Scottish Journal of Political Economy, Vol. 54, No. 2, May 2007

THE CASE FOR PAY-AS-YOU-GO PENSIONS IN A SERVICE ECONOMY

Bas van Groezen, Lex Meijdam** and Harrie A. A. Verbon***

Abstract

The elderly consume more labour-intensive services than young individuals. This makes them vulnerable to rising costs of services due to higher wages, which can be caused by increased capital accumulation. This paper shows that in a model with a service sector, the golden-rule capital stock is lower and dynamic inefficiency is more likely to occur than in the conventional one-sector model. This implies that in many cases, a positive Pay-As-You-Go tax maximises long-run welfare in a service economy. Calculations based on data from the United Kingdom and the Netherlands show that the long-run optimal degree of funding coincides with the current situation in these countries.

I Introduction

The prospective age wave that many countries are confronted with has put pension reform high on the political agenda. As most countries mainly rely on public Pay-As-You-Go (PAYG) schemes, one of the most prominent reform options involves the move to a more funded pension system (see e.g. Economic Policy Committee, 2002, for Europe). In the United States a shift to a fully funded system of individual mandatory accounts is even advocated (see e.g. Feldstein, 1996). The central argument for such a pension reform is that decreasing the PAYG-scheme stimulates savings, which implies a higher capital stock. As the reward for capital is on average higher than the rate of population and wage growth, which can be considered the implicit return of an unfunded scheme, such a policy will increase lifetime income of the current young and future generations, and thus entails substantial welfare gains as long as the interest rate exceeds the rate of economic growth (see e.g. Lindbeck and Persson, 2003).[1]

As is well known, switching to a more funded scheme also involves short-run transition costs, so that this policy cannot be enacted in a Pareto-improving way

*Utrecht University
**Tilburg University

[1] Other arguments in favour of a shift to a more funded scheme are the reduction of the distortionary effects of PAYG-taxes on labour supply (see e.g. Feldstein, 2005) and the stimulation of economic growth if capital accumulation involves positive externalities (see e.g. Gyárfás and Marquardt, 2001).

(see Verbon, 1988, and Breyer, 1989). In this paper, we disregard the transition problem and focus on the alleged long-run gains of such pension reform. We argue that if the elderly consume relatively more labour-intensive services (as budget surveys show they do) instead of capital-intensive goods as young consumers do, the general-equilibrium effects of higher wages and lower interest rates that accompany a (partial) transition to a funded system can lead to an unbalanced consumption pattern over life, and may well cause lifetime utility to fall rather than rise. In particular, a point often overlooked in pension discussions is that the higher wages make services more expensive, and harm the consumption possibilities of individuals when old. If individuals prefer a smooth utility profile over their life, they will tend to save even more following the transition, thus exacerbating the adverse price effect, which creates a larger bias towards dynamic inefficiency (Diamond, 1965).

In this paper, we explicitly take a service sector into account. As we only consider long-run effects, we define optimality as the situation in which steady-state utility is maximised, i.e. the golden rule. It appears that the capital stock at which long-run utility is maximised is lower than in a one-sector model. This implies that from a long-run point of view, the PAYG-public pension tax corresponding to the golden rule rate is higher. By using data on the consumption of services for young and old individuals, we produce estimates of this PAYG-tax in a two-sector model, which we compare with estimates resulting from a one-sector model. Our calculations indicate that the tax rates corresponding to the golden rule in a two-sector economy are 2–4% gross domestic product (GDP) points higher than in a one-sector economy. If individuals prefer a smooth consumption over their life, this PAYG-tax rate is positive whatever the expected life-length, and the fertility rate of the population. So, if long-run utility is to be optimised, complete funding of public pensions is certainly not what one should do. In fact, our calculations suggest that the current level of the PAYG-tax in countries like the United Kingdom, the United States, and the Netherlands may be quite close to the golden-rule value.

The rest of the paper is organised as follows: Section II presents a model of a closed economy consisting of two overlapping generations, two sectors and four markets. In Section III, we analyse the golden rule for this model and the effects of the sector structure. Furthermore, we calculate the tax rate that maximises long-run utility and show how it varies for different intensities of service demand by the elderly. Section IV deals with the effects of a switch to a more funded pension scheme and presents some calculations. Section V concludes.

II The Model

We use a simple two-OLG model of a closed economy. Population at time t (N_t) grows at rate n, so $N_{t+1} = (1+n)N_t$. Firms are fully competitive and maximise profits. Following Baumol (1967), we distinguish between two sectors of production: a commodity and a service sector. In the *commodity sector* (labelled Y), homogeneous goods are produced that either serve as consumption or investment good, according to a CRTS-production function $F(K_t, L_t^Y)$, K_t being the capital stock (which is assumed to depreciate completely in one period) and L_t^Y the number of people employed in the commodity sector. In competitive factor markets, the wage will be $w_t = \partial F(K_t, L_t^Y)/\partial L_t^Y$ and the interest rate $r_t = \partial F(K_t, L_t^Y)/\partial K_t$. In the *service sector* (labelled D), labour is the only factor of production.[2] One unit of labour translates into one service, so the total provision of services equals total labour supply in this sector (L_t^D). The price of services in terms of commodities, p_t, is therefore equal to the wage in this sector. As labour is homogeneous and perfectly mobile across sectors, there is only one wage, so $p_t = w_t$.

An individual dies at the onset of old age with probability $1-\varepsilon$ and lives throughout old age with probability ε, during which (s)he is retired. In the first period of life, (s)he inelastically supplies one unit of labour[3] and earns a wage income, which is taxed by the government at rate τ for the purpose of running a PAYG-scheme that pays a public pension benefit η_t to every retired person alive in period t. Hence, the government budget constraint is given by

$$\varepsilon\eta_t = \tau(1+n)w_t. \tag{1}$$

Young individuals partly spend their net income on commodities and services, and save for old age. These savings are invested in annuities or through an actuarially fair pension fund, so savings in this model reflect the funded part of old-age pensions. As only a fraction ε of young savers born at time t survives to period $t+1$, the assets of those who deceased fall to surviving contemporaries. The total return on the savings is therefore $\frac{r_{t+1}}{\varepsilon}$. The individual budget constraints in the two periods of life are given by

$$c_t^{\mathrm{y}} + p_t d_t^{\mathrm{y}} = (1-\tau)w_t - s_t, \tag{2a}$$

$$c_{t+1}^{\mathrm{o}} + p_{t+1} d_{t+1}^{\mathrm{o}} = \frac{r_{t+1}}{\varepsilon} s_t + \eta_{t+1}, \tag{2b}$$

where c^{y} (c^{o}) is the number of commodities consumed when young (old), d^{y} (d^{o}) is the number of services used when young (old).

Given these constraints, individuals maximise their expected lifetime utility represented by

$$\begin{aligned} EU_t = {} & \frac{(1-\mu)(c_t^{\mathrm{y}})^{1-\theta} + \mu(d_t^{\mathrm{y}})^{1-\theta}}{1-\theta} \\ & + \gamma\varepsilon \frac{(1-\lambda)(c_{t+1}^{\mathrm{o}})^{1-\theta} + \lambda(d_{t+1}^{\mathrm{o}})^{1-\theta}}{1-\theta} \\ \equiv {} & u_t^{\mathrm{y}} + \gamma\varepsilon u_{t+1}^{\mathrm{o}}, \end{aligned} \tag{3}$$

[2] These are both personal services (domestic services such as gardeners, cleaners, housekeepers, butlers, and recreation and cultural services) and social services like nurses and physicians. Note that these are not services like bank services, which extensively use high-tech goods like computers.

[3] Allowing for a positive elastisicty of labour supply would weaken our results. However, the exact value is unclear as a vast body of empirical studies (as summarised by e.g. Blundell and MaCurdy, 1999) finds a wide variety of estimated values, many of them close to zero.

where u^{y} (u^{o}) is felicity when young (old) as defined by the second equality sign, γ is the private discount factor, μ (λ) denotes the preference for services when young (old), and $\frac{1}{\theta} > 0$ is the elasticity of substitution. Maximising equation (3) subject to equations (2a) and (2b) results in the following first-order conditions

$$\frac{c^{\mathrm{o}}_{t+1}}{c^{\mathrm{y}}_t} = \frac{r_{t+1}\Gamma_{t+1}}{\varepsilon}, \tag{4a}$$

$$\frac{d^{\mathrm{y}}_t}{c^{\mathrm{y}}_t} = \frac{\Omega_t}{w_t}, \tag{4b}$$

$$\frac{d^{\mathrm{o}}_{t+1}}{c^{\mathrm{y}}_t} = \frac{r_{t+1}\Delta_{t+1}}{\varepsilon w_{t+1}}, \tag{4c}$$

with

$$\Omega_t \equiv \left(\frac{\mu}{1-\mu}\right)^{\frac{1}{\theta}} w_t^{\frac{\theta-1}{\theta}},\ \Gamma_{t+1} \equiv \varepsilon\left(\frac{\gamma(1-\lambda)}{1-\mu}\right)^{\frac{1}{\theta}} r_{t+1}^{\frac{1-\theta}{\theta}},$$

and

$$\Delta_{t+1} \equiv \varepsilon\left(\frac{\gamma\lambda}{1-\mu}\right)^{\frac{1}{\theta}}\left(\frac{r_{t+1}}{w_{t+1}}\right)^{\frac{1-\theta}{\theta}}.$$

These conditions show that the ratio of old-age commodity consumption over young-age commodity consumption depends on the interest rate r_{t+1}, whereas the ratio of old-age services over young-age commodity consumption is determined by the interest rate divided by the price of services, $\frac{r_{t+1}}{w_{t+1}}$ (unless they are complements, i.e. $\theta \to \infty$). Combining equations (4a–c) with equations (1) and (2a and b) gives the individual demand and saving functions

$$c^{\mathrm{y}}_t = \frac{I_t}{1+\Omega_t+\Gamma_{t+1}+\Delta_{t+1}}, \tag{5}$$

$$d^{\mathrm{y}}_t = \frac{\Omega_t}{w_t}\left(\frac{I_t}{1+\Omega_t+\Gamma_{t+1}+\Delta_{t+1}}\right), \tag{6}$$

$$c^{\mathrm{o}}_{t+1} = \frac{r_{t+1}\Gamma_{t+1}}{\varepsilon}\left(\frac{I_t}{1+\Omega_t+\Gamma_{t+1}+\Delta_{t+1}}\right), \tag{7}$$

$$d^{\mathrm{o}}_{t+1} = \frac{r_{t+1}\Delta_{t+1}}{\varepsilon w_{t+1}}\left(\frac{I_t}{1+\Omega_t+\Gamma_{t+1}+\Delta_{t+1}}\right), \tag{8}$$

$$s_t = \frac{(1-\tau)w_t(\Gamma_{t+1}+\Delta_{t+1})}{1+\Omega_t+\Gamma_{t+1}+\Delta_{t+1}} - \frac{\tau(1+n)w_{t+1}(1+\Omega_t)}{r_{t+1}(1+\Omega_t+\Gamma_{t+1}+\Delta_{t+1})}, \tag{9}$$

where $I_t \equiv (1-\tau)w_t + \frac{\tau(1+n)w_{t+1}}{r_{t+1}}$ is the lifetime income.

The model comprises four markets, which are all characterised by fully flexible prices and therefore simultaneously in equilibrium at each point in time.

The commodity market

Demand for commodities comes from young individuals who spend part of their after-tax wage income on commodities, both as consumption and investment, and from retired individuals. Accordingly, the equilibrium condition is $F(K_t, L_t^Y) = N_t(c_t^y + s_t) + \varepsilon N_{t-1} c_t^o = N_t w_t (1 - \tau - d_t^y) + \varepsilon N_{t-1} c_t^o$, or

$$f(k_t, l_t^Y) = w_t(1 - \tau - d_t^y) + \frac{\varepsilon}{1+n} c_t^o, \tag{10}$$

with $k_t \equiv K_t/N_t$ and $l_t^Y \equiv L_t^Y/N_t$.

The services market

As described before, one service requires the input of exactly one employee, so the total provision of services is L_t^D. Equilibrium in the services market at time t is thus given by $L_t^D = N_t d_t^y + \varepsilon N_{t-1} d_t^o$, or

$$l_t^D = d_t^y + \frac{\varepsilon d_t^o}{1+n}, \tag{11}$$

with $l_t^D \equiv L_t^D/N_t$ the fraction of workers employed in the service sector.

The labour market

The total labour force at time t consists of all young individuals, who each inelastically supply one unit of labour in one of the two sectors. Equilibrium in the labour market therefore implies that $N_t = L_t^Y + L_t^D$, i.e.

$$1 = l_t^Y + l_t^D. \tag{12}$$

The capital market

Aggregate savings are invested domestically; the capital market clears when $N_t s_t = K_{t+1}$ holds, or

$$s_t = (1+n)k_{t+1}. \tag{13}$$

Equations (10)–(13) with the individual demand functions determine the market outcome.

III The Golden Rule

In this section, we will derive the golden-rule conditions for the two-sector economy (i.e. the equilibrium where steady-state utility is maximal) and compare it with the conventional one-sector economy. As is well known, the market outcome generally does not coincide with the golden rule, which will be demonstrated with some numerical examples. In these cases, the government can bring the economy to the golden rule by implementing a PAYG-scheme.

In steady state,[4] the economy's resource constraint with respect to commodities in per capita terms is given by

$$c \equiv c^{y} + \frac{\varepsilon}{1+n}c^{o} = f(k, l^{Y}) - k(1+n),$$

For the service sector, the constraint is

$$d \equiv d^{y} + \frac{\varepsilon}{1+n}d^{o} = 1 - l^{Y}.$$

Maximising steady-state utility subject to these resource constraints gives the following first-order conditions which characterise the golden rule (see Appendix A for the derivation),

$$\frac{\partial f(k, l^{Y})}{\partial k} = 1 + n, \tag{14a}$$

$$\frac{\partial f(k, l^{Y})}{\partial l^{Y}} = \left(\frac{c}{d}\right)^{\theta} \frac{\mu[1-\varepsilon/(\beta_2(1+n))]^{1-\theta} + \gamma\varepsilon\lambda(\beta_2)^{\theta-1}}{(1-\mu)[1-\varepsilon/(\beta_1(1+n))]^{1-\theta} + \gamma\varepsilon(1-\lambda)(\beta_1)^{\theta-1}}, \tag{14b}$$

with

$$\beta_1 \equiv \frac{\varepsilon}{1+n} + \left(\frac{1-\mu}{(1+n)\gamma(1-\lambda)}\right)^{\frac{1}{\theta}}$$

and

$$\beta_2 \equiv \frac{\varepsilon}{1+n} + \left(\frac{\mu}{(1+n)\gamma\lambda}\right)^{\frac{1}{\theta}}.$$

Solving these two conditions gives the golden-rule level of the capital stock and number of employees in the commodity sector.

Proposition 1: In the two-sector model, the golden rule capital stock is lower than in a one-sector model.

Proof: Condition (14a) says that the marginal product of capital equals the sum of the rate of depreciation and population growth, which is also the case for the golden rule in a one-sector economy. Denoting the golden-rule capital stock in a one-sector economy by K^1 and in a two-sector economy by K^2, this implies that the capital–labour ratio in the commodity sector of the two-sector economy (K_t^2/L_t^Y) should be equal to the capital–labour ratio in a one-sector model (K_t^1/N_t). As $L_t^Y < N_t$, it follows that $K_t^2 < K_t^1$. ■

This suggests that the extent to which the market outcome differs from the golden rule depends on the sector structure. Figure 1a and b shows that this is indeed the case. They show the equilibrium capital stock per capita in a market

[4] Omitting the time subscript denotes the steady-state value of the variable.

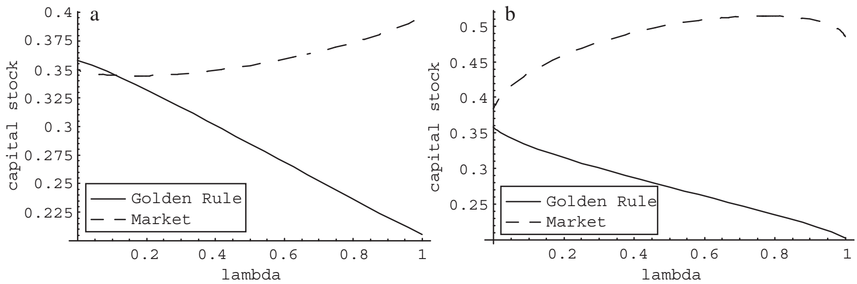

Figure 1. (a) $\theta = 0.9$. (b) $\theta = 4/3$.

setting (with $\tau = 0$) and the capital stock that matches the golden rule.[5] In Figure 1a, $\theta = 0.9$, so individuals have a rather high intertemporal substitution elasticity.[6] The capital stock in the market setting is below that of the golden rule if service consumption is small (low values of λ). In other words, the economy is dynamically efficient. For values of λ higher than 0.12, the opposite holds: the market outcome generates a capital stock that is too high compared with the level at which steady-state utility is maximal. The higher the value of λ, the greater this difference is. The same holds to a greater extent if individuals have a lower intertemporal substitution elasticity, as shown in Figure 1b where $\theta = 4/3$.[7]

To correct the market outcome, the government can use an intergenerational redistribution scheme, which affects private savings. The golden rule is attained when the PAYG-tax is set at such a level (τ^{GR}) that the resulting market outcome is characterised by an interest rate equal to $1+n$ [which follows from equation (14a)]. If the production function is $f(k) = k^{\alpha}$, $\mu = n = 0$ and $\varepsilon = 1$, this tax rate is given by

$$\tau^{GR} = \frac{\gamma^{1/\theta}(1-\lambda)^{1/\theta} + (1-\alpha)(\gamma\lambda)^{1/\theta}\left(\alpha^{\alpha/(\alpha-1)}/(1-\alpha)\right)^{(1-\theta)/\theta}) - \alpha}{(1-\alpha)(1+\gamma^{1/\theta}(1-\lambda)^{1/\theta} + (\gamma\lambda)^{1/\theta}\left(\alpha^{\alpha/(\alpha-1)}/(1-\alpha)\right)^{(1-\theta)/\theta})}. \tag{15}$$

Figure 2 shows this tax as a function of λ.

This figure shows that for $\theta = 0.9$ and small values of λ, the tax rate for which the market outcome coincides with the golden rule is negative, so the economy is dynamically efficient if $\tau = 0$. However, for the empirically minimum value of $\theta = 4/3$, the tax rate matching the golden rule is non-negative in all cases. The

[5] These and all next figures are based on the following production function: $Y_t = K_t^{0.3}(2L_t^Y)^{0.7}$, $n = 0$, $\varepsilon = 1$ and $\mu = 0$. Furthermore, $\gamma = 0.75$; assuming one period to be 25–30 years, the latter value corresponds to an annual discount rate of 0.01, consistent with the empirical evidence of Hurd (1989).

[6] For $\mu = 0$, the elasticity of substitution between goods and services is the intertemporal substitution elasticity.

[7] This is the value found by Blundell *et al.* (1994).

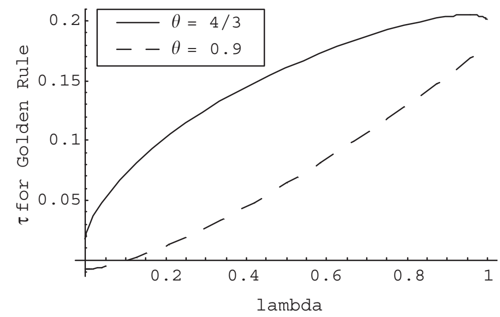

Figure 2. Pay-As-You-Go-tax matching the golden rule.

tax for the one-sector economy ($\lambda = 0$) is always lower than for a two-sector economy ($\lambda > 0$).

IV SWITCHING TO A MORE FUNDED PENSION SYSTEM

It is well known that if the economy is dynamically efficient, reducing the PAYG-tax leads – *ceteris paribus* – to a higher steady-state lifetime income and utility. This also holds for a two-sector model as described above. However, the point at which the golden rule is attained, so that a further reduction of unfunded pensions will no longer be beneficial in the long run, differs. To demonstrate the effect of the sector structure on the consequences of decreasing the PAYG-scheme, consider two extreme cases: one in which both young and elderly agents only demand commodities ($\mu = \lambda = 0$), which reflects the conventional one-sector economy, and one in which elderly demand only services ($\lambda = 1$).[8] In the latter case, the total demand for commodities at time t is given by $N_t(c_t^y + s_t)$, which, according to equation (2a), boils down to $N_t(1 - \tau)w_t$. Equilibrium in the commodity market [equation (10)] and labour market [equation (12)] gives the total amount of labour employed in the commodity sector, which is constant and equal to $l_t^Y = (1 - \alpha)(1 - \tau)$, also outside the steady state. The individual number of services that each person uses when old is then given by[9]

$$d_t^o = \alpha + \tau - \alpha\tau. \tag{16}$$

Although the individual demand for old-age consumption will generally depend on the substitution elasticity, the services that will be supplied to a retired person are determined only by the production elasticity α and the PAYG-tax τ.[10] Apparently, individuals will not increase their old-age services following a

[8] Again, we take $\varepsilon = 1$ and $n = 0$ for convenience, and the production of commodities is $f(k) = k^\alpha$.

[9] Commodity demand per young individual is $c_t^y + s_t = (1 - \tau)w_t = (1 - \tau)(1 - \alpha)k_t^\alpha(l_t^Y)^{-\alpha}$. Output per young individual is $k_t^\alpha(l_t^Y)^{1-\alpha}$, so equilibrium in this market implies $l_t^Y = (1 - \tau)(1 - \alpha)$. This combined with equation (12) gives $l_t^D = d_t^o = 1 - (1 - \tau)(1 - \alpha)$.

[10] This result is due to the Cobb–Douglas specification of production. For more general specifications, the parameter θ will affect the equilibrium quantity of d_t^o as well, but the positive relation between τ and d_t^o remains.

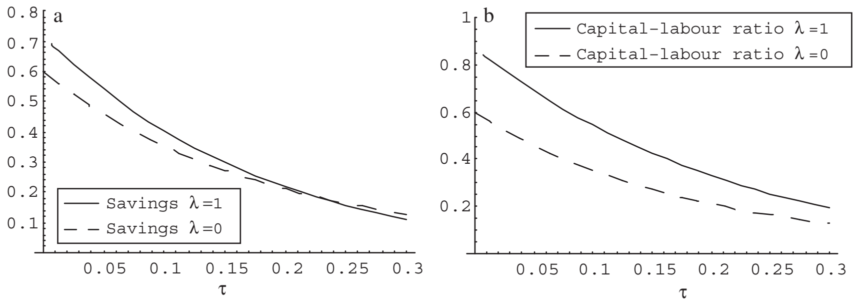

Figure 3. (a) Savings. (b) Capital–labour ratio.

decrease of the tax rate τ whatever their inclination to smooth consumption over their life. As a result, felicity when old will unambiguously fall if the tax rate decreases. The reason for this is that in a two-sector economy, savings react more heavily to a decrease of the PAYG-tax compared with a one-sector economy, as can be seen from Figure 3a.[11] Consequently, the general-equilibrium effects are much stronger. Figure 3b shows the capital–labour ratio in the commodity sector, which determines the wage, price of services and interest rate.

It is clear that for a certain value of the PAYG-tax, the wage is higher and the interest rate lower in a two-sector economy as compared with a one-sector economy, implying a lower rate of return to savings and more expensive services when retired. Figure 4a and b shows the corresponding individual felicity when young (u^y) and old (u^o), for a one-sector economy and a two-sector economy, respectively.

We can see that with a lower PAYG-tax the felicity of an individual will be lower during his old age. However, the decrease in u^o due to a shift to more funding is stronger if $\lambda = 1$, i.e. if the elderly consume services. On the other hand, with a lower PAYG-tax, felicity during youth (u^y) will be higher.[12] The relatively low level of consumption during old age strongly increases marginal utility when old compared with marginal utility when young. Individuals therefore have a strong incentive to save more in order to reduce the gap between felicity when young and old. As a result, in case of a decrease of the PAYG-tax, they increase their savings more than the rise of their net wage, so that eventually consumption in both periods of life decreases.

Of course, the effects of changing the PAYG-tax not only depend on the value of λ but also on the intertemporal elasticity of substitution. If the unfunded pension scheme is small, individuals who prefer a smooth consumption pattern over their lifetime will try to carry over a large part of their high income when young to their old age by saving. When general-equilibrium effects prevent such consumption smoothing, the government needs to run a more

[11] In Figures 3a, b and 4a, b, $\theta = 4/3$.

[12] Except in the two-sector economy ($\lambda = 1$) where for low values of the tax rate, both young-age and old-age felicity will decrease.

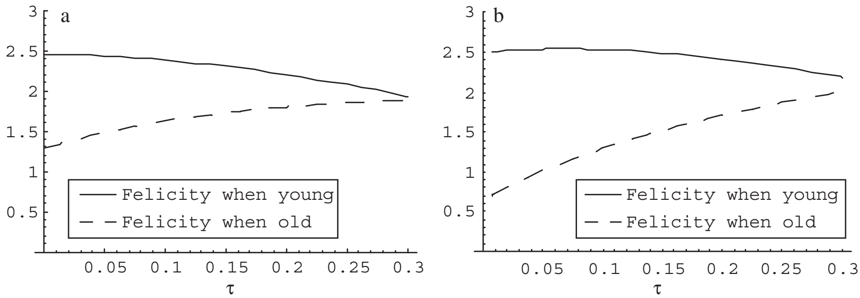

Figure 4. (a) $\mu = \lambda = 0$. (b) $\mu = 0$, $\lambda = 1$.

generous PAYG-scheme to correct for the dynamically inefficient high savings. This will especially be the case in a two-sector economy with different spending patterns of generations.

Calculations

The analysis above points at the higher theoretical probability of a dynamically inefficient economy in a two-sector model. However, the likelihood and importance of this effect are basically an empirical issue, to which we turn now.

To get an idea of the empirical importance of making a distinction between two sectors in the economy, we use detailed data that are obtained from two questionnaire surveys, which report household expenditures on many different commodities and services.[13] From this, we can calculate the expenditures on services (that approximate the very labour-intensive services as used in the two-sector model) as the share of total expenditures, which is reported in Table 1, and thus obtain a 'guesstimate' of actual values of λ and μ, after which calculations such as those underlying Figure 2 produce an estimate of the PAYG-tax at which the market outcome coincides with the golden rule. Obviously, if this value is positive, the economy without PAYG-scheme is dynamically inefficient and a complete shift to funded pensions is not optimal. Notice from Table 1 that the 'real-world' values of λ and μ must be rather small. Although individuals use more services when they are old than when they are young, spending on services is at most 10% of total individual spending. Yet, as we will show, taking account of this service sector has a considerable positive effect on the optimal tax rate.

For the calculations we take both the working and the retirement period to be 30 years.[14] We therefore need an average expenditure share for the young

[13] The data are from the *ONS Family Spending* 2000–2001 carried out by the Office for National Statistics in the United Kingdom and the Dutch *CBS Budgetonderzoek* for the year 1994.

[14] We seem to overestimate the length of the pension period and to underestimate the length of the working life. However, labour participation of workers older than 55 years of age has decreased dramatically over the last 20 years, to $<50\%$, so the majority of these individuals does not work.

Table 1
Spending on services as % of total spending

United Kingdom		the Netherlands	
Age	Share (%)	Age	Share (%)
0–30	4.8	0–54	4.0
30–50	6.3		
50–65	6.3	55–65	4.9
65–75	8.8	65–75	6.0
75+	10.2	75+	9.0

generation (those aged 0–55) and the retired generation (aged 55 and above). The value of the first one will be taken equal to 4.8%, the average for the age groups 0–54 in both countries. The expenditure share for the retired generation is a weighted average of the categories older than 55, where the weights are the population shares in this age group. As the population is growing older, the older categories (who spend more on services) get a higher weight in the future. We will consider the year 2000 and 2050. The population shares are calculated using the medium variant of the United Nations' *World Population Prospects* (2000). For instance, in the Netherlands, 26% of the individuals in age group 55+ is 75 years or older in 2000, whereas this number will rise to 39% in 2050. The average service expenditure share for the retired is thus calculated to be 7.3% in 2000 and 7.6% in 2050.

Longevity is captured by the probability ε to 'survive' the first period and live throughout the entire second period, which is as long as the first one. Life expectancy tables show that people who have reached the age of 55 (and therefore have worked for 25–30 years) are expected to live another 25–30 years, which matches our assumption of a retirement age of 55. The parameter ε is then the probability that someone who enters the labour market (aged around 25) reaches the age of 55, which is equal to 0.94 in 2002 and will be 0.99 in 2050.[15]

As for the fertility rate, we take $n=0$ and -0.1, respectively, which corresponds to 2.1 (1.9) children per woman.[16] The combination of a high value of ε and a low value of n is then indicative for an aged society, which many economies will be in a few decades from now.

Table 2 gives the 'golden-rule' tax rates in the one and two-sector economy, for three different values of θ,[17] and for different combinations of longevity and population growth.

[15] See life expectancy tables taken from the Dutch Statistical Office (CBS) for 2002 and 2050.

[16] An average number of children of 2.1 per woman corresponds to a constant population size. Many industrialised countries will have a lower number in 2050: Germany and Italy 1.6, Japan and the Netherlands 1.8, United Kingdom 1.9 and only the USA will have a number of 2.1. Taking $n=-0.1$ corresponds to 1.9 children per woman.

[17] Empirical studies to date find relatively large values of θ: Mankiw *et al.* (1985), for instance, report values of 2.5 and higher.

Table 2
'Golden-rule' PAYG-scheme (% of GDP)

	$\varepsilon = 0.94$						$\varepsilon = 0.99$					
	$n = 0$			$n = -0.1$			$n = 0$			$n = -0.1$		
θ	0.9	$\frac{4}{3}$	2.5	0.9	$\frac{4}{3}$	2.5	0.9	$\frac{4}{3}$	2.5	0.9	$\frac{4}{3}$	2.5
No services	− 2.3	0.3	2.7	− 2.7	0.9	4.5	− 1.0	1.5	4.0	− 1.4	2.3	5.9
Services	0.1	3.1	5.9	− 0.2	3.9	7.9	1.4	4.4	7.2	1.1	5.3	9.3

Note: PAYG, Pay-As-You-Go; GDP, gross domestic product.

In nearly all cases, the table shows a positive PAYG-tax at which the golden rule is attained, implying that also in the one-sector economy, dynamic inefficiency is likely to occur. Of course, this is due to the two-period structure we apply and resembles the result found by Diamond (1965). Moreover, for high values of θ individuals show a strong desire to smooth consumption over the life cycle. As a result, savings will be high for a low interest rate in both the one-sector and the two-sector model, and the economy is more likely to be dynamically inefficient in the long run.

From this table, we can deduce the effect of an increase in life expectancy (ε) and a decrease in fertility (n), as well as the effect of the sector structure. It is clear that in all cases an increase in life expectancy will lead to a higher PAYG-tax. If people expect to live longer, they will save more in order to reach the preferred consumption level during old age. A higher tax rate is then needed to correct for the decreasing rate of interest following the increase in savings.

Regarding the effect of a decline in fertility, Table 2 shows that it gives rise to a higher PAYG-tax rate as long as the intertemporal elasticity of substitution is below unity ($\frac{1}{\theta} < 1$). This holds in both the one-sector and the two-sector model. Only if individuals can easily substitute young-age consumption for old-age consumption, so that there is less need to save more for old-age consumption after a decrease in the rate of interest, a decline in fertility warrants a lower PAYG-tax.

Finally, and most importantly, Table 2 illustrates the effect of the sector structure. Although the values of λ and μ that can be calculated from the actual expenditure ratios are not dramatically large, it appears that in all cases allowing for service demand leads to an optimal PAYG-tax that is more than 2%-GDP points above the value that results from applying the conventional one-sector model. We see that the tax can be negative, warranting a complete funding of the pension system, especially in the conventional one-sector model, but only in cases where the intertemporal elasticity of consumption is very high ($\theta = 0.9$). However, in most cases (especially in the two-sector model) the PAYG-tax matching the golden rule is positive. It can even get as get as high as 9.3%-GDP.

If $\varepsilon = 0.94$ and $n = 0$ corresponds to present demography, this tax rate varies from 3.1%-GDP to 5.9%-GDP with $\frac{1}{\theta} < 1$, which is at or below the current values in many countries. For example, actual public pension spending is somewhere between 5% and 6% of GDP in the United Kingdom, the

Netherlands, and the United States. However, as for the longer term $\varepsilon = 0.99$ and $n = -0.1$ can be considered to be the relevant values, the long-run optimal level of unfunded pension expenditures in a two-sector economy as a percentage of GDP is larger than the current percentage in these countries.

If anything can be concluded from these tentative calculations, it is certainly not that completely funding public pensions is the optimal solution in the long run to the financing problems of PAYG-financed pension schemes in ageing economies. On the other hand, these calculations should not be interpreted as implying that a reform of the pension scheme is not necessary. As the old-age dependency ratio ($\frac{\varepsilon}{1+n}$) rises from 0.94 to 1.1, it is evident that the benefit per retired individual will have to decrease even if the public pension expenditures as percentage of GDP rise a few points.

V Conclusion

This paper showed that in an economy with a service sector, the golden rule capital stock is lower than in the conventional model with only one commodity. This leads to a different conclusion regarding the saving rate at which long-run utility is maximal, and the corresponding size of the PAYG-pension scheme. As is well known, a pension reform which involves a significant shift to a more funded scheme generates general-equilibrium effects that corrode the purchasing power of the retired. Using a conventional one-sector model may well lead to the conclusion that this effect is not particularly strong, and such a reform is desirable from a long-run perspective. However, elderly individuals need relatively more labour-intensive services than the young, which intensifies the negative impact of the general-equilibrium effects on their consumption possibilities. Taking this effect into account implies that the economy is more likely to become dynamically inefficient if the PAYG-scheme is significantly reduced. We showed in this paper that, for plausible parameter values, the conclusion about the desirability of (completely) cutting down the PAYG-scheme in a two-sector model can be quite the reverse of what is often concluded from a conventional one-sector model. Maintaining unfunded public pensions expenditures of about the current percentage of GDP in the United Kingdom and the Netherlands appears advisable in order to guarantee a balanced consumption profile between young and old individuals in society. However, as the old-age dependency ratio will rise sharply due to ageing, this still implies a significant decrease in public pension benefits.

Our results apply to a closed economy. For a small open economy characterised by perfect capital mobility, a complete transition to a funded pension scheme would imply long-run welfare gains. But if such reforms take place simultaneously in many countries, as seems to be occurring nowadays, factor prices will change nevertheless, and the analysis presented in this paper is more applicable. Furthermore, we only focused on the long run. If instead we applied a social-welfare function that also takes the utility of current generations into account, the case for a PAYG-scheme becomes even stronger.

Appendix A

In steady state, the economy's resource constraint with respect to commodities in per capita terms is given by

$$c \equiv c^{\mathrm{y}} + \frac{\varepsilon}{1+n} c^{\mathrm{o}} = f(k, l^{Y}) - k(1+n). \tag{A1}$$

For the service sector, the constraint is

$$d \equiv d^{\mathrm{y}} + \frac{\varepsilon}{1+n} d^{\mathrm{o}} = 1 - l^{Y}. \tag{A2}$$

The golden rule is characterised by maximising steady-state (expected) utility

$$EU = \frac{(1-\mu)(c^{\mathrm{y}})^{1-\theta} + \mu(d^{\mathrm{y}})^{1-\theta}}{1-\theta} + \gamma\varepsilon \frac{(1-\lambda)(c^{\mathrm{o}})^{1-\theta} + \lambda(d^{\mathrm{o}})^{1-\theta}}{1-\theta}, \tag{A3}$$

subject to equations (A1) and (A2). For a particular value of k and l^{Y}, this leads to the following first-order conditions concerning the consumption of commodities and services

$$\frac{c^{\mathrm{y}}}{c^{\mathrm{o}}} = \left(\frac{1-\mu}{(1+n)\gamma(1-\lambda)}\right)^{1/\theta},$$
$$\frac{d^{\mathrm{y}}}{d^{\mathrm{o}}} = \left(\frac{\mu}{(1+n)\gamma\lambda}\right)^{1/\theta}.$$

Combining these conditions with equations (A1) and (A2) gives $c = \beta_1 c^{\mathrm{o}}$ and $d = \beta_2 d^{\mathrm{o}}$, with $\beta_1 \equiv \frac{\varepsilon}{1+n} + \left(\frac{1-\mu}{(1+n)\gamma(1-\lambda)}\right)^{1/\theta}$ and $\beta_2 \equiv \frac{\varepsilon}{1+n} + \left(\frac{\mu}{(1+n)\gamma\lambda}\right)^{1/\theta}$. The utility function (A3) can now be reformulated in terms of per capita consumption

$$V = \frac{(1-\mu)[c(1-\varepsilon/(\beta_1(1+n)))]^{1-\theta}}{1-\theta} + \frac{\mu[d(1-\varepsilon/(\beta_2(1+n)))]^{1-\theta}}{1-\theta} + \gamma\varepsilon\frac{(1-\lambda)(c/\beta_1)^{1-\theta}}{1-\theta} + \gamma\varepsilon\frac{\lambda(d/\beta_2)^{1-\theta}}{1-\theta}.$$

Maximising this indirect utility function subject to the resource constraints (A1) and (A2) gives the first-order conditions for the (per capita) capital stock and labour in the commodity sector

$$\frac{\partial f(k, l^{Y})}{\partial k} = 1+n,$$
$$\frac{\partial f(k, l^{Y})}{\partial l^{Y}} = \left(\frac{c}{d}\right)^{\theta} \frac{\mu[1-\varepsilon/(\beta_2(1+n))]^{1-\theta} + \gamma\varepsilon\lambda(\beta_2)^{\theta-1}}{(1-\mu)[1-\varepsilon/(\beta_1(1+n))]^{1-\theta} + \gamma\varepsilon(1-\lambda)(\beta_1)^{\theta-1}}.$$

References

Baumol, W. J. (1967). Macroeconomics of unbalanced growth: the anatomy of urban crisis. *American Economic Review*, **57**, pp. 415–26.

Blundell, R., Browning, M. and Meghir, C. (1994). Consumer demand and the life-cycle allocation of household expenditure. *Review of Economic Studies*, **61**, pp. 57–80.

Blundell, R. and MaCurdy, T. (1999). *Labor Supply: A Review of Alternative Approaches Handbook of Labor Economics*. Amsterdam: Elsevier Science.
Breyer, F. (1989). On the intergenerational pareto efficiency of Pay-As-You-Go financed pension systems. *Journal of Institutional and Theoretical Economics*, **145**, pp. 643–58.
Diamond, P. A. (1965). National debt in a neoclassical growth model. *American Economic Review*, **55**, pp. 1126–50.
Economic Policy Committee. (2002). *Annual Report on Structural Reforms*, European Commission, Economic Papers No. 167.
Feldstein, M. (1996) . The missing piece in policy analysis: social security reform. American Economic Review, Papers and Proceedings.
Feldstein, M. (2005) . Structural reform of social security. NBER Working Paper 11098.
Gyárfás, G. and Marquardt, M. (2001). Pareto improving transition from a Pay-As-You-Go to fully funded pension system in a model of endogenous growth. *Journal of Population Economics*, **14**, pp. 445–53.
Hurd, M. (1989). Mortality risks and bequests. *Econometrica*, **57**, pp. 779–813.
Lindbeck, A. and Persson, M. (2003). The gains from pension reform. *Journal of Economic Literature*, **49**, pp. 74–112.
Mankiw, G., Rotemberg, J. and Summers, L. (1985). Intertemporal substitution in macroeconomics. *Quarterly Journal of Economics*, **100**, 225–51.
Verbon, H. A. A. (1988). Conversion policies for public policies in a small open economy. In B. Gustafsson and N. A. Klevmarken (eds), *The Political Economy of Social Security*. Amsterdam: Elsevier Science.

Date of receipt of final manuscript: 13 July 2006