Variable annuities with financial risk and longevity risk in the decumulation phase of Dutch DC products*

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August 2020

Abstract

This paper develops a general framework in which the stock market risk, interest rate risk, inflation risk and longevity risk of variable pension products can be quantified. The paper starts by analyzing the risk of a basic pension product taking into account these risk factors. Under the Koijen et al. (2010) model, we show that interest rate risk can be substantial for the participant and we derive the interest rate risk hedge. Under the additional assumption of a model proposed by Lee-Carter we conclude that financial market risk dominates longevity risk. The paper ends by showing that the results can be generalized for a wide variety of pension products. A pension product which includes smoothing of financial shocks reduces the year on year volatility substantially compared to a basic variable annuity.

Keywords: variable pension products, equity risk, interest rate risk, inflation risk and longevity risk.

*We thank Marcus Haveman, for the useful input and feedback in this project.

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1 Introduction

Worldwide, guaranteed pension products in the decumulation phase are under discussion, see Balter et al. (2018). The European Insurance and Occupational Pensions Authority (EIOPA) has furthermore challenged the sustainability of such products. This has resulted in proposals for a Pan European Personal Pension Product (PEPP) available to all European inhabitants, which would no longer aim at guaranteed pension income. In the implementation process, some lessons might be learned from the recent developments and challenges the Dutch pension system has faced and indeed still faces. In Dutch Defined Contribution (DC) schemes, the option is nowadays available to take investment risks to decumulate pension capital using life-long variable annuities that carry both financial market risk and longevity risk. Pension products covered by this new legislation - 'Wet Verbeterde Premieregeling' (WVP) - can offer a significantly higher first pension payment compared to a pension product without equity exposure.[1]. However, this comes at the cost of uncertainty in the pension income.

As of 2019, pension providers in the Netherlands are required to show the pension income distribution (in nominal and real terms) to a retiree using a uniform economic scenario set, based on Koijen, Nijman and Werker (KNW, 2010). However, not all risks are covered by this legislation. For example, macro longevity risk is not included in the KNW scenario set. This implies that pension providers can leave this risk in the pool without quantifying the increased risk to the participants. Based on the introduction of some stylized pension products, the present paper will provide guidelines on how to interpret the risks associated with these WVP pension products. We take risks related to stocks, bonds, inflation and longevity into account step by step. This project is timely in showing how pension providers can communicate to participants with respect to micro and macro longevity risk since it is not yet included in prescribed consumer information. We extend this information so that all future retirees can make a well-informed choice between a fixed and variable annuity.

To analyze the risks in these variable annuity pension products, this paper develops an inte-

[1] Currently the market for variable annuities in the Netherlands is relatively small. In 2017, 6% of 15.000 retirees chose the variable annuity according to the Dutch Financial Authority (AFM), see https://www.afm.nl/nl-nl/nieuws/2019/apr/wvp-onderzoek. However, this market is expected to grow rapidly. More retirees are faced with the decision for a variable annuity; currently 1.3 million employees participate 'in premie- of kapitaalregelingen' corresponding to 20% of pension plan participants. The number of participants in these schemes has doubled compared to 2009, and the percentage is expected to grow from 6% to 19% according to the AFM.

grated framework in which we can quantify the financial and longevity risks for a wide variety of variable annuities. We develop a framework in which we can quantify the risk of pension products under almost any financial model, and in which we can take into account stock market risk, interest rate risk, inflation risk and longevity risk. Finally, we develop some intuition which product features can be attractive for different type of participants, although we do not perform a formal welfare comparison between pension products.

This paper uses the money pot approach, in which we model the problem as if we set aside a fraction of accumulated pension wealth for the consumption at each age, to analyze pension products. This method of modeling pension products is inspired by Balter and Werker (2020). This methodology has several advantages. In particular, it gives the participant intuition for product characteristics, such as the assumed interest rate and smoothing of financial shocks. Within this approach, incorporating economic risk factors (i.e. financial market risk, longevity risk) is a matter of scaling the money pots.

To be able to obtain analytical expressions for the pension income distribution, and to provide intuition for these expressions, we start the analysis by quantifying the risks of a basic variable annuity (i.e. no smoothing, no guaranteed benefit level) in a Black and Scholes financial market. In contrast to Balter and Werker (2020) we assume an unknown date of death, which we model using deterministic (cohort) survival probabilities. We will present the risks associated with WVP pension products in line with the mandatory communication of pension providers in the Netherlands. This means that the risks associated with pension products will be communicated via a pessimistic, expected and optimistic scenario[2].

Then we extend the analysis of Balter and Werker (2020) by analyzing the risk of pension products under the KNW model. This is the underlying model of the scenario set prescribed by the regulator. On the one hand this enables us to quantify the risk of pension products in a setting with time-varying stock returns and add interest rate risk and inflation risk as economic risk factors. On the other hand this enables us to quantify the risk of pension products in line with Dutch legislation.

Using a Lee Carter (1992) model, De Waegenaere et al. (2019) outline how to compute the implications of a one year micro and macro longevity shock both for pension funds and for WVP contracts. They abstain from financial market risk. We extend their setting by integrating both risk factors, in a multi-period setting to present the pension income distribution under financial market risk and longevity risk. Then we can quantify the additional risk of macro longevity in a basic

[2]This reflects the 5%, 50% and 95% quantile respectively.

variable annuity that currently only quantifies financial market risk.

We repeat the analysis for a wide variety of pension products, for example products that incorporate smoothing, a guaranteed benefit level and high-low, where the retiree can get a higher pension income in the first years of retirement. We also analyze the impact of smoothing of financial shocks.

The first main finding of this paper is that financial market risk dominates the macro longevity risk in variable annuities. However, the stand-alone longevity risk can be substantial in fixed annuities. The pension income for a variable annuity with an asset allocation of 35 % with or without longevity insurance is approximately similar in a 5 % quantile in the absence of an insurance premium. The pension income for a fixed annuity without longevity insurance is 2.6 % lower than a fixed annuity with longevity insurance in a 5 % quantile fifteen years after retirement in the absence of an insurance premium. The second main finding is the quantifiability of the financial market risk for a wide variety of pension products, when the financial market model is able to generate scenarios for stock, bond and inflation returns. In particular, we show the implications of different strategies regarding interest rate risk in variable annuities. We show that a yearly rebalancing strategy adds significant uncertainty to the pension income stream, whereas a perfect hedge yields almost the same pension income stream as in the basic setting. The third finding is that smoothing of financial market risk, as proposed by Balter and Werker (2020), reduces the year on year volatility of pension income. This can be an attractive pension product for an agent exhibiting habit formation. We conclude that the year on year volatility for a pension product with an asset allocation of 35 % and smoothing period of ten years is 1.2 %, whereas the year on year volatility for a pension product giving the same expected income without smoothing is 3.1 %.

Our work is related to Bonekamp et al. (2017) and Balter and Werker (2020), who show how to incorporate stock market risk in (WVP) variable annuities. The analysis was done in a simple financial market model without interest rate risk and inflation. They show that for a variable annuity with 35% equity exposure and maximum assumed interest rate the expected pension income stream is 14% higher than in a fixed annuity. In WVP pension products, macro longevity risk has been quantified by De Waegenaere et al. (2019), under the assumption of a Lee Carter model, as a one-year longevity shock. The focus on a one-year longevity shock implies that their paper did not quantify the aggregate macro longevity risk for pension payments more than one period in the future. They find for a fixed annuity that a one year longevity shock can lead to a drop of approximately 1.1% in pension income in a 2.5% quantile for a participant at retirement age without risk sharing. Balter et al. (2019) calculated macro longevity risk based on multiple historical realized updates in the longevity tables. It is questionable how informative these historical updates are for future longevity shocks. They calculate that a participant at retirement age in 2007 would incur a drop of approximately 10% in pension income in 2010 based on an increased life expectancy. They find an even sharper decrease in pension income for the updates in 2013 and 2016. Steenkamp (2016) shows that the pension income of a fixed annuity without longevity insurance in a 5% percentile at the age of 120 is 2.5 percentage point lower in replacement rate compared to a fixed annuity with longevity insured for a 5% premium under a model assumed by the Dutch Actuarial Society.

This paper also fits in a broader literature on (variable) annuities. Davidoff et al. (2005) derive conditions under which it is optimal to annuitize pension wealth. Koijen et al. (2011) show that adding equity exposure to the annuity enhances welfare in many cases. Balter et al. (2018) present where and to what extent variable annuities are available internationally, and describe an international transition from guaranteed pension products to variable annuities. For example, Horneff et al. (2015) show that variable annuities, in the form of Guaranteed Minimum Withdrawal Benefit, are one of the most rapidly-growing financial innovations in the US and preferred over fixed annuities. Chen et al. (2015) also identify that a retiree might have a preference for a variable pension product with guarantees. Maurer et al. (2013) show that in many cases it can be attractive for participants to bear the systematic longevity risk in variable annuities themselves, since insuring is costly because of Solvency requirements. Boon et al. (2019) find similar results with respect to bearing the longevity risk in Group Self-Annuitization products.

The structure of this paper is as follows. Section 2 starts by quantifying the risk of variable annuities in a standard financial market model. In section 3 we develop a general framework in which we can express the risk of pension products under any financial market model. In section 4 we extend this analysis by taking into account (macro) longevity risk as a risk factor in these variable annuities. In section 5 we show that the results from earlier sections can be generalized for a wide variety of variable annuities (high-low, smoothing and product with guaranteed benefit level). Section 6 provides some conclusions.

2 Pension income distribution with stock market risk

In this section we present the pension income distribution for a basic variable annuity, taking into account financial market risk only. Balter and Werker (2020) and Bonekamp et al. (2017) provide an elaborate description of the setting assuming a deterministic date of death. This section starts by taking into account deterministic (cohort) survival probabilities. In the empirical results, we use gender neutral survival probabilities from the Dutch Actuarial Society (AG, 2018).[3]

At retirement age T, a person has accumulated pension wealth W_T. This person can decide on the investment mix w. In the Black and Scholes financial market there exists an asset with return the risk free rate r and an asset with an uncertain return. The return of this asset is log-normal with mean $r + \lambda\sigma - \frac{1}{2}\sigma^2$, volatility σ and sharpe ratio λ. We define a maximum age L for this person. We allocate the available pension wealth to $L-T$ money pots, that we denote by $V_h(T)$. We define $V_h(T)$ as the part of W_T that we set aside for consumption at age $T+h$. The assumed interest rate AIR determines the allocation to money pots, through the allocation rule.[4] The higher the AIR is, a larger fraction of W_T will be allocated to $V_h(T)$ for smaller h. This is typically referred to as front loading. We allow the assumed interest rate to be horizon-dependent (AIR_h). The assumed interest rate and the money pot allocation satisfies

$$V_h(T) = W_T \cdot \frac{p_h(T)\text{exp}(-h \cdot AIR_h)}{\sum_{k=0}^{L-T-1} p_k(T)\text{exp}(-k \cdot AIR_h)} \tag{1}$$

where $p_h(T)$ is the probability that a person is h years from retirement age still alive. The return on money pot $V_h(T)$, h periods from retirement age onwards, follows a log-normal distribution with mean $h(r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2)$ and variance $hw^2\sigma^2$. This implies the expectation and quantiles for the pension income distribution.

[3]For computational simplicity, a maximum age of 100 will be assumed. In all figures the pension income stream will be presented until the age of 90. Note that this way of presenting the pension income distribution is in line with the communication of pension providers.

[4]The AIR can be written as the sum of the risk free rate and a parameter often referred to as fixed decrease in the Dutch policy discussions.

$$
\begin{aligned}
E_t(V_h(T+h)) &= V_h(T) \cdot \exp(h \cdot (r + w\lambda\sigma)) \cdot \left(\frac{1}{p_h(T)}\right) \\
&= W_T \cdot \frac{\exp(-h \cdot AIR_h)\exp(h \cdot (r + w\lambda\sigma))}{\sum_{k=0}^{L-T-1} p_k(T)\exp(-k \cdot AIR_h)}
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
Q_t^{\alpha}(V_h(T+h)) &= V_h(T) \cdot \exp\left(h \cdot (r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2) + z_\alpha\sqrt{h}w\sigma\right) \cdot \left(\frac{1}{p_h(T)}\right) \\
&= W_T \cdot \frac{\exp(-h \cdot AIR_h)\exp\left(h \cdot (r + w\lambda\sigma - \frac{1}{2}w^2\sigma^2) + z_\alpha\sqrt{h}w\sigma\right)}{\sum_{k=0}^{L-T-1} p_k(T)\exp(-k \cdot AIR_h)}
\end{aligned} \tag{3}
$$

Formula (2) shows that, given a constant asset allocation (w) in order to obtain a flat consumption pattern in expectation, the AIR should be set as

$$
AIR_{mean} = r + w\lambda\sigma \tag{4}
$$

Note that the AIR in (4) is independent of the horizon. In the Dutch setting there is a restriction on the assumed interest rate set by the regulator such that in expectation the pension income stream is non-decreasing.

We can plot the pension income distribution of a basic variable annuity with w=35 % and AIR=2.01 % calculated in line with (4). Table 1 reports the parameter assumptions.[5]

Name	Parameter	Value
Retirement age	T	67
Pension wealth	W_T	233.000
Risk free rate	r	0.43 %
Volatility stock returns	σ	16.75 %
Excess stock returns	$\lambda\sigma$	4.52 %

Table 1: Overview parameter values

In line with Dutch pension providers we scale the pension income to a monthly basis. We consider an individual[6] that has accumulated a deterministic amount of wealth at retirement age.

[5] We set the parameters in such a way to create equivalence with the current calibration of the KNW model prescribed by the regulator.

[6] For simplicity we assume that this individual does not have a partner to be able to abstract from partner pension.

Figure 1 presents the pension income stream of this participant from retirement age onwards.[7]

Observe that we get a constant expected pension income stream by setting the assumed interest

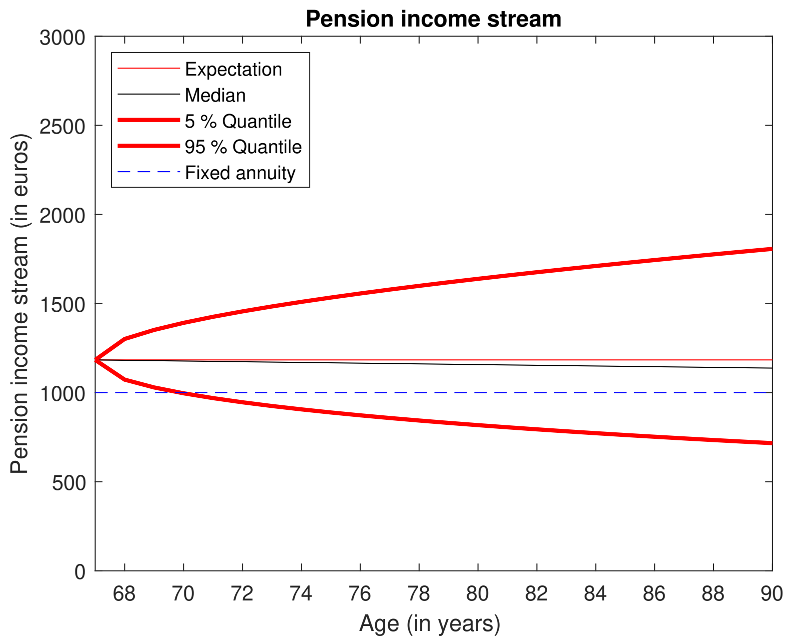


Figure 1: Pension income distribution in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 % and parameter assumptions in line with Table 1.

rate in line with (4). This implies that the median pension income is decreasing, as a small probability on a high pension income stream heavily influences the expected pension income stream in the skewed distribution (log-normal). Grebenchtchikova et al. (2017) present conditions under which this variable annuity is optimal assuming an individual with constant relative risk aversion preferences. Figure 1 shows that the variable annuity has a significant higher mean payment compared to a fixed annuity. As a rule of thumb, we approximate (1) with h=0 and calculate this higher pension income (as a percentage) by multiplication of the duration (approximately 10) with the difference between AIR and risk free rate (i.e. $10 \cdot 1.58\% = 15.8\%$).

Assume that we choose the assumed interest rate equal to the risk free rate ($AIR = r$). Then the first pension payment is equal to that of a fixed annuity. The pension income grows in expectation by the excess returns $w\lambda\sigma$. Figure 2 shows this result.

[7] In line with communication of Dutch pension providers, the income from the state pension is not presented in these Figures.

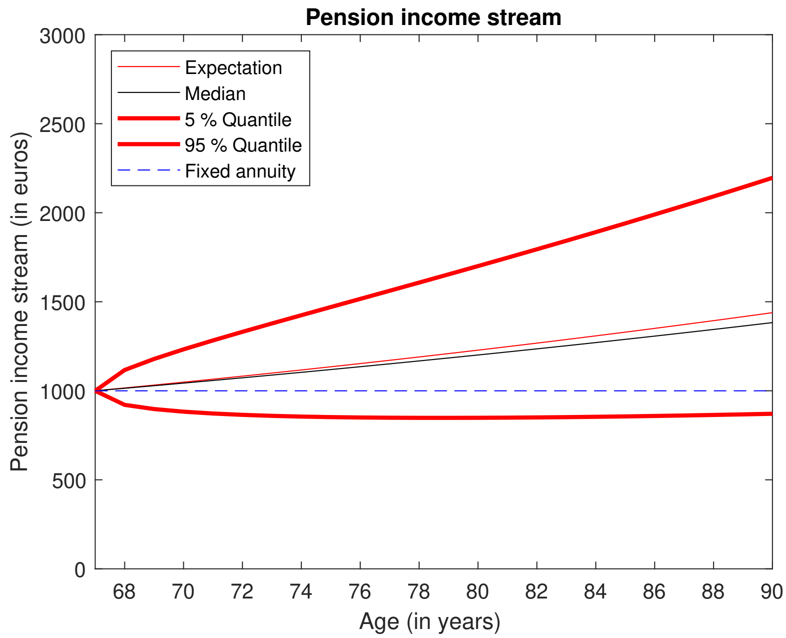


Figure 2: Pension income distribution in a Black and Scholes financial market with w=35 %, AIR=r=0.43 % and parameter assumptions in line with Table 1.

In sections 3 and 4 we will focus on a pension product with constant asset allocation taking into account interest rate risk, inflation risk and/or longevity risk. We compare this to the basic variable annuity in Figure 1. In section 5 we will discuss the risks for several non-basic pension products, a high-low pension product, a pension product with a guaranteed benefit level and a pension product incorporating smoothing of financial shocks.

3 Pension income distribution with interest rate and inflation risk

In this section we analyze the risk of pension products in line with the Koijen et al. (2010) model (KNW model). This is the underlying model prescribed by the Dutch regulator and Dutch pension providers need to quantify the risk of pension products in line with this model. However, the framework that we introduce in this section to calculate the risk of pension products is applicable to any financial market model as long as the model can generate scenarios for stock, bond and inflation returns.

One of the major changes compared to the Black-Scholes model in Section 2 is that we no longer assume a constant interest rate. In linear affine models the yield y_t^h, which is a function of the state variables X_t and $A(h)$ and $B(h)$ (see Brennan and Xia (2002)), satisfies

$$y_t^h = \frac{-A(h) - B(h)X_t}{h} \tag{5}$$

To allocate the wealth to money pots accordingly, defined in (1), we replace the constant interest rate in (4) by the yield as follows

$$AIR_h = y_0^h + w\lambda\sigma \tag{6}$$

We define a recursive relation for each money pot to calculate the pension income stream. We let this recursive relation run over horizon j, where obviously $1 \leq j \leq h$. This is presented in a general form, independent of the scenario set, for different investment strategies.

Investment strategy 1 (Time varying stock returns, one year bonds): We start to extend the results by taking into account a time varying return on stocks S and we invest $1-w$ recursively in a one year bond. The price of a nominal zero coupon bond at time t with a single payout at time $t+h$ is defined as $P(X_t, t, h)$.

$$\begin{aligned} V_h(T+j) = \Bigg(\Bigg\{ & w \cdot V_h(T+j-1) \cdot \underbrace{\frac{S_j^i(X_j)}{S_{j-1}^i(X_{j-1})}}_{\text{stock return}} \\ & + (1-w) \cdot V_h(T+j-1) \cdot \underbrace{\frac{1}{P(X_{j-1}^i, j-1, 1)}}_{\text{one year bond return}} \Bigg\}\Bigg) \cdot \left(\frac{1}{p_1(T+j-1)}\right) \end{aligned} \tag{7}$$

For the application in this project, we insert the stochastic scenarios for stocks from the KNW model. In the prescribed parameterization of the model there is an increasing (expected) return on stocks, due to an increasing (expected) nominal instantaneous interest rate. We calculate the (one year) bond prices implied by the state variables from the model.

Investment strategy 2 (Partial interest rate hedge): We invest $1 - w$ of each money pot needed h periods from retirement onwards in a bond with maturity h years. The corresponding price is $P(X_{j-1}, j-1, h)$ at time $j-1$. After one year, at time j, the price of this bond is equal to $P(X_j, j, h-1)$. This we incorporate as follows by changing the 'one year bond return' in (7) to a matching portfolio

$$\frac{P(X_j^i, j, h-j)}{P(X_{j-1}^i, j-1, h+1-j)} \tag{8}$$

Under this investment strategy we are still exposed to interest rate risk via the stock market, therefore, we refer to this as partial interest rate hedge.

Investment strategy 3 (Full interest rate hedge): If we were to hedge all the interest rate risk we should invest each money pot $V_h(T)$ in a bond with maturity h. Additionally, we allocate a fraction w towards the stock market and we have a short position of fraction w in a one year bond. The hedge is not completely perfect due to a discretization effect, since marginal differences arise between a one year bond and the nominal instantaneous interest rate. We calculate the pension income stream by letting the following recursive relation run over j, where obviously $1 \leq j \leq h$.

$$\begin{aligned} V_h(T+j) = \Bigg(\Bigg\{ & \underbrace{V_h(T+j-1) \cdot \frac{P(X_j^i, j, h-j)}{P(X_{j-1}^i, j-1, h+1-j)}}_{\text{return on matching bonds}} \\ & + \underbrace{w \cdot V_h(T+j-1) \cdot \left(\frac{S_j^i(X_j)}{S_{j-1}^i(X_{j-1})} - \frac{1}{P(X_{j-1}, j-1, 1)} \right)}_{\text{excess return on short position}} \Bigg\}\Bigg) \cdot \\ & \left(\frac{1}{p_1(T+j-1)} \right) \end{aligned} \tag{9}$$

Pension income in real terms: We add inflation Π to the model. In our application, inflation is only relevant to convert the pension income stream from nominal to real terms. This implies that we abstain from investment options in inflation linked bonds. We do this by multiplying the general recursive relation, defined in (7) by

$$\left\{ \frac{\Pi_j^i(X_j)}{\Pi_{j-1}^i(X_{j-1})} \right\}^{-1} \tag{10}$$

We will take w=35 $\%$, assumed interest rate in line with (6) and the parameters and state variables in the model are those from the third quarter of 2019. Schotman et al. (2020) used this scenario set to make welfare comparisons among different pension products, where they extend Bonekamp et al. (2017) by taking into account interest rate risk as well. We refer to the pension product described in section 2 as basic in the upcoming figures.

In Figure 3 we extend the setting by taking into account time varying stock returns and invest $1-w$ in one year bonds.

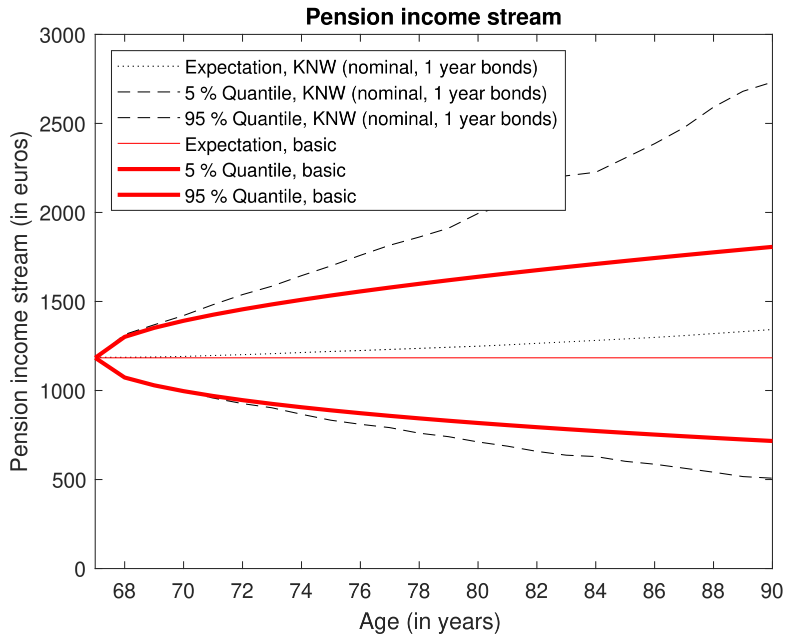


Figure 3: Pension income distribution in a KNW financial market with w=35 %, AIR_h in line with (6) and the setting extended by time varying expected stock returns and one year bond returns in line with (7). This is defined as investment strategy 1. The pension income distribution of the basic variable annuity is calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 % and parameter assumptions in line with Table 1.

Observe that we calibrated the risk free rate from Table 1 such that the first pension income in the KNW setting is similar as in the Black and Scholes setting. Figure 3 shows that the expected pension income stream is no longer constant but increasing. This is because the (nominal) instantaneous interest rate and the return on one year (nominal) bonds are expected to increase over time. This in comparison with the yields of the initial term structure, that we used in the allocation of the accumulated pension wealth towards money pots.

The volatility of stock returns in the KNW model is 16.75 %[8], where the parameters are based on Draper (2014) and prescribed by the Dutch regulator. This is identical to the volatility of stock returns in the Black Scholes setting (see Table 1).

[8] $\sigma_S = \sqrt{\sigma^2_{S(1)} + \sigma^2_{S(2)} + \sigma^2_{S(3)} + \sigma^2_{S(4)}} = \sqrt{(-0.53)^2 + (-0.76)^2 + (-2.1)^2 + (16.59)^2} \approx 16.75\%$

In Figure 4 we extend the setting by considering investing in the bond portfolio that partially hedges the interest rate risk.

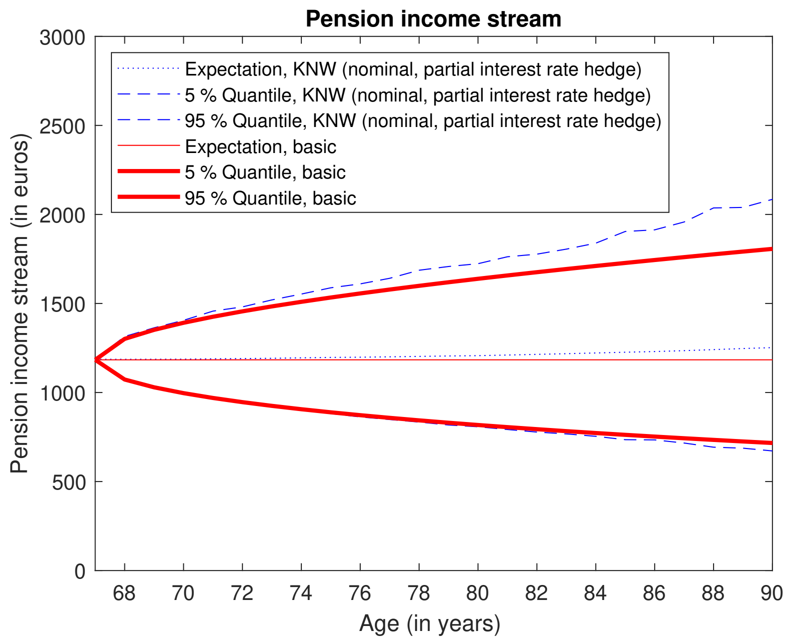


Figure 4: Pension income distribution in a KNW financial market with w=35 %, AIR_h in line with (6), setting extended by partial interest rate hedge by replacing the one year bond return in (7) by the matching portfolio of (8). This is defined as investment strategy 2. The pension income distribution of the basic variable annuity is calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 % and parameter assumptions in line with Table 1.

Figure 4 shows a small increase in the expected pension income. However, there is a smaller increase compared to the previous setting. This is because the bond portfolio will not profit from the expected increase in the return of one year bonds, since this is matched. Still, there is some interest rate exposure via the allocation towards the stock market because of the nominal instantaneous interest rate. Therefore, we refer to this setting as partial interest rate hedge.

In Figure 5 we extend the setting also hedging against changes in the nominal instantaneous interest rate. Therefore, we refer to this setting as full interest rate hedge.

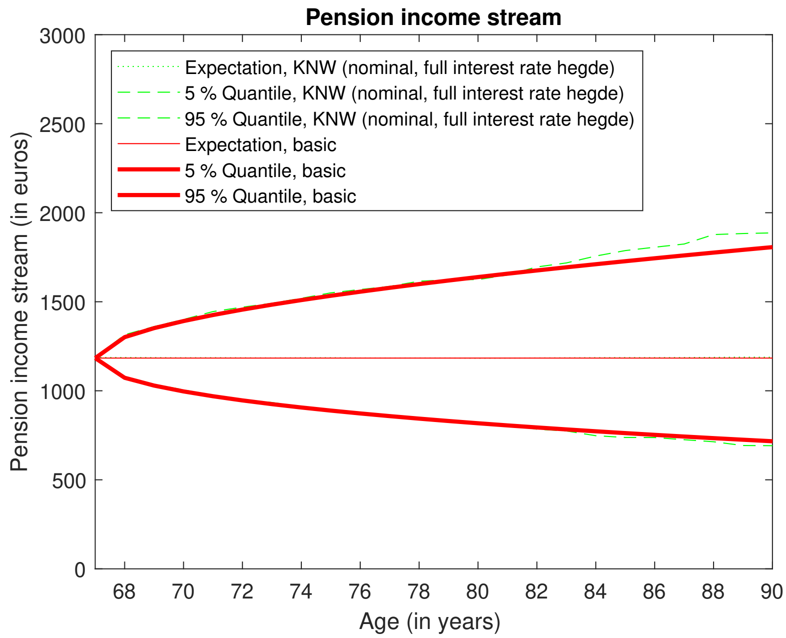


Figure 5: Pension income distribution in a KNW financial market with w=35 %, AIR_h in line with (6), setting extended by full interest rate risk hedge in line with (9). This is defined as investment strategy 3. The pension income distribution of the basic variable annuity is calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 % and parameter assumptions in line with Table 1.

Figure 5 shows that we can get a constant pension income stream under the KNW model in expectation. This result more or less overlaps with the Black and Scholes setting. The conditions under which such a pension income distribution is optimal, assuming CRRA preferences, is available in Grebenchtchikova et al. (2017).

In Figure 6 we express the pension income stream in real terms to determine the purchasing power of the pension income. This we do by multiplying inflation (9) with the pension income stream following the full interest rate hedge strategy (10).

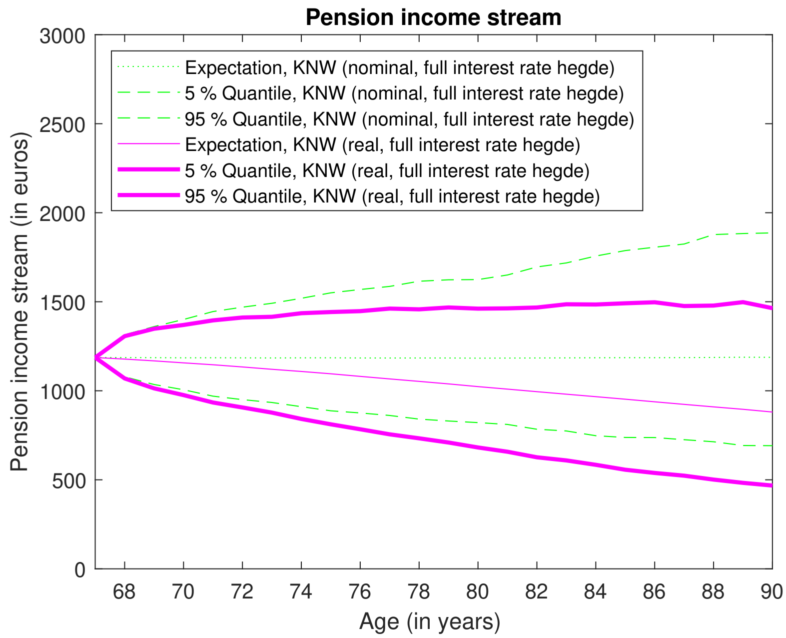


Figure 6: Pension income distribution in a KNW financial market with w=35 %, AIR_h in line with (6), setting extended by comparing the pension income following the full interest rate hedge strategy in nominal (9) and real terms, where we need to multiply by (10).

Figure 6 shows that in nominal terms the expected pension income is constant, whereas in real terms this is no longer the case. In particular, we observe a sharp decrease in real expected pension income. Therefore, Figure 6 shows that communication in nominal or real terms, which is mandated for Dutch consumers, really matters.

Although it is currently not required, a different goal could be to require a constant pension income in real terms. We can achieve this by lowering the assumed interest rate in (1). Then we will end up with a horizon-dependent assumed interest rate, since the expected inflation is increasing over time. A different way to achieve the constant pension income stream in real terms, is by defining an investment strategy with allocation towards real bonds. This we will not present because perfect inflation linked bonds are not available for the Netherlands.

4 Pension income distribution with longevity risk

In this section we will take into account longevity risk. We distinguish between micro and macro longevity risk. Micro longevity risk quantifies the risk related to uncertainty in the time of death if survival probabilities are known with certainty while macro longevity risk is due to uncertain future survival probabilities (Hari et al., 2008). Macro longevity risk (changes in survival probabilities) is fundamentally different because it does not diversify. We assume that longevity risk is shared within the own age group[9]. We also assume that longevity risk is uncorrelated with financial market risk described in the previous sections.

Throughout this section we assume a pension fund that has $J(T)$ participants at retirement age T. There is no new inflow. This means that we plot the pension income stream of a participant at retirement age in this (simplified) pension fund[10]. In subsection 4.1 we will quantify the micro longevity risk, shared within the own age group, for different number of participants in the pension fund and different asset allocations. In subsection 4.2 we will quantify the macro longevity risk, shared withing the own age group, for different asset allocations. In this setting, the size of the pool is irrelevant since macro longevity risk does not diversify, in contrast to micro longevity risk. Therefore, we assume that the pool of participants is large enough such that micro longevity risk is diversified.

In section 2 we were able to present the pension income stream using analytical solutions. In this section, we need simulation and use 10.000 scenarios. To keep the setting as simple as possible, we return to the Black and Scholes setting of section 2.

[9] We do not take into account risk sharing mechanisms.

[10] To clarify, this set up implies that the pool becomes smaller over time.

4.1 Pension income distribution with micro longevity risk

In De Waegenaere et al. (2019) the one-year implications of a micro longevity shock for a fixed annuity is calculated under several risk sharing mechanisms and different compositions and sizes of the pension fund. Their recommendation is to share this risk over all participants in the pension fund (accumulation and retirement phase), since this risk diversifies for a large group of participants.

We extend the setting of De Waegenaere et al. (2019) by taking into account multiple periods. We start by simulating the remaining number of participants $J(T+j)$, with age $T+j$, in the pool recursively from the binomial distribution.

$$J(T+1) \sim \text{BIN}\Big(J(T+j-1), p_1(T+j-1)\Big) \tag{11}$$

Then we can simulate the pension income stream taking into account financial market risk and micro longevity risk shared within the own age group by the following iterative relation for $1 \leq j \leq h$, where $R^{\text{fin}}(T+j-1)$ is the financial return at age $T+j-1$ and $J(T+j)$ is simulated from (11).

$$V_h(T+j) = \left\{ V_h(T+j-1) \cdot R^{\text{fin}}(T+j-1) \cdot \frac{1}{p_1(T+j-1)} \right\} \cdot \underbrace{\frac{p_1(T+j-1) \cdot J(T+j-1)}{J(T+j)}}_{R^{\text{micro}}(T+j)} \tag{12}$$

Observe that for a large pool of participants, the variable denoting the micro longevity risk $R^{\text{micro}}(T+j)$, is close to 1. Observe that we can easily take into account interest rate and inflation risk by inserting for $R^{\text{fin}}(T+j-1)$ in (12) what we have derived in Section 3.

The pension income stream is presented for different asset allocation strategies w and different sizes of the pool of participants $J(T)$ under the assumption of no new inflow, as defined in Table 2. Please note the scale difference on the vertical axis for an annuity with w=0 $\%$ (Figure 7-8 versus an annuity with w=35 $\%$ (Figure 9-10).

	w	AIR	$J(T)$
Figure 7	w=0 %	AIR=r	$J(T)$=500
Figure 8	w=0 %	AIR=r	$J(T)$=2.500
Figure 9	w=35 %	AIR=2.01 %	$J(T)$=500
Figure 10	w=35 %	AIR=2.01 %	$J(T)$=2.500

Table 2: Overview of different cases financial market risk and micro longevity risk.

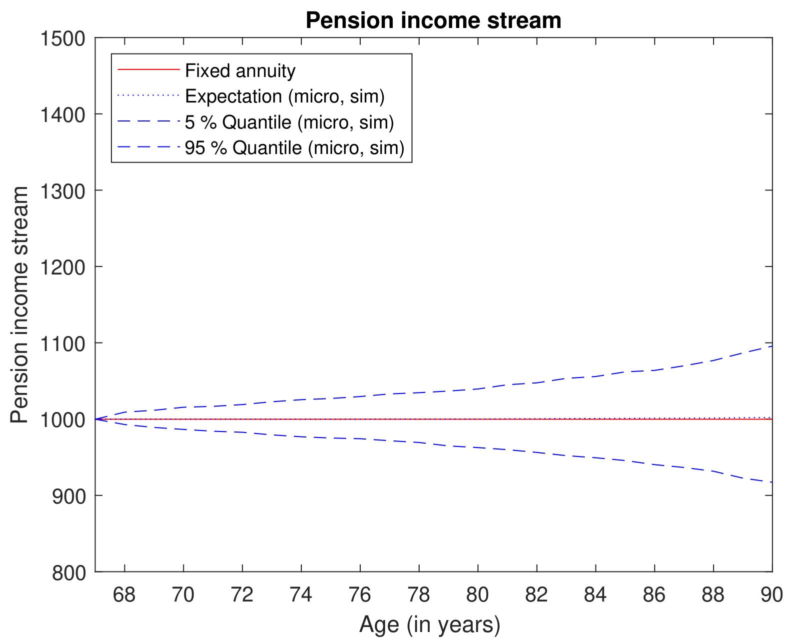


Figure 7: Simulated pension income distribution in a Black and Scholes financial market with w=0 %, AIR=r=0.43 %, J(T)=500 and parameter assumptions in line with Table 1. This is compared to the pension income distribution of a basic variable annuity calculated in a Black and Scholes financial market with w=0 %, AIR=r=0.43 %, parameter assumptions in line with Table 1, implicitly assuming J(T)=∞.

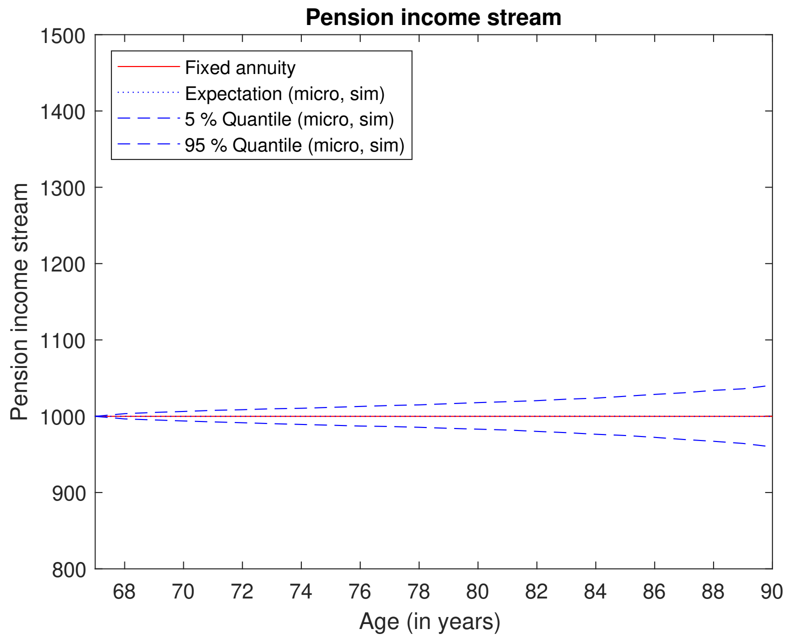


Figure 8: Simulated pension income distribution in a Black and Scholes financial market with w=0 %, AIR=r=0.43 %, $J(T)$=2.500 and parameter assumptions in line with Table 1. This is compared to the pension income distribution of a basic variable annuity calculated in a Black and Scholes financial market with w=0 %, AIR=r=0.43 %, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞.

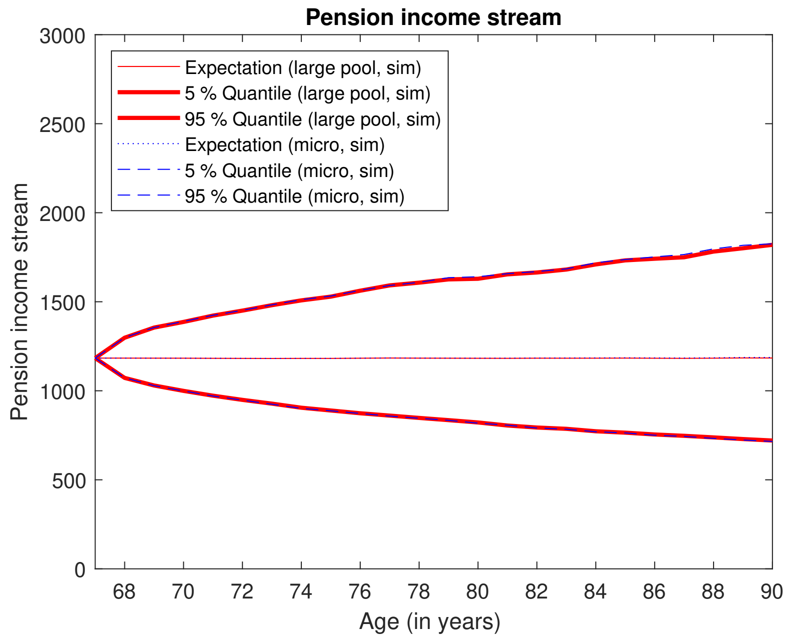


Figure 9: Simulated pension income distribution in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 %, $J(T)$=500 and parameter assumptions in line with Table 1. This is compared to the pension income distribution of a basic variable annuity calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 %, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞.

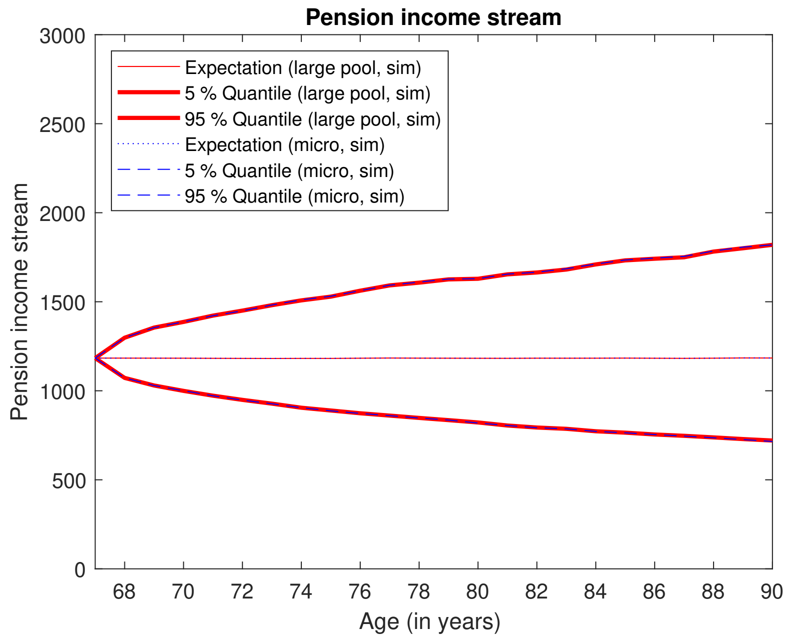


Figure 10: Simulated pension income distribution in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 %, $J(T)$=2.500 and parameter assumptions in line with Table 1. This is compared to the pension income distribution of a basic variable annuity calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 %, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞.

In case the participant opts for the fixed annuity, micro longevity risk can be substantial in small pool of participants (with no new inflow). In case the variable annuity is chosen by the participant, already for a small pool of participants the micro longevity risk is dominated by the financial risk. The group of participants that currently have a variable annuity is still relatively small (Hers et al., 2019).

4.2 Pension income distribution with macro longevity risk

It is well known that in the last century the population is becoming older, a trend that is expected to continue in the near future. According to AG (2018), a current 65 year old man (woman) has a remaining life expectancy of 20.3 (23.1) years. In 2069, a 65 year old man (woman) is projected to have a remaining life expectancy of 25.6 (28.0) years. This is a significant increase, though obviously these expectations are uncertain. A real life example is an increased mortality rate because of COVID-19, that might lead to an overall decrease in future life expectancy. For the variable annuities considered in this setting it is important to take into account the uncertainty concerning the aggregate life expectancy's of retirees.

In this paper a Lee Carter (1992) model will be assumed. This is a somewhat simpler model than the version used by AG, though nevertheless a highly accepted longevity model in literature. However, we cannot guarantee that in the near future longevity shocks will be realized that will be outside the scope of this model. Richards et al. (2014) quantify longevity risk by simulating data from the current calibration of the longevity model. The longevity trend risk is then measured by re-estimating the model including the simulated data. De Waegenaere et al. (2019) use a similar approach in analyzing macro longevity risk. Their paper only took a one year period into account for a fixed annuity. Abstracting from equity exposure the implications of a one year shock lead to a drop of approximately 1.1 % in pension income in a 2.5 % quantile for a participant at retirement age without risk sharing. Balter et al. (2019), however, argue that using historical data[11] from the Danish setting the macro longevity risk can be significant in variable annuities. This should be interpreted as an (extremely negative) scenario that has occurred in the past and is not informative regarding expected future longevity shocks. Still, they calculate for a participant at retirement age in 2007 with an annuity bearing longevity risk a decrease in pension income of more than 10 % after the updates in 2010, 2013 and 2016. Piggot et al. (2005) calculate the longevity risk in group self annuitization products (GSA) and use the changed expectation adjustment (CEA) factor to adjust for mortality changes. This factor has the interpretation of the changed value of the annuity based on a mortality update. Steenkamp (2016) shows under the AG model that the pension income of a fixed annuity in a 5 % quantile is 2.5 percentage point lower in replacement rate at the age of 120 if the longevity risk is not insured.

The present paper chooses not to recalibrate the parameters from the Lee-Carter model based on new years of simulated data, in order to be in line with the modelling approach of financial mar-

[11] Therefore, Balter et al. (2019) deviate for obvious reasons from Richards et al. (2014).

ket risk. We also do not allow for revision of the KNW model and the input parameters. In reality every five years a Committee Parameters reconsiders the financial market model and calibration to be in line with financial market expectations at that point in time. In that sense we deviate from the actuarial literature described above.

The pension income stream can be defined by the following recursive relation where $1 \leq j \leq h$, where $p_1^{\text{old}}(T+j-1)$ and $p_1^{\text{new}}(T+j-1)$ represent the one year survival probability at age $T+j-1$ under the old and new survival table respectively.

$$V_h(T+j) = \left\{ V_h(T+j-1) \cdot R^{\text{fin}}(T+j-1) \cdot \frac{1}{p_1^{\text{old}}(T+j-1)} \right\} \cdot \underbrace{\left(\frac{p_1^{\text{old}}(T+j-1)}{p_1^{\text{new}}(T+j-1)} \right)}_{R^{\text{macro}}} \quad (13)$$

In the Appendix a description of the Lee-Carter model can be found, where we present calibration results. Observe that in expectation the variable denoting the macro longevity risk $R^{\text{macro}}(T+j)$ is close to 1. This has the interpretation that ex ante we do not expect deviations from the best estimate of future survival probabilities. Observe that we can easily take into account interest rate risk and inflation risk by inserting for $R^{\text{fin}}(T+j-1)$ in (12) what we have derived in Section 3.

Therefore, we can present the pension income stream taking into account financial market risk and macro longevity risk shared within the own age group[12]. The pension income stream will be calculated for an equity exposure of w=0% and w=35% in Figure 11 and 12 respectively.

[12]In Figure 11 and Figure 12 a pension product in which the macro longevity risk is borne by the pension provider is compared to a pension product in which this risk is borne by the pool of participants. If the macro longevity risk is borne by the pension provider a premium needs to be paid. For simplicity, we abstract from this by setting it equal to 0 %.

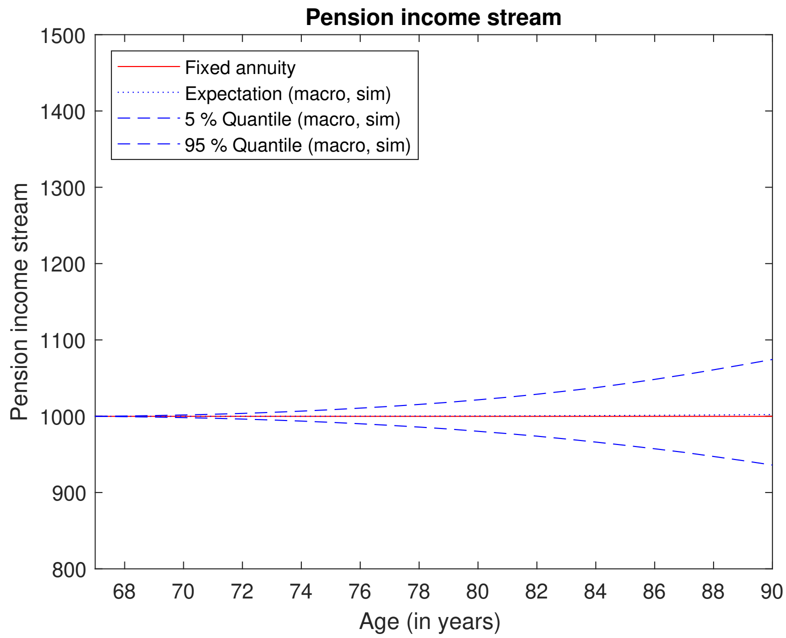


Figure 11: Simulated pension income distribution in a Black and Scholes financial market with w=0 %, AIR=r=0.43 %, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞ and including macro longevity risk. In the Lee Carter model, we have calibrated the expected life improvements and volatility of life improvements with C=-1.8979 and $\sigma_k = 2.3198$ respectively. This is compared to the pension income distribution of a basic variable annuity calculated in a Black and Scholes financial market with w=0 %, AIR=r=0.43 %, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞.

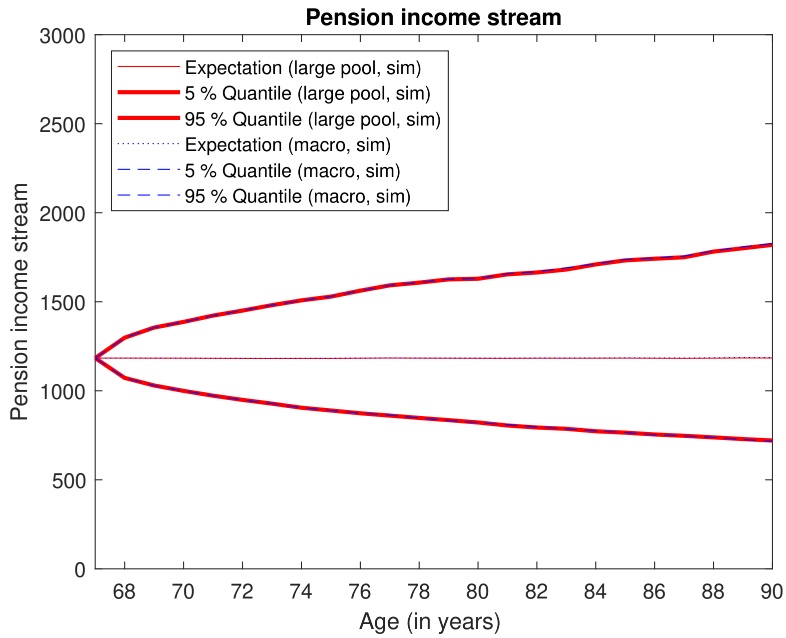


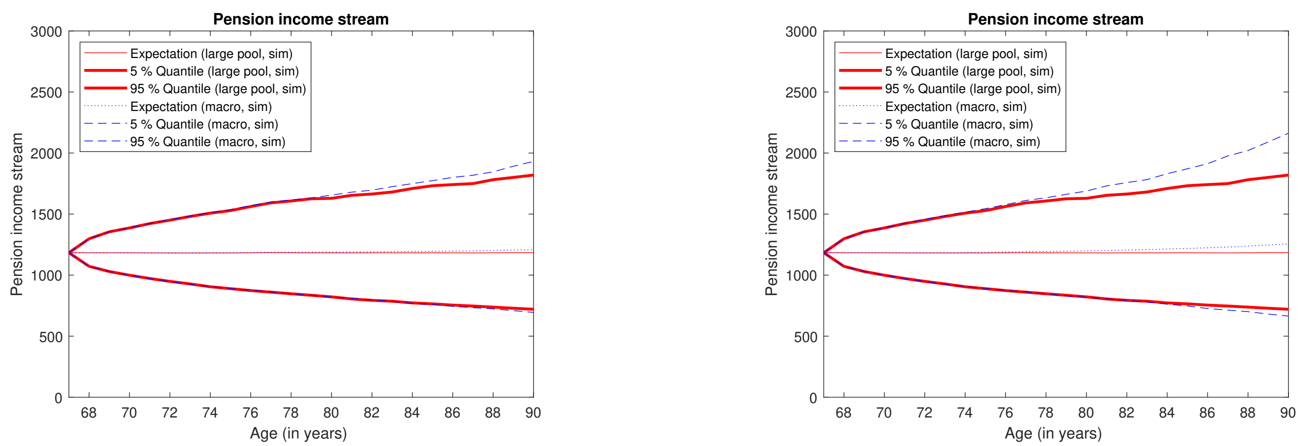
Figure 12: Simulated pension income distribution in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 %, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞ and including macro longevity risk. In the Lee Carter model, we have calibrated the expected life improvements and volatility of life improvements with C=-1.8979 and $\sigma_k = 2.3198$ respectively. This is compared to the pension income distribution of a basic variable annuity calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 %, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞.

Figure 12 shows that for larger equity exposure, the financial market risk dominates the quantiles for the pension income distribution[13]. However, the stand-alone longevity risk is substantial (see Figure 11). Gielen and De Waegenaere (2014) quantify the cost of insuring longevity risk, within the Solvency framework, as 4.6 % in excess of the fair annuity price. We can easily extend Figure 11 and 12 with a premium for insuring longevity risk around 4.6 % and conclude that bearing this risk is attractive for the participant.

[13]Overlapping quantiles obviously do not exclude that in individual scenarios where financial returns are in line with the expectation, an increase in the life expectancy can lead to a significant decrease in the pension income.

4.3 Pension income distribution with macro longevity risk, robustness

The previous analysis was based on a Lee-Carter model with historically calibrated expected longevity improvements and calibrated volatility of historical changes in survival probabilities. Of course, there could be much larger changes and we want to investigate the robustness of our results to the estimates. We therefore also run simulations with much higher volatility of the survival probabilities. We quantify quantify the macro longevity risk by assuming a higher volatility of life improvements, for example $\tilde{\sigma_k} = 3 \cdot \sigma_k$, $\tilde{\sigma_k} = 5 \cdot \sigma_k$. This is presented in Figure $13a$ and $13b$ for an asset allocation of w=35 $\%$, abstracting from micro longevity risk. Then we see that longevity risk becomes a more important risk factor, although marginal differences can be observed in the 5 $\%$ quantile.



(a) Volatility of life improvements in Lee Carter set to $\tilde{\sigma_k} = 3 \cdot \sigma_k$.

(b) Volatility of life improvements in Lee Carter set to $\tilde{\sigma_k} = 5 \cdot \sigma_k$.

Figure 13: Simulated pension income distribution in a Black and Scholes financial market with w=35 $\%$, AIR=AIR_{mean}=2.01 $\%$, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞ and including macro longevity risk. We have increased the volatility in the Lee Carter model by $\tilde{\sigma_k} = 3 \cdot \sigma_k$ in Figure $13a$ and by $\tilde{\sigma_k} = 5 \cdot \sigma_k$ in Figure $13b$. This is compared to the pension income distribution of a basic variable annuity calculated in a Black and Scholes financial market with w=35 $\%$, AIR=AIR_{mean}=2.01 $\%$, parameter assumptions in line with Table 1, implicitly assuming $J(T)$=∞.

5 Extensions pension income distribution to additional product features

In this section we present an analysis of the dynamics of a high-low pension product. Also, we discuss a pension product with guaranteed benefit level, which is a combination of a variable and fixed annuity. We end this subsection with a description of a pension product incorporating smoothing of financial shocks.

5.1 High-low pension product

In a high-low pension product the retiree can get a higher pension income in the first years of retirement. As a consequence, at older ages the retiree receives less pension income compared to a basic variable annuity. In the Dutch institutional setting, there are two important restrictions in a high-low pension product.

1. A high-low construction implies that there is some variation between the highest and lowest pension income. The difference between the highest and lowest pension income is maximized at 100:75. This implies that the lowest payment should be at least 75 $\%$ of the highest payment.
2. The period that you can acquire a higher pension income stream is not maximized. However, for practical reasons pension providers will typically set this to 5 or 10 years.

To allocate the wealth to money pots, defined in (1), we should introduce some notation. We define $\mathbb{1}_{low}$ as an indicator function with value 1 if we are in the low period and with value 0 if we are in the high period. z_h will be a function of the length of the high period and the relative difference in pension income between the high and low period. We assume that the lowest pension income is to be 75 $\%$ of the highest pension income. Then we define $z_h \mathbb{1}_{low}$ as follows, where the derivation can be found in the Appendix.

$$z_h \mathbb{1}_{low} = -\frac{1}{h} \log(0.75) \tag{14}$$

After this the pension income distribution can be easily calculated with (2) and (3) by increasing the assumed interest rate in (4) with $z_h \mathbb{1}_{low}$ that changes the allocation towards money pots in (1).

$$\text{AIR}_h = r + w\lambda\sigma + z_h \mathbb{1}_{low} \tag{15}$$

We assume w=35%, assumed interest rate in line with (15) and a product where the pension income is lower after 10 years. Figure 14 illustrates this.

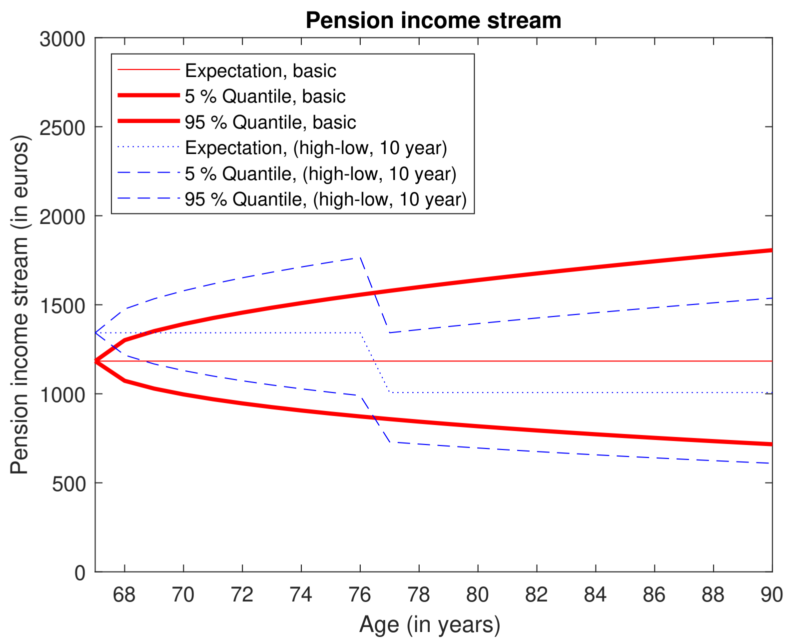


Figure 14: Pension income distribution of high-low product with lower income after 10 years, in a Black and Scholes financial market with w=35 %, AIR in line with (15) and parameter assumptions in line with Table 1. The pension income distribution of the basic variable annuity is calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 % and parameter assumptions in line with Table 1.

Figure 14 shows that the expected pension income for a high-low pension product is no longer constant. The shorter the high period, the higher the pension income is compared to a basic variable annuity. As a consequence, in later years the complete pension income distribution (high-low) is lower compared to a basic variable annuity.

5.2 Pension product with guaranteed benefit level

Worldwide, guaranteed pension products are under discussion. However, Cochrane (2007) derives that for an agent exhibiting a subsistence level a pension product with a guaranteed benefit level is optimal. In the setting of Van Bilsen et al. (2019), the reference level leads to a demand for guarantees. Also, Chen et al. (2015) conclude that a variable annuity with a guaranteed component can be optimal. Horneff et al. (2015) show the existence of variable annuities with a guaranteed income in the US setting. Calvet et al. (2019) show that the focus on the guaranteed component in a financial product can increase the overall allocation towards the stock market.

In the Dutch setting, everyone (in principle) implicitly has a guaranteed pension income via the state pension of roughly 1.000 euros a month. The exact amount depends on the composition of the household. In addition to this, some pension providers offer a pension product with guaranteed benefit level where they allocate the accumulated pension wealth as a linear combination over a fixed and variable annuity. The provider allocates a fraction $1 - v$ to a fixed annuity and a fraction v to a variable annuity. Hence, the participant can increase the guaranteed component by decreasing v in line with his/her own preferences. Then the pension income distribution can be calculated, using (2) and (3), by making a linear combination of a fixed annuity with AIR=r and a variable annuity with AIR, for example, in line with (4). We assume r=0.43 % and $\lambda\sigma$=4.52 % as was defined in Table 1.

We take a pension product with guaranteed benefit level with v=35 % and w=100 %, where w refers to the fraction allocated to stocks in the variable part of the pension product. To get a constant pension income stream in expectation, the assumed interest rate in the variable part is 4.95 %, in line with (4). This pension product is compared to the basic variable annuity with v=100 %, w=35 % and an assumed interest rate of 2.01 % in line with (4). Figure 15 shows the results.[14]

[14] In Figure 15 the results will be presented until the maximum age of 100. We do this in order to show explicitly that the pension income of a pension product with guaranteed benefit level is never below the guarantee.

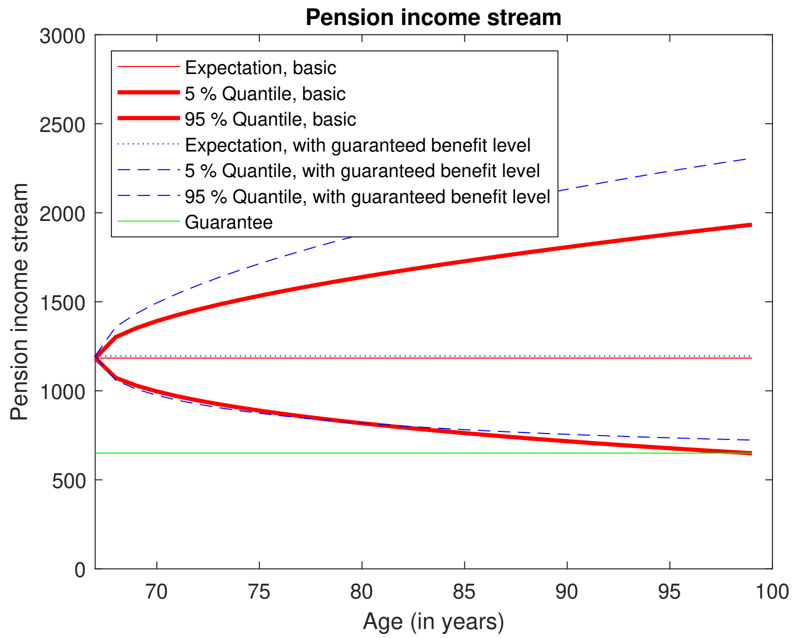
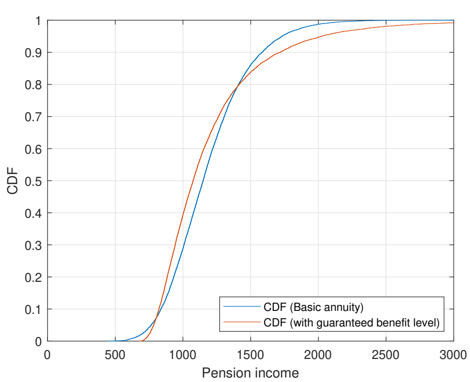


Figure 15: Pension income distribution of a product with guaranteed minimum benefit level, in a Black and Scholes financial market with v=35 %, w=100 %, $AIR = AIR_{mean} = 4.95\%$ and parameter assumptions in line with Table 1. The pension income distribution of the basic variable annuity is calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 % and parameter assumptions in line with Table 1.

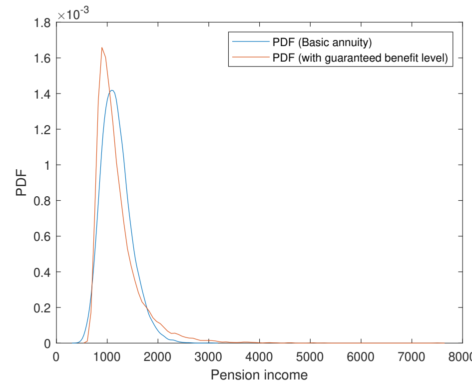
A pension product with guaranteed benefit level obviously leads to a guaranteed component in the pension product as can be seen from Figure 15. This yields that a pessimistic scenario (and lower quantiles) in a product with guaranteed benefit level can never be lower as the guaranteed component. This is a clear advantage compared to a basic variable annuity, in which the participant can end up below the guaranteed component. For example, the basic variable annuity is in 29 (118) scenarios out of 10.000 below the guarantee at the age of 80 (85).

Since a pension product with guaranteed benefit level does not need to rebalance between stocks and bonds, the optimistic pension income exceeds the optimistic pension income in a basic variable annuity. Also, notice that this pension product yields in expectation a higher pension income compared to a basic variable annuity. The difference is approximately 1 %.

The above suggests that a pension product with guaranteed benefit level is always better than a similar basic variable annuity, in terms of v and w. This is obviously not the case and we will illustrate this by showing the probability distribution function (PDF) and the cumulative distribution function (CDF) of the pension income at the age of 85 in Figure 16b and 16b respectively.



(a) Cumulative Distribution Function of pension income at the age of 85 of pension products presented in Figure 15.



(b) Probability Distribution Function of pension income at the age of 85 of pension products presented in Figure 15.

The CDF in Figure $16a$ shows that in approximately the 8 $\%$ quantile until the 80 $\%$ quantile the basic variable annuity yields a higher pension income compared to the product with guaranteed benefit level at the age of 85. Among other things, this implies that at the age of 85 the median pension income of a basic variable annuity is above the median pension income of a pension product with guaranteed benefit level. Similar conclusions can be drawn from the PDF in Figure $16b$.

5.3 Smoothing of financial shocks in pension product

Balter and Werker (2020) describe a pension product which includes smoothing of financial shocks. Smoothing of financial shocks in a pension product reduces the year on year volatility of pension income. This can be attractive for a participant exhibiting habit formation (see for example Van Bilsen et al. (2019)).

In line with Balter and Werker (2020) we define N as the smoothing period. In each time period we need to determine the remaining investment horizon of all money pots. If this remaining investment horizon is shorter than the smoothing period, we will not have the asset allocation w for this money pot. Instead, the asset allocation will be a fraction of w. This fraction is $\frac{1+h-j}{N}$, where h refers to the horizon of the money pot and j indicates the time. Generalizing, we can present the asset allocation at time $j-1$, for a pension income stream h periods from retirement age onwards, as

$$w_{j-1}(h) = w \cdot \text{min}\left(1, \frac{1+h-j}{N}\right),\ j = 1, ..., h \tag{16}$$

which shows that we will end up with a horizon dependent asset allocation strategy. This means that we no longer have a constant asset allocation in the retirement phase. We can calculate[15] the expectation of the pension income distribution as follows

$$E_t(V_h(T+h)) = V_h(T)\text{exp}\left(\sum_{j=1}^{h}\left(r + w_{j-1}(h)\lambda\sigma\right)\right) \cdot \left(\frac{1}{p_h(T)}\right) \tag{17}$$

When we insert the definition of $V_h(T)$, defined in (1), in (17) we see that the pension income stream is constant in expectation if the assumed interest rate satisfies the following.

$$AIR_h = r + \lambda\sigma \cdot \frac{1}{h}\sum_{j=1}^{h} w_{j-1}(h) \tag{18}$$

[15]The return on money pot $V_h(T)$, h periods from retirement age onwards, still follows a log-normal distribution. The mean and variance of the log return are $\sum_{j=1}^{h} r + w_{j-1}(h)\lambda\sigma - \frac{1}{2}w_{j-1}^2(h)\sigma^2$ and $\sum_{j=1}^{h} w_{j-1}^2(h)\sigma^2$ respectively.

Figure 17 presents the horizon dependent assumed interest rate that will give a constant expected pension income in (17).

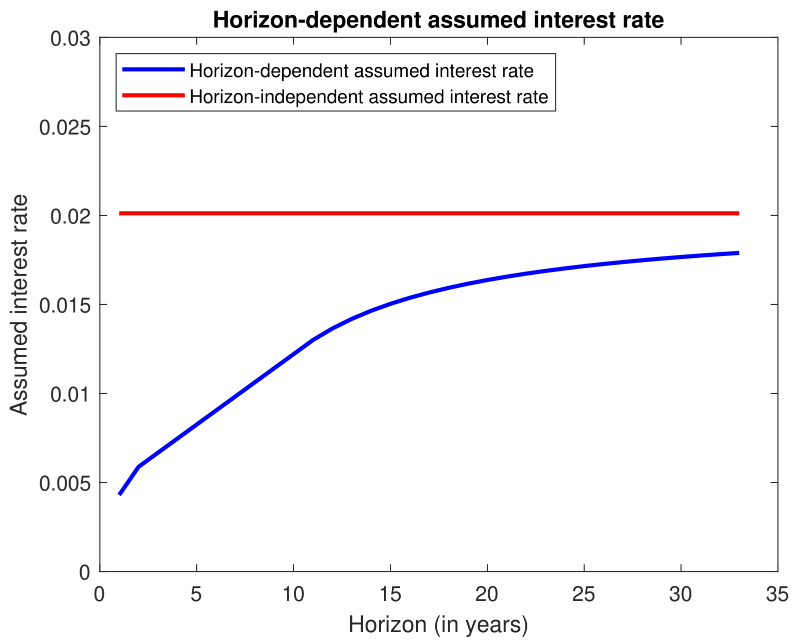


Figure 17: Horizon-dependent assumed interest rate per money pot from (18), with w=35 %, N=10 and parameter assumptions in line with Table 1.

Figure 17 shows that the assumed interest rate, defined in (18), increases in the horizon. The intuition is that for pension payments further in the future a higher assumed interest rate can be used, because a higher equity exposure is chosen for these money pots since there is more time left to smooth financial shocks.

Also, the assumed interest rate in the case of smoothing of financial shocks is for any horizon below the level of the assumed interest rate in the case without smoothing. At some point in time, when the money pot tends to expire we need to reduce the equity exposure (in line with (16)). If this is a money pot for a payment with a long horizon, we are able to approach the assumed interest rate in the case without smoothing. Figure 17 shows this, where we add a horizontal line of the assumed interest rate of 2.01 % in the case without smoothing of financial shocks.

A pension provider can calculate the expected pension income stream, to act in accordance with the law, by choosing the assumed interest rate from (18) and calculate the expectation of the pension income stream in line with (17).

The pension income streams (with and without smoothing of financial shocks) are presented in Figure 18.

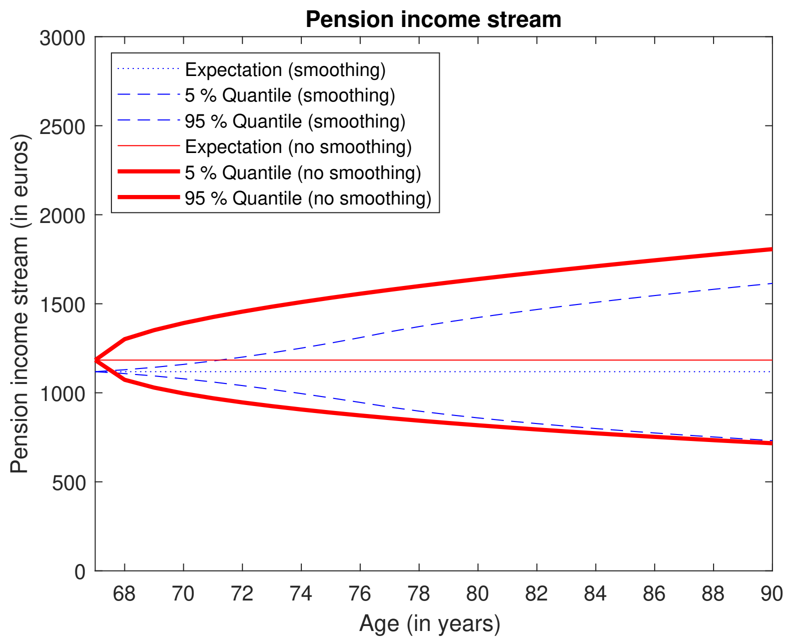


Figure 18: Pension income distribution of a product which includes smoothing, in a Black and Scholes financial market with w=35 %, N=10, AIR is horizon dependent in line with (18) and parameter assumptions in line with Table 1. The pension income distribution of the product without smoothing (=basic variable annuity) is calculated in a Black and Scholes financial market with w=35 %, AIR=AIR_{mean}=2.01 % and parameter assumptions in line with Table 1.

In Figure 18 we show that the retiree will have a lower first initial pension income compared to the case without smoothing. The pension income stream is constant in expectation, which is prescribed by the Dutch law.

We also determine the constant asset allocation w (which includes smoothing, implying a horizon dependent asset allocation) which leads to the same expected pension income as the constant asset allocation $\tilde{w}$. We will do this in 3 steps as described below.

$$\sum_{k=0}^{L-T-1} p_k(T)\exp\Big(-\sum_{j=1}^{k} r + w_{j-1}(k)\lambda\sigma \Big) = \sum_{k=0}^{L-T-1} p_k(T)\exp\Big(-k(r+\tilde{w}\lambda\sigma) \Big) \tag{19}$$

Step 1: Assume an asset allocation w and smoothing period N.
Step 2: Then we calculate the term on the left hand side of (19).
Step 3: Equation (19) then enables us to identify the constant asset allocation without smoothing, $\tilde{w}$ that yields the same constant expected pension income stream, as the horizon dependent asset allocation implied by the smoothing mechanism.

Equation (19) can be derived if we set the expected pension income stream which includes smoothing (where we assume a w) (17) equal to the expected pension income stream with constant asset allocation $\tilde{w}$ (2). Then we need to solve for $\tilde{w}$.

w	0 %	35 %	66 %
$\tilde{w}$ (N=5, r=0.43 %, $\lambda\sigma$=4.52 %)	0 %	28.99 %	54.01 %
$\tilde{w}$ (N=10, r=0.43 %, $\lambda\sigma$=4.52 %)	0 %	22.93 %	42.38 %

Table 3: The relation between the asset allocation which includes smoothing (with w and N) and constant asset allocation $\tilde{w}$ such that the expected pension income stream from (17) and (2) respectively overlaps.

For example, presented in Table 3, an asset allocation strategy of 35 % with a smoothing period of 10 years, implies a similar constant expected consumption stream compared to an asset allocation strategy of 22.93 %. Figure 19 presents this graphically. Observe that (1), using the asset allocation $\tilde{w}$, defines the first pension payment for these products.

Figure 19 shows that with smoothing of financial shocks the yearly fluctuations in pension income are less compared to a pension product without smoothing of financial shocks. We define the year on year volatility of a pension product as the average yearly change in pension income in absolute terms, in line with Association of Insurers (2017). We quantify the year on year volatility

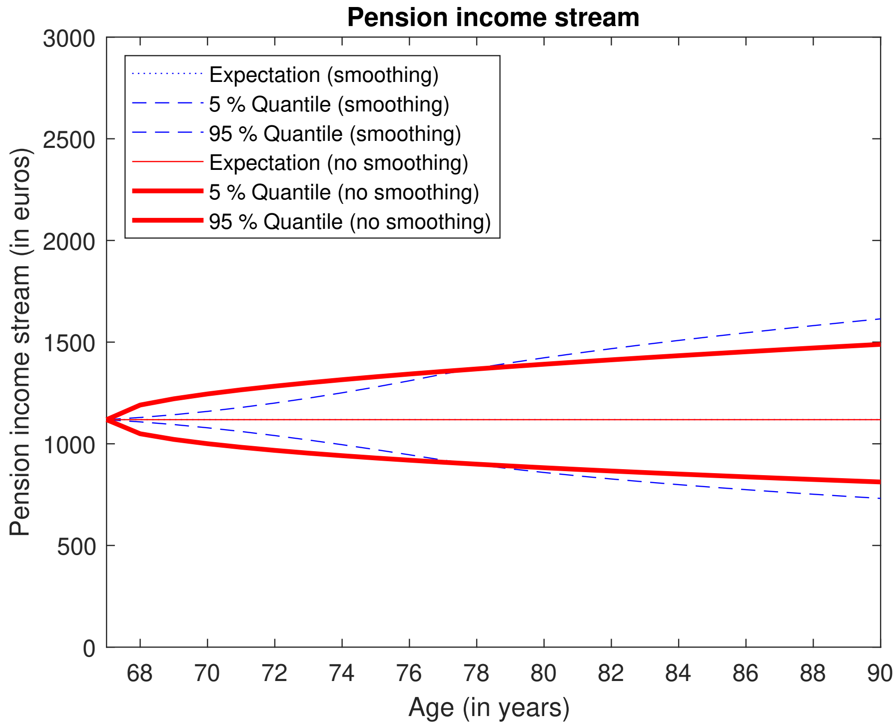


Figure 19: Pension income distribution of a product which includes smoothing, in a Black and Scholes financial market with w=35 %, N=10, AIR is horizon dependent in line with (18) and parameter assumptions in line with Table 1. The pension income distribution of the product without smoothing (=basic variable annuity) is calculated in a Black and Scholes financial market with w=22.93 %, AIR=AIR_{mean}=1.45 % and parameter assumptions in line with Table 1.

of a pension product with an asset allocation of 35 % and smoothing period of 10 years to be 1.2 %. The year on year volatility of pension product with asset allocation 22.93 % without smoothing is 3.1 %. Although these products yield a similar pension income in expectation, the year on year volatility for a product with smoothing is substantially lower. However, in absolute terms the risk will be higher at later ages in the case which includes smoothing of financial shocks. It is a matter of individual preferences which pension product is preferred by a participant. For example, a participant exhibiting habit formation preferences, a pension product incorporating a smoothing mechanism can be attractive since it reduces the year on year fluctuations in pension income.

In principle, we can extend the analysis of the non basic pension products by taking into account time varying stock returns, interest rate risk, inflation and longevity risk. However, this will not add additional insights to the results presented in earlier sections.

6 Conclusion

We have quantified the riskiness of a basic variable annuity with stock market exposure. We have extended this setting by interest rate risk in line with the KNW model, the underlying model of the scenario set prescribed by the Dutch regulator. We have shown that with a full interest rate hedge, the participant faces similar risks as in the Black and Scholes setting taking into account equity risk only. Also, we have derived the horizon-dependent assumed interest rate under which we were able to get a constant pension income in expectation in nominal terms. Furthermore, we can quantify the pension income stream in real terms.

The additional risk for pension products when taking into account (macro) longevity risk depends on the asset allocation in the product. For a fixed annuity the longevity risk can be substantial, though the first year shock is smaller than in De Waegenaere et al. (2018). In a 5 % quantile the pension income is 2.6 % lower fifteen years after retirement neglecting the insurance premium. For a variable annuity taking equity exposure, financial risk will dominate the longevity risk. Roughly spoken, the 5 % quantile is overlapping for a variable annuity with and without longevity insurance. By assuming a higher volatility on future life improvements than a one factor Lee-Carter model would calibrate based on historical data, we show that macro longevity risk becomes a more important risk factor, though the additional risk in a 5% quantile is marginal ($<$ 0.4 % fifteen years after retirement). Assuming that the Lee-Carter model is the true longevity model and a realistic cost of insuring longevity risk of 4.6% (Gielen and De Waegenaere (2014)), it is attractive for participants and pension providers to leave the longevity risk in the pool of participants. Steenkamp (2016) find a similar conclusion for a fixed annuity with longevity risk. However, as stated before, we cannot guarantee that in the near future no longevity shocks will be realized that are beyond the scope of this model as in Balter et al. (2019).

A wide variety of Dutch pension products, several discussed in Balter and Werker (2020) as well, can be written in the methodology used in this paper. Although not explicitly presented, we argue that for these pension products the financial market risk dominates the longevity risk, given that there will be some equity exposure. For pension products incorporating smoothing of financial shocks it is possible, depending on the length of the smoothing period, to reduce the year on year volatility of pension income by a factor three compared to a basic variable annuity. This is in line with the results of Balter and Werker (2020) and Bonekamp et al. (2017).

Appendix

In this Appendix we present mathematical derivations. We separate the derivations for each section.

Derivations in section 4

Lee-Carter model

Before we can discuss the implications of a macro longevity shock, we should first discuss the Lee-Carter model.In this model, one year survival probabilities (in continuous time) can be calculated from the force of mortality $q_{x,t}$ as follows.

$$p_{x,t} = \exp(-q_{x,t}) \tag{20}$$

The (natural logarithm) of the force of mortality can be described as follows, where $\sum_x \beta_x = 1$ and $\sum_t \kappa_t = 0$ to ensure uniqueness.

$$\ln(q_{x,t}) = \alpha_x + \beta_x \kappa_t \tag{21}$$

The vectors for α_x, β_x and κ_t will be estimated via (22), where $m_{x,t}$ represents the central death rate between ages x and $x+1$ at time t with $\zeta_{x,t}$ Gaussian i.i.d.

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \zeta_{x,t} \tag{22}$$

If we denote the first year of the sample with t_0 and the last year of data with $t_{\text{intermediate}}$, we obtain for α_x, the following estimate.

$$\alpha_x = \frac{1}{t_{\text{intermediate}} - t_0 + 1} \sum_t \ln(m_{x,t}) \tag{23}$$

Now we perform a singular value decomposition (SVD) on $\ln(m_{x,t}) - \alpha_x \iota$ as follows, where $\iota = (1, 1, ..., 1)^T$.

$$\text{SVD}(\ln(m_{x,t}) - \alpha_x \iota) = \lambda_1 U_{x,1} V_{t,1} + \lambda_2 U_{x,2} V_{t,2} + ... + \lambda_k U_{x,k} V_{t,k} \tag{24}$$

Then we can estimate β_x, κ_t, where the scaling is needed to be in line with the constraints $\sum_x \beta_x = 1$ and $\sum_t \kappa_t = 0$.

$$\beta_x = \frac{1}{\sum_x U_{x,1}} U_{x,1} \tag{25}$$

$$\kappa_t = \lambda_1 V_{t,1} \sum_x U_{x,1} \tag{26}$$

Next, we should calibrate σ_k and C in the random walk formula for κ_{t+1} in (27), where η_{t+1} has the standard normal distribution.

$$\kappa_{t+1} = \kappa_t + C + \sigma_k \eta_{t+1} \tag{27}$$

Now we are able to present the evolutions of κ_t over time (i.e. the best estimate and the corresponding quantiles). Note that we are extrapolating for h periods.

$$\kappa_{t+h}^{\text{be}} = \kappa_t + Ch \tag{28}$$

$$\kappa_{t+h}^{\text{up}} = \kappa_t + Ch + z_\alpha \sigma_k \sqrt{h} \tag{29}$$

$$\kappa_{t+h}^{\text{down}} = \kappa_t + Ch - z_\alpha \sigma_k \sqrt{h} \tag{30}$$

Substituting (21) in (20) and replacing t by $t+h$, we can write $p_{x,t+h}$ as follows.

$$p_{x,t+h} = \exp(-q_{x,t+h}) = \exp(-\exp(\alpha_x + \beta_x \kappa_{t+h})) \tag{31}$$

Now, we have the tools to construct the best estimate (be) and the quantiles (up and down) for the future survival probabilities.

$$p_{x,t+h}^{\text{be}} = \exp(-\exp(\alpha_x + \beta_x \kappa_{t+h}^{\text{be}})) \tag{32}$$

$$p_{x,t+h}^{\text{up}} = \exp(-\exp(\alpha_x + \beta_x \kappa_{t+h}^{\text{up}})) \tag{33}$$

$$p_{x,t+h}^{\text{down}} = \exp(-\exp(\alpha_x + \beta_x \kappa_{t+h}^{\text{down}})) \tag{34}$$

Discussing the Lee-Carter model enables us to explain the implications of a macro longevity shock. As a remark we should state that since we do not take into account the $\zeta_{x,t}$ in the calibration we are underestimating the variance.

An overview of the α_x, β_x and κ_t is presented, based on gender neutral survival probabilities is presented below. Note that we will do a minor adjustment in the Lee-Carter calibration of α_x to create equivalence with the mortality rates of the Actuarial Society (AG). We solve the following equation for α_x.

$$\alpha_x + \beta_x(\kappa_t + iC) = \log(q_x^{\text{AG}}) \tag{35}$$

In (35) we have to insert for i=3. This is because we have the data from the Human Mortality Database until 2016 and use the AG (expected) mortality rates from 2019 in line with the assumption that the annuity is purchased in this year.

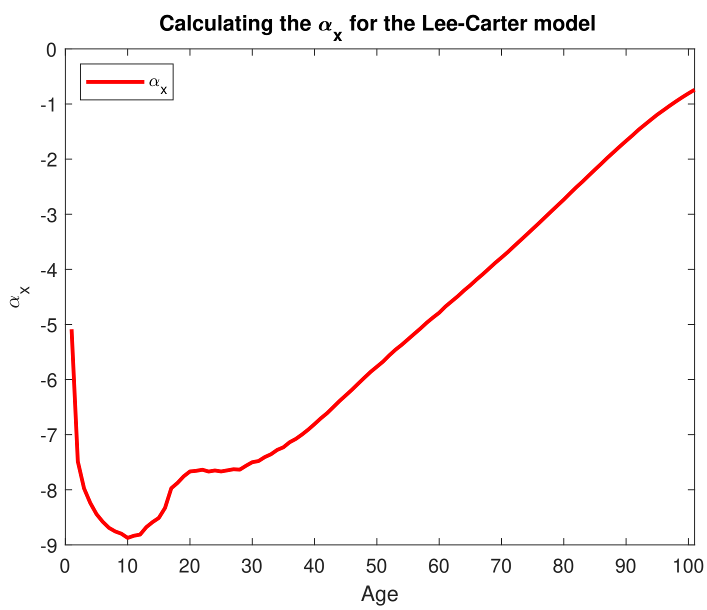


Figure 20: The calibration of α_x in the Lee Carter model based on gender neutral data from the Human Mortality Database with sample period 1970-2016. The α_x is marginally adjusted to create equivalence with the survival probabilities of the Dutch Actuarial Society as described in (35).

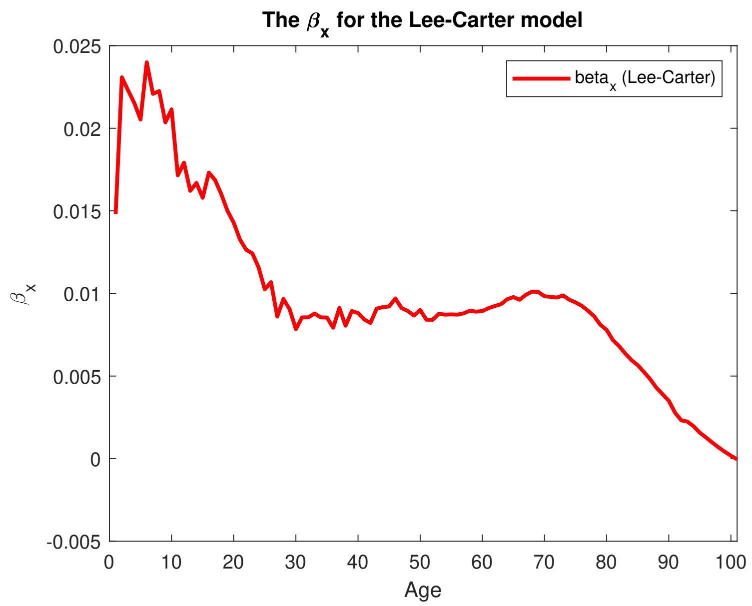


Figure 21: The calibration of β_x in the Lee Carter model based on gender neutral data from the Human Mortality Database with sample period 1970-2016.

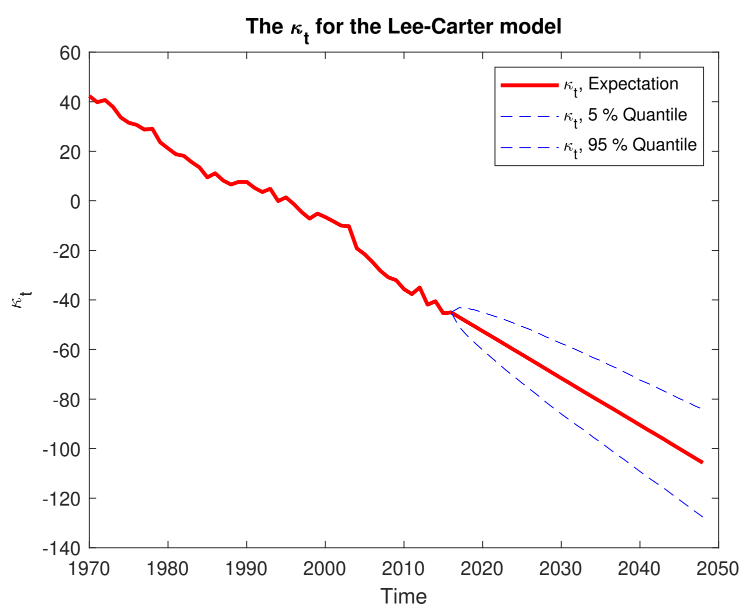


Figure 22: The calibration of κ_t in the Lee Carter model based on gender neutral data from the Human Mortality Database with sample period 1970-2016. We have also included simulated values of κ_t from (27), with C=-1.8979 and $\sigma_k = 2.3198$.

For σ_k, the volatility of life improvements, we have estimated a value of 2.3198. For C, the expected life improvement over time, we have estimated a value of -1.8979. This is based on gender neutral data from the Human Mortality database (Netherlands), where the sample period is taken from 1970-2016.

In the specific setting of section 4.2, we have to calculate each year the survival probabilities under the old and new survival table as is defined below.

$$\begin{aligned} P^{\text{old}}(x) &= \text{exp}(-\text{exp}(\alpha_x + \beta_x\{\kappa_t + iC\})) \\ P^{\text{new}}(x) &= \text{exp}(-\text{exp}(\alpha_x + \beta_x\{\kappa_{t+1} + (i-1)C\})) \end{aligned} \tag{36}$$

For the same reason as in (35) we will use i=3.

Derivations in section 5

The unknown $z_h \mathbb{1}_{low}$ is for simplicity derived in the setting of a fixed annuity. Therefore, the pension income stream, incorporating high low is presented as follows.

$$\begin{aligned} E_t(V_h^{\text{high-low, fixed}}(T+h)) &= W_T \frac{p_h(T)\text{exp}(-h(r + z_h \mathbb{1}_{low}))}{\sum_{k=0}^{L-1} p_k(T)\text{exp}(-k(r + z_k \mathbb{1}_{low}))} \cdot \text{exp}(hr) \cdot \left(\frac{1}{p_h(T)}\right) \\ &= W_T \frac{\text{exp}(-h \cdot z_h \mathbb{1}_{low})}{\sum_{k=0}^{L-1} p_k(T)\text{exp}(-k(r + z_k \mathbb{1}_{low}))} \end{aligned} \tag{37}$$

To get a constant pension income stream in the lower period, we need to make this z_h (and we did that already) horizon dependent. This is done as follows.

$$z_h \mathbb{1}_{low} = \frac{1}{h} \cdot Z \mathbb{1}_{low} \tag{38}$$

By inserting 38, we can simplify the expected pension income stream from 37.

$$E_t(V_h^{\text{high-low, fixed}}(T+h)) = W_T \frac{\text{exp}(-Z \cdot \mathbb{1}_{low})}{\sum_{k=0}^{L-1} p_k(T)\text{exp}(-k(r + z_k \cdot \mathbb{1}_{low}))} \tag{39}$$

Now note that we can write the fraction (i.e. difference between highest and lowest pension income) as follows in this setting.

$$\begin{aligned} \text{Difference high-low} &= \text{exp}(-Z \cdot \mathbb{1}_{low}) \\ -\text{log}(\text{Difference high-low}) &= Z \cdot \mathbb{1}_{low} \end{aligned} \tag{40}$$

So, we can also write as follows.

$$
\begin{aligned}
z_h \mathbb{1}_{low} &= \frac{1}{h} Z \mathbb{1}_{low} \\
&= -\frac{1}{h} \log(\text{Difference high-low})
\end{aligned}
\tag{41}
$$

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