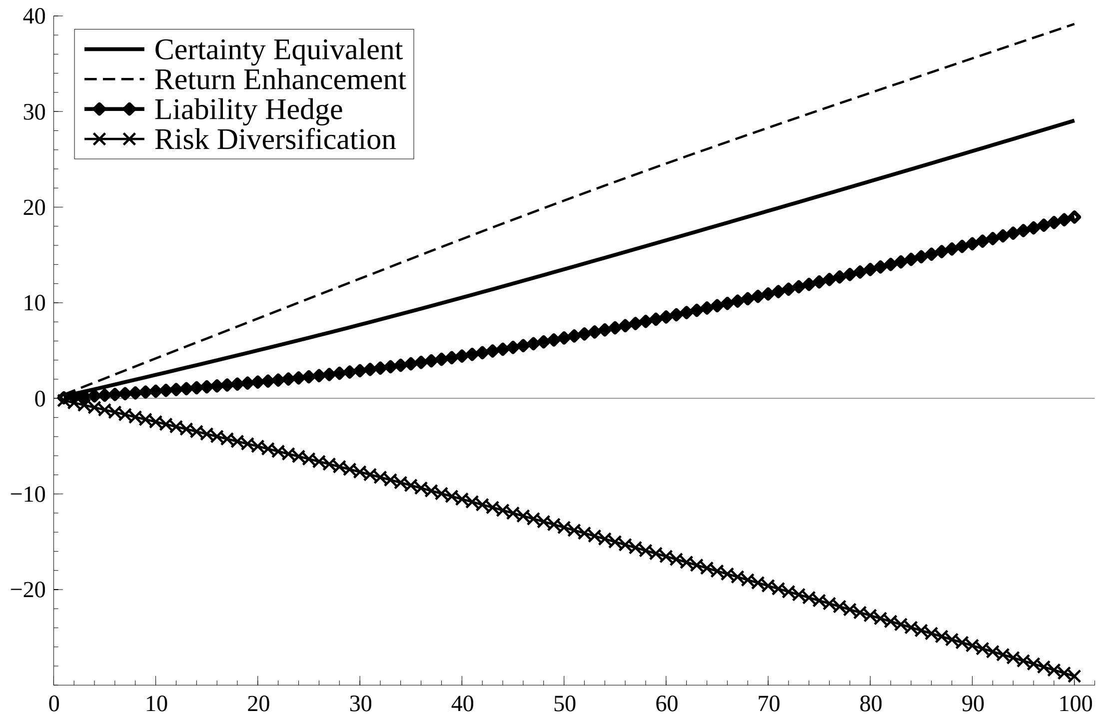

Figure 9: Certainty Equivalent

Certainty equivalent in simple terms for a k-holding period investor, which indicates the percentage increase in the intial funding ratio the investor with risky liabilities requires as a lump sum for investing in a particular suboptimal portfolio, instead of the optimal portfolio choice in an "asset-liability" context with stocks, treasuries, credits, commodities, real estate and hedge funds. The log of the certainty equivalent is based on (17).