Secular Stagnation? Growth, Asset Returns and Welfare in the Next Decades - The Model

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- Summers (2014): “Secular stagnation”:

  1. Low growth
  2. Low (negative) rates of return
  3. Different policies

Demographic Change and Secular Stagnation

- One reason for secular stagnation: Demographic developments
- Demographic forecasts for the US:

	$\frac{\textit{working age population}}{\textit{total adult population}}$	$\frac{\textit{retired population}}{\textit{working age population}}$
year 2010	82.0%	22.0%
year 2030	73.0%	38.0%

- Macroeconomic implications: low growth, decreasing capital productivity, decreasing asset returns (asset market meltdown hypothesis)

Research Questions

1. How large are effects on output?
2. Implications for life-time utility?
3. How strongly do
   - Wages (human capital returns) increase?
   - (Differential) asset returns decrease?
   - Is there a stronger reduction of the risk-free rate?
4. Policy implications: Role of human capital, *quality* of labor?

- Back of the envelope:

$$Y/N = (K/N)^{\alpha}(L/N)^{1-\alpha}, \ \alpha = 0.33$$

$$\widehat{L/N} = 73/82 - 1 = -10\% \ \Leftrightarrow \ \widehat{Y/N} = (1-\alpha)\widehat{L/N} = -7.3\%$$

Approach

- Exogenous driving force: demographic change
- Quantitative overlapping generations (Auerbach & Kotlikoff 1987):
  - General equilibrium
  - Production economy
  - Uninsurable idiosyncratic risks: (i) earnings and (ii) investment
  - Endogenous portfolio choice: equity, bonds, human capital
  - Risky human capital (cf. Krebs 2003)
  - Risky equity (cf. Angeletos 2007)
  - (pay-as-you-go pension system)
- Predict growth, wages and asset returns
- Compute welfare effects

Related Literature

Model Outline

- Households hold
  - physical capital K - risky (r^K)
  - human capital H - risky (r^H)

    Age dependent depreciation
  - bonds B - risk-free (r^f)
- Firms employ
  - K
  - H
- Bonds in zero-net supply $B = 0$

Results

Preview

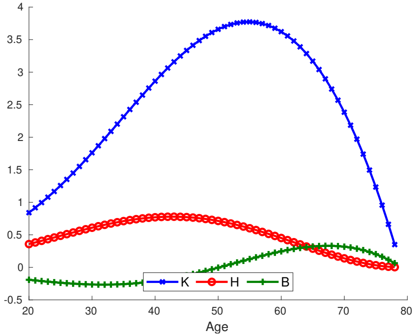
- Cross-sectional profiles
- Aggregate dynamics & welfare:
  - Baseline model
  - Constant human capital shares

Table: First and Second Stage Parameters

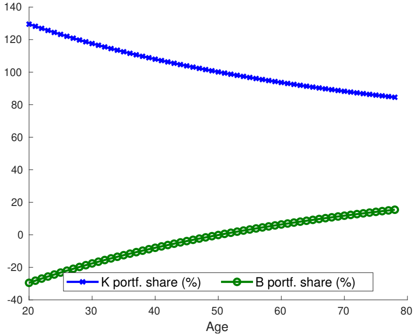
Parameter	Value	Target	Target Source, Comment
Preferences			
Elasticity inter-temp. substit., ξ	0.5	1^{st} stage	Bansal and Yaron (2004)
Time discount factor: β	1.046	$K/Y = 2.86$	NIPA
Relative risk aversion: θ	3.99	$\mathbb{E}[r^K] - r^f = 2.23\%$	Bank lending rates in 1960s
Budgets			
Endowment: $\{h_0, k_0\}$	$\{1.0, 0.0\}$	1^{st} stage	normalization
Technology			
Capital share: α	0.36	1^{st} stage	wage share (NIPA)
Technological progress: g	0.018	1^{st} stage	TFP growth (NIPA)
Depreciation rate K: δ_0^K	0.079	$\mathbb{E}[r^K] = 4.73\%$	PST
~~TFP Scaling factor: ψ~~	~~1~~		
TFP shock std. (mean is 1): σ_ζ	0.048	$std(r^k) = 8.4\%$ (std. of stock returns)	PST
Human Capital			
Depreciation rate h: $\{\chi_0, \chi_1\}$	$\{0.095, 0.0037\}$	Labor income profile	PSID
Hum cap. shock std. (mean is 0) σ_η	0.51	mean return on human capital $= 0.12$	Krebs (2003)

Source: Baseline model: The target year is 2010. *Notes*: RZ $\hat{=}$ Rajan and Zingales (1995). PST $\hat{=}$ Piazzesi, Schneider, and Tuzel (2007). We target the average of the post-Second World War risk-free rates of PST and Shiller (2015).

Cross-Sectional Profiles

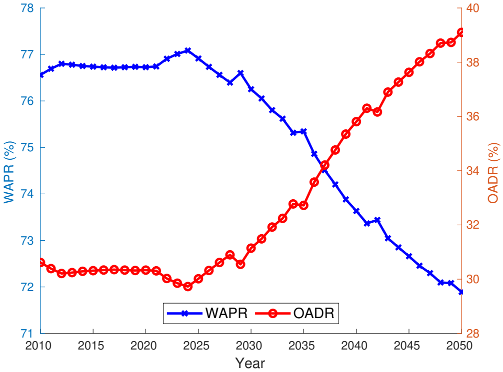


Assets

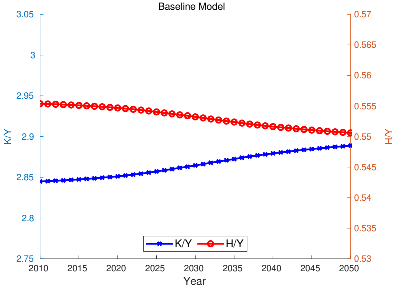


Financial Portfolio Shares

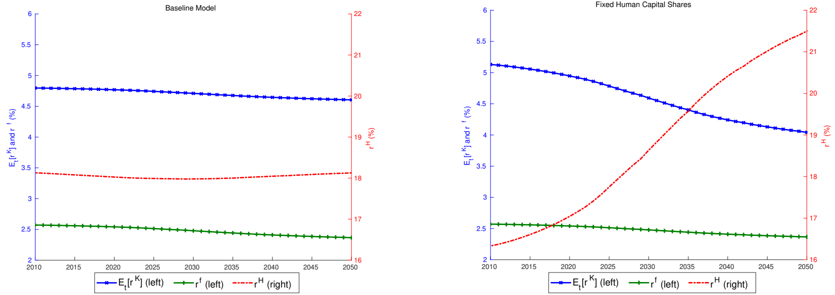
Aggregate Dynamics: Demography



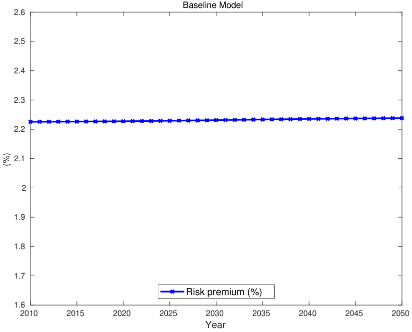
Aggregate Dynamics: Capital-Output-Ratios

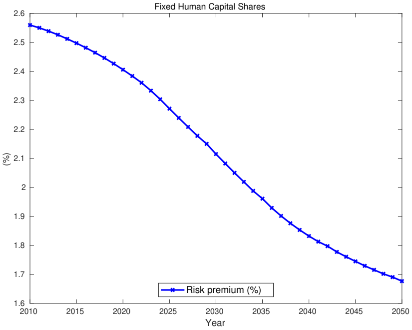


Aggregate Dynamics: Rates of Return



Aggregate Dynamics: Risk Premium





Results: Aggregate Dynamics

Summary

	r^f	r^s	$\{r_j^h\}_{avrg}$	$EP := r^s - r^f$
Baseline Model				
2010 (in %)	2.57	4.80	18.13	2.23
2050 (in %)	2.37	4.60	18.13	2.24
$\Delta_{\{2050-2010\}}$ (in %p)	-0.21	-0.19	0.00	+0.01
Constant Human Capital Shares				
$\Delta_{\{2050-2010\}}$ (in %p)	-1.09	-0.21	+5.17	-0.88

End

Thank you!

- Indices
  - t - period
  - j - age, from $j = 0$ to $J_t + 1$
  - i - household

Model: Households

Budget Constraint

- Indices
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  - j - age, from $j = 0$ to $J_t + 1$
  - i - household
- Households face consumption $c_{t,j,i}$ & portfolio decision
  - bonds $b_{t,j,i}$ with gross return $1 + r_t^f$
  - physical capital / equity $k_{t,j,i}$ with gross return $\pi_{t,j,i}(k_{t,j,i})$

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  - human capital $h_{t,j,i}$ with earnings $(1 - \tau_t)\omega_j r_t^H$
  - $\omega_j \in [0, 1]$ fraction of time working and social contribution rate τ_t
  - laborable human capital: $\ell_{t,j,i} = \omega_j \cdot h_{t,j,i}$
  - investment into human capital $e_{t,j,i}$

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- Dynamic budget constraint:

$$b_{t+1,j+1,i} + k_{t+1,j+1,i} = b_{t,j,i}(1 + r_t^f) + \pi_{t,j,i}(k_{t,j,i}) + h_{t,j,i}(1 - \tau_t)\omega_j r_t^H + (1 - \omega_j)p_t - c_{t,j,i} - e_{t,j,i}$$

- Every household i owns a firm producing

$$y_{t,j,i} = \zeta_{t,j,i} k_{t,j,i}^{\alpha} (\Upsilon_t \ell_{t,j,i}^D)^{1-\alpha},$$

$\Upsilon_t = \Upsilon_{t-1} \cdot (1+g)$: technology level

$\zeta_{t,j,i} \sim \mathcal{D}(1, \sigma^{\zeta})$

$\ell_{t,j,i}^D$: labor demand, costs r_t^H, determined after $\zeta_{t,j,i}$

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- Profit maximization

$$\pi_{t,j,i}(k_{t,j,i}) = \max_{\ell} y_{t,j,i} - r^h_t \ell^D_{t,j,i} + (1-\delta^K) k_{t,j,i} \tag{1}$$

- Labor demand:

$$\ell_{t,j,i}^{D} = (1-\alpha)^{\frac{1}{\alpha}} \Upsilon_t^{\frac{1-\alpha}{\alpha}} \left(\frac{\zeta_{t,j,i}}{r_t^H} \right)^{\frac{1}{\alpha}} k_{t,j,i}$$

- Income from physical capital:

$$\pi_{t,j,i}(k_{t,j,i}) = (1 + r_{t,j,i}^{K}) k_{t,j,i}$$

with idiosyncratic equity rate

$$r_{t,j,i}^{K} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \left(\Upsilon_t^{1-\alpha} \zeta_{t,j,i} (r_t^H)^{\alpha-1} \right)^{\frac{1}{\alpha}} - \delta^K$$

Model: Households

Human Capital Accumulation and Pension Stock

- Human capital accumulation:

$$h_{t+1,j+1,i} = h_{t,j,i} \cdot (1 - \delta_j^h + \eta_{t,j,i}) + \tilde{e}_{t,j,i}^h, \qquad h_{t,j,i} \geq 0 \;\; \forall \; t, j, i,$$

$$\tilde{e}_{t,j,i}^h \equiv e_{t,j,i}^h / \Upsilon_t \quad \eta_{t,j,i} \sim \mathcal{D}(0, \sigma^\eta)$$

$$\delta_j^h = -\chi_0 + \exp(\chi_1 \cdot j) + H_t^\gamma, \quad \chi_0 > 0, \chi_1 \geq 0$$

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- Pension stock $ps_{t,j,i}$

$$ps_{t,j} = \frac{p_t}{1 + r_t^f} + \frac{p_{t+1}}{(1 + r_t^f)(1 + r_{t+1}^f)} + \cdots + \frac{p_{t+J_{t-j}+1-j}}{\prod_{s=0}^{J_{t-j}+1-j}(1 + r_{t+s}^f)}$$
$$\Rightarrow ps_{t+1,j+1} = ps_{t,j}(1 + r_t^f) - (1 - \omega_j)p_j$$

No aggregate risk $\rightarrow$ sequence of risk-free rates known and certain

Combine (de-trended ($\widetilde{quantity}_t = quantity_t/\Upsilon_t$)) budget constraints

$$\underbrace{\tilde{b}_{t+1,j+1,i} + \tilde{k}_{t+1,j+1,i} + \frac{h_{t+1,j+1,i}}{1+g} + \widetilde{ps}_{t+1,j+1,i}}_{\tilde{w}_{t+1,j+1,i}} =$$

$$\frac{1}{1+g}\Bigg(\tilde{b}_{t,j,i}(1+r_t^f) + \tilde{k}_{t,j,i}(1+r_{t,j,i}^K) +$$

$$\underbrace{h_{t,j,i}\left(1+(1-\tau_t)\omega_j \tilde{r}_t^H - \delta_j^H + \eta_{t,j,i}\right) + \widetilde{ps}_{t,j}(1+r_t^f)}_{\tilde{w}_{t,j,i}(1+\hat{r}_{t,j,i})} - \tilde{c}_{t,j,i}\Bigg),$$

Model: Households

Transformations

Define "cash-on-hand": $\tilde{x}_{t,j,i} := \tilde{w}_{t,j,i} \cdot (1 + \widehat{r}_{t,j,i})$,

$$\tilde{x}_{t+1,j+1,i} = \frac{1}{1+g} \cdot \{\tilde{x}_{t,j,i} - \tilde{c}_{t,j,i}\} \cdot (1 + \widehat{r}_{t+1,j+1,i})$$

$$\widehat{r}_{t+1,j+1,i} := r^f_{t+1} + \widehat{\alpha}^K_{t+1,j+1,i} \cdot (r^K_{t+1,j+1,i} - r^f_{t+1}) + \widehat{\alpha}^H_{t+1,j+1,i} \cdot (\widehat{r}^H_{t+1} - r^f_{t+1}),$$

$$\widehat{\alpha}^K_{t+1,j+1,i} := \frac{\tilde{k}_{t+1,j+1,i}}{\tilde{w}_{t+1,j+1,i}}, \qquad \widehat{\alpha}^H_{t+1,j+1,i} := \frac{h_{t+1,j+1,i}/(1+g)}{\tilde{w}_{t+1}},$$

$$\widehat{\alpha}^f_{t+1,j+1,i} := \frac{\tilde{b}_{t+1,j+1,i} + \tilde{ps}_{t+1,j+1,i}}{\tilde{w}_{t+1}} = 1 - \widehat{\alpha}^K_{t+1,j+1,i} - \widehat{\alpha}^H_{t+1,j+1,i}$$

$$\widehat{r}^h_{t+1} := (1+g) \cdot (\tilde{r}^H_{t+1} + 1 - \delta^h_j + \eta_{t+1}) - 1$$

hh prob

Model: Households

Policy rules

- Optimal portfolio shares solve

$$\max_{\alpha^K_{t,j}\geq 0,\alpha^H_{t,j}\geq 0} -\mathbb{E}\left[(1+\widehat{r}'_{t,j,i})^{1-\theta}\right]$$

- Policy function for consumption:

$$\tilde{c} = m \cdot \tilde{x}$$

where

$$m_{t,j} = \frac{(\beta^\gamma \cdot \mathbb{E}[(1+\widehat{r}')^{1-\theta}])^{\frac{1}{1-\theta-\gamma}} \cdot m'_{t,j}}{1+(\beta^\gamma \cdot \mathbb{E}[(1+\widehat{r}')^{1-\theta}])^{\frac{1}{1-\theta-\gamma}} \cdot m'_{t,j}}.$$

- Portfolio shares and and $m_{t,j}$ are same across all households of same age j

Model: Demographics

- Individuals are "born" at age 20
- Fixed life expectancy: $J_t + \phi_t$
  - e.g. 77.4 is $J_t = 77 - 20 = 57$ and $\phi_t = 0.4$

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- law of motion of population mass:

$$N_{t,j} = \begin{cases} N_{t-1,j-1} & \text{for } j \in \{q : 0 \leq q \leq J_{t-q}\} \\ newborns_t & \text{for } j = 0 \end{cases}$$

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- Demographic transition: changing $J_t + \phi_t$ and $newborns_t$

Model: Households

Preferences

- Epstein/Zin-preferences:

$$u_{t,j,i} = \left[c_{t,j,i}^{\frac{1-\theta}{\gamma}} + \beta \cdot \left(\mathbb{E}[u_{t+1,j+1,i}^{1-\theta}] \right)^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{1-\theta}}$$

- Expectations taken w.r.t. idiosyncratic earnings and investment risk
- Final age: $u_{t+J_t+1,J_t,i} = \phi_t \cdot c_{t+J_t+1,J_t,i}$

Model: Government

- $B_{t+1} = B_t(1 + r_t^f) + D_t$
- D_t: public primary debt, exogenous

Model: Market Clearing

- Wealth distribution in one j group does not matter for aggregation.

  Reason:

  1. Consumption linear in cash-on-hand
  2. m and $\hat{\alpha}$'s independent of idionsyncratic variables

- $'$ indicates next period's variables

- Risk-free market

$$\widetilde{B}^{D\prime} + \widetilde{PS} = \frac{1}{1+g} \cdot \sum_j N(j) \cdot \widetilde{X}_j \cdot (1 - m(j)) \cdot (1 - \widehat{\alpha}^{K\prime}(j) - \widehat{\alpha}^{H\prime}(j))$$

$$\widetilde{B}^{S\prime} = \frac{1}{1+g} \left(\widetilde{B}^S \cdot (1 + r^f(t)) \right)$$

$$\widetilde{PS} = \sum_j N(j) \cdot \left(\sum_{s=0}^{J_{t-j}+1-j} (1+g)^s (1 - \omega_{j+s}) \tilde{p}_{t+s} \prod_{q=0}^{s} \left(1 + r^f_{t+q} \right)^{-1} \right)$$

Model: Market Clearing

- Labor market

$$L^{S\prime}(t) = \sum_j N(j) \cdot \widetilde{X}_j(\cdot) \cdot (1 - m(j, \cdot)) \cdot \hat{\alpha}^{H\prime}(j, \cdot) \cdot \omega_j$$

$$L^{D\prime} = (1-\alpha)^{\frac{1}{\alpha}} \mathbb{E}[\zeta^{\frac{1}{\alpha}}](\tilde{r}^{H\prime})^{-\frac{1}{\alpha}} \widetilde{K}'$$

- Pension payments covered by social security contributions

$$\tau(t)\tilde{r}^H(t)L(t) = \sum_j N_j \cdot (1 - \omega_j) \cdot \tilde{p}_t$$

$$\tilde{p}_t = \rho_t \cdot \frac{(1-\tau_t) \cdot \tilde{r}^H_t \cdot L_t}{\sum_j N_{t,j} \cdot \omega_{t,j}} = \rho_t \cdot \text{ net avg. labor income}$$

ρ_t - replacement rate and τ_t - labor contribution rate

Appendix: household problem

$$v(\tilde{x}_{t,j,i}) = \max_{\tilde{c},\tilde{x}',\widehat{\alpha}^{k\prime},\widehat{\alpha}^{h\prime}} \left\{ \tilde{c}_{t,j,i}^{\frac{1-\theta}{\gamma}} + \widehat{\beta} \cdot (\mathbb{E}[v'(\tilde{x}'_{t,j,i}, \Phi')^{1-\theta}])^{\frac{1}{\gamma}} \right\}^{\frac{\gamma}{1-\theta}}$$

$$\text{s.t. } \tilde{x}'_{t,j,i} = \frac{1}{1+g} \cdot (\tilde{x}_{t,j,i} - \tilde{c}_{t,j,i}) \cdot (1 + \widehat{r}'_{t,j,i}), \ \tilde{x}_0 > 0 \text{ given}$$

$$\hat{r}'_{t,j,i} \equiv r^{f'} + \hat{\alpha}^{k'}_{t,j,i} \cdot (r^{k'}_{t,j,i} - r^{f'}) + \hat{\alpha}^{h'}_{t,j,i} \cdot \left(\hat{r}^{h'}_{t,j,i} - r^{f'}\right)$$

$$\rho'_t \quad \text{given}$$

$$\eta_{t,j,i} \sim \mathcal{D}(0, \sigma^2_\eta)$$

$$\zeta_{t,j,i} \sim \mathcal{D}(1, \sigma^2_\zeta),$$

$$\widehat{\beta} = \beta \cdot (1+g)^{\frac{1-\theta}{\gamma}}$$

back

Summary: Neat Model

- Rich model
  - "3.5" assets
  - Labor quantity and quality
  - PAYG pension system
  - transition
- Computationally not intensive

Our Contribution to Related Literature

- Draw from and contribute to two strands of literature:
  1. Demography, growth & welfare: Börsch-Supan, Ludwig & Winter (2006), Attanasio, Kitao & Violante (2007), Krueger & Ludwig (2007), Ludwig, Schelkle & Vogel (2012), and many others
  2. Demography & asset prices: Abel (2001, 2003), Poterba (2001), Brooks (2004), Geanakoplos, Magill, and Quinzii (2004), Ang and Maddaloni (2005)
- Our contribution:
  - Endogenous asset pricing
  - Risky and risk-free returns
  - Human capital: important adjustment channel to demographic change
  - Large scale OLG, preserving computational tractability

Go back