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Abstract

We study risk sharing between generations in a variety of realistic collective funded pension schemes, where pension benefits and contributions may depend on the funding ratio and the asset returns. The collective pension schemes organizing the intergenerational risk sharing are optimised with respect to the risk allocation rules, the asset allocation rules and the generosity of pension benefits. Using contingent claims valuation method, we calculate the market value of the transfers between generations. We perform a welfare comparison between the collective plans and the optimal individual pension scheme without risk sharing. We show that well-structured intergenerational risk sharing is a zero-sum game in market value terms, but welfare-enhancing above the optimal individual benchmark. To some extent, even initially underfunded collective funds may provide higher utilities than the optimal individual benchmark.

Keywords: fair value, value-based generational accounting, intergenerational risk sharing.
JEL codes: G13, H55

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1 Introduction

It is well known from literature that optimally designed intergenerational risk sharing (IRS) is potentially welfare improving. However, the private market fails to provide insurance products based on IRS because human capital is non-tradable and current generations cannot sign contracts with future generations. Governments can implement institutional arrangements that enable generations to share risk among each other (see for a general exposition Gordon and Varian (1988) and Shiller (1999)). There are several ways to institutionalize IRS and to complete the market, such as fiscal policy and public debt management (Fisher (1983), Gale (1990), Bohn (2003)), monetary policy (Weiss 1979), Pay-As-You-Go social security programs (Merton (1983), Enders and Lapan (1982), Krueger and Kubler (2005)). Some recent papers look at the role of funded pension systems in facilitating IRS (Gollier (2006), Teulings and De Vries (2006)). The long term nature of pension funds allows smoothing of risk over time. Therefore the current and future participants of collective schemes are able to share shocks in asset returns and labor income. Important questions are: whether or not IRS is desirable in funded pension schemes and what is the optimal design regarding risk sharing rules?

In this paper, we analyze IRS within a number of realistic collective funded pension schemes. We look at a pure DB plan, a DC-like plan, and two hybrid schemes, which all four are stylized versions of actual pension schemes operating in public sector or industry pension funds in countries like the US, UK, Canada and the Netherlands[1]. The key feature of the collective schemes is that they are able to smooth shocks over and beyond the lifetime of any single generation. Surpluses or deficits in the funding process are shared between young, old and future generations by adjusting either contributions, or benefit levels or a combination of them, which lead to intergenerational transfers. Mandatory participation backed by appropriate government legislation makes this long run smoothing possible as future generations cannot opt out when they are confronted with a low initial level of funding. There are a few modelling differences between our paper and the aforementioned literatures. Gollier (2006) models the IRS via an insurance company where shareholders provide external capital (equity) as buffer. Teulings and De Vries (2006) model the IRS via collective pension funds by assuming that individuals can borrow against their future labor income and to invest before entering the labor market. We do not make these assumptions, i.e. plan sponsors are absent, and individuals are borrowing constrained.

In the paper, we perform a static optimization of the pension structure. The risk allocation rules specify who of the stakeholders, when, and to what extent is taking part in risk-bearing.

[1] In the United Kingdom and Canada, pension benefits are indexed with inflation until the maximum levels (e.g. 5%) are reached. In the Netherlands, in 2005, over 90% of the Dutch pension funds participants are covered by DB schemes with conditional indexation, where the magnitude of indexation depends on the financial position of the pension fund. (Source: Pension Monitor, DNB, www.dnb.nl). The 'Collective Hybrid' scheme modeled in this paper resembles these practices.

We optimize the risk allocation rules and the investment portfolios simultaneously, according to the expected utility of lifetime consumptions of a newly entry cohort. To avoid a cohort specific optimization, the pension structures and asset allocation rules are chosen at an initial time and are fixed for the entire period under consideration. Comparing with the expected utility of participating an optimal individual pension scheme, we show that the collective schemes could be designed to be welfare improving vis-a-vis the individual scheme due to the possibilities of intergenerational risk sharing. We also show that pension plans with flexibility in both contributions and benefit levels to absorb funding residue are the most preferred in welfare terms and these plans outperform plans that allow only adjustments in either contributions or benefit levels. This finding confirms Van Hemert (2005), who shows that the optimal intergenerational risk sharing in social security is neither a pure DB type nor a DC type, but a state contingent hybrid scheme.

We use the contingent claims analysis to value intergenerational risk sharing and transfers, building on Bader and Gold (2002), Blake (1998), Chapman et al. (2001) and De Jong (2003). This results in the method of value-based generational accounting (Ponds (2003)). We start off by defining the economy and a model for the returns on assets and liabilities. The asset return model implies a deflator that can be used to value future cash flows such as contributions, benefits and net asset transfers. Ex ante, a pension deal is fair when the net economic value of the transfers is zero. Ex post, a generation might be the net receiver or net payer. For each pension deal, we calculate the market value of the positive and negative transfers.

The most important analysis in the paper is the welfare comparison of various schemes, where we use the optimal individual scheme as the benchmark. Pension schemes that lead to smoother and higher consumption levels are ranked higher in utility terms. The recognition of the welfare aspects of risk-sharing within pension funds is important. An analysis in terms of only economic value may easily lead to the spurious conclusion, as argued for example by Exley (2004), that a pension fund is not relevant. Indeed, our collective pension schemes are a zero-sum game in value-terms, however they are potentially a positive-sum game in welfare terms.

Our result has an important implication related to the social security reforms world wide. Due to the increasing demographic pressure, many countries are gradually reducing (or planning to reduce) the PAYG social security and promoting the individual DC saving schemes (see for instance recent discussions of Feldstein (2005), Diamond and Orszag (2005)). Our result shows that the individual DC scheme is not the optimal funded scheme, even in a frictionless world with rational and sophisticated individuals. Collective funded schemes with well organized intergenerational risk sharing mechanisms can be better choices from a welfare perspective.

We want to stress a few important maintained assumptions of the paper. In our modeled collective schemes, the sponsors don't have an obligation to capture losses. Instead, investment and other risks are borne by the overlapping generations of pension plan members. We assume mandatory participation and a stationary composition of participants profile. We also make a number of simplifying assumptions on the economic environment. Firstly, we assume that a constant real interest rate exists in the economy. Secondly, we assume that the wage inflation is identical to the price inflation, and we model a flat real wage profile for all plan participants, without modeling the real wage uncertainties[2]. Finally, we assume that the risk allocation rules and asset allocation rules are invariable once they are chosen. We leave the effects of switching pension scheme outside the scope of this paper, but also here one may recognize its welfare enhancing potential.

The paper is organized as follows: Section 2 introduces the structure of the economy and the optimal individual schemes as our benchmark. Section 3 introduces collective pension arrangements that allow for intergenerational risk sharing. Section 4 develops the tools for valuation and welfare analysis. Section 5 presents and discusses the results of the analysis, and Section 6 concludes.

2 Financial market and the optimal individual pension arrangement

2.1 Financial markets

In this section we distinguish two economic environments, namely with and without the real risk-free assets. All the variables throughout this paper are expressed in real terms, i.e. scaled by the price level.

2.1.1 Financial market with real risk free assets

Under our baseline economy, price index linked bonds (ILBs) with all maturities are available. For simplicity, we assume for now that real interest rates, r, are non-stochastic. Real stock (equity) prices follow a geometric Brownian motion with drift:

$$dE_t/E_t = \mu_E dt + \sigma_E dZ_{E,t} \tag{1}$$

The real Sharpe ratio of the investment portfolio equals

$$\lambda_E = \frac{\mu_E - r}{\sigma_E} \tag{2}$$

The investment portfolios are a mix of equity and index linked bonds, with portfolio weight x

[2] With stochastic wage rate, wage indexed pension schemes resemble wage linked bonds, which are clearly not available in the markets. Wage rate uncertainties can be shared among generations via the funded schemes, in the same way as the investment risks. The arguments provided in the literature to organize IRS could be applied in this situation as well, which implies a further welfare enhancing potential of IRS.

for equities. The real value dynamics of this asset portfolio are given by

$$dA_t/A_t = [r + x\sigma_E\lambda_E]dt + x\sigma_E dZ_t \tag{3}$$

In shorthand, we will sometimes denote the real asset price dynamics as $dA_t = R_t A_t$, where R_t is the right hand side of equation (3). Both the expected real return and the volatility increase linearly with the fraction x invested in equities.

The stochastic discount factor M_t in this economy is the deflator for (real) risky cash flows, which in our model evolves according to

$$dM_t/M_t = -rdt - \lambda_E dZ_t \tag{4}$$

2.1.2 Financial market without real risk free asset

As an alternative environment, we also consider portfolios of equities and nominal bonds. We adopt the structure of the economy in Brennan and Xia (2002). Their model has four underlying risk factors: stock returns, expected inflation, unexpected inflation, and real interest rate shocks. Inflation is modeled as the sum of expected inflation π_t and an independent unexpected inflation shock. The expected inflation follows a mean reverting process

$$d\pi_t = q(\bar{\pi} - \pi_t)dt + \sigma_\pi dZ_{\pi,t} \tag{5}$$

The price level changes according to

$$d\Pi_t/\Pi_t = \pi_t dt + \sigma_u dZ_{u,t} \tag{6}$$

where the last term is the unexpected inflation. The corresponding Brownian motions are summarized in $\begin{bmatrix} Z_{E,t} & Z_{\pi,t} & Z_{u,t} \end{bmatrix}$ with correlation matrix $\rho \equiv \begin{bmatrix} 1, \rho_{E,\pi}, 0; & \rho_{E,\pi}, 1, 0; & 0, 0, 1 \end{bmatrix}$.

The stochastic discount factor M_t in this economy is the deflator for (real) risky cash flows, which in our model evolves according to

$$dM_t/M_t = -rdt - \kappa_E dZ_t - \kappa_\pi dZ_{\pi,t} - \kappa_u dZ_{u,t} \tag{7}$$

where $\kappa_t \equiv [\kappa_E, \kappa_\pi, \kappa_u]' = \rho^{-1}\lambda$ and λ is the vector of real market prices of risk.

The time t real price for a nominal bond with term-to-maturity τ is given by

$$P_t(\tau) = \exp\{A(\tau) - B(\tau)\pi_t\} \tag{8}$$

where $B(\tau) = (1 - e^{-q\tau})/q$ and $A(\tau) = B(\tau)\overline{\pi} - (\overline{\pi} + r - \sigma_u\lambda_u)\tau + \frac{1}{2}\sigma_\pi^2 \int_0^\tau B^2(s)ds$. The real return on the nominal bonds are given by

$$dP_t(\tau)/P_t(\tau) = [r - B(\tau)\sigma_\pi\lambda_\pi - \sigma_u\lambda_u] - B(\tau)\sigma_\pi dZ_\pi - \sigma_u dZ_u \tag{9}$$

The real interest rate, r, is well defined, but the risk free asset is not available. Investors invest in equities and nominal bonds. The return on portfolios of nominal bonds and equities follows the same logic as before

$$\begin{aligned} dA_t/A_t &= [r + x\sigma_E\lambda_E + (1-x)(-B(\tau)\sigma_\pi\lambda_\pi - \sigma_u\lambda_u)]dt \\ &+ x\sigma_E dZ_t - (1-x)B(\tau)\sigma_\pi dZ_\pi - (1-x)\sigma_u dZ_u \end{aligned} \tag{10}$$

2.1.3 Model calibration

Table 1 shows the default values for the model parameters to be used in the calculations. The parameters are partly based on estimates reported in Brennan and Xia (2002), but some values are superseded to numbers of our own preference. For example, we set the average inflation at 2%, in line with the informal ECB target for the euro area. We also assume the real interest rate is constant and equals 2%. Finally, the expected real return on equity is assumed to be 6%, implying an equity premium of 4%, which is in line with the long run estimates in Fama and French (2002).

2.2 The optimal individual schemes

Before model the collective schemes in Section 3, we describe two individual DC schemes, which serve as the benchmark(s) for the analysis of collective schemes. The first scheme is the flexible contribution scheme, where the individual can optimally choose his consumption level and portfolio composition at any time throughout his live. The other scheme is the fixed contribution scheme, where the contribution is a fixed fraction of labor income, so as to capture the general practices in the real world. Throughout the paper, we assume that the only savings of the investor are in the pension. Furthermore no state pension is available after retirement.

2.2.1 The flexible contribution scheme

The flexible contribution scheme (*DCflexible*) implements the optimal life cycle consumption and portfolio choice of an individual investor under no-borrowing and short sales constraints. The individual starts working at age 25 and retires at 65, which is the mandatory retirement age. For simplicity, we assume the agents die at age 80. During the working period (from age 25 to 65, i.e. $0 < t < 40 = R$), the individual earns a flat real labor income y_t. It is assumed that wage inflation is identical to price inflation. The individual chooses his consumption optimally and invests the rest of his income in the financial markets. After retirement the individual consumes his pension wealth out of this scheme. The individual is free to balance his portfolio, x_t (percentage in equities), during the whole life period, but subjected to the short sale constraint and the borrowing constraint, i.e. $0 \leq x_t \leq 1$. Furthermore, we assume that there is no state pension income available during the retirement period. The individual's consumption before and after retirement are denoted by c_t. The contributions are flexible, which are $p_t = y_t - c_t$ $(0 < t < 40 = R)$.

Formally the individual's consumption and portfolio choice problem is characterized by the following preference and subject to the pension wealth dynamics

$$\max_{\{0 \leq x_t \leq 1, c_t\}} E_0 \left[\int_0^T e^{-\delta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right] \tag{11}$$

$$s.t. \quad dW_t^{DCflexi} = [W_t^{DCflexi}(r + x_t(\mu - r)) + y_t - c_t]dt + \sigma x_t W_t^{DCflexi} dZ_t \tag{12}$$

$$W_0^{DCflexi} = 0 \tag{13}$$

where $T = 55$ and $W_t^{DCflexi}$ is the accumulated financial wealth.

Dynamic programming method is used to solve the individual scheme. After retirement, the optimal solution is the standard myopic portfolio, as first shown in Merton (1969).

$$x^* = \frac{1}{\gamma} \frac{\mu - r}{\sigma^2}, \qquad c_t^* = \frac{W_t^{DCflexi}}{g(0, T-t)}, \qquad (R \leq t < T) \tag{14}$$

where $g(0, T-t) = \frac{1}{F}\left(1 - \exp\left(-F\left(T-t\right)\right)\right)$, and $F = \frac{\delta - r(1-\gamma)}{\gamma} - \frac{(1-\gamma)(\mu-r)^2}{\gamma^2 \sigma^2}$. The optimal solution before retirement is not available due to the short sales and the borrowing constraints. We use the numerical procedure presented by Carroll (2006) to solve the consumption and portfolio policies backwards, for both financial market settings.

2.2.2 The fixed contribution scheme

The fixed contribution scheme (*DCfixed*) differs slightly from the flexible one in that the individual must contribute a fixed fraction of his labor income into the individual pension scheme and consumes the rest of his income. A fixed contribution rate, m, is chosen at the beginning of his career, such that the utility of life time consumption is optimal. The periodic contributions are denoted by $p_t = m y_t$. The individual's consumption before and after retirement is $c_t = (1 - m) y_t$ (for $t < R$) and $c_t = b_t$ (for $t \geq R$) respectively. This scheme is more realistic than the flexible DC scheme, capturing the common practice in real world. Poterba, Rauh, Venti and Wise (2005) study a fixed contribution scheme as described here and how different asset allocation strategies, including the lifecycle strategies, affect the expected utility of wealth at retirement. They show that the empirical lifecycle strategy provides the highest expected utility for $\gamma = 4$ investor, in a DCfixed scheme (Table 6 of their paper).

Formally the individual's consumption and portfolio choice problem is characterized by the following preference and subject to the pension wealth dynamics

$$\max_{\{m,0\leq x_t\leq 1,b_t\}} E_0\left[\int_0^T e^{-\delta t}\frac{c_t^{1-\gamma}}{1-\gamma}dt\right] \quad (15)$$

$$s.t. \quad dW_t^{DCfixed} = [W_t^{DCfixed}(r+x_t(\mu-r))+my_t-b_t]dt+\sigma x_t W_t^{DCfixed}dZ_t \quad (16)$$

$$W_0^{DCfixed} = 0 \quad (17)$$

The optimal consumption and portfolio choice after retirement is identical to the flexible contribution case. The analytical solution before retirement is not available due to the short sale constraint and borrowing constraint. However, without the constraints, the optimal individual portfolio choice before retirement is the leveraged myopic portfolio, as shown in Campbell and Viceira (2002):

$$x_t = \frac{1}{\gamma}\frac{\mu-r}{\sigma^2}\frac{W_t^{DCfixed}+PVC_t}{W_t^{DCfixed}} \quad (18)$$

where PVC_t denotes the present value of the future pension contributions, $PVC_t = m\int_t^R e^{-rt}y_s ds$ (before retirement) and $PVC_t = 0$ (after retirement). The present value of the contributions is therefore a fraction of the investor's human capital. Appendix 1 gives the analytical solution of the optimal contribution rate, m^*, if the investor were not borrowing nor short sales constrained.

Since the investor is borrowing and short sales constrained, the optimal portfolio[3] can be closely approximated by: $x_t^{app} = \min(\max(0,x_t),1)$. Figure 5 shows the average patterns of x_t^{app} for $\gamma = 3,5,8$. The approximated optimal contribution rate before retirement is found by numerical search. The approximated expected utility is concave in the contribution rate, meaning that over-saving and under-saving are both not desirable.

3 Optimal collective pension arrangements

In this section, we model the returns on investment assets and the evolution of the liabilities of the four collective schemes. We consider the steady state behavior of several pension funds where funding risk is borne by the plan members via adjustments in contribution and/or benefit. In the practice the contribution and benefit policy rules of such pension funds are based on the surplus or deficit of the funds. We assume stationary pension funds, where the inflow and outflow of members guarantee a stable composition of members as to age and sex. Mandatory participation is required

[3] Grossman and Vila (1992) show that the optimal portfolio is not the myopic portfolio with bounds, $\min(\max(0,x_t),1)$, but affected by the possibility that the constraints may bind in the future.

by law. We assume each age cohort consists of identical individuals, whose preference and labor income profile are the same as described in the previous section. There are 55 age cohorts (25–80 year-olds) coexisting in the pension fund at any point in time.

3.1 General setup

The collective schemes aim at an average salary DB scheme, wherein the real benefit is a weighted average of the past labor income during the whole working period. The weight is called the pension accrual rate, denoted by k (say 2% per year over salary). At the individual level, the accrued pension right b_t and the pension income during retirement b_R are then determined by

$$b_t = \begin{cases} \int_0^t k y_s ds, & 0 < t \leq R \\ \int_0^R k y_s ds, & R < t < T \end{cases} \tag{19}$$

where $t = 0$ denotes age 25, and $t = R$ denotes age 65. Since the real benefits are guaranteed, the nominal benefits are fully indexed with inflation. The ratio of pension benefit to the final salary, b_R/y_R , is called the replacement rate.

These pension benefits are financed by the contributions, $p_t = m y_t$. The required contribution rate, m, is solved from the following present value budget constraint:

$$\int_0^R \exp(-rs) m y_s ds = \int_R^T \exp(-rs) b_R ds \tag{20}$$

There is a one-to-one relationship between the contribution rate and the accrual rate. The higher pension ambition, the higher contributions are required. Without investment uncertainty, each generation finances their own pension. For instance, if annual salary is flat at 30000 euro, and $k = 2.06\%$, and assuming a fixed real risk free rate r =2%, this gives b =24763 euro (or 82.5% as replacement rate), and p = 5237 euro (or equivalently m = 17.5%) of the salary. In practice the board of trustees decide upon the level of accrual rate, taking into account the preference of the participants. Here we assume the board of trustees optimally chooses the accrual rate together with the risk allocation rule and investment policy, based on the preference of a representative participant (see Section 4.1).

The pension liability of each participant (or each generation), which is called the accrued benefit obligation (abo), is the present value of the accrued benefits:

$$abo_t = \int_R^T e^{-r(s-t)} b_t ds \tag{21}$$

On the aggregate level, the total liability of the fund ABO_t equals the sum over all individuals (or all age cohorts). In the stationary pension fund with fixed target benefits, ABO_t is a constant.

$$ABO_t = \sum_{ind} abo_t \tag{22}$$

The aggregate asset follows the dynamics:

$$dA_t = \left[A_t\left(r + x\left(\mu - r\right)\right) + P_t - B_t\right]dt + \sigma A_t x dZ_t \tag{23}$$

where x denotes the portfolio weight in risky asset, and P_t and B_t denote the collective contributions and benefits, which will be defined shortly per scheme. Furthermore, we denote P_t^* and B_t^* as the targeted contribution and benefit level, if all contributions are invested in the risk free assets. P_t^* and B_t^* are defined by (19) and (20), for any chosen accural rate. Notice that we use capital letters to represent the amount at the fund level (P_t, B_t, ABO_t, A_t), whereas the small letters to represent the amount at individual level (p_t, b_t, abo_t, a_t).

3.2 Risk allocation rules

Except for investing all the contributions in the risk free asset, the index lined bonds in this case, there is always the risk of mismatch between the accumulated asset and the liability for the targeted DB scheme. The aggregate surplus in the fund is defined by

$$S_t = A_t - ABO_t \tag{24}$$

We present four risk allocation rules using four pension schemes. The risk allocation rule specifies who of the stakeholders, when, and to what extent is taking part in risk-bearing. In a nutshell, the contributions or benefit (or indexations) are adjusted according to the risk allocation rules. The additional contributions and/or additional benefits above the target amount depend on the funding residual:

$$\begin{aligned} P_t - P^* &= -\boldsymbol{\alpha} S_t \\ B_t - B^* &= \boldsymbol{\beta} S_t \end{aligned}$$

where the bold symbols $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are to be specified in each pension deals below.

In general, the dynamics of the surplus in the collective schemes can be described by[4]

[4]This expression is derived with the help of an alternative definition of the accrued liability ABO, which is the present value of future benefits, minus the yet to be paid premiums:

$$ABO_t = \int_t^{\infty} e^{-r(s-t)} B_s^* ds - \int_t^{R} e^{-r(s-t)} P_s^* ds,$$

$$dS_t = [(P_t - P^*) - (B_t - B^*)]\, dt + R_t A_t - (r + i_t)\, ABO_t dt \tag{25}$$

where the additional indexation i_t is to be specified below (see 3.2.2). This equation is very intuitive: the change in the fund's surplus is determined by the difference between expected and actual cash flows, the asset returns, minus the increase in the accrued benefits due to the additional indexation and the cost of capital (the real interest rate). The following table summaries the differences among the collective schemes to be discussed.

CDB	$\boldsymbol{\alpha > 0}$ and $\boldsymbol{\beta = 0}$
CDC	$\boldsymbol{\alpha = 0}$ and $\boldsymbol{\beta > 0}$
CLinear, CHybrid	$\boldsymbol{\alpha > 0}$ and $\boldsymbol{\beta > 0}$

3.2.1 Collective DB

In the traditional collective defined benefit scheme (CDB), real benefits are defined by (19) and guaranteed, but contributions are flexible. Hence $\boldsymbol{\alpha > 0}$ and $\boldsymbol{\beta = 0}$. In the CDB scheme, contributions vary in order to absorb the funding mismatch S_t. The contribution policy can be formulated as

$$P_t = P^* - \alpha S_t \qquad \text{with} \qquad r \leq \alpha \leq 1 \tag{26}$$

The slope coefficient $\alpha \in [r, 1]$[5] determines the speed of absorbing the funding risks. $\alpha = 1$ implies that funding surplus is immediately fully absorbed. This response is difficult or infeasible when the funding deficit is large. When α approaches the risk free rate r, each generation in each period absorbs only the 'interest' accrued on their funding surplus and passes the principal into the infinite future. A lower value of α implies shifting the funding mismatch into future, and eventually shared across generations. Therefore the lower the value of α the higher the degree of intergenerational risk sharing.

The dynamics of ABO_t without additional indexation can be described by

$$dABO_t = r \cdot ABO_t dt + P_t^* dt - B_t^* dt$$

The dynamics of ABO_t with additional indexation (see 3.2.2) can be described by

$$dABO_t = (r + i) \cdot ABO_t dt + P_t^* dt - B_t^* dt$$

The dynamic of surplus is then given by

$$dS_t = dA_t - dABO_t$$

[5] To get stability over time (i.e., non-explosive surplus values), we need to assume that $\alpha \geq r$.

The pension fund optimally chooses $\{\alpha, k, x\}$ on the behalf of the entry cohort to optimize the their life time utility (11).

3.2.2 Collective DC

The collective defined contribution scheme adjusts pension indexations[6] (hence all future benefits) in stead of contributions in order to absorb the funding surplus. In contrast to the CDB scheme where the unconditional full indexation is guaranteed, in the CDC scheme indexations are adjustable. This is achieved in the form of an additional indexation, i_t, over and above the realized inflation rate. More specifically, every period the accrued liability as well as the accrued pension right are adjusted by the additional indexation

$$dABO_t = i_t ABO_t dt \tag{27}$$

$$dB_t = i_t B_t dt \tag{28}$$

Note that the additional indexation rate applies only to the accrued benefits. In this way, the mismatch risk in the funding process is borne by all members with accrued benefits by a suitable adjustment in the offered indexation. When the fund is overfunded, all accrued real liability is increased by i_t percent, that is all accrued future real benefits are increased by i_t percent. The additional indexation rate is used to (partly) close the balance of the fund at the end of the period. The additional indexation rate can be solved from the expression below

$$i_t ABO_t = \beta S_t \tag{29}$$

The slope coefficient $\beta \in [r, 1]$[7] determines the speed of absorbing the funding risks. $\beta = 1$ implies that funding surplus is immediately fully absorbed[8]. A lower value of β implies shifting certain funding mismatch into future, and eventually shared across generations. Therefore the lower the value of β the higher the degree of intergenerational risk sharing. When β approaches the risk free rate r, each generation in each period absorbs only the 'interest' accrued on their funding surplus and passes the principal into infinite future.

[6] Indexation is a technique to adjust income payments by means of a price index. This to keep up the purchasing power of the public after inflation. (Wikipedia)

[7] To get stability over time (i.e., non-explosive surplus values), we need to assume that $\beta \geq r$.

[8] This is what happens in the individual DC schemes, where pension liability always equals the accumulated asset by construction.

3.2.3 Collective Linear

The Collective Linear scheme adjusts both contributions and current-period benefits simultaneously to absorb the funding residual, thus $\boldsymbol{\alpha}>\mathbf{0}$ and $\boldsymbol{\beta}>\mathbf{0}$. The additional contribution and benefit are linearly related to the funding mismatch S_t. More specifically, a fraction, α, of the funding residual is shared among employees and a fraction, β, of the funding residual is shared among retirees. In total, $(\alpha+\beta)\,S_t$ is absorbed.

$$P_t = P^* - \alpha S_t \quad \text{for employees} \tag{30}$$

$$B_t = B^* + \beta S_t \quad \text{for retirees} \tag{31}$$

with $r \leq \alpha \leq 1$ and $r \leq \beta \leq 1$.

3.3 Collective Hybrid

Now we turn to a hybrid scheme which is composed of the components of the CDB scheme and the CDC scheme. Contributions and indexations are adjusted simultaneously up to certain limits to exclude unrealistic extremes. More specifically, this hybrid scheme offers caps and floors on the additional indexation, i_t, and the additional contribution P_t^{add}. Figure 2 depicts the structure of this scheme. This hybrid scheme resembles the practices in the Netherlands and UK.

The additional indexation is limited between plus and minus of the long run inflation rate, $[-\overline{\pi},+\overline{\pi}]$, where $\overline{\pi}=2\%$ in our setting. Within the limits, the additional indexation is a linear function of the funding risk, measured by S_t/ABO_t. The slope coefficient β determines the speed of absorbing the funding risks. A lower adjustment speed corresponds to a larger degree of intergenerational risk sharing, which allows shifting more funding risks over time and across generations. The liability and benefit level evolve in the same way as in CDC scheme. The additional contribution is limited between plus and minus twice of the cost price contribution. Here we assume the workers accept a maximum contribution equal to $3P^*$. They buy this protection by giving up the possibility that in good times the contribution rate may fall below zero. Similarly to the additional contribution in CDB scheme, P_t^{add} is a linear function of the funding risk S_t/ABO_t within the limits, i.e. $P_t^{add}=\alpha\frac{S_t}{ABO_t}=-2P^*/\left(\overline{\pi}/\beta\right)\frac{S_t}{ABO_t}$. However, the slope is time varying and depends on the choice of β. Lower value of β leads to longer period of risk absorbing and hence larger degree of intergenerational risk sharing.

$$i_t = \min\left(\max\left(-\overline{\pi}, \beta\frac{S_t}{ABO_t}\right), \overline{\pi}\right) \tag{32}$$

$$P_t^{add} = \min\left(\max\left(-2P^*, -\frac{2P^*}{\overline{\pi}/\beta}\frac{S_t}{ABO_t}\right), 2P^*\right) \tag{33}$$

$$P_t = P_t^{add} + P^* \tag{34}$$

$$dABO_t = i_t ABO_t dt \quad \text{and } dB_t = i_t B_t dt. \tag{35}$$

4 Evaluations of pension deals

4.1 Pension fund optimization problem

In this section we optimize the pension design from a new-entry-cohort's point of view, in order to compare with the individual scheme benchmark (*DCflexible*). The pension schemes are characterized by the adjustment coefficients (α, β) and the accrual rate k. The specification of the utility function is the same as the individual preference (11). The optimal share invested in stocks portfolios is denoted by x, which is not cohort specific nor state dependent. Since the pension fund is also borrowing constrained and short-sale constrained, the constrained portfolio choice thus requires $0 \leq x \leq 1$. The contributions and benefits (p_t, b_t) follow directly from the risk allocation rules in different schemes. Formally, the objective of pension fund is to optimize the utility of consumptions of the 25-year-old entry cohort, by optimally choosing $\{x, \alpha, \beta, k\}$. The optimization problem then becomes

$$\max_{\{0\leq x\leq 1,\alpha,\beta,k\}} U = E_0\left[\int_0^T e^{-\delta t}\frac{c_t^{1-\gamma}}{1-\gamma}dt\right] \tag{36}$$

$$\text{with} \quad c_t = y_t - p_t \quad \text{for employees} \tag{37}$$

$$c_t = b_t \quad \text{for retirees} \tag{38}$$

subject to the wealth dynamics (23), the liability dynamics (27), the surplus dynamics (25), and the portfolio choice constraint.

Since the decisions $\{x, \alpha, \beta, k\}$ are made at an initial time and remain time invariant, the problem is a static problem and can be solved using simulation and standard grid search. When changing one control variable while keeping the others constant, the maximized expected utility is smooth and concave in that control variable, which ensures that the global optimal is found.

In the following subsections we evaluate the optimized collective schemes in welfare terms and in economic value terms.

4.2 Welfare analysis

For the ease of comparison, we translate the achieved utility levels into the certainty equivalent consumption (CEC). The CEC of each scheme can be easily backed out from the following equality

$$\max U = \int_0^T e^{-\delta t} \frac{(CEC)^{1-\gamma}}{1-\gamma} dt \tag{39}$$

Let CEC^{DCflex} denote the welfare level achieved by the optimal individual scheme with flexible contributions. The ratio $\frac{CEC}{CEC^{DCflex}}$ shows the relative welfare gain or loss of the other schemes to DCflexible.

4.3 Market value of pension deals

We also look at all the cash flows generated by the pension deals and value these using the deflator method. This assumes all the risks are tradable. For an individual member this may be difficult once he stepped into a pension contract, but a market valuation is useful ex-ante when deciding which pension offer to accept, or when transferring pensions from one fund to another. From the point of view of the fund member, the market value of the pension deal is the value of the actual benefits minus the premiums. For a member with retirement date R and life expectancy T, this can be modeled as

$$NPV = \mathsf{E}\left[-\int_0^R M_t p_t dt + \int_R^T M_t b_t dt\right] \tag{40}$$

where p_t and b_t are the actual premiums paid and benefits received. The participants may contribute more or less than the actual enjoyed benefits. From an ex-ante point of view, the pension scheme is a fair deal if the net present value (40) is zero.

An alternative way of calculating the present value of pension deals is to look at the remaining balance, a_T, left in the notional cohort account at the end of life.

$$NPV = \mathsf{E}\left[M_T a_T\right] \tag{41}$$

$$= \mathsf{E}\left[M_T \left(a_T\right)^{+}\right] + \mathsf{E}\left[M_T \left(a_T\right)^{-}\right] = call - put \tag{42}$$

The remaining balance in the optimal individual DC schemes are zero. The individual consumes all the remaining asset in the last period as formulated in the dynamic programming approach. However, in the collective schemes, participants may shift the funding mismatches beyond one's life time, leaving surplus or deficit in the notional cohort account. $a_T > 0$ indicates the positive transfers from this generation to earlier and future generations in the scheme. A positive transfer towards future generations happens when surplus is left over unconsumed. A positive transfer towards older generations happens when existing deficit has been made up by this generation. Similarly a negative transfer means the current generation receives positive cashflows (protections) from other generations. Therefore the intergenerational transfers take place in both directions, in contrast with the one-way transfer from young to old as in the Pay As You Go system.

The value of the positive and negative transfers can be seen as the value of a call and a put option respectively. The current generation writes a call option to the other generations, and holds a put option from the other generations. When the value of call equals the value of put, the pension deal is a fair deal in value terms ex ante.

5 Results

5.1 Welfare analysis

Table 2 (in a market with ILB and stocks) and Table 4 (in a market with nominal bonds and stocks) show the main results of the paper. The tables show the optimized schemes and the achieved welfare in terms of *CEC*. All schemes are initially fully funded.

Firstly, well-structured intergenerational risk sharing is welfare enhancing once we compare the collective schemes to the optimal individual scheme (*DCflexible*). The ratio $\frac{CEC}{CEC^{DCflex}}$ indicates the welfare gain or loss relative to the optimal individual scheme. For instance in Table 2, for participants with a reasonable risk aversion ($\gamma = 5$), the CLinear scheme provides a welfare gain of 2.3%. The CHybrid scheme generates a welfare gain of 0.8%. The collective DB scheme has a marginal welfare loss of 0.3%. The CDC scheme shows a welfare loss of 2.7% relative to the optimal individual scheme. However, CDC slightly outperforms its individual counterpart (DC-fixed). Notice that DCfixed induces a 2.8% welfare cost due to the fixed contribution profile. If the more realistic DCfixed scheme were used as the benchmark, the hybrid collective schemes would significantly outperform the DCfixed by 3–6%.

The benefits of intergenerational risk sharing are reflected in several aspects. First, consumptions could be smoothed because the investment shocks can be absorbed beyond one's lifetime and shared by more generations. Second, intergenerational risk sharing reduces the effective risk aversion of participants, which results in higher stakes in risky assets. For instance, the CLinear participants with $\gamma = 5$ invest nearly 100% in equities, which is much higher than the DC individual schemes (Figure 5). This finding confirms the results of Gollier (2006) that intergenerational risk sharing increases the demand for risky investment. Participants are more capable to exploit the positive risk premiums in stocks and nominal bonds.

Remember that the collective schemes are restricted to be time invariant across different states of the economy. We expect that the welfare gains can be further improved if allowing state dependent portfolio choices and structure designs.

Two remarks should be mentioned alongside the theoretical comparison. First, the benchmark scheme, DCflexible, can hardly be implemented by individuals, due to the behavior problems and insufficient financial sophistication. On the contrary the collective schemes reflect the real world practices. Second, various transaction costs and management fees are not considered in our paper. In real world, the difference in costs is substantial between individual schemes and the collective schemes. Poterba, Rauh, Venti and Wise (2005) report that weighted average expense ratio of the lifecycle funds in U.S. for individual schemes (close to DCfixed in this paper) is 80 basis points per year. Whereas the industry pension schemes in the Netherlands are *20–30* basis points per year.

Secondly, it is shown that more efficient risk sharing can be achieved by using more risk absorbers. Through adjusting both contribution and benefit policies, the CLinear (also CHybrid) scheme spreads the funding surplus over all generations of participants. Whereas the CDB (CDC) scheme uses only contribution (indexation) policy as the risk absorber, hence only workers (retirees) are involved in risk sharing. CDB and CDC represent two extremes in risk allocation perspective. It is well known that it is never efficient to fully insure one category of agents in the economy unless they have a infinite risk aversion. The hybrid schemes allocate the risks more efficiently among all generations, hence reducing the costs of risk taking and resulting in welfare improvements. This result has important implications for the current trend of shifting from DB to (individual) DC schemes.

Thirdly, the less risk averse agent is more inclined to take more risk, and benefits more from the intergenerational risk sharing arrangement. The welfare gain of a $\gamma = 3$ investor is 3.6% in CLinear scheme, whereas it reduces to 0.9% for a $\gamma = 8$ investor.

5.2 Characteristics of the optimal collective schemes

Table 2 and 4 also show the optimal structures with parameters $\{k, \alpha, \beta, x\}$ for two alternative financial markets. We compare the schemes in two dimensions: cross scheme and cross a set of risk aversion values.

First, the cross schemes comparison. The desired level of contribution (or accrual rate) and portfolio choice vary significantly cross schemes. The desired contribution rate in CDC is the lowest among the collective schemes, requiring 11–13% of annual salary, which is close to the individual DCfixed contribution scheme. The contribution rate in CDB is the highest, ranging from 16–18% depending on the risk aversion. The portfolio choice of CDC scheme is less aggressive than the other collective schemes. The risky portfolio weight of CDC ranges from 30% to 80% depending on the risk aversion parameter, whereas the risky share of CDB ranges from 40% to 100%. The CHybrid is a hybrid form of CDC and CDB, therefore sitting in the middle. The CLinear scheme combines a moderate level of contribution with the most aggressive investment portfolio, namely 60–100% in stocks. This shows that intergenerational risk sharing makes the aggressive investment more attractive.

Second, as the degree of risk aversion increases from 3 to 8, three changes are observed: *i*) the portfolios become less risky, *ii*) the level of contribution rate increases, and *iii*) the schemes rely more on intergenerational risk sharing (by choosing lower values for α and β), and . However, the changes in portfolio compositions are more dramatic than the changes in the contribution rates and the scheme structures.

5.3 Market valuation of the pension deals

Table 5, 6 and 7 show the market valuation of the intergenerational transfers. The market value of each pension scheme is presented in columns 'actual contribution' and 'actual benefit'. The 'actual contributions' and 'actual benefits' are the present values of what 25-year-old entry cohort contributes to and receives from the pension fund respectively. The market value of the intergenerational transfers (from current generation to other generations) are shown in columns 'positive transfers' and 'negative transfers', that is, max(transfer, 0) and min(transfer,0), which are the value of a call and a put option respectively. In most of the cases, the value of the options are large (between 0.5 to 1.25 times of annual salary), indicating a substantial amount of transfers.

The call and put option can be regarded as contracts written between current generation and all other generations. The new-entry-cohort writes a call option to the other generations, and holds a put option from the other generations. In other words, they sell a part of the upward potential to other generation in exchange for some downward protection from other generations. When the value of call equals the value of put, i.e. the value of net transfers are zero, the pension deal is a fair deal in value terms ex ante. Indeed as shown in Table 5, 6 and 7, the net transfers are zero ex ante. The only exception is the CHybrid scheme, which induces slight positive net transfers due to the specified collar structure (Figure 2).

The value of the calls and puts would be exactly zero if all assets were invested in Index Linked Bonds. This is no surprise, as with this investment strategy the cost price contribution is exactly the right price to guarantee the benefits. The optimal portfolios are not the risk free assets of course. When all schemes adopt the same contribution rate (17.5%) and the same portfolio (50% ILB and 50% stocks), Table 5 shows that the CDB scheme results in a higher value of transfers than other schemes, indicating that guaranteeing the retirement benefits calls for larger transfers between the generations. Table 6 shows that, when all schemes adopt the optimal structures (as in Table 2), the market values of transfers are further affected by the level of contribution and the asset mix. The optimal CDB scheme results in the highest value of transfers (1.25) whereas the optimal CDC scheme results in the lowest value of transfers (0.5).

Figure 3 plots the {5%, 50% 95%} quantiles of the normalized consumptions, c_t/y_t, of four collective plans, with the scheme parameters corresponding to Table 5. The distributions of consumption indicate that the contribution reductions are more frequent (and also larger) than contribution increases. This is due to the equity premium. However, this does not add value to the pension deal: the lower contributions occur in scenarios where the equity returns are high, but the deflators in such scenarios are low. Therefore, the present values of the actual contribution in CDC, CDB and CLinear equal the cost-price contribution.

5.4 Extension and Robustness checks

5.4.1 Without risk free asset

In a market setting where ILB is not available, but nominal bonds are available, investing in nominal bonds is the obvious alternative although inflation risks could not be hedged perfectly by this portfolio. Replacing ILB with nominal bonds in the optimal asset mix does not change the overall picture of the main results. To approximate the returns on an actively managed bond portfolio, which may include various coupon-bonds with different term-to-maturity and different coupon rates, we use the return process of a zero-coupon bond, with term-to-maturity equals the (modified) duration of the bond portfolio. Here we assume that the duration of the bond portfolio is 7 years. The results in Table 4 are very comparable with the results shown in Table 2. The intuitions from section 5.1 hold. Furthermore, we investigate how the assumed risk premium affects the utility outcome. When the inflation risk premiums are put to zero $[\lambda_\pi = 0, \lambda_u = 0]$, the welfare levels are reduced to what achieved in the market with ILB (Table 2).

5.4.2 Reduced equity premium

Table 8 shows the optimal schemes and the welfare evaluations when equity premium is reduced to from 4% to 3%. The results are in line with our baseline results (Table 2). However the welfare levels are lower and the relative welfare gains are smaller. For instance, the welfare gain for $\gamma = 5$ investor is reduced from 2.3% to 0.8% in terms of the certainty equivalent consumption.

5.4.3 Initially underfunded schemes and overfunded schemes

Figure 4 shows the quantile plots of the normalized funding mismatch, S_t/ABO_t, of four collective plans with 50/50 asset mix. Funding surpluses are more often (and larger in size) than funding deficits. However, the under-funded situations worry us most. Figure 6 shows the underfunding frequencies of the four optimal schemes as described in Table 2. In the short run the frequency of 90%- (and 80%-) underfunding is roughly 20% (10%). In the long run the frequency of underfunding is dramatically reduced. This indicates that intergenerational risk sharing becomes more sustainable in the long run. Table 9 shows the welfare gains and losses of entering the initially underfunded schemes (with $FR_0 = 0.8$ and $FR_0 = 0.9$ respectively), in a market with index linked bonds and stocks. The new cohort joining an under-funded collective scheme is not necessarily worse off in welfare terms, comparing with the optimal individual benchmark. For instance, the welfare of CLinear is 100.9% (and 99.2%) of that of the optimal individual scheme when FR_0 drops to 0.9 and 0.8 initially. Within certain range of the funding ratio, it is possible for well-structured collective pension schemes to absorb funding deficits by intergenerational risk sharing and meanwhile enhance the welfare for her participants.

Table 10 shows the market valuation of the intergenerational transfers when the collective schemes are initially underfunded, in a market with the risk free asset (ILB). The net transfers ex ante are non-zero (column 6). The net transfers are potentially large and positive, meaning the entry cohort makes up a large part of the initial deficits, by either higher contributions (CDB, CHybrid, CLinear) or lower benefits (CDC). The net transfers are proportional to the degree of under funding. For instance, the value of net transfers double when FR_0 is reduced from 0.9 to 0.8.

Initially overfunded schemes are also reported in Table 9 and 10. The market values of the net transfers starting with $FR_0 = 1.1$ are mirror images of $FR_0 = 0.9$. Van Bommel (2006) argues that intergenerational risk sharing can not sustainable when the fund is overfunded, since the living generations might renegotiate and raid the surplus. However in his paper, agents only share downside risks, but not participate in profit sharing. On the contrary, our paper allows for downside risk sharing as well as upside profit sharing. The overfunded schemes could be sustainable when the profit-sharing is provided in a desirable manner.

5.4.4 Schemes under the solvency constraint

The solvency constraint requires that, the probability of 80% reduction in funding ratio within next period should be small (e.g., lower than 2%). That is, $\Pr\left[FR_{t+1} < FR_t \cdot 80\%\right] < 2\%$. The solvency constraint thus set an upper bound on the portfolio share in risky assets. Let's denote the upper limit as $x^{solv}_{80\%}$, then the constrained portfolio choice is $0 \leq x^{\mathrm{col}\,l} \leq x^{solv}_{80\%}$, instead of $0 \leq x^{\mathrm{col}\,l} \leq 1$ in the baseline setup. In our setup $x^{solv}_{80\%} = 87\%$ (84%) for market with (without) the risk free assets. More details are given in Appendix 2. When such constraint is imposed, the constraint is binding for the optimal CDB and CLinear schemes. Table 12 shows the welfare gains and losses when an additional solvency constraint is imposed in the optimal collective schemes as in Table 2 (and 4 respectively) for $\gamma = 5$. The collective schemes are initially fully funded, $FR_0 = 1$. The welfare levels are slightly reduced. For CLinear scheme, the welfare gain is reduced from 2.3% to 1.8% in market with real bonds, and 2.7% to 2.5% in market with nominal bonds.

6 Conclusion

We have used the institutional setting of real existing pension funds where risk-bearing is intergenerational of nature in order to study the welfare aspects of intergenerational risk sharing. Typically for these collective pension plans is that pension benefits and/or pension contributions may depend on the funding ratio (the ratio of assets to liabilities of the fund). Adjustments in contributions and benefits can be seen as contingent claims. We optimized the explicit risk allocation rules, which specify who of the stakeholders, when, and to what extent is taking part in risk-bearing. The method of value-based generational accounting is employed to model the transfers of value between generations in a pension fund based on intergenerational risk-sharing. We have found that a pension deal may be characterized as being ex-ante fair for a generation when the economic value of underwriting downside risk in case of a deficit position of the pension fund is exactly matched by the economic value of holding a claim on a surplus position of the pension fund. These contingent claims typically take the form of adjustments in both benefits and contributions.

Collective pension scheme are a zero-sum game in value terms, however they are potentially a positive-sum game in welfare terms. We showed that well-designed funded schemes with intergenerational risk sharing are welfare improving over and above the fully optimal individual DC scheme by 1–4% in terms of the certainty equivalent consumption. Introducing more sources of risk apart from the modelled investment risks and inflation risk, like labor income risk and real rate of interest rate risk, might further strengthen the welfare-enhancing potential of intergenerational risk sharing via collective schemes compared to the optimal individual scheme.

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A Appendix

A.1 Solution methods for DCfixed scheme without constrains

This appendix solves the DCfixed scheme without borrowing and short sale constraints in close form. During the working period, the individual saves the fixed fraction of their labor income in each period, and optimizes the portfolio without constraints. After retirement the individual draws down the pension income from the accumulated financial capital, optimally choosing both consumption (i.e. income drawn down) and portfolio.

The optimal consumption and portfolio choice during the retired period is the same as the DCflexible case. During the working period, the individual's problem can be solved backwards in two steps. First maximize the accumulated wealth on the retirement date by optimizing portfolio, taking the contribution profile as given. Second, optimize the contribution rate at the beginning of his career, $t = 0$.

$$
\begin{aligned}
J(t_0) &= \max_{m} \max_{\{x_t\}_{t=0}^{T}\{c_t\}_{t=R}^{T}} E_0\left[\int_0^R \beta^t \frac{((1-m)\,y)^{1-\gamma}}{1-\gamma}dt + \int_R^T \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}dt\right] \\
&= \max_{m}\left\{\max_{\{x_t\}_{t=0}^{R}} \int_0^R \beta^t \frac{((1-m)\,y)^{1-\gamma}}{1-\gamma}dt + \max_{\{x_t,c_t\}_{t=R}^{T}} E_0\left[\int_R^T \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}dt\right]\right\} \\
&= \max_{m}\left\{\int_0^R \beta^t \frac{((1-m)\,y)^{1-\gamma}}{1-\gamma}dt + \max_{\{x_t\}_{t=0}^{R}} E_0\left[\beta^R \frac{f(T-R)^{\gamma} W_R(m)^{1-\gamma}}{1-\gamma}\right]\right\} \qquad (43)
\end{aligned}
$$

s.t. the wealth dynamics

$$
\begin{aligned}
dW_t &= [W_t(r + x_t(\mu - r)) + m y_t]dt + \sigma W_t x_t dZ_t, \qquad 0 \le t \le R \qquad (44) \\
W_0 &= 0
\end{aligned}
$$

where $f(\tau) = \exp\{-\left[r\,(1-1/\gamma) + \lambda_E^2\,(1-1/\gamma)\,/\,(2\gamma)\right]\tau\}$

Step 1, for any given level of contribution rate, m, solve the following optimization problem (i.e optimizing terminal wealth, with labor income y_t) ,

$$
\begin{aligned}
&\max_{\{x_t\}_{t=0}^{R}} E_0\left[\frac{W_R(m)^{1-\gamma}}{1-\gamma}\right] \qquad (45) \\
&s.t. \\
dW_t &= [W_t(r + x_t(\mu - r)) + m y_t]dt + \sigma W_t x_t dZ_t
\end{aligned}
$$

This problem is identical to the one optimizing terminal wealth, but with a down-scaled labor income my_t. Without the constraints, the optimal individual portfolio choice before retirement is the leveraged myopic portfolio:

$$x_t^* = \frac{1}{\gamma}\frac{\mu - r}{\sigma^2}\frac{W_t + PVC_t}{W_t} \tag{46}$$

where PVC_t denotes the present value of the future pension contributions, $PVC_t = m\int_t^R e^{-rt} y_s ds$ (before retirement) and $PVC_t = 0$ (after retirement). x_t^* does not depend on m.

Step 2, optimize over m,

$$\max_m \left\{ \int_0^R \beta^t \frac{((1-m)\,y)^{1-\gamma}}{1-\gamma} dt + E_0\left[\beta^R \frac{f(T-R)^\gamma W_R^*(m)^{1-\gamma}}{1-\gamma}\right]\right\} \tag{47}$$

where $W_R^*(m)$ is the optimal retirement wealth when the optimal investment strategy x_t^* is implemented.

When we solve the problem (45) using the martingale approach, it can be shown that the optimal retirement wealth is given by

$$W_R^*(m) = \frac{W_0}{f(R)} M_R^{-1/\gamma} = \frac{PVC_0(m)}{f(R)} M_R^{-1/\gamma}$$

The first order condition w.r.t. m is

$$\begin{aligned}(1-m)^{-\gamma}\int_0^R e^{-\delta t} y_t^{1-\gamma} dt &= m^{-\gamma}\left(\int_0^R e^{-rt} y_t dt\right)^{1-\gamma} e^{-\delta R} f(T-R)^\gamma f(R)^\gamma \\ m^* &= \frac{Y^{1/\gamma}}{X^{1/\gamma} + Y^{1/\gamma}}\end{aligned}$$

where $X = \int_0^R e^{-\delta t} y_t^{1-\gamma} dt$, and $Y = \left(\int_0^R e^{-rt} y_t dt\right)^{1-\gamma} e^{-\delta R} f(T-R)^\gamma f(R)^\gamma$.

In a special case where labor income is flat, $y_t = y$, as assumed in this paper, the optimal contribution rates for different risk aversion parameter are:

γ	3	5	8
$m^{*,unconstrained}$	6%	7.14%	7.65%

A.2 Additional solvency constraint

The solvency constraint requires that, the probability of 80% reduction in funding ratio within next period should be small (e.g., lower than 2.5%). That is, $\Pr\left[FR_{t+1} < FR_t \cdot 80\%\right] < 2\%$.

$$\Pr\left[\frac{A_{t+1}}{ABO_{t+1}} < \frac{A_t}{ABO_t} \cdot \chi\right] < 0.02 \tag{48}$$

where the lower bound of the funding ratio is denoted by $\chi = 0.8$, and we assume that $A_t = ABO_t$.

Plug in $A_{t+1} = A_t \exp(R_{t+1})$, $ABO_{t+1} = ABO_t$, then (48) is equivalent to

$$\Pr\left[R_{t+1} < \log \chi\right] < 0.02 \tag{49}$$

with

$$R_{t+1} = r + x\sigma_E\lambda_E - \frac{1}{2}x^2\sigma_E^2 + x\sigma_E\left(\sqrt{1-\rho_{s,\pi}^2}\varepsilon_{t+1}^E + \rho_{s,\pi}\varepsilon_{t+1}^{\pi}\right) \tag{50}$$

The probability of underfunding requirement is translated into a cumulative probability function

$$\begin{aligned} N\left(\frac{\log\chi - \left(r + x\sigma_E\lambda_E - \frac{1}{2}x^2\sigma_E^2\right)}{x\sigma_E}\right) &< 0.02 \\ \frac{\log\chi - \left(r + x\sigma_E\lambda_E - \frac{1}{2}x^2\sigma_E^2\right)}{x\sigma_E} &< -2.05 \end{aligned}$$

This inequality bounds the range for the portfolio weight x. The upper limit in a market with nominal bonds is solved in a similar way. Let's denote the upper limit as x_{χ}^{solv}, then the constrained portfolio choice is $0 \leq x^{\mathrm{col}\,l} \leq x_{\chi}^{solv}$, instead of $0 \leq x^{\mathrm{col}\,l} \leq 1$ in the baseline setup. $x_{80\%}^{solv} = 87\%$ (84%) for market with (without) the risk free assets.

Table 1: Default parameters for the stochastic models

parameter	value
μ	0.06
r	0.02
δ	0.04
$\bar{\pi}$	0.02
a	0.2
σ	0.15
σ_π	0.01
σ_u	0.01
λ_π	-0.2
λ_u	-0.2
$\rho_{E\pi}$	-0.1

Table 2: The optimal collective schemes in a market with real bonds and stocks. The scheme design parameters $\{m, k, \alpha, \beta, x\}$ are optimally chosen. (m, k) denote the contribution rate and the corresponding accrual rate; (α, β) control the speed of risk absorbing; (x) denotes the portfolio weight in equities. CEC/y denotes the normalized welfare level achieved by the optimal individual schemes, in terms of annual salary y. The ratio $\frac{CEC}{CEC^{DCflex}}$ shows the relative welfare gain or loss of the other schemes to DCflexible.

γ		DCflexible	DCfixed	CDB	CDC	CHybrid	CLinear
3	CEC/Y	0.908	0.886	0.926	0.887	0.931	0.941
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.6%	102.0%	97.6%	102.5%	103.6%
	(m, k)	-	11.0%	16.6%, 1.96%	11.4%, 1.34%	14.8%, 1.75%	14.0%, 1.65%
	α	-	-	0.07	-	-	0.06
	β	-	-	-	0.11	0.03	0.02
	x	-	-	100%	81%	100%	100%
5	CEC/Y	0.892	0.867	0.889	0.868	0.899	0.912
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.2%	99.7%	97.3%	100.8%	102.3%
	(m, k)	-	12.8%	16.6%, 1.96%	13.1%, 1.55%	16.6%, 1.96%	14.0%, 1.65%
	α	-	-	0.05	-	-	0.04
	β	-	-	-	0.11	0.02	0.02
	x	-	-	96%	49%	76%	100%
8	CEC/Y	0.876	0.854	0.862	0.852	0.869	0.888
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.7%	98.4%	97.3%	99.2%	100.9%
	(m, k)	-	14.1%	18.3%, 2.17%	14.0%, 1.65%	17.5%, 2.06%	15.7%, 1.86%
	α	-	-	0.04	-	-	0.04
	β	-	-	-	0.09	0.02	0.02
	x	-	-	67%	31%	48%	76%

Table 3: The collective schemes with equal contribution rate or accrual rate, i.e. $m = 17.5\%$, $k = 2.06\%$, but other scheme parameters $\{\alpha, \beta, x\}$ are optimally chosen in a market with real bonds and stocks. The contribution rates are fixed at 17.5%, so that the consumption would be flat over life time if all contributions were invested in risk free asset. (α, β) control the speed of risk absorbing; (x) denotes the portfolio weight in equities. CEC/y denotes the normalized welfare level achieved by the optimal individual schemes, in terms of annual salary y. The ratio $\frac{CEC}{CEC^{DCflex}}$ shows the relative welfare gain or loss of the other schemes to DCflexible.

γ		DCflexible	DCfixed	CDB	CDC	CHybrid	CLinear
3	CEC/Y	0.908	0.851	0.926	0.853	0.923	0.94
	$\frac{CEC}{CEC^{DCflex}}$	100%	93.7%	101.9%	93.9%	101.6%	103.5%
	(m, k)	-	17.5%	17.5%, 2.06%	17.5%, 2.06%	17.5%, 2.06%	17.5%, 2.06%
	α	-	-	0.06	-	-	0.06
	β	-	-	-	0.11	0.02	0.02
	x	-	-	100%	100%	100%	100%
5	CEC/Y	0.892	0.839	0.889	0.841	0.896	0.905
	$\frac{CEC}{CEC^{DCflex}}$	100%	94.1%	99.8%	94.3%	100.5%	101.5%
	(m, k)	-	17.5%	17.5%, 2.06%	17.5%, 2.06%	17.5%, 2.06%	17.5%, 2.06%
	α	-	-	0.05	-	-	0.05
	β	-	-	-	0.11	0.02	0.02
	x	-	-	94%	51%	80%	100%
8	CEC/Y	0.876	0.833	0.865	0.835	0.869	0.875
	$\frac{CEC}{CEC^{DCflex}}$	100%	95.1%	98.7%	95.3%	99.3%	99.9%
	(m, k)	-	17.5%	17.5%, 2.06%	17.5%, 2.06%	17.5%, 2.06%	17.5%, 2.06%
	α	-	-	0.04	-	-	0.04
	β	-	-	-	0.11	0.02	0.02
	x	-	-	70%	32%	51%	80%

Table 4: The optimal collective schemes in a market with nominal bonds and stocks. The scheme design parameters $\{m, k, \alpha, \beta, x\}$ are optimally chosen. (m, k) denote the contribution rate and the corresponding accrual rate; (α, β) control the speed of risk absorbing; (x) denotes the portfolio weight in equities. CEC/y denotes the normalized welfare level achieved by the optimal individual schemes, in terms of annual salary y. The ratio $\frac{CEC}{CEC^{DCflex}}$ shows the relative welfare gain or loss of the other schemes to DCflexible.

γ		DCflexible	DCfixed	CDB	CDC	CHybrid	CLinear
3	CEC/Y	0.904	0.887	0.924	0.892	0.929	0.938
	$\frac{CEC}{CEC^{DCflex}}$	100%	98.2%	102.1%	98.4%	102.6%	103.7%
	(m,k)	-	10.7%	16.6%, 1.96%	11.4%, 1.34%	15.7%, 1.86%	14.0%, 1.65%
	α	-	-	0.07	-	-	0.06
	β	-	-	-	0.1	0.03	0.02
	x	-	-	97%	62%	78%	100%
5	CEC/Y	0.887	0.868	0.895	0.879	0.909	0.912
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.8%	100.9%	99%	102.3%	102.7%
	(m,k)	-	11.7%	17.5%, 2.06%	11.4%, 1.34%	15.7%, 1.86%	14%, 1.65%
	α	-	-	0.07	-	-	0.04
	β	-	-	-	0.09	0.03	0.02
	x	-	-	61%	40%	55%	94%
8	CEC/Y	0.874	0.854	0.875	0.865	0.887	0.891
	$\frac{CEC}{CEC^{DCflex}}$	100%	98,1%	100.1%	99.0%	101.5%	101.9%
	(m,k)	-	12.8%	18.3%, 2.17%	13.1%, 1.55%	16.6%, 1.96%	14.8%, 1.75%
	α	-	-	0.06	-	-	0.04
	β	-	-	-	0.1	0.02	0.02
	x	-	-	43%	28%	41%	63%

Table 5: Valuation of the intergenerational transfers in a market with real bonds and stocks. The scheme parameters are not optimal but set as follows: the portfolio consists 50% ILB and 50% stocks; 17.5% contribution rate; the values for α and β are as given in Table 3 for $\gamma = 5$. The market values are expressed in terms of gross salary of 30000 euro.

$\gamma = 5$	contr. rate m	act. Contrib	act. Benefit	pos. transfer	neg. transfer
	(1)	(2)	(3)	(4)	(5)
CDB	17.5%	4.85	4.85	0.85	0.84
CDC	17.5%	4.86	4.85	0.58	0.58
CH	17.5%	4.90	4.88	0.64	0.63
CL	17.5%	4.86	4.85	0.74	0.74

Table 6: Valuation of the intergenerational transfers in a market with real bonds and stocks. The scheme parameters are set at the scheme-specific optimal levels as in Table 2 for $\gamma = 5$.The market values are expressed in terms of gross salary of 30000 euro.

$\gamma = 5$	contr. rate m	act. Contrib	act. Benefit	pos. transfer	neg. transfer
	(1)	(2)	(3)	(4)	(5)
CDB	16.6%	4.62	4.61	1.26	1.25
CDC	13.1%	3.64	3.64	0.52	0.51
CH	16.6%	4.71	4.64	0.91	0.84
CL	14.0%	3.89	3.88	0.77	0.77

Table 7: Valuation of the intergenerational transfers in a market with real bonds and stocks. The scheme parameters are set at the scheme-specific sub-optimal levels as in Table 3 for $\gamma = 5$, as the contribution rates being fixed at 17.5%. The market values are expressed in terms of gross salary of 30000 euro.

$\gamma = 5$	contr. rate m	act. Contrib	act. Benefit	pos. transfer	neg. transfer
	(1)	(2)	(3)	(4)	(5)
CDB	17.5%	4.85	4.85	1.29	1.29
CDC	17.5%	4.85	4.85	0.71	0.72
CH	17.5%	4.96	4.89	1.01	0.94
CL	17.5%	4.85	4.85	1.01	1.01

Table 8: Robustness checking: with lower equity premium, so that the expected return of equity is $\mu = 5\%$ instead of 6% as in the default setting. The optimal collective schemes, where $\{m, k, \alpha, \beta, x\}$ are optimally chosen, in a market with real bonds and stocks. (m, k) denote the contribution rate and the corresponding accrual rate; (α, β) control the speed of risk absorbing; (x) denotes the portfolio weight in equities. CEC/y denotes the normalized welfare level achieved by the optimal individual schemes, in terms of annual salary y. The ratio $\frac{CEC}{CEC^{DCflex}}$ shows the relative welfare gain or loss of the other schemes to DCflexible.

γ		DCflexible	DCfixed	CDB	CDC	CHybrid	CLinear
3	CEC/Y	0.882	0.863	0.885	0.865	0.886	0.897
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.7%	100.3%	97.8%	100.5%	101.7%
	(m,k)	-	12.4%	16.6%, 1.96%	12%, 1.42%	14.8%, 1.75%	13.8%, 1.65%
	α	-	-	0.08	-	-	0.075
	β	-	-	-	0.08	0.05	0.02
	x	-	-	92%	62%	82%	100%
5	CEC/Y	0.869	0.852	0.862	0.850	0.870	0.876
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.9%	99.2%	97.8%	100%	100.8%
	(m,k)	-	14%	17.5%, 2.06%	13.4%, 1.58%	16.6%, 1.96%	14.3%, 1.69%
	α	-	-	0.065	-	-	0.055
	β	-	-	-	0.08	0.025	0.025
	x	-	-	62%	37%	57%	88%
8	CEC/Y	0.859	0.842	0.846	0.842	0.851	0.860
	$\frac{CEC}{CEC^{DCflex}}$	100%	98.0%	98.5%	98.0%	99.1%	100.1%
	(m,k)	-	15%	18.9%, 2.22%	15.7%, 1.85%	18.3%, 2.16%	15.7%, 1.85%
	α	-	-	0.05	-	-	0.045
	β	-	-	-	0.11	0.02	0.02
	x	-	-	45%	24%	37%	62%

Table 9: Cross scheme comparison with initially underfunded schemes ($FR_0 = 0.8$ and $FR_0 = 0.9$ respectively) in two alternative financial markets. The optimal scheme designs are as showns in Table 2 and 4 for $\gamma = 5$. $\frac{CEC}{CEC^{DCflex}}$ shows the relative welfare gain or loss to the optimal individual scheme. (The individual schemes are always fully funded by construction.)

		DCflexible	DCfixed	CDB	CDC	CHybrid	CLinear
		(1)	(2)	(3)	(4)	(5)	(6)
with Index Linked Bonds and Stocks							
$FR_0 = 1.1$	CEC/y	0.892	0.867	0.909	0.870	0.918	0.927
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.2%	101.9%	97.6%	103.0%	104.0%
$FR_0 = 0.9$	CEC/y	-	-	0.867	0.860	0.877	0.896
	$\frac{CEC}{CEC^{DCflex}}$	-	-	97.4%	96.6%	98.5%	100.7%
$FR_0 = 0.8$	CEC/Y	-	-	0.845	0.855	0.855	0.881
	$\frac{CEC}{CEC^{DCflex}}$	-	-	94.8%	95.9%	95.0%	98.8%
with Nominal Bonds and Stocks							
$FR_0 = 0.9$	CEC/y	0.887	0.868	0.868	0.874	0.886	0.896
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.8%	97.7%	98.4%	99.8%	100.9%
$FR_0 = 0.8$	CEC/Y	-	-	0.838	0.865	0.862	0.881
	$\frac{CEC}{CEC^{DCflex}}$	-	-	94.4%	97.5%	97.1%	99.3%

Table 10: Valuation of the intergenerational transfers when the collective schemes are initially underfunded ($FR_0 = 0.8$, $FR_0 = 0.9$) or overfunded ($FR_0 = 1.1$), in a market with the risk free asset (ILB). The schemes parameters are set according to Table 2, $\gamma = 5$. The market values are expressed in terms of gross salary of 30000 euro. The stars indicate the cost-price contribution or benefit levels.

$\gamma = 5$	contr. rate m	act. Contrib	act. Benefit	pos. transfer	neg. transfer	net transfer
	(1)	(2)	(3)	(4)	(5)	(6)
$FR_0 = 1.1$						
CDB	16.6%	4.05	4.61*	1.02	1.58	-0.56
CDC	13.1%	3.64*	3.79	0.14	0.30	-0.15
CH	16.6%	4.32	4.78	0.70	1.16	-0.46
CL	14.0%	3.55	3.91	0.62	0.98	-0.36
$FR_0 = 0.9$						
CDB	16.6%	5.16	4.61*	1.53	0.98	0.55
CDC	13.1%	3.64*	3.49	0.27	0.12	0.15
CH	16.6%	5.08	4.51	1.16	0.59	0.57
CL	14.0%	4.21	3.86	0.94	0.60	0.35
$FR_0 = 0.8$						
CDB	16.6%	5.71	4.61*	1.85	0.75	1.10
CDC	13.1%	3.64*	3.34	0.37	0.07	0.30
CH	16.6%	5.47	4.38	1.47	0.38	1.09
CL	14.0%	4.53	3.84	1.14	0.45	0.70

Table 11: Valuation of the intergenerational transfers when the collective schemes are initially underfunded ($FR_0 = 0.8$ and $FR_0 = 0.9$ respectively), in a market with the risk free asset (ILB). The contribution rates are fixed at 17.5%. The investment portfolio is set according to Table 3, $\gamma = 5$. The market values are expressed in terms of gross salary of 30000 euro.

$\gamma = 5$	contr. rate m	act. Contrib	act. Benefit	pos. transfer	neg. transfer	net transfer
	(1)	(2)	(3)	(4)	(5)	(6)
$FR_0 = 0.9$						
CDB	17.5%	5.45	4.85	1.58	1.00	0.58
CDC	17.5%	4.85	4.65	0.38	0.18	0.21
CH	17.5%	5.37	4.74	1.28	0.66	0.62
CL	17.5%	5.32	4.83	1.24	0.77	0.47
$FR_0 = 0.8$						
CDB	17.5%	6.02	4.85	1.94	0.76	1.17
CDC	17.5%	4.85	4.45	0.50	0.09	0.41
CH	17.5%	5.77	4.61	1.61	0.44	1.16
CL	17.5%	5.75	4.82	1.52	0.57	0.95

Table 12: The optimal scheme designs under additional solvency constraints, in two alternative financial markets. The collective schemes are initially fully funded, $FR_0 = 1$. The scheme designs are based on Table 2 and 4, for $\gamma = 5$, respectively. CEC/y denotes the normalized welfare level achieved by the optimal individual schemes, in terms of annual salary y. The ratio $\frac{CEC}{CEC^{DCflex}}$ shows the relative welfare gain or loss of the other schemes to DCflexible.

Market		DCflexible	DCfixed	CDB	CDC	CHybrid	CLinear
Nom. Bond	CEC/Y	0.887	0.868	0.895	0.879	0.909	0.910
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.8%	100.9%	99.1%	102.4%	102.5%
Real Bond	CEC/Y	0.892	0.867	0.889	0.868	0.899	0.908
	$\frac{CEC}{CEC^{DCflex}}$	100%	97.2%	99.7%	97.3%	100.8%	101.8%

Figure 1: Linear premium policies

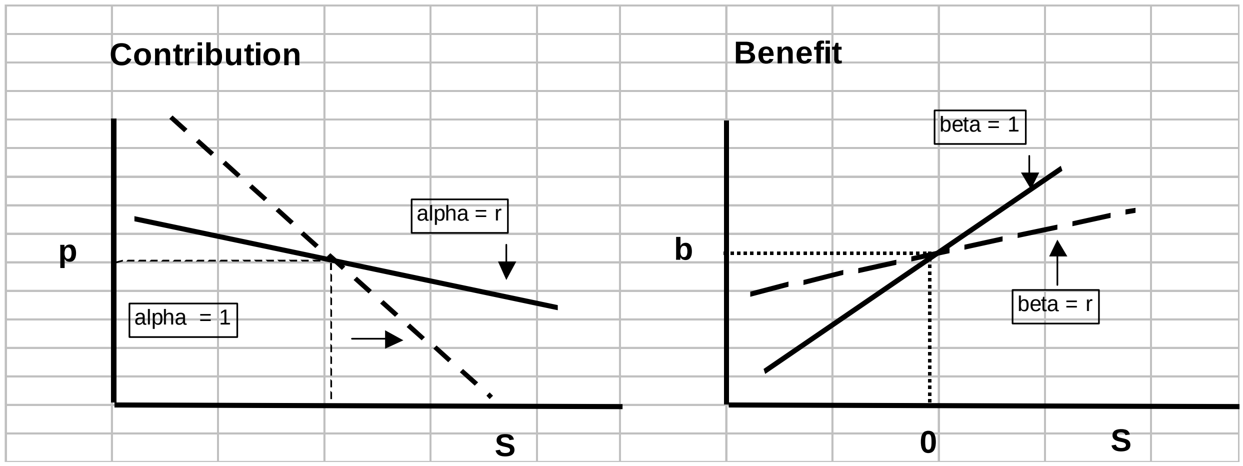

Figure 2: collective hybrid policies (CHybrid). The horizontal axis shows the surplus ratio, S_t/ABO_t; The vertical axis shows the additional contribution and the additional indexation respectively.

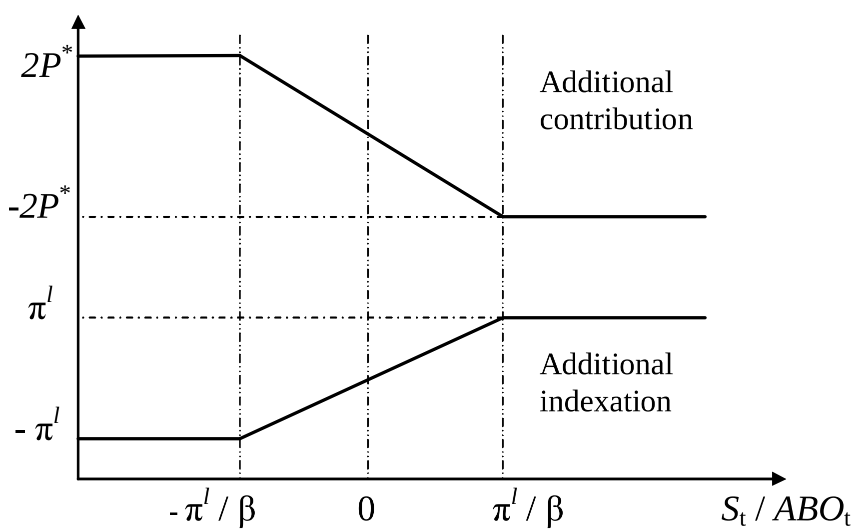

Figure 3: The 5%, 50%, 95% quantile plots of normalized consumptions (c_t/y_t) offered by the collective schemes, with the scheme parameters as following: 50% ILB and 50% stocks; 17.5% contribution rate; the values for α and β are as given in Table 3 for $\gamma = 5$.

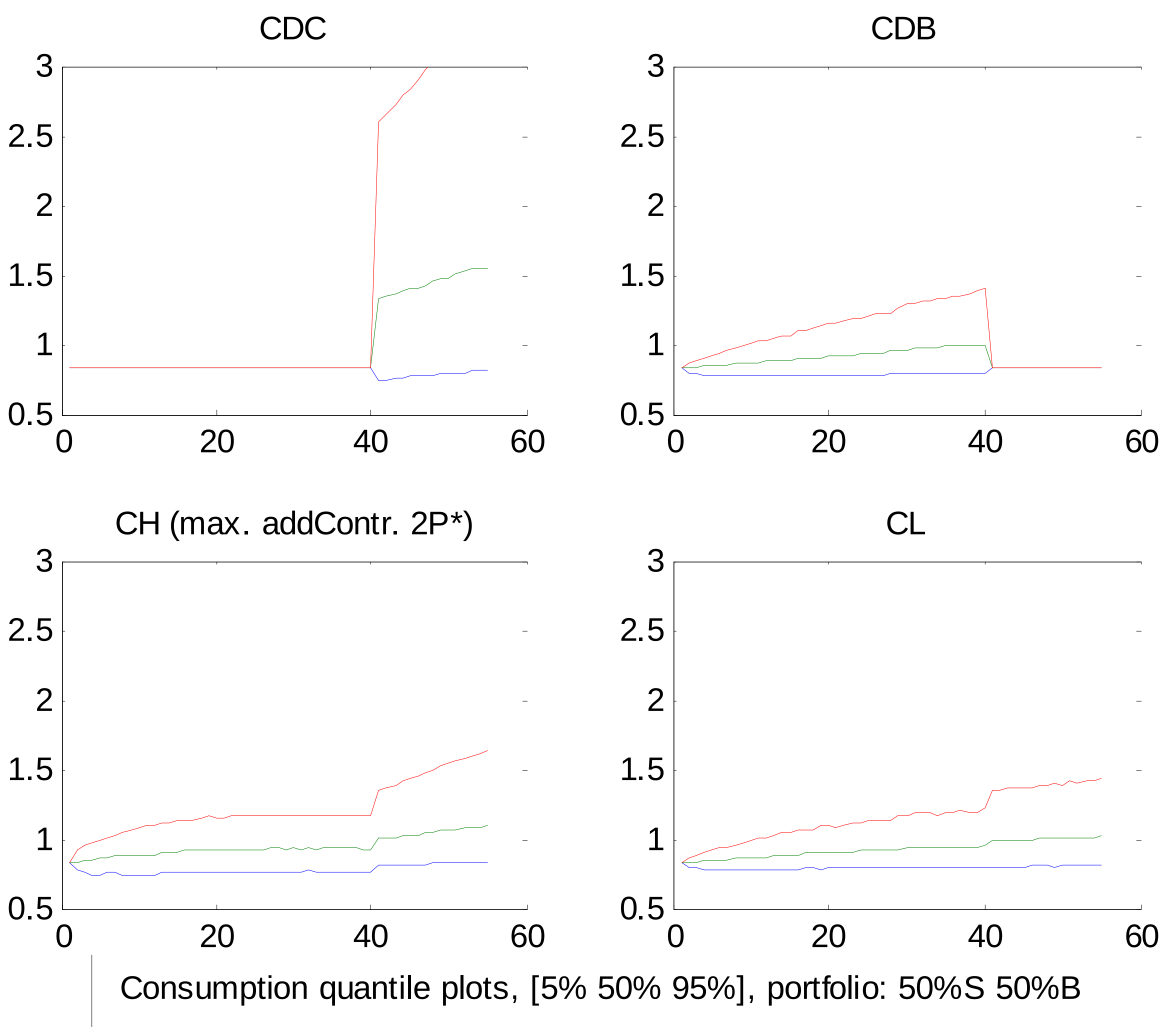

Figure 4: The 5%-, 50%-, 95%-quantile plots of surplus ratio of the fund, i.e. S_t/ABO_t,generated by the collective schemes, with the scheme parameters as following: 50% ILB and 50% stocks; 17.5% contribution rate; the values for α and β are as given in Table 3 for $\gamma = 5$.

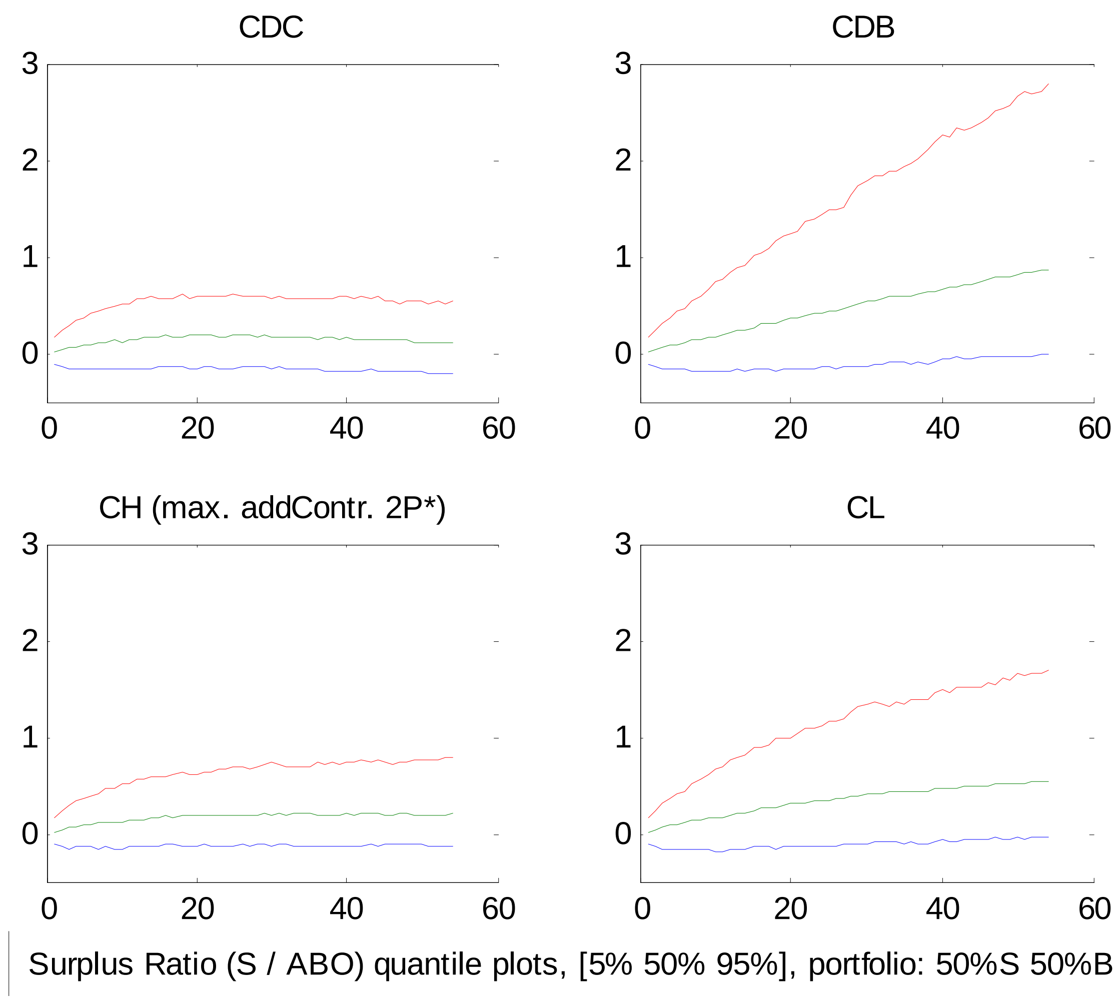

Figure 5: The average profile of the life-cycle optimal portfolio weights in stocks of the individual schemes (Left: DCflexible; Right: DCfiexed), in a market with risk free assets and stocks. Early in life the portfolio is borrowing constrained.

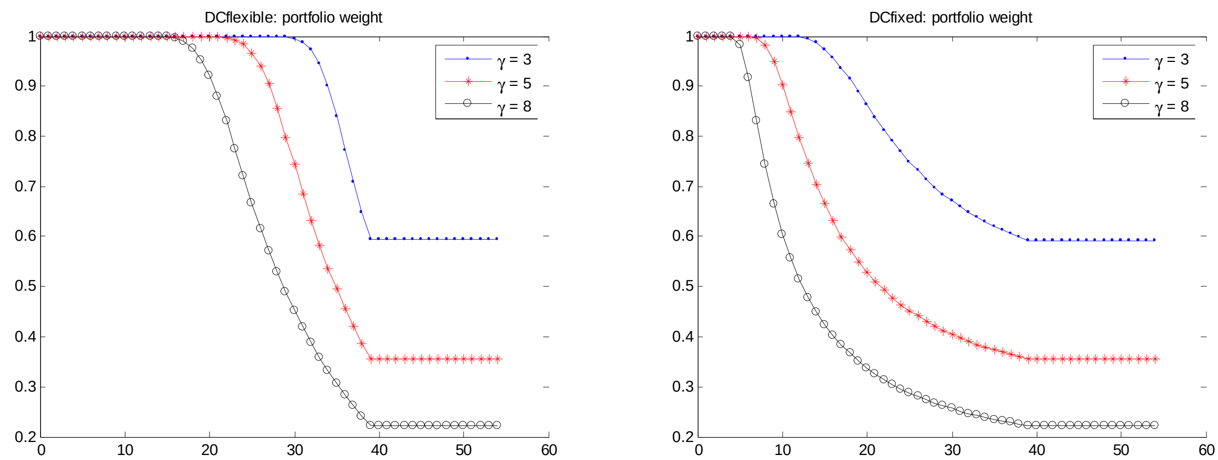

Figure 6: The underfunding frequency of the optimal schemes characterized in Table 2.